Proceedings of the Fifth International Mathematics Education and Society Conference

João Filipe Matos, Paola Valero and Keiko Yasukawa (Editors)

Neuza Pedro and Patricia Perry (Collaborators)

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Centro de Investigação em Educação, Universidade de Lisboa
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MES 5 Conference Logo

The conference logo *is a composition based on a photograph of a tree taken in the centre of Lisbon. The tree and its branches are being looked up as a metaphor of growth, expansion and diversity.*

MES 5 International Organising Team

João Filipe Matos, Paola Valero and Keiko Yasukawa

MES 5 Local Organising Team

Neuza Pedro, Madalena Santos, Nélia Amado, Susana Carreira, João Filipe Matos

MES 5 International Advisory Team

Candia Morgan, Dimitris Chassapis, Eric Gutstein, Karin Brodie, Mamokgethi Setati,
Ole Skovsmose, Peter Gates, Robyn Zevenbergen, Tony Cotton, Valeria Carvalho.

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INTRODUCTION

The First International Conference on Mathematics Education and Society took place in Nottingham, Great Britain, in September 1998. The Second Conference was held in Montechoro, Portugal, in March 2000. The Third Conference took place in Helsingør, Denmark, in March 2002. The Fourth Conference was held in Queensland, Australia, in July 2005. On all occasions, people from around the world had the opportunity of sharing their ideas, perspectives and reflections concerning the social, political, cultural and ethical dimensions of mathematics education and mathematics education research that take place in diverse contexts. As a result of the success of these four meetings, it was decided to have a fifth conference in Portugal. As a trial of an international and cross-institutional collaboration in conference organisation, an international organising team of Joao Filipe Matos (University of Lisbon, Portugal), Paola Valero (Aalborg University, Denmark) and Keiko Yasukawa (University of Technology Sydney, Australia), took the lead in the planning of the conference. Together with a Local Organising Team with members from the University of Lisbon and the University of Algarve, and with the International Advisory Team, it was possible to set up the Fifth International Conference on Mathematics Education and Society in the city of Albufeira, in February 2008.

The conference has been promoted and sponsored by the Centro de Investigação em Educação at the Universidade de Lisboa, the Department of Education, Learning and Philosophy at Aalborg University, and the Faculty of Education at the University of Technology Sydney. Other institutions such as the Fundação Ciência e Tecnologia and the Reitoria da Universidade de Lisboa also sponsored this meeting.

AIMS OF MES 5

Education is becoming more and more politicised throughout the world. Mathematics education is a key focus in the politics of education. Mathematics qualifications remain an accepted gatekeeper to further education and employment opportunities. Thus, defining success in mathematics becomes a way of controlling people's pathways in work and life generally. Mathematics education has also tended to contribute to the reproduction of an inequitable society through undemocratic and exclusive pedagogical practices, which portray mathematics and mathematics education as absolute, authoritarian disciplines. The fact that particular mathematics education and research practices can have such significant impact on the type of society we live in suggests that different mathematics education and research practices could have equally significant but more socially just impact on society. There is a need for uncovering and examining the social, cultural and political dimensions of mathematics education; for disseminating research that explores those dimensions; for addressing methodological issues of that type of research; for planning international co-operation in the area; and for developing a strong activist

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research community interested in transforming mathematics education as an agent and practice for, rather than against, social justice.

The MES 5 Conference aims to bring together mathematics educators around the world to provide such a forum as well as to offer a platform on which to build future collaborative activity.

CONFERENCE PROGRAMME

The conference was organised bearing in mind the importance of generating a continuing dialogue and reflection among the participants. There was a range of activities directed towards the aim of generating this sustained discussion:

Opening plenary panel: The Advance and Consolidation of Research on Mathematics Education and Society

The conference took place 10 years after the first conference of Mathematics Education and Society that was held in Nottingham. Between then and now, there have been three MES Conferences – in Portugal, Denmark and Australia. Organisers of each of the past MES Conferences were invited to reflect on how the Conferences have influenced and shaped questions and directions of research in mathematics education and society, and what they saw as some of the goals for us in this Conference.

Panelists: Joao Filipe Matos, University of Lisbon, Portugal; Candia Morgan, Institute of Education - University of London, UK; and Paola Valero, Aalborg University, Denmark.

Moderator: Keiko Yasukawa, University of Technology Sydney, Australia.

Plenary addresses and reactions

The four invited keynote speakers were asked to address a topic of relevance to the conference, building on their current research. They held 50-minutes presentations. Each presentation was followed by 10-minutes reactions by two mathematics educators and teachers.

The four plenaries were:

- “Reinventing” Freire: Mathematics Education for Social Transformation by Eric Gutstein, University of Illinois-Chicago, USA.
- Describing teacher change: Interactions between teacher moves and learner contributions by Karin Brodie, Wits University, South Africa.
- Equity-in-Quality: Towards a Theoretical Framework by Murad Jurdak, American University of Beirut, Lebanon.
- Order of the World or Order of the Social – a Wittgensteinian Conception of Mathematics and its Importance for Research in Mathematics Education by Ole Ravn Christensen, Aalborg University, Denmark.

Working groups

Groups, set at the beginning of the conference, discussed the plenary lecture and the reactions. Each discussion group produced a brief report detailing key questions or issues to be addressed by the speaker and reactors in a plenary response session.

Plenary response session

In these sessions, one during each day of the conference, there was an opportunity to bring back to the whole conference group the questions and concerns of each working group, and to have a further comment by the plenary speaker and reactors.

Symposia

Six symposia proposals were accepted after review of the organising committee. Each symposium had three hours in total to engage participants in a reflection of a particular topic of interest for the conference.

The symposia were:

A. The social transformation as object of studies and practices, by Maria do Carmo S. Domite, Maria Cecília Fantinato, Wanderleya Nara Gonçalves Costa and Mônica Mesquita.

B. Social Theory and research in MES, by Jill Adler, Jeff Evans, Peter Gates, Clive Kanes, Stephen Lerman, Candia Morgan, Anna Tsatsaroni and Robyn Zevenbergen.

C. No teacher candidate left behind – a study of the largest mathematics alternative certification program in the United States, by Andrew Brantlinger and Laurel Cooley.

D. A symposium on "Opening the research text: Critical insights and in(ter)ventions into mathematics education", by Kathleen Nolan, Elizabeth de Freitas, Tony Brown, Ole Ravn Christensen, Paul Ernest, Shana Graham, Eric Gutstein, Stephen Lerman, Diana Stentoft and Paola Valero.

E. The production of legitimate text and the stratification of achievement in mathematics classrooms, by Eva Jablonka, Uwe Gellert, Christine Knipping and David A. Reid.

F. Learning mathematics and competences: bringing together three theoretical perspectives, by João Filipe Matos, Madalena Santos, Elsa Fernandes, Susana Carreira, Margarida Belchior, Neuza Pedro, Helena Gerardo, Mônica Mesquita, Alexandre Pais, Ana Sofia Alves, Teresa Faria, Teresa Silva, Nélia Amado and Isabel Amorim.

Paper discussion sessions

After peer review of all paper submissions, the organising committee accepted 34 papers for presentation and discussion during the conference. The full text of accepted papers was posted on the conference website and published in these conference proceedings. These projects generally centred around the presenters' current research work.

Project discussion sessions

After peer review of project submissions, there were six accepted project discussion sessions. Discussion papers were posted in the conference's website and published in the conference proceedings.

Agora

Inspired on the Greek tradition of a "popular political assembly" taking place in a public, open space such as the market place, it was decided to have two informal, evening discussion sessions about the future of MES.

Networking

Within the programme there were slots dedicated to informal networking among participants.

Concluding panel

This time the conference organisers proposed to have a last, concluding panel with all the plenary speakers in order to discuss dilemmas and questions that have emerged during the whole conference. The panel was led by Stephen Lerman.

	Sat 16	Sun 17	Mon 18	Tue 19	Wed 20	Thu 21
9:00		Plenary 1 *E. Gutstein* W. groups Plenary response	Plenary 2 *K. Brodie* W. groups Plenary response	Plenary 3 *M. Jurdak* W. groups Plenary response	Plenary 4 *O. Christensen* W. groups Plenary response	Closing Plenary Panel Closing
12:30		Lunch	Lunch	Lunch	Lunch	Lunch
14:00 16:00	Registration & reception	Papers / Project discussion	Papers / Project discussion	Visit to the Community Centre	Papers / Project discussion	Departure
18:30	Opening Panel	Symposia *A, B, C, D*	Symposia *A, B, E, F*		Symposia *C, D, E, F*	
20:00	Dinner	Dinner	Dinner Agora 1	Dinner at the Community Centre		

THE REVIEW PROCESS AND PROCEEDINGS

All of the papers published in these Proceedings were peer reviewed by two experienced mathematics education researchers before publication. Strict guidelines were followed to ensure that the papers had a significant contribution to make to the field, and were based on sound literature review and methodology. The production of the Proceedings was possible through the cooperation of many of the Conference participants who offered their time to peer review papers. The challenges faced by some of our conference participants from language backgrounds other than English to write their paper in English are acknowledged and appreciated, as well as the time of some generous reviewers who provided support for language correction.

PARTICIPANTS

In this occasion there were 66 participants from 16 countries: Australia, Brazil, Canada, Denmark, Germany, Greece, Iran, Israel, Mozambique, Portugal, Republic of Lebanon, South Africa, Spain, Sweden, United Kingdom, United States of America.

ACKNOLWEDGEMENTS

Finally, we would like to especially thank the support of three women who worked with us in the realisation of this conference. Thanks to Neuza Pedro, from the Universidade de Lisboa, for her support in all the administration of the conference. Thanks to Patricia Perry, from the Universidad Pedagógica Nacional de Colombia who demonstrated super-human efforts in undertaking the final formatting and preparation of the papers for publication. Thanks to Sanne Almenborg from Aalborg University in developing and managing the conference website.

An electronic file of all individual papers as well as of the whole proceedings is available at http://www.mes5.learning.aau.dk/

Lisbon, Aalborg and Sydney, January 30th, 2008

João Filipe Matos, Paola Valero and Keiko Yasukawa

PLENARY AND REACTION PAPERS

REINVENTING FREIRE: MATHEMATICS EDUCATION FOR SOCIAL TRANSFORMATION

Eric (Rico) Gutstein[1]

University of Illinois

For Paulo Freire, education was a necessary part of the political process of changing society. Mathematics education can play that role, supporting young people to read and write their worlds with mathematics as a key analytical means. In urban Chicago, our mathematics work in a social-justice-oriented high school of low-income African American and Latino students attempts to reclaim Freire's purpose. In this paper[2], I describe our praxis—teaching, learning, and research in mathematics education which involves teachers and the students themselves in collaborative efforts. We focus on preparing both the youth and adults to participate in social movements and political change.

> No oppressive order could permit the oppressed to begin to question: Why? (Freire, 1970/1998, p. 67).

Paulo Freire left us many things from which to learn, and several of them have particular meaning for my work. First and foremost is that education needs explicitly needs to be in the service of the people of the world, standing firmly with the oppressed against capitalism and neoliberalism (1994, 1998, 2004). Though Freire acknowledged the limitations of education by itself in changing society by itself, he believed it played an important, essential role in social transformation. For him, education needed to be for liberation rather than for domination and submission which is how it functions in urban US schools (and elsewhere). His terms, *reading the world*, or developing a deep sociopolitical consciousness of relations of power and the genesis of structural oppression, and *writing the world*, or taking one's own destiny into one's hands one's own destiny to make history, are useful ways of understanding education in relation to changing the world.

Second, he argued that teachers and students need to be partners, learning from and working with each other, in various movements for liberation, including in anti-colonial, independence struggles (e.g., Guinea-Bissau and Cape Verde, Freire, 1978), adult literacy campaigns (e.g., Brazil, Nicaragua, and Grenada, Freire, 1994), and school settings for children and youth (e.g., São Paulo, and the United States, Freire, 1993; Freire & Macedo, 1987). Teachers need to see students as allies in common struggles for social justice; this perspective echoes the long history and tradition within the US of African American education for liberation (Anderson, 1988; Bond, 1934/1966; Du Bois, 1935; Perry, 2003; Provenzo, 2002; Siddle Walker, 1996; Woodson, 1933/1990).

Third, he saw history "as possibility." By this, he meant that "the future does not make us. We make ourselves in the struggle to make it" (Freire, 2004, p. 34). He wrote repeatedly that humans are *conditioned* by structural and institutional forces,

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but not *determined*, and the fact that we are able to be conscious of that conditioning means that we can transcend it (1994, 1998, 2004, 2007). He believed that humans are "unfinished," and our unfinishedness implies our constant search for deeper understanding and social (and individual) transformation.

Fourth, Freire had a deep appreciation for what he called "popular knowledge," that is, the knowledge of the "popular classes" (1978, 1994). His writings are full of vignettes of how he learned from workers, peasants, fisherpeople, and other so-called "ordinary" people. He came to understand the limitations of his own class position, to understand the meaning of "class suicide" (1978), and to appreciate the perspectives, analyses, values, beliefs, and hopes of the oppressed (1994). In discussing his early work with culture circles in his home city of Recife, he wrote:

> In the beginning of my work, my surprise in the face of the critical positions assumed by these unschooled workers arose from the perception that I had up to that time that these were positions held exclusively by university students. My surprise had its origin in my own class position, increased by my university training—perhaps, to be more accurate, I should say by my elitist university training. (Freire, 1978, pp. 116-117)

Freire believed that educational programs needed to tap into and build on this community knowledge. He argued that "the starting point for organizing the program content of education or political action must be the present, existential, concrete situation, reflecting the aspirations of the people" (1970/1998, p. 76). In his work, these contexts were uncovered by studying the people's *generative themes* (the dialectical interrelationship between key social contradictions in their lives and how they understand and interact with them, Freire, 1970/1998). Teachers in Brazil (and elsewhere, including in the US) have successfully developed curriculum from these themes (Freire, 1993; Gandin, 2002; O'Cadiz, Wong, & Torres, 1999).

Finally, although Freire's work covered many other ideas, currently the most relevant to me, and perhaps the most important, was that political experiences are essential to develop political consciousness, and this conscientization is key to learning to read—or to do mathematics. When people are involved in political struggles and social movements, their engagement can lead to a deeper understanding of relations of power and how they can affect the course of history. Furthermore, and dialectically, this increased awareness can then lead people to become more involved and committed to transforming society. In particular, since he was involved in literacy campaigns in which people were learning to read both the *word* (acquire textual literacy) and the *world*, a fundamental issue was that of motivation—why should people learn to read, or, in our case, to do mathematics?

Freire's experience taught him that this motivation was directly related to how people politically understood the social necessity to be literate. This was a key lesson from his work in Brazil, Guinea-Bissau, Chile, and Nicaragua (Freire, 1970/1998, 1978, 1994; Freire & Macedo, 1987). Commenting on where they chose to conduct literacy campaigns before the 1964 Brazilian coup (which exiled him), Freire (1978) wrote, “between acting in an area where popular consciousness was still buried and one where popular rebellion was visible, we did not hesitate in choosing the second” (p. 111). It was clear to him even then of the relationship of political engagement to the demand for literacy: “In Brazil…literacy in rural areas…made sense only to those within the peasant population who were involved in situations of conflict and who saw within them one more tool for their struggle” (p. 112). Others shared this view. Describing a 1975 UNESCO study on literacy, Freire (Freire & Macedo, 1987) wrote: “the relative success of literacy campaigns evaluated by UNESCO depended on their relation to the revolutionary transformations of societies in which the literacy campaigns took place” (p. 108). Freire’s analysis of why that was so has to do with how people develop their sociopolitical consciousness through writing the world, or, in other words, acting politically to change society. He described the process in Nicaragua after the Sandinista revolution:

> Literacy in the case of Nicaragua started to take place as soon as the people took their history into their own hands. Taking history into your own hands precedes taking up the alphabet. Anyone who takes history into his or her own hands can easily take up the alphabet. The process of literacy is much easier than the process of taking history into your own hands, since this entails the “rewriting” of your society. In Nicaragua the people rewrote their society before reading the word. (pp. 106-107)

How is this relevant to mathematics education in an US urban context? Freire worked mainly with adults, who were volunteers, in economically developing countries, on literacy campaigns, with no high-stakes tests, and the freedom to design curriculum from learners’ generative themes. In contrast, we are working with youth in a Chicago public high school, who are mostly not volunteers, in a so-called “advanced” capitalist country, on mathematics, with plenty of high-stakes tests, and, mostly with a mandated curriculum that is irrelevant to our students’ lives. What can we learn from his experiences and from those who have tried to actualize in practice his theory and principles, and how can we apply this to our contexts?

Before addressing this, I want to make a few points. First, this is not to “import” Freire. He consistently argued that that this was not possible, that the particularity of local contexts was fundamental. As he put it, “In order to follow me it is essential not to follow me!” (Freire & Faundez, 1992, p. 30). A central idea of Marxism that I draw on is that external conditions are only that—conditions—while internal contradictions or dynamics are the basis for development. As Mao Zedong (1937/1971) wrote, “In a suitable temperature an egg changes into a chicken, but no temperature can change a stone into a chicken, because each has a different basis” (p. 89). In other words, reinventing Freire means to concretely analyze the concrete conditions in front of one. Second, Freire did not write much about certain contradictions in his own country, let alone in the US. A number of African American scholars (e.g., Haymes, 2002; Ladson-Billings, 1997; Murrell, 1997), while upholding Freire, questioned his lack of deep analysis of questions of race in Brazil—the country with the second largest number of people of African descent on the planet. No one doubted Freire's commitment to anti-racist politics, but his overall focus on issues of social class and oppression and lack of attention to racialization seem problematic. At the very least, in the US, we have to turn elsewhere on these issues. For me, the record of African American education for liberation is a major source of inspiration, theoretical clarity, history, and practical direction (Anderson, 1988; Bond, 1934/1966; Harding, 1990; Perry, 2003; Watkins, 2001; Woodson, 1933/1990). Third, Freire's treatment of gender issues has also been critiqued (Ellsworth, 1989), or at the very least, extended (Weiler, 1991), although some, like bell Hooks (1994) and Freire himself (1994, 2004), made clear that his weaknesses just reinforced the notion of human unfinishedness. Finally, one last point is that Freire's historical harsh critique of capitalism and his scathing condemnations of neoliberalism in his later writings (1994, 1998, 2004) dealt much more with ideological aspects than with structural ones. That is, Freire wrote very little about political economy, which, for me, is essential to understand our current sociopolitical contexts. Those points notwithstanding, as we have tried to comprehend the settings in which Freire wrote, to grasp the larger principles and how they emerged from those situations and in turn guided his efforts, his work been extremely important in providing a political orientation for our work in urban US contexts.

THE CONTEXT OF MATHEMATICS EDUCATION IN THE US TODAY

The educational situation in the US today, particularly that of mathematics (and science and technology) education, cannot be understood outside of the larger, global political situation. From the perspective of the US government and capital, there is a near-crisis. Their analysis is that the US is in danger of losing its dominant hold on the global economy (while maintaining its military might). A rash of influential policy documents have recently come out, with disaster-evoking titles (*Rising Above the Gathering Storm, The Looming Work Force Crisis, Tough Choices or Tough Times,* and *America's Perfect Storm*). The primary theme in the reports is that the US may be unable to maintain its global economic supremacy due to its underprepared workforce and poorly performing student body in the face of the "*billions of new competitors* [who] *are challenging America's economic leadership"* (Dept. of Education, 2006, p. 4). The imagined solution to what is framed as a national problem is to increase productivity through innovation; upgrade the US workforce; and step up mathematics, science, and technology education. The central policy scheme is the *American Competitiveness Initiative* (ACI) which President Bush unveiled in 2006. He later signed the *America Competes Act* in August 2007 which partially codified the ACI and put large resources into the initiatives.

Although the ACI has several components, the most relevant here is the major focus on mathematics education as a central part of the solution. As the logic goes, improving mathematics education will help improve productivity and that will raise the standard of living of the US people (National Academies, 2006). However, recent history shows that productivity increases in the US have benefited only the wealthiest, not the majority. Over the last 40 years, real wages stagnated, while productivity markedly increased. "The typical, or median, workers' hourly wage was just 8.9% higher in 2005 than in 1979…. In contrast, productivity has grown by 67% since 1979" (Economic Policy Institute, 2007). More recently, "from 1980 to 2004, while U.S. gross domestic product per person rose by almost two-thirds, the wages of the average worker fell after adjusting for inflation" (Tabb, 2007, p. 20). Economist and New York Times columnist Paul Krugman (2004) added, about income inequality in the US more generally:

> According to estimates by the economists Thomas Piketty and Emmanuel Saez—confirmed by data from the Congressional Budget Office—between 1973 and 2000 the average real income of the bottom 90 percent of American taxpayers actually fell by 7 percent. Meanwhile, the income of the top 1 percent rose by 148 percent, the income of the top 0.1 percent rose by 343 percent and the income of the top 0.01 percent rose 599 percent. (Those numbers exclude capital gains, so they're not an artifact of the stock-market bubble.)

If the past is any indication of the future, the productivity increases that the ACI may create will not likely not go to the majority of the US people, but will accrue to the richest as they have in the recent past. Thus, the ACI goal to boost mathematics education is a way to serve capital by developing mathematical adept professionals who will help the US produce its way to continued world economic dominance and greater prosperity—for the wealthiest in the country. Nothing in it advocates for the transformation to a more socially just planet (Gutstein, in press).

The ACI is a particular and current manifestation of positioning mathematics education to serve capital in the US, but education in the service of the status quo and profitability for the financial and corporate elite of the country is nothing new. Freire (1978), citing Marx, pointed out that capital's use of the means of production and workers' labor power to produce commodities with high exchange value had a potential ally in education: "Education in the service of this lucrative combination obviously cannot have as its objective to reveal the alienating character of the process. What it must do, therefore, is to hide it, reducing education to the mere transference of know-how, seen as neutral" (p. 109).

In opposition to this role, Freire wrote about the changes taking place in Guinea-Bissau, right after liberation from Portugal, in which education was intended to support the transition of society to one which supported humanity and social justice: "In the society seeking to reconstruct itself along socialist lines, on the contrary, basing itself on the new material reality which is taking shape, education should be preeminently revealing and critical" (p. 109). Although we are far from having socialism in the US, and the general problems of democracy and socialism were not solved in the 20th century, Freire's words about education are just as meaningful for our preparation today in the US as they were for a newly independent country in Africa emerging from over 500 of colonial occupation. This is a potential role for mathematics education, in urban schools populated by low-income students of color in the US. I now turn to our Chicago experiences in trying to reframe mathematics education.

TEACHING MATHEMATICS FOR SOCIAL JUSTICE: PROVIDING OPPORTUNITIES FOR POLITICAL EXPERIENCE

I have worked with public schools in Chicago for the last 14 years, first for 10 years with an elementary school I call Rivera, in a low-income Mexican immigrant community where I taught 7th and 8th grade mathematics for about four years (just one class, as part of how I defined my work as a university professor). For the past four years, I have been working with a new high school in a similar community whose students are 30% African American and 70% Latino, mainly of Mexican descent, essentially all of whom are low income. That school is the Greater Lawndale/Little Village School for Social Justice (called "Sojo" by most), and I support the mathematics teachers, work with students, participate in developing social justice mathematics curriculum, and co-teach the social justice mathematics projects (which range from a few days to two weeks). In both settings, I have studied the process as it unfolds, with students and teachers as coresearchers (Gutstein et al., 2007; Sia & Gutstein, 2008).

Briefly, I understand social justice mathematics education to be when teachers and students work together to provide students the opportunity to read and write the world with mathematics (Gutstein, 2006). These ideas owe much to my interpretation of Freire's work and of the history and tradition of African American liberatory education. My goals include that students learn both mathematics and about the world. They should develop deep sociopolitical consciousness of their immediate and broader contexts and also acquire a sense of social agency, that is, see themselves as capable of changing the world. In the process, they should develop strong cultural and social identities, to be rooted in who they are as a people and to develop the confidence to stand up for their beliefs. They should learn rich mathematics so that they have opportunities to study, pursue meaningful lives, and support themselves, families, and communities, but even more, so that they can use mathematics to fight injustice and improve society. (Our data suggest that developing sociopolitical awareness and a deeper understanding of injustice demands mathematical sophistication and maturity). And finally, we want students to change their orientations toward mathematics, to realize that it has real meaning in life and can specifically be used to read and write the world.

I am well aware that at Sojo and Rivera, I am an outsider to the communities, languages, and cultures. However, I am a close outsider because I have a good deal of life experience in such communities, and I consciously try to stand in solidarity with the people there. Also, I am a anti-Zionist Jew with the memory of the Holocaust and anti-Semitic racism in my being, and thus have some empathy for other people's suffering. Nonetheless, attempting to teach for social justice is complicated enough, and to attempt to do so while teaching "other people's children" (Delpit, 1988)—

crossing lines of social class, race, age, gender, culture, language, ethnicity, experience—is even more complex. Although I do not have space here, this is important to my story (see Gutstein, 2006).

There is evidence from our work that the above goals can be partially realized—that is, youth can *begin* the process of reading and writing the world with mathematics, while learning rich mathematics—but the work is complicated, slow, and difficult (which I say more about below). A positive outcome is that Sojo's 11th graders (the class on which we have focused, about 90 students) have normalized learning mathematics for social justice. When they were 10th graders, we conducted focus group interviews with about two thirds of the class. We proposed to them that we do a mathematics project about neighborhood *displacement*, which has politically related but distinct specific meanings to the school's two populations. It means gentrification in North Lawndale (the Black community) and exclusion from the country altogether in South Lawndale (the Mexican immigrant community). During the interviews, we described mathematics as a weapon in the struggle for social justice. No student expressed surprise, and all students we interviewed except one reported being interested and said they wanted to do the project.

We suggest this is so for several reasons. Sojo has an explicit mission statement about social justice, although that means very different things, in theory and practice, to different teachers, parents, and students. Nonetheless, the class I refer to here has intermittently completed social justice mathematics projects since the week they started school. Although we have only spent perhaps 15% of our total time, on three or four projects a year, they have been evidently been sufficient meaningful and memorable to students that none reported it as unusual to hear that particular framing of mathematics.

When students were 9th graders (2005-2006, the year Sojo opened), they completed a project on racial profiling with which many students were familiar or had personal experience. Part of their work was to simulate (with their calculators' random number generators) the number of supposedly random traffic stops police made in an area for which we had the real data. Before we began, we explained that we would "use mathematics to check up on the police" to verify if they were conducting unbiased stops. Our framing was explicit: to use mathematics to collect and analyze data to evaluate police behavior, to pose other questions and possible further investigations, and to fight for social justice.

This year (2007-2008, their 11th grade), we started out school with a two-week project about the criminalization of youth of color, specifically about the *Jena Six*, six African American male high school students from Jena, Louisiana, a small town in the southern US. In December 2006, they were initially charged with attempted murder in a schoolyard fight that developed out of a racist incident a few months earlier. The first of the six (Mychal Bell) was tried and convicted by an all-white jury in June 2007 and was awaiting sentencing in September 2007, the beginning of the school year. The focus question was: Given Jena's demographics (85.6% white, with 2,154 adults as of the 2000 census), what was the probability of randomly choosing an all-white (12-person) jury for Mychal Bell? The project contributed to students taking action—they walked out of school and organized an impromptu protest on a nearby corner on Mychal Bell's sentencing day. This happened when students discovered that a local Black college was holding a rally, but the school principal was unable to satisfy students' demands for a bus to take them there.

When students were 10th graders, they completed a project, "Reading Hurricane Katrina with Mathematics." It began: "This is an investigation into Hurricane Katrina. The main question we are asking is: What story can mathematics tell us about what happened with Hurricane Katrina—and who did it happen to and why?" We had students look at pictures of displaced people, all African American who had nowhere to stay but in the "Superdome" (a sports stadium), and asked. "This picture looks like only African Americans were in the Superdome, so maybe only African Americans lived in New Orleans. Is that true? Or maybe they were the only ones who stayed/got left behind? We will investigate these questions." They used a very confusing graph from the New York Times to answer a series of questions. The final part of the assignment was:

> Now that you've done all this investigation, it's time to pull together the story that your data tell. Write a good, solid essay explaining your analysis of Hurricane Katrina on the people of New Orleans. You *must* use mathematical arguments from your work here and create one (or more) *well-labeled* graphs to present your data/mathematical arguments. Here are some questions to help you:
>
> a) What data are most convincing and what do they tell you? Why are these data convincing to you?
>
> b) How do the data help explain the story? Could there be other explanations?
>
> c) What other data would you need to know or do you want to know? What questions do *you* have?

Students' essays were emotional, strongly worded, and uneven. Although the mathematics was essentially proportional reasoning and not overly challenging, making sense of the graph was quite difficult. Students had to understand that the ratio of *poor-African-Americans-with-no-car* to *poor-whites-with-no-car* was relevant and key to arguing why more African Americans were stranded in New Orleans versus whites and why their percentage was higher. For the most part, students used mathematics to argue their points, although there were weaknesses and errors. I include some student data here to give a sense of the issues and perceptions of Sojo students. Guadalupe, a Latina student, wrote in her essay:

> It was 3.2 times more likely for a black person to be poor than a white person in New Orleans. The question we really need to ask is did the African American people get left behind for their skin color?…. A large percent of the people that got left behind were

> poor & Black. 14 to 1 ratio of not having a car for poor Black person vs. a poor white person. It was 8 more times as likely for a Black person [regardless of income] not to have a car.

And Jermane, an African American male wrote:

> The most convincing piece of data was that it was 3.2x more likely to be a poor black in New Orleans than white. This basically tells me that it is more likely for you to see a poor black than a poor white [in the Superdome]. This data is the most convincing because I saw a lot of this on TV. This helps to tell the story because when you look at the pictures, that is all you see.

Mirella, a Latina, after doing a lot of careful mathematical analysis is her paper, wrote:

> Mathematics helped me make a realistic picture, how many black people were left behind because they didn't have any type of transportation. I think using math was an amazing way [of] dealing with huge problems around the world. You could use mathematics for almost anything in the everyday world. Projects like this keep people aware [of] what's going on.

Virginia, also Latina, made a few points that were mathematically not quite accurate: "Another ratio that is very unfair is that for every one white household, there are 14 black households w/out no car [this is actually the ratio of poor Blacks w/out cars to poor whites w/out cars]. So there are more black people in New Orleans, but still there are more whites w/ cars" [actually true, but she may have confounded rates with actual numbers].

She continued, suggesting that despite some confusion, she learned from the project:

> From this project, I found out that in New Orleans, racism is going on. I really never thought of there being such things around. This opened my eyes to see that math helps us find REAL percentages of what really happens. This showed me that I don't need no one to come tell me and lie to me about who are being left behind when I can do it myself.

As a final note, Jeronda, an African American female wrote in anger about President Bush and captured a very common sentiment expressed by Blacks in the US at the time:

> Bush don't like Black people, forget him. If Bush cared about us Blacks, the Hurricane Katrina victims wouldn't be in the predicament they are in now. According to the project we did, "Reading Hurricane Katrina with Mathematics," mostly Blacks were left behind. Fewer whites than Blacks and others were left behind…. Suffer hell! With NO help, no money, no food, no transportation, etc. You can tell Bush don't care. Forget Bush. Elect Bush OUT!

Although we can point to possibilities, as well as actual achievements in our work, there have been (and continue to be) significant difficulties and obstacles. I cannot give them the space they demand, but that is certainly not to minimize the challenges (see Gutstein, 2006 for further discussion). Probably the most significant issue we have faced is how to reconcile the contradiction between, on the one hand, having a mostly mandated curriculum and a set of high-stakes assessments (e.g., the ACT exam), and, on the other hand, trying to develop —and teach— mathematics for social justice curriculum that builds on students' generative themes. An alternative framing is: How can we build on students' community (popular) knowledge while simultaneously supporting the development of critical (mathematical) knowledge and classical (academic) knowledge (Gutstein, 2007)?

This dialectic has numerous complexities. First, no social justice mathematics curriculum exists. There are several collections of "mathematics for social justice" projects, units, and lessons (e.g., Gutstein & Peterson, 2005; Mistrik & Thul, 2004; Shan & Bailey, 1991; Stocker, 2006; Thul, 2004; Vatter, 1996), but no actual curriculum that is cogent and cohesive. Second, a *published* mathematics for social justice curriculum is an oxymoron in a sense because it could not stem from students' generative themes, although one could develop a general framework for creating such a mathematics curriculum (none exists). Third, building curriculum from students' lives and knowledge is extremely difficult and time consuming (witness the 10-step process in the Porto Alegre, Brazil, *Citizen Schools Project*, Gandin, 2002). Fourth, writing rich mathematics curriculum in general is daunting. To create each of the National Council of Teachers of Mathematics-based curriculum in the US (there are 13 "reform" curricula) took millions of dollars and years of work. One "solution" to these quandaries has been to use rich mathematics curricula and interjecting, as coherently as possible, social justice mathematics projects, while working with students to co-create an environment that supports political relationships between students and teacher (Gutstein, 2006). But much remains to be done. At Sojo, we are ambitiously planning to teach a senior-level mathematics class next year (2008-9) that will blend pre-calculus and quantitative literacy with the social justice contexts to be determined collectively with students. In effect, we will try to move from our current 85-15 ratio of "standards-based mathematics" to social justice mathematics and reverse that proportion so that 85 to 90 percent of the contexts are ones in which students investigate their social realities.

A second major difficulty is the complexity of teaching —as opposed to developing— social justice mathematics curriculum. The literature in mathematics education is clear that even experienced teachers cannot easily teach good mathematics curricula in ways the developers intended (Fennema & Scott Nelson, 1997). One should expect that teaching mathematics for social justice would be even harder given the interdisciplinary complexity and the background knowledge teachers need. If we accept Freire's (1978) formulation of the relation of political experiences to conscientization, then teachers themselves will need to develop, in myriad ways, the necessary sociopolitical consciousness to teach for social justice and to build political relationships with students (Gutstein, 2008). At Sojo, the mathematics teachers are dedicated, but inexperienced and young. They have had to learn to teach, to teach mathematics, to teach a standards-based curriculum (new units each year), and to teach mathematics for social justice, all in the context of a new, complex school. None of it has been easy.

A third challenge has been to support students in using mathematics to present and mathematically defend their views and analyses about social justice issues. For example, we would like them to argue using mathematics (as one of multiple criteria) whether Mychal Bell's jury selection was fair or biased. This has not been easy. We acknowledge that teaching students in general to use mathematical argumentation and justification is not simple, especially given their lack of experience in doing so in elementary school (which we surmise). As evidenced by our data from a 2006 project (when students were 9th graders, Gutstein, 2007), students reported feeling strongly that there were things they could do to effect change —but almost none of their "action" suggestions used mathematics (Gutstein et al., 2007). In other words, students might be learning to *write* the world —but not necessarily *with* mathematics. This contrasts with our data from multiple sources which showed that students were beginning to *read* the world with mathematics. This raises the question of whether it matters if students advocate for specifically mathematical ways to change the world, as long as they make sense of social reality with mathematics and act as historical actors in whatever ways they see fit. We are not settled on this but plan to continue to work with students to defend their ideas mathematically and with words.

CONCLUSION

How does students' participation in learning mathematics for social justice relate to political experiences that can lead to political consciousness? The mathematics education of Sojo youth, I believe, has generally been divorced from their concrete reality since they began school at age five. We are all aware of the perennial question students ask mathematics teachers: "When am I ever going to need this?" The vague answer is usually along the lines of "you will need this in the future," or "when you get to college." Engagement, commitment, perseverance, and motivation in learning mathematics, and more generally, in school, clearly matter. Why should students who have been excluded, marginalized, criminalized, and discriminated against spend the time and effort to commit to school? Perry (2003) raised this question about African American education, though she could also have been talking about other people of color:

> Why become literate in contemporary America? Why become proficient in reading and writing?… Why work hard at school, or at anything else for that matter, if these activities are not inextricably linked to and address one's status as a member of a historically oppressed people? (p. 19).

Her response is powerful:

> *The Autobiography of Malcolm X* takes up these questions and provides an answer, *the* answer that has become embedded in African Americans' collective consciousness and

> narrative tradition: Read and write yourself into freedom! Read and write to assert your identity as an act of resistance, as a political act, for racial uplift, so you can lead your people well in the struggle for liberation! (p. 19)

Framing mathematics as a weapon in the struggle for social justice is a way of explicitly and intentionally politicizing mathematics education in particular, and school in general. Freire (1978) wrote of the clear choice between working with people whose consciousness "was still buried…[as opposed to] where popular rebellion was visible," (p. 111). Because one cannot force rebellion, one has to create opportunities for youth to be involved in political experiences and social movements, in appropriate and various ways, both directly and vicariously. In this case, the social justice mathematics projects at Sojo and Rivera become a form of political experience, fitting for youth in urban US schools. In 10 years of teaching social justice mathematics projects, although not all students loved them or were enthralled, some things are clear. I never heard a student ask, "When will I ever need this math?"

Overall, students have consistently been more engaged in the projects than in any other mathematics, regardless of context or content. This is because the projects are, in Perry's (2003) words, "linked to and address one's status as a member of a historically oppressed people." They position learning mathematics as a liberatory tool that provides students deeper understandings of Hurricane Katrina, racial profiling, conditions of immigrant agricultural workers, disparity in mortgage rejection rates, wealth inequality, the cost of the Iraq War, the impact of different world map projections, gentrification in their neighborhoods, and many other issues that we studied. These matter to the Sojo and Rivera students because of the righteous anger and powerful sense of justice that they bring with them into the classroom due to their location in an oppressive society. Furthermore, the projects at times give them ways to begin to see themselves as agents of change, whether through demonstrating in support of the Jena Six, attending city hall hearings about displacement, or just coming to know that, although they have the capacity to do so, their prior schooling has not prepared them to read and write the world (Gutstein, 2006).

Some students at Sojo are beginning to understand that they have been profoundly "miseducated" (Woodson, 1933/1990) in US schools that prepare them mostly for low-skill service-sector jobs, prison, the military, or the grave (Lipman, 2004). In North Lawndale (the African American community), the mathematics is grim—two males for every three females, because, to quote one male student from the community, "all the brothers [Black men] are locked up or in the ground." This realization of an incomplete, inaccurate education—which one student, Charles, wrote about on a project that examined world map projections, "It makes me feel like I was lied to all these years"—is related to what a farmer in post-colonial Guinea-Bissau told Freire: "Before [liberation], we did not know that we knew. Now we know that we knew. Because we today know that we knew, we can know even more"

(Freire & Macedo, 1987, p. 114). Rigoberto spoke to this well, also on the map project:

> I feel as if someone was trying to take advantage of me as a student. That just because I am a student, that I should believe anything that I was told…. The project really made me think more about the information being provided in many schools. I now start to question the material being taught. I really enjoyed doing the project because now I can think about maps and their differences. I can also see the differences in peoples' different point of views. I can imagine what other people think and see about how the globe and world really is.

This broader consciousness, gained through the political experiences of learning to read and write the world with mathematics, will be necessary for Rigoberto, other Sojo students, and youth like them to become effective change agents in the larger historical motion. Their mathematics education *can* contribute to this process. I end this paper with a quote from Rogelio, who was a part of our co-researcher team until he moved from the community:

> Before this [participation in the research project], everything was like a black and white picture. I just went to school, like I was a student soldier doing the same thing over and over, just went to school, came home, did my homework and didn't care about anything. But now, when I started doing this, everything started getting full with color, being understanding and getting ideas and not just learning the same things, but pushing it to the limit into what a person can do and actually understanding what's really going on in the world.

NOTES

1. Correspondence concerning this manuscript should be addressed to the author at 1040 W. Harrison St., M/C 147, Chicago, IL 60607, USA, or to gutstein@uic.edu

2. I would like to acknowledge the students at "Rivera" Elementary School and the teachers and students at the Greater Lawndale/Little Village School for Social Justice in Chicago, especially the student co-research teams. The work and knowledge production I describe here regarding the schools was genuinely collaborative.

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REINVENTING SCHOOL?

REACTION TO ERIC GUTSTEIN'S "REINVENTING FREIRE: MATHEMATICS EDUCATION FOR SOCIAL TRANSFORMATION"

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During my first years as a teacher I had the chance to stumble into some of Paulo Freire's writings. My ideas about education at that time were oscillating between didactic knowledge and pedagogical knowledge, which provided me a very professional but narrow view of my role as a teacher. I felt comfortable. I was enjoying my first years of teaching, with lots of new things to do, and plenty of didactic ideas to implement in the classroom, in order to allow success to my students. But problems and contradictions started to arise. For example, the fact that some of the nice and worked plans that I prepared to my classes failed because of strange things that aren't supposed to happen in a classroom. Things that had to do with the presence of thirty children with wills, fears, desires, problems, families, that definitively weren't the ones I imagined when preparing the class, in the comfort of my home. I started in a very intimate way, to realize that education is more than just teaching the right thing to a bunch of children. And the reading of Paulo Freire offered me the language to start thinking more clearly about the entire educational dimension that surpasses didactic knowledge and school. Freire helped me to realize that education, per se, is an empty word. Education implies cultural communication and a dialogical relation between teacher and students. But continuing the readings of his works I went further. I situated my work as a teacher in the world, in a society, which I progressively started to look with a critical stance. From then I couldn't ever separate my role as a teacher from my role as a human being. I started to be politicized about education.

So reading and having the opportunity to comment on this paper that describes an educational experience substantiated in the ideas and work of Paulo Freire, was a joy and an opportunity to see work of this kind in action in a contemporary space. But I am getting old, and my first years as teacher are history, as is my confidence in school education. So I would like to raise some doubts and to put some questions concerning the implementation of a critical mathematics education in a classroom within a modern capitalist society. But first, a short comment to the most relevant aspects of Gutstein's paper.

Like I said, bringing Paulo Freire's ideas and work to the current scenery of mathematics education is one of the important aspects of the paper. According to Freire (1998), the universal ethic of educators is to educate, to have political clarity regarding their stance (which implies an interpretation of man and the world), and to engage dialogically with their students. That political dimension should be present in every act of education, because, like Freire said, education is politics and an act of

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indignation against the injustices of the world. Education has the double role of making possible the conscientization of dehumanizing structures and practices present in everyday life, but also the transformation of that same reality. Those two aspects are very strong in Gutstein's work.

Secondly, Gutstein's has a preoccupation with connecting scientific research to people's real problems. The way Gutstein and his colleagues worked in school from *generative* problems, that emerged from real concerns of students and community, is based on the importance for teachers "to see students as allies in common struggles for social justice" (Gutstein, 2008, p. 3). But, since Gutstein is also a researcher in university, we could also say that this work is an example of research where the researcher sees people as allies and not as objects of study.

Finally, there is the possibility for students to do a different mathematics, a mathematics that opens up the possibilities for world scrutiny, and not just the truth as given by the logic of mathematics that assesses a given, real world. The projects were specifically designed to use mathematics as a vehicle to become aware of racial, economic, gender, or other discrepancies or inequalities. The question was very clear: how can we use mathematics to promote social justice?

Being a teacher in a basic Portuguese school (from 7th to 9th), after reading Gutstein's paper I immediately began thinking about my school. It's a typical Portuguese low-middle class school, with children from different backgrounds, some descendents from African, Brazilian and Eastern Europe countries, but mainly Portuguese students, not much different from the majority of Portuguese urban schools. It's also a very old school and it's literally falling apart, with scarce places to child to play. Aesthetically speaking, it's a decrepit school. During last an atmosphere of discontentment emerged between the teachers due to several central political measures that changed the condition of the profession. Among the teachers prevailed several dogmas on education, concerning its purposes, the pedagogical relation with the learners and so on, most of then very far from Freire's ideas. But the most severe aspect of the school reality is the huge gap between students and teachers. They are far from being allies; there are, if not adversaries, no more than neighbours. So, inspired by the central question made by Gutstein (2003) - how might teaching for social justice in a regular school be different? – I started to wonder about the possibilities and constraints of implementing or, using Gutstein (2008) words, normalizing a curriculum based on critical mathematics education for social justice in a (all) regular schools. But doubts appeared. Let me start by taking a look at the difficulties pointed by Gutstein.

In his other work (Gutstein, 2003), Gutstein goes further describing his experience in implementing a critical mathematics education in a Latino, urban school. Here Gutstein enumerates some problems and constraints that we could face when trying to carry out one initiative in mathematics education for social transformation in a school. Those are related with:

- Creating and implementing a curriculum with generative themes involving critical mathematics education. Many teachers don't have the time, the knowledge or the will to do that. According to Gutstein (2007), in some way you need to be a super teacher, that is, a teacher that simultaneously havs knowledge to create rich mathematics curricula and skills to successfully teach in urban schools.
- The pressure to learn the mathematics of the standard curriculum, or, as Gutstein (2007) named, the classical knowledge, that will be essential to students' approval in the high-stakes tests.
- The roots of mathematics education as a field stem from mathematics and psychology, and researchers have historically focused more on cognition than on sociocultural contexts.
- The common notion that mathematics is an "objective" science that is neutral and context free. To most people it sounds strange to talk about mathematics and social justice.
- The character of school. Children are not voluntarees, they are forced to go to school, and they learn that school is more a space and a time they have to surpass to be someone, and not a place to criticize or go to discuss their problems.
- In the capitalist society, education is market oriented. Like Gutstein (2008) says: "The ACI is a particular and current manifestation of positioning mathematics education to serve capital in the US" (p. 8).

  The disciplinary society that normalize and accommodate all the possible agents of transformation (like teachers). Like Gutstein (2003) says "educational practices that involve students in discussions and actions that critique sources of knowledge, question institutional practices, and run counter to norms and power structures within society are potentially problematic and can threaten schools and authority. Teachers put themselves at genuine risk by raising such issues" (p. 41).

Then we can say that there are difficulties related with power (the disciplinary society), ideology (psychology versus sociology), epistemology (nature of mathematics), economy (high-stakes tests, capitalist society), and students' expectations about school. All those dimensions clearly influence and condition the implementation of a critical mathematical education in the classroom.

Gutstein managed to overcome some of those difficulties. The question of ideology and epistemology is well resolved since the curriculum developed is clearly socially relevant . Schooling continues to be obligatory, but, as mentioned by Gutstein (2008), students stop asking questions like "When am I ever going to need this?" Those projects in which they were involved were, per se, meaningful. The economic question was problematized in the classroom through the development of a social justice oriented curriculum, but the high-stake tests remain (as does the capitalist society). Finally the question of power and the fact that education is part of a state apparatus to govern children's souls, as put by Popkewitz (2002). This problem is apparently overcome by Gutstein due to the specific conditions under which he implemented this initiative: particular students were interested in common problems making it possible to construct a curriculum based on generative themes, a school that gave him space for developing this project, a group of teachers motivated and engaged on doing mathematics for social justice. But I ask the question (as Gutstein does): will the results will be as optimistic in the majority of regular schools? What are possible scenarios for implementing mathematics education for social justice in ordinary schools, with normal conditions, heterogeneous students and teachers resigned to the status quo?

Taking teachers as an example, Covaleskie (1993) argued that the institutional arrangements, in ways no one quite seems able to pin down, make even the most able and intellectual of the teachers tone down their teaching to the level of the approved curriculum materials. Many teachers have personal interest in real political, economic, and social issues which they leave at the classroom door. Seeing their job as controlling their students, they seek to do this through control of the curriculum. Then it is not a "personal" problem of teachers, but a "structural" problem, having to do with the mechanisms and discourses to govern people that constantly disable us to work in transformative ways. The question is how to connect the implementation of critical mathematics education in schools that, as mentioned by Gutstein, need to be for liberation rather than for domination and submission, with the social role carried out by school in regulating and governing population (Walkerdine, 1994; Rose, 1999; Popkewitz, 2002)?

Taking my school as example, we can imagine all the constraints I felt when trying to implement critical mathematics education in the classroom. Although the Portuguese curriculum explicitly mention the importance of working with student topics of mathematics and society, it is content orientated and the high-stakes tests are always present, putting pressure on teachers and students to be glued to specifically mathematical content. That corrupts any possible change. As a teacher I feel that the only thing I can do is to confront students with the reality, with the contradictions I feel, and doing so, contribute to politicizing mathematics education in particular, and school in general. But I am conscious that little transformation has been made.

Like I mentioned before about my school, all the discourses and practices present in my school are disciplinary mechanisms that progressively constrain any well-meaning initiative. For one reason or the other: the initiative is seen as something isolated and become marginal, or is absorbed in practices that are far from promoting change, or integral to the system.

The work of Gutstein clearly shows that under certain circumstances, it is possible for teachers to promote more equitable classrooms, "helping students explicitly and consciously use mathematics itself as a tool to understand and analyse the injustices in society" (2003, p. 69). But my point is that, when we try to expand or institutionalize critical mathematics education in school, all the attempts of transformation is *compromised* by the structure of a society that obviously has other goals for education. Even Gutstein (2003) admits this when he says “but one cannot easily know how our 2 years together helped students develop more as agents, nor in fact, whether helping them do so will contribute to justice in society.” (p. 69) In fact, all the problems remain intact: school as an obligatory institution, in a capitalist society, with plenty of high-stakes tests, and with a mandatory curriculum that is irrelevant to students’ lives.

I am not saying that experiences like the one Gutstein developed aren’t important. They certainly were important to those people he worked with, in a particular context, and, through the dissemination of his work, will definitely inspire others to carry on initiatives for social justice in the classroom. But the particularity of the case made me more pessimistic about implementing critical mathematics education in the classroom. Not only because nothing has changed in the core difficulties pointed by Gutstein, but also because I feel that creating the idea that a critical mathematics education could be implemented in the regular school will contribute to the normalization of topics of mathematics for social justice in such a way that allows everything else (the main problems of our society) to remain the same. Assuming that a critical mathematics education is possible in school and can contribute to social transformation within our contemporary society, is taking the risk of normalizing those practices and integrating them in the school discourse, that is, as mentioned by Gutstein, the economical discourse. Like Freire (1998) said “the elites are anxious to maintain the status quo by allowing only superficial transformations designed to prevent any real change in their power of prescription” (p. 508). So we are always facing the danger of being deceived, as we think that we are struggling against oppression, when in fact we are being allowed by the dominant class to do so, just to cool down the rebellion. As I said before, the core things, like assessment and school, are here to stay.

What I want to highlight is that there are traps in trying to educate children to be participative, critical, active, socially competent. Like Freire mentions, nobody educates anybody, men are educated in communion, so there is a problem when we stipulate that education should form some kind of citizen, though we may have a view of what the right citizen is. We must fight the idea that to educate is to fill out people with something they lack: whether it is knowledge, self-esteem, skills, or criticism, activism or participation. On other hand, what should we do with the children who aren’t active, participative, critical or socially competent? This is, what do we do with the young peoplewho don’t achieve through education that level that we all (teachers and society) desire. They will be excluded. Exclusion begins when we create those categories, those goals based on someone’s opinion of what education means, or the characteristics of the educated person. So critical mathematics education should be dealt with care when trying to stipulate goals or concerns for mathematics education in order not to get trapped in the same artifice that leads to social exclusion and injustices, those very aspects of education that critical mathematics education criticizes.

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DESCRIBING TEACHER CHANGE: INTERACTIONS BETWEEN TEACHER MOVES AND LEARNER CONTRIBUTIONS

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This paper focuses on a teacher's changing practice in the context of curriculum change in South Africa. The teacher taught in a low socio-economic status school and worked to engage and develop learners' mathematical reasoning. Using a range of analytic tools, I show that his pedagogy was responsive to learners and enacted a number of key aspects of the new curriculum. At the same time he maintained a number of 'traditional' practices and there were strong continuities in his teaching across 'traditional' and 'reform' contexts. The paper shows that a key issue for teaching in this classroom was learners' very weak mathematical knowledge, which was made visible by the teacher's approaches and which simultaneously constrained his teaching. The paper argues for complexifying our notions of teacher change, and that issues such as the interaction between learner knowledge and pedagogy be taken into account if we are not to exacerbate existing divides among rich and poor contexts.

In this paper I draw together two strands of research that have occupied me for the past ten years. These are a concern with how mathematics teachers develop their practice, particularly in the context of curriculum reforms[1] and a methodological concern with how to describe mathematics teaching practices. To date, I have addressed these two concerns in different socio-economic contexts of teaching, informed by a strong concern for equity and social justice in mathematics teaching, particularly in an era of mathematics curriculum reform. In this paper, I present a case study of a teacher teaching in a school that serves poor learners whose mathematical achievement is low. I draw on a range of tools to describe his developing practice and the successes and challenges that the new curriculum presents to him and his learners.

DESCRIBING PRACTICE

Practices involve patterned, coordinated regularities of action directed towards particular goals or goods (MacIntyre, 1981; Scribner & Cole, 1981). Practices are simultaneously practical and more than practical as they involve particular forms of knowledge, skills and technologies to achieve the goals of the practice (Cochran-Smith & Lytle, 1999; Scribner & Cole, 1981). Practices are always located in historical and social contexts that give structure and meaning to the practice and situate the goals and technologies of the practice. Thus "practice is always social practice" (Wenger, 1998, p. 47), and practices involve social and power relations among people and interests (Kemmis, 2005). In the case of classrooms, classroom discourse is always co-produced between teacher, learners and their social contexts.

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For MacIntyre a practice is a means whereby goods and standards of excellence internal to the practice are realised and "human powers to achieve excellence … are systematically extended" (MacIntyre, 1981, p.175). Thus learning is central to a practice, because as social goods and goals shift, so to do the means to achieve them. According to Wenger (1998) practice entails community, meaning and learning; practices learn and people, both teachers and learners, learn in practice. Given the complex nature of practices, how to describe them, is an ongoing methodological concern for researchers. It is inevitable that we need to foreground some aspects of practice, while backgrounding others (Lerman, 2001), particularly in presenting our research. At the same time, as we do this, it is important not to sanitise practice, and to try to present at least some of the real "noise" of classrooms (Skovsmose & Valero, 2003). There is no doubt that we need a range of methodological tools and lenses to capture the complexity of classrooms.

TEACHER CHANGE

Since learning and development are key to practice, it is also important to be able to capture changes in practice. This is especially the case in the context of curriculum reforms, where mathematics teachers are being encouraged to shift their teaching practices in ways that support a different kind of mathematics learning – the development of conceptual and reasoned understandings on the parts of learners rather than the procedural learning that takes place in many classrooms (Ball & Bass, 2003; Kilpatrick, Swafford, & Findell, 2001). Supporting learners' reasoning suggests kinds of classroom interaction where learners discuss their reasoning with each other and their teacher, and where learners and teachers communicate and justify their mathematical ideas to each other.

The research on teacher change in contexts of reform tends to make two major claims. The first is that teachers, the world over, struggle to change their practices in the direction of reform-oriented teaching (Brodie, Lelliott, & Davis, 2002; Hayes, Mills, Christie, & Lingard, 2006; Kitchen, DePree, Celedon-Pattichis, & Brinkerhoff, 2007; Sugrue, 1997; Tabulawa, 1998; Tatto, 1999). In a historical study of United States schools over the last century, Cuban (1993) shows that, through a number of reform movements, curricular and pedagogical aspects of reforms rarely took hold in classrooms and that major changes were usually in interpersonal relationships between teacher and learners. A second outcome of research on reform teaching is to describe models of exemplary reform teaching, making the claim that such teaching is possible, albeit with many challenges, illuminating different approaches to reform teaching and showing how the challenges can be overcome. Such cases come mainly from well-resourced contexts (Boaler, 1997; Boaler & Humphreys, 2005; Chazan & Ball, 1999; Hayes et al., 2006; Heaton, 2000; Lampert, 2001; Staples, 2007). Some research takes the middle road, presenting more textured descriptions of points of difficulty for teaching and when, how and why teaching in reform-oriented ways breaks down (Gamoran Sherin, 2002). In my own work with colleagues and students, we have shown that some aspects of reform practice are easier for teachers to work with, for example selecting tasks of higher cognitive demand (Modau & Brodie, 2008) while others are more difficult, for example interacting with students while maintaining the level of task demand (Jina & Brodie, 2008; Modau & Brodie, 2008). We have also argued that adopting reforms requires teachers to coordinate a range of new practices and to think about their practice in new ways. Such coordination is an immense task and means that teachers' taken-for-granted practices might break down in the face of new practices (Slonimsky & Brodie, 2006). It is thus highly likely that teachers attempting to work with reforms may resort to traditional practices, more or less deliberately (Brodie, 2007a).

So in describing changing practice, it is important to capture some of the 'noise' of change, that reform-oriented teaching can and should include some traditional practices. Such practices are both an integral part of reform teaching as well as possibilities for further development. Methodological tools should be able to capture this complexity as well as distinguish key moments of and for change.

REFORM AND EQUITY

There has been much debate as to whether current mathematics reforms can be a mechanism for ensuring more equitable participation and achievement in mathematics (see Brodie, 2006b, for a summary of these debates). Empirical evidence in well-resourced countries is beginning to show that reforms do mitigate achievement gaps between marginalised and other learners (Boaler, 1997; Hayes et al., 2006; Kitchen et al., 2007; Schoenfeld, 2002). However, as mentioned above, the evidence also shows that implementation is not widespread and in fact it is likely that implementation of reforms is inequitably distributed (Kitchen et al., 2007). Particularly in African contexts, issues of resources, including big classes and few materials, teacher confidence and knowledge, and support for teachers, can be major barriers to developing new ways of teaching (Tabulawa, 1998; Tatto, 1999). If reforms are successful in promoting equity and if they are not taken up in less-resourced countries, then existing divides between rich and poor countries are likely to be exacerbated.

In what follows, I will present a case study of a teacher, called Mr. Peters in this paper, in a Johannesburg school with learners of low socio-economic status. All of the learners are black and their parents and caregivers work in menial jobs or are unemployed. The school has old furniture, intermittent electricity, peeling paint and broken windows. There is gang activity in the area, and learners are often assaulted on their way to and from school. During the period of the research, an armed robbery was committed against a teacher on the school premises, by a former learner at the school. This research focuses on Mr. Peters' grade 10 class of 45 learners. Through learner interviews and classroom observations, the learners' mathematical knowledge was established to be at least two years below grade level. Mr. Peters' mathematical knowledge, established through an interview, was very strong. He was enrolled in a post-graduate degree programme at Wits University at the time of the study, and was one of five teachers who chose to participate in this study and formed a purposive sample for the study. Mr. Peters had learned much about the new curriculum during his studies but this was the first time that he was attempting to systematically shift some of his practices. He can thus be characterized as new to reform-oriented teaching, as were his learners.

Two weeks of Mr. Peters' lessons were observed and videotaped. In the first week, Mr. Peters taught from his usual syllabus in his usual way. In the second week, Mr. Peters worked to develop mathematical reasoning among his learners and to listen to their developing reasoning as he interacted with them. Using a range of analytic tools, I will show that in parts, his pedagogy was responsive to his learners and enacted a number of key aspects of reform mathematics. Definite shifts from his prior teaching to his teaching of mathematical reasoning could be discerned. At the same time there were aspects of continuity in his practice and he maintained both positive and negative aspects of his prior practice.

TASKS

A key aspect of reform-oriented practice is choosing tasks that allow for conceptual thinking, reasoned justification and communication of mathematical ideas. Mr. Peters' choice of tasks shows interesting shifts and continuities across the two weeks. In the first week he chose standard textbook tasks on the topic of factorising differences of two squares and spent substantial time teaching learners the procedure for factorising these. He chose examples of varying difficulty, and each time went through the procedure again with learners. More conceptually, he consistently asked learners to 'test' their answers, showing them that they could evaluate their own answers by multiplying. He also spent some time asking learners if $a^2 + b^2$ could factorise, and getting them to generate possible factors and then multiply to test whether they did give $a^2 + b^2$. So while Mr. Peters chose standard textbook tasks and taught procedures, he did attempt to support learners to make links between the different ways of writing expressions and to recognise why the difference of two squares could factorise and the sum could not. Even though Mr. Peters did have some conceptual goals for these lessons, an analysis of his questions and his moves (see below) show that when learners did not respond as he wished, he often, but not always, resorted to funneling (Bauersfeld, 1988) them towards the correct answer.

For his teaching in the second week, Mr. Peters worked with the other Grade 10 teacher in the study to develop tasks that would engage learners in reasoning mathematically. Their first task was:

i. Someone says that $x^2 + 1$ cannot equal zero for x a real number. Do you agree with her/him? Justify your answer.

ii. What is the minimum value of $x^2 + 1$?

In this task learners can reason empirically by trying out numbers in the expression x^2+1 and noticing that x^2 will always give a number greater than or equal to zero. They can also reason theoretically by drawing on the property that as a perfect square, x^2 will always have to be greater than or equal to zero and therefore $x^2 + 1$ will always be positive and have minimum value 1. In planning the task, Mr. Peters expected predominantly empirical reasoning from learners and hoped that through the class discussion he could build on their empirical reasoning to develop their theoretical reasoning. The task is cognitively demanding, what Stein et al. (2000) describe as "doing mathematics", because it requires non-algorithmic thinking, self-monitoring and exploring mathematical relationships. Research has shown that when teachers choose tasks of higher demand, the task demands often decline during interactions with learners (Modau & Brodie, 2008; Stein et al., 2000). I will show below that as Mr. Peters worked with learners on this task, he both interacted more openly with learners to keep the task demands high and also resorted to more constrained interaction and funneling. These different interaction patterns occurred in response to different kinds of learner contributions.

A central argument of this paper is that in both weeks, Mr. Peters was very aware of and responsive to learner errors, but that his shifts in interaction in the second week supported the public expression of more learner errors, particularly what I have called 'basic errors', which are errors that would not be expected at a particular grade level. I will show that Mr. Peters responded differently to two different kinds of error. One response to what I have called 'appropriate errors', which are errors that could be expected at this grade level as learner grapple with new concepts, was to develop new tasks, which addressed the errors, and the underlying misconceptions, more directly. So in week one, when learners were struggling to see $(x + y)^2$ as a perfect square, he had them substitute a range of values into the expressions $x + y$, $(x + y)^2$ and $x^2 + y^2$. In week two, when learners claimed that $-x$ is a negative number, he had them answer the question: Are the following expressions less than, greater than or equal to zero: x; $-2x$; x^2; $-x^2$; $(x + 1)^2$ and $-(x + 2)^2$. Again, we can see differences in the task demands, with those in the first week requiring only empirical reasoning, while those in the second week required a combination of both empirical and theoretical reasoning. So in developing new tasks to address learners' errors, Mr. Peters maintained the level of task demands in relation to the preceding tasks in each week.

In the following sections I show how Mr. Peters' interaction patterns shifted over the two weeks, in response to particular learner contributions.

TEACHER MOVES

More than 30 years ago Sinclair and Coulthard (1975) and Mehan (1979) identified a key structure of classroom discourse, the Initiation-Response-Feedback/Evaluation (IRF/E) exchange structure. The teacher makes an *initiation* move, a learner *responds*, the teacher provides *feedback* or *evaluates* the learner response and then moves on to a new *initiation*. Mehan calls this basic structure a *sequence*. Often, the feedback/evaluation and subsequent initiation moves are combined into one turn, and sometimes the feedback/evaluation is absent or implicit. This gives rise to an *extended sequence* of initiation-response pairs, where the repeated initiation works to achieve the response the teacher is looking for. When this response is achieved, the teacher positively evaluates the response and the extended sequence ends.

Neither Sinclair and Coulthard nor Mehan evaluated the consequences of the IRF/E structure. Other researchers have argued that it may have both positive and negative consequences for learning. Much research has shown that because teachers tend to ask questions to which they already know the answers (Edwards & Mercer, 1987) and to 'funnel' learners' responses toward the answers that they want (Bauersfeld, 1988), space for genuine learner contributions and classroom conversations are limited. At the same time, it is very difficult for teachers to move away from this structure (Wells, 1999) and so, in trying to understand a range of practices, it is important to try to understand the benefits that it affords. Whether the IRE has positive or negative consequences for learning depends on the nature of the elicitation and evaluation moves, which in turn influence the depth and extent of learners' responses.

I developed a set of codes to describe the function of teacher utterances as they initiate and evaluate. When looking at how teachers interact with learners' contributions, a key code is *follow up*, which is when the teacher picks up on a contribution made by a learner, either immediately preceding or some time earlier. The teacher could ask for clarification or elaboration, ask a question or challenge the learner. Usually there is explicit reference to the idea, but there does not have to be. Usually the idea is in the public space, but it does not have to be; for example when a teacher asks a learner to share an idea that she saw previously in the learner's work. Repeating a contribution counts as *follow up* if it functions to solicit more discussion in relation to the learner's contribution. An initial coding of my data showed that there were a large number of *follow up* moves which functioned differently, so I further divided this category into different kinds of *follow up*. The five subcategories of follow up are described in Table 1.

Insert	The teacher adds something in response to the learner's contribution. She can elaborate on it, correct it, answer a question, suggest something, make a link etc.
Elicit	While following up on a contribution, the teacher tries to get something from the learner. She elicits something else to work on learner's idea. Elicit moves can sometimes narrow the contributions in the same way as funneling.
Press	The teacher pushes or probes the learner for more on their idea, to clarify, justify or explain more clearly. The teacher does this by asking the learner to explain more, by asking why the learner thinks s/he is correct, or by asking a specific question that relates to the learner's idea and pushes for something more.

Maintain	The teacher maintains the contribution in the public realm for further consideration. She can repeat the idea, ask others for comment, or merely indicate that the learner should continue talking.
Confirm	The teacher confirms that s/he has heard the learner correctly. There should be some evidence that the teacher is not sure what s/he has heard from the learner, otherwise it could be press.

Table 1: Subcategories of Follow Up

These codes are informed by various concepts in the literature. *Elicit* is closest to Edwards and Mercer's (1987) "repeated questions imply wrong answers" or Bauersfeld's (1980) "funneling", which, the authors argue, can constrain rather than enable learner thinking. *Press* is a category that comes from descriptions of reform pedagogy (Kazemi & Stipek, 2001), where the teacher wants to give the learners a chance to articulate and hence deepen their thinking, and/or wants to make sure that other learners gain access to their colleagues thinking. *Elicit* and *press* moves can sometimes seem similar to each other, they are distinguished in similar ways to how Wood (1994) distinguishes focusing from funneling – a press move orients towards the learners' thinking, rather than towards a solution. *Maintain* is similar to "social scaffolding" (Nathan & Knuth, 2003), and supports the process of learners' articulating their contributions, rather than the mathematics itself. It is also similar to revoicing (O'Connor & Michaels, 1996) and often involves a repetition or rephrasing of the learner's contribution which keeps the idea in the public realm for further consideration. *Insert* describes instances when the teacher gives information to learners as a follow up to what they had said. This category is motivated by a similar rationale to that of Lobato et al (2005), that teachers cannot avoid "telling" and that used appropriately, inserting or explaining is part of any teacher's repertoire.

The categories *confirm*[2]*, press, elicit* and *insert* all function to maintain learner contributions. The main difference between *maintain* and *confirm* and the other three codes is that *maintain* and *confirm* are more neutral, confirming the accuracy of what the teacher has heard or maintaining the contribution very similarly to how the learner said it. The moves can be arranged on a continuum of less to more intervention as follows: *confirm* is where the teacher makes very little intervention, she merely tries to establish what the learner said; *maintain* is where the teacher makes very little intervention, rather she repeats the contribution, in order to keep it going, either for later intervention or transformation, or for other learners to do something with the contribution; *press* tries to get the learner to transform her own contribution; *elicit* tries to get learners to transform a contribution by contributing something else; and *insert* is where the teacher transforms the contribution by making her own mathematical contribution. So *press* and *maintain* might be considered to be more "reform-oriented" moves while *insert* and *elicit* are more "traditional".

Table 2 gives the distributions of follow up moves in the two weeks of videotaped lessons in Mr. Peters' Grade 10 classroom. It should be noted that the percentage of *follow up* moves actually declined from 82% in week 1 to 68% in week 2, suggesting that follow up in itself does not indicate more responsive teaching. However, the distributions of the different kinds of follow up moves do suggest a different kind of interaction in the two weeks.

	Elicit	Insert	Maintain	Press	Confirm
Week 1	48	19	27	4	2
Week 2	23	24	30	20	4

Table 2: Distributions of teacher moves in Mr. Peters' lessons (percents)

The table shows a substantial increase in *press* moves and a decrease in *elicit* moves in Week 2. There were slight increases in *insert* and *maintain* moves. The four main moves were more evenly distributed in the second week than in the first. The table shows that while Mr. Peters shifted from *elicit* moves to *press* moves, a shift indicative of reform pedagogy, he still did do a lot of *eliciting* in Week 2, as well as *inserting* and *maintaining*. This resonates with a finding by Boaler and Brodie (2004) that teachers using reform curriculum materials in the United States asked significantly fewer recall type questions than those using traditional curricula, although they still asked a substantial number of these questions. So some shifts to reform pedagogy are evident, as well as some continuities with traditional practice.

A qualitative analysis of sections of discourse where Mr. Peters used the different kinds of moves, suggests further similarities and differences in his teaching approach during the two weeks. This will be discussed below, in relation to the discussion on learner contributions.

LEARNER CONTRIBUTIONS

An important part of teaching mathematical reasoning is to support learners to voice their mathematical thinking and reasoning, nascent or flawed as it might be. One of the key challenges identified in the research is how to respond appropriately to learner contributions, to engage learners' thinking and take it forward (Ball & Bass, 2003; Heaton, 2000; Lampert, 2001). In my previous research, I identified two issues that faced teachers attempting to shift towards reform teaching: supporting learners to participate, and when they did, dealing with the many mistakes that they made (Brodie, 1999, 2000).

In traditional mathematics pedagogy teachers tend to work with the categories of "right" and "wrong". They affirm correct answers and methods and negatively evaluate incorrect ones (Boaler, 1997; Davis, 1997). Teacher engagement with incorrect answers and methods aims for the production of correct answers, rather than an understanding of why the answers are incorrect and why the learners might be making errors. Moreover, there is always the possibility of a correct response masking a misconception (Nesher, 1987), or of learners producing what they think the teacher wants to hear (Bauersfeld, 1988).

When teachers go beyond traditional teaching, and actively engage with learner ideas in order to develop conceptual links, promote discussion and develop mathematical reasoning, they are often confronted by a range of learner contributions. These contributions might be correct, incorrect or partially correct, well or poorly expressed, relevant or not relevant to the task or discussion, and productive or unproductive for further conversation and development of mathematical ideas. Interacting with a range of learner contributions makes teachers' decisions about how to proceed and when and how to evaluate learner thinking far more complex. I therefore developed a coding scheme to categorise learner contributions in my study. These are described, with examples, in Table 3 (the examples are in response to the task: what is the minimum value of $x^2 + 1$).

Basic Error	An error not expected at the particular grade level. Indicates that the learner is not struggling with the concepts that the task is intended to develop, but rather with other concepts that are necessary for completing the task, and have been taught in previous years.	$x^2 + 1 = 2x^2$
Appropriate Error	An incorrect contribution expected at the particular grade level in relation to the task.	-x is a negative number
Missing Information	Correct but incomplete and occurs when a learner presents some of the information required by the task, but not all of it.	x^2 is always greater than zero
Partial Insight	Learner is grappling with an important idea, which is not quite complete, nor correct, but shows insight into the task.	As you substitute lower numbers, the value of $x^2 + 1$ decreases.
Complete Correct	Provide an adequate answer to the task or question.	For $x^2 + 1$ equal to zero, x^2 must be equal to -1. But the square of any number cannot be negative
Beyond Task	Are related to the task or topic of the lesson but go beyond the immediate task and/or make some interesting connections between ideas.	The square root of -1 squared $[(\sqrt{-1})^2]$ equals -1, and then you say -1 + 1, then you get 0

Table 3: Learner contributions: description and examples

Table 4 gives the distributions of learner contributions in the two weeks of videotaped lessons in Mr. Peters Grade 10 classroom.

	BE	AE	MI	PI	CC	BT	O
Week 1	13	12	7	3	64	0	2
Week 2	22	19	11	8	34	3	4

Table 4: Distributions of learner contributions in Mr. Peters' lessons (percents)

This distribution shows a substantial decrease in complete correct contributions from the first to the second week, a substantial increase in basic errors and a small increase in the other contributions. It is notable and highly significant that in the four other classrooms in my study, almost no basic errors were seen, even in the one other classroom where learners' knowledge was also weak. While this suggests that Mr. Peters may be a teacher who supports the expression of and responds to errors, it also suggests that some forms of reform pedagogy can accentuate the visibility of errors, particularly basic errors. The fact that basic errors became increasingly evident in week two can be accounted for both by the very weak knowledge of the learners together with Mr. Peters' teaching practices, which allowed these errors to enter into the public arena. I will show below that the increase in basic errors in week two can be partially accounted for by Mr. Peters' changing teacher moves and that the larger number of complete, correct contributions in the first week can be attributed to his funneling (Bauersfeld, 1988) of learners' answers towards correct answers. Also significant is the wider range of other contributions in the second week: in particular more partial insights and beyond task contributions. Although the percentage of these is still small, a qualitative analysis suggests that the shift in Mr. Peters' practices does account for the wider range of contributions, and that learners with weak mathematical knowledge can be supported to make these kinds of contributions (see Brodie, 2006b, for a comparison with learners with strong mathematical knowledge).

RELATIONSHIPS BETWEEN TASKS, TEACHER MOVES AND LEARNER CONTRIBUTIONS

The relationships between tasks, teacher moves and learner contributions are complex and so I will only be able to give a small taste of some of these relationships here. A first point to note is that tasks and teacher moves together support particular learner contributions, and learner contributions support particular teacher moves (and tasks in Mr. Peters' case). I have shown above that the tasks in week two required more conceptual mathematical reasoning from learners than those in week one. I will show below that the ways in which Mr. Peters interacted with learners in relation to the tasks both supported and responded to the wider range of learner contributions in week two.

In the extract below, taken from week one, learners were trying to factorise $a^2 + b^2$. In lines 12-20 below Tebogo made a suggestion: (-a – b)(a – b) and Mr. Peters wrote it on the board and asked if he tested it. In lines 21-30, the teacher led the class through testing Tebogo's suggestion and in lines 31-38 he confirmed with them that the test showed that Tebogo's suggestion did not work.

12	Tebogo:	Eh Sir, eh, I got negative a and negative b		
13	Mr. Peters:	Did you test it? (*pause*) Negative a	Follow up	Elicit
14	Tebogo:	Negative b		
15	Mr. Peters:	In the bracket	Follow up	Insert
16	Learner:	Yes Sir, and I said a negative b		
17	Mr. Peters:	And you said a negative	Follow up	Maintain
18	Learner:	b		
19	Mr. Peters:	And you tested it	Follow up	Elicit
20	Learner:	Yes, Sir		
21	Mr. Peters:	Let's see what you saying. You said minus a, its minus a times a, what is minus a times a yes, Pumzile	Follow up	Insert
22	Learner:	Minus a (*inaudible*)		
23	Mr. Peters:	What's negative a times positive a, Pumzile	Follow up	Elicit
24	Pumzile:	Negative a sir		
25	Mr. Peters:	Negative a	Follow up	Elicit
26	Learners:	Negative a squared		
27	Mr. Peters:	What is negative a (*pause*) multiplied by positive a	Follow up	Elicit
28	Learners:	Negative a squared		
29	Mr. Peters:	Mary, you awake (*learners laugh*) yes Mary	Follow up	Elicit
30	Mary:	Negative a squared		
31	Mr. Peters:	Negative a squared, what is, have we got negative a squared here	Follow up	Elicit

32	Learners:	No		
33	Learner	Sir		
34	Mr. Peters:	Have we got negative a squared, answer the question	Follow up	Elicit
35	Learners	No, Sir		
36	Mr. Peters:	So can these be the factors, already you, you running into problems, (*pause*) so our test, does our test work	Follow up	Elicit
37	Learners	No sir		
38	Mr. Peters:	Its not working. (*pause*) Who wants to try? Yes	Follow up	Elicit

Tebogo's contribution was an appropriate error because he was grappling with the task demands of factorising and testing $a^2 + b^2$. Mr. Peters did not correct the error, he wrote it on the board for the class to see and asked whether Tebogo had tested his solution. Although Tebogo said he had, Mr. Peters asked other learners to test it with him. Mr. Peters led the learners through the test, asking constraining questions. Pumzile made a basic error, in multiplying –a by a, which was quickly corrected by Mr. Peters using elicit moves and the other learners responding. Mr. Peters then used another series of elicit moves to make the point that they don't have $-a^2$ in the original expression so the test does not work and the factors are incorrect. So although Mr. Peters followed up learners' contributions, he did so in constrained ways, with mainly elicit moves in order to correct both the basic errors and the appropriate error.

This pattern of interaction was similar throughout week one and also appeared during week two, although not as often. In week two, it occurred mainly when Mr. Peters worked to correct learners' basic errors. However, other kinds of interaction emerged as well in week two, where Mr. Peters spent more time *pressing* on learner errors, trying to help learners to transform their own reasoning. The extract below begins with Mr. Peters writing a solution on the board that most learners had written in their group discussions the previous day, that $x^2 + 1$ could not equal zero because x^2 and 1 are unlike terms and cannot be added. This is an appropriate error, because it shows learners grappling with current knowledge to address the task.

3	Mr. Peters:	Grace and Rethabile. And most of you belong in this group (*writes Grace and Rethabile's solution*). Grace, do you want to say something about that? What were you thinking? What were the reasons that you (*inaudible*)	Follow up	Press

4	Grace:	Sir, because the x squared plus one ne sir, you can never get the 0 because it can't be because they unlike terms. You can only get, the answers only gonna be x squared plus one, that's the only thing that we saw because there's no other answer or anything else.		
5	Mr. Peters:	How do you relate this to the answer not being zero? Because you say there it's true, the answer won't be zero, because x squared plus one is equal to x squared plus one. You say they're unlike terms. Why can't the answer never be zero, using that explanation you are giving us?	Follow up	Press
6	Grace:	*(sighs and pinches Rethabile)*		
7	Mr. Peters:	Rethabile, do you wanna help her	Follow up	Press
8	Rethabile:	Yes, sir.		
9	Mr. Peters:	Come, let's talk about it.	Direct	
10	Rethabile:	Sir, what we wrote here, I was going to say that the x squared is an unknown value and the one is a real number, sir. So making it an unknown number and a real number and both unlike terms, they cannot be, you cannot get a zero, sir. You can only get x squared plus one.		
11	Mr. Peters:	It can only end up x squared plus one	Follow up	Maintain
12	Rethabile:	Yes, sir. There's nothing else that we can get, sir. But the zero, sir.		
13	Mr. Peters:	So you can't get a value, you can't get a value	Follow up	Maintain
14	Rethabile:	That's how far we got sir		
15	Mr. Peters:	Come, Lerato, lets listen so you can contribute.	Direct	
16	Mr. Peters:	So it will only give you x squared plus one, it won't give you another value, zero. Will it give us the value of one? Will it give us the value of two? x squared plus one.	Follow up	Press
17	Rethabile:	It will give us only one, sir, because x is equal		

		to one, sir.		
18	Mr. Peters:	Is x equal to one	Follow up	Elicit
19	Grace:	Yes, sir.		
20	Mr. Peters:	How do you know x is equal to one	Follow up	Press
21	Learners:	(mutters)		
22	Grace:	Not always, sir, because		
23	Mr. Peters:	Shhh. Wait, let's hear this. Let's give her a chance.	Direct	
24	Grace:	Sir, not always sir, because, this time we dealing with a one, sir. That's why we saying x squared equals to one, sir. Because, that's how I see my x equals to one, sir. Because, a value of one, only for this thing, sir		

In the above extract, Mr. Peters both maintained and pressed a number of times on Grace and Rethabile's solution. In the transcript we see Mr. Peters pressing for explanations (lines 3, 5, 9), maintaining the girls' claim (lines 11 and 13) and trying to press them more specifically to think about the expression (lines 5 and 16). His press moves ranged from being very general (line 3) to more specific (line 16). Even this final question which could have enabled learners to think about $x^2 + 1$ as taking several values depending on the value of x, did not help. In fact it led Rethabile directly into a basic error, that x is equal to 1. Mr. Peters' response to her basic error was to try to understand it, using and elicit and press move. He continued to do this for a few turns after the extract and then he moved to correct the error with a sequence of constrained elicit moves, similar to his approach in week one. Once he corrected this (and a number of other basic errors that came up immediately afterwards), he came back to the discussion of the appropriate error.

In all the cases of appropriate errors in week 2, Mr. Peters' response was to work on them in two ways. First, he kept them in the public arena for discussion for some time and tried to get learners to justify and clarify their thinking. He did not always move to teaching the correct answer as he did with basic errors and when he did so, he worked less procedurally. Second, he planned new tasks, which he hoped would help learners with their errors (which he did in week 1 as well). However, each time Mr. Peters began a discussion on an appropriate error a host of basic errors came up. Mr. Peters dealt with these basic errors relatively quickly, although he took more time to try to understand them than in week 1, and then came back to the discussion of the appropriate errors.

Mr. Peters' responses to the other kinds of contributions were similar. In the case of missing information contributions he maintained and pressed, and then moved to complete them with elicit and insert moves, similar to how he worked with basic errors. With partial insights, he maintained and pressed and kept with them, similarly to appropriate errors. In the case of the beyond task contributions he developed a conversation using all the moves (Brodie, 2007b).

CONCLUSIONS

The two extracts discussed above were chosen to maximise differences in Mr. Peters' teaching across the two weeks. At first glance these two extracts appear to suggest very different teaching in the two weeks. However, tables 2 and 4, together with the task analysis shows that although there were some major differences across the two weeks, there were also similarities. One similarity that emerged in the qualitative analysis was that Mr. Peters consistently noticed and engaged learners' appropriate errors and as he did so, basic errors emerged. A major difference was in how he engaged both the appropriate and basic errors in each week. In week one, he elicited correct responses from other learners while in week two, he pressed the learners concerned on their errors in order to understand them and to try to get the learners to transform their own thinking. This partially accounts for the predominance of basic errors in week 2. In both weeks he privileged appropriate errors, working on basic errors towards complete, correct solutions and then coming back to the appropriate error that generated the basic errors. However, in week two, he did this with more emphasis on getting the learners to focus on and transform their own and each other's errors. In both weeks he addressed appropriate errors by developing new tasks. In week one these tasks remained at an empirical level, even as Mr. Peters wanted learners to make a generalisation across them, while in week two the tasks required that learners work between the empirical and the theoretical.

This analysis resonates with other work (Boaler & Brodie, 2004; Hiebert & Wearne, 1993) which shows that as teachers take on some elements of reform, much of their teaching continues to look traditional. Boaler and Brodie show that a first level change is the press move or question, whereas developing conceptual questions is a more difficult skill for teachers to develop. In the case of Mr. Peters, the major shift was from elicit to press moves. This shift, together with a different approach to the tasks, did enable a broader range of learner contributions and teacher responses to these contributions[3]. The analysis suggests that focusing on specific instances in teaching, rather than finding ways to code for the bigger picture, can mislead and might account for the predominant finding that there is very little change among teachers adopting reforms. If we are serious about finding even small changes in practice and suggesting ways in which these can become bigger changes, we need to develop analytic tools at a range of levels.

CONTINUING ISSUES

In this paper I have used a number of analytic tools to describe Mr. Peters' practices as he begins to adopt reform-oriented practices in his classroom and I have shown how what we see of his practice depends on how we look. I claimed at the beginning of the paper that I would show some of the "noise" (Skovsmose & Valero, 2003) that emerges from this kind of analysis. This noise occurs on two levels. First there is the noise argued for in the previous section, which suggests that we will never see clean reform or traditional practices and that our tools for analysing teacher change needs to take this into account. Second there is the noise of this particular classroom, a classroom of mathematically weak learners, who make many errors that, ideally, they should not be making. This is a reality for teachers in similar classrooms and complicates the practice of supporting learners' reasoning through communication and discussion. While Heaton (2000) and Staples (2007) argue that part of a reform teacher's role is developing learner contributions to a point where they can contribute to ongoing discussion, it is not trivial to see how this can be done with so many basic errors in one lesson. The fact that in Mr. Peters case the basic errors increased in relation to his shift in teaching, i.e. more were allowed to become public in the classroom, suggests a double demand of this kind of teaching.

In presenting some of the noise of changing classrooms, I have of course ignored other noisiness, in particular how the social and racial contexts of the school, the teacher and the learners play out in the classroom. I have not looked at particular learner-learner interactions, nor the emotional consequences of a shift of approach for teacher and learners (see Brodie, 2006a, for the beginnings of a discussion). While these are important, what I want to argue in this paper is that the bigger contextual discussions play out in classrooms in particular ways and that we need to think through how to help teachers think about them. This research, as well as my previous research suggests that one of the ways that poverty plays out in classrooms is through weak mathematical knowledge of learners (see also Fleisch, 2007) and we need to deal with this in the context of reform.

A second issue is whether and how complex social practices can be described with the tools that I have suggested here. In particular, how do codes that break down teaching and learning into turns of talk and individual contributions, come together to create an understanding of practice. It might seem reductionist to analyse classrooms in this way, first to code and then looking for relationships between the codes. However, as I have argued above, these codes form a first-level description of pedagogy and show trends across teachers. They need to work together, as well as with qualitative analyses to help develop stronger descriptions. In particular, if they are used in the qualitative analysis, as I have done above, then the codes develop a liveliness in relation to particular classrooms and are able to tell us a story both across classrooms and for each classroom on its own terms.

NOTES

1. I use the terms "new curriculum" (South African) and "curriculum reforms" (international) interchangeably. Although there are subtle differences in 'reform' movements in different countries, for the purposes of this paper, 'reform' mathematics visions are similar enough across countries to be referred to as one concept.

2. There were very few confirm moves in any of my data so the analysis focuses on the other four moves.

3. Elsewhere, I have discussed the limitations of the press move (Brodie, in press).

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MAKING SENSE OF MR. PETER CLASSROOM

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This is the third MES Conference in which I participate. I'm very pleased to react to Karin's paper, but I feel also that it is a great responsibility. I must share something with you: this is my first English paper too. To grow as researcher and as person, I could not reject this challenge, and I must thank it publicly to João Filipe, who never gave up of challenging me in my research trajectory, and also to the organizers of MES for giving me such an opportunity.

WHO AM I?

I'm here because I'm a member of the Research Project's team Learning, Mathematics and Technology, whose leader is João Filipe Matos. I finished my masters degree, three years ago. In my thesis I wrote about learning as social practice and the professional development of primary school teachers using a situated learning approach.

My reaction to Karin's paper is done from my own point of view: someone who lives in Portugal, with both a specific trajectory and a historic and socio-cultural context, with also my own way of seeing and living; this means, with a precise way of participating in the world.

I react from the reality where I live now: a country from the Northern hemisphere that is part of the European Union. I am teacher, teacher educator and pedagogue engaged in teachers' pedagogical associations, where the participating teachers live their professional role as one way of engaging socially and politically in the Portuguese society. Their main principles are students' direct democratic participation and the collaborative organization of learning work to promote communication circuits among students.

My academic background is a generalist one: as former primary teacher, with a license in Sciences of Education and specialised in Pedagogical Supervision. So, I can say I was also math teacher. As a learner I was a very enthusiastic math learner: my relation with math was always a very good one – to solve math tasks was for me always like to solve a strategic game, a very playful challenging way.

Now I'm temporarily working in the Ministry of Education. I'm part of a team in charge of supporting teachers in the use of ICT in education and the curriculum development.

WHERE ARE WE?

Painting a very impressionistic picture, with a large brush, about the current Portuguese educational context, for those of you who have not heard so much about it in the last three years …

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 51 - 55).

During the last two years, we have had a government with majority of the socialist Party in the Parliament. This government is making the second major reform movement in the Portuguese society after our revolution, that started the 25th of April 1974. The first reforms after the revolution, as a result of the social movements that grew up in 1974 against the dictatorship, where made in a democratic sense of participation and of opening the society in general. Actual reforms are made in the context of the European Union, and the goals are such as the control of the public administration budget, to reduce the deficit of the national budget according to the needs of the European currency, the euro.

These constrains in the education domain added with the benchmarking studies between countries all over the world, where PISA comparatives results are the most important ones, are the determinant factors that dictate educational reforms.

These reforms in the educational system are happening in fields like teachers' careers, teachers' assessment (the learners' assessment will have impact in teachers assessment - a very new thing in Portugal) and also in the school management and school board. The government is also preparing huge investments related with ICT for schools: equipment, networks, availability of digital contents, teacher education, …

All these are clearly neoliberal oriented reforms. Michael Apple talked about them eight years ago, at MES 2, here in Algarve. We live nowadays with these reforms as if they were inevitable in the current European context.

These reforms are taking place in Portugal, the European country where there is the largest gap between rich and poor people, where 45% of the youngest, in the range of 18 and 24 years old, do not complete secondary school, where 10 % of the primary school pupils have to repeat at least one school year.

Nevertheless there are measures that clearly benefit families' and children' lives, mostly those from primary schools: now children stay more time in schools with organised activities by the local authorities after class time; they start to learn the first foreign language, the English. Mainly for primary schools, national teacher education programs for Portuguese, Mathematic and Experimental Sciences are being developed. Teachers, for the first time, are supervised in their classrooms during teaching time – similar with what is reported in this paper. This is the answer to the low results of the Portuguese learners in PISA tests. Consequences of these reforms are: teachers are obliged to spend more time in schools, after classes. They feel their role is more bureaucratized and they feel also much pressure over their shoulder.

As you can imagine we live times of great contradictions, both at national level with some social agitation – union supported – and at school level where the implications of these measures can be felt in the relationships between teachers and in the ways they work together.

MAKING SENSE OF MR. PETER'S CLASSROOM

Having told you all those things about our country, I started to think directly about my reaction on Karin's paper … and about how little I know about her country.

I don't know the orientation of the educational reforms in South Africa, I know very little about its history and about their pedagogical traditions. I know that it is a country in the southern hemisphere, where political and social movements had fought against the apartheid regime and they succeeded in eradicating it. I can imagine that there are being done great efforts of empowerment of the poorest populations and of education and training for all.

I thought this reaction could be an opportunity to know a little more about the relationship of the educational, research and social movements in South Africa.

This paper's presentation brought me thoughts at different levels: at global and national political level, and how the national reforms are related with the PISA studies; how the national reforms are implemented at the local level, schools' and classrooms' level – what is the relationship between the teachers and international comparative studies? Are they pressured about the results of their learners? How do they feel and what do they think about these international studies? What is the thinking of the mathematics' educators' movements in South Africa about this subject and how do they relate it with the efforts of empowering the poorest?

We all make choices about our participation in social movements at those different levels, both at an individual and social dimension. We all are immersed in socio-cultural contexts with specific features. Those contexts and their public can condition the way we express ourselves, the way we work and also the way we do research. Each of us has our own trajectory, a concept used by Wenger (1998). In our own trajectories we cross and participate in different kinds of communities of practice. Even at the same historical moment we participate in different communities: each one has their own practice. We draw in relation with each of the social groups where we live in different kinds of social belonging: some in a fuller way, some in a more peripheral way.

Describing practice is Karin's main concern, and she tried to do it in her paper. She did it in a very concrete way and, by reading the paper, I could understand what was happening in Mr. Peters' classroom. But, like other practices, this is also a very complex one. At this level the question could be: what is the intentionality of describing classroom practice?

Karin made her choices to describe us this specific teaching practice, in the on going movement of her trajectory as researcher: she made video recording of the interactions between the teacher and the learners, when they worked a syllabus topic (squares) over two weeks. As she reports the change on the teacher's intentionality, from the first week to the second week, there was a source of changes in the nature of the proposed tasks and in the way the teacher dealt in the public arena with the learners' expression of their thinking and reasoning.

Karin presents us with an interesting and fine analysis of what happened in these two weeks, but she interrogates also the way she did it – how to describe practice in a more rich and fruitful way? As Lave (1991) defines transparency: we only can see through the window, what the window's glass allows us to see; we cannot see through the wall and it is just beside the window. So the paper raises a lot of questions and questions are as important as answers.

FURTHER QUESTIONS

From my point of view, another way of describing those practices could let us perceive other subtle features.

From a situated learning point of view, the one that I know the best, we can raise questions about the kind of community of practice this classroom represents and about what kinds of practices are developed there. In this community we can observe different kinds of trajectories that meet each other in the same place, this classroom: the teacher's trajectory, the researcher's trajectory and the learners' trajectories.

a) What practices are being developed in this classroom: the teacher's practice of teaching? The learners' practice of learning? The researcher's practice of researching? How are these three practice related? Do they have a shared repertoire? What is their mutual engagement? Do they have a belonging sense?

b) What kind of artefacts is used in this classroom: teacher's discourse? The mathematical concepts and expressions? The blackboard and chalk? The paper? The learner's notebooks and pencils? Learner's discourses? The furniture and his distributions in the classroom? The videotape recorder? The sessions' transcriptions made by the researcher? The categorisation of the interactions made by the researcher?

c) What kind of identities is this community contributing to develop: the one from the teacher, those from the learners or the one from the researcher?

d) What meanings are there shared about the practice? What kind of trajectories are being developed there?

e) What answers do we find in this paper for these questions? Are they satisfactory answers for us? Why?

And I can go on asking…

Each practice in which we participate occurs and can be changed in a specific historic, political and socio-cultural context.

f) What external factors, historical and socio-cultural ones, affect the practice of that social group? And their other conceptual elements (meaning, identity, community) how are they also affected?

Only one more question:

g) What implications had this research for these learners and for the teacher?

And finally, all this can drive us to the question:

h) How do we deal, as researchers, with the methodological implications of the theories?

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EQUITY-IN-QUALITY: TOWARDS A THEORETICAL FRAMEWORK

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The paper addresses the concepts of equity and quality as they apply to mathematics education and argues that the two concepts of equity and quality are interdependent and are only meaningfully understood in a specific socio-cultural context. The paper argues that meaningful comparisons across different socio-cultural contexts can be achieved by focusing on the relationships of the two concepts to each other and to contextual factors. To underline the interdependence of equity and quality and their relationships to contextual factors the paper introduces a framework based on activity theory and activity system as developed by Engeström. The last part of the paper uses data from TIMSS 2003 to demonstrate empirically the relationship between equity and quality, and their relations to contextual factors.

INTRODUCTION

Context

Educational equity and quality are not only research issues which cut across different disciplines but are presently, as evidenced by the annual reports of the United Nations Development Program (UNDP), major determinants of socio-economic and human development in both industrial and developing countries. The status and role of mathematics, a subject which has long enjoyed a privileged status in school curricula worldwide due to its perceived role in science and technology, render equity and quality in mathematics education at the heart of human development. This is reflected by the governments' relatively large investments in improving the quality of mathematics education and extending it to marginalized and underprivileged groups.

Mathematics was described as a filter and gateway to the professions and science and technology. Research in the last four decades has focused on the identification of inequities in mathematics education, the factors that contribute to them (gender, socio-economic class, ethnicity, location, special needs…) and the contexts (school, national, global) that impact equity and social justice, and the modalities through which teachers and schools deal with such inequities. The attention given to issues of equity and quality in mathematics education is reflected by recent books on the subject (Atweh, Forgasz, & Nebres, 2001; Burton, 2003; Secada & Byrd-Adajian, 1995; Valero & Zevenbergen, 2004) and comparative studies based on international or regional mathematics achievement databases (Hanushek & Luque, 2003; PISA, 2005; Jurdak, 2006). However, there is a need for more theoretical and comparative studies for a better understanding of the complexities of the equity issues in mathematics education.

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 56 - 73).

Numerous calls and proposals have been made, and many projects implemented, to improve quality in math education. The impact of such efforts on the quality of the learning outcomes has, though positive in many instances, created disparities which in fact increased, and even created, inequities in math education. The risk that math education quality enhancement may result in different levels of mathematical literacy, and consequently increase the potential of marginalizing certain individuals and groups in the same society, has become a real concern.

The growing roles of globalization and Information and Communication Technology (ICT) have increased the tension between equity and quality in mathematics education. The demands of the global economy have increased the gap between developed and developing countries and thus made equity in mathematics education not only a within-country phenomenon but also a global one. On the other hand the disparities in access to and ownership of ICT which has become an essential tool for quality improvement in mathematics education rendered the developing countries at a disadvantage in benefiting equitably from quality improvement in mathematics education.

To demonstrate the different conceptions of equity and quality and the tensions between them, I have selected four quotations from the research literature in mathematics education for the purpose of illustration and discussion.

Quotation 1: Inside and Outside School

> "This study examines the computational strategies of ten young street vendors in Beirut by describing, comparing, and analyzing the computational strategies used in solving three types of problems in two settings: transactions in the workplace, word problems, and computation exercises in a school-like setting. The results indicate that vendors' use of semantically-based mental computational strategies was more predominant in transactions and word problems than in computation exercises whereas written school-like computational strategies were used more frequently in computation exercises than in word problems and transactions. There was clear evidence of more effective use of logico-mathematical properties in transactions and word problems than in computation exercises. Moreover, the success rate associated with each of transactions and word problems was much higher than that associated with computation exercises." (Jurdak, 1999, p. 155)

Do the street vendors have a better "quality" in their use of mental computational strategies? Did their disadvantage as far as access to school affect their opportunity to learn mathematics beyond the context of their work?

Quotation 2: In the Same Classroom

> "In this paper I explore the structuring of English children into learning and life trajectories and the part that mathematics has in this process. Using case reports of two ten-year olds in their final year of primary school education, I examine how broader family social milieu impact upon mathematics learning trajectories. Stacey and Edward

live not far from one another in a city in the midlands of England and have been in the same class from age 5 to 11 yet their social distance is considerable. Through the mobilization of various classed and classifying responses to school mathematics they have developed two very different perspectives on the value of mathematical study. This examination of mathematical marginalization and misrecognised meritocracy raises questions about the extent to which teachers can disrupt such processes." (Noyes, 2007, p. 35)

Is the quality of mathematics learning affected by factors outside the school control (such as family social milieu), even for students who have been in the same school and in the same class for six years? Is the social distance a determinant of the quality of mathematics learning regardless of equal opportunities in school?

Quotation 3: Inside and Outside a Country

"In this paper, I discuss some links between mathematics education and democracy, what these links could imply to what and how we teach, and the issues that arise from trying to further these links. I first suggest three links between mathematics education and democracy formulated on the basis of experiences in Denmark, in particular: learning to relate to authorities' use of mathematics, learning to act in a democracy, and developing a democratic classroom culture. The first two are discussed in relation to narratives from real life, with a focus on the tensions which they reveal. From the discussion following the first narrative, it is clear that what is a competency in one context may not be so in another. This is supported by the second narrative which also questions what is most relevant to students in South Africa and thereby gives rise to the formulation of a fourth connection between democracy and mathematics education, related to issues of access. The third narrative informs a discussion of what it means to be critical. It also continues to address the potential tension between wanting to promote students' critical skills and a democratic classroom culture versus wanting to support students in learning what others have developed and what is required in order to succeed in the schooling system…". (Christiansen, 2007, p. 49)

Is it the case that what is valued as significant mathematics learning in one context is perceived as irrelevant and may be offensive in another context? Are the criteria by which we judge the quality of mathematics universal? Consequently, what is the basis for comparing the quality of mathematics learning across countries?

Quotation 4: Across Countries

"With these findings in mind, case studies from eleven countries provide insights into how both rich and developing nations have tackled the quality issue. Four of the eleven – Canada, Cuba, Finland and the Republic of Korea – have achieved high standards of education quality, as measured by international tests. The Republic of Korea is ranked first for science and third for mathematics in PISA, Canada comes second for reading and Finland has the highest overall scores, while in Cuba students' average performance topped countries in the region surveyed in 2002 by OREALC1/UNESCO." (UNESCO, 2005, p. 13)

> "Several common strands emerge in the four high performing countries. All hold the teaching profession in high regard and support it with investment in training. There is policy continuity over time and a strong, explicit vision of education's objectives." (UNESCO, 2005, p. 14)

How could such four countries, in four different continents and at varying distances from each other economically, socially, culturally, and politically, have achieved 'high standards of education quality'?

The questions that were posed on the quotations do not have easy answers. One might say that these quotations are eclectic summaries of larger papers or that the questions are pointed to suggest certain answers. Despite all of this, the fact is that we do not have reasonable answers to such disturbing questions. At least these questions point to a problem manifested in our lack of sufficiently adequate conceptions of quality and equity and the relationship between them. I hypothesize that the discrepancies we tried to underline in the previous questions concerning equity and quality in mathematics education can not be adequately explained by the conceptual model of the school as a production system. In the next section, we introduce the school as a production system and demonstrate how these discrepancies relate to the conceptions of equity and quality in it.

EQUITY AND QUALITY IN A PRODUCTION SYSTEM

A well known conceptual framework is that of the school as a productive system (in the industrial sense) where education within schools is viewed as the transformation of inputs into outputs through school processes within a social context. Figure 1 represents the model of the school education as a system (PISA, 2005). The notions of equity and quality that are presented in this section assume the system model.

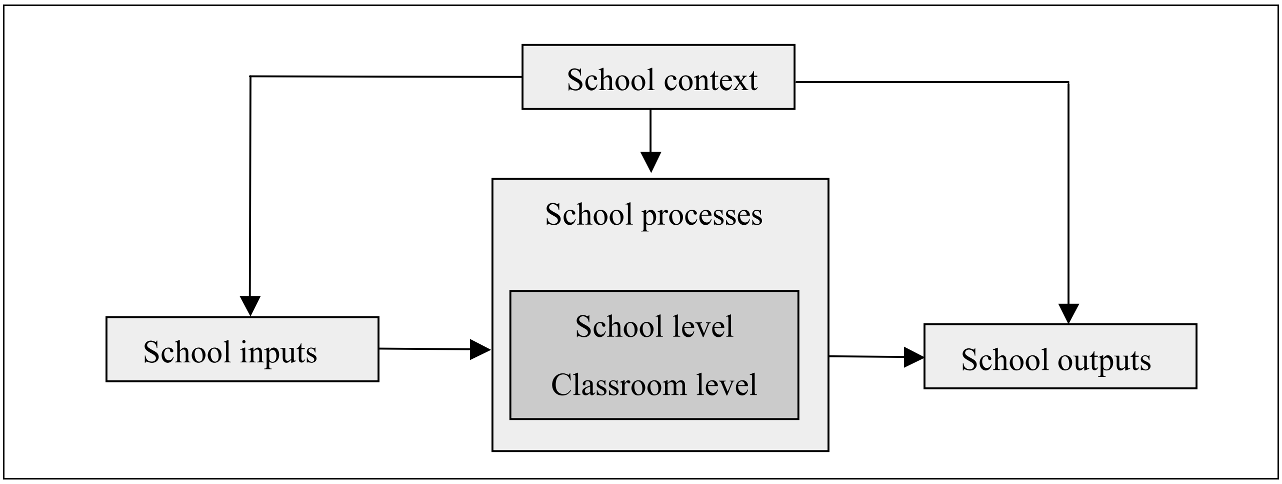

Figure 1. Model of how schools function

Equity

Educational equity is a fundamental concept which has its bases in ideology, sociology, epistemology, and psychology. It is not surprising therefore that educational equity has assumed different meanings over the years (Sriraman, 2007). Both the concept "equity" and its label have been challenged lately by many researchers who proposed "social justice" as an alternative on philosophical and ideological grounds (Burton, 2003).

Berne and Stiefel (1984) proposed a framework for school systems. The framework consists of three components: Targets of *equity concerns* (gender, socioeconomic status, ethnicity, disability status...), *objects of equity* (access, resources, and outputs), and *principles of equity* (principles to analyze equity across individuals, regions, countries…). Berne and Stiefel (1984) provided three different principles- horizontal equity, vertical equity, equal opportunity. Horizontal equity requires students who are equally situated be equally treated by ensuring that they experience similar levels of human and material resources and hopefully achieve similar outcomes. Vertical equity requires differentiation of provision of resources according to individual characteristics in the sense that students who are differently situated would be provided with unique resources (e.g. support programs) to achieve similar results. Equal Educational Opportunity (EEQ) is based on the notion that all students should be given equal chances to succeed. This requires that students should have access to resources that equalizes their starting point and to provide the conditions to allow the possibility of success to all.

This framework seems to be applicable to mathematics education with the equity concerns and equity objects defined to suit mathematics education. The equity concerns in education (gender, socioeconomic status, ethnicity, disability status...) apply to mathematics education. Objects of equity in mathematics education may differ somewhat from those of general education and include access to and participation in mathematics education, continuation in studying mathematics education, and achievement. The three principles of equity apply to mathematics education and we have many examples of policy and school practices that follow the principle of horizontal equity, vertical equity, or equal opportunity principle.

Quality

There are different definitions of quality in education on different philosophical, psychological, social, and discipline specific perspectives. Quality is closely related to our conceptions of learning. Sfard (1998) proposed that learning theories fall under two learning metaphors, acquisition and participation. In the acquisition metaphor, the individual mind is viewed as a container and thus learning is a matter of acquisition of knowledge and outcomes which are realized in the process of transfer. In the participation model, learning is viewed as a process of participation in cultural practices and shared activities and the emphasis is on the process of knowing and on participating in it, rather than on products such as knowledge and outcomes.

The quality of the output is at the core of the quality of the school as a production system. Three variations of quality in the production system are often cited. The first is the productivity view, which translates in the case of mathematics education to saying that the quality of mathematics education depends on the degree of the attainment of the desired outcomes. The second is the instrumental view which assumes that the quality of mathematics education is contingent on the optimal selection of inputs, processes, and contexts that increases the chances of improving performance on outcomes. The third perspective is the efficiency view which defines quality in terms of achieving the highest output at the lowest possible cost.

Re-visiting the Quotations from the Perspective of the Production System

The discrepancies in the conceptions of equity and quality in the four episodes do not seem to be satisfactorily explained by the school as a production system. Quotation 1 illustrates that the production system does not adequately explain the superior performance in computational strategies of young street vendors, compared to students since it is not capable of explaining learning mathematics in a social context.

In quotation 2, seemingly, Stacey and Edward had equal opportunities to learn mathematics but have different valuation of their mathematics learning because of the difference in their cultural capital due to differences in family social milieu. Thus the seemingly equitable inputs and processes in the school did not result in comparable quality of their mathematics learning trajectories. Thus, even in the same school differences in quality, due to social factors, can not be accounted for by the school as a production system.

Episode 3 illustrates the difference in conception of quality in two different cultures. What is valued as mathematics goal in Denmark (learning to relate to authorities' use of mathematics, learning to act in a democracy, and developing a democratic classroom culture) is not considered valuable in South Africa which has a hard-earned democratic political system. This difference in the democracy-related goals of mathematics education reflects different conceptions of quality attributed to ideological factors not accounted for in the school production system framework.

Episode 4 illustrates that quality, even if is narrowly defined as the performance on achievement test, is not necessarily dependent on material resources of the country but rather on cultural values (holding the teaching profession in high regard and support it with investment in training) and the political system and vision (policy continuity over time and a strong, explicit vision of education's objectives).

Comments on Equity and Quality in the School as a Production System

The issue with the production system is that it does not capture the complexity of the social, cultural, and political contexts of mathematics education. First, the school context in the production system has a one-way contribution to the system (Figure 1) and does not encompass the broader social-cultural context. Second, the system is not cognizant of the community of learners and the cultural capital they bring to the learning process. Third, placing so much emphasis on the quality of the outcomes is likely to make it a closed system with limited responsiveness to change and innovation because its ultimate aim is in improving the productivity and the efficiency of the system. Fourth, the ability of the system to manipulate inputs and processes seemingly makes it responsive to equity concerns. However, this responsiveness remains constrained to surface and macro level indicators such as access, resources, and processes and does not extend to socially and culturally equity concerns of individual students.

I suggest that the former apparent discrepancies in conceptions of quality and equity and the relationship between them emanate from two sources. First, equity and quality in mathematics education are aspects of a complex social-cultural-political activity, and second, the absence of a theoretical framework that captures the nature of mathematics education as a social-cultural-political activity. We suggest a theoretical framework that will hopefully reduce the complexity of the equity–quality issues and consequently enhance our understanding of them. This framework is based on activity theory as developed by Leont'ev (1981) and activity system as developed by Engeström (1987).

ACTIVITY THEORY AND MATHEMATICS EDUCATION

Because the production model does not seem to capture the nature of mathematics education as a social-cultural-political activity, we propose the activity system model as an alternative model. We first introduce activity theory (Leont'ev, 1981) on the basis of which the construct of activity system (Engeström, 1987) was built. Then we demonstrate how we can look at mathematics education as an activity system.

Activity theory

Activity theory was developed by Leont'ev (1981). He defined activity as:

> "…the unit of life that is mediated by mental reflection. The real function of this unit is to orient the subjects in the world of objects. In other words, activity is not a reaction or aggregate of reactions, but a system with its own structure, its own internal transformations, and its own development." (p. 46).

A central assertion of activity theory is that our knowledge of the world is mediated by our interaction with it, and thus, human behavior and thinking occur within meaningful contexts as people conduct purposeful goal-directed activities. This theory strongly advocates socially organized human activity as the major unit of analysis in psychological studies rather than mind or behavior.

Leont'ev (1981) identified several interrelated levels or abstractions in theory of activity. Each level is associated with a special type of unit. The first most general level is associated with the unit of activity that deals with specific real activities such as work, play, and learning. The second level of analysis focuses on the unit of a goal-directed action that is the process subordinated to a conscious goal. The third level of analysis is associated with the unit of operation or the conditions under which the action is carried out. Operations help actualize the general goal to make it more concrete.

Human activity can be realized in two forms: "mental" activity or internal activity and practical objective or external activity (Leont'ev, 1981). The fundamental and primary form of human activity is external and practical. This form of activity brings humans into practical contact with objects thus redirecting, changing and enriching this activity. The internal plane of activity is formed as a result of internalizing external processes.

> "Internalization is the transition in which external processes with external, material objects are transformed into processes that take place at the mental level, the level of consciousness" (Zinchencho & Gordon, 1981, p. 74).

Three types of actions in mental activities had been identified: perceptual, mnemonic, and cognitive (Zinchencho & Gordon, 1981). Perceptual actions are those by which the human being maintains contact with the environment. They are initiated by stimuli from the environment and enriched on the basis of prior experience. Mnemonic actions refer to actions, which involve recognition, reconstruction, or recall (Piaget & Inhelder as cited in Zinchencho & Gordon, 1981). Cognitive actions involve thinking in terms of images of real objective processes (Gal'perin cited in Zinchencho & Gordon, 1981).

Activity System

Engeström (1987) developed the construct of activity system to describe and account for the collective human activity in the broad historical-cultural-social contexts. Figure 2 is a schematic diagram of the activity structure as developed by Engeström (1999).

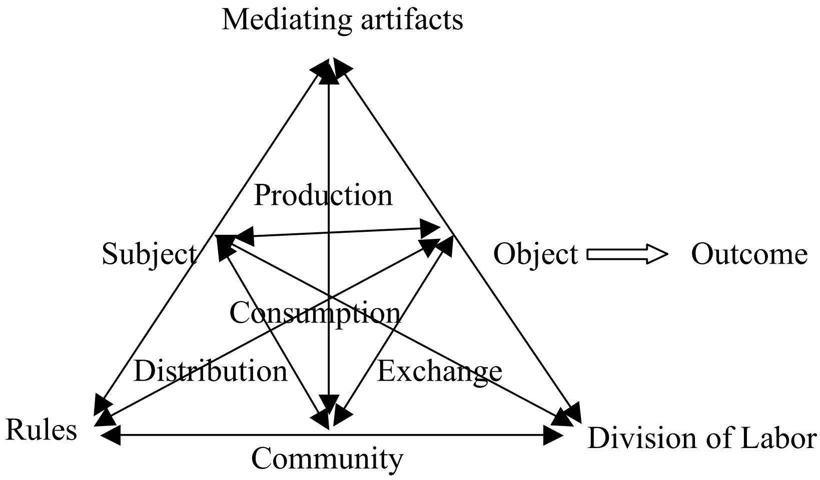

Figure 2. A schematic diagram of the activity system (Engeström, 1999

In the model the subject refers to an individual or a group in an organization. The object is the problem space targeted by the activity of the organization and this goal – object is transformed into outcomes with the help of mediating artifacts which consist of physical and symbolic, external and internal mediating instruments, including both tools and signs.

The community represents those individuals and /or subgroups that share the same general object and, as part of that organization, define themselves as distinct from other communities. The rules are the explicit and implicit regulations, norms, and conventions to regulate and control the actions and the interactions within the activity. Finally, the division of labor refers to both the division of tasks between members of the community and to the division of power and authority within the activity.

Mathematics Education as an Activity System

Figure 3 is a schematic diagram of mathematics education if viewed as an activity system at the classroom level. It is to be noted that the model of activity system may be used to describe and analyze mathematics education at different levels: Classroom, school and state or national level. In the next paragraph we illustrate how the activity system may be used to describe mathematics education at the classroom level.

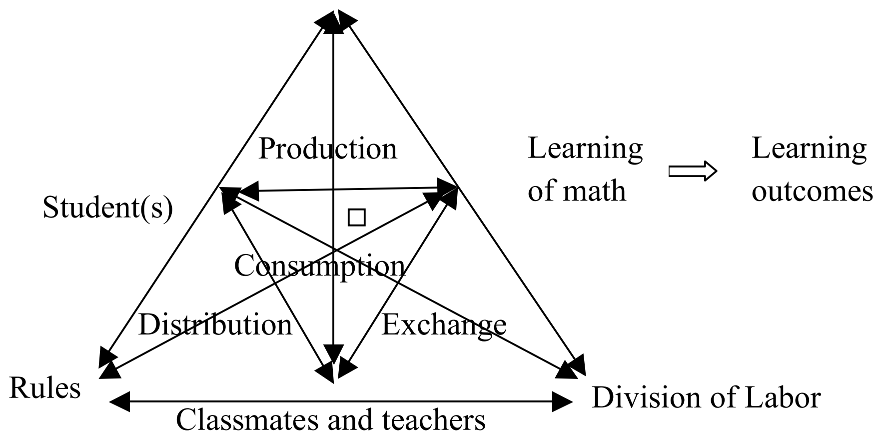

Figure 3. Mathematics education as an activity at the classroom level

The object of mathematics education as an activity system is the learning of mathematics, both at the individual and group levels. The learning of mathematics is transformed into learning outcomes by the help of the mediating artifacts which are used in the classroom and include mathematical and non-mathematical physical tools such as the computer or symbolic tools like language and mathematical symbols. The community which consists of those individuals which share the same object of learning mathematics includes the students in the class as well as the teacher. The division of labor refers to division of tasks as well as division of authority among the students and teachers while trying to achieve the object of the activity. The rules consist of explicit regulations of the school as well as implicit school and wider-scale social norms and conventions.

The school system can be viewed as an activity system in which the classroom activity systems are sub-systems. Thus the elements of the school activity system will be expanded to include the respective elements in the classroom system. For example, the community in the school activity system includes all students in different grades in the school who share the same object of learning mathematics. This hierarchical structure applies also to the national level, which can be viewed as an activity system in which school activity systems are sub-systems.

Having compared and contrasted the production and activity systems of mathematics education, we compare and contrast the concepts of equity and quality in the two systems and re-visit the four quotations from the perspective of the activity system.

EQUITY AND QUALITY IN THE ACTIVITY SYSTEM

In this section, we present the conceptions of equity, quality, and the relationship between them from the perspective of the activity system model.

Equity

As a descriptive framework, the activity system helps in identifying and rationalizing the source of inequities. For one thing, the activity system may help in rationalizing inequity concerns that have been identified so far (gender, socioeconomic status, ethnicity, disability status...) as well as in identifying additional equity concerns. For example, in the activity system framework, gender as an equity concern may be viewed as an unfair distribution (the triangle in Figure 3, whose vertices are student, community, and rules) of resources in the classroom. In the activity system, rules include, among other things, social norms and conventions. So, inequities that are associated with gender in the classroom are the result of carry-over from the cultural context.

The activity system framework widens the scope of equity concerns in mathematics education. In the production model, the equity concerns were limited to access, resources, and outputs. In the activity system framework, equity concerns are widened to include equitable participation in the processes which result from the tri-lateral interactions among the nodes of the system and these are the production, distribution, exchange, and consumption of knowledge (Figure 3). For example, language as a mediating artifact in the uneven production of knowledge in the learning of mathematics may be an equity concern because it is a factor which may discriminate among students coming from socio-economic strata of society. In the same way, the use of technology in teaching and learning of mathematics may be viewed as an equity concern. Along the same lines, additional equity concerns may arise from discrepancies in the exchange process in the activity system (the triangle whose vertices are division of labor, community, and object). The discrepancy between the goals of mathematics teaching and the expectations of the community may result in new equity concern.

Quality

Quality in the activity system is closely related to the knowledge creation metaphor of learning which differs from the other two metaphors: the acquisition metaphor and the participative metaphor (Paavpla et al., 2004). According to Paavpla (2004) the ultimate aim of the knowledge creation models (including Engeström's activity system) is the development of innovative knowledge communities through learning:

> Learning is not conceptualized through processes occurring in individuals' minds, or through processes of participation in social practices. Learning is understood as a collaborative effort directed toward developing some mediated artifacts, broadly defined as including knowledge, ideas, practices, and material or conceptual artifacts. The interaction among different forms of knowledge or between knowledge and other activities is emphasized as a requirement for this kind of innovativeness in learning and knowledge creation. (p. 569)

In the case of activity system, Engestrom (1999) introduced the model of expansive learning in work teams which is based on a learning cycle with seven stages. The learning cycle in expansive learning starts from some dialectical tension between the different nodes in the activity system and stabilizes with the re-conceptualization of the activity system in relation to the participants' relation to the shared objects, mediating artifacts, rules, and /or division of labor. A new tension between the nodes of the system will eventually lead to a new process of adaptation. Thus not only the activity system is transformed but more importantly the activity system will be "expanded" by creating new activity systems. Consequently the quality of an activity system is dependent on the responsiveness of the system to expand and create new activity systems that meet the emerging needs of the community.

An example of expansive learning in the context of mathematics education is in order. Let us look at the reform activity in mathematics education that took place during the last 15 years. Of course this activity took different forms in different countries and communities. Let us consider a certain country where there was a need to change the object of mathematics education as embodied in the mathematics curriculum. Through debate and criticism of the existing system, the community arrived at new shared expected outcomes of mathematics learning. Soon after, a tension is expected to be created between the expectations of learning outcomes (say, mathematical sense) and the mediated artifacts (say, methods of teaching which do not promote such expectations). If sensitive to change and improvement, the educational system responds to the tension between outcomes and mediated artifacts not only by re-conceptualizing the mathematics education system, but also by creating a new activity system (such as a new teacher education). On the other hand if it fails to respond to such tensions, the system will not improve and consequently will not create new activity systems. It should be noted that inequities, being sources of tensions and conflicts in the activity system, may act as factors which trigger the process of change and improvement of quality in the system.

The Activity System and the Social-Cultural-Political Nature of Math Education

The criterion of quality of mathematics education from the perspective of an activity system does not reside in the quality of its output (learning outcomes) or in the quality of the inputs or the processes of the system but rather in: a) the ability of the system as whole to respond to emerging needs by re-conceptualizing the relationships of the participants (learners) to the elements of the activity system (improvement), and b) by creating new activity systems (innovation).

The dialectical relationship between equity and equality in the activity system seems to capture the social-cultural-political nature of mathematics education. From the perspectives of activity system, the inequities that appear in the system because of social, cultural, or political reasons act as de-stabilizing factors, thus producing tensions which, according to expansive learning, will render the system more responsive to social-cultural-political concerns of mathematics education. This responsiveness takes the form of re-structuring the system to address these inequities.

Re-visiting the Quotations from the Perspective of Activity System

In this section we re-examine the quotations from the perspective of the activity system to find out whether this system, compared to the production system, contributes to a better understanding of the discrepancies we identified earlier. In Quotation 1, the discrepancies regarding equity and quality between street vendors and students may be accounted for, from the perspective of activity theory, by the observation that equity and quality are not comparable in the two cases since the street vendors and students are operating in two different activity systems. In the case of vendors, the workplace activity system consists of subjects (vendors) who are working in a community of other vendors and customers whose object is selling and buying produce, using all mediated artifacts (calculations and other physical tools), utilizing agreed upon division of labor, and operating within the rules of the local market and the acceptable social norms and conventions. On the other hand, the school activity system consists of a community of students and teachers whose object in the mathematics classroom is the learning of mathematics, using mediated artifacts and division of labor determined and limited by the school, and operating within the rules and policies of the school and social conventions of the larger school community.

In Quotation 2, the fact the equal opportunities to learn mathematics afforded to Stacey and Edward did not lead to a comparable valuation of their mathematics learning may be accounted for by the impact of social-cultural capital (rules)and the relation of each of Stacey and Edward to the object of learning mathematics.

In Quotation 3, the difference in conception of quality in the two cultures of Denmark and South Africa may be also explained by the activity system framework. What is valued as desirable object for learning mathematics in Denmark (learning to relate to authorities' use of mathematics, learning to act in a democracy, and developing a democratic classroom culture) is not considered a valuable outcome of the activity of learning mathematics in South Africa.

In Quotation 4, the four countries – Canada, Cuba, Finland and the Republic of Korea –have achieved, according to UNESCO, high standards of education quality, which was attributed to the fact that these countries shared some cultural similarities (holding the teaching profession in high regard and support it with investment in training) as well as political similarities (policy continuity over time and a strong, explicit vision of education's objectives). However, what was considered as a quality factor (policy continuity) from the perspective of the production system (quality of learning outcomes) is considered as a liability from the perspective of activity system since it constrains the ability of the system to adapt and innovate.

RELATIOSHIP BETWEEN EQUITY AND QUALITY: AN EXAMPLE FROM TIMSS 2003

The relationship between equity and quality is a complex one in the production system and even much more so in the activity system. I shall present an example to illustrate an approach to investigating the relationship between equity and quality taken from the Trends in International Mathematics and Science Study TIMSS 2003, which is modeled after the production system. Forty eight countries participated in TIMSS 2003 of which eight Arab countries participated at the eighth grade. Jurdak (2006) conducted a study commissioned by UNESCO to identify and compare the effect of student-level variables, teacher-level variables and school-level variables on mathematics achievement of Grade 8 students in the eight Arab countries which participated in TIMSS 2003. The TIMSS database was the source of data for the two statistical analyses that were done: 1) The variance component analysis was done to compare the variance accounted for by the school as a random variable; and, 2) stepwise multiple regression with the student, teacher, and school background variables as predictors and the Average Mathematics Plausible Score as dependent variable. The percentage of variance in mathematics achievement (a measure of quality) accounted for by a variable is an indicator of equity regarding that variable. For example, the between-school variance indicates the size of variation among schools. The larger the between-school variance in mathematics achievement, the larger the extent to which schools contribute to overall performance differences, and hence to potential inequity among schools in this country.

Between-school variation

The percentage of between-school variation to total variance in mathematics achievement by country is shown in Figure 4. Lebanon and Egypt have the highest percentage between-school variation among the Arab countries in math achievement due to school (Figure 4). This suggests that the school in Lebanon and Egypt contributes more than other Arab countries to the variation in student mathematics achievement. Consequently in these two countries the variation in school quality (an inequity factor) contributes more to quality variation in mathematics education.

Variations accounted for by student-, teacher-, and school-level variables

Figure 5 represents the percentage of variance in mathematics achievement accounted for by student, teacher, school variables and by country. Compared to teacher-level and school-level variables, the student-level variables' relative contribution to the within-country variance in mathematics achievement was the highest in all countries except Bahrain. Again the lowest relative contribution to within-country variance in mathematics achievement came from teacher-level variables for all countries except in Bahrain.

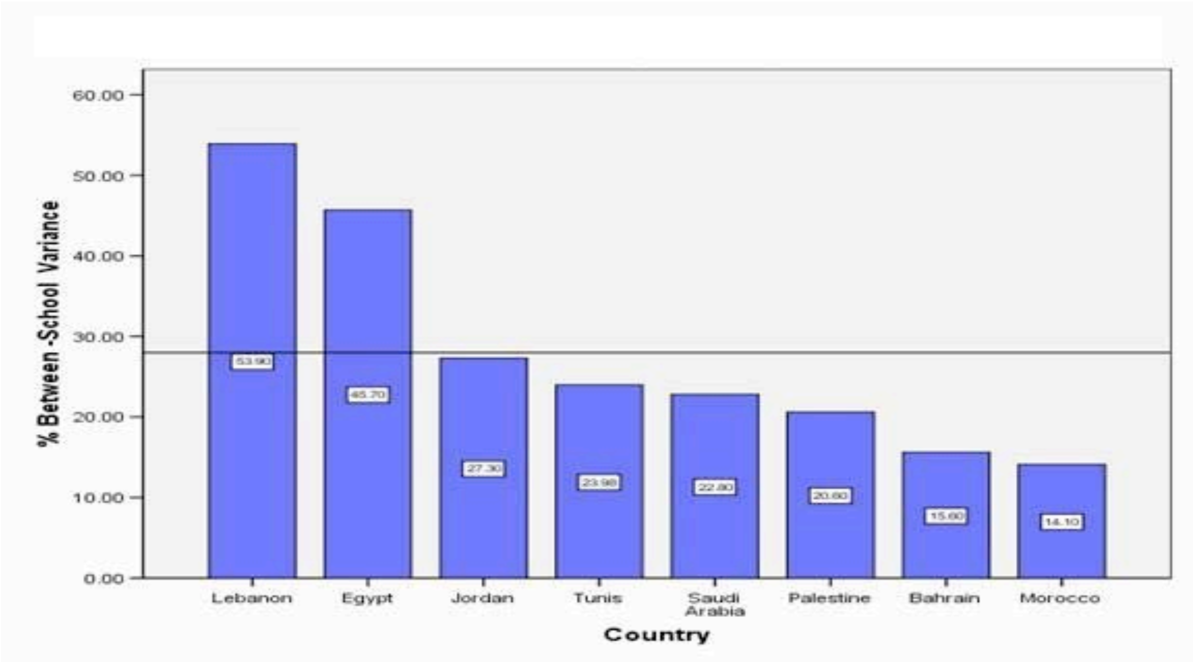

Figure 4. % of total variance in mathematics achievement accounted for by school

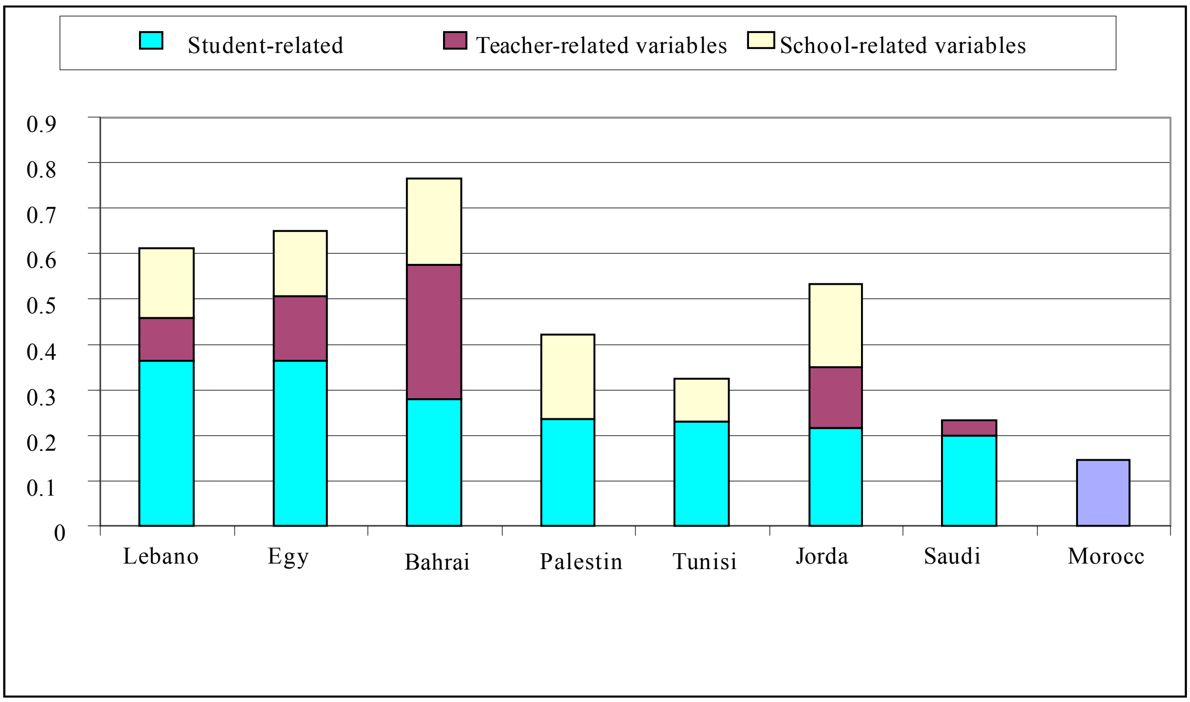

Figure 5. Proportion of variance in mathematics achievement accounted for by student, teacher, school variables and by country

Comparisons of variations in mathematics achievement accounted for by individual student- level variables

Figure 6 indicates that in the seven of the eight countries, an affective student-level variable entitled “Index of Self-Confidence in Learning Mathematics” entered first in the stepwise regression analysis and consequently accounted for the largest proportion of variance in mathematics achievement. The variable “Index of Self-Confidence in Learning Mathematics” is defined by TIMSS 2003 as “student perceives that he/she usually does well in mathematics, mathematics is easier for him/ her than for many of classmates, mathematics is one of his/her strengths, and perceives that he /she learns things quickly in mathematics”. One of two student-level variables related to student home environment ("Parents Highest Education Level” or “Students’ Educational Aspirations Relative to Parents Educational Level”) entered second in the regression equation in seven of the eight countries. It seems that each of the factors of self-confidence in learning and parental educational level impact mathematics achievement differentially and thus may act as contributors to inequity in mathematics achievement.

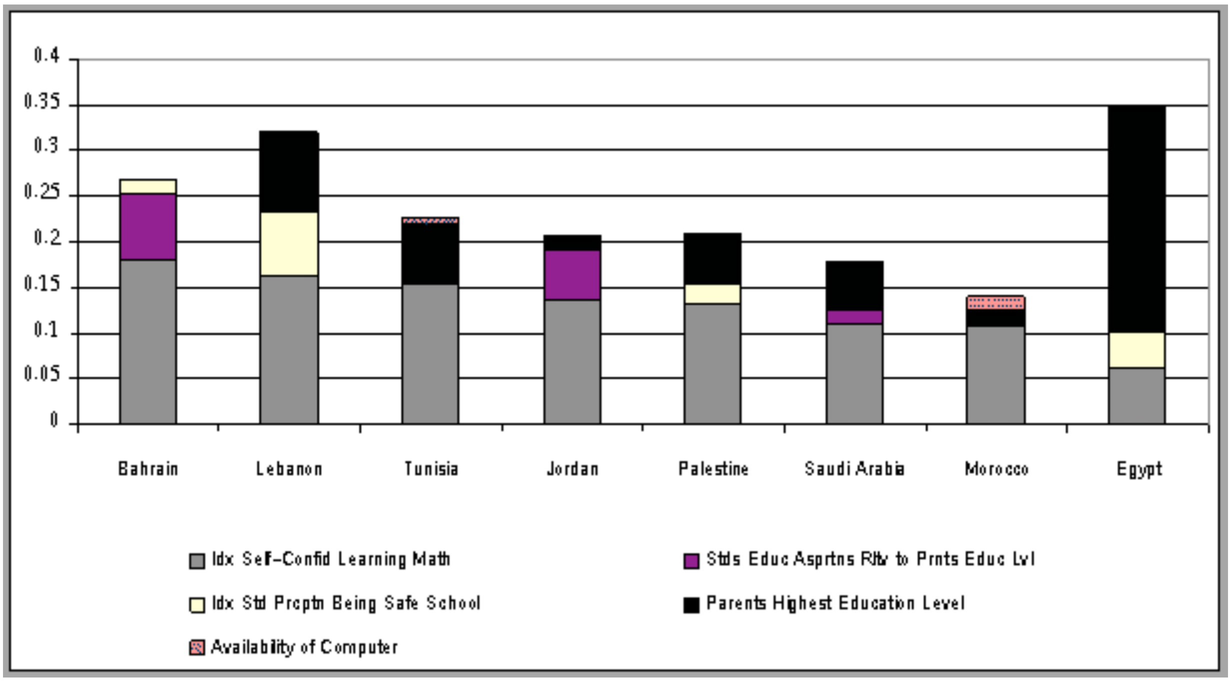

Figure 6. Proportion of Total Variance in Mathematics Achievement Accounted for by Student-level Variables by Country

Comparisons of variations in mathematics achievement accounted for by individual teacher- level variables

The impact of teacher-level variables was quite limited. Teacher-related variables had an impact on mathematics achievement in only five of the eight countries. In those five countries in which one or more teacher-level variable entered the regression equation, three such variables seem to compete for the first place in the order of entry of the variables: “Index of Principals' Perception of School Climate”, “Index of Mathematics Teachers' Perception of Safety in the Schools”, and “Index of Teachers’ Reports on Teaching Mathematics Classes with Few or No Limitations on Instruction due to Student Factors.

Comparisons of variations in mathematics achievement accounted for by individual school- level variables

Figure 7 shows the proportion of variance in mathematics achievement accounted for by each of the mathematics school-level variables in each of the eight Arab countries. Figure 7 shows that in two countries (Morocco, Saudi Arabia), no school-level variable entered the stepwise multiple regression, indicating the weak contribution of school-level variables to mathematics achievement in those two countries. In the six countries in which one or more school-level variable entered the regression equation, variable “Index of Principals' Perception of School Climate” entered first in the regression equation in all countries except Bahrain. Only in Egypt and Jordan, a second variable entered the regression equation and in both of them, this variable was “Trends in Index of Availability of School Resources for Mathematics Instruction”.

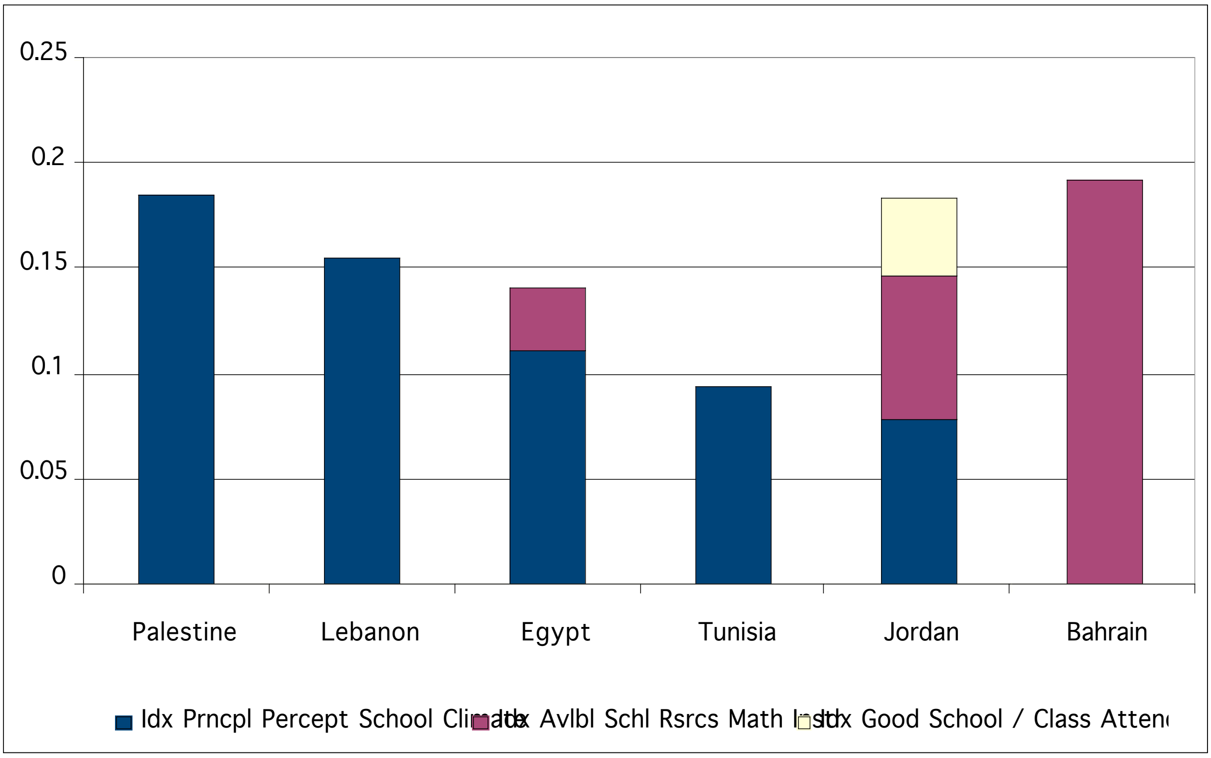

Figure 7. Proportion of total variance in mathematics achievement accounted for by school-level variables and country

The common factors that seem to impact the quality of mathematics education as measured by an achievement score in the eight Arab countries are: Self-confidence in learning mathematics, parental level of education, and student educational aspiration relative to parent’s level of education, school climate, and availability of school resources for mathematics instruction. By virtue of producing differential impact on mathematics achievement, these factors are potential inequity-producing factors.

The activity system provides a way to relate each of these factors to the four processes in the activity system: Production, distribution, exchange, and consumption. The self-confidence in learning mathematics, though an individual affective aspect is nevertheless, an attitude that is formed during production of mathematics learning in the classroom i.e. in the triangle formed by subject, object, and mediated artifacts. This puts a special responsibility on the mediating effect of the teacher and related methodology as well as on the targeted learning outcomes. In a similar manner, school climate belongs to the school as a whole i.e. the school activity system. On the other hand, the factors related to parental education and availability of school resources for mathematics instruction are specially affected by the distribution process of the system i.e. in the triangle formed by subject, community, and rules. The latter being a conduit to policies and social norms and expectations.

In conclusion, the activity system framework seems to have comparative advantages for research, policy making, and professional practice. For research, the activity system, being a theoretical framework based on learning by creation as compared to learning by acquisition and by participation, provides a lens with a broader perspective of equity and quality in mathematics education. For policy–makers it provides a framework to see the complexity of quality and equity in the mathematics education as a social-cultural system. For teachers, it provides a framework to analyze the opportunities, challenges, and limitations of their professional practice.

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RESPONSE TO: EQUITY-IN-QUALITY: TOWARDS A THEORETICAL FRAMEWORK

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With respect for the situatedness of the eight Arab countries under discussion in Jurdak's paper - Bahrain, Egypt, Jordan, Lebanon, Morocco, Palestine, Saudi Arabia, Tunisia - and acknowledging my unfamiliarity with the education and other conditions there, I have chosen to fashion this response in philosophical terms, focusing on the theoretical underpinnings of Jurdak's argument.

The paper offers a theoretical framework – "activity theory" – as a means of elucidating the complex relationship between "equity and quality" in mathematics education. The production model of equity (which identifies the targets, objects and principles of equity and rests on an input/output metaphor of knowledge production) is limited in its conceptualizing of the interconnectedness of equity and quality in education. The activity model, argues Jurdak, widens the scope of objects of equity to include "mediating artifacts", such as language, and other objects and distributions of labor that govern the system, such as processes of production, distribution, exchange, and consumption of knowledge. Quality in the activity model promotes a "knowledge creation" metaphor instead of an acquisition or participation metaphor. "Learning is not conceptualized through processes occurring in individual's minds, or through processes of participation in social practices. Learning is understood as a collaborative effort directed toward developing some mediating artifacts, broadly defined as including knowledge, ideas, practices, and material or conceptual artifacts." (Paavpla, 2004, 569. Quoted in Jurdak, p. 11). This theoretical approach seems promising when considering the discursive nature of learning, and when accepting the crucial role of language and representation in mediating and in part constituting the regimes of truth that are legitimated in/through schools. The question is whether this approach is the most insightful (or the most effective) for examining the intersections of equity and quality, and to what extent this approach can be operationalized to actually address the concerns we have about these intersections. Based on my reading of Jurdak's paper, this approach seems both powerful and limited in its current application.

Jurdak samples the research literature to make evident the many different ways in which differences emerge in/through education. He considers four cases: (1) differences between street vendors and students in terms of problem solving skills, (2) differences between students within the same classroom regarding attitudes and opportunities due in part to "family social milieu", (3) differences between national curriculum based on current access to the global market, and finally (4) political and cultural differences between countries which perform in the top ten on international tests. In each case, these differences correlate to particular lived experiences of inequity, while problematizing pat notions of "quality" education. For instance, the

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 74 - 77).

different "cultural capital" that students might bring to the same classroom, could override (and often does) any measure of the quality of teaching, and thereby determines the student learning trajectory and later opportunities, choices and actions. The relationship between quality and equity is thus problematic, and dependent on context and purpose.

The four examples that Jurdak uses are all radically different themselves, and require close examination, and yet the strategy of collecting these under one umbrella, and interrogating them in terms of equity and quality, demands that the reader look for sameness amongst them. The reader is asked to consider the various conditions of inequity, to note the problematic or situated measures of quality, and to consider the variable nature of the relationship between equity and quality. And yet the paper aims to generate a theory that explains all four instances. Jurdak selects the term "equity" over "social justice" and in doing so, taps into a particular line of inquiry and philosophical thinking. The two terms are affiliated with different political approaches to problems of justice, and selecting one over the other indicates a particular vision of these problems. Perhaps the term equity suits the particular application he makes of the theory – that being to describe the situated notion of quality in particular Arab countries in relation to the variance of student performance on *standardized tests*. The sameness of the standardized test - the assumption that from the same test one can infer truths about radically different students – is used to further his argument. Although Jurdak acknowledges that defining quality in terms of performance on achievement tests is inadequate or "narrow" (p. 4), he still chooses to make his argument using this data.

Jurdak uses the TIMSS 2003 data to explore the theoretical implications of activity theory, but the theory seems to be burdened by this particular application, despite the compelling statistical correlations that he traces. Arguments based on student performance scores are always constrained by the implicit theoretical frameworks embedded in the testing industry. Jurdak's analysis must rely on the categories of the TIMSS 2003 test in determining the intersections of equity and quality. For instance, he relies on the TIMSS variable "Index of Self-Confidence in Learning Mathematics" in studying the variation of student achievement. Through stepwise regression analysis he finds that this variable accounted for the largest proportion of variance in mathematics achievement between schools within each of the eight countries, indicating how important affective student-level variables are. The data about this variable, however, are obtained from the TIMSS student questionnaires in which students are asked to what extent the student "perceives that he/she usually does well in mathematics, mathematics is easier for him/her than for many of their classmates, mathematics is one of his/her strengths, and perceives that he/she learns things quickly in mathematics" (p. 15) This variable paints a particular portrait of mathematics and of student achievement. First, the assumption is that student confidence is more relevant than, for instance, other affective factors such as passion, risk-taking, adventure, or the aesthetic. Second, the questionnaire's emphasis on confidence entrenches a particular vision of student achievement in terms of ease (how quickly do you learn it? Do you do well in it?) instead of questions about engagement (Does it challenge you? Do you strive to solve mathematical problems? Do you play mathematically? Do you question mathematical assertions? Do you argue your opinions based on mathematical analysis?), questions that might help focus student and teacher attention on the adventure of learning mathematics. The questionnaire data (and the *activity* of asking students to answer questions) contains and conveys and constructs particular kinds of subject positions by prescribing what the students are able to say.

As Jurdak explains, activity system theory is responsive to innovation and cycles of expansion (and contraction). With the mediating influence of artifacts, "A new tension between the nodes of the system will eventually lead to a new process of adaptation"(p. 11). It seems to follow that by relying extensively on the TIMSS 2003 data and re-inscribing its assumptions into the system of education research, and thereby legitimating its assumptions about mathematics learning, the system (in which we are included) is impacted in ways that may not suit the equity aims of Jurdak. In other words, if we apply the activity systems model to Jurdak's text, and consider our own inclusion in an activity system that impacts students, we have to be critical and sensitive to the ways in which our own mediating artifacts (the TIMSS data) lead to adaptations (or not) of the education system. I'm not sure if the TIMSS data is an outcome or a mediating artifact, or both, but it is circulating within the activity system of mathematics education research and the activity system of policy, and ultimately, mediates inside the classroom as well. The new system generated through the mediating influence of the artifacts is not necessarily better in terms of quality. There is no guarantee that the new system has a better quality, since the participants to whom it is responding are *not* necessarily, I would argue, the students – whose voices are barely heard through the inadequate questionnaire data. Responsiveness of a system to expand and create new activity systems that "meet the emerging needs of the community" will not by necessity improve quality, since not all members of the community will be given voice or power to speak in any given system. For instance, one could argue that the participants with the loudest voices in a system that mobilizes global education statistics are those of the policy makers and test makers. It is important to interrogate the ways our research might be serving the emerging needs of a massive testing industry.

Jurdak acknowledges that TIMSS 2003 data is modeled on the Production system. His statistical analysis is extremely insightful for many reasons, and compelling in terms of the correlations he is able to identify, although he is obliged to submit to the TIMSS categories of "teacher-level" categories and "student-level" categories when it is evident, if based only on the specific questions of the questionnaire, that it is almost impossible to disentangle these, as he points out in his conclusion. Ultimately, his analysis points to affective factors and school culture factors as contributing significantly to school variance, which begs the question that qualitative researchers have been asking for decades – shouldn't we be following the actors? If affect and culture are the key aspects, then shouldn't we be immersed in these contexts in order to make meaning of them? Might our insight into the relationship between equity and quality be more nuanced if we supplement this statistical analysis with grounded ethnographic studies of the participants? As Bruno Latour asks, in his own version of activity theory (Actor-network theory (ANT)), might we gain insight into the emergent mechanisms and mediating artifacts of an activity by following the actors on the ground and watching closely the human details of mediation? Despite the supposed descriptive role of statistics in this current application, I am left feeling somewhat unsatisfied with the application of activity system theory to the TIMSS data. The statistical analysis produces new understanding, but it doesn't seem to need the social theory that frames it. I am left thinking that understanding activity requires additional detailed descriptions of the sort that Latour recommends: "What ANT does is that it keeps asking the following question: Since every sociologist loads things into social ties to give them enough weight to account for their durability and extension, why not do this explicitly instead of doing it on the sly? Its slogan, 'Follow the actors', becomes, 'Follow the actors in weaving through things they have added to social skills so as to render more durable the constantly shifting interactions.' (Latour, 2005, p. 68). If we aim to understand activity, Latour suggests that the voices of the actors must be heard, and the actions of the actors traced in detailed descriptions and narratives. In terms of a theoretical framework, Latour suggests that activity is better understood through ANT, an acronym that conjures the kind of work of the sociologist, who painstakingly studies his or her subject, like a "blind, myopic, workaholic, trail-sniffing, and collective traveller" (Latour, 2005, p. 9).

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ORDER OF THE WORLD OR ORDER OF THE SOCIAL – A WITTGENSTEINIAN CONCEPTION OF MATHEMATICS AND ITS IMPORTANCE FOR RESEARCH IN MATHEMATICS EDUCATION

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In this article the connection between the philosophy of mathematics and mathematics education is discussed. Special focus is on the implications of different conceptions of the nature and importance of mathematics. The argument will be made that the later Wittgenstein presents us with an unreservedly social interpretation of mathematics that favours a certain direction for our research on mathematics education. According to this interpretation, mathematics could be considered to be constituted exclusively in complex social processes, in which case any conception of it mirroring a pre-existing world of mathematical objects is rejected. To contrast with the Wittgensteinian position, a Platonist position is presented and the two philosophical positions are discussed in relation to their significance for mathematics education.

INTRODUCTION

In the 20th century, many philosophers of mathematics turned their focus on highly technical discussions on the foundations of mathematics, thereby indirectly closing the field down for engaging in close cooperation with other fields of research. These foundational discussions may have been fruitful from a mathematical point of view but a side effect has been the absence of a place in which philosophical questions about much broader issues relating to mathematics could be discussed. Interest in understanding and thinking about the role played by mathematics in society from a philosophical perspective seems to have suffered, as have continued reflections on the relation between the philosophy of mathematics and mathematics education.

Several authors have, however, continuously discussed such interconnections in the overlapping space between mathematics education and the philosophy of mathematics and made important contributions to our conception of this connection (see for example Ernest (1993)). Recently there has been a related interest in the connections between mathematics education, the philosophy of mathematics, and the sociology of mathematics (Kerkhove & Bendegem, 2007), as well as important research on the use of philosophical ideas on mathematics in planning mathematics curricula (François & Bendegem, 2007).

In this article, we shall attempt a similar exploration into the connections between the philosophy of mathematics and mathematics education. Here, we shall be especially concerned with the question of the nature and importance of mathematics and its significance for thinking about mathematics education.

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Considerations about the nature and importance of mathematics define a particular space of connection between the two research fields. In this space, central questions for both researchers in mathematics education and the philosophy of mathematics can be posed: “What role does mathematics play in society?”; “Why is something at this level of abstraction given such a prominent position among the sciences?”; “Why is mathematics considered to be so important that it holds a central position at all levels of curricula all around the world?” These are just some of the questions one could imagine to be relevant for both philosophers of mathematics and mathematics education researchers.

The focus of this article will be two partial investigations into this space of research. Here we shall attempt to investigate a) How Wittgenstein’s conception of mathematics presents to us a purely social interpretation of mathematics and thereby brings about the best possible foundation for an analysis of the social nature of and role played by mathematics in society, and b) How considerable differences in philosophical standpoints on the nature of mathematics could influence our research in mathematics education.

In connection with the first enquiry, we shall develop an in-depth description of Wittgenstein’s conception of mathematics and attempt to explain how it presents to us the most profoundly social account of mathematics imaginable. In the second enquiry we shall investigate the implications of interpreting the nature of mathematics, in turn, from a Wittgensteinian perspective and then an opposing Platonist conception.

It seems to me that there are many good reasons to bring a Wittgensteinian perspective to the fore. First of all, Wittgenstein is thought by many to be the most inventive philosopher of the 20th century, and even his opponents often acknowledge that his position is one of those that must be contested in a variety of research areas if one wants to successfully make different claims to his (for an example from the philosophy of mathematics, see Katz (1998)). Another reason is that he offers a new approach to understanding the social nature of mathematics that differs from the one expounded by for example Imre Lakatos, in his otherwise groundbreaking considerations about the quasi-empirical nature of mathematical objects and the social processes involved in mathematical discovery (Lakatos, 1981). Thereby Wittgenstein seems to me an important figure in discussions on the nature and significance of mathematics in society that aim to clarify and widen our insight into the social role played by mathematics.

In the following section, we start out by contrasting the Wittgensteinian position by considering an entirely contrary idea about the nature and importance of mathematics, to which we shall refer as a Platonist conception. In this conception mathematics is part of the fundamental structure of the world – uninfluenced by the doings of human beings – and only through acquiring knowledge of mathematics is it possible to lay bare the fundamental characteristics of the world. After briefly outlining the main ideas of this Platonist conception in the philosophy of mathematics, we shall consider at some length the Wittgensteinian ideas about what characterises mathematics.

In the final section of the paper, it will be discussed how these two divergent philosophical interpretations of mathematics present to us different visions for thinking about mathematics education and why we should pay a special interest to Wittgenstein's findings on the nature and importance of mathematics in society.

MATHEMATICS AS THE ORDER OF THE WORLD

In the Western history of philosophical thought about mathematics, one position surpasses all others as a sort of crest or initial theory as to what mathematics is all about. I am of course referring to Plato's position, and it is necessary for the task we have set ourselves here to clarify what is meant by a Platonist conception of mathematics. We shall develop such key features of a Platonist conception, not by directly studying what Plato himself has said on the subject, but instead primarily by looking at what is normally referred to as Platonism in the contemporary philosophy of mathematics. In addition we shall briefly look at some of the historical roots and the persistence of this conception of mathematics.

We shall concentrate on three aspects of a Platonist philosophy of mathematics – that people who delve into mathematics are discovering facts about an already existing mathematical reality; that this mathematical realm is an extremely well-structured entity and finally the idea that mathematics should be considered a hidden but very important order of the world.

Pre-existing mathematics

The conception of mathematics connected with Plato's philosophy is often referred to as *Platonism*. Platonism, in the widest sense of the word, refers to the idea that there exists a mathematical reality that mathematical theories seek to uncover. Instead of Platonism, therefore, discussions often concern the slightly broader school of *realism*. Mathematics is assumed to have a genuine field of objects. Like physics analyses the physical nature, biology analyses plants and animals, and geology analyses rock formations, so mathematics analyses geometrical shapes, numbers, and whatever else that belongs among mathematical objects in a similar fashion.

This 'in a similar fashion' should be considered in more depth, because while physics, biology, and geology can employ empirical methods, the objects of mathematics are not accessible to our senses. They exist outside of time and space, which makes them empirically inaccessible. But humankind possesses a capability other than the senses for understanding, namely reason. When cultivated in its sublime form, this capability gives access to the objects of mathematics. Thus Platonism claims the existence of an eternal mathematical world of objects and holds that our reason can reveal truths about this world. These truths are then also eternal and necessary, thanks to the immutability of mathematical objects.

This may sound rather prodigious, but let us look at Platonism in a more everyday version by way of an example. It has been proven, and the proof can be found in Euclid among others, that there are an infinite number of primes. It is not proven, however, that there are infinitely many prime twins, i.e. ordered pairs of the form 3-5, 5-7, 11-13, 17-19, …, 450797-450799, … etc. We know that the density of primes decreases up through the scale.[1] And when primes are farther and farther apart, it is conceivable that the occurrence of prime twins will cease altogether at some point. The statement that there are infinitely many prime twins is, however, neither proved nor disproved.

How does this problem look from the point of view of Platonism? The sentence "there are infinitely many prime twins" has to be either true or false. It states something about mathematical reality. Mathematics has simply been unsuccessful so far in mapping out this particular part of mathematical reality. Such altogether sensible formulations represent a Platonist way of thinking about mathematics.

Platonist and non-Platonist perspectives will have implications for the way mathematical activities are viewed and interpreted. A Platonist will see such activity as an exploration of a hitherto unknown world, discovering more and more truths about an already existing – eternal and immutable – mathematical world. Another perspective, to which we will return later, holds mathematics to be a human construction. We 'build up' mathematics from the ground. Both perspectives seem to have something of importance to say about mathematics but this also poses a dilemma in our thinking about mathematics. In *What is Mathematics, Really?* (1997) Ruben Hersh points out that it is perfectly normal for mathematicians to be Platonists on weekdays but not during the weekend. By this he means that mathematicians go about their work practice as though they were uncovering truths about a mathematical reality. It is only in more detached moments, e.g. when they are asked about their work, that many mathematicians distance themselves from the somewhat peculiar contention that mathematical objects which we can neither see nor touch should exist in reality. Nevertheless it is certainly no exaggeration that Platonism is quite a common perspective today, among mathematicians as well as other people.

The Grand Structure of Mathematics

While Platonism in this general sense is expressive of a comparatively 'simple' idea about the reality of mathematics, Plato's own Platonism is somewhat more complex and we shall omit a thorough look into Plato's own version of Platonism here. It is however not without reason that he should hold such a high position in the philosophy of mathematics. Plato lived from 427 to 347 BC, much of that time in Athens, and he had many disciples after founding the school that he named "the Academy".[2] Above the entrance of the Academy was inscribed, "Let none ignorant of geometry enter here", and this headline was in many ways exemplary of the importance Plato ascribed to the mathematical training of his students.

At the Academy Plato's students and other thinkers laid some of the groundwork for Euclid's *Elements* in the following century and thereby nurtured the idea that mathematics is a world of unity – an entity of truths which can be represented as an ordered entity. It has been speculated that Euclid must have attended Plato's academy for him to have achieved such profound insight into, among other things, the mathematical work of Eudoxus and Theaitetos.

Euclid (c. 300 BC) is considered the pioneer with regard to the axiomatisation of mathematics. In Euclid's *Elements* geometry is decisively presented as a unified entity. Euclid managed to join together much of the known mathematics at the time into one grand structure of interconnected theorems and proofs that rested on only five basic axioms in addition to the definitions of the basic geometric concepts like 'point', 'line' and 'plane'. The five axioms stood for thousands of years as the foundation of mathematics.[3]

These axioms are followed by a long chain of proofs and theorems leading to more proofs of more theorems etc. You only use the theorems that have already been acknowledged as true for the proof of new theorems all based on the truth of the axioms. This construction of interconnected theorems goes on for thirteen books starting with the first theorem dealing with equilateral triangles and finishing with theorems 13-17 in Book 13 of the *Elements* that deal with the regular polyhedra. There are only five of these regular polyhedra (among them the cube) and they are also known as the "Platonic bodies" as Plato refers to them as the elements and building stones of the world in his dialogue called *Timaeus*. There is no question that Euclid carefully chose to begin Book 1 with a theorem on regular figures and to finish the entire work of his mathematical structure with his theorems on spatial regular figures. In this way, his work is in line with the basic idea within a Platonist conception of mathematics, where mathematics is considered a fundamental structure of the world that human beings can explore through the gradual buildup of mathematical theorems from a few rational and self-evident axioms.

Hence, the idea of the axiomatisation of mathematics can be thought of as a natural part of a Platonist conception of mathematics. I have made this idea a focal point here because it is important to be aware that this is a particular way of representing mathematics that could be contested, as we shall see later on. It is also a very dominant and influential way of understanding mathematics with a very long history. Even though the Euclidean system only deals with geometry, it has been a paradigmatic example of how real mathematics is to be represented and presented which has had a deep influence on our thinking about mathematics. At the same time, the work of Euclid reveals to us how mathematics in the Platonist framework is considered in some way or another to be the order of the world – in Plato's outline the buildings blocks of the universe, namely the elements. This leads us on to the last focal point of the Platonist conception.

The Order of the World

We have considered two aspects of what we refer to here as a Platonist conception of mathematics – the pre-existing reality of mathematics and thereby its independence from human activities, and the idea that mathematics is a unified structure of knowledge. Let us finally add one more feature to the Platonist conception.

Much later in Western history, the Platonist conception of mathematics is still a force to be reckoned with, and as we reach the renaissance breakthrough of early modern science, the idea is formed that the divine construction of the universe is hidden and written in mathematics. In order to illustrate this continued adherence to a Platonist conception of the nature of mathematics, we could consider how Kepler (1571-1630) – one of the superstars of the breakthrough of modern science – was deeply inspired by Greek philosophy, and like the Pythagoreans and Platonists he had no doubt that a mathematical reasoning was needed to fathom the construction of the heavenly spheres of the universe. In his first principal work, *Mysterium Cosmographicum* (1596), the Pythagorean-Platonist influence is impossible to miss in his arguments for the number of planets. He favours the explanation that there is exactly six planets in the solar system because there are five regular polyhedra in addition to the sphere (Field, 1988).

Today one might claim that such 'rationalistic' explanations about the cosmos are long gone, but the role we attribute to mathematics in fathoming the world does not seem to have waned. Let us finish our outline of a Platonist conception of mathematics by rephrasing a famous quote of Galileo that seems to me to decisively explain the idea that mathematics is the fundamental logic of the world.

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze, but the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth. (Galileo cited in Crosby, 1997, p. 240).

According to Galileo the universe has been thought out, written down, and put in front of us – just like a book. And this book is not written in Hebrew or Aramaic – as many former generations might have held – but in mathematics. Mathematics, according to Galileo is not just afforded the power of an effective instrument in discovering the world, rather it is the way the world basically is. In this way we can trace a long history of interpreting mathematics as the logic of the world. Whether God was believed to have installed the mathematical edifice and left it for us to discover, or the faith in an architect God had gradually been left behind in modern times, the idea that mathematics is the construction blueprint for the fundamental workings of the universe has never really been abandoned.

MATHEMATICS AS THE ORDER OF THE SOCIAL WORLD

If one is to present a contrast to the Platonist conception of mathematics, several 20th century philosophers come to mind. Brouwer, Hilbert, Dummett and many others have each in their way launched a critique of a Platonist conception of mathematics. Their ideas build on Kant's outline of mathematics and epistemology and in different ways contrast a Platonist conception of mathematics, be it from an intuitionist approach or a more constructivist approach. From inside the field of mathematics education Paul Ernest (1993) and Reuben Hersh (1997), have both argued at length about the social nature of mathematics. These writers in the space between the philosophy of mathematics and mathematics education have corroborated the theory that many of the historically important conceptions of mathematics are 'inhuman' in stressing the other-worldly character of mathematics and its independence from human interference. They conclude from detailed studies and building on Lakatos' ideas on mathematics that mathematics as a construction of knowledge invented by humans resembles the empirical sciences.

Social interactions do play an important part in these philosophies of mathematics, but they probably do not represent the full step towards conceiving of mathematics as a purely social enterprise – a point to which we shall return later on. It could of course be that it is an impossible, or even an unimaginable endeavour to describe mathematics as a purely social enterprise. If mathematics works more or less like physics and geology each with their own objects to study, it might be inconceivable to suggest an interpretation of mathematics that is built only on human social interactions.

As mentioned earlier, we shall be paying special attention to Ludwig Wittgenstein's (1889-1951) ideas about mathematics in what follows, as we have set forward the task to explore how his conception of mathematics is the most thoroughly social and human-centered explanation possible. Wittgenstein is a peculiar character in the philosophy of mathematics. His explicit writings on the nature and foundation of mathematics have been judged insignificant by many authors, on the account that Wittgenstein did not grasp the actual content of the foundational debate going on in the 1930s and 1940s. It has been said that Wittgenstein never completed his writings on mathematics, and that we only have fragments of a position which are insufficient to characterise his writings as an important contribution to the philosophy of mathematics (Shanker, 1986, p. 2). However, it is unthinkable that one of the greatest philosophers of last century should have nothing of importance to say about a topic on which he spent years of research.

Different issues do make it difficult to approach Wittgenstein's writings on mathematics. First of all, Wittgenstein's work does not present its audience with an all-inclusive theory. Instead, small and more or less independent paragraphs consisting of thought experiments, problems, explanations and rhetorical questions comprise his texts. As Wittgenstein himself explains, he is not interested in doing the thinking for the reader, but to make the reader think (Wittgenstein, 1994, p. 32). This is an issue closely related to his proposition that certain of our basic convictions about mathematics be subjected to a thorough rethinking. Secondly, his thoughts on mathematics, as presented in the posthumously published *Remarks on the Foundation of Mathematics* (1935-36), are not easily derived from this text alone, as it draws on his general philosophy of language from his principal work, *Philosophical Investigations* (1936).

In this later period of his writing, it is fair to say that he developed a groundbreaking conception of mathematics that posed a challenge to all earlier theories on the nature of mathematics.[4] He tries to explain to his reader how our general conception of mathematics is in several ways misleading. He sets the task to let the "fly out of the bottle" that is to dissolve our entanglements in excessively metaphysical, philosophical and theoretical uses of our everyday practised use of mathematical symbols. In what follows we shall develop some of the basic concepts from Wittgenstein's general philosophical vocabulary, before we attempt to understand how we might think radically different thoughts about mathematics compared to the Platonist approach.

Language-Games and Family Resemblances

Wittgenstein's early philosophy is contained in his first major work *Tractatus Logico Philosophicus* from 1922, a work that attempts to illustrate how language, beneath the surface of our everyday use of it, has a strict logical structure. By his 'later' writings, one refers to the ideas he came to hold during his last years. More precisely, this is the period from 1929, when he started to be sceptical about the concept of language presented in *Tractatus*, a scepticism he held until his death in 1951 (Gefwert, 1998, pp. 7-8). In order to get a clear conception of the philosophy of mathematics promoted by Wittgenstein during this period, it is essential to understand the new notion of language that he presented.

Whereas his former principal work *Tractatus* expressed a formal theory with its system of propositions, his later principal work *Philosophical Investigations* is deliberately written in an informal style. The aim is no longer to give a formal theory of the way in which we can uniquely express ourselves through language, but rather to emphasise the complexity of ways in which our language functions. Therefore, *Philosophical Investigations* consists of small paragraphs in no strictly determined order. It is formed as a discussion between Wittgenstein and his (imaginary) opponent, who often expresses Wittgenstein's earlier thinking from *Tractatus*. This style emphasises the later Wittgenstein's thesis that everyday language is the foundation of meaningful use of language.

According to the theory on language and meaning presented in *Tractatus,* language supposedly works as a medium to depict relations between objects, and words gain their meaning through this reference to objects. In questioning this understanding of the correlation between words and objects, Wittgenstein sets the agenda for a rejection of theories of language that focus on the descriptional features of language. Instead, he favours the opinion that words and sentences gain their meaning from the contexts in which we make use of them. An example of this is the sentence “Can she walk?”. Uttered by an uncle it could mean whether or not his niece has taken her first steps, but asked to a doctor the meaning could be one of concern for a victim of an accident. Whereas in *Tractatus*, Wittgenstein had claimed that a sentence has a fixed meaning in the composition of its constituent parts, he now holds that there is no single, fixed meaning of a sentence. The circumstances under which a sentence is uttered – the use made of the sentence – is what fixes its meaning.

Wittgenstein therefore introduces the notion of ‘language-games’. It is meant to underline the fact that speaking a language is part of an activity or life form (Wittgenstein, 1997, p. 11e [23]). Language in its totality is also called ‘the language-game’ from time to time. Examples of language-games are to command and to act; to describe something; to talk about an event; to make jokes; to solve equations; and so on. Language works as a number of tools with which we can perform a vast variety of different actions. Wittgenstein shows this variety of forms in which we use language meaningfully and maintains that this is in sharp contrast to the traditional interpretation of meaning and language of which his earlier publication *Tractatus* is an example.

> It is interesting to compare the multiplicity of the tools in language and of the ways they are used, the multiplicity of kinds of word and sentence, with what logicians have said about the structure of language. (Including the author of the Tractatus Logico-Philosophicus.) (Wittgenstein, 1997, p. 12e [23])

Wittgenstein does not try to give an explanation catching the ‘essence’ of language as he did earlier in *Tractatus*. On the contrary, he refutes any notion that there exists a common characteristic for the class of activities we call language. The phenomenon we call language consists of a multiplicity of uses and there is no essential common characteristic between these uses (Wittgenstein, 1997, p. 31e [65]). Instead Wittgenstein proposes that the meaning of words and sentences flow from their use in human practice, i.e., in our language-games, and hence he maintains that the meaning of a word does not have any real existence as a physical, mental or ideal object. The adequate approach to finding the meaning of a word is an analysis of the use of the word in the appropriate language-games. Therefore the main theme of Wittgenstein’s later writings has been expressed as follows: The meaning of a word is its use.

The relation between different language-games is indicated by the term ‘games’. The variety of activities we call games have no obligatory common characteristics. Some games include the use of a round ball; others include a board, and so on. The partial similarities that might be between two or more language-games are what Wittgenstein calls ‘family resemblances’ (Wittgenstein, 1997, pp. 31e-32e [66-67]).

> I cannot characterise these resemblances better than by the word “family resemblances”; for the various resemblances which exist between different members of a family: height,

> facial features, eye colour, walk, temper, etc. etc., overlap and cross each other in exactly this way. – And I would say: the 'games' make up a family. (My translation from Wittgenstein, 1994, p. 67 [67])

This concept reflects the lack of essence or a unique common feature in the activities we call games. All there is, is a complicated net of familiarities overlapping each other. With the concept of family resemblance, he refutes the idea of commonality among things that we categorise together (e.g. games or mathematics) and the idea that exact definitions of words are necessary in order for them to have meaning (e.g. of the word 'game').

On Wittgenstein's account, the quest for definitions of essences is in vain. To understand a word of our language is simply to be able to use the word according to certain 'rules' attached to the language-games in which the word is embedded. The extension of a word has no exact limits. Wittgenstein does not reject that we can, to a certain degree, specify limits to the extension of a word, but his point is that a word's lack of such limits has never worried us when we have used it in practice (Wittgenstein, 1997, pp. 32e-33e [68]).

Rule-following

The foregoing considerations will become clearer when we analyse the important notion of following a rule. Wittgenstein's considerations on rule-following are meant as an elaboration of the 'meaning-is-use' conception of language. With these considerations, the aim is to explain that to use a language is to follow rules. With his remarks on rule-following, Wittgenstein tries to show us that the idea we normally attach to following a rule (and as we shall see for doing inferences in mathematics) is basically wrong. This is the idea of there being 'bodies of meaning' underlying and determining the use and extension of a word or a rule (Shanker, 1987, p. 16).

Along with this idea goes the thought that when someone has grasped the rule, it must be followed in a certain way. When we speak of following a rule, we think of some guidance, which we are to follow precisely in order to do things the right way. Wittgenstein rejects this idea, as he maintains that rules in themselves cannot explain to us how to follow them. Whether a rule is presented to us through a formula, a signpost or something else, it is always possible to interpret these signs differently from that which we call the correct way to follow them (Malcolm, 1986, p. 158). We simply apply them as we do as a consequence of practice. With endless practice through exemplars in the use of equations we finally become very certain of how to manipulate them. What logically compels us to follow the rule 'add 2' in the way we do is that following the shared rule is itself the criterion for understanding the rule. It is we who, through our practice, determine what is to count as the correct way to follow the rule.

A most important aspect of Wittgenstein's comments on rule-following is the inability for a single individual to 'fix the meaning of a rule' (Malcolm, 1986, p. 156). For there to be a difference between following a rule and believing that one is following the rule, there has to be some external criteria by which this difference can be established. If a single individual were to try and fix the meaning of a rule, what they believed to be the correct application of the rule would never be challenged. But then nothing they could possibly do would ever count as a wrong application of the rule. There would be no difference between believing one followed the rule and actually following the rule. This ultimately leads to the rule losing its meaning, as there are no criteria for what defines the wrong and the right applications (Malcolm, 1986, p.156). Thus the activity of rule-following requires a community in relation to which it can be determined whether the rule in question is followed according to normal practice. Practice is therefore seen as the necessary condition for establishing meaning and rules, where a practice is understood as a community of rule-followers who have had the same kind of training and therefore agree on the implications of certain rules.

According to Wittgenstein, we can be absolutely certain of how to use a rule, but still not be able to give ultimate reasons for following the rules as we do (Wittgenstein, 1979, p. 39e [307]). To understand a rule is parallel to the understanding of a word discussed above. You do not *have* something in your mind or elsewhere when you understand a word or a rule; but rather you are able to *do* something. To understand a word or a rule is comparable to mastering a technique. You are simply able to use the rule or the word within a language-game in accordance with its established use. This agreement is the bedrock of our explanations, because it constitutes the possibility of language. Without this agreement there would be no rules. This agreement in 'doing the same' is an example of what Wittgenstein calls a 'form of life', which is what we must accept as 'the given' that escapes explanation (Wittgenstein, 1997, p. 226e).

The Language-Games of Mathematics

By introducing some of the basic concepts of Wittgenstein's philosophical vocabulary, we have already touched upon his conception of the nature of mathematics. According to Wittgenstein's interpretation of language, it would appear that mathematics is considered a network of different language-games that share family resemblances. According to Wittgenstein, mathematical objects (equations, functions etc.) do not stand for anything. Instead, and as a consequence of his conception of language, they acquire their meaning from the rules we attach to them and according to which we use them; how we calculate with them. By a 'calculation' Wittgenstein therefore means a certain procedure for manipulating mathematical objects, e.g. deriving one equation from another, according to certain rules (Wrigley, 1986 (II), p. 186). The nature of mathematics in Wittgenstein's terminology is therefore: 'Mathematics consists entirely of calculations' (Wrigley, 1986 (II), p. 186).

The calculation conception of mathematics is a radical and entirely new approach to the traditionally important question: What is the nature of mathematical propositions? Wittgenstein holds the view that it is wrong to think of mathematics as consisting of a body of propositions having a meaning in themselves, as this easily leads us to think that these propositions were somehow there before we constructed them. In other words, he emphasises that the nature of mathematics consists in different techniques of calculations, rather than a body of true propositions (Wittgenstein, 1978, p. 365 [VII-8]).

This means that Wittgenstein does not see the theorems of mathematics as self-explanatory. Wittgenstein says that a mathematical proposition is connected to its proof like the surface of a body is connected to the body itself. Thus, proof and proposition in mathematics are intrinsically tied together. If we have no proof for a certain mathematical proposition, as until recently was the case with respect to Fermat's Last Theorem, it is actually wrong for us to call it a proposition. A mathematical 'conjecture' is the appropriate term, as we have not yet established any rules to govern its use in the mathematics. With this distinction, Wittgenstein wants to show us that a mathematical conjecture is a stimulus for our constructions, and not a meaningful proposition in need of a proof to confirm its truth or falsity.

> A mathematician is of course guided by associations, by certain analogies with the previous system. After all, I do not claim that it is wrong or illegitimate if anyone concerns himself with Fermat's Last Theorem. Not at all! If e.g. I have a method for looking at integers that satisfy the equation **Error! Objects cannot be created from editing field codes.**, then the formula **Error! Objects cannot be created from editing field codes.** may stimulate me. I may let a formula stimulate me. Thus I shall say, here there is a stimulus – but not a question. Mathematical problems are always such stimuli. (Wittgenstein, taken from Shanker, 1987, p. 113)

The proof gives meaning to its resulting proposition, and it would be misguided to say that the proof has changed the meaning of the conjecture. The conjecture is meaningless, as it has no place within the meaningful frame of a language-game. It is a stimulus and not a question, because a question presupposes that we have a method for answering it. Fermat's Last Theorem was a stimulus to all of us, as we associated this conjecture with the case of $n = 2$. But only after Andrew Weyl's proof was it legitimate to talk about this theorem as a meaningful mathematical proposition, in Wittgenstein's interpretation.

Proofs and Experiments

The close connection between mathematical proof and mathematical proposition illustrates an important aspect of the language-games we call mathematics. Whereas the natural sciences have each their own type of objects on which to perform experiments, mathematics is the practice where we lay down the grammatical rules for description in the sciences and many other language-games.

> Let us remember that in mathematics we are convinced of grammatical propositions; so the expression, the result, of our being convinced is that we accept a rule.
>
> I am trying to say something like this: even if the proved mathematical proposition seems to point to a reality outside itself, still it is only the expression of acceptance of a new

measure (of reality). Thus we take the constructability (provability) of this symbol (that is, of the mathematical proposition) as sign that we are to transform symbols in such and such a way. (Wittgenstein, 1978, pp. 162-163 [III-26-27])

The construction of a proof convinces us of the proposition, but it does so in the normative sense of our accepting a new measure of reality. This acceptance determines what makes sense to say and what does not. Because it is a grammatical proposition, the mathematical proposition cannot be refuted by an experiment. It has nothing to do with empirical matters whatsoever. In contrast to empirical propositions, it makes no sense to doubt the mathematical proposition, as doubt has been excluded from it, because we use it as a grammatical rule. This means that mathematics and the natural sciences consist of language-games of completely different natures.

> We feel that mathematics stands on a pedestal – this pedestal it has because of a particular role that its propositions play in our language games.
>
> What is proved by a mathematical proof is set up as an internal relation and withdrawn from doubt. (Wittgenstein, 1978, p. 363 [VII-6])

Hence, Wittgenstein very strictly maintains a distinction between proofs in mathematics and experiments in the sciences. Proofs distinguish themselves by being just that type of technique from which doubt is logically excluded. The distinction between our use of the concepts 'proof' and 'experiment' therefore indicates that mathematics has an important characteristic that it does not share with the natural sciences. And Wittgenstein's maintaining how very different the language-games of mathematics function compared to those of science also makes it clear that the perception of mathematics he presents to us is different from the social constructivist conception mentioned earlier on.

Mathematics as Conventional Measures

Wittgenstein's account of the questions concerning the genesis and growth of mathematical knowledge is basically of a conventional nature. His explanation is based on our freedom to invent new rules of grammar to follow (Shanker, 1986, p. 21). Sometimes this consists in constructing new links between 'old' mathematical concepts, and sometimes it consists in the construction of entirely new mathematical systems. Mathematics continually forms new rules and extends the old network of mathematics (Wittgenstein, 1978, p. 99). The construction of complex numbers is just one example of this.

Wittgenstein holds that mathematical conventions are concerned with the creation of new systems of representations.

> What I want to say is: mathematics as such is always measure, not thing measured. (Wittgenstein, 1978, p. 201 [III-75])

Mathematical forms of representation provide us with standards for representation in our description of the world. They are measures in the sense that they set up the rules of grammar through which we can describe something. Wittgenstein exemplifies the role played by grammatical rules with Einstein's use of Bolyai-Lobatchevskian geometry in his Theory of Relativity. Einstein's use of this alternative geometry (as opposed to the use of Euclidean geometry) is seen as an application of an alternative system of mathematical rules that decides the grammar for describing phenomena (Shanker, 1987, p. 270). Mathematical systems are thus seen as different grids or structures by which we measure or describe the world. According to Wittgenstein, some rules of grammar are presupposed in any description of reality (Shanker, 1987, p. 318), and in the sciences, mathematics is presupposed in this way.

In connection to Wittgenstein's conventionalist account of mathematics, it seems obvious to ask whether the development of mathematics is arbitrary or not. Wittgenstein comments on this aspect of the development in the following passage.

> But then doesn't it [mathematics] need a sanction for this? Can it extend the network arbitrarily? Well, I could say: a mathematician is always inventing new forms of description. Some, stimulated by practical needs, others, from aesthetic needs, - and yet others in a variety of ways. And here imagine a landscape gardener designing paths for the layout of a garden; it may well be that he draws them on a drawing-board merely as ornamental strips without the slightest thought of someone's sometime walking on them. (Wittgenstein, 1978, p. 99 [167])

Wittgenstein's answer is that the development of mathematics is arbitrary in so far as there is nothing in reality which compels or necessitates us to develop mathematics as we do. In another sense, we are, however, always guided in developing new mathematics. An established tradition can guide our mathematical constructions, and the trial against 'the facts of nature' in the sciences often generates new forms of mathematical representations. Thus, we have reason to construct mathematics in certain directions, but this does not mean that these reasons must somehow be justified by a correlation between reality and mathematical forms of representation (Shanker, 1987, p. 319). The mathematical forms of representation are autonomous in the sense that their meaning consists in our use of the grammatical rules within the mathematical system. If 'facts of nature' or objectives we pursue suggest that we develop new forms of representation, these are not in any sense more true forms of representation, but simply new grammatical rules by which we can describe our surroundings.

Inference as Rule-following

To corroborate his view of mathematics, Wittgenstein knows that he has to convincingly explain the act of logical inference involved in mathematics as a natural part of his theory.

> These considerations bring us up to the problem: In what sense is logic something sublime? (Wittgenstein, 1997, p. 42e [89])

Wittgenstein wants to show that there is no such thing as an ultra-experience of some sort of reality, to which inference must obey (Wittgenstein, 1978, p. 40 [8]). What he argues against in passages like this one is the conception of mathematics known from for example as Logicism and the logical positivists in general. Their position has also been called a conventionalist position, because they held that by convention we agree on certain basic self-evident truths in logic and mathematics from which all remaining truths can be tautologically derived. Many writers have conflated Wittgenstein's theory with the conventionalism of the logical positivists, but the very different conceptions of mathematical growth reveal the error of such interpretations (Shanker, 1986, p. 20). Wittgenstein, as opposed to the logical positivists, avoids the difficulty raised against conventionalist theories by Poincaré, who questioned how there could be new discoveries in mathematics if it consists entirely of tautologies. Mathematics has nothing to do with tautologies, in the later Wittgenstein's opinion, but rather one could say that mathematics is built up by conventions as opposed to tautologies. Wittgenstein would ask, from what source do these imminent consequences of the rules of inference stem in a conventionalist theory (Wrigley, 1986 (I), p. 362)?

On Wittgenstein's account, the meaning of symbols depends exclusively on the use we make of them. Nothing is hidden beneath the surface of our practice, and it is therefore misleading to think of mathematics as mechanically following self-evident rules, as did the logical positivists with their notion of mathematics as tautologies. Instead, Wittgenstein's conception of inference is an application of his thoughts on rule-following. On the process of inferring he says,

> There is nothing occult about this process; it is a derivation of one sentence from another according to a rule; … (Wittgenstein, 1978, p. 39 [I-6])

Inference takes place as a transformation of our expressions according to some paradigm (language-game) and the right way of performing this transformation is the accordance with a convention or use (Wittgenstein, 1978, p. 41 [I-9]).

Hence, the theory of rule-following is also applied to the mathematical practice; i.e., to the different techniques of calculation. This means that logical inference in mathematics simply amounts to sufficient practising within the accepted practice of mathematics. For example, the rule 'add 2' does not in itself explain how it is to be used. If one knows the meaning of the rule, one would know how to use it. But as discussed earlier on, according to Wittgenstein, meaning is use; that is, understanding the rule is the capability of using the rule in accordance with mathematical practice.

It is through our mathematical practice that we determine what is to count as being in compliance with the rule in question. That mathematics is embedded in human practice means that the rules we learn to follow in mathematics are not of a kind which we can apply without thought, as logical positivists understood them. Machines can act in a rule-bound fashion if they are programmed to do so, but they cannot perform calculations, as they are not capable of justifying their application of the rule. Rule-following is about doing things for a reason, which is only possible for creatures having will, who can set up goals to pursue.

Rejection of Platonism

Wittgenstein's philosophy of mathematics can be seen as an attempt to make us abandon all aspects of the Platonist view of mathematics that has prevailed since the time of the ancient Greeks. Platonism holds that mathematical objects and relations between those exist independently of human practice and of humans' capability of discovering these mathematical facts. In the Platonist conception, there are bodies of meaning lurking around beneath the surface of the mathematical symbols used within the mathematical practice, and Wittgenstein would dismiss the possibility of such meaning that did not stem from our use of the symbols. His critique of the nature of logical inference as truth-preserving also stresses the fact that he sees no reason to believe that mathematical objects exist independently of us and have relations for us to discover.

> The mathematician is an inventor, not a discoverer. (Wittgenstein, 1978, p. 99 [I-167])

Mathematics has often been considered to stem from worlds of ideas or basic intuitions etc., which has nothing to do with the actual practice of mathematics from which mathematical calculations gain their meaning. Wittgenstein therefore claims that not even natural numbers exist or refer to something independent of our language-games, as for example the inborn pre-linguistic mathematical intuitions proposed by Brouwer (Körner, 1986, p. 122). We must resist such unfounded but ever present temptation to idealise the words and numbers of our language.

> Counting (and this means: counting like this) is a technique that is employed daily in the most various operations of our lives. And that is why we learn to count as we do: with endless practice, with merciless exactitude; that is why it is inexorably insisted that we shall all say "two" after "one", "three" after "two" and so on. – But is this counting only a use, then; isn't there also some truth corresponding to this sequence?" The truth is that counting has proved to pay. – "Then do you want to say that 'being true' means: being usable (or useful)?" – No, not that; but that it can't be said of the series of natural numbers – any more than of our language – that it is true, but: that it is usable, and, above all, it is used. (Wittgenstein, 1978, pp. 37-38 [I-4])

From this quotation we see how the concept of truth can only be used within a language-game, and Wittgenstein emphasises that the things we actually do are the ultimate foundation of language as well as mathematics. Calculations are techniques embedded in human practice and hence dependent upon the teaching of mathematics and all the other connections it has to human life forms.

Hence, we see that the philosophical investigations that Wittgenstein performs with respect to mathematics do not consist of deriving the epistemological source of mathematical knowledge, as mathematics is a part of our life form and cannot be meaningfully talked about as true or false, absolute or fallible. Neither does he try to justify our mathematical knowledge. Instead, these investigations are an attempt to discern from the interpretations and metaphors that surround them the actual content of the different practices of calculations. Wittgenstein uses the terms 'prose' and 'calculation' to signify the philosophical problems of a linguistic nature attached to a mathematical practice and the actual activity in the practice, respectively (Gefwert, 1998, p. 236).

Let us here end this presentation of Wittgenstein's conception of mathematics with a summary of its most important features. Mathematics is considered a group of language-games that share family resemblances and has incorporated into them different types of calculations. The growth and genesis of mathematical knowledge stem from our freedom to construct new mathematical structures. These mathematical structures are forms of representations through which we can describe the world. Derivations within a mathematical structure are therefore certain by virtue of being grammatical constructions.

In this way Wittgenstein accomplishes a full-scale rearrangement of the Platonist conception of mathematics. Mathematics is a social practice, and not just in the 'normal' sense that people work together on mathematics; but in the sense that they establish the very meaning of mathematical objects and processes. Doing mathematics is a rule-following activity, but following these rules is not a mechanical action. Rather it is an exclusively human activity to interpret how a rule is to be applied under given circumstances.

THE IMPLICATIONS OF DIFFERENT PHILOSOPHICAL POSITIONS

Two investigations were outlined at the beginning of the paper, and by now we have completed the task of showing how Wittgenstein's conception of mathematics seems to be the most socially founded explanation about the nature and importance of mathematics imaginable. In addition we have contrasted Wittgenstein's position to a classical Platonist conception of mathematics and are now in a good position to examine what different visions they bring to the table for thinking about mathematics education.

The very different types of answers to the question about the nature of mathematics seem to me to implicate certain differences in the possible direction of our thinking about mathematics education. Different types of visions for research agendas come to the fore, depending on the philosophical position one adheres to. In what follows I will attempt to discuss two themes – case studies if you like – within the field of mathematics education research, in order to highlight the differences that philosophical considerations about mathematics can effect in the standpoints taken on educational issues. These visions are not easily derived in a unilinear fashion but nonetheless, differing philosophical standpoints lead to different ways of talking about mathematics that could be of vast import in practice. Firstly we shall discuss these differences in relation to discussions on the learning processes of mathematics and subsequently to discussions on the content of mathematics education.

Learning mathematics

If there actually exists a mathematical world of truths before any human being has thought about mathematics, then mathematics education is concerned with bringing students to see the logical necessity they possess deep within them – or at least that is how Plato himself thought of learning mathematics, namely as recollection. We can get an idea about what he means by looking at the dialogue *Meno*, in which Socrates discusses virtues with Meno, who was a student of Gorgias, one of the worst sophists imaginable in Plato and Socrates' world. Halfway through the dialogue between Socrates and Meno, the former continues the dialogue with a slave boy who never learned mathematics, by way of example. In the course of the dialogue, the boy achieves some mathematical insight. Socrates sets the problem to find a square whose area is double that of a given square. The boy succeeds in solving the problem once Socrates has put him on the right track by way of rational dialogue.

In this way, Socrates demonstrates that the unskilled slave boy and hence all human beings have some ability to think rationally about mathematical ideas, and thus can be said to have innate ideas about the organisation of the world. According to Socrates, he had not offered the boy any information about mathematical coherences. He would only ask guiding questions. Thereby Socrates suggests that we have a foreknowledge of the correlation between mathematical ideas and objects, such as squares and lines. We only need to be reminded of this knowledge. The learning of mathematics is then merely a question of recollecting knowledge that we all already possess. And in order to recollect, we just need to think rationally and engage in dialogue about mathematical subjects.

The discussion of mathematics in *Meno* is just one example that illustrates how to think about a range of other circumstances in life, e.g. what is the good thing to do, what is just, and what is true about reality. It is no coincidence, however, that Plato employs mathematics to demonstrate these conditions regarding cognition. In the Platonist framework, mathematics is the discipline that affords insight into, and training to think rationally about the world of Ideas in general (here we recollect the words over the portal of his Academy).

For the Platonist another issue concerns the proper medium for the learning processes of mathematics. As mathematics is basically about a world of abstract objects, pure thinking is all that is needed to access this world. Drawings might be helpful tools in learning mathematics for the untrained mind – for example the slave boy – but they are always only secondary to the abstractions of the mind.

> Although they make use of the visible forms and reason about them, they are thinking not of these, but of the ideals which they resemble; not of the figures which they draw, but of the absolute square and the absolute diameter, and so on […] they are really seeking to behold the things themselves, which can only be seen with the eye of the mind? (Plato, The Republic, Book VII)

Drawings or imprecise communication about mathematics suffers from not meeting the criteria found to be the nature of the mathematical world of objects.

We may imagine a Wittgensteinian arguing against this line of reasoning. If there is nothing but human rule-following involved in doing mathematics, it becomes clear that only one thing really matters in learning mathematics, namely training within a community of practitioners doing mathematics. Mathematics has its foundation – just as the natural language – in our everyday use of the terms, signs etc. involved in the use of mathematics, in the numerous language-games that involve mathematics in one way or another. Nothing is hidden beneath the surface of the mathematical calculations of signs involved in the practised language games, and in this way there are no rational short cuts or speedy aha-experiences that will help gain an immediate insight into the world of mathematics. In the case of the slave boy, the Wittgensteinian could even argue that the reason why the slave boy is actually getting along quite well in discussing mathematical arguments is because in his everyday practices or language-games – concerning very worldly things – he has had a great deal of out-of-school-training in mathematics. Mathematics is part of his language-games and his life form.

On the second point – regarding the proper ways to represent mathematical arguments – the Wittgensteinian would hold that there are no metaphysical reasons for demanding that a 'pure' mathematical proof be as abstract as possible, and there are no reasons to claim that a graphical proof should be of a lesser nature than a thought experiment. Even an axiomatised line of proofs and theorems towards a certain mathematical proposition does not in principle take higher ground. Wittgenstein would argue that a Euclidean proof with all of its graphics or even a completely graphical argument is every bit as valid as any other means of proving mathematical propositions.

A consequence of the rationalistic conception favoured by the Platonist might be that it fosters an environment where it is more prestigious to be able to calculate in your mind than to use 'techniques' for calculating. If the logical deductions of mathematics are something sublime that you may or may not have an extraordinary insight into, then you had better be seen to manage it without the use of such earthly measures as rough workings, drawings or helpful rules to guide your way. In Wittgenstein's interpretation of mathematics, this downplay of techniques in calculations and an excessive tribute shown to 'the eye of the mind' betrays the nature of mathematics.

What is the relevance of Socrates' account to contemporary mathematics education? We do see diverse degrees of aptitude for learning mathematics among students in every classroom around the world. In other words, there seems to be certain obstacles that may impede the learning of mathematics. How will the Platonist and Wittgensteinian account for this experience?

In a Platonists conception of mathematics it would be perfectly sensible to suggest that not all humans alike have the same inborn skills required to learn mathematics at a highly abstract level. In *The Republic Book III,* in his discussions with Glaucon, Plato himself explains how different people are born with different abilities – some with a soul that is best suited for the work of the hand, some with a soul of gold best suited for thinking, and hence more prepared from the outset to learn mathematics and take political decisions. At times the centre of attention in mathematics education research has been the attributes of the individual learner. This research might be focused on the cognitive structures of the individual or the IQ of a student. It might be the case that IQ measures indicate that a student is unlikely to be successful in learning mathematics, and it might even be suggested that the capability of the individual is connected to the DNA support structure for doing abstract thinking.

To understand the Wittgensteinian vision about obstacles in learning mathematics, we must first of all divert our attention from the individual. Mathematics is the least individualised thing one can imagine, from a Wittgensteinian perspective, because it is necessarily a community of practitioners that settle on the meaning of the symbols used. The individual is always only the secondary bearer of mathematics. Primary for understanding educational practices is the socialisation into the language games of mathematics. These are interrelational human activities where the meanings of signs, operations, proofs, drawings etc. are determined and continuously negotiated. Singling out one individual's obstacles in learning mathematics is therefore in principle nothing to do with the difficulty of the subject of mathematics. No matter what the subject, training and continued use within a supporting community should in principle be enough for anybody to learn any conceivable mathematics as it is 'only' a game of rule-following.

Hence, the Wittgensteinian would think of the approach to learning mathematics and the obstacles associated with this process a purely social matter. What matters here is not some preset ordering of each individuals mind, the gene pool one has received or the like. Obstacles in learning mathematics is rather to be explained in terms of the social inclusion and exclusion processes at the micro-didactical level – for example in a class room – or explained through social background, which naturally has an effect on the preparedness and discipline a pupil or student is able to muster. In principle – according to a Wittgensteinian conception of mathematics – there is nothing difficult or special about the topic of mathematics that should leave us baffled with the task of learning its many facets. In a sense it is just as easy or difficult as learning any other language, were it not for the 'prose' we attribute to the learning of mathematics. Relentless practice in a supporting environment is what it takes to be good at mathematics and perhaps eventually a mathematical 'genius'. The social agenda of course sets its rigid limits to what any individual is likely to achieve in mathematical skills.

This line of reasoning about the learning processes of mathematics reveals the importance of understanding how the sociology of mathematics and sociological approaches in general can help to disclose who will be successful in learning mathematics. It opens up the research agenda for conceptualising why some learn and some do not, from a sociological point of view. It is the interaction between people in relation to mathematical learning – formal as well as informal – that will decide how well one is acculturated with the rules of the game. Here a distribution of power can be witnessed, where some are singled out as those who have the skills necessary and some as those who lack them. In the Wittgensteinian account, it takes a safe environment for years and years – not a sharp mind – to learn mathematics for the uninitiated; an environment where, for example, making mistakes is allowed, motivation is present etc.

The content of mathematics education

Let us turn to another field of consideration within mathematics education. The two different philosophies of mathematics discussed seem to be at odds when discussions turn to the content of mathematics.

What would be a rational content for the Platonist to suggest for educational purposes? For the Platonist the foundation of mathematics, its axiomatic foundation, the logical progress from natural numbers to rational numbers and so forth could be an organising principle. The world of mathematics is organised, and the more organised the content for education is presented, the easier it should be for the student to realise and clearly see the world of mathematics. This could mean, for example, that some fundamental operations should necessarily be learned before other parts of the mathematical building up, and this could be used as an argument in favour of a step-by-step approach in mathematics education.

So it would seem – as already outlined in section 2 – that an orderly presentation of geometry along the lines of the Euclidian construction would fit the Platonist conception. And indeed these are some of the guidelines that have been brought down through the generations during which the Euclidean paradigm has had such an immense influence. This actually represents how much mathematical practice has been exercised since the time of Plato. The many new developments in mathematics in modern times have of course changed the content of mathematics education. But it is nonetheless important to remember that the ruler and the pair of compasses were still the main components in the mathematics curricula until just a few decades ago – at least in the Danish curricula. It would appear that they were there because of the tradition set by Euclid. Apparently, they figured in Euclid's *Elements* in the first place because they could produce what was thought to be the heavenly forms of infinitude, namely the straight line and the circle. The second and the third axioms of Euclidean geometry exactly points out that these are the constructions that are allowed in geometry (see the endnote 3).

The Wittgensteinian would not oppose the use of Euclidean geometry in mathematics curricula. But they would denounce the idea that the content is anything to do with heavenly spheres and the like. They would also reject any notion that this was the only and necessary way of doing mathematics, since there are so many other ways mathematics might have turned out. In this way the Wittgensteinian opens the door to thinking about topics like ethno-mathematics, or the possibility in general that mathematics is not something universal – there is no such thing as 'the' mathematics needed. It is perfectly conceivable that different cultures might find that different mathematical curricula suited them and their way of living, whereas others might even do without mathematics in the traditional sense. Wittgenstein's philosophy does not really offer much guidance in this respect, in that it does not point to a particular mathematical structure as the essence of mathematics. Rather Wittgenstein tends to suggest that there is no essence to be found, as explained earlier on.

Through his claim about the absence of any sacred 'essence' in mathematics, he is naturally saying a great deal about how we might think of the planning of mathematics education. The learning of mathematics is easily divorced from a purely abstract entity of mathematical propositions in the Wittgensteinian conception. It might not be functional analysis in complete abstraction in which university students should be trained. It could be a completely different field of mathematical investigation, perhaps connected to a highly complex practical situation where mathematics is used in a cross disciplinary setting?

Applications of mathematics are in many educational traditions divorced from the teaching of the mathematical system itself. This altogether fits in with the ideas of Platonism. Plato himself disputed his contemporary mathematicians for being too occupied with the practical uses of mathematics instead of pure reasoning.

> They have in view practice only, and are always speaking in a narrow and ridiculous manner, of squaring and extending and applying and the like --they confuse the necessities of geometry with those of daily life; whereas knowledge is the real object of the whole science. (Plato, The Republic, Book VII)

Plato is obviously not happy with the geometricians of his time. They seem to him to be too engaged with the problems of everyday life and they even lower themselves to apply mathematics!

In the Wittgensteinian conception, mathematics should be thought of as a toolbox, a network of techniques for representation and measure. There is nothing to disqualify the use of practical settings for learning mathematics – rather it seems to be a natural component of any mathematics curricula.

CONCLUSIONS

We have discussed how Wittgenstein's conception of mathematics is a thoroughly social conception, in the sense that it refuses any rationalistic order of reasoning or world of mathematical objects of any sort to enter into our description of the nature of mathematics. In this way he presents us with a vision for doing research in mathematics education that represents the learning processes of mathematics as purely social. I have argued that his philosophical perspective favours a sociological foundation for doing research in mathematics education.

In addition to this we have discussed how Wittgenstein establishes a novel understanding of what it means for mathematics to be a social construction. In recent years approaches called 'Humanist' or 'Social' mathematics have developed, that in a sense have the same agenda as Wittgenstein, namely to effectively show how mathematics is a social construction. But the social constructivist schools have focused on an epistemological critique of former 'absolutist' schools in the philosophy of mathematics. They consider mathematics to be essentially subject to historical changes and mathematical knowledge to be fallible, not absolute (Hersh, 1997, p. 22). They think, in accordance with Lakatos, that it is impossible to find a foundation of mathematics that can secure mathematical knowledge.

Despite the vast difference between the Platonist and the Wittgensteinian conceptions, they may actually be said to agree that mathematics is more absolute than fallible – but for completely different reasons. For the Platonist mathematics is absolute because it is out there before we get to it, while to the Wittgensteinian, it is absolute because it serves the same purpose as the meter in Paris – a measure that is the result of a socio-historical development over many centuries. Questioning whether it is true or not is nonsensical because it is precisely a measure. Despite their basic ideas often overlapping, Wittgenstein therefore differs from the social constructivists in that he does bring another perspective to the nature of mathematics; one that excludes any possibility of talking about the fallibility of mathematics. Instead he points to language-philosophical conclusions that certainly seem worthwhile in establishing an opposition to many Platonist elements in our thinking about mathematics.

NOTES

1. See The Prime Pages – Prime number research, records and resources for elaboration: (http://primes.utm.edu).

2. Plato's academy existed for no less than 900 years before the Roman emperor Justinian abolished it in 529 AD.

3. The five axioms in a modern version look like this: 1) A straight line segment can be drawn joining any two points. 2) Any straight line segment can be extended indefinitely in a straight line. 3) Given any straight line segment, a circle can be drawn having the segment as radius and one endpoint as center. 4) All right angles are congruent. 5) If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. This postulate is equivalent to what is known as the parallel postulate. (Source: Weisstein, Eric W. "Euclid's Postulates." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/EuclidsPostulates.html).

4. Shanker has called it 'a turning point in the philosophy of mathematics' in his extensive book on the subject (Shanker, 1987).

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WITTGENSTEIN IN SUPPORT OF A SOCIAL AGENDA IN MATHEMATICS EDUCATION. A REACTION TO OLE RAVN CHRISTENSEN

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INTRODUCTION

Wittgenstein's conception of mathematics as constituted exclusively through social-historical development supports any kind of discourse about mathematics and mathematics education in which the "truth" of mathematics is of secondary importance. To Wittgenstein, mathematics appears as a network of different *language-games with family resemblances* thus subordinating discussions about the fallibility or infallibility of mathematical propositions. These games consist in procedures for the manipulation of – language: *meaning of language* is attached to the use we make of this language according to the rules that have been established for the games.

As Christensen argues, this position ('meaning-is-use') has significant consequences for the ways in which researchers in mathematics education may look at their field of research. Attention is diverted from the individual learner: language-games in mathematics appear as interrelational activities of teachers and students that are involved in using – according to certain rules – mathematical signs, terms, drawings, expressions and the like. A sociological gaze is promoted by which these collective activities as well as the underlying rules become open to examination.

A second consequence is related to the never-ending dispute about the relevance and function of practical settings (the 'non-mathematical context') for mathematics curricula. According to Christensen's interpretation, mathematics might be thought of as a network of techniques for representation and measure, thus qualifying practical settings as an essential component of school mathematics. Again, this is asking for a sociological point of view from which the 'mathematics in practical settings' can be analysed – in order to go beyond the common simplistic arguments for applications in mathematics education.

In my reaction to this plenary address, I am going to seize Christensen's suggestions by trying to elaborate, although sketchy, sociological positions, from which 'mathematics in practical settings' (1) and the 'rules of the game' (2) can be investigated.

1 PITFALLS OF THE PRACTICAL SETTINGS

In many countries, school mathematics curricula do not aim exclusively at introducing the students into formal mathematics but embrace, in some cases with top priority, applications and use of mathematics in mundane (mostly economic)

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situations. If, according to Wittgenstein, the meaning of mathematics is attached to the use we make of it, then the mundane situations become incorporated, at least partly, into what counts as 'school mathematics' as a knowledge domain. However, since the relationship between mathematics and the mundane is not as simple as it may appear at first glance, this is a terrain full of pitfalls (Damerow, 2007; Davis & Hersh, 1986; Dowling, 1998; Jablonka & Gellert, 2007; Keitel, Kotzmann & Skovsmose, 1993; Skovmose, 1998).

The mundane is not free from mathematics. Mathematics has penetrated many if not most parts of our lives. By its abstract consideration of number, space, time, pattern, structure mathematics has gained an enormous descriptive, predictive and prescriptive power: state salaries, social benefits, political decisions rely on mathematical extrapolations of data; mathematics-based communication technologies have already changed the habits and styles of private conversations. What has been called 'our time-space-money-system' is based on an underlying mathematical abstraction process that shapes society and exerts considerable influence on our everyday lives. This process, termed *mathematisation*, results in an increasing formalisation of the mundane. In many cases, however, the underlying mathematical abstraction is hardly visible, because it is "crystallised" in all kind of technologies, including social technologies. These technologies function as black boxes: nobody needs to reflect the underlying mathematical abstractions any more. This *implicit* mode of presence of mathematics goes often unnoticed, and so does the mathematics in the mundane.

There is nothing to disqualify the use of practical settings for learning mathematics, as Christensen argues. In fact, without inclusion of practical settings in mathematics curricula, only knowledge of the coherent, neutral, apolitical, "clean and uncontaminated" side of mathematics is distributed to students in mathematics classes. Where the other side of mathematics is a substantial component of mathematical classroom activities, mathematics is used in an essentially different way. In Wittgenstein's terms, students who are introduced to the political side of mathematics take part in a different language-game, with different rules and different meanings.

For making use of mathematics in practical settings, mathematics as a pure technique of representation and measure runs the risk of misrepresentation and overconfidence – as can be observed in many examples of tasks provided by the didactic conception of 'mathematical modelling'. A social conception of mathematics is needed, in which the social is not narrowed to ways of knowledge generation or to the issue of meaning. Mathematics is, of course, a result of socio-historical development, but this development is not restricted to signs and symbols; it is also about intentions and interests. Can Wittgenstein help us out, here?

2 THE RULES OF THE GAME

The learning of mathematics can be understood, according to Wittgenstein, as becoming accustomed to the rules of this particular language-game. By regarding the underlying rules of the game, attention is diverted from individual minds and towards the social complex of the classroom community of students and a mathematics teacher. As Christensen suggests, it might be interesting to delve into the socially relevant issues of equity and access to the rules of the game; of power and distribution of knowledge; of evaluation and assessment.

Within the context of the learning of mathematics, the language game of mathematics is always embedded in the specific institutionalised or non-institutionalised frame in which the learning occurs. In schools, the learning of mathematics is highly institutionalised and follows a rather unique and traditionally perpetuated set of rules. Mathematics classroom activity can be understood as a language-game apart.

Since, in the context of learning, the language-game of mathematics is embedded in the language-game established as teacher-student and student-student interaction in the mathematics classroom, it is subordinated to the principles of the latter. Whoever is involved in learning mathematics in school needs to follow the rules that govern the course of interaction and the production of legitimate text of mathematics classrooms.

This issue has attracted the attention of researchers in mathematics education. A growing number of these adhere to the theoretical framework provided by Bernstein (1996), formulated within the sociology of education, in which the focus is on how macro-sociological structures are engrained in classroom practice and how classroom practice acts as a reproducing device of social structures. One central component of this framework is the analytical distinction of two rules: the recognition rule and the realization rule. The command of these two rules is said to be necessary for legitimate participation in subject matter classroom interaction. Before students are able to produce ('to realize') legitimate text – within the language-game of mathematics education –, they need to recognise the specific classification principles established in institutionalised learning. Apparently, access to the recognition rule is neither evenly distributed to students with different social backgrounds, nor provided by educational practice.

Wittgenstein's concept of the language-game is supportive of making us aware, that students' differential success in mathematics education cannot be explained fully by intra-psychological differences. However, as Wittgenstein is dealing with the development and meaning of *mathematics*, and not primarily with institutionalised forms of learning mathematics, his explanation seems, to me, rather a form of inspiration than of conceptual fundament. For instance, consider his proposition (as cited by Christensen in the context of a rejection of Platonism): "The mathematician is an inventor, not a discoverer." (Wittgenstein, 1978, p. 99) We might ask whether we can substitute 'the mathematician' by 'the student' learning mathematics in school. Is the student an inventor, or is s/he a discoverer? Perhaps it is more sensible to regard the student as a discoverer. The mathematician might actually be inventing and establishing new rules of the language-game. As Platonism is rejected, this can be considered a process of creation. Contrary to the mathematician the student is not creating any new rules. This is particularly important, since the language-game of mathematics is embedded in the language-game that is legitimately exercised in mathematics classes. However, the language-game of mathematics classes is based on rules, which refer to the socio-history of schooling rather than of mathematics. As these rules remain to a large extend implicit, it is left to the students to *discover* them. Due to the established power differences between teachers and students, students' inventive scope is subject to severe restrictions. Wittgenstein modified, the successful student is a discoverer of the rules of the language-game exercised in the mathematics class.

A CONCLUDING REMARK

Wittgenstein's social interpretation of mathematics proves to be significant and inspiring for research in mathematics education, in fact. Owing to the theoretical character of his philosophical position any direct and straightforward 'use' of it for matters of research in mathematics education is, however, hardly feasible. Research in mathematics education that draws on Wittgenstein's conception of mathematics needs to explicate exactly how his position can be made use of. Again, as meaning-is-use, the process of this very explanation is the mechanism by which the meaning of Wittgenstein's position for research in mathematics education is generated.

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SYMPSOSIA

SOCIAL THEORY AND RESEARCH IN MES

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AIMS OF THE SYMPOSIUM

In this symposium we intend to explore the ways in which theoretical frameworks drawn from the field of sociology may and do inform research in mathematics education. We will consider the various ways in which current research by MES participants makes use of theory associated with, for example, the work of Bernstein, Bourdieu or Foucault and will discuss how such frameworks can shape our research questions and methodologies and form a basis for change in mathematics education. The symposium will also consider the importance and influence of the various ideological stances of researchers in identifying research questions, designing research projects and interpreting research data.

RATIONALE FOR RELEVANCE TO MES

The MES community encompasses a variety of theoretical and methodological approaches to research in the area of mathematics education and society. This is perhaps inevitable within a multidisciplinary field such as mathematics education and much is to be gained from the contributions of these different perspectives. There is a danger, however, of 'cherry picking' ideas from theories arising within diverse disciplines such as sociology, psychology or anthropology without following through the implications of the theoretical frameworks within which the ideas were originally situated. While this can lead to useful insights, its explanatory and predictive power tends to be limited and it is unlikely to provide adequate support for action to effect change.

Our interest in proposing this symposium is to develop a fuller understanding of the contribution that sociological theories make to work in mathematics education and to consider what might be the characteristics of a framework with strong potential to support our understanding of mathematics education within its societal and political context. In particular, we aim to:

1. appreciate the influence that social/sociological perspectives have had on the development (and activity) of mathematics education as a field of research;
2. ask questions that we believe are significant in searching for a more principled, sociologically informed framework. These questions include:

   • What issues does a given framework sensitise us to - and what does it not allow us to address?

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- What kind of research questions does it enable us to raise about mathematics education within its social context?
- How does it support us to explore issues in mathematics education systematically?
- Does it provide us with new or alternative insights into important issues in mathematics education?
- Does it provide a basis for formulating proposals for change in mathematics education?

PROPOSED CONDUCT OF THE SYMPOSIUM

There will be two sessions of the symposium. The first session will centre around discussion of the question "Why use sociological frames?" Reading(s) will be provided in advance through the MES website.[1] The second session will take as its starting point current work in mathematics education that makes use of sociological theories. A common theme in both sessions will be a 'case study' of a research problem or practical situation within mathematics education. Participants will work with chosen theoretical constructs to formulate research questions and to design approaches to data collection and analysis.

NOTE

1. The readings have yet to be finally agreed but current suggestions are:

Nash, R.: 2002, 'Numbers and Narratives: further reflections on the sociology of education'. *British Journal of Sociology of Education 23*(3), 397-412.

Nash, R.: 2006, 'Bernstein and the explanation of social disparities in education: a realist critique of the socio-linguistic thesis'. *British Journal of Sociology of Education 27*(5), 539-553.

NO TEACHER CANDIDATE LEFT BEHIND - A STUDY OF THE LARGEST MATHEMATICS ALTERNATIVE CERTIFICATION PROGRAM IN THE UNITED STATES

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Researchers from MetroMath@CUNY are conducting a three-year study of the mathematics component of the largest alternative certification program in the United States. The study is multifaceted, with a classroom-based component that includes videotape, fieldnotes, and interviews from 120 lesson observations and a larger systemic component that includes over 500 in-depth surveys and 30 in-depth interviews of alternative route math teachers in NYC. The research adds to our understandings of mathematics education in low SES urban schools and provides insights on the effects of alternative teacher preparation pathways on urban school systems in the United States.

INTRODUCTION AND RELEVANCE TO MES

The recruitment and retention of qualified mathematics teachers is a pressing issue facing high-poverty urban schools in the United States (Boyd, Lankford, Grossman, Loeb, & Wykoff, 2006; Darling-Hammond, Holtzman, Gatlin, & Vasquez Heilig, 2005; Howard, 2003). Alternative certification (AC) programs increase the pool of teacher candidates by allowing recruits to become teachers of record with minimal pre-service training. AC programs have emerged largely as a response to the lack of traditionally trained teacher candidates willing to work in lower SES urban schools (Haberman, 1991; Ingersoll, 2004).

Despite the salience of this issue, little research looks specifically at the mathematics teacher candidates recruited and trained by alternative certification (AC) programs. An online ERIC search or search of other databases shows less than thirty results for AC mathematics teachers. Of these thirty articles, the majority are not peer-reviewed. The lack of research in this area is due, in part, to the difficulty of the undertaking; considerable researcher collaboration is required to study large-scale AC programs and access to data on teacher backgrounds and student achievement is restricted and difficult to acquire.

Most of what the research community knows about AC mathematics teachers comes from large-scale studies that compare AC teachers to control populations of either traditionally certified (TC) teachers or the general teaching population (Darling-Hammond, 2000; Goldhaber & Brewer, 2000; Lackzo-Kerr & Berliner, 2002). These studies investigate the relationship between indicators of teacher quality (e.g., certification status, subject matter coursework, teaching experience) and student achievement. That is, this research speaks to the comparative effectiveness of AC and TC mathematics teachers as measured by student mathematics (and literacy)

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achievement. Taken as a whole, these studies essentially show that similarly experienced AC and TC teachers of mathematics are equally effective. However, no set of qualitative studies exist to complement this research base. We know very little about the nature of teaching and learning in the lower SES urban classrooms. While AC teachers may be bright and energetic, it may also be that both AC and TC teachers are similarly unprepared or ineffective to teach in lower SES urban schools.

NO NYC MATHEMATICS TEACHER LEFT BEHIND

The New York City Teaching Fellows (NYCTF) program is currently the largest AC program in the U.S. Founded in 2000, NYCTF is a joint program between the New York City and States Departments of Education. The program is designed to provide teachers for "high needs" urban schools in "hard to staff" disciplines such as mathematics. In 2005, the NYCTF program alone provided over 300 new mathematics teachers for NYC schools, over 60% of all new math teachers entering the NYC public school system. The effect of this program is particularly powerful in low SES urban schools that hire most of them. In sum, the differential impact of the NYCTF program on urban math education will be felt for years to come in NYC and other U. S. cities that look at the NYCTF program as worthy of replication.

Like other AC programs, NYCTF is a quick route to teaching. Each summer, over 300 "mathematics Fellows" begin their education in an intensive summer program centered at one of four area colleges. A large portion – over 80% – enter the program having completed minimal college math coursework and are therefore required to attend an additional two weeks of a "mathematics immersion" session. Mathematics immersion reintroduces the Fellows to topics in secondary math and primes them for a math content exam they are required to pass. The six-week session that follows exposes the Fellows to a range of theoretical (e.g., multiculturalism) and practical educational topics (e.g., classroom management). The Fellows also complete 40-80 hours of fieldwork in summer classrooms. In August, successful candidates (most of them) are given transitional licenses and most find jobs teaching in high-needs NYC middle and high schools for the fall. Because they have yet to complete state requirements for full certification, they spend the next two years taking Master's coursework in the evenings while teaching mathematics full time.

MICRO AND MACRO METHODOLOGIES

The MetroMath project is a 3-year study of the mathematics component of the NYCTF program that uses both micro and macro lenses. The micro perspective is comprised primarily of classrooms observations of eight case study mathematics Fellows. These Fellows are currently in their 2nd and 3rd years of teaching and each has been observed fifteen to twenty times over the past year and a half. We currently have over 200 hours of classroom observations where one researcher videotapes and a second writes up detailed field notes of the observed lesson. We also conduct a follow-up interview with the mathematics Fellow to understand the lesson from her or his perspective.

The micro component of our study allows us to examine the Fellows' instruction and their relationships with the urban students they serve. Some research suggests that AC teachers may be less prepared mathematically and pedagogically than traditionally prepared teachers (Johnson, Birekeland, & Peske, 2005). Moreover, given that most Fellows end up in schools that serve low SES urban students of color, an important aspect of the NYCTF program is how it prepares Fellows to teach students in the most racially, culturally, and economically diverse district in the U.S. Other researchers posit that AC teachers appear to lack requisite understandings of the urban youth they teach in order for them to develop positive, or caring, relationships (Ng, 2003; Reyes, 2003). By collecting extensive fieldnote, videotape, and interview data, we address these and other issues.

In order to facilitate comparisons across our eight case study Fellows, we follow Stigler and Hiebert (1999) in constructing "lesson overviews" of each classroom observation. The lesson overview is a structured abstract of each lesson. The lesson overviews follow the "workshop" model of instruction that the NYC Department of Education promotes and attempts to enforce. In theory at least, the workshop model promotes student-centered learning, providing less time for lectures and more time for collaborative student learning than traditional U. S. lessons.

The macro view of the MetroMath study comes from analyses of extensive surveys of two annual cohorts of the mathematics Fellows (approximately 300 per year). The Fellows are surveyed prior to becoming the teacher of record to determine their beliefs and ideas about urban classrooms and then one year later after having been in the classroom. In order to complement the survey, and classroom observational data, we have also conducted individual interviews with over 30 mathematics Fellows. In these in-depth interviews, we ask the Fellows to reflect on their training in the NYCTF program, their beliefs about mathematics instruction, and their understandings of urban students' lived experiences and cultures. In sum, this detailed and large-scale data has allowed us to begin to add to a growing body of research on alternative certification and urban mathematics education.

RESULTS

This presentation will include the demographic and educational backgrounds of the two cohorts and an account of the development of the first cohort over a one-year period in terms of their instruction, their professional identities and their future plans. We also report on variations (or lack thereof) in the nature and structure of mathematics lessons in our case study classrooms and how this relates the typical U. S. lesson as documented by Stigler and Hiebert (1999). Finally, we will present initial results on the mathematics Fellows' emerging identities as teachers of mathematics and of urban youth.

Some Highlights from the Data Analysis

Mathematics Fellows are drawn from a broader geographic range than the majority of new teachers in NY State. Most are not teaching in the communities they grew up in. For approximately two-thirds of the Fellows, teaching math in a low SES school is a first serious career out of college. Even at the outset of the program, approximately one-half of the Fellows state that they plan to leave NYC public schools within five years. Thus, for many, urban mathematics teaching is resumé building for graduate school or more prestigious careers. In addition, we find that their math backgrounds are not as extensive as had been anticipated at the inception of the NYCTF program in 2000; less than 20% of the 2007 cohort majored or minored in college mathematics and a number of Fellows have not taken math since high school. Their attachment to mathematics is often fleeting; over 30% of survey takers indicated that they would have preferred to teach a subject other than mathematics.

While the Master's coursework appears to provide Fellows with some opportunity to develop their mathematics and pedagogical knowledge, it is less clear that they are developing the requisite understandings of urban youth and their communities. Our study indicates that the constraints of working on a Master's while teaching full time, decreases opportunities to develop relationships with the urban youth they teach. In interviews, many discuss the social distance between their urban students and themselves, having generally come from comparatively privileged backgrounds. Some note that urban students, in contrast to the often privileged students they went to school with, need more explicit forms of discipline and motivation in order to effectively engage with mathematics.

The analysis of the Lesson Overviews shows that it is clear that the workshop instructional model, mandated for use by all teachers prior to 2003-04 (Traub, 2003), has been widely implemented. Over 80% of the surveyed 2nd year math Fellows claim to be using the workshop model to teach their lessons – often while disliking it. As we've observed, seven of the eight teachers involved in our case studies also use this model. However, despite the widespread use of the workshop model – which purported to be a "standards-based" and "student-centered" model –our case study classrooms have remained largely teacher-centered. That is, while the workshop model theoretically was designed to promote interactive and creative student learning, in practice the teachers maintain relatively tight control of how students learn and practice school mathematics.

THE SYMPOSIUM

Dr. Brantlinger and Dr. Cooley will jointly present results from the survey data, case studies and lesson overviews and focus on the Fellows preparation for teaching and developing understandings of their urban students' lives and communities. This joint presentation will take about half of the symposium time. Following this, a question and answer period will follow, including feedback and discussion with participants about the effectiveness of AC mathematics programs in urban school systems.

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THE SOCIAL TRANSFORMATION AS OBJECT OF STUDIES AND PRACTICES

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A "nation's narrative" (Hall, 2003, p. 52) represents common experiences that provide sense and consolidate a nation. The members of a community share this narrative, linking their daily lives with a national fate. In Brazilian narrative, the presence of some socio-cultural groups was weaken, as well its importance. An integrative discourse of assimilation of races and knowledge was constructed that helps, too, to dissimulate facts, to obtain/handle a social peace through an adequacy of roles played along the History. However, due to the action of some groups connected to the minorities fighting[1], a nation's counter-narrative has been engendered that allows the perception that, after some centuries, psychological, political and economical Brazilian miscegenation did not occur, only biological one.

The Group of Studies and Researches in Etnomathematics[2] at the University of São Paulo, GEPEm, has produced some researches that contribute for the constitution of this counter-narrative, when we point that there is not recognition and respect to the particularities of learning and ways of being of diverse Brazilian socio-cultural segments. Our production make clear that, in scholarship space, some groups are segregated by means of discourses and attitudes able to hurt their dignity – by comparing historical, aesthetic, familiar and social knowledge and values which take as referees the dominant culture of European matrix imposed to them by curriculum. So, the GEPEm searches not only to contribute to the constitution, establishment and strengthen of discursive practices, but too of no discursive practices that may lead to the acceptance and reinforcement of different ways of being, thinking, educating and do mathematics. In fact, acting together with indigenous people, afro-descendents, young and older people included later in scholar system, together with rural population and the urban marginalized one, GEPEm members turned itself to: a) situations of knowledge production, b) situations of learning and teaching, c) teacher education, d) curriculum, and e) evaluation. In this playing, GEPEm puts problems over power questions, citizenship, tension between universal and particular knowledge, ethnic-racial interactions, among others. Such a wide actuation, at the side of so diverse socio-cultural groups, has demanded the adoption of theoretical contributions coming from various areas – such as education, sociology, anthropology, linguistics, philosophy, history and studies of imaginary. So, GEPEm's members have taken support at some authors/thinkers as Foucault, Hall, Spengler, Durand, D'Ambrosio, Freire and others.

From this situation, GEPEm's production not only elevate, acknowledge and enrich diverse socio-cultural groups as well their knowledge, remarkably the ones that identify with measurements, counting, temporal and spatial localization, among

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others. Our works also try to insert these groups and knowledge within a net of strategic discourses, not letting that may play dispersive roles in a network, but applying to the accentuation of their particularities and adequacy. In this sense, we try to reinforce the self-esteem of marginalized groups, strength cultural roots, give value to daily knowledge, to focus different ways of being, thinking and educating, but, overall, to utilize the teaching of Mathematics to come to the reinforcement of "minorities" strategies to fight for what they consider as important. In order to make known the group work, we prepared a "mosaic" based on works that have been developed by some of its members and that we intend to present at MES 5. Some of them are mentioned below.

Silva (Vanisio L.) questions the relationship between educators and afro-descendents students. He makes a retrospective of Brazilian history under a perspective that may reveal the existence and the origins of the tensions that take place in socio-cultural and fighting relations by conquering power in Brazilian scholar and extra-scholar environments. From this situation, as well as the interest of afro-descendents movements in the sense that the specificities of slaves descendents acquire respect in scholar environment, Silva asks: Which are the feelings, the rationality and the logic present at the acting of the mathematics teacher faced with the necessity of consideration of the afro-descendent culture in scholar environment? His objective is to question present postures and practices and to contribute to its modification in order that they come to enrich the historical and political knowledge and trajectories of Brazilian afro-descendents.

Fantinato (Maria Cecilia C. B.) has pretended to understand the relations between mathematical knowledge constructed by young and mature workers in a low earning community, in their daily life and mathematical scholar knowledge. The research made accentuated the preeminence of socioeconomic aspects in processes of construction/representation/utilization of mathematical knowledge in an urban context, pointing this aspect as a significant factor of identity, transcending cultural factors exclusively.

Santana (Ivanilde C.) directs herself to teachers that act next to young and mature people having as objective to apprehend the methods they use in searching for the development of geometrical thinking of their pupils. Together with this, she analyses the relation between teachers and students along the learning process trying to detect some moments in which are created pedagogical situations that permit to the eleven the applying of his previous knowledge.

Jesus (Claudio L.) noted that etnomathematics present in practices of indigenous professionals education at Xingu – place where live together seventeen Brazilian indigenous nations. In this case, focusing the acquisition and (re)creation of mathematical knowledge – specially of quantitative, special and temporal relations – he suggested the assumption of actions and positions to be adopted by the educators involved with multicultural teaching.

Oliveira (Cristiane C.) investigates the adequacy of Durand's Theory of Imaginary (1996) as a theoretic-methodological proposal for mathematical Education. As Durand argues, myths exist to "explain, know, understand, and treat with social, cultural and imagistic environment ". So understood, myths appear in the primal triangle person-nature-other obeying to the human pulsation of transcendence, as says D'Ambrosio. He also argues that myth situates itself in the matema, inside the etymology of the word itself mathematics. Based in such authors, as well as in approximations between the mitemas and the matemas, Oliveira makes use of the proposal of Myth Critics to throw a new sight to the history of Brazilian mathematical education. In doing this, she exposes original considerations over a character of this history and shows that the scenery of researches in Mathematical Education gets wider with the approach of the studies of imaginary.

Costa (Wanderleya N.G.) also approaches herself of imaginary studies by means of the subsequent question: How mathematical knowlege of indigenous people relate to their myths? She observes that foundational myths develop their senses under the form of knowledge, laws, values, rites, and so on, transferring part of the significance of their pattern inclusively to their mathematical creations. Having these observations as basis, she analyses comparatively the mythical cosmology of the A'uwe-xavante - an indigenous brazilian people - with that of greek people and occidental/Christian ones, bringing to light not only different (ethno)mathematics, but too identities, subjection forms, disciplinary methods, discursive practices as no-discursive ones, among others.

Mesquita (Mônica M. B.) follows her production by recognizing the complex interactions and cultural and multicultural interpenetrations occurring in the learning process. Based on this, and also on empirical data that emphazise spatial notions, she points to the importance of the other in the contemporaneous discourse, as well as the needing for researchers of evincing the links among theory, practices, power of corporification and ethic of identification.

Santana (Diana P. F.) questions the relations between universal and particular knowledge, in order to understand how a specially type of production, thinking and/or mathematical activity, intimately linked to a cultural and social practice, determined and internal in a group, break with the borders of this grouping and comes to be an "universal" patrimony. Freitas (Regina S. A.) takes as object of investigation/reflection her own playing with children of 5th grade of basic school in urban environment. She investigates the possibilities that are generated when the docent develops his classes on the basis of a dialogue with students, trying to construct a wider comprehension about these individuals and their previous knowledge. On her turn, Bezerra (Keli M.) investigates the mathematical teachers representations about the primordial knowledge of the pupil – built up in his daily making/ knowing – searching to identify how this knowledge is conceived, and how it has been taken into account in the mathematics teaching, and, facing it, she discuss the role of mathematical educator in the relationship teaching-learning.

Santos (Eliane Costa) examines the rationality presented in geometrical constructions that appear in Kente tissues (or textiles), a culture of people of Ghana, as well the possible interactions with Etnomathematics. She searches, with this, significant ways for the teaching/learning of afro-descendents at public schools in Salvador, Bahia.

Souza (Régis L.L.) intents to answer to the coming question: Is it possible to understand the implications of Continuously Teacher Education courses over the transformations of docent practices? His aim is to institute a reflexive discussion about these courses, in order to observe its interrelations, intentions and significations as spaces of experiences changing amidst professionals of the education area. To come to this, he puts his grounds on Etnomathematics as a proposition that tries to give voice and place to the different socio-cultural groups on the basis of appreciation of knowledge that teachers and students create in their daily tasks. A similar work is that one of Silva (Paulo S. P.), that has as locus a poor and isolated place in Brazil, with a political trajectory plenty of troubles, where it has gained priority give more power and control to the eleven over his own apprenticeship.

Finally, Domite (Maria do Carmo S.) has actuated at the coordination of teacher education courses for indigenous teachers of five different ethnic groups, as well as at the coordination of GEPEm itself. Further, she undertakes researches about mathematical teacher education. Domite detaches the focus of formative process of teachers as unique social and intellectual individuals to a perspective in which they ally to other individuals, the students, respecting the culture they bring considering their socio-cultural proveniences. In this sense, Domite research tries to become a medium for the generation of a structural change in the ambit of teacher education, denouncing that if pupils are not totally aside of the proposals of teacher education, ation, they are not understood as protagonists in this process. She, then, suggests that the socio-cultural knowledge of eleven be contemplated in teacher education, processes.

NOTES

1. Minorities not in quantitative sense but in terms of power and representation.

2. Abreu, Rodrigo – Bezerra, Keli – Chieus Júnior, Gilberto – Coelho, Sonia – Conrado, Andréia – Costa, Wanderleya – Crevatin Rita de Cássia – D'Ambrosio, Ubiratan – Domingues, Kátia – Domite, Maria do Carmo – Fantinato, Maria Cecília – Ferreira, Rogério – Freitas, Regina – Jesus, Cláudio – Kumayama, Hideo – Martins, Adriano – Mesquita, Mônica – Oliveira, Cristiane – Ribeiro, Esmeralda – Ribeiro, José – Ribeiro, Uilson – Sabba, Cláudia – Santana, Diana – Santos, Beneval – Santos, Eliane – Silva, Paulo – Silva, Vanisio – Souza, Clécio – Souza, Régis.

THE PRODUCTION OF LEGITIMATE TEXT AND THE STRATIFICATION OF ACHIEVEMENT IN MATHEMATICS CLASSROOMS

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AIMS OF THE SYMPOSIUM

In school, the stratification of achievement is based on the students' differential production of what counts as a legitimate text in mathematics classrooms. The symposium will be concerned with a language of description that accounts for the construction, maintenance or mitigation of unequal attainment on the micro-level of classroom practice. The discussion is intended to focus on the interface between the theoretical and the empirical, that is, on an external language of description (an "observation language"), which may help to organise accounts of classroom practice.

RATIONALE

Teachers and students in mathematics classrooms quickly come to know which students perform well in mathematics and which do not. This occurs even in classrooms where selection processes are intended to produce homogeneous classes and in contexts where the students are together for the first time. We regard the disparity in students' achievement as a social construction in the context of the social practices of the mathematics classroom and as unrelated to the students' cognitive dispositions.

From this perspective, our interest is in the discursive and interactional mechanisms that provoke a stratification of achievement within classrooms. Methodologically, we aim at a *constructive description* (Dowling 1998; Dowling in press), construed as an organising of social practices observed within the mathematics classroom. As far as our theoretical position is concerned, we are influenced by the work of Bernstein (1990, 1996) and its *heretical misreading* by Dowling (in press).

It is the former who claims that classroom discourse is structured by implicit rules to which not all students have direct – and equal – access and which constitute disparity in achievement. On the part of the learner, these rules translate into recognition and realisation rules. Command of the recognition rule is important for being able to *locate* classroom discourse, that is, to distinguish the speciality of the context with respect to its domain of practice, its degree of discursive saturation etc. Command of the realisation rule is important for the *production* of legitimate text, that is, how meanings are to be put together (cf. Bernstein, 1996, p. 32).

We take *legitimate text* as comprising two facets: the instructional and the regulative, although this distinction is theoretically not as clear, as it appears prima facie. However, we can take the instructional facet of legitimate text as referring to the

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(school) mathematical part of the discourse, e.g. what is valued as a mathematical argument; and the regulative facet as referring to the order of the social interaction, e.g. which ways of turn-taking or solving tasks (with or without discussing) are accepted. In this view, for a student to produce legitimate text in the mathematics classroom requires an acceptable exteriorisation of mathematical thought in the form of spoken or written language *and* the demonstration of a comportment that is conformable with the social classroom norms. What is acceptable and conformable is acquired within the mathematics classroom. Teachers differ in the ways in which they provide access for the students to the organising principles of the regulative and the instructional discourse so that some practices are of advantage/ disadvantage for distinct groups of students.

When confronting the theoretical constructs that we partially outlined above with accounts of classroom practice (see Knipping et al., MES5 paper), we encountered the following four questions, which seemed of particular interest to us:

(1) Bernstein (1990, 1996) distinguishes *classification* and *framing* as the principles that translate *power relations* between different categories of groups, class, race etc and *control relations* within forms of interaction into the space of classroom practice. Dowling (in press) contends that three of these four concepts were redundant. In our analyses of classroom interaction, we find strong classification connected to strong framing, and weak classification to weak framing. Is there still a potential – in terms of an external language of description – of distinguishing between classification and framing? Are the distinctions between power and control and between classification and framing of use for empirical analyses of classroom practice?

(2) Bernstein claims pedagogic discourse to be the principle, which leads to the embedding of instructional discourse in regulative discourse, "to create one text, to create *one* discourse." (1996, p. 46) Can we find an unambiguous empirical interpretation of the concepts of instructional and regulative discourse or turns this distinction out to be a mystifying artefact? Can we find an empirical interpretation for the *embedding* of discourses?

(3) How do teachers actually introduce students to the production of legitimate text? Are there distinct groups of students who benefit from these introductions? Who *could* benefit if this practice were different?

(4) At which moment in the course of a teaching unit or of a school year, on which occasion, do teachers provide an insight into the criteria along which the stratification of attainment within the mathematics classroom is achieved – if they do at all?

HOW THE SYMPOSIUM WILL BE CONDUCTED

The symposium is planned to occur over two sessions. In the first session, we will present some videotaped scenes from Canadian and German mathematics classrooms. These extracts are chosen from a data corpus, which comprises videotapes from all mathematics lessons of students' first six weeks in secondary/middle/junior high school. During that six weeks, a group of students that is together for the first time develops into a class that is hierarchically differentiated with respect to 'mathematical ability'. We will present our first developments of an external language of description by analysing the selected classroom scenes. Participants of the symposium will be invited to react on and to discuss our exposition.

In the second session, the discussion will be expanded, first, in order to fathom the potential and the limitation of the external language of description. Second, another classroom scene will be presented. The participants of the symposium are invited to conduct analyses of the scene.

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LEARNING MATHEMATICS AND COMPETENCES: BRINGING TOGETHER THREE THEORETICAL PERSPECTIVES

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The symposium aims to bring into analysis and discussion a possibility of articulating and integrating theoretical concepts from Activity Theory, Situated Learning and Critical Mathematics Education in order to contribute to understanding learning in practices where mathematics and technology seem to be relevant.

This symposium emerges from the on-going work of the Project LEARN. This Project is one of the activities of the Technology, Mathematics and Society Learning Research Group of the Centre for Research in Education at the University of Lisbon. The research programme of the Group aims to bring together elements of three theoretical perspectives – Activity Theory, Situated Learning and Critical Mathematics Education – in order to illuminate forms of learning within a wide range of settings and activity having a relevant dimension of technology and/or mathematics.

The Group developed out of the informal meetings of a few researchers concerned with the need to find powerful theories that help us to analyze learning. From a small group of four people discussing five years ago the foundations of situated learning and making sense of the contribution of the idea of learning as participation in communities of practice, we became a larger group of 15 with different background and a variety of experiences aiming to put forward a theoretical framework (drawing on the three domains of theoretical developments referred above) and formulate guidelines and scenarios for learning mathematics and technology.

As oldtimers acting as coordinators of smaller groups and the newcomers being induced into the field of research in education, the Group started growing through a rather difficult challenge of interrogating the theories and provoking the emergence of bridged, links and contradictions that help us to make each next step towards a solid framework – our main goal.

Why choosing those three theoretical approaches? The historicity of each one in the last few years thrown us into a situation that seemed to be inescapable. Activity Theory (specially 3rd generation as coined by Engeström, 2001) had a go for some of us as it seems to be a rather strong framework to address the ways how people acting in the world transform objects into outcomes and, in doing so, learn by transforming themselves. The ways how mediating artefacts are included in the activity and

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contribute to shaping and sustaining the evolving actions of participants is one of the issues to be addressed in the project.

For some of us, the very notion of 'learning as participation in communities of practice' (Lave & Wenger, 1991; Wenger, 1998) is for a number of years at the heart of what learning is about. Recognizing the situated character of learning as an integral part of practice, allows us to look into forms of participation in practices that illuminate the key role of the shared repertoires in communities of practice and the importance of considering identity in the framework.

Because we assume that both mathematics, mathematics education and technology are not neutral and aseptic domains of practice, and because we see education as a political act, we bring into the discussion a social, political and ethical dimension of learning mathematics and we challenge the technological determinism inherent to what Bauchspies, Croissant, and Restivo (2006) call 'technological fix' – the idea that the accumulation of technology will solve social problems. Recognizing the complexity of the actual society, we interrogate the conditions and the phenomena of the social world that constitute western mathematics as it is teach in schools as places shaped to maintain the neo-liberal social system we live in. Critical theory in mathematics education plays a rather important role in our research as a background against which we critically address issues in learning mathematics in various practices and the implications of considering the social world read and written with mathematics (Gutstein, 2006; Skovsmose, 2006).

In the symposium, members of the Group will report on the ongoing work of the projects we are developing and will illustrate some of the ways in which we have been working to bring the various theoretical perspectives to bear together on the research field and the ways how we accommodate our daily practices to research. We will invite participants to join our conversations.

After an introduction to the Group aims we will give an overview of the issues we find more relevant for our purposes in the three theoretical perspectives – Activity Theory, Situated Learning and Critical Mathematics Education. Using our research questions as entry points, we will illustrate one of the ways through which we try to bring together and put in dialogue the various perspectives. This illustration will take the form of a 'conversation' about pupils and adults' learning — and about our own learning as researchers. Participants will be invited to join us in discussing key issues focusing on the themes of learning, activity and critical participation.

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A SYMPOSIUM ON *OPENING THE RESEARCH TEXT: CRITICAL INSIGHTS AND IN(TER)VENTIONS INTO MATHEMATICS EDUCATION*

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Opening the Research Text: Critical Insights and In(ter)ventions into Mathematics Education (de Freitas & Nolan, 2008) is a new mathematics education research text that poses difficult questions about school mathematics, educational research, and power. What makes this research text unique is its approach to interweaving theory and practice by alternating core research chapters with arts-based response chapters, or "insights and in(ter)ventions". Through poetry, collage, fiction or lived experience narratives, the response chapters function as a "lived theory" where the ideas being advocated in the author's research text are lived out through diverse and multiple forms. This symposium is designed to actively and critically engage the audience in an introduction to, and dialogue on, this innovative and provocative edited collection.

AIMS OF SYMPOSIUM

This symposium aims to engage the audience in a critical discussion on a new and provocative book in the field of mathematics education. *Opening the Research Text: Critical Insights and In(ter)ventions into Mathematics Education* is a recently published edited collection in Springer's Mathematics Education Library (2008, volume 46). The book uses multiple lenses to explore the political context of school mathematics, focusing less on the "situated" nature of learning, and more on the power relations that structure learning experiences within dominant educational discourses. *Opening the Research Text* represents an important contribution to the expansion of the research field because of its unremitting interrogation of "common sense" practices. Contributors in this volume were asked to take risks in their writing, to critique and disrupt cherished notions embedded in the field, and to "speak truth to power" by whatever means they deemed necessary. The result is a collection that is sometimes uncomfortable to read; a collection that troubles many taken-for-granted assumptions about mathematics education and research.

The research text is comprised of two types of contributions: core chapters and response chapters. The core chapters in this collection pose difficult questions about school mathematics, educational research, and power. These core chapters tackle the political facets of mathematics education using different theoretical lenses to examine the ways that power relations constitute, and are constituted by, the cultural practices

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of mathematics education. The response chapters in this collection open up the research texts and offer diverse reading practices that are both inventions and interpretations of the core chapters. These “insights and in(ter)ventions” function as linking responses and “what if not” exercises, building on and extending the discussions found in each core chapter. The intent of such an approach—each core chapter being followed by two or three response chapters—is to entice readers into engaging the research text and actively pursuing its implications. Each response is meant to complement the research paper by triggering critical reflection, dialogue and action. The response chapters— the “insights and in(ter)ventions”— are not intended as authoritative applications of the more theoretical research text, but rather as suggestive multi-directional supplements that challenge reader ambivalence and engender dialogue. Response formats in this collection include narrative, fiction, teacher self-study, collage, concept maps, graphic novels, conversations, and poetic renderings, often highlighting the messiness and ambiguity of the research/writing process— a process that is often only acknowledged by its absence in a “finished” text.

The diverse response formats in the collection return again and again to the struggles and challenges of theory-practice transitions in mathematics education research. What research calls for in theory, on the one hand, often seems elusive in practice. What is needed is a “lived theory” where the ideas being advocated in the author’s research text are lived out through diverse and multiple forms. The arts-based response chapters, together with the core chapters, draw attention to the methods by which we might interweave theory and practice in new styles of research-writing.

This symposium, which turns the pages of *Opening the Research Text,* aims to create more openings than closures, and will do so by actively and critically engaging the audience in multiple ways, thereby reaching different people differently. In creating a dialogic text in which a variety of voices and positions are enacted, and in relying on arts-based forms of representation that fly in the face of more traditional forms of research text, the result is a research-based text that breaks the norm in mathematics education.

RELEVANCE OF SYMPOSIUM

The poststructuralist philosophy that informs *Opening the Research Text* demands a vigilant critique of the cultural habits associated with mathematics education - a critique sustained indefinitely because the job of interrogating the status quo is never complete, and readers are not allowed, or at least not encouraged, to settle for easy answers and comforting narratives. The writing in this collection is therefore “sous rature” (under erasure) (Derrida, 1976, p. 3) in the sense that the authors’ claims to research truth or insight are subject to the same inquisitive probing that each has tactically deployed as a means of examining school mathematics. This kind of deconstruction is the hallmark of poststructuralism– building on the strengths of structuralist programs (in this case, traditional semiotics, psychology and Marxist-inspired sociology), but employing these theoretical frameworks reflexively and without reliance on positivist epistemologies.

The provocative contributions to Opening the Research Text reflect current interest in the political and cultural underpinnings of mathematics education. With 22 contributors including both established researchers and newcomers, this innovative research-oriented volume challenges traditional theories and "comforting narratives" of pedagogy through realistic, non-linear scenarios reflecting the ambiguities and power relationships of the classroom. By alternating research chapters with inventive responses (including poetry, concept mapping, graphic novel, and collage), the book present theoretical as well as practice-based possibilities in areas as diverse as arts-based inquiry and social justice pedagogy, all in relation to mathematics education. Consistent with the philosophies and goals of the Mathematics Education and Society conference, this symposium will inspire the audience to:

- Rethink the accessibility and impact of their classroom work.
- Consider the value of poststructuralist strategies to curriculum theory.
- Explore alternate research paradigms in mathematics education.
- Trace the intersections of power, economics and mathematics.
- Critically examine the discourse of school mathematics and policy documents.
- Engage in self-study, writing their own stories of insight and in(ter)vention.

SYMPOSIUM PLAN

The symposium will include presentations by the authors of three core chapters (Brown; Christensen, Stentoft & Valero; Ernest) along with presentations by the response authors for these chapters (de Freitas; Graham; Nolan). To begin the symposium, the editors (de Freitas & Nolan) will provide a brief overview of the book—the content, organization, philosophies, and theoretical underpinnings. Each core chapter author will then give a brief presentation on the nature of her/his research study along with emergent understandings and implications for mathematics education. Following each core chapter presentation, the response author will discuss both the content and form of her "insight and in(ter)vention" into the core chapter research text. The response author will draw on her own classroom practice and/or research to convey how and why the response text was constructed—as a poem, personal experience narrative, concept map, collage, conversation or other arts-based format.

Our intent for this symposium is the same as that for the book— to open up the research text by inviting the audience/reader to speculate on the implications of the research for their own practice and the community at large. In addition, we hope that the symposium and book will inspire scholars in the field (both new and more established ones), who are searching for fresh theoretical perspectives that point to different ways of communicating and interpreting their research, to play with form in/as content and to consider tangible alternative strategies for writing research texts. And, in the process, we are always mindful of the socio-political perspectives that must always be at the heart of the enterprise.

Opening the Research Text asks teachers, researchers and scholars to add to the dialogue that is transforming the mathematics education field. This symposium will also add to that critical dialogue.

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PROJECT DISCUSSION PAPERS

CONCEPTUALISING IMPROVEMENT IN CURRICULUM REFORM: AGAINST CONSENSUS

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Many new curriculum initiatives are predicated on a supposition that we might be able to agree on what constitutes improvement to an earlier regime. This paper contrasts mathematics curriculum reform in England and the USA and how their administrative enactment is understood. The paper argues that the aspirations of reform to achieve consensus are sometimes predicated on time and context specific assumptions of improvement that do not apply across all situations. By using theoretical ideas from contemporary political theory the paper argues that the reform movements might be understood as ideologies that squeeze out alternative perspectives and needs through defining improvement in overly specific ways.

INTRODUCTION

Research in mathematics education is often predicated on identifying deficiencies in current practices as part of a rationale for implementing a new approach. Hargreaves (1996, p. 5) has suggested that educational research must demonstrate "conclusively that if teachers change their practice from x to y there will be a significant and enduring improvement in teaching and learning... ". Hence a history of research might be characterised as a series of papers and books, with many arguing the case for some sort of improvement. Yet looking back at any one time it is not easy to argue how we might assess the nature of the improvement that has been achieved over any given period of time. Meanwhile, teacher biographies are typically characterised by engagements with a number of teaching approaches throughout any one career. Each shift from one to another entails mathematics being framed in a slightly different way that perhaps results in a different teaching style and, perhaps also, a different conception of mathematics. Elements derived from each phase feed into composite experience and contribute to that teacher's mode of practice and emergent, and perhaps convergent, professional identity. These elements might be attributed variously to fashions in school practices, learning theories, assessment preferences, career phase of the individual teacher, etc. The shifts in teaching approach would normally be locally negotiated on the basis of some supposed improvement on the previous model. The term "improvement", however, can be understood in many different ways and resists stability across time, space and circumstances. Nevertheless, many new curriculum initiatives are predicated on a supposition that we might be able to agree on what constitutes improvement to an earlier regime.

This paper contrasts mathematics curriculum reform in England and the US and argues that the aspirations of reform to achieve consensus are sometimes predicated on time and context specific assumptions of improvement that do not apply across all

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situations. Rather teachers and researchers see the reform movement as a broadly agreeable structure with which they can identify and that enables them to join in a collective effort. Here governance is achieved through supposed common sense but where the quest for consensus suppresses the expression of alternative needs.

OBJECTIVES AND PURPOSES

This paper concerns the sort of identifications that teachers might have with successive curriculum initiatives. How might adjustments to practice be understood when a teacher is confronted by new discursive styles being applied to his practices? The paper theorises such identifications through the notion of dialectical materialism in which the world shapes itself around the descriptions made of it. Such themes have implications for how we think about initiatives designed to work at creating consensus in teaching approaches. I question the efficacy of research agenda predicated on encouraging teachers to align themselves with a particular model or philosophy of practice. In particular, I suggest that within any curriculum implementation both teacher self-perception and the curriculum itself are reconstituted such that any supposed convergence to an end-point is disrupted. I offer an alternative conception of change in recognising in the words of Lather (2003, p. 262) that "we move to a future which is unforeseeable from the perspective of what is given or even conceivable within our present conceptual frameworks".

PERSPECTIVES REFORM AS IDEOLOGICAL IMPLEMENTATION

US reform in mathematics is typically defined in relation to NCTM guidelines and are for many teachers seen as the transition from a transmission to a constructivist pedagogical approach, characterised by "genuine mathematical problems for students to solve" (Lloyd, 1999, p. 228) and a focus on "conceptual understanding" (Wilson & Goldenberg, 1998, p. 269). Such reform, however, does not offer a trajectory with universal appeal or applicability across the world or, I assume, even within the USA. The "inquiry" methods associated with constructivist reform, characterised by greater learner and teacher autonomy, would be less acceptable in many Eastern or Pacific cultures where curricula, teacher/student roles and the collective good are defined differently. Further, the alleged autonomy understood within the "reform" agenda conflicts with the reality teachers have come to accept in other Western countries, assessed as they are through legislative documentation and recognised through the filter of their compliance with this. In the UK, for example, student centred pedagogies emphasising problem-solving, investigations and project work dominated curriculum reform agendas some thirty years ago, but a more recent backlash resulted in prescribed curricula for both teachers and pupils in which student centred approaches have become tightly structured. Thus conceptions of improvement are very much a function of the country, or even local community, in which they apply and the state of affairs prevalent there. And it is this sense of contingency that underpins this present paper's focus on adjustments to new paradigms. I also draw on another study in which I provided an account of how trainee and new teachers in the

UK begin to include official curriculum descriptors into accounts of their own practices as they move through the accreditation process. The conception of identity introduced here, however, does not necessarily favour compliance with the dominant group. Conceptions of self are governed by a tussle between achieving personal aspirations and meeting external demands. The task of socialisation entails the teacher gradually introducing social/official language into her self-descriptions. She becomes increasingly implicated in official accounts of her practice as she begins to recognise herself in such accounts and to describe herself in those terms. And in so doing she loses aspects of her earlier, perhaps more personal, conception of self. Teachers saw this as necessary from the point of view of their accreditation as teachers yet found the discourse highly prescriptive, albeit a form of prescription that released them from the need to make so many content decisions in a curriculum area where often they had in the past lacked confidence in their own capabilities. The research perspective offered in this paper similarly attempts to weigh up the relative advantages of achieving personal aspirations or fulfilling external demands, rather than supposing external demand is to be favoured. Thus my attempt in this paper is to resist describing curriculum development from the point of view of how teachers align themselves or not with an overarching rationale or model such as reform. Alignment by a teacher with a new curriculum is not in itself necessarily to be viewed as success, since improvements are a function of the ideological stance being assumed. Both teacher and curriculum change through any curriculum initiative, as do the parameters through which those changes are understood.

THEORETICAL FRAMEWORK

By using theoretical ideas from some neo-Marxist writers the paper shows how the reform movements might be understood as ideologies that squeeze out alternative perspectives through defining improvement in specific ways. Mouffe (2005) has strenuously resisted the idea of human progress as being shaped by ideals relevant across all communities. Mathematics education, for example, would be seen as culturally dependent with each cultural perspective predicating an alternative conception of mathematics. Laclau (2005) has rejected the notion of the “people” as a collective actor, and by extension the possibility of a research “community” or a set of governments being able to define a common interest with regard to the purposes of school mathematics. Instead he has examined the nature and logics of the formation of collective identities and suggested that such collectives be seen as being held together through identifications with specific populist demands. Althusser (1971) focuses on how the individual understands herself through ideological filters. That is, the individual recognises herself in some discourses but not others. For example, an individual American teacher may truly believe that she is subscribing to reform agenda and following such approaches in her practice, whether or not others see it this way. But, there is always a gap in this identification, a distance between the person and the story in which she sees herself. This gap stays there. Althusser is not persuaded by consensual aspirations where difficulties are ironed out. And surely some American teachers are sceptical about reform projecting them to the top of international league tables or even that everyone will agree with the content of that ambition. Time does not necessarily make alternatives more attractive or comprehensible. Althusser sees the supposition that you could get to a consensual ideal beyond conflicting ideologies as the biggest ideology of all. Finally, Rancière (2004) examines how particular ways of understanding life, and the cultural forms that prevail, are functions of time- and culture-specific conditions of possibility. Here we are alerted to the possibility that successive cultural forms derive their meaning from earlier cultural configurations rather than from any supposed underlying truth of practice. The individual's immersion in successive ideologies of practice might be understood as a task of crafting the various ideologies together into a functional whole in some more or less personal way, rather than being immersed in one distinct ideology rather than in another. Rancière (2004, p. 50) argues:

> The visibility of a form of expression as an artistic form depends on a historically constituted regime of perception and intelligibility. This does not mean that it becomes invisible with the emergence of a new regime. … At a given point in time, several regimes coexist and intermingle in the works themselves.

Similarly, mathematics teaching schemes become a function of the history into which they are being inserted, but a history different for each individual according to how the individual has accessed this history through a variety of alternative cultural forms. Alternative discursive forms are alternative forms of life and cannot readily be compared side by side. The parameters through which these discursive forms are understood are time and experience dependent. And such parameters derive from successive manifestations of the ideological filters that govern teachers' and researchers' participation in life.

DATA SOURCES AND EVIDENCE

This paper is primarily theoretical/discursive. It draws however on an empirical study, which itself was an offshoot of a major Gatsby funded initiative based on trialling Mathematics in Context materials, based on a philosophy of Realistic Mathematics Education, in British schools. The empirical study was a smaller scale pilot for a project, funded in the following year by the UK Economic and Social Research Council and was designed to track the shifting perceptions of the teachers involved in the larger study. In particular, the pilot study asked how the teachers accommodated their exposure to a new paradigm within their existing conceptions of practice. It tracked the teachers through their first year of participation and sought to document changes to the ways in which they accounted for their practice with reference to old and new paradigms.

CONCLUSIONS

If we accept the analogy of approaches to mathematics teaching with what Rancière terms artistic forms, we also take on the parallel notion of mathematics, and more specifically mathematics curricula, being articulated through cultural regimes, or particular systems of conditions of possibility. Laclau and Mouffe progress this position suggesting that human identity might be better understood as an amalgam of partial identifications with co-existing ideologies. Who I am, or my teacher identity, is a function of how I draw on elements from the alternative discourses in which I am immersed. But links between immediate tools and broader conceptions may be transitory or unreliable. Consequently, I suggest that what might be seen by governments and researchers as the long march to improve standards through major holistic change, might better be understood as a succession of ideological changes which resist a unified conception of what improvement might be. My argument is that mathematics education research and development, should seek to recognise difference in teachers' understandings, experiences and context of action and assist them in making informed professional judgements about how their practice might be developed *in situ*, rather than supposing that external evaluative judgements should be based on movement to a consensually preferred conception of teaching.

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COGNITIVE ACADEMIC LANGUAGE PROFICIENCY IN PRIMARY MATHEMATICS CLASSROOMS

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THEORETICAL CONSIDERATIONS: CALP, REGISTER, AND CODE

Language factors play a double role in the learning of mathematics at school. First, the subject, school mathematics, is developed mostly by means of spoken language. Second, students are introduced, although often implicitly, to the linguistic features of the language in which mathematics is taught. The command of a specific kind of language proficiency is a condition for becoming a prosperous student in the mathematics classroom. As mathematics education is increasingly emphasizing argumentation and reasoning, this issue is of vital importance. However, a precise and detailed description of this specific kind of language proficiency is still lacking.

It would be too simple an answer to take the linguistic demands that students face in mathematics education as a conglomerate of 'good German', or 'good English', and mathematical terminology. Students need to develop a complex linguistic competency in order to participate successfully in the mathematics class. On one hand, this competency is bound to the particular situation in which it is developed, that is, the particular interaction of a teacher and the students in the mathematics classroom. On the other hand, this interaction is linguistically orientated at a model of an exalted form of speech, which, according to Jim Cummins (e.g. 2000), is characterized by its tendency to be lexically and grammatically similar to writing, and which Cummins, referring to the underlying mental processes, has termed cognitive academic language proficiency (CALP):

> Oral classroom discussions do not involve reading and writing directly, but they do reflect the degree of students' access to and command of literate or academic register of language. This is why CALP can be defined as expertise in understanding and using literacy-related aspects of language. (Cummins 2000, p. 70)

In this perspective, which has been developed in the frame of bilingual education, being educated, and being mathematically educated as part of it, is first of all a matter of language proficiency. It is highly important for access to and success in higher educational institutions to show this particular proficiency.

In mathematics education, a subject-specific occurrence of CALP can be observed. In schools, mathematics-specific CALP is recurring on the particular vocabulary made of school mathematical concepts, and the respective symbolism, as well as on linguistic devices that aim at rendering a text coherent. By mentioning the notions of literacy and academic *register*, Cummins emphasizes that the concept of CALP is not restricted to syntactic features of language but includes semantic and structural aspects that refer to the social and institutional particularities of classrooms and

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schools – a perspective which has been developed extensively by linguist Michael Halliday (e.g., 1978).

It is well within the scope of this characterisation to take CALP, and particularly the mathematics-specific variation of it, as an expression of structural relationships between elements such as position, power, status, and control. A consideration of these elements, and of the relationships between them, draws the attention to a *socio*linguistic, rather than linguistic, theorisation of academic language proficiency within classrooms. Such a focus would be typical for sociologists of education. The work of Basil Bernstein (1990, 1996) on *codes* is well appreciated in this respect. For Bernstein, code is not a metaphor but a "regulative principle, which selects and integrates relevant meanings, the forms of their realizations and their evoking contexts" (1990, p. 101). Bernstein's code theory "draws attention to the relations between macro power relations and micro practices of transmission, acquisition and evaluation and the positioning and oppositioning to which these practices give rise" (1990, pp. 118-119). In this view, the practice of mathematics education in school is regulated by implicit rules. Although mastery of these rules is a precondition for success, not all of the students are familiar with them.

According to Bernstein, relations of power and control correspond to the codification of what, and what not, pertains to a particular discourse, and how this discourse is produced and legitimated by classroom talk. Mathematics as a school subject is characterized by internal boundaries between mathematical domains, such as geometry, arithmetic, chance and data. In each domain, specific forms of discourse are appropriate, with respect not only to terminology and symbolism but also to typical forms of concept formation, abstraction and generalisation. Externally, school mathematics is thematically insulated (1) from other school subjects as well as (2) from everyday knowledge and out-of-school experience. There is empirical evidence that this second insulation is particularly difficult to capture for children starting school. However, not only for first graders, it is extremely important to recognise and internalise what counts as relevant contribution to classroom talk and how out-of-school experience can be brought into the classroom. Those, who do not become familiar with these insulations, who (in Bernstein's terms) do not command the *recognition rule* of classroom discourse, cannot contribute substantially to classroom talk and are threatened with failure.

INVESTIGATING SCHOOL STARTERS' CALP

Since subject-specific academic language proficiency is regarded as of crucial importance for participation in mathematics classes *from the beginning of schooling on*, a detailed description of its linguistic characteristics would be useful for supporting students' acquisition of CALP. However, theoretical and empirical work on CALP has mainly been done in the area of first and second language acquisition. Few studies have drawn on linguistic elements of mathematical language proficiency, such as the use of 'equal' (Warren 2006) or relational complexity between 'more' and ‘bigger’ (Halford 1993), but a comprehensive categorisation of linguistic elements is still lacking. Yet the setting up of such a catalogue of linguistic means is a problematic endeavour, because of tensions between theory-inspired collections of criteria and rather heterogeneous pedagogic practices in mathematics education.

Nevertheless, without exposure of CALP elements, mathematics education remains a socio-linguistically invisible practice, and discriminates all those with low CALP. Hence, I am currently approximating this issue by means of a descriptive-analytic categorisation of school starters’ mathematical academic language proficiency. The strategy is to engage school starters in a narration that is explicitly drawing on their mathematical linguistic resources. On the grounds of these narrations, the linguistic variety in terms of syntactic and semantic elements and of text structure is analysed, aiming at a multidimensional categorisation of school starters’ mathematical academic language proficiency.

First analyses show, that the school starters’ (audio-taped and transcribed) narrations differ substantially in length, focus, and language use. Not unexpectedly, it is viable to identify a variety of CALP elements. However, the most striking result is, that students’ differential production of school mathematical speech is apparently dominated by their differential command of the *recognition rule*. Bernstein’s theoretical claim is empirically manifest. This is particularly evident in the case of some school starters who, over a period of two weeks, have repeatedly responded to the prompt to narrate a mathematical story. The same six-parts picture story has been presented to them four times, showing a girl and a boy constructing towers with building bricks. The prompt has been given in identical word order: *Please, look at the six pictures and tell me a detailed mathematical story of what is happening there!* No feedback has been given to the children.

However, some children “improved” their mathematical narration. Apparently, these children developed a keen sense of the challenging situation and of the kind of production of speech that is favoured – without having received any response to their first narrations. This observation is overtly supported by the case of one seven-year-old girl that, after telling the story for the third time, approaches the interviewer:

Girl:	But, actually, I didn’t really tell a *mathematical* [emphasised] story.
Interviewer:	You want to tell the story once more?
Girl:	Um, yeah, … [starts telling the story]

This girl did not only develop a sense of what the interviewer tries to achieve. In her fourth version of the story she uses more, and more frequently, elements that meet the demands of mathematics-specific academic language proficiency in school. Hence, by improving her command of the recognition rule, she was able to demonstrate a higher level of proficiency with respect to mathematical academic language.

Thus, the command of the recognition rule and the quality of mathematical academic language production are closely related. From a socio-linguistic perspective, command of the recognition rule can be interpreted as *a fundamental part* of CALP. If this is a sound interpretation, then two consequences can be sketched:

- Any attempt to investigate students' subject-specific CALP will face the problem of the differential command of the recognition rule blurring or generating the differences that a linguistic analysis of students' speech may detail.
- Students' acquisition of subject-specific CALP will be significantly enhanced, when not only the linguistic elements are considered a matter of instruction, but also the socio-linguistic codification of discourse.

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DEVELOPING A COMPLEX MATHEMATICAL LEARNING COMMUNITY: (RE)CONSIDERATIONS OF LEARNING/TEACHING EXPERIENCES

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INTRODUCTION

In my experiences as a student and as a high school teacher of mathematics I understood mathematics as infallible knowledge (an object) that was best transferred from teacher to student through lecture, practice, memorization, and regurgitation – a learning/teaching process that could be verified by testing. I learned mathematics using such positivistic ('traditional') perspectives and practices and I thought I had to teach mathematics in this very way because "one's teaching methods tend to reflect one's own history as a student" (Davis, Sumara & Luce-Kaplar, 2000, p. 94). However, through graduate courses and teaching experiences as a Masters student at the University of Regina, I am developing situated social constructivist perspectives and practices for learning/teaching mathematics. In considering a self-study research methodology it is necessary to explore and publicize teaching perspectives and practices with the goal of reframing beliefs and/or practice through collaboration (Samaras & Freese, 2006). I will share my recent experiences in focussing on counting emergent mathematics, while working with preservice middle years teachers, and my (re)considerations of learning/teaching as a result of this focus. I invite your feedback and hope you join me in this session by contributing personal teaching stories about challenges in developing mathematical learning communities. Through sharing and discussing such stories I hope we can collaboratively explore how teaching/learning communities might be developed.

COUNTING EMERGENT MATHEMATICS

I was first drawn to the idea of counting emergent mathematics while reading "Who Counts What as Math? Emergent and Assigned Mathematics Problems in a Project-based Classroom" by Stevens (2000). In this chapter, Stevens describes his observations and experiences as a researcher within a seventh grade project-based mathematics classroom. He warns of "the institutional invisibility of emergent mathematics" (p. 134) by suggesting that when students work in small groups, most of the mathematical questions/ideas that emerge stay within the confines of the group. As a result, teachable moments related to such situated questions/ideas are often missed by the teacher. Questions/ideas remain ignored, hidden, or forgotten - they never make it from the group to the public arena for whole-class discussion because

> only infrequently are teachers at the right time and the right place to participate pedagogically in emergent problems. In light of this, students themselves should have a

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greater role in identifying and circulating those problems that emerge in projects as prospectively mathematically relevant. (Stevens, 2000, p. 137)

Stevens recommends that students be encouraged to share emergent mathematics questions/ideas with the whole class and that teachers need organizational and assessment devices for keeping emergent mathematics in the foreground. He argues that the students he observed already had "a relatively well developed notion of what counted as mathematics around traditional forms and social functions – as what appeared on tests, on work sheets, in textbooks, or in standard mathematical orthography" (p. 135). Therefore, students did not bother to publicize emergent mathematical questions/ideas that might lead to deeper understandings/interpretations because they did not realize their value.

As a high school mathematics teacher it was similarly my experience that only the mathematics I graded was perceived by my students as valuable so Stevens' recommendations resonated within me. I began to (re)consider how learning/teaching experiences within my classroom might improve by bringing emergent mathematics to the foreground and making it worthy/count. My desire to focus on emergent mathematics continued to strengthen in the fall semester of 2006 when I volunteered as a teaching assistant in an experimental mathematics course (EMTH 290AA) designed for teaching preservice middle years teachers in non-authoritarian ways. Students were provided with mathematical experiences which differed from most of their previous school experiences. They participated in collaborative and independent activities including a problem-based geometry construction project and an investigative statistics computer project (which I co-authored with Dr. Kathleen Nolan). While observing and interacting with students and reflecting on their journal entries, I learned that many students were frustrated during these experiences, partially because they had unresolved difficulties with mathematical concepts and connections they would be responsible for teaching. I perceived that project related activities stimulated small group conversations and provided opportunities for students' mathematical (mis)interpretations and (mis)understandings to surface. However, rarely did students' questions/concerns about mathematical understandings and interpretations ever proceed to the public domain for whole class discussion. Davis and Simmt (2003) suggest "concepts and understandings must be made to stumble across one another. Without these neighboring interactions, the mathematics classroom cannot become a mathematics community" (p. 12). These words resonated within me too since sometimes I felt *the blind were leading the blind* during EMTH 290AA small group activities; group members did not seem capable of expanding and enriching emergent mathematical ideas for their peers and I could not be with groups simultaneously to facilitate/guide/coach deeper discussions of mathematical ideas. Through this experience I also began to (re)consider how students and teachers, as a whole/community, might enhance learning.

(RE)CONSIDERATIONS OF LEARNING/TEACHING EXPERIENCES

By June 2007, through completion of thesis and ethics proposals, I expressed my (re)considerations. As a sessional lecturer for EMTH 290AA in the 2007 fall semester, I intended to focus on students' emergent mathematical *thoughts* (questions, concerns, ideas, understandings, interpretations, and connections) that arose within the classroom community as a result of statistics and geometry project-based mathematical experiences. I suggested I would encourage students to bring private (individual or small group) mathematical thoughts into the public domain by writing their thoughts on displayed chart paper, as per Steven's recommendation. I hoped seeing a peer generated list of mathematical thoughts would compel students to contribute to the list. I wanted students to then select from a variety of ways to participate in the development of mathematical interpretations and understandings by utilizing personal strengths/abilities in researching and presenting mathematical thoughts they perceived interesting. For example, I proposed students might inquire into the history of a mathematical concept, or depict (draw, sculpt, and etcetera) a concept or connections between concepts, or develop an activity to show how manipulatives can be used to enhance understanding of a concept, or find and explain useful concept related computer activities. I suggested students might even build on initial presentations so as to expand and deepen understandings/interpretations of mathematical concepts and connections – possibly critiquing presentations and providing alternative perspectives (histories, depictions, activities, and computer demonstrations).

In August 2007, after granted approval to collect data for my thesis through teaching EMTH 290AA, it was as though I had forgotten the breadth of my proposed ideas. Rather than (re)reading my thesis and ethics proposals, and fully pursuing my intentions, I looked at the past syllabus and prepared to teach EMTH 290AA as I previously observed while volunteering. I would begin with the statistics project and develop/adapt some proscriptive activities/assignments to introduce/review statistical concepts necessary for successful completion of the project. I would mark the assignments, project, and related journal entries[1] - then I would follow the same learning/teaching process for the geometry project. I neglected to consider how students might participate in the various ways described above and how I might count such participation but I did remember to focus on emergent mathematics.

In the first class I explained to students that my research entailed logging/writing, on displayed chart paper, *their* mathematical thoughts that would likely emerge through participation in class activities/assignments. I hung the paper on the wall. As the first month passed it became obvious, from lack of logging, that students had difficulty identifying emergent mathematical thoughts. One student even wrote in her journal that she was unable to recognize relevant thoughts and I agreed with her – for the time being. Sometimes, during group presentations, *I* noticed students struggle to clearly express understandings and interpretations of mathematical concepts so *I* was able to identify and write a few thoughts on the chart. For instance, while returning to her desk from a group presentation one student remarked, “I think the mode is just a different type of average.” In response to hearing her I wrote on the chart ‘What is an average? Is the mode just a different type of average?’ In focussing on emergent mathematical thoughts I was learning to better listen to students and to identify situated mathematical thoughts. However, students were not (re)visiting the displayed thoughts; exploration of thoughts was unnecessary for it wasn’t connected to grading.

Inadvertently an emergent mathematical problem, that could not be ignored, arose from students having to collect data and find measures of central tendency for their project. Many students could not use their data for the set task because it was qualitative, nominal, or prematurely grouped. Questions had to be revised and data recollected for successful completion of the project. It occurred to me that I could purposely plan such difficulties in future projects so students would have to investigate particular problems. For part of the geometry project I decided to have students complete a 3-dimensional model of a geometrically architectural building using only paper and adhesives. I surmised the difficulty of making curved surfaces from flat paper might link some students’ projects to calculus and the idea of limits.

While sharing my insights with a colleague it was suggested that preservice teachers, in evolving from student to teacher, might also benefit from perusing the middle years curriculum, preparing lessons/activities, and teaching geometry project related concepts to class peers. This idea coincided with my understanding that students’ presentations provided me opportunities to identify mathematical thoughts. Thus, I planned for students to teach the remaining classes and in the introductory class for the geometry project students were divided into grade level groups to search the curriculum and present major geometry and measurement concepts. This exercise seemed successful as I was able to easily identify more thoughts during this class than in the past. That night I decided it might be interesting to see if students would (re)visit thoughts by counting related discussions. I posted the identified thoughts on WebCT (an electronic university discussion forum only accessible to EMTH 290AA students) and asked students to participate weekly, for the few remaining weeks, in at least one discussion a week for an unconditional 10%. To my surprise, not only did some students begin to investigate questions and share ideas, they began to ask their own questions. Consequently, I offered [on Tuesday November 6] students the choice to opt out of the geometry project and into a WebCT project.

I will share this project and student responses with you and I invite you to bring, for discussion, stories of other ways you have tried to build mathematical communities within your classrooms.

NOTE

1. Journal entries involved reflections on: class activities; participation of peers or self; and mathematical concepts.

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STUDENTS' DISPOSITION FOR DE-CONTEXTUALISED AND ALGEBRAIC (SYMBOL-BASED) REASONING IN RELATION TO THEIR SOCIO-ECONOMIC AND CULTURAL BACKGROUND IN MOZAMBIQUE

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BACKGROUND

Algebra is an important topic because it is relevant to learn topics like functions, trigonometry, geometry, analysis and calculus. In Mozambique, mathematics is compulsory until 10^{th} grade and optional from grade 11 to 12, from where the learners who intend to follow linguistics, literature, law, medicine, history and geography have to choose option A (which does not have mathematics) and others who intend to follow technical studies like architecture, engineering, geology, agriculture, farming, statistics or pure mathematics have to choose option B or C which include mathematics. Available data on students´ enrolment show that the majority of learners prefer the option without mathematics. However, algebra is taught from 8^{th} grade on that means for learners who are at least 12-13 years old. From 8^{th} to 10^{th} grade algebra consists of: the concept of equation, linear equations and inequalities with one unknown, word problems conducing to linear equations and systems of linear equations, inequalities with two unknowns, quadratic equations, exponential equations, logarithmic equations, and trigonometric equations. In upper secondary school algebra aims at preparing learners to achieve in set theory and formal logic, analytic geometry, statistics and probability, advanced trigonometry, calculus and combinatorial analysis.

As stated by Usiskin (1997), algebra as a language can be characterized by the following five major aspects: (i) unknowns, (ii) formulas, (iii) generalised patterns, (iv) placeholders, and (v) relationships. Wheeler (1996) describes algebra as a symbolic system (to describe patterns and relationships without the need for the use of ordinary language); a calculus (among its primary elementary uses is the computation of numerical solutions to problems) and a representational system (tables, graphs from where can be extracted and interpreted the needed or presented information what usually plays a large role in the mathematisation of situations and experiences).

MacGregor & Stacey (1993), analysing test items responses, present a brief overview of students' developing competence in four essential basic algebraic skills: (i) recognising what operation relates two quantities; (ii) using algebraic notation to write an expression; (iii) interpreting an equation and (iv) writing an equation. MacGregor & Stacey suggest several origins of difficulties in learning to use algebraic notation, including: (1) intuitive assumptions and sensible, pragmatic

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reasoning about an unfamiliar notation system; (2) analogies with symbol systems used in everyday life, in other parts of mathematics or in other schools subjects; (3) Interference from new learning in mathematics and (4) poorly-designed and misleading teaching materials.

In fact, in the literature (Küchemann, 1981 and MacGregor & Stacey, 1997) it can be found that when learning algebra:

(i) Students frequently base interpretations of letters and algebraic expressions on intuition and guessing, on analogies with other symbol systems they know (for example associating *m* to metro, *l* to litre, *k* to kilo).

(ii) Students' misinterpretations lead to difficulties in making sense of algebra and may persist for several years if not recognised and corrected;

(iii) At all year-levels there were some students who seemed to be unable to deal with precise distinctions between letters and their referents as it is necessary for a proper understanding of algebra;

(iv) When algebraic concepts and methods are not used in other parts of the mathematics curriculum students forget them and the notation for expressing them.

Meaney (2002) states:

> Within every mathematics classroom there is an intersection between the culture which surrounds mathematics and the way that it is taught and the culture which forms students' backgrounds. When there are large differences between what is valued in these cultures, this intersection resembles a clash rather than a successful symbiosis. (p. 167)

The second part of Meaney's statement corroborates with learning theories such as that of 'cognitive apprenticeship' which emphasised the need for mathematical activity to begin by being 'embedded in a familiar activity' (Brown et al., 1989, p. 37 cited by Meaney, 2002, p. 177). This gains particular importance when taking into account that Mozambique is a multilingual and multicultural society where the language of instruction is Portuguese but the mother tongue for only about 6% of the population, which means that the majority of students in their everyday communication utilise Mozambican languages.

AIM OF THE STUDY

The purpose of the study is to explore students' disposition for de-contextualised and symbol-based reasoning in relation to their socio-economic and cultural background in the specific context of Mozambique. It aims to find ways to help secondary school pupils´ to understand algebraic concepts and look for ways, relevant for students, to introduce these concepts. To this end, the following questions are proposed to guide the study:

> (i) What difficulties do students experience in translating problems from everyday language into algebraic language?

(ii) What are the constraints and affordances in interpreting algebraic problems or problems asking for logical reasoning in relation to students' social and cultural background? In particular, are the problems linked to their mother tongue, to their socio-economic background or to both?

CONCEPTUAL FRAMEWORK

Although Mozambique is a multilingual society the medium of instruction is Portuguese. It is important to identify an adequate conceptual framework, especially, because the purpose of the study is to explore students' disposition for de-contextualised and symbol-based reasoning in relation to their socio-economic and cultural background. A classroom can be viewed as a social context in which mathematical knowledge is negotiated and constructed (Bauersfeld, 1992; Cobb, 1986 quoted by Atweh et al., 1998, p. 63). It is in the classroom where the teachers and students interact and share perceptions in specific circumstances creating a socio-cultural context. Assuming the social nature of mathematical knowledge construction in classrooms, the language tool is required. Halliday (1985) states that:

> [L]anguage also varies according to the function it is being made to serve; what people are actually doing, in the course of which there is talking or writing involved; who the people that are taking part in whatever is going on (in what statuses and roles they are appearing); and what exactly the language is achieving, or being used to achieve, in the process. These three variables (what is going on; who is taking part; and what role the language is playing) are referred to as FIELD, TENOR and MODE; and they collectively determine the functional variety, or register, of language that is being used. (p. 44)

Holliday's model is appropriate for the purpose of this study because it allows identifying the type of discourse used. The study will try to interpret the data from the perspective of these levels.

MATERIALS AND METHODS

Research Design. The main instruments to be used in this study are written tests for the research question (i) and semi-structured interviews for question (ii). The written tests will comprise a set of questions (such as the example given below) covering problem solving skills using Usiskin's (1997) characterisation of algebra considered above. This includes the three levels (1) understanding of ordinary language, (2) understanding of the presence of algebra in a given word problem and (3) the ability to translate an everyday problem into algebraic language.

> A Linguistic conducted a survey of 100 families to determine the popularity of three local languages: E-Makhuwa[1], Chi-Chone[2], and Chi-Changana[3]. The results were as follows: 42 families spoke E-Makhuwa, 48 families spoke Chi-Chone, 41 families spoke Chi-Changana, 15 families spoke both E-Makhuwa and Chi-Chone, 17 families spoke both Chi-Chone and Chi-Changana, 18 families spoke both E-Makhuwa and Chi-Changana, and 10 families spoke the three languages. Determine the number of families who spoke none of the three. (Adapted from Hanna S., & Saber J. 1971)

This is a problem of set theory and formal logic but it requires also algebraic knowledge: unknown and relationships among variables. This question allows exploring the levels considered above via written text and semi-structured interviews to understand students' constraints or affordances in interpreting it.

Sampling. The target population for the study will consist of 8th to 10th grades lower secondary school pupils in Mozambique. The schools involved in the study will be randomly selected taking into account geographic and socio-economic criteria. Within the schools, the classes to be involved in the study will also be randomly selected and it will comprises about 36 classes of about 40 to 50 pupils each, from 3 provinces representing the main regions of the country (south, centre and north) and the different socio-economic origins. In these provinces/regions, the referred classes will be selected in schools from an urban area (2 public and 1 private schools) and a rural area (1 public school). In each school the sample for the written test will involve students from 2 classes of each grade, comprising a total of about 1800.

Instruments. The study will use a combination of qualitative and quantitative data-gathering instruments assuming that, as stated by authors such as Lawrenz & McCreath (1988) and Miles & Huberman (1994), (i) the combination can enable a confirmation or corroboration of each other via triangulation; (ii) it makes possible an elaboration or development of analysis, providing richer detail, that is, the results of the first method can inform the second's sampling; (iii) quantitative methods can ´persuade´ the reader trough de-emphasizing individual judgment and stressing results that can be generalized while the qualitative research methods persuades trough rich strategic comparisons across subjects, thereby overcoming abstraction inherent in quantitative studies; (iv) during analysis, quantitative data can help, by showing the generality of specific observations, and verifying or casting new light on qualitative findings. And the qualitative, on the other side, data can help validating, interpreting, clarifying and illustrating quantitative findings. According to Schumacher & McMillan (1993), tests, when used alone, have certain disadvantages and since with such instruments there is no possibility of asking subject clarifying questions immediately after a particular opinion has been given and also are static and give no information about the stability and dynamics of the subjects. Thus, to compensate for these disadvantages, a semi-structured interview will be developed by the researcher in order to identify and further explore the roots of the specific errors and misunderstandings in the use of algebraic language or its absence. Thus, after collecting data from the written tests two pupils (the best and the worst) in each tested classroom will be interviewed. With the test, a short questionnaire will be given to the students which will help to identify the socio-economic background and the language used in the everyday context of the participants.

Data Analysis. A descriptive statistics (frequency/percentage distribution of score data will be presented and discussed the percentage of correct responses that students have given to all questions in the problem areas; measures of central tendency (means) and measures of relationship (correlation) and inferential statistics (testing for mean differences in knowledge scores through correlated t-test) will be used. The students' explanations to the tests results will be explored in a qualitative perspective, using Halliday's framework as a language of description. The interviews will also provide additional information about the students' socio-economic background and language used in everyday context.

NOTES

1. One of the African languages spoken in Mozambique.

2. One of the African languages spoken in Mozambique.

3. One of the African languages spoken in Mozambique.

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THE INTERPRETATIONAL SPACE IN THE CURRICULUM: INTENTIONS AND INTERPRETATIONS OF MATHEMATICAL REASONING IN THE NEW CURRICULUM FOR SECONDARY SCHOOLS IN MOZAMBIQUE

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BACKGROUND

In general, Mozambican students are not exposed to approaches where they can interact with teachers and peers. Fagilde (2001) argued that student don't have opportunities to speak in the classroom. Their participation is limited to isolated words, agree with the teachers statements or complete sentences initiated by the teachers. Accordingly, they encounter difficulties to contribute in a classroom discussion. Usually they use ordinary language, quasi-mathematical language to avoid sophisticated language, gesture and unspoken but shared assumptions (Pirie, 1998). This environment, where students are not stimulated to express their thoughts, to justify what they do, does not promote the evolution of students' reasoning. In such a classroom culture, that is common in many Mozambican classrooms, it is not surprising to find the difficulties students face to verbalize their thoughts, to reason about the concepts and relations between them or to ask or think about the reasons for applying a determined procedure or method in solving problems.

As a result of Mozambican teachers' participation in international conferences, and through discussions between them, there are already some teachers, who step by step are giving more space to the students, creating opportunities for discussions with and between students in the classroom.

In addition, the National curriculum in Mozambique is being changed. This change has started from primary school and gradually moved to the secondary school.

THE MOZAMBICAN CURRICULUM

Interpretations of the term curriculum have been subject to many changes, resulting in an extension of meaning not only including the intended learning outcomes (as manifested in curriculum documents), but also "hidden" unintentional effects and the ways teachers interpret curriculum documents (Jackson, 1992). Robitaille et al. (1993) make a useful distinction between three levels of the Curriculum: the *intended* curriculum, which is reflected in curriculum guides, course outlines, syllabi and textbooks adopted by the educational system, the *implemented* curriculum that echoes what actually is taught in the classroom and the attained curriculum, that express what students actually have learned. In the present project the term National Curriculum refers to the document that encompasses the subjects, the knowledge,

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skills and understanding required in each subject and how students' progress is to be assessed and reported, that is to the intended curriculum.

Within the framework of the National Curriculum, schools are free to plan and organize teaching and learning in the way that best meets the needs of their students. Therefore, the implementation of the curriculum depends on the teacher's beliefs and knowledge and his(her) interpretation of the National texts. Accordingly, s(he) selects the topics, methods and the textbooks. This selection may not be consistent with the intended curriculum.

The intended curriculum is a guide for the teacher. Mathematics and methodological aspects valorized by the curriculum may be expressed in each of its components such content, assessment and methodology. For instances, if the curriculum valorize the development of mathematical reasoning in the classroom, one would expect to find the concept "reasoning" or evidence of issues, proposed activities or methodological instructions that may promote mathematical reasoning. In addition, one would expect that the proposed instruction for students' assessments considers and expresses the reasoning processes.

The curricula in Mozambique, in the colonial era and in the three decades after the independence, were a body of knowledge that was presented by the teacher to silent students that have the mission of reproducing the exercises as the teacher did. It did not emphasise features such as explanation, justification and argumentation that are the core of mathematical reasoning in the classroom. In 1996 new curricula have been gradually introduced in Mozambican primary school, and in 2004 an intermediary curriculum for Grade 8 has ran in some pilot schools. The new curriculum from Grade 8 will be introduced in 2008 in all Mozambican secondary schools and gradually in each following year in the subsequent grade. These curricula, in contrast to the old one, promote a learner-centered approach and are competency based. They emphasise, to some extent, the value of mathematical reasoning in the learning and teaching process.

The Mozambican programs are in general divided in seven sections: the introduction to the discipline, general aims, content, specific aims, detailed topics, methodological suggestions and assessment. In addition, occasionally there is a lesson plan in the intermediary program for grade 8. So it can be expected to find lesson plans also in the other upcoming new programs.

THE NARROWING OF THE INTERPRETATIONAL SPACE

The new curriculum in Mozambique reflects an international trend towards taking into account new insights into the way students learn and retain knowledge. However, studies focussing on classroom interaction show that even in countries where a more "learner centred" approach has been advocated, the space for students' involvement seems to be limited. The practice of justifying statements, explaining solutions and procedures and arguing about alternatives, a practice that aims at contributing to the development of students' independent mathematical reasoning skills, is constrained by principles of classroom communications. Jablonka (2002), in a study of competently taught 8th grade mathematics classrooms in Hong Kong, Germany and the U.S.A, finds some principles that amount to the restriction of students' mathematical reasoning. For example, reasoning among the students exclusively occurred in situations, in which they realised that their results differed. Also, the teachers showed a tendency to give reasons only in case of incorrect solutions. The study of Brodie, Lelliott and Davis (2002) shows some restrictions in the ways in which teachers have taken up learner-centred practices after an in-service programme in South Africa. At the beginning, the teachers had a tendency to adopt only superficial features of the program.

Teacher's qualification and interpretation of curriculum documents, classroom size, shortage of resources, financial problems that lead teachers to teach in more than one school, lack of time to think thoroughly about their lessons and methods, are some of the constraints that may be added to the obstacles found by researchers, for the implementation of the curriculum in Mozambique and other underdeveloped countries.

THE AIM OF THE STUDY

As was said before, the introduction of aspects related to fostering students' mathematical reasoning in the Mozambican curricula is very recent. Therefore, it is important to monitor and evaluate the process of implementation in order to provide institutions involved in the process with a base for accompanying actions.

The present study aims to focus on the extent, forms and functions of mathematical reasoning in Mozambican classrooms.

METHODOLOGY

Aspects valorized in the curriculum may influence the way teachers behave in the classroom. Therefore, it is essential to investigate what is stated in the curriculum about mathematical reasoning. For that reason, the study will be carried out in two dimensions: one refers to the intended and the other to the implemented curriculum in Mozambican secondary schools. On the one hand, I will analyze the Mozambican programs to seek the extent to which it puts across the relevance of argumentation, justification, explanation and other features of reasoning in the classroom. On the other hand, I will observe lessons during a semester and interview teachers. In the classroom, I will take mathematical reasoning represented by instances of teacher or students utterances that contain giving reasons for statements, explanations for any method or process used for solving problems, arguments expressing agreement or disagreement with peers' claims and resolutions of tasks. I hope the collected data will form a base for analyzing the requirements expressed in the curricula and how teachers interpret and are influenced by the curriculum and how they implement these requirements.

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IN BETWEEN REALITY AND UTOPIA: A SOCIO-POLITICAL RESEARCH AGENDA FOR MATHEMATICS EDUCATION IN SITUATIONS OF CONFLICT AND POVERTY [1]

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> As I walked up the hill, I started feeling the bad smells because in that shantytown there was no sewage system. Terrible. When I got there, four colleagues welcomed me and showed me my classroom. It didn't have a floor, just the bare ground. There were 45 children. They hadn't got a teacher. The children were in terrible conditions, dirty, extremely dirty. The room stank. I approached and started asking their names. At least I am going to make a list, I thought, and give them some recommendations about their personal care, their look, how we are going to organize the classroom, because all of them were like packed in a corner. Next day, when I came, they at least knew I was going to be their teacher. It was a first grade class. The children haven't had any previous school immersion experience. I had to start from scratch and give them some introduction, but fast because I also had to prepare them to read, write and calculate. There were older children with plenty of problems. That day I started noticing their reality when I called the list. I called for somebody. He is not here. Why didn't he come? Does somebody know why he didn't come? Yes. Last night his father came back home extremely drunk and beat the whole family, so they haven't slept. And I don't know how many similar situations were frequent. Harsh. Very harsh. That moved me deeply. I started to realize that life was not easy. I started to feel that I had to do something, that those children had been put in my hands and that I had to help them. And basically, the only thing I could give them was affection. I particularly remember Daniel, who was so good for doing calculations but could not read. He dropped school soon after I came but I saw him around in the neighbourhood. I always wondered about him. I still remember many of those kids. I have them here in my heart despite that it has been now twenty years since I first met them.

This is Mercedes, a Colombian teacher, describing her first teaching experience in a public school in a shantytown at the outskirts of Bogotá (Valero, 2002). Mercedes' words are not unique but represent the everyday experience of many teachers in Colombia and in many other parts of the world where similar life conditions characterize teachers' and students' contexts. Mercedes' words also represent the tension that many educators experience, between a belief in one's work making a difference and contributing towards a better world, and the realization of the crude and harsh reality in which one's efforts are embedded. Following Diana Jaramillo's words (Jaramillo, Torres, & Villamil, 2006), the work of teachers where the harshness of life is evident invites conceiving educational activity as a constant move between reality and utopia. Utopias and realities can be of different kinds depending

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of the contexts from where they are experienced and dreamt. Here I am referring to what is possible to imagine about mathematics education practices (and their role in people's lives) from situations where severe conflict and poverty clearly permeate mathematics classrooms.

My intention in this paper is to contend that if mathematics education research is committed to the understanding and betterment of mathematics education practices and their contribution to equity in education, in society and in the world in general, then research has to address mathematics education in situations of conflict and poverty. To do so, I will shortly point to the lack of studies addressing mathematics education in such contexts and the need for them. I will then examine some of the notions involved in such an endeavour and I will end by pointing to some of the elements of a socio-political research agenda addressing the issue.

BEYOND "PROTOTYPICAL" CLASSROOMS

In the last two decades there has been a growth in research viewing mathematics education practices from cultural, social, an political perspectives. The "strong social turn" (Lerman, 2006) has brought more attention to how and why mathematics education practices operate as inclusion/exclusion mechanisms for particular groups of students. That body of research has been carried out mainly in developed countries and illuminates the problems of students at the margins in relation to the main dominant Western, white culture. Existing research has illuminated the experiences of students and teachers in mathematics classrooms for children who do not succeed in mathematics on the grounds of their ethnicity, gender, language, ability, and social class. Some understanding of how mathematics education practices can relate to processes of inclusion/exclusion in developed countries such as the USA, Australia and many European countries have been possible thanks to the growth of this type of research.

Few research studies in developing countries, however, address these issues in general, and very seldom do they study mathematics education in marginal situations. With few exceptions of studies in South Africa, Brazil, Guatemala, Malawi and Peru (Adler, 2001; Kitchen, 2001; Knijnik, 2007; Mwakapenda, 2002; Secada, Cueto, & Andrade, 2003; R Vithal, 2003), the international research community has little idea of what happens with the teaching and learning of mathematics for marginal students in the developing world, particularly for those living in cases of severe or extreme poverty, and accentuated conflict.

I assume that generating understandings about mathematics education in these situations is a desirable aim for a research field that, in its growth and consolidation has sought not only to comprehend the complexity of teaching and learning phenomena but also to contribute with educational alternatives for improving mathematics teaching and learning. It is desirable because there are large proportions of children in this world who live in harsh conditions and for whom schooling and (mathematics) learning could/could not be a means of either making possible a betterment of material life conditions, or of bringing some kind of stabilizing, coming over and even reconstructing life possibilities when they have been literally crushed in and by political conflict.

By saying this I am not adhering to a narrative that attributes (mathematics) education and mathematics education researchers a redemptive role of the marginal, poor, displaced, and prosecuted children of the world. I agree with Popkewitz's (2002) critique of the construction of an educational narrative that gives (mathematics) education the role of saving the minds, souls and lives of children through attributing mathematics education some kind of saving grace associated with an assumption on the power of mathematical knowledge. Rather, I build on an interest in advancing an understanding of mathematics education from a perspective that views them as social and political practices, and for which researching such situation can bring important insights about the constitutive relation between the social and political context and the practices of mathematics teaching and learning in those contexts (see Christensen, Stentoft, & Valero, 2007; Valero, 2007).

Mathematics education research based on the "prototypical classrooms" where less than the 10% of school children in the world experience mathematics education (Skovsmose, 2006) has provided important insights into the nature of school mathematics learning and teaching. For me it is time to go beyond and challenge those insights by enlarging the focus of interest and the sites of research of the field. It could be that in this way we are able to generate new imaginaries that allow us seeing mathematics education and its role in society in ways that we have not seen before.

SITUATIONS OF POVERTY AND CONFLICT

Poverty, defined in terms of a lack of access to material, social and cultural resources, is a difficult concept pointing to a hard reality. So is conflict, which can at least be understood as an open clash of values and world-views held by different groups of people. A meticulous examination of these notions and the realities where they appear is not possible here. Suffice to say that there exists extensive literature addressing the connection between the two and education. When reviewing the literature, associated terms —which I have already used— feature also evidently: exclusion, marginalization, segregation, violence, all of these of different kind and intensity. All these are intricately connected and are necessary to understand what it means to live in such situation.

Situations where poverty and conflict exist are not the exclusivity of the developing world where poverty and conflict —with all the associated concepts— are endemic and extensive. They also exist in the midst of the developed world. As stated previously, most research published on issues of equity and (mathematics) education focuses on marginal groups of girls or boys, ethnic and linguistic minorities, working class students, etc. However, notice that being at the margins in the developed world can mean something quite different from being at the margins in the developed world. The experience of severe or abject poverty and of open violent conflict in (civil) war situations is materially and symbolically different, and impact individuals in different ways. While for some it may still be possible to dream a future, for some others it is even difficult to imagine a life. Such differences may also have concrete consequences for learning and teaching, and for how the researcher conceptualizes them.

ELEMENTS OF A RESEARCH AGENDA

It seems to me that a central point at stake for researching mathematics education in situations of poverty and conflict is the way in which we theorize —and empirically document and analyze— the connection between mathematics learning and its context. In "prototypical" classrooms, the context is assumed to play no role. Teaching and learning processes in mathematics, children's mathematical thinking or teachers' instructional practice can easily be researched independently of what "surrounds" them. When the focus moves to situations of learning evidently affected by its context, the same neutrality cannot be assumed (Renuka Vithal & Valero, 2003). Here I present three points, which I see as fundamental in advancing research in situations of poverty and conflict.

The theories that have been used to study mathematics learning build on a fundamental assumption of continuity and of progression in the flow of interactions and thinking leading to learning: the material world of the learner, the stimuli and interactions, and the conditions for thinking are assumed to exist and to be available for the learner. Definitions of learning as a *process* reflect these assumptions. When "learning" is studied in a situation characterized by drastic change, sudden destruction or intermittent and disruptive provision of material and human resources, the concepts and language that we have available for naming learning seem not to be adequate. When they are simply applied without further examination the result has often been the creation of deficit discourses on the learners or the teachers. Following that, students in poverty and in conflict will all be extremely cognitive deficient. However, such view has been empirically challenged and therefore is far from acceptable (Ginsburg, 1997). The question then becomes how can (mathematics) "learning" be redefined as to provide a better language to grasp the conditions and characteristics of thinking in situations where continuity and progression cannot be assumed. Children and human beings continue to think and to cognize even school mathematics, but probably in ways that we have not carefully considered before.

Socio-cultural theories of learning resolve the problem of the role of the macro-social world in individual thinking by formulating the thesis that cultural tools and artefacts mediate the relationship between the individual, his/her thinking and his/her cultural environment. Neo-piagetian theories focusing on the role of the social world and the social interaction and individual learning formulate the thesis that, in social interactions where learning takes place, the macro-social world enters individual thinking through social marking, and the evoking of social representations (Abreu, 2000). In other words, it is assumed that the macro-social world enters the micro-social world of mathematics learning interactions in some kind of symbolic, fuzzy way. However, it seems to me that the missing teacher, the leaking roof, the bare dusty floor, the blue beaten arms of a child, or the lack of food are more materially present in situations of poverty and conflict than the "symbolic, mediational presence" that these theories assume. The macro-context and all its harshness are vividly present —sometimes almost physically present— in many classrooms. If this is the case, research that re-conceptualizes the impact of the macro-social world in the micro-social world is necessary.

Finally and as a consequence of the previous two points, it is important to develop theoretical tools and corresponding analysis strategies that allow grasping the complexity of the way in which mathematics education practices occur and gain meaning in both micro-contexts and macro-contexts, and where poverty and conflict are constitutive elements of those practices. Viewing mathematics education as a network of socio-political practices (Valero, 2007) could be a way of providing a broader landscape for understanding the multiplicity of forces involved in forging "mathematics learning" in contexts fraught with violent disruption and abject resourcelessness.

It is my hope that concerted research efforts in different parts of the world, between researchers and teachers, could in the future offer conceptualizations of mathematics education that address the realities that teachers like Mercedes experience.

NOTES

1. This paper is a modified version from the paper prepared for presentation at the Symposium on the Occasion of the 100th Anniversary of ICMI, to be held in Rome in March 2008. It is based on the work I am initiating together with Gloria García and Francisco Camelo from the Universidad Pedagógica Nacional of Colombia.

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REGULAR PAPERS

THE EFFECTS OF REPRESENTATIONAL SYSTEMS ON THE LEARNING OF STATISTICS BETWEEN GREEK PRIMARY SCHOOL STUDENTS AND IMMIGRANTS

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The main objective of this study is to contribute to the understanding of the role of the four different types of representations and translations in statistical problem solving (SPS) in Greek primary school. Specifically, this study investigates the abilities of 3rd, 5th and 6th grade primary school indigenous students and immigrants in using representations of basic statistical concepts and in moving from one representation to another. The samples of the studies consisted of students of primary schools in Greece. The results of this study reveal that indigenous students have not acquired sufficient abilities for transformation from one representation system to another. Results reveal the differential effects of each form of representation on the performance of the two groups of students and the improvement of performance with age of indigenous students.

INTRODUCTION AND THEORETICAL FRAMEWORK

The notion of representations

In the field of statistics learning and instruction, representations play an important role as an aid for supporting reflection and as a means in communicating statistical ideas. Furthermore the NCTM's Principles and Standards for School Mathematics (2000) document include a new process standard that addresses representations and stress the importance of the use of multiple representations in statistical learning.

A representation is defined as any configuration of characters, images, concrete objects etc., that can symbolize or "represent" something else (Goldin, 1998). Representations have been classified into two interrelated classes: external and internal (Goldin, 1998). Internal representations refer to mental images corresponding to internal formulations that we construct of reality. External representations concern the external symbolic organizations representing externally a certain mathematical reality. In this study the term "representations" is interpreted as the "external" tools used for representing statistical ideas such as tables and graphs (Confrey & Smith, 1991). By a translation process, we mean the psychological processes involving the moving from one mode of representation to another (Janvier, 1987). Several researchers in the last two decades addressed the critical problem of translation between and within representations, and emphasized the importance of moving among multiple representations and connecting those (Gagatsis & Elia, 2004). Duval (2002) claimed that the conversion of a mathematical concept from one representation to another is a presupposition for successful problem solving. According to Elia and Gagatsis (2006) the role of representations in mathematical

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understanding and learning is a central issue of the teaching of mathematics. The most important aspect of this issue refers to the diversity of representations for the same mathematical concept, the connection between them and the conversion from one mode of representation to others. Gagatsis and Shiakalli (2004) and Ainsworth (2006) suggest that different representations of the same concept complement each other and contribute to a more global and deeper understanding of it.

The understanding of a mathematical concept presupposes the ability to recognise the concept when it is presented with a series of qualitatively different representation systems, the ability to flexibly handle the concept in the specific representation systems and finally, the ability to translate the concept from one system to another (Lesh, Post & Behr, 1987). In statistical education, the interest focuses both on the various types of representation and on the translations between them.

This study intends to shed light on the role of different modes of representation on the understanding of some basic concepts in statistics. The study was designed to explore primary school students' performance in using multiple representations of statistical concepts with emphasis on the effects exerted on performance and on the relations among the various conversion abilities from one representation to another both by the age of the students (Anastasiadou, Elia & Gagatsis, 2007) and between indigenous students and immigrants.

The situation in Greece

At this point it is needed to give the situation in Greece in relation to immigrants' students because the cultural and ethnic elements are fundamentals to students' adjustment and progress in Education, Since the 1980s Greek society was one of the most homogeneous societies racially, culturally and linguistically. From that time until now there has been a continual but fluctuating influx of "foreigners" of contrasting characteristics: economic immigrants from ex-soviet union countries after the collapse of that state, new refugees (Greeks originating from the Pontos) from ex-soviet countries, economic immigrants and political refugees from mostly eastern islamic countries. This mainly economic immigration created a new reality of inequality of multilinguil, multiculturality in a country, or rather in a nation-state remarkably homogeinic linguistically and culturally (Kogidou, Tressou & Tsiakalos, 1997). In recent years Greek society has faced a particular challenge: *to create the right educational conditions for Greeks returning from abroad, foreign immigrants, romanies and muslims.* The de facto multiculturalism (Anastasiadou, 2007) which now describes the Greek society, as it does other countries, dictates the necessity to take on board these new approaches in education, society, in interstate relations and in cooperation since Greek society despite its multicultural character, continues to function with the logic of assimilation (Centre of Intercultural Education, 1998). In the field of education the adoption of the policy of assimilation means that it continues to have a monolingual and monocultural approach in order that every pupil is helped to acquire competence in the dominant language and the dominant culture. The attendance of children in Greek school with a different cultural or linguistic expression is seen as a problem and must be discouraged. However, the problem is focused on the inability of those children to see the official language at school since it is thought that the learning of the official language is the basic ticket for their assimilation and academic achievement.

METHOD

Participants

The sample of the study involved 220 third grade indigenous students and 178 immigrants, 225 fifth grade indigenous students and 216 immigrants and 229 sixth grade indigenous students and 218 immigrants from primary schools in four regions of Western Macedonia. Below we briefly describe the content of teaching that students receive on statistics in the third, fifth and sixth grade of primary school according to the Greek curriculum, in order to give some information on students' prior knowledge.

The content of statistics in the third, fifth and sixth grade

According to the curriculum, third grade primary school students are taught to: record data, portray data through the relevant graphic and tabular representations, make assumptions and predictions regarding the results of the relevant actions, and reach the relevant conclusions based on the data. Additionally, according to the aims of the curriculum, third grade students must be able to: perceive the concepts of chance and probability, as well as the relationship between them, detect probable events, calculate the frequency of events and categorise the relevant statistical data and create the relevant tables.

Fifth grade students have often come across the terms "mean value" and "average" in math problems and can perhaps understand their meanings intuitively. According to the curriculum, fifth grade primary school students are taught: the meanings of the terms "average" and "mean value", how to read simple statistical tables and charts, how to use charts in order to present specific statistical data and how to empirically interpret the meaning of research. According to the aims of the curriculum, fifth grade students are expected to: understand the meaning and process of finding the average of the numbers provided; know the concepts of research, research population, research sample and research conclusions; know the basic steps that are required for conducting research, which include recording statistical data, sorting data, working out the absolute and relative frequency, graphic representations, calculating the average and formulating predictions and conclusions. Students themselves are required to gather and present statistical data that are drawn from their school and wider social environment.

In sixth grade students are taught to: record data, read simple statistical tables and charts, portray data through relevant graphic and tabular representations, read a table, extract information from it and convert it to a verbal or tabular representation, calculate the average and formulate predictions and conclusions. Additionally, according to the aims of the curriculum, sixth grade students must be able to: work out the absolute and relative frequency, calculate the frequency of events and categorise the relevant statistical data, construct the relevant tables and bar or pie or histogram charts, understand the meaning and process of finding the average of the numbers provided.

Tasks and variables

A test was developed and administered to the students of the three grades. The test consisted of 6 tasks on frequency tables, bar charts and their application to solving everyday problems. These 6 tasks can be divided into three groups of two "equivalent" problems in difficulty from the mathematical point of view. In particular, the first task gives some information in verbal form and students are required to give the graphic form of this information (bar chart) (V1vg), while the second task gives information of the same kind in verbal form and requires its transformation into tabular form (V2vt). The second task is the following: "The values that follow represent the height of six children: Maria 100cm, Nicos 120cm, Kostas 132cm, John 140cm, Ann 114cm. Represent these data on a table." The third task involves reading a table (see Table 1) of the frequency of students' grades, extracting information from it and giving an interpretation in verbal form (V3tv).

Grade	6	7	8	9	10
Frequency	1	3	4	6	5

Table 1. The table included in the third task of the test

The fourth task involves reading a bar chart, extracting information from it and giving an interpretation in verbal form (V4gv). The fifth task involves reading a bar chart, extracting information from it and converting it to a tabular representation (V5gt). The sixth task involves reading a frequency and relative frequency table, extracting information from it and converting it to a bar chart (V6tg).

Right and wrong or no answers were scored as 1 and 0, respectively. Students' responses to the tasks comprise the variables of the study which were codified by an uppercase V (variable), followed by the number indicating the exercise number. Following is the letter that signifies the type of initial representation (e.g. r=representation, t=table, g=graphic, v=verbal) and, lastly, comes the letter that signifies the type of final representation.

Data analysis

For the analysis of the collected data the similarity statistical method (Lerman, 1981) was conducted using a computer software called C.H.I.C. (Classification Hiérarchique, Implicative et Cohésitive) (Bodin, Coutourier & Gras, 2000). This method of analysis determines the similarity connections of the variables. In particular, the similarity analysis is a classification method which aims to identify in a set V of variables, thicker and thicker partitions of V, established in an ascending manner. These partitions, when fit together, are represented in a hierarchically constructed diagram (tree) using a similarity statistical criterion among the variables. The similarity is defined by the cross-comparison between a group V of the variables and a group E of the individuals (or objects). This kind of analysis allows for the researcher to study and interpret in terms of typology and decreasing similarity, clusters of variables which are established at particular levels of the diagram and can be opposed to others, in the same levels. It should be noted that statistical similarities do not necessarily imply logical or cognitive similarities. The red horizontal lines represent significant relations of similarity.

RESULTS

Descriptive results

Table 2 presents the success rates of third, fifth and sixth grade indigenous students and immigrants in all types of conversions. The results show that older indigenous students performed better than younger ones over all types of tasks but there was only a slight improvement in success rate between younger and oldest immigrants' students. Further more the success rates between the two groups of students are differentiating. The success rate of indigenous students are much higher that the immigrants ones in the all the three grades.

Students' success in each grade varies across the different conversion tasks. Considering the lowest and the highest percentage in each grade, this variation decreases with age: third grade indigenous students, 22-36% and immigrants, 10-18%; fifth grade indigenous students, 48-59% and immigrants, 12-18%; sixth grade indigenous students, 75-82% and immigrants, 15-22%. These findings indicate that maturation and instruction of statistics help students carry out conversions of statistical concepts more successfully and treat representations more flexibly.

Tasks	Type of translation	Third grade success rate of indigenous students (%)	Third grade success rate of immigrants (%)	Fifth grade success rate of indigenous students (%)	Fifth grade success rate of immigrants (%)	Sixth grade success rate of indigenous students (%)	Sixth grade success rate of immigrants (%)
V1vg	Verbal - Graphic	33.15%,	11.65%	54.67%	14.298%	82.3%	15.13%
V2vt	Verbal - Tabular	23.5%	10.54%	62.3%	15.73%	76.5%	15.86%
V3tv	Tabular - Verbal	22.2%	10.32%	60.4%	11.74%	75.8%	15.32%
V4gv	Graphic - Verbal	30.16%	11.87%	48.4%	12.09%	79.1%	14.7%
V5gt	Graphic- Tabular	24.4%	13.45%	53.7%	13.68%	74.6%	17.73
V6tg	Tabular- Graphic	35.56%	17.92%	59.2%	18.36%	79.4%	21.83%

Table 2. Success rates of indigenous students and immigrants in the tasks

In order to examine in a more comprehensive way the differences between 3rd, 5th and 6th grade **indigenous students and immigrants** with regard to their performance in the various tasks and the interrelation of their responses, a comparison was made between similarity diagrams 1, 2 and 3 concerning the 3rd, 5th and 6th grade respectively.

Similarity analysis results

The similarity diagrams in this study concern the data of each grade separately, and allow for the arrangement of students' responses (V1vg, V2vt, V3tv, V4gv, V5gt, V6tg) to the tasks into groups according to their homogeneity.

Two clusters (Cluster A and B) of variables are identified in the left similarity diagram of third grade indigenous students' responses as shown in Figure 1. The strongest similarity occurs between variables V1vg and V6tg in Cluster A. It is suggested that indigenous students employed similar processes to construct a graph based on information given verbally or in a table. The similarity connection of the variables V1vg and V6tg to the variable V4gv reveals students' consistency as regards their performance in constructing a graph and their performance in drawing information from the graph and interpreting it verbally. Cluster B consists of the variables V2vt, V5gt and V3tv. It is suggested that students dealt consistently with the tasks that required the construction of a table based on information given in verbal or in graphic form, as well as, with the task involving the verbal interpretation of its data.

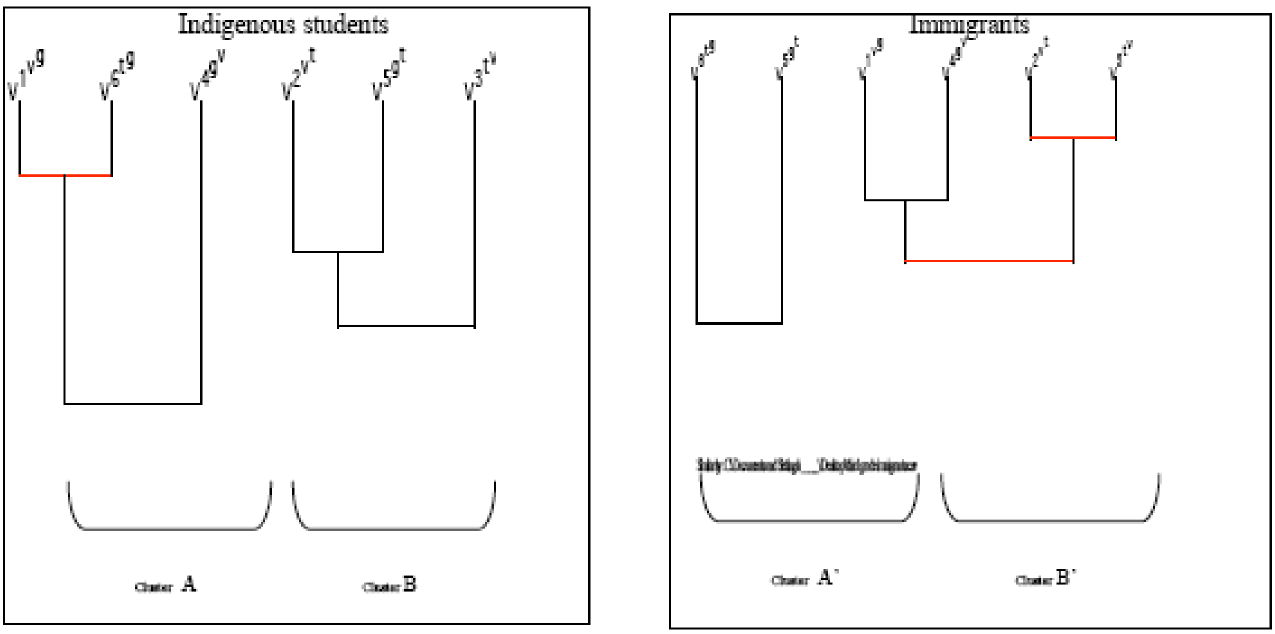

Figure 1. Similarity diagram of third grade indigenous students' and immigrants responses

The formation of the two distinct clusters indicates that indigenous students dealt differently with conversions requiring the construction of a graph or the verbal interpretation of a graph (V1vg, V6tg, V4gv), relatively to the conversions involving the creation of a frequency table or a verbal description of the data given on a table (V2vt, V5gt, V3tv). This suggests that students in third grade treated the graphic and the tabular representations in isolation. Students' higher success rates at the tasks of the first cluster (V1vg: 33.15%, V6tg: 35.56%, V4gv: 30.16%) relatively to the tasks of the second cluster (V2vt: 23.5%, V5gt: 24.4%, V3tv: 22.2%) indicate their greater difficulty in tackling the second group of tasks and provide further support to the above assertions.

Two clusters (Cluster A' and B') of variables are identified in the right similarity diagram of third grade immigrants students' responses as shown in Figure 1. The strongest similarity occurs between variables V2tv and V3tv in Cluster B'. It is suggested that immigrants employed similar processes to construct a table based on information given verbally or to explain verbally the table elements. The similarity connection of the variables V2tv and V3tv to the variables V4gv and V1vg reveals students' consistency as regards their performance in constructing a graph and their performance in drawing information from the graph and interpreting it verbally and students' consistency as regards their performance in constructing a table and their performance in drawing information from the table and interpreting it verbally.

The formation of the two distinct clusters indicates that immigrants dealt differently with verbal conversions (V1vg, V4gv, V2vt, V3tv), relatively to the conversions involving the creation of a frequency table or a graph of the data given on a graph or a table (V6tg, V5gt). This suggests that immigrants in third grade treated the verbal representations in isolation. Students' higher success rates at the tasks of the second cluster (V6tg: 17.92%, V5gt: 13.45%) relatively to the tasks of the first cluster (V1vg: 11.65%, V4gv: 11.27%, V2vt: 10.54%, V3tv: 10.32%) indicate their greater difficulty in tackling the first group of tasks with verbal conversions and provide further support to the above assertions.

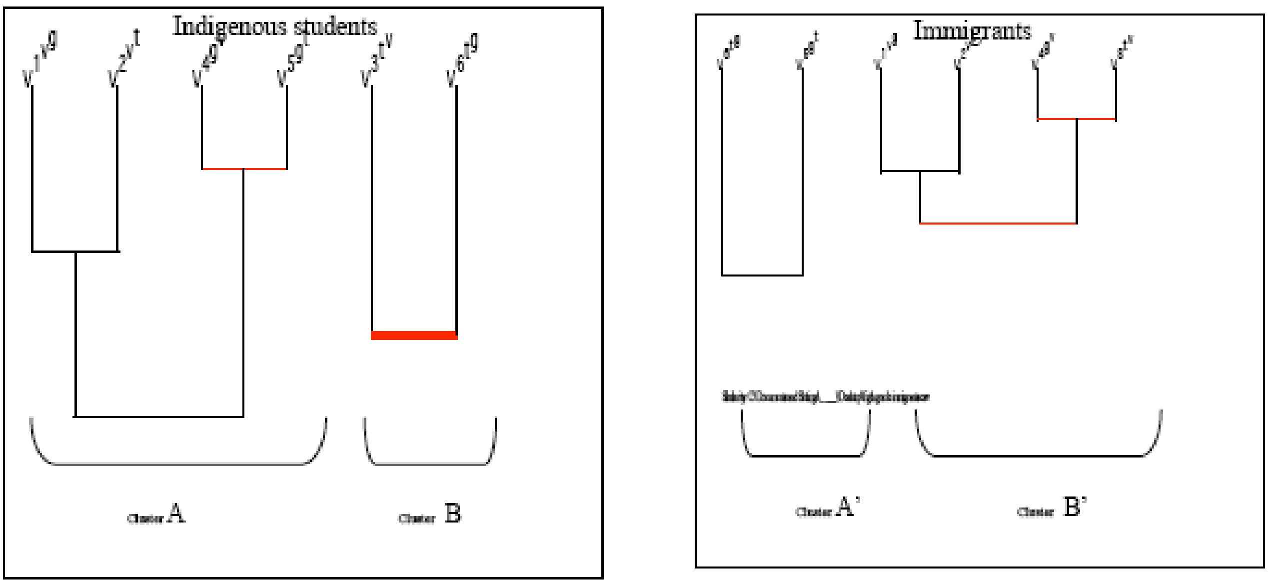

Figure 2. Similarity diagram of fifth grade indigenous students' and immigrants responses

The left similarity diagram of the fifth grade indigenous students' responses, illustrated in Figure 2, involves three pairs of variables (V1vg-V2vt, V4gv-V5gt, V3tv-V6tg). This grouping suggests that students dealt similarly with the conversions involving the same initial representation, which are verbal form, graph and table. Thus, the initial representation of the task had an effect on the conversion or interpretation processes employed by the fifth grade students. The similarity cluster (Cluster B) of the variables including the table as a starting representation (V3tv-V6tg) is disconnected from the other similarity pairs which form a joint cluster (Cluster A), indicating students' compartmentalized ways of handling frequency tables and the other forms of representation, i.e. graph and text.

The right similarity diagram of the fifth grade immigrants' responses, illustrated in Figure 2, involves three pairs of variables (V6tg-V5gt, V1vg-V2vt, V4gv-V3tv). The similarity cluster (Cluster A') (V6tg-V6gt) is disconnected from the other similarity pairs which form a joint cluster (Cluster B'), indicating immigrants' compartmentalized ways of handling verbal conversions and the other forms of conversions, i.e. graphical to tabular and tabular to graphical.

The strongest similarity in the similarity diagram of the sixth grade students' responses, illustrated in Figure 3, occurs between the variables V1vg and V6tg. This similarity reveals sixth grade students' consistency in their processes when constructing graphs on the basis of verbal or tabular representations. Students' responses to the other tasks are interwoven in the similarity diagram, indicating students' coherence in dealing with the corresponding conversions irrespectively of their initial or target representation. Students' high success rates at all of the tasks of the test ranging from 74.6% to 82.3% provide further evidence for this assertion.

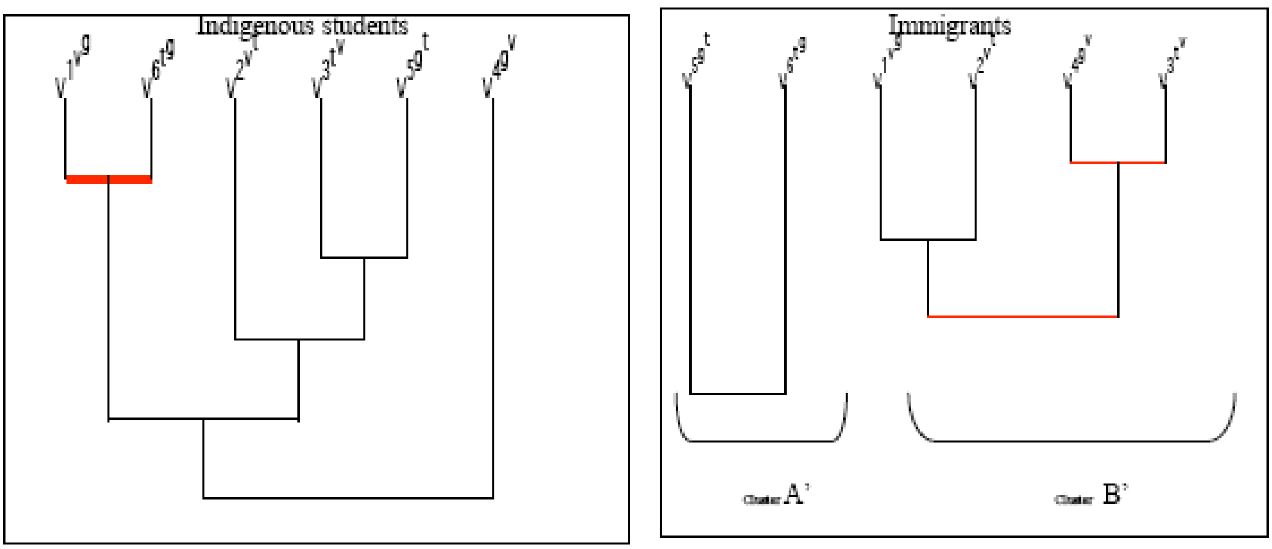

Figure 3. Similarity diagram of sixth grade students' responses

The right similarity diagram of the sixth grade immigrants' responses, illustrated in Figure 3, involves three pairs of variables (V5gt-V6tg, V1vg-V2vt, V4gv-V3tv). This grouping suggests that immigrants' responses are similar with the grouping in Figure

2 concerning immigrants' responses in fifth grade in conversions involving the same initial representation, which is verbal form, graph and table.

CONCLUSIONS

Representations are considered to be extremely important with respect to cognitive processes in developing statistical concepts. The main contribution of the present study is the identification of Greek indigenous students' and immigrants abilities to handle various representations, and to translate among representations related to the same statistical relationship across three age levels in primary education. Indigenous students' success was found to increase with age. Moreover, the three similarity diagrams clearly showed the different ways in which third, fifth and sixth grade students dealt with tasks involving different representations of statistical concepts.

These findings show that despite the improvement of students' performance from third to fifth grade, students in both grades encountered difficulties in the understanding of statistical concepts and more specifically in moving flexibly from one representation to another. Lack of connections among different modes of representations indicates the difficulty in handling two or more representations in mathematical tasks. This incompetence is the main feature of the phenomenon of compartmentalization in representations, which was detected in both third and fifth grade students (Duval, 2002). The phenomenon of compartmentalization in representations had also appeared with immigrants in the all three grades. The main difference of immigrants in relation to indigenous students is that there was no improvement of performance with age.

This phenomenon did not appear in the performance of sixth grade indigenous students. Their success was found to be independent of the initial or the target representation of the tasks. The basic problem with immigrants is the understanding of the verbal representations as an initial and final mode and this due to the general difficulties that immigrants face with the Greek language.

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CONTRADICTIONS IN MATHEMATICAL MODELLING ACTIVITIES FROM A CRITICAL MATHEMATICS EDUCATION PERSPECTIVE [1]

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In this article, I present an experience that took place during a Mathematics I course proffered to undergraduate geography students. The context is the presentation of a mathematical modelling project developed by a group of students enrolled in the course. The objective of the article is to apply an analysis, using Activity Theory, of apparent contradictions between students' views regarding the use of mathematics in geography, the guidance they received regarding the development of the mathematical modelling project, and the words of the group members during the final presentation of their project. The comments made by students during these different instances, which may appear contradictory, can be re-interpreted when analyzed in a broader activity, from the perspective of Activity Theory.

INTRODUCTION

The experience I describe here took place in the first semester of 2006 in a Mathematics I course offered to undergraduate students of the geography program at the Federal University of Minas Gerais (UFMG). Specifically, it refers to a presentation of a mathematical modelling project developed by one group of students enrolled in the course.

The objective of the article is to apply an analysis, using Activity Theory, of apparent contradictions with respect to three aspects (or at three instances): 1) students' views regarding the use of mathematics in the field of geography (to create the science of geography, to describe and discuss geographical phenomena, and as a required course within the undergraduate geography program); 2) the guidance they received regarding the development of the mathematical modelling project; and 3) the statements made by the group members during the final presentation of their project.

With the aim of presenting students' views regarding the use of mathematics in the field of geography, I begin by describing some characteristics of the Mathematics I course as well as of the students enrolled in it. In this way, I present the space in which the mathematical modelling project was developed. In the third section, I lay the theoretical groundwork for the perspective of mathematical modelling adopted and describe how it was carried out, emphasizing the guidance the students received regarding the development of the project. The project developed by one group of students is presented in the fourth section.

Some of the comments made by students during these different instances which appear, in principle, to be contradictory, can be re-interpreted when analyzed in a

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broader activity, from the perspective of Activity Theory. In the fifth section, I attempt to apply such an analysis, suggesting possibilities for this re-interpretation.

THE COURSE AND ITS STUDENTS

Until March, 2006, Mathematics I was among the required courses for the undergraduate program in Geography at the UFMG. Based on the need for new pedagogical projects for the program to meet new curriculum guidelines for elementary teacher education (BRASIL, 2002), the course became optional. I was assigned by the Mathematics Department to teach the course in the first semester of 2006.

The mathematics contents planned for the course included functions, derivatives, and notions of integral. However, in the interest of developing the classes in accordance with the pace of the students, not all of this content was covered in 2006. With few exceptions, geography students generally have a history of poor relationships with mathematics, complaining of learning difficulties, traumatic past experiences, and disappointment regarding the requirement to take the course, having believed they were finally free from mathematics.

When I inquired about the role of Mathematics I within the Geography program, the students reacted with indignation. In addition to their reports of difficulties with mathematics in previous academic experiences, they argued that geography pertains to the Human Sciences, and as such, cannot be constructed in terms of mathematical arguments. At times they confused the use of mathematics in geography with a positivist approach to the latter, and armed with critiques of Positivism, they questioned the validity of its use. Students required to repeat the course stated that they failed to see the applicability of Mathematics I in their program. Thus, in the students' opinion, Mathematics I should not be included in the curriculum of the Geography program. According to the coordinator of the program at that time,

> students always complained that they were unable to see mathematics in a practical way in their academic future (connection with other courses) and their professional futures. Remaining distant from the reality of geography, mathematics loses it meaning, in their opinions (MAGALHÃES JR., 2006).

And, in fact, the Mathematics I course was removed from the required curriculum of the Geography program.

Regarding the activities developed in the first semester of 2006, due to a study that was in the phase of data collection at that time (ARAÚJO & PINTO, 2004) [2], I proposed the development of *milieus of learning* with computers within which the students were invited into a *landscape of investigation* (ALRØ & SKOVSMOSE, 2002). But since the course was being offered for the last time, and based on a positive previous experience (ARAÚJO, 2004), I could not miss the opportunity to propose the development of mathematical modelling projects, described in greater detail in the following section.

THE DEVELOPMENT OF MATHEMATICAL MODELLING PROJECTS

Mathematical modelling has stood out among current perspectives in mathematics education. In general terms, it can be understood as the utilization of mathematics to resolve real problems. When applied in the classroom, this approach takes on special forms, depending on the educational context, the professionals involved, and the profile of the students, among other factors.

Bassanezi (2002), for example, understands mathematical modelling – whether as a scientific method or a teaching and learning strategy – as the "art of transforming problems from reality into mathematical problems and resolving them through interpretation of their solutions in the language of the real world" (p. 16). For Barbosa (2001), "modelling is a milieu of learning in which students are invited to question and/or investigate, by means of mathematics, situations with reference in reality" (p. 31).

In the Mathematics I course, I sought to put into practice an understanding of mathematical modelling as

> an approach, by means of mathematics, to a non-mathematical problem based in reality, or to a non-mathematical situation based in reality, chosen by groups of students in such a way that questions of Critical Mathematics Education form the basis for the development of the work (ARAÚJO, 2002, p. 39).

Within this perspective, there are some explicit characteristics of the milieu of learning that I seek to put into effect when I propose the development of mathematical modelling projects, including working in groups, and basing the work on Critical Mathematics Education.

According to Skovsmose (1994), the main concern of Critical Mathematics Education is the development of *mathemacy*, which is an extension to mathematics of the problematizing and liberating conception of education proposed by Freire (1970). A similar concept – *matheracy* – has also been discussed by D'Ambrosio (1999). In *mathemacy*, the objective is not to merely develop the ability to carry out mathematical calculations, but also to promote the critical participation of students/citizens in society, discussing political, economic, and environmental issues in which mathematics serves as a technological support. In this case, critique is directed at mathematics itself, as well its use in society, the concern thus extending beyond the teaching and learning of mathematics.

The development of the modelling project in the Mathematics I course began with the discussion of a text (ARAÚJO, 2006). In this text, I present my understanding of mathematical modelling and suggestions for topics that should be considered in the "research proposal" to be written by the groups. At the same time, students were asked to think about themes for their projects and about the formation of groups to develop them.

In the following class, themes and groups were defined through a long process of negotiation. In the first semester of 2006, each group ended up with approximately seven members, and the themes chosen were the following: the transposition of the São Francisco River (two groups formed, one to address physical aspects and the other social aspects); physical impacts of the implantation of hydroelectric dams; socio-cultural aspects of the Linha Verde (Green Line) freeway construction project in Belo Horizonte; Campus 2000: consequences for transportation in the UFMG; climate myths; solar energy.

Once the themes had been defined, each group elaborated a work plan which I evaluated and returned to the group. In this evaluation, I encouraged them to describe in detail all the steps to be followed during the development of the project, as well as the definition of the focus of the research. I also sought to raise questions regarding how mathematics would be used in the project.

After the projects had been approved, the groups began to carry them out, holding meetings during and outside of class. They presented partial reports on their progress each month, and based on these reports, each group received guidance and suggestions - my own as well as from the entire class - regarding how to proceed. During each of these advisory sessions, I sought to take into account the concerns of Critical Mathematics Education.

At the end of the semester, all the groups made an oral presentation of their project to the class (which were videotaped), and handed in a written version of the project. One project, in particular, attracted my attention because of the group's careful treatment of the mathematical information. This project is considered in greater detail in the section that follows.

FINAL PRESENTATION OF THE PROJECT "TRANSPOSITION OF THE SÃO FRANCISCO RIVER: PHYSICAL ASPECTS"

The theme of the group's project was "physical aspects of the transposition of the São Francisco River". The group's choice of theme portrays, at the same time, the relation with their field of interest, geography, and their interest in a controversial subject, the transposition of the São Francisco River [3]. The objective of the project was to analyze whether or not the rainfall in a given region along the course of the river would be sufficient to compensate for the amount of water that would be diverted as a result of the transposition.

This small report demonstrates the possibility for using mathematics (**quantity** of rainfall and diverted water) to discuss a problem from geography (quantity of **rainfall**) in a critical manner (questioning the environmental consequences). Thus, a mathematical modelling project was proposed that could be approached from a Critical Mathematics Education perspective.

In the written report [4], the group reported that, after agreeing on the objective of the research (analyze the quantity of rainfall . . .), they began to consider what mathematical model to use, and decided, without much justification, to adopt a periodic function. This choice may have resulted from the students' knowledge regarding the behaviour of rainfall, but it may also have been influenced by the subject discussed in class, which would exemplify what Araújo and Barbosa (2005) call the inverse strategy in the modelling process. However, data that the group had gathered, relating to the rainfall in a given region along the course of the river, seemed not to fulfil a periodic function. Then, the solution that the group found was to re-group data in such a way that they fit a mathematical model represented by a periodic function.

In my point of view, this is an example of what Skovsmose (1994) calls the *formatting power of mathematics*. The author defends the thesis that mathematics is used to format reality. According to this thesis, part of our reality is projected by means of mathematical models. One example of this is the Human Development Index (HDI): based on mathematical models, a number from zero to one is associated with every city or locale. Based on this index, the government, for example, decides how to distribute funds to achieve a given objective. A city with an HDI near 1, because of their relatively high rating, might not be selected to receive funds that could resolve some of their problems. Thus, mathematical models are used to create a "real situation" that did not exist before. Critical Mathematics Education questions this power with which mathematics is imbued.

In the case of the group of geography students, it appears to me that the data relating to the precipitation in the region they chose were formatted by a periodic model.

Moreover, the group appeared to construct certainties regarding the mathematical discussion they developed. In the written work, they state that the development of the modelling project was important for the group to agree "that the science of geography needs mathematical analysis **to prove** environmental or social impacts" (my emphasis).

Such statements reinforce what Borba and Skovsmose (1997) call *the ideology of certainty of mathematics*. According to these authors, the ideology of certainty sustains the character of neutrality of this science, imbuing it with the power of the holder of the definitive argument in various debates in society. Thus, mathematics is considered in the presentation of political decisions, for example, suggesting that the decision taken represents the best path to follow, without leaving room for counter-arguments, thus characterizing its use as a *language of power*. Combating the ideology of certainty is one of the objectives of Critical Mathematics Education.

On the other hand, as mentioned in section "The Course and Its Students", these same students are uncompromising critics of the use of mathematics to discuss social issues. They believe the exactness of mathematics to be insufficient to account for the complexity involved in subjects from the Human Sciences. In addition, the development of the modelling projects was guided in such a way that the ideas of Critical Mathematics Education were considered, as described in section "The

Development of Mathematical Modelling Projects". In other words, I, the professor, expected the students to question the use of mathematics to generate certainties regarding the transposition of the São Francisco River, and to use mathematics as **one** way (and not **the** way) to understand the situation, and not to format the information they gathered. What was happening with the group? Could we say that the group was contradictory in the development of this mathematical modelling project?

APPARENT CONTRADICTIONS: APPLYING AN ANALYSIS

To say that there were contradictions in the group's work during the three instances discussed here could be understood colloquially. In this case, we would say that there are incoherencies or conflicts in their statements. Understood in this manner, our evaluation of the group could have a negative connotation: the students were not very sure about what they were doing.

On the other hand, contradiction is a key concept in Activity Theory, as according to Engeström (1987), internal contradictions are "the source of dynamics and development in human activity." The word "activity", despite being part of our everyday vocabulary, is also the central concept to Activity Theory, which has its origins in the historical-cultural school of Soviet psychology, whose principle representative is Vygotsky.

Activity Theory considers activity as the basic unit of human development. According to Leont'ev (1978), it is born of the process of reciprocal transformations between subject and object. In his own words,

> Activity is a molar, not an additive unit of the life of the physical, material subject. In a narrower sense, that is, at the psychological level, it is a unit of life, mediated by psychic reflection, the real function of which is that it orients the subject in the objective world. In other words, activity is not a reaction and not a totality of reactions but a system that has structure, its own internal transitions and transformations, its own development. (p. 50).

According to this reference, the contradictions emerge from the duality of human activity, as a production of society, in general, and as a specific production within an activity. This duality is the result of the relation between the individual and society (ROTH, 2004).

How, then, can we re-interpret the procedures of the group of geography students at the three instances presented, according to this perspective?

One possibility is to understand the development of the mathematical modelling project as an activity. If that were the case, according to Leont'ev (1978), it should have a motive, a need that drives it. Interpreting it in this way, the object of the activity of the group of geography students (the subjects of the activity) was the transposition of the São Francisco River; and what drove them, their motive, was their questioning regarding the real need and conditions of the river for this procedure. Thus, mathematics could have been used, in a critical manner, as part of the analysis developed by the group.

On the other hand, the mathematical modelling project was one of the tasks to be used to evaluate students for the Mathematics I course; and if we interpret the course as an activity in which the students were subjects (together with the professor), perhaps their motive was to pass the course. Understood in this way, a whole set of values traditionally associated with school mathematics can come into play and influence the students' procedures during the development of the modelling project. For example, the students, who had a history of problems with mathematics in school, may want to show the professor that they now have command of this powerful tool and no longer view it with disregard.

It thus appears that isolated assignments that take into account questions raised by Critical Mathematics Education may have little influence on students who live in a social world that, in general, values the power of mathematics and rarely questions the reality it constructs.

We can, however, re-interpret the procedures of the group of geography students in a direction opposite that of the preceding analysis. Understood in this way, what was in principle an action within a broader activity aimed at passing the Mathematics I course became a new activity that took on its own life, acquired its own motive, and in this way, more closely approximated my initial intention of discussing problems or situations from geography, by means of mathematics, from a Critical Mathematics Education perspective.

I believe that it is important to develop work from a Critical Mathematics Education perspective in specific situations (in a classroom, for example), but aiming to extend this discussion to society. According to Activity Theory, the individual, as a social being, is influenced by the values, conceptions, traditions, etc., that are part of society, but at the same time, he/she has the power to change these values, acting (critically) in this same society.

NOTES

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2. Research project developed with the support of the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) and FAPEMIG, both Brazilian governmental agencies that support scientific research.

3. The São Francisco River is the most important river in the Brazilian Northeast, the driest region in the country. The fertile areas along the river contrast with the rest of the region, dominated by caatinga. For years, there has been talk in Brazil of diverting the waters of the river to other areas of the northeast. However, the river has been suffering from pollution and silting, and it is not known whether it would withstand such a procedure. It is also known that there are political interests involved. In summary, it is a very controversial topic.

4. The complete reference of the group's written work is not presented to preserve their identity.

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ARE MY STUDENTS ENGAGED WITH CRITICAL MATHEMATICS EDUCATION?

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Students enrolled in a pre-service Primary-Middle Years teacher education program at the University of South Australia undertake two integrated mathematics and science education courses. Many of the learning topics within the courses introduce students to a critical perspective of mathematics and science education. The author is a member of the team that constructs and teaches the courses and has recently commenced a PhD program with a critical mathematics education focus. Reading in the field of critical mathematics education has raised many questions about this focus in each course. This paper draws from Paul Ernest, Marilyn Frankenstein, Betty Johnston, Gelsa Knijnik and Ole Skovsmose to explore one of these questions, are students undertaking these courses really engaged in critical mathematics education?

Ernest (2001), Frankenstein (1998, 2000), Johnston (1994), Knijnik (1998, 2002), Skovsmose (1994, 2005) and Vithal (2003) have all written in the field of critical mathematics education. Each presents a particular construct for critical mathematics education. This paper comes from grappling with the concept of critical mathematics education, elements that constitute each of its various constructs and reflecting upon how these elements are part of learning topics within the integrated courses. The paper has four parts; a discussion about the role of critical mathematics education and four elements drawn from its various constructs, a description of the student and course context, a discussion about how one of the learning topics engages students with critical mathematics education and a conclusion.

CRITICAL MATHEMATICS EDUCATION

Critical mathematics education comes from the need to foreground the role of mathematics education in educating for citizenship and empowerment. Skovsmose (2005, p. 3) puts the view, "To acknowledge the critical nature of mathematics education, including all the uncertainties related to this subject, is a characteristic of *critical mathematics education*." Learning mathematics may result in empowerment, citizenship and democratic participation or it may result in disempowerment, marginalisation and exclusion. Critical mathematics education and ethnomathematics are two developments where some versions aim to implement a more equitable mathematics education and mathematics education for democracy and citizenship.

Critical mathematics education challenges the nature of mathematical knowledge, the purpose of mathematics education and, in turn, challenges mathematics curricula and classroom practices. Ernest (2001), Frankenstein (1998, 2000), Knijnik (1998, 2002), Skovsmose (1994, 2005) and Vithal (2003) amongst others argue that education policies, curricula and teaching practices present mathematics as an absolute, neutral, unquestioned body of knowledge that has little to do with the socio-political

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environments we live in and does little to develop students as participating, thinking citizens.

Ernest (2001, p. 278) argues that critical mathematics education is about applying a critical attitude to mathematics and its teaching. As such, a key aim for critical mathematics education is for "students to think mathematically, use it in their lives to empower themselves both personally and as citizens, and appreciate its role in history, culture and in the contemporary world" (Ernest, 2001, p. 285). Skovsmose (1994, p. 16) states to be critical "means to draw attention to a critical situation or a progression in a crisis, to identify it, to try to grasp it, to understand it and to react to it." In his philosophy for critical mathematics education, Skovsmose (1994) focuses on the role and formatting power of mathematics in highly technological societies and how these are often hidden and left unquestioned. Mathematics has made positive contributions to society but has created what Skovsmose (1994) and Skovsmose and Nielson (1996) describe as crises with respect to inequitable distribution and access to resources, democratic participation and acting ethically. Skovsmose (2005, pp. 4-5) further develops the notion of problematic outcomes from mathematics education with his discussion about the concerns of mathematics education, particularly the concern of globalisation and ghettoising and the emergence of a Fourth World.

The ideals and values that underpin critical mathematics education as discussed by Skovsmose and others support the principles of the Primary-Middle Years Program.[1] The ideals and values also support the teaching team in two ways. Firstly, through mathematics and science, engage students with issues from the perspectives of social justice, equity, sustainability and ethical action. Secondly, through interactive, collaborative learning topics, model practices that can implemented in classrooms.

The literature around critical mathematics education presents a complex, evolving concept with a range of views about what constitutes critical mathematics education. Johnson (1994) introduced the idea of critical numeracy as a way to make meaning with mathematics that extends beyond the functional or utilitarian application of mathematics to include socio-historical perspectives and the potential to explore the power relations when applying mathematics. Frankenstein (1998, 2000) developed the idea of criticalmathematics literacy based on interrogating the use of number and statistics in particular to interpret and challenge inequalities in society. Her criticalmathematics literacy curriculum framework is built around the four goals about understanding mathematics, the mathematics of political knowledge, politics of mathematical knowledge and the politics of knowledge (Frankenstein 1998, p. 1).

Mathemacy was first introduced by Skovsmose (1994) as a key competence within critical mathematics education that mirrors Freire's competence of literacy for democracy. Mathemacy is a competence with mathematics to interpret social life, to act in a world structured by mathematics (Skovsmose 1998, p. 200). It draws together three ways of knowing; mathematical, technical and reflective knowing with reflective knowing being the potential catalyst for critical awareness. Ernest (2001, pp. 285–286) describes what a critical mathematics education should encourage through five points of awareness or understanding that in many ways reflect the goals of politics of mathematical knowledge and mathematics of political knowledge (Frankenstein 1998) and reflect the ideas of mathemacy and mathematics in action with respect to science and technology (Skovsmose 1994, 1998, 2005).

Through her work with the Movimento dos Sem-Terra, Knijnik (1998, 2002) developed an ethnomathematics approach to empower students and community members. The approach identifies and recovers popular mathematical knowledge drawn from personal and community activities. The popular mathematics is shared and decoded. Knowledge is also acquired from academic mathematics and comparisons established between the popular and academic knowledge (Knijnik 1998, p. 189). A criticism of ethnomathematics is that while community practices are acknowledged and valued, the popular mathematics is glorified and exacerbates cultural relativism (Vithal & Skovsmose, 1997). Knijnik's ethnomathematics approach addresses this criticism as the power relations produced by the confrontation between popular and academic mathematics become the centre of comparison and analysis and as such opens up a larger world view.

Grappling with each of the above constructs for critical mathematics education has highlighted elements common to most constructs and others particular to a few constructs. Given the length of this paper, I have chosen four elements as the basis for answering my question. My intent is not to fragment elements in ways that loses the relationships between them nor do I claim to be exhaustive with my identification. The elements chosen are: authentic, interdisciplinary learning experiences; landscapes of investigations; the role of mathematics in socio-political contexts; and reflection, critique and action.

Authentic, interdisciplinary learning experiences

Engaging learners with authentic, interdisciplinary experiences is a common theme across the constructs of critical mathematics education. Examples of practice from each author engage learners with socio-political contexts that provide opportunities to problematise the application of mathematics and maximise drawing upon other disciplines. Frankenstein (1998) argues that mathematics is made more accessible through real-life contexts through an interdisciplinary mathematics and social studies curriculum. The ethnomathematics approach developed by Knijnik (1998) identifies, decodes and shares people's popular mathematics practices and continually engages them with the socio-political contexts from which they are drawn. Ernest (2001, pp. 289-291) offers examples drawn from social, political and environmental aspects of everyday life that have a local or a more global perspective as rich contexts to engage learners with critical mathematics education. This is also a characteristic of the project work described by Skovsmose (1994) and Arlø & Skovsmose (2002).

Landscapes of investigations

Closely connected to authentic, interdisciplinary learning experiences are landscapes of investigation. Landscapes of investigation engage students with questions that challenge and questions that lead to explanations, situations that offer more than one answer, support student directed investigation, enable students to develop their mathematics as part of their investigation, and encourage teachers to work in a risk zone where questions, directions and outcomes may change (Skovsmose, 2001a). The idea of landscapes of investigation is reflected in Frankenstein's criticalmathematics literacy framework and Knijnik's ethnomathematics approach. Frankenstein (1998) promotes engaging students with deep and complicated questions, open-ended investigations where students pose their own questions and create their own problems. Knijnik (1998) supports students to share, decode and compare their mathematical practices and explore thinking behind these practices.

The role of mathematics in socio-political contexts

A key tenet of critical mathematics education is that mathematics is applied in contexts of particular social, economic and political values, ideologies and vested interests. Johnston (1994, p. 35) argues that unless one asks, 'in whose interest is mathematics used?' and acts in response to this, one cannot be critically numerate. Frankenstein (1998) argues that students need to develop an understanding of the mathematics of political knowledge and the politics of mathematical knowledge. The former refers to how mathematics can be used to understand institutional structures in society while the latter argues that applying mathematics is not neutral and is underpinned with political positions or perspectives. Knijnik (1998) makes the power relations between popular and legitimised mathematics the centrepiece of her ethnomathematics approach to uncover the vested interests and impact of mathematics on societies. Skovsmose (1994, 2001b) puts the view that mathematical models are at the centre of the formatting power of mathematics in highly technological societies. Mathematical models are developed in the contexts of social, economic and political interests or as a result of further technological development. The interests behind and the impact of applying mathematics are often hidden and rarely critiqued leaving power with those who develop, understand and apply mathematical models.

Reflection, critique and action

Reflection, critique and taking action are connected themes across the various constructions of critical mathematics education. Arlø and Skovsmose (2002) highlight the relationship between reflection and action and include reflection as a key component of landscapes of investigation. They describe three different aspects of reflection; what can be addressed by reflection, who is carrying out the reflection and the context in which the reflection is carried out (Arlø and Skovsmose 2002, p. 184). Mathemacy has reflective knowing as one its three competencies with the task of being a catalyst for critical awareness and this could lead to criticising systems established by means of mathematics (Skovsmose, 1994, p. 125). Reflective knowing enables one to identify the nature of political and economic understandings that direct the application of mathematics and enables one to question the choice of mathematics, the accuracy with which it is being applied, the reliability of the results and how the application of mathematics relates to broader contexts.

STUDENT AND COURSE CONTEXT

Critical mathematics education is context bound (Skovsmose, 2005, p. 3). For example, Vithal (2003) writes from the context of post apartheid South Africa, Frankenstein (1998, 2000) writes from the context of marginalised, urban adults in the United States, Knijnik (1998, 2002) writes from the context of Brazilian landless people while Skovsmose (1994) writes from a Danish context. Given this, I will briefly describe key characteristics of the students enrolled in the two integrated mathematics and science education courses and the courses themselves.

The Primary and Middle Years program commenced in 2005 and develops students as generalist years 3 to 9 teachers with two areas of curriculum specialisation for teaching in the middle years (years 6 to 9). The program and courses within it are constructed around a set of seven guiding principles.[1] Two of these principles, the principle concerned with equity and social justice and the principle concerned with sustainability reflect the general aim of critical mathematics education.

Key student characteristics

Two student characteristics in particular impact on the structure and learning content of the two integrated courses. Firstly, the diversity of students enrolled in the program is not representative of the wide cultural and soci-economic diversity found in South Australian schools. A challenge for the program is to develop students' understanding and appreciation of the diverse range of cultures, values and socio-economic situations within school communities as well as prepare them to confidently and competently cater for the breadth of social, emotional and academic needs of the students they will teach in their future classrooms. Secondly, the vast majority of students in the program have a narrow view of mathematics and lack confidence to "do" and learn mathematics. This impacts on their self image as early career generalist teachers of mathematics (Paige, Chartres & Rowell, 2004). Most believe there is 'one mathematics,' dominated by number, rules, techniques and correct answers. The ideas of a mathematics education for democracy and citizenship and that mathematics is not neutral are foreign to students.

The structure of mathematics and science education courses

The two courses integrate mathematics and science education to maximise the opportunity to engage students with social justice principles and ethical concerns and to connect these to students' life-worlds. Integration has two meanings with respect to the courses. The first meaning refers to learning topics that integrate aspects of mathematics and science. The second meaning refers to integrating pedagogy and research drawn from both mathematics and science education. Key characteristics of the integrated courses include a workshop structure that employs interactive practices, students working collaboratively and authentic assessment tasks. Learning topics in each course may employ a solely mathematics vehicle, e.g. chance, a solely science vehicle, e.g. invertebrates, employ an integrated focus such as sorting and classifying or focus on an interdisciplinary issue. The notion of critical numeracy (Johnston 1994) and arguments for a socio-political science education (Hodson 2003) informed the teaching team's early ideas about implementing a critical education focus, particularly with ecological and social sustainability in mind (Chartres, Lloyd & Paige, 2003; Paige, Lloyd & Chartres, 2005).

The main aim of the two courses is to develop students' understanding of curriculum and their ability to teach mathematics and science and to plan for, resource and assess years 3 to 9 student learning. While an interdisciplinary approach with a critical mathematics and science focus enriches the courses it places tension on prioritising outcomes. This tension is visible when students undertake their practicums or school placements. The tension between aims, experiences and teaching approaches that reflect a critical mathematics education and those of a more traditional approach to mathematics is well documented by Skovsmose (1994) and Vithal (2003).

CRITICAL MATHEMATICS EDUCATION AND A LEARNING TOPIC

'Data and living sustainability' and 'A place in time' are two learning topics that have a critical mathematics education focus. Both engage students with the issue of sustainability. Through the topic 'Data and living sustainability' students identify their personal actions in living sustainably and then focus on water use from a personal and a global perspective. Currently South Australia is in drought and communities are implementing stringent water restrictions. One outcome for this learning topic sees students plan, undertake and critique strategic action to decrease their water footprint. The topic 'Data and living sustainability' has much to offer this paper but the context of working with data is well documented by Frankenstein (1998, 2001) and the context of resources (energy) is used as an example of project work by Skovsmose (1994). As such the discussion about how students engage with critical mathematics education is drawn from the learning topic 'A place in time.' My analysis of 'A place in time' identifies several aspects that support students to engage with critical mathematics education and two aspects that need further development.

'A place in time' was first developed from the need for students to familiarise themselves with their new university environment. It also presented an opportunity to engage students with a learning topic that modelled an interdisciplinary approach and introduce them to a critical perspective for mathematics and science education. The program, courses and campus were new to both students and academic staff when the topic was first taught. 'A place in time' has since proved to be a powerful learning topic for subsequent years' students. Two strengths of this learning topic with respect to critical mathematics education are the real-life context (Ernest, 2001; Skovsmose,

1994) and deep, complex questions (Frankenstein, 1998) that situate student learning. Students focus their investigation of ecological sustainability on a campus that is undergoing continuous redevelopment and provides opportunities for students to experience the impact of human activity first hand. Deep questions, "Where are we?" "Who and what was here, is here, maybe here in the future?" "What is the impact of human activity?" "Which aspects of the physical and biological environments would you prefer to be here in fifty years time and what action is required for this to happen?" frame the students' investigations.

A further strength of 'A Place in time' with respect to critical mathematics (and science) education is its interdisciplinary approach. Student pairs begin by exploring many campus locations with respect to the built and natural environment and from this choose a mature tree and its surrounds as their adopted space – their 'place in time.' Over three, three-hour workshops and out of class tasks, students use a mathematics lens, a science lens and an environmental lens to investigate aspects of the physical, biological, and socio-political dimensions of their adopted space. The mathematics lens supports students to use aspects of measurement, pattern and data handling to explore the location, size and characteristics of the physical, biological and social environments. The science lens supports students to investigate the surrounding physical environment through weather and soils and to investigate the biological environment through plant and animal structure, habitat and plant–animal relationships. The environmental lens introduces students to the ideas of future scenarios and sensory experiences as a means to explore change over time and the impact of human activity as part of this change. Here, interdisciplinary means using each lens to build a connected understanding of sustainability and explore key concepts that underpin sustainability including, interdependence, intergenerational and intragenerational fairness, equity within and between species, stewardship and ethical action.

'A place in time' reflects many characteristics of a landscape of investigation including, working collaboratively, posing questions, finding ways to answer these questions and building mathematical understanding as part of the investigation. Many student questions early in the topic are typically 'how' and 'what if' questions (Skovsmose, 1994, 2001a) where they negotiate and make decisions about the mathematics they apply and how they apply it and about the direction of their investigations. For example, questions like, "How can we measure the height and size of canopy of our tree?" "Which is the best kind of map to represent our place?" "How can we show directions on an aural map?" "What is the best way to sample who or what visits our space?" "How can we sort and represent the natural and manufactured items found in our space?" see students coming to grips with familiar and unfamiliar measuring strategies, making decisions about data they collect and how to work with this data. In some ways this approach places student democratic action as part of directing the learning experience (Skovsmose, 1994). Part of my role is to encourage students to share and decode their popular knowledge before inquiring about the academic mathematics to solve a problem. This strategy is more successful for students who are prepared to take risks. While this strategy is sometimes sabotaged by students' lack of confidence with mathematics and their view of one correct way, it does highlight that learning mathematics can be developed through an investigation (Skovsmose, 2001a). The approach also reflects Frankenstein's goal of understanding the mathematics and, in part, Knijnik's ethnomathematics approach but falls short of focussing on the power relations between the students' popular mathematics and the academic mathematics (Knijnik, 1998) or engaging with mathematics of political knowledge (Frankenstein, 1998).

The role of mathematics in a socio-political context is the most problematic aspect of 'A Place in time' with respect to critical mathematics education. Students apply mathematics to investigate their space and identify issues around human impact but they only superficially explore the socio-political milieu that controls their space. For example, the futures scenario asks students to consider what their space was like in the past and the impact of past and current human activity on their space. It also asks them to use their investigations to frame a preferred future for their space including what will be there and how it is used from an ecological sustainable perspective. Finally, the futures scenario asks students to describe the actions required to realise their preferred future. This is not extended to actually taking action where students would encounter the socio-political milieu more explicitly. That is, the topic does not explicitly engage students with the mathematics of political knowledge (Frankenstein, 1998) or with mathematics in action (Skovsmose, 2001b). Incorporating opportunities for students to investigate and critique how mathematics is used to plan for ecologically sustainable development on campus and critique the underlying economic and political interests would strengthen the critical mathematics education focus. So too would opportunities for students to participate in the planning process, share the results of their investigations and argue their views.

Students' reflections and critiques are focussed on their use of mathematics. For example, students reflect on and critique the appropriateness and reliability of the mathematics and strategies they choose over the three workshops. At times, they critique and provide feedback about the strategies and findings of their peers. This often happens informally during a debriefing period and more formally when students use a draft another pair's summative findings to locate and explore this pair's place in time. Reflection and critique is also a key part of an assessment task. The assessment task has two parts. Firstly, student pairs produce a pamphlet using each lens to describe their adopted space, its inhabitants and visitors, the impact of human activity, and a futures scenario. Secondly, each student produces an overview of how his/her investigation has impacted on personal perceptions of the adopted space, raised issues about sustainability and informed the suggested actions to preserve the adopted space. Finally, students describe what they consider to be the strengths of implementing a topic like 'A place in time' in a primary or middle years classroom and the challenges they may encounter when doing so. Broadening the topic to engage students with the socio-political milieu as described above may also provide opportunities to shift student critique towards using mathematics to critique the plans and actions of others rather than solely critique their application of mathematics.

CONCLUSION

Are my students engaged with critical mathematics education? The learning topics ‘A place in time’ and ‘Data and sustainability’ challenge students to consider the notion of critical mathematics education, a mathematics education that can empower and leads to citizenship and democratic participation. Such learning topics engage students with real issues drawn from their life worlds in a way that suggests the application of mathematics is not neutral and supports them to experience how their application of mathematics informs their decisions and actions. Having said this, ‘A place in time’ has some way to go to truly engage students with the socio-political milieu that is the landscape of critical mathematics education.

In closing, both learning topics draw on the idea of exemplarity (Skovsmose, 1994, p. 75). From a citizenship perspective, students engage with issues drawn from local communities that are examples for future investigations of similar issues. From the teaching perspective, students have participated in learning topics they can reflect on, adapt, refocus and implement as aspects of critical mathematics education in their future years 3 to 9 classrooms. It is my hope they do so.

NOTES

1. The University of South Australia’s Primary and Middle Years teacher education program is structured on seven principles. Namely, social justice and equity, futures thinking, sustainability (Education for one world), education for community living (Placed-Based Education), well being and relationships development, professional competence, and program and course delivery that reflects and models the first six principles.

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SOCIO-CULTURAL INFLUENCES ON CHILDREN'S CONCEPTIONS OF CHANCE AND PROBABILITY

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This study investigates conceptions of chance and probability held by children who live and attend schools in different socio-cultural settings in Greece and Jordan.[1] Different socio-cultural settings attribute different values in those social activities that the concepts of chance and probability are rooted and referred to. Furthermore, people conceive differently world's control by God and hence causality of everyday life events due to their different religious beliefs and their associated social values. Our research evidence suggests that religious beliefs and social values may be considered as an important influence on children's conceptions of chance and probability conforming or contradicting their mathematics teaching.

INTRODUCTION

Mathematics education as cultural induction has been well researched over the last twenty years (e.g., Bishop, 1988, Seah & Bishop, 2002, Wilson, 1986) and this research clearly shows that values are an integral part of mathematics learning and teaching. As Bishop (2001: 347) put it:

> "values exist on all levels of human relationships. On the individual level, learners have their own preferences and abilities that predispose them to value certain activities more than others. In the classroom, values are inherent in the negotiation of meanings between teacher and students and among the students themselves. At the institutional level, we enter the political world. Here, members of organisations engage in debates about both deep and superficial issues, including priorities in determining local curricula, schedules, teaching approaches, and so on. The larger political scene is at the societal level, where powerful institutions determine national and state priorities for mathematics curricula, teacher-preparation requirements, and other issues. Finally, at the cultural level, the very sources of knowledge, beliefs, and language influence our values in mathematics education. Further, different cultures influence values in different ways. Cultures do not all share the same values."

It may be claimed, therefore, that values also shape children's mathematical thinking as school learners. In this context mathematics learning and teaching may not be considered culture-free – needless to say value-free – as is the case of mathematical knowledge itself. Mathematics learning and teaching are culturally situated and mathematical knowledge is culturally based, embedding social and cultural values, in an overt or hidden way.

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THE 'EMBEDDED VALUES' IN MATHEMATICS

Bishop (1988) has described three pairs of complementary sets of values created during the development of modern mathematics. These values are thereafter carried by, and transferred through, the school mathematics all over the world, thus being indispensable of the dominant/Western culture of mathematics education.

The first pair of values is "*rationalism*", involving ideas such as logical and hypothetical reasoning and "*objectism*", involving ideas such as symbolising and concretising ideas. The second pair of values includes the value of "*control*", involving the security that mathematics offers through its rules and their applications to non-mathematical situations and the value of "*progress*" characterising mathematical knowledge and its development through a continuous search of mathematicians for change and alternatives. The third pair of values embedded in mathematics includes the value of "*openness*" related to the public verification of mathematical knowledge by published proofs and demonstrations; through these proofs and demonstrations mathematics is considered to be open to examination by anybody having the necessary prerequisite knowledge. Complementary to the value of "openness" is the value of "*mystery*" covering the sources of mathematical ideas as well as their results.

If these six values are associated to mathematics as a school subject commonly taught around the world, what about individual concepts, symbols, practices and products of the mathematical activity?

Each mathematical concept, symbol, practice and product can be seen as having two aspects. One aspect relates to its mathematical meaning that is acquired by a particular mathematical theory in which it is embedded. The other aspect of the same mathematical construct relates to the values associated by the people employing it in their everyday activities and/or by various communities using mathematics in their practices. This second aspect is socially and historically determined, since one mathematical construct can be valued in one context and de-valued in another while its value in the same context can change over time due to social changes.

Mathematical constructs, just as many other mental constructs, are used to describe and manipulate real world situations. Any such description or manipulation of a real world situation may be considered as a micro-theory of that particular aspect of the real world; in such a case any implicated mathematical statement may be interpreted as a statement about that aspect of reality. The addition on integers for instance, may be employed to and considered as a reasonable micro-theory of a particular financial transaction. Thus, any mathematical statement implied by that particular use of addition to a particular financial transaction is interpreted as a statement about that particular aspect of social reality.

Real world situations, however, acquire their meanings by implicated human activities, that are always intentional, therefore meaningful. Consequently, real world situations and their representations, as well as associated human activities bear meanings that are never value-free. The mathematical constructs used to describe and/or manipulate real world situations reflect these particular value-laden meanings and, at the same time, contribute to the valuing of meanings imposing thus on the real world situations the values of the dominant culture of mathematics.

CHILDREN'S CONCEPTIONS OF CHANCE AND PROBABILITY

Since the first study of Piaget and Inhelder (1975) on the development of the idea of chance in children, literature on children's conceptions of chance and probability as well as on the influence of teaching on these conceptions has considerably grown. A concise review of the research on probabilistic thinking, as well as on the learning and teaching of probability during the 50 years from the 1950s onwards is provided by Jones & Thornton (2005).

Although, an overview of this literature is not feasible in our paper, an overall comment is necessary. The powerful impact of Piagetian perspective through its guiding principles of psychologisation and individualisation seems to have promoted a context-free approach of the study of children's conceptions of chance and probability, as well as of their learning and teaching in schools. The knowledge acquisition of chance and probability has not been problematised from a socio-cultural point of view, in spite of the historical reviews which underline the influential role played by philosophical ideas on the formation and development of these concepts and their relative ones (e.g. Batanero et al., 2005). Consequently, contextual aspects as represented by social, cultural and political factors has not been given considerable attention, in contrast to factors related to either psychological or instrumental aspects of learning and teaching concepts of chance and probability, as well as probabilistic thinking.

Few cross-cultural psychology studies approaching probabilistic thinking mainly from a decision-making approach (e.g. Lau & Ranyard, 2005; Wright & Phillips, 1980) seem not to have influenced the relevant research in mathematics education.

SOCIO-CULTURAL INFLUENCES ON CHILDREN'S CONCEPTIONS OF CHANCE AND PROBABILITY: A RESEARCH EVIDENCE

An exemption of the mainstream research on children's conceptions of chance and probability is the study of Amir and Williams (1999) investigating cultural influences on the thinking about "chance" and "luck" of 11-12 year old children in England, partly of Asian origin, in relation to their stories about these concepts, their religious beliefs about certain events, their use of language expressing probability and their experience of games of chance and other probabilistic phenomena. A main conclusion of this study is that language, beliefs and experience of children, as components of their culture, influence their informal knowledge of probability while a significant proportion of children (Muslim more so than Christian) revealed superstitions and attributed outcomes of chance events to God. The researchers claim that verbal ability of children incorporating a large part of cultural differences accounted for the differences observed in probabilistic thinking.

The rationale and results of this research have influenced our study's approach; the research reported here was carried out in two stages. In the first, descriptive stage data collected using questionnaires from 6th grade children (10-11 year old) in (a) two elementary schools in Amman, Jordan (72 pupils of Arabian origin, Arabian speakers and Muslims), (b) three schools in Thessaloniki, Greece (75 pupils of Greek origin, Greek speakers and Christians) and (c) one Arabian school of the Palestinian community in Thessaloniki, Greece (11 pupils of which one parent is of Arabian origin, Greek and Arabian speakers and Muslims). At the 6th grade of the elementary school in both countries children had little formal learning on the subject of probability, thus making it easier to induce them to express their informal ideas.

The questionnaire included open questions based on ideas from the relevant literature (e.g. Amir & Williams, 1999; Konold, 1989; Shaughnessy, 1981), intended to elicit children's conceptions of chance and probability. The first part of the questionnaire included questions focusing on the meanings ascribed by the children to the words, which refer to chance and probability. In the second part, children were asked to attribute the cause of an unexpected event happened to them to one of the following: chance, probability, fate or destiny, God's will or to any other causality, making three choices according to their first, second or third thought. The last part of the questionnaire selected children's personal data such as age, gender, parents' origin, religion and languages spoken at home.

In the second, explanatory stage of our research we investigated the conceptions of chance and probability of six children selected from each school, through semi-structured interviews. These interviews further discussed with children questions included in the questionnaire, aiming at the illumination of their thinking on chance and probability, their precision in understanding words expressing chance (e.g., random, possible, frequent, rare, etc.) and their relevant experiences, especially games of chance they played. Particular attention was given to their attributions of unexpected events in relation to their religion beliefs, especially their beliefs about the role of God in the world, as well as their relevant superstitions, if any.

Preliminary observations and conclusions generated by the analysis of the evidence collected both by questionnaires and interviews are outlined in the following.

Children's conceptions of chance and probability

Children's answers to the relevant questions, summarised in the following tables, reveal similarities but also remarkable differences regarding their conceptions of chance and probability, that may be related to their different socio-cultural contexts.

Chance is identified by children with	Jordan N = 72		Greece N = 86	
Sudden or unexpected	36	50%	40	46%
Luck	1	1%	20	23%
Unwanted	2	3%	11	13%
Not planned	25	35%		
Happening once	6	8%		
Unclear	2	3%		
Error			12	14%
Unknown			3	4%
Note. Percentages in this table are column percentages.				

Table 1. Children's conceptions of chance

Most children in both countries (in Greece irrespective of their origin) adopting a similar conception identify chance with the sudden or the unexpected event. However, a considerable number of children living in Jordan consider chance to mean something that is not planned, a conception very close to the previous one, in contrast to an equally considerable number of children living in Greece who conceive chance as luck, unwanted or error. In their interviews, children illuminate these conceptions of chance by offering examples from their everyday experiences, as for instance are "meeting unexpectedly a friend", "finding something in the street", "witnessing an accident", "making a mistake in exams". In contrast, they report "a planned trip", "an intended something", "a justified action", "something difficult to be happen" as a non-random event.

Striking similarities are also found in conceptions of probability held by children in both countries. Most children identify probability with uncertainty, while a remarkable percentage of children living in Greece (irrespective of their origin) interpret probability as a hope for an event to happen. In contrast, children living in Jordan adopt either a mathematically referring meaning associating probability with a percentage rather influenced by school instruction or a meaning of choice between options. The later interpretation of probability is close to the meaning of opportunity given by a small number of children living in Greece. Children illuminate these conceptions of probability by offering examples from their everyday life experiences, as for instance weather forecasts, fulfilment of wishes, football game results. In addition to these examples, children living in Jordan offered examples of probability referring on war incidents or peace agreements.

Probability is identified by children with	Jordan N = 72		Greece N = 86	
Uncertainty	40	56%	41	48%
Hope to happen	6	8%	35	41%
May be happen	6	8%	5	6%
Percentage	15	21%		
Choice	5	7%		
Opportunity			5	6%
Note. Percentages in this table are column percentages.				

Table 2: Children's conceptions of probability

Overall children's conceptions of chance and probability seems to stem from intuitions and feelings about variation, certainty, what might or what cannot happen. Conceptions, which have also been recorded and analysed in relevant research literature (e.g. Amir & Williams, 1999; Konold, 1989; Shaughnessy, 1981), however not examined in relationship to different socio-cultural contexts.

Children's attributions of random events

As mentioned, children were asked to attribute the cause of an unexpected event happened to them to one of the following: chance, probability, fate or destiny, God's will or to any other causality, making three choices according to their first, second or third thought. Their answers are summarised in Table 3. We have to underline that none of the children attributed an unexpected event to fate, superstition or other natural causality, a finding inconsistent with the results reported in the research of Amir & Williams (1999).

It is evident that causalities attributed to an unexpected event by children living in Jordan differ significantly from those of children living in Greece. The formers are mainly referred to God's will (65% in a first thought) while the later is related to chance (56% in a first thought).

This finding may be ascribed to the different socio-cultural contexts of the children's life. In a few words, the Arab-Jordanian context is characterised by a strong influence of the Muslim religion and by a widespread consequent personal and social ethos. In the case of chance and probability, Muslim doctrine asserts that God controls the life span and life story and especially the fortune of every person. Thus, Muslims when referring to the future usually qualify any predictions of what will come to pass with the phrase "Insha'Allah" (if God willed it), recognising that human knowledge of the future is limited and that all (that may or may not happen) are under God's control. In their interviews children affirmed these beliefs when questioned about their religion beliefs, especially about God's role in the control of natural phenomena and human affairs. At the same time, the social practices which provide the ground for the emergence of chance and probability (initially as informal conceptions and finally as formal mathematical constructions), such as are gambling or betting, are not only undervalued, but they are prohibited in Muslim societies. However, they are these practices which brought forth the concepts of chance and probability during the 15th century in Western Europe and provided later on the ground for the theory of probability. A mathematical theory which valued and gained its scientific status, as developing capitalism extended the existing fields of its applications and introduced new ones, as for instance are insurance, pensions etc.

On the contrary, Greek context is determined in most aspects by the defining elements of the developed capitalism of Western Europe. Although religiously homogeneous, Greek society is not socio-culturally uniform, and from our research's viewpoint it is a rather contradictory one. Modern beliefs and life styles are prevailing but they co-exist with conservative Christian religious beliefs and practices, supporting a relevant personal and social ethos characterising particular population groups. For example, gambling and betting are widespread social practices however disapproved by the strict Christian doctrine and its followers.

	Chance		Probability as frequency		God's will	
Children in Jordan attribute an unexpected event to. N = 72						
in a 1st thought	20	28%	5	7%	**47**	**65%**
in a 2nd thought	12	17%	**45**	**62%**	15	21%
in a 3rd thought	**40**	**56%**	19	26%	13	18%
Children in Greece attribute an unexpected event to. N = 86						
in a 1st thought	**48**	**56%**	16	19%	22	25%
in a 2nd thought	10	12%	**43**	**50%**	33	38%
in a 3rd thought	27	31%	26	31%	**33**	**38%**
Note. Percentages in this table are column percentages.						

Table 3. Children's attributions of an unexpected event

However, significant differences between children in Greece independent of their origin (Greek or Arabian) and religion (Christians or Muslims) were not found in their attributions of an unexpected event. Furthermore, a relatively high percentage of them attributed an unexpected event happened to God's will (25% in a first and 38% in a second and third thought).

Searching for explanation various questions were posed to the children living in Greece during their interview (18 children of Greek origin/Christians and 6 of Arabian origin/Muslims). The following is a specimen question. Seven specific events in various contexts were described (success in school exams, rainy weather, ace in throwing a dice, a football game score, head in tossing a coin, beginning of a war, a road accident) and children were asked to attribute each one to chance, human action or God's will. The children's answers are shown in Table 4.

	Children Greek origin/Christians N = 18			Children Arabian origin/Muslims N = 6		
Attributions / Events	chance	human action	God's will	chance	human action	God's will
Success in school exams	6	12		1	5	
Rainy weather tomorrow	3		15			6
Ace in throwing a dice	18			5	1	
A win in football	6	12		2	4	
Head in tossing a coin	15	3		5	1	
Beginning of a war	3	15			3	3
A road accident	6	9	3	2	4	
Note. Numbers in this table are frequencies.						

Table 4. Children's attributions of specific events

Considering proportionately the frequencies of children's answers it may be certified that no remarkable differences exist between the two groups.

Thus, according to the evidence drawn from the interviews about religion beliefs and practices of the children and their families, the following claim may be put forward. The considerable number of Greek origin children who attribute an unexpected event to God's will are strongly religious and they assign anything that happens in the world (natural and social) to God's control. That is, strong religion beliefs either Christian or Muslim influence in a rather equal extent the conceptions of chance and probability, therefore the probabilistic thinking of people.

However, as several insights provided by interviews of the children indicated, beliefs in God's will and probabilistic thinking may be compatible in some cases leaving space to the formation of chance and probability conceptions. This happens whenever children believe that God intervenes in natural phenomena and human affairs in some cases considered important according to the religious doctrine. In other cases God leaves them to chance or to human initiatives. This can explain the differences in the children's attributions of an unexpected event to chance, probability or God's will, even in cases they hold strong religious beliefs.

CONCLUDING REMARKS

Adopting and modifying the idea of categorising knowledge on the basis of its social genesis proposed by Billet (1998), we may claim that the thinking and feeling involved in valuing take place in the context of a:

- Macro-genetic development of the historically derived mathematical knowledge, where cultures (whether intellectual, societal or institutional) form and validate values. It is on this level that the above mentioned dominant/Western mathematical values described by Bishop (1988) were created and are reproduced by academics, researchers, teacher educators, etc.

- Meso-genetic development of knowledge, where the historically derived mathematical knowledge is transformed by cultural needs and norms guiding practice. On this level mathematics curricula and philosophies of mathematics education, mathematics textbooks, students' evaluation and assessment practices, ethos of school mathematics teachers, etc. value aspects of mathematical knowledge.

- Micro-genetic development of knowledge, where everyday mathematics practices and the knowledge negotiation between teachers and students are shaped by/and shape values and norms of teachers and students alike.

- Ontogenetic development of knowledge, in which individuals internalise the socially determined mathematical knowledge through their participation in multiple overlapping communities. It is on this level that personal experiences of learning and using mathematics (in formal, informal and non-formal settings) as well as the relationships, if any, between in-school and out-of-school mathematics value mathematical knowledge.

At this ontogenetic level of mathematical knowledge our research may be considered as a first attempt to trace influences of socio-cultural values on children's conceptions of chance and probability. The underlying notion of our research presumes that values incorporate and determine a person's stance towards what is desirable by using criteria established on both feelings and thoughts. Therefore, a person's values about particular mathematical concepts, symbols, practices and products of mathematical activity crucially shape one's own knowledge and furthermore thinking.

NOTE

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CRITICAL MATHEMATICAL EDUCATION AND STS STUDIES: APPROACHES TO DISCUSS A RESEARCH

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Students of mathematical subjects in Brazilian universities show many learning difficulties and professors complain about the education they received in previous years. Official documentation presents suggestions that range from the usage of mathematical modeling and project work among others. Critical Mathematics Education (CME) and the studies of Science Technology and Society (STS Studies) are approaches that enable working with the contents of mathematics and other sciences in an integrated way with a critical and thoughtful vision. We have discussed the data collected in a research performed with High School students regarding these approaches and point out the possibilities of inserting context into mathematics teaching.

INTRODUCTION

Mathematics teaching in Brazil at any level is in crisis. This assertion is not original, nor recent since for decades we have been detecting learning problems in our students and proposing solutions that invoke many theories and different methodological approaches to support the proposals. In recent years, there is an increase in offerings of courses at undergraduate level, especially in private institutions, but also there was admission of students with scholarships and the ones who were admitted by means of the quota policy implemented by some public universities. Facing this situation, mathematics university professors started looking for the reasons to be blamed for the difficulties faced in teaching subjects such as Differential and Integral Calculus, the latter being the villain among all students of the courses of Exact Sciences.

One of the most recent complaints of professors is directed at teaching in the earlier educational levels, i.e., the mathematics that is not being taught properly in high school and the last years of junior high. On the other hand, teachers in those levels complain about the difficulties that students bring from the earlier school grades and in an attempt to reach the root of the problem everybody protests about the mathematical bases of the teachers in charge of the first four grades.

Besides students coming from regular high school courses, students coming from Adults and Young Adults Education – who in a few years learn all the contents that should have been developed in regular courses in more years, that due to personal, professional and mainly financial reasons they could not take - are also candidates to places in higher education courses.

Due to this scenario that in some points is very similar to other countries, it is important to think about possible approaches that may modify, even if in a small

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scale, a situation that afflicts professors, takes away the motivation of students and has economical, political and social consequences.

In this paper we present some common aspects of Critical Mathematics Education (CME) and the studies of Science, Technology and Society (STS Studies) with the objective of reflecting about the possibilities offered by these approaches to help students and professors to overcome somehow the difficulties of the process of teaching and learning mathematics.

THE OFFICIAL SOLUTIONS: PARAMETERS AND CURRICULUM GUIDELINES FOR UNDERGRADUATE COURSES

When we talk about suggestions for mathematics education, what immediately comes to our minds is the Brazilian National Curriculum Parameters (Parâmetros Curriculares Nacionais, PCNs) for School Education (Ministério da Educação e Cultura [MEC], 1998, 2000). These documents present a summary of what has been studied in terms of teaching and learning in any curricular component. We intend to mention here only the elements related to mathematics and specifically the topics that align with the perspectives in which we are focusing and are close to the mathematics of life and society.

For Elementary School, the PCNs indicate as general objectives among other items to "identify mathematical knowledge as a means to understand and transform the world that surrounds them" and "provide systematic observations of quantitative and qualitative aspects of reality". (MEC, 1998, pp. 47-48). For high school, the PCNs indicate among other objectives, the application of mathematical knowledge to "many different situations - using it in the interpretation of science, in technological activity and everyday activities" (MEC, 2000, p. 42). Also the development of skills related to the socio-cultural context demonstrating the need to "develop the ability to use mathematics in the interpretation of and intervention in reality." (MEC, 2000, p. 46).

In order to update the document, the Fundamental Education Agency (Secretaria de Educação Básica) of the Ministry of Education prepared the Curricular Guidance for High School (MEC, 2006), in which we found work suggestions concerning mathematics:

> In recent years, studies in mathematical education have also shown that an idea for making mathematics more effective in schools is that of mathematical modeling that can be understood as an ability to transform real world problems into mathematical problems and solve them, interpreting their solutions in real world language. (p. 84).

Further on after considering that this modeling has connections with problem resolution, this document also indicates that:

> Articulated with the mathematical modeling idea is the alternative of working with projects. One project may simulate the creation of organization strategies of school knowledge when integrating different disciplinary learning. It may be triggered from a

> very specific problem or from something more general, from a set of interrelated themes or questions. But above all it must have as its priority the study of a theme that is of the students' interest in a way that it promotes social interaction and reflection about problems that are part of their reality. (p. 85)

Initially PCNs had raised many questionings since not all approaches were familiar to teachers. Therefore many new clarifying documents focused more on content and with new references to studies at schools became necessary. Mathematics teaching formation courses started to spread the PCN and discuss its guidance at the same time teachers were receiving also new work guidance in the same courses (MEC, 2001) that point out competencies and skills to be developed by future teachers and professionals such as the establishment of relations between mathematics and other areas of knowledge, the intelligence about current issues and the understanding of the impact of the solutions found in a global and social context.

Taking into consideration all these elements pointed out by the official documentation, one might think that applying the suggestions proposed would provide a mathematics education committed to reality and other areas of knowledge, developed with the help of methodologies well-known to professors. However how do these educators understand the work done with modeling? Is there only one single accepted definition for this approach? Working with projects and with modeling are distinctive proposals? How can the future teacher determine the relations between mathematics and the other areas of knowledge if he or she does not receive an overview of those areas during his or hers education? And how can the impact of the solutions found for the proposed problems be measured and criticized?

We do not assume that any of the approaches discussed below will be able by themselves to give the answers to so many inquiries, but we will present some of the results of a study performed with high school students. We hope that their analysis may show the path to news experiences in classroom that take into account ideas from Critical Mathematics Education and STS Studies.

CRITICAL MATHEMATICS EDUCATION AND STS STUDIES

In Brazil Critical Mathematics Education is a perspective that has been promoted by means of papers and books by the Danish researcher Ole Skovsmose. In his own words,

> [...] critical mathematics education is concerned about the different possible roles which mathematics education could play in a particular socio-political setting. (Skovsmose, 2007, p. 74)

Borba and Skovsmose (2001) discuss the ideology of certainty in mathematics, mentioning the absolutist basis of this ideology employed by society when mathematics is needed to economical or political decision-making and justified by the purity, impartiality and trustworthiness of this science. Skovsmose (2007) also mentions the fact that mathematics has a formatting power, for instance when mathematical models are used to manage economy and built indexes are the basis for decision-making.

Skovsmose (2000) proposes also the creation of landscapes for inquiry involving environments that support research works in which students explore real problems reflecting about them.

Transversal themes, interdisciplinary studies, some methodological approaches proposed by PCNs, such as mathematical modeling itself and the work with projects can be approached with a critical view. However it is necessary to take into consideration the education of teachers in regard to this kind of activity. How are these aspects being dealt with in initial or continuous education courses? We have to consider that many mathematics teachers performing in Basic Education do not have postgraduate degrees and some of them do not even have full undergraduate degrees in mathematics. How can we make these ideas accessible to them? This is a challenge that is becoming permanent especially if we observe that in official tests such as the National High School Examination, the text of the questions show an interdisciplinary view.

But there is also another focus that may be involved in the development of the critical thought and the relation of mathematics with the real world and other sciences: the STS studies. Developed in the mid 1960s and in the 1970s, Science Technology and Society Studies emerged as a response to the doubts raised by the scientific and technological advancements generated since the 19th century. When questioning current paradigms over science with the spreading of Kuhn's ideas and reconsidering the role of technology, ecological and pacifistic social movements have exposed the consequences of a disordered growth of science and technology and the dangers to social well-being (Auler, 2003; Nascimento & von Linsingen, 2006).

In educational terms, the STS focus is present especially through science teaching where there is focus on real and current subjects and problems such as the environment, natural resources, space exploration, the cloning of living beings etc.

As for mathematics teaching, the experiences with STS studies are few in Brazil with highlight to some that have been approached in interdisciplinary combined projects. (Angotti & Auth, 2001; Pinheiro, 2005). Nevertheless we see possibilities of discussing the historical development of mathematics and its influence in the development of society as well as in its destruction, the discussions about the harmful use of data in mathematical and statistical models, the debate about the power of exclusion of mathematics by the ones who own its knowledge among other aspects that have already been pointed out by the Critical Mathematics Education (Cury & Bazzo, 2001). This way it will be possible to show students that mathematics is a human construction and inquire about the "absolute certainties" of this science, helping them to make decisions about problems in which mathematical contents are involved.

Pinheiro (2005) highlights the recommendations of the high school PCNs to create a critical, ethical citizen integrated to the labor world and able to keep continuously learning. However the author herself mentions that it is hard to create a critical citizen with the teaching and learning currently employed and that it is an urgent need to think of new ways of working. She suggested the adoption of the ideas of CME and STS studies, despite the fact that there are no “tested formulas”.

According to this gathering of ideas from Critical Mathematics Education and STS focus, we believe that there are possibilities for new experiences in mathematics teaching under these approaches. But how could these ideas be introduced to classrooms? How are the perceptions of students in regard to mathematics, sciences and society? In order to explain these issues, we present below part of a study performed with high school students

SOME DATA OF AN INVESTIGATION

Data here presented are part of a master degree research with the objective of identifying the opinion of students about the subject mathematics, evaluating the relations established between mathematics and sciences and analyzing students’ view about the relation between mathematical topics and everyday events. The study was performed among students of the second and third grades of high school in a public school of the Greater Porto Alegre, Brazil. In every step of the research we will highlight only the elements connected to the topic approached in this paper.

Initially 143 students of the second grade were submitted to a questionnaire with open questions. We pointed out the 23 students who stated that mathematics is the subject with which they least identify themselves and we will point out their answers to two of the questions. In the first one, students were requested to expose their opinion about the subject mathematics. The answers were separated in classes, according to the key idea of the students: mathematics is difficult (74%), it is useless (9%) and it deals with too many numbers and calculations (13%). The other question of the research asked whether the student could see any relation between the contents studied in the subject and everyday facts of his/her life. In case of an affirmative answer, the student was asked to exemplify. Forty-eight per cent of the students said that there was no relation between the subject and their regular lives. Twenty-six per cent mentioned a relation with money or trade business and twenty-two per cent answered yes, but gave no reason.

Due to these results we considered that we should go deeper into these topics and on the second step of the investigation performed on the next scholar year, we applied a new questionnaire to the third grade high school class that had participated in the first step as the kickoff of the activities we developed with those students. This time it was a multiple-choice questionnaire. From the questions presented, we point out here three of them. The first one used some of the answers gave by students in the previous questionnaire and asked their opinion about mathematics as a subject. More than fifty per cent of the students considered that it is a difficult subject, that does not bring much meaning and most of the time is useless in everyday activities.

Then we tried to investigate how students see the relation between sciences and mathematics. In this question, 65% of students think that sciences use mathematics only for calculations to corroborate numerical data. We believe that these students have only been using given formulas in their physics, chemistry or biology classes and did not see any further relation among the many sciences.

The third question tried again to detect the role of mathematics in the lives of students from the answers given by them previously. The sum of the percentage of those who consider it important for trade business and to deal with money and the ones who mentioned its importance to economy and finances is 74% of the students, demonstrating that for them these are the most important uses of the subject. It is important to highlight here that most of these students face financial problems and many of them already work especially in sales.

After applying the questionnaire we brought to this class four texts taken from newspaper, magazines or the Internet, with current topics in which mathematics is present. Each group was asked to read a piece of text allocated to them and answer questions referring to the topic approached. We chose to report here the activity related to a text about health. People live everyday surrounded by quantitative data such as medicine instructions, weight and height tables, blood component measurements, etc. We wanted to verify if students were able to understand text information and solve problems related to the data.

Two groups of students worked with this activity, a group of four students and another of three. The groups had a tape measure and a pair of scales. First we asked them what is necessary to calculate the Body Mass Index (BMI) and students demonstrated to have an understanding of it. Next we asked them to calculate the BMI of each member of the group and classify the person according to the table that was part of the text: thin, normal, overweight and obese. In this case students faced many difficulties in the calculations. Besides that, one of the groups instead of dividing the weight by the square of the height, divided it by the double of the height. This kind of mistake seems to be common because many students, even high school ones, still use to multiply the base by the exponent of the power.

After performing these activities, the class watched a documentary, Einstein's Equation of Life and Death, with the objective of bringing mathematics closer to sciences since the students did not seem to view any relations between these areas.

Then we applied a questionnaire with three open questions in which the students were asked to comment on the role of mathematics in society and sciences and the need of mathematical contents in their future and the possibility of understanding and discussing everyday topics involving mathematics. After a general evaluation of the responses of the students, we observed that they:

a) recognized the role of mathematics in the lives of human beings;

b) had a superficial overview about the relation between mathematics and daily facts, seeing no connection between the contents studied at school and the mathematical elements present in daily actions even after performing activities that involved the analysis of tables and graphs, calculation of interests and BMI.

c) believed that they need the mathematics learned at school to pass official examinations, university entrance exam and the four basic operation on an day-to-day basis;

d) considered it possible to understand and discuss topics that involved mathematics shown in communication media, only if they comprised contents already learned at school.

Finally we interviewed six of the students to gain a deeper understanding of their opinions. They corroborated the ideas supported previously that mathematics is difficult and only deals with numbers and calculations. They did not relate the mathematics they study with the mathematics used in their daily lives, saying "it is not the same mathematics". Even having enjoyed the activities proposed and the opportunity of learning the life and work of Einstein, students kept their opinion that mathematics classes are "always the same, correction of exercises, calculations and tests".

INQUIRY ABOUT THE STUDY DATA UNDER THE PERSPECTIVES OF CME AND STS STUDIES

In the above report we demonstrated that the students of that suburb school group do not see any application of the contents studied in classroom in their daily activities or in topics mentioned in communication media. As we analyzed their answers to the BMI problem, we noticed that they were interested in the subject since young adults in Brazil worry very much about their body image and want their bodies to be according to the current fashion trends: thin and with well defined muscles. However when performing the calculations, not only in this activity but in all the four ones developed in class, they committed mistakes that made it almost impossible for them to answer the final questions of the task, since they needed coherent results in order to be able to talk about them. This shows us that the suggestion of working critically with problems and situations presented in mathematics classes, we cannot avoid the need of teaching content and methods that are inherent in this science. When commenting on the perspective for Mathematics Education, Matos (2005, p. 8), considers that the learning process of this science must include the appropriation of modes of understanding of the day-to-day mathematics but that

> Does not mean to set aside the concern about knowledge and the use of mathematical instruments and tools, but to transform the idea of school mathematics.

The debates about the BMI issue proposed to the students who participated in our research could include elements of Critical Mathematics Education and STS studies, since we could question current beauty standards and disorders caused by them, such as anorexia and bulimia, and also diseases resulting from bad nutrition and related to the excessive ingestion of salt or fat. We could also discuss the BMI itself, check its origin, the presuppositions in which it is based since it is an index almost unanimously accepted by doctors and nutritionists. We could even discuss the mathematics embedded in the packages of light and diet products whose tables with measurement units do not correspond to the ones we commonly use at home (a spoonful, a cup), and that some times may lead the user to an unbalanced nourishment. All these topics of discussion would meet the suggestions of the PCNs of integrating the knowledge of the school subject and give priority to themes that would interest the student and would also help students to create a critical sense about the data they found in their day-to-day activities.

There are many mathematical contents adjoining the BMI issue presented to the students, but it is necessary to prepare teachers to discuss them. Why are some contents emphasized in school books and programs? And particularly: why the others are not emphasized? (Matos, 2005). In Brazil one of the structured themes indicated by the PCNs is statistical literacy, but mathematics teacher education courses still give little emphasis to the teaching of probability and statistics (Viali, 2007). But information handling is essential to understand and whenever necessary, criticize and form an opinion about the data presented in communications media such as the result of opinion polls, for instance. Why do teachers avoid this topic? We may think that it may be difficult to show data without approaching the merit of the question, it is easier to keep the faith of an impartiality in mathematics.

We believe that it is important to spread the ideas of our research and the doubts that it brought so that more colleagues will get involved and engaged in debates about the theme and give some new suggestions to contextualize mathematics teaching.

FINAL CONSIDERATIONS

Wedege (1999) considers that mathematics is a contextualized activity and makes a distinction between two meanings of the term "context": task context and situation context. In the first case she considers that

> ´Context` representing reality in tasks, word problems, examples, textbooks, teaching materials, is closest to the linguistic fundamental meaning. (p. 206).

In another definition the term refers to a context to learn, use and know mathematics, such as the school or workplace or even the context of mathematical education as the educational system itself. To clarify the distinction, Wedege (1999) exemplifies it with a Swedish study in which students worked a day-to-day problem that consisted of determining the cost of sending a letter by mail in mathematics class and in social studies class. According to it, "the task-context is the same but the problem is solved in two different situation contexts." (p. 207).

When we talk about contextualizing mathematics teaching, the situation context is the mathematics classroom. Considering the research above described, students were working in a contextualized task in which reality was represented in the problems they had to solve. But how far have mathematics teachers contextualized their teaching? We believe that it is not enough to employ problems that have elements of reality or semi-reality, which Skovsmose (2000) understands as a situation built for learning purposes and that simulates reality. It is necessary that the problem makes sense to the student and especially that he/she will be able to think about it not only mathematically, but also under a critical perspective, questioning the text, the data and possible answers.

Therefore we would be following the suggestions of the PCNs, introducing an interdisciplinary view of the tasks we have proposed to students and helping them to have an education involving contemporary issues that have to do with their communities as well as with their personal lives.

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TEACHER EDUCATION AND CULTURE: UNDERSTANDING AND ASKING FOR CHANGES

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The purpose of this research lies in displacing the issue that was brought up - with much seriousness - about the (mathematics) teacher education in a perspective centered almost only in the formative process of the teachers while social/intellectual subject of his/her actions for a perspective allied to the culture that each student brings inside of him/herself. In this sense, the formation of teachers herein reflected - from an ethnomathematics perspective - can be recognized as a way to generate a structural change in the scope of the formation of teachers - or, at the least, to denounce that the student has not been completely out of the proposals of teacher education, but neither is quite on the target.

INTRODUCTION

> The school will treat all equally. However they ARE NOT ALIKE. Because of this, for some it will be enough what the school gives them; for others it will not. Some will triumph others will fail. This triumph will confirm those to whom society supplied the means to triumph. And the failure will usually confirm the disdain toward those society conditioned as inferior. (Nidelcoff, 1978, p. 25)

This text presents partial results of the research[1] on the subject of "Teacher education and culture: understanding and asking for changes" carried out in the period of 2004-2006. The research questions were born from an almost personal feeling of dissatisfaction and perplexity.

It is well known that a teacher's experience of a teacher education is a confluence of what happens in the teacher's individual/professional life and in his/her life in a group, that involves all the school community, the formers of curriculum, among others. However, when a reflection is directed upon teacher education from the point of view of ethnomathematics, the second aspect of the confluence - the "others" that constitute the group – acquires special value and the educator's contextualized knowledge becomes more intensely the central focus.

In this perspective, the research developed here has as central pre-occupation – the students' knowledge in relation teachers' transformation –, in the formation of teachers while line of research in (mathematics) education. In truth, the purpose here intended is that one of contemplate and reflect, in teachers educational processes, the students' and the teachers' socio-cultural vision, bringing them to reflect upon cultural diversity. In other words, the teachers' vision does not need uniquely to be constructed from the academic intellectual culture, from the dominant culture.

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FOCUS OF INTEREST: TEACHER EDUCATION

The focus of interest of this paper is in what regards the teacher education from the perspective of ethnomathematics. And, specially, it wants to call the researchers attention to the fact that in the immense volume of inquiries in this field of studies "the student is not completely aside of the proposals of teacher education, but neither is he/she within." (Domite, 2000, p. 44).

Various models have been proposed - sufficiently consistent and well constituted in terms of teacher transformation -, amongst some are not directly addressed to the teacher education as the social subject of his/her action and, therefore, still characterize the transmission type that leads, somewhat, to an impositive attitude; others are especially centred in the teacher as a constituted subject, centred in the types of transformation processes and in the formative dynamics itself (Cooney, 1999; Fiorentini, 1998; LLinares, 1995; Ponte, 1994; Schön, 1987; Shulman, 1986; Zeichner, 1993).

One of the axial themes that has guided the most current discussions is that of the reflective professor. The original ideas of the reflective practice come from Schön, since the 80s, who has discussed ways of operationalizing the reflection in action and the reflection on action. In some way, the movement appeared in the opposite direction of the idea that a teacher educator transmits amounts of information pre-established and began to reorient, in terms of world, the scholars' discussions of education reform and teacher education. The conceptions that guide the reflective formation of the teachers emphasize that the teacher education must have as a main goal the reflective self-development of the teacher, that is, to form teachers who learn to form themselves when facing affective-intellectuals problems of the pedagogical practice and of the educators reasoning.

As mentioned, looking in a specific way at the orientations proposed by teacher educators, there is very little concern in addressing it to a connection with students' knowledge. Certainly, these educational designers consider that the knowledge of the students must be in the formation proposals, but it is not clear how they have made it explicit.

Anyway, it is worthy pointing out here that some initiatives have been developed joined to the students' knowledge. The study group called "to grant reason to the student", formed by a sub-group of educators involved with reflective formation (Schön, 1992), studies the teachers who investigate the reasons that make the students say and express certain things. The vision of teaching and knowledge of the educator who "grants reason to the student" indicates that the student's knowledge has been part of formation proposals – it has been emphasized that the teacher should recognize and value the intuitive, experimental, daily knowledge of the pupil, as for example, looking to understand how a student "knows to change money, but does not know how to add numbers" (Schön, 1992).

Further, on mathematics educators directed toward teacher education focussing the student's information and his/her learning processes, it is worthwhile to highlight the reflections of Beatriz D'Ambrosio (1996). When emphasizing some characteristics to be incorporated by the mathematics teachers in face of the current curricular reforms, D'Ambrosio accentuates that one of helping "our students to establish a positive relationship with mathematics". In order to do this, she gives value to turning attention toward the previous knowledge of the student claiming that:

> The main ingredient of the teacher's decision regarding the direction of the classes and the student's learning is the discovery, by the teacher, of the student's knowledge. The student comes to the educational process with a wealth of experiences. The teaching of mathematics (and, in fact, of most of the school disciplines) is not based any longer on the structure of the discipline, but on the contrary, it is based on the student's knowledge. For that the teacher needs to organize the work in the classroom in a way that elicits the student's knowledge so that this knowledge can be analyzed. It is also important to create activities that will lead the student to seek in his/her experiences knowledge already formed" (D'Ambrosio, B., 1996).

In truth, rare are the inquiries that take into account the student's knowledge in teachers' transformation processes. It seems that everything happens as if a large parcel of the teacher educators were attentive and concordant with this issue, but the configuration of the majority of the proposals developed by them does not disclose direct incidence in terms of this orientation.

About teacher education in terms of the student's knowledge and ethnomathematics, we situated in Brazil educational history two projects on teacher education that have as central focus the student - the proposals by Freire and D'Ambrosio - that have not only called the teacher's attention to the worth and role of culture in the learning and teaching processes but have also sustained the idea that the students can not be developed in an isolated way, deprived of cultural identity.

The great search by Freire was, on one hand, to bring the teacher "to take as reference for learning the reality itself of the people", with the concern in seeing such reality related to in 'generating words' and represented in the 'coding' that is analyzed and discussed with this people (Freire, 1980). On the other hand, he tried to make the teacher turns him/herself toward his/her students and, through dialogue, tries to learn with them.

D'Ambrosio, in turn, has brought the teachers to realize that one of the biggest historical distortions has been to identify mathematics only with the European thinking, in particular in its origins, with the Greek thinking and, then, to situate the several contributions of diverse cultures to the formation of the contemporary mathematical thinking. It is here, in this appeal of D'Ambrosio, the germ of Ethnomathematics, study field worldly inaugurated by him.

In any event here is being brought up a question that can be taken as generating the discussion in general: “Can teacher education, as a practice of recovering the student’s culture, transform/reduce the segregating function of the school education?”

JUSTIFICATION

The research in question is justified by the fact that the lack of reference about the dialogue with the students (previous) knowledge in classroom - to a large extent of the studies on Teacher education - can make the teachers lose the possibility of: (a) activating focuses of dignity and self-esteem in those they want to call for (school) knowledge and, (b) activating interactive forces for classroom situations.

In truth, it seems to be reproduced in the scope of teacher education the tendency of the so-called traditional school in treating the students as if they were all equal, to consider that they know and are developed in the same way.

However, it may be recognized, on one hand, that the great majority of those who are involved with teacher education have clear that the critique that the school treats uniformly all the pupils as equal stands there for a long time, a consideration of socio-political-economic order, linked to the problematic of education and power, education and ideology and education and culture (Nidelcoff, 1978). On the other hand, when it is proposed to consider the limits and the interfaces among mathematical education, culture and teacher education, it may be easily noticed how difficult it is to place at one side teacher education and at another cultural issues - as well as it is not easy to provoke the deconstruction of the students evaluative neutrality.

This research paper is also justified by the fact of going after the theoretician-methodological option of the research in ethnomathematics, based on the ethnographic experience, trying to perceive the “other group”, from the angle of its logic, searching to understand it in its own rationality and terms. In general, in the scope of the research in ethnomathematics, the researcher experiments some estrangement and tension process since the quantitative and spatial relations observed in the investigated group – as long as it is not centered any longer exclusively in the explanations of the researcher’s society group - reveal many times, to him/her, disarticulated and, in general, a process of re-signification and analysis of the same ones calls for the creation of categories that involve articulation between mathematics and other areas of the knowledge as history, myths, economy, among others. Truly, such relations ask for articulation in a non-disciplinary dimension of knowledge, but rather in a transdisciplinary one.

This work results upon the approach of reflectiveness – from Giddens *point of view* – connected to the understanding of the influence of the etnomathematics movement. According to Giddens, any political principle or methodological purpose, even that ones filled with good intentions and contents, might tend to fail if there are not concrete subjects to proceed to the analysis and to the reflections upon them.

Moreover, these subjects are responsible for verifying the possibilities of such principles or purposes of becoming concrete, suggesting adaptations, allowing a new vigor in the fulfillment of new objectives. In other words, the balance between the theory and the fragmented practices has to be reflected upon, so that, in its continuous joining and confrontations of them, new alternatives be created.

In terms of learning-teaching, in turn, we could say that ethnomathematics suggests to the teacher to bring forth ways of reasoning, measuring, counting, drawing conclusions from the students, as well as searching to understand how culture is developed and reinforces the learning issues.

In fact, when the concern of an ethnomathematics study is the pedagogy of the mathematics, the attention has been situated around legitimizing the knowledge and information of the students born from experiences constructed in their own environments and to study possibilities to deal with the learning that comes from outside the school and from the school. In this sense, with the discussion of ethnomathematics, what is intended is to help the teacher "to establish cultural models of belief, thought and behavior" (Fasheh, 1997, p. 98), in the sense of not only reflecting upon the potential of pedagogical work that takes into account the knowing of the students but also the learning, by the school, more significant and that would give more power and dominion to the student over his/her own learning.

From the exposed, one may say that the central question of this research can thus be delineated: is it possible to recognize the interfaces between mathematical education, culture and teacher education in order to better understand the connections between ethnomathematics and teacher education?

METHOD

Several can be the reasons for the justification of the chosen path and method employed in an academic educational research, but the research question, in general, determines the most adequate way. In this sense, the research of the qualitative type is justified here because the procedures that are involved in such style can lead, in some way, to the recognition of mini-processes of thought by the mathematics teachers in the sense of acknowledging students' previous knowledge, as well as the relations that involve teacher and students.

In truth, when regarding an inquiry that tries to understand the "other" through his/her practice, in special in the educational scope, it is more consensual among the researchers that the entanglement with which everything is developed turns the isolation of the involved variables difficult – the treatment characteristic of the quantitative methods - and, mainly, a more clear, objective indication of those ones responsible for determined effect.

The great desire with this research is to take as groundings the principles of the participant research, characteristic form of the Popular Educational movements (Brandão, 1986; Freire, 1980) - an action resulting from an integrative process involving the individual, the school and the social context, actualized in a critical and transforming way. Thus, the attention will be wholly directed toward the subjects of research, the social conditions, the more or less intuitive "explanations" and the personal interpretations, loaded with emotion and the researcher's own elaboration.

In general, both in terms of gathering facts and analysis, the intention is to take into account at least two basic aspects: (a) the teachers will be not positioned in a null stage of reality knowledge – on the contrary, the starting point comes from the already existing conditions, that is, of a prior practice of the researcher and theirs, in a way to understand the need for change and, (b) when analyzing the facts, it will be attempted to establish the mediations and contradictions of the questions that constitute the investigated problematic matters, in order to overcome the ingenuous analysis of its first and previous impressions.

OBJECTIVES

These are the main objectives of the research in question:

- to bring the teachers to appreciate and to legitimize the (previous) knowledge and information of the students;
- to bring the teachers to understand always more the advantages in taking into account the student's culture in the process of teaching and learning mathematics;
- to understand grasp the possible connections between Ethnomathematics and the movement of Teacher Education while research areas and,
- to better understand what the ethnomathematics scholars would like to see in the movement of teacher education.

THE RESEARCH: IT'S DESCRIPTION

From the considered, regarding to the research progress it may be noticed an approximation to the studies of Paulo Freire, who was chosen as a central theoretician to answer the questionings here formulated, especially because his reflections have been dedicated to the exploration and the legitimating of the knowledge of the "other" and of the student who, in general, is formed and conformed within determinate relations of power. The greater intention is to activate the perception of teacher educators and teachers on their own unfamiliarity about who are their students, what do they know and how they know about these students, in order to propitiate one another speech, another way of seeing and of being teacher educators, in order to create opportunities of educators' transformation.

It is worth to highlight here that, in this search to incorporate the knowledge of the students in order to operate the dynamics of teacher education, it is present the expectation of always recognizing the student's needs and not on the opposite way. In other words, the dialogue between the need to develop the teacher education "with" the students and the theoretical-practical instruments of the school system must constitute a dialectical process – none of the poles of this tension must dominate the other.

With these concerns and since one of the basic presuppositions of ethnomathematics is in focusing, identifying and legitimizing the quantitative and spatial relations based on the knowledge of the "other", the research proposal in the scope of teacher education consists of: a) recognizing how much teachers are aware of the movement and literature on Teachers education in the educational field; b) searching an understanding of the conceptions of the teachers and researchers on (school) education and culture and, c) to problematizing processes that emerge in the social reality of a classroom, in which the knowledge of the pupil becomes (by force of circumstances) the axis of the teacher's concern.

To direct a systematic analysis on the concern of teachers in taking into account the (cultural) knowledge of the students - as well as of the other items mentioned - this research tries to collect information on the basis of two proposals. The first proposal was constituted of interviewing mathematics teachers, in service and postgraduates, supported by questions about teacher education and the main characteristics that, we teachers, need to have and develop when we decide to place as the centre of the teaching-learning process the feelings, attitudes, opinions, culture and previous knowledge of our students. The second proposal, and here is the focus of this research, was to **request the manifestation of the investigated individuals, based on the confrontation with a situation that is distinct from those of regular standards. The prepared script is as follows:**

- How would you go forward and continue the lessons like these that were presented to teacher Mário and teacher Janaína (two real cases). That is, in a first moment you are the teacher Mário and in a second the teacher Janaína, teachers who offered "to start the lesson with the speech of the students"…

First case: The teacher (Mário) begins, in one of his 5th grades, a conversation with his students on the calculating division, by asking:

Teacher: How do you calculate 125 divided by 8?

José, student who sells bubblegum at the traffic sign downtown, starts speaking:

José: We are more or less 10 "guys", almost all day long, some boys and some girls. Then, we divide like this: more for the girls, who are more responsible than the boys, more for the taller ones than the smaller ones".

Teacher: Give us an example, José. For example, how was the partition yesterday or the day before?

José: Ah! Like this ... there were 4 girls, one of which is small; 6 were tall boys and 2 more or less small. Then we were 12 and the gums were 60. Then, it was given half and half, a little more for the girls. The small girl ended up with 3 and the others with 6 or 7, I do not remember well... The boys...

Now you[2]: …

Second case: the teacher (Janaina) asks to the group of pupils of 4º semester of the course of education of adults:

Teacher: What do you know about percentage? How do you do the calculation of a percentage?

Luiz[3]: Even today I needed to make a calculation... 35% of 195 and I did like this... 19 + 19 + 19 and then plus 9,5. It's 30 plus 27 ... more or less 10.

Teacher: How did you get 19? Tell us a little about your way of calculating.

Luiz: Ah! I do not know why I did it like this... every time that percentage appears I divide by ten because somebody taught me this way, and I add the times that it appears... like this... 30% I add three times, 40% I add four times.

Teacher: And how did you get 9,5? Tell me the way you thought to do this.

Luiz: I know that one has to divide by two when it is 25% or 35% or 45%, but I do not know why I do this

Now you[4]: …

FROM THE ANALYSIS

In a general way, in the context of traditional formation of the mathematics teacher, as in his or her material and cultural conditions of work, it is usual to emphasize an evaluative regulation, certification and standardization of teaching behavior processes. According to Giroux, these processes occur "in spite of the creation of conditions for the sensitive, ethical and political roles that they are supposed to play/act, as public intellectuals enrolled/involved in tasks of bringing up students for a responsible and critical citizenship" (1985, p. 85).

So far, it was analysed some of the questions related to the second moment of the research - referring to the emergent situations in the classroom – with the examination of 25 answers. Surely, such examination, as a reflexive reading, was accomplished in a serious and reflective manner, but not so profoundly as it would be desirable to do in order to more deeply understand the different cultural, social and pedagogical points of view of the different mathematics teachers. Further insights can be gained from the analysis of the data.

Among the 25 in-service mathematics teachers, 14 are public school teachers with more than 10 years of experience, 6 with less than 10 years (3 of them also in private schools) and the last 5 are also postgraduates, 3 of them effective teachers in public education.

When sketching a theoretical picture for analytical purposes, it was possible to perceive that the recognition of the teachers, regarding themselves, as teachers in such presented lessons, happens in three axis: (a) the first axis refers to the teacher's desire of transforming the real situation into an exercise or mathematical problem (he/she is looking towards the teaching of mathematical content); (b) the second contemplates the reflective and interrogative teacher, and, (c) the third is represented by beliefs, values and power relations allied to pedagogical practice.

In general, the analysis of the pedagogical-practical situations brought some evidences on the types and kinds of attitudes that teachers seem to have incorporated and some subsidies to understand what they need in order to become people who recognize and legitimize the students' (cultural) knowledge.

It was possible to observe that, on one hand, teachers acquire identity in one determined school grade as active participant subjects of a part of knowing. This seems to occur after some years of teaching mathematics in different schools or in the same school, in different grades. Nothing changes, everyone teaches similarly the same things, little is obtained significantly in terms of conquests and innovations in the courses. It seems that the teacher does some kind of appropriating of a way of being professional in service, capable of contributing to education in terms of (mathematical) content with increasing worth and power, but almost nothing in terms of more open proposals that allow educators to formulate questions, to develop their own projects, to reflect on themselves as thinkers.

On the other hand, teachers seem to become more easily sensitive with discussions on key-notions for the "new" mathematics teacher that is always in construction: an individual who reflects on his/her reality with critical and constructive spirit, searching for solutions, betting on the collective reflection as renewals sources. In other words, the teacher revealed him/herself as capable of innovating and producing knowledge, taking the education of mathematics as objective, and occupies the position of one who can propose innovations, question mathematics teaching practices and suggest new paths.

In a very abridged way thus, some considerations from this first attempt of analysis can be highlighted:

- the mathematics teacher's education allied to the knowledge of the students is not incompatible with their formation on how to teach mathematics to children, teenagers and adults; on the contrary, it may be one of the aspects that can involve them;
- the mathematics teacher education allied to the knowledge of the students can help teachers on realizing that what we teach, or what we don't teach, is much more than a day in our students' lives; it is something that lasts forever in their lives and in the lives of those they interact with.

Finally, it could be said that, on one hand, a first look at these questions shows that there is a lot of work to be done and that their consideration can both inform our research techniques and also lead us to criticize our practice in teacher education. On the other hand, we can affirm that the national discussions in education can interpret the actions of the students and teachers, but they cannot significantly understand their social and cultural identities… in order to incorporate them in a national curriculum.

NOTES

1. Developed by first author. The second author has taken part in a section of data analysis in this research.

2. In this moment, the participant of the research should begin speaking, and the data of research were collected.

3. The student.

4. In this moment, the participant of the research should begin speaking, and the data of research were collected.

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TEACHERS' PRACTICE UNDER THE ETHNOMATHEMATICAL PERSPECTIVE: A STUDY CASE IN YOUNG AND ADULT EDUCATION

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This paper discusses the results of a case study, which investigated a middle school maths teacher's practice under an ethnomathematical perspective when teaching a group of students from young and adult education. Taking into account the dialogical interactions among the many and varied shapes of mathematical knowledge, this study tried to analyse not only the possibilities of teaching grounded on an ethnomathematical posture, but also the development of knowledge learning processes in a young and adult education classroom. It is pointed out that a long and continuous process of knowledge legitimation, for both teachers and students, characterizes teachers' practice under this perspective.

INTRODUCTION

This article aims at delivering the results of a research named *Ethnomathematics and EJA teachers´ continuous education: construction of dialogical spaces among varied mathematical knowledges*, which looked into the role of a Mathematics teacher under the ethnomathematics perspective in a young and adult education classroom[1].

First, this text will analyze some of the contributions of ethnomathematics to the field of young and adult education. Then, we present the methodologycal aspects, which helped us build this research. The third part of this article will show the results of this investigation, focusing dialogue disposition, the ability to establish relations and the experience of automony**.** The last part, the final considerations will indicate that an ethnomathematical approach in teachers' practice favours the process of knowledge legitimacy as a dual carriageway.

ETHNOMATHEMATICS AND YOUNG AND ADULT EDUCATION

Ethnomathematics is a branch in research and study within Mathematics Education, which has several contributions to offer to young and adult education.

Ethnomathematics, as a research area aimed at representing/ percieving/ shaping spatial and qualitative relations of diverse cultural forms, has been offering theoretical grounds to understand the many ways different sociocultural groups reason their mathematical knowledges. As for these students, the young and the adults ones, these knowledges are built along their domestic and professional lives and their previous academical experiences.

Most students who belong to young and adult education classes portray cultural roots, which have been socially, economically and culturally marginalized. The ethnomathematical proposal not only acknowledges these experiences, but also

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allows a new perspective on the student: one who can develop mathematical knowledge. Therefore, it stimulates the *cultural dignity retrieval*, which is related to the ethnomathematics "political dimension" (D'Ambrosio, 2001). It is one of the most meaningful contributions to the researches in ethnomathematics towards pedagogycal practices for young and adult students.

One of the challenges of young and adult education has been to work with a sociocultural group marked by several kinds of diversity, such as of age, religion, race, origin, and yet share of one or more experiences of social exclusion. Another challenge to researches in ethnomathematics which aim at investigating young and adults mathematical knowledges is the academic environment itself for many reasons: it does not stimulate students individual knowledges; it had rashily excluded them from its surroundings and consequently mining their self-esteem as learners.

According to D'Ambrosio (2001), ethnomathematics has an "educational dimension". However, the relations between ethnomathematics and the educational field have not been able to avoid conflicts as ethomatemathics welcome multiple forms of quantitative and spatial representations of the world and this concept clashes with the idea of a single, universal mathematics offered by schools' homogenized curriculum. As a result, very few practical indications have been made towards an ethnomathematical pedagogical program (Santos, 2004). Working under an ethnomathematical perspective means to deal with contradictions between the homogenous academic mathematics and the diverse mathematical knowledges present in the classroom[2].

In order to accomplish such goal, and approaching the same problem by a different angle, in this research ethnomathetics was used as the theoretical means to establish dialogue between the several mathematical knowledges mediated by the teacher.

In the young and adult educational context, richness and cultural complex dymanics are found among the different and diverse types of knowledges. In previous research (Fantinato, 2003), it was observed the usage of different calculation procedures among young and adult students in order to confirm a result. For example, informal procedures were used merely for the confirmation of a mathematical result, because any other procedure, such as one of academic type, couldn't guarantee the required accuracy. These peculiar systems of reasoning are barely noticed to outsiders as they take an invisible character. They coexist with others and are considered, by these young/adult students as more *suitable* to the educational environment in general, these practices result from past or present experiences at educational surroundings, what Fonseca (2001) has named "reminisces of schooling mathematics".

However, it is important to keep in mind that the mathematics teacher stands for the oficial mathematics image in the classroom. This person holds a knowledge considered *superior* students daily knowledge due to its priviledged social position in our society. This uneven *status* position interferes in the relations among different types of knowledges, which take part in the classroom cultural dynamics. When voicing students' knowledges, the dialogic attitude of the teacher entails an awareness of the mythical status of *his* math and the depreciation of *other* math as an effort to reverse this difference.

Therefore, ethnomathematics area has a lot to contribute to the development of sensibility approach towards the knowledge of the *other*. The research tried to indicate how a teacher's dialogic generous attitude could contribute to building bridges between mathematical diverse knowledges in young and adult education classroom contexts.

THE RESEARCH

To the development of this research, we are going to present the results of a *case study* based on a math teacher who deals with young and adult education, who had been taking part in an ongoing teacher development program on *ethnomathematical approach.* The choice for this methodology is due to the need to deepen comprehension on the cultural dialogue knowledge this teacher sets with his students – young and adult students, in relation to the mathematical knowledges found in these multicultural groups. It is also justified by the distinctiveness of this teacher who has been reviewing his practice grounded on the ethnomathematical studies.

The theoretical and methodological dimension used in this research presupposed the consideration of all participants in the dialogue between the diverse mathematical knowledges in the classroom. Teachers' and students' mathematical knowledges, as much as the teacher's awareness of his pedagogical choices, his transformation process from an ethnomathematical point of view have all been studied. It is a qualitative research, in which both UFF's ethnomathematical study and research group[3] and the teacher have been objects of study.

The methodological procedures included interviews, made at different moments of the process, with the professional chosen. There were also interviews with some of the students from the selected group, participative observation of the math classes in young and adult education groups and scrutiny of the logbook notes. We analyzed the teacher's pedagogical support documents, the written documents produced by the students of this teacher throughout the proposed math activities and the group meetings diary.

André Luiz Gils is a 40-year-old math teacher who has nearly twenty years teaching experience in private and public middle high and junior high schools. He is one of the founders of *UFF's Ethnomathematical Group* and he has been actively working with young and adult education for six years; due to André's specific characteristics, he was chosen to be subject of study. His professional path and practice fulfilled the objectives of the exploration. Besides, he also offered unique and specific characteristics proper to a case study. (Stake, 1992).

The chosen group of students were André's belonged to second UP[4] from Block I, in CIEP Anita Malfatti, a municipal school placed in Campo Grande, West Rio de

Janeiro.When we started the classroom's observations, they were at the second half of 2005. These classes always took place every Monday night. The investigation data, on André's pedagogical practice in the classroom, were carried out by the two researchers and recorded on logbooks. In the school, there were also interviews with some students from the selected group, a little before classes started, in the cafeteria, or in their classroom.

TEACHERS' PRACTICE UNDER THE ETHNOMATHEMATICAL PERSPECTIVE

Here, some research results will be presented, particularly in terms of André's pedagogical practice in the classroom, noticeable from classes' observation, his own reports and from students' point of view as well. Considering his ethnomathematical teaching practice, three aspects stood out: dialogue disposition, ability to establish relations and the experience of autonomy.

Dialogue disposition

One of the main goals of ethnomathematics is to listen to the voices of the subjects who belong to the group, which was selected to be observed, that is, the legitimacy of the other's knowledge and the way they interprete reality (Domite, 2005). An ethnomathematical attitude supposes a disposition to dialogue, an attitude of respect to differences. The routine in André's classroom is built on permanent dialogue with his students.

One of the strategies used to get class started is to motivate students raising previous knowledges on the topic to be studied. "Who has already heard about fractions?" Students jump to answer eagerly, giving various examples from their daily lives: "book recipes have lots of them" (female student); "in paint gallons" (male student); "cooking Jell-O, add a certain amount of water, a certain amount of gelatin" (female student). Little by little, taking advantage of students' participation, André keeps building his class.

Besides thought provoking their participation, André recurrently legitimates their contribution, such as with; "It all brings the idea of fraction. What is more, you said you had not a single clue about it. Who would wonder? Perfect!" Students seem to feel at ease to convey their opinions and to question, unafraid of mistakes. Clara, one of his students admits "(…) he has a very caring peculiar teaching ability. He explains and makes us at ease in his class." Léa, another student, states: "what I don't understand …I have enough freedom to ask him, because I know he'll gladly answer and help us".

André's class is not silent, many of them take part in the class - some from their free will, others because they were asked. Students, share this soothe, open atmosphere, and identify it as one the characteristics of his class. "He is the best teacher of this school. If you ask him anything, he answers. Some teachers don't even let students ask a single question".

Students also point out he is always ready to clear the questions they might have, even when class is over, as we can notice in the statements below:

> He can see what we don't understand, do you understand? He talks to us and manages to explain attentively and when class finishes and he is required, he is available as well. He never says no when it comes to teach us, can you see? I guess it is important, isn't it?
>
> Whenever I can't understand I… go to him … and ask him and he clears what I hadn't understood, and then, it is much clearer to me.

While spurring the group during the dialogical exposition, André keeps walking around the room, trying to observe the individual ways to handle questions, asking them to explain their argument. André's pedagogical practice attends some of Paulo Freire's (1974) *dialogical* ground characteristics, which he himself refers to as *an opening to the other, modesty, faith* in men and reciprocal *confidence*. His disposition to dialogue has also broadened his ability and capacity to understand his students. In his words:

> The chance to be teaching at EJA is also the chance to learn. There, I learn a lot. I learn to make electrical wiring, the proper usage of paint, I learn how to use plaster, I learn how to make concrete. Things I never imagined I would possibly learn. I was not raised for that but I learned because I wanted to and because I realized these were all knowledges as well.

The teacher's ethnomathematical attitude seems to favor this availability to dialogue with knowledges different from his, not only to legitimate them, but also to learn with them, under the belief that these mathematical alternatives can change the way math is conceived as well, as Barton (2004) points out. The multicultural mosaic conditions in EJA's classrooms (De Vargas, 2003) seem to contribute to this sort of sensibility of the dialogical teacher for other forms of mathematical representation of the world.

Ability to establish relations

Another characteristic of André's teaching practice is to establish many types of relations. André is always searching for relations between academic and day-to-day knowledges. Sometimes, these relations are set from spontaneous situations brought in by the students. Some other times, they belong to the role of didactical strategies used to teach a certain mathematical topic. In the situation described below, the teacher seeks a connection between daily and educational knowledge:

> André asks the students, "Has anybody else brought any more packages?" They hand in toothpaste, soap bars, shampoo, and ketchup ones. A student says, "I went to the market and got some cookies", and André takes that one as well. Another student makes this comment: "Well, it is an easier way to learn". André goes to the board and writes *equivalence = the same amount.* He then gets two of the packages they brought and ask if they are equivalent. The teacher keeps showing the different packages and setting the comparisons. At a certain point, he shows two small packages, the ketchup one and the

> mayonnaise one and asks if they are equivalent. Léa, a student, disagrees. He queries, "Are you sure?" Then, he checks both packages' weighs and gets surprised. He realizes they indeed didn't really have the same weigh. The teacher asks how she reached that conclusion. Léa says the red one has more because it is a bit bigger. The researcher asks if it was a naked eye measurement, and Léa bounces her head positively.

In the story above, the teacher the teacher used props from his adult students' ordinary life and relied on their active participation. However, what catches the eye is his unassuming attitude when he admits he might have made a mistake towards the student's observation and the acknowledgment of how Léa managed to evaluate the weighs of the packages.

This same student ackowledges the importance of her every day life being a topic in the classroom and points clearly that the teacher's approach contributes to her mathematical learning process.

> He works.... he works with our every day life, you know. He teaches us math... asking us to take… packages of beans, rice, to teacher fractions. Things which are going to make it easier for us to learn… our learning, which is something from our daily life… then… it makes it easy… and it really does, because when I had to learn it when I was a child, I was twelve or thirteen, I just couldn't learn it, but the way he taught, it was very easy, you know. Get a 5-kilo bag, divide in, in five of one kilo, it was easy for me, you know. Then, I think that his math is related to our life, to our domestic life… it makes things easier.

The student emphasizes that what makes learning easier does not come from the topics chosen by the teacher, but by his approach which is linked to concret situations, from their reality, and to the same extend acknowledging a level of autonomy in André due to the way he conducts his class.

> It is not easier to teach this way, he found a way, a technique to make us learn, because we live it. It does make it easier. Yes, it does. It is way easier to learn fractions with beans, rice, sugar because it is my routine. It is easier than learning with numbers alone without an example...

André is always working under interdisciplinary approach, establishing relations among other areas of knowledge without missing his target, the teaching of mathematics. One day, when he was teaching the different ways to divide a square from a symmetric axis, a student makes this remark, "The geography class has an axis that divides Earth, an imaginary axis." Immediately, André goes to the board, draws a circle with an axis in the middle, and says that it was the symmetric axis and it was related not only to the idea of balance, but also to the idea of fraction. The teacher applied a piece of information from geography, building his class from another of his personal didactic characteristics – the permanent link done among the different mathematical subject matters, and in this specific situation, the connection between geometric and arithmetical concepts.

André is aware of his interdisciplinary attitude in the classroom as he points out:

> I can draw a bridge between mathematics and geography, science and history. It all feels natural. I can see the link between them without any difficulty.

Although André's classes are always planned to focus on a certain schooling math, his dialogical procedures, looking for contextualized connections among the different subjects, *bring the mathematical concepts into life* (Monteiro, 2004) and make comprehensive in their relations with the varied aspect of their lives. According to Ubiratan D'Ambrosio: "ethnomathematics is rarely detached from other cultural demonstrations, such as art and religion. Ethnomathematics fits within a multicultural and a holistic concept of education" (D'Ambrosio, 2001, p. 44).

Autonomy experience

André is a teacher who exercises his autonomy in the classroom, especially in the groups he deals with young and adult education. He says he feels "freer to handle things there rather than he would in a private institution". He feels like innovating and this freedom gives him a professional fluffiest feeling. He explains it himself:

> In young and adult education, routine is scarce. Each daily situation is new and it is at the same time motivating.. they renew themselves, their looks are different, the experiences are different, so we always have a goal, we have the topic to teach, but how is it is going to be performed, is related to the expectancies they bring day to day, on that specific day. That is, it is not closed, it is not definite.

This exercise of his autonomy seems to be related to the room André gives his students, the chances he gives them to show their own knowledges. It means that, when respecting his students' knowledges he is also respecting his own as a math teacher.

During André's class, his autonomy exercise comes out in his choice not to use materials such as a didactic book, previous remarks nor even other objects known for their pedagogical usage. We have been able to witness André's creativity when explaining a new concept. Elements such as the chalk box, rubber bands, door hinges, the pen, students' props were easily taken into as teaching resources, if regarded suitable for the class.

The beginning of each class was stressed by a review on the previous class and at its end, the topic to be studied in the coming class was announced, followed by a request of some sort of material, such as news report, packages or even cut outs to be used.

For this teacher, the experience of autonomy in the classroom implies voicing and giving autonomy to his students. It also involves ignoring the compartmentalization of school subjects in order to broaden the limits of academic math, in a constant exercise of creativity.

VALIDATING KNOWLEDGES IN A DUAL CARRIAGEWAY

This paper has handed the results of a case study of a math teacher who works under an ethnomathematical perspective, analyzing his role as mediator among the different knowledges present in the young and adults' classroom. It has also highlighted the characteristics of an ethnomathematical practice (Santos, 2004); open to dialogues, to the ability to establish multiple relations and the exercise of autonomy.

An ethnomathematical perspective, according to many writers, engages the process of knowledges legitimacy of specific groups, in a way to make visible *invisible* and *frozen* knowledges, specially with groups either in social exclusion situation or subordinated to social, cultural and economical capital (Knijnik, 1996). In other words, an ethnomathematical perspective engages the process of *knowledges legitimacy*. The results of this research have signed that ethnomathematical attitudes in the classroom configures a *dual carriageway* validation. That is, voicing students and their knowledges, André's schooling knowledges (Tardif, 2002) are also being legitimated as well. This hypothesis gets body when he talks about ethnomathematics and its influence over his pedagogical practice.

> Maybe I already had a tendency towards an ethnomathematics attitude but I just didn't know it was named that. So, in fact, ethnomathematics didn't show me a new perspective, it wasn't that. It just helped me to *support what I already had in mind* (…) (my own practice) has changed because today I am far more aware of it.

The ideas developed in this article aim at taking ethnomathematics and education relations in terms of teachers' pedagogical practice and teacher development programs into a deeper level, specially the one related to young and adult education.

NOTES

1. The research counted on the participation of a undergraduate research student, Rosana Kelly dos Santos, supported by CNPq (A Brazilian Grant Institution).

2. Most likely for this reason, most of the researches on ethnomathematics have been focusing on specific ethnographic groups (such as indigenous nations, professionals groups in general, among others) who share knowledges, techniques that we can make a parallel with math.

3. Research and study group on ethnomathematics coordinated by the writer of this article.

4. UP stands for *Progression Unit Groups.*

5. EJA – For Educação de Jovens e Adultos (meaning Young and Adults Education, in Portuguese).

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RETHINKING SUCCESS AND FAILURE IN MATHEMATICS LEARNING: THE ROLE OF PARTICIPATION[1]

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This paper analyses and discusses the notion of participation in a community of practice and its role on the rethinking of the notion of success and failure in the learning of mathematics, taking as support for the discussion some episodes collected on a vocational school were a group of young boys wanted to became blacksmiths.

INTRODUCTION

Before starting school life children learn many things, participating with their parents and friends in important activities for their lives and for their family's lives. Learning occurs meanwhile children participate in ongoing activities of their families and the activities purpose and the reasons for learning are obvious for the children.

After starting school life children continue learning a lot of important things, out of school.

But during many years this reality had been ignored or had not aroused researchers interest that dedicated themselves quite exclusively to the understanding of learning in school contexts focusing themselves "on observable aspects of individuals or on mental representations (that are assumed as reflecting or indicating experience" (Matos, 1999, p. 3)

In 1988, Jean Lave, with her book *Cognition in Practice,* brought in changes on cognitive and transfer (of mathematical learning) theories.

In 1991, Lave and Wenger presented a 'new conception' of learning arguing that to understand learning it is important to shift "the analytic focus from individual as learner to learning as participation in the social world, and from the concept of cognitive process to the more-encompassing view of social practice." (Lave and Wenger, 1991, p. 43)

Learning central feature is legitimate peripheral participation that is, according to Lave and Wenger, (1991) "the process by which newcomers become part of a community of practice." (p. 29) To belong to a community of practice "does imply participation in an activity system about which participants share understandings concerning what they are doing and what that means in their lives and for their activities." (p. 98)

Also Wenger (1998) discusses the notion of participation: "Participation refers to a process of taking part and also to the relations with others that reflect this process." (p. 55) Participation refers not only to local events of engagement in certain activities but to surrounding processes of being active participants on the practice of

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 237 - 247).

community and to processes of building identities in relation to those communities. Such participation shapes, not only what we do, but also who we are and the way we interpret what we do. It also shapes communities in which we participate; "the potential transformative goes both ways. Indeed, our ability to shape the practice of our communities is an important aspect of our experience of participation." (p. 57)

To analyse participation in the practice of the community become important in order to discuss and understand learning as a phenomenon that emerges from participation in social practice. In this paper we will analyse the participation of a group of young boys in a community of blacksmith apprentices. Our trajectory on this paper will be to present the boys and what they had been doing to become blacksmiths. Then to characterize participation in the practice of blacksmiths apprentices' community, discussing aspects such as: negotiating the meaning, motives to participate, ways in which practice in constituted, forms of participation and patterns of the practice. Finally we will discuss the main ideas emerged from this paper pointing out the situated character of learning in communities of practice and suggesting implications for school learning.

TO BECOME A BLACKSMITH

The vocational school 'CAMPO' proposed a course of blacksmith technical to the community. A group of youths, between 16 and 21 years old, had subscribed the initiative, by very different reasons.

Starting classes, life of blacksmith apprentices - all boys - gained a set of routines. They have to attend different classes (from 9h to 18h) with an interruption for lunch (from 13 to 14h). Students had usually one morning or evening with the same theoretical classes (as they call it, such as Mathematics, English, Technical Design, Computing, etc.). 'Blacksmith practices' was the exception. They had this subject every Friday or Saturday alternatively in a 'real' blacksmith's workshop.

Apprentices started to work individually, training several soldering techniques, but quickly they were doing different things with different times and methods. After learning how to sold, the blacksmith master proposed them to build a dust-pan. The second project proposed by the blacksmith master was a bench. Initially apprentices drew the object they will build on blacksmith's workshop in Technical Design classes. The dust-pan and the bench had been drawing on that class. When apprentices noticed that they could propose their own projects to the blacksmith master they quit drawing in Technical Design classes and drew it mentally. Many times apprentices drew part of the object they were building on the bench, with the aim of analysing and discussing with the blacksmith master any aspect of the work. This communication technique had been started by the blacksmith master.

Frequently blacksmith master and apprentices discussed their work communicating with support on schemes drew over the bench. These schemes usually showed one of the views of part of the object they were building. It was not a perfect drawing, but it was always in proportion and communication between apprentices and the master was sustained by the use of these drawings. Eventually apprentices added any line to the scheme and questioned the master or explained their ideas using the draw more than oral language.

On this context it was not frequent the dialogue between apprentices, despite they were working on group. This fact is related with the machines noise, characteristic of the blacksmith's workshop that makes audition difficult. But blacksmith's workshop has its own way of communication - using as support the object scheme.

Blacksmith master circulated between apprentices and observed them attentively all the time, but only interfered if one of the apprentices posed a question or if he saw them using the machines in a dangerous way. In this situation he showed how to do it correctly.

Mathematics class was a traditional one.[2]

PARTICIPATION IN A BLACKSMITH APPRENTICES COMMUNITY

To characterize participation in this practice, informed by situated learning theories, we had been analysing some aspects of it, such as negotiation of the meaning, motives to participate in different activities[3] of practice, ways in which practice in constituted, forms of participation and patterns of the practice.

To Participate in Different Spaces of the Practice

When apprentices started blacksmith practices they had a particular space to work, inside the blacksmith's workshop. It was a corner on the 2^{nd} floor limited by a balcony. Sometimes, they need to use some machines that were out of this space and timidly they frequented that space that does not belong to them. It was the space where professional blacksmiths worked. It was common to see apprentices observing professional blacksmith working and vice-versa.

In a beautiful Saturday ('The Special Saturday') the blacksmith owner asked apprentices to help professional blacksmith to finish a work whose time of delivery will not be accomplished without apprentices help. In other words, apprentices were not only explicitly authorized to frequent all the space of the blacksmith's workshop but also to participate in blacksmith practice for a task that corresponds to a need of the workshop. From this moment apprentices did not leave this space. The boundary crossing and the entrance in the space of professional blacksmith practice not only give apprentices autonomy in relation to the space, but also favoured, in a natural way, the interactions between apprentices and professional blacksmith, that started to comment apprentices work.

The situation above related was, for the workshop owner, only a way to solve his own time of delivery problem, for apprentices was a crucial moment of that activity. It was the moment in which they secured all space of the blacksmith workshop, but this corresponds to another conquest equally (or maybe more) important - to see themselves and to be seen as able to performance a more all-embracing task and more responsibility, in interaction with professional blacksmiths of that workshop. It was also, from this Saturday that, on Abreu's initiative - one of the most active apprentices - apprentices started to propose, to Mr. António, the execution of their own projects.

The Negotiation of the Meaning in Different Practices

Since the beginning of apprentices' blacksmith practice they were in the presence of two different practices on the same physical space. These practices were well demarked by physical boundaries, behaviours, ways of talking, by identity features evidenced in interactions, on mutual dependencies, on implicit comprehensions in moments where the dialogue seems not to exist. More than to ask if they were in presence of two communities of practice (Lave and Wenger, 1991) it is important to characterise the way students/apprentices create the continuity between both practices - support to talk about participation in different (although with overlapped elements) communities of practice. In spite of apprentices had a clearly peripheral position in relation to the professional blacksmith practice, the authority of the workshop owner legitimated their participation on the blacksmith community. This aspect give them access to a set of relations, ways of being, acting, doing, dialoguing that apprentices will not have access if those 'Special Saturday' did not happen. Apprentices were explicitly and intentionally authorized to participate in an organized practice in patterns and vocational terms, 'larger' when we think in its positioning on life trajectories of apprentices, opening possibilities and potentialities of apprentices engagement on the blacksmith professional practice.

Another element that had contributed, in a decisive manner, for the blacksmith school practice integration in the blacksmith professional practice was the fact that Abreu brought some pieces of an old griller and asked the master if he could rebuild it in spite of building the window proposed by the master. From this phase of implicit negotiation between the blacksmith master and Abreu (by initiative of the second), all the group became aware that they could bring their own projects to the workshop and that they did not need to wait for the master proposals. Despite the negotiation was between two elements of the community, all others assumed that negotiation as being their own negotiation.

"The engagement in practice has patterns, but it is the production of such patterns anew that gives rise an experience of meaning" (Wenger, 1998, p. 52). In blacksmith activity one of the aspects of extreme importance is visualization, but master António did not make it explicit to apprentices. When master António communicated with them about what they were building, he had drawn a scheme of the object. Apprentices started timidly drawing some more lines on the scheme and using these schemes to explain, with few words, why they adopted certain process instead of another. Few weeks later, apprentices used this process commonly to communicate with master and with the all the members of the community. In other words, it was through repetition of a certain pattern that apprentices negotiated the meaning of visualization on that practice.

"Negotiation of the meaning is a process that is shaped by multiple elements and affects these elements. As a result, this negotiation, constantly changes the situation to which it gives meaning and affects all the participants." (Wenger, 1998, p. 54)

In fact, below examples 'Special Saturday' and 'the griller brought by Abreu' show that nothing was left as it was before. In the first case, apprentices had access to the work and comments of professional blacksmiths working on the workshop. And this had changed the dynamic of that space. The second case also showed that negotiation of a meaning affected all the participants and had completely changed the situation. In both cases, apprentices gained power with the negotiation of the meaning.

To be a blacksmith is required a specific way of looking to the objects they have to build. The ability to interpret the object building in blacksmith workshop reflects the relation that blacksmith and the object has in practice. Apprentices contribute for the negotiation of the meaning because they are members of the community and because they transport with them stories of participation on that practice. In a similar way, objects built contribute for this process reflecting aspects of the practice that had been 'frozen' in themselves and had been fixed on their forms. Blacksmith apprentices, as community members, incorporate a long process that Wenger (1998) called participation. Similarly, the objects as artefacts of a certain practice incorporate a long process that Wenger (1998) called reification. It is on the convergence of these two processes that negotiation of the meaning occurs.

As a pair, participation and reification, refers to the fundamental duality for the negotiation of the meaning (Wenger, 1998). Participation and reification are not mutually exclusive. They are intrinsic and complementary elements in the negotiation of the meaning. On blacksmith workshop, it is participating that apprentices build objects that, by its turn, are reifications of that practice. The negotiation of the meaning occurs on this process. On their practice apprentices expressed their way of belonging and their identities as members of the community of blacksmith apprentices but progressively manifesting their belonging to the blacksmith community (in which they want to become). For instance, when they started blacksmith activity they always used overall, maybe because they did not feel such as blacksmiths and the needed something that identified them (for themselves and for others) as blacksmiths. In summer, almost in the end of the course, some of them did not use the overall completely dressed. This fact can mean that these apprentices did not need anymore clothes to see themselves and to be seen as blacksmiths.

In mathematics classes it happened several moments of negotiation of the meaning, namely in relation to the kind of communication that was possible on that context, to the kind of attitudes acceptable on that activity and also to the institution expectations, namely in relation to the kind of student that was expected they were. It was through the repetition of several moments of communication that students had becoming aware of the kind of answers that teacher approves. For instance, when a student answered teacher and he ignored the answer, student did know that that answer was not correct or that it was not appropriated to that context.

In relation to mathematical contents there was little visibility of eventual negotiation of meaning. When we had been on the mathematics classroom we had not observed any discussion, between teacher and students or between students, concerning a certain mathematical content. Therefore opportunities to the negotiation of the meaning were reduced.

Mathematical contents were presented by teacher to students in a reified way and this difficult the negotiation of mathematical meaning. "An excessive emphasis on formalism without corresponding levels of participation or conversely a neglect of explanation and formal structure can easily result in an experience of meaninglessness." (Wenger, 1998, p. 67)

Motives to Participate

To become a blacksmith had been a common motive, to participate, for all blacksmith apprentices and this motive was decisive in the way how students had participated in both the activities of practice analysed in this work. Nevertheless, there are other nature motives, in the different apprentices and that we grouped as follows: (1) the present – all apprentices approached to a moment in their lives that they felt necessity to wrap themselves in a transition; (2) the future imagination. Here we can distinguish three groups – the larger group that sees in the profession a social and professional identity that will allow them to reach ways of life that they desire. The minority group, that sees in profession an intermediate step to a 'more elevated' professional life project. And another minority group that seems to be not quite sure of the professional identity that desires.

How the Practice is Constituted and What are Forms of Participation?

On the blacksmith activity dynamic we can identify three different phases of participation. The first one – the beginning – that is characterized by the existence of two distinct practices – the real practice and the blacksmith pedagogical practice. Blacksmith apprentices' participation in the real practice is characterized by non-participation[4]. But non-participation had fundamental importance on the learning of ways of participation on blacksmith learning activity. Apprentices had a peripheral participation in relation to blacksmith real practice. The second phase – the appropriation of physical space. On this phase there was only one practice – blacksmith practice. Apprentices had had a legitimate peripheral participation on the real blacksmith practice. The third phase – full participation – where there was only a practice and blacksmith apprentices had a legitimate participation towards full participation. It has been important that apprentices learned to participate, but for that is has been necessary that they had access to participation cause this access is a fundamental condition to the practice. There was an 'evolution' on apprentices' identity. They left seeing themselves as students to seeing themselves as blacksmiths.

Students have learned to participate in mathematical activity when they engaged on it. They have learned what type of answers to give to the teacher, how to show (to the teacher) that they are interested in (looking for or pretending looking for on the copy-book a similar task), there is, they have learned to be students of that mathematics class. All students had, on that activity, a full participation, seeing that all of them saw themselves and were seen as 'from within' that activity. Nevertheless, existed students with a bigger or smaller degree of legitimacy in agreement with they had better learned the kind of participation to have on that class.

Patterns of Practice

Patterns that have emerged from the analyses of apprentices' participation on that practice can be characterized in order to three aspects: discourse, ways of working and knowledge. Discourse between apprentices and between apprentice and the master on blacksmith activity can be characterized by the use of few words. Apprentices were not used to translate verbally their work, neither to talk explicitly about it in detail. This kind of discourse did not belong to that activity of practice. On mathematics class the discourse between students is to joke or to 'blow' the answer to a colleague. Discourse between teacher and students had several styles according to the moment: if the moment was naturalness, the discourse was informal, when they were solving tasks discourse was characterized by question/answer. When teacher wanted to maintain order on the class he talked in a loud voice.

According to Lave and Wenger (1991) discourse is a resource where visibility and invisibility are in constant interaction. Discourse must be invisible so that the problem to be solved can be engaged, that is, can become visible. The challenge is to make the discourse invisible. In many mathematics classes, discourse is invisible for some (few) students but visible for others (the great majority). If we want that discourse becomes invisible it is important to create opportunities for the negotiation of the meaning.

Students were able to recognize the context and specificities of mathematics (for instance, through teacher discourse) but they didn't have access to the realization rules (Bernstein, 2000) seeing that they do not realize it. To participate in one activity that somebody described is not only to translate the description to an incorporated experience, but to renegotiate its meaning in a new context (Wenger, 1998), there is, students in mathematics class should renegotiate the meaning of learning mathematics, the meaning of look for solutions for the problems they have to solve, the mathematical meaning of contents they were using, and, in fact, they didn't have space to that renegotiation. To be able to say and to be able to do are not equivalent things. To know a mathematical formula (a concept or mathematical idea reification) can make us believe that we have completely understood the process that it describes. But exploring ideas only intuitively, without moments of reification, can make things too much disconnected and that will lead to a meaninglessness experience.

In relation to ways of working in blacksmith activity apprentices worked most of the time on pairs. On mathematics activity, most of the time students and teacher worked in a big group. It was unusual to see students solving problems on pairs or individually.

In relation to the knowledge, on blacksmith activity, we had identified some patterns such as: visualization (we have discussed before on this paper), measurement unit (that is millimetre), the tolerance margin of error (one millimetre) and mathematical proceedings (as we will see below).

The Dustcart

Vasco and Abreu were building a dustcart. They had over the bench where they were working, a piece of aluminium, a ruler, a set-square, a scriber, a piece of iron (used to trace out straight lines and a tape measure.

Vasco was drawing several lines on the piece of aluminium as we can see on the figure 1.

Researcher: Why did you trace out these lines (pointing to the trapezium diagonals)

Vasco: To find the middle of these one (pointing to the parallel sides of the trapezium).

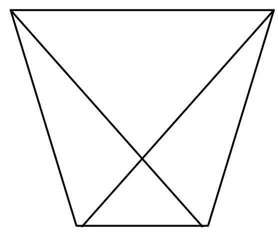

Figure 1

Researcher: The middle?

Vasco: Yes (and he drew a perpendicular line to the parallel sides of the trapezium through meeting point of diagonals).

Researcher: How do you know that this point is the middle point of parallels sides?

Abreu takes a tape measure and measures the bigger side (of parallel lines) of the trapezium. After that he measures the distance between one of the vertexes of the trapezium and the 'middle point' defined by the perpendicular line that Vasco drew on the bigger side. He didn't talk. He only looked for me.

Researcher: Yes. But... why?

Vasco: Because we always do like that. All of us do it like that (referring to the old-timers of the community).

The above episode shows the use of an important geometrical property (the meeting-point of the diagonals of a parallelogram or of an isosceles trapezium belongs also to the mediatrix of opposite sides or of parallel sides respectively) that apprentices did not know theoretically.

This property is not valid to other kind of quadrilateral, but in fact, I have not seen apprentices neither blacksmiths (old-timers) working with it. For the type of things they do on blacksmith practice, this 'rule' always works. Apprentices used it because they learned it with 'insiders' of the community that it worked. Mediator tools (such as blacksmith tools, mathematics, etc) invisibility allowed apprentices to focus on and to learn on in the visibility of their aim – the building of the dustcart.

CONCLUSION

In blacksmith apprentices community of practice, apprentices had strong motives to participate on several activities of practice that composed the practice of that community. Those motives, namely the motive 'to want to become something' was determinant in the way students participated, namely, in activities with a more scholar logic, such as mathematics. What motives have students to participate in the different activities that constitute the scholar practice of Public School? In which they want to become? Which communities they want to belong?

All these boys already had failure in school mathematics and in general in school. Suddenly, with a traditional class, very similar to the kind of class they had in public school, all the students had success on mathematics. Why was this change? The motivation for the success in mathematics comes from the fact that they want to become blacksmiths and for that they have to be approved in all subjects of the course. Probably, till then, most of these students had the label of 'incompetent' in mathematics. To change the emphasis from 'ability' to 'to belong' or 'to become' suggests a redefinition in the way of looking to 'success' and 'failure' in mathematics class (Boaler, William and Zevenbergen, 2000).

These boys are the kind of students that Skovsmose (2003) called 'disposable[5]'. Mathematics Education, in a certain sense, prepares some groups to be the 'disposable'. It's important to understand how failure in mathematics can be part of the process of nomination of people as 'disposable' and also understand that the one who are considered 'disposable' in public school can not be 'disposable' in another situation, also scholar, but with stronger motivations than those that public school allows themselves to develop.

Non-participation on blacksmith activity reveals itself fundamental in the learning of forms of participation. Non-participation in public school is, usually, punished. It's important that non-participation be decriminalised on mathematics class, seeing that it is a crucial element on the learning of forms of participation.

To have success on the learning through participation on activities of a community includes the development of an identity on that community and that development is related with and depends of the identity of the person in another activities of the community and in other communities to which the person belongs.

Success or failure cannot be understood, only in terms of knowledge and skills brought by the student to the situation. This individual attributes have to be considered in relation with social arrangements and resources in which individuals interact (Greeno, Eckert, Stucky, Sachs and Wenger, 1999).

NOTES

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2. When you read this sentence you have imagined a traditional mathematics class. What I have been observing was quite similar to your imagination. The only difference is that tables are arranged in a U form.

3. The different activities that constitute the practice of that community take in from two activity systems with enough different logics. Some of those activities are strongly related with production logic ('Blacksmith Practice' and 'Traineeship' subjects) in spite of being integrated in the curriculum of a vocational school. Other activities have school logic (Applied Mathematics, Technical Drawing, English, etc, subjects).

4. On Wenger (1998) sense.

5. Skovsmose (2003) comments about groups of people that can be enfolded in or affected by mathematics education. These groups are: constructors, operators, consumers and disposables. On this perspective disposable are the one who are not 'necessary' to the informative economie.

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IS MATHEMATICS LEARNING A PROCESS OF ENCULTURATION OR A PROCESS OF ACCULTURATION?

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This study aims to identify the degree to which mathematics learning approximates either to a process of enculturation or to a process of acculturation. We examine the practice of a secondary mathematics teacher, in terms of Bishop's notion of 'aculturator-teacher', and the impact of this practice over the students' affect. We conclude by discussing some pedagogical implications resulting from the study.

INTRODUCTION

For a number of researchers (e.g. Bishop, 1988; Clarkson, FitzSimons & Seah, 1999, Knijnik, 2002), most people tend to see mathematics as a culture and value-free discipline. These authors suggest, each one in their way, that failures and difficulties regarding mathematics at school are usually ascribed to the students' cognitive attributes, or to the quality of the teaching to which they are submitted. Thus, social aspects, and especially cultural aspects, have received insufficient consideration in the teaching and learning of mathematics. From the 1980's, however, one can observe a gradual change in the teaching of mathematics, in particular in countries notably marked by a multiethnic population (e.g. Abreu, Bishop & Presmeg, 2002), concerning socio-cultural issues. Keitel, Damerow, Bishop & Gerdes (1989) show how the social dimension has been affecting mathematics education research, and consequently, clarifying the cultural nature of mathematical knowing. According to Bishop (1997), such a dimension stimulates research at five main levels: a) *individual level,* which is concerned with the personal learning both in and out-of-classrooms; b) *pedagogical level*, which is concerned with the social interactions in mathematics classroom; c) *institutional level,* which is concerned with the social norms and interactions within schools, which influence the teaching of mathematics in classrooms; d) *societal level* which is concerned with the relationships between mathematics education and society; e) *cultural level,* which is concerned with the relationships between mathematics education and the historical-cultural context of the society. In relation to the last level, Bishop (2002) argues that situations of cultural conflicts strongly involve emotional and affective reactions by the students. This has led him to an interest in exploring relationships between affect and culture, in terms of teachers' values and students' affect.

Based on the literature of anthropology, Bishop has introduced the concepts of *enculturation* (Bishop, 1988) and *acculturation* (Bishop, 2002) in mathematics education. He argues that both concepts are intensively linked to the teachers' values in relation to mathematics. Further, the author suggests that mathematics education can rely on experiences of enculturation or acculturation, influencing thus the affective dimension of the students' learning. Enculturation is the induction, by a

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 248 - 258).

particular cultural group, of young people into their culture, whereas acculturation refers to the induction into an outside culture by an outside agent. Often one of the contact cultures is dominant, regardless of whether or not such dominance is intended.

From the above premises, we report on a study whose aim is to identify the degree to which school mathematics learning can be thought either as a process of enculturation or as a process of acculturation. In doing so, we examine the practice of a secondary mathematics teacher, in terms of Bishop's notion of *acculturator-teacher*, as well as the impact of this practice over the students' affect. Our basic theoretical assumptions will be firstly presented[2], followed by our methodology and data analysis. We conclude by discussing some pedagogical implications resulting from the study.

CULTURE AND AFFECT IN MATHEMATICS EDUCATION

Much has been discussed about the assumption that mathematics has a cultural history and that different cultural histories can produce different mathematics. In the context of mathematics education this assumption has been treated through distinct approaches and foci. For Bishop (1988), mathematics is a *pan-human* phenomenon in the sense that it consists of six fundamental activities that seem to be employed by a number of cultural groups already studied. These activities are: counting, locating, measuring, designing, playing and explaining. Bishop's ideas about the pan-cultural nature of mathematical activity were developed prior to his later educational analysis using the perspective of enculturation. This perspective presupposes the existence of a cultural consonance/harmony between school mathematics and the culture the student brings from home. However, along the development of his works (e.g. Bishop, 1994) the author re-evaluates his premises aiming at the understanding of cultural conflicts. From these, Bishop turns his research towards the hypothesis that mathematics education may not be a process of enculturation, but instead a process of acculturation.

Within a psychological perspective, the studies developed in Brazil by Terezinha Nunes, Analúcia Schliemann and David Carraher during the years from 1980 to 1995 demonstrate that very poor children from some Brazilian villages can make complex calculations about money, commercial costs and profit, and so on, without being able to solve mathematically isomorphic problems in school (Nunes & colleagues, 1993). According to Nunes (1992), the fact that mathematical knowing can be learnt out-of-school by diverse cultural groups, brings important contributions to the analysis of the process of the teaching and learning of mathematics at schools. From the perspective of ethnomathematics, D'Ambrósio (1997) argues that mathematics education has witnessed significant transformations, most of them due to the fact that it is embedded in cultural diversity, many times not taken into account at schools. Knijnik (2002) discusses the power relationships involved in the school curriculum. She suggests that mathematical inclusion and exclusion can be understood as results of curriculum choices like what types of mathematical knowledge, what cultural values and principles are considered legitimate to be part of the school. One of the most significant influences of ethnomathematics in education, says Bishop (2006), is related to values and beliefs; it makes us realize that any mathematical activity involves values, beliefs and personal choices. Concerning situated learning perspectives, Lave's (1996) studies with some communities of practice raised fundamental educational questions about the application of school mathematics techniques in out-of-school practices. These studies indicate that the process of learning mathematical strategies and decision-making procedures are part of who is 'becoming' in that practice. People's identities are developed in participating in a socio-cultural practice and, in this sense, learning is seen as developing in practices. According to this approach, such communities and formal schooling are not distinguishable in what concerns the modes of learning. However, learning viewed as changes in participation and formation of identities within communities of practice still represents, in our view, a real educational challenge in regard to school mathematics.

In relation to affect several researchers (e.g. McLeod, 1992; Zan, Brown, Evans & Hannula, 2006) have been emphasizing the fundamental role of the affective dimension in the process of teaching and learning mathematics. McLeod (*ibid*) discusses the existence of three main aspects related to affect, which should be considered in mathematics education: *beliefs*, developed by the students about mathematics; *emotions*, provoking perturbations and blockings, leading to the students to experience positive and negative sentiments concerning their learning; and *attitudes*, developed by the students in regard to the discipline. DeBellis & Goldin (2006 in Zan et al., 2006) add a fourth element in the research of affect in mathematics education: *values,* and propose a tetraedrical model of affective representation, in which each vertice (beliefs, emotions, attitudes and values) interact both with all other vertices and with the individual. As far as we know, it is in Bishop's works that we find the most expressive elaboration of the relationship between culture and affect, in particular, culture and values. For this reason we will take his definition of values as being *beliefs-in-action*: for him, our values are revealed when we make choices; this is when we express elements of our system of beliefs (Bishop, 2002). Bishop's elaboration on culture and affect is based on the notions of enculturation and acculturation in mathematics education, already described in the introduction, as well as the concept of cultural conflicts. Since these notions constitute the core of our study, the next section is dedicated to them and their impact on students' learning in terms of affect.

MATHEMATICS ENCULTURATION AND ACCULTURATION

In developing the concept of mathematics enculturation Bishop (1988) argues that a child does not receive the culture as if it is an abstract entity; cultural learning at school is not a mere unilateral process that goes from the teacher to the student.

According to him, mathematics enculturation in a classroom should have as its target the initiation of the students into the conceptualization, symbolization and values of mathematics culture. And this process is interpersonal; it is interactive among people. In this sense, says Bishop, mathematics enculturation is not different from any other enculturation, and mathematics classrooms should be a propitious environment for mathematics enculturation. In a similar way, Lerman (2006) says that (mathematics) enculturation is a process of getting used to mathematics. Thus, he associates mathematics enculturation to a process of becoming mathematical, and discusses some research that indicates how becoming mathematical can mean different things in different modes of teaching. Later on, Bishop (2002) recognizes that we should not disregard the existence of cultural conflicts generated in classrooms, provoking also the process of mathematics acculturation. He discusses two distinct conceptions of cultural conflicts: a) conflict as a mere aspect of differences and mismatches; b) conflict as an aspect of explicit cultural interactions between the opposite parts. According to him, by adopting the second conception we would have an alternative and reciprocal development of both conflict and consensus, resulting continuously in a 'healthy' alternation between dissonances and consonances. At this stage, the author's studies focus not on the student individually, but in the process of acculturation per se, and in the role of those ones who we could define as being acculturators. In doing so, he observes apprentices (in general) during their experiences of cultural conflicts, and explores, in particular, how the process of acculturation affects the students' actions in multiethnic classrooms. From these observations, Bishop (2002) raises a more radical hypothesis: "…all mathematics education is a process of acculturation…every learner experiences cultural conflict in that process. However, cultural conflict need not be conceptualised exclusively in a negative way…" (p. 192). In what concerns the teacher's role, the author suggests that the teacher is the most important agent of acculturation in mathematics education. He considers two types of acculturator-teacher. The first involves school mathematics and daily-life mathematics: an acculturator-teacher would be the teacher who keeps the exclusivity between these two cultures (school mathematics and daily-life mathematics), who does not make any reference to any mathematical knowledge out-of-school, and who is not able to do anything with this knowledge even knowing that the students may have it. The second type of acculturator-teacher alludes to the institutionalized power of the teacher. In this case, an acculturator-teacher is the teacher who exercises his hierarchical power over the students in a negative way, i. e. by imposing to them what s/he wants through her/his privileged power and position legitimated by the educational institution and system. In both cases, says Bishop, the resulting cultural conflicts, although containing a cognitive component are infused with emotional and affective traces/nuances indicating deeper and more fundamental aspects than can be accounted for from a cognitive perspective. Returning to Lerman's above-mentioned work we could conjecture that the process of mathematics accultuturation, as described by Bishop, might not result in a process of becoming mathematical.

Frade (2007) provides a thoughtful contribution for the issue built over Luciano Meira's (Meira & Lins, 2006) *island metaphor*. In presenting a reconceptualization for the traditional dichotomy between 'the theoretical' and 'the practical', Meira associates the former to a person's life (natives living in an island), whereas the latter is associated with a representation of this person's life (explorers coming to the island and drawing a map). The researcher points out that there is no good reason to think that the natives do not 'theorize' about the explorers' *forms of life* at the moment they are mapping the island. According to Meira's approach cultural conflicts arise inevitably both when the natives begin to live the lives 'imposed' by the map, and when they visit the homeland of the explorers and question the rationale for the map. Based on this, Frade (ibid) has elaborated an interpretation of the island metaphor to mathematics education, in which teachers and students are supposed to belong to two different cultures – with one dominating culture, that of the teachers. She proposes to think that the island corresponds to a mathematics classroom within a *strongly classified curriculum* (using Bernstein's terms) in which 'children-natives' live a great part of their lives. The mathematics 'teacher-explorers' 'impose' on them a map which includes the *vertical discourse* of mathematics – via *recontextualization* – and some established social and mathematical norms, which the children-natives are supposed to share and to follow. The teacher-explores' homeland would correspond to the 'mathland'. Cultural conflicts arise, for example, when students question the rationale for this map or when they feel themselves to be 'outsiders' in mathland. Frade concludes that whatever the correspondence between Meira's metaphor and mathematics education, it should suggest a kind of 'dominator-dominated' relationship between teachers and students, inviting us to a reflection about the character of mathematics education in terms of humanity.

In an attempt to humanize the imbalanced relationship between the culture of the teachers and the culture of the students Bishop (2002) proposes a reconceptualization for mathematics learning environments based, to a great extent, on Gee's (1996) theoretical construct of *borderland discourse.* This would correspond to the area of intersection between the students' primary and secondary discourses. The primary discourse refers to the discourse learnt and used within the family, at home or surrounding groups. The secondary discourse is related to traditions passed forward by generations through time, aiming at the learning of behaviors in external environments to us. So, this discourse is considered more institutional or formal than the primary one. The potential oppressive character of the process of acculturation leads Bishop to propose that the intentional mathematics acculturation of a young person becomes a *cultural production* in which schools should be the place where the primary discourse of the students' families and communities meet the secondary discourse of mathematics community. In this place the co-construction of meanings, values and cultural practices would occur; it is where Popkewitz's (1999 in Bishop 2002) notion of *productive power* could be developed. According to this notion people are not seen as 'owners' of the power, but instead as mediators of the systems of knowledge and rules from which the power derives.

CONTEXT AND METHODOLOGY

The research was carried out in a Brazilian urban secondary school. The subjects involved were 31 students (17 girls and 14 boys) of a Year 6 class (ages approximately 11) and their mathematics teacher Ana. This class was not a multiethnic class, though we can say that it was characterized by a cultural diversity concerning the children's socio-economical position. Ana was a novice teacher and has taught as a temporary teacher in this school during a period of two years. Data were collected by: a) audio and video recorder of a sequence of mathematics lessons, b) audio recorder of interviews with some students, c) audio and written register of observations in class. The second author of this paper – Diogo – started his observations in the class seven days after starting the recording of the data by audio and video. This 'entrance' of Diogo in the class was aimed at developing familiarirty with both Ana and the students. By this time, Diogo had registered his observations only in writing. After this period, a sequence of twelve lessons about fractions and geometry were recorded in audio and video. The observations in class were focused on Ana's practice, especially on the mathematical and affective interactions between her and her students, aiming at identifying processes of mathematics enculturation and/or acculturation. They were also aimed to identify any characteristics of an acculturator-teacher in Ana's practice, according to Bishop's parameters. Attempting to search for evidence of cultural conflicts, some students were also asked to participate in interviews in small groups after the observations in class. Throughout his time in the class Diogo interacted with Ana and the students, participating effectively in the classroom activities and clarifying the student's doubts when requested by them.

ANALYSIS AND DISCUSSION

In relation to the observations of the lessons, we have identified that the secondary discourse, i. e. the academic mathematical discourse was predominant. The discourse of the *borderland discourse* we expected to find as something 'funding' the academic mathematics and the students' daily-life (primary discourse) was not identified during the lessons. On the other hand, we did not find any evidence that the students question this; they seemed completely adapted to and involved in the secondary discourse led by Ana. The potential cultural conflicts resulting from the gap between the secondary and primary discourses were not apparent; they did not have any stimulus to become explicit in the cultural interactions between the opposite parts: the culture of the teacher and the culture of the students. Moreover, the students have shown understanding of and effectively participated within the secondary discourse as it had been the only available or permissible discourse. We conclude that the students were able to attribute meaning to the mathematics learnt in classroom exclusively through the secondary discourse.

Two illustrations of the preponderance of this discourse in Ana's class are the following: in a certain lesson she was explaining the equivalence of fractions using a text prepared to the students by another mathematics teacher of the school. In relation to adding fractions, the text stimulated the work with equivalent fractions attempting to avoid students' rote learning of rules, but instead getting them used to the concept of equivalence. However, Ana opted to work with this topic in an abstract way; through the practical rule: *"... calculate the lcm* [least common multiple]*, divide by the down and multiply by the up"*. We are not saying that this method is not useful. What we claim in the terms of teaching in question is that the initiation of adding fractions should dispose of didactical resources and use a discourse closer to the students' primary discourses, likely tangible materials associated with daily-life contexts. An interesting utterance of the teacher addressed to the students drew our attention: "*I think this method* [the practical rule] *is easier. For this reason we are going to use it.*" Despite the friendly tone of Ana's voice, her utterance sounds authoritative; for she does not give any chance to her students to experience other forms of adding fractions and impose to them the method she thinks is better. This attitude of Ana and her preference for a discourse that is exclusively academic reveals some of her values in relation to mathematics. And this can be due to the fact that she was a novice teacher, bringing with her a discourse impregnated from years of undergraduate mathematical studies, which relies mostly on the secondary discourse.

In another lesson, Ana taught geometry to the students, using a sort of game as a didactical resource. On this day a teaching practice student was in her class and helped her in the conduction of this activity. The game consisted of a sheet of paper containing drawings of sixteen plane figures (e.g. triangles, rectangles, and so on). In order to play the game the students were divided in pairs and each pair received one of those sheets of paper in which one figure should be chosen. The game's rule was the following: each pair should discover the figure chosen by another pair. To this end, it was needed that the students had to elaborate questions related to the figures to be discovered, e.g. 'does this figure have right angles?', 'parallel sides?' Such an activity had, in our view, a great potential in exploring bridges between the secondary and primary discourses. We had the expectation to identify elements of a borderland discourse but this was not possible. All the activity with the game as well as Ana's and her students' mathematical systematization, related to both the definitions and the properties of the plane figures, were supported by the secondary discourse.

The two lessons described contain evidence of a process of acculturation, using Bishop's framework. Indeed, Ana can be considered as an acculturator-teacher, since we could identify in her practice two of Bishop's characteristics of an acculturator-teacher. However, this does not imply a negative influence in the students' affective dimension in relation to Ana nor in relation to their learning (at least, the learning Ana seemed to expect from them!). Either in the lessons or in the interviews, we noted that they nourished a positive affect for Ana. Examples:

Diogo: Every time you need or have any doubt, does the teacher [Ana] help you?

Leonardo: Always!

Diogo: She is a nice teacher, isn't she?

Leonardo: Yes!

Lucas: Yes! She helps, but sometimes I am [still] in doubt. Then, I ask her to explain [again] and she explains.

Our conjecture is that this affect might unfold into a very positive relationship between the students and their learning. All students interviewed by Diogo have expressed more or less explicitly their pleasure to learn mathematics and a satisfaction in completing a task. Examples:

Tainah: It [mathematics] is my favorite discipline, I adore it!

David: Mathematics, for me, is a good discipline. When I grow up I want to be a mathematics teacher. [This surprised us as this student showed much difficulty with mathematics in the lessons observed].

Lucas: When is a thing [in a task] that I don't know I stay there [persisting] and I feel even sweating! But when I see it [the response] is right, then I am delighted!

On the other hand, the two characteristics of an acculturator-teacher identified in Ana's practice do seem to contribute to a development of the students' beliefs that mathematics learning at school and mathematics learning at daily-life are two distinct practices that are insulated from each other. Examples:

Diogo: You said that you use fractions at home. Here, in the school, do you learn and use fractions as you do at home?

Tainah: Here [at school] is the same language! For fractions [at home], we give examples of food. Here [at school] we learn with sticks and at home we add other things.

Diogo: And where do you think is easier to learn mathematics, at home or at school?

Tainah: Are different learning!

Diogo: Could you please explain this better?

Tainah: At home we learn calculating quantity, money, millimeters, and at school we start calculating with natural numbers!

Leonardo: The carpenter is a person who knows mathematics I think, angles.

Diogo: Does he know this mathematics you learn at school?

Leonardo: No!

Lucas: No!

We suggest that this episode captures moments of cultural conflicts lived by the three students. In fact, desipte Tainah's statement that the language of fractions is the same at school and at home, she seems to be convinced that learning mathematics in these two contexts is different. She can be revealing another belief concerning an insulation between these two different cultures when she suggests that the learning of fractions at home is related to daily-life and practical uses, whereas at school it passes by an abstraction that moves the practical use of the subject to the mere use of number calculations. Leonardo and Lucas seem to share with Tainah a similar belief of insulation between mathematics in and out-of-school: the carpenter knows mathematics, but a mathematics that definitely has nothing to do with that of school.

FINAL COMMENTS

Considering the specific context of this study we can say that the students have experienced a process of mathematics *acculturation*. However, our research results indicate that such a process may not be necessarily negative, at least in terms of the students' affect in regard to both their teacher and their learning. The affective way – tender and supportive – by which Ana has assisted her students is an evidence of this assumption. On the other hand, the same results suggest that Ana's values concerning the exclusivity of the secondary discourse in class do seem to consolidate a negative belief among the students of a conflicting/insulated dialogue between mathematical practices in- and out-of-school. We conjecture that this might result in an obstacle for the students from experiencing a process of becoming mathematical as we understand it (a question of continuing concern to us for future investigations). We agree with Bishop when he says that teachers should stimulate interactions in classrooms in which cultural conflicts might become explicit, and consequently, objects of negotiation and co-construction of meanings between the opposite parts. In doing so, teachers could make the boundaries between mathematics culture and that of the students more permeable, allowing them not only to cross these boundaries, but also to circulate in and to explore dialogues between these two non-insulated cultures.

At the moment we are observing the practice of a second teacher with the same research purposes and using the same methodological procedures as the study in Ana's class. The preliminary results of this second observation together with those related in this paper suggest to us that, with some revision, Bishop's notion of the processes of mathematical enculturation and acculturation can better account for a relationship between teachers' culture/values and students' affect. We conjecture that teachers' practices are characterized by other relevant aspects, such as the supportive and affective ways in which the teachers assist their students, that go beyond the scope of the notion of acculturator-teachers.

NOTES

1. Supported by FAPEMIG and CNPq.

2. Due to lack of space this paper will not present a wide review of the literature in mathematics education, related to culture and affect; it will be restricted to the presentation of some academic ideas that we believe are sufficient to give a sense to and report on our study.

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REGULAR PAPERS (CONTINUATION)

QUANTITATIVE FORM IN ARGUMENTS

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Changing the form in which numerical information is presented can help us make sense of quantities whose significance we cannot grasp. Changing the form through basic calculations can allow us to feel the impact of those quantities through better understandings. Further, knowing the most meaningful quantitative form in which to express information is necessary in order to understand what's going on, and to make arguments for changing what's going on. This paper will develop these general ideas, including specific examples of arguments with quantitative evidence and discussions of the various ways in which that evidence can be presented, and the consequences of those various presentations.

INTRODUCTION

Many people argue that too much data about the injustices in our world make us numb to the realities of those situations in people's lives. Wasserman's editorial cartoon about the Israeli occupation of Palestine makes this point:

However, I argue that we do need to know the meaning of the numbers describing our realities in order to deepen our understandings of our world. One of my proudest academic moments was when *The Nation* published an edited version of the following letter I wrote responding to an article by Howard Zinn in which he argues that the numerical descriptions of the deaths from the USA war on Afghanistan can obscure those horrors.

February 11, 2002

To the Editor:

Howard Zinn's article ("The Others," February 11) is a powerful reminder of the horrors that are perpetrated in the world on all the days in addition to 9/11. I, too, cried as I saw the portraits of the 9/11 victims. I, too, was crying not only for them, and not only for the victims of the wars the USA and other powers create, but also for the millions who die every year because of economic terrorism—from unsafe working conditions that kill 'by accident', to unjust working conditions that result in death from preventable causes such as hunger.

Binu Mathew reports in Z magazine (January 2002) that at this point in time the original death toll of 8,000, caused by the Bhopal gas leak at Union Carbide's factory, has increased to 20,000, growing every month by 10-15 people succumbing to exposure-related diseases. Union Carbide management delayed sounding the public siren for 15 hours, and continues to obstruct full revelations which would have helped decrease some of this horrific toll. I, too, wondered, if detailed, in-depth TV and newspaper

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 261 - 271).

portraits of these victims, and of, say, the 12 million children who die from hunger every year (Food First Backgrounder, “12 Myths about Hunger,” Summer 1998), would wake up our collective consciousness.

Zinn makes another important point that I stress with my Quantitative Reasoning classes at the College of Public and Community Service (University of Massachusetts/Boston): statistical data can distance us from a deep empathy and understanding of the conditions of people’s lives. Of course, the data are important because they reveal the institutional structure of those conditions. But, also, quantitatively confident and knowledgeable people can use those data to deepen their connections to humanity. Those 12 million children are dying faster than we can speak their names.

In essence, the quantitative point of my letter is about the form in which we put numerical information. Changing the form can help us make sense of quantities whose significance we cannot grasp. Changing the form through basic calculations can allow us to feel the impact of those quantities through better understandings. Further, knowing the most effective form in which to present those quantities in arguing for creating a just world, is an important skill to teach in a criticalmathematical literacy curriculum. I would go so far as to argue that knowing the most meaningful quantitative form in which to express information is necessary in order to understand what’s going on.

CHANGING THE FORM TO UNDERSTAND LARGE QUANTITIES

In the *Globe Magazine* article “Playing with Billions” (Denison, 2002) the author did not use my favorite ways of making sense of the size of one billion. Try to guess, without calculating, the answers to the first two questions to get a sense of how little sense we have about the meaning of large numbers.

- About how long, at the non-stop rate of one number per second, would it take to count to one billion?
- About how long, at the non-stop rate of $1000 per hour, would it take to spend $1,000,000,000?

And, the *Globe Magazine* article certainly did not use a more political and real meaning of these numbers by asking us to think about such questions as the following: What human services programs we could fund from various items in the military budget of the USA. In 1997, was “a B-2 bomber worth more than twice the $800 million currently being saved by cutting 150,000 disabled children with insufficiently severe disabilities? Is $248 billion for the military and $31 billion for education a proper balance in the use of federal funds?” (Herman, 1997, p. 43)

Domestic Program	Military Program
Home-heating assistance for low-income families ($1 billion)	Cost of 1 Arleigh Burke destroyer ($1 billion)
Raise Pell grants to $3000 ($1.7 billion)	Cost of 1 B-2 bomber ($2.1 billion)

Head Start for young children ($4.3 billion)	Cost for 1 Seawolf attack submarine ($4.3 billion)
Drug prevention programs ($2.2 billion)	Request for F-22 fighter program ($2.2 billion)

Key domestic programs vs. Major military programs. Fiscal 1998 Budget (Center for Defense Information, quoted in Herman, 1997, p. 43)

Further, other seemingly large numbers are not so large when placed in context, in a different quantitative form. Dean Baker (2005) even argues that without proper context, people's lack of quantitative understanding results in their voting against their own interests. For example, in the mid-90's a Kaiser poll found that 40 percent of people ranked welfare for those with low incomes as one of the two largest items in the federal budget—the number, even according to the government statistics that mislead in a way that underestimates human services, was under 4 percent. Baker blames this on the way these numbers are reported--$16 billion sounds gigantic, but in context it is only 0.6 percent of total federal spending. He feels that when people have such an exaggerated view of current welfare spending, they are unlikely to support increases in programs for those with low incomes.

QUANTITATIVE FORM IN ARGUMENT

One consideration in understanding, evaluating and constructing arguments whose claims are supported by quantitative evidence, is the form in which this evidence is presented. Is it clear, or is it misleading? Is it powerful, or is it likely to be ignored? In this section of the chapter, I am going to focus on examples in which the latter question is explored. Below are a number of examples from my curriculum which illustrate various ways I ask students to reflect upon how the forms of quantitative data affect the meanings we take from information.

(1) We discuss the table below from *The Nation* (1991): is the numerical form of this counter to the first Bush's claim about our former wars, in particular our war on Vietnam, powerful? What might be more powerful forms in which to express the numbers supporting this counter-argument?

WHAT DOES HE MEAN?

"*Our troops…will not be asked to fight with one hand tied hehind their back*."

—President Bush, national address, January 16 [1991]

Tons of bombs dropped on Vietnam by the U.S.: 4,600,000[1]

Tonnage dropped on Cambodia and Laos: 2,000,000[2]

Tonnage dropped by the Allies in World War II: 3,000,000[3]

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1 Jim Harrison, "Air War in Vietnam," recent conference paper

2 Jim Harrison, "Air War in Vietnam," recent conference paper

3 Howard Zinn, author, *A People's History of the United States*

Gallons of Agent Orange sprayed: 11,200,000[4]
Gallons of other herbicides: 8,000,000[5]
Tons of napalm dropped: 400,000[6]
Bomb craters: 25,000,000[7]

(2) As part of understanding Helen Keller's argument below, students are asked to discuss how she uses numbers to support her claim, to evaluate whether that support makes her claim convincing, and to reflect on the form of her data—why she sometimes uses fractions, other times uses whole numbers, and whether there are alternative ways of presenting her quantitative evidence that would strengthen her argument.

> In 1911, Helen Keller wrote to a suffragist in England: "You ask for votes for women. What good can votes do when ten-elevenths of the land of Great Britain belongs to 200,000 people and only one-eleventh of the land belongs to the other 40,000,000 people? Have your men with their millions of votes freed themselves from this injustice? (Quoted in Zinn, 1995, p. 337)

(3) In another argument we study about the globalization of workers in the garment trades, Gonzalez (1995, p. 148) compares the Leslie Fay company's Honduran workers' earnings with the company's sales receipts. Specifically, he expresses the quantities as a "grand total of $300" that all assembly line workers in Honduras cost Leslie Fay for one day (and he includes the information that this comes from 120 workers paid $2.50 per day), and $40,000 in retail sales that the company takes from the skirts those workers make in that one day. He does not say how many skirts they each make (another quantity which could reveal a different aspect of the exploitation). He does not calculate the workers' yearly pay and compare it to the yearly retail sales of those Leslie Fay skirts.

One of the points brought out on our discussions is that if he had just focused on the average worker's daily pay and the average amount the company made from selling the skirts she made, he would have compared $2.50 to $333. We speculate that he felt, even though the numbers for one worker are in the same proportion as the numbers for 120 workers, the $333 figure would not appear as outrageous to the reader as the $40,000. Also, most readers can relate to $40,000 in terms of their yearly income—most would have a yearly income that is somewhere between half and double that figure. Since Leslie Fay is grossing that amount of money in one day, that adds to the outrageousness Gonzalez wants us to feel. If he had calculated the yearly sales—$14,600,000—since most readers could not deeply grasp the meaning of such a giant number, its impact would have been less than when readers can think

4 Jim Harrison, "Air War in Vietnam," recent conference paper

5 Jim Harrison, "Air War in Vietnam," recent conference paper

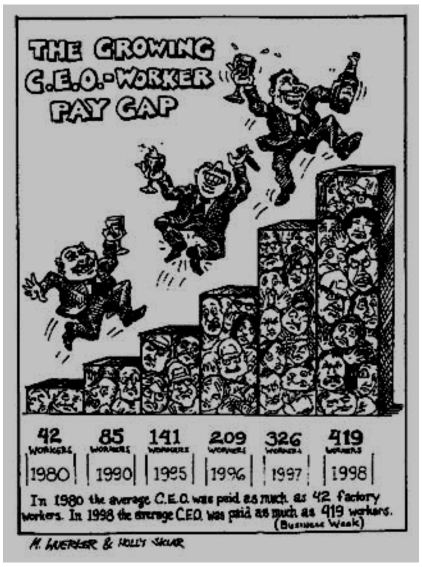
6 Jim Harrison, "Air War in Vietnam," recent conference paper

7 Marilyn Young, *The Vietnam Wars*

"Leslie Fay makes more in one day than I make in a year!" And, even minimum-wage workers in the USA could think, "In one day, they are paying their workers less than I make in half and hour—could it cost that much less to live decently in Honduras?"

(4) Following is an argument supported by quantitative evidence, and four forms of similar quantitative information that could have been used to support the claim. Students are asked to identify the claim, the reasoning and the evidence Wuerker and Sklar are making in their editorial graph below. Then we compare the different forms of the quantitative evidence given in each of the related arguments. Students reflect on which quantitative form provides the most powerful support for the claim? What other kinds of numerical data would further strengthen the claim? What data or reasoning would a counter-argument present?

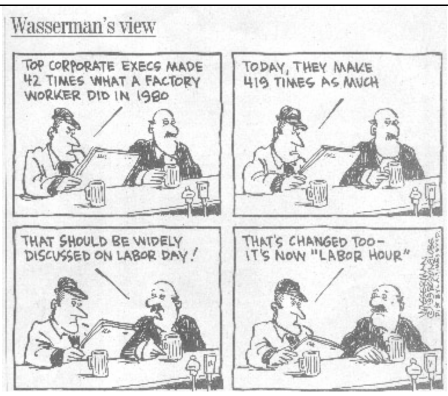
(a) The graphic argument from Sklar (1999):

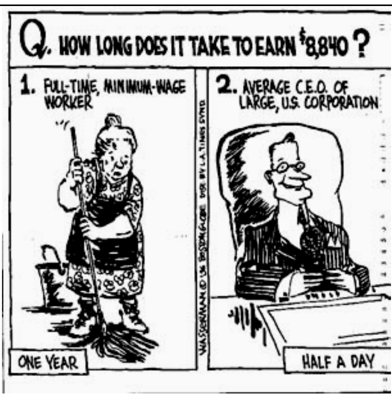


(b) Another presentation of similar quantitative evidence from Jackson (2001): If the minimum wage had risen at the same level pace as executive pay since 1990, it would be $25.50 an hour, not $5.15; if average pay for production workers had risen at the same level as CEO pay since 1990, the annual salary would be $120,491, not $24,668.

(c) Another presentation from United for a Fair Economy (Sklar, 1999. p. 63): "If the real 555-foot Washington Monument reflects average 1998 CEO pay, then a scaled-down replica representing average worker pay would be just 16 inches tall—5 inches shorter than in 1997. Back in 1980, the Workers Monument was over 13 feet tall—reflecting a CEO-worker wage gap of 42 to 1."

(d) and (e) Two presentations from Wasserman (1996):





QUANTITATIVE FORM IN ARTISTS' ARGUMENTS

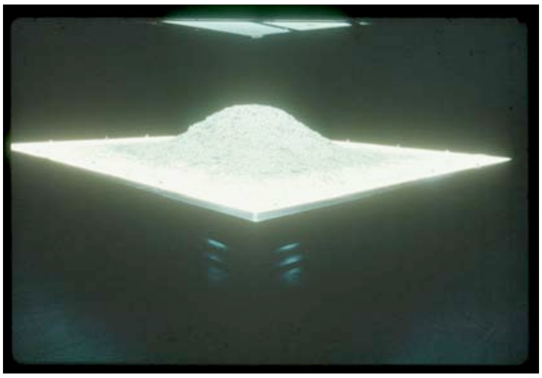
In addition to various written and graphical forms of quantitative information, the arts present different kinds of opportunities for people to understand and use quantities in arguments. As Toni Morrison states: "Data is not wisdom, is not knowledge." (Quoted in Caiani, 1996, p. 3) Caiani goes on to add:

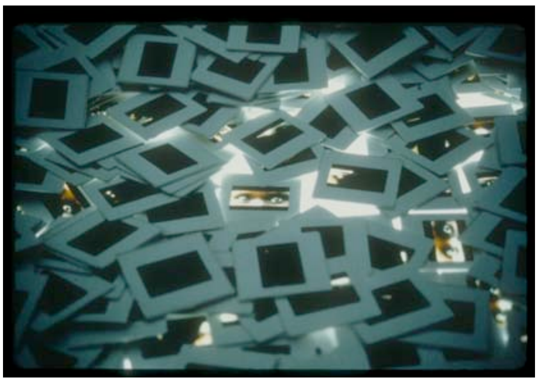
> In contrast to stories told in a living language filled with images taken from the human world, facts, statistics, data and bits of information, valuable as they are, slide in and out of memory without fully engaging sustained, powerful connections to the whole being. Data are important, are necessary, but not all by themselves, not alone. Analysis and facts are not able to give a face, eyes, a body to the suffering, joy, love, anguish of the people … Art, I am convinced (as indeed have been many scientists like Pascal and Einstein), is at least as necessary as the sciences in grasping reality if we are ever to effect the change we seek in our long struggle to be human. …Our efforts to make a better world through a narrow, reductive, isolated, scientific method which relies on the accumulation of data and its business-like interpretation, will fail. (p. 3)

The following examples of art encode quantitative information in ways that make us understand—what does this amount mean? The numbers are the data of our world—our wars; the art allows us to understand the quantities in ways we could not understand from the numbers alone.

"'The other Vietnam Memorial' (by artist Chris Burden, USA) refers to the famous memorial in Washington, D.C. by artist Maya Lin which lists the names of 57,939 Americans killed during the Vietnam War. In this work, Burden etched 3,000,000 names onto a monumental structure that resembles a Rolodex standing on its end. These names represent the approximate number of Vietnamese people killed during U.S. involvement in the Vietnam War, many of whom are unknown. Burden reconstructed a symbolic record of their deaths by generating variations of 4000 names taken from Vietnamese telephone books. By using the form of a common desktop object used to organize professional and social contacts, Burden makes a pointed statement about the unrecognized loss of Vietnamese lives." (notes from the Museum of Contemporary Art in Chicago, IL)

Another artist, Alfredo Jaar (born in Chile, works in New York City), went to Rwanda in 1994 to try to understand and represent the slaughter of "possibly a million Tutsis and moderate Hutus" during three months of Prime Minister Jean Kambanda's term. "Even after 3000 [photographic] images, Jaar considered the tragedy to be unrepresentable. He found it necessary to speak with the people, recording their feelings, words and ideas….In Jaar's Galerie Lelong installation, a table containing a million slides is the repetition of a single image, The Eyes of Gutete Emerita." The text about her reads: "Gutete Emerita, 30 years old, is standing in front of the church. Dressed in modest, worn clothing, her hair is hidden in a faded pink cotton kerchief. She was attending mass in the church when the massacre began. Killed with machetes in front of her eyes were her husband Tito Kahinamura (40), and her two sons Muhoza (10) and Matriigari (7). Somehow, she managed to escape with her daughter Marie-Louise Unumararunga (12), and hid in the swamp for 3 weeks, only coming out at night for food. When she speaks about her lost family, she gestures to corpses on the ground, rotting in the African sun." The art review ends with a comment about the numbers: " The statistical remoteness of the number 1,000,000 acquires an objective presence, and through the eyes of Gutete Emerita, we witness the deaths, one by one, as single personal occurrences" (Rockwell, 1998).






ADDITIONAL CONSIDERATIONS ABOUT QUANTITIES IN ARGUMENTS

In addition to quantitative form, there are other kinds of considerations about quantities that are important in understanding, evaluating and making powerful arguments that challenge the global imperialism that has trickled down into every corner of our world. In addition to the more typical ways of classifying various misleading statistical accounts (like mistaking correlation for causation), I think two overarching questions are important to consider: What are the political, as opposed to scientific/mathematical, aspects involved in the data presented? Is the measure chosen the most accurate way of describing or analyzing the situation, or, in other words is the correct answer being given to the wrong question?

It is important to understand which aspect of quantitative evidence is mathematical fact and which is political, and therefore, subject to debate. Much data is presented as if they were neutral descriptions of reality, masking the political choices that produced the data. For example, once the government determines which categories of workers count as part of the labor force, and which categories of workers count as unemployed, rewriting that information in percent form is a mathematical algorithm for which there is only one answer and about which it does not make sense to argue. The politics about which we can, and I would argue, should, argue comes in decisions made by the government such as to count part-time workers who want full-time work, as fully employed, and to not count workers who have looked for over a year and not found a job, as unemployed.

It is also important to determine which quantitative measure gives the most accurate picture of a particular issue. For example, the ultra-conservative Heritage Foundation argues that our progressive federal income tax is terribly unfair because of facts such as in 1997 the top 1 percent income group paid 33.6 percent of all federal income taxes, while the share of all taxes paid by the bottom 60 percent was only 5.5 percent. But, a left (and even a liberal) perspective would argue that is the wrong measure by which to judge tax equity. It makes much more sense to look at what happens to the share of total pre-tax and total after-tax income. That picture reveals only a small progressivity: a slight upward shift for the bottom four quintiles, and a slight downward shift for the top income quintile. Ellen Frank (2002) then argues

> If one believes that Ken Lay deserved no less than the $100 million he collected from Enron last year, while the burger-flippers and office cleaners of America deserve no more than the $6.50 an hour they collect, then a progressive tax would seem immoral. But if one believes that incomes are determined by race, gender, connections, power, luck and (occasionally) fraud, then redistribution through the tax system is a moral imperative. (p. 44)

Frank goes on to discuss the impact of other kinds of federal taxes, such as Social Security taxes (which are capped at $90,000), excise taxes and so on which are regressive, as well as state and local regressive levies like sales taxes. She hypothesizes that adding all these taxes together would "almost certainly find that the U. S. tax system, as a whole, is not progressive at all" (p. 44).

CONCLUSION

In "Scenes from the Inferno," Alexander Cockburn (1989) wrote about some of the realities behind the so-called worldwide triumph of capitalism. One of his illustrations is particularly relevant to understanding how the wrong quantitative measure has real consequences in people's lives. He relates how in some neighborhoods of Santiago, Chile, "the diet of 77 to 80 percent of the people does not have sufficient calories and proteins, by internationally established standards, to sustain life." Under Pinochet, the dictator of Chile during that country's period of 'triumphant capitalism,' malnutrition was measured in relation to a person's weight and height, in contrast to the usual comparison of weight and age. "So a stunted child is not counted as malnourished, and thus is not eligible for food supplements, because her weight falls within an acceptable range for her height" (p. 510). I argue that the overarching goal underlying a criticalmathematical literacy curriculum is to explore the connections between understanding the outrageousness of collecting such statistics, and struggling to change the outrageousness of such conditions.

This paper, however, focused on the importance of the form in which the measures are actually expressed. I argue for the importance of criticalmathematical literacy curricula also consisting of reflections on questions such as whether it is more powerful to state that "The wealthiest 1 percent of Americans control about 38 percent of America's wealth" (Jackson, 1999) or that in the United States, "The richest 1 percent owns more than the poorest 92 percent combined" (Food First, 1998). Part of struggling to change our world in the direction of more justice is knowing how to clearly and powerfully communicate the outrageousness.

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Note: A version of this paper, complete with extensive endnotes, appears on the conference website.

ENACTING IDENTITY THROUGH NARRATIVE: INTERRUPTING THE PROCEDURAL DISCOURSE IN MATHEMATICS CLASSROOMS

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This paper emerges from a research project designed to explore the complexities of mathematics teacher identity. Four mathematics teachers were studied in their classrooms during one semester. Notes and transcripts from the class observations were analyzed using a discourse analysis framework. The discourse was treated as cultural data regarding identity in the math classroom. Analysis focused on the use of the narrative register in classroom discourse, and on how this register was contextualized within the dominant procedural discourse.

MATHEMATICS TEACHER IDENTITY

When procedural tasks govern classroom discourse, an "identity of mastery" tends to govern subjectivity. Many mathematics teachers, however, disrupt this seamless identity by performing against mastery in both deliberate and unanticipated ways. Teacher narratives, for instance, often erupt unbidden in classroom discourse, disrupt the procedural sequence of blackboard instruction, and function as embodied performances of socio-historical identity. Personal narratives which are grounded in the embodied experiences of the teacher often constitute teacher identity in terms of vulnerability and contingency.

When teachers shift back and forth between procedural discourse and personal narrative they enact radically different identities. The procedural discourse enacts a mastery identity ("I" as machine) while the personal narrative enacts a vulnerable identity ("I" as embodied self). At the same time, each discourse is inscribed with normative messages about the legitimacy of particular subject positions in the classroom; the procedural discourse positions the teacher as expert ("I" as authority) and the personal narrative positions the teacher according to race, gender, ethnicity and class ("I" as member of the community). This last facet is often conveyed implicitly in the personal narratives that the teacher "chooses" to share. For instance, stories about incidents witnessed as one drove to work that day communicate messages about one's socio-economic position within the community. If one accepts that these personal narratives "teach" students about socio-cultural positioning while in math classrooms - often without teacher awareness that they are doing so - then the need to study the role of personal narratives in relation to the dominant procedural discourse becomes essential.

In this paper I examine when, how and why mathematics teachers shift between the procedural and the personal narrative registers. The shift between procedural and personal narrative register is almost always awkward because of the radically

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different subject positions constituted through the two discourses. Indeed, the two discourses are so radically displaced from each other, it's difficult to imagine the bridging or blending that might create a cohesive discourse that includes them both. It is this apparent incomprehensibility which is the focus of this paper. My aim is to show how the personal narratives are actually used to enforce the legitimacy of procedural discourse.

Four high school mathematics teachers were studied in their classrooms during one semester. Notes and transcripts from the class observations were analyzed using a discourse analysis framework. The discourse was treated as cultural data regarding identity and community in the math classroom. Analysis focused on the use of the narrative register in classroom discourse, and on how this register was contextualized within the dominant procedural discourse.

PATTERNS OF DISCOURSE

A discourse analysis framework (Fairclough, 2003) was used to examine the transcripts from the classroom observations. The data was first coded for shifts between three kinds of discourse: mathematical (inquiry, procedural, conceptual), administrative (assessment, management, school issues), and contextual (personal narrative, anecdote, metaphor, application). These were then subdivided into eight registers: (1) procedural (2) conceptual (3) inquiry questions (4) personal narrative (5) anecdotal (6) metaphoric (7) classroom management (8) school business. The transcripts were analyzed for occurrences of and transitions between these registers. I use the term register to refer to a subset of the many genres that characterize conventions of spoken language. In this paper, the term register refers to different modes of address in spoken discourse. A register enacts rules or conventionalized practices of language use. Because of my focus on procedure and narrative, I will define only these two registers and leave the others alone.

The procedural register is highly abstract and depersonalized. It contains almost no traces of personal presence. Personal narrative, on the other hand, positions the speaker and explicitly constructs a social identity. My analysis focused on the juxtaposition between procedural and personal narrative registers. This paper examines the relationship between these two registers, focusing on the way that teachers blend or join the registers. Narrative embodiments within the realm of procedural discourse can be seen as an attempt to introduce a sense of self or identity. The personal narrative register presents the speaker as a situated body in a socio-cultural and physical world. The discourse within the narrative is understood to be of a different ontological status than the discourse within the procedural register. Each register seems to interrupt the other because they seem so mutually incomprehensible. I argue, however, that the two function alongside each other in co-constituting the subject position of the teacher. I argue that the process of suturing an identity across these two radically different registers must be read in terms of the dominant procedural discourse.

Shifting registers or frames is a complex linguistic capacity. When a speaker moves abruptly from one form of address to another, they often enact two radically different and sometimes mutually incomprehensible forms of subjectivity. The shift involves more than two specialized sets of vocabulary in two different discourses. Each register has its own social grammar, and each thereby interpellates a distinct subject or identity. Both the speaker and listener are constituted through the form of address. The meaning of an utterance is constructed within the social grammar and the implied power relations of the register. The personal narrative register often enacts power relations of intimacy, exposure and vulnerability. The listener is addressed as a confidant. Refusal to be recognized as personal acquaintance would disrupt the power relation conveyed within the register. The act of personal narrative is one of vulnerability but it is also an act of centering. The story teller offers personal anecdote and seemingly "exposes" a private world, but she or he also demands that the listener recognize (by listening) her status as a person with a life history, and, in some cases, her status as an agent of change or action. The following are a few facets of subjectivity that are constituted through the personal narrative register: (1) vulnerability (2) agency (3) social position/presence, and (4) temporality. In contrast, the procedural register is characterized by rigorous rule following and the imperative mode, and thereby constitutes facets of subjectivity that are radically different. In the procedural register, both speaker and listener are addressed in terms of: (1) proficiency (2) compliance (3) abstraction/absence (4) atemporality.

WHY TELL A STORY?

While it is possible to read registers in terms of purpose, and it seems as though most discourse occurs for a purpose, it is essential that we recognize the difference between overtly purposeful discourse – like that found in the procedural register – and what is often apparently non-instrumental discourse – like that found in personal narrative discourse. Although both registers in the classroom can be read through the lens of strategy, as can the "informal chattiness" of employees in other work environments (Fairclough, 2003, p. 72), the personal narrative can also appear entirely without purpose. Speakers often spontaneously erupt into story, without any premeditation. Although one could argue that stories are told when a speaker deems it fortuitous or appropriate, it's important to recognize the apparent spontaneity of story in particular instances, and to note that this perceived spontaneity marks the narrative as a kind of psychoanalytic trace. Narratives often erupt unbidden, disrupt our attempts to present ourselves in a professional or other way, and thereby indicate facets of our subject position which we enact without conscious intention. In this project, the narratives were not invited in an interview, but simply emerged spontaneously in the classroom. This research method allowed for a more accurate study of how narrative functioned in the classroom discourse. The approach allowed me to focus on what stories are doing for the participants, and on what stories are designed to do in this context. In contrast to a more deliberate collection of stories through interviews, this approach examined how narratives function ethnomethodologically.

Recent research in mathematics education calls for further study on how narrative intersects with mathematics learning (Povey & Burton et al., 2004; de Freitas, 2004; Drake, Spillane & Hufferd-Ackles, 2001; Doxiadis, 2003). Doxiadis argues that a new vision of 'doing mathematics' might also incorporate the telling of mathematics experiences in a narrative mode, a kind of "paramathematics" that situates mathematics in story (Doxiadis, 2004). Povey & Burton suggest that such narratives are crucial for making sense of why so many "fail in their attempts to learn mathematics and, in particular, why so many of these unsuccessful learners are predominantly found in particular communities." (Povey & Burton et al., 2004, p. 43). But none of these researchers have examined the role of story in the mathematics classroom as a form of identity enactment, nor the ways that teacher stories disrupt procedural discourse in classroom practice.

THE PARTICIPANTS AND THE STORIES

The four participants were Roy, Janet, Mark and Leslie. They teach in a rural school in Canada. The school population is over 95% white. It is not a high-needs school, but there is significant socio-economic diversity within the student population. Classes are each 80 minutes long, and class size is about 30 students. Each participant was observed on five occasions. In this paper, I focus on Janet and Leslie, each of whom employed narrative in their mathematics classrooms in different ways.

Janet

Janet had been teaching for 13 years. She taught a grade 10 academic class. During the five observations, Janet shared only one personal narrative with the class. She spent more time on classroom management talk than the other participants. She used very unsituated examples in her instruction (emphasis on sign manipulation), but she did try on occasion to give motivating contexts for the mathematical material ("Suppose you want to build a roof"). Her lessons were delivered using overhead projector, digital projector and blackboard. The students copied notes as she revealed them on the screen. She spoke to the whole class as she disclosed the written material and diagrams. Whole class instruction was followed by individual work. Below is the only personal narrative that Janet shared while she was observed. I have included the procedural discourse that contextualizes the narrative, so that the reader can see the way that Janet shifted back and forth between the registers.

[After 4 minutes of requests for "hats off" and "put your calculators away", and 3 minutes of announcements about the day's agenda, she asks the students to complete two calculations on the blackboard. The first calculation is 231 x 25. The second calculation is 42360/20. She prepares her slides while they settle down and attempt the calculations.]

Janet: Alright, has everybody finished the first problem? Ok. Let's just take a look at number 1. Because it's not necessarily easier but a little less mess than that other one. Alright. Is it a fair assumption that by the time you've hit grade 10 academic math you should be able to multiply those out.

Students: Yes.

Janet: Ok. Perfect, so what do you do? Where do you start?

Students: The bottom right.

Janet: The bottom right? Times each of the top. Right, you're going to get zero there. And then you go down to your next row. You put a place holder for that one and you start the next one and so on. Is that what everybody did? Used placeholders and worked it all out? What did we get for an answer?

[Most of the class responds]

Janet: 7350? How many got it? Excellent. Ok. That is encouraging. Alright, now there's a method to my madness in having you do this today. Ah, twofold. Obviously you should be able to multiply and divide in grade 10. But you would be surprised at how many people can't. I remember, I think I may have told this story before, I went to get gas. This happened last summer. And I gave the person like, I don't know, it cost 35 dollars to fill my car and I gave them 50 and they couldn't figure it out. And it was weird, because like within a 2 week span every time I went to buy something somewhere, groceries, I kind of ran into the same problem. People, if the machine wasn't working right or you gave them like, say it came to 20 dollars and 46 cents and you gave them $25.46, it threw them off. They didn't know what to do. That sort of thing. So it was kind of enlightening to me that, you know, not everybody is getting these basic math skills. So I'm glad and impressed that you guys can multiply. So, that's a good thing. The real reason I had you do this is, because how long did it take you just to do problem number 1?"

Student: A long time.

Janet: A while. If I said now that you can use your calculators, how long would it take you?

Students: seconds

Janet: Or less, right? So, if you do not bring a calculator every day for trigonometry, you'll be doing that all class. I'm serious. So, the method to my madness is, you will bring a calculator every day after seeing how much torture it is trying to multiply and divide those numbers out.

Janet's story is explicitly functional. First, the story is the explanation as to why she is asking them to perform calculations "long hand". It functions as an explanation for the mathematical task. Her story recounts an experience that has made her worry about basic numeracy skills in the local population. She says, “I think I may have told this story before”, pointing to its ritual status as a story of moral significance. Stories are repeated in this way when they function as parables or moral lessons. By pointing out the repetition, she reminds the students that this story is essentially a lesson about life. The story also functions implicitly as a lesson about numeracy and socio-economic positioning. As a moral lesson, the story warns the students that, without numeracy skills, they are no better than the unskilled gas attendants and grocery cashiers whom she encounters. Her grade ten class will be divided the following year into university track students (to be found in Roy’s class) and college or other bound students. The story is a way of reminding them of the material consequences of their performance in school mathematics.

Janet shifts between the narrative discourse and the procedural by saying “The real reason I had you do this ….” thereby bracketing the narrative into a separate enclave of less value (as it turns out, the story is a red herring, because she actually wants them to rely on calculators during trigonometry). But her emphasis on procedure is mirrored in the story she chooses to present to the class, a story about procedural mastery of multiplication. Note that she is concerned that her encounters at the gas station and elsewhere point to others’ inability to perform mental math calculations, and yet she asks the students to perform longhand calculations. The disconnect between the story and the task is compounded when she states that her “real” purpose is to help them see the value of calculators, since the story admonishes those who become too reliant on calculating machines. The contradictions regarding her stated purpose underscore the many different functions of the story.

Janet’s personal narrative is highly disembodied. Her story conveys no intimacy and no vulnerability on her part, but rather functions as a lesson about social position and subjectification. As such, it breaks with one of the usual aspects of personal narrative – to convey the vulnerable “I”. She is the authority in the room and uses the story to communicate both her power as moral judge and the power of school mathematics to determine socio-economic status. The paradox of her own subject status is in her disembodiment. She enacts or performs an identity of mastery which she uses to control the students. Her personal narrative functions to enforce her control, by pointing to the ramifications for students if they do not submit to the rules of the discourse. Thus, her desire to control the students is implicated and given larger cultural significance through the narrative performance.

Leslie

Leslie had been teaching for 7 years. She taught a grade 11 “Life mathematics” course. Poor attendance was an issue. During the five observations, Leslie taught from the front of the classroom, writing examples on the blackboard and asking the students for answers. Her questions shifted back and forth between personal questions about student experiences (both related and unrelated to “Life Mathematics”) and questions about factual or procedural issues in the content. She struggled to keep student attention. She frequently introduced personal anecdotes into her talk, often about her family. The following is one paradigmatic story in Leslie's classroom discourse.

[After 7 minutes of administrative discourse (exam schedule), Leslie asks "Ok! How many go to the races in the summer? You know the races?"]

Student: What races?

Leslie: Horse races. When you're looking at the horse race, what do you see on the board? When they have the horses all …

[An announcement interrupts the class]

[Students walk in late]

Leslie: Guys, class starts at 8:50. If you're gonna be late, you'd better bring a note. You missed my speech about how there's 11 days left and to make sure you're here on time and all the rest.

Student: I didn't even think I was going to, like, make it. I was ready to go back to sleep.

Leslie: We don't need all the stories. Ok, um, Brad, you were saying that the horse races, again. I'm not 100% accurate on this because I don't go to the horse races but I have been there a time or two. Ok, what would this mean?

[Leslie writes on the board 1:100. She continues to question the students about the ways in which odds in favour of a horse winning is calculated. Then she talks about odds in favour and against in the abstract case. Twice more she refers to horse racing and mentions that she doesn't know much about them: "I don't know, anybody, who goes to the races a lot? Mike? What are the numbers usually like?" and then "Oh. Anybody a horse person? I'm not so I can't really say I've seen those numbers on there when I go to the exhibition, to walk through there, the horse races." A few students speculate about the numbers (no use of hands to mark the right to speak).]

Leslie: Yeah. Anyway, like I say. I 'm not up on horse racing to tell you, but I do know that those numbers are telling what are the odds, and the greater the odds, the higher the payout. I do know that much. My husband has a good friend who's big into horse racing. They went away to school one year, they went down to the states where it's cool and they went to the horse races one night, and there was a horse there who the guy remembered being back [in his hometown], years ago, and what a good horse it was. So, they had like 20 bucks each and decided to throw it on to see what happens and the odds were very high that he would not win. And, so anyway, they won. They won like 250 bucks each or something like that. When you're students away somewhere that's a lot of money.

[An individual student near the front says something to her and they talk while the rest of the class cannot hear. Leslie then begins writing on the board. The students then begin recording what she is writing]

Student: What is the word right after …?

[Leslie proceeds to talk to the entire class about the procedure for calculating and recognizing odds and probability. Her description of the rules for representing probability and odds are somewhat confused as she speaks: "When we're looking at probability, it was always a fraction. It was always like 1 out of 10 or 2 out of 6 … ok each of them would be a number between zero and 1 and they would be written as a ratio…"]

Leslie tells stories about her family. She presents herself as a mother and wife. Her anecdotes convey her preferences and her knowledge about everyday matters. A number of utterances communicate her unfamiliarity or lack of knowledge with the rules of horse racing. These anecdotes set up her vulnerability as someone who is willing to learn from her students. The course she teaches lends itself to the personal narrative register, because of the curriculum resources, but nonetheless she might have used stories about others instead of her own family. In this example, when she shifts to the personal narrative register, her confidence about what she knows increases. She is demonstrating that knowledge gained from personal experience is more reliable than other kinds of knowledge. Unfortunately, she fails to say "odds against" in the utterance: "… the greater the odds, the higher the payout," and thereby suggests the opposite and incorrect correlation between odds and pay-out. But it may be that the actual mathematics embedded in the story is less important than the other messages embedded in the story. Leslie shifts to the personal narrative register in order to present herself as socially and physically positioned.

The story recounted in this excerpt is about her husband's experience "away" (in another country) as a student without money, making a bet on a long shot because of the connection between the horse and his home town. It's not clear why she decides to tell the story, except to entertain the students or perhaps reveal a "real life" situation where evaluating odds matters in terms of money. The narrative, like the many anecdotes about burgers and other local items, presents her as an individual with a particular life history. It is also a story about going away to school, which is an experience associated with academic success, particularly in this rural community, and even more so in the "Life Mathematics" class. Much of the talk in this course pertains to making, saving and judiciously spending money. There are chapters in the textbook about balancing a cheque book, assessing bank loans, and how to raise money to buy a car. And yet the story seems to convey the very opposite of a moral lesson about frugality and sound money sense. In the story, her husband and his friend enjoy the privilege of a post-secondary education and the accompanying pleasures of making high-risk and playful gestures with their money. She closes the story for the class with an evaluation of the value of the winnings. When she shifts back to the procedural register she prefaces it with "And these are definitely not based on fact, these are just examples," thereby highlighting the radical disjuncture between the two registers. The classroom examples are inauthentic, whereas the story of her husband's successful gambling represents "the real".

Leslie enacts the vulnerable "I" to create a relationship of trust with her students. Her story also positions her as the wife of someone who is from the community (an important issue of membership in rural contexts) and whose husband is sufficiently privileged to pursue post-secondary education in another country. The story about gambling on the horses represents a disruption of the dominant cultural message of frugality and balance found throughout the "Life Mathematics" course. Because the students in this course have low socio-economic status, relative to the rest of the school and the surrounding area, the story addresses them in terms of class structure and privilege. When this story is told within this context, it tacitly conveys normative messages about access and opportunity denied to these students. The "real" of this story, in the context of apparently "real" life mathematics, communicates to the students the inequity of their own subject position. Obviously this is only one possible reading, and there are a multitude of others - as is the nature of interpretation - but it seems crucial that we recognize this possible reading and begin to grapple with the ways in which these students might internalize the messages within these kinds of stories.

CLOSING REMARKS

Narrative research is both descriptive and explanatory, attending to the "storied" nature of participants' lives (Clandinin & Connelly, 2000). Teachers' stories are "stories to live by", and teacher identity is mapped onto school culture through the telling and re-telling of these stories (Clandinin & Connelly, 1995). Teachers modify and multiply these life stories as they negotiate their position on the professional teaching landscape. Narrative is therefore a personal site for identity construction. My focus here has been on the function of these stories in the mathematics classroom – on what these stories do when used in context.

Janet and Leslie use personal narrative differently in their classroom discourse. Janet uses only one dispassionate and disembodied narrative to impart a moral lesson about socio-economic status and schools, and to entrench an "identity of mastery" in school mathematics. While Leslie presents an embodied presence throughout the procedural discourse, and offers family stories that implicitly contradict the cultural messages of the curriculum. By employing a close textual analysis, I have tried to show how different forms of identity are enacted in these personal narratives, and that these stories often function to enforce the legitimacy of the dominant procedural discourse.

I have argued that teacher identity is enacted in the classroom through various discursive registers. The mastery identity is enacted through procedural discourse and represents the dominant identity in this study. When teachers shift back and forth between procedural discourse and personal narrative they perform radically different identities. The "I" of machine talk is suddenly juxtaposed with the vulnerable "I" of the embodied self. At the same time, each enactment conveys normative messages about the legitimacy of particular subject positions in the classroom. In particular, the personal narrative positions the teacher according to race, gender, ethnicity and class (the "I" as member of the community) and thereby addresses the students in these terms. This embodied aspect of narrative is what makes narrative such a powerful form of pedagogy. Teacher personal narratives "teach" students about socio-cultural positioning within math classrooms. I have shown that one possible reading of these narratives reveals their role in communicating messages about socio-economic status in relation to school mathematics. In each case, the personal narratives functioned to enforce the legitimacy of the dominant procedural discourse. I am not suggesting that we discourage teachers form telling these stories, but rather that we attend to the nuanced meanings that are embedded in them, and examine their relation to other discursive registers. If school mathematics is often less about understanding and more about habit (Tate & Rousseau, 2002), then we need to interrogate the classroom practices by which those habits are inscribed onto identity.

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TOWARD A METHODOLOGY FOR EXPLORING MATHEMATICS PRESERVICE TEACHERS' LEARNING FROM A SOCIOCULTURAL PERSPECTIVE

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This paper[1] describes a methodological procedure for characterizing preservice teachers' learning from a sociocultural perspective. The procedure involves interpreting some aspects of Wenger's theory of social learning, adapting them to the preservice teacher training, and making this adaptation operational for coding and analysing audio recordings of a group of preservice teachers working at home. An example of a research result obtained using this procedure is presented.

Sociocultural views provide new ways of conceptualising the process of becoming a teacher (e.g., Borko, 2004; Lerman, 2001; Llinares & Krainer, 2006, p. 439). Some researchers have explored preservice teachers' learning from this perspective (e.g., Gómez, 2006; Graven & Aurbaugh, 2003; Llinares & Krainer, 2006) and suggest that training programs based on it promote learning (Kilpatrick, 2003, p. 7; Lieberman, 2000; Little, 2002, p. 917). However, it is not clear how to make these learning theories operational from a methodological point of view. The researcher must examine the learning processes from a broader perspective and include many aspects of the participants' behaviour that are usually not taken into account in more cognitive approaches to learning.

I tackled these methodological issues in a research project that explored the didactical knowledge development of preservice teachers in a methods course (Gómez, 2007). One of the studies in this project focused on the learning processes of a group of preservice teachers working at home on the tasks assigned in class (Gómez & Rico, 2007). I decided to explore and characterize this group's learning over the academic year based on some aspects of Wenger's theory of social learning (Wenger, 1998).

The preservice teachers were organized in groups. They performed several tasks during the course that involved the analysis of a mathematical topic taking into account the topic's multiple didactical meanings (Gómez & Rico, 2004). For each task, each group worked at home and then gave a presentation to the class using transparencies. I asked the members of one group to allow me to audio record their interaction as they prepared their presentations for the course. This group, of four male students, had the quadratic function as its topic of study. Eight meetings were recorded, producing 18 hours of recording.

My problem was then to design instruments that would allow me to code and analyse the transcriptions of the recordings in terms of Wenger's theory of social learning. In what follows, I first describe the features of Wenger's theory on which I based the inquiry. Then, I present the methodological procedure I established to code and

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analyse the audio recordings based on that theory. Finally I provide an example of one of the results of this analysis.

LEARNING AS A SOCIAL PRACTICE

Wenger's social theory of learning is based on four notions: meaning, practice, community and identity. Wenger introduces meaning as a way of talking about our (changing) ability—individually and collectively—to experience our life and the world as meaningful. The negotiation of meaning emerges from the interaction of two processes: participation, the process in which we establish relationships with other people, define our way of belonging to the communities in which we engage on some enterprises, and develop our identity; and reification, the process of giving form to our experience by producing objects that congeal this experience into "thingness". Every community produces abstractions, tools, symbols, stories, terms and concepts that reify some of the practice in congealed form. For Wenger, practice is a way of talking about the shared historical and social resources, frameworks, and perspectives that can sustain mutual engagement in action. Practice is the source of community coherence and the process through which we experience the world meaningfully. It does not exist in the abstract; it exists because people engage in actions whose meanings are negotiated. A community of practice represents the smallest unit of analysis in which one can include the negotiation of meaning as a mechanism of learning. It is a way of talking about the social configurations in which our enterprises are defined as worth pursuing and our participation is recognizable as competence. The idea of a community of practice is based on three notions: mutual engagement, joint enterprise and shared repertoire. The notion of identity is introduced as a way of talking about how learning changes who we are and creates personal histories of becoming in the context of our communities. Learning as social practice can be characterized by the three notions shaping the community of practice: learning in practice implies mutual engagement in the search for a joint enterprise with a shared repertoire. That is, learning emerges to the extent that (a) different forms of mutual commitment evolve; (b) the enterprise is understood and refined; and (c) a shared repertoire, style and discourse are developed.

FROM SOME ASPECTS OF THE THEORY TO DIMENSIONS AND CATEGORIES OF ANALYSIS

The methodological problem lay in the design of instruments for coding and analysing the audio recordings in terms of the three dimensions that characterize the emergence of learning as a social practice. The instruments should satisfy at least two conditions: to ensure both the relevance of the issues that might emerge concerning the group's learning and the completeness of the inquiry and analysis. Furthermore, the instruments should enable the interpretation of results to focus on the theory and "produce well-grounded assertions regarding social practice and learning" (Little, 2002, p. 920). The first step in this process was the construction of a set categories of analysis based on the theory. These categories were to link the central notions of the theory in which I was interested and the code set that would determine the instrument for exploring, selecting and articulating the information available. These categories emerged from a detailed and purposeful reading of the theory. After an initial review of the transcriptions of the audio recordings, I interpreted and selected notions and aspects of the theory based on the information on the audio recording. In this way, I produced several versions of a list of categories until the list was consistent with and meaningful to the information. The following final list emerged:

- *Mutual engagement*: environment, identities, relationships, and meaning.
- *Joint enterprise*: external conditions, discourse, enterprise, and responsibilities.
- *Shared repertoire*: working routines and resources for the negotiation of meaning.

Keeping in mind the meaning of the categories within the theory, I identified a set of questions that characterized the categories and suited both the phenomena I wished to study and the information available. I identified and articulated these questions in a cyclical process in which, while coding the information with a given version of the questions, I corrected, deleted and added new questions to the list. Figure 1 shows the final version of the questions for the dimension of mutual engagement. These questions are framed in terms of the performance of the group of preservice teachers studied. They organize the code set that I will introduce below.

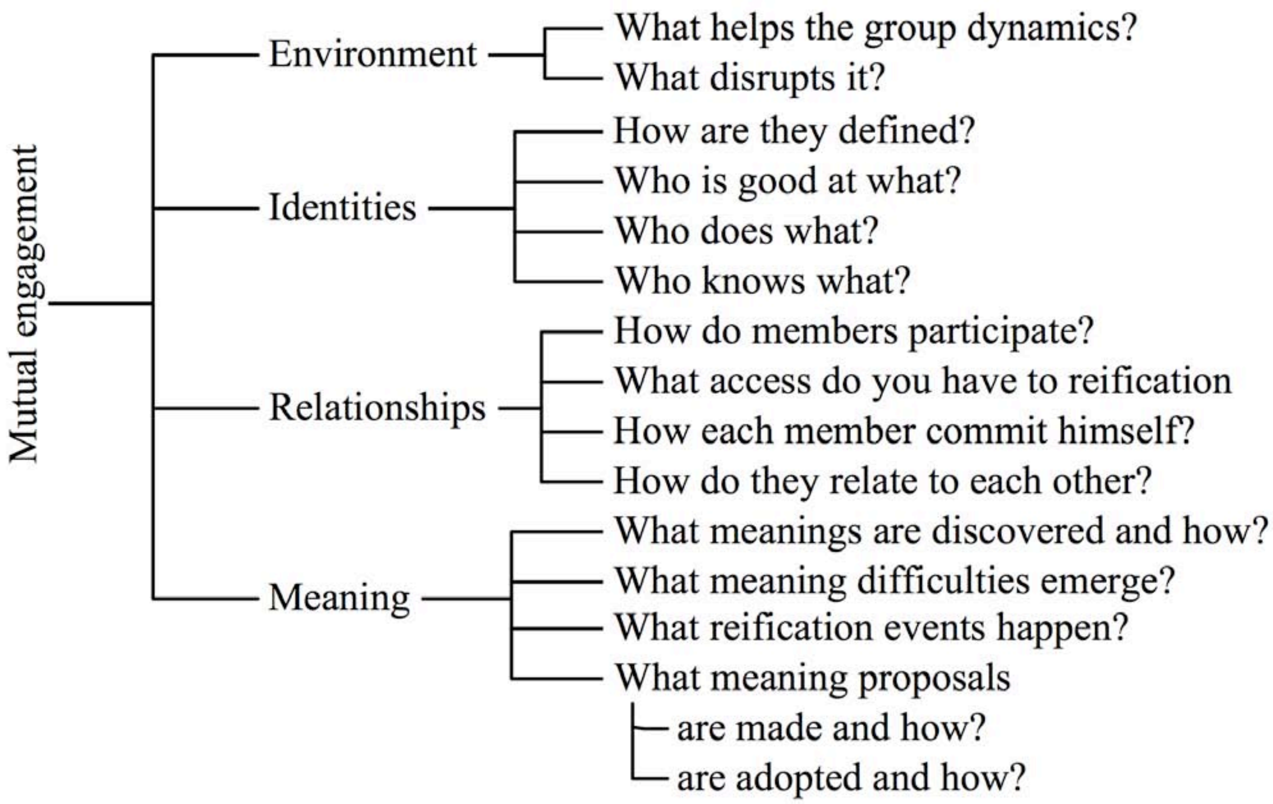

Figure 1. Questions for the dimension of mutual engagement

CODE SET

I developed a preliminary set of codes starting from the questions described above. This code system evolved in the process of coding the transcriptions. For instance, after coding the transcriptions of the first two sessions, I observed the need to introduce a code in the "external conditions" category of the mutual engagement dimension. The purpose of this code was to identify episodes in which the members referred to the way the task at hand was formulated. This external condition affected their performance. The final code set contained 94 codes. Table 1 presents some examples of the codes. Their meaning establishes the characteristics of the episodes to which the code is assigned.

Code	*Meaning*
Teaching experience	At least one member refers to his teaching experience
Who is good at what?	The group identifies a member as competent for a task or that member proposes himself as such
Discussion steering	One of the members organizes or steers the group discussion towards a particular issue
Meaning confusion	There is evidence of confusion in one or more members with respect to the meaning of a given issue
Commentaries on transparencies	The group refers to the educator's written commentaries on its transparencies
What is valued?	Evidence of aspects of the work or the discussion that are valued by the group
What are the working routines?	Working routines are established within the group
Complexity of the conceptual structure	References are made to the complexity of the conceptual structure[2]
Connections	References are made to connections among representation systems

Table 1. Examples of codes

CODING PROCESS

In the coding process, I identified, registered and characterized the episodes. An episode is a segment of the transcription, of variable length, that contains statements from one of the members or an exchange of statements between several group members. Its coherence as an episode derives from its treatment of one idea or message. Thus, some episodes both refer to a particular idea and form part of a larger episode that refers to a more general idea. More than one code may be assigned to an episode.

I produced a database for registering the results of the coding process. Each record contained all of the characteristics of a given episode – code pair, as well as a comment for that episode. I also made notes that described my interpretation of the interaction and identified its most relevant aspects. The following is an example of an episode that I coded with the codes corresponding to personal relationships, leader and complementary participation. In this episode, one of the members, whose performance represents complementary participation, addresses the leader's authoritarian attitude. I assigned the following commentary to this episode: "Again, there is tension: they criticize the leader explicitly. 'He knows everything because he teaches'":

P1: Now, he is a specialist. Since he teaches, he now thinks that everything is clear.

After coding, there were 7,412 records in the database. These correspond to 2,606 episodes (since several codes could be assigned to a given episode). Figure 2 shows the coding process I have described.

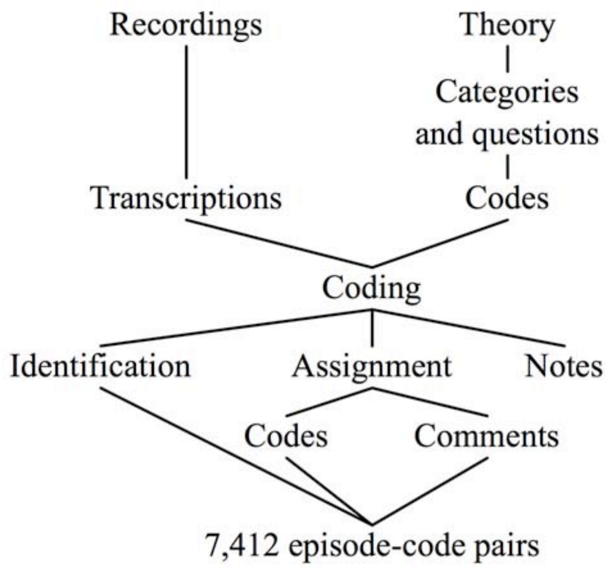

Figure 2. Coding process

The database design allowed me to produce and organize my comments on the episodes. From there, I summarized the transcriptions of each session. The summaries enabled me to identify the most relevant issues. These *issues* represented my characterization of the group's interaction. This list of issues was produced by synthesizing the episode-code pairs, taking into account the theory (through the categories and the questions) and the additional information that I registered during the coding (comments and notes). Figure 3 shows a diagram of this process.

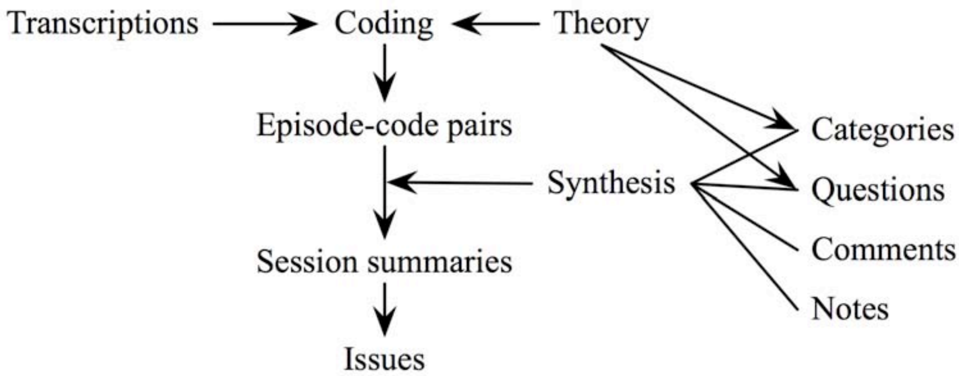

Figure 3. Identifying the issues

For example, the analysis in Figure 3 shows that one of the issues was the fact that the group had a leader and that his performance determined several aspects of the learning process. I thus had to characterize the leader and his relationship to the other members of the group. I summarized the list of issues in a set of phrases (role of leader, role of comments on transparencies, importance of connections among systems of representation, etc.), which in turn summarized the 950.5 minutes of the original audio recordings.

I identified 32 issues. The following are the issues corresponding to the dimension mutual engagement:

- *Environment*: teaching experience, practice course, and textbooks
- *Leader*: characterization of leader, complementary participation
- *History of the tension in the group*
- *Meaning*: search for meaning, meaning confusion, meaning conflicts and resolution, evaluation: a story of meaning conflict, meaning discovery, reification events

I obtained the list of relevant issues through a process of synthesis. Figure 4 shows a diagram of this process in terms of the databases used.

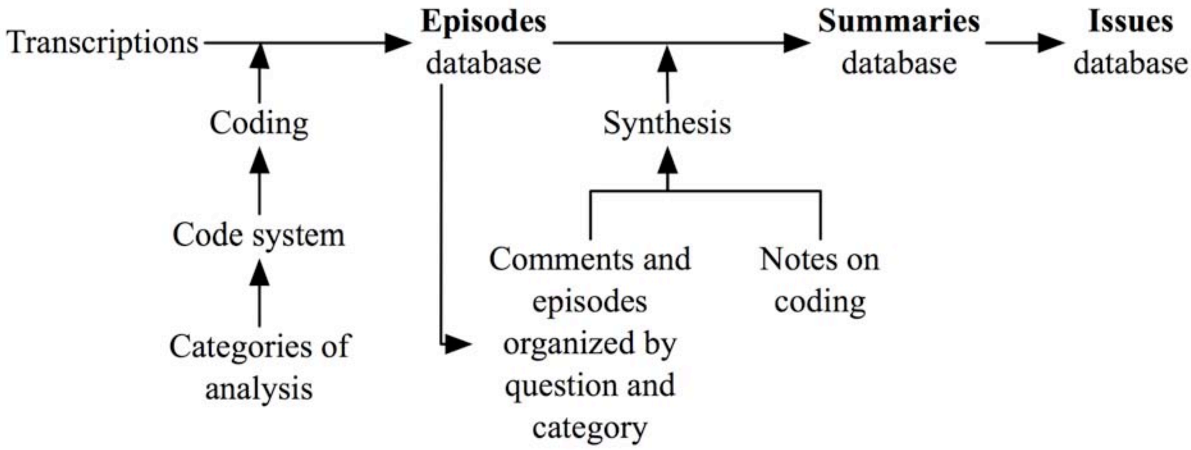

Figure 4. Synthesis: From transcriptions to issues

ANALYSIS

The issues database was the starting point for a process of analysis. For each issue, I wanted to (a) describe the issue, identifying its main characteristics and (b) identify the most representative episodes of those characteristics in order to provide evidence for the issue's characterization. To achieve these goals, I had to solve a new methodological problem. A given issue (e.g., the characterization of the leader and of his relationships with the other members of the group) could involve more than one code. Furthermore, for each code there might be a high number of records in the episodes database. For instance, the code "complementary participation" was assigned to 55 episodes and the code "meaning search" to 475. During the coding process, it was not possible to identify which episodes would become representative, since at that point I did not know the issues I needed to analyse. The problem was thus to design a procedure that would allow me to select those episodes.

I designed a new database with the information contained in the summaries described in the previous section. For each topic in a summary, a record of the database was created containing the dimension, category and codes corresponding to that topic. 754 records were created in this database. To select the representative episodes to characterize a given issue, I implemented the following procedure:

1. identification of the statements in the summaries related to the issue
2. identification of the codes related to the issue
3. search for all episodes related to the issue (by code and by comment)
4. review of the list of episodes based on related comments: first selection of episodes and assignment of categories for its characterization
5. review of the transcriptions of the selected episodes: new selection and assignment of categories
6. final selection of the representative episodes, and
7. description of the issue in terms of the characteristics identified.

GRAPHIC SIGNIFICANCE OF THE PARAMETERS

This section provides a brief glance at one of the study's results[3]. The graphic meaning of the parameters of the symbolic forms was discussed in the session on preparing the didactic unit. Up to this time, the meaning of the connections between symbolic and graphic systems of representation had been general. The specificity of these connections (with respect to the parameters) arose from the need to design in detail the activities that would be proposed to the students in the sessions to make up the didactic unit. Tackling this problem generated confusion and made explicit some of the difficulties that the preservice teachers encountered in the mathematical handling of their topic. These difficulties became evident in their use of the graphic significance of the parameters of symbolic forms.

The doubts and confusion on this topic can be seen in the following episode, in which questions arose about the role of the parameters in locating the intersections of the function with the *x*-axis:

P4: So, the points of intersection with the x-axis influence the other coefficients of the function. Don't they?

P2: Yes, but.

P3: Wait.

P4: Let's see.

P3: What are you trying to say?

P4: Bartolo is saying... Bartolo is saying that, when you have just seen the general characteristics…, such as, for example, the intervals of increase and decrease, these depend on the lead coefficient, as it says here. That's what you're saying.

P4: Then, I say the same thing that is being said about the lead coefficient; when you see the points of interaction, you will have to say how they influence all of the other coefficients. Because here is the influence. Because in the other one, it's true that they influence all of them. In the points of intersection, all three have influence. Don't they?

When the group reflected on the role of parameter *a* in the expression $f(x) = ax^2 + bx + c$, they concluded that all of the characteristics of the graph of the function depended on this parameter [2]. But, as is natural, they encountered the greatest difficulties with the meaning of parameter *b*. These difficulties appeared at the beginning of the session, when one of the members asked explicitly about the graphic meaning of this parameter. In discussing this topic, they decided that this parameter alone had no influence. The group then reverted to the algebraic consideration to focus the graphic meaning of the parameter in its influence on where the function intersected with the x-axis. Finally, they established that this parameter influenced the horizontal translation of the vertex, but they did not realise that this influence was linear, while the effect on the vertical position of the vertex was quadratic:

P2: When the sign of the coefficient of x is negative, the thing is translated…, always to the right, I think.

P3: ( ) would be x -... Let's see; if it's negative, it is to the right. The positive… (Several people talk at the same time).

P2: The positive to the left. Yay! That's it. There you have it. ( ) the b. (Several people talk at the same time).

P4: If it's negative, it's to the right.

In the end, some of the members did not understand the details of the discussion, and the confusion was not clarified in the group, although the didactic unit contained activities that tackled the problem:

P2: $x^2 - 1$.

P3: You understand, don't you?

P1: No, I don't. ( ).

DISCUSSION

The methodological issues and procedures involved in mathematics education research are not usually described in detail. Detailed descriptions are usually left to doctoral dissertations and in many cases refer to methodologies already developed. However, in this study, the problem was twofold. First, it was necessary to interpret Wenger's theory of learning as social practice in the context of mathematics preservice teacher training. Second, this interpretation had to be made operational: I needed to design coding and processes for analysing the information available.

My purpose was not to identify some episodes that could exemplify some aspects of the group's learning in terms of Wenger's theory. Rather, it was to give specific meaning to the ideas that articulate learning in communities of practice in the context of the initial training of high school mathematics teachers and to design instruments for codification and analysis of this complexity. This kind of procedure was time-consuming, but it enabled me to tackle a large body of data systematically and obtain results whose validity was based on the procedure itself.

The results show a complexity behind the in-class presentations of the groups of future teachers and their projects that is inherent to the development of a community of practice. By analysing this complexity systematically and in detail, I identified and characterised many aspects of the social learning of a group of future teachers. The level of detail that this methodology allows makes these characterisations interesting and important in themselves. They illuminate dimensions of the initial training of high school mathematics teachers that often remain opaque in the research literature. For example, they enabled me to understand the processes of negotiation of meaning that materialised in the transparencies and in the group's final project. They also revealed the different positions of the participants, their questions and confusion, the conflicts they had to face and resolve, and the plans and techniques they developed to complete the tasks they were assigned. Finally, the in-depth analysis of the transcriptions illuminates the group's progress in its commitment to the joint construction of the meanings that its members believed necessary to satisfy both the requirements of the course and their interest in becoming mathematics teachers.

NOTES

1. This work was partially supported by Project SEJ2005-07364/EDUC of the Ministry of Science and Technology.

2. The examples of codes that follow refer to particular concepts and procedures in the methods course. They formed an important part of the group's shared repertoire.

3. For ease of reading, I have not included the references to the location of the episodes that support these claims. For instance, each sentence in this paragraph is a statement that has at least one representative episode supporting it.

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DISCOUNTING IRAQI DEATHS: A SOCIETAL AND EDUCATIONAL DISGRACE

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In the absence of any attempt by the occupying forces in Iraqi to document numbers of Iraqi deaths, three studies have produced consistent evidence that these deaths number in the hundreds of thousands. Rejection and acceptance of these data are aligned with political positions, despite the fact that they constitute the most scientific information available. Views expressed by journalists, academics, and politicians show pervasive misunderstandings of basic statistical concepts and methods, an indictment of mathematics education. Ignorance about, and indifference to, the scale of human suffering in Iraq constitute an indictment of our societies.

DATA

> "We don't do body counts," says America's soldier-in-chief, Tommy Franks. That's a damn shame. (Chernus, 2003)

In a study reported by Roberts, Lafta, Garfield, Khudhairi, and Burnham (2004) the estimate of excess mortality of Iraqis *from all causes* during the 17.8 months after the 2003 invasion of Iraq was 98,000, with a 95% confidence interval of 8,000–194,000 (figures rounded). In a later study (Burnham, Lafta, Doocy, & Roberts, 2006) the corresponding estimate was 654,965, with 95% confidence interval of 392,979 – 942,636. A longer report (Burnham, Doocy, Dzeng, Lafta, & Roberts, 2006) provides more detail and context. In both these studies, the estimate of excess mortality was calculated using the difference between mortality after the invasion, based on interviews of a carefully selected sample, and baseline mortality prior to it.

In September, 2007, a British company, Opinion Research Business (ORB), with experience of polling in Iraq, reported on a poll in which a representative sample of 1,499 adults aged 18+ were asked "How many members of your household, if any, have died as a result of the conflict in Iraq since 2003 (i.e. as a result of violence rather than a natural death such as old age)? Please note that I mean those who were actually living under your roof." On the basis of their data, the point estimate for deaths as a result of the conflict was 1,220,580, with a 95% confidence interval of 733,158 – 1,446,063 (Opinion Research Business, 2007).

REACTIONS TO THESE REPORTS

> Yes, 'n' how many deaths will it take till he knows
>
> That too many people have died? (Bob Dylan, 1962)

In the case of the first two studies, which were reported in *The Lancet*, the estimates were widely contested in the media, and dismissed as not credible by government leaders. I watched on television President Bush being questioned about the second

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 293 - 301).

Lancet study. He said that he did not consider the 2006 report credible, that the methodology had been "pretty well discredited" and that he stood by the number 30,000 that he had cited previously. He referred to the estimate in the second Lancet report as "600,000, or whatever they guessed at" (White House, 2006).

A simple search on the Internet will produce many political commentaries on the two reports, in many cases predictable given the stance of the authors and/or the publications (e.g., Hitchens, 2006; Moore, 2006).

By contrast to the rather extensive media coverage of the two studies, the ORB poll received almost none. According to MediaLens (2007), four days after the findings were announced, the poll had been mentioned in just one national UK newspaper – ironically, the pro-war Observer. There don't appear to have been any reports in US newspapers except for the Los Angeles Times (Susman, 2007). MediaLens further reported that the BBC's *Newsnight* programme may have been alone in providing TV broadcast coverage, 34 seconds of it as follows:

> More than a million Iraqis have been killed since the invasion in 2003, according to the British polling company ORB. The study's likely to fuel controversy over the true, human cost of the war. It's significantly up on the previous highest estimate of 650,000 deaths published by the Lancet last October. At the time, the Iraqi government described that figure as 'ridiculously high'. The independent Iraqi [sic] Body Count group puts the current total at closer to 75,000. (MediaLens, 2007)

Comprehensive summaries of discussions around these studies, including criticisms and rebuttals, very fully documented, can be found on Wikipedia under the headings "Lancet surveys of mortality before and after the 2003 invasion of Iraq" and "ORB survey of casualties of the Iraq War".

MEDIA BEHAVIOR

> Mr. Garlasco says now that he had not read the paper at the time and calls his quote in the *Post* "really unfortunate." He says he told the reporter, "I haven't read it. I haven't seen it. I don't know anything about it, so I shouldn't comment on it." But, Mr. Garlasco continues, "like any good journalist, he got me to.". (Guterman, 2005)

An interesting aspect of media treatment of technical reports is their appeal to accessible experts, whom they typically quote briefly. Divergence in the opinions cited is typical of what happens when statistical experts give opinions on a complex study. There is an irony in that, reporting on studies based on sampling, there is no mention of the implicit samplings whereby the experts quoted are a sample, probably a convenience sample. Secondly, the short quotations are, almost inevitably, sampled on the basis of the journalists' subjective criteria, from longer and more nuanced statements. Interactions with the experts are usually one-shot deals and it is not uncommon for the experts to want to clarify or correct statements attributed to them, but such an opportunity is rarely afforded.

The British group, MediaLens, doggedly pursued a number of reports following the Lancet publications. Their harrying of Mary Dejevsky, senior leader writer on foreign affairs for the London newspaper *The Independent,* is particularly revealing (see Mukhopadhyay & Greer (2007) for a summary and MediaLens (2005a) for the full account).

IGNORANCE, DAMNED IGNORANCE, AND STATISTICAL IGNORANCE

> Among those polled for the AP survey, however, the median estimate of Iraqi deaths was 9,890. (Associated Press, 2007)

The above refers to a poll carried out in February 2007 in the US. At that time, just over 3,100 US troops had been killed in Iraq. The median estimate for this number among those polled was about 3,000. However, the same accuracy was not found for estimations of Iraqi deaths (see above).

As an example of the "democratization of stupidity" that discussion groups on the Internet afford, consider the following:[1]

> That Lancet study is poorly done. The actual range of estimated civilian deaths was something on the order of about 10,000-100,000. That is a wide range that lends NO credence to the 100,000 number being selected over the 10,000 number. It was a politically biased article and never should have made it to print, at least in the form it was written.

After another contributor pointed out that the report gave a 95% confidence interval of 8,000–194,000 with a point estimate of 98,000 the original contributor persisted as follows:

> I didn't bother to look it up because the range was so varied. My point was in a range so large there is no way to pick one number over the other. That the article was flawed is true and that it should not have been published is true.

It is interesting to compare this inanity with the work of a sophisticated writer (Kaplan, 2004). He quotes from the first Lancet report: "We estimate there were 98,000 extra deaths (95% CI 8,000 – 194,000) during the post-war period." and comments as follows:

> Readers who are accustomed to perusing statistical documents know what the set of numbers in the parentheses means. For the other 99.9 percent of you, I'll spell it out in plain English – which, disturbingly, the study never does. It means that the authors are 95 percent confident that the war-caused deaths totaled some number between 8,000 and 194,000. (The number cited in plain language – 98,000 – is roughly at the halfway point in this absurdly vast range.) This isn't an estimate. It's a dart board.

On the evidence of this statement, it seems doubtful that this journalist with a PhD in Political Science understands what a 95% confidence interval means (or that in a scientific journal it is not customary to explain a standard technique).

Another noticeable phenomenon is the number of commentators who prefer to back their own subjective estimates, sometimes based on some data, against data reported in a highly regarded peer-reviewed journal and by an established polling company. For example, the mathematician John Allen Paulos, well known for his books, including *A Mathematician Reads the Newspaper* (Paulos, 1996), wrote in the British newspaper, *The Guardian* (MediaLens, 2005b):

> Given the conditions in Iraq, the sample clusters were not only small, but sometimes not random either... So what's the real number? My personal assessment, and it's only that, is that the number is somewhat more than the IBC's[2] confirmed total, but considerably less than the Lancet figure of 100,000.

The most widespread fundamental misinterpretation relates to comparison with other estimates of casualties, in particular those from IBC. When President Bush mentioned a figure of 30,000, it was probably based on this source. However, the methodology used by IBC, and what they measure, is fundamentally different:

> Iraq Body Count (IBC) records the violent civilian deaths that have resulted from the 2003 military intervention in Iraq. Its public database includes deaths caused by US-led coalition forces and paramilitary or criminal attacks by others.
>
> IBC's documentary evidence is drawn from crosschecked media reports of violent events leading to the death of civilians, or of bodies being found, and is supplemented by the careful review and integration of hospital, morgue, NGO and official figures.

MediaLens (2007) reported:

> IBC only collects records of violent civilian deaths reported by two different (mainly Western) media sources operating in Iraq. Epidemiologists report that this type of study typically captures around 5 per cent of deaths during high levels of violence, such as exists in Iraq. By contrast, the Lancet studies provide figures for all deaths – violent and non-violent, civilian and military, reported and unreported.

Nevertheless, IBC argues on *a priori* grounds that the Johns Hopkins estimates could not be accurate. Zamparini (2007) relates the following:

> The Toronto Star informed us today: "The death toll could be twice our number, but it could not possibly be 10 times higher," he [John Sloboda, professor of psychology at Keele University, and a co-founder of IBC] told me, referring to the other studies.

and comments as follows:

> **Question:** How can a professor of psychology who collects Iraqi deaths through media reports possibly know what the death toll could be?

We may be guilty of "the soft bigotry of low expectations" in relation to President Bush, but pervasively in media accounts, people who should know better indiscriminately lump together the IBC numbers, the estimates from the Lancet studies and ORB poll, and other numbers derived by a variety of methods. Just look at the 34-second *Newsnight* report quoted above, which implies that the IBC number and the ORB estimate are comparable. Reynolds (2006) concludes an "analysis" entitled "Huge gaps between Iraq death estimates" thus:

> We are then left with the estimate from this report [the second Lancet study] and the various counts by other groups. The figures are now even more divergent than they were.

ON SCIENTIFIC PUBLISHING

> … egregious politicization of what is supposed to be an objective and scientific journal (Washington Post editorial, June 23, 2005, referring to *The Lancet*)

It is to be expected that the reaction to politically loaded reports of almost all people (including me – and you) will be influenced by their political views. This reality fundamentally challenges the notion of conclusions being reached, at least partly, on the basis of scientific evidence. Many criticisms of the reports claim that the political views of the authors and of the editor of *The Lancet* discredit the data. At least in the case of Les Roberts, the authors are opposed to the invasion and occupation of Iraq, as is the editor of *The Lancet*, Richard Horton (and as, indeed, is the author of the present paper). What are the implications? Are people with such views considered incapable of carrying out studies of this sort and having the findings taken seriously? Such a position rests on the myth of science and mathematics being value-free, ethically neutral, and apolitical. It is worth remembering that *The Lancet* is one of the most highly respected scientific journals, and that papers published in it are subject to the most stringent peer review. Apparently, however, it should not deal with deaths in war when those deaths are, to a considerable extent, caused by "us". Consider the editorial opinion of the Washington Post quoted above. Why is it unreasonable that a journal serving a profession whose members take an oath to protect human life should raise issues about the avoidable killing of human beings? As Horton (2006) states:

> … if we were talking about the risk of smoking to the population and we published research demonstrating the impact of tobacco on mortality, few would dispute the message or the importance of scientists and medical journals in being actively engaged in a public debate. For Iraq, violence is the public-health priority right now. It is a proper subject for science and it is a proper subject for a medical journal to comment on.

IMPLICATIONS FOR MATHEMATICS EDUCATION

> This is a great discovery, education is politics! After that, when a teacher discovers that he or she is a politician, too, the teacher has to ask "What kind of politics am I doing in the classroom?" (Freire, 1987, p. 46)

The lack of "statistical empathy" (Mukhopadhyay & Greer, 2007) documented in this paper is a societal and educational disgrace. It reflects the success of regimes that work hard to keep their subjects in docile ignorance, with the connivance of most of the media/entertainment industry. It also reflects the historical pattern of reducing others (slaves, indigenous peoples, colonized peoples) to subhuman status as a justification for barbaric acts against them.

It is easy to perceive a chronic lack of the analytical tools that mathematics education ought to equip people with, a particular manifestation of what Macedo (2000, p. 5) calls “education for stupidification”. As Chomsky (Macedo, 2000, p. 24) stated: “The goal is to keep people from asking questions that matter about important issues that directly affect them and others”.

People, in general, have a weak understanding of numerical data, especially how to interpret large numbers. Lack of numeracy is compounded by a lack of understanding of basic statistical principles such as sampling variation, randomness, margin of error. The 2008 presidential race in the US has already started and the media are full of terms such as “statistical dead heat” which (I am prepared to bet) only a tiny fraction of the electorate understands. Shouldn't such understanding be part of what is considered an adequate mathematical education?

Technological advances mean that the amount of information available is swamping people's intellectual and analytical tools for making sense of, and critically evaluating, opposing claims. The irony of the information age is that the increase in information leaves us not knowing what to believe and not believe. Mathematics education should promote skepticism in the face of conflicting information, combined with tools for making better-informed judgments.

Given that this paper is mostly based on material from the Internet, it's appropriate to state some of the ways in which I try to reach a judgment, given the political position I have already declared. While a full treatment of this issue would need (and deserves) another paper, some principles and criteria that I applied in judging what I accessed are the following:

- Judge the standing of the author(s) and the place of publication. For example, a paper in *The Lancet* by investigators with scientific credentials has a higher standing than one in a political magazine by a polemicist – which is not to say that the political position of the authors and/or the editor of a journal should not be taken into account.

- In the case of an issue like that discussed in this paper, if an author shows clear lack of understanding of statistical issues, their analysis is automatically suspect (though it is still possible to consider their conclusions reasonable).

- Triangulate where possible. For example, the consistency of the ORB data with those of the Lancet studies carries weight and follows the basic scientific principle of replication. On the other hand, information on how US citizens estimate deaths in Iraq is based on a single poll.

- Give more credence to analyses that mention, and provide sources for, contrary views. Thus, while Wikipedia is often regarded as a dubiously reliable source, I judge its articles referred to earlier to be excellent, for the reasons stated.

• Try to situate the situation under scrutiny in historical context. The methodology used in the Lancet studies had earlier been used for mortality estimates in war zones such as Darfur and the Congo (Horton, 2006) and the results of those studies were not challenged by politicians.

In this paper, I have not, in any depth, attempted to relate the example discussed to the theoretical frameworks that have been developed to expose the essentially political nature of mathematics education and the ways in which mathematics frames how people think about political and societal issues. The case I have focussed on, in many ways, speaks for itself. One comment I will add is that I suspect that history will document how the neoconservative regime that gained power during the Bush presidency developed the arts, science, and apparatus of propaganda, media manipulation, and the manufacture of consent to unprecedented levels.

Ubiratan D'Ambrosio (2003) has challenged mathematicians and mathematics educators to accept their ethical responsibilities as the world's population seeks survival with dignity. I do not know how many mathematicians, but the number is surely not insignificant, who contributed with varying degrees of direct involvement to the deaths of Iraqis, or how many are currently working in the United States to develop the next generation of weapons of mass destruction. Mathematicians profess a profound interest in truth, and proving Fermat's Last Theorem is an intellectual achievement of the highest order, but citizens knowing truth about the consequences of the actions of their governments in destroying the lives of fellow human beings should also be a concern. Those with a concern for truth, as it directly impacts people's lives, should speak out in a world in which those who can pay the most get to decide what the truth is.

NOTES

1. Retrieved on 13 November 2007 from: www.sport-groups.com/board/nextpost/93676/0.

2. IBC is the acronym for Iraq Body Count, an organization that monitors recorded deaths in Iraq.

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TOWARD A THEORY OF MAESTHETICS: PRELIMINARY CONSIDERATIONS OF THE DESIRABILITY OF BRINGING AN AESTHETIC PERSPECTIVE TO MATHEMATICS, EDUCATION AND SOCIETY

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This paper makes an argument for the recognition of 'maesthetics', a neologism for a perspective on the nature, teaching and learning of mathematics that emphasizes the human and aesthetic aspects of the discipline. A case is made for the need for alternative perspectives given the numerous detrimental effects at both the individual and societal levels of the 'standard' conception of the enterprise of mathematics teaching. Maesthetics is seen to have many features in common with ethnomathematics, with the essential difference lying in its focus on the individual rather than the societal level of activity. Existing work from a range of scholarly fields that might contribute to the foundations of such an enterprise are noted. Some characteristics are suggested and examples are given.

CAVEAT LECTOR

In the spirit of forewarning the reader, it is perhaps as well to say at its beginning, that this paper is not, from the perspective of conventional scholarly publication, orthodox. It attempts to stay within the very reasonable stylistic recommendations of the editors but it strays a long way from the conventional norms of a focused, coherent, tightly argued and precisely documented argument easily placed with respect to context and methodology. Given that one of the major aims of the paper is to suggest that mathematics educators badly need to broaden their conceptions of what their enterprise attempts to do, this may not be entirely negative. Using the terminology of the philosopher and historian of science, Thomas Kuhn, the paper might be considered a plea for the consideration of a different, or at least another, alternative 'paradigm' for the nature, teaching and learning of mathematics. It is clear that this aim is ambitious and that this particular statement represents an early stage of development. Another ambition, in the spirit of the view sometimes attributed to Karl Popper, namely that, 'the purpose of a professor is to provoke', is to generate discussion

THE PLACE OF MATHEMATICS EDUCATION IN SOCIETAL CHANGE

A panoptic and interesting perspective from which to consider the place of mathematics with respect to society and education was articulated by the distinguished political philosopher, Isaiah Berlin, in an address he gave on being awarded the first Agnelli Prize in 1988. In the opening paragraphs of this address that was later published under the title "On the Pursuit of the Ideal" (1991) he wrote:

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University. (pp. 302 - 311).

> There are, in my view, two factors that, above all others, have shaped human history in this century. One is the development of the natural sciences and technology, certainly the greatest success story of our time - to this, great and mounting attention has been paid from all quarters. The other, without doubt, consists in the great ideological storms that have altered the lives of virtually all mankind: the Russian Revolution and its aftermath - totalitarian tyrannies of both right and left and the explosions of nationalism, racism, and, in places, religious bigotry, which, interestingly enough, not one among the most perceptive thinkers of the nineteenth century had ever predicted. … When our descendants, in two or three centuries' time (if mankind should survive until then), comes to look at our age, it is these two phenomena that will, I think, be held to be the outstanding characteristics of our century, the most demanding of explanation and analysis. …. The goals and motives that drive human action must be looked at in the light of all that we know and understand, their roots and growth, their essence, and above all their validity, must be examined with every critical resource that we have. (Berlin, 1990, pp. 1-2)

Berlin's exhortation is powerful, and one obvious place to start a mathematics educator's explanation and analysis of 'the goals and motives that drive human action' is with a consideration of how our discipline is connected to the logical geography (to use a phrase of another late Oxford philosopher, Gilbert Ryle) of science and technology and ideological storms. I want to contend that we have been - consciously or not - exceptionally close to the intersection of these two forces.

No informed consideration of the development of twentieth century science and technology could do anything other than recognize the centrality of mathematical concepts and structures. Mathematics has provided not just the backbone for science and technology but also, in the 'information age', its nervous system as well (Davis & Hersh, 1986). On the educational side of our mandate the responsibility is perhaps less sharply outlined. However, I think it not a very large step in the era of 'multicultural' societies to argue, even in jurisdictions where there are no explicit statements to that effect, that one of the major aims of the educational enterprise is to teach people to coexist peacefully. And if we have, as I claim above, been close to the centre of things, what has our contribution been? At the end of some grand, Berlinesque, reckoning of moral responsibility for the development of the modern era, do mathematics teachers and mathematicians find themselves aligned with the forces of light or of darkness? This is clearly an impossible question to answer definitively, and even a cursory consideration would generate a large number of examples of conflicting contributions. The case for the prosecution - that is to say, arguing for the position that mathematics education as conventionally manifested, is 'guilty' of contributing in an essentially negative way to historical development - would, I think, be very strong.

THE EXPERIENCE OF MATHEMATICS EDUCATION

To sharpen this image let us imagine a 'revised' edition of Berlin's vision. Two decades later the ideological storms have intensified considerably, and the parlous realities of environmental degradation are swiftly becoming more evident (Homer-Dixon, 2006; Monbiot, 2006) as the most pressing front in the world of science and technology. Keeping these contemporary realities in mind and switching our imaginary gaze to some 'typical' classroom where pliable young minds are being introduced to a range of 'languages' that, at least in theory, will assist them in future interactions with others and the world. If, according to all the best techniques of statistical sampling, our selected classroom is highly 'representative', how confident might we be that the mathematical component of their day is going to be positive? From this perspective perhaps the case for the prosecution suggested above becomes easier to imagine because the 'client satisfaction' levels among mathematics learners have historically been and continue to be very low (Tobias, 1993). One way in which these views are reflected are in research studies that ask respondents to share their images of mathematicians. The historical trends in this exercise are of concern. Forty years ago British psychologist, Liam Hudson found that English schoolboys found the Mathematician to be "even colder, duller and less imaginative than the Physicist." (Hudson, 1970, p. 48). Some thirty years later Susan Picker and John Berry (2000) found that school children from several countries had consistently negative images of mathematicians and that it was not uncommon for their drawings to incorporate aspects of violence.

Research carried out by Nardi and Steward (2003) with lower-form secondary school students in England gave some unusually clear insights into the sources of pupil dissatisfaction. In their paper they captured some elements of the discontent with the acronymic "T.I.R.E.D." with the five factors being, respectively, Tedium, Isolation, Rote learning, Elitism and Depersonalisation. The skill with which this piece of work was carried out - it is not easy to gain the confidence of this age group in a school setting - and the well written paper that reported it contributed to a significant sense of authority. In our current age of 'globalization' there was also a strong feeling - paralleling the findings of Picker and Berry who found a high degree of consensus across geographic boundaries - that the views of the English adolescents were essentially isomorphic to those of their age cohort in many other parts of the world. The fact that some particular teaching enterprise is not getting good reviews with adolescents is not, in itself, cause for concern. Should, however, their unease reflect a disconnection with fundamental human values the issue would need to be taken much more seriously. If many of our children see our subject as one that is narrow, cold, boring, dehumanized and mechanistic we cannot realistically expect them to function at a high level in a world that is increasingly built on this discipline. Hidden just under the surface of Nardi and Steward's sensitive report of quiet disaffection is one almost completely ignored dimension of contemporary mathematics education. The majority of responsible educators are, rightly, very concerned about the depressingly high percentage of learners who fail quite conspicuously to gain any significant level of competence in mathematics. What gets very little attention is that even those who are 'successful' by the conventions of the enterprise are very frequently taking away a thin and brittle version of the subject. They may have jumped all the requisite hoops necessary to obtain a credit, but it would be an optimistic employer who would expect any degree of comfort with anything outside a narrowly proscribed range of applicability. Nor would the chance of their seeing the discipline as anything beyond a collection of techniques be very large.

FROM MUSIC TO MAESTHETICS

To gain some perspective on this issue let us consider some parallels between mathematics and music. Despite the fact that it is frequently noted that the two areas have many common features, their public presences are almost diametrically opposed. For most people mathematics is exclusively identified with a particular type of institutional setting and rituals. For music the corresponding 'school' structures represent a very small subset of the total disciplinary presence in the culture. There certainly are courses and examinations, but they are associated much more often with free choice and they are relatively unimportant for most citizens who may or may not choose to participate in some way with a cornucopia of musical offerings. Some people find that the string quartet meets all of their musical needs but other have a dazzling range of other choices including folk, jazz, choral, country, blues, and orchestral. The situation in mathematics is not quite the equivalent of 'string quartets or nothing', but it certainly is in that direction.

So what might be done to try to move mathematics in the direction of music in the sense of broadening the range of styles? If this could be done, might it, at least for some learners, some of the time, lead to more satisfying experiences in mathematics education? I believe that the answer to both of these questions is yes, and in the remaining pages of this paper I would like to outline what at least one of these alternatives might look like. For reasons that will emerge shortly I propose to give the name 'maesthetics' to this particular approach to mathematics and its teaching and learning with the intention of clearly marking the fact that it is a merging of math and aesthetics. Its conception has been influenced in a constructive way by the conscious attempt at combating the 'fatigue' of 'T.I.R.E.D. ness'.

MAESTHETICS AS A STYLE OF MATHEMATICS

As a first central characteristic of maesthetics let us stress its human element. Unlike classical, or orthodox mathematics where texts and symbols are omnipresent and creative thinkers almost invisible (note the 'D' for Depersonalised in T.I.R.E.D.), in maesthetics the role of the creative individual is to be stressed. The elitism mentioned by the students interviewed by Nardi and Steward is a reflection of the common view built into conventional mathematics teaching that it is a pursuit, at anything other than a functional level, for a small minority. The distribution of attitudes toward mathematics is, in the conventional view, quite naturally highly skewed, with only a few capable of a full appreciation of the subject. Maesthetics will proceed from the assumption that mathematics, like music, is open for active appreciation, perhaps in many different forms by most people. A second central characteristic is a pervasive sense of play. In the spirit of Huizinga's *homo ludens* we will encourage students of maesthetics to shuck off the utilitarian shackles of conventional mathematics. There are two reasons for thinking that this is not entirely unrealistic. The first is the exceptional impact of one outstanding writer in the field of recreational mathematics, Martin Gardner. For almost three decades Gardner wrote a column in *Scientific American* called *Mathematical Games* and it is now almost commonplace to have prominent research mathematicians in their autobiographical writing credit their passion for mathematics to their early exposure to Gardner's columns. The second is the quite remarkable attraction of a class of mathematical puzzles and games for a very wide spectrum of the population. In recent years it has been the logic puzzle, Sudoku, but it would seem that every decade has its particular example. The '15' puzzle fascinated one generation just as Mr. Rubik's contraption did another.

FOUNDATIONS FOR MAESTHETICS - EVOLUTIONARY & CULTURAL

One of the most popular approaches to the development of new perspectives across a number of academic fields in recent decades has been the technique of looking at a discipline from an evolutionary perspective. The most high profile of these investigators have worked in fields like biology (Wilson, 1975), psychology (Barkow, Cosmides & Tooby, 1995) and linguistics (Pinker, 2002). Perhaps more surprisingly, evolutionary speculation has been the centre of much activity in fields such as history (Smail, 2007), music (Levitin, 2007), and literary theory (Gottschall & Wilson, 2005). There has been research carried out from this perspective with respect to mathematics, particularly near its boundary with developmental psychology (Butterworth, 1999; Dehaene, 1997; and Devlin, 2000). The most ambitious effort in this sub-field was the attempt from a philosophical/linguistic perspective by Lakoff and Nunez (2000) to account for the generation of mathematics by an 'embodied mind'. The speculations of the distinguished mathematician Saunders Mac Lane (1986) about the origins of mathematics in "human cultural activities" might also be placed in this category.

With respect to generating foundational ideas for maesthetics, however, the richest source of ideas is the work of the American anthropologist, Ellen Dissanayake. In three significant books published over a period of fifteen years Dissanayake delved deeply into the role of art in human culture and cognition. In the second of these publications, *Homo aestheticus* (1995) she proposed that humans are by their nature, aesthetic beings. She makes this claim having examined many situations where humans are predisposed to have a sensitivity to, and attraction toward, concepts like symmetry, pattern and balance. The neologism, 'maesthetics' (and the related idea of *homo maestheticus*) is derived directly from this work by extending the observation that these same predispositions can be seen as fundamental to the generation of mathematical ideas.

Dissanayake's documentation of the pattern-rich decorative and artistic production of many human groups brings her very close to the work of ethnomathematicians like Zazlavsky (1979), Eglash (1999) and Gerdes (1998). And this, in turn, points to one potential benefit of developing another 'alternative perspective' on mathematical activity. Ethnomathematics has been a rich and fascinating field of work for a small group of talented scholars (D'Ambrosio, 1985; Joseph, 1991; Powell & Frankenstein, 1997) over the last three decades. However, when it has been seen as a 'replacement' for the conventional perspective criticism has been savage. If it was to be seen as one of a range of 'supplementary' emphases, it might be accepted more easily.

FOUNDATIONS FOR MAESTHETICS - PRECURSORS AND PIONEERS

There are many teachers, researchers and curriculum developers whose work would, at least in part, be consistent with a maesthetic perspective. In this closing section I point briefly to some of these sources. The world of recreational mathematics has already been mentioned. From the viewpoint of the philosophy of education generally, the work of Alfred North Whitehead stands out (1926, 1967). His idea of the 'rhythms' of education, with its identification of the cycle of 'romance, precision and generalization' is particularly apt. One way of characterizing maesthetics would be to note that, from this Whiteheadian perspective, it accentuates the stage of romance. Conventional curricula have a great deal of 'precision' work, a little generalization, and almost no romance. In previous work (Higginson, 1999) I have suggested that mathematics educators would have been much wiser to follow the process orientation of Whitehead rather than committing themselves to the sharp edges of the certainty and logic obsessed Bertrand Russell. Among contemporary philosophers of mathematics education Paul Ernest's work (1991) is especially stimulating. His model of ideologies of mathematics education could be adapted relatively easily to support the development of a theory of maesthetics. [It is interesting to note how few curriculum materials have incorporated Ernest's message in any explicit sense. For an interesting counter example see Roulet's *Math Towers* (2007)]. The emphasis on the visual in maesthetics could draw on the rich source of examples in works like those of Alsina & Nelson (2006). Bruner's ideas of the tripartite forms of representation provides a good underpinning for an emphasis on the 'iconic' and the 'enactive' as well as the 'symbolic' which is the almost exclusive form of conventional texts. This also provides a direct bridge to the visual potential of new information technology and Whitehead's rhythms mentioned above. The potential for giving 'human glimpses' of mathematics and mathematicians is greatly increased with the evolution of the internet. Short visits to the websites of Tao, Demaine, duSautoy, and Conway would do much to counter any view that mathematicians are a boring, self-obsessed and inarticulate group.

It would be possible to interpret much of at least the early years of the 'investigative' era of secondary mathematics education in the United Kingdom (Boaler, 1997) in maesthetic terms. Other rich sources include the classic *Starting Points* (Banwell, Saunders, & Tahta, 1972) and Orton's fine collection (1999) on *Pattern in the Teaching and Learning of Mathematics*.

The math/art interface has been extensively mined historically (Emmer, 1993) and some of that literature is consistent with maesthetics. The very active 'Bridges' group has produced some very interesting math/art materials in the Proceedings documents from their annual meeting (Sarhangi & Moody, 2005).

Several members of the Mathematics, Science and Technology Education Group at Queen's University have been involved in projects which have had - without it explicitly being articulated that way - a maesthetic foundation. These include a Vision Statement Project, *Tomorrow's Mathematics Classroom*, that looked at the implications for teachers and students of considering mathematics to be at tool, a language and an art. The book, *Creative Mathematics* by Upitis, Phillips and Higginson (1997) documented the work of a gifted elementary teacher using a variety of innovative ("constructive aesthetic") teaching approaches. More recently, the doctoral work of Nathalie Sinclair (2006) has been published (in extended form) by Teachers College Press under the title, *Mathematics and Beauty: Aesthetic Approaches to Teaching Children*. Many of the ideas in this paper are considered at greater length and from different perspectives in an edited collection entitled *Mathematics and the Aesthetic: New Approaches to an Ancient Affinity* (Sinclair, Pimm & Higginson, 2006), especially Higginson (2006).

EPILOGUE

In the spirit of the visionary artist: from deep in the bowels of one of the great American novels of our age.

> We have an idea, some of us, that's taking shape. A new sort of collegium. Closer contact, minimal structure. We may teach Latin as a spoken language. We may teach mathematics as an art form like poetry or music. We will teach subjects that people don't realize they need to know. All this will happen somewhere in the hinterland. Don DeLillo, *Underworld,* (1997), p. 675.

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LANDLESS PEASANTS OF SOUTHERN BRAZIL AND MATHEMATICS EDUCATION: A STUDY OF THREE DIFFERENT LANGUAGE GAMES

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This paper aims to discuss issues related to mathematics education and culture. Based in the work developed by the author with the Brazilian Landless Movement, it discusses a theoretical background that can give support to the field of Ethnomathematics. It is rooted in a Post-Modern perspective in its connections with Poststructuralist theorizations – more specifically, those associated with the work of Michel Foucault – and the ideas of the "Later Wittgenstein", which corresponds to his book "Philosophical Investigations". Three different language games, each of them associated with a specific form of life, are shown: a language game belonging to the mathematics of the Landless peasants' form of life, another one which is part of a urban sawmill men's form of life and a third, found at Western Eurocentric school's form of life, even considering that all of them have family resemblances.

INTRODUCTION

This paper[1] aims to discuss issues related to mathematics education and culture, taking as an empirical base for the discussion the work developed by the author with the Brazilian Landless Movement (Knijnik, 2006, 2007). Its theoretical support is rooted in a Post-Modern perspective in its connections with Poststructuralist theorizations – more specifically those associated with the work of Michel Foucault – and the ideas of the "Later Wittgenstein", which corresponds to his book "Philosophical Investigations". According to such theorizations, I have considered that Ethnomathematics may consist of a toolbox[2], which allows analyzing: a) the Eurocentric discourses of academic and school mathematics; b) the effects of truth produced by such discourses; c) issues of difference in mathematics education, considering the centrality of culture and the power relations that institute it; d) the language games that constitutes different mathematics associated to distinct forms of life.

We are facing issues connected to politics of knowledge, to the dispute around the definition of which knowledges are included and which excluded in the schooling processes. This dispute is marked by power-knowledge relations, which ultimately legitimate and are the legitimizer of some discourses, which interdict others, precisely those that are about the knowledges, the rationalities, the values, the beliefs of cultural groups we place in the position of "the others".

One should then ask how a single rationality among other rationalities, for example, the rules by which individuals and cultures deal with space, time and quantification processes – all that which Western civilization associates with the notion of

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mathematics – became a "truth", the only "truth" that could be accepted as mathematics in the school curriculum. What is at stake here is to problematize the sovereignty of Modern rationality, which despises all other rationalities associated with "other" forms of life; the existence of a single mathematics – "the official one" – with its Eurocentric bias and its rules marked by abstraction and formalism. To be more precise, we must say that this "official" mathematics – the academic one – is composed by a set of branches, a set of language games, as shown by Ernest (1991), including all those associated with "pure mathematics" and "applied mathematics". The school mathematics – the traditional set of knowledges taught at school – inherits at least part of the formal and abstract grammar that constitutes academic mathematics, through pedagogical recontextualized processes, in Bernstein's words (1996). In summary, it can be said that the set of language games of academic mathematics, as well as those of school mathematics, offers a "dream of order, regularity, repeatability and control (…) and with it the idea of a "pure", disembodied reason" (Rotman, 1993, p. 194).

THEORETICAL FRAMEWORK

The issues briefly shown above lead us to Ludwig Wittgenstein's ideas presented in his book "Philosophical Investigations" (2004) in which he criticized not only his earlier work (presented in Tractatus) but also "the whole tradition to which it belongs" (Glock, 1996, p. 25), "the foundationist schools, and dwells at length upon knowing as a process in mathematics" (Ernest, 1991, p. 31).

In shaping a new philosophy of mathematics – social constructivism – Ernest refers to Wittgenstein as one of the philosophers who considers knowledge not only as a product, giving "great weight to knowing and the development of knowledge" (*ibidem*, p. 90). He considers that "social constructivism employs a conventionalist justification for mathematical knowledge" (*ibidem*, p. 64), assuming that "the basis of mathematical knowledge is linguistic knowledge, conventions and rules, and language is a social construction" (*ibidem*, p. 42). Ernest argues that his philosophical perspective "assumes a unique natural language" showing that "an alternative (i.e. different) mathematics could result" (*ibidem*, p. 64) as a consequence of this position. Mentioning the work of Alan Bishop as an evidence of different mathematics (*ibidem*, p. 67), Ernest will say that "such evidence of cultural relativism strengthens rather than weakens the case in favour of social constructivism" (*ibidem*, p. 64).

These ideas are strongly connected to the ethnomathematics thinking presented in this paper. In fact, viewing mathematics "not as a body of truths about abstract entities, but as part of human practice" (Glock, 1996, p. 24), the philosopher's work gives us tools for thinking about rationality as forged from social practices of a form of life, which implies considering it as "invention", as "construction" (Condé, 2004, p. 29). Moreover, with the support of the philosopher's ideas – and using the expressions that he coined – one can admit the existence of distinct mathematics – distinct ethnomathematics, in D'Ambrosio words[3]. The basis of this statement can be found in the argument that these different mathematics – in Wittgenstein's words, different language games – are part of different forms of life, a term conceived by the "Second Wittgenstein" as "stress[ing] the intertwining of culture, world-view and language" (Glock, 1996, p. 124), as "patterns in the weave of our life" Glock (1996, p. 129)).

In Wittgenstein's late work, especially in the new conception of language presented by the philosopher, Condé (1998, 2004) argues about the crucial role of the notion of use:

> In such work, use is directly connected to the concept of meaning (…) the meaning is determined by the use we make of the words in our ordinary language. (…) The meaning of a word is given based on the use we make of it in different situations and contexts. (…) the meaning is determined by the use. (*ibidem*, p. 47)

It is in this sense that this notion of use, according to Wittgenstein, is considered pragmatic, not "essentialist". Meaning is determined by the use of words and such a use obeys rules, which are themselves produced in social practices, constituting language games. As pointed out by Condé (1998, p. 91) "the notion of language games involves not only expressions, but also the activities with which these expressions are linked". Language games are produced based on sets of rules (that are rooted in social practices), each of them constituting a specific grammar. So, the grammar that marks each language game is itself a social institution. Moreover, authors like Spaniol (apud Condé, 1998, p. 110) argue that "the grammar constitutes the logic itself, the grammar is the logic. (…) It is impossible to analyze the logic without considering the language".

From what was briefly explained here it follows that different language games, each of them having by a specific grammar with its own set of rules, constitutes a specific logic. This rationale drives us to admit that there is more than a single language game: there are different language games. Is there some kind of relationship between them? If the answer is positive, how does it operate? The response to these questions is given by the "Second Wittgenstein" through the notion of family resemblances. The philosopher would say (as shown in aphorisms 66 and 67 of Philosophical Investigations) that language games form "a complicated network of similarities overlapping and criss-crossing: sometimes overall similarities, sometimes similarities of detail" (Wittgenstein, 2004, p. 320) and adds:

> I can think of no better expression to characterize these similarities than family resemblances; for the various resemblances between members of a family? Build, features, colour of eyes, gait, temperament, etc. etc overlap and criss-cross in the same way – and I shall say: 'games' form a family.

Operating with the ideas of the "Second Wittgenstein" in the context of the struggle for land in the south of Brazil leads us to assume the existence of three different mathematics: a mathematics produced by a form of life associated with MST peasants, another one produced by a form of life of the urban sawmill men and a third, produced by a form of life found in the Western Eurocentric school, even considering that all of them have family resemblances.

LANDLESS' LANGUAGE GAME OF *CUBAGEM OF WOOD* AND TWO OTHER LANGUAGE GAMES OF DIFFERENT MATHEMATICS

Cubagem of wood (in Portuguese Cubagem da madeira) – to calculate "how many cubics[4] there are in a truck load" – is a common practice in the Landless' culture. The peasants perform it when it is necessary to build houses or animal shelters in camps and settlements and to purchase or sell planks, i.e., "in our negotiations with the sawmill men", as said one MST member. Throughout my work with MST groups I have realized the importance they give to such language games, which are part of their form of life. In teacher education courses and at settlement schools, particularly, I have found great interest in discussing that practice, constituted by a specific grammar, a specific set of rules.

During my work with that group of students I found that they were expecting me to help them learn more about the grammar that marks the *cubagem of wood'* language game. But it was expected that I also assume another role. In consonance with the Sector of Education pedagogical guidelines they aimed to acquire the school mathematics knowledge – the one called by them "book mathematics". Avoiding a naïve perspective, they were aware of the social importance of such a set of language games and the need to learn its specific grammar as part of their struggle.

Even considering the theoretical difficulties involved in translating language games, it is important to express Roseli's method using the words and the syntax of the school mathematics' language games, which we are more familiar with. I am aware that in doing so some (or maybe most of the) specificities that constitute the Landless' form of life which produced the cubagem of wood language game are suppressed. So, it can be said that Roseli's method became a "hostage" of the school mathematics language game when it is said that "her" method basically involves two steps: the first, to identify, by modelling, a tree trunk with a cylinder whose circumference coincides with that of the middle part of the trunk, and the second, identification, also by modelling the cylinder in a quadrangular prism, whose measure on the side is one fourth of the perimeter of the cylinder base. Thus, Roseli's Method for "cubagem" of wood finds, as trunk volume, the volume of the quadrangular prism whose side of the base was obtained by determining the fourth part of a circumference. This, in turn, corresponds to the cylinder base, obtained by modelling from the initially given tree trunk. Roseli explained "her" method[5] step by step, as she pointed to the different parts of the trunk involved in the process. This narrative triggered the study on cubagem of wood which we developed from then on.

During the discussion of Roseli's method there were students who immediately related its grammar to the one that constitutes the land cubação language game called by the group Jorge's method, which was studied before (Knijnik, 1997). In fact, both grammars have one rule in common: the identification process which associates a cylinder base (in the case of cubagem of wood) or a quadrilateral (in the case of land cubação) with a square. The relationship established by the group between both language games was an interesting pedagogical issue linked to what Wittgenstein called family resemblances.

This notion of Wittgenstein can be helpful in understanding another language game which emerged in the pedagogical process. At some point in the discussions, a two-student dialogue produced a shift in the debate about Roseli's method.

Jorge: The measurement process that I know is almost the same [as Roseli's method], except that we measure at the narrow end of the wood.

Ildemar: The point is that the right thing would be to do it in the middle. But the purchasers do not want to buy a piece that will fall away, if they want it for square wood[6] or things like that. They will want a piece that goes from here to there [which goes from one end to the other of the log]. Those chips that are produced will only be for burning.

According to these students, there were urban sawmill men who did not use the "middle of the log" as reference, considering only its narrower end, since they were interested in obtaining whole planks.[7] For this purpose a different rule of calculation was introduced, conforming a specific grammar, which leads to a new language game, different from Roseli's. The sawmill men's method was mentioned by most of the group as being practiced at sawmills in the urban areas close to their communities. We found that we were dealing with a language game which is part of a specific form of life, different from that of the Landless' peasant.

But the pedagogical process was not circumscribed to Roseli' method and to that of the sawmill men. One of the language games that constitutes the "book mathematics" – precisely that linked to the calculation of a pyramid trunk's volume -- with its specific rules was also analyzed. Moreover, the family resemblances of the three language games were emphasized. The work involved studying the modelling process of Roseli's Method and learning mathematical tools such as the relations between a cubic meter and its multiples. In different situations the results of calculating the "amount of wood" obtained by Roseli's method were compared empirically to the volume of the cylinder produced by "her" method, which would correspond to a better approach to the total quantity of wood of the trunk, reckoning not only the part useful to obtain "whole planks". The group also found that the results of Roseli's method minimize those obtained using the cylinder volume. The group showed particular interest in learning "the formulas of book mathematics" connected to the discussion we were holding. In learning how to calculate volumes of the cylinder and rectangular prisms the group was dealing with part of the Western Eurocentric school mathematics' set of language games.[8]

Bringing those three language games into the mathematics class enabled the group to go further in the appropriation of their specific rules and this led them to learn more about the Landless peasant cubagem of wood practiced in their communities. When the family resemblances of those language games were analyzed, the students were able to identify the "remnants" of wood that were produced by Roseli's method, which were even greater when the initial measure of the log circumference was determined at the "narrow end", as considered by the sawmill men's method. So, in this case, the wood not used for making planks could be useful for other purposes and therefore, in given situations, it should also be included in the accountancy of their calculations.

Summing up, it can be said that learning about different mathematics and their family resemblances allowed the peasant students to broaden not only their mathematical world, but also their ways of seeing the complex social relations involved in different forms of life and those different language games associated with them.

SOME CLOSING WORDS

I would like to end saying that the issues I attempted to discuss here are no more than provisional, unmarked by hopes for certainty, in the sense given by Stronach and Maclure (1997). I agree with them when they say that we must recognize and try to work within the necessary failure of methodology's hope for certainty, and its dream of finding an innocent language in which to represent, without exploiting or distorting, the voices and ways of knowing of its subaltern 'subjects'" (*ibidem*, p. 4). The ideas I brought to this paper are inspired by this position.

NOTES

1. The paper is a condensed version of the article "Mathematics education and the Brazilian Landless Movement: three different mathematics in the context of the struggle for social justice" (Knijnik, 2007).

2. In considering the Ethnomathematics' perspective as a theoretical tool-box I am following Gilles Deleuze who argues that "a theory is exactly like a box of tools. It has nothing to do with the signifier. It must be useful. It must function. And not for itself. (...). We don't revise a theory, but construct new ones (...). A theory does not totalize; it is an instrument for multiplication and it also multiplies itself." (Bouchard, 1977, p. 208)

3. In fact, D'Ambrosio (2001) considers that each branch of academic mathematics shapes an ethnomathematics; school mathematics is an ethnomathematics and also the ways in which specific cultural groups – like the Brazilian Landless peasant – deal with numbers, space, measurement, etc are considered different ethnomathematics.

4. The terms "cúbicos" and "cúbicos de madeira" are used in the Brazilian rural areas to mean cubic meters of wood. The term "metros de madeira", in English, "meters of wood", is also used.

5. Several students referred to the use of Roseli's Method in their communities. The so called Roseli's method had already been identified in fieldwork previously performed in the south of Brazil (Klüsener & Knijnik, 1986) and it was also practiced in the state of Acre, in the north of the country (Mattos, Nepstod & Vieira, 1992).

6. At that time, some students used the expression "square wood" to refer to a wooden plank.

7. Taking into account the remarks made before, concerning the "translation" issues from one language game to another, it could be said that the sawmill men's method consists of calculating the volume of a quadrangular prism whose height is given by the tree trunk. The quadrangular base, however, different from Roseli's method, is obtained by the inscription of a square with a maximum side at the log base, considered as a circle.

8. The group questioned the possibility of using the rules they had studied in the context of both Roseli's method and the sawmill men's method in other peasant practices. One of the students mentioned that the rules he learned could be used in planning the construction of silos for crop storage, at the time one of the main goals of his comrades to render the settlement economically feasible. In some way, we can say that he is identifying family resemblances, in Wittgenstein's words, between these different language games.

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THE EMERGENCE OF DISPARITY IN PERFORMANCE IN MATHEMATICS CLASSROOMS

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In this discussion paper we consider questions related to research which attempts to consider structural elements such as social and family backgrounds, language and nationality and how these factors influence the internal dynamics of the mathematics classroom and students' achievement. We discuss difficulties encountered when trying to study the discursive and interactive dynamics that produce disparities in mathematical classrooms while taking into account socio-cultural structures of our societies. In addition we ask what role an inter-cultural comparative approach could play in studying these complex relationships. We explore the extent to which broad social categories (like "class") are useful in classroom research and difficulties in locating a suitable theoretical framework for comparative work addressing both classroom dynamics and socio-cultural structures.

Disparity in achievement is one of the major concerns in mathematics education research studies. Some studies, for instance, examine the mathematics achievement of *large groups* and relate differences in achievement to social categories such as nationality, language, and class. PISA, TIMSS, and other large scale assessment studies fall into this group. In these studies, the social categories are present as independent variables. Other studies examine mathematical activity in classrooms and the *interactions* between and among students and teachers. These studies do not consider social categories as input-variables only, but consider the social dynamics within mathematics classrooms, which might amount to successful or unsuccessful participation of students. Finally, there is a large body of work that focuses on individuals' differences in *understanding* of mathematics.

In this discussion paper we will consider questions related to research which attempts to consider structural elements such as social and family backgrounds, language and nationality and how these factors influence the internal dynamics of the mathematics classroom and students' achievement.

Research that focuses on "socio-cultural contexts" as for example Abreu (2000) and others (see below) have conducted research at these two levels, but such research has normally focussed on a particular socio-cultural group, as for example sugar cane farming families or street children in Brazil. In our research and in this discussion paper we are considering especially the challenge of a socio-anthropological perspective when the focus is on comparisons between socio-cultural groups. Our on-going comparative research into the construction of social disparity in mathematics classrooms in (rural) Canada and (urban) Germany will be used to ground the discussion in actual events and circumstances.

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BACKGROUND OF OUR RESEARCH

Teachers and students in mathematics classrooms quickly come to know which students perform well in mathematics and which do not. This occurs even in classrooms where selection processes are intended to produce homogenous classes and in contexts where the students are together for the first time. In selective school systems, disparity in performance is both a goal and an effect; the practice of streaming makes this evident. But even within different streams in which students are supposed to be starting at a comparable level, differences in performance are detected within a short time. Likewise, inclusive (unstreamed) school systems accept and produce divergence of achievement.

That such differences are observed so quickly may appear as "natural" in cultural systems (such as Canada's and Germany's) where "natural ability" is part of the implicit theory about how children learn mathematics. However, cross-cultural comparative research (Stevenson & Stigler 1992, Azuma 1998) has revealed that other cultural systems (notably Japan's and China's) explain performance through other implicit theories and that different implicit theories may influence students' success in school mathematics. This suggests that it would be useful to investigate the emergence of disparities from a theoretical perspective that examines their social construction in the context of the social practices of the mathematics classroom. Further, an intercultural comparative approach seems promising as it reveals implicit theories of different classroom cultures and institutional systems.

Further, large-scale quantitative studies show that there are gaps in achievement and that those gaps are wider in some contexts than in others. For example, the gap in Germany is huge compared to many other countries (Bos et al. 2004). The mathematics achievement gap is generally defined along lines of students' class, gender, ethnicity, language of instruction and immigrant backgrounds. In the case of Germany, a number of factors have been found to be related to differences in the mathematics achievement of students. The selection or streaming of students into different school types, family structure, immigrant background, and gender have been shown to influence students success in schools (Baumert / Schümer 2001).

In Canada, for example, the provincial variation in mathematics performance is striking. Three provinces, Quebec, Alberta and British Columbia usually have scores higher than other Canadian provinces and the Atlantic provinces (Nova Scotia, New Brunswick, Prince Edward Island and Newfoundland) usually have scores lower than other Canadian provinces (Robitaille & Taylor 2000; Bussière, Cartwright & Knighton, 2004).

The discussion we would like to induce by this article is twofold: 1) How we can study the discursive and interactive dynamics that produce disparities in mathematical classrooms (the "micro-perspective"), while taking into account socio-cultural structures of our societies (the "macro perspective")? 2) What is the role that an inter-cultural comparative approach can play in studying these complex relationships?

In this article we will begin by presenting studies that have investigated the existence and emergence of educational disadvantage. Most of these studies have been based on sociological theories that assume that teaching is less free than one might think, but restricted by structural elements such that actions of students and teachers are influenced by factors that have their origin “outside” the classroom. Two theoretical perspectives from sociology that have been useful in qualitative research into differences in mathematical achievement are those of Bourdieu (1991) and Bernsein (1990). We will then present our own current research on the emergence of disparity in mathematical classrooms, which is also embedded in a sociological theoretical framework, based on Bernstein. We will outline the focus of our study, its methodological and theoretical approach, which will lead us then to some theoretical considerations and finally to an important discussion. Empirical results of our own study and other research indicate the complex dynamics of “inner” and “outer” factors and how difficult it is to determine simple “outer” factors that might lead to the exclusion of some students. It seems necessary to reflect critically on what shapes the dynamics of the emergence of disparity within mathematical classrooms and how we can research these complex interplays.

STUDIES ON THE EMERGENCE OF EDUCATIONAL DISADVANTAGE

The following studies are a sampling of those that illustrate the many subtle and indirect ways that schools produce (rather than reproduce) class identities, which has been a focus of empirical research based on critical sociological theory over the last twenty years (Arnot et al., 2003).

Teese (2000) found significant quantitative relationships between students’ socio-economic background and their success in mathematics final exams in the state of Victoria, Australia. He explains these relationships by a qualitative analysis within a theoretical framework based on Bourdieu (1991, 1992). He argues that this discriminating potential is implicit in a curriculum which raises cognitive demands over successive levels of mathematics by calling more and more on embedded scholastic attitudes and behaviours. Teese observes that the choice of content, the relative stress placed on different tasks, the compression of the content and the pace of teaching are based on the implicit view of an ideal student, who is “the young scholar-intellectual” (Teese, 2000, p. 4). Zevenbergen (2001) finds in her research that the linguistic habitus of Australian middle-class students works as cultural capital as in school – at least at the discursive level – the discursive practices are close to practices that are common in middle-class families.

Cooper and Dunne (2000) investigate how students with different socio-economic backgrounds react to word and context problems. They analysed large sets of data from the Key Stage 2 Tests for 10-11-year-old students in England. The study documents that students of families where the parents do manual work have significantly lower achievement. Cooper and Dunne use the work of Bourdieu (1990, 1994) and Bernstein's framework (1990, 1996) to explain their results. The researchers find that these students tend to misinterpret the problems and to solve them with their everyday knowledge, which means that their mathematical competence is systematically underestimated in the tests.

Boaler (2000) illustrates through her analyses of interviews with grade nine students from groups with different achievement levels that it is not only the difference between "everyday" and mathematical discourse that makes it difficult for students to give meaning to mathematical tasks, but that students in London feel disconnected in demographically inhomogeneous classes. These studies are important for our research as they try to identify mechanisms that can explain if and how structural elements can be found in classroom interactions.

There is also research that does not focus on mathematics teaching but is of relevance for our study, as classroom interactions are analysed from the perspective of sociological theory.

Bourne (1992, 2003) investigates urban schools in Great Britain where students with low educational and social backgrounds show higher achievement than students with similar backgrounds in other schools. Using Bernstein's concepts of "vertical" and "horizontal" discourse, Bourne shows, through micro-analyses of the discourse in English lessons in an elementary school with bilingual students, how the teacher manages to arrive at a normal distribution of performance. She illustrates how the teacher attains this goal by student attributions of "natural ability" in verbal and nonverbal interactions.

Morais and Miranda (1996) investigate if students are familiar with the assessment and evaluation criteria of their teachers and if they can use these criteria to mark solutions of their peers, in the context of science teaching in grade 5 in Portugal. Making use of a theoretical framework based on Bernstein's work, the authors see relationships between students' achievement and their family and social background, the expectations of the teacher, and the explicitness of these criteria in the classroom.

THE EMERGENCE OF DISPARITY WITHIN THE FIRST WEEKS OF SCHOOL: OUR STUDY

In our research we study how teachers and students come to know which students perform well in mathematics and which do not within the first weeks of school. Our research focus is on mathematics classrooms at the first grade after primary school, where in a new school the students and the teacher are together for the first time. We compare classrooms in two countries, Germany and Canada, which differ in the degree of streaming in their school systems. In Germany, the students are in grade 5 or 7, when they move to the Gymnasium, the Hauptschule, Realschule, or Gesamtschule. In Canada, the students are in grade 6 or 7, when they move from elementary school to junior high school (middle school). We will include classrooms from Sweden in our study in 2008. There the students will be in their first year after compulsory school in grade 10.

Beginning from a sociological perspective, we study empirically the interactions that may produce disparities in these mathematical classrooms. Our central research question is:

Which discursive and interactional mechanisms provoke a stratification of achievement within the mathematics classroom? What are the characteristics of these mechanisms in relatively homogeneous and in heterogeneous groups, in socially advantaged and disadvantaged groups?

As we have outlined above, the research literature and our own theoretical perspectives support the hypothesis that disparities in achievement, as perceived by the teacher and students in the classroom, may reflect external factors, as well as the internal dynamics of the classroom. In developing our methodology and designing the studies that constitute this research program we have attempted to look at the emergence of disparity in a number of ways that capture both internal classroom dynamics and external factors.

First, data are gathered at data sites across and within two national contexts: Berlin and Hamburg, Germany and Nova Scotia, Canada (Northern Sweden will be added in autumn 2008). In each context, data are gathered in two types of schools: in Germany in two Gymnasien and two Hauptschulen, in Canada in public schools and a private school. The choice of two types of schools in each context is guided by consideration of the most obvious systemic difference that might account for the difference in disparity. Most schools in Germany are, from grade 5 or 7 on, selective. At that grade level, students go to different kinds of schools, Gymnasium, Realschule, Hauptschule and Gesamtschule, which lead to different future educational and professional opportunities. In Nova Scotia, in contrast, an inclusive approach is the official public school policy, at least up to grade 10, when some streaming occurs in mathematics and science. If this systemic difference contributes to the emergence of disparity in classrooms, a comparative approach in the two contexts should make it possible to explain this contribution.

Second, the methods used to gather data focus on the emergence of disparity both through the interactions within the classroom and from an outer perspective using sociological factors. It is important to study the emergence of disparity from within the classroom because that is where the teachers' and students' knowledge of differences between students is constructed. To follow classroom interactions, video recordings are made at the start of the school year and continue for six weeks. In addition, copies of written work handed in to the teacher or marked and handed back by the teacher are collected, as such documents also form a part of the communicative interaction in the classroom. The teachers are interviewed twice, once before the beginning of the school year and once at the conclusion of the classroom observations. The initial interview focuses on the teachers' expectations of the incoming class and their history of encountering diversity in their classes. The final interview focuses on the teachers' emerging view of the students in the class. Groups of about six students are also interviewed, beginning in the fifth week of observations. The focus of these interviews is the students' perceptions of, and accounting for, the diversity in mathematics achievement in the classroom.

To gather data from the outer perspective, a questionnaire is administered to students to collect standard measures (e.g., socio-economic indicators, educational resources in the home, etc.) that have been found to be correlated with mathematical achievement in large scale studies. If necessary, individual interviews are conducted to provide more detailed data or to account for anomalous data.

DISCUSSION

At first glance finding out how the students in a mathematics classroom fit into the categories that have been found useful in large scale studies seems an obvious way to relate classroom interactions to social structures of our societies. Large scale research suggests that achievement is related to socio-cultural factors such as social class, gender, ethnicity, language of instruction, parents' education and occupation, and immigrant background. However, when we apply these categories to the complexity of students' backgrounds in a mathematics classroom in rural Nova Scotia, it becomes apparent that these categories are problematic.

Difficulties arise in a number of ways. Most pragmatically, it is not clear that students in grade 6 (11 years old) can provide accurate information about their parents' backgrounds, education and occupations. What does one make of a claim that a mother with only a high school education is working as a substitute teacher (an occupation that normally requires six years of post-secondary education)? Follow-up interviews can provide some clarification, but in some cases students simply do not know about their parents' backgrounds in the detail we might like. Interviewing parents could provide these details, but even then there are other difficulties inherent in the use of these categories.

Large scale studies necessarily group together occupations into broad categories. But even precise labels can be misleading. Several students in our study reported that their fathers are carpenters, but the degree of skill (and mathematics use) in carpentry can vary. Descriptions such as "stairway carpenter" and "worker in a carpentry workshop" may indicate access to very different skills. If the broad categories used in large scale studies are deconstructed to individual occupations' descriptions, in what sense are we relating classroom based research to large scale studies?

A related issue is the way in which the other parents' occupation is considered. In large scale studies assumptions are made as to the importance or status of occupations and only one parent's occupation is considered. In traditional societies in which the father is the main wage earner this may be valid, but in our study such an approach denies important realities. For example, in three families where the father is a carpenter, the mother is absent in one case, stays home with the children in another, and is a university professor in the third.

Class is a very important category in sociological research, however comparative research like ours can reveal ways in which it may be a problematic category. For outside observers who are used to thinking in terms of class, a different context can reveal differences in indicators of class not only in the context being observed but also in the reference contexts of the observers. For example, in discussions between Knipping (raised in Germany) and an English research assistant surprising differences occurred in the class identifications of students based on their observations in the classroom and the characteristics used as markers of class. This indicates the importance of some sort of additional measures of social-cultural background beyond observers' classifications (as e.g. questionnaires or interviews), but it also indicates the lack of universality in the category "class" which makes it unclear whether a statement such as "Social class is a predictor of mathematics achievement." means the same if this is said about German students or Canadian students.

These issues bring us back to our question: 1) How can we study the discursive and interactive dynamics that produce disparities in mathematical classrooms (the "micro-perspective"), while taking into account socio-cultural structures of our societies (the "macro perspective"). How can we answer this question if such methodological attempts to bring the "macro" and "micro" perspective together seem to fail? The problem seems to be the suggested starting point. Instead of beginning with categories identified in large scale studies as predictors of mathematics achievement could we instead start at the micro level? Could we use the disparities that do emerge in a classroom as a starting point for identifying aspects of the larger socio-cultural context that are significant?

Starting with the micro context requires a theoretical framework that allows us to describe the emergence of disparity in terms of the relationship between everyday knowledge derived from the larger socio-cultural context and school knowledge that is related to success in classrooms. Bernstein's theoretical framework is promising as a starting point, but also brings with it some difficulties.

THEORETICAL FRAMEWORKS FOR DESCRIBING THE EMERGENCE OF DISPARITY

Bernstein's theory of pedagogic discourse is concerned with the production, distribution and reproduction of knowledge and how this knowledge is related to structurally determined power relations. Bernstein (1971) introduced the terms "classification" and "framing" to distinguish two different systems of "educational knowledge codes". According to his analyses curricula and pedagogy can be characterised by these codes.

“Classification” refers to the curriculum, areas of knowledge, or what is taught. Strong classification means that strong boundaries between subjects are maintained. For example, a traditional mathematics curriculum has strong boundaries as few connections are made to other disciplines. A project-based mathematics curriculum, on the other hand, has weaker boundaries, as mathematics teaching is integrated with other subject areas inside or outside school. This means that everyday knowledge and subject knowledge are less separated. According to Bernstein, weak or strong boundaries establish a relation to everyday knowledge and therefore students' contributions in class can appear more or less appropriate. Teachers' evaluations of students' achievement will reflect their decisions about what areas of knowledge are to be selected, how these areas are related within the subject, and how they are related to other subject areas inside or outside school.

“Framing” refers to pedagogy, that is, to the how of teaching. Strong framing is linked to explicitness of the social rules. Weak framing is indicated by implicitness of the rules.

Though classification and framing are theoretically distinguished (which is helpful for purposes of analysis), these different systems overlap in classrooms and so together create the conditions of learning and the grounds for evaluation. In Bernstein's model, almost by definition, certain combinations of framing and classification exclude some students.

In addition classroom discourse is structured by implicit rules to which not all students have equal access. These rules allow for the recognition of legitimate classroom discourse and also the production or realisation of such discourse. As access to these rules are related to both mathematical achievement and to socio-cultural background they could provide a basis for connecting the micro with the macro starting with analysis of classroom activities in terms of classification, framing and recognition and realization rules.

While this theoretical framework seems to offer many of the features we require for working at the micro and macro levels together, it is not without problems, both practical and theoretical.

Practically, in our analyses of classroom interaction, we find strong classification mostly connected to strong framing, and weak classification to weak framing. This suggests that the distinction between classification and framing may not be as useful as we expected. Dowling (in press) argues that this is the case.

Within classroom discourse one can distinguish between instructional and regulative discourse. Instructional discourse refers to the (school) mathematical part of the discourse, e.g. what is valued as a mathematical argument; hence it is related to classification. Regulative discourse refers to the structuring of social interaction, e.g. which ways of turn-taking or working with others are acceptable; hence it is related to framing. Bernstein claims pedagogic discourse to be the process which leads to the embedding of instructional discourse in regulative discourse, “to create one text, to create *one* discourse” (1996, p. 46). But it is not clear if the distinction between instructional and regulative discourse is empirically observable, nor that the embedding of one discourse in another can be detected.

These issues of the theoretical basis for comparative research connecting macro and micro contexts is the subject of Jablonka, Gellert, Knipping & Reid (2008, MES symposium).

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SOCIAL AND DIDACTICAL ASPECTS OF ENGAGEMENT IN INNOVATIVE LEARNING AND TEACHING METHODS – THE CASE OF RUTH

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Mathematics educators emphasize the need to change mathematics teaching and learning. Despite the fact that great endeavors are invested for that matter, changes are not as widespread as was expected. Working with prospective teachers, we enable them to experience various innovative teaching approaches, hoping that they will decide to implement these methods in their future classes. However, we realized that prospective teachers express resistance towards changes. In this paper we present the case study of Ruth, a prospective teacher who was engaged in learning via a computerized project-based learning approach. Through Ruth's reflection on her experiences we realized that her resistance can be attributed to the social norms she adopted, as a result of her past experience as school student.

INTRODUCTION

The last two decades were characterized by the intensive calls for employing reforms in mathematics education (e.g. NCTM's standards, 2000). It is anticipated that the teachers should be the ones that put the innovative approaches into practice. However, real modifications are not as extensive as was expected (Stigler & Hiebert, 1999). A variety of explanations can be suggested in order to explain this "stagnation". One possible explanation might be related to what Desforges (1995) had found in his review of literature: teachers are satisfied with their practices and do not tend to question educational processes. They often disregard data that is inconsistent with their beliefs and practice and tend to avoid new experiences. Instead, they prefer to stick only to those practices that match their existing system of beliefs. A question to be asked is: What are the factors that cause this tendency and why is it so widespread? Since teachers' beliefs are influenced, among others, by social and cultural norms and it is within these contexts that teachers make sense of their role (Lee, 2005), we believe that in order to be able to answer the above question teachers' beliefs and their relation to the social and cultural environment have to be examined.

Prospective teachers (PTs) begin their training with explicit beliefs regarding various issues that concern teaching and learning (Tilema, 1995). These beliefs are result of their past experience as school students (Lee, 2005). Consequently, it is reasonable to assume that PTs' beliefs regarding teaching and learning resemble those of their teachers. PTs are the next generation of teachers. In order to be able to teach in the spirit of the reform, they have to be convinced that these teaching methods are beneficial. We believe that one of the ways to enable them to recognize the benefits of innovative teaching approaches is by involving them in challenging their existing

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beliefs and examining the adequacy of these beliefs to the changes that are recommended by the reform. In this paper we describe an experiment made with a group of PTs, aimed at making their beliefs about teaching and learning mathematics explicit, and assist them in examining the compatibility of these beliefs to the spirit of the reform. We present the case study of Ruth, a member of the PTs group that were engaged in a Computerized Project-Based Learning (CPBL) activities (Krajcik, Czerniak and Berger, 1999). We chose to focus on Ruth from two reasons: first, her expressed beliefs reflected the common beliefs of the majority of the study participants; second, comparatively to the other PTs in the class she demonstrated outstanding self expressive abilities.

THEORETICAL BACKGROUND

This paper describes and discusses the expressed beliefs of Ruth while engaging in CPBL. These beliefs relate to teaching and learning of mathematics and reflect the social environment in which Ruth and her classmates were educated. The theoretical framework of this paper focuses on the meaning of ‘system of beliefs’, and on social and sociomathematical norms, which are among the constituents of such a system.

System of beliefs. Beliefs are perceptions and attitudes towards a certain reality. According to Tilema (1998), a system of beliefs does not require external approval. The influence of beliefs is strongest on the meanings which people attribute to occurrences, and on activities they choose to carry out. PTs hold beliefs regarding various aspects relating to teaching and learning, among them: their role as teachers, students’ learning processes, curriculum suitability, and so forth (Van-Dijk, 1998). Their beliefs reflect their values in terms of what is “desirable”. These beliefs are result of thousands of hours in an “apprenticeship of observation”, which inspire school students’ perception regarding teaching and learning (Lortie, 1975). Unfortunately pre-existing beliefs about teaching, learning and subject matter are resistant to change (Foss & Kleinsasser, 1996; Lee, 2005), consequently, PTs graduate the university holding the same beliefs with which they arrived (Kagan, 1992). Namely, PTs’ personal beliefs and images are not affected by their training practice and generally remain unchanged. They tend to utilize the information they are exposed to during their training mainly to strengthen their existing beliefs and perceptions. That means that the topics that are being presented in teacher education programs are subjected to interpretations according to PTs’ pre-existing beliefs (Tilema, 1998). Those interpretations also affect their performance in class (Kagan, 1992), since they rely on their own subjective theories of teaching or on what they believe will work in class. For example, many PTs believe that teachers ‘deliver’ knowledge to their students, and learning means memorizing of contents (Richardson, 1996). Their memories of themselves as learners influence their expectations of their future students as well as their views regarding “proper” teaching strategies. The image they possess regarding “good teaching” relates to the kind of teachers they see themselves becoming. As a consequence PTs tend to exhibit conservative teaching, replicating their own teachers.

Social and socio-mathematical norms. The theme of classroom norms has been addressed by various researchers in recent years. Cultural and social processes are integral to mathematical activity, and the culture of the mathematics classroom, is central to the development of mathematical disposition among students and bring change in mathematical beliefs (Yackel and Cobb, 1996). Yackel and Cobb (*ibid*) distinguished between general classroom social norms (for example: the need to explain or justify) and norms that are specific to the mathematical activities of the students, termed as socio-mathematical norms (for example: what counts as mathematically efficient, mathematically sophisticated, mathematically elegant, acceptable mathematical explanation and justification). The teachers' and the students' beliefs serve as key factors for negotiating classroom norms. The teacher-students verbal interactions provide the opportunity to negotiate the socio-mathematical norms, which are continually regenerated and modified, and might differ substantially from one classroom to another. Yackel and Cobb (*ibid*) suggested that there is a reflexive relationship between beliefs and classroom norms: the student beliefs influence the classroom norms and those norms, in turn, influence the beliefs of students.

Various classroom norms and socio-mathematical norms develop in various settings, in accordance with the acceptable teaching/learning approaches. In each class different classroom norms are established via the students-teachers interactions. In our class of PTs the teaching/learning approach was based on inquiry activities via CPBL. In order to enable the PTs to appreciate the benefits of this method, we supported them in developing self-awareness to their pre-existing beliefs and challenge the adequacy of those beliefs to the new setting.

THE STUDY

The learning environment. This paper presents the case study of Ruth who participated in an annual method course which focuses on theories and didactical methods implemented in teaching and learning geometry and algebra in junior high-school. 25 college students (8 male and 17 female students) in their third year of studying towards a B.A. degree in mathematics education attended the course. This course was the second method course they were participating. The CPBL approach was one of the main teaching/learning methods discussed in the course. The PTs used dynamic geometrical software in the various stages of their work on the project. During the engagement in the project the PTs were asked to write a portfolio describing their experiences and reflect on them.

In order to clarify to the PTs the principles of the CPBL approach and what we believe should be the phases of the work, we presented a ready-made project which was based on Morgan's theorem (Watanabe, Hanson & Nowosielski, 1996). This theorem is a mathematical discovery of a middle school student, which occurred while Morgan's teacher engaged his class in an inquiry assignment. The PTs had experienced CPBL approach, which included the following phases (Lavy & Shriki, 2003): (1) Solving a given geometrical problem which served as a starting point for the project; (2) Using the "what if not?" (WIN) strategy (Brown & Walters, 1990) for creating various new problem situations on the basis of the given problem; (3) Choosing one of the new problem situations and posing as many relevant questions as possible; (4) Concentrating on one of the posed questions and looking for suitable strategies in order to solve it; (5) Raising assumptions and verifying/refuting them; (6) Generalizing findings and drawing conclusions; (7) Repeating stages 3-6, up to the point in which the student decided that the project has been exhausted. The research data included: (a) transcripts of videotapes of all the class sessions; (b) two written questionnaires; (c) students' portfolios that included a detailed description of the various phases of the project and reflection on the process; (d) informal interviews. During the class sessions the PTs raised their questions and doubts, asked for their classmates' advice, and presented their work.

Methods. We focused on Ruth, one of the PTs, and used case study methods (Stake, 1995) for analyzing her system of beliefs during the various phases of the project. Ruth was chosen since she tended to be more reflective than the other students in class. As a consequence, her portfolio was rich and detailed in comparison to others. The methods for analysis included use of an analytical model for analyzing data to identify critical events. In order to clarify and elaborate some of Ruth's written reflections we interviewed her after each phase of the project.

RESULTS AND DISCUSSION

At the beginning of the research we assumed that the PTs would face some difficulties while trying to internalize the various aspects associated with the unfamiliar CPBL approach, but we could not anticipate their source and nature. Analyzing Ruth's portfolio and her interviews revealed that the source of those difficulties can be attributed to Ruth's tendency to interpret her new experiences through her pre-existing beliefs (Tilema, 1998). Namely, she failed in her attempts to challenge the compatibility of her existing beliefs with the new information gained while engaging in the project. During the process of work on the project we could identify several types of existing beliefs. These beliefs related mainly to learning and marginally to teaching and to our educational system.

Although Ruth was an average student, she had a significant contribution to the class discussions, in which various aspects concerning the project were elucidated. She often used to ask for further clarifications regarding issues raised by her other class mates and teacher. At the beginning of the process Ruth was motivated by her wish to discover new mathematical regularity, saying: *"I want to be like Morgan, I want to discover a new regularity*". At the initial phases of the project Ruth decided to focus on a problem situation in which she changed two of the problem original attributes. After a period of time, during which she kept on looking for regularities, she had managed to find only marginal discoveries. In what follows are some of her reflections during the various phases of work on the project.

By the end of the first class session in which we explained the constituents of the project and demonstrated Morgan's work, Ruth wrote in her portfolio:

> "At the beginning I asked myself whether there is any connection between what we ought to teach in school and what we have to do in this project. No one at school will ever let us teach in that manner. Schools do not welcome such an approach. So at the beginning I was not enthusiastic at all, until I heard about Morgan and his discovery. Only then I felt like I really want to do that - to explore and discover".

First glance at the above excerpt raise the question: why Ruth felt resistance towards the whole idea of the project, even before starting to work on it? It should be mentioned that this resistance was expressed in most of the study participants' portfolios. It appears that the PTs were intimidated by the new and unfamiliar learning approach. The excuse Ruth provided for justifying her reservation relied on external factor – schools will not welcome such teaching approach. According to Ruth's beliefs, schools represent the authority which determines what and how to teach. Moreover, she believes that schools are not open to new teaching ideas. These beliefs imply on how the educational system is conceived by Ruth – rigid and conformist. Ruth's perception of the educational system, stem from her past experience as school student (Lortie, 1975). Ruth's beliefs were so dominant that they instinctively 'withheld' her motivation to experience innovative approach to learning and teaching. However, the case of Morgan stimulated and motivated Ruth to reconsider her resistance and she was willing to take an active part in the new experience. The fact that a 9th grade student (Morgan) succeeded in discovering new mathematical regularity, added a competitive dimension to her system of decision-making, and she was determined to succeed ("*explore and discover*"). Ruth began working on the project with a great enthusiasm. After the second phase of the project, in which she had to use the WIN strategy for creating various new problem situations on the basis of the given problem, she wrote:

> "After I wrote the list of various new problem situations I felt good as if I was going to discover something new in mathematics – I really love it!".

Despite her initial resistance and doubts the actual engagement in the project reinforced her enthusiasm, believing she was capable of discovering new mathematical regularity. This process was accompanied with a strong emotional reaction ("*I really love it!"*).

After the 4th phase (concentrating on one of the posed questions and looking for suitable strategies in order to solve it) Ruth reflected:

> "The work was very interesting and challenging. At the beginning I felt a sense of anxiety, afraid I would choose to concentrate on an 'inappropriate' attributes, and it would be a waste of time. But shortly after I felt confident and it was clear to me that I

> will gain something meaningful from this project. I believe I will discover a new regularity".

From the above excerpt we can learn that during the work on the project Ruth was emotionally rater than rationally involved. It can be inferred from her use of words such as: love (in the previous excerpt) anxiety, afraid and confident but no references to rational expressions, that might indicate that the process was also rationally examined. While at the beginning of the process Ruth 'blocked' herself even from considering any involvement in the project justifying it by excuses that relate to school system, when she began to work on the project, she found this process to be interesting and challenging. The new feelings were accompanied by a sense of anxiety which was originated by her exiting beliefs referring to waste of time which might cause by the choosing of inappropriate attributes.

When we asked Ruth to clarify what she meant by concentrating on '*an 'inappropriate' attributes and it would be a waste of time*' she said: '*it is like I am standing in a junction from which several directions are possible. I have to decide which one is the desirable road to choose… I mean in which road I will manage to find an interesting regularity. If I will not succeed in choosing the proper way – it will be a waste of time since it will end up with no results*'. From these utterances we can see that Ruth believes that mathematical assignment must end up with a result or product. She could not realize that one can learn merely from the engagement in the process itself. These expressed beliefs reflect the socio-mathematical norms (Yackel and Cobb, 1996) Ruth internalized as a school student. During the 5th phase she wrote:

> "Sometimes during the work on the project I felt a lack of motivation. Perhaps it is because I am not used to activities of this kind. During my school years I was asked to prove existing mathematical regularities, and now we are asked to do something different, something that we are not used to – to discover something new".

Ruth's further involvement in the project raises some conflict feelings. On one hand "*I felt confident and it was clear to me that I will gain something meaningful from this project"*, and on the other "*I felt a lack of motivation"*. Ruth justifies her lack of motivation using her past experience as school student: "*During my school years I was asked to prove existing mathematical regularities"*. Proving existing and unquestionable regularities is what Ruth experienced as a student, and consequently she beliefs this is the proper way to learn and teach mathematics. In the project she had to deal with a new learning approach, in which she had to look for a new regularity and then prove it. The different approach decreased her motivation and raised negative and ambiguous feelings towards the process. One might say that Ruth's resistance rose because she did not discover something meaningful. However, it is still points to the fact that Ruth was focused on the final products rather than the process she was involved in.

In her final reflection Ruth wrote:

> "…Contrarily to what I had said before I must say that when I observe and examine what I had gone through during the work on the project, I realize that only a minor part of the sessions contributed to my professional growth. As part of my educational duties I have to teach in various classes. I don't know yet how to teach and handle class situations in the traditional way, and you expect that I will adopt and implement innovative teaching approaches which I do not see their relevance to my work".

In the above excerpt Ruth refers to two existing beliefs. First, Ruth perceives professional development as a process in which she will be 'equipped' with tools and methods that will assist her in handling class properly. Second, she raises crucial points: she wants first to experience and gain confidence in teaching in the traditional way and only then to consider innovative teaching approaches.

We asked Ruth to clarify what she meant by saying: "*I don't know yet how to teach and handle class situations in the traditional way".* According to Ruth, 'traditional' ways are:

> "When I was a student, all mathematics lessons were handled in the same routine – the teacher explained the new topic, and then provided a solved example. Afterwards, we were asked to solve several related exercises. The teacher solved some of the problems in class, and usually asked us 'to help' her. This is what I perceived as 'traditional way'".

Ruth's reply is consistent with Richardson (1996) according to which PTs believe that the teacher's role is to deliver knowledge. Ruth describes the mathematics lessons as a well known routine, with no surprises or unexpected occurrences.

To summarize, we can trace Ruth's shifting from enthusiasm to frustration during her work on the project. Ruth started the project with a rigid set of classroom beliefs regarding teaching, learning and school functioning: school has its rules regarding 'proper' teaching methods, and inquiry-based learning is not part of them. The case of 'Morgan's theorem' made Ruth to temporarily distract from her existing classroom norms and to open her mind to the thought that if a young student had the ability to discover a new formula, so can she. When Ruth felt that she was about to uncover a new mathematical regularity, she was willing to consider a new perspectives regarding learning according to which: students are able to discover new mathematical regularities and not just prove existing formulae; teachers are not the only source of knowledge. However, as a result of unfulfilled self-expectations, Ruth's enthusiasm began to fade. When Ruth faced a situation in which she did not mange to discover any meaningful regularity she started to feel that she was not accomplishing her aim. This situation resulted in a retreat to her pre-existing beliefs (Foss & Kleinsasser, 1996), and enabled her to justify her failure. In fact, she did not take responsibility for her lack of success. Instead of searching for new directions of inquiry, or analyze the process in a rational manner, Ruth withdrew and relied on her existing system of beliefs as an 'alibi' for her lack of success. In fact, she 'justified' her failure by the fact that she was not familiar with this kind of learning, and by the fact that it was not the way she believes school students should learn. During Ruth's engagement in the project it can be observed that she tried to 'confront' her pre-existing beliefs concerning teaching, learning and the educational system with the new reality to which she was exposed. She emphasized the final product rather than the process itself and the challenging of her pre-existing beliefs was emotional rather than rational.

CONCLUDING REMARKS

Teacher education programs have a slight influence on PTs' perceptions regarding teaching and learning (Kagan, 1992, Tilema, 1998). It appears that one of the reasons for that phenomenon is the fact that PTs' systems of beliefs do not require external approval (Lamm, 2000). In order to be able to change, PTs have first to acknowledge the need to change, namely - to be convinced that alternative teaching methods have the potential to stimulate better learning and understanding of mathematics. In addition, they have to overcome the instinctive human tendency to resist changes. For that matter they have to challenge the adequacy of their pre-existing beliefs to the new information they are facing. This internal process of examination is social, cultural, environmental and personal dependent.

Ruth's beliefs reflect the educational system she was educated in over the years, as well as the common excepted norms of our society. Lacking teaching experience, most of her expressed beliefs concern the learning of mathematics and only marginally the teaching of mathematics and the educational system. The prominent beliefs of Ruth's reflect several characteristics of our society:

(i) Since the educational system is a conservative organization, educational changes are hard to accomplish.

(ii) We live in a competitive society, therefore integrating a competitive dimension to the process of learning might increase its attractiveness.

(iii) Our society puts an emphasis on the final product rather than the process that goes along with its accomplishment. This is consistent with the former (*ii*), since in competitive society achievements are measured and assessed merely by the quality of the final product.

(iv) The final product should be achieved rapidly. Failing to get the desirable product implies on the failure of the entire process.

(v) Coping with challenges is difficult. Hence, there should be social awareness to the importance of setting challenges and learning to cope with them.

(vi) There is a tendency to avoid taking personal responsibility. Instead, failure is often attributed to external factors.

Considering the above, as teacher educators we may ask ourselves whether it is possible to bring any change into the educational system. Since social norms are very dominant and serve as inhibitors to the accomplishment of this change, PTs have difficulties in internalizing the need to change the teaching approaches. It appears that we have to find ways and means for overcoming this barrier, and break the cycle. We believe that continuous engagement in profound examination of self-beliefs regarding teaching and learning might increase the plausibility of challenging the PTs' beliefs. A further research is needed in order to examine the nature of their resistance to implement innovative teaching/learning approaches and its relation to the accepted norms of the society in which the PTs are practicing. Identifying these relations might pave the road to the desirable change.

In order to educate a new generation of teachers who will appreciate innovative teaching approaches, we have to create learning environments which will encourage PTs' openness and provide them with the required support needed for recognizing the advantages of such approaches.

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RELEVANCE AND ACCESS IN UNDERGRADUATE MATHEMATICS: USING DISCOURSE ANALYSIS TO STUDY MATHEMATICS TEXTS

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Despite considerable efforts to promote access to Mathematics and to improve the mathematical performance of all students in South Africa, the reality is that fourteen years after our first democratic elections the experiences of students remain inequitable. In this paper I report on the use of discourse analysis to study the text of a mathematics problem which is used in a first-year university Mathematics course designed to give students access to tertiary study in Science. I use the method and tools of Gee (2005) to identify and explain (a) how the text presents the activity of answering a mathematics problem, and (b) how the text may position the student. I argue that this analysis raises questions about the concepts of relevance and access in undergraduate mathematics.

INTRODUCTION

Mathematics education reform in South Africa

Reform in mathematics education in South Africa since 1994 has been influenced both by reform initiatives elsewhere in the world and by the particular needs of the developing country. For example, the social justice agenda has focused on the urgent need to redress the inequities of the past, while an economic agenda has promoted the need to accelerate economic development. Reform has been characterised by calls to promote access to the subject of Mathematics and to increase the relevance of the subject for students. One way in which the term "relevance" has been contextualised in mathematics classrooms is through the use of mathematics problems with real-world contexts, which I refer to as "real-world problems" in this paper. Tertiary institutions have responded to the challenge of providing "access" by establishing foundation programmes and extended curricula, designed to provide access for students who have been educationally disadvantaged in the school system and who are identified as having the potential to pursue further studies.

Furthermore, mathematics education reform in South Africa has taken place within the setting of often rapid changes in the education system as a whole, and in wider society in general. This is a complex setting; in the past educational experience, language, class, race and poverty were often conflated, yet ongoing change, for example in the schooling system, means that these relationships are not as clear as in the past (see for example Bangeni & Kapp, 2007).

Despite the varied attempts to improve mathematics education in South Africa since 1994, empirical data shows that the experiences of students currently completing school remain different and inequitable (South African Human Rights Commission,

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 340 - 351).

2006). Furthermore, while students may gain access to study at tertiary institutions, the success rates at these institutions, particularly in Science and Engineering, remain poor (Scott, Yeld & Hendry, 2005).

From a theoretical perspective there is a wealth of work, particularly on school mathematics, that problematises attempts to make mathematics relevant (see for example Ensor, 1997). Of particular interest in this paper is recent work arguing that the use of real-world problems may prevent access to both the real-world and to the study of mathematics (see for example Dowling, 1996). There are also suggestions that certain students may be marginalised from school word problems (Cooper & Dunne, 2000; Tobias, 2006). Furthermore, certain theorists have problematised the notion of access itself. From the perspective of learning mathematics, Baker (2005) argues that formal education actually conflicts with an access agenda; while pedagogic practices may be aimed at increasing access, there is a wealth of literature suggesting that they in fact privilege certain students and reproduce social difference. This is confirmed by the research of Lerman and Zevenbergen (2004) and Cotton and Hardy (2004).

This paper

In providing the setting for this paper I have described the dynamic and challenging landscape in which mathematics education is both practised and researched in South Africa. The work presented in this paper is one attempt to investigate and explain some of the challenges. This work is located in a first-year university Mathematics course at a South African university (which I will refer to as the "Course" from this point). This Course forms part of an extended curriculum programme which is specifically designed to provide students disadvantaged by the schooling system with access to tertiary studies in Science. The majority of the students taking this Course are Black and Coloured students. The Course is taught in English, which is an additional language for more than half of the students (Visser, 2006). The Course material contains a number of real-world problems, where by "real-world" I mean "everything that has to do with nature, society or culture, including everyday life as well as school and university subjects or scientific or scholarly disciplines different from Mathematics" (The International Commission for Mathematics Instruction, 2002, p.230). I am involved in this Course in various ways; as lecturer, course convener, student advisor, and more recently as researcher.

In this paper I present the analysis of three related texts which form part of the Course material used in this access Course. I use Gee's (2005) method for discourse analysis as well as his concepts of the seven building tasks, intertextuality, situated meaning, Discourse and social language to link the features of the texts to wider social practices. These concepts are used to identify the enacted activities and identities in the texts, and to examine how the texts may position the student. This work forms part of a larger study in which I am investigating the nature of a selection of real-world problems from the Course, and examining how the practices used by students to solve these problems may be enabling or constraining.

CONCEPTUAL FRAMEWORK

Language and Mathematics as Social Practices

In this study I have adopted a perspective that language-use is a social practice (Gee, 2005). From this perspective, language is not value-free and simply a grammar and set of rules for how to use this grammar. Rather, language is linked to the context in which it is used, and language forms take on meaning in particular contexts. Consistent with this perspective on language is the view of Mathematics as social practice. Mathematics is not viewed as skills-based and divorced from contexts, but is learned and used in social contexts (Baker, 1996).

Discourse and text

I use a broad conception of the notion of discourse, as suggested by Gee (2005) who uses the term "Discourse" to include both language and non-language forms of discourse. He argues that in a social setting we use language, behaviours, actions, tools, etc. to recognise ourselves and others as belonging to a particular group or set of practices, or Discourse. At the same time we give meaning to that Discourse by reproducing or transforming it. Discourses are not mutually exclusive and fixed, but can overlap, can be contested and can change over time.

From this perspective a written text may be part of one or more Discourses, and hence may be overlapping, dynamic, and contested. Furthermore, a text has a history. In the context of education, Apple (1996) argues that, as an artefact of curriculum, a text cannot be neutral. Herbel-Eisenmann and Wagner (2007) argue that in constructing a text, a writer makes conscious and unconscious choices, and that textbooks as examples of text have agency with respect to how they can structure relationships. It should be noted here that my interest in this paper is in the text as a social artefact. I regard the detailed analysis of the text as one possible way to study how users are positioned by the text, and I am interested in the possibility for a text to reproduce or indeed produce social practice. In this paper I do not deal with how the text is used in practice, but this forms part of my wider study.

METHODOLOGY

Given the assumption that text as an aspect of Discourse is a social practice, how does one relate features of written text to the wider social practices? Discourse analytic frameworks have been used to study how texts construct roles for and position users of mathematics texts (Herbel-Eisenmann & Wagner, 2007; Herbel-Eisenmann, 2007) Bennie (in press) has used Fairclough's three-dimensional framework as a model for linking the written text of a mathematics problem to wider discursive and social practices.

In this paper I present the analysis of text using Gee's (2005) method for discourse analysis and a selection of his analytic tools. The initial analysis of the features of the text is done in two ways, as suggested by Gee (2005, p.54-58). "Form-function" analysis allows me to focus on the meanings communicated by particular textual features, for example, the layout, repetition, naming, etc. "Language-context" analysis enables me to study the specific meanings that different language forms take on in a particular context. Gee (2005, p.59) notes that in a specific context, a language form will take on a particular meaning, called a "situated meaning". Certain features associated with a language form will be grouped together in a pattern, a pattern that a specific group of people find significant.

Secondly, Gee's (2005) concept of the "seven building tasks" provides me with a systematic way to investigate how the textual features give meaning to the text. He argues that when we use language we build a "reality" by building seven things, which I give in italics in the following description. Gee claims that a situation in which language is used will involve *activities* in which people take on certain *identities*, develop *relationships* with one another and use certain *sign systems and forms of knowledge*. In such a situation certain things are given *status* and people and things take on meaning and *significance* and are *connected* or not connected to one another. I make use of Gee's (2005) list of 26 questions about the seven building tasks; he argues that the extent of the convergence of the answers to these 26 questions can be used as one measure of the validity of the analysis.

Thirdly, how does one explain this "reality" in the context of wider social practice? Gee (2005) claims that different people will have differential access to identities and activities, and different value and status will be assigned to these identities and activities. Gee provides a number of tools that can be used to study how this may be done through language, two of which I have already described, namely "Discourse" and "situated meaning". In this analysis I also use the concept of "intertextuality"; Gee (2005) argues that when we use language our words may reference other texts, either directly by quoting or indirectly by alluding to them. Lastly, the term "social language" is used by Gee (2005) to refer to the language aspects of a Discourse.

THE TEXTS

The focus of the analysis is on the mathematical problem in Figure 2 (which I will refer to as the "car problem" from this point). Students on the Course are required to solve this problem during an afternoon workshop session in which they work in self-selected groups of four to five students. When analysing the car problem it was decided to include two other related texts in the analysis; the textbox located immediately prior to the car problem in the Course material (see Figure 1), and the worked solutions for the car problem (see Figure 3). These worked solutions are provided to students a few days after tackling the problems in the workshop session. The three texts analysed in this paper (Figures 1 to 3) are given below in the order in which students encounter them.

The following questions are related rates problems. These MUST be set up correctly. Follow these steps for **EVERY** question:

1. Draw a diagram and define variables.
2. Write down what is given, using the correct notation.
3. Write down what is to be found.
4. Write down a formula linking the variables.
5. Differentiate and complete the question.

Figure 1. The textbox (Workshop 15, 2007 Resource Book)

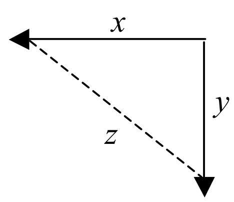
2. Two cars start moving from the same point. One travels south at 100km/h and the other travels west at 75km/h. At what rate is the distance between the cars increasing two hours later? (Let the distance between the cars after a time *t* be *z* km).

Figure 2. The car problem (Workshop 15, 2007 Resource Book)

Let x = distance covered by car A

Let y = distance covered by car B

Let z = distance between car A and car B



Given: $\frac{dx}{dt} = 75$ and $\frac{dy}{dt} = 100$

To Find: $\frac{dz}{dt}$ when $t = 2$ hours

$x^2 + y^2 = z^2$ (Pyth)

$\therefore 2x.\frac{dx}{dt} + 2y.\frac{dy}{dt} = 2z.\frac{dz}{dt}$

When $t = 2$ hours, $x = 150$ km and $y = 200$ km and $z = \sqrt{150^2 + 200^2} = 250$ km

$\therefore 150 \times 75 + 200 \times 100 = 250.\frac{dz}{dt}$

So $\frac{dz}{dt} = \frac{1}{250}(150 \times 75 + 200 \times 100) = 125$ km/h

Figure 3. The worked solutions for the car problem (Solutions to Workshop 15, 2007)

FINDINGS

In this section I identify and describe the activity enacted in the three texts. I then explore the enacted identities in the texts and how the activity may position the student. I link the texts, via the concept of intertextuality to other texts, and via situated meaning, social language and Discourse to wider social practices. I present the textual evidence for my claims (although space prevents me from providing all the detail).

In relating the three texts to wider social practices I have named certain Discourses, for example, the "Access Discourse" and the "Everyday Discourse". This process of naming and classifying has the effect of fixing these Discourses in time and space and of setting up boundaries. Yet, by definition, Discourses are overlapping, changing, and may vary across communities. This dilemma is noted by Moschkovich (2007) in her discussion of the naming of what she calls "Discourse practices". Furthermore, my classification is influenced by my understanding of the setting of the study, and unavoidably reflects some value judgements about this setting. Acknowledging these difficulties, I have thus attempted to make explicit the criteria I have used in naming the Discourses used in the analysis.

I use the term "First-year Undergraduate Mathematics Discourse" to describe the mathematics currently studied by first year undergraduate students in mainstream mathematics courses, for example, at the institution at which this study is being conducted. This Discourse can be linked to a wider "Undergraduate Mathematics Discourse", which describes the practices we would like students to participate in during their undergraduate study in Mathematics. I use the term "Calculus Reform Discourse" to describe a particular approach to teaching undergraduate calculus. This approach emphasises understanding of concepts, flexibility in moving between different representations, and the solving of problems with everyday and disciplinary contexts. The "Access Discourse" is the term I use to describe the actions, beliefs, etc. associated with attempts to provide students with access to tertiary study. My naming of the "School Mathematical Word Problem Discourse" is based on the work of Gerofsky (1996) who classifies school word problems as a particular genre. My use of the term "Everyday Discourse" is based on the work of Moschkovich (2007, p.27) who gives the name "everyday" to the practices that adults and children engage in out of school (or out of university for this study) and out of professional mathematics.

The enacted activity

I argue that the main activity enacted in the three texts is the solving of "related rates problems". This activity is announced in the form of a statement in the first sentence of the textbox in Figure 1, and is given value by its framing in the textbox. The text of the car problem can thus be linked via intertextuality to a particular set of problems in the First-year Undergraduate Mathematics Discourse. The term "related rates problems" takes on a particular situated meaning in this Discourse and refers to group of problems that share certain characteristics, for example, they are structured in a certain way, they deal with the concept of rate of change, and they can be solved using certain procedures.

The sub-activity of solving a related rates problem can be identified in sentence 2 of the textbox; it involves "setting up" the problem "correctly" by following five steps. This sub-activity is given value as it is communicated as an instruction to the student in sentences 2 and 3, and through the use of upper case and bold letters for emphasis, for example "MUST" and "**EVERY**". These instructions can be linked via intertextuality to the text of undergraduate calculus textbooks (texts that form part of the First-year Undergraduate Mathematics Discourse), where these steps for solving related rates problems are commonly presented. The word "correctly" in sentence 2 of the textbox takes on a situated meaning in this Discourse and is given meaning by the five steps provided in the textbox.

The five steps in the textbox are in the form of instructions on different actions to be performed in solving the car problem. Certain words and phrases in these steps take on situated meanings, and are given meaning implicitly by their link to "related rates problems" and more explicitly by their link to the text of the worked solutions to the car problem (Figure 3). For example, the "diagram" required in step 1 is a **mathematical** diagram, and the "correct notation" in step 2 means using the appropriate mathematical notation for rates of change. Furthermore, certain phrases and words used in the textbox, for example, "variable" (steps 1 and 4), "formula" (step 4) and "differentiate" (step 5) are words that take on certain meanings in an Everyday Discourse, but also form part of the social language of the First-year Undergraduate Mathematics Discourse.

The car problem may appear at first to be unstructured, in the sense that the student is required to answer one question (posed in sentence 3), the solving of which requires the student to perform a number of unspecified interim steps. Yet the presence of the textbox prior to the car problem provides the student with structure for solving the problem. Instructions on problem-solving steps for solving related rates problems such as those given in the textbox are a common feature of undergraduate calculus textbooks. Yet there are certain features of these particular texts that set them apart from those traditionally found in these textbooks. Firstly, sentence 4 of the car problem contains a hint to the student about how to assign variables. Secondly, the textbox contains repeated reminders to the student about how to proceed, for example, with the use of the words "MUST", "**EVERY**", and "<u>Write down</u>". I argue that these features link the texts to the Access Discourse, as students are provided with additional reminders and support.

What does the text indicate about the nature of the activity identified as solving "related rates problems"? Firstly, the classification of the group of problems as "related-rates problems" suggests that certain problems in this Discourse can be grouped together according to their characteristics. Secondly, the insistence that the student follow the given five steps when solving the "related rates problems" suggests (a) that a systematic problem-solving approach is valued, and (b) that all the problems can be and should be solved in the same way. Thirdly, the five steps in the textbox, together with the worked solutions for the car problem, make it possible to identify that certain types of activity are valued, for example, presenting a full, neat, and systematic written solution, converting flexibly between different mathematical representations, carrying out certain mathematical procedures like differentiating, and having some conceptual understanding of the notion of instantaneous rate of change. The emphasis on both conceptual understanding and procedural proficiency can be linked to what is valued in the Calculus Reform Discourse.

The context for the car problem is the motion of cars, suggesting a link to the Everyday Discourse. However, as argued so far, the focus of the question is on solving a "related rates problem" within the First-year Undergraduate Mathematics Discourse. So the car problem should be read in a particular way, as only certain aspects of the Everyday Discourse are valued. A number of features of the text reinforce this argument. Firstly, there is a resistance to naming the cars and the starting point of the motion, suggesting that some of the detail about the real-world context is not important. Secondly, it is highly unlikely in an everyday setting that two cars would drive at a constant speed and at right-angles to one another. These features of the text of the car problem link this problem to a group of texts that belong to the School Mathematical Word Problem Discourse. In her discussion of the school mathematical word problem genre (which in Gee's terms would be the social language of a wider Discourse), Gerofsky (1996, p.40) argues that these word problems only "pretend *that* a particular story situation exists" and that the story in such a problem has no truth value.

This location of the car problem text in the School Mathematical Word Problem Discourse is further reinforced by a study of the use of tenses in the text. There is a lack of consistency in the tenses of the car problem, which Gerofsky (1996) argues is a common feature of the genre. Furthermore, the composition of the problem is also typical of this genre; the first two sentences provide the "set-up" for the story and the "information" needed for solving the problem, and this is followed by a question in sentence 3 (Gerofsky, 1996, p.37). Gerofsky claims that in some cases the "set-up" is not essential to the problem solving. I argue that in the car problem, the cars could quite easily have been replaced by runners, cyclists, or walkers, with the problem remaining a "related rates problem".

The enacted identities

I begin by discussing how the "successful student" may be positioned, that is, the student who solves the car problem as required by the textbox and the worked solutions. Firstly, the successful student is positioned as a student who has access to the First-year Undergraduate Mathematics Discourse. Such a student will have access to the situated meanings in the text, for example, knowing the pattern associated with “related rates problems” and hence being able to recognise these problems in the Course material. S/he will also have access to the situated meaning of terms such as “correctly”, “diagram”, “correct notation”. The use of terms with particular meanings in the social language of the First-year Undergraduate Mathematics Discourse, for example, “variables” and “differentiate” positions the student as someone who understands and can use this social language.

Secondly, in order to arrive at a correct answer the successful student is required to demonstrate both conceptual understanding and proficiency in mathematical procedures such as differentiation, which can be related to the Calculus Reform Discourse. Thirdly, the successful student is required to deal with the real-world context of the cars appropriately, that is, s/he must choose only those aspects of the Everyday Discourse that are appropriate for a problem that is located in the First-year Undergraduate Mathematics Discourse. The successful student thus needs access to the assumptions of the School Mathematical Word Problem Discourse.

Continuing on the theme of the successful student, one could possibly argue that the presence of the problem-solving steps in the textbox construct this student as a problem-solver in mathematics, and as a student who can solve a real-world problem by mathematising it and using appropriate mathematical tools. However, I argue that certain features of the text may construct an identity for the student that conflicts with the identity of the successful student described so far. Firstly, the student is instructed to solve the “related rates problem” in a certain way, when the method presented is not the only possible method for solving the car problem. Secondly, the student is repeatedly reminded (with textual features such as upper case letters, bold and underlining) to follow these steps. Thirdly the student is presented with a hint for starting out with the problem (sentence 4 of the car problem). I argue that these textual features construct the student (a) as someone who needs help solving the car problem, and (b) as a student who does not usually do what is required when instructed to follow given problem-solving steps. The student is thus positioned as an “access student”.

CONCLUSION

This analysis is restricted to the car problem and the two associated texts. Yet a detailed discourse analysis of these three texts raises a number of questions related to the possible positioning of the student and the concepts of access and relevance. These questions need to be considered in the wider study in which I am investigating the practices used by students when solving the car problem as well as other real-world problems.

Regarding the concept of access, I begin by asking two questions: “Does classifying problems in a group as ‘related-rates problems’ and suggesting that they can all be solved in the same way promote access to the Undergraduate Mathematics Discourse?” and “Does providing the student with problem-solving steps and a hint for getting started on the problem promote access to the Undergraduate Mathematics

Discourse?" Secondly I ask, "Who has access to the social language of the First-year Undergraduate Mathematics Discourse?" Research evidence suggests that students learning in English as an additional language may be at a disadvantage as they face the dual challenge of mastering the language of instruction as well as the social language of Mathematics (Setati, 2005; Barton et al. 2005).

Thirdly, I ask, "Who has access to the assumptions of the School Mathematical Word Problem Discourse?" I have argued that while the problem may draw on aspects of the Everyday Discourse, only certain aspects of this Discourse are regarded as being important, and these are the aspects required to solve a "related rates problem" located in the First-year Undergraduate Mathematics Discourse. The term "relevance" thus takes on a particular meaning, that is, as relevance in this particular Discourse. I ask, therefore, "Who has access to what is relevant in this Discourse?" This question needs to be asked, given claims by Cooper and Dunne (2000) that successful performance on school word problems can be linked to social class. Given the varied nature of schooling in South Africa, as described in the introduction to this paper, it is possible that those students entering tertiary institutions will have had very different experiences of school mathematical word problems.

Lastly, I ask, "Do the successive reminders to the student and the positioning of the student as an access student promote access to the Undergraduate Mathematics Discourse?" Assuming that students have agency with respect to the identities that they choose to inhabit, it is possible that some students may make a conscious choice not to follow the repeated instructions in the car problem.

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ARE ALL STUDIES ON EQUITY IN SCHOOL MATHEMATICS EQUAL?

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Researching what might lead to more equitable outcomes, across diverse social groups, especially since disadvantaged groups consistently achieve less than children from more advantaged backgrounds in many countries, has become quite common in recent years. In this paper we carry out a literature review of a selection of that body of research. We construct a model for analysis and place the selection on the model. We discuss our findings and conjecture what is important for future research in the domain.

INTRODUCTION

During the participation of the first author as a co-investigator on a research project funded by the Australian Research Council (Principal Investigators: R. Zevenbergen, P. Renshaw & S. Lerman) and as critical friend on a second such grant (Principal Investigators: P. Sullivan, J. Mousley & R. Zevenbergen) it became clear that there are very different orientations to research on achieving equity in mathematics teaching and learning. Looking beyond these two projects, we became interested in the following questions: how are researchers in the mathematics education community researching equity; what theoretical frameworks are being drawn upon and/or developed; what approaches to the issue are being adopted; and what do we know, at this stage? It is the intention of this paper to review and classify the research literature in the expectation that some critical insights into these questions will emerge from the analysis to inform future research and to enable a conversation between these different orientations.

Research on teaching and learning mathematics has traditionally drawn largely on either psychology or on mathematics itself as intellectual resources for framing that work (Kilpatrick, 1992). Student 'failure' to produce adequate levels of performance can then be ascribed to inadequacies of the pupil her or himself. Oversimplifying perhaps, one could say that these views see either the failing student's development as a deviation from a norm determined by psychological studies or that mathematical thinking is inherently difficult and accessible only to those with a propensity towards the subject, an assumption of innate mathematical ability. The leading research group in the field, formed in 1976, the International Group for the Psychology of Mathematics Education (PME) saw it as natural to locate its research within psychology, as the only field that seemed to offer the intellectual resources to investigate issues of learning. In 2005 the constitution was changed, although not the name, to recognise other theoretical resources as of equal value for research. One major argument put forward in support of the change was the recognition that issues of equity cannot be adequately explained within mainstream psychology. The

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 352 - 361).

founding of the Mathematics Education and Society group was clearly a response to the domination of psychological paradigms in mainstream mathematics education.

With a growing recognition of the significance of the role of the teacher, along with the emergence of competence modes (Bernstein, 1990) in education in general, came a focus on forms of pedagogy, on the effects of stratification of pupils into classes based on levels of achievement, and on the recontextualisation of mathematics into the school mathematics curriculum as at least mediators, if not determinants, of students' experiences and hence of who fails and why (Lerman & Tsatsaroni, 1998). Sociological and socio-cultural theories have now become key intellectual resources in studies of pedagogy and classroom interactions and, in particular, analyses of success and failure in school mathematics (e.g. Dowling, 1998; Lerman & Tsatsaroni, 1998; Cooper & Dunne, 2000; Lerman, 2000; Tsatsaroni, Lerman & Xu, 2003; Watson & Mason, 2004).

As research on equity in mathematical achievement has developed and spread geographically, so too have the approaches researchers have taken. In analysing why some students succeed and others fail, and indeed why there is such a correlation between low socio-economic status and low mathematical achievement, researchers have focused on different aspects of the pedagogic relationship. On the one hand, many are convinced that the way mathematics *per se* is taught and the kinds of mathematical activities that are set up for students are the key to improving equity, whilst others see the social organisation of learning to be the key. At the same time, the research methods that researchers bring to their studies vary, from those that are focused on what works, often in a design and improve model, to those who find direction in particular theoretical orientations, either in the methodological design of their study or in the development of tools for analysis, most particularly from sociology.

In this paper we present a model of those approaches with a view to mapping the field and enabling a conversation between different perspectives. Clearly equity is of serious concern to mathematics teachers and researchers, given the kudos that comes with a certificate in mathematics. The challenges to assumptions about mathematical ability have been many and varied, from projects aimed at specific groups disadvantaged by the education system (e.g. Bob Moses' Algebra Project) to critiques of the hegemony of academic mathematics and mathematicians (D'Ambrosio's ethnomathematics programme) and to post-modern analyses (Walshaw, 2004; Nolan & De Freitas, forthcoming). We suggest it is timely to examine how researchers approach their studies and identify some key questions.

CONSTRUCTING A MODEL

Based on the first author's engagement with the two research projects in Australia an initial model was formed as a working hypothesis by constructing two perpendicular coordinate axes (Figure 1) drawing on the broad characterisation of the approaches researchers seem to have taken to the study of equitable achievement in school mathematics set out above. A vertical axis of ‘Focus of Study’ formed one element in the model, with the two extremes being ‘Mathematics *per se*’ and ‘Mathematics in its social context’. ‘Methodology’ formed the horizontal axis of the model with the positions of theory-informed and pragmatic as the two extremes. At this stage it was by no means clear whether the axes, with an indication that distance along them might be of significance, would prove to be the most appropriate structure. The next stage was to carry out a literature search, with suitable limits set, and develop a structure for analysis enabling us to place the studies on the model, whilst at the same time allowing the model to change and develop as criteria for making decisions about the location of those studies were framed.

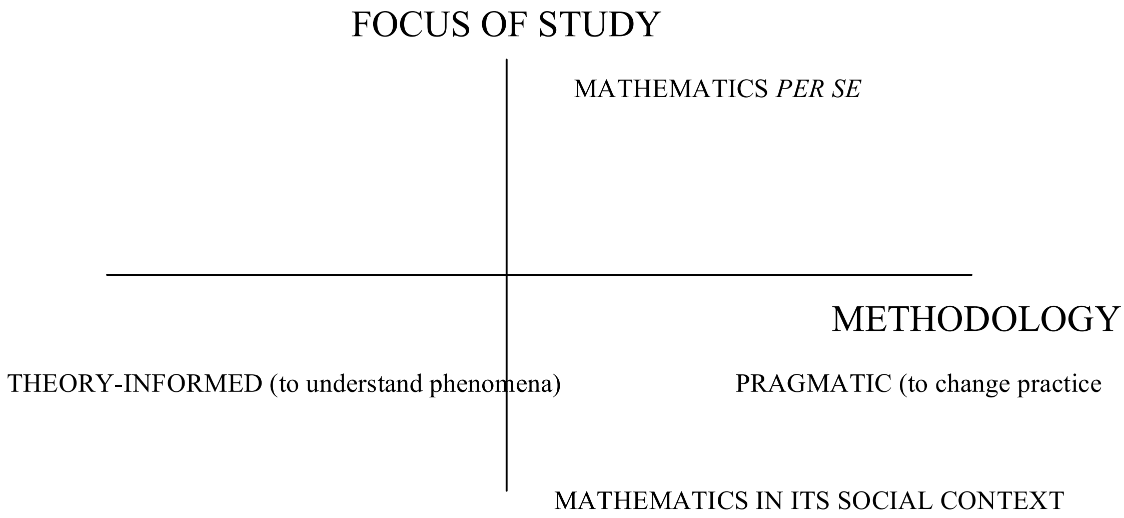

Figure 1. Initial model of research on equity

At this first stage of analysis we decided to restrict our study to those research projects that either initiated an intervention aimed at improving the equity of achievement in school mathematics or projects carried out to observe particular innovations or current situations in school mathematics and analyse them in equity terms, perhaps amongst other aspects. It may be appropriate at a later stage to look also at these theoretical studies on equity. We decided to work with articles rather than books and, since it is usual to find a number of articles from any one study, we restricted our choice to one on each study. We selected 11 articles (from a total of 27) that represented those studies and they form the empirical field for our analysis. We certainly do not claim to have exhausted the field. They are listed in Table 1, with full references at the end of the paper.

1	Boaler, J. (2002). Paying the price for “sugar and spice”: Shifting the analytical lens in equity research.
2	Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity.
3	Lubienski, S. (2002). Research, reform, and equity in U.S. mathematics education.

4	White, D. Y. (2003). Promoting productive mathematical classroom discourse with diverse students.
5	Kitchen, R. (2003). Getting real about mathematics education reform in high-poverty communities.
6	Sztajn, P. (2003). Adapting reform ideas in different mathematics classrooms: Beliefs beyond mathematics.
7	Zevenbergen, R. (2005). The construction of a mathematical *habitus*: implications of ability grouping in the middle years
8	Zevenbergen, R. & Lerman, S. (2006) Using ICTs to support numeracy learning across diverse settings.
9	Forgasz, H. (2006). Teachers, equity, and computers for secondary mathematics.
10	Sullivan, P., Mousley, J., & Zevenbergen, R. (2006). Teacher actions to maximize mathematics learning opportunities in heterogeneous classrooms.
11	Clements, D., H., & Sarama, J. (2007). Effects of a preschool mathematics curriculum: Summative research on the Building Blocks Project.

Table 1. Studies on Equity

Empirical studies (Intervention and Observational Studies)

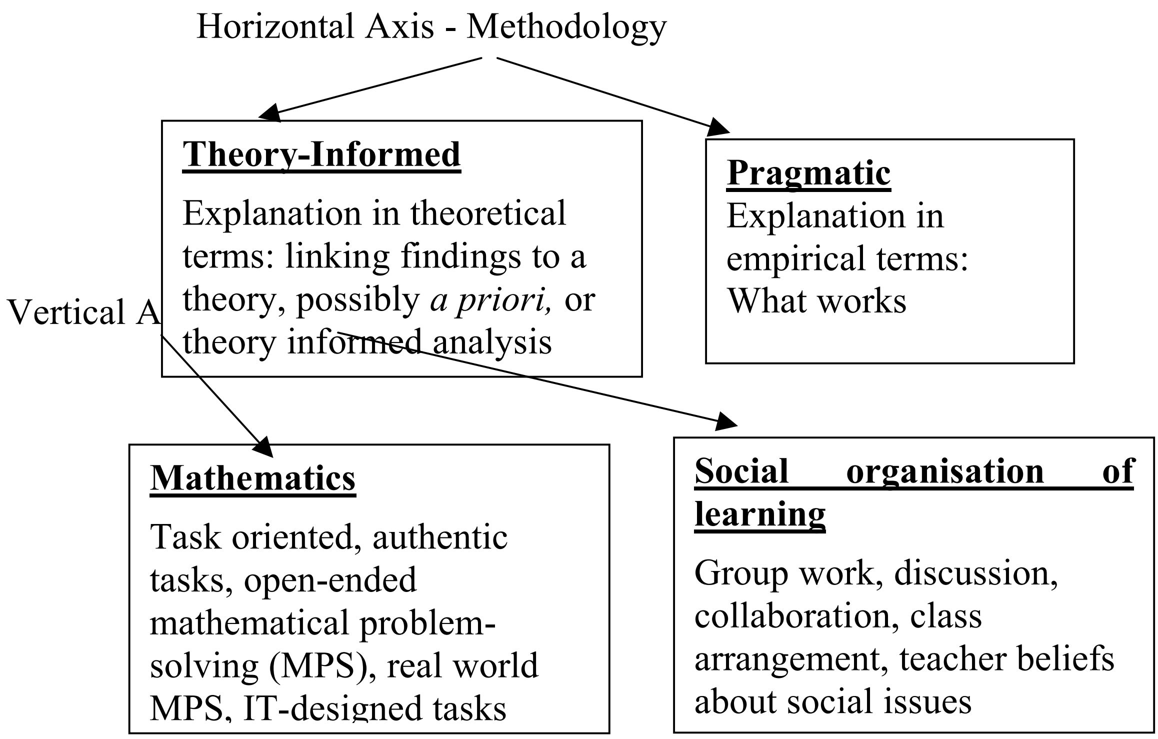

Figure 2. Decision tree

The above diagram (Figure 2) presents the final version of the decision tree that we used to classify the studies in terms of the two axes. In our view, as researchers, the criteria we used for decisions (recognition and realisation rules) should be made as explicit as possible within the constraints of a paper, in order both for our peers to evaluate and critique those decisions and to facilitate the extension of studies such as these to other intellectual domains

We now give two examples of the analysis and hence the decisions we made regarding the selected papers. The first (Sullivan et al., 2006, number 10 in Table 1) is an intervention and the second (Zevenbergen, 2005, number 7 in Table 1) is a study of an existing situation, what we have called an observation. We offer selective quotes from the articles and the conclusions we have drawn.

(i) Sullivan, P., Mousley, J., & Zevenbergen, R. (2006). Teacher actions to maximize mathematics learning opportunities in heterogeneous classrooms

- Lesson focused
- "We examine three teacher actions that address the mathematical goals: using open-ended tasks, preparing prompts to support students experiencing difficulty, and posing extension tasks to students who finish the set tasks quickly; as well as actions that address the socio-mathematical goals by making classroom processes explicit" (p. 117)
- The article contains a deep description of an observed lesson planned by researchers and teacher. "…the intent was to stimulate engagement in the mathematical ideas inherent in the tasks, including creating opportunities for visualisation and for identifying patterns, and recognising the possibility of transfer or extension to other spatial or design situations." (p. 128)
- "In the lesson the students undertook tasks that were substantial and that involved engagement with significant mathematical ideas. The students worked actively, creatively, and individually in drawing the shapes or buildings using isometric representations, rather than listening to explanations by the teachers. The engagement seemed directly due to the open-ended nature of the task, and the opportunities for the students to respond creatively. It also seems to us that the prompts that the teacher used scaffolded students so that they could engage with the task and the mathematics." (p. 138)
- Focus on the mathematics, together with an awareness of social factors
- Pragmatic explanation of outcomes

(ii) Zevenbergen, R. (2005). The construction of a mathematical *habitus*: implications of ability grouping in the middle years

- "Using the theoretical tools offered through the writings of the French sociologist, Pierre Bourdieu, I propose, drawing on the notions of field and *habitus*, that the practice of ability grouping helps to reproduce the status quo, and can be detrimental to goals of social justice." (p. 608) Theoretically driven.

• 96 students of years 9 and 10, high through to low achieving students in secondary schools from six Australian schools were interviewed

• "Through the practices within the field, ability-grouping constructs a *habitus* that either includes or excludes students from the subject. The higherstreamed students made it clear that they have a greater sense of belonging to their classrooms than their peers in the lower streams. They are more positive about mathematics, and have a greater sense of being able to achieve in the subject. Their peers in the lower streams recognize the more restrictive experiences of schools mathematics in terms of pedagogy, assessment, and classroom ethos, and this comes to constitute a predisposition to think in particular ways in relation to school mathematics." (p. 617)

• Focus on ability grouping, i.e. social organization of the classroom

Finally, we present the location of the articles selected onto the model, in Figure 3.

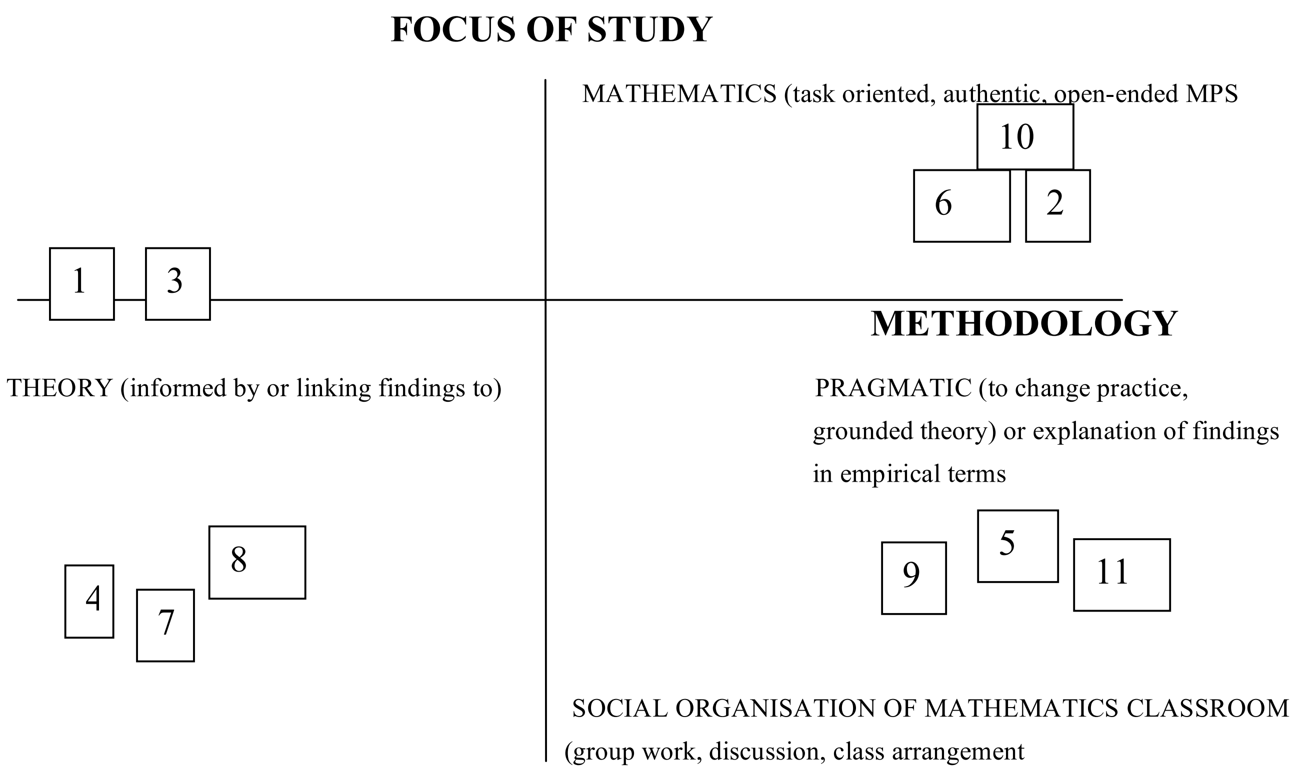

Figure 3. Locating the studies on the model

DISCUSSION

We first make a few comments on the location of the articles on the model. We subsequently conjecture what the features of the particular arrangement might suggest, within the constraints of the sample size and the selection of research studies.

We find that we have placed articles in each of the quadrants and that they are evenly spread, with a small preference for what we have termed a pragmatic approach to the analysis and the same slight tendency towards a focus on the social organisation of the mathematics classroom. But these differences are too small, especially when taking into account the sample size, to be significant.

We found relevance in where the articles were placed in the quadrants in relation to the axes with the two articles numbered 1 and 3 that were to be placed in the quadrant 'Theory/Mathematics *per se*'. All the articles except these two were relatively strongly categorised as representing the approach of both axes. Our reasoning was different in each of these cases. Article 1 was rich in empirical data and the analysis drew on a number of theoretical resources, including sociology (both Bernstein and Bourdieu) and communities of practice theory. The author argued both for the kinds of mathematical activities that were used by the teachers and for the forms of social organisation of the classroom as being factors that distinguished the two settings being examined. We therefore placed it in the theory section but neutrally between mathematics *per se* and social organisation of the mathematics classroom. Article 3 examined national data on performance in mathematics in the USA to identify possible links between enquiry-based mathematics and disadvantaged groups. The focus here was on the curriculum and associated forms of pedagogy and not on either mathematics *per se* nor the social organisation of the mathematics classroom. At the same time the analysis drew on Bernstein's theories and hence was clearly in the theoretical domain.

We have not found it possible, nor useful, to rank the articles along the axes in any other instances.

In relation to the horizontal axis it is not surprising to find a range of research methods, and indeed philosophies of research methods, in education and its sub-fields; in our case in mathematics education. In a previous study (Tsatsaroni et al., 2003) we examined the state of the research community of mathematics education through an analysis of research publications over a 12 year period in the two leading mathematics education journals (*Journal for Research in Mathematics Education* (JRME) and *Educational Studies in Mathematics* (ESM)) and the Proceedings of the International Group for the Psychology of Mathematics Education (PME). We wrote:

> It is… clear that reference to some theory is a feature that the researcher has explicitly and perhaps routinely demonstrated in the process of empirical investigation. In particular, theory appears to be informing the empirical in the majority of papers (65.5% in ESM, 71.7% in JRME, 79.1% in PME…). Our data also suggests that 76.3%, 73.9% and 76.4% for ESM, JRME & PME respectively are content with applying the theory rather than engaging with it in any other way... Here it is also interesting to observe that this feature appears to be constant, as we have not observed in the data any change of pattern. One can hypothesise that we have a case of a research field/community where *to be seen to be doing research using and applying theory* seems to be the main pattern. In terms of the methods of inquiry used, if our initial interpretations are sound, the drive towards balancing qualitative and quantitative types of inquiry in the three examined sites of research publication might indicate the adoption of a pragmatic attitude towards

research, consistent with the overall tendency in the social sciences towards a new positivity in the ‘90s and beyond, as described in the relevant literature. (p. 32)

In that study we found also that the theories upon which researchers drew for their analyses ranged widely with an increasing tendency towards what we might call the social turn (Lerman, 2000).

The present study places a higher percentage of papers in the category of those research productions that do not draw explicitly on a theory. We conjecture that this might be because of an increasing tendency towards positivity within all educational research communities in the years since we carried out the above research. Certainly the political pressure to focus on ‘what works’ in educational research in the USA will have had an influence. Furthermore, within the community we are seeing an increasing application of design science (e.g. Cobb, 2006) which is, at heart, a pragmatic approach to analysing why things happen as they do in education. These two points may be connected.

In relation to the vertical axis, researchers are evenly divided between, on the one hand, a conviction that mathematical activities other than those most often found in textbooks and classrooms can provide opportunities for all students to successfully learn mathematics and, on the other hand, those who look to the way the classroom is socially organised, or perhaps we might better say how the interactions between students and between students and the teacher are organised, for features that make a difference in who succeeds.

For our part, we are concerned that we in the community have explanations for why particular interventions seem to be successful in improving equity, or why particular features of classrooms lead to more equitable outcomes. That is, we are concerned with both what works and why it works. Social science research in general and educational research in particular recognises that what works in one place, with one teacher or department, at one time, is unlikely to transfer to another situation in a reliable manner. We are convinced that if we have some good explanations for why a specific intervention or feature works it may be translatable into different circumstances by different teachers at different times, as it may be adaptable in its application to those circumstances. Some theories provide a rich explanatory tool; in research reported above, we have found Bernstein’s theories to be just such a framework (see also Morgan, Tsatsaroni & Lerman, 2002). In other approaches explanations are perhaps harder to come by.

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'MATHS WITH SAM AND ALEX': A DISCUSSION OF CHOICE, CONTROL AND CONFIDENCE

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> I can only answer the question 'what am I to do'? if I can answer the prior question 'of what story or stories do I find myself part of' (MacIntyre, 1981, p. 201).

In this paper, I draw on poststructural and feminist epistemologies to analyse the narratives of two student teachers on a primary education degree. Specifically I look to discuss the competing discourses of the masculine mathematician and the feminine primary school teacher. The initial purpose of the paper is to deconstruct themes which are prevalent within mathematical discourse; a further aim is to explore the representation of narratives within educational texts.

INTRODUCTION

Over recent years many educational researchers have begun to question and thus to challenge the dominant discourses that bound their practice. With regards to mathematics, it has been noted that postmodernism (and poststructuralism) can serve to disarticulate the absolutist version of the subject that predominates current writing and potentially restricts movement around the subject (Walshaw, 2004; Walkerdine, 1990). The project which I discuss in this paper seeks to 'open up spaces that allow us to think about how our world may be changed' (Cotton, 2002, p. 1); specifically it builds on the work of Mendick (2005, 2006), Walshaw (2004, 2005) and Povey & Angier (2006) in its aim to explore both the mathematical discourse and the subsequent representation of student teachers' narratives within educational texts. In particular the study seeks to delve into the discourse of primary school mathematics, and to use the competing notions of the feminine primary school teacher and the masculine mathematician as an arc of analysis; the 'truths' surrounding both gendered routines have been explored by Walkerdine (1989) though not solely from the perspective of the teachers. Thus I am discussing a topic (primary school mathematics) that, notwithstanding Walkerdine's notable work, has been identified as a neglected area of research especially with regards to its underlying philosophies (Bibby 2002); furthermore as the primary school is UK pupils' first point of exposure to compulsory education, I am engaging with research at the source, a fundamentally important space to explore.[1]

The two participants whose narratives I re-imagine in the discussion below were interviewed for approximately 30 minutes as part of an exploratory phase of a larger research project; these conversations took place during the first few months of a three year Primary Education degree course which is based in a university in a city in the North of England. The reasons for choosing to discuss excerpts from these particular student teachers are two-fold; in the first instance both of their narratives demonstrate participation in apparent 'truths' of masculinity and of femininity, as such the

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 362 - 375).

different discourses provide a fascinating contrast as there are stark similarities between their narratives yet their 'chosen' positioning with mathematics is quite different. Secondly, there are many parallels between the participants' backgrounds. Both Alex and Sam achieved a grade D at GCSE and as a consequence they both attended an access course in order to gain their place upon the degree course, furthermore they also come from very similar social backgrounds and are of a similar age; however there is at least one 'important' difference to note - their respective genders.[2]

In order to analyse their narratives I will follow such luminaries as Lacan, Derrida and Foucault and propose that meaning is ambiguous and is created through language (MacLure, 1994, 2003). In particular I borrow from the French 'post-structural/post-modern' philosophers Jacques Derrida and Michel Foucault where the term 'discourse' refers to 'practices that systematically form the objects of which they speak' (Foucault, 1972, p. 49). In this, I am viewing 'self-conception not as an individual's personal and private cognitive structure but as discourse about the self' (Gergen, 2001, p. 247); and again 'narratives do not reflect so much as they create the sense of what is true" (Gergen, p. 249); though from a post-structural perspective 'no single truth is possible because reality is neither singular nor regular' (Taylor, 2001, p. 12).[3]

In the first instance, and explicitly drawing on Foucauldian analysis, I have sought to draw out themes from the data; subsequently I have examined these themes for overt and covert illustrations of power relations, throughout looking for absences and silences within the narratives (Carabine, 2001). The three themes I have chosen to analyse are control, choice and confidence. All have been discussed elsewhere with regards to mathematics (see for example Walkerdine 1989, Mendick 2006, Boaler 1997 or Hardy 2006) and all have been directly associated with masculinity and femininity. For example, when discussing power and conflict relations in the classroom, Walkerdine (1989, p. 36) comments that 'this powerful illusion of choice and control over one's destiny is therefore centrally implicated in the concept of rational argument'. Walkerdine is seeking to expose the 'fantasy' of choice and control and in doing so has highlighted not only their relation to masculinity, but also that these themes are particularly pertinent within poststructural analysis; it is this epistemological approach that can allow us to explore the power relations the student teacher will be both subject of and subjected to; i.e. the chooser and the chosen; the controller and the controlled. The final C word I have chosen to analyse is confidence which again is relevant to the arc of analysis under which I operate. The apparent 'truth' of the feminine lack of confidence has been well cited within various academic texts (Boaler, 1997) and furthermore 'confidence' is a word found deep within current government educational discourse (Hardy, 2006); thus from a Foucauldian perspective, it is relevant to explore the use of this word within the current episteme.[4]

With regards to both the presentation and analysis of the text it is worth reiterating that I aim to work both inside and outside prominent discourse in order to 'destabilise the old us/them oppositions' (MacLure, 1994, p. 284). Thus some aspects of the discussion are presented in a recognisable format, such as structuring the paper around the three themes, whereas other aspects of the text are less familiar; for example the inclusion of endnotes and the absence of certain language from the text. It is this space which intends to offer the opportunity to 'make us sceptical about beliefs concerning truth, knowledge, power, the self and language' (Flax, 1990, p. 41).[5]

As a final point I wish to take note of my approach to conducting the interview and writing the analysis, as it was certainly not to be 'objective'. To use Foucault's language, I chose to reject 'that… which gives absolute priority to the observing subject' (Foucault, 1970: p. xv) and instead I draw on feminist epistemologies (Fontana & Frey 2005, Bryman 2004), and as far as I can, remove the 'other' from the interview situation. I attempt to do this by way of dissolving the inevitable asymmetrical relationship (MacLure, 1993) that arises from the hierarchical interview. Furthermore, I realise that as 'I' am always going to be 'part' of the interview and the analysis I will acknowledge my'self' in its role where necessary and use endnotes to do this. Thus I could argue that the interviews I discuss in this text are valid and reliable, in the sense that the analysis is offered openly and honestly and invites questions and reanalysis of both the interviews and the interviewer, and accordingly not only are the stories of the participants told, in some respect, so is mine.[6]

MATHS AND CONTROL

Alex: I seem to have a brilliant understanding of it and it wasn't from my teachers, where did I get it from?...I'd have found my own way of doing maths and you know found my own way type of thing… If I do something I have to do it right and it better be the best and I push myself and I push myself and I think that's what I did… I'd take my textbook home and try and look at things and I tried to teach myself things in the textbook that I haven't covered.

Sam: I mean, don't get me wrong, I tried hard but, erm, I got moved down from the top set to do, like, the intermediate paper rather than the higher. Erm, and I think when I got moved down I stopped trying…from my experiences within school it felt like recital, it felt like, erm ((sniffs)) remembering these facts to get through a test…I mean fair enough I did have to re-sit the exam but I mean it was a lot of concepts that, erm, I'd never come across before grading or I hadn't come across for a long time

In the sense that the 'language installs the dimension of truth…even as it excludes all guarantee of truth' (Lacan, quoted in Spivak, 1976, p. 1xiii), it could be said that the above text implies that to 'do' mathematics is to be in 'control'. To explore this with specific illustrations I would first like to argue that a desire for understanding could be seen as a need for control, as devoid of understanding we no longer have ownership of the situation. Both Alex and Sam discuss the necessity of learning mathematics through understanding, which contrasts sharply to their school experiences which seem to be predominated by rote learning. It has been documented elsewhere that a 'quest for understanding' (Boaler, 1997, p. 111) is a familiar discourse surrounding mathematics (Mendick, 2006) and furthermore it has been spoken into being as a feminine trait (Boaler, 1997). In addition Sigurdson and Olsen (1992) and Sigurdson et al. (1994) have noted that teachers tend to rely upon procedural methods over proceptual thinking; many teachers in fact will over use rote teaching if they believe a child to lack mathematical understanding (Bibby, 2001). We should of course note that 'understanding' can have a variety of meanings (Watson, 2002). However here is not the place to define the word but to state that further exploration with Alex or Sam could lead to their definition of understanding; in this text, it is the need for it rather than what it is, that is relevant.[7]

The second instance of control I wish to discuss plays upon another familiar narrative of the subject - 'to do mathematics is to be right'. Here Alex could be seen to be adopting a masculine discourse (Walkerdine, 1989, 1990; Mendick, 2006) since in addition to implying control 'to be right' proposes a subject that is proper, rational and precise as opposed to a subject which is erroneous, emotional or fuzzy. This use of oppositions within our narratives is something which I am deliberately recognising as they are not only natural to our spoken language but are also widespread within educational research and writing (MacLure, 2003) (the previous paragraph is also constructed around such a conflict); thus in acknowledging these binaries, I am seeking to use post-structuralist paradigms to deconstruct such prevalent representations of language. Furthermore, as a consequence of their popularity, it is these binaries that continue to perpetuate such constructions of mathematics as rational and masculine and thus othering irrationality (or emotion) as feminine and non-mathematician in the process. Walkerdine (1989, 1990) and Mendick (2006) have both thoroughly investigated such uses of oppositional language within mathematics and found the discourse to be highly gendered; their work justifies and epitomises the conviction that 'I can only answer the question 'what am I to do'? if I can answer the prior question 'of what story or stories do I find myself part of' (MacIntyre, 1981, p. 201). As a final point with regards to 'being right', it is worth noting that both of the participants' narratives appear to be constrained and consequently both are deficient of freedom and space; furthermore, and using Foucauldian principles, one could argue that the concept of constraint is prevalent within mathematics as a subject, the teaching of mathematics and the government documentation concerning school mathematics (for example, the National Numeracy Strategy (DfEE, 1999)). Here I am not arguing that constraints remove power from the individual, but that the power is within the system and that by acting within constraints, we are in fact performing in a matter which is appropriate or 'right'.[8]

The final case within this section that I wish to draw attention to is where control is used to justify mathematical attainment; it could be said that both of the above narratives demonstrate this position, though for rather contrasting purposes. In stating that 'brilliant understanding … wasn't from teachers' Alex seeks to remain in control by removing the teacher from the discourse, furthermore we see Alex frequently seeking ownership of the mathematics and constantly striving to 'finding my own way' (this is noted elsewhere in the transcript); Sam however attempts to stay in control of the situation by using the teacher (and the 'ability' grouped class) as a reason for 'failing' with mathematics. This again mirrors the discourse of mathematics, masculinity and the powerful, active, self (Walkerdine, 1989); if I do well it is down to me, but if I fail the fault lies with someone or something else.[9]

MATHS AND CHOICE

Sam: cos obviously you want to be seen as cool and things like that and you mess around at the best of them so I probably wouldn't have been as inclined in school to ask for help…I remember being challenged in the top group ((sniffs)), erm, but no, erm, the second group, it was easier to mess around because I sort of knew what I was doing work wise…I played quite a bit of sport as well at the time so maths wasn't really top of my agenda, erm, and neither was science to be fair, erm, I'm not very scientifically minded…it's not that I don't enjoy maths, I'm just, it's not my strong point, erm, I'm better with language and things like art and music and things like that.

Alex: No, I've always enjoyed maths, always enjoyed maths, erm, I think it was just the actual day of my exam because I'd been predicted a much higher grade, erm, and I did the higher paper, erm and I, I know, roughly, I can't remember the exact, the exact exam but I know around the time of my GCSEs 'cos I was ill but I couldn't stay off, erm, and to be honest I didn't revise that much…[in reference to English] like I say, I can handle it and I don't mind it but just maths is, I feel as though there's more of a connection between me and maths and it seems to be easier for me to understand…I think a lot of it is like positive and negative, it's, I think some of it is in your attitude towards maths…everybody's certainly got the ability to be good at maths but it's whether they choose to or whether they choose not to.[10]

Building upon the notion of control, this section explores how in order to be successful at mathematics one has to 'choose' to 'do' it. Though from a Foucauldian perspective I wish to remain sceptical about the choices that people actually have; the argument is that choice does not belong with the person, but within social systems and power relations (Foucault, 1978, 1980).

In the first instance both Alex and Sam describe opting out of mathematics and choosing not to achieve at it, Sam with the justification that 'to do maths is to be unpopular' (as explored in great depth by Mendick (2005, 2006)) and Alex in defence of the attainment of a grade D at GCSE; as discussed previously this may also imply their need to be in control of the situation. Sam takes the conception of 'opting out' further by identifying with a feminine discourse and defining sport, art, language and music as distinct from mathematics (all bar sport are discussed further by Mendick (2006)); Sam is relating to the non-mathematical discourse - the 'other', whereas Alex (the more 'confident' maths performer – see the next section) describes similar oppositions but through the narrative is placed in the opposing camp.[11]

The next point of interest I wish to draw attention to is that Alex theorises that everyone can be good at maths, 'it's whether they choose to', thus we could argue that to achieve with mathematics, one must choose to 'do' it. The idea that 'it is in your attitude' serves as a contrast to Sam's discourse of 'maths people/non-maths people' (Mendick, 2006, p. 60); however an alternative reading of the situation, which is possible in post-structural research, would be that as a self positioned person who is good at maths, Alex has become the norm, and thus is not placed as academically gifted or different.[12]

At this point, and drawing together this and my earlier analysis, I wish to note that the distinction between choice and control is rather fuzzy; in fact they can be directly linked by using the last line of Alex's narrative. Here the implication is that we are ultimately in control of what we do, how we understand things and what we learn (not the teacher); it could suggest that (as before) this is where Alex wishes to be – a person who is in control and who makes the choices. Though to reiterate previous arguments, to accept as true that people have choice and control is no more or less than a powerful fantasy (Walkerdine, 1989).[13]

MATHS AND CONFIDENCE

Sam: I've got a D at GCSE and that's it ((chuckles)), I'm not a very mathematical person…maths isn't my strong point it's something that I'm a bit, like, cautious of…[with reference to maths] I know that other subjects will be my stronger point but obviously as I progress through this course my confidence is going to grow with it.

Alex: I'm probably one of the few who really enjoy maths ((chuckles))…[in reference to maths lessons] Absolutely loved them! I was in group one, I was in group one in all of them and I loved maths. I was, I was a maths geek…[in describing the mathematics class on the access course] I felt as though I was able to be myself and there was just no judgement and that made a big difference because I did feel as though I was going to be judged…with maths I'm constantly relating it to things and it's always in my head to be honest ((chuckles))

In this final section of analysis, I wish to emphasise the role that 'confidence' can play in shaping our mathematical 'identities'. It has been noted elsewhere that confidence seems to be the word *de jour* within government documentation and that the use of confidence is often confused with competence (Hardy, 2006); for example if we take the government statement that ‘the best specialist teachers have a more confident command of mathematics’ (DfEE, 1998, p. 72) we could easily replace the word confidence with competence, and furthermore perhaps this exchange may be more ‘appropriate’. It is interesting that Sam has also attached importance to this intangible ‘confidence’ as in the narrative it appears to supersede an actual ability or attainment. From a Foucauldian perspective it is important to highlight this ‘regime of truth… that is, the types of discourse it accepts and makes function as true’ (Foucault, 1980, p. 31) and explore its relation to current educational practice; moreover from my poststructural position, it is essential to note that confidence is acting as neither a solely liberating nor oppressive power.[14]

Continuing to explore the use of confidence and ‘approach(ing) language as a topic’ (Taylor, 2001, p. 15), I wish to argue that the narrative implies that Alex is confident, Alex belongs to mathematics and Alex is a maths person; moreover the phrase ‘one of the few’, suggests Alex as different or special, which is often how mathematicians are positioned within discourse (Mendick, 2006). This contradicts an earlier analysis in this piece of writing where Alex has been positioned as the norm, however this reading of the situation does highlight the instability and ambiguity that exists within language (MacLure, 2003) and the ‘multiplicity of meanings’ (MacLure, 2003, p. 12) that can be produced. As a further point, I do not believe that this reading entirely contradicts the analysis in the previous section where I have suggested that Alex implies that everyone can be good at mathematics, as according to the narrative it is the attitude (or possibly the subsequent success) and not the ability that makes someone different or special. A further reading of both situations could be that in the former text, Alex takes a feminine position as somebody who is ‘nothing special’, as perhaps this is the discourse that it is socially acceptable to produce, however in the narrative above Alex is identifying with the strong, successful and masculine mathematician; a position that Alex can also perform and who is special after all. To use Gergen’s description of conceptions of self, we can say that Alex has been ‘storied’ into several positions with regards to the learning of mathematics, or to paraphrase Walkerdine (1989, 1990), Alex has taken up various roles in multiple fantasies or fictions.[15]

In contrast to Alex, Sam is cautious of mathematics and is placed outside of the subject (as discussed in the previous section); the position of non-maths person is taken, and as a consequence an unhelpful opposition is created. Hence it could be written that ‘confidence’ (and thus masculinity) is present within Alex’s narrative but it is absent from Sam’s; furthermore we could declare that the former status may be as a result of Alex’s self-positioning as good at mathematics (or vice versa); though we should also note that being placed in a top set is probably something which Alex had neither choice in nor control over; as discussed earlier ‘individuals are the vehicles of power and not its point of application’ (Foucault, 1980, p. 98). For Sam of course we could apply the converse arguments and assign the performance to a feminine position.[16]

Another interpretation to consider is that Alex further demonstrates the value of confidence through the frequent references to judgement. To be seen to be allowed to ‘be myself’ appears to be an important state of being within the narrative and could imply that currently (and probably since attending the access course) Alex is experiencing the feeling of acceptance, or the space for acceptance is present; as a consequence Alex could be said to possess a confidence from within. It may also suggest that previously there has been a period when there has not been this space or acceptance within the mathematics classroom; (other dialogue from the interview implies that at times this was the situation during school).[17]

Finally, the notion that confidence (or any conception) is not stable is further demonstrated in Sam’s belief that ‘my confidence is going to grow’; thus Sam is currently ‘outside’ of the ‘confident maths person’ but with space to manoeuvre. Alternatively, perhaps this text is simply Sam producing a socially acceptable position with regards to masculinity; though conversely the narrative still has its feminine positioning, as once again it attaches importance to confidence and not competence. As before this demonstrates the various roles that we perform within multiple fantasies or fictions (Walkerdine, 1989, 1990).

CONCLUSION

In the text above I have chosen to explore discourses of mathematics; in doing so I have attempted to force open a space within language and text, particularly with regards to gendered ways of being. My purpose was to highlight that as we act within various fictions (Walkerdine 1989, 1990), ‘anyone’ can perform as masculine or as feminine. I have demonstrated this through the narratives of Alex and Sam as they both position themselves within various gendered constructs, as they work within the competing discourses of the masculine mathematician and the feminine primary school teacher.[18]

Throughout, I have also sought to show that control, choice and confidence are all themes which are very much bound to mathematical discourses and performances, and that these themes are neither oppressive nor liberating. In doing so, I have intentionally not only discussed the differences but also the similarities that are spoken into being within the two participants’ narratives; this is an attempt to work outside of educational discourse that in the past has relied upon using the apparent differences between data to formulate arguments and conjectures (Walkerdine, 1989).[19]

To summarise the parallels and variations, the difference that is most striking is Alex’s self-positioning as a ‘maths person’ whilst Sam’s placing is outside of the discourse of ‘maths person’; this is of course regardless of their similar actual mathematical attainment. In fact we could maintain that Alex has chosen to opt into mathematics, whilst Sam is currently opting out, perhaps through some unstable conception of confidence. However with regards to similarities both participants seek to be in control of mathematics, which is demonstrated firstly by their desire to learn through understanding, but also through their relationship between their perceived levels of control over their own mathematical achievement.[20]

Another purpose of my work is that it invites exploration and further analysis, particularly with regards to my style of writing. In the first instance the themes of control, choice and confidence, clearly overlap and whether my presentation limits or enhances the analysis and writing is certainly open for debate; moreover the first two constructs (choice and control) perhaps could have been analysed as one or even presented in their own paper aside from confidence. In addition, and returning to the subject of gender, you may have noticed the absence of gender specific pronouns from the text. By ignoring the gender of the participant I have attempted to force open the gender of the situation, as influenced by Mendick (2006) who drew on the work of Connell (1987), I am using gender as a verb as opposed to a noun. Whether such 'tricks' inhibit or liberate the participants, the analysis, or the construction of meaning is another point that exposes itself for contestation. You may for example choose to argue that I have 'subtracted the feminine' (as discourse has previously done, Walkerdine (1989)) as I have also chosen to ignore the gender of the majority of academics from which I have drawn reference and inspiration.[21]

To conclude, it is worth reiterating the purpose of this paper. As I state in the opening paragraph, it was always my intention to 'explore both the mathematical discourse and the subsequent representation of student teachers' narratives within educational texts'; it was my aim to 'work both inside and outside the hegemonic discourse' (MacLure, 1994, p. 291) as I 'insist on occupying the ground that one knows not to exist' (MacLure, 1994, p. 291); whether those aims are at all met is a question with which I leave the reader.[22]

NOTES

1. I always find the first paragraph one of the most difficult to write; I struggle to find the words to begin, words which efficiently and intelligently state the purpose of my project and justify its being. The words have a point to prove and yet I still get sucked into the pleasure of playing with language, 'delve into discourse', and 'arc of analysis' being two examples of my playful intentions. This paragraph also contains the names of many influential mathematical educators from whom I both borrow and admire; perhaps I felt the need to get these names in early, especially as they may be the people stood in front of me when (or if) I come to make this presentation. Moving on from this cynicism, I use 'I' frequently in this paragraph which is deliberate (and quite feminist) of course. It turned out to be one of my favourite paragraphs in the end.

2. I love the use of the word 're-imagine' and intensely dislike the use of the word 'two-fold', thus I am unsure why I have left the latter in – discuss? In this paragraph I

also begin shrouding certain words in scare quotes - 'truths', 'chosen' and 'important'; this is of course to contest the meaning of these words. The ambiguity of 'truth' and 'chosen' fit fairly straightforwardly into poststructural paradigms, whereas 'important' is shrouded because I am asking the reader to decide if it is actually important or not. Notice of course that I have chosen the pseudonyms Alex and Sam for the reason that these could be male or female names; this paragraph is where the gender games begin.

3. Here I wish to note the distinction I give Derrida and Foucault; I even give them first names and gender identities. The only other person, who has a gender identity throughout the text is me (though I did consider writing under non gender specific pseudonyms or masculine anagrams of my own name). In the end I thought it better to own up to who I am.

4. I dislike spelling out how I conducted the research as in reality it was quite subtle and never felt over controlled or contrived, yet literally spelling out my actions does; I also think the reader should be able to find there own way on some matters – so I have given some guidance, though perhaps not enough for some tastes. Again I am aware of the language I use, such as 'the final C word'. I guess it's quite silly to use all C words as your themes; I have become a tabloid journalist. Also in terms of language I begin to use my favourite connecting words and phrases: in the first instance subsequently; furthermore etc, these are all over my writing.

5. Three important things to note: inside and outside; the reference to MacLure; the mention of space.

6. Note the use of 'attempt' here, as that is all I can do. 'Other' and 'I' are important throughout and so is the notion of my'self' – of course this again plays on the use of English language, something which I seem to insist on doing. As an aside I am particularly fond of the last sentence.

7. Note the use of language, e.g. 'spoken into being' 'do', 'surrounding'; – quite deliberate (as it is throughout). I should also note that I have a particular interest in understanding and rote learning; one of the most frustrating parts of my job (as a PGCE Secondary Mathematics Tutor) is trying to convince students that their pupils may benefit from being given the opportunity to understand the mathematics.

8. The link between 'control' and to 'be right' is perhaps open for contestation, as you can probably be in control of something without actually being right and vice versa. Perhaps the last section relating to constraints is worthy of more discussion and exploration. I start my binary crusade in this paragraph also.

9. The beginning and ending of this paragraph are worth noting. 'The final case I wish to…' I realise that there are endless cases you could analyse; this is just where I stop. I finish the section with quite a simple and direct use of language, perhaps to make a prominent statement. I should also note that 'finding my own way' is probably something I can relate to, or have had to relate to in my own studies.

10. I should note that I reversed the order of Sam and Alex's transcripts, so as not to give either participant priority.

11. I could have gone into more detail with regards to the unpopular image of maths.

12. I guess I relate to Alex here. I would like to believe that everyone can be good at maths and in my self-deprecating manner I would never had defined myself as special, even if I recognised that not many people seem to be good at maths.

13. This is the shortest section of the three themes – which may perhaps suggest that I should have written about control and choice together. I love the word fuzzy here – it conjures up all sort of images or even emotions. The 'no more or less' is interesting, as throughout I try to say that something is not better or worse than anything else.

14. presented confidently' – what value do these statements? I don't care whether you think I am confident -'was it good', 'was it weak'? If confident is what I have to be, I'm afraid I'm screwed! Some personal history there. Moving on to other things, 'appropriate' – I use this word tentatively. 'Du jour' – I'm trying to learn to speak French. 'Intangible' – I like this; again it shows my contempt for the value attached to confidence.

15. I felt it was important to highlight possible contradictions in my work, and to let these stay in the text – after all I am not searching for a 'single truth'. Other people may disagree and feel it devalues the analysis. 'I do not believe' stating firmly that it is my opinion. There is also some interesting analysis about what is socially acceptable and again this could have been explored more.

16. 'Cautious of maths' – I really like this phrase.

17. Again I can relate to Alex here. I intensely dislike feeling as if I am being judged by people, or not being given the opportunity to 'be myself' – whatever that is (in a poststructural multiple self type of way).

18. 'Force open a space' – I like this phrase. 'Anyone' is used tentatively as I am questioning the use of the word.

19. 'Bound' – nice word.

20. I'm not sure if this paragraph should be in the conclusion; as it is a summary of the analysis maybe it belongs in the main body of work. Oh and for the record, I have a degree in mathematics but I wouldn't class myself as a maths person. 'Parallel' – nice maths word slipped in there – hold on a minute, what about the declaration in my previous sentence; I might have just contradicted myself in this endnote.

21. Re-analysis and further exploration is of course invited; multiple readings of the text are very much encouraged. This paragraph is also the first time that I am explicit about the lack of reference to the gender of the participants, though it was alluded to in the opening paragraph, and was specifically mentioned in an earlier endnote. There is an argument that I should have been upfront about this from the start; I left this declaration until the end as I wondered whether it would be noticed and I wondered

whether the reader would find the language too disjointed (as writing without reference to gendered pronouns will have restricted the language that I can use). It may simply be a 'trick', though it does highlight the power of language.

22. Endings are difficult as well as beginnings (refer to endnote 1), thus I attempt to reconnect with the reader. I also felt it was quite fitting to use references from MacLure, whose work was influential in forming the analysis and writing behind this paper. Overall I have enjoyed writing these endnotes; it has been a pleasant experience to deliberately write without reference or academic rigour and feels akin to a film director's DVD commentary (which you can of course turn off if you choose). I seem to have used the endnotes to write myself into the work and to critique the writing, which I hope the reader has welcomed (if not just turn it off).

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BROCCOLI AND MATHEMATICS: STUDENTS' SOCIAL REPRESENTATIONS ABOUT MATHEMATICS

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Social representations play an essential role in students' performances and school achievement. Collaborative work can act as a mediating tool in order to change them. Assuming an interpretative/qualitative approach, inspired in ethnographic methods, an action-research project was developed in an 8th grade class. Its main goals were to understand how collaborative work contributes to change students' social representations about mathematics, and to illuminate the impact of this process in students' performances and school achievement. The analysis of two cases illuminate the potentialities of collaborative work to promote more positive social representations about mathematics and students' competencies.

INTRODUCTION

In a multicultural society, like the Portuguese one, classrooms are getting more multicultural too. Portugal changed from being an emigration country, until the mid-seventies, into an immigration country (César & Oliveira, 2005). Thus, the education became less focused in a mainstream social class with its own ways of acting and values. It had to adapt itself to cultural diversity and to the challenges, but also to the richness, it brought into schools (César, 2007). Therefore, education, in general, and mathematics education, in particular, need to investigate and spread practices that are well adapted to all and every student's characteristics and needs (César, 2007; César & Oliveira, 2005), allowing them to construct their life projects. But this also means changing teachers' practices, and the evaluation system, assuming that the formal educational scenarios must be spaces and times that facilitate the emergence of social interactions, namely peer interactions, promoting students' academic self-esteem (César, 2007; César & Oliveira, 2005; Perret-Clermont, Pontecorvo, Resnik, Zittoun, & Burge, 2004; Renshaw, 2004).

The social representations that students develop about mathematics are constructed since when they were born, when they start to establish their first social contacts with the others, and they go on being developed under the influence of school, family, friends, and *media* (Abreu & Gorgorió, 2007; Piscarreta, 2002; Piscarreta & César, 2004). When they start school many students are already convinced that mathematics is only understandable for very bright people and that they will never be able to learn it and to succeed. Thus, changing students' social representations about mathematics is a first and essential step in order to promote their mathematical performances.

The problem is addressed in this study is the negative social representations about mathematics developed by many students. The research question addressed in this

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paper is: How do students' social representations change from the beginning of the school year until its end when they experience collaborative work practices?

In order to address this problem and questions we developed a study in Portugal, during the school year 2006/2007, that allow us to understand how mathematics is seen by 8th grade students (n = 21) and how changing teachers' practices, namely implementing collaborative work in the class, contributes to change these students' social representation about mathematics and their mathematical performances.

THEORETICAL BACKGROUND

The theoretical framework is based on a historical-cultural approach (Vygotsky, 1932/1978) and it is focused in collaborative work (Perret-Clermont et al., 2004), namely peer interactions, and in dialogism (Renshaw, 2004), conceived as tools that mediate students' mathematical achievement and the development of students' (mathematical) competencies (César, 2007). It is also based on social representations (Abreu & Gorgorió, 2007; Moscovici, 2000) and on the role they play in the learning process, namely in mathematics, learnt in formal educational scenarios (César, 1998, 2007; César & Oliveira, 2005).

The social representation construct was coined in the sociological theory of Durkheim. This author claims for the distinction between the individual social representation and the collective social representation (Piscarreta, 2002; Piscarreta & César, 2004; Ramos, 2003). But Moscovici (2000) was an essential author in order to stress the dynamic and multi-faced character of social representations that he defines as

> "systems of values, ideas and practices with a two-fold function; first, to establish an order which will enable individuals to orientate themselves in their material and social world and to master it; secondly, to enable communication to take place amongst members of a community by providing them with a code for social exchange and a code for naming and classifying unambiguously the various aspects of their world and their individual and group history". (p. 12)

He claims that social representations are the result of the interaction and communication between the individuals. The social representations are not constructed in an empty social context, but since the first social contacts that we establish with the others, and the society.

According to Moscovici (2000), social representations are a dynamic construct. As stated by Abreu and Gorgorió (2007), "social representation theorists agree that there can exist more than one social representation to interpret the same phenomenon. Social representations are not only plural but dynamically created and transformed" (p. 3). Thus, the role of social representations is to "organise and interpret the reality, giving a meaning to the information and trying to explain the surrounding situations, mediating behaviours and social relations which allow us to differentiate the diverse social groups." (Piscarreta, 2002, p. 26).

Studying and understanding students' social representations about mathematics allows to realise them but also to change them through practices, namely implementing collaborative work, tasks that are elaborated in order to allow for establishing connections between students knowledge and mathematics knowledge, and a differentiated process of evaluation (César, 1998, 2007). Abreu and Gorgorió (2007) claim that "the notion of social representation can offer useful insights into understanding practices of teaching and learning" (p. 1).

Working collaboratively, namely in peers, there is a change in teacher's and students' role, since the learning process is focused in the students (César, 2007; César & Oliveira, 2005; Perret-Clermont et al., 2004; Renshaw, 2004). The students assume a dynamic role in the classroom once they become more autonomous and responsible for their learning process. They negotiate meanings, roles, arguments, constructing what Perret-Clermont (2004) calls thinking spaces. Thus, they reconstructed their knowledge and their identity (César, 2007), namely by being part of a community of learning (Lave & Wenger, 1991), and becoming legitimate participants instead of peripheral ones (César, 2007).

METHOD

This work is based on the *Interaction and Knowledge* project which studies and promotes collaborative work in formal educational scenarios (5th to 12th grade). This project was implemented during 12 years and had three levels: (1) micro-analysis; (2) action-research; (3) case studies (for more details see César, 2007; Hamido & César, in press). This research is part of the action-research level of this project. Due to the collaborative work that was implemented during the whole school year, this class became a learning community (Lave & Wenger, 1991).

This research was developed in an 8th grade class, in a school in the surroundings of Lisbon. At the beginning of the school year there were 28 students in this class. During the school year some students changed to another class and others drop out school. Thus, by the end of the school year there were 21 students which are the ones we consider as participants in the study. We also consider as participants the co-teacher-researcher and two observers (one supervisor and a colleague that also acted as trainee teacher). We analyse the productions of two students, a male and a female, whose names were changed in order to cover their identification data. These two students are paradigmatic cases as similar changes in their social representations about mathematics were also observed in many other students from this study.

Data were collected through a task inspired by projective techniques, questionnaires (at the beginning of the school year and in the last week of the school year), participant observation (registered in a researcher's diary, and including his own reflections about his practices) and students' protocols.

According to Carvalho and César (1996) and Piscarreta (2002), tasks inspired by projective techniques (TIPT) are an effective way to study social representations about mathematics since they allow understanding the socio-cognitive and socio-affective dimensions. The task inspired by projective techniques was presented at the beginning of each school period as we aimed at illuminating the evolution/changing process that could emerge. Thus, we could illuminate this process for each one of the students and confront it with his/her mathematical performances. In tasks inspired by projective techniques instructions play a main role. In this case, each student received a white A4 sheet and they had to draw or write what was mathematics for him/her. But in order to better understand the importance of the instructions and how they shape students' answers, these instructions were slightly change: (1) in the first term (TIPT 1), it was said and was written in the blackboard "Write or draw what is mathematics for you."; (2) in the second (TIPT 2) and third (TIPT 3) tasks it was said and was written in the blackboard "Draw and write what mathematics for you".

RESULTS

This research aims at understanding the contributes of collaborative work to change students' social representation about mathematics. Thus, we decided to analyse students' social representations at the beginning and at the end of the school year, after working collaboratively during the school year. In the first task inspired by projective techniques all the students wrote what mathematics represented for them. But in the second and third tasks, in which we changed the instructions, students not only drew but also wrote what represented mathematics for them. This illuminates the essential role played by instructions and how students' performances change according to them. Thus, when analysing the results it is not only the nature of the tasks that must be made explicit and discussed but also the working instructions that were given. And when we aim at changing teachers' practices we must reflect upon the nature of the tasks and about the working instructions. Inaccurate instructions can change the results.

We are going to analyse Anna's and Peter's social representations about mathematics and the changing process that emerged from collaborative practices. Anna is a girl with presented a fair performance in mathematics (Level 3, in a scale from 1 to 5, being Level 5 the top mark) and she called herself as a fair student because " I'm distracted (…) and [teacher] thinks that I'm talker". In her first task inspired by projective techniques, she wrote:

> " For me, mathematics is an area in which we should develop the logic. All schools have mathematics as a subject, in any part of the world. Because there is mathematics everywhere and without it may not be able to enter at the studies or a job that we aim. And even if we don't like it, we have to learn it…" (Anna, TIPT 1, September 2006).

As we can see, Anna recognised the importance of mathematics in students' future and its role in the world. These are both social representations that are spread through the *media*, teachers' discourses, families and friends, as pointed out in other studies (Piscarreta, 2002). Nevertheless, this student expressed the idea that we must learn it whenever we like it or not, which stresses its compulsory character. It also has the implicit assumption that there are many students that do not like mathematics, but they still have to learn it. Moreover, she recognises that most of students' future is connected to mathematics success – or failure – and this illuminates a point that was also claimed by many authors: mathematics shows high percentages of underachievement but is also one of the most selective subjects (César, 2007).

For Anna, doing and liking mathematics are two separated issues. In the first questionnaire (Q1, September 2006) when we asked her if she liked mathematics, she wrote that she liked it "More or less, I know I must learn it". Once again, whenever liking it or not, learning it is unavoidable. Thus, the question, for her, is not about feelings – liking it, or not – but about the huge need to learn it if one aims at succeeding in his/her studies and, moreover, in his/her professional life.

In the third task inspired by projective techniques (TIPT 3), this student used a metaphor - the broccoli and the mathematics - establishing the connection between her home culture and the academic culture. This unusual metaphor illuminates her creativity and sense of humour, revealing the ability to connect her daily life and mathematics.

Making explicit some of the graphic features of her drawing, we can add that she draw the broccoli on the top of the sheet. According to Bédard (2005), drawing in that part of the sheet means the head, the intellect and

Figure 1 – Anna's drawing in TIPT 3 (June 2007)

the imagination. It is characteristic of a person who is curious and who enjoys discovering. According to the students' questionnaire, we see that she would like to be a photographer, because she likes "to observe the beautiful landscape that the world has". She sees mathematics as being part of the world, but not as a beautiful landscape, at least for most of the students. After drawing the broccoli she wrote a short sentence clarifying the meaning of her drawing:

> For me, mathematics is like broccoli, many people don't like it, some of them only [eat them] when it is really needed, and some others really like it.

The analysis of the three TIPT illuminate that there was a change in her social representation about mathematics. The didactic contract, including the implementation of collaborative work, was an essential mediating tool for this change, as claimed by César (2007) and by César and Oliveira (2005). As Anna accounted in the final questionnaire (Q2, June 2007), working in peers should be continued in the next school year since "it was by working like this [in peers] that I was able to learn mathematics, and thus I started to like it". She also adds that "Now it is a good thing!".

Anna's case illuminates a positive change in her social representations about mathematics. At the beginning of the school year she felt she should learn mathematics but she did not like it. She only knew it was needed for her future. But at the end of the school year, after working collaboratively, she was able to give a meaning to her mathematics knowledge. She called it "really learning it" (Q2, June 2007), and that was what made possible for her to be able to like mathematics. As Moscovici (2000) claimed, the social representations are dynamic and they are influenced not only by the social context but also by the interactions with others. Thus, collaborative work can play a mediating role changing negative social representations about mathematics into more positive ones.

Peter is a boy with a higher performance in mathematics (level 4). He described himself as a good student in mathematics because he did not have "high grades nor negative grades" (Q1, September 2006). He considered mathematics as a subject which he liked less than others because he liked "much more the others subjects".

In the first task inspired by projective techniques he wrote:

> " For me mathematics is a very important subject for our future and it is needed for our good development". (Peter, TIPT 1, September 2006)

Once again this student focuses himself in the importance of mathematics for students' future and on its contributions to students' "good development", rephrasing many of the teachers' and *media* discourses. Each time there are international studies ranking students' mathematical performances (and the Portuguese ones were quite low in TIMMS and in PISA), or national exams with high percentages of failures in mathematics, these kinds of discourse appear again and are repeated over and over. Thus, they are also appropriated by students who use them to convince themselves that even hating mathematics they still need to make an effort and learn it.

In the third task inspired by projective techniques, Peter chooses to express his opinion by drawing. He filled all the sheet with materials and designations connected to the mathematics, many of them learnt at school and clearly connected to some mathematical contents.

Figure 2 – Peter's drawing in TIPT 3 (June 2007)

The student drew several triangles, connected to Pythagoras' theorem. According to Bédard (2005) "this symbol represents the knowledge" (p. 20). Peter also used many geometrical symbols, namely square forms. Other interesting aspect in his drawing is that he drew stars in two specific places: next to the Pythagoras' theorem and to "Matemática" [It means mathematics, in Portuguese]. As stressed by Bédard (2005), drawing stars means that we can not live only in the present, but we aim at "being a star", in the future, which means succeeding. Thus, this drawing also illuminates that this student aimed at showing higher performances in mathematics and at succeeding in his future studies and career.

At the end of the school year, he wrote:

> "Mathematics is a challenging subject and no matter how much we want it, or not, it appears in our day life, even when we do not realise it. It is something that may bother us for being difficult but it is something that hides interesting facts and many mysteries, that makes our head turn over and over to find out the solution". (Peter, TIPT 3, June 2007)

Comparing both tasks inspired by projective techniques we can see that the importance of mathematics is present in both of them. But in the end of the school year mathematics is seen as a challenge, but also as something he will be able to solve, even if this means a lot of effort. He is able to see himself as being successful in mathematics, and he is also able to point out some of its beauty.

We must add that at the beginning of the school year Peter was not convinced that working in peers was such a good idea. But as time went by and he realised that the practices were so much focused in students' contributions he got more engaged in the activities. He started to like the challenges that were presented by the teacher and by his colleagues. Thus, when we asked him, in the questionnaire (Q2, June 2007), if he liked working collaboratively, he answered positively accounting that "I think that we learned more quickly".

In short, if both students claimed that mathematics is an essential subject for their future life as students and workers, they also state that recognising its need does not mean liking it. Working collaboratively, namely in peers, allowed them to learn mathematics and to become more confident about their mathematical competencies. This was an essential step in order to allow them liking mathematics and also to become more confident about their success b their mathematical performances.

FINAL REMARKS

The knowledge appropriation, and the mobilisation and the development of (mathematical) competencies are complex processes shaped by many elements, namely the nature of the tasks, the working instructions, the type of interactions established in the classroom and the didactic contract negotiated between the teacher and the students.

We illuminated that knowing and studying students' social representation about mathematics is an essential step if we aim at promoting students' mathematical performances. As Abreu and Gorgorió (2007) claim, "social representations are not just something one uses to inform one's practices, but something that becomes part of one's reality (p. 2). Thus, understanding why mathematics is so much rejected and feared by many students allows us to use forms of mediation during practices like the nature of the mathematical tasks and/or collaborative work, facilitating students' promotion of more positive social representations about mathematics, and their achievement in this subject.

Implementing collaboratively work in the classroom allowed students to change their social representations about mathematics and to develop more plastic and dynamic social representations. This contributed to the development of students' mathematical competencies, and also to promote their positive academic self-esteem. Moreover, knowing students' social representations about mathematics allowed us to deal with cognitive, social and emotional features of the learning process, adopting an ecological and systemic approach to learning that favours students' development and helps them to engage in life projects connected to mathematics.

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ACTIVITY, ARTEFACTS AND POWER: CONTRIBUTION OF ACTIVITY THEORY AND SITUATED LEARNING TO THE ANALYSIS OF ARTEFACTS IN MATHEMATICAL THINKING IN PRACTICE[1]

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This paper explores and discusses the contribution that key concepts of activity theory and situated learning can offer to the analysis of the use of a mediating artefact – the calculator – in the distribution or power and control in social practices, using insights coming from the empirical field studying ardinas' practice[2] in Cape Verde. Drawing on activity theory and situated learning, we present the key concepts used to discuss the situated role of artefacts in the practice. The analysis of the use of a calculator by some ardinas in moments of the selling process opens room to discuss artefacts as resources for power and the relationships between power, technology of practice and shared repertoire.

INTRODUCTION

Artefacts gain relevance when we seek to understand learning as a phenomenon emergent from participation in social practices. In fact, the dimension of the relations between resources or artefacts and power introduced by Giddens (1996) should be considered; resources are means through which social power is exerted. Thus, power should not be presented as a resource but as depending on the use of resources. On the other hand, artefacts constitute a technology of the practice or resources (Lave & Wenger, 1991) which include the repertoire of the practice (Wenger, 1998). Artefacts as part of the historical trace left by the reproduction cycles reveal the productive as well as the reproductive character of those cycles and the contribution to the constitution of the practice over time. Thus, "understanding the technology of the practice is more than learning to use tools; it is a way to connect with the history of the practice and to participate more directly in its cultural life" (Lave & Wenger, 1991, p. 101). In the following we will explore and discuss the contribution that key concepts of activity theory and situated learning can offer to the analysis of the use of a mediating artefact – the calculator – in the distribution or power and control in social practices, using insights coming from the empirical field studying *ardinas*' practice in Cape Verde.

CONCEPTS IN ACTION IN THE ANALYSIS

For Leontiev (1978), activity is a molar unit, not an additive unit in the life of the person but a system with its own structure, its transitions and its internal transformations, its own development. Here we can identify non-adding elements linked to central concepts: activity (linked to motives), action (linked to goals) and

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operation (linked to conditions). The motive of the activity is intimately connected to the need felt by the individual – the form responding to that need. Activity may involve different processes – actions – that aim to produce certain results intimately related to the activity and in this way directing the activity. Action can be made concrete in different ways and forms – operations – according to the conditions available but always making sense in terms of the goal that is supposed to achieve.

In these terms, two methodological implications emerge: (i) activity cannot be reduced to a set of simpler stand-alone additive parts or processes, and (ii) its structural and functional unit can only be examined looking at the phenomenon in its active state. This perspective allows us, to identify elements of the activity and, to say that those elements have only a potential character, neither deterministic nor definitive, as activity can only be realized through a dynamic, transformative process of development.

Activity, artefacts and power

We will use central concepts from activity theory (Engeström, 1999; 2001) but concentrating on the artefacts, and in particular the links with the idea of mediation, one of the key concepts of the historical-cultural approaches.

A central principle of activity theory is that "a collective, artefact-mediated and object-oriented activity system, seen in its network relations to other activity systems, is taken as the prime unit of analysis" (Engeström, 2001, p. 136), the activity systems realizing and reproducing themselves through generation of actions and operations. Both individual and collective actions–conscious and intentional–and the automated operations they eventually become are fully understandable when considered against the entire activity system. This hierarchical structure of an activity system is permeated by its multi-voicedness character as participants in their different positions go along with their narratives carrying its history engraved in the mediating artefacts and rules. It is relevant to bring in here the notion of 'ongoing activity' coined by Jean Lave (1988) when she refers to activity as she readdresses our attention to the strongly fluid and dynamic character of activity. We need to keep present the local character of activity, developing here and now, with the resources and the constraints present in the situation for the actors in place. The ongoing character of activity seems consistent with Leontiev's view that activity should be analyzed in its active state.

Referring to the non-definitive nature of activity, Engeström (1999) considers that none of the usual dichotomised forms of characterising artefacts are useful–tools and signs, or external and internal. In both cases it is the internal nature of the artefacts that is of interest, independently of the system of activity where they are used. Given the dynamic nature of systems of activity, artefacts' functions and uses are in constant transformation and, accordingly, the elements that seem to be internal in a certain moment can be externalized in the next one. Similarly, external processes can on one occasion be internalized. In any case, a dialectic relationship pervades the internal-external localizing of activity flux. Freezing or splitting the two processes seems to be a poor basis for understanding artefacts. Engeström (1999) proposes a differentiation in regard to the uses of artefacts:

> The first type is *what* artefacts, used to identify and describe objects. The second type is *how* artefacts, used to guide and direct processes and procedures on, within or between objects. The third type is *why* artefacts, used to diagnose and explain the properties and behaviour of objects. Finally the fourth type is *where to* artefacts, used to envision the future state or potential development of objects, including institutions and social systems. (Engeström, 1999 p. 382, italic in the original)

A strong claim of the mediating role of artefacts is highlighted by Holland and Lave (2001) as they look with particular attention to the power of inscription of cultural forms–a notion close to Cole's conceptualization of cultural artefact; Holland and Lave (2001) explicitly discuss "the materiality of cultural artefacts" (p. 63), the artefacts assuming an obvious material dimension and a conceptual or ideal aspect, an intentionality, whose substance is embedded in the world of its uses.

On the other hand, resources (or artefacts) can be conceptualized as ways of transformative relations which are incorporated in the production and reproduction of social practices (Giddens, 1996). This means that resources are intimately connected to power, be it seen in a broad sense as an ability that transforms activity or in a more specific sense of domination or ability to intervene. Resources are the basis and the vehicles of power. Given that resources are equally structural components of social systems, they become also the means through which the structures of domination are reproduced. It is within this framework that we can consider that exerting power is not a type of action; power is instantiated in action while a regular and routine phenomenon. In this sense, power is not a resource but it depends on resources.

THE EMPIRICAL FIELD: ARDINAS' PRACTICE AT CAPE VERD

The participants observed were *ardinas*–boys aged between 12 and 17 years, half of them in 5th to 7th grade at school, who sold newspapers in the streets of Praia, the capital of the Republic of Cape Verde in Africa. The two weekly newspapers existing in Cape Verde during the time of the study were written in Portuguese, the official language[3]. The number of *ardinas* who sold the newspapers during the study varied from 19 to 32, all of them with no formal link to the institutions that owned the newspapers. However, selling newspapers in the city of Praia was done only in this way (on the street and by *ardinas*), so totally dependent upon the availability and interest of the *ardinas*.

There was no external sign that could identify an *ardina* in the street, except for the fact that he was carrying a number of newspapers under his arm. However, they were careful in their presentation and image on the selling days as this represented an important issue to gain access to certain places (eg. in state departments in Praia). Most of the *ardinas* were motivated by the idea of getting some money to help their families and some were in this practice for a few years. These boys lived in the city of Praia or in the nearby small village of St Martinho.

The work of the *ardinas* developed in three phases: (i) receiving the newspapers from the agencies; (ii) selling them on the street; (iii) paying back the money to the newspaper agencies. The organisation of these phases was necessarily connected to the instructions of the agencies but the *ardinas* positioned themselves in that organisation in their own way.

In 1998 there was only one weekly newspaper available, *O Tempo*. Every Friday morning, as soon as the newspapers were printed, at the door of *O Tempo* the newspapers were delivered to Disidori, the man who was accountable to the newspaper agency and responsible for the whole process of selling and returning back the unsold newspapers. Disidori distributed immediately a number of newspapers to each *ardina* (between 50 and 150 units each) and wrote down a list to record the *ardinas* names and the number of newspapers given to each one.

After receiving the newspapers the *ardinas* run quickly to the usual areas for selling in the city, some trying to maintain their own selling place in the street. However, those places varied during the day according to the rhythm of selling and the rhythm of the activities in the city.

The price for one newspaper was 100 *escudos*[4] and, when the *ardinas* finished selling, they had to pay back to Disidori 87.5 escudos per newspaper sold, these amounts being defined by the newspaper administration.

Some time after the distribution of the newspapers, Disidori used to go to the central Square in Praia carrying with him a set of newspapers for those *ardinas* who were in school, and hence came later to the selling, or for those who sold out very quickly and asked for more newspapers. The central Square had a strategic role for the *ardinas* to integrate the newspaper selling into the socio-economic life of the city and it was the main point of convergence of the boys at several moments during the day: (i) as a selling place; (ii) as a lunchtime resting place, and (iii) when they finished selling and came to pay back.

As the *ardinas* finished the selling they started showing up for payment. Each *ardina* approached Disidori, saying how many newspapers were left; Disidori made the calculation with his calculator and showed the result on the calculator screen to the *ardina* who then gave him the money. Sometimes the more experienced *ardinas* made their own calculation with their own calculator or Disidori's. Usually, several operations were in progress: some of the *ardinas* were counting the newspapers left for returning, others were counting and organising the money, giving back newspapers to Disidori, observing the processes or giving the money to Disidori. The environment could seem confusing at a first glance as there were several boys present and a lot of money changing from hand to hand but when we observed in detail we understood that everything was running in a certain order and that this allowed each *ardina* to see what was going on with the calculations – their own or those of the others.

This phase represented an important moment in the selling practice. The *ardinas* exchanged stories of the day, they had face-to-face discussions, and they organized the moment of making the final account with Disidori. It was also a very rich opportunity to observe how the *ardinas* interpreted and solved their problems. It is worth noticing the total absence of any attempt to make their calculation strategies explicit either through verbal explanation, deliberately showing, checking processes or anything we could classify as some sort of mathematical conversation.

In 1999–the second phase of field work–a second newspaper *O Espaço* came into the market being sold at the same time as *O Tempo* which provoked profound modifications in the whole structure and organization of selling. For instance, Disidori moved to this new newspaper with a new position. The majority of the *ardinas* were involved in selling both newspapers. New rules were in action: for each newspaper sold the *ardinas* would receive 20 *escudos.* But now they had to go to the *O Tempo*'s agency to receive the newspapers and go back there after selling in order to pay for the newspapers sold. For the new *O Tempo* the payment routine was similar to the previous year in the Square, but now with Manu–an experienced *ardina* playing the role of Disidori. This boy started helping Disidori in his relations to the *ardinas*, in particular in the distribution as well as in the support and control of what was going on in the Square including the reception of payment at the end of the day (all of this related only to *O Tempo*). At the end of the day Disidori used to go to Manu's home to make the final accounts checking the journals left and the money paid.

Although the second author was present during the whole process of distribution and selling, the payment phase in the Square was one of the best settings to collect data. There were plenty of opportunities to talk with the *ardinas* in a quite natural way videotaping informal interviews whose guidelines were mainly directed by the topics the *ardinas* wanted to talk about or the problems they were discussing among themselves.

THE POWER OF THE CALCULATOR AS A MEDIATING ARTEFACT

The *ardinas* use a variety of resources in the everyday practice of selling newspapers in the street. The forms of use and role of artefacts was one of the foci in our study its importance coming from the fact that in order to understand and characterize the processes of calculating-in-action it was relevant to identify the artefacts in use and to understand how those artefacts are constituted while structuring resources. We concentrated on who used the artefacts, how power was embodied in their use according to the emergent goals and motives in the activity. In doing so, we tried to highlight the way artefacts, as historical and mediating tools, were present in the ways of acting and thinking. The focus of analysis here is the use of the pocket calculator in certain moments of the practice of selling newspapers in the street.

In several moments of the selling practice a calculator was used by Disidori, Manu and some of the other *ardinas*. A rather important issue here was that the calculator was one of the few tools present in the practice of the *ardinas* that was associated to mathematics[5]. The calculator was not an artefact present in the everyday life of most people and not used in school. Some people selling goods in shops used it but not in traditional selling such as street markets. It was a technological tool with a quite restricted and limited circulation and use in people daily practices in Cape Verde and in general it was associated to specific technical domains outside the range of the mainstream of the population.

This study took place in a period of the life in Cape Verde when 'electronic technology' was seen by most people as 'magic' and 'automatic', something that people in general did not really understand but that carried a strong degree of power and rightness, and a social connotation to 'serious and important matter'. The calculator is placed within the category of desired and socially valued objects, although with a quite restricted access. Therefore, it was unnatural for *ardinas* to use the calculator when acting as *ardinas*. And in fact although the calculator was used everyday in the practice of selling, its use by the *ardinas* was very limited both in frequency and in forms of use.

The control mediated power of the calculator

In 1998 the manipulation of the calculator was mainly associated to the phase of payment. The forms of use of the calculator adopted by Disidori in the practice focused on its role as a mediating artefact in his interaction with the *ardinas* towards the control of the selling process. When paying back to Disidori the *ardinas* were immersed in a routine orchestrated by Disidori's actions, and the calculator was an element always present and visible to the *ardina* he was addressing. It was through the mediation of the calculator that Disidori 'organized' his interaction with the *ardina*–a *how* artefact–turning possible for each *ardina* to see his particular situation represented.

He slowly typed in the numbers and operations:

X (newspapers delivered)

–

Y (newspapers not sold)

* 87.50

=

Z (to pay)

Figure 1. Final calculations taking place

At the same time, he was naming loudly each one, he transformed the calculator into a kind of 'guarantee'–a *what* artefact–because in using it he described the specific selling situation of a particular *ardina*. While providing a certain degree of access to the *ardina* to follow the whole process of calculating, Disidori was offering a way not only of showing that he was not cheating–which entails a demonstration of the power of his managing role in the game–but was also introducing signs of care and honesty that Disidori found important for the development of the autonomy of the *ardinas*–the calculator playing the role of a *where to* artefact. Sharing the process (carefully taking each step of calculation) with the *ardina* clearly suggested Disidori's will to reinforce confidence and transparency in relationships. The correctness and rigour of the results brought in by the calculator was reinforcing the power of Disidori to apply the rules of selling and contribute to legitimate his positioning in the practice.

When finishing the process of computation Disidori used to say out loud the amount that the *ardina* had to pay showing explicitly to him the calculator screen. The calculator was extensively used as medium for communication that allowed Disidori to offer to the *ardinas* different forms of representation of the amounts and this was important to those *ardinas* less experienced in school mathematics. In the frame of the social meanings associated to the technological tools (e.g. belief in the infallibility of the results produced) the calculator presented here the characteristics of a *why* artefact–if the numbers inserted were correct, the result would be correct.

However, the calculator was not present only as a resource for communication for Disidori. While organizing themselves in order to make the final accounts with Disidori, some of the older and more experienced *ardinas* used the calculator (their own or borrowing it from Disidori) for self-control, checking their previous mental computations. Thus, the calculator had here a status of a tool for confirmation of the final selling situation–a *why* artefact. Also important was the fact that using it in this way was also an affirmation of autonomy of the *ardinas* as it avoided a long interaction with Disidori. Again, the use of the calculator embodied power as it conveyed some degree of authority to the less experienced *ardinas*, being a *where to* artefact, a symbol of a particular *ardina* positioning and power.

The way Disidori had used the calculator gave rise to a variety of learning opportunities for the *ardinas*. For example, they enlarged their repertoire of forms of naming and representing numbers and operations and they had support in learning to respect hierarchy as the calculator was used by the *ardinas* with more and more autonomy according to their experience of selling.

The perception that each *ardina* had of their selling behaviour was mediated by subjective interpretation of the situations and the feedback that Disidori gave through several signs. On the other hand, the almost individual nature that framed the payment phase was reinforced by the fact that Disidori used his selling records in a very private way and gave a central role to the calculator in the interaction with the *ardinas*. In addition, as the state of the account of each *ardina* was reconstructed for himself alone, and because the screen of the calculator was only visible to Disidori and the *ardina*, it was not possible in this process to compare his selling to that of another one. Finally, the way Disidori used the calculator reinforced the power of the institutional organization of selling while keeping the *ardinas* 'dependent' on the way they are considered by 'authority'.

There is another side of the use of the calculator in the practice. The transparency embodied in the form of use adopted by Disidori during payment gave visibility to those *ardina* who was involved in the interaction during payment, but keeping invisible the specific forms of computation which produced the final account. Therefore, in relation to mathematical thinking inherent to the computations, the calculator was not transparent at all. The visibility was given mostly to the sequence of actions and this was obscuring the processes behind the results. It was apparent that the calculator had no impact in the ways the *ardinas* calculated, and it did not mediate their thinking in the computations. The *ardinas* who had almost no familiarity with the calculator, when challenged by the field observer to use it as a support to solve a question related with some selling situation, always started by trying to calculate their profit and they were not able to explore any other type of manipulation.

In 1999, the calculator became less of a resource for control and a marker of power. Manu used it mainly for his own calculations and not to interact with the *ardinas*, thus giving less visibility and less organizing role to the calculator[6]. At the same time, the close and affective relationships of Manu with the other *ardinas* made it difficult to develop the formality and the rituals that were the norm when Disidori was in charge. An additional issue was the smaller need to use the calculator given the amounts involved: 20 escudos of profit (instead of 12,5 escudos in 1998) and 80 escudos to pay back (instead of 87,5). As now most of the *ardinas* received 25 newspapers and sold them all, there was less variation in the situations. The calculator was a tool for computation rarely used and there was no evidence of its mediating role as artefact as it was noticed in 1998.

In terms of the whole activity in place, we should finish this section pointing out (i) the reinforcement of the hierarchical structure of the selling activity within the social world where it takes place (not only dealing with the cultural status that age represents but also with experience and responsibility as the *ardinas* are able to transform their object of activity into new forms along the time); (ii) the forms of talk and signs that become part of the repertoire of the practice of talking about the calculating-in-selling – decimal point (drop), operations (times, more), big numbers; (iii) the understanding of profit in the sequence of actions that allow earnings to happen (the remaining money after paying back which is not matter for verbalization with Disidori); and (iv) the place and status of the common *ardina* in the hierarchy of power positions in the chain involved in the selling.

ARTEFACTS AND RESOURCES: HOW POWER PERMEATES THE TECHNOLOGY OF THE PRACTICE AND THE SHARED REPERTOIRE

The calculator was clearly the tool used more systematically by Disidori, in the moment of payment with all the *ardinas*. It was the best support for the dialogue with them reinforcing on one side the individual character that Disidori gave to participation of each *ardina*, and on the other side the visibility of his power and authority. Within this framework, the calculator assumed essentially a role of reproduction as it was used in order to regulate the participation of the *ardinas* and sustaining the established social order–strongly marked by hierarchy and empty of argumentative elements.

The way the artefact was used, while a mathematical-based tool, had a structuring role in the activity mainly in 'paying back' phase. It reinforced the importance of the act of paying back and the role of authority, and regulated the *ardinas*' participation. However, the regulation made possible with the artefact did not come from the artefact itself but from the way it became present in the everyday and the power attached functions added by participants in the practice. The process of regulation was in accordance to aspects connected to the social world that framed the resources available, the activity, and the people who organized, managed and acted on it. The calculator in the hands of Disidori was introduced as a reified object, with associated strong social meanings, although serving the power and legitimating the actions of someone who did not in fact belong to the community of practice of the *ardinas* but was an officer of the institutional power. The calculator as artefact in the practice had an effect mainly reproductive of the social world.

We can conclude that this artefact, although not being appropriated in its totality by the *ardinas*, was a resource that had an important role in structuring their activity. There was a shared repertoire within the practice of the *ardinas* that reflected the attached nature of power of the calculator in the framework of the activity of selling. However, the repertoire used to compute-in-action was of a different nature[7] as it was based on elements (i) that emerged, on one side, in the structure of the broader social world where the activity was developed and, on the other side, in how people intervening in the activity coordinated it; and (ii) that reflected the motives that where behind their participation in the practice. What really was structuring the activity of the *ardina* was the 'gain' (called for the need of the *ardina*) while what was structuring the computation-in-action was 'paying back' (being honest). Perhaps we could hypothesise that there are sources of power and legitimation of the practice in both fields of the practice of selling and the issue is on the problems of border crossing.

It is the articulation of participation and reification within the practice that allows and orients the construction or re-construction of artefacts with potentialities of going on functioning as resources for new needs that could emerge in the evolution of the responsibilities of the participants. As Engeström (1999) puts it, the functions and use of artefacts are in a constant fluidity and transformation that goes along with the development of the activity. In this sense, the artefacts are not something fixed and external to the practices but are in the development of the practices; its usefulness is not revealed in the characteristics identified independently of its use in the practices where they are put in action. Artefacts are artefacts-in-the-practice; they should be understood in interaction with the forms of use that users develop in those practices.

The concept of technology of the practice introduced by Lave & Wenger (1991) refers to the set of elements, artefacts, which people act with, associating a certain practice to the existence of a particular technology. It is the cultural nature of artefacts which carry part of the cultural heritage and historicity of the practice and relate their use to matters of power and access in the context of the discussion of the problematic character of the reproduction of a practice. The notion of technology suggests a dynamic stability, accepting renovation and transformation while based in the history of the practice. Entering in a new space of participation is thus associated with learning about its history and its technology. The technology is appropriated by participants in forms that serve their needs and goals as well as the opening of affordances to the emergence of power and its manifestations. The calculator as a technological mediating artefact becomes a constitutive part of the practice and provides access to new forms of participant.

In addition, the idea of shared repertoire (Wenger, 1998) includes both a set of forms of doing, taking and acting, and a group of people who share them as resources. The very word 'repertoire' leads us to aspects different from those related to technology and closer to forms of talking, acting and doing, and to stories people tell and share. This refers to a broader spectrum beyond action that presupposes an audience to whom and with whom one acts. Such a collective entity shares constraints and affordances which involve action and interaction. And it is in that process that meanings, power and positioning are negotiated, reproduced and constructed. The notion of shared repertoire directs the attention to the dynamics of using, constructing and sharing certain resources and calls for a view of people as collective constructers and thus collectors of their own constructions. In doing so, it localizes knowledge on the collective and on the circumstances where the collective produces knowledge, uses and reproduces it. The shared repertoire, reflecting the coherence of the practice, emerges as a source of the coherence of the community of practice (Wenger, 1998).

The shared repertoire of a community of practice is permanently in construction *via* participation and side by side with reification. The technology of the practice includes the reified aspects of the practice that almost shadow the role of the practitioners in its construction, while maintaining a memory of its development. Shared repertoire and technology of the practice can be conceptualized as complementary. Focusing on each one gives visibility to particular aspects, in one way to the process of construction (e.g. what facilitates or restricts the access to participation) and in another way related to the history (which allows or restricts access to meanings, comprehension and to the practice itself). In both concepts the key idea of participation is present and it is through participation that one contributes to construction and has access to history as sources of power.

The various artefacts analyzed in the original study (such as the calculator, the recording table, the school algorithms, certain forms of talk, etc.) (Santos, 2004) were present in the everyday of the *ardinas* as reifications; they made part of the technology of the practice of selling newspapers in Praia that every newcomer faced. The ways they were used in the practice gave visibility and reinforced the institutional order inherent to the social world where the *ardinas* activity was taking place. Through interaction with such artefacts the *ardinas* gained access to certain aspects of the practice of selling, sharing meanings of the inherent social world. The artefacts situated in the practice of selling constituted structuring resources although with less direct impact on the strategies of computations of the *ardinas* which were appropriate to their participation in the selling. Such a contribution was rather more visible in the forms of talking and in the social meanings developed by the *ardinas*. The mediating character of the calculator as an artefact in the mathematical-thinking-in-action of the *ardinas* was revealed in strong association to social meanings.

NOTES

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2. *Ardina* is the Portuguese word to refer to kids who used to sell newspapers on the street.

3. Although Portuguese is the official language, the actual language in use in everyday is the Creole.

4. 100 Escudos corresponds to €50.

5. A tool not so strongly interpret as mathematical was the table used by Disidori to record the number of newspapers distributed, sold and returned by each *ardina*. An analysis of the use of this artifact is developed in Santos & Matos (in press).

6. Manu's major structuring artefact was a very detailed table he used as a record of the selling and which was always available during his interaction with the *ardinas*. See Santos & Matos (in press) for an extended discussion of the mediating roles of both the calculator and the table.

7. In Santos (2004) central structuring elements for the *ardinas'* computation during the selling activity were identified: (i) the monetary system; (ii) the selling price of each newspaper (100 escudos); (iii) the gain for each sold newspaper (12,50 or 20 escudos); (iv) the way how the newspapers were delivered (in groups of 25).

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MULTICULTURALISM, HISTORY OF MATHEMATICS AND SCHOOLBOOK OF THE THIRD CLASS IN PRIMARY SCHOOL IN GREECE

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During the last decades the movement that supports the cultural and historical dimension for the learning and teaching of mathematics has set on the stage of the international level the necessity of using elements from the global and local history of mathematical concepts. The communities of teachers of mathematics of the Secondary education but also some groups of Primary school teachers agree with the new possibilities that the introduction of the above dimensions in the teaching of mathematics offers. The big problem that does not permit them to often use all these possibilities is the curriculum and the schoolbooks they use, which demand the development of capacities of techniques for doing the operations and the algorithms that do not permit teachers and consequently students the substantial understanding of mathematical concepts. The fact is that the mathematical education, the books, the training of teachers does not cultivate such approaches. In the core of all of the parameters mentioned above is an extreme formalism. We believe that the introduction at the undergraduate level in the Universities, in the context of the pedagogical Institutes of Training of teachers, in the curricula and in the school textbooks of all classes could change the approach of the teaching of mathematical concepts.

In our presentation we are going to present one example of introduction of elements from the history of mathematics in the context of the school book and of the teaching of mathematics for the third class of Primary education in Greece. Our intervention contains examples from geometry and arithmetic.

The introduction of cultural and historical elements for the teaching of mathematics is a fact for many educational systems all over the world. Our main objective is to exchange with other colleagues from other countries the best practices of all these efforts and gradually develop a kind of paradigm of the best practices which could positively influence the future decisions.

INTRODUCTION

During the last decades many people working in the field of mathematics education accept the thesis that it is very important to show first of all to the teachers of the subject and consequently to their students the way that mathematical thinking and the different applications developed in different civilizations and in different historical periods. This could be another instrument for the teaching and could permit a wider understanding of the concepts embodied in mathematics. The International Congress of History of Science in Mexico City, in 2001 was under the theme of the Cultural

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 398 - 405).

Diversity, and the main characteristic of this Congress was that the History that shows the diversity, rather than the universality of mathematical and scientific development adds a very exciting dimension in the subject because saying that Mathematics has a history can change the image this science has in the eyes of teachers. We can not summarize mathematics as a final product but as a procedure in time and in space. The use of historical elements of a notion or a technique can offer to teachers and also to their students two important occasions for working in two dimensions (Cerquetti et al., 1997).

First, working in contexts of different historical time where they leave the status of a stereotype knowledge which has come from the skies and put in its place a knowledge constructed by the humanity: mathematical notions were not always here but they have been constructed and changed during history for resolving problems and being instruments of response to important questions. By saying that mathematics has a history you give a constructivist image, which goes against the dogmatic impressions for this science.

Second, working in different geographical and cultural areas, because the history of mathematics permits to re-establish the human construction in the context of the different cultures and civilizations: mathematics is Babylonian, Egyptian, Greek and Arabic, Hindu and Chinese, Mayan and Incas, that of the middle ages, of Renaissance or that of the 17^{th} – 18^{th} century. Saying that mathematics has a history permits a multicultural approach to this science.

The history that interests teachers is epistemological, conceptual and historico-cultural and before coming to be a teaching instrument for this subject it should pass through the training of teachers of Primary and Secondary Education. The purposes of this introduction in the training and teaching of mathematics are various and are fixed more or less in the following (Fauvel & Van Maanen, 2000):

- to humanize mathematics,
- to put mathematical knowledge in the context of a culture,
- to give to students the opportunity to change their beliefs for the subject,
- to find and analyze epistemological obstacles and notions that are not very well understood by the teachers and consequently by the students,
- to show that mathematics has a history and was influenced by cultural and social parameters,
- to be another interdisciplinary project that could be studied with the students,
- to develop and enrich mathematical knowledge included in the curriculum.

The ways and strategies used for this introduction in the training and teaching of the subject are: histories, construction of activities, construction of exercises, reproduction of manuscripts, portraits, biographies, interdisciplinary projects, use of primary sources, use of new technologies etc. (Barbin, 1987).

The main characteristics of this kind of intervention in mathematics curricula, training of teachers, textbooks etc. could be a way of working against an ethnocentric view and an entry into different cultures by valuing the different styles and branches of the mathematical activity.

The broadening of perspectives, that the historical study could give us could also give a new impetus to teachers and to their students to search their own background and culture as well as within the cultures of others and (hope) try to understand that what is found is a part of a global heritage rather than merely a national or regional one.

Mathematics is text, artefacts, inscriptions, instruments, books and technical devices that have been developed in different places in different historical periods for particular reasons and the understanding of these reasons can help all of us to relate mathematical ideas to something more global than simply their own immediate environment (Fauvel & Van Maanen, 2000). Integrating elements from the history of mathematics of other cultures and traditions in the schoolbooks of the discipline in an operative manner is one of the major tasks of the mathematical community because in this way we offer to all students the possibility to study all these. In the following we will discuss our effort to integrate historical elements and methods in the schoolbook of the third class of primary school in Greece.

The methods proposed should be in an operative way integrated in the text and help the student to do an operation and to understand a mathematical concept. Our history is not personality based but problem based, which is an important parameter of what we call "Mathematics of Nature and Life". Some of the basic principles of this approach about the teaching of mathematics are the following: it is very important and special attention shall be given to the content of the didactical situations and problems used for the introduction and application of mathematical concepts. Contents and situations must be pleasant and come from the pupils' world and from their preexistent knowledge. They are interested in reporting nature, culture and the history of mathematics (Lemonidis, 2005).

THE CULTURAL AND HISTORICAL ELEMENTS FROM THE SCHOOL BOOK OF THE THIRD CLASS (8-9 YEARS OLD) OF PRIMARY SCHOOL IN GREECE

In Greece, all students use the same schoolbook for mathematics and all other subjects offer every year by the Ministry of Education and under the responsibility of the Pedagogical Institute.

In the following we are going to present paradigms from the schoolbook, the students' exercise books and the book for the teacher (Lemonidis et al., 2006 a, b).

Tangram

In chapter 3 we can see the introduction of Tangram. In this example students observe and find the same figures in different positions on the page level. Via this they are coming to recognize congruent figures in the context of the Chinese mathematical culture. In chapter 42 they can construct the tangram with a carton patron, and they can exercise themselves and play.

The context in which Tangram is introduced is the study of geometrical figures and solids. The objectives and the didactical instructions are that pupils already know the names of the basic geometrical figures and solids. The objective of this intervention is to re-establish the main characteristics of these figures and to reinforce their knowledge. The main goal is that the pupils at the end of this procedure should recognise and isolate a figure from a synthesis of geometrical figures. Another objective is to make pupils to recognize figures in different positions and not only in the prototype position. It is very important for students to discover empirically the relations between the figures. In this direction we can develop activities that permit to use known figures for the production of new ones.

In the exercise book in the activity 2 pupils construct two paper triangles and regard them as two pieces of tangram. They need to put them together and to figure other schemes, the square and the isosceles triangle.

After almost 40 chapters in the context of the study of Puzzles, pavement and mosaics (ch. 42) we use once again the Tangram for approaching the application of optical procedures of analysis and synthesis of geometrical figures. All these activities develop the optical geometrical capacities which are extremely useful. They introduce pupils to the concept of surface to a pre-area study situation and go further to study the properties of the figures.

In this context we can make reference to China civilization and its mathematical tradition.

Multiplication and Division

In the chapter of multiplication and division (ch. 6) we introduce the Pythagorean table. The pupils discover how to read in the Pythagorean table that contains multiplies of numbers by using the lines and columns. We ask them to find some multiplications. They observe the evolution of the multiplications by 1, 2, 5 and 10. We ask them to find how many times and where we can see, for example, the number 24 and to write down the different multiplications.

We give elements on Pythagoras life and activities and on the different kinds of numbers he developed: pairs, impairs, square numbers, triangular numbers and of course his very well known theorem. We propose the use of an interdisciplinary activity by using the field of history and geography.

Numbers to 3.000

In chapter 14, we are working on numbers to 3.000 by introducing the Roman arithmetical system. We are showing to pupils that except the symbols of monads (1, 10, 100 and 1000) there are symbols for the numbers (5, 50 and 500). This is an evolution which does not permit the repetition of symbols like other ancient arithmetical systems. The Roman system goes beyond M (1000). By confronting it with the Hindo-Arabic origin numerals in use today we found that it has basic disadvantages: it has a lot of symbols, repetition of the same symbol and non existence of number zero.

Introduction of the “Greek multiplication” in teaching

In the chapter 29, we continue the practice of pupils on multiplication with the aid of squared paper. We guide them to pass from the figure of the squared paper, where the factors of the product are separated by 10, in the corresponding table of multiplication. This kind of multiplication is called “Greek multiplication”. It was presented for by Eutocius - a Greek scholiast of 6th century A.C. (Mugler, 1972). The Greek multiplication is a good way to introduce pupils to the algorithm of multiplication in use today.

In pupils’ book in the first activity students work in groups and we ask them to cut and construct on the squared paper a rectangle with 24 x 35 squares. When they cut the rectangle we ask them to find how many squares they have in the rectangle. Students give different solutions and we advise them to trace the rectangle, and to figure squares and rectangles and by this way to find the number of squares. By discussing, the teacher, show them how to trace and to separate their rectangle with the way shown in the book: to establish a table with 20 and 4 on the two lines and 30 and 5 on the two columns. We ask them to compare this table with one which could be constructed with the logic of the previous course (a table with 10, 10 and 4 on the lines and 10, 10, 10 and 5 on the columns). At the end we introduce the way by which we can multiply two two-digit numbers by the “Greek multiplication”.

The objective of the activity 3 is to make pupils practice in using the table, to analyze two digit numbers and to calculate the products.

In the exercise book we propose the multiplication of two digit number with one digit number and we ask in activity 2 to complete the products in the table. The objective is to observe that two products represented side by side result by multiplications in two cells of the table. In the following activity (no. 3) we have a multiplication of two digit and three digit numbers and the objective is to make them practice by analyzing numbers and use the distribution property of multiplication to addition.

In the following chapter we introduce the algorithm of multiplication and one of the goals is to interpret the products of the algorithm that come from the multiplication of the multiplier with the multiplicand by using as a base the multiplication table and to consider them as the shorter expression of the products of the multiplication table.

“Greek multiplication” is very useful to students by using a table for analyzing the multiplicand and the multiplier. The two factors of the multiplication are analyzed in units, tenths, hundreds etc. in the columns of the table. We put the analyses of the multiplicand in the columns and of the multiplier in the lines. We propose to put them in these places because there is a correspondence with the classical and typical algorithm in which we put the big number – multiplicand- up and the small – multiplier-down and multiply the down number with the up one.

Eutocius used to multiply by a diminishing series. He multiplied the biggest numbers and afterwards the smaller ones. In the typical algorithm multiplication must be done from right to left, the digits were put by increased series.

In this book the "Greek multiplication" is used as an introductory phase for the introduction of the typical algorithm used today. In more details in the first phase we give to pupils problems for multiplying multidigit numbers to develop atypical methods for the calculation. The "Greek multiplication" is introduced via a geometrical context, with situations of measurement of surfaces in a squared paper. Pupils measure surfaces of squares and rectangles by a small square as unit. For the most quickly and easiest calculation of the surfaces they use the table of multiplication. Via this way they use the "Greek multiplication" with the help of the table. By the use of the "Greek multiplication" we can give explanations to the typical algorithm. The goal is that pupils can understand the way of production of the products and the place-value of numerals of the factors of multiplication.

The above examples are included to encourage teachers to find different methods from history for doing calculations which have applications that are both modern and appropriate.

Decimal fractions and decimal numbers

In chapter 35 we have the decimal fractions and decimal numbers, where we make a reference to the historical appearance of decimal numbers by writing that fractions were known from Antiquity. They were used by Egyptians in the 2^{nd} millennium B.C. In Europe mathematicians used them in contrast with the decimal numbers. Al-Kashi, an Arabic origin mathematician (died on 1429 A.C.) was the first who introduced the theory of decimal fractions and established that operations can be done by the same way as integers. We have to wait until the 16^{th} century, when Simon Stevin (1548-1620) a Flemish mathematician introduced the way of writing the decimal numbers with the following way: 5,237 were written 5 0 2 1 3 2 7 3.

He has noticed that this number is equivalent with 5 + 2/10 + 3/100 + 7/1000 or 5.237/1000.

Numbers to 7.000

In chapter 40, for approaching numbers to 7.000, we use the arithmetical-alphabetical number system from ancient Greece

In the context of pupil's book, in activity 1, we have the introduction of the ancient Greek arithmetical-alphabetical system. This is not an arithmetical system with place – value numerals. In this system they have not a notion or symbol of zero. The goal is not to learn this system but to experiment with it and to use it, by transforming numbers from one system to another. Pupils are invited also to make comparisons with the Roman arithmetical system.

Motifs

In chapter 48 we introduce and practice arithmetical and geometrical motives and especially in activity 3 we use Pascal's triangle which is a very well known arithmetical triangle and has been used to resolve problems on combinatorics and probabilities.

INSTEAD OF EPILOGUE

These examples help us to see the different aspects of mathematics and its development can be discussed when a problem-based approach is enhanced by adding a multicultural dimension to the teaching of mathematics. These could help students to see ways in which differences in historical periods, geographic location, culture and beliefs have influenced developments in mathematics (Fauvel & Van Maanen, 2000). In turn, they may help students to understand better the concepts of operations such as multiplication etc. This approach allows students and teachers to think of mathematics as a discipline of continuous reflection and action influenced by thoughtfulness, reasoning, known procedures, intuitiveness, experimentation and application of practical situations as is always the teaching context of the subject in the Primary school education.

In the teaching of mathematics in the third class of Primary school in Greece there are opportunities for introducing aspects of the history of mathematics through examples (Roman arithmetical system, ancient Greek alphabetical number system, Greek multiplication, tangram, Pascal's triangle, decimal fractions and decimal numbers, Pythagorean table) from different cultural perspectives (Nikolantonakis, 2005). Pupils and teachers are able to see how one culture or one group of people, or one geographical area has influenced another or added to understanding already gained in a different setting. They can see that mathematics is a human enterprise and not a constantly upward movement towards perfection.

Studying and understanding the methods that other groups of people have developed in response to their needs may well help students to identify the particular characteristics of the method being taught to them and thus better understand a particular concept. It opens up the possibilities of comparisons and the recognition of diversity and enables us to see that an exchange of ideas can be made from the security of a mutual concern to explore mathematical concepts in the environment of the people who use it.

Multiculturalism is the identification and celebration of diversity, the respecting and valuing of the work of others, the recognition of different contexts, needs and purposes, and the realization that each society makes and has made important contributions to the body of knowledge that we call mathematics. Given this view, the inclusion of a multicultural dimension in our teaching of mathematics makes a significant contribution to humanist and democratic traditions in mathematics education.

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THEORY-PRACTICE TRANSITIONS AND DIS/POSITIONS IN SECONDARY MATHEMATICS TEACHER EDUCATION

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One challenge for mathematics teachers and teacher education programs lies in translating socio-cultural theories into pedagogical practices. The study described in this paper endeavours to address the challenges faced by secondary mathematics pre-service teachers as they negotiate the transitions between/within university curriculum courses and their classroom field experiences as interns. This paper presents several theory-practice transitions, including those of the researcher, as it seeks to dis/position the power and tradition at the centre of secondary mathematics teacher education.

INTRODUCTION

Mathematics education research documents a range of personal, political, and social factors that influence the development of mathematics teachers and their pedagogical identities (Goos, 2005; Kaartinen, 2003; Lerman, 2005; Walshaw, 2005). If these factors are viewed through socio-cultural and poststructural lenses, several possible dimensions of a mathematics teacher's pedagogical identity can be revealed. However, even in acknowledging the multiple dimensions of identity, these "collections of stories about persons" (Sfard & Prusak, 2005, p. 16) as mathematics teachers are still strongly influenced by the conservative power of school tradition and culture. According to several researchers (Jaworski & Gellert, 2003; Lerman, 2005), traditional textbook and teacher-directed approaches still dominate mathematics classroom practices because of a number of socio-cultural issues relating to institutional structures, the perceived nature of mathematics, acceptable styles of interaction, and personal epistemological beliefs. These issues are often neither trivial nor overt in the lives of pre-service teachers but, instead, embedded within personal and professional ideologies at work in the classroom.

According to Kaartinen (2003), one challenge for mathematics teachers and teacher education programs lies in translating socio-cultural theories into pedagogical practices such that a reformed participatory approach to learning mathematics is stressed. In other words, instead of emphasizing traditional modes of interaction between teacher, student, textbook, and standardized tests, a participatory approach emphasizes social interaction and discourse, student problem posing and solving, distributed expertise, and the recognition of collective meaning making. But why has it proven so difficult to realize such participatory approaches in mathematics classrooms (Nolan, 2007)? Perhaps the reason lies in how participatory approaches fly in the face of typical classroom and teacher concerns, such as covering large amounts of curriculum and demanding that all students attain similar levels of 'understanding' and 'expertise' within the same pre-given timeframe.

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The study described in this paper embodied the characteristics of participatory approaches in an attempt to address the challenges faced by secondary mathematics pre-service teachers as they negotiate the transitions between/within university curriculum courses and their classroom field experiences as interns. By viewing the mathematics classroom through socio-cultural and poststructural lenses, the study aimed to understand more about theory-practice transitions through a deliberate effort to ground theory in the practice of innovative instruction and assessment in mathematics classrooms.

THEORETICAL FRAMEWORK AND PURPOSE OF STUDY

While there are a range of theoretical landscapes for describing and understanding how/when/if learning occurs, socio-cultural views of learning are being drawn upon more and more by educators and researchers due to an increasing belief that learning embodies social, political, historical and personal dimensions. To explore learning through socio-cultural lenses means to open the nature(s) of learning to scrutiny by (1) viewing learning as situated with/in the social interactions of members of a social group (Bauersfeld, 1988), (2) understanding cognition to be both in the minds of individuals and distributed across communities of practice (Eames & Bell, 2005; Wenger, 1998), (3) exploring how particular practices of schooling are implicated in the constitution of teacher and student identities (Walshaw, 2005) and, (4) exploring how meaning is negotiated through the cultural tools (especially language) that operate within school discursive practices (Lerman, 1994). In addition, research with/in a poststructural framework can highlight the importance of a critical mathematics education (Skovsmose & Borba, 2004) by drawing attention to assumptions that remain unquestioned while highlighting possible alternative images of mathematics practices and discourses (Nolan, 2007).

In contrast to research that merely directs attention to the lack of innovative instruction and assessment approaches being used by secondary mathematics pre-service teachers (i.e. a description of *what is happening*), this critical study incorporated a more constructive and feedback-oriented approach to understanding the conditions necessary for pre-service teachers to try such innovative approaches (i.e. an exploration of *what is possible*). In order to better understand what is necessary for new approaches to gain access to, and hopefully change the face of, the mathematics classroom, this study exemplified a progressive intervention into what might otherwise function as traditional mathematics classroom situations.

DESCRIPTION OF STUDY

The aim of the study was to mentor pre-service secondary mathematics teachers as they negotiated transitions from the theories of a university curriculum course to the practices of the classroom. The question posed in the study was: What happens in a secondary mathematics classroom when pre-service teachers who have been introduced to alternative and innovative instruction and assessment strategies in a university-based curriculum course attempt to realize the strategies in practice? Since this question was explored throughout the pre-service teachers' internship semester, the research study was really about viewing the mathematics classroom as a curriculum laboratory (Vithal, 2000) where these new ideas could be tried under the guidance of experienced cooperating teachers and a mentoring teacher educator.

The research study was designed as a case study to investigate the experiences of three pre-service teachers during their internship in secondary school mathematics classrooms. The study emerged out of a recognized disconnect between the theory of a university-based curriculum course on alternative instruction and assessment and the practical implementation of these ideas in mathematics classrooms. The university curriculum course focused on studying the theory and practice of alternative instruction and assessment strategies such as problem-based learning (PBL), technology-integrated pedagogy, portfolio assessment, journal writing, student interviews, and self-assessment. The strategies clearly represented a paradigm shift in mathematics teaching and learning for the pre-service teachers enrolled in the course. Their perceptions of what it means to know, to teach, and to learn mathematics did not readily enable (let alone encourage) them to integrate these new and different ideas into practice. In fact, as the instructor, I encountered substantial student resistance to the course based in their perceptions of the "reality" of mathematics classrooms, curricula, and students. From my perspective as both a teacher educator and researcher, I sought to design a means to assist pre-service teachers as they negotiated their way through the theory/practice transitions— ways to enable teachers to resist the strong current of tradition once inside the classroom walls. Desirable transitions between theory and practice demand fluid movements between university and school, including a more reflective and mutually supportive relationship between practicing teachers, teacher educators, and pre-service teachers.

THE RESEARCH "PLAN"

Acting in the capacity as both the researcher in this study and the instructor for the university curriculum course, I wanted to make a deliberate effort to re/position the course discussions, assignments, and learnings into secondary mathematics internship classrooms. My main criterion for selection of the case study pre-service teachers was that they were willing to make an effort to incorporate alternative instruction and assessment practices into their internship classroom. The methods chosen to gather data in the study were such that the pre-service teachers' beliefs, concerns, and practices could be brought to light. The "planned" methods included individual interviews with the three pre-service teachers (monthly), focus group discussions with the pre-service teachers and their cooperating teachers (monthly), and maintaining an ongoing reflective artefact in the form of a written journal or a weblog. By audio-taping and then transcribing the interviews and discussions (and also by keeping a researcher's journal), I sought to identify the challenges or questions encountered by the interns, along with general thoughts and feelings regarding the research conversations. In addition to these formal methods for data collection, my commitment to an on-going mentorship approach meant that I planned to maintain regular contact with the interns throughout the semester through individual conversations (in person, via telephone, and e-mail).

After selecting the three case study pre-service teachers, my "plan" of research included the following sequence of events: (1) meet with each of the pre-service teachers individually to discuss the instructional and assessment strategies they preferred to try in their classroom, (2) work with each of them to create a tentative plan for implementing these strategies into their internship semester, (3) mentor the pre-service teachers along the way to help them with the implementation, and (4) interview them to understand the challenges and successes of the theory-practice transitions they were experiencing. As will be described in the next section of this paper, such research "plans" did not come to fruition for a variety of reasons.

THE RESEARCH "REALITY"

The carefully outlined plan of research described in the previous section can be viewed through a socio-cultural lens itself. As it turned out, the research plan of exploring pre-service teachers' theory-practice transitions had to be modified quite dramatically because of social, political, and cultural factors that came into play while attempting to "realize my research agenda". It is at this point that this paper itself becomes one in transition— branching into two paths of discussion. One path remains focused on the theory-practice transitions of the three secondary mathematics interns, while another path moves the discussion in the direction of the theory-practice transitions experienced by the researcher. The intent of this paper is to briefly describe the topography of these two distinct paths, but each has also been written about in greater detail in separate papers (Nolan, *forthcoming 'a'*; Nolan, *forthcoming 'b'*).

THEORY-PRACTICE TRANSITIONS: FOCUS ON RESEARCHER

It is not unusual for a research process to change, regardless of how carefully the methods and methodologies are considered in advance and contextualized to the question at hand. I expect, however, that researchers seldom write about what they planned to do, but didn't. Writing reflexively about the changes and modifications to the research process is, I believe, more common in postmodern projects where one seeks to highlight the tentative and in-flux nature of research. There are two issues I wish to bring forth and briefly discuss with respect to researcher (and research process) theory-practice transitions (Nolan, *forthcoming 'a'*). It could be said that both of these issues emerged from a naïve notion of the in/visibility of the researcher.

In/visibility: An agenda in disguise

In the first few pages of their book, Brown & Jones (2001) caution postmodern researchers with an emancipatory quest that "[a]ny emancipatory perspective presupposes values which cannot be agreed upon universally or permanently. If we fight for something we are always working against someone else's interests and there are difficulties in creating a robustly moral perspective that will be seen as better by everyone" (p. 4). My research agenda was naïvely presented from the perspective of a concerned teacher educator trying to learn from her students how to *do things better* in a mathematics curriculum and instruction course. In reality, however, I believe that underlying my research plan was a desire to learn from my students how I could *convince* them that alternative instruction and assessment (as described and modeled in my curriculum and instruction course) IS the way to *do things better*. In a sense, then, my research agenda was disguised as an open exploration of how to improve the teaching and learning of mathematics, while it was really quite closed after all. Brown et al. (2007) would basically agree with this self-critique of my research agenda since these authors "question the efficacy of a research agenda predicated on encouraging teachers to align themselves with a particular model or philosophy of practice" (p. 184) in the name of 'improvement'. In other words, even though my agenda seemed focused on a goal of expanding their repertoire of instruction and assessment strategies, it seems as if what I *really* wanted (and expected) was for them to buy into the notion that my ideology is better than theirs! Ritchie and Wilson (2000) propose that change will not happen as long as we, as teacher educators, believe we can *do* or *give* something to our pre-service teachers that will 'emancipate' them from cultural narratives that tie them to traditional practices and views on knowledge (p. 180).

In/visibility: Stepping back

A second key theory-practice transition of the researcher and research process occurred during the first step of my four-step planned research process (outlined previously). Perhaps I waited too long to implement this first step because, by the time I met with each of the case study pre-service teachers to discuss the instructional and assessment strategies they wanted to implement in their classroom, they had already spent considerable time with their co-operating teachers "learning" about all of the limitations they would experience in the secondary mathematics classroom. They learned, among other things, that they would not be able to experiment much with alternative strategies because the curriculum was too full and time was too short. Hence, my first meeting with the pre-service teachers, with the co-operating teachers at their sides, was one of intense angst for me. I could say very little that would be taken seriously; I could feel their gaze saying to me that my ideas belonged in the ivory tower milieu, where theory resides on the surface and practice falls through the cracks.

During this first meeting with interns and their co-operating teachers, I found myself being very cautious in how I introduced and described the goals of the research project. I prepared myself in advance of the meeting to tread softly and speak quietly about my goals, and to tone down my criticality. *I* was confident that implementing alternative instruction and assessment strategies would create opportunities for the currently unsuccessful mathematics student to experience and demonstrate mathematics knowledge in diversely legitimate ways. However, when speaking with cooperating teachers and interns, I wanted to be cautious in how I advocated for changes in mathematics teaching and learning. I felt that, ultimately, my open and critical expression of a desire to *change* practice reflects dissatisfaction with *current* practice. How was I to express such a dissatisfaction with current practice without alienating myself and the research project from the practicing teachers and interns? As part of the conversation, one cooperating teacher responded to my call for more student-centred problem solving by saying, "I tried teaching in more constructivist ways where the students try to solve the problems on their own, but the students said they preferred it if I just did an example first and then they could follow it to do more." I felt strongly that this teacher was *explaining away* the obstacle of student resistance to alternative ways of learning mathematics by claiming that students do not, in fact, learn better through these more participatory approaches and that they prefer the way things are done now (Nolan, 2006). I wanted to express my conviction that students have learned to play the rules of the game over many years and so it is expected that they would resist changing the rules and/or the game without understanding why, but I remained silent. In remaining silent, I know that I took a step back from my research agenda.

THEORY-PRACTICE TRANSITIONS: FOCUS ON SECONDARY MATHEMATICS INTERNS

As discussed in Nolan (*forthcoming 'b'*), the data from two interviews and a focus group session with the case study pre-service teachers was initially read with the intent of understanding the dynamics of transforming university course curriculum theory into mathematics classroom practice and the role these transitions play in shaping one's identity as a mathematics teacher. In doing so, however, issues came to the fore that demanded attention and deconstruction.

Adopting Bourdieu's concept of "dispositional harmonization", Noyes (2004) defines habitus as "a set of dispositions fashioned in the peculiar social milieu(s) in which they originated but that also, through the outworking of those dispositions, restructure the social space" (p. 246). I refer to the themes constructed out of the data as dis/positions because they have been fashioned out of the classroom culture and student experience but they are, in the words of Bourdieu, "a structuring structure" (Bourdieu, 1984, as cited in Noyes, 2004, p. 246); that is, they are not merely constituted, but they also have the potential to constitute. In other words, I believe that if the pre-service teachers could critically analyze and reflect on these dis/positions then there would be potential to restructure the space from which they originated. The research "reality", however, was such that this key step of critical reflection (over such a short period of time) did not occur and so the dis/positions continued to structure the milieu, functioning almost seamlessly as regulative discourses within secondary mathematics classrooms. As Lerman & Zevenbergen (2004) state:

> … the regulative discourse, which remains invisible, manifests the criteria by which students are judged as complying or not with the cultural order… It is through the regulative discourse that the instructional discourse gains its internal logic—how the interactions and content are framed, sequenced, and delivered. For any curriculum, including mathematics, the regulative discourse contains ideological elements which are often unknown or unrecognized by the participants. (p. 33)

Through the analysis of interview and focus group transcript data, several dis/positions were identified as regulative discourses operating in the secondary mathematics classrooms of the interns involved in the study. The themes of these dis/positions are: *time* constraints, discomfort with *innovation*, the culture of *tests*, traditional classroom *structures*, student *discipline* and management, and the labelling of mathematics '*strength'*. It would be too daunting a task to describe these regulative discourses in this brief paper; they are de/constructed in considerable detail in Nolan (*forthcoming 'b'*).

EDUCATIONAL SIGNIFICANCE OF STUDY AND FUTURE DIRECTIONS

As mentioned previously, this study arose out of a perceived necessity to design ways to assist pre-service teachers in negotiating the theory-practice transitions in becoming a secondary mathematics teacher. The design and purpose of the study corresponded well with, and responded directly to, ideas and recommendations emerging out of recent research on teacher education and induction:

- deconstruct the myths inherent in perceptions of mathematics and what it means to teach and learn it which are rooted in a belief that *learning* mathematics is about memorizing rules and procedures and so it follows that teaching mathematics is showing how to memorize and execute them effectively and efficiently. As Jaworski & Gellert (2003) remind us: "How mathematics itself is perceived… emerges as central to the way it is taught" (p. 845).
- study the induction and mentoring of novice teachers through educative mentoring by, and joint reflection between, exemplary support teachers, tutors, and teacher educators (Feiman-Nemser, 2001; Jaworski & Gellert, 2003)
- resist describing theory and practice in dichotomous language when it is more valuable to "consider theory and practice… as reflexively connected elements of knowledgeable activity" (Jaworski & Gellert, 2003, p. 832).

This was the theory and purpose of the study; the practice was, indeed, a different story. This paper can be said to be in transition because the study itself is in transition. In one extension of this research initiative, I propose to conduct case studies with pre-service teachers spanning a four-year time period. The study will focus not only on their internship semester (fourth year) but also on their pre-internship semester (third year) and their first two years of teaching. This extension to the study is anticipated to be of significance since recent studies (Muis, 2004) have drawn attention to the fact that few longitudinal studies have been conducted into the potential impact of teacher education on challenging and changing teachers' beliefs and practices regarding reformed practices in the teaching and learning of mathematics.

Additionally, in the spirit of a confession to designing the study as a "my-ideology-is-better-than-yours" approach, it is worth revisiting the nature of a study that jumps to practical solutions without sufficient theoretical explorations of the problem(s). In retrospect, it seems apparent that I designed the study to propose a clear and obvious solution (alternative instruction and assessment strategies) to what seemed to be a very clear and obvious problem: that the "exercise paradigm" (Skovsmose, 2008) currently visible in secondary mathematics classrooms is simply not working to engage students in the learning of mathematics. In other words, the goal of the study was surreptitiously embedded in a move to replace one ideology (viewed by this researcher as 'the problem') with another one (viewed as 'the solution')—where the 'solution' ideology is perhaps just as suspect in the long run as the current 'problem' ideology. Brown et al (2007) discuss Althusser's conviction that believing there could be one 'consensus' ideology that works for everyone is the most mistaken ideology of all (p. 187). Hence, a very different extension of this research initiative—an appendage, so to speak— involves stepping back from classroom practice solutions to focus more on theory in/of mathematics teacher education and the study of teacher identity. *AFTERmath Education* (the new study underway) works to reveal and deconstruct the *A*porias and *F*issures in/for *T*eacher *E*ducation *R*esearch and *M*athematics *E*ducation. This theoretical appendage to the body of classroom practice research holds promise for delving into mathematics teacher education from new perspectives— exploring the doubts and cracks in pre-service teachers' images of what it means to know, to teach, and to learn mathematics, with a goal to understanding more about the cultural and discursive landscapes of schooling that currently work to maintain the power of dominant school traditions and regulative discourses in mathematics classrooms.

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IF SCHOOL IS LIKE THIS, THERE IS NOTHING WE CAN DO: SOME THOUGHTS[1]

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Our goal is to share some of our thoughts concerning society, school and mathematics. We try to understand how school mathematics is more related with governance instead of knowledge. The real important thing in school is not to assure the formation of citizens with knowledge, but the formation of citizens that operate in the broad society. Using the Foucault's language, people who will be simultaneously docile and useful. The knowledge, in that sense, is only an alibi to the construction of the desirable subject. Taking that point of view we make some considerations about the role of the teacher and the researcher concerned with social change, trying to highlight the gap between the critique and the struggle to change the reality, which we criticize, in order not just to affirm the capitalist system, but transform it.

SCHOOL AND MATHEMATICS IN SCHOOL: THE KNOWLEDGE AS AN ALIBI

School and Society

The expression "knowledge as an alibi" is not ours. It belongs to Roberto Baldino, a Brazilian researcher. Baldino (1998a) wishes to address to the fact that knowledge appears in school only as an excuse to education. The real important thing in school is not to assure the formation of citizens with knowledge that allow them to be participative agents in society, but the formation of citizens that contribute to the normalized society. Using the Foucault (2004) language, schools form people who will be simultaneously docile and useful. The knowledge, in that sense, is only an alibi to the construction of the desirable subject.

And how is the society in which we live in? Well, it is the capitalist society, described by Marx more than one hundred years ago, and, basically, consists on the fetishes of merchandising and the growing of capital, that originates different huge disparities of richness all around the world, because of the generation of surplus value. Although all the disparities caused by the capitalist system, the strong force of it is the total exclusion of possibility. The capitalist system, since the decline of the Soviet block, as became the only and in our days, the "natural" way of living in society. Because of that, we are losing the imagination to think alternative ways of living. In fact, it has disappeared completely from our official political life all kinds of divergent struggle and, although with some exceptions, it also disappeared the possibility of imagining another way of producing and different ways of living from the one that became hegemonic. It is very common, when we start to criticize the system, at the point where we realized that it is a system that generates poverty, inequality and huge doses of welfare that feeds apathy, we face the criticism of the

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 416 - 423).

question "How could we live in another way?" or even worse, if we sunk ourselves in the critique and realise that is not possible to change the system within the system, we could ended up on imobilism. So, most of the time, like is made in our days by the leftist political wing, we continued to criticize the system, but happily living and, inevitably, perpetuating the capitalist system.

School is part of the capitalist society in which we live in, and, inevitably, although all the changing power that usually are addressed to it, will reproduce that kind of society. Inevitably? Well, we are not sure that school, only by itself, could be a force for the change of society; simply by the fact that school, like other institutions are part of that society. But, by criticizing society and school, we could at least have the honesty to say: as a professor committed to the education of people in a school, I am a hypocrite. But there are ways of going beyond hypocrisy. But those are the ways of struggle.

The role of school

In our days education has became disciplinarized, schoolarized, academic-ized. Like mention by Stoer et al. (2001), schoolarization has become the only legitimacy modality of thinking education. In school, education is a disciplinary device (Foucault, 2003), which fabricates the individual. Following Foucault is thought, school has became one of the bigger modern disciplinary centres of the body. Obligatory in the modern societies, in school we are introduced to the disciplinary society, via academically recognized knowledge, by the way we submit our bodies and minds to the training devices (it exists in school a huge load of corporal discipline, whether is in the space organization, or in rules as norms about what is considered to be good and bad behaviours). It is in school that the human being, no longer a person but a student, become to understand the hierarchy of behaviours and knowledge, by the means of the creation of classificatory systems that limited them, integrate and exclude them.

Roberto Baldino (1998a, 1998b) has been trying to go further on the task of showing that is in school that the individual learn how to participate in the process of capitalist production. Taking as a starting point the school evaluation system that allowed the selection by means of failure, Baldino shows how in the social practices that occur in school, students, teachers and other employees, they all participate in the transformation of the students is working force, initially poor and without qualification, in a bigger value merchandise, sold by a price that, we all expect, will be greater in the future.

In that way, school is a place where the potentiation of the work force could occur, representing the students in this process the role of the workers (as a social being actively engaged in the process of potentiation) and the role of the capitalists (as owners of a merchandise – its own work force – whose value will increase). The working force will only belong to the student in the end of the course if he or she will get approved. If not, the work that he or she developed during years of studying will be appropriated by those who get the diploma:

> Only those who finish the course and succeed *appropriate* of the work made by all. With their work, students produce a value. Those who fail or abandon the course, even in the last year of it, only get a small fraction of the work they have produced; this incomplete graduation by itself ensures the student a better work. When the student graduates, he/she will assume other structural position as effective owner of the produced value. As it is, he/she appropriates of all the work, inclusively the signification work made by him/her and his/hers colleagues, which also includes those who abandoned. (Baldino, 1998a, p. 8)

School is far from being the place of education. By the contrary, it has developed the function of reducing, dominated and suffocate education, by the ways of reinscribe it on the interior of the state apparatus. Like mentioned before, knowledge in school works as an alibi. An alibi to the formation of the subject needed to the modern society.

The purpose of mathematics in school

In official documents it is easy to understand the four normally explicit arguments that justified the teaching of mathematics in school: because it is useful to the student in his daily life; because is an important cultural heritage that must be preserved; the argument that the study of mathematics develops psychological skills; and final, the most divulgated in the recent years, because it contributes to the development of participative citizens in a socio-political perspective.

All of those arguments are a decoy. It is easy to deconstruct them and show how fragile and incoherent they are, especial the socio-political argument (Pais, 2005). Mathematics appears in school with other surreptitious functions. Let´s analyse them by the eye of the foucaultian notion of power.

The first idea to retain is that power constitutes rather than constrains the subject. The relations of power, that are not just hierarchy relations but, above all, microrelations (Foucault, 2004), constitute the subjectivity of the capitalist society. On a basic level, power for Foucault is the capacity that something has to alter the other's conduct. In that sense, school mathematics exercises a power that differentiated them from other school subjects. In a study by Gomes & Queirós (1999), the authors tried to understand in which ways the options of the students are, in most of the cases, determined by the results on mathematics during the schoolarity. As mention in the study, the performance on mathematics has been a decisive criterion to select students, especially in what concerns to access to job marked by its technical or scientifically dimension: "those who have bad results on mathematics will be discourage on taking a career on engineering or a course on sciences. Implicitly, mathematics became decisive on the professional options of many students" (p. 8). Mathematics ended up exercising a power in the sense that takes the students to set on a profession, that will be part of their life, not by motives of willing or competence, but by a presumable incapacity to have success in a particular school subject.

If, on the one hand, school mathematics influenced the professional choice of the individual, it is not just where it shows its power as a school subject. It is already well documented in studies about the emotional aspects of school mathematics, like for instance the research made by Ilda Couto Lopes (1993) about the affective aspects involve in the school mathematic activity of students, that the failure in that subject is motive of discontentment and disorientation to many students. The bad experiences with mathematics tends to be more traumatic to the student, because he or she feels the importance that mathematics has in curriculum, to his future, and also to his social image (not knowing mathematics is not know how to think rationally).

On the other hand, not to achieve success in mathematics catalogues the student as incompetent, inapt to the demands of a society more and more technological. The numbers of failure demonstrate it, as well as the scholar abandon which every year, in Portugal, sweeps to the dust-bin of social exclusion 15 000 to 17 000 students.

The power of school mathematics is immerse, in the sense that it shapes us to something, formats us, alters our conduct, changes our life aspirations, causes us emotional harassment and familiar conflicts. Mathematics education, as a part of the school system, tried to imbue children in a world that doesn't belong to them, in a world where the valued knowledge is the scientifical one, and the exercise of power is a system.

Here we need a very important notion from the work of Foucault. School mathematics acts like a device that makes behaviour elements to be reinforced or punished, as they adhere to the rule or note. Or, using Foucault's terminology, like a normalization device. As said by Popkewitz (2002), who takes into account the work of Foucault on education:

> [the mathematical curriculum] is an inscription devise that makes the child legible and administrable. The mathematics curriculum embodies rules and standards of reason that order how judgments are made, conclusions drawn, rectification proposed, and the fields of existence made managealble and predictable. (p. 36)

From these lines emerge another purpose for mathematics' teaching. An implicit and tolerated argument but, by that, very powerful: teaching mathematics to approximate the student to the norm, to normalize, by putting in motion mechanisms that penetrated the bodies, the gesture and the behaviour. Normalizing is associated with the governance of the people, by "controlling their multiplicities, use them at maximum, maximizing the utility of their work and activity, thanks to a system of power susceptible of control them" (Foucault, 2004, p. 105). Basically, "making grow at the same time the docility and the utility of all elements of the system" (p. 180).

AND NOW? WHAT CAN WE DO AS TEACHERS AND AS RESEARCHERS?

The example we will describe next is paradigmatic on many researchers on mathematics education. Basically, it consists on knowing and admitting that the big problem of scholar mathematics' failure lies outside the didactic of mathematics, understood has the research for theory or tools for better teaching and learning mathematics, and, paradoxically, continuing to research only the didactic or psychological aspects of school mathematics.

In the last ten years in Portugal the debate about mathematics education has been very inflamed, due to the shock between different ways of understanding education and the role of mathematics in school. In a country were the failure in mathematics is very severe, several ideas about how to surpass this crisis have been flourish in the last years, most of theme impregnated on ideology, and conveying conservative measures like an increase of examination, a focus on pure mathematics contents, and a more authoritarian power to the teacher in the classroom. Those measures are not taking into account considerations about the predisposition of students to learn mathematics, or other social aspects of mathematics learning. It is just taken for granted that all students will naturally learn mathematics if the curriculum is good, and the teacher efficient.

Many mathematics educators have criticize this ideas, and, based on decades of mathematics education research, made a much more lucid and complete analyse of the problem. Taking as example João Pedro da Ponte, one of the most prominent math educators in Portugal, he developed a very exhaustive description of the problem of the failure in mathematics and its causes. In one sentence he resume what he thinks it is the reason why mathematics is a discipline condemned to failure:

> The fundamental reason why we have failure on mathematics is because this discipline is socially conceived precisely to lead to failure. It is a result of the function that is attributed to mathematics in the educative system and interiorized by all the intervenient in the learning process. In truth, the big role of mathematics is to serve as a selection instrument of students. (1994, p. 5)

In another text, Ponte (2003) pointed out the «crisis» in society as the first cause for the failure. Doing that, Ponte take into consideration all the social and political dimension involved in the problem of failure in school mathematics, showing a more deep knowledge of the problem. However, when, in the same text, Ponte pointed the possible resolutions to the causes he identified for the failure none of them focus on the so call «crisis in society», neither in the fact that, like he said, failure in mathematics is a result of the mathematics' function in school.

That happens because Ponte assumes that the problems that most of the teachers feel related to the failure of the students on mathematics are not problems that can be put on the field of mathematics education research. By this way, we have teachers feeling the problems, and also researchers describing the problem as something "social" or "political", and simultaneously are forbidden to think or research about them. At least, in the mathematics education field.

The idea is that, although we have knowledge of the importance of the social context, that is often characterized has being on crisis, it is not taking into account in the mathematics education research, because it is not specific of that field. In that way, the analysis of the social situation that are identified has problematic is excluded from the research. Because of this, a lot of the scientific results on math education are results made to an abstract society, that convey the idea that if the crisis in the society will be solve, then those are the results that will allowed us to achieve the success on mathematics. But who will solve this crisis if not us that identified society has being in crisis?[2]

The assumption that the social and political issues that influenced the mathematics' teaching should remain outside the mathematics education's research, because that is not their vocation, it is related with the process of specialization characteristic of modern science, and the will to delimitate the field of investigation, in order to be able to achieve the status of scientific area (Pais, 2005). With the specialization becomes difficult to the scientist to understand the several problems that worried people on a global and local level. Paulo Freire (1998), describes very well the way in which a scientist specialized on a given area looses his capacity of thinking in broader ways:

> Distinct from specialities, to which we are not opposed, specialisms narrow the area of knowledge in such a way that the so-called "specialists" become generally incapable of thinking. Because they have lost the vision of the whole which their "specialty" is only one dimension, they cannot even think correctly in the area of their specialization. (p. 516)

Also Baldino(1998a)

> Maybe they say that Mathematics Educaton cannot, because its lack of tools, open certain questions that are seen as being of the field of sociology. This argument is, at least, curious, because it supposes that those "certain questions" are present in the classroom, and doing so admiting that the teacher is efectively dealing with them, simoultanously, preach that he cannot think them, or refer to them. That make us think that, may be, the obliteration of discussion are taking care in order to allowed the teacher to exercise some particular solution of that questions, with more efficacy as less he knows what do do. (p. 5)

From here it follows that is convenient to the system that the teacher continued to effectively teaching their classes, investing more and more in their didactic formation and the researcher to continue to investigate problems and strategies to improve the mathematics teaching and learning. The fact that society is in crisis, or the feeling that the problems of failure in mathematics are situated in another sphere, are not part of the practice or research. That desresponsability, by one side, absolve the teacher and the researcher and, by other side, guarantee the absence of critique inside the system.

We must understand that is favourable to our society that the teacher recognize himself as someone who is concerned with the development of the capitalist society, than, on the opposite, like someone who supports that society. It would be unbearable if the teacher understands himself as someone responsible for the state of the same society that he criticizes. So, understand himself as an element that personally criticizes the capitalist society and exercising in their daily work what he thinks will make the difference, it turns out integrating the big capitalist machinery. His work, as a teacher, has taken place already conditioned by a set of disciplinary mechanisms, which shape it in order to maintain the system moving. His work happens on an episteme (Foucault, 2002), or under the effect of a system of reason that will conduct not to the transformation of the system, but to its affirmation.

SOME WORDS

Firstly, let us researchers concern with social change, do our research on what we consider to be the core of the problem: the capitalist system. By studying mathematics and school mathematics under the light of the broad capitalist society, with its discourses and mechanisms, we could develop a critique on how scholar mathematics is so implicit in that system that any possibility of local change is rapidly and tacitly rejected or "phagocyted" by the system. Secondly, as teachers, if we want to make a difference and put an end to our hypocrisy, we have to struggle. Struggle in the classroom, struggle in the school, in the community and with the state. Just because being critical and being a teacher, in our days, is a contradictory task.

NOTES

1. This paper was prepared within the activities of Project LEARN: Technology, Mathematics and Society (funded by Fundação Ciência e Tecnologia (FCT), contract no. PTDC/CED/65800/2006.

2. If all researchers act this way, they are an innocuous piece on social change. This is to say that our work as researchers is not changing the world, but just to identify a problem in a specific field scenario and studied it. Ubiratan D'Ambrosio and Paulo Freire said the opposite, that our work as researchers is subordinated to a higher goal.

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A PLURALISTIC VIEW OF CRITICAL MATHEMATICS

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What counts as critical mathematics (CM)?What mathematical and political messages characterize CM and how explicit do they have to be? How do the social context of instruction and the marginal positioning of students figure into whether a pedagogical practice is CM and politically engaging? We argue a pluralistic conception of CM. We posit that what counts as CM depends as much as on the students—their sociopolitical and institutional positions and their goals—as the explicitly political nature of the curriculum. Our discussion points to problems with what we believe are current, overly narrow theorizations of CM.

In this paper, we outline a perspective of critical mathematics that admits varied forms, each characterized by attention both to mathematics learning and teaching and to social critique, contributing to movements for greater social justice. What varies among the forms is the balance between mathematics and critique or what is highlighted and what is implicit. Further, we contend that this perspective implies a pluralistic, nuanced yet political, view of what constitutes critical mathematics.

To illustrate our perspective, we present and analyze two sets of data that instantiate distinctive forms of critical mathematics. These data come from classroom-based investigations where we independently taught and collected data. The first data set comes from an entry-level, college mathematics course with students from historically marginalized communities of color. Traditional measures of mathematical achievement positioned these students as mathematically "deficient" and even incapable of pursuing mathematics beyond college-level algebra. While younger, the urban high school students in the second context were not dissimilar from those in the college context. All had failed a geometry course previously and most did poorly on a pre-examination. Being students of color from communities of lower socio-economic status, many in both settings had internalized hegemonic and academically destructive messages about themselves.

Based on our presentation and analyses of these data, we will discuss the extent to which the instructional settings from which they emerged evidence critical mathematics and elaborate on how they represent social critique in the service of social justice. To distinguish the two approaches to critical mathematics that our data suggests, we consider the first approach CM1 and the second approach CM2.

CM1: ACCESS TO ACADEMIC MATHEMATICS AND THE IMPLICIT POLITICAL MESSAGE THAT CARRIES

An objective of critical mathematics ought to be to engage students, socially marginalized in their societies, in cognitively demanding mathematics in ways that help them succeed in learning that which dominant ideology and schooling practices

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position them to believe they are incapable. Such opportunities for learning have been integral to struggles of socially excluded sectors (D'Ambrosio, 2001; Gerdes, 1997; Moses & Cobb, 2001).

In 2007, the people of the US celebrated the fiftieth anniversary of the landmark racial integration of Central High School in Little Rock, Arkansas. In September of 1957, defying a federal court order, armed National Guardsmen along with an angry, racist mob blocked the precedent-setting path of nine, 15-year-old African American students who attempted to attend the all European American high school. After a three-week standoff, President Dwight D. Eisenhower, embarrassed by the negative coverage in the world press, ordered the US Army forces to escort the black teenagers to school and put down the mob. These teenagers continued to suffer a litany of verbal and physical assaults. Why did they endure the suffering? Melba Patillo Beals explained precisely what integration meant to her and the other eight students: "These people had language labs. They had typewriters.". Furthermore, she says, "We didn't go to Central to sit beside white people, as if they had some magic dust or something. I would not risk my life to sit next to white people. No, no, no, no. We went to Central for *opportunity*".[1]

Forty years after the Little Rock events, racial integration of schools was overshadowed by a powerful counter-imagery. The entering class at the University of Texas Law School contained only two non-white students. This was applauded by Lino A. Graglia, the A. Dalton Cross Professor of Law at the University of Texas, who observed that African American and Latino students "are not academically competitive with whites" and that they belonged to a "culture that seems not to encourage achievement. Failure is not looked upon with disgrace." Notions that people of color are culturally or genetically academically incapable permeate dominant discourses, both popular and learned (see, for instance, Herrnstein & Murray, 1994). At the end of 2007, the world-renowned geneticist and Nobel Laureate, James Watson opined that blacks are less intelligent than whites[2]. The struggle against such dominant-ideological narratives and for learning opportunities continues.

A challenge for critical mathematics educators is to counter hegemonic narratives about who can do mathematics and to reconstruct the role of mathematics in the struggle to empower learners whose mathematical powers have been underdeveloped. This challenge demands a critical mathematics curriculum that inverts traditional pedagogical structures of instruction so that the teaching of mathematics takes second place to the development of students' mathematical ideas, heuristics, and reasoning. This inversion is what Gattegno (1987) calls "the subordination of teaching to learning."

This pedagogical notion and the historic struggle for learning opportunities inform the course from which the CM1 data come. The course —pre-precalculus— was designed for students who entered the university underprepared for university-level courses in mathematics. The students were African Americans and Latinos from economically impoverished school communities in close proximity to the university campus. The university considered these students as “at-risk” of not succeeding academically and required that they attend a six-week summer program designed to equip them with study and other academic skills that would help them succeed in their university studies. The pedagogy of this course was trangressive in that students participated in challenging, cognitively demanding mathematics[3] —mathematics that had previously been “off limits.” Doing serious mathematics allowed students to challenge dominant ideological messages they had internalized about their own ability —and the ability of people like them— to do academic mathematics.

The following investigation from a unit on functions is typical of the course’s mathematical demand and attempt to challenge students’ internalized hegemonic sense of themselves as mathematics learners. In this investigation with graphing calculators, students were presented with mathematical rather than contextual situations to explore. They later related their investigations to contextual situations. In this unit, they worked in small groups to investigate several families of curves of single-variable functions. They had to distinguish among the independent variable, constants, and parameters to know what to vary in function such as $l(x) = e^{-kx}$. Other functions they investigated include these:

a) $f(x) = ax^n + b$

b) $g(x) = \frac{a}{x^n} + b$

c) $h(x) = a\sin x$

d) $j(x) = \cos(wx + d)$

By combining these functions and adding parameters, students crafted new functions and constructed function expressions to correspond to given curves. For example, students attempted to determine the symbolic algebraic description of a function whose graph corresponds to the shape of the curve depicted below in Figure 1.

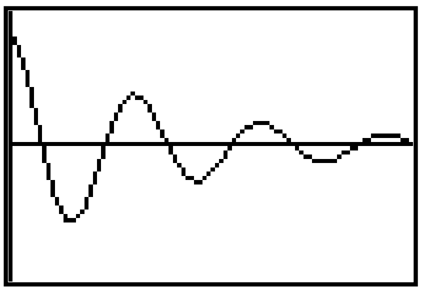

Figure 1. A curve that represents damped harmonic oscillation

Afterward, students related their function expression to the physical context of a damped harmonic oscillator such as a mass at the end of a spring (a vibrating spring) or a mass at the end of a cord (a pendulum). Finally, they explored questions such as the ones below aimed to increase their awareness of relationships between the curve, function expression, and movement of a pendulum affected by friction:

1. For the function $f(t) = ae^{-kt}\cos(bt + c)$, where f(t) is the displacement (in inches) of a mass from its rest position, t is the elapsed time (in seconds), and a, k, b, and c are parameters, determine parameters values that give a good fit to the following data:

Time	Displacement
0	10
4.5	9.2
9.4	8.9
13.9	8.3

2. At what time does the pendulum first pass its resting position?
3. At that time, what is the speed of the pendulum?

At the end of the investigation, students wrote reports, summarizing their findings. We discuss and present the excerpts from one student's report. This student, Rohan, as a condition of his acceptance to the university, attended the six-week summer program because the university considered him an "at-risk" student because of his educational background, his intended participation in the university's football (soccer) program and his weekend free-lance job as a disc jockey (DJ).

In his report, Rohan first states a conclusion of his investigation: that the curve in Figure 1 "represents the distance a pendulum travels over time." He then declares that his goal is "to find a function whose graph resembles" the curve and begins his exposition by reviewing his knowledge of the functions he believes are involved in the curve, namely some form of $\cos x$ and e^x. He next provides illustrations of their graphs and discusses each of these functions.

Concerning the cosine function, he shows in his report the effect of multiplying the independent variable by a constant, $\cos bx$. After illustrating how different values of the parameter *b* affect the period of the cosine curve, he explores the effects of multiplying the dependent variable —$\cos bx$— by a constant, *a*, and explains that the value of the parameter *a* affects the amplitude of the cosine curve. Reflecting on the graph of the curve in Figure 1, he notices that it tends to be a dampening, oscillating function of *x* and suggests that the graph results when the cosine function is multiplied by some decreasing function.

After some thought, the decreasing function of *x* that Rohan chooses to dampen the $\cos bx$ curve with is e^{-kx}, assuming $k > 0$. He notes that e^{-kx} decreases as *x* increases and that as the value of *k* increases the graph of e^{-kx} "decreases proportionally slower and approaches the *x*-axis." He then speculates that "[s]ince e^{-kx} is a decreasing function whose graph decreases slowly along the *x*-axis as the value for '*k*' gets smaller, and the graph of the distance a pendulum travels over time is a decreasing cosine graph along the x-axis, I modify the function $f(x) = a\cos bx$ to $f(x) = ae^{-kx}\cos bx$." In earlier sections of his report, he demonstrated his awareness of the effects that the parameters a, b, and k exert on the behavior of his modified $f(x)$. With a suitable choice of parameter values and with a restriction of the domain of $f(x)$ to $0 \leq x \leq 15$,[4] he states his discovery that the graph of the function $f(x) = 5e^{-\frac{1}{32}x}\cos 4x$ resembles the one given in Figure 1.

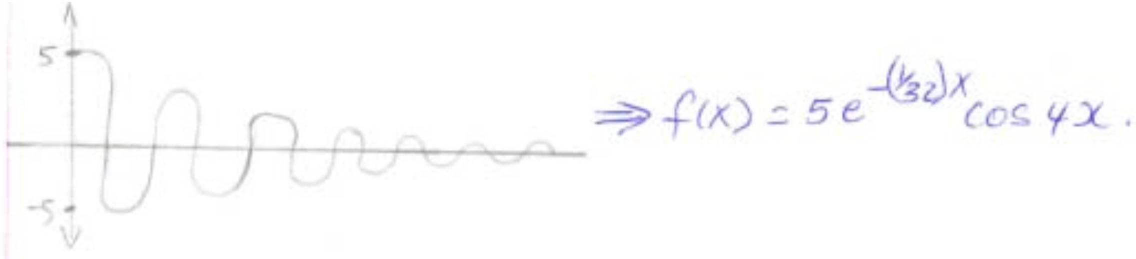

Figure 2. Rohan's graphical and algebraic representation of the curve given in Fig. 1

In the investigation, Rohan and his peers worked as mathematicians, generating knowledge that was new to them. His description of his discovery resembles the writing of mathematicians. It follows a rather linear, logical order. He carefully states is conclusion, then discusses what he knows, and illustrates his knowledge appropriately with both algebraic and corresponding graphical examples. Characteristic of writings in textbooks and professional journals, he reveals neither the affective states he experienced during his struggle nor the messy, perhaps even non-methodical, scratch-pad work in which he engaged. Those affective and messy experiences remain private.

This raises an important instructional point. It is valuable to point out to students the contrast between the messy affective and cognitive experiences of their work and the emotionless, logical exposition of their laboratory reports. This may serve to demystify mathematics textbooks and to make students cognizant that the struggle of discovery is likewise usually missing from the narratives they read in mathematics texts.

Among other issues, this example illustrates the power of a graphing calculator to allow the study of functions and physical situations that educators traditionally consider beyond the mathematical reach of pre-precalculus students. Typically, students in advanced calculus study this function as the solution to a problem in physics that, applying Hooke's Law and Newton's Second Law, leads to a second-order linear differential equation that models the movement of a vibrating spring or a swinging pendulum affected by friction.

Just as important, this report exemplifies how graphing calculator and writing are tools for educating one's awareness. By experimenting and focusing his attention, Rohan informed his awareness of the effects of varying the values of parameters of $f(x) = ae^{-kx}\cos bx$ and the corresponding graphical changes. Aided by a graphing calculator, he constructed a function whose graph resembles the one in Figure 1. Writing the report prompted him to articulate connections between information embodied in algebraic and graphical representations of functions. The connections he expressed are displays of his mathematical awareness. Moreover, to inform and persuade his audience, writing obligated him to arrange his awarenesses in coherent, logical order that possibly led him to other insights.

Pedagogical and ideological issues arise from examining both the content and process of CM1. Pedagogically, the course combined the use of graphing calculators and transactional writing to promote dialogue and reflection with challenging, cognitively demanding mathematical situations. These were crucial for engaging students to focus their attention and become aware of mathematical features of functions. The activities of the course engaged students in examining functions in context as well as context in functions. It challenged their internalized beliefs about themselves as students. The pedagogy that informed this work indicates an ideological position concerning the education of students' awareness in contextual and mathematical situations. This pedagogy responds to traditional instructional practices about which one can ask the following question: What is the view of students of color that positions lecture and rote practice as the central mode of instruction and that holds students hostage to the minutia of textbook discourse?

CM2: CRITICAL MATHEMATICS WITH AN EXPLICIT POLITICAL MESSAGE

The second example of critical mathematics, what we label CM2, is more explicitly political and, less academically focused, than the CM1 example. This CM2 example comes from a geometry course taught by the second author (Brantlinger, 2007). The course took place in a night program at an urban high school that served lower SES students. All but one of the 27 night course students was African American or Latino and all were making up credit for past failure in geometry. Many were pushed into the night school program for various "offenses" including truancy, gang involvement, and teen pregnancy. In this lesson, the instructor asked students to interpret a relatively complicated statistical chart that summarized data on the distribution of recess by the racial make-up of public elementary schools in Chicago in 1999. As Figure 3 shows, there was an inverse relationship between race and recess: as the student-of-color population increased the amount of recess time decreased precipitously.

In contrast to student engagement in standards-based reform activities that comprised four-fifths of the night course curriculum, the politicized and racialized CM2 activity appeared to engage several previously disengaged students. That is, the "Race and Recess" activity resonated with some—though not all—of the night school students in ways the "apolitical" reform mathematics activities had not resonated to date.

At the same time, there was considerable mathematical confusion surrounding the chart. While it was clear that students had little past experience with data analysis of this type, the videotaped data suggests there was a tension between students' firsthand knowledge of segregated schooling and the existence of "whiter" public schools in Chicago reported in the chart. This tension appears to have contributed to problems students had interpreting the chart. For instance, Osvaldo first read and then evaluated the chart stating, "percent white students, it throws it off." Shortly thereafter, when asked to provide a mathematical interpretation, rather than directly referencing the chart, he drew on his own perceptions of racialized socioeconomic distinctions, stating, "the white people have better jobs and stuff —they live in a better community and so they can afford to go to [schools with recess]." Later in the activity, Osvaldo asked the instructor if schools that were majority white actually existed in Chicago. While it could be that he was more interested in discussing the political than the mathematical, there appeared to be a disjuncture between Osvaldo's past experiences in schools with small white populations and the real world data that documented the existence of whiter schools in Chicago. Osvaldo was not alone in his confusion. Only two mathematically stronger students —Sonny and Princess— were initially able to provide an interpretation that the instructor saw as sufficiently mathematical. As an example of a mathematical interpretation, Sonny stated, "it's like where there's white people there's recess."

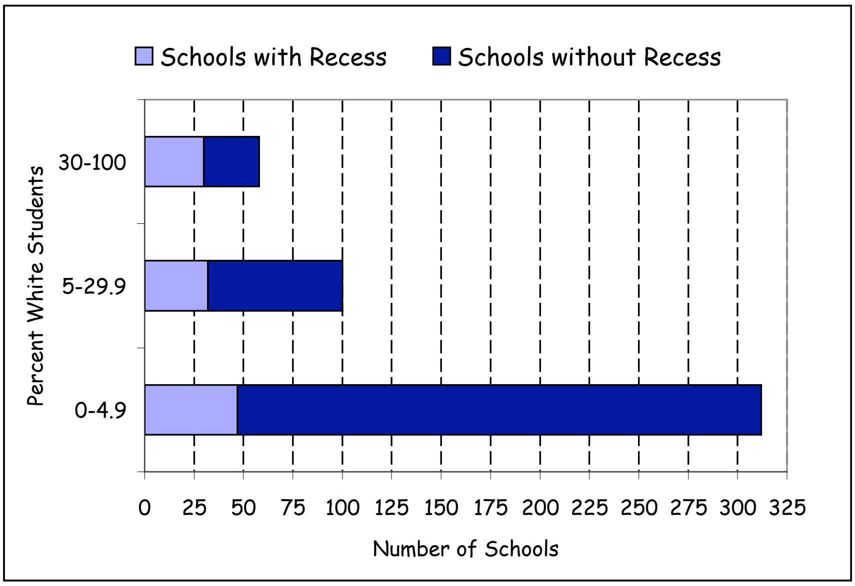

Figure 3. Race & Recess Chart from Pardo (1999)

For his part, the instructor hesitated to let the rest of the class off the hook by providing, or allowing Sonny or Princess to provide, the mathematical interpretation for them. However, after spending the first fifteen minutes of the activity vainly insisting that the majority of the class students indicate that they understood the mathematical message of the chart, he provided them with his own mathematical interpretation. He did so primarily because, at this point in the lesson, many students followed Osvaldo's lead in discussing their lived experiences in a segregated society and essentially bypassed the mathematics of the chart. In particular, after being prompted for a mathematical interpretation, Princess, an African American student, shouted, “the school is mostly black and so they don’t have recess! Black kids can’t go outside!” Dino, also African American, shifted the conversation away from Princess’ more mathematical explanation when he responded stating, “they cause too much damn trouble, that’s why they can’t have recess.” This launched a flurry of back and forth about the source of differential racial access to recess: problematic student behaviors, social inequality, or racist school policies.

In the ensuing conversation, a number of students gave their opinions and reacted to those put forth by their peers. Princess and Kampton, both black, pointed out that white students, such as those at Columbine High School, could “blow up shit” and still have recess, while black students were often punished for minor infractions. (It was the case that many students were bitter about being pushed into the night program from what could be seen as minor infractions.) In contrast, Dino continued to insist that black students did not behave well enough to deserve recess. He said, “they banned recess at our school cause they [i.e., African American students] was cussing up the classroom.” This provoked a negative reaction from other, mostly African-American students. At the same time, if Dino’s statement can be seen as fundamentally hegemonic (it blamed the victim) there was no clear articulation of a counter-hegemonic perspective (that blamed racist institutions) on the part of his classmates. Instead, many students discussed their particular experiences (i.e., whether or not they had gotten recess at their elementary school) and, at other times, shot each other’s differing ideas down. To be clear, the instructor was pleased with the politicized conversation of race and recess in this CM2 activity even as he worried about the lack of a clear mathematical focus.

In sum, this CM2 lesson posed three related tensions, namely, tensions between: (1) the critical and mathematical goals of the activity, (2) the teacher valuing student contributions and stating his own “official” mathematical and political perspectives, and (3) the teacher including more data analysis activities and covering more esoteric topics required by the official geometry curriculum. There were successes as well, at least, from a CM perspective. First, the CM2 challenged urban students expectations of what content is considered appropriate for school mathematics and, more generally, school. Second, several previously disengaged students (e.g., Osvaldo, Dino, Kampton) as well as previously engaged students (e.g., Sonny, Princess) expressed interest—even excitement—about this explicitly political activity. Many night students protested when the instructor ended the politicized whole class discussion and instead asked them to write down their own opinions of the fairness of the Chicago school system. To be clear, there was also student resistance throughout this and subsequent CM2 activities. For example, at the end of this lesson, Efrain complained that CM2 was “not what we’re here for” and Lucee added, “it’s goofy.” Other, less vocal, students quietly resisted the activity in various ways (e.g., putting their heads down, passing notes). With this example in mind, we might conclude that while CM2 provides a rupture in status quo schooling, it may not be –or is not currently– the panacea for the lack of mathematical engagement and understanding exhibited by many students of color in lower SES urban schools.

CONCLUSION

Our two examples raise questions about the nature of critical mathematics, its pedagogy, and its content; and what these means for student learning, for student, teacher, and researcher empowerment, and for social justice. Particular questions we consider are: How does the teaching context such as historical, national, and institutional influence critical possibilities and, ultimately, what counts as CM? How are students, and particularly marginalized students, positioned in CM lessons and how do they respond to that? Why are we not doing CM —CM2 in particular— with privileged students? What messages (e.g., about the nature of mathematics, mathematics learning, marginalized students, and their communities) are being sent to students in CM1 and CM2 activities? What language or discursive registers (students' vernacular, academic mathematics) or modalities (iconic, indexical, symbolic) are privileged in different versions of CM and how do they affect students' access proficiency in academic mathematics and their participation in movements for social justice?

Politically, questioning and interrogating dominant pedagogical practices remains an important task of critical mathematics educators. Critical mathematics educators may argue for the primacy of so-called real-world, politically oriented applications, what we call CM2 in this paper, for involving underrepresented groups in mathematics and for advancing social justice (Gutstein, 2005). Though this position merits serious attention, nonetheless, it is important that critical mathematics educators create narratives that are alternative to the idea that mathematics is inaccessible and the misconceptions about its inherent nature and about who can do mathematics. CM educators should not be satisfied with engaging historically marginalized students in politicized investigations of injustices (e.g., wage distributions) if they do not have access to academic mathematics. Students who have inherited the academic space opened by Melba Patillo Beals and her fellow students should not be educated under a practice that may unwittingly support an ideology that posits the oppressed as incapable of being motivated by the abstract nature mathematics.

NOTES

1. http://teacher.scholastic.com/barrier/hwyf/mpbstory/melchat.htm, emphasis added.

2. http://www.timesonline.co.uk/tol/news/uk/article2677098.ece

3. For a discussion of what we mean by "challenging mathematics" and for other examples, see Powell et al. (in press).

4. Although Rohan writes the interval as $0 \leq x \geq 15$, in an interview, he demonstrated that he knows well how to use inequality notion to indicate intervals on the real line.

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SITUATED DECISION MAKING IN MATHEMATICS EDUCATION

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Mathematics education is a social phenomenon, not like a distinct part of the society which could be influenced by social events, but as an evolving and active part of a whole body of society. Non-stopping interaction between changing world, education and mathematics, and the failure of traditional, static programs in mathematics education, emerges the need for more intelligent and efficient decisions to increase desired effects of changes, reduce unwilled ones or/and change the undesired effects to desired and positive effects. These are subtle decisions, which should be situated and dynamic and driven in the entire system of mathematics education .We call them Wise Decisions or WDS. This paper considers the identifications and the important concerns of WDS.

INTRODUCTION

We can not stop social influence on mathematics education. As a matter of fact mathematics education is a social phenomenon, not like a distinct part of the society which could be influenced by social events, but as an evolving and dynamic part of a whole body of society. As Popkewitz(1998) says: " our social conditions contain a host of elements that interact in ways that are never fully specified, predetermined, anticipated, or willed". Brookes also argues that: "The recognition that the world is changing rapidly casts doubt on any programme which depends on rigidly defined propositions embodied in a static educational theory not capable of responding to environmental change." It could be claimed that any static decision made for the sake of mathematics education improvement, distinct from social and cultural concerns, leads to failure.

The relation between society, culture and education is not a one way road. Social and cultural attitudes influence education and consequently, any change in educational attitudes and approaches affects society and culture.

Non-stopping interaction between changing world, education and mathematics, and the failure of traditional, static programs in mathematics education, does mean that the role of human decision for designing successful plans has reduced, but it means that we need more intelligent and efficient decisions to increase desired effects of changes, reduce negative, unwilled ones or/and change the undesired effects to desired and positive effects. In this way, we may suppose that: "*what affect mathematics education, might be changed to a facility or even a need in it"*.

Any mutual interrelationship have desired and undesired effects. For example, development of technology and computer science which leads to widespread usage of computers at society affects students' attitudes, behaviour and even mental abilities.

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 434 - 443).

This social fact could be used as a useful or even necessary tool for development of education. A good example is CAS (computer algebra systems).(positive application of a social fact) On the other hand, this useful equipment may be even harmful for education in another place, like a small village whose students are not even familiar to basic computer skills.(undesired effect).Of course, improving students' computer skills changes the situation to a positive one, but this is not the immediate sequent of technology in an unskilled society, it needs wise decisions and actions to change the situation to our desired form.

WISE DECISIONS

How can social and cultural factors which influence mathematics education be distinguished and how can be these effects changed to desired ones? The answer is not always as simple as aforementioned situations.

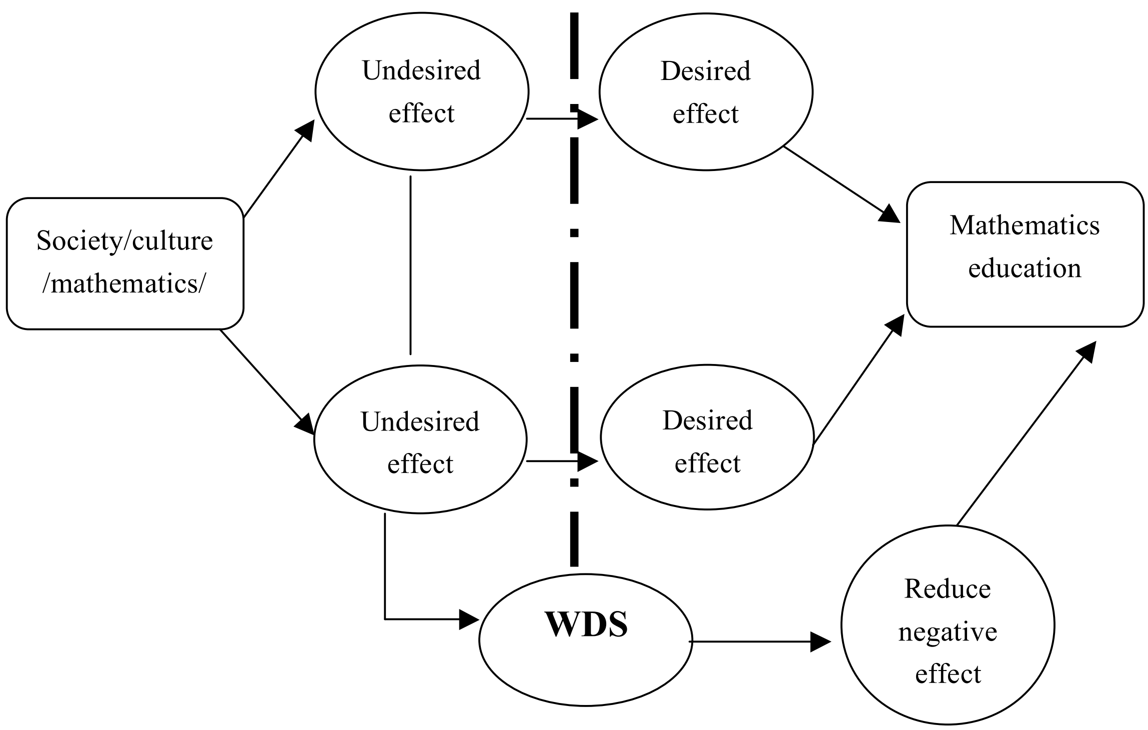

Figure 1. Deriving desired effects from undesired situations

Clarifying the most effective factors and making the best decisions for filtering or changing the negative effects is time-dependent. These are subtle and complicated decisions. We call them *"wise decisions"*. These decisions must be dynamic and situated because of the dynamic nature of society at whole and education as a part of it. Wise decision system is not only responsible to answer the general questions like whether "mathematics curriculum should be global or local" or "which content is more suitable for lesson books" or even "how culture affects education'. Of course these are important questions but what specifies *wise decisions system (WDS)* is the dynamic and adapting nature of them. WDS deals with other important questions like: "How can we make the most benefit from unpredictable international, cultural, social or/and ethnical effects, even if they are not willed, in our educational system?", "How can we reduce undesired negative effects on mathematics education or probably change them to positive effects?" or "How should we make situated and emergent decisions in unpredictable and fast-changing situations?". WDS frequently gets feedback from existing situations and reorganizes itself.

Feedback loops between WDS components needs communication. Detecting the reflected data from each feedback loop, gathering the useful information and making the best decisions due to the situation needs expert manner of well- trained human participants.

Figure 1 shows the filtering function of WDS in changing the undesired effects on mathematics education to desired effects.

MAIN CONCERNS OF WISE DECISIONS

In "MAPS" model introduced by Hingginson Mathematics education is supposed to be a point whose position changes each time among the inner space, edges or vertexes of a polygon. Vertexes are "M" for mathematics, "A" for anthropology, "P" for psychology and "S" for society, as affecting factors on mathematics education. We claim that this point never locates exactly on edges or vertex but only inside the polygon. There are fuzzy boundaries between the affecting factors on mathematics education. How ever, we improve and use these factors as the aspects which might be concerned in "wise decisions".

The important concerns of wise decisions seem to be:

1. Mathematics
2. Psychology
3. Anthropology
4. International trends
5. Society and Culture

We have added the "international trends" as an affecting factor on mathematics education, because approaches and attitudes in Mathematics and mathematics education differ in many aspects in international societies. There might be even many contradictions between mathematicians and mathematics educators' views, but both, the international accepted results of studies in mathematics education and universal Mathematics introduced by mathematicians, affect the mathematics education in a society.

The link between these items is "education" which has caused more interaction between the affecting factors on mathematics education. Even in content- oriented approaches, such as "new mathematics", no content of mathematics could be applied in lesson books without being" chosen" or "modified". On the other hand, the recent "new-new mathematics" which was strongly criticized because of paying less attention to the basics of mathematics in education, still needed some mathematics to be taught in so called" new-new" methods.

Kemmis and McTaggart(1998), have argued that in the case of mathematics education and cultures, the main criterion to be employed for choosing a culture and then improving it, should be " rationality and justice".

However, these items should be in conformity with "rationality" and "justice", i.e. any wise decision considering the main concerns of mathematics education (mathematics, society, values...) should pass "rationality" and "justice" filters.

For example, intending values in mathematics education should not contradict to rationality. On the other hand, Conformity with national trends is rational but it should be justified through social, cultural and anthropological concerns of a region or a nation. Whatsoever, the frequent balance and interaction between "rationality" and "justice" would be the final passing filter for any decision, and is mostly due to human conscious.

WISE DECISION IDENTIFICATIONS

The lack of wise decision system (WDS), results in undesired effects on the system of mathematics education. International trends and decisions in mathematics education impose themselves to local educational system. Any conflict between adopted curriculum from international societies in one hand, and culture of a region in the other hand, makes embarrassing situations for teachers and students.

In the case of poor decisions, some solutions and decisions might be dictated to each part of the local educational system, including teachers and students. These would be, as Clements and Elerton declare, *top-down* decisions. But it is not always easy to separate the decision makers from the processes of decision making and any separation would be artificial. In fact, each part of the mathematics education system could be a potential decision maker. In real situations, the entire system of mathematics education (researchers, administrators, teachers, students...) deals with frequently renewed problems and solutions. The events and processes in human life at whole and mathematics education system in particular, are characterized by complex nonlinear dynamics - they arise, evolve and disappear as a direct result of interaction of many interwoven factors. These are the same factors which influence human decision-making in WDS. WDS dictates no certain solution, nor even problem, but it can aid decision making by providing information relevant to the decision and to the decision makers.

According to Vladimir Dimitrov, decision making requires division and separation. The decision maker needs to extract out of the available information (related to a decision situation under concern) at least three independent constituents:

(1) set of alternatives to choose from;

(2) set of criteria to satisfy;

(3) a goal (or set of goals) to achieve.

After analyzing the above constituents, a specific procedure is sought in an attempt to connect them in an 'optimal' way. These are general characteristics in any domain of decision making including mathematics education. Many goals and sub goals as well as alternatives for decision making, in the case of mathematics education might be identified, the main criterion for justifying decisions, as was argued before, might be rationality and justice. WDS includes all these constituents but specifically it provides facilities for decision making in real dynamic and unpredictable situations. In such conditions, there might be few alternatives to choose from and even the goals might be emerged in situation.

As a matter of fact, in real situations, as Vladimir Dimitrov declares, *decision emergences* are needed rather than decision making. He says: "*Decision emerging* does not require division and separation - on the contrary, it depends crucially on the ability of the *decision-initiators* (persons or groups responsible for initiating the process of decision-emerging) to fully experience decision situations. The emergence is never in past or in future. Decision emerges *now* - in parallel with the act of experiencing the unfoldment of life. That is why the personal (or group) awareness (alertness, vigilance) is a vital factor in decision emergence."

Similarly, in-time decisions of teachers emerge from in-time interaction between teachers and students. WDS is responsible to prepare and train teachers for confronting decision emergences.

Briefly, WDS should provide following capabilities for each part of the system:

1. situateded Decision (action in time.)
2. prediction
3. communication and interaction
4. wareness
5. self organizing and adapting nature

Communication between parts of WDS is of great importance. Mathematics education researchers, mathematicians, administrators, teachers and … should frequently interchange information and experiences to be aware of the recent problematic of mathematics education and suggesting solutions. Communication increases the knowledge of each part. By enough amount of information and knowledge, prediction is possible.

Figure 2 represents a model for the communication between some components of mathematics education system; there are certainly much more interacting components in the system of mathematics education.

It may seem that this diagram represents an ideal and impractical situation. Of course the links between *modern approaches* and *academic societies* is not the same as *modern approaches* and *students*. In fact there may be no direct link between *modern* *approaches* and *student,* but this connection is not either impossible. So the connecting links between components might be assumed weighted. WDS is to improve the links of communication between the components whose interaction have the most influence on mathematics education.

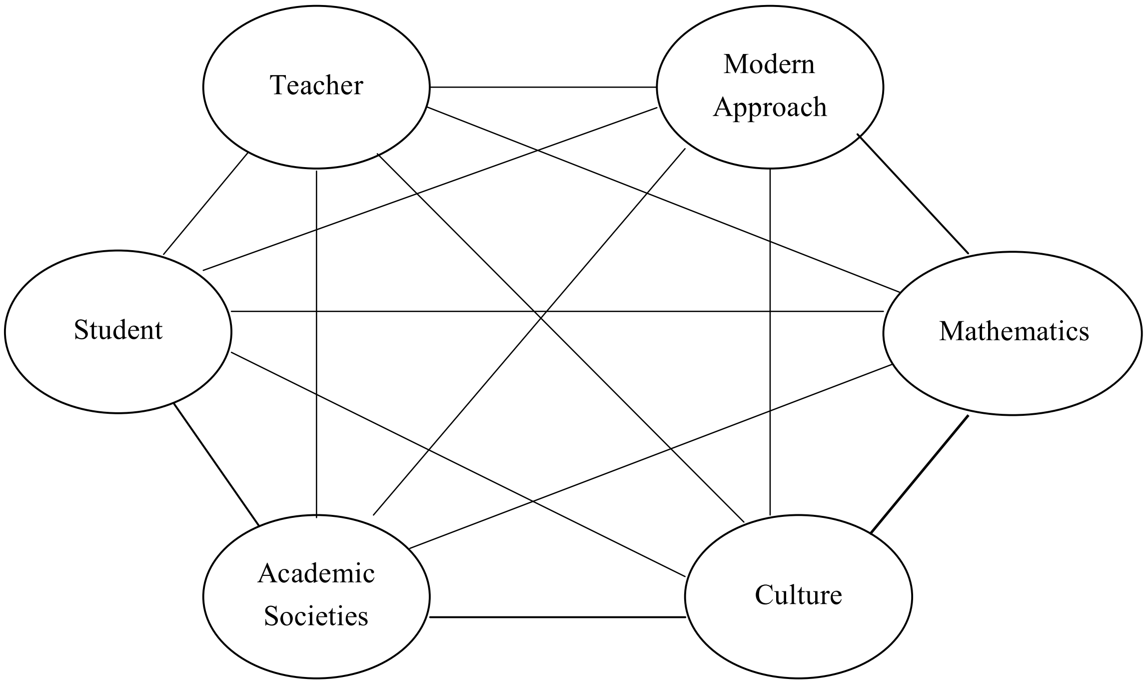

Figure 2. Diagram of WDS communication

ILLUSTRATING EXAMPLE

Now we consider one of the main concerns of mathematics education, i.e. "mathematics for all" and "mathematics for elite" and the local and international possible reactions, decisions and possible problems related to these matters, as an example. Meanwhile, we show that most of related decisions are not wise decisions in the case of local situations.

C1- International trend: equity in mathematics education, non- Elite mathematics

> **Decision**: international conferences, meetings, plans (UNESCO report in 1984; NCTM standards, Mathematics for all; designing research for developing non-elite curriculum).
>
> **WD**: attempting to internationalize the results of studies, seeking for more communication ways between different countries.

C2- Academic Mathematics: mathematics knowledge has decreased among college students.

> **Possibility**: most students have not been qualified enough in mathematics before entering universities and colleges./ They are not interested in mathematics.

Decision: more and more students should be failed in final exams of universities to take the matter serious.

Decision: examinations should be easier in order that more students could pass the exams.

WD: setting more communication opportunities between mathematics professors and mathematics education experts, seeking for the roots, trying to modify students' attitudes. (Communication, situated action)

C3- Educational administration: the majority of the high school students show little interest in mathematics.

Possibility: Teachers can not motivate students to learn mathematics.

Possibility: Students are not successful in mathematics examinations.

WD: setting more in- service teacher training courses to inform teachers from newest approaches of mathematics education and the results of researches. (i.e. Mathematics for all, developing non-elite curriculum(communication)/ changing the format of school examinations(studies shows that there might be a relation between elitist views on education and achievement tests.)

C4- Teachers: most high school students are not interested in mathematics/ few students are able to use their mathematics knowledge in real life/mathematics is science of elite

Decision: taking more difficult exams in order that the majority non- elite try their best to pass the final exams.

Decision: taking easier exams in order to satisfy school principals and students' parents by the results of the exams.

Decision: do nothing at all and waiting for top-down decisions

WD: consulting with experts, getting information about equity in mathematics or non-elite mathematics from different resources, trying to modify students' attitudes toward mathematics. - One simple way is to tell some stories about the mistakes that great mathematicians have done in history .This may break the huge wall of the *elite* which does not allow students to learn mathematics. (Situated action)

C5- Local mathematics education researchers: awareness of recent educational reforms, mathematics for all. / Mathematics should not be concerned as an elitist discipline any more.

Decision: Doing their own studies without any concern of the realities in schools of Iran.

Decision: Doing something about the teachers' and students' attitudes toward mathematics as a solution for increasing the interest in mathematics, among the students.

WD: Giving consult to educational administrations, / conducting pre/in- service teacher training courses to inform teachers./ being a mediation between international and local societies of education./ designing and conducting studies related to this subject and reporting the results to all.

C6- Parents and students: Mathematics is necessary for students' future progress in society / mathematics is the science of **elite**

Decision: parents blame their children for being not talent in mathematics and force them to try hard, / parents encourage their children to attend in out-school institutions for increasing their mathematics knowledge.

Decision: students try their best/ they become disappointed. They feel that they are not able to learn mathematics and give up.

WD: getting consult from experts in the field of mathematics education through communication

Obviously, some links in WDS communication network are more significant than the others. For instance, the connections:

C2
↕
C1 ↔ C5 ↔ C6
↕
C3

might be more significant than the connection: C1 ↔ C6 ↔ C2.

Many Iranian teachers and students think very high about mathematics. A recent study in the district 1 of Tehran education department shows that about 80% of mathematics teachers believe that "*Mathematics is the science of Elite"* (C4). Iranian students and educational administrators believe more or less the same (C1 and C3). This belief affects teachers' behaviour. Also, teachers' beliefs about students' ability and learning greatly influence their instructional practices. Teachers with this attitude do not attend to make decisions for changing the top- down imposed elitist mathematics program for the benefit of the non-elite majority. In these circumstances situated decisions are meaningless. By this view, students who are not Elite (nearly most of them), do not deserve themselves to be qualified in school mathematics and do not try hard, and educational administrators do not make serious decisions for the benefit of the majority *non-Elite*, because it is useless! Parents also do nothing except blaming children for being not talent and hard working. (C6)

The problem that "Mathematics *for Elite*" has caused in education has been an international concern since 1980s' and is not a new one. (C1)

D'Ambrosio (1985,1989) have argued that, in the past, school mathematics has been an elitist affair, especially suited for the preparation of middle- class males for prestigious professions such as engineering and natural sciences. According to Clements and Elerton, what is needed, D'Amborsio (1984, 1994) has argued, is a totally new approach whereby different mathematics curricula are developed, always with the specific needs of existing group and potential learners in mind. The NCTM Curriculum and Evaluation Standards for School Mathematics advocates "mathematics for all" as a central idea in education reform. The Draft for "Standards 2000' from the NCTM (NCTM 98) calls for increased equity by exposing all students, not just the elite, to challenging mathematics. (Decisions on C1) In-service Teacher Training is a suitable way of informing teachers from recent problematic of mathematics education and equipping them with suitable techniques for making decision in-time.(WDS)

In short, many of decisions in mathematics education of Iran are not wise decisions. Teachers are obliged to execute the top-down decisions in their classrooms. For this reason they have little opportunity to make decisions and act due to the occurred situations on their own authority. Almost the entire Teachers 'in-service training in Iran deals with developing teachers' scientific or professional knowledge domain. Training courses give little information about recent educational approaches in the world. They are not even in touch with local academic societies and mathematicians. (Lack of communication) Mathematics education is a new branch in universities of Iran. Most of the academic studies in this field reflect little about the reality of mathematics education in Iranian schools. The main concerns and purposes of the academic studies in mathematics education are determined by international trends rather than by local situations. (Lack of communication)

The first and the most important step for establishing a network of WDS in Iran seems to be communication. Mathematics education researchers should play the role of a media and a coordinator between international trends, and local societies of education. Teachers should be trained and equipped for deciding and acting in unpredictable, dynamic conditions by their own authority. Students will need to be more equipped to generate and work with their own accounts of the realities they face rather than rely too heavily on the accounts provided by their elders. Curriculum designers, meanwhile, perhaps need to be motivated more by needs and possibilities, based on movement from existing practices.

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TEACHERS AS PARTNERS FOR DESIGNING PROFESSIONAL DEVELOPMENT PROGRAMS

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The recent reforms in mathematics education generate the necessity for teachers to attend professional development programs. In order to increase the teachers' motivation to attend such programs it is essential to adjust their contents to the teachers' needs. These needs are culturally and socially dependent; hence a certain program which is successful in one country might not be appropriate in another. While designing professional development programs it is important to consider the needs of the population to whom it is intended. The current study was designed in order to explore the needs of Israeli mathematics teachers for professional development, assuming that considering their urgent needs would help us in designing relevant programs.

INTRODUCTION

Mathematics education has undergone significant changes in recent years. The NCTM's Standards (2000) had motivated educators to develop frameworks and approaches aimed at supporting the professional development (PD) of teachers, assuming that such a development will result in improving pupils' learning and understanding of mathematics. Consequently, various extensive professional development programs (PDPs) were initiated, bearing the shared vision of the 'new teacher'. Unfortunately, many of these programs did not yield the expected results.

Various factors are associated with the accomplishment of the goals of the reform. Most of them depend on the teachers' ability to change (Borko, 2004). Teacher change necessitates extensive support and guidance, and opportunities for PD. As PDPs' designers we often face difficulties, not to mention frustration, in bringing teachers to recognize the need for attending PDPs and changing their practice. This implies to the need to rethink the aims and the methods of such programs.

CONTEXTUAL FRAMEWORK

The study was performed within the framework of Israel National Center for Mathematics Education ("Kesher Cham"). The Center provides in-service mathematics teachers with various opportunities for PD (short/long term programs, on-line courses, conferences and seminars). During the last few years we noticed a constant reduction in the attendance of teachers to most of our programs. Talking with teachers about the phenomenon, we realized that teachers feel distressed and frustrated. Many of them express the need to modify some of the contents and the teaching methods, however they are not convinced that the suggested PDPs provide satisfactory answers to their needs. PDPs usually rely on what the designers believe might help teachers expand their mathematical and didactical knowledge and what

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sort of knowledge teachers need. These decisions are made on the basis of field experience and the research literature. The problem with designing PDPs based on research literature is the fact that the roles and needs of teachers are culturally and socially embedded, and teachers' perspectives regarding their profession are affected by the way teaching is perceived by their societies (Calderhead & Shorrock, 1997). Namely, the accumulated shared knowledge regarding PDPs is actually based on experiences gained through working with specific communities of teachers. This knowledge should be synthesized considering local, social and cultural aspects, in order to adjust it to local needs. Following that perspective, and based on Little's (1993) claim that PDPs should explicitly consider the experiences of teachers, we initiated a study that enabled us to learn about the Israeli mathematics teachers' actual needs, with the intention of implementing the results to new PDPs. This paper describes our findings regarding what the study participants find as essential for their PD.

THEORETICAL BACKGROUND

Teacher educators and school reformers are paying considerable attention to the influence of effective PDPs on the improvements of teaching. In this section we present a brief literature survey concerning PD of teachers. We distinguish between PD, as an internal process teachers experience through their entire professional lives, and PDP as an external framework aimed at supporting their PD.

Professional development. PD is viewed as an essential mechanism for teachers to improve their knowledge and expertise, in order to enhance pupils' learning and achievement (Guskey & Huberman, 1995). PD means changes over time in behavior, knowledge, images and perceptions (Kagan, 1992). The process of PD enables teachers to review, renew and extend their commitment as change agents to the purpose of teaching, and by which they acquire and develop critically the knowledge, skill and emotional intelligence that is required for good teaching (Day, 1999). This process is a long-term learning by its nature, and is based on self reflection on experiences (Brown & MacIntyre, 1993). The self examination enables teachers to strengthen their knowledge about the subject matter, about different pedagogies and about the ways pupils learn, as well as supporting them in applying these kinds of knowledge in practice (ACME, 2006), in order to generate better learning opportunities for their pupils.

Professional development programs. PDPs play an essential role in successful education reform. The enhancement of students' achievement is dependent upon responsive teachers, who are active participants in on-going, high quality PD activities (Guskey & Huberman, 1995). According to the NCLB forum for teachers preparation institutions (2003), PDPs should include, among others, activities that (i) improve and increase teachers' knowledge of the academic subjects the teachers teach, and enable teachers to become highly qualified; (ii) give teachers the knowledge and skills to provide pupils with the opportunity to achieve highly academic standards; (iii) improve classroom management skills; (iv) advance teacher understanding of effective instructional strategies. The types of teacher knowledge mentioned above are consistent with Shulman's (1987) suggested framework for discussing teachers' knowledge: content knowledge, pedagogical knowledge, curriculum knowledge, knowledge of learners, knowledge of educational contexts, and knowledge of educational ends.

Phases of professional development. PD is a gradual process that occurs in teachers' professional lives. At any stage in their careers, teachers move forwards and backwards between phases for reasons associated with personal history, psychological and social factors (Day, 1999). Therefore, the meaningful variable – teachers evolve over cycles during their careers – should be taken into consideration while planning PDPs. Huberman (1989) developed a schematic model of cycles of teaching career. The implication of these cycles is that teachers have different aims and dilemmas at various moments in their professional lives. Consequently, their desire to acquire more information, knowledge, expertise, and technical competence will vary accordingly. The phases teachers go through while developing professionally generate the necessity for adjusting PDPs to their needs, according to their current phase of development.

Teachers as partners for designing professional development programs. Knowles (1990) uses the term "Andragogy" to describe the process of engaging adult learners in learning experiences. According to Knowles, adults want to know why they need to learn a certain subject and they are most interested in learning subjects that have direct and immediate relevance to their job or personal life. Since adults are self-directed and expect to take responsibility for their decisions, they need to be involved in the planning and evaluation of their instruction. According to Wilson & Berne (1999), in order for PDPs to affect teaching, they have to consider teachers' experiences and use them as a basis for designing the learning activities. Moreover, PD may be viewed differently in diverse settings (Scribner, 1999). There is no single model or form of PDP which is better than the others. Therefore teachers must evaluate which PDP model would best serve their needs, beliefs and practices.

The current study aimed at listening to the Israeli teachers' voice, viewing them as partners for designing PDPs, and hoping that it might increase their motivation to take an active part in such programs and consequently internalize their contents (Knowles et al. 1998). Following that, we examined the responses of a group of Israeli mathematics teachers to the question: what do they find as essential for their PD.

THE STUDY

The present study has an explorative nature and it uses quantitative as well as qualitative methods. No preliminary hypotheses were assumed. In what follows we describe the phases of the study, the main research tools and the methods of the data analysis.

Phases of the study. The study was performed within the frame of the Israel National Center for Mathematics Education, and was implemented in three main phases:

At the *first phase* 17 experienced teachers (with experience ranging from 2 to 22 years) were asked to write as many statements as possible for describing their perceptions regarding the meaning of: i. Professional development; ii. Good mathematics teaching and iii. Meaningful learning. In order to identify what teachers perceive as the most meaningful characteristics of teaching, learning and PD, we followed the process of analytic induction (Goetz & LeCompte, 1984). Within this process we reviewed the entire corpus of data gained from the teachers' statements in order to identify themes and patterns and generate initial assertions regarding their perceptions of PD, teaching and learning. The statements were then categorized according to Shulman's (1987) categories of knowledge. In addition, we found that part of the teachers' statements related to communication and self development. The categories and the attribution of the statements to them were validated by these 17 teachers. Due to space limitations, the obtained results of the first phase are not included in this paper.

At the *second phase*, the statements were used for generating two-part Likert-type questionnaire. The statements that were included in the questionnaire were the most frequent in each category (more than two thirds of the participants referred to them). The first part included statements regarding teaching and learning, and the second part included statements regarding PD. The questionnaire was given to another 43 teachers, with various number years of teaching experience (see Table 1 below). Referring to the second part of the questionnaire, these teachers were asked to rank each statement according to its essentiality for their PD needs and explain the ranking.

At the *third phase*, ten representative teachers (out of the 43 teachers) were interviewed (two from each group, according to number of years of experience). The purpose of the interviews was to gain deeper understandings regarding the teachers' PD needs.

Research tools. In this paper we describe two of the research tools – the questionnaire and the interviews.

(i) The *questionnaire*. The questionnaire was anonymous and was comprised of two sections and some informative details relating to the teachers' teaching experience. The first section included 15 statements referring to aspects that concern teaching and learning, and the second section included 15 statements referring to PD. The statements in each section were randomly assigned. In the scope of this paper we relate only to results obtained for the second section of the questionnaire. The questionnaire was a Likert-type scale with the choices: 1. most relevant; 2. relevant; 3. fairly relevant; 4. less relevant; 5. not relevant. The 15 statements that referred to PD were:

S1. Strengthening my confidence in viewing myself as a proficient mathematics teacher; S2. Developing my ability to cope with my limitations; S3. Developing my ability to deal with conflicts that concern my relations with pupils; S4. Developing openness for changing my ways of instruction; S5. Developing my reflective skills; S6. Changing my perceptions regarding teaching and learning; S7. Enriching my knowledge regarding theories that relate to teaching and learning; S8. Changing my teaching methods; S9. Developing my ability to write papers which describe my teaching experiences; S10. Expanding my knowledge regarding the way pupils perceive various concepts; S11. Expanding my knowledge regarding the way pupils construct their mathematical knowledge; S12.Developing my ability to cooperate with my colleagues; S13. Enriching my mathematical knowledge; S14. Developing my ability to implement mathematical inquiry activities; S15. Developing my ability to implement an educational research.

The categories of the statements were: self development (S1, S2, S4, S5, S9, S14, S15); pedagogical knowledge (S6, S7, S8); knowledge about pupils (S10, S11); inter-personal communication (S3, S12); and content knowledge (S13).

(ii) The *interviews*. In order to gain deeper insights regarding the teachers' PD needs we interviewed 10 of the 43 teachers. As was mentioned, within the questionnaire the teachers were asked to explain the ranking they attached to each statement. From each group of teachers (see Table 1) we interviewed two of the teachers who provided the most extensive explanations. The interviews were open, and we asked the interviewers to relate to any issue that appears to them relevant to PD.

The subjects. At the second phase, the questionnaires were filled by 43 mathematics teachers, teaching middle and high-school pupils. Table 1 presents the distribution of the study subjects according to number years of teaching experience.

Years of experience	1-5	6-10	11-15	16-20	21+	Total
Number of teachers	7	6	9	10	11	43

Table 1. Informative data regarding number of years of teaching experience

Data analysis. The study is both qualitative and quantitative; therefore we used data analysis methods from both research paradigms. Analytic induction (Goetz & LeCompte, 1984) was applied for the first and the third phases, and the non-parametric Mann-Whitney Test for the second phase, in order to compare between groups. The comparison was made according to various factors associated with teaching experience and former participation in in-service PDPs. In this paper we focus only on the comparison according to number of years of teaching experience, since it turned out to be the most significant discerned factor among groups.

RESULTS AND DISCUSSION

In this section we present and discuss the issues which we found to be relevant for mathematics teachers regarding PD and PDP. In particular, we focus on the relation between the ranking of a statement as most relevant (MR) and number of years of teaching experience (YTE). It should be noted that we do not assume that statements which were not rated as MR is regarded by the teachers as unimportant.

Results and discussion of the second phase of the study

General Findings. Table 2 summarizes the results. The first row (1) refers to the number of the statement. At the second row (2), appears the percentage of teachers (out of 43) who find each statement as MR for PD (e.g. in row no. 2, 30.2% referred to S1 as MR). In rows 3-7 appears the distribution of percentage of teachers who find each statement as MR, according to their YTE (e.g. in row 6, 3 teachers from the group of 10 teachers with 16-20 YTE designated S1 as MR (30%)). In addition, in each of the rows (2-7), the numbers in bold designate the highest percentages obtained for each group.

1	Statement	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15
2	MR(%) YTE	30.2	23.3	34.9	48.8	41.9	18.6	46.5	23.3	4.7	**69.8**	**69.8**	30.2	**69.8**	30.2	14
3	1-5 (N=7)	0	14.3	28.6	**71.4**	**71.4**	28.6	57.1	28.6	0	**85.7**	**85.7**	28.6	57.1	42.9	28.6
4	6-10 (N=6)	16.7	50	33.3	50	16.7	16.7	50	16.7	0	50	**66.7**	33.3	**83.3**	16.7	0
5	11-15 (N=9)	44.4	0	22.2	33.3	33.3	11.1	55.6	33.3	11.1	**66.7**	55.6	11.1	**77.8**	33.3	0
6	16-20 (N=10)	30	0	20	60	60	10	50	30	10	**80**	**80**	25	60	10	40
7	21+ (N=11)	45.5	54.5	**63.6**	36.4	27.3	27.3	27.3	9.1	0	**63.6**	**63.6**	54.5	**72.7**	45.5	0

Table 2. The frequencies of ranking a statement as most relevant (MR) according to number of years of teaching experience (YTE)

From Row 1 of Table 2 one can realize that most of the study participants perceive knowledge about pupils (S10, S11) and content knowledge (S13) to be the MR for their PD. The least relevant are engagement in research and writing papers (S15, S9). It appears that the study participants are more concerned with psychological aspects (the way in which pupils comprehend and process information, and consequently build their knowledge), than with pedagogical ones. In order to be able to explain the findings we examined the syllabi of several pre-service teachers training programs intended for qualifying middle and high-school mathematics teachers. We found that students are required to attend at least two one-semester courses dealing with educational/social/cognitive psychology. However, these courses are general in their essence, and are not specific to mathematics. Most of the suggested pedagogical courses deal with the development of teaching skills and didactical methods. Not likewise methods courses, which are subject matter oriented, the courses aimed at developing teaching skills are not always specific to the teaching of mathematics. We also examined some PDPs intended for in-service mathematics teachers. Most programs focus on assimilating new contents of the subject matter, other focus on the integration of innovative teaching methods and tools (including computer software), and the minority concern with general mathematical education issues. We realized then that mathematics teachers are not provided with sufficient opportunities to acquire suitable knowledge regarding the various aspects concerning pupils' thinking and apprehending.

Relations between MR and YTE. Using the non-parametric Mann-Whitney Test we compared between the five groups of teachers (see Table 1), trying to identify differences between the groups regarding their perception of PD. In the scope of this paper we focus on findings relating to the most prominent differences between beginners and highly experienced teachers. As to the beginners, we found that most of them (71.4%) express a need for self development (S4, S5) while most of the highly experienced (63.3%) expressed a need for developing inter-personal communication (S3).

The fact that the beginners are more interested in self development than the others is consistent with Huberman's (1989) model of career cycle. In their initial steps as teachers they are mainly engaged in 'internal observations' of themselves as proficient teachers, struggling to consolidate their world view regarding teaching methods. For that matter, they need to develop their reflective skills and to be open minded. Highly experienced teachers, according to Huberman, are more 'mechanical', self-accepting and exhibit resistance to innovations. Consequently, as can be seen from Table 2, they neither express a significant need for developing their reflective skills nor a need for developing openness to changes. As to S3, Huberman's model does not provide a framework for explaining the gap between the two groups. We refer to this gap within the analysis of the third phase.

Results and discussion of the third phase of the study

We present excerpts from the interviews that concern the issues discussed above. We mainly focus on teachers' responses relating to cognitive aspects of knowledge about pupils, aspects of self-development, and aspects of communication.

Excerpts concerning knowledge about pupils (S10 and S11). Sara, a teacher with 3 YTE said: "*... Frequently I have to teach the same subject* [to low and high achievers] *but in different levels…While I was a student, my method course instructor talked about the necessity to adjust the materials to the level of the pupils. But what does it really mean? …What actually discern between pupils – I don't deeply understand. What is going 'in the head' of one pupil that is not in the other? Do they* *think differently? How? Do they have different images? Why?...When I talk, for example, about a parallelogram, in what way every pupil perceive it?*".

Rebecca, a teacher with 23 YTE said: *"... I can tell which teaching method is going to work in what level, and what problem should be posed in every class. But I gained these insights from my experience. I developed some theories about the way in which pupils think and process knowledge, and also about the source of their difficulties. I am curious whether my theories resemble those which are based on research, but we never discussed it formally in PDPs ".*

Both Sara and Rebecca wish to acquire knowledge about pupils. However, it stems from two different perspectives. While Sara believes such knowledge can assist her in becoming a better teacher, Rebecca does not sense that it would be beneficial for her in terms of improving her teaching. Rebecca is curious about whether her own theories, which are based on her experience, suit the research literature. This is in line with Hubermans' (1989) model, according to which beginning teachers are trying to define their professional goals, while experienced teachers tend to stick to what they know.

Excerpts concerning self reflection and openness (S4 and S5). Gail, in her second year of teaching said: "*When I was a pre-service teacher…they* [her instructors] *observed me while teaching and together we analyzed my lessons…Now that I am on my own, I find myself doing what I was preached not to do– lecturing…Why do I do that? How can I change it? What should I do differently? I know I have to do things differently, but I need someone…to direct me in seeing my successes and failures, to help me help myself in realizing what is going on".*

Rebecca said: "*Examining my professional life, I believe that I was sensitive to pupils' needs, and gradually constructed my knowledge about teaching methods and how to adjust them to various learning styles. For years I examined how things are 'working' in my class, and gained many insights...I don't think I need to attend a PDP in which a young teacher educator would tell me ...what should be done at class".*

Gail is frustrated from her inability to be reflective, although she did not use the word "reflection" explicitly. She feels that she does things 'wrong' but she keeps doing that because she is not sure how to do them differently. Rebecca, on the other hand, believes she knows everything about teaching and learning, and that this knowledge was acquired through her 'trial and error' experience. In fact, teaching experience seems to her central in consolidating a world view regarding effective teaching that she refuses to accept young educator's advices. This perspective is compatible with the description of highly experienced teachers in Huberman's (1989) model: they are self-accepting and resistant to innovations.

Excerpts concerning conflicts with pupils (S3). Sara said: *"The most enjoyable part of my work is my relations with the pupils. I think that because I am young, they feel* *like they can talk with me about things that are going in their lives. I am not just their mathematics teacher, but also kind of a friend".*

Eva, a teacher with 27 YTE said: "*When I started to teach 27 years ago, the pupils were different. They respected teachers. I didn't have to spend so much time and efforts dealing with discipline. This is a new generation of pupils. They are not disciplined and they show no respect to us not only as their teachers but also as human beings".*

The differences between Sara and Eva are prominent. While Sara, as a young woman, enjoys the company of the pupils, and loves the idea that they regard her as their friend, Eva is frustrated. She does not know how to communicate with her pupils. Similar to Huberman's (1989) model, Eva is nostalgic, and leans on her memories. However, her past memories of her relationship with the pupils do not refer to 'friendship' but to 'mutual respect'. This difference in perception reflects the social and the cultural changes that the Israeli society had undergone, regarding the status and position of teachers.

Implications for designing local teacher educational programs. From the above results the following recommendations are raised: 1. Knowledge about pupils is essential for all teachers regardless their YTE. Teachers do not have sufficient opportunities to acquire knowledge regarding the way pupils perceive various mathematical concepts and the way pupils construct their mathematical knowledge and understandings. 2. All teachers find the enrichment of their mathematical knowledge as very important component of any PDP. Though most PDPs include this component, it is important to consider the initial knowledge of the teachers, taking into account their previous experience. 3. While designing PDPs special attention should be given to the fact that YTE is a meaningful factor distinguishing between groups of teachers. For example, PDPs designed for beginners should focus on developing the teachers' ability to reflect on their various experiences in class, and help them to consolidate their beliefs and views regarding teaching and learning, since these beliefs and views will affect their practice (Richardson, 1996). Highly experienced teachers should be provided with opportunities for developing abilities to communicate with the 'new generation' of pupils, in order to assist them to overcome their fatigue and frustration.

CONCLUDING REMARKS

Teachers' needs are culturally and socially embedded and are affected by the way teaching is perceived by their societies (Calderhead & Shorrock, 1997). Therefore, PDPs which are based mainly on research literature might not fully address the needs of a certain community of teachers. The needs of teachers, as members of the society, should be evaluated considering cultural and social aspects of the community in which they are functioning. It should be remembered that adults want to know why they need to learn specific subject and they are most interested in learning subjects that have direct and immediate relevance to their job or personal life (Knowles, 1990). Thus, the teachers' voice is important for evaluating their real needs and for understanding what they perceive as essential for their PD. Relating to teachers as partners for designing PD programs might increase their motivation to take an active part in such programs and internalize their contents (Knowles et al., 1998). Providing suitable answers to variety of teachers is a real challenge for PDPs' designers.

A further research is needed in order to be able to explain why teachers are not interested in sharing their knowledge, as can be concluded from the low rate of S9 and S12. Teachers also show a little interest in carrying out action research (S15) for building their own knowledge. Instead, they prefer to acquire knowledge by learning from PDPs' instructors.

Further research is also needed in order to be able to answer questions like: How to construct an approach to PD, one that takes comprehensive perspectives on the relations between PDPs and the improvement of teaching and learning (Ball & Cohen, 1999)? How do teachers implement their acquired knowledge and develop it along the years? What kind of support teachers need in order to become life-long learners?

Research aimed at providing answers to these questions should consider the nature of the local society and culture, in order to generate an optimal compatibility between the contents of the PDPs and the needs of the target population. It would be interesting to compare between teachers' needs all over the world, relating it to social and cultural differences.

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BATIKS: HOW TO LEARN MATHEMATICS A DIFFERENT WAY AND IN A PARTICULAR SCENARIO

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The Dance School community is a minority but it is valued by society. This is a vocational school but students also attend subjects from the mainstream educational system. This research took place in a 9th grade class already that was used to work collaboratively. It assumed an interpretative/qualitative approach, based in an action-research project, inspired in ethnographic methods. This study was based on 2 research projects: Interaction and Knowledge (IK) and IDMAMIM. A microproject was implemented in order to elaborate batiks, using the school's name. Results illuminated students' (re)actions and accounts related to this microproject, to Mathematics' classes, namely mathematical performances. They also illuminate how important this microproject was to students' engagement and to their achievement.

INTRODUCTION

In Portugal there are minority cultures that are rejected by society and others that are recognized and appreciated. The Dance School is a minority culture that is socially valued. It is a vocational and artistic school, where only students that want to become dancers (and are potentially very talented dancers) have place. In order to make part of these community students need to succeed in dance examinations.

In the Dance School everything is organized around dance and the artistic subjects: timetables, classrooms, and so on. Mathematics plays a secondary role. The selective subjects are classic and modern dance techniques. Although students know that they cannot ignore academic subjects because they need to succeed on them too, in order to continue their dream, they also state that their favourite subjects and the ones they spend most of their time and effort are the ones directly related to dance. Thus, to learn and to teach mathematics in this scenario is different than teaching it in a mainstream school and it is a new and stimulating challenge to the teacher.

As we live on a multicultural society and we claim that intercultural practices are needed, we promoted an intercultural (and interdisciplinary) microproject based on Cape Verde culture. Cape Verde community is the biggest ethnic minority in Portugal and their uses, ways of being, reasoning or approaching mathematical tasks are not socially recognised and valued. Thus, gathering mathematical learning and valuing this culture seemed a promising enterprise.

Thus, the problem we were studying was the lack of significance that mathematics used to have for the Dance School students. The research questions we were addressing were: (1) What are the contributions of an intercultural microproject, associated to collaborative work, in order to facilitate students' mathematical

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meaning construction and mathematical knowledge appropriation?; (2) What are the contributions of an intercultural microproject in order to develop students' citizenship, namely their respect and valorisation of usually less valued minority cultures, like the one from Cape Verde?

THEORETICAL BACKGROUND

There is an enormous diversity of definitions of culture. Many authors tried to define culture and presented different claims. Nieto (2002) defined culture as:

> "(…) the ever-changing values, traditions, social and political relationships, and worldview created and shared by a group of people bound together by a combination of factors (which can include a common history, geographic location, language, social class, and/or religion), and how these are transformed by those who share them." (p. 53)

Thus, at school we can find an enormous diversity of cultures. Not only origin cultures but also many others, including the school's culture. In many cases this culture is so far away from students' cultures that they focus their energies on others directions (Säljö, 2004). Thus, it is important to find a way of promote interactions among the different cultures which are part of a particular school. This illuminates the need of an intercultural education, namely in mathematics (D'Ambrósio, 2002; Favilli, César, & Oliveras, 2004; Peres, 2000; Powell & Frankenstein, 1997). In 1991, Ouellet already stressed that the intercultural education was not only for the minorities but also for majority groups, based on the comprehension of each other, on the communication among them, and on the promotion of interactions. Intercultural education also include citizenship education, namely through mathematics (Skovsmose, 1998, 2005).

In the Dance School, we tried to contribute to students' citizenship education during the classes and also through the microproject. We tried to show some elements of a minority culture that is highly represented in the Portuguese society, relating this culture with school mathematics. We aimed at facilitating students' recognition of the value of Cape Verde culture and its mediation role in order to learn mathematics (Teles, 2005; Teles & César, 2005, 2006a, 2006b, 2007). Some authors argued that intercultural microprojects related to handicraft activities support an intercultural approach, giving a cultural dimension to the learning process, contributing to academic achievement (César & Azeiteiro, 2002; César, Mendes, & Azeiteiro, 2003; Favilli, 2000; Favilli, César, & Oliveras, 2003; Favilli, Oliveras, & César, 2003). They illuminated the potential of intercultural and interdisciplinary microprojects in order to promote mathematical knowledge appropriation, but also to mobilise/develop students' competencies, including social and emotional ones.

During the whole school year these 9th grade students worked collaboratively in mathematics classes. They worked in dyads, discussing their reasoning and solving strategies, helping each other, and co-constructing their knowledge (Teles, 2005; Teles & César, 2005, 2006a, 2006b). Thus, the microproject was part of a coherent didactic contract implemented during the whole school year and negotiated with students on the beginning of the year. Collaborative work was studied and promoted in other studies and stated to be a facilitator for students' knowledge appropriation when it was part of a negotiated and coherent didactic contract (César, 1998, 2007; César & Santos, 2006; Schubauer-Leoni & Perret-Clermont, 1997).

A community of learning emerged from the practices that took place in mathematics classes as there was a mutual engagement, a joint enterprise and a shared repertoire (César, 2007; Wenger, 1998). The nature of the mathematical tasks assumed a relevant role on that process (César, Oliveira, & Teles, 2004). Their social marking was essential to students' engagement (Doise & Mugny, 1981), promoting their participation in the solving strategies and during the general class discussion.

Hummel (1979) stated that "culture and education are intensively linked as verse and reverse of the same reality. It is impossible determinate where the educational ends and the cultural starts and it would be nonsense separate them" (p. 234). Thus, teachers' role also includes contributing to create bridges among cultures and education, promoting diverse and intercultural learning experiences.

METHOD

This research is an interpretative/qualitative study, inspired in ethnographic methods and based in two research projects: *IK* and *IDMAMIM*. The first one was developed during 12 years and its main goal was to study and implement social interactions in formal educational scenarios. *IDMAMIM* project was developed in some towns of Spain (Granada), Italy (Pisa) and Portugal (Lisbon). The two main goals of this project were to identify didactic needs to develop intercultural Mathematics Education, and to elaborate intercultural didactic materials.

This study was an intercultural and interdisciplinary microproject also engaging the mathematics, drawing, Portuguese, and history teachers. The mathematics teacher was also the researcher. Students elaborated batiks based in the school's name (EDCN). Later on the batiks' elaboration process was used to explore some mathematical contents such as direct and inverse proportionality. These students worked collaboratively during the whole school year and they developed this microproject in 4-students groups after being used to work in dyads in mathematics classes. To explore direct proportionality we used tasks based on the first day of the batiks' elaboration process. In that day students needed to make a paste with flour, water and lime. They had a recipe referring to the ingredients needs for 500g of dry cotton. But they only had pieces of dry cotton that weighed 80g/90g. So they needed to calculate some proportions in order to know the quantities they needed to make the paste. Through those calculations we explored direct proportionality notion and its properties. Inverse proportionality was explored through what students did on the third day: the tainting process. Each student chose two colours to his/her batiks and only two students chose the black colour. In another school, where batiks were elaborated during the previous year, only one group of students chose the black colour and all ink was used on it. Inverse proportionality was explored through this situation, supposing that the number of black batiks could be changed but the quantity of ink could not. In this paper we focus our analysis on the batiks' elaboration process and on the first day after that, when we began exploring the mathematical contents based in the batiks' elaboration.

This research was developed with sixteen 9th grade students from the Dance School. As the Dance School is a small vocational and artistic school, these were all the students attending the 9th grade.

Data were collected through participant observation (audio and/or video taped), questionnaires (students and teachers), interviews (six students chose as main informants and the drawing teacher), several documents, and students' protocols.

The participant observation took place all over the school year and was registered in the teacher/researcher's diary. The audio and video tapes used in this paper were taped in May. Students answered to questionnaires in the beginning of the school year (September), in the beginning of the second term (January) and in the end of the school year (June). The teachers only answered in the end of the school year. All the interviews also took place after the end of the school year (July). The documents were mainly collected in the beginning and at the end of the school year, and students' protocols were collected during the whole school year.

We created six inductive categories based on an in-depth and successive content analysis: school's culture, interdisciplinarity, didactic contract, leadership, argumentation and mathematical knowledge appropriation. In this paper we focus in the microproject, students' work, and how students used their own experience to appropriate mathematical knowledge.

RESULTS

Promoting this kind of project in the Dance School was a great challenge to the teacher/researcher and it was also a great pleasure. Students were deeply engaged in the tasks related to the intercultural and interdisciplinary microproject. For instance, they have a very hectic life, as they have classes all mornings, afternoons and part of the evening and, sometimes, when they have shows, they also have the rehearsals. But they asked the dance teachers for permission to enter a bit later in their class – something unusual and usually forbidden – in order to participate on the 2nd day of batiks' elaboration, as this part had to be in an extra class time. Thus, the batiks' elaboration illuminated the need of a great organization and depended on students' motivation, responsibility and engagement. At the end of the school year these were some students' voices accounting for what they learned through the batiks:

> I think that... I think that it is easier and more interesting to learn. Because many people don't like Mathematics (Madalena, I., p. 4).
>
> Because it is nice and funny. And we can know some cultures from other countries (Carlota, I., p. 5).

In order to elaborate the batiks students needed to make templates with the school name (EDCN). The school's name is deeply connected to this community's identity and the Dance School culture is a very powerful one. Thus, producing the templates and knowing that there would be an exhibition at the end of the school year contributed for students' engagement. These templates were made in mathematics and drawing classes, and the two teachers worked together.

During the batiks' elaboration, students had a paper with a detailed description of the steps they needed to do. On the first day, students had to do a paste with flour, water and lime. But they needed to adapt the receipt to their own conditions using direct proportionality. This is illustrated in Figure 1 which shows the computations of one of the groups, using direct proportionality.

Weigh of dry cotton: 70g

Weigh of small glass: 29g Weigh of great glass: 46g

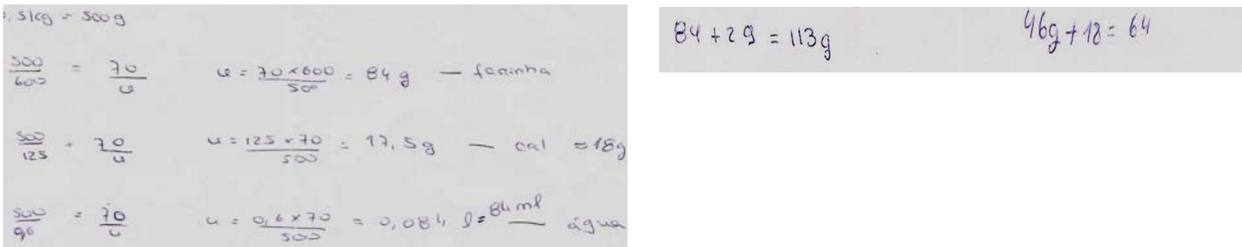

Figure 1. Students' computations (direct proportionality)

In mathematics classes after the batiks' elaboration the teacher/researcher proposed some tasks based on the microproject and what students did. The first task began with a question where students should explain they needed to keep the proportionality among all the ingredients of the first paste (1st day). And they explained it as we can see through the following example:

> Dyad's answer: We think that all ingredients should be divided proportionally in order to the paste to be consistent. If it doesn't happen, the paste doesn't be well done and thus we never could to construct batik.

The teacher/researcher elaborated the task of direct proportionality based on the relation between the wraps' weight and the quantities of each ingredient for the 1st paste. Students had to calculate proportions and drew graphs. They calculated the quantities of flour, water and lime when the wraps' weight was changed and they discussed about what happened. Students also needed to calculate the value of the constant of proportionality and discussed about its meaning on that particular context. Through the relation between the quantity of flour and the wraps' weight, students could establish the analytic expression of the function (quantity of flour depends on the wraps' weight). Finally, students represented graphically that function, and they discussed its form. After solving the task, some students went to the blackboard, answered to the questions and explained their dyad's reasoning and solving strategies. At the end, they did a synthesis of the characteristics of a direct proportionality relation. The same kind of practices was used in order to explore the inverse proportionality.

As we stated before, students work collaboratively during the whole school year. The following photos were registered after the batiks' elaboration and it illuminates that students were all engaged in the task, that they helped each other.

Figure 2. Students solving a task based on the batiks' elaboration process

Although some students did not like the contents related to functions, no one rejected these mathematical tasks and they all worked together in their resolution. Students' enthusiasm and pride was visible because they were learning mathematics based on their own work and on a meaningful task, they performed from bottom to top, something that they liked to do, as Salvador accounted:

> Yes, I think because with batiks, I say, we pl… we learn playing. (…) We have fun elaborating batiks. And we need to make calculations to elaborate batiks. This way, we learn, we develop mathematics doing something we like (Salvador, I., p. 4).

During the classes after the batiks' elaboration it was common to listen to students' argumentations based on what they did, on their own experience, as we can see on Célia and Eduardo interaction:

23	Célia:	Quantity from the recipe: 500g. Then, flour's quantity: 600g.
24	Eduardo:	600g.
25	Célia:	Grams. Lime: 125g. Now, 0,6l of water.
26	Eduardo:	Lime. 125g of lime.
27	Célia:	Is it 600 ml?
28	Eduardo:	Quantity... (...)
29	Célia:	Teacher... it is better we do...
30	Eduardo:	No teacher, it's ok. I remember that... the result was 90 something.
31	Célia:	I'll ask the teacher.
32	Eduardo:	There is no need.

Eduardo was the less competent peer in mathematics in this dyad (César, 1998, 2007; César & Santos, 2006; Vygotsky, 1932/1978). Célia loved mathematics. She had always experienced success in this subject and she loved to help her colleagues who experienced some difficulties learning it. But, sometimes she needed her teachers' support to feel secure, as we can observe in this episode (Talks 29 and 31). But Eduardo remembered what he did during the batiks' elaboration and he felt so confident that he did not want Célia to ask for the teacher's help, he wanted to solve the task on their own (Talks 30 and 32). One of the features of the didactic contract is that students should discuss between themselves before asking for the teacher's help (César, 1998; César & Santos, 2006).

In the next three classes, students discussed direct and inverse proportionality based on their own work and experience.

At the end of the school year, students and the mathematics, drawing, Portuguese, and history teachers organised an exhibition. Students exhibit their arts works based on the school name, the batiks applied on t-shirts, the templates, mathematics tasks, Portuguese contributions (Cape Verde song lyrics in *Creole*) and history contributions (the origins of the batiks).

FINAL REMARKS

The microproject developed with those students was a rich learning experience for them and for us, as teacher/researcher. As Salvador accounted, they learned mathematics doing something they enjoyed, by a funny way, but also through a meaningful task. Madalena and Carlota also underlined the importance of learning mathematics through an interesting and easier way. At the same time, as Carlota focused, it was a great opportunity to know more about another culture. Through this microproject students also experienced the relations between mathematics and practical activities, relating manual and mental work.

Students worked together, trying to achieve a common goal: to elaborate the batiks. They collaborated in a project that was their own project, and that they had to elaborate from bottom to top, making up their mind and assuming the consequent responsibilities about their choices. On the other hand, having the school's name on the templates facilitated the (co)construction of a sense of identity related to this microproject, as the Dance School culture is a very robust one and students who are part of this school usually chose to be there and love being there.

The collaborative work that was developed in mathematics classes since the beginning of the school year and the interactive inter-play (César, 2007) that was part of this school culture were a great help to this microproject success. Students worked as a team, and they were used to help each other, and to collaborative work, usually not in academic subjects but in dance subjects. A show means a lot of individual work but also a lot of collaborative work.

The nature of the mathematical tasks solved after the batiks' elaboration was crucial to students' engagement and for the appropriation of mathematical contents. We could see as Eduardo relied on his experience in order to solve the mathematical tasks and to discuss them with Célia. Thus, even if some students had stated before that they did not like functions, that they were difficult and they could not understand them, when they solved tasks related to functions but also to the batiks' elaboration, they were able to appropriate the mathematical knowledge and to give a meaning to that knowledge. Thus, they were able to overcome the barriers they felt in the previous school year, when they studied functions for the first time.

At last but not least, this microproject was an opportunity to contact with a culture students did not know well, to use mathematics in a real context and to learn mathematical contents through their own experience. It was a striking experience to students and to us all, as teachers and researchers.

NOTE

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THE ASSESSMENT DISCOURSE OF TEACHERS' TEXTBOOKS IN PRIMARY SCHOOL MATHEMATICS

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Considering language as discourse, this chapter attempts to analyze the assessment discourse of mathematics supplement textbooks for teachers. Official texts of curricula are examined focusing on assessment in primary school mathematics. Our analysis shows similarities in characteristics of textbooks' language and to discourses their producers appeal for their positions' justification, as inconsistence and ambiguities within the official discourse, which make obvious that positions and practices should be submitted to scrutiny and clarification and that teachers' training programs on alternative assessment methods should be planned.

INTRODUCTION

This study adopts the theoretical premises of Halliday (1985) as well as Critical Discourse Analysis (C.D.A.) by Fairclough (1989, 1995) and Kress (1989) and aims to investigate the picture of assessment constructed within the discourse of mathematics textbooks supplement for teachers, as well as the role assigned to the teacher. The picture of assessment thus constructed is examined with regard to the meaning attributed to it, the functions and purposes it serves, the forms and criteria highlighted. The language features of texts that contribute to it is also examined.

In order to comprehend the texts of mathematics textbooks supplement for teachers it is necessary to examine the frame of discourse within which those texts are produced (Fairclough, 1989). In our study due to the limitations of the proceedings we only consider the official texts of curricula currently in effect related to assessment in education focusing on mathematics at primary school.

Also the ways in which the producers of the texts justify their positions and practices they highlight and any tensions within the official discourse are considered.

UNIFIED INTEGRATED CURRICULUM (U.I.C.) AND MATHEMATICS CURRICULUM

The writing of mathematics schoolbooks, texts of which are analyzed is based on the Unified Integrated Curriculum and Mathematics Curriculum (U.I.C., 2002).

As editors argue, the Unified Integrated Curriculum (U.I.C.) is an attempt of united planning of educational system which aims to contribute to the development of student's personality and to his/her harmonious integration to the society. It highlights the need for a more essential connection between school education and the work market, the need for connection between knowledge and daily life and it proposes the thematic teaching approach. As regards assessment of student a definition is not given in U.I.C., but basic principles and its characteristics are

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 465 - 474).

referred (U.I.C., p. 18): the assessment is continuous and purposeful, concerns not only the acquisition of knowledge but also the acquisition of skilfulnesses, the formation of attitudes, values and behaviour, and presupposes criteria and objectives which are specific and clear. Assessment has to be characterized by transparency, reliability, validity and objectivity. It has also take into account the particular needs of each student and their personal way of learning. The student has to be involved in the process of assessment so that to acquire skilfulnesses of self-assessment.

The main purpose of student's assessment, as it is defined by U.I.C. is the feedback of teaching and the detection of learning gaps for the purpose of the improvement in school education and the progress of student. According to U.I.C. the assessment contributes to the satisfaction of students' needs, the development of his/her cognitive and meta-cognitive skilfulnesses. The strengthening of learning motives constitutes also a great objective of teaching and assessment. The pedagogic function of assessment is dissociated from the selective function and the descriptive assessment is highlighted especially for the first classes in primary school. The descriptive assessment is proposed to be combined with traditional numerical assessment in last classes.

The assessment is related directly with the objectives of teaching and learning and for this reason we consider necessary to mention the objectives of teaching as they defined by the mathematics curriculum (U.I.C., p. 311): The acquisition of basic knowledge and skilfulnesses, the development of mathematic language as important mean of communication, the understanding of basic mathematic methods, the acquaintance with the process of reasoning and prove, the development of solving problem ability, the understanding of historical dimension of mathematic science and the formation of positive attitude to mathematics are highlighted as essential objectives of mathematics teaching. According to the new mathematics curriculum the teaching approach owes to rely on principles of constructivism and discovering-investigative learning. It proposes the thematic teaching approach and the team-work teaching. It puts emphasis on procedures of learning and not only on its product. This position is justified to modern views about mathematics teaching and learning. According to these views "mathematics is the product and the activity through which the result is produced." (U.I.C., p. 367).

Examining the official discourse of assessment in curricula currently in effect in Greece, we notice that there is an agreement between official discourse and researchers' discourse regarding the pedagogical dimension of assessment and its objectives.

The forms of assessment that are highlighted in the U.I.C. are: diagnostic, formative and summative assessment. Pedagogical function of assessment is strengthened with a great variety of assessment methods among which are proposed: Written and oral close and open-ended questions, semi-structured dialogue, projects, systematic observation, portfolio, self-assessment, peer-assessment, combination of different methods not only do they aim to effective detection of knowledge acquired, but to the ability of knowledge management and application too. Official texts draw on the researchers' discourse to justify their positions, yet at the same time considerable tension is apparent in educational policy about assessment.

Today in the assessment system for the primary school in Greece, the numerical assessment coexists with the descriptive assessment. These forms of assessment contradict each other, since the descriptive assessment clearly has pedagogical aims and is linked to the individual norm-referenced assessment and individual-specific teaching, whilst the numerical assessment grades and categorises students and follows the classroom norm promoting comparison and competition.

An other contradiction is the fact that today, though descriptive assessment is legislated for all classes of Greek primary school it isn't applied by teachers in practice.

With regard to the descriptive assessment and the alternative methods of assessment which the official texts suggest, explicit instructions to teachers on how to apply them in practice are not provided. With regard the criteria of assessment official texts give a hint, but do not provide explicit instructions on which criteria and how to use them.

Considering fundamental principles of assessment in the U.I.C., according to which "assessment of students not only concerns knowledge that has been acquired, but also the acquaintance of skillfulnesses, the formation of attitudes, values and behaviors" (U.I.C., p. 18) we see contradictions when consider the aims of mathematics curriculum that seem to concern mostly the field of knowledge.

For materialization of curriculum objectives printed educational material is used which consists of a Book for the Student, a Copybook for the Student and a Supplement Textbook for the Teacher. The Book for the Student beyond the texts of cognitive content of subject includes work-sheets related to the subject of each unit. The Copybook for the Student includes work-sheets for practice and aims to an extensive and in-depth knowledge of each unit. The Supplement Textbook for the Teacher includes extensive directions about the didactic methodology, a teaching timetable of each unit, suggested activities, the required material and instructions about the assessment.

The U.I.C. highlights as priority of education the provision of equal opportunities and possibilities of learning for all students having as ulterior purpose the bluntness of social inequalities and the maintenance of social cohesion (U.I.C., p. 7). For all that, today there is one and unique schoolbook for each subject and an undifferentiated curriculum for all Greek students, despite the fact that the Law has established the introduction of more than one schoolbook (N.2525/97) in whatever subject it is considered as necessary. It worths mentioning that the schoolbooks which are based on the U.I.C. are used in the classroom during the second school year.

METHODOLOGY

We considered the texts of teachers' textbook supplement for mathematics that relate to assessment.

A method based on Halliday's "Functional Grammar" (Halliday, 1985) and on the interpretative techniques of Fairclough's Critical Discourse Analysis (Fairclough 1992, 1989, Kress, 1989, Hodge & Kress, 1993) was used to analyse these official texts. Thus characteristics of the texts' language were examined and the functions - "ideational", "interpersonal" and "textual" - (Halliday, 1973) that they carry for the speaker/author and the reader/listener were interpretated.

The "ideational" aspects of texts to be analysed relate to the picture constructed for mathematics assessment. They were analysed primarily through the examination of types of processes (Halliday, 1985, pp. 101-131) that take place in the discourses under examination and are related to the task of assessment, the type of logical subjects, ie the human or inanimate actors of these processes (*ibid*, pp. 32-37). The presence of a human being in a text or the absence thereof, and the use of inanimate abstract nouns as actors of the processes, was examined via the use of passive voice and nominalisations that relate to social and ideological aspects of language (Fairclough, 1989, 1992, Hodge & Kress, 1993).

The teacher's role constructed for the task of assessment, the relationship between writer and reader and the degree of the teacher's autonomy concerns the "interpersonal" function of language. It was analysed through the use of personal pronouns, modes and the text's modality (Halliday, 1985, p. 86; Fairclough, 1989, p. 129; 1992, p. 159 on modality, and Kress, 1989, on their interpretation).

The structure of official texts as a whole and the type of text that are related to the "textual" function were examined through the type of "themes" (Halliday, 1985) that dominate the text.

Drawing on the theory of C.D.A. by Fairclough, "member resources" were also examined which the producers of texts draw on, in order to justify their positions and practices.

THE DISCOURSE OF TEACHERS' TEXTBOOKS FOR ASSESSMENT IN MATHEMATICS

Today's teachers' textbooks report on assessment in separate chapter in opposition to previous ones that mentioned to assessment in few paragraphs. Thus new textbooks lay greater weight on assessment. Basic instructive principles that previews textbooks proposed (emphasis on problem solving, need for learning to be connected with real situations of daily life, need of respect for the individuality of a child and adaptation to characteristics of class and type of school, stress on processes of learning and not only on its product, active participation of student in correction of his/her written work's errors) are maintained in new mathematics textbooks. In addition alternative methods of assessment are introduced and particular emphasis is laid on handling and turning to account of error in order that children acquire metagnostic abilities, "learn how to learn".

The analysis of its discourse showed not only similarities but also differences with regard to the picture of assessment that is constructed within the texts of teachers' mathematics textbooks for all classes in primary school[1].

The definition of assessment's notion isn't given in textbooks, but the nature of assessment is defined implicitly through its purposes, functions and characteristics. In texts it is noticed unanimity with the discourse of researchers as to notion and purposes of assessment that make it necessary. The purposes of assessment are diagnosis of students' needs (diagnostic function) and feedback of teaching (feedback function). That is, its purpose is the advance of knowledge and "not performance" (Mathematics for Primary School, Textbook Supplement for Teacher, B' class, p. 19). Thereby, its character is considered clearly pedagogic.

As regards the basic principles of assessment, some of them which are pointed out in the U.I.C., are stressed in new textbooks:

> Assessment doesn't concern only knowledge but also skillfulness… (Mathematics for Primary School, Textbook Supplement for Teacher, B' class, p. 15)
>
> …the sheet of assessment relies on the objectives of each unit and on particular needs of students…students are involved in the assessment process" (Mathematics for Primary School, Textbooks Supplement for Teacher, E' class, p. 17)
>
> assessment constitutes part of daily school work (Mathematics for Primary School, Textbook Supplement for Teacher, A' class, p. 163).

Regarding the criteria of assessment only in the textbook of F' class is mentioned that criteria of assessment are mainly the interest and effort that a student makes not the result. As it is mentioned the above criteria must be acknowledged to the students.

As for the forms of assessment formative and summative are mainly highlighted. After the analysis of students' errors in revising tests at the end of each period remedial teaching is proposed. As for the initial diagnostic assessment only the textbooks for B' and E' classes (products of the same team of authorship) make an indirect reference to it by the statement:

> The teacher doesn't wait for revisioning chapters to discover the children who have difficulties or gaps…doesn't go to the next lesson if children don't have prerequisite knowledge and skillfulness... The teacher finds out if the children have prerequisite knowledge and skillfulness through assessment in order to go to the next lesson. (Mathematics for Primary School, Textbook Supplement for Teacher, B' class, p. 15)

In addition the textbook for D' class specifies the basic knowledge which is necessary for working out of the new chapter. This knowledge is checked by the teacher with one or more questions or activities which are cited at the beginning of the chapter in the Book for the Student.

As far as the methods of assessment is concerned the consideration of the texts showed that given information about assessment mainly concerns the revising tests of assessment included in the teacher's textbook and the way of their correction. More concretely textbooks for A' and C' classes (products of the same team of authorship) focus to the revising test for each period and to the scale of objectives and notions as well as to the way of their use. The scale of objectives and notions is suggested to be used: 1. by the teacher during the teaching to observe students' behaviors and ascertain to what extend they comprehend taught notions. 2. by students for self assessment and peer assessment. (Mathematics for Primary School, Textbook Supplement for Teacher, A' class, p. 163). In textbooks for B' and E' classes, as it is pointed out, self assessment and peer assessment are realized in the context of revising chapters and "constitute basic elements of students' assessment" (Mathematics for Primary School, Textbook Supplement for Teacher, B' class, p. 15). In addition in the textbook for D' class the two paragraphs which refer to assessment concern the use of assessment's sheets in the textbook at the end of each period, as well as the form of self assessment and peer assessment on teams work and the form of teacher's self assessment. Alternative methods of assessment that are highlighted in the U.I.C. such as portfolio are not highlighted in teachers' textbooks except in the textbook for F' class. In this book, though various methods are described (not formal methods: oral questions and answers of students, observation of team-working by the teacher, formal methods: written questions, exercises, problems in the Copybook for the Student and revising tests, projects and portfolio) their use is canceled by the statement:

> Generally assessment is conceptualized in two levels: In a daily basis formative assessment takes place…which is applied during the teaching through the "Questions for self control and discussion"… In a second level, at the end of each unit, summative assessment takes place through the test…The results from the two levels of assessment will be useful for students' assessment as well as teaching assessment and schoolbook's assessment generally …(Mathematics for Primary School, Textbook Supplement for Teacher, F class, p. 14)
>
> Specific questions for not formal methods by which the teacher gathers information about what a student has learned are concluded in the Book for the Student. They are the"Questions for self control and discussion". (Mathematics for Primary School, Textbook Supplement for Teacher, F' class, p. 15)

In the above statement assessment seems to be faced as a process not continuous but broken away from the remaining educational processes. It worths mentioning that "Questions for self control and discussion" at the end of each chapter conclude two or three questions of the form "right-wrong" and require from students to explain by their examples the terms which they were taught in this chapter. The latent conception in this statement is that students' answers to these questions are sufficient indications for the comprehension of student. It is about an assumption on which traditional discourse of assessment relies. According to this assumption the teacher can arrive at a precise conclusion about the comprehension of student and his/her way of thought. (see, Morgan, 2000).

Considering the areas of objectives which are assessed by the proposed tests and forms of assessment in teachers' textbooks, inconsistency is found between textbooks and U.I.C. is found out because in tests of assessment cognitive objectives and skilfulnesses which concern problem solving and performance of processes mainly are assessed. In the teachers' textbooks for B', E' and D' classes forms for self-assessment on team-cooperative skilfulnesses are also proposed.

As regards to autonomy which teachers' textbooks give to teachers as to the duty of assessment, a degree of freedom is given as to the use of proposed tests. Teacher can use them "unedited or in his/her judgment" (Mathematics for Primary School, Textbook Supplement for Teacher, D' class, p. 21) choosing exercises from a proposed test of the textbook for his/her class or each student (Mathematics for Primary School, Textbook Supplement for Teacher, F' class) or construct a test of his/her own with the presupposition that it does not deviate from the predetermined objectives of the unit (Mathematics for Primary School, Textbook Supplement for Teacher, B' class). According the textbook for B' class the teacher has also the freedom to modify the teaching time of each chapter.

Within the discourse of teachers' textbooks the role constructed for the teacher is that of the local enforcer who is obliged to follow the official policy on assessment, yet, at the same time, ought to take into account his/her students' needs. He/she takes the initiative in modifying, to a degree, the proposed assessment practices, depending on the students' individual situation, yet taking care not to stray far from the general frame of assessment that the official discourse shapes.

By investigating the sources of their arguments, it became apparent that the producers of teachers' textbooks mainly draw on the discourse of researchers with respect to the concept, objectives and necessity of assessment. Some times they found their statements on the official discourse of curricula.

As for the main linguistic features used in the teachers' textbooks which are linked to "ideational" and "interpersonal" functions of language, essential similarities were observed. Specifically:

The processes that dominate are "attributive" processes (Halliday, 1985) when it comes to the nature of assessment and its methods. "Existential" processes are used when it comes to the existence of tests of assessment which are included in the teacher's textbook. In these cases the use of nominalizations is a usual phenomenon. "Material" processes are used when instructions are given about the use of assessment methods and then the teacher is the logical subject of the process that is identified with the grammatical subject, then active voice is used. When the teacher is stated explicitly as the agent or he/she is omitted but his/her identification is implicit, passive syntax is used. Statements are expressed by positive clause except for the use of some negatives which connect intertextually the text with previous texts to which the writer had access. The use of negatives can also indicate preexisted views of teachers about the nature of assessment as well as established practices for the use of its methods.

The formulation of statements in third person, the absence of personal pronouns in first and second person as well as the use of nominalizated nouns (“assessment”, “revisioning lessons”, “tests”, etc.) in the place of actors give a typical style to the text and contribute to the creation of distance between the writer and the reader constructing a typical relation between them. In some cases the use of verbs in the first plural person is noticed which (in Greek language) refers to the personal pronoun “we”. The pronoun “we” has sometimes “exclusive” meaning, sometimes “inclusive” (see Fairclough, 1989, p. 127). In the first case it is indicated that the team of authors speak with the voice of scientists’ community where they belong to, that guarantees the validity of their statements. The use of modal “must” and of phrases as “we owe…”, “it is necessary” intimates obligation of observance of proposed process by the teacher. In the phrases “it is better”, “it is advisable”, “it is very important”, “it is a good tool”, “the proposed activities correspond clearly with the objectives”, “we suggest”, “the authorship followed the valid specifications of Pedagogic Institute” the writer’s intention of persuading the reader to apply the instructions which are proposed is revealed. The relation that is constructed between the writer and the reader is asymmetrical relation of power: The writer is the provider of knowledge and vehicle of instructions, whilst the reader is constructed as a person who isn’t informed of the way of assessment and for this reason he/she needs guidance. In some cases (the phenomenon is often in the teachers’ textbooks for B’ and E’ classes which are products of the same team of authors) it is noticed use of the first plural person “we” which has “inclusive” meaning, and then the intention of reader’s identification with the position of writer and consequently active participation in the activities which are proposed. It has to do with attempt of the writer to decrease the distance between him/her and the reader so that more favorable conditions are created for the performance of these activities. The use of technical/scientific vocabulary such “portfolio”, “self assessment”, “peer assessment” and the absence of further explanations suppose that the reader can comprehend the statement of the text and apply the instructions. The clauses are declarative intimating the asymmetrical relation of power between the writer and the reader. The tense that is used is the present in the indicative mood which expresses the certainty of writer for his/her statements. The modality of certainty is dominant.

As for the textual aspects of teacher’s textbooks, the thematic choices that the producers of texts make, and information or instructions which they give without making distinction between them, construct texts as descriptive, informative, but also a handbook with instructions about the assessment. In some cases it is noticed justification for statements by arguments and causal turn isn’t consistent in all texts. The choice of causative and deductive conjunctions as well as the apposition of statements that relate with causality is more often in the textbook for first class (A') in primary school constructing the text argumentative beyond informative.

CONCLUSIONS

Within the official texts, we see two dominant discourses in conflict with each other. On one hand we find the pedagogical discourse which is encoded in words and phrases of pedagogical content, and on the other hand the educational-political discourse which expresses the government's intentions for education and is encoded in the syntax of official texts.

The main characteristics of official texts are similar to those of scientific texts: tendency to impersonal expression that adds objectivity to the statements in the text, absence of personal syntax achieved with the use of nominalizations and passive syntax, use of processes that concern relationships when it comes to the nature and methods of assessment, use of simple present in the indicative that expresses the modality of certainty, specialized technical/scientific vocabulary (see Kress, 1989). Analysis of texts shows contradictions and ambiguities within the official discourse, lack of unified approach as to proposed practices of assessment in the classes of primary school, inconsistency between curriculum and textbooks. This fact obviously poses the need to critically consider and clarify the values and proposed practices. Simultaneously, considering the difficulty in applying alternative methods of assessment, as researches point out (Black & Wiliam, 1998, Kahn, 2000, Vlachou 2007) as well as tensions that it causes to teachers (Broadfoot, 1998, Lyons, 1998), it is evident that teachers' training programmes on assessment and how to apply alternative methods should be planned.

NOTE

1. Author's note: Attendance at Primary Education lasts for six years and includes six classes: A', B', C', D', E', F'. Children are admitted at the age of 6.

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CHILDREN TALK ABOUT MATHEMATICS ASSESSMENT

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This paper reports on the second phase of a study in which Year 7 children speak about standardised compulsory state-wide Numeracy Tests. While accepting these tests as an unavoidable part of their schooling, the children developed strategies to cope with the multiple pressures the tests created. Their views reflect their understandings of the test as defining both mathematics, and mathematical competence. They speak of an assessment regime with the power to create anxiety, undermine their mathematical confidence and construct their mathematical identities, and contemplate changes to that might both improve the quality of information gathered, and enhance their wellbeing.

GIVING CHILDREN A VOICE IN MATHEMATICS ASSESSMENT

Children's powerlessness and vulnerability in the face of traumatising situations such as war, poverty, natural disasters, and child labour are well-recognised worldwide. The less dramatic yet nonetheless significant impacts on children's lives of the prosaic and apparently 'everyday' nature of compulsory, standardised assessment practices in their schooling receive far less attention and are not well understood, despite the profound life consequences. Recent developments in mathematics education take little account of children's rights to participate in all matters affecting their lives, as enshrined in Article 12 of the UN Convention on the Rights of the Child. Within the discourse of raising standards in mathematics in many countries and the introduction of increasingly prescriptive mathematics curricula to address perceived shortcomings, the performance of teachers and schools is undergoing intensifying scrutiny, and children's mathematical achievement as measured against clearly-defined and measurable standards is becoming a core focus of the curriculum. In spite of their status as the primary stake-holders in our education systems, children are not considered to be critical partners in deciding what those standards might be. It is assumed that children have nothing to offer debates about what mathematics they must learn, how they might most effectively learn it, and how their learning might best be assessed. Such exclusion positions children as passive and deficient recipients within coercive, adultcentric cultures of expertise.

Assessment is a process with social consequences (Broadfoot, 2002; Morgan, 2000). Annual, compulsory state-wide Year 3, 5 and 7 Numeracy Tests were introduced in Queensland over a decade ago but no research has been undertaken to assess the effects of this policy on the social dimensions of children's learning of mathematics. This method of assessment is poised to become part of a national testing regime which will be extended to include numeracy tests at Years 4, 6 and 9. This paper reports on a research study based on the approach that implementation of any major social policy should be subjected to *social impact assessment,* defined by Taylor et al

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 475 - 484).

(1995) as “a process of research, planning and management of change arising from policies and projects” (p. 1). These authors see social impact assessment as underpinned by critical theory, thus “the ‘enabling’ practice of social assessment becomes an essential part of a more complete response to social change” (p. 57). The introduction of compulsory assessment regimes into schooling constitutes ‘social change’ yet there seems to be no recognition at any level of a need to assess the social outcomes of such change either in its planning or implementation phases, and even less of ‘enablement’ of children as participants in the change process.

Sinclair Taylor (2000) argues, “Empowerment is about shifting the balance of power from service providers and scrutinizers across political, community and institutional levels to recipients… Giving children a say in their schooling, or any matters affecting them, gives them a stake in the process…giving children a voice in decision making makes them visible…” (p. 32). Ernest (2004) also highlights the importance of involving the research subject as a co-researcher. Accordingly, this research seeks to empower a group of children in their learning of mathematics by providing them with a platform from which to speak about how standardised state-wide numeracy assessment impacts upon their lives not only as individual learners, but as social beings - members of families and peer cohorts. The research enables their views to be shared with parents, teachers and the state educational assessment authorities, raising awareness of the social issues for children in high stakes mandatory numeracy testing.

RESEARCH DESIGN

Research Phase One: The research began in 2005 as a school community initiated project focussing on participants’ experiences of the Year 5 Numeracy Test in Queensland, Australia. It was based on the premise that affording key stakeholders in mathematics education - children, parents, teachers and school administrators - a say in standardised state-wide numeracy assessment is an empowering process given the insights gained and communication created. The first phase of the research indicated three distinct negative impacts of the test on children. These were:

(1) *Adverse effects on student and family wellbeing*. Participants’ reports revealed a significant proportion of children experienced mild to severe stress and some suffered severe feelings of disappointment, loss of confidence and decline in mathematical self-belief as a result of the Test process; participants also reported extra pressure on families created by the Test, including managing children’s stress and disparities between siblings’ and friends’ Test performances.

(2) *Limitations of information gained from the Test.* Participants all reported that they found the multiple-choice test results yielded little specific or useful information about the children’s mathematical skills and knowledge in the absence of the children’s completed Test papers or the original Test questions. Discrepancies between children’s school and Test results were particularly difficult to explain for all groups of participants. Given the perceived importance and significance of the Test

results, this lack of information caused considerable frustration and consternation, particularly for parents and children.

(3) *Adverse effects of the Test on delivery of Year 5 mathematics programmes***.** Teachers and school managers reported that the Test had a significant effect on the content and pedagogy of Year 5 mathematics programmes, and that the school's focus on interdisciplinary teaching and learning of mathematics and the development of the attributes of lifelong learners including creative thinking and innovative problem solving was both undermined and compromised by the Test. Children were aware of the interruption to their everyday mathematics programmes that weeks of practising for the Test caused, and that answering multiple choice questions was not how they were used to being assessed – it was a skill they had to learn.

The Phase 1 research report stated that the Test created significant difficulties for participants which needed to be addressed. Participants suggested a range of possible changes to the Test design, administration, and reporting procedures to better manage these issues. These included: modifying the multiple-choice format to allow for children to demonstrate their mathematical working, thinking, and reasoning; reporting results alongside children's responses to the questions; granting greater choice to schools and children about Test participation; rescheduling the Test to lessen the impact on classroom programmes and maximise the usefulness of the results, and reporting the results to children and parents in a non-comparative format.

Participants' responses pointed to an overall desire for a greater sense of choice and ownership in the Test design and administration process, and better communication about the significance and implications of the Test procedure and results, firstly between Education Queensland, the Queensland Studies Authority, and schools, and secondly between school management, teachers, children, and families. It suggested that further community level research of this nature needed to be undertaken to explore the social impacts of standardised testing in mathematics across a range of school types, children's ages and from a range of geographical regions.

On reading a copy of the report, the Queensland Studies Authority responded:

> "There are many myths in school communities about the Years 3, 5 and 7 Tests, and we appreciate your attempt to understand how the Year 5 test effected [sic] a particular school… the nature of standardised testing and the legislation regarding the Years 3, 5 and 7 tests does constrain what we are able to do and your report highlights the need to educate teachers about the whole process, from item demands to standardised testing in general. With this knowledge they may be better able to communicate with students and parents in a more positive way to reduce the 'adverse effects' of the test".[1]

Research Phase Two: Encouraging comments about the value of the research from participants at the PME29 Conference presentation of a paper (Walls, 2006) in which the research was described, prompted the school administrators and the fifteen Year 5 children involved in Phase 1 of the research to suggest an extension of the study to include the 2007 Year 7 Numeracy Test. They wished to investigate whether communicating with students and parents ‘in a more positive way’ at the school level as the Queensland Studies Authority letter suggested might offset some of the adverse impacts of the test.

This paper reports on the outcomes of such intervention by focussing on children’s views of the Year 7 Test[2]. For the Phase 2 of the research, eight of the children from the Year 5 study who had remained at the school agreed to continue. Eleven more Year 7 children opted to participate in the 2007 study. To gather their views about the test, the children were clustered into focus groups. Each group spent thirty minutes talking together with the researcher acting as conversation facilitator. Based on the most frequent topics of conversation in the earlier phase of the research, a list of questions was prepared to guide these semi-structured conversations. The conversations enabled the children to voice their experiences of the Test, their beliefs and opinions about it, and their suggestions for positive change.

FINDINGS

The children revealed the Test’s impacts through their accounts of personal thoughts and feelings, conversations between their peers, conversations with their families, and interactions with their teachers. These revealed that while the children responded in diverse ways, there were common themes in their responses.

Although a number of the children reported feeling less nervous about the Year 7 Test than they had about the Year 5 Test, feelings of anxiety and apprehension were still a common feature of their conversations. The build-up to the Test was a particularly anxious time for most of the children as these comments illustrate.

Teagan: I was nervous…probably because it’s so important. And I worry about doing well and that.

Bella: The night before the test I was kinda shaky because I hadn’t done one before…I didn’t know what to expect. (Bella had moved to Queensland)

Simon: I was a bit nervous coming into the test. I didn’t know how I was going to go because you don’t know what’s in the Test and you don’t get given a second study of it or anything.

Some were less agitated about the test.

Talia: During the test I felt relaxed and a bit excited.

Cheyenne: Lots of kids get really worried. There’s a couple of kids around school that do really care about the mark and stuff. If they don’t get a decent mark they get really upset. I just genuinely don’t care. There’s no point in doing this, it doesn’t count on your report card or anything.

These comments indicate that (un)familiarity with the test, knowing what to expect, the perceived importance of the test results, and ‘caring’ about personal achievement were key factors in children’s comfort levels. The children’s talk about how their teachers had prepared them for the Test demonstrated tensions that were created.

Teagan: [The teacher] said "It's for High School. If you do well you'll have a good mark, this is for the future" and stuff…She said "Take your time, check your work over and over", and stuff.

Ari: Miss Bartell said "Don't get tricked up with the questions – think them through."

Talia: I was a bit nervous, but I was alright because we got told a couple of weeks before so it gave me time…it's a big thing and you're always going to try to do your best

Some children reported additional pressure exerted by parents' or siblings' expectations, illustrating the ways in which sitting the Test was a 'social' event.

Perry: My Dad said, "Do well or I'll bash you"… If I get a bad mark then my [younger] brother will think, "Well, that's cool"… Mum pays me to do it. [If] I don't get a good mark she'd probably get at me.

Cheyenne: I've got an older brother and he does really well at everything. It's like, "How come you can't do this?"

Teagan: On the way to school in the car Mum said "Don't forget to check your work" the whole way. "Just make sure of your answers, take your time, there's no prize for finishing first." …We got in the car and Mum's like, "How did you go? Do you think you did well?" and I'm like, "I don't know. I never know." I hope I do well.

Talia: Mum was having a joke, "If you don't do well, you're not coming home".

The following exchange showed that for some children, the Test created intense feelings of disempowerment, resentment and anger which they expressed amongst their peers.

Perry: Kids said they were going to rip the test up when they get it back…Some kids came up to me and said, "Are you doing the Test? Do you hate the teachers?"

Chas: Yeah, blowing up the buildings.

These alarming responses illustrate the capacity of assessments such as the Test to coerce, expose, exclude and alienate. The children talked of the strategies they used to contend with this potentially threatening event in their lives.

Cheyenne: Some kids try to get suspended or something so they won't have to sit the test.

Chas: Zac said, "I'm just going to guess everything."

Teagan: I tried to pretend it was normal work so I didn't have a brain overload sort of thing.

Such comments illustrate typical social responses to potentially disempowering situations: avoidance (getting suspended), subversion (I'm just going to guess everything) and resignation (pretending it is normal work).

In response to the findings of Phase 1 of the research, the teachers had attempted to explain the purpose of the test to the children, hoping this might allay their fears. This message was not consistently received by the children, however, as these very different explanations illustrate:

Teagan: The Year 7 Tests are for the government so they know how well people are achieving at different schools and so they can help with the funding and give extra money to the schools that need extra help.

Britta: They test us to see if our brains are functioning properly.

Ari: Like when you get older, like you go on to have a job, your boss will have a look and if you did well he'll probably accept you more than other people.

This confusion, exacerbated by peer discussion about what the results of the test really meant, clearly created apprehension for children where popular beliefs about the import of the test had outweighed the school's message.

Bella: The teacher was stressed out because people kept saying that if you don't get good marks you won't get into High School.

Talia: Kids were just trying to make other kids nervous.

Bart: To make everyone else feel bad about it.

When asked whether they would have liked more information about the test, there was general agreement.

Sally: I can imagine every kid would. Because it's a mystery and you kind of want to know more about it.

An important consideration for many of the children was the nature and difficulty of the Test questions. The Year 7 teachers said that they had found some of the questions very difficult and were not sure of the correct answers themselves.

Cheyenne: We need questions that we understand… I didn't even understand most of them. The first question was about flags. Is it a rhombus or trapezium? I didn't know what a rhombus was.

Bart: Some of the questions I didn't get how they were.

Ari: Some of them were really tricky…with the flags, they were talking about "parallel" and that tricked me.

Eric: There was the silhouette one. That was a killer.

Perry: I hated the one with the square and with the area. That really sucked.

As these comments illustrate, for many of the children the Test was an alienating experience for its inaccessibility and incomprehensibility. In speaking of "tricky", “killer”, “sucked” and “hated” the children were describing the Test as something which violates. Lack of mathematical vocabulary as much as lack of mathematical understanding seemed to be a barrier for some of these children. For others, perceived lack of preparation for the questions was an issue.

Lim: In our class we hadn’t started to you use the 360 degree protractor yet, so it was hard just getting used to that.

Mark: Some [questions] were just plain weird, like we hadn’t done it before.

When asked about the kinds of questions that they would prefer, the children made useful suggestions as the following examples show:

Perry: Adding up the points on your video game.

Teagan: I would probably put questions that you use in real life, like how to use your money and make good deals.

‘Real’ questions for the children were clearly what they would have preferred, for their familiarity, relevance and authenticity. The children talked about whether they thought the test was an effective method of finding out what they knew and could do in mathematics. The fact that they could not show how they had reached their answers was concerning for some children.

Lim: [In class mathematics assessments] When you figure out the shapes and you use the formula, you get three marks for the formula. [In the Test] You just gotta have the right answer, ‘cause it’s not about the work.

Teagan: Sometimes you had gone to a lot of trouble to get that answer and the computer or whatever marks it, can’t see that.

The children made suggestions for improvements in the way the Test worked.

Talia: Maybe they should have a box under each question for how you did it.

Cheyenne: I think the best way is to do it [mathematics assessment] in class when the teacher is there and everyone is sort of laid back, because we normally perform our best then.

These comments speak of the children’s wish to increase the chances of their mathematical thinking being understood, and of their need for greater comfort by making the test a more ‘natural’ (less pressured) part of everyday classroom life.

For some children, there was satisfaction in sitting the Test. This seemed to be derived not from gaining a greater understanding of their mathematical learning but from the assurance of feeling *prepared*, and *knowing where they ‘stood’* compared with others, as captured in these comments:

Dante: Didn’t really feel anything different ‘cause we were pretty much prepared… Pretty easy to me. I do well on those Maths and English tests.

Mark: It makes you feel better when you know you are good and other kids aren’t so good at it.

DISCUSSION

The Test was a socially embedded event with social consequences. The processes by which each child constructed meaning from the Test were individual and complex, yet commonality can be found within the sample group. Where the children believed the Test to be personally significant, that is, that it might affect their future chances or show up their inadequacies, their stress levels about the Test were increased. Those who said they enjoyed the Test were those who were usually comfortable with mathematics in class, were able to finish within the given time, and felt confident they would score highly. The competitive aspects of the Test seemed to appeal to them. Those who were less sure of their mathematical capabilities, who were not able to understand or complete questions in the given time, or who were not confident that they would score highly reported feeling either resigned to their 'fate' or very negative about the test. The Test was seen as a "mystery" and the children wanted more information. Significantly, none of the children's comments suggested that the Test was a powerful tool in enhancing their *learning* of mathematics.

The children's conversations reveal that while some children reported feeling *relaxed* about the Test, for most the Test was an unpleasant and unwelcome intrusion in their lives. Their teachers' attempts to offset the adverse effects of the Test were largely unsuccessful; many of the issues revealed in the Year 5 Test research had remained and for some had been intensified as the children grew older and had become more aware of the sense of disempowerment the test engendered.

Cotton (2004) discusses the ways in which assessment in mathematics education acts to produce self-evidential truths enshrined in practice. We can see from the children's accounts that they view the Test as an event with clearly-defined rules of engagement: Participation is compulsory for all children in Queensland; each child must complete the Test in silence and without help from others; answers' only (mostly multiple choice) must be produced in response to 'questions'; the same questions are used for all children assuming 'one size fits all'; an unknown and invisible external authority designs (and marks) the questions; the questions are quiz-like - they are usually disconnected from one another and presented at random, and the questions have to be answered within a very limited time. These rules created *pressure* for children in six distinct ways: (i) forcing each child to participate whether s/he wanted to or not (compulsion/coercion) (ii) pushing individual children to perform beyond their natural and comfortable pace (too fast) (iii) pushing individual children to perform beyond their current state of skill and knowledge (cognitive inaccessibility) (iv) calling for only one 'correct' response (creating a high risk situation through the chance of being 'wrong') (v) exposing the child to public scrutiny with potential for being labelled a 'failure' or 'success' by parent, teacher, siblings, classmates or self or any combination of these (creating the chance of being shown up and/or ridiculed and/or excluded from high school), and (vi) banning or discouraging the use of materials other than those designated by the rules of the Test, peer discussion, and checking of answers with others (reducing support and increasing likelihood of error).

These multiple sources of pressure can have a cumulative effect. For some children, this pressure seemed to contribute to their dislike and fear of the Test, and for others, excitement. The kinds of pressure exerted by the Test have been found to be deeply embedded in mathematics pedagogical practices in many classrooms around the world, (TIMSS Video Mathematics Research Group, 2003) and form either an explicit or implicit part of everyday classroom routines. The Test therefore defines, mirrors and reinforces accepted cultural practices in mathematics education.

CONCLUSION

There is universality in these children's responses. While each child speaks of his or her unique experiences of one particular Test, as Pollard and Filer note, "the stories of children are likely to resonate with the experiences of others because they provide examples of fundamental processes in human experience, through which individuals develop and act in society" (p. 267). In showing how the Test acted as an invisible powerful moral authority that compelled them to participate in an activity whose results they variously perceived to be *important* yet potentially damaging, the children's accounts remind us of what it is to be a child making sense of a complex world of schooling, mathematics, family, peers and 'self'. Within their testimonies, mathematics classrooms in general, and the Test in particular are never presented as spaces where children become active architects of their own mathematical learning.

The research alerts us to some *social* consequences of leaving children out of decision-making about their education, but this exclusion may also have *educational* effects. Munns & Woodward for example demonstrate significant connections between children's self-assessment and their engagement in learning. It can be argued that we have entered a renewed era of Authoritarianism in education (Law, 2007) in which control is being exercised over children through increasingly prescriptive assessment regimes that seek to *judge*, *classify*, *rank* and *differentiate* them. Law asks "to what extent should children be encouraged to think for themselves and make their own judgements?" (p. 15) noting the connections between freedom of speech, democracy, and Liberal views of educating. He argues that while more research needs to be done, a growing body of evidence suggests that *thinking for themselves* is good for children academically, socially and emotionally (p. 39).

This study concludes that if mathematics education in general and its assessment practices such as the Year 7 Numeracy Test in particular are to serve our children's best interests, mathematics educators need to actively solicit and take into account children's views. Further research is needed.

NOTES

1. Year 7 in Australia is the final year of primary schooling. Children in Year 7 are aged twelve or thirteen years.

2. Personal Communication: Letter from Anna van Hoof, Assistant Director Testing and Analysis, Queensland Studies Authority, 3 Feb, 2006.

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"DOWN IN THE DARK ZONE": TEACHER IDENTITY AND COMPULSORY STANDARDISED MATHEMATICS ASSESSMENT

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In this paper, teachers of Year 5 and Year 7 students teachers at one state primary school in Queensland, Australia, share their thoughts and experiences of the mandatory standardised state Numeracy Tests, noting the tensions, dilemmas and challenges these tests pose for teachers involved in children's mathematical learning. Using Foucault's theories of discourse and Lacan's theories of identity the paper contemplates the ways in which mathematics teachers are inscribed and constituted within the standardised test process.

TEACHERS, MATHEMATICS, ASSESSMENT AND DISCOURSE

As I sat and observed the 2007 Year 7 Test in one state primary school in North Queensland, Australia, I was struck by the ways in which this kind of test *produces* teachers of mathematics. Firstly, it was the teachers' job to ensure that the physical space (the school staffroom) was appropriately organised. The teachers supervised the arrangement of the furniture, ensuring that desks were well spaced to minimise interaction between students. They then directed students, many of whom appeared nervous, to their seats. Secondly, the teachers explained the test rules. They described how the papers were be handed out and collected, how the answers were to be entered on the papers and how each section was to be timed. They instructed the students to 'read the paper carefully', to 'check, and recheck' their work, and to 'take their time'. They then asked the students if there were any questions. One student asked, "What will happen if we fail the test?" One of the teachers assured the student that the test "means nothing" and advised the students to "just relax" and "do their best". Thirdly, the teachers administered the test, distributing the test papers, and ensuring children had entered their personal details correctly. The children were then instructed to begin. Each child worked in silence. From time to time a teacher would read aloud from a prepared transcript to guide the children through the various sections of the test. The teachers remained distant from the students except to read to them a word or sentence where the students requested help. In one section of the test the teachers read mental calculation questions aloud in rapid succession. No other assistance or explanations from the teacher were permitted. Towards the end of the test, two boys who flouted the 'silence' rule were quickly separated. There was palpable relief from both students and teachers when the two-hour test was finished.

This scene was familiar in more ways than one. It occurred to me that what I had just witnessed bore all the hallmarks of a military operation. Using this metaphor I began to consider the ways in which teachers and students were constituted within the apparatus of this standardised Test. Acting on orders from above the teachers became

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commanders in the field. Facing their compliant and well-ordered troops, they marshalled their student/combatants for the assault. Assigned a (solo) mission, the students were trained, briefed, primed and cautioned. As if this were an engagement in hostilities, the teachers advised students to *read* the situation carefully, *check* their positions, and to *restrain* themselves from rushing precipitously into unknown territory, speaking of the test as they might a danger zone fraught with ambushes and traps. Much as soldiers are expected to blindly obey, students' obedience to the test's rules was strictly policed by their commanding officers. Students who failed to carry out orders were summarily placed in solitary confinement. In the manner of troops kept in the dark about dangerous manoeuvres masterminded by their superiors, the students were told that the test 'meant nothing', even though everything about "Operation Big Test" was indicating otherwise. The atmosphere was one of excited tension and potential menace. For teachers and students alike, conscientious objection, desertion or absences without official leave were out of the question. In using the phrase 'just do your best' the teachers both acknowledged and accepted the inevitability of casualties - some students might survive this assignment, triumph even, but others would 'fall'.

Using this kind of metaphor to make new sense of a familiar event in order to see the ways in which teachers, students and the test were acting and being produced as 'identities' may seem out of place in an academic research paper, but as Foucault (1977) has convincingly illustrated through comparisons between prisons, hospitals, armies and schools, discursive practices – those practices we not only speak about but *call into existence*, *allow* and *enable* through our speaking of them – cross institutional boundaries and constitute the interactive relationships of power found in mega-organisations, such as armies and education systems, that characterise much of contemporary social life. Within these social discursive spaces, the identities of individual actors such as teachers and pupils for example, are made and remade.

Many researchers have used the concept of identity to examine what happens for individuals in mathematics education settings. *Identity* is by no means a straightforward concept. Schifter (1996) views teacher identity as multifaceted and formed through professional narratives constructed in practice. Holland, Lachiotte, Skinner & Cain (1998) make a distinction between *figurative* and *positional* identities, the former described as something generic, desired and imagined, and the latter more specific, located and relational. A focus on the situated and social nature of identity in education as something individuals build within communities of practice is emphasised by Wenger (1998) and Boaler, Wiliam & Zevenbergen (2000). Hogden and Johnson (2004) and Van Zoest & Bohl (2005) incorporate specific teacher knowledge, enactment in social situations, and cognitive engagement into their theories and analysis of the development of mathematics teachers' identities.

Mendick (2006) is uncomfortable with the word *identity*, explaining that, "identity' sounds too certain and singular, as if it already exists rather than being in a process of formation" (p. 23) preferring to speak of 'identity work' or 'identification' (from

Hall, 1991). These terms capture the nuanced, mutable and 'lived' nature of identity as situated, as in constant process, as both psychic and relational, and as represented in narrative. Mendick believes that "'identity work' positions our choices as producing us, rather than being produced by us" (p. 23).

Lacan's psychoanalytic theories of subjectivity (1977, 2002) are useful for the ways in which they supplement Mendick's inclusion of the psychic. Lacan points to the desire of the *self* to be 'present' as a *secure identity*. Where many post-modern theorists perceive identity as the product of deliberative, social, conscious constructions of realities of self including 'self as *mathematics teacher* or *learner of mathematics*, Lacan was more interested in the work of the subconscious, believing that it is in the interplay of what he termed the Symbolic, Imaginary and Real psychic registers that identity hovers as something always in the making, something formed and form*ing* in its own seeking of itself.

Inspired by Walshaw's (2004) analysis of learners in which she attempts to weave together postmodern social theories of identity production and Lacan's theories of self, I use a combination of theories in this paper to explore teacher identity in children's mathematical schooling; I look in particular at the ways in which teachers' identities were both challenged and confirmed by the Year 5 and Year 7 Numeracy Test. The paper uses data from a research project that began in 2005 and was extended in 2007. The administrators of a large suburban school in Queensland Australia were aware of challenges the Queensland Studies Authority (QSA) Numeracy Tests presented for teachers, students and parents and requested research assistance in gathering data and compiling a report to present to the QSA, believing a community-based, bottom up approach to ongoing development in education to be essential. Four Year 5 teachers and four Year 7 teachers were involved in the study in which they shared their views and experiences of the Numeracy Tests. Shortly after the test had been administered, the teachers talked about the test in small groups and their conversations were recorded and transcribed.

In contemplating their experiences of the test, the teachers were defining and reflecting their identities as teachers. Some teachers like Karen, Allie and Col had been teaching Year 5 and Year 7 students for many years and were therefore very familiar with the tests, but for teachers like Jemma and Ralph, this was their first experience. In the following analysis of selected excerpts from the teachers' conversations, I look at the ways in which the teachers were both *inscribed* from without and *became* from within, through the productive and signifying processes of engagement with the test.

Mathematics teachers as coaches

One of the first issues the teachers raised in their conversations was how they had prepared the children for the test. They spoke about preparation as something they believed they must do but problematic since the test was an unknown.

Karen: You have to prepare them, but you don't want to scare them. You try to prepare the kids for what's in the test. As you prepare, you think, that in the past there has been a big measurement component, and operations with the calculator. And this year what is the emphasis? - spatial knowledge! It's like "we're testing only the boys this year because you want to see how they go on spatial knowledge." For us to prepare, because of where [the test] comes in the year, we haven't covered all the concepts so you try to pick up what possibly will be there and spatial knowledge [in the past] has only been fairly small [proportion of the test].

Jemma: This was my first time doing the test. I didn't really know how to prepare them. In the few weeks before you do a lot of practice tests. I probably worked with number more than anything else presuming that this is such a large part of it.

Ralph: You want your kids to do the best that they can and you try to help them out and you do your preparation…it's good to do them in the multiple choice format that they have, just to familiarise the kids.

Col: We talk about pressure on the kids but also on the teachers as well, because if we don't, you know… we've got to prepare for this because you're not giving the kids the opportunity that they, you know, to show their best and that sort of stuff…

Dan: You've just got to make sure that you've covered everything, so the kids aren't surprised… it's a stress to make sure that you've covered everything, and you've covered it well enough…we did a lot of practice, especially the numeracy one, I found a really good book and it had quite a bit of multiplication, colouring in bubbles, and I felt that it really helped them in terms of that stuff.

In the discursive practices surrounding the test, the teachers both acted and spoke of themselves as *coaches* whose job it was to ensure that the children were well-prepared without 'scaring' them. They attempted to anticipate the content of the test based on their knowledge of previous tests. They did their best to 'cover' all the mathematical concepts that were likely to be included. Because the test was not in a format that the teachers usually used to assess mathematics in their classrooms, the teachers trained their students for the test through practice so the children would be fit to sit the real test, and 'do their best'. The teachers regarded their attempts to prepare the children as being thwarted by the unseen writers of the test whose agendas were at odds with their own.

Here we can discern not only Lacan's Symbolic psychic register at play, in which the "Big Other" of the test itself – its questions, its instructions and its statistical results – both allowed and constrained teacher actions, but also the Foucaultian notion of discourse as 'productive' – hence teachers are produced as trainers, kept in the dark about the content of the test, shut out of the process of test design, and distinctly separated from governmental assessment authorities.

Mathematics teachers as mathematical experts

The test was as much a test of the teachers' mathematical skills as it was the children's. Both the Year 5 and Year 7 teachers found that they were sometimes struggling to determine the correct answers for test questions, as these excerpts show.

Allie: Even in the angles [one] as adults we all had huge discussions about that didn't we?...You know the one with the angles.

Dan: It was worded really strangely this year I felt, and the kids got really confused with how it was worded… they had to do the "insides" (angles).

Kate: Some of it was ambiguous.

Jemma: I found a lot of the questions were trick questions rather than just doing straight operations. I was thinking that these twelve-year-olds were asked to do some horrendous problem solving.

Ralph: I enjoy maths but I didn't enjoy all those spatial questions, probably because I hadn't covered as much of that type of stuff.… It's testing comprehension rather than mathematical skill.

Karen: It makes you wonder what is the purpose of the test. Are we trying to find the gifted kid, therefore we throw in the hairy question, or are we trying to find whether the kids in Queensland are at a standard?

Lee: We think, "Well if we teachers don't know, how are the children expected to know?"

A Lacanian analysis looks to the ways in which the teachers reacted when confounded by some questions in the test. Their need for clarification had led to discussions among the staff to establish the 'right' answers. Teachers, they seemed to be saying, should be experts. What is too difficult for the teacher must be too difficult for children. To maintain (self)perceptions of mathematical competence, some teachers had looked to external causes such as 'strange wording' 'horrendous problem solving' or 'ambiguity' to explain their own difficulties in answering the questions. Although it may have crossed their minds, none of the teachers suggested that their own mathematical content knowledge may have been insufficient, since his would have undermined the security of their identities as 'experts'.

Mathematics teachers as program managers

The test altered the teachers' mathematics programs, and was thus viewed as an intrusion into their everyday role in deciding what mathematics the children should learn and how they should learn it.

Col: To me it rearranges my teaching format…I used to do chunks of things, so you know in the fourth term might have been the measurement or it might have been heavy in that, and therefore you've changed your whole way of teaching because you've got to do bits of everything to ensure that they get a range of choice.

Allie: In a way this term has been totally modified because you've been trying to teach the kids the things that they need to know for the test that's in August... You're disadvantaging them by not doing it…It has really altered, you know, almost like a whole term of nothingness…

As *teachers* they seemed to believe that they were conferred the responsibility and the freedom to design and implement mathematics programs as they saw fit. The test created dissonances by reconfiguring this function. Teaching children 'things that they need to know for the test' – teaching *to* the test, in other words - was rationalised as 'ensuring the children were not disadvantaged', even though in so doing the teachers' best laid programs were reduced to 'nothingness' or 'bits of everything'.

Mathematics teachers as pedagogues

The test ran counter to what the teachers said they viewed as best practice in teaching and assessment of mathematics.

Yolanda: I find it very frustrating that we test them that way when it's not what we teach, we teach them to talk and discuss, that's how they work now…we teach them to find information now. There's no way you can have it all in your head…

Karen: When we are teaching, you get a mark for the formula and a mark for the process, and a mark for the answer. The answer is the least important part, whereas understanding the process is more important.

Contemporary approaches to teaching and learning mathematics espoused in current curricula were reflected in the teachers' descriptions of their own teaching. Speaking as those who know best how mathematics should be taught and learned, the teachers were able to criticise the test for the ways in which it undermined exemplary practice. In this way the test enabled the teachers to define and clarify their pedagogical positions.

Mathematics teachers as mediators

The teachers also spoke of themselves as intermediaries between the children, the test authorities, and parents. They were particularly concerned about the reaction of the parents and saw their identities as 'competent teachers' under threat if the marks were not what the parents expected. They had developed a self-protective strategy for such a contingency – the role of the mediator. Both children and parents were told by the teachers that the test 'means nothing'.

Karen: I think parents don't really understand the test and they put too much value on it. Parents will come in and say "Can you explain this to me?" The biggest most important thing to say to them is [their child] may have got very close to the correct answer, they may have actually known what they were doing, but if they colour in the wrong bubble, they are wrong… He understands, he knew to divide, but he made a 'boof head' mistake.

Ralph: I marked what the kids got out of 49 but I'm sort of reluctant to relay that to the parents…

Mathematics teachers as professionals

The teachers saw their own assessment of the children's mathematics as more accurate, indeed more 'truthful' than that of the test. This placed them in a difficult position when the test results were at odds with their professional judgement.

Dan: I just think it puts a lot of unnecessary stress on the kids…I have a much better idea [of children's mathematical abilities] myself, of what I see every day …it's not a true reflection. I don't really see the purpose in it apart from, I suppose it's a good way…the next reporting system to go back to parents to show where their kids are up to…as I said, I don't think it's totally accurate so is that a good thing?

Karen: Sometimes it is a shock. If I think it's no reflection on the child's ability I'll just tell them that…I had a kid with full-on 'flu sitting the test. What do you do? So when he got his test back he was crying. I remember him in particular because he was so upset and he was so bright. I said, "Don't worry about this test, mate."

As professionals, the teachers were caught between seeing the merits of the test as a 'good way' to report children's achievement to parents because of its supposed objectivity, and their view of the test as an invalid form of assessment. They trusted their judgements about the children because they were based on professional observations of children's mathematical skills that the test could not 'see'. However, the underlying notion that a truth about the children's mathematical skills was there to be discerned and judged by a trained and experienced professional was reinforced rather than disturbed by the test.

'Good' mathematics teachers

Identifying as the 'good' mathematics teacher was another significant theme.

Jemma: That's a concern of mine that [the results] will come back and they will be down in the dark zone (referring to the shaded part on the scale of results that indicates results below the average or national benchmark). I'm not worried in myself because I think I do a good job, but I don't actually know. Maybe I'm not doing a good job. We know that there are some teachers around that really aren't good teachers and this test would probably pick that up…I just know that if my kids don't do well I would have thought, "What am I doing wrong?"

Karen: It's not meant to judge the teacher…Those kids bring so much to the test anyway, so if it was a really bad teacher, those kids could still do well. A really good teacher is a teacher where the kids are doing really great assignment work, they're doing great art…

Ralph: I guess the pressure comes in the results. I was confident that I prepared them well for it, but I haven't had to sit down next to somebody and compared my class and seen whether my kids were above, average or below compared to other classes.

Jemma's distinction between 'thinking' and 'knowing' that she is a good teacher, is telling. It was insufficient for Jemma to think she was a good teacher; she needed some external measure such as the test to tell her for sure. Only then would she really 'know'. The test therefore served as a reliable gauge of performance. For this reason it was worrying for teachers when the children's marks on the test differed from the marks they received in classroom-based assessment tasks because it created a tension between *thinking* and *knowing* they were good teachers. Any discrepancy in the 'truths' that the assessment results told about the child were also difficult for children and parents to reconcile. The teachers were then faced with a choice between either explaining away the test results as invalid, or admitting they were not good teachers. For the teachers it was a relief when the test results were consistent with their own judgements since their desire to be seen as good teachers was fulfilled. Allie's remarks illustrate this very clearly.

Allie: [The test results] came out pretty much exactly the same as [the children's] record cards. It's nice when that happens. I've had a couple of parents say, "Oh it was nice to see that they were saying the same thing." It doesn't normally happen that way. Normally [the test results] look much worse. Maybe I'm getting better at it. But there was one [surprise result] - I rang the parents up and I said basically, "I don't know where this [result] came from." And they said, "We don't care anyway because this makes no sense to us." They know what their children are like. It's not [the 'right result]…so we go, "Oh well, it's an anomaly".

Allie's comment, "Maybe I'm getting better at it," suggests that she believes that a good teacher is one who is able to make judgements about children's mathematical abilities that closely align with the official, objective, standardised and therefore more 'truthful' judgement of the numeracy test. Where test results 'make no sense', it is a relief for the teacher when parents view the result as 'an anomaly' rather than a consequence of poor teaching. Ralph took 'good teaching' a step further:

Ralph: Another point is, when it comes to the federal government's interest in performance-based pay… a national standardised test, something like that, will be a guide to performance based…

Karen: Can you imagine how we would be teaching? We would be teaching straight multiple choice. Oh my God, it would be so boring.

DISCUSSION AND CONCLUSION

I set out to examine the part the Years 5 and 7 Numeracy tests play in teacher identity. From the teachers' conversations there is evidence that the test challenged as well as reinforced teachers' perceptions of themselves as teachers of mathematics. The teachers spoke of the format of the test as undermining the ways that they usually taught mathematics. Standardised testing has long been criticised for the way it channels teachers' actions, particularly where classroom practice becomes dominated by 'teaching to the test'. There is clear evidence that this cohort of teachers treated the test not only as a measure of the children's capabilities in mathematics, but also of their own teaching of the subject. Even though school policy dictated that a child's progress should be monitored and reported using a suite of assessment results gathered over time and using 'authentic' assessment methods, the capacity of the test to define, shape and reflect teacher identity subverted the school's attempts to offset what teachers described as the untrustworthiness of the test results.

'Teaching to the test' can be seen as a response that is tightly tied to teacher identity. This analysis shows that the 'identity work' involved in teachers' engagement with the test is complex. Caught between multiple views of themselves as teachers of mathematics within the defining apparatus of the test, the teachers struggled to reconcile their beliefs about best practice involving processes, understanding and discussion rather than correct answers, the test as an authoritative judge of children's mathematical capabilities, their desire to be seen as 'good' teachers as shown by positive test results, and the need/wish to support children to do their best. In the end, despite its disruption to teachers' mathematics programs and the difficulties it created for teachers in mediating between parents and children, the test was vested with authority as much from teachers' choice to go along with it for the part it played in their 'identification' as teachers of mathematics, as from the intrinsic power it wielded as an externally imposed form of assessment. In Mendick's (postmodern) view, such a choice *produced* the teachers rather than being produced by them, in other words, the teachers were 'choosing' to behave only in the teacherly ways that the test allowed and/or demanded. From a Lacanian perspective, the choice was tied to the teachers' desire for a secure identity. While they may have railed against aspects of the test for the troubles it caused, in so doing, the teachers were producing themselves as professionals, as mathematically competent experts, and as 'good' mathematics teachers. The test symbolised a commanding authority, made them 'real' as mathematics teachers, and at the same time opened spaces for their visualising themselves as ideal teachers in ideal mathematics classrooms.

I will now return briefly to the military metaphor I used at the beginning of this paper to reframe the identification I observed in the test procedure. The ease with which I was able to translate the teacher/pupil actions and interactions into a military context suggests that identity is as much about the social and psychical aspects of relationship, interaction and techniques of power as it is about local needs, specific events or specialised knowledge. In other words, teaching and learning mathematics has little to do with it. If this is so, the test is only marginally concerned with the teaching and learning of mathematics and primarily serves, intentionally or not, to establish and maintain an accepted 'order of mathematical identity' of which teachers' identities, in a continuous process of formation, are a (self)recognised and recognisable part.

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THE BENEFITS OF ADULTS LEARNING NUMERACY

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We examine the benefits of adult numeracy learning in the current Australian context by drawing on Schuller's framework for analysing the benefits of learning in terms of three capitals: human capital, social capital and identity capital. We argue that although the current adult education policy framework in Australia is biased towards the achievement of only one of the three capitals – human capital, the practices of experienced adult educators help to extend the benefits of learning to encompass identity and social capital benefits. We take a case study of a numeracy workshop in an Adult Basic Education (ABE) program in Australia to show how one teacher exemplifies teaching practice that despite the policy gap, helps her learners reap a range of benefits from their numeracy learning.

INTRODUCTION

Interpreting the goals of adult education and training primarily in terms of labour market outcomes has been the norm among public policy makers in Australia since the early 1990s. Adult Basic Education (ABE) programs on the other hand, have been characterised as 'second chance' education for those who were not successful in their experiences in formal schooling or who did not have the opportunity to fully participate in schooling as a child. For many ABE learners, the benefits of participation in literacy and numeracy classes are not only linked to the acquisition of skills for employment; they are linked to broader social and personal benefits such as inclusion and participation in a social network, and increased self-confidence.

In 2006 we were funded for a project on "Sharing Innovative Best Practice in Adult Literacy and Numeracy" to research and produce a DVD resource with case studies of teaching practices of experienced adult literacy and numeracy teachers. The project was motivated by awareness among experienced ABE practitioners and those of us involved in teacher development that much of the ABE practices that are founded on important social justice principles was at a risk of being "lost". This was because those who have developed these practices were nearing retirement, and there was an absence of policies that supported any genuine renewal of these broader dimensions of ABE.

In the course of our project we met with a number of experienced ABE teachers, some identified by us and others by their practitioner colleagues. The interviews with the teachers and their learners and observations of their classes revealed some common attributes among the teachers. These were that 1) these teachers can articulate a pedagogy that is critically grounded in their experience and philosophy about teaching; 2) these teachers see connectedness with the learners and their lives as central in their roles; and 3) their philosophies are observable and demonstrable in

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their actual teaching practice. The views of the learners in the classes showed that they were benefiting in more ways than just skills acquisition.

In this paper we will focus on one of the case studies of our project - a numeracy workshop in an ABE program. We will look at what the teacher and the learners say about the class, and examine this using the framework developed by Schuller (2004). The framework provides a way of looking at learning by focussing on three types of benefits: human capital, social capital and identity capital. We will then look at how a teacher's understanding of effective practice might be connected to the production of these capitals in their classroom.

The rest of the paper is organised as follows. We will first provide an overview of Schuller's framework, some of the related studies and Australian policy contexts in adult numeracy. We will introduce some frameworks for thinking about teaching practice and pedagogy that we will draw on to examine the case study. We will outline the case study and our findings from it. Finally we will draw some conclusions about the links between the types of benefits that can be reaped from adult numeracy learning and their relationship to the teacher's conception of effective practice.

SCHULLER'S THREE CAPITALS FRAMEWORK

Schuller (2004) developed a framework for examining the benefits of learning in terms of producing three types of assets: human capital, social capital and identity capital. Schuller's framework arose from his research group's UK investigation of the positive benefits of learning that went beyond the economic benefits to the individual, and to their increased capacity to engage in civic life and to their lives within social networks and communities.

Schuller defines human capital in terms of "knowledge and skills possessed by individuals, which enable them to function effectively in economic and social life" (2004, p. 14). Since the early 1990s, Australian adult education and training policies have been increasingly narrowly focussed on the economic benefits of training and education. For adult literacy and numeracy, this has meant a lack of renewal of an Australian language, literacy and numeracy policy that looks at the broader social benefits of adult literacy and numeracy education (Wickert and McGuirk, 2005; Sanguinetti, 2007; Balatti, et al, 2006). The earlier focus on participation and access, and basic education as a 'right of all citizens to develop their literacy and numeracy in an increasingly complex society' was the mission for ABE (ACAL, 1989, p. 1). They are not visible in the current policy environment. Consistent with the broader human capital focus on education and training of the Australian Government, adult literacy and numeracy is currently located within the "training and skills" section within the Commonwealth Department of Education, Science and Training (DEST). Within DEST jurisdiction, there are two programs that directly involve adult literacy and numeracy. They are the Language, Literacy and Numeracy Programme (LLNP) and the Workplace English Language and Literacy (WELL) Programme. Both of these are funded through open tendering by public and private registered training organisations. The LLNP is focussed on job seekers (DEST, 2006) and the WELL on employees needing language, literacy and/ or numeracy skills development in their workplace (DEST, n.d.).

While the human capital outcomes are not disputed as important, measuring the benefits of adult literacy and numeracy learning purely in terms of economic benefits has been critiqued by researchers and practitioners (see for example, Balatti et al, 2006; ACAL, 2006). The research by Black et al (2006) showed that participation in adult literacy and numeracy programs brought about a wide range of social capital outcomes. They found that even for those who participated as part of a labour market program, there was a wide range of social capital generated that included the strengthening of family ties, inclusion in civic life, developing new friendships, or effectiveness in workplace teams. These outcomes match Schuller's use of the term social capital which is linked to "networks and norms which enable people to contribute effectively to common goals" (Putnam, 2000, in Schuller, 2004, p. 17). In fact, what Balatti, et al (2006) claim is that without the social networks that were developed through participation the participants' success in employment would be limited. Thus, the learning of job-related literacy and numeracy skills is intimately related with the social capital gains in the programs.

Related to both of the above is the concept of identity capital which Schuller describes as "characteristics of the individual that define his or her outlook and self-image" (2004, p. 20). In numeracy, and mathematics education broadly, there have been numerous studies about mathematics anxiety and related concerns (Tobias, 1978; Evans, 2000) that is related to the notion of identity capital. A recent study of adult numeracy learners in the UK further education system by Swain (2005) revealed that a significant majority of participants in his study felt that they had "changed as a person in some way through learning maths" (p. 8), including "greater independence and autonomy" (p. 8). In an Australian study of integrated literacy and numeracy learning in the education and training sector, Wickert and McGuirk (2005) found that the workers participating in the programs experienced improved self-confidence, not only in their workplace roles but in their roles in the community. The findings on the social capital outcomes from the study by Balatti et al (2006) of adult literacy and numeracy courses show how the building of social networks in the class is related to the building of identity capital:

> They are a practice field and they are bridges. As far as possible, the learning environment is controlled through specific group norms and pedagogical practices to allow for students to generate new resources, that is, to learn. Resources may be new skills, new attitudes and beliefs about self and others, new ways of interaction and new links and connections. For many, the networks are a new and safe environment in which to play out new aspects of identity and practise new skills. Within these networks, social capital outcomes are experienced. (p. 38)

Schuller (2004) not only sees the three capitals as interdependent, but moreover he does not try to draw clear boundaries between them. He illustrates the interaction between the three in the triangular diagram shown in Figure 1:

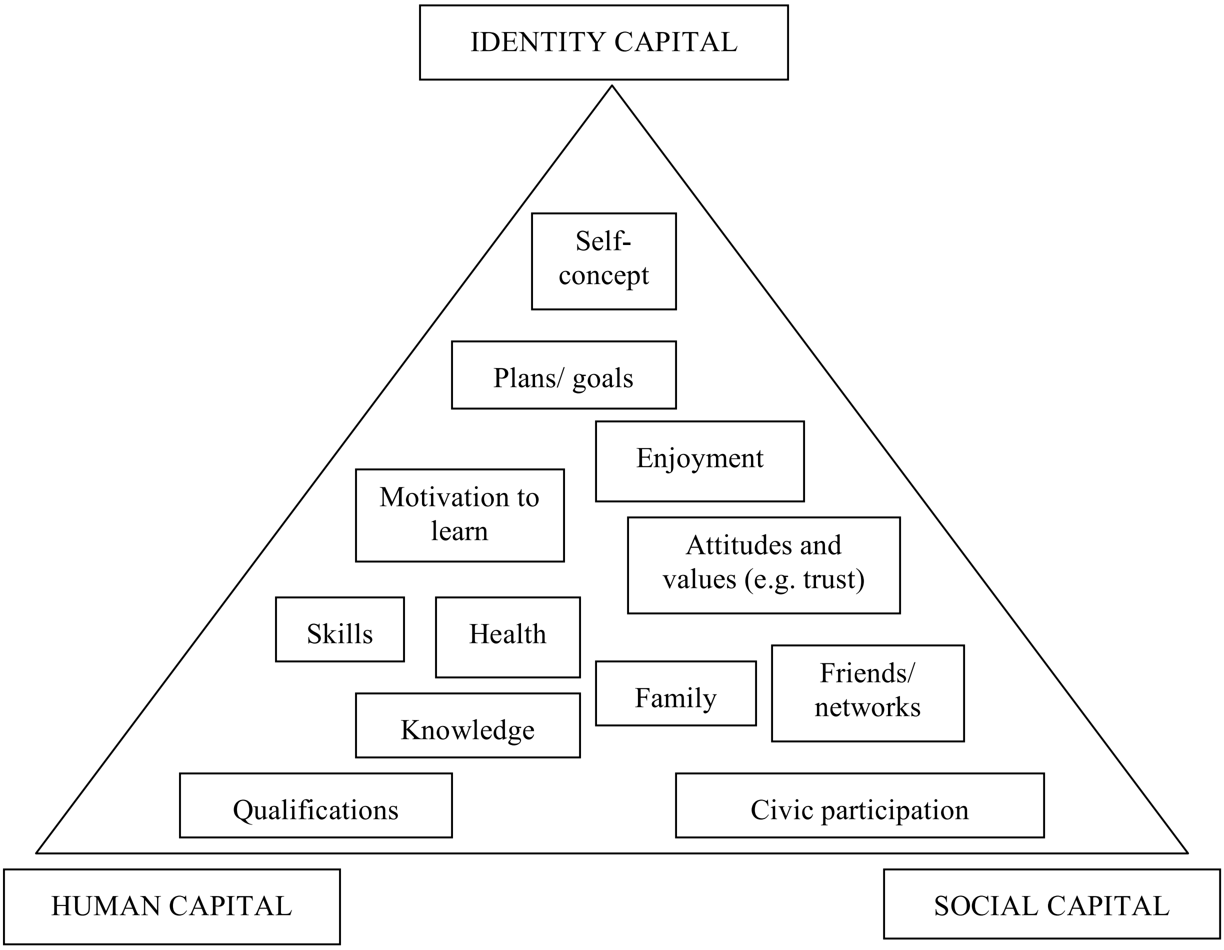

Figure 1. "Conceptualisation of the wider benefits of learning" (Schuller, 2004, p. 13)

The following case study is used to illustrate how the three capitals are generated within an adult numeracy workshop in an ABE program.

TEACHING PRACTICE AND PEDAGOGY

There are different ways in which we might try to examine a teacher's practice or pedagogy. McGuirk (2001) undertook a snapshot study of adult literacy and numeracy practices. She found that a majority of teachers had as their central concern to develop tailored materials and resources that met the individual needs of the learners, and that they regarded adult literacy and numeracy as a fundamental human right. From this strongly shared belief, however, there was a diversity of theoretical influences on each of the teachers that were studied. In the project that we were involved in, there was also no one theory or method that teachers espoused. None of the teachers made explicit reference to a particular theory or school of thought. Rather we found a range of strongly articulated philosophies and principles about their own roles, ethics and relationships with learners.

To interpret the different practices we examined, we found the formulation of Kemmis (2000) useful; he argues that practices can be examined in terms of five different aspects:

> 1. the individual performances, events and effects which constitute practice as it is viewed from the 'objective', external perspective of an outsider
>
> 2. the wider social and material conditions and interactions which constitute practice as it is viewed from the 'objective', external perspective of an outsider
>
> 3. the intentions, meanings and values which constitute practice as it is viewed from the 'subjective', internal perspective of individual practitioners themselves
>
> 4. the language, discourses and traditions which constitute practice as it is viewed from the 'subjective', internal social perspective of members of the participant's own discourse community who must represent … practices in order to talk about and develop them …
>
> 5. the change and evolution of practice – taking into account all four of the aspects of practice just mentioned – which comes into view when it is understood as reflexively restructured and transformed over time – in its historical dimensions (pp. 1 – 2).

A novice teacher who looks at the minimum qualification standard for adult education and training in the current Australian adult education and training policy contexts might come away thinking that teaching practice is defined by a long list of atomized competencies. On the other hand, our findings from this research suggest that experienced teachers' practices are much richer than what can be expressed in these terms. Indeed, the teachers whose practices we studied were distinctive in the way they were able to portray an authentic and coherent picture of their practice that acknowledged tensions in the different aspects of what they did as teachers. Thus rather than practices driven by some objective performativity measures or some externally defined and objectively recognized standards that are the core of the first two notions of practice in Kemmis' list, what we saw most strongly was the third aspect, of practices shaped strongly by subjective judgments which in turn were strongly informed by and sensitive to the teacher's relationships with their learners, past and present.

Thus the study of practice in this project has been what Kemmis (2000) says, "a study of connections – of many different kinds of communicative, productive and organizational relationships between people in socially, historically – and discursively – constituted media of language (discourse), work and power – all of which must be understood dynamically and relationally" (Kemmis 2000, p. 6). The emphasis on making connections is one that is also expressed by Noddings (2003) who talks about teaching as "relational practice":

> Teaching is thoroughly relational, and many of its goods are relational: the feeling of safety in a thoughtful teacher's classroom, a growing intellectual enthusiasm in both

teacher and student, the challenge and satisfaction shared by both in engaging new material, the awakening sense (for both) that teaching and life are never-ending moral quests (p. 249).

Noddings expands on the notion of relational practice by emphasizing that "care and trust" are fundamental to this (2003, p. 250). Neither Kemmis nor Noddings refers to any methods for achieving connections or relationships in teaching. According to Kumaravadivelu (2003), trying to identify specific methods is not a meaningful way of thinking about how to teach. Instead he provides a framework for thinking about teaching practice that he calls *postmethod pedagogy* (Kumaravadivelu, 2003). Postmethod pedagogy is a three-dimensional system consisting of:

- the parameter of particularity which is opposed to the notion that there can be an established method with a generic set of theoretical principles and a generic set of classroom practices;
- practicality, that is, a teacher generated theory of practice, a practical theory;
- the possibility of tapping into the sociopolitical consciousness that participants bring with them as a catalyst for continual identity formation and social transformation (Kumaravadivelu, 2003).

In postmethod pedagogy, there is no method or theory of practice that is applicable for all teachers at all times. Rather, a teacher's practice or pedagogy is the system that they themselves generate that gives meaning to what they do in response to particular contexts, with particular learner groups that are connected to their moral sensibilities as well as theories and principles that have meaning to them in those particular situations. Thus in looking at our case study, we will not be looking for particular skills or methods that the teacher might exhibit, or particular theories or principles they espouse. Rather, we will be looking for the holistic picture they present of their teaching approach, and how that is connected to their beliefs and their classroom practice.

CASE STUDY OF AN ADULT NUMERACY TEACHER AND HER CLASS

The case study is of a numeracy workshop run by the teacher Anne (pseudonym). Anne's class takes place in an ABE Unit in a college of the public Technical and Further Education (TAFE) system. She has six adult learners, four of whom are Australian born and have had very interrupted learning, and the other two are from other countries. She describes her class to us in the following way:

Anne: They have poor literacy skills as well as poor numeracy skills. Their understanding of basic concepts in general is very poor, … their numeracy concepts are very poor. They're mostly dependent learners so they're not capable of independent learning on their own at the moment. They sometimes lack living skills, which impedes learning.

Here Anne signals what might be associated with the concept of identity capital - their need to increase their own independence in their learning. In doing that each of Anne's students needs a lot of one – on - one help throughout their classes. They are also in the process of developing the basic concepts that might help them to become skilled enough to participate in job seeking and employment. Furthermore, Anne links gaps in what might be categorised as their social capital (living skills) as key factors in impeding their learning.

According to Anne, the needs of the six learners are varied. Some have very fragmented educational backgrounds, while others might have had some schooling but in completely different cultural and social contexts to what they are experiencing now. Anne devises both group and individualised tasks, and spends considerable time working one-on-one with the learners in her class. In response to a question about her teaching approach, she says:

Anne: The really important thing you've got to know is to know your students; know what their learning difficulties are or where their strengths are and work on those. Know what's happening in their lives so you can link into that. …So it's really important to listen to the students, listen to their stories and that helps you to understand where they're coming from and what's happening with them at the moment. Then I try and link the resources and the learning to that so their learning is relevant to them.

On the day of the class that we observed, Anne spent some "informal time" speaking to the students outside of the classroom for 5 to 10 minutes before the class was due to begin. The conversation was initiated by Anne asking the learners how they were and what they had been doing, and listening to where they had been on the weekend. For Anne, this was not an incidental moment, or something determined by some social convention; she explained that this was part of her plan to build a relationship with students:

Anne: I think one of the really important things that underpin my teaching is the fact that I'm teaching students and I'm not teaching the syllabus.

Interviewer: So how did you arrive at this? What brought all this together?

Anne: A love of people, just a wanting to know and understand how people react to things and how they live their lives. I'm really interested in my students and I let them know that.

At no time during the interview, did Anne name grand theories of adult education or pedagogy. However, she expressed her beliefs and approach confidently, reflectively and with clarity. These beliefs and approach were demonstrated in her class when she taught a session on the topic of measurement. She elicited what the learners knew about the topic, allowed learners to share personal stories that they wanted to tell about the topic, and used those stories to teach the concepts. She used a number of fun and easy to use measurement tools, and where possible used an active kinaesthetic approach to the tasks she set. While keeping the class together by using the common topic of measurement, her measurement tasks were individualised according to the current abilities that Anne assessed in each of the learners.

The learners were visibly engaged in the class, some working more independently than others in their individualised tasks of measuring objects and distances in the room, while others sought help from each other or from Anne. The learners all said that they were enjoying learning in the course, for example:

Learner 1: Well I enjoy all parts of maps and that. I'm quite good at measuring things and that …

Learner 2: I found the maths really good … very helpful as well … with different adding up and money management and all those sort of things

In response to a question about their motivation for attending the class, not all of them mentioned employment outcomes, or as their only reason. There was a mix of aspirations related to improvements in their negotiation of their everyday life, further study and employment. For example:

Learner 1: Well hopefully it will mean you know a better education and that for me .. maybe get another job or something.

Learner 2: Well not having a job you've got to keep yourself motivated so I had to come to TAFE to learn you know, to better yourself. .. I'm actually trying to look for more for the gardening side of work because I was actually doing that before and I'm trying to get back into it.

Learner 3: Now I want to study computer and then maths and then English and writing and listening … Because I need always [in] my daily life.

Learner 5: This course for me … is for future is very important for me. Because … I immigrate here so I got my family here. I'm going to live here now. I'm Australian resident and I love the country … So I … want to get through to English reading and writing, a little bit computer ... for future, you know every job you need that.

The learners talked about Anne as being "helpful", "friendly", "explains things well". And they valued the learning environment, and working with each other:

Learner 3: They [are] very good, very good. We have a very friendly, very nice persons, ... Very important. I'm very comfortable.

Learner 5: So since I come to this class I meet all these people … like so good for me. … I know these people … they're my friend[s]. When I come to school I'm happy you know.

Within this small class, the teacher Anne and the six learners were building assets that were linked to human capital – skills and knowledge for further study and employment; social capital - friendships and trusting relationships among the class members including the teacher, and for some, greater capacity for civic participation in a new country; and identity capital - motivation to learn, enjoyment, planning and goal setting, and a sense of feeling valued by the teacher and each other.

TEACHING AS A RELATIONAL ACTIVITY

The case study of the numeracy workshop might be characterised as a "humanistic", "learner-centred", or "communicative" classroom. What struck us most strongly for us as researchers of one teacher's teaching practice was the consistency between her espoused views of teaching and her practice, in particular, the importance she placed on her relationship with the learners and her classroom practice. It was also noticeable that the benefits of learning in terms of human capital, social capital and identity capital were not only were being achieved, but were being interlinked throughout the learning environment set up by the teacher. There were learners who could articulate the links they saw between the skills they were learning and increased participation in their community or "daily life". Several commented on how the course helped them to "better themselves" and to begin to set goals for themselves in further learning and employment.

While the teacher described her learners as being "dependent" learners, and the learners acknowledged the help they were receiving from her, the relationship was not one of enforcing dependency. Rather the teacher responded to their needs in order to help them grow and develop their skills as well as their self-concept, including independence and self-confidence. She created an environment of care and friendships so that the learners could a class - what Balatti et al (2006) called a "practice field" or a "bridge" to wider social participation. There was no single method or theory that was used by the teacher that could explain the benefits that the learners in this class were gaining from the numeracy workshop. However, the notion of teaching as a "relational practice" (Noddings, 2003) resonated strongly in the teacher's practice; the coherence of her pedagogical approach reflected what Kumaravadivelu calls postmethod pedagogy, where the teacher's practice is critically informed by, challenged by and responsive to the particular situational contexts in which a teacher finds themselves.

CONCLUSION

We have presented a small case study of an adult numeracy class and examined some of the possibilities for learners to gain a range of benefits from their learning, benefits that Schuller (2004) calls human capital, social capital and identity capital. We suggested that the possibilities of realising these benefits were linked to the teacher's understanding and demonstration of teaching as a relational practice, that is, of making connections with the learners and their individual lives, and a pedagogy that is strongly influenced by what the teacher sees as the learners' individual learning needs and goals.

The case study is too small to draw any general conclusions. However, in a policy context where adult literacy and numeracy education is seen only from the perspective of economic gains, and teaching as an application of an externally determined list of competencies, this case study is an important reminder of the potentials that learning carries, and the potentials that teachers have to help learners to gain rich benefits from their learning.

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THE DILEMMAS OF INDIGENOUS EDUCATION: THE PASSION FOR IGNORANCE

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Within the Australian context there is a significant gap between those students who perform well in the standardised national testing schemes and those who do not. Those most at risk of poor performance are indigenous students, and those students who live in remote and isolated communities. Recent years has seen a growth in research projects that seek to address the falling performance of indigenous students. These projects are founded on particular ideologies. This paper explores the dilemmas of working in this field of educational reform. The imperative of educational success is juxtaposed with the implications of cultural subversion.

> The passion for ignorance: having ignorance allows us to lead the lives we do (Willis, 2002).

I was remarkably taken by these words of Sue Willis (2002) as she wrote of the anomalies that appear in the teaching of counting. Her examples showed how despite evidence to suggest that the taken-for-granted approaches in the teaching processes associated with counting were violated by what children could do, this was largely ignored. This ignorance, she argued, allowed educators to get on with their job of teaching. In her case, she found that some Indigenous students could recognise collections of objects without counting and yet they were not seen as counters since they could not do the one-to-one correspondence that is the marker of the successful counter. One-to-one counting and rational counting strategies are clearly identified in many curriculum documents and testing regimes as the marker of a successful counter and yet some children did not demonstrate these key principles of counting but could tell how many were in a collection. For students in the latter group, they were frequently constructed as failed learners since they did not undertake what were seen as prescribed indicators for successful counting despite the fact that they could recognise how many items were in a group. For Willis, what was key to her argument was that as educators we fail to recognise anomalies such as these as they violate our taken-for-granted wisdom in how mathematical ideas (in this case, counting) are developed. In so doing, it helps to preserve the status quo, enables teachers to move in ways set by syllabus documents and not need to look outside the box. In these cases, the counting process as defined by curriculum becomes a regime of truth that allows educators to ignore violations of those truths. In so doing, there is a risk of positioning those violating the taken-for-granted norms to be positioned as marginal. In contrast, by recognising that the process does not work for all children means that something needed to be reconsidered with regard to such regimes of truth. However, school wisdom has not prepared educators for how to deal with such examples or where to move with these issues as they appear. Being ignorant of these anomalies enables the educator to avoid such issues and move on with the task of teaching. In

J.F. Matos, P. Valero & K. Yasukawa (Eds.) (2008). *Proceedings of the Fifth International Mathematics Education and Society Conference*. Lisbon: Centro de Investigação em Educação, Universidade de Lisboa – Department of Education, Learning and Philosophy, Aalborg University, (pp. 505 - 514).

many cases, teachers are ill-prepared to deal with anomalies such as those cited by Willis and the task of teaching is made much easier by ignoring such abhorrent patterns. However, the key question then becomes one of "To what effect does such ignorance have on the labelling of, and subsequent success of, students who violate curriculum norms".

The focus of this paper is to analyse contemporary approaches to Indigenous mathematics education within Australia. Despite considerable money being spent on interventions and research into ways to improve the educational outcomes for Indigenous Australian learners, there has been limited success. In terms of education, there have been some identified case studies that go against the national trends, but overall, the educational outcomes for Indigenous Australians are very poor. The issues are very complex and cannot be reduced to simple innovations in curriculum or pedagogy since issues of culture, health, geography, absenteeism and language impact on the potential for school learning alongside the overall relevance of western curriculum to the lives of Indigenous learners and their communities.

In this paper, I explore issues of Indigenous success in mainstream schooling as identified in the current testing regimes across the nation. This is supplemented with a discussion on the dilemmas of working in this field.

INDIGENOUS AUSTRALIA: A BACKGROUND

Within the Australian context, one of the most important and pervading educational issues is the education of the Indigenous children. Raging a much more silent apartheid than South Africa, Australia (as a nation) has an appalling record in the treatment of our indigenous people. 2007 makes the 40th anniversary of the referendum in which Indigenous Australians were recognised as a people who could have voting rights. Prior to this time, they did not appear on census collections or had the right to vote. In the following 40 years, there have been little gains for Indigenous people – their life expectancy is 17 years less than non-indigenous people with the largest differences in death patterns occurring between the ages of 35-44 and 45-54 where indigenous people were 5 times more likely to die than non-indigenous people (Trewin, 2005). Issues of employment and standard of living are other markers of the issues confronted by Indigenous Australians. Indigenous people (13%) were 2 ½ times more likely to be unemployed than non-indigenous people (4.6%) and the gross household income for Indigenous families was 59% of that of non-indigenous families (Trewin, 2005). Approximately 20% of Indigenous males have been arrested and/or served time in jail and represent 24% of the national prison population (Australia Bureau of Statistics, 2007). Adjusting for age and other factors, indigenous people were 13 times more likely to be incarcerated than non-indigenous people in 2006 (Australia Bureau of Statistics, 2007).

In terms of education, Trewin (2005) cites that Indigenous people represent only 1% of the higher education population even though they are approximately 3% of the overall population. Within these figures, women (63%) represented the greater proportion of indigenous participation (Australian Bureau of Statistics, 2004). Most indigenous students attend public schools – 88% (Australian Bureau of Statistics, 2004). In 2002, 18% of indigenous people over 18 had completed Year 12 (non indigenous is 44%) but these figures vary with demographics – 24% in city areas, 18% in regional areas, and 14% in remote areas. Similar declines in non-school qualifications are evident across the demography. The retention rate of Indigenous students from yr 7/8 to year 12 in 2002 was 38% in comparison with non-indigenous students of 76% (Australian Bureau of Statistics, 2004).

To be successful in white Australia is quite a task for Indigenous Australians when such a range of issues impact on their lifestyles. It is not my intention in this paper to discuss the reasons proposed by researchers (and politicians, welfare agencies. historians and so on) as to why these conditions exist. Suffice to say, for an international audience, the conditions for Indigenous Australians make success in schools a considerable challenge. When issues of health, the justice system and poverty are combined with the relevance of a Western mathematics curriculum being imposed through education systems, the potential for success becomes more evasive.

In the following sections, I raise a number of quandaries that surround working in Indigenous education in Australia. These are moral and educational dilemmas that confront the researcher when working with communities.

DILEMMA ONE: THE DILEMMA OF SUCCESS

While the use of national testing is fraught with difficulties – practical and ideological - the trends in the data arising from these testing regimes show some noteworthy trends in achievement in numeracy. Further, the question as to whether or not the tests actually test for numeracy or mathematics is another contentious issue that cannot be addressed here. Suffice for this paper is that the test is of some aspect of mathematical thinking and learning.

The tests are based on benchmarks which were set where it was envisaged that 80% of Australian students should reach the nominated standard in Years 3, 5 and 7. There is some difference in the education provided by the different states but for the purposes here, they need not be considered. In considering the data in Table One, it becomes very obvious that there is a trend to suggest that as Indigenous students progress through schooling, their achievement in numeracy decreases with each year in schooling in comparison to their non-indigenous peers. The national differences indicate a trend where the percentage of non-indigenous performances remains at or above the set benchmark but the performance of non-indigenous students declines with an increasing difference of almost 10% each two years so that by Year 7 less than half of the Indigenous students are reaching the national benchmarks.

	Year 3			Year 5			Year 7		
State	**All**	**Indig**	***diff***	**All**	**Indig**	**diff**	**All**	**Indig**	**diff**
NSW	95.4	87.6	7.8	91.7	75.4	16.3	75.8	44.5	31.3
Vic	95.5	91.8	3.7	95.4	89.5	5.9	86.9	66.5	20.4
Qld	92.7	78.9	13.8	88.1	65.8	22.3	83.2	54.5	28.7
SA	92.6	74.5	18.1	90.1	69.8	20.3	85.7	55.8	29.9
WA	90.2	64.8	25.4	85.9	51.6	34.3	84.3	46.8	37.5
Tas	91.2	82.4	8.8	89.1	78.7	10.4	80.5	66.4	14.1
NT	86.2	68	18.2	69.6	35.1	34.5	64.8	24.9	39.9
ACT	94.6	92.8	1.8	93.2	81.4	11.8	88.1	62.6	25.5
Aust	94.1	80.4	13.7	90.8	66.5	24.3	81.8	48.8	33

Table One. Percentage of students achieving the numeracy benchmark by state, 2005 (MCEETYA, 2006)

The data in this table raise a number of serious concerns for educators. First is the increasing gap in performance between students. Regardless of the state, the gap in performance increases in each state. However, in some states, the scores and increasing gap is of grave concern. By Year 7, in most states, Indigenous students are well below the 80% benchmark, with most states scoring around 50%. Remembering that this benchmark is a minimal standard, these low scores are troubling.

The context of Australian education needs some explication. Many of the scores in Table One indicate further concerns. In the states of Queensland, Northern Territory, Western Australia and South Australia, there a many Indigenous peoples who live in remote and isolated areas. This geographic isolation compounds their access to education but also impacts on their cultural isolation. These data suggest that the geographical location has a significant impact on the numeracy outcomes of learners. In this case, it would appear that there is a 20% to almost 40% difference in the outcomes between students in metro areas compared with their peers in remote areas of Australia. Separating out issues of indigeneity and geographical location is not possible, but these data allude to a link between the two factors, that is, that for many indigenous students who live in remote locations, their opportunities for success in numeracy are severely limited.

While these figures have come from the national results in 2005, the persistence of the issue can be seen in Table Two. By comparing the data over a period of six years (since the time the data were collected), the perseverance of the problem can be seen.

	Year 3			Year 5			Year 7		
	All stud	Indig	Diff	All stud	Indig	Diff	All stud	Indig	Diff
2000	92.7	73.7	19	89.6	62.8	26.8	-	-	
2001	93.9	80.2	13.7	89.6	63.2	26.4	82	48.6	33.4
2002	92.8	77.6	15.2	90	65.6	24.4	83.5	51.9	31.6
2003	94.2	80.5	13.7	90.8	67.6	23.2	81.3	49.3	32
2004	93.7	79.2	14.5	91.2	69.4	21.8	82.1	51.9	30.2
2005	94.1	80.4	13.7	90.8	66.5	24.3	81.8	48.8	33

Table Two: Percentage of students achieving numeracy benchmarks by subgroup (MCEETYA, 2006)

These data indicate that there is a constant trend for the performance within a year level and the increasing difference across year levels over time. The national data over time is relatively stable suggesting that the problem is entrenched and that the 2005 may represent consistent trends.

Recognising the stability and depth of the issue of under-achievement in numeracy for Indigenous students requires educators to move from positions ignorance to one that recogises the failure of the system to cater for the learning needs of these students. Clearly the practices that have been enacted in the past have met with considerable failure. Holding on to failed practices ensure reproduction of poor outcomes. The position taken by Willis when confronted with behaviours that violated the taken-for-granted practices upon which school mathematics enabled a reconceptualisation of thinking mathematically. By gaining insights into other ways of thinking and working mathematically provides a platform for a new agenda in the learning of mathematics by Indigenous communities.

When considering success in school mathematics for Indigenous students, particularly those in remote areas, the dilemma becomes one of the prioritising of mathematics and the implications of learning mathematics in those communities. How relevant are particular forms of knowing mathematics, what forms or aspects of the mathematics curriculum are needed or should be included in curriculum for the students or should the expectation be one where they are exposed to the same curriculum as their urban counter parts.

DILEMMA TWO: INCLUSION OF INDIGENOUS CULTURES IN SCHOOL MATHEMATICS

Is mathematics an endeavour in its own right? If so, how does this view rest with cultures or social groups for whom the practices of school mathematics are not part of their cultural systems. If school mathematics is taken to be a social practice, then it has been shaped by history, culture and practices so as to take a particular form, and value different aspects and outcomes. The dilemma then becomes one of aligning two very different social practices – that of school mathematic with that which those that students bring from their homes and communities. In some cases, there are elements of Indigenous cultures that will be align with the practices of school mathematics but there will differences that have been shaped by the contexts in which they have developed. The task of education thus becomes one of bridging between the different ontology so that students can come to learn school mathematics. Part of this learning is as much about mathematics as it is the social practice of mathematics.

In their comprehensive study of community numeracy practices, researchers (Rennie, Wallace, Falk, & Wignell, 2006) reported a number of disjunctions between community and school practices. They contend that community activities are dominated by cultural knowledges whereas school is dominated by curriculum knowledge. They argue that the knowledge in the community 'was not written and it was never forgotten'… 'knowledge in schools was largely of the text book kind … it was written and recorded' (p.10). These differences pose conundrums for school mathematics where the social practices are often about writing and recording, and where the textbook often takes a prime position. In working in Indigenous communities the process of recording knowledge becomes one that may be antithetical to indigenous ways of working.

Harris (1990) in her studies of Indigenous concepts noted that some indigenous people talk of the "everywhen" and that there is a stronger focus on history rather than forward looking. Similarly, others have noted that indigenous community knowledge rarely focuses on the future whereas schools talk about the future – going to work, preparing for Year 12 and so on (Rennie, Wallace, Falk, & Wignell, 2006). Community knowledge and activities were strongly connected to people, place and artifacts in that community whereas school activities were centred around the teacher and students. In coming to learn mathematics, this sense of time and place create another conundrum for teachers as they reconcile whose sense of time and place takes priority, whose sense is the legitimate account, and how differences can be reconciled or addressed.

The ethnomathematical approaches that seek to identify the 'frozen mathematics' (Gerdes, 1986) in cultural activities has been taken up in the work of Harris (1990; 1992) who sought to identify the mathematics in Aboriginal activities. In her keynote address, she sought to identify the hidden mathematics in Aboriginal artworks. Such an approach has been heavily criticized by Dowling (1993) who argued that this type of work subjugates the indigenous activity (spiritual representations) for a mathematical activity. The intentions of approaches such as Harris' have to identify the mathematics in cultural activities in order to provide legitimation of those activities so as to illustrate the indigenous people are capable of undertaking the mathematics of school activities.

In contrast, the work of Watson (1988) has taken a more grounded approach so that the researchers work with communities to develop a both-ways approach to learning where the cultures of both the Indigenous communities AND school mathematics are integrated into learning. Watson, working with the Yirrakala community identified indigenous ways of knowing and these became a legitimate part of the school curriculum. For example, mapping processes undertaken by the Yolgnu people were part of the curriculum. Their 'singing the land, signing the land' was a way that they mapped their lands where significant historical and cultural events were used to mark the land.

In considering the two very different cultural systems, the dilemma becomes one where the researcher and educator need to consider the balance between the two cultural systems, which has greater priority and how to move a curriculum forward.

DILEMMA THREE: LEARNING AS A PROCESS

Learning for many indigenous students is experiential and occurs in contexts where the skills and knowledges would be needed such as hunting or shopping. Learning school mathematics is premised on particular epistemological foundations so that coming to know is shaped by particular views of learning and pedagogy. In many cases, the traditional learning of mathematics is an individual endeavour that is often taught in highly competitive environments. School learning is largely an individualist endeavour where independence rather than collaboration are valued. (Rennie, Wallace, Falk, & Wignell, 2006). In contrast, learning for Indigenous students was a shared responsibility for all of those people involved in the activity (Rennie, Wallace, Falk, & Wignell, 2006).

The role of questioning is an integral part of the teaching process in mathematics classrooms. Early research on questioning highlighted the different forms of questioning where most questions are low level recall questions. Culturally, the questions that are posed by teachers are premised on the notion of checking students knowledge so that the teacher usually knows the answer and is 'testing' students' understandings. In contrast, for many indigenous students, questions are posed as authentic situations where they are seeking to find out information – "how do I get to x?" so that the posing of questions commonly used in mathematics are antithetical to the role of questioning in their communities.

In Indigenous community settings, the passing on of knowledge is through oral stories where elders pass on knowledge to the younger generations. Coming to understand whether the younger generations have understood the teachings is through questioning but not the ways that are posed in Western classrooms. Questions posed by the young to the elders serves as an indicator to the elders of how much has been understood. This role of questioning is very different from that of the classroom.

In considering the learning environment, the dilemma becomes one of enculturation. Should the learning environment embrace the indigenous ways of working an knowing or is the learning environment a process of enculturation into Western ways of working.

DILEMMA FOUR: DEVELOPING AN IDENTITY

For many students, coming to develop an identity is about being a member of a community (or communities). For indigenous students, coming to learning western mathematics presents a rupture between the lifeworlds of the students and the school. It is widely recognized that identity is critical to learning. Coming to foster an identity of a mathematics learner for indigenous students often requires a significant cultural shift. This shift is significant as it is about lifeworlds where representations of quality and relationships are central to the lifeworlds of Indigenous people is juxtaposed with the quantification lifeworlds of Western cultures. Developing an identity of a mathematics learner requires a

DILEMMA FIVE: TEACHER QUALITY

Often remote communities are hard to staff and attract fresh graduates who have little teaching experience and frequently have not worked within indigenous families or remote communities. (Cooper, Baturo, & Warren, 2005). While such attributes could be a strength in that there has been little opportunity for preconceptions to have developed, the experiences of teaching, planning and assessment have been limited for beginning teachers. As such, it is most likely that such teachers will need considerable support in their transition into classrooms from their employing authority.

A WAY FORWARD

In this final section of the paper, a way forward is proposed. Drawing on the considerable work that has been undertaken in building towards equitable mathematics classrooms, a number of strategies are framed. IN considering the significant dilemmas confronted by educational researchers working with Indigenous people, the reliance on ignorance to sustain existing practice must be addressed. The dilemmas noted in the previous section provide some indication of the considerations that must be made by those in the field. They are not necessarily dichotomies and clearly working with communities is critical. Their involvement in determining their forms and outcomes of education is taken for granted. While it has not been possible to raise the intricacies of many of the issues at the chalk face, the dilemmas raise considerations that need to be thought through carefully when working with Indigenous people where the intention is to raise the performance of their children. More specific changes need to be considered with respect to shifts in reforms and innovations. These are outlined in Table Three below.

	Current	**Shift to**
Pedagogy	Teacher centred, teacher directed	Student centred, culturally directed
	Skills based, rote and drill, low level	High quality, deep learning
Expectations of learners	Low expectations	High expectations
Assessment	What learners don't know	What learners do know
Curriculum knowledge	Built on Western knowledge structures	Incorporates indigenous ways of knowing
Curriculum processes	Hierarchical knowledge	Build from Indigenous knowledge
Ways of knowing	Linear	Networked
Language	Anglo-centric	ESL approach, language immersion, scaffolding

Table Three. Shifts needed in Reforming Mathematics

Returning to the original comment made by Willis with regard to ignorance enabling the continuation of taken-for-granted practice, the dilemmas raised in the paper pose challenges to some areas of educational reform that need to be considered if ignorance is to be addressed and the success of Indigenous learners is promoted.

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