STANDARDS, MARKETS, AND INEQUALITY

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Thinking Relationally

It is an unfortunate but true that there is not a long tradition within the mainstream of mathematics education of both critically and rigorously examining the connections between mathematics as an area of study and the larger relations of unequal economic, political, and cultural power. A number of scholars and activists throughout the world–some of whom are in this room--have attempted to build such a tradition of critical work. While I have written elsewhere about some of the ways in which recent “reforms” in mathematics education may result in increasing inequalities (see, e.g., Apple, 1999), I want to contribute to the development of such critical work by focussing on the larger context in which mathematics education operates. I want to critically examine the current context of educational “reforms”, a context that is structured by neo-liberal and neo-conservative movements. Without an examination of these movements and the ideological tendencies that characterize them, I do not believe that we will be able to adequately understand the limits and possibilities of a more democratic and critical education. In an essay of this length, I can only outline the tendencies that are currently structuring the terrain on which we operate. But, I want to give a picture of the social movements and ideological mobilizations that unfortunately are currently gaining even more power in education and the larger society in general. As something of an “outsider” to mathematics education, I hope that this examination of this larger picture provides sufficient detail for you to make the connections to specific movements, debates, and tensions within mathematics education in particular.

Right Turn

In his influential history of curriculum debates, Herbert Kliebard has documented that educational issues have consistently involved major conflicts and compromises among groups with competing visions of “legitimate” knowledge, what counts as “good” teaching and learning, and what is a “just” society (Kliebard, 1986). While I believe neither that these competing visions have ever had equal holds on the imagination of educators or the general citizenry nor that they have ever had equal power to effect their visions, it is still clear that no analysis of education can be fully serious without placing at its very core a sensitivity to the ongoing struggles that constantly shape the terrain on which the curriculum operates.

Today is no different than in the past. A “new” set of compromises, a new alliance and new power bloc has been formed that has increasing influence in education and all things social. This power bloc combines multiple fractions of capital who are committed to neo-liberal marketized solutions to educational problems, neo-conservative intellectuals who want a “return” to higher standards and a “common culture,” authoritarian populist religious fundamentalists who are deeply worried about secularity and the preservation of their own traditions, and particular fractions of the professionally oriented new middle class who are committed to the ideology and techniques of accountability, measurement, and “management.” While there are clear tensions and conflicts within this alliance, in general its overall aims are in providing the educational conditions believed necessary both for increasing international competitiveness, profit, and discipline and for returning us to a romanticized past of the “ideal” home, family, and school (Apple, 2000; Apple, 1996).

In essence, the new alliance--what I have elsewhere called “conservative modernization” (Apple, 1996)--has integrated education into a wider set of ideological commitments. The objectives in education are the same as those which guide its economic and social welfare goals. They include the dramatic expansion of that eloquent fiction, the free market; the drastic reduction of government responsibility for social needs; the reinforcement of intensely competitive structures of mobility both inside and outside the school; the lowering of people's expectations for economic security; the "disciplining" of culture and the body; and the popularization of what is clearly a form of Social Darwinist thinking, as the popularity only a few years ago of The Bell Curve (Herrnstein and Murray, 1994) with its claim that people of color, poor people, and women are genetically deficient so obviously and distressingly indicates.

The seemingly contradictory discourse of competition, markets, and choice on the one hand and accountability, performance objectives, standards, national testing, and national curriculum have created such a din that it is hard to hear anything else. As I have shown in Cultural Politics and Education (Apple, 1996), these tendencies actually oddly reinforce each other and help cement conservative educational positions into our daily lives.

While lamentable, the changes that are occurring present an exceptional opportunity for critical investigations. Here, I am not speaking of merely the accumulation of studies to promote the academic careers of researchers, although the accumulation of serious studies is not unimportant. Rather, I am suggesting that in a time of radical social and educational change it is crucial to document the processes and effects of the various and sometimes contradictory elements of the forces of conservative modernization and of the ways in which they are mediated, compromised with, accepted, used in different ways by different groups for their own purposes, and/or struggled over in the policies and practices of people's daily educational lives (Ransom, 1995, p.427). I shall want to give a sense of how this might be happening in current "reforms" such as marketization and national curricula and national testing in this essay.

New Markets, Old Traditions

Behind a good deal of the New Right's emerging discursive ensemble was a position that emphasized "a culturalist construction of the nation as a (threatened) haven for white (Christian) traditions and values" (Gillborn, 1997a, p.2). This involved the construction of an imagined national past that is at least partly mythologized, and then employing it to castigate the present. Gary McCulloch argues that the nature of the historical images of schooling has changed. Dominant imagery of education as being "safe, domesticated, and progressive" (that is, as leading toward progress and social/personal improvement) has shifted to become "threatening, estranged, and regressive" (McCulloch, 1997, p.80). The past is no longer the source of stability, but a mark of failure, disappointment, and loss. This is seen most vividly in the attacks on the "progressive orthodoxy" that supposedly now reigns supreme in classrooms in many nations.

For example, in England--though much the same is echoed in the United States, Australia, and elsewhere--Michael Jones, the political editor of The Sunday Times, recalls the primary school of his day.

> Primary school was a happy time for me. About 40 of us sat at fixed wooden desks with ink wells and moved from them only with grudging permission. Teacher sat in a higher desk in front of us and moved only to the blackboard. She smelled of scent and inspired awe. (Quoted in McCulloch, 1997, p.78)

The mix of metaphors invoking discipline, scent (visceral and almost "natural"), and awe is fascinating. But he goes on, lamenting the past 30 years of "reform" that transformed primary schools. Speaking of his own children's experience, Jones says:

> My children spent their primary years in a showplace school where they were allowed to wander around at will, develop their real individuality and dodge the 3Rs. It was all for the best, we were assured. But it was not. (Quoted in McCulloch, 1997, p.78).

For Jones, the "dogmatic orthodoxy" of progressive education "had led directly to educational and social decline." Only the rightist reforms instituted in the 1990s could halt and then reverse this decline (McCulloch, 1997, p.78). Only then could the imagined past return.

Much the same is being said my own side of the Atlantic. These sentiments are echoed in the public pronouncements of such figures as William Bennett, E.D. Hirsch, Jr., and others, all of whom seem to believe that progressivism is now in the dominant position in educational policy and practice and has destroyed a valued past. All of them believe that only by tightening control over curriculum and teaching (and students, of course), restoring "our" lost traditions, making education more disciplined and competitive as they are certain it was in the past--only then can we have effective schools. These figures are joined by others who have similar criticisms, but instead turn to a different past for a different future. Their past is less that of scent and awe and authority, but one of market "freedom." For them, nothing can be accomplished--even the restoration of awe and authority--without setting the market loose on schools so as to ensure that only "good" ones survive.

We should understand that these policies are radical transformations. If they had come from the other side of the political spectrum, they would have been ridiculed in many ways, given the ideological tendencies in our nations. Further, not only are these policies based on a romanticized pastoral past, these reforms have not been notable for their grounding in research findings. Indeed, when research has been used, it has often either served as a rhetoric of justification for preconceived beliefs about the supposed efficacy of markets or regimes of tight accountability or they have been based--as in the case of Chubb and Moe's much publicized work on the benefits of marketization in education (Chubb and Moe, 1990)--on quite flawed research (see, e.g., Whitty, 1997).

Yet, no matter how radical some of these proposed "reforms" are and no matter how weak the empirical basis of their support, they have now redefined the terrain of debate of all things educational. After years of conservative attacks and mobilizations, it has become clear that "ideas that were once deemed fanciful, unworkable--or just plain extreme" are now increasingly being seen as commonsense (Gillborn, 1997b, p.357).

Tactically, the reconstruction of commonsense that has been accomplished has proven to be extremely effective. For example, there are clear discursive strategies being employed here, ones that are characterized by "plain speaking" and speaking in a language that "everyone can understand." (I do not wish to be wholly negative about this. The importance of these things is something many "progressive" educators have yet to understand.) These strategies also involve not only presenting one's own position as "commonsense," but also usually tacitly implying that there is something of a conspiracy among one's opponents to deny the truth or to say only that which is "fashionable" (Gillborn, 1997b, p.353). As Gillborn notes,

> This is a powerful technique. First, it assumes that there are no <u>genuine</u> arguments against the chosen position; any opposing views are thereby positioned as false, insincere or self-serving. Second, the technique presents the speaker as someone brave or honest enough to speak the (previously) unspeakable. Hence, the moral high ground is assumed and opponents are further denigrated. (Gillborn, 1997b, p.353)

It is hard to miss these characteristics in some of the conservative literature such as Herrnstein and Murray's (1994) publicizing of the unthinkable "truth" about genetics and intelligence or E.D. Hirsch's (1996) latest "tough" discussion of the destruction of "serious" schooling by progressive educators.

<u>Markets and Performance</u>

Let us take as an example of the ways in which all this operates one element of the conservative restoration--the neo-liberal claim that the invisible hand of the market will inexorably lead to better schools. As Roger Dale reminds us, "the market" acts as a metaphor rather than an explicit guide for action. It is not denotative, but connotative. Thus, it must itself be "marketed" to those who will exist in it and live with its effects (Roger Dale, quoted in Menter, et al, 1997, p.27). Markets are marketed, are made legitimate, by a depoliticizing strategy. They are said to be natural and neutral, and governed by effort and merit. And those opposed to them are by definition, hence, also opposed to effort and merit. Markets, as well, are supposedly less subject to political interference and the weight of bureaucratic procedures. Plus, they are grounded in the rational choices of individual actors (Menter, et al, 1997, p.27). Thus, markets and the guarantee of rewards for effort and merit are to be coupled together to produce "neutral," yet positive, results. Mechanisms, hence, must be put into place that give evidence of entrepreneurial efficiency and effectiveness. This coupling of markets and mechanisms for the generation of evidence of performance is exactly what has occurred. Whether it works is open to question.

In what is perhaps the most comprehensive critical review of all of the evidence on marketization, Geoff Whitty cautions us not to mistake rhetoric for reality. After examining research from a number of countries, Whitty argues that while advocates of marketized "choice" plans assume that competition will enhance the efficiency and responsiveness of schools, as well as give disadvantaged children opportunities that they currently do not have, this may be a false hope (Whitty, 1997, p.58). These hopes are not now being realized and are unlikely to be realized in the future "in the context of broader policies that do nothing to challenge deeper social and cultural inequalities" (Whitty, 1997, p.58). As he goes on to say, "Atomized decision-making in a highly stratified society may appear to give everyone equal opportunities, but transforming responsibility for decision-making from the public to the private sphere can actually reduce the scope of collective action to improve the quality of education for all" (p.58). When this is connected to the fact that, as I shall show shortly, in practice neo-liberal policies involving market "solutions" may actually serve to reproduce--not subvert--traditional hierarchies of class and race, this should give us reason to pause (Whitty, 1997; Whitty, Edwards, and Gewirtz, 1993; Whitty, Power, and Halpin, 1998; Apple, 1996).

Thus, rather than taking neo-liberal claims at face value, we should want ask about their hidden effects that are too often invisible in the rhetoric and metaphors of their proponents. Given the limitations of what one can say in an essay of this length, I shall select a few issues that have been given less attention than they deserve, but on which there is now significant research..

The English experience is useful here, especially since Chubb and Moe (1990) rely so heavily on it. In England, the 1993 Education Act documents the state's commitment to marketization. Governing bodies of local educational authorities (LEAs) were mandated to formally consider "going GM" (that is, opting out of the local school system's control and entering into the competitive market) every year (Power, Halpin, and Fitz, 1994, p.27). Thus, the weight of the state stands behind the press towards neo-liberal reforms there.[1] Yet, rather than leading to curriculum responsiveness and diversification, the competitive market has not created much that is different from the traditional models so firmly entrenched in schools today (Power, Halpin, and Fitz, 1994, p.39). Nor has it radically altered the relations of inequality that characterize schooling.

In their own extensive analyses of the effects of marketized reforms "on the ground," Ball and his colleagues point to some of the reasons why we need to be quite cautious here. As they document, in these situations educational principles and values are often compromised such that commercial issues become more important in curriculum design and resource allocation (Ball, Bowe, and Gewirtz, 1994, p.19). For instance, the coupling of markets with the demand for and publication of performance indicators such as "examination league tables" in England has meant that schools are increasingly looking for ways to attract "motivated" parents with "able " children. In this way, schools are able to enhance their relative position in local systems of competition. This represents a subtle, but crucial shift in emphasis--one that is not openly discussed as often as it should be--from student needs to student performance and from what the school does for the student to what the student does for the school. This is also accompanied too uncomfortably often by a shift of resources away from students who are labelled as having special needs or learning difficulties, with some of these needed resources now being shifted to marketing and public relations.

[1]. Whether there have been significant changes in this regard given the victory by "New Labour" over the Conservatives remains to be seen, although the outlook is not necessarily good in many ways. Certain aspects of neo-liberal and neo-conservative policies have already been accepted by Labour, such as the acceptance of stringent cost controls on spending put in place by the previous Conservative government and an aggressive focus on "raising standards" in association with strict performance indicators. See for example, Ken Jones (1999).

"Special needs" students are not only expensive, but deflate test scores on those all important league tables.

Not only does this make it difficult to "manage public impressions," but it also makes it difficult to attract the "best" and most academically talented teachers (Ball, Bowe, and Gewirtz, 1994, pp.17-19). The entire enterprise does, however, establish a new metric and a new set of goals based on a constant striving to win the market game. What this means is of considerable import, not only in terms of its effects on daily school life but in the ways it signifies a transformation of what counts as a good society and a responsible citizen. Let me say something about this generally.

Drawing on Kliebard's significant historical work, I noted earlier that behind all educational proposals are visions of a just society and a good student. The neo-liberal reforms I have been discussing construct this in a particular way. While the defining characteristic of neo-liberalism is largely based on the central tenets of classical liberalism, in particular classic economic liberalism, there are crucial differences between classical liberalism and neo-liberalism. These differences are absolutely essential in understanding the politics of education and the transformations education is currently undergoing. Mark Olssen clearly details these differences in the following passage. It is worth quoting in its entirety.

> Whereas classical liberalism represents a negative conception of state power in that the individual was to be taken as an object to be freed from the interventions of the state, neo-liberalism has come to represent a positive conception of the state's role in creating the appropriate market by providing the conditions, laws and institutions necessary for its operation. In classical liberalism, the individual is characterized as having an autonomous human nature and can practice freedom. In neo-liberalism the state seeks to create an individual who is an enterprising and competitive entrepreneur. In the classical model the theoretical aim of the state was to limit and minimize its role based on postulates which included universal egoism (the self-interested individual); invisible hand theory which dictated that the interests of the individual were also the interests of the society as a whole; and the political maxim of laissez-faire. In the shift from classical liberalism to neo-liberalism, then, there is a

> further element added, for such a shift involves a change in subject position from "homo economicus," who naturally behaves out of self-interest and is relatively detached from the state, to "manipulatable man," who is created by the state and who is continually encouraged to be "perpetually responsive." It is not that the conception of the self-interested subject is replaced or done away with by the new ideals of "neo-liberalism," but that in an age of universal welfare, the perceived possibilities of slothful indolence create necessities for new forms of vigilance, surveillance, "performance appraisal" and of forms of control generally. In this model the state has taken it upon itself to keep us all up to the mark. The state will see to it that each one makes a "continual enterprise of ourselves"...in what seems to be a process of "governing without governing." (Olssen, 1996, p.340)

The results of Ball and his colleagues' research document how the state does indeed do this, enhancing that odd combination of marketized individualism and control through constant and comparative public assessment. Widely publicized league tables determine one's relative value in the educational marketplace. Only those schools with rising performance indicators are worthy. And only those students who can "make a continual enterprise of themselves" can keep such schools going in the "correct" direction. Yet, while these issues are important, they fail to fully illuminate some of the other mechanisms through which <u>differential</u> effects are produced by neo-liberal reforms. Here, class issues come to the fore in ways that Ball, Bowe, and Gewirtz (1994) make clear.

Middle class parents are clearly the most advantaged in this kind of cultural assemblage, and not only as we saw because the principals of schools seek them out. Middle class parents have become quite skilled, in general, in exploiting market mechanisms in education and in bringing their social, economic, and cultural capital to bear on them. "Middle class parents are more likely to have the knowledge, skills and contacts to decode and manipulate what are increasingly complex and deregulated systems of choice and recruitment. The more deregulation, the more possibility of informal procedures being employed. The middle class also, on the whole, are more able to move their children around the system" (Ball, Bowe, and Gewirtz, 1994, p.19). That class and race intersect and interact in complex ways means that--even though we need to be clear that marketized systems in education often expressly have their conscious and unconscious raison d'etre in a fear of "the other" and often express a racialization of educational policy--the differential results will "naturally" be decidedly raced as well as classed.[2]

Economic and social capital can be converted into cultural capital in various ways. In marketized plans, more affluent parents often have more flexible hours and can visit multiple schools. They have cars--often more than one--and can afford driving their children across town to attend a "better" school. They can as well provide the hidden cultural resources such as camps and after school programs (dance, music, computer classes, etc.) that give their children an "ease," a "style," that seems "natural" and acts as a set of cultural resources. Their previous stock of social capital--who they know, their "comfort" in social encounters with educational officials--is an unseen but powerful storehouse of resources. Thus, more affluent parents are more likely to have the informal knowledge and skill--what Bourdieu would call the habitus (Bourdieu, 1984)--to be able to decode and use marketized forms to their own benefit. This sense of what might be called "confidence"--which is itself the result of past choices that tacitly but no less powerfully depend on the economic resources to actually have had the ability to make economic choices--is the unseen capital that underpins their ability to negotiate marketized forms and "work the system" through sets of informal cultural rules (Ball, Bowe, and Gewirtz, 1994, pp.20-22).

Of course, it needs to be said that working class, poor, and/or immigrant parents are not skill-less in this regard, by any means. (After all, it requires an immense amount of skill, courage, and social and cultural resources to survive under exploitative and depressing material conditions. Thus, collective bonds, informal networks and contacts, and an ability to work the system are developed in quite nuanced, intelligent, and often impressive ways here.) However, the match between the historically grounded habitus expected in schools and in its actors and those of more affluent parents, combined with the material resources available to more affluent parents, usually leads to a successful conversion of economic and social capital

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[2]. See the discussion of the racial state in Omi and Winant (1994) and the analyses of race and representation in McCarthy and Crichlow (1994).

into cultural capital (see Bourdieu, 1996). And this is exactly what is happening in England, the United States, and elsewhere (see, e.g., Lauder and Hughes, 1999).

These empirical findings are made more understandable in terms of Pierre Bourdieu's analysis of the relative weight given to cultural capital as part of mobility strategies today (Bourdieu, 1996). The rise in importance of cultural capital infiltrates all institutions in such a way that there is a relative movement away from the <u>direct</u> reproduction of class privilege (where power is transmitted largely within families through economic property) to <u>school-mediated</u> forms of class privilege. Here, "the bequeathal of privilege is simultaneously effectuated and transfigured by the intercession of educational institutions" (Wacquant, 1996, p.xiii). This is <u>not</u> a conspiracy; it is not "conscious" in the ways we normally use that concept. Rather it is the result of a long chain of relatively autonomous connections between differentially accumulated economic, social, and cultural capital operating at the level of daily events as we make our respective ways in the world, including as we saw in the world of school choice.

Thus, while not taking an unyieldingly determinist position, Bourdieu argues that a class habitus tends to reproduce the conditions of its own reproduction "unconsciously." It does this by producing a relatively coherent and systematically <u>characteristic</u> set of seemingly natural and unconscious strategies--in essence, ways of understanding and acting on the world that act as forms of cultural capital that can be and are employed to protect and enhance one's status in a social field of power. He aptly compares this similarity of habitus across class actors to handwriting.

> Just as the acquired disposition we call "handwriting," that is a particular way of forming letters, always produces the same "writing"--that is, graphic lines that despite differences in size, matter, and color related to writing surface (sheet of paper or blackboard) and implement (pencil, pen, or chalk), that is despite differences in vehicles for the action, have an immediately recognizable affinity of style or a family resemblance--the practices of a single agent, or, more broadly, the practices of all agents endowed with similar habitus, owe the affinity of style that makes each a metaphor for the others to the fact that they are the products of the implementation in

different fields of the same schemata of perception, thought, and action. (Bourdieu, 1996, p.273)

This very connection of habitus across fields of power--the ease of bringing one's economic, social, and cultural resources to bear on "markets"--enables a comfort between markets and self that characterizes the middle class actor here. This constantly produces differential effects. These effects are not neutral, no matter what the advocates of neo-liberalism suggest. Rather, they are themselves the results of a particular kind of morality. Unlike the conditions of what might best be called "thick morality" where principles of the common good are the ethical basis for adjudicating policies and practices, markets are grounded in aggregative principles. They are constituted out of the sum of individual good and choices. "Founded on individual and property rights that enable citizens to address problems of interdependence via exchange," they offer a prime example of "thin morality" by generating both hierarchy and division based on competitive individualism (Ball, Bowe, and Gewirtz, 1994, p.24). And in this competition, the general outline of the winners and losers has been identified empirically.

National Curriculum and National Testing

I showed in the previous section that there are connections between at least two dynamics operating in neo-liberal reforms, "free" markets and increased surveillance. This can be seen in the fact that in many contexts, marketization has been accompanied by a set of particular policies for "producers," for those professionals working within education. These policies have been strongly regulatory. As in the case of the linkage between national tests and performance indicators published as league tables, they have been organized around a concern for external supervision, regulation, and external judgement of performance (Menter, et al., 1997, p.8). This concern for external supervision and regulation is not only connected with a strong mistrust of "producers" (e.g., teachers) and to the need for ensuring that people continually make enterprises out of themselves. It is also clearly linked both to the neo-conservative sense of a need to "return" to a lost past of high standards, discipline, awe, and "real" knowledge and to the professional middle class's own ability to carve out a sphere of authority within the state for its own commitment to management techniques and efficiency.

There has been a shift in the relationship between the state and "professionals." In essence, the move toward a small strong state that is increasingly guided by market needs seems inevitably to bring with it reduced professional power and status (Menter, et al., 1997, p.57). Managerialism takes center stage here.

Managerialism is largely charged with "bringing about the cultural transformation that shifts professional identities in order to make them more responsive to client demand and external judgement" (Menter, et al., 1997, p.9). It aims to justify and to have people internalize fundamental alterations in professional practices. It both harnesses energy and discourages dissent (Menter, et al., 1997, p.9).

There is no necessary contradiction between a general set of marketizing and deregulating interests and processes--such as voucher and choice plans--and a set of enhanced regulatory processes--such as plans for national curricula and national testing. "The regulatory form permits the state to maintain 'steerage' over the aims and processes of education from within the market mechanism" (Menter, et al., 1997, p.24). Such steerage has often been vested in such things as national standards, national curricula, and national testing. Forms of all of these are being pushed for in the United States currently and are the subject of considerable controversy, some of which cuts across ideological lines and shows some of the tensions within the different elements contained under the umbrella of the conservative restoration.

I have argued elsewhere that paradoxically a national curriculum and especially a national testing program are the first and most essential steps toward increased marketization. They actually provide the mechanisms for comparative data that "consumers" need to make markets work as markets (Apple, 1996). Without these mechanisms, there is no comparative base of information for "choice." Yet, we do not have to argue about these regulatory forms in a vacuum. Like the neo-liberal markets I discussed in the previous section, they too have been instituted in England; and, once again, there is important research available that can and must make us duly cautious in going down this path.

One might want to claim that a set of national standards, national curricula, and national tests would provide the conditions for "thick morality." After all, such regulatory reforms are supposedly based on shared values and common sentiments that also create social spaces in which common issues of concern can be debated and made subject to moral interrogation (Ball, Bowe, and Gewirtz, 1994, p.23). Yet, what counts as the "common," and how and by whom it is actually determined, is rather more thin than thick.

It is the case that while the national curriculum now so solidly in place in England and Wales is clearly prescriptive, it has not always proven to be the kind of straight-jacket it has often been made out to be. As a number of researchers have documented, it is not only possible that policies and legislative mandates are interpreted and adapted, but it seems inevitable. Thus, the national curriculum is "not so much being 'implemented' in schools as being 'recrated,' not so much 'reproduced,' as 'produced' (Power, Halpin, and Fitz, 1994, p.38).

In general, it is nearly a truism that there is no simplistic linear model of policy formation, distribution, and implementation. There are always complex mediations at each level of the process. There is a complex politics that goes on within each group and between these groups and external forces in the formulation of policy, in its being written up as a legislative mandate, in its distribution, and in its reception at the level of practice (Ransom, 1995, p.436). Thus, the state may legislate changes in curriculum, evaluation, or policy (which is itself produced through conflict, compromise, and political maneuvering), but policy writers and curriculum writers may be unable to control the meanings and implementations of their texts. All texts are "leaky" documents. They are subject to "recontextualization" at every stage of the process (Ransom, 1995, p.436).

However, this general principle may be just a bit too romantic. None of this occurs on a level playing field. As with market plans, there are very real differences in power in one's ability to influence, mediate, transform, or reject a policy or a regulatory process. Granted, it is important to recognize that a "state control model"--with its assumption of top-down linearity--is much too simplistic and that the possibility of human agency and influence is always there. However, having said this, this should not imply that such agency and influence will be powerful (Ransom, 1995, p.437).

The case of national curriculum and national testing in England and Wales documents the tensions in these two accounts. It was the case that the national curriculum that was first legislated and then imposed there, was indeed struggled over. It was originally too detailed and too specific, and, hence, was subject to major transformations at the national, community, school, and then classroom levels. However, even though the national curriculum was subject to conflict, mediation, and some transformation of its content, organization, and its invasive and immensely time consuming forms of evaluation, its utter power is demonstrated in its radical reconfiguration of the very process of knowledge selection, organization, and assessment. It changed the entire terrain of education radically. Its subject divisions "provide more constraint than scope for discretion." The "standard attainment targets" that have been mandated cement these constraints in place. "The imposition of national testing locks the national curriculum in place as the dominant framework of teachers' work whatever opportunities teachers may take to evade or reshape it" (Richard Hatcher and Barry Troyna quoted in Ransom, 1995, p.438).

Thus, it is not sufficient to state that the world of education is complex and has multiple influences. The purpose of any serious analysis is to go beyond such overly broad conclusions. Rather, we need to "discriminate degrees of influence in the world," to weigh the relative efficacy of the factors involved. Hence, although it is clear that while the national curriculum and national tests that now exist in England and Wales have come about because of a complex interplay of forces and influences, it is equally clear that "state control has the upper hand" (Ransom, 1995, p.438).

The national curricula and national tests did generate conflict about issues. They did partly lead to the creation of social spaces for moral questions to get asked. (Of course, these moral questions had been asked all along by dispossessed groups.) Thus, it was clear to many people that the creation of mandatory and reductive tests that emphasized memory and decontextualized abstraction pulled the national curriculum in a particular direction--that of encouraging a selective educational market in which elite students and elite schools with a wide range of resources would be well (if narrowly) served (O'Hear, 1994, p.66). Diverse groups of people argued that the such reductive, detailed, and simplistic paper and pencil tests "had the potential to do enormous damage," a situation that was made even worse because the tests were so onerous in terms of time and record keeping (O'Hear, 1994, pp.55-56). Teachers had a good deal of support when as a group they decided to boycott the administration of the test in a remarkable act of public protest. This also led to serious questioning of the arbitrary, inflexible, and overly prescriptive national curriculum. While the curriculum is still inherently problematic and the assessment system does still contain numerous dangerous and onerous elements within it, organized activity against them did have an impact (O'Hear, 1994, pp.56-57).

Yet, unfortunately, the story does not end there. By the mid-1990s, even with the government's partial retreat on such regulatory forms as its program of constant and reductive testing, it had become clearer by the year that the development of testing and the specification of content had been "hijacked" by those who were ideologically committed to traditional pedagogies and to the idea of more rigorous selection (O'Hear, 1994, p.68). The residual effects are both material and ideological. They include a continuing emphasis on trying to provide the "rigor [that is] missing in the practice of most teachers,...judging progress solely by what is testable in tests of this kind" and the development of a "very hostile view of the accountability of teachers" that was seen as "part of a wider thrust of policy to take away professional control of public services and establish so called consumer control through a market structure" (O'Hear, 1994, pp.65-66).

The authors of an extremely thorough review of recent assessment programs instituted in England and Wales provide a summary of what has happened. Gipps and Murphy argue that it has become increasingly obvious that the national assessment program attached to the national curriculum is more and more dominated by traditional models of testing and the assumptions about teaching and learning that lie behind them. At the same time, equity issues are becoming much less visible (Gipps and Murphy, 1994, p.209). In the calculus of values now in place in the regulatory state, efficiency, speed, and cost control replace more substantive concerns about social and educational justice. The pressure to get tests in place rapidly has meant that "the speed of test development is so great, and the curriculum and assessment changes so regular, that [there is] little time to carry out detailed analyses and trialing to ensure that the tests are as fair as possible to all groups" (Gipps and Murphy, 1994, p.209). The conditions for "thin morality"--in which the competitive individual of the market dominates and social justice will somehow take care of itself--are re-produced here. The combination of the neo-liberal market and the regulatory state, then, does indeed "work." However, it works in ways in which the metaphors of free market, merit, and effort hide the differential reality that is produced.

Basil Bernstein's discussion of the general principles by which knowledge and policies ("texts") move from one arena to another is useful in understanding this. As Bernstein reminds us, when talkin about educational change there are three fields with which we must be concerned. Each field has its own rules of access, regulation, privilege, and special interests: 1) the field of "production" where new knowledge is constructed; 2) the field of "reproduction" where pedagogy and curriculum are actually enacted in schools; and, between these other two, 3) the "recontextualizing" field where discourses from the field of production are appropriated and then transformed into pedagogic discourse and recommendations (Bernstein, 1990; Bernstein, 1996). This appropriation and recontextualization of knowledge for educational purposes is itself governed by two sets of principles. The first--de-location--implies that there is always a <u>selective</u> appropriation of knowledge and discourse from the field of production. The second--re-location--points to the fact that when knowledge and discourse from the field of production is pulled within the recontextualizing field, it is subject to ideological transformations due to the various specialized and/or political interests whose conflicts structure the recontextualizing field (Evans and Penney, 1995).

A good example of this, one that confirms Gipps and Murphy's analysis of the dynamics of national curricula and national testing during their more recent iterations, is found in the process by which the content and organization of the mandated national curriculum in physical education were struggled over and ultimately formed in England. In this instance, a working group of academics both within and outside the field of physical education, headmasters of private and state-supported schools, well known athletes, and business leaders (but <u>no</u> teachers) was formed.

The original curriculum policies that arose from the groups were relatively mixed educationally and ideologically, taking account of the field of production of knowledge within physical education. That is, they contained both progressive elements and elements of the conservative restoration, as well as academic perspectives within the specialized fields from the university. However, as these made their way from report to recommendations and then from recommendations to action, they steadily came closer to restorational principles. An emphasis on efficiency, basic skills and performance testing, on the social control of the body, and on competitive norms ultimately won out. Like the middle class capturing of the market discussed earlier, this too was not a conspiracy. Rather, it was the result of a process of "overdetermination." That is, it was not due to an imposition of these norms, but to a combination of interests in the recontextualizing field--an economic context in which public spending was under severe scrutiny and cost savings had to be sought everywhere, government officials who were opposed to "frills" and consistently intervened to institute only a selection of the recommendations (conservative ones that did <u>not</u> come from "professional academics" preferably), ideological attacks on critical, progressive or child-centered approaches to physical education, and a predominant discourse of "being pragmatic." These came together in the recontextualizing field and helped insure in practice that conservative principles would be reinscribed in policies and mandates, and that critical forms were seen as too ideological, too costly, or too impractical (Evans and Penney, 1995, pp.41-42). "Standards" were upheld; critical voices were heard, but ultimately to little effect; the norms of competitive performance were made central and employed as regulatory devices. Regulatory devices served to privilege specific groups in much the same way as did markets. Thus goes democracy in education.

Conclusion

In this relatively brief essay, I have been rather ambitious. I have raised serious questions about current educational "reform" efforts now underway in a number of nations. I have used research on the English experience(s) to document some of the hidden differential effects of two connected strategies--neo-liberal inspired market proposals and neo-liberal, neo-conservative, and middle class managerial inspired regulatory proposals. Taking a key from Herbert Kliebard's powerful historical analyses, I have described how different interests with different educational and social visions compete for dominion in the social field of power surrounding educational policy and practice. In the process, I have documented some of the complexities and imbalances in this field of power. These complexities and imbalances result in "thin" rather than "thick" morality and in the reproduction of both dominant pedagogical and curricular forms and ideologies and the social privileges that accompany them.

Having said this, however, I want to point to a hidden paradox in what I have done. Even though much of my own and others' research recently has been on the conservative restoration, there are dangers in such a focus of which we should be aware. Research on the history, politics, and practices of rightist social and educational movements and "reforms" has enabled us to show the contradictions and unequal effects of such policies and practices. It has enabled the rearticulation of claims to social justice on the basis of solid evidence. This is all to the good. However, in the process, one of the latent effects has been the gradual framing of educational issues largely in terms of the conservative agenda. The very categories themselves--markets, choice, national curricula, national testing, standards--bring the debate onto the terrain established by neo-liberals and neo-conservatives. The analysis of "what is" has led to a neglect of "what might be." Thus, there has been a withering of substantive large scale discussions of feasible alternatives to neo-liberal and neo-conservative visions, policies, and practices, ones that would move well beyond them (Seddon, 1997, pp.165-166).

Because of this, at least part of our task may be politically and conceptually complex, but it can be said simply. In the long term, we need to "develop a political project that is both local yet generalizable, systematic without making Eurocentric, masculinist claims to essential and universal truths about human subjects" (Luke, 1995, pp.vi-vii). Another part of our task, though, must be and is more proximate, more appropriately educational. Defensible, articulate, and fully fleshed out alternative progressive policies and practices in curriculum, teaching, and evaluation need to be developed and made widely available.

While, in Democratic Schools, James Beane and I have brought together a number of such examples for a larger educational audience (Apple and Beane, 1995; Apple and Beane, 1999), so much more needs to done. Of course, we are not starting anew in any of this. The history of democratically and critically oriented educational reforms in all of our nations is filled with examples, with resources of hope. Sometimes we can go forward by looking back, by recapturing what the criticisms of past iterations of current rhetorical “reforms” have been, and by rediscovering a valued set of traditions of educational criticism and educational action that have always tried to keep the vast river of democracy flowing. We will not find all of the answers by looking at our past, but we will re-connect with and stand on the shoulders of educators whose lives were spent in struggle against some of the very same ideological forces we face today.

Although crucial, it is then not enough, as I have done in this essay, to deconstruct the policies of conservative modernization in education. Neo-liberals and neo-conservatives have shown how important changes in commonsense are in the struggle for education. It is our task to collectively help rebuild it by reestablishing a sense that “thick” morality, and a “thick” democracy, are truly possible today. There is political and practical work that needs to be done. If we do not do it, who will?

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Schooled and Community numeracies; understanding social factors and 'under-achievement' in numeracy.

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Abstract

This is a discussion of research in the 'Schooled and Community numeracies' focus within the Leverhulme funded Low Educational Achievement in Numeracy Research Programme. The intentions of the research in this focus are to seek explanations for underachievement in numeracy that derive from understandings of mathematics as social. We wanted to understand why some children apparently cope easily with informal numeracy practices whilst others struggle with formal numeracies. We wanted to investigate boundaries children face or which are constructed between home and schooled numeracy practices. The paper will initially consider some of the conceptual and methodological issues that have arisen in the research. Work done in a pilot study will be used to throw further light on these issues and possible implications for both research and schooling will be raised.

1 Introduction

This paper is a discussion of one part of the Leverhulme Numeracy Research Programme (1997 – 2002) investigating Low Educational Achievement in Numeracy in the United Kingdom. The intentions of the programme are to contribute to: understanding of the critical points in progression in primary mathematics; knowledge of how classroom practices affect attainment; identifying training and intervention strategies; understanding teacher change; understanding the effects of social factors. This paper will concentrate on the latter aspects, which form focus 4 of the programme on *'Schooled and Community Numeracies'*. The team of researchers working on this focus are Dave Baker from the University of Brighton, and Brian Street and Alison Tomlin, from Kings College, London. The intentions of the research in this focus are to seek explanations for underachievement in numeracy that derive from understandings of mathematics as social. The reasoning behind this approach is that research on, and educational policy for, raising achievement in numeracy has recently mainly focused on aspects such as teacher subject knowledge, pedagogy (pace, style, whole class teaching, setting, calculators, homework), schools (leadership, effective management, policies), and educational structures (assessment regimes and systems, Local Education Authorities). Consideration of the effect of social factors on the other hand have been marginalised and even rejected despite serious research exposing their significance. For instance:

> "The data from the National Child Development Survey (1991) show that there is a strong relationship between children's performance in maths and reading tests between the ages of six and eight and their parents' earnings,

> with the children of higher earning parents performing better." (Machin, 1999 p 19).

and

> "The British Cohort Study shows that at 22 months the children of parents in social classes one or two with higher education levels are already 14 percentage points higher up on the educational-development distribution than children whose parents are in social classes four and five and have low educational achievement." (HM Treasury, March 99, p 29)

Macro visible social factors like poverty clearly do play a large role in educational achievement. In terms of numeracy we wanted to focus on less conspicuous social factors. We wanted to understand why some children apparently cope easily with formal numeracy practices whilst others struggle often in vain to handle those practices. We wanted to investigate boundaries and barriers children face between formal and informal numeracy practices and between home and school numeracy practices. The construction and maintenance of such boundaries and barriers are also considered.

The paper will initially unpack some of the conceptual and methodological issues that have arisen in the research so far. Work done in a pilot study will be used to throw further light on these issues. Implications for research and for teaching will conclude the paper.

2 Conceptual issues

The objective of looking at mathematics as 'social' is to understand and describe different meanings that pupils bring to their encounters with schooled maths/numeracy and thereby to contribute to explanations for the underachievement of many in schooled maths. This includes developing a language of description and refining lenses for viewing maths practices in social contexts that might complement and enrich the current tools for description available in the field. The concepts we needed to clarify were therefore: understandings of numeracy as social; the nature of community and schooled numeracies: and home/community/school relationships. A great deal of work in the field of 'social literacies' has addressed many of these issues. One dimension of the research is to consider how far this social literacies work (cf. Street, 1996) can be applied to the field of mathematics education.

The first aspect we needed to clarify was the social in mathematics and numeracy. (c.f. Baker BCME 99). A social perspective on mathematics does not entail simply privileging everyday or 'ethnic' mathematics or treating everything as 'social', which can be rather vacuous. Nor does it entail pronouncing on the ontological status of mathematics. Rather it provides a vehicle for an exploratory inquiry into what follows from considering mathematics as social practice. In my research, (Baker,1999), into this question I found that understanding numeracy or mathematics as 'social' tends to be understood in strikingly different ways in educational contexts. The dominant model was a narrow view of the social in numeracy. It was expressed in terms of interactions with others or of numeracy being seen as useful or functional. I proposed a different, broader view, which saw the social in terms of ideology and discourse, power relations, values, beliefs, social relations and social institutions, (Baker, 1999). Here, 'values and beliefs' feature in choices made and in contexts in which numeracy is sited. The contexts of home and school are very different and I wish to understand the extent to which the numeracy practices sited within them are different. By 'social relations' I mean our views of ourselves and others and how we represent ourselves to each other in terms of numeracy practices. We take on various roles and identities. We therefore position ourselves as insiders or outsiders and feel either part of the community of practices or alienated from that community. 'Social institutions' and procedures are to do with issues of control, legitimacy, status and privileging of some practices over others in mathematics. This results in accepted and dominant paradigms and procedures, which remain uncontested. A narrow view which is based on an autonomous model of numeracy as described by Baker and Street, (1996) and notions of accepted pedagogy and curriculum (cf. DfEE, 1999c), leads to blaming failure or underachievement in numeracy on the teacher, the child or the home and seeing them in some sense in deficit: the teacher in terms of her subject knowledge or her use of ineffective teaching practices; the child in her lack of skills, knowledge and understandings; and the home as lacking adequate understanding of schooled numeracy to support their children (Freebody and Ludwig, 1996). Ideology underpins all these perceptions and views which are revealed in the discourses (cf. Gee) that occur around them. The broader social model, which makes the epistemological and ideological explicit (Baker and Street 1996), provides different ways of viewing and understanding underachievement and could lead to policies that go beyond access and empowerment towards transformations of curriculum and pedagogy. Instead of viewing underachievement in terms of deficit in dominant practices the model accepts social notions of difference and multiple practices and seek to represent and build upon informal numeracy practices and funds of knowledge, (Moll, 1992).

One concept that flows through the above discussion and through our work is that of numeracy practices. As has been discussed elsewhere, (Baker, 1996), and in parallel to literacy practices, (cf. Street, 1996), we see numeracy practices as more than behaviours that occur when people do maths. We propose that numeracy practices include the conceptualisations, the discourse, the values and beliefs and the social relations that surround these activities as well as the context in which they are sited. The concept of numeracy practices is grounded in the broad notion of the social in maths and is a central concept in our research. It provides a language of description and a lens through which to view practices in different contexts, and leads to an acceptance of multiple numeracies, each one framed and sited in the context in which it occurs. To complement this concept we use the more contained idea of numeracy events, in parallel to literacy events (cf. Street, 96), to denote any occasion when mathematics is used, (Baker, 1996).

We are particularly interested in relationships between home/community numeracy and schooled numeracy practices. These relationships are about ways these practices are the same or different, the boundaries between them and the ways they are viewed.   Work on the former can be seen in the work of Massingila et al (1996), Baker (1996) or Abreu (1995). It is argued that these numeracy practices are different because of the context, the values or the discourses in which they are sited. It is suggested in these articles that no set of practices is superior but that they are different. On the other hand, it is also clear that each is constructed and viewed quite differently. Schools and educational policy privilege schooled numeracy over home practices. They see the relationships between them as unequal with the role of homes subservient to that of schools and the boundaries between them clearly delineated. Homes are places where, if possible, the numeracy practices of the school are to be practised and reinforced. Homework is set by the schools and the role of the home is to assist the children's schooled numeracy activities. In an article in the Education Guardian about the National Numeracy Strategy, (NNS), Ebbutt (1999, p 2) wrote:

> "The National Numeracy Strategy promotes informing parents fully, so that mathematics at home can support mathematics in the classroom"

In a document on the National Numeracy Strategy the DfEE states:

> "An important part of the NNS is that parents are involved and well informed about their children's learning at school. Before parents can help their children effectively with mathematics, they need to understand something of how mathematics is taught in school".   (DfEE 1999a).

Formal education views homes as homogeneous with a model of the 'normal' family. Brown (1999) says in his analysis of the IMPACT programme:

> “Initiatives such as IMPACT (Merttens et al, 1990) are frequently presented as a unified programme aimed at a homogeneous group” and “a relatively homogeneous image of ‘parents’ appears to be shared by teachers”.

Tasks set for the children to do at home are based on this assumed homogeneity and on the needs of the school. The DfEE (1999b, p 20) provide sample tasks to be tackled in homes. These assume parental involvement which may not be appropriate for all homes or may even be rejected by some parents. For example, a parent at Mountford School on the pilot project, (cf. account below), when questioned about homework and home based tasks said:

> “the Government has got it wrong. Children have other things to do at home. Maybe they should stay at school for 20 min and finish it off. Then go home and play. My dad used to ask me if I had any. I would say no so I could go out and play”. (6 July 99)

An alternative view which sees the home as possible sites of rich educational resources or as ‘funds of knowledge’ (Moll, 1992), are not seriously considered. Yet this might in the long term have much to offer as a possible strategy to raise achievement in mathematics.

3 Methodological Issues

In seeking to investigate differences between schooled and home numeracies and other relationships between home and school we needed to study events in classrooms and in schools in some depth. In the light of this we based our work on detailed case studies. This threw up some key methodological issues: firstly, selection of schools and individual children; secondly, accessing home practices; and thirdly, obtaining parental agreement for opportunities to engage with the families in home numeracy practices. The development of a language of description for identifying and categorising mathematics and numeracy practices in both schooled and community contexts is an issue as well as deciding what precise practices and data the research would seek and collect. We needed to develop case studies of home practices and decide on the balance of interviewing and observing. Some of these issues are discussed below others are raised in the account of the pilot fieldwork.

The sites for the case studies have been chosen to be contrastive, 'telling' cases (Mitchell 1984). The criteria for selection enable us to cover, where possible, the main dimensions of suspected heterogeneity in the population. We selected three schools according to social features frequently cited as significant for achievement in schools. These were location (Freebody et al 1996), ethnicity (Jones, 1998), and relative affluence (Machin, 1999). One of the schools is in a mainly 'white', affluent suburb, the second has a ‘white’ socially deprived catchment area, and the third is in a mixed urban area attended by predominantly ‘black’ children. Four children will be chosen initially from one class in each of the three schools, a total of 12 children. The children will be recruited from reception classes, as nearest in social influence to the home environment, and followed through reception to years 1 and 2. The children will be selected in consultation with the teacher. To suit our contrastive methodology we will seek children with the greatest differences in home/school relationships. This may prove difficult in practice and we may use attainment as an indicator of these differences and to give us a contrast in the successful or relatively unsuccessful negotiation of boundaries and barriers between home and schooled numeracies. However, a substantial constraint on selection will be the agreement or non-agreement of parents/carers to be part of the research.

Data will include: field notes from observations of school lessons, home numeracy activities and 'community' numeracy practices and events amongst pupils from these schools; collections of work and texts used in those contexts, including official curriculum documents, course documents, 'homework', teacher feedback materials; documents regarding home/ school links; audio-recorded interviews with teachers, parents and pupils. We are also drawing upon documents on home/school relationships from the NNS. The balance of school and home visits will need to be flexible as it will build reflexively on previous visits and will depend on access to homes.

4 Pilot fieldwork.

The purpose of the research was to investigate relationships between home and schooled numeracy practices. Our concerns about conceptual and methodological issues led us to try ideas and methods in a pilot phase in order to hone our concepts and methodological principles. In particular we wanted to try and get access to homes and schools to alert us to possible strategies and problems for such visits and to think about the responses of parents and teachers to the research. In particular we wanted to refine our methods of data collection and observation in both classroom and homes in the light of such responses. What follows is a description of the pilot work we have been doing together with a vignette from the pilot to illustrate conceptual and methodological issues.

In the pilot phase in 1998/9, we worked for 6 months with children aged 4 to 5 years old in a primary school serving a 'white' socially deprived housing estate with high unemployment. We have observed and participated in classroom activities, mainly associated with numeracy but unavoidably also associated with literacy. One feature of reception classes such as this is that children are used to a number of adults being present as assistants and the researcher is inevitably drawn into participation, not just observation. We have particularly been interested to focus on classroom activities that might also be similar to those in homes e.g. playing games or using money. Following these classroom observations, our aim was to track selected children into their communities, through contacts with parents made with the help of classroom teachers. During the pilot we were able to visit a child and his mother at home after school. These sessions were arranged through the class teacher but in fact other children were keen to have us visit their homes as well. Occasions such as these have helped us explore methodological and ethical issues involved in such research. In these cases we used an open and unstructured approach to the investigations and sought issues that would shed light on the further enquiries which we plan to conduct in this and other schools over the next three years. We are developing methods that will enable us to investigate issues such as numeracy practices in the home, relationships between home and schooled numeracy, the extent to which schooled numeracy impacts on the home and vice-versa. For example, a focus on the use of games in homes and schools may provide insights into the numeracy practices of the children and to differences between those in homes and schools.

An instance of this kind occurred when I was asked by the class teacher to work with a group on literacy using an game on a number track, like 'Snakes and Ladders'. The track was in the form of a winding snake marked out in square-like sections. In each square section there was a letter. There was a starting square and a finishing square. The first child to reach this end square was the winner. The game involved the children taking turns rolling a die which had from 1 to 6 dots on each face. Players moved their piece the number of squares indicated by the die along the track, beginning from the starting square. When they landed on a square with a letter they had to find something around them beginning with the letter in the square. The next time they rolled the die, they had to start counting from where they had landed previously. The children hardly engaged with the literacy aspects of the game at all. They found the activity hard not only because the literacy skills of finding a word beginning with the identified letter were difficult for them but because they struggled with concepts contained in the game playing and numeracy aspects of the task. Game playing required acceptance and understandings of turn taking, rolling a die, the purpose of the rolling of a die, ways of relating the sign on the die to amount of movement along the track, which direction to move along the track, where to start the count from, which parts of the track counted as a unit, and an interest in or cultural affinity with winning. They had to accept that they had arrived at a square on the number track and had to stay there till their next turn. They frequently wanted to handle and play with their pieces between turns. Notions of numbers and a number track were not obvious to them either. They had problems recognising and counting the dots on the die and relating that number of dots to a number which they had to retain and use to move along the track. This suggested that the numeracy practices that the teacher wanted to draw on for this task included those associated with game playing and with number tracks. One could represent this as a lack of skills and the children as in deficit. Alternatively we could describe this as not being part of the children's cultural assumptions about numeracy practices. The teacher's unspoken assumptions about the children's familiarity or otherwise with game playing and with the numeracy aspects of the tasks may help to explain the children's difficulties or even so-called under-achievements.

The children had met similar games and number tracks in the classroom before. However, it was not clear how this related to their number practices out of school at home or in the community. We decided to investigate the use of games and number tracks at home. When we subsequently visited the home of one of the children it was clear that they had never played games like snakes and ladders that used many of these game playing, die or number track concepts. This is not to say that they did not play other games. The 4 year old child spent over an hour playing a fantasy game with a model of Godzilla, a very large two legged dinosaur-like creature that was about 40 cm tall. On the ground next to its feet were 3 cm high cars and people made to scale. This model had a wide range of numeracy practices hidden in it including, comparisons of heights, power, size and scale, language as well as fantasy and creativity. When presented with Snakes and Ladders at home he showed little interest. His numeracy practices at home were different from those at school. His practices were not necessarily less valid, less interesting or less powerful. They were simply different. However, the assumptions of schooling are that children are exposed to game and number track numeracy practices at home and that homes where children do not meet such activities are in deficit. In this case it meant that a child from this particular home was unable to engage fully in the literacy task set for him because for him home and school practices were clearly at odds with each other. That is, the boundaries between these practices were very substantial for some children and could be less so for others.

Experiences in the pilot study established some methodological practices that we would need to follow. For example we may have to engage with the children fully in their activities in the classroom, and not remain detached from these interactions. From the pilot it seemed that access to homes may remain problematic but could be achieved using both the teacher and the children as a means of introduction. However, longer periods of access over three years as is intended in the project may depend more on being seen to provide homes with aspects of formal educational value in return, (Civil, 1999). Access to schools and teachers seems less problematic. They have remained positive about the work we are doing, mainly because home/school links are currently high on their agendas, although they may request feedback relating to these agendas in return. In the case of the pilot school they have welcomed us back next year for the substantial phase of the project.

Issues about home/school relationships may revolve more around teachers' expectations and images of homes. Some teachers, in parallel with others in education, seem to have a deficit model of homes. They see some homes as unable to contribute significantly or effectively to their children's education and do not see possibilities for building on 'funds of knowledge' from the home communities. Teachers seem to have conflicting expectations of homes. On the one hand they have this deficit model of homes and see homes particularly from low socio-economic backgrounds as unable to support the kinds of numeracy practices they would welcome in their classes. On the other hand, they assume that children's numeracy practices outside the school are the same as of those in formal schooling so that games like snakes and ladders can be used without further explanation. Alternatively, some reject home practices altogether as in the deficit model (cf. Rowland, 1999), either irrelevant or non-existent and do not allow them to influence their curriculum or pedagogy. In a parallel way, parents/homes and community expectations of schooling are that children will learn formal numeracy practices at school and that children's informal or non-schooled numeracy practices will not be of value in the classroom. The differences between these views of what counts as numeracy and the practices associated with it may go someway to explaining difficulties some children have in moving from home to school. The pilot confirmed and extended our concepts of numeracy practices as contextually defined and framed and our belief that there are many different numeracy practices. It also confirmed our models that relationships between homes and schooling are asymmetric with schooled numeracy having a high status and home practices being marginalised.

5 Implications

When researching social factors and classroom practices there are strong expectations and desires for answers to practical classroom issues. This can result in the neglect of research and theoretical issues that may be of value and interest. At this stage in our research we do yet think we have answers, we are still investigating the situation. However, where there are some initial implications we see them as relating to teaching; research and policy.

For researchers there are different ways of seeing achievement and underachievement in numeracy. From one perspective there is a deficit model where children and homes need to be enabled by teachers to learn dominant numeracy skills. A different social perspective provides understanding of social aspects of numeracies and multiple numeracy practices, the importance of context, social relations and ideology, and social factors in schooled and home/ community numeracies. Instead of the dominance of deficit and hierarchical models of numeracy practices this perspective proposes notions of difference, multiple practices and of funds of knowledge.

In terms of detailed teaching, pedagogical and curricular implications of a broad view of the social in maths remain a complex question. Our reading of research literature suggests that a move to acceptance of maths as social does not necessarily result in changes in ways of teaching maths. There is not a 1-1 causal relationship between epistemological models and pedagogy. We are, however, suggesting, that broader views of the social in maths can lead to greater understandings of classroom interactions and though such understandings to changes in classroom practices. What our data do tell us is that relationships between home and school are complex and the extension of schooled numeracy into homes through homework or parent evenings, though encouraged in official policy statements, may be problematic. Instead a commitment to making use of the funds of knowledge in homes, acceptance of the value of home numeracy practices, investing resources and energy into identifying and understanding such funds of knowledge, in and out of school experiences, may have more to offer curricula teachers and schools than has previously been accepted.

Official government concerns about the access to powerful knowledge such as schooled numeracy, particularly for children from educationally disadvantaged homes, may have to be challenged and replaced by transformations in curriculum and pedagogy rather than only in homes. One implication of viewing mathematics in general and numeracy in particular in this way may be that *both* home and schooling have to change if we are to have any substantive and long lasting affect on achievements in schooled numeracy.

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Neurone-Z, Philosophy of the Mind and Symptom

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ABSTRACT

Attempts to locate mathematics inside the human brain have fascinated certain educators. Which neurone connections are responsible for the mathematical operations and abstractions? Once the gene that assures the formation of such connections is located, many teaching problems may be looked at under a new light. Foundations of teaching methods and academic control of research on mathematics education may be justified. The paper shows why such a scientific discovery and the theory it supports are anxiously awaited and seized as definitive by many of its readers.

Neurones, genes and mathematics

In spite of almost a century of efforts dedicated to mathematics instruction, the following question has not as yet received a satisfactory answer: why some like and learn mathematics while others, the majority, hate it, venerate it as difficult and apparently never manage to learn it? What form of intelligence is this, "mathematics"? How is this intelligence distributed in the population and is transmitted from one generation to another?

For Hypocrate (460-379 AD) intelligence resided in the brain, for Aristotle (335 AD) in the heart. In recent decades, due to techniques of computerised tomography and magnetic resonance imaging, we have witnessed an explosion in the production of knowledge about the working of the human brain.[2]

> "The challenge of cognitive neuroscience is to describe the relationship between the brain and the mind, *i. e.*, to reveal how structural neural elements are driven into the psychological activity that results in perception and cognition. (...) The neural correlates of higher mental functions, such as language or mathematical reasoning need to be sought directly in awake humans" [Levänen, 1997:19].

Following this approach the eminent British mathematics educator David Tall describes a connectionist model of the brain[3] and concludes that:

> "(...) the broad action-process-object-scheme (APOS) has a natural biological underpinning. (...) APOS theory seems to have a deep underpinning in biological structure. (...) In counting there is the action of repeating the number words and beginning to accompany this by pointing at objects in turn. Later various learning sequences set up neuronal connections in the brain, routinizing the procedure, seeing it as a process when it is realized that different orders of counting the same set give the same number, and then "encapsulating" the process into the concept of number" [Tall, 1999:112, 114].

The greater or lesser facility to learn mathematics can then be explained by the greater or lesser readiness of the brain to establish neuronal connections. However, in order to explain the distribution and transmission of such an ability among the population, it is necessary to invoke another science: genetics. It had its basis launched by Darwin who published *The Origin of Species* in 1859. It was founded by Mandel in 1866 as a sub-field of biological knowledge. Two years before, Herbert Spencer (1820-1903) had reformulated Darwin's concept of *natural selection* and invented the expression *survival of the fittest.* Such an apparently innocent shift allowed him

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[2] See, for instance, Milestones in Neuroscience Research, *http://faculty.washington.edu/chudler/hist.html*

[3] "The human brain, at least when healthy and mature, is a system of neuronal networks that are both interconnected among themselves (hence the name 'Connectionism') and conjoined to the sensory input systems and the behavioral cum reactive output systems" [Lyons, 1995:lxii].

to extend Darwin's theory to the social domain, inaugurating what is now called *Social Darwinism*. This theory asserts the hereditary transmission of social qualities; in one word, the fittest become richer. In 1883, Francis Galton (1822-1911) moved one step forward: he introduced the word "eugenics" and suggested the improving the human race by selected breading. Eugenics would certainly include the higher mental processes such as mathematical ability among its aims. [4]

Chromosomes were discovered by Walter Flemming in 1882; the genes that determine the individual characters were spotted in the chromosomes by Thomas Morgan in 1910; DNA was discovered as the hereditary material in 1944 by Oswald Averey and Maclyn McCarty; and the genetic code was deciphered by Francois Jacob and Jacques Monod em 1965 [5]. Since then, genetics has thrived. The Human Genome Project, started in 1990, enrols more than five thousand scientists from the US, Europe and Japan and aims at identifying and determining the location in the chromosomes of all human genes. Such knowledge will have an enormous impact on the understanding of physiological processes of the human organism, on health and the cure of diseases, and hence on medicine in general.[6] Among its effects, The Human Genome Project retrieves the hopes of social Darwinism.

"A biologist from Princeton (...) Joe Tsien has published the results of his research (on mice) in the periodical Nature, August 2, (1999). The research has proved that the gene NR2B is fundamental in the control of brain ability to associate one event to another, a basic learning skill. (...) It is known that humans possess a corresponding gene but its effect on intelligence has not yet been determined. (...) The discovery opens the way to future use of genetic manipulation in the treatment of human beings" [Moon, 1999, 56].

On the other hand, "the American neurologist Steven Pinker asserts that language is 'congenial to humans as the trunk to the elephant' and that therefore there must be a 'grammatical gene' " [Kurz, 1997: 196]. Then it becomes natural to inquire about the existence of a corresponding mathematical gene.

The neurone-Z theory

Recently the scientist S. Zanati, of the International Center of Brain Injury (ICBI) succeeded in determining the gene responsible for the development of neurone-Z in human beings. As is well known, the Neurone-Z is in fact a group of neurones that are activated whenever a mathematical kind of operation is performed. The neurone-Z has been named after the Planet–X, a never-observed hypothetical planet whose existence is deduced from certain abnormalities in the orbits of Uranus, Neptune and Pluto. The existence of neurone-Z had been postulated by neuro-educators of ICBI in order to explain the difference between people who know and love mathematics and those who do not know and seem to never be able to learn it. Neurone-Z would be responsible for the elementary local operations, such as the question "*Who is the father of John's son?*" whose answer is so simple for some and so difficult, almost impossible for others. Neurone-Z would chiefly be responsible for the operational synthesis described by Piaget and re-stated by the APOS theory as the "encapsulation" of processes as objects. Instances of such an encapsulation are: synthesising the numerator and denominator to form the concept of fraction, synthesising the three quantities involved in the concept of percentage, synthesising the operators of multiplication and sign-change to form the concept of integers, synthesising the independent and depend variables to form the concept of function, synthesising direct and inverse operations such as differentiation and anti-differentiation, etc.

Since the discovery of gene-Z, located in chromosome 17, responsible for the development of neurone-Z, it has been possible do determine that only 10 to 15 per cent of men and 5 to 8 per cent of women are neurone-Z carriers. The gender distribution of neurone-Z in the population also explains why the majority of mathematicians are men. Studies are being carried out in the US to determine the percentage of carriers among the African American and Chicano minorities. It is

[4] I am indebted to Pedro Garcia Duarte for remarks on this paragraph.

[5] See, for instance, Genome Research Project. http://dev99.advanced.org/28920/

[6] See, for instance *Programa Educacional em Multimídia na Internet*, Escola Paulista de Medicina ,Universidade Federal De São Paulo, *http://www.epm.br/ge/capa.htm*

suspected that it is considerably smaller than among the white population. In fact "the American scientists Richard Herrnstein and Charles Murray, in the study called *The Bell Curve*, had already created a correlation between race, genes, and intelligence coefficient that excluded American Negroes from the cognitive elite" [Kurz, 1997: 196]. From the teaching perspective, individuals who are not neurone-Z carriers have generally been designated names such as "low-achievers", "risk students", students with "special difficulties" and of "average intelligence".

"Mathematics teaching in elementary school (...) should be conceived for students who, in great majority, will not go beyond high school. (...) It should be construed and executed aiming at students of current or average intelligence, with no special gift for mathematics. Ability for mathematics is like ability for music: both can be naturally found in a minority of students, both can be stimulated to a certain extent in a small percentage of students, but at elementary school level, students in evident majority, do not have learning ability neither for mathematics nor for music" [Nachbin, 1981:18].

The discovery of neurone–Z provides an immediate explanation for why so many people find mathematics difficult and feel aversion towards it. This is due to the great effort that non-carriers have to make in order to produce adequate answers using non-specialised alternative neurones. These students have to develop rote methods and blind-rule searching. *May I cut above and below? Only if the sign is times. If it is plus or minus I may not*. A non-carrier child will face difficulties operating with ordinary fractions and will tend to replace them by decimal fractions that can be performed on calculating machines, for which only non-specialised neurones suffice. In college, non carriers will face great difficulties in anti-differentiation and will tend to rely on tables of integrals and systematically use integration by parts even when this method obviously fails, because it is a routine kind of operation that can be carried out by alternative neurones. The absence of neurone-Z makes it almost impossible to form the concept of function, so that, when it comes to applying differential calculus to solve extreme-value problems, students experience a discontinuity in the level of difficulty of the calculus course: to find the function to be differentiated is almost impossible for non-carriers. Also in the definition of the limit of a sequence, for instance, in absence of neurone-Z the student gets stuck at the first part (*for every epsilon there is an N*) obsessed with this N without connecting it with the property that it is supposed to satisfy (*for every n>N the absolute value of the difference is less than epsilon*).

The need to memorise more and more rules ends up producing dissatisfaction and finally aversion. It has been noted that this difficulty is much greater than usually assumed: "In general, encapsulating processes to become objects is considered to be extremely difficult (...) and not very many pedagogical strategies have been effective in helping students do this in situations such as functions and cosets" [Dubinski et all, 1999:100]. The reason for this is now clear: the difficulty is genetically rooted.

However, from the point of view of education, the most noticeable consequence is that current traditional teaching methods can now be justified, both from cognitive and political perspectives. Here is a characterisation of such methods.

"Classical instructional approach is characterized by curricula that is taught directly, systematically, and incrementally in small structured and guided steps that progress from basic to more complex learning; instruction focused on specific academic content (not process or outcomes); repetition, practice, and memorization used to derive automaticity; and students receive immediate feedback and correction" [7].

From the cognitive point of view, this method is justified by the argument that the most talented students (i.e. neurone-Z carriers) will be able to form operational synthesis among the several partial and local acquisitions and will emerge as those who are mathematically gifted. The political justification rests on the argument that even those who are not carriers will have the opportunity and the time to learn how to use their alternative neurones in order to produce correct answers by rote learning and *ad hoc* rules. These will take advantage of an apparently high standard curriculum to increase the sing-value of their certificates. From the NZ-theory, traditional teaching can now be justified as a genetically based social agreement.

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[7] *Texas Public Foundation's Policy Action Update*, volume 3, nº. 12, May, 1999.

The teacher should stand in front of the class, explain the subject matter on an overhead or blackboard in such a way that carriers will understand immediately. Matrix disposition of children in the room will assure equal opportunity to all. Children must stay quiet and silent because the establishment of neuronal connections require serenity of the body. Copybooks and spreadsheets must receive blue marks so that parents will not be upset by the eventual absence of neurone-Z in their children. Grades should be limited to results, so as not to impose impossible methods on non-carriers. This model should be reinforced by the media whenever a classroom is shown on TV.

Insofar as traditional teaching assumes a scientific character, just like any science, it indicates its ideological past; that is, the dispute that it came to settle. Such a past is formed by the fight against alternative teaching methodologies such as “new math”, “group work”, “whole math” and the like, generally called “fuzzy math”. Supporters of such methods did not believe in the existence of neurone-Z. They assumed that mathematics was not the specific function of a group of neurones, so they assumed that it could be learned equally by all, as long as they went through adequate controlled experiences. Constructivism, stemming from Piaget's experiences was, perhaps, the most persistent of such ideologies. The NCTM Standards are their last offspring. Here is a nice summary of such methods.

“In 'standards-based' math programs students direct their own learning; work in groups to teach one another; construct their own math language, facts, and computations; are not taught or required to memorize facts or formulas; are taught to use calculators as the first and primary form of computation; and, are taught that deriving correct solutions lacks importance” [ibid.].

In such teaching strategies and curricular directives, only neurone-Z carriers have a chance of responding and finding their way through it towards knowledge. The discovery of neurone -Z reveals the extreme cruelty of imposing new methods on non-carrier children. “Fuzzy math has been shown to hurt children academically, specially disadvantaged and minority students” [ibid.]. In such methods these students feel completely at loss. During group work, for instance, the operation of suspending one’s own point of view in order to explain the reasoning of somebody else, the simple act of listening to somebody else’s opinion or entering into effective dialogue is almost impossible for a non-carrier. Therefore the parents, among whom the percentage of neurone-Z carriers is as low as among the population in general, rightly react against teaching reform attempts. They demand routine exercises that children can do by repeating until they assimilate it by drilling. They refuse to accept that a method of work or thinking may be imposed on their children and demand that only the final answers of the exercises be graded. The child who worked with decimals and instead of 49/20 has found 2,45 must score because “it is correct”. Parents know very well that operating with ordinary fractions is a tremendous task in terms of non-specialised neurones.

Current school systems can now be justified also as adequate social contracts specially designed to take into account a heterogeneous distribution of neurone-Z among the population. Strict pass/fail rules based on precise testing should be enforced, so that the ghost of failure will press carriers to work hard and do their best. However, what has now been proved, has always been implicitly admitted, namely that the majority of the students will not succeed no matter how much effort they make, because they do not have a neurone-Z. Therefore the school system has carefully developed a way to give a chance to non-carriers. “What to do for low achieving students? (...) Provide these children extra assistance , summer school if need be (...) and then promote” [Bracey, 1999]. Via the subsidiary promotional criteria, the school apparatus can cope with the reality of an heterogeneous population: strict pass/fail rules make up the façade but a scheme of social promotion should be set up in the back yard. The school system has rightly developed itself into a hierarchical elitist system well suited to the upper classes (carriers) as well as an equal-opportunity democratic system suited to the lower classes (non-carriers).

The discovery of neurone-Z sanctions this social contract: the class split in school can now be genetically justified. Up to now, such an arrangement between social classes could not be fully revealed. The interests of non carriers could not be stated overtly. “Social promotion”, meaning the practice of promoting a youth based on age and “sitting time” rather than on acquired skills, offers a too explicit support of non carriers’ interests. It risks revealing the whole plot. So U.S. federal funds have been targeted to schools doing away with this practice. Interests of non-carriers for promotion had to be disguised as an opposition to “retention” defined as the use of failure as a pedagogical technique. Retention had to be condemned either emotionally as a “disaster”, as having “significant negative emotional outcomes”, or economically as “the increased cost of retaining lots of children”, or cognitively as having an “impact on education achieving” [Bracey, 1999]. With the discovery of neurone-Z, retention can now be bluntly condemned on the basis that it simply breaks the social contract, depriving non-carriers of the accorded chance they must have in the school credit system.

The enforcement of the school social agreement has already been submitted to court. “The lawsuit (...) alleges that Tempo high school did not properly teach Jonathan Govias chemistry, math and physics (...) Without the necessary knowledge he expected to have obtained in high school he had to withdraw from the (engineering) course” [8]. The lawsuit began in 1995 and had not come to trial in July 1999. The discovery of neurone-Z inverts the cases. A simple DNA exam will reveal if the student is a carrier or not. If not, the law suit may be directed against the engineering school, holding it responsible for failing to provide Jonathan with opportunities of rote learning strategies through which he succeeded in elementary and high school. The engineering school will have violated the social pact.

The neurone-Z theory also explains other forms of student behaviour. When facing a different point of view, non-carriers, unable to argue back, tend to see opposition not only to his/her ideas but also to his/her self personally. Violence is then a natural reaction. Thoughts of non-carriers about political issues tends to be oversimplified: if the Berlin Wall fell it is because capitalism is better; I voted for the Right because the Left would be worse; if guns kill, forbid guns; if ethics is missing in schools, introduce classes about ethics.

However, on one point the Neurone-Z theory has not brought comfort to the American population. Recent international comparative studies[9] have shown that mathematical abilities of children in the U.S. are situated just below the international average, behind the abilities of some Asian and Eastern European nations, including Japan. Independent studies show similar U.S. deficiencies with respect to China [Ma, 1999] and Germany [Stigler & Hiebert, 1999]. In the face of such results, educators who did not believe in the existence of Neurone-Z suggested and tried to implement reforms in curriculum and teaching methods. Those who never lost faith that the Neurone-Z would some day be discovered rejected such changes and stressed the need to reinforce current traditional teaching methods. The Japanese lead has been most stirring in U.S. While neurone-Z believers refer to the innovations as “fuzzy math”, reformers refer to traditional teaching as “parrot math”. The debate is known as the “math war” [O’Brien, 1999]. With the discovery of neurone-Z, the matter can finally be settled. If the percentage of neurone-Z carriers among the population in China and Japan is in fact greater than among the American White population, Chinese and Japanese will forever perform better, unless inter-racial marriages are promoted.

[8] The National Post (Canada) Wednesday, July 14, 1999.

[9] For instance, the Third International Matheamtics and Science Study (TIMSS) tested half a million students from 41 nations during 1995, sponsored by the International Association for the Evaluation of Educational Achievement (IEA), http://ustimss.msu.edu/

A decisive test

The reader is probably anxious to know whether s/he is an NZ-carrier as well as her/his offspring. So here is a decisive test. If you believed this story about neuron-Z then you are certainly not an NZ-carrier. The international Centre of Brain Injury never existed, S. Zanati is an anagram of a well known ideology, and 17 is my phone extension number (at ICBI, of course).

The presentation of neurone-Z theory has raised considerable excitement in the audiences that have been introduced to it [10]. Some reacted with anger, others showed some sympathy. People who have read the draft of the paper tended to ignore the above paragraph and attribute the theory's authorship to me. One colleague with some knowledge in neuroscience told me that I was not sufficiently qualified in neuroscience to produce such an involved theory. Another one complained that the theory was flawed: "*How can a group of neurones be 'absent'? These guys are nuts.*" Nevertheless, NZ-theory proved itself highly credible. Why? "Two things contribute in order to really deceive the reader: the text composition is very "official" and the fact that the whole presentation is enclosed by bibliographical references that, in fact, do not exactly refer to the theory but whose frequency in the text is sufficient for the reader to process them as if they were so" [11]

However, there should be something beyond the mere form of the composition causing the commotion. We can invoke at least three arguments. The formulation of NZ-theory presented here, corresponds to one of the three ways of facing a myth [Barthes, 1985]. We may receive the impact of the myth, acting as if NZ existed as most people do, or we may attempt to avoid the myth's effect. In this case, we may either try to decipher the myth, arguing against the possibility of a neuronal solution to the mind-body problem, or we may proceed as we did here, taking the myth "seriously" and obliging it to confess itself plainly. From another point of view, we can say that the hypothetical theory completes an ideological field by exhibiting its *central subject* [Althusser, 1976], a group of neurones in the name of which long established intellectual vertical (Father/son) and horizontal (brotherhood) social relations can be justified. The process of social exclusion can then be justified as a process of natural selection through school with the survival of the fittest. The theory "identifies, in the ideological building, the element that represents its own impossibility" [Zizek, 1990:158]. Still from another point of view we can say that NZ-theory realizes what Hegel calls the *dialectical moment* of the concept development. It means the self-suppression of finite determinations characterising traditional teaching and their passage into their opposites [Hegel,1994:343]. However, two questions remain:

1. Why so many people take the NZ-theory for granted and some cheer for it?
2. Why, in spite of knowing it is a fake and has fascist motives, people continue to behave as if they believed it?

1) In fact, NZ-theory is just a crude formulation of the general old mind-body problem [Lyons, 1995 [12]]. For a long time, philosophers and scientists have been looking for the psychophysical connection that would explain how the brain produces consciousness. Up to now, the solution is dismaying: the mind-body problem is a *non-mysterious mystery*: "I would like to suggest that the nature of the psychophysical connection has a full and non-mysterious explanation in a certain science, but that this science is inaccessible to us as a matter of principle" [Mcginn, 19095:284].

So, after almost three hundred pages, philosophers profess faith in a future inaccessible science, a mystery that would dissipate the mystery. However they do not abandon their belief in the physical objectivity of this pair of abstractions: "mind" and "body". "Everything physical has a purely physical explanation" [ibid, 282]. New light may be shed on the problem if we take up a Marx-Lacan-Zizek line of reasoning about how abstractions are formed. "As a rule, the most general

[10] VI ERMAC, Departamento de Matemática da Universidade Estadual de Londrina, PR, September 1st. 1999, I Encontro Brasiliense de Educação Matemática, Brasília DF, September 19, 1999, XIX Semana da Matemática, UNESP, Rio Claro, October 8, 1999.
[11] From a reaction to a draft of this paper by Eduardo N. Baldino, e-mail in October 13, 1999.
[12] This book was indicated to me by Romulo Lins after the reading the first draft of this paper.

abstraction arises only in the midst of the richest possible concrete development, where one thing appears as common to many, to all" [Marx, 1973:104]. Marx was studying the problem of commodities, one of which is money. Whenever two owners meet in the market in order to exchange their commodities, a *real abstraction* takes place. Their commodities are reduced to their exchange-values regardless of their empirical particular shapes (use-values). This real abstraction is taken as the departure point of an epistemological reflection:

"Before thought could arrive at the concept of a purely *quantitative* determination, the *sine qua non* of modern science of nature, the pure quantity was already in action in money, this commodity that makes it possible to measure the value of all others, whatever their particular qualitative determination be. (...) The repressed *social* dimension of his act [of exchange] emerges next under the form of its contrary, the universal reason focused on the observation of *nature* (the net of categories of 'pure reason' as the conceptual framework of natural sciences). (...) *'Real abstraction' is the unconscious of the [Kantian] transcendental subject*, the support of the objective-universal scientific knowledge. (...)" [Zizek, 1988:171].

This author quotes Alfred Sohn-Rethel who stated the effect of real abstractions on the way of thinking in still sharper words: "The abstraction of exchange is not thought itself, but it has the *form* of thought" [Rethel, 1970 in Zizek, 1988:137]. This is a possible definition of the unconscious as a form of thought exterior and irreducible to the content of thought. The unconscious is "Another Scene, exterior to thought, in which the form of thought is articulated beforehand" [Zizek, 1988:174].

We may then understand why, once the abstractions "mind" or "consciousness" and "body" are formed, people cannot avoid thinking of them in terms of cause and effect. Their (unconscious) form of thinking has already prepared the answers for which they make the questions. "Every human behaviour is reducible to the function of neuronal nets. It is obvious that the whole human being is inside the skull-box, otherwise we would be forced to believe in God. And what is inside the skull box can be explained because it is a finite amount of matter" [13] [note 13]. This thinking only needs questions to complete it, because the answers are already determined by its form. It is clear, then, how NZ-theory came to fill a need: it provided the right questions for previously established answers: *what is the percentage of carriers among the population? How to identify talented students? How to keep America beautiful?* A colleague who was enthusiastic about the theory showed some distress when he realised that it was a fake. *How else could we explain individual differences?* he complained. Yes, given a certain *form of thought*, unconsciously established from *real abstractions*, there is, in fact, no other explanation.

2) The persistence of the faith in NZ-theory even after the machination has been revealed, relates to the persistence of the mind-body problem. How are these abstractions, "mind" and "body", formed and why do people insist on their objectivity? In order to explain this, we have to take a turn through money. Money has a physical, tangible body formerly expressed in gold coins, then in paper bills and now-days in plastic cards. Washed out coins, torn bills and broken cards do not lose their money value. Money has a sublime indestructible material body granted by some *symbolic authority*, generally a "bank". Money has a *bodiless body*. This sort of abstraction is a consequence of everyday real exchange. It is easily made by the most simple and illiterate of all creatures.

"In a certain relation, the same happens with man. Since he does not come to the world with a mirror as the Fichtean philosopher, I - Myself, in principle he only recognises himself in another man. It is only in his relation with the man Paul, as a man equal to him, that the man Peter relates to himself as a man. This is why it is Paul, with his skin and hair, in his Paul's body, that is worth Peter as the form of appearance of the male gender" [Marx, 1976:586].

It is in front of a mirror that one day Paul pointed to Peter's image and told him: *<u>That</u> <u>one</u> <u>is</u> <u>you</u>. That, is the one whom you should love, because that is the one I love*. At this moment a double identification occurred. On the one hand, Peter identified himself with the image in the mirror as how he should look in order to be loved by Paul. This is the imaginary *identification*, in the sense that this word has on "identity card" for instance. On the other hand, Peter identified himself with Paul's point of view: *I have to look at myself the way he does because it is from his*

[13] Idem note 11.

stand point that I deserve love. This is the symbolic or ideological identification, in the sense of "identify with". The symbolic identification is the builder of the unconscious. Peter has to believe that he is developing his own opinions, not copying Paul's.

Peter attempts to imitate something about Paul that is impossible to imitate: his thinking. He also tries to keep his image distinct from Paul's so as to build his own identity. The effort to copy Paul's *mind* and distinguish his from Paul's *body* is what constitutes Peter as a human subject integrated in a social-symbolic field. Then mind and body are assumed to be objectively "*common to many, to all*". The complete Cartesian formula of the interplay between the two identifications would be: *I think* (like you), *hence I am* ((in love with) that one in the mirror). Minds are equalised insofar as bodies are differentiated.

It is not the mind-body relation that has to be explained. What has to be explained is how this relation became a problem. How is it possible to ignore the different origins and developments of these two abstractions? How is it possible to throw both inside the mirror as part of the objective world? The answer is: by work of the unconscious. "This combined game between the imaginary and symbolic identifications, under the domination of the symbolic identification, constitutes the mechanism through which the subject is integrated in a given social-symbolic field" [Zizek, 1990:138]. The social-symbolic field lies entirely inside the mirror, the place where Peter is equal to Paul, where both can speak objectively about each other, about the world and nature. It is the place of science, of *statements* where, according to Sohn-Rethel [1970], thought already has a *form*, determined by the abstraction of exchange. The authors of scientific statements dwell on this side, outside the mirror, the place of *enunciation*, they dwell in the "Other Scene", the scene ignored by the objective scientific statements. "The Freudian concept of unconscious is (...) of an *entity whose existence implies a certain not-to-know*. (...) Its ontological consistency relies from end to end on a certain unknowing" [Zizek, 1988:176]. It is refusal to know about the symbolic authority that presided over the two identifications, the authority who says "*hence*", a mark that precedes the "I think" and the "I am"; it is a refusal to know about "la zone d'un savoir létal" [ibid.] that sustains the objectivity of the mind-body problem.

Classical political economy focused on money, unable to decipher where its "intrinsic" properties come from. They were first attributed to physical properties of gold. Modern philosophy of mind focuses on the mind-body relation only to conclude that it is a mystery to be deciphered by an inaccessible science. Mind is to body just as the bodiless character of money is to coins. This abstraction shapes people's (unconscious) thinking in the modern capitalistic society. It is from this perspective that NZ-theory derives its fascination effect. Whatever cannot be put into such a form, cannot be stated; whatever can be stated into such a form, fulfils it and becomes permanent, even if it is a fake.

The mystery is expressed in the form of thought assuming that consciousness is first with respect to language: "Surely language and the propositional attitudes are more complex and advanced evolutionary achievements than the mere possession of consciousness by a physical organism" [Mcginn, 1995:287]. This form of thought assumes that there is a direct route to mind, independent of language. The problem then consists of filling this form of thought with adequate questions so that the discourse can go on, repeating itself in several stages. What is Rodin's *Le Penseur* doing? What is it like to be a bat? Are we made of matter or soul-stuff? [Lyons, 1995: LIII;159;133]. In the end, a mystery...

Such a persistence in focusing on an abstract problem as if it were an objective one, evident in the debate about the philosophy of mind, is reproduced in small scale with the NZ-theory. In order to explain this persistence, we have to resort to the concept of *symptom*.

"The 'symptom' in the strict sense is this particular element that denies the Universal of which it is a part. Instead of functioning as an 'insufficient realisation' of this universality and being a remainder to be abolished by its ulterior radicalisation, (the symptom) functions rather as a constitutive moment of it" [Zizek, 1988:176].

NZ-theory sets up an ideological realm through some promises of happiness. Difficulties in mathematics can be explained, some people will be relieved from the suffering of being submitted to certain methods, the social pact can be sustained, and so on. Then, on a second level, some disturbing consequences emerge. DNA tests would replace exams, genetic manipulation would replace efforts to learn. Then, on a third level, the stage is open to social Darwinism, eugenics, controlled breeding, race improvement. Finally, what seemed a remainder, an excess that could be avoided by an adequate conduction of the process, reveals itself as the convergence point, the attractor that is actually constitutive of the whole story: a shrilling cry in unison: *Heil*. This was the real point of attraction for all readers, the empty spot necessary for the hypnotic fascination effect.

It is useless to interpret the symptom in terms of statements because the unconscious that supports the symptom is structured as the language of enunciation, the language of the Other Scene, precisely the scene that the form of thought refuses to incorporate. One reader explicitly recognised that he chose not-to-know. "The theory is so credible that it generates a very emotional response, which the paragraph on the decisive test is not sufficient to dismantle. The reader chooses to file this paragraph in the 'not understood, think of it later' folder " [14].

Interpreting a symptom does not help a psychoanalytical patient, just as explanations do not help mathematics students. The subject has to elaborate the meaning of the symptoms and deal with his/her *refusal to know* [Cabral, 1997]. Why is this so difficult?

"Why, in spite of its interpretation the symptom does not disappear? Why does it persevere? Lacan's answer is naturally *jouissance* [15]. The symptom is not just a ciphered message, it is also a way for the subject to organise his *jouissance* – this is why even after a complete interpretation, the subject is not ready to renounce his/her symptom" [Zizek, 1990:209].

It is certainly not due to lack of NZ !

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[15] We follow the tradition and use the French word in italics since the corresponding English word "enjoyance" is not in the dictionary.

[16] We are indebted to **Jerry Becker** for many references that he has been kindly furnishing us through his circular e-mails.

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All change, no change:
shifting discourses in the official rhetoric of primary school mathematics.

Abstract

This paper explores some of the critical tensions within the discourses of mathematics education in the primary setting in England. These tensions, I suggest, are always present and have never been fully resolved although at key moments there may have been partial resolutions in key texts. These partial resolutions are exemplified by the manner in which primary teachers and the mathematics they are expected to teach are construed in these key texts. I argue that particular discursive formations may, at different historical moments, be spotlit and privileged through rehearsal and reformation or be darkened, hidden and ignored or derided. Whether spotlit or darkened, however, the discourses are never completely silenced.

In England current government rhetoric is placing blame for the perceived problems in primary education upon the reforms it identifies with The Plowden Report (1967) and its subsequent influence on teacher training. Both rhetoric and policy seek to expunge the language of Plowden from schools and to re-cast teaching as a technical craft. These moves have been criticised for decontextualising and deproblematising teachers and teaching. This paper examines the roots of some of the ideas and vocabularies which are currently being branded with the unfashionable 'Plowden' label and asks whether such discourses might not have a longer pedigree.

Before I attempt to unpack some of these critical tensions within the discourses of primary mathematics teaching it is important to say that this paper is conjectural and incomplete. What I want to attempt is an exploration of the tensions that exist historically and which continue to be played out in the discourses of primary mathematics education and the lives of the teachers who toil in that arena. I use discourse here in a broadly Foucaudian sense to mean historically located versions of power/knowledge that become established as 'common sense'; such discourses are seen as historical artefacts (see for example Ball, 1990:2). I also make some use of linguistic analysis (Fairclough, 1995). These two analyses (the Foucauldian and linguistic) are used in conjunction to strengthen each other, *the concern is with the task rather than theoretical purism or conceptual niceties* (Ball, 1994:2).

The key texts that have been selected for this exploration are the 1931 Hadow Report, the 1979 HMI (Her Majesty's Inspectorate) publication 'Mathematics 5-11: A handbook of suggestions', The Cockcroft Report (1982), and a brace of reports that go together to make up the work of the Numeracy Task Force (NTF) (1998a, 1998b) the committee which launched the National Numeracy Strategy (NNS) on the back of the National Numeracy Project (NNP)[1]. An important reason for selecting these texts is the fact that the reports were written by committee; as such they represent, not just one person's views but a mosaic of a range of views held at the time of writing. This implies not only that the multiple discourses that were current at the time have a good chance of some representation within the texts; but also that the ways in which these are privileged within the texts will be a reflection of the zeitgeist. By taking each report in turn I hope to develop a story which charts the different discourses as they ebb and flow with the varying economic fortunes and value shifts that shape the changing political map. More detailed discussion of the socio-historical contexts within which they are sited will have to wait for another, wider discussion (see for example Lawton, 1994).

The Hadow Report (1931) has been chosen as a starting point as it encapsulates the recurrent contradictions that have dogged the teaching of primary school maths down the years. Part of this tension can be seen as residing in varying conceptions of what constitutes good or effective pedagogic and didactic practices and whether these can be consistent across all areas of the curriculum; or whether mathematics requires a different treatment. Another tension, which interacts strongly with the first, pertains to how mathematics should be taught; this depends largely on how the epistemology of mathematics is understood. One alternative is to hold that maths exists as a body of knowledge 'out there' which can be 'transmitted' in some way to learners (a product or absolutist view); the other extreme holds that it is something that is constructed by the child - either through their 'discovery' or in joint construction (a process or constructivist view) (Ernest, 1991).

With regard to teaching generally the Hadow report presages Plowden, portraying children as active learners:

> It has been maintained that teaching by subjects is a mode of instruction which, though it may be appropriate for older boys and girls, who have themselves

[1] A brief description of the purposes and context for each of these reports is given in Appendix A

> developed specialised interests, and who are ready to follow the major intellectual pursuits of mankind (sic) along the lines of their logical development, does not always correspond with the child's unsystematised but eager interest in the people and things of a world still new to him. We agree with this view, and we think that what is needed therefore, is a new orientation of school instruction which shall bring it into closer correlation with the natural movements of children's minds. (p101)

Young children are portrayed as having 'natural' ways of thinking that are spontaneous and unconfined by traditional subject boundaries. While there remain in Hadow the strong moral and 'civilising' overtones that will be written out in Plowden this doesn't appear to dilute the strength of the encouragement towards an integrated curriculum (topic work) however contradictory this may now seem.

Within this movement towards topic work Hadow expresses a general desire to broaden the mathematics curriculum:

> There is a general agreement among our witnesses that too much time is given to arithmetic in primary schools, ... At present it is usual for about one-fifth of the total time-table to be allotted to arithmetic, while the subject matter is confined almost exclusively to purely arithmetical topics, and rarely includes any geometry apart from a little mensuration. (p175)

However, in a more detailed section dedicated to the mathematics to be taught, the discourses relating to children's 'natural' ways of working and a broader curriculum are overwhelmed by more 'traditional' discourses about mathematics and mathematics teaching:

> The aim in the primary school is to secure ability and readiness in using the processes, and this can only be attained by much practice of them, both oral and written. (p176)

The reader's attention is engaged directly through the inclusive use of the active voice. The task of the teacher is to '*secure a thorough mastery of these basic operations without devoting too much time to them, and without creating a distaste for the subject*'. There is no suggestion that topic work might be useful here, rather the emphasis is on rote learning; the need to use knowledge to work out number facts is judged to be *superfluous ... mental mechanisation*.

> It is essential that these fundamental processes of arithmetic shall become automatic before the child leaves the primary school. Unless he can add,

> subtract, multiply and divide accurately, quickly, and without hesitation, his future progress will be severely handicapped. (p176)

The focus is clearly on the need to acquire skills - divorced from understanding - and an authoritarian/ transmission teaching style. The tension between the child as active and interested with attendant 'progressive' pedagogies and the need to know and learn with attendant traditional transmissive pedagogies is unacknowledged but firmly entrenched at all levels in Hadow. We will return to this last extract later - it presages the more recent words of David Blunkett. The mathematics curriculum here retains its characteristic as a distinctive body of knowledge. This characteristic, which is suppressed in Plowden, is reasserted in the 1979 HMI document and, to a progressively greater extent, the reports that followed.

The Plowden Report (1967), while not a major focus here, cannot be completely ignored. The moves away from a segregated curriculum begun in Hadow find full expression here. The teaching style recommended is less authoritarian and more democratic (Ernest, 1986:17); the teacher is portrayed as a facilitator, stepping with skill and understanding into children's 'natural' learning. Such natural learning which proceeds from 'natural' play can be encouraged by the provision of stimulating experiences (notions of naturalness are critiqued elsewhere, see for example Walkerdine, 1984). Interestingly Plowden avoids too much talk of maths pointing the reader instead to The Schools Council's Curriculum Bulletin No. 1 (1965). This draws heavily on Piaget among others. In seeking to redefine educational purpose in the mathematics curriculum the Schools Council quote Dienes:

> It is suggested that we shift the emphasis from teaching to learning mathematics, from our experiences to the children's, in fact from our world to their world. (p10)

The extent to which the moves suggested in Plowden were ever really taken up has been questioned (for example: Galton, 1999) but at the time the perception quickly grew that teachers were no longer teaching (see for example the debate in the Black Papers: Cox and Dyson, 1969a; 1969b; and 1970). During the 1970s, following the Black Papers, Her Majesty's Inspectorate (HMI) and the Department for Education and Science (DES) went into a period of crisis (for a detailed examination see for example: Lawrence, 1992). While HMI were still concerned with keeping curriculum control with the schools and a process model of maths, the DES started to pull towards the acquisition of skills and a more centrally controlled 'product' oriented curriculum. This conflict over who would define and control the curriculum saw the DES start to gain the upper hand and a bureaucratic model of curriculum emerge which saw fulfilment in the National Curriculum and NNS.

During this conflict HMI moved towards a specified curriculum in an attempt to avoid more extreme DES calls for central curriculum controls. **Mathematics 5-11: A Handbook of Suggestions** (HMI, 1979) sets out objectives which are summarised in Appendix One; half the objectives are stated in terms of skills development ('the ability to...') and the other half emphasising understanding (develop, recognise, understand, appreciate). The mode of teaching is a tempered form of that advocated in Curriculum Bulletin No. 1 and the teacher is given a more directive role:

> The belief that children should be enabled to discover important mathematical ideas for themselves has been developed over many years .... Discovery methods are a sound approach when they are used to lead children to acquire a deeper understanding of the processes involved and more enthusiasm for the subject. Of course, neither teachers nor children have the time or skill to ensure that children discover everything, but, if their attitudes to the subject are to be positive, it is important that each should have sufficient experience of personal discovery. (p1)

That the tension is epistemological and, perhaps, dialectical is recognised by HMI:

> The achievement of a proper balance between teaching for understanding and teaching for skill is currently the subject of much debate, and complete agreement is very unlikely as fundamental questions of educational aims are involved. But it is unnecessary to set understanding and basic skills against one another since they are complementary: both are needed. (p17)

Note that, while this has moved on from Hadow where understanding seemed unimportant, skills and understanding remain complementary rather than mutually constitutive. Later, in the 1990s, the National Numeracy Project would seek to address this tension although its attempts would be repolarised by the political rhetoric around (though not the spirit of) the National Numeracy Strategy. Unfortunately the DES, continuing to move closer to a centralised curriculum, were ceasing to communicate with HMI (Chitty, 1989).

In the HMI Handbook of Suggestions (1979) teachers are communicated with directly and given explicit instructions about what they need to do and why; the tone is conciliatory, treading a path between the polarised 'progressive' Plowden and 'traditional' camps. Engaging with the thorny question of learning multiplication tables and in what might be seen as a more yielding 'Hadowesque' voice, they say:

> A sound knowledge and recall of multiplication facts should be acquired if children are to be able to perform anything beyond the simplest calculations involving multiplication and division. Teachers should encourage memorisations and recognise the importance of precision when there is the need. Random recall is vital but this by no means implies *rote* memorisation. (p23, emphasis in original)

A major difference between the foregoing reports and the **Cockcroft Report** (1982) is its remit to look at the mathematical needs of employers rather than directly at schools and the curriculum. The expectation seems to have been that the committee would toe the DES line and encourage greater central control of the curriculum and more authoritarian teaching styles. The report, however, fails to deliver what might have been expected given the climate at the time, taking instead a more 'HMI' line (unsurprising since it was largely written by the secretary: a leading maths HMI).

That Cockcroft felt able to speak with such a personal voice may be partly explained by the fact that the expected employer outrage that was the motivating force for the committee's work was found to be a myth: most employers reported satisfaction with the mathematical abilities of their young employees (paragraph 46). Although there are aspects of the DES's bureaucratisation that the report seems in accord with, the writers appear more interested in embracing HMI concerns about purposes for doing maths, problem solving, and developing enjoyment and a positive attitude: a stance that might be considered 'technological pragmatist' (Ernest, 1991). Cockcroft resists DES pressure to make curriculum control an issue for central government. In this respect perhaps their most DES resistant comment is that 'there are some teachers who would wish us to indicate a definitive style for the teaching of mathematics, but we do not believe that this is either desirable or possible' (242).

Cockcroft also draws on a view of mathematics as a discipline ('*a major intellectual pursuit'*) and provides a more technical view of maths; but like HMI, treads a line between developing knowledge and understanding: so although there is still a need to provide plenty of practice (229) so that long-term memory can become effective (237), teachers need to teach at the right pace: too fast and '*understanding is not able to develop*', too slow and '*pupils can become bored and disenchanted*' (230).

Again, like the earlier HMI document there is a recognition of complexity. Teaching is said to comprise three elements: facts and skills, conceptual structures, and general strategies and appreciation (240). Each of these '*involve distinct aspects of teaching* *and require separate attention*' and again skills and understanding are cast as separate rather than mutually supportive. Deriving a picture of the roles and sensibilities of the primary school teacher becomes increasingly difficult as we move into Cockcroft (and beyond) and reports are increasingly written in the passive voice and are couched in terms of desirable outcomes. Cockcroft's list of things the teacher *needs* to do and *should* do is at once long and inferred and brooks no discussion. The way it is written, with no reference to the teacher makes engaging with the text difficult:

> The primary mathematics curriculum should enrich children's' aesthetic and linguistic experience, provide them with the means of exploring their environment and develop their powers of logical thought, in addition to equipping them with the numerical skills which will be a powerful tool for later work and study. (287)

The qualities an ideal teacher might possess and what their actions might be are left to be inferred. This begs the question 'how am I supposed to achieve that?' and makes answering such 'how's imperative.

Cockcroft and The Handbook of Suggestions represent an ironic resolution of the HMI/DES conflict which was largely ignored as the political agenda moved 'DESwards' (Chitty, 1989). This resolution or compromise built by both HMI (1979) and Cockcroft in an attempt to enable teachers to maintain control over the curriculum was shattered first by the National Curriculum and now, further by the National Numeracy Strategy.

The National Numeracy Strategy and Numeracy Task Force (NTF) (1998) bring us full circle and yet still embody earlier contradictions and ambiguities. While not completely throwing out everything that happened between 1931 and 1996 there is broadly a return to the 'non-Plowden' themes of Hadow. Much emphasis is placed on consistency and efficiency turning teachers into a homogenous group and teaching into a technical craft (Reynolds, 1998). There is no sense of dialogue; the tone is one of moral coercion:

> Mathematical concepts are abstract, and children can find them difficult to grasp. Effective teachers often first illustrate these concepts in a context that makes sense to their pupils, carefully choosing demonstrations and explanations that draw out the underlying mathematical concept. (1998a:37)

The virtues of teachers demonstrating and explaining are extolled in earlier reports, the most significant change is the loss of dialogue between text and reader - the inclusive, illustrative anecdotes of Plowden and Curriculum Bulletin No 1 have gone and the imperative that comes through in the overall tone of the NTF report is even more urgent than Cockcroft.

Long-standing concerns about teacher subject knowledge are addressed although again this is not new, however, the more authoritarian voice lends the comment new force:

> ... teachers need a deep and interconnected knowledge of the subject, relevant to the primary curriculum and pupils' later development. Teachers need to understand what they are teaching before they can begin to consider how to pass on knowledge of the subject to their pupils. (1998a:30)

The ideal teacher is now a mathematics specialist (although there is no attempt to define what this is - a person with a maths degree? A' Level?):

> ... The best specialist teachers have a more confident command of mathematics, which they use to maintain a brisk pace to lessons and to set high expectations, which are invariably met by pupils. (1998a:72)

Unlike all previous reports, this one has no compunction about telling teachers how to do their jobs. The rather dictatorial tone is justified by its basis in 'research and inspection evidence' (although the degree to which this is justified has been disputed: Brown, Askew, Baker, Denvir & Millett, 1998). Other forms of learning or the danger of turning children off maths are not entertained. The level of precision in the detail given paints a picture, not of the teacher as artist engaging with children and guiding their development as in Plowden etc., but of teacher as technician, scientifically providing:

> ... a brisk pace whilst allowing some time for pupils to reflect on what they have learnt, with objectives clearly stated at the beginning and key points summarised at the end. Beyond the daily lessons, a little extra time - about 115 minutes per class each week - is needed to talk with individual pupils about their progress. (1998a:33)

Despite these increasingly trenchant comments and the move back towards maths teaching as a process of skill and fact transmission, the cross curricular work of Hadow, Plowden et al is still said to be important. It is to be undertaken to '*provide an opportunity to consolidate, practise and extend what they have learned in mathematics lessons*' (1998a:38). This contrasts antithetically with the reasons for its genesis: to provide a curriculum unfettered by subject boundaries and therefore more in tune with *the natural movements* of children's minds.

Discussion.

Reading across all these documents shifts in the values of education and the epistemological understanding of mathematics can be traced. The variety of discourses form a patchwork of stances, practices, constructions and so on. At different times different discourses are illuminated by lights of varying focus and intensity leaving other discourses darkened. When the spotlight moves those ‘darkened’ discourses don’t go away, they hover in the shadows waiting for the light to come their way again. At this macro level, the variety of discourses reflects a range of values and beliefs.

My suggestion is that individuals reflect the mosaic-like nature of the discourses that exist at a macro level. That teachers hold multiple beliefs, some of which may be logically inconsistent, is increasingly recognised (see for example Andrews & Hatch, 1999). Running counter to this conception are detailed efforts, like those of Ernest (1991:138-9), to categorise values and beliefs. Such categorisation reinforces perceptions that individuals make use of singular discourses. This habit of categorisation and simplification carries with it an implication that views held by individuals can be polarised - that there is no room within one view for the beliefs of someone who may hold different views. While it is certainly true that beliefs tend to be strongly held and passionately defended, an individual may suppress arguments they might deploy in another context to strengthen their argument (this is generally done unconsciously).

Polarised and singular views of beliefs may be under attack within the research community. However, ahistorical mathematics courses in Initial Teacher Education may act to compound these monocular tendencies, by delivering mathematics (and mathematics teaching) as a content package divorced from underpinning values.

In the background, through all the political pushing and pulling is the primary school teacher whose career may well span several discursive shifts. As we have seen, different settlements or partial resolutions are privileged at different times. As well as participating in and being influenced by different discursive climates, teachers are also constituted by other things - not least autobiographies which may not hold maths in positive lights. Their interpretations of these tensions are unheard in policyland. Suggestions that teachers hold monolithic representations of mathematics and mathematics teaching are simplistic and other, more inclusive conceptions have been suggested (Brown, McNamara, Hanley & Jones, 1999). Perhaps hope lies with committee written reports which allow space for multiple representations.

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APPENDIX A - Changes in the English Education System: a brief and incomplete guide to the reports and documents referred to in this paper.

Date and Document	**Context and purpose**
1931 Hadow Report	Written for an audience of civil servants, Hadow marks the splitting of education into two phases: primary (5-11 years) and secondary (11- leaving age).
1965 School's Council-Curriculum Bulletin No1	Written primarily for teachers and teacher educators *'to assist individual teachers to reach their own decisions about changes and developments in the curriculum'*. (p.vii)
1967 Plowden Report	A report written primarily for an audience of civil servants, it was seen by the author partly as an evaluation of the earlier Hadow report. The move is broadly towards a more child-centred approach and teachers as facilitators. Attention is directed to Curriculum Bulletin No1 for subject guidance.
1979 HMI 'Handbook of Suggestions'	Written primarily for teachers and teacher educators at a time of increasing debate about who should have control over the curriculum, teachers or the state. This document errs in favour of the teachers but was swimming against the prevailing political tide.
1982 Cockcroft Report	This was a report written primarily for an audience of civil servants but with teachers and teacher educators clearly in mind. Despite clear political wishes for increasing curriculum control to be handed over to the state (DFE), the report fails to encourage movement in that direction. To the annoyance of government, its impact on teacher training and curriculum development was strong.
1988 Introduction of the National Curriculum	The 'who controls the curriculum' debate finally ended with the introduction of the National Curriculum. This document lays down the curriculum to be taught to all children. It is an entitlement curriculum set out by levels (rather than ages).
1996 Setting up of the National Numeracy Project	As a response to moral panic (generated by TIMMS) the National Numeracy Project was set up by the Conservative government to improve levels of numeracy in 12 local authorities. The success of this project was to be measured by a series of tests over five years.

1998 Preliminary and Final reports of the Numeracy Task Force (NTF)	With the change of administration in 1997, the NTF was set up to advise on changing the limited Numeracy Project into a National Numeracy Strategy.
1999 Introduction of the National Numeracy Strategy	The NNP is transformed into the NNS and is given out to all schools with a ‘Framework for Numeracy’ as the key documentation going into schools.

The Construction of Identity in Secondary Mathematics Education

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Abstract

Drawing on data from 120 interviews with secondary schools students of mathematics aged from 14 to 18 in England and the United States, this paper argues that young people's developing identities are an important and neglected factor in success at secondary school mathematics. Students in both countries believe mathematics to be rigid and inflexible, and in particular, that it is a subject that leaves no room for negotiation of meaning. However, while the lack of opportunity for understanding mathematics was important, a much more salient factor in determining students' attitudes towards mathematics was that they did not see success at mathematics as in any way relevant to their developing identities, except insofar as success at mathematics allowed access to future education and careers.

Introduction

One of the more persistent and widespread problems in mathematics education is that many students who are successful in mathematics give up the subject as soon as they are able to do so, even though they are aware of the limitations this places on future careers. A variety of studies has sought to understand this phenomenon through a range of psychological viewpoints including attribution theory, locus of control and role modelling. Such studies have been useful in shifting the emphasis away from models of ability, but have tended not address the phenomenon as an explicitly *social* one. This paper represents an attempt to understand why some students will continue with their studies in senior mathematics, while others do not. We take the notion of "identity" as critical to our analysis. We contend that students who develop a sense of identity which resonates with the discourse of mathematics are more likely to continue with their studies than their peers who do not develop such a sense of identity. Critical to this proposal is the understanding of the processes through which students develop such a sense of who they are in relation to mathematics.

Psychological Studies on Identity

Most studies of identity formation have been grounded in psychological discourses —see Erikson's (1968) theory of identity development for example. Such theories posit that in the early stages there is a lack of awareness of an individual's identity in relation to a social or cultural group. As children enter the adolescent years they become more aware of who they are within the boundaries of a group and as such begin to explore the group mores. As they become more aware of the nature of their group identity in relation to other groups (in terms of race, class, ethnicity, work, gender, etc), they become more committed and secure within their chosen group.

Such theories are based on the age/stage ideology where it is posited that students will identify more with their group as they age and mature. In contrast, other theories adopt a more social psychological approach. For Tajfel & Turner (1986) for example, the social aspects of identity formation are central. Within this approach, there is a greater emphasis on the person's sense of belonging to a group and the resultant feelings of security and other associated attitudes that represent belonging to a group. In part, belonging to a group is a seen to be a key component of a sense of self and the developing self-concept whereby members develop a keen sense of the value of the group and group membership and as a consequence derive considerable self-esteem from belonging to a particular group. In identifying the effects of identity in young adolescents Roberts, Phinney, Masse, & Chen (1999) have suggested that members who have a positive view of their group tend to have high self-esteem while those who have a negative view of their group have a low self-esteem and several other studies have suggested that there is a correlation between identity with a group and self esteem.

In attempting to define and measure "identity", such discourses have identified three components – a sense of belonging to a group; a sense of achievement within the norms of the group; and particular behaviours associated with belonging to a particular group – which are seen to represent key aspects of identity. These components provide indicators of key aspects to consider when theorising identity from a psychological standpoint. However, in order to understand how these attributes become manifested, it is important to consider the social contexts within which such attributes are developed.

Mathematics as a Community of Practice

In contrast to the psychological theories of identity, we propose to take a sociological approach in which we consider how students interact with their social environment and how the two elements, the individual and the group are mutually constitutive of identity. In their extensive work with communities of practice, Lave and Wenger have argued persuasively that learning is a social practice through which we come to know who we are (Lave, 1992; Lave & Wenger, 1991; Wenger, 1998). Rather than seeing learning as a process that takes place 'within' an individual', Lave and Wenger argue that it is only through social processes and shared experiences that people gain a sense of self and meaning. Lave and Wenger also reposition identity as a function of participation in different communities – they argue that people do not have one identity, but different identities that are more or less salient in different situations. Thus identity is not represented as stable, consistent or life-long but *dynamic* and *situated*. Through their works, Lave and Wenger have systematically explored the intersection of community, practice, meaning and identity. For us, this seems to be a more productive means through which we can come to understand not only how students come to learn mathematics, but more broadly in that through participating in a community of practice – in this case mathematics – they come to learn mathematics and acquire a sense of who they are as learners within the social practice of mathematics.

Through studying how students learn about mathematics, they also learn how to make sense of learning mathematics and sense of themselves. In his study of claims officers, Wenger (1998) argued that:

> They learn how not to learn and keep their shoulders bent and their fingers busy, to follow the rules and ignore the rules. They learn how to engage and disengage, accept and resist, as well as how to keep a sense of themselves in spite of the status of their occupation. They learn how to weave together their work and private lives. They learn how to find little joys and how to deal with being depressed. What they learn and don't learn makes sense only as part of an identity, which is as big as the world and as small as their computer screens, and which subsumes the skills they acquire and gives them meaning. They *become* claims processors. (pp. 40-41, emphasis in original)

We would argue that the same can be said for students of mathematics. As they are compelled to sit in a mathematics classroom for a significant period of their school life, they come to learn how to participate in that context – they learn when to respond, when to resist, how to appear busy but avoid work. They learn how to cope with the embarrassment, the joy, the cajoling. They learn how the actions in the classroom have meaning and how some of the actions of teachers, texts and students take on substantially different meanings for themselves and others. They learn how to *be* a mathematics student. They develop a sense of who they are as learners within this context, a context which may be very different from other subjects within the school context and beyond the school context. The mathematics student that they see themselves to be may be very different from other students in the same classroom. Similarly, the student that they see themselves as in the mathematics classroom, may be very different from the student they see themselves as in other subject classrooms.

There are very few studies in mathematics education that explore the construction of identity in and through the practices of mathematics (Boaler, in press). Wenger (1998, p. 47) defines practice as a process of doing within a "historical and social context that gives structure and meaning to what we do…[such that] practice is always a social practice." Practices include both the explicit and implicit; what is said and what is left unsaid. It includes language, artefacts, tools, symbols, rules, along with less obvious aspects including unspoken conventions and rules, assumptions and world views. All of these practices come to make up what it is to be a participant and member of a particular community of practice – in this case, a mathematics student.

It is through the practices within a community of practice (ie secondary school mathematics) that students develop a coherent sense of what it is to be a member of that community. Students attempt to make sense of the community, and in so doing, develop a sense of self in relation to that community of practice. For some students, there is a greater synergy and sense of belonging whereas for others, there is a sense of rejection and hence little sense of identity within the community of practice. Like all communities of practice, the mathematics classroom has developed over a period of time—what is perhaps most remarkable about this particular community of practice is how little it has changed in most countries over the last hundred years.

For most students, mathematics continues to be a teacher-dominated practice, with a substantial amount of self-directed work undertaken from either a text-book, board work or individual worksheets. It has been heavily reliant on formal pencil-and-paper testing, particularly in the secondary school. Students come to learn what it is to be a mathematics student through these practices. While there have been notable changes over periods of time, there are equally periods where there has been little change thus suggesting that as a community of practice, mathematics is neither fixed nor transitory. Rather, some features are relatively constant, while others can change.

A recent study of the impact of teachers' classroom practices on identity (Reay and Wiliam 1999) examined primary school students' perceptions of themselves and how the teacher's assessment practices were influential in developing a sense of self. They argue that assessment practices are critical in shaping the way that students come to understand who they are within and beyond the confines of their classrooms, providing both explicit and implicit feedback to students as to their potential to become a member of this community. However, the information is not taken 'at face value'. Instead, students *negotiate* an identity within that community of. In American studies of assessment effects, Donald (1985) posits that assessment practices feed directly in the construction of a range of subjectivity in an insidious manner so as to appear to be a normal and natural process.

In this paper, we examine the practices of secondary school mathematics teaching from the perspectives of the students in order to understand how they construct a sense of themselves in relation to mathematics. In mathematics classrooms, students learn more than the mathematics—they learn what it is like to *be* a member of that community of practice, and whether or not they want to become participants. Learning is a social activity which encompasses the relations between people and knowing. The 'old timers' (the teachers) through their actions and talk convey a sense of what it is to be a member of this community of practice. This can be in terms of the ways in which one works mathematically, how one talks and how one presents to outsiders. Newcomers (students) observe and evaluate the actions of their teachers and the practices within the discipline and decide – either consciously or unconsciously – whether or not they want to become members of this community. This paper explores how students come to make sense of who they are as learners in relation to the community of practice of mathematics students.

Method

The data reported in this paper comes from two studies. In the first, from the United States, one of the us (JB) interviewed 48 students in Advanced Placement (AP) calculus classes in 6 Northern Californian public schools in order to investigate the nature of confidence in mathematics. In the second, from the United Kingdom, 72 students from six schools were interviewed about a range of issues related to their mathematics classrooms (see Boaler, Wiliam & Brown, 2000 for further details).

The Mathematics Classroom Environment.

The students were asked to describe their mathematics lessons, and interviewers engaged students in conversation about the different features they described. The students in the two countries reported a sequence of pedagogical practices that was remarkably consistent. This may be characterised by the following students' description:

> Basically, throughout my experience, we go to class and the teachers lecture, go over the material and show us exactly how to do the problems, cover the subjects that they're teaching and after the teacher's finished teaching if we ask questions and sort of like clear up anything that we don't know and then homework will be assigned to us that day then we go home and do it. (Brad, Cherry school[1])

The students all described teachers reviewing homework, explaining methods at the board and assigning questions to students. Students of two of the US teachers, both women, at Grape and Orange high schools, said that they were encouraged to work on questions collaboratively. Students of the other four US teachers described mathematics classes as individual environments in which they received few opportunities to discuss work.

The mathematics textbooks in the US schools all presented the fundamental theorem of calculus, expanded upon the different concepts underlying the domain and demonstrated procedures that could be used to solve problems. Students would then be led through a series of questions that required them to practice the different procedures. In four of the US schools (Apple, Lemon, Lime & Cherry) and four of the UK schools (Alder, Fir, Redwood and Willow) teachers asked students to practice textbook procedures for a large part of each lesson. In the other two US schools, (Grape and Orange) students spent lesson time discussing the different questions, as a class, and in student groups, while in one of the other two UK schools (Cedar) students worked in small groups, and in the other (Hazel), they worked mainly individually on a series of activities programmed by the teacher

The students' reported beliefs about the nature of mathematics and learning varied according to the extent of mathematical discussion in their classes, with students from the two US discussion-based classes presenting a completely different perspective on mathematics and learning. In the four US schools that encouraged individual work, the 32 students unanimously described mathematics as a procedural, rule-bound subject, and this view was shared by most of the students in the UK schools. These views were held irrespective of gender, confidence levels and prior levels of attainment. Students described mathematics as absolute, concrete and always having one right answer:

> There's definitely a right answer to it. The other subjects like English and stuff that really have no right answer so I have to think about it. (Kim, Apple school)

> In the English I was relaxed, maths I wasn't at all. It's just like, cause there's always got to a definite answer, it's not so much opinions and stuff. It's not any opinion, so I felt a bit more pressure to do well in that, and everyone was saying like 'it's so useful' and it's what at job interviews they're always going to look for, so… (Jane, Firtree school)

> It’s because maths is different from other subjects. You have to know the facts and remember them, […] remember the rules and stuff, remember which way goes that way and there’s just a lot to remember. (Fiona, Willow School)

> It’s all about the formulas. If you know how to use it then you’ve got it made. Even if you don’t quite understand the concept, if you’re able to figure out all the parts of the formula, if you have the formula then you can do it. (Lori, Lime school)

> I used to enjoy it, but I don't enjoy it any more because I don't understand it. I don't understand what I'm doing, so if I was to move down [to a lower ‘set’] I probably would enjoy it. But I enjoy it when I can actually do it, but when I don't understand it I just get really annoyed with it. (Alison, Firtree school)

> S: It's the only class, where there will be a right or wrong answer, there's a way to get the right answer.
>
> C: I see it more as procedures and solving one problem at a time. It's hard for me to see how it relates to everyday things, so I don't really get the big picture a lot of the time. (Susanna & Cathy, Lemon school)

> You have to memorize these little steps, there's always an equation to solve something and you have to memorize stuff in the equation to get the answer and there's like a lot of different procedures. (Vicky, Lime school)

The students in the four US schools and the six UK schools presented a remarkably consistent picture of their classroom experiences as working through problems with one, non-negotiable answer and concomitantly they regarded mathematics as a series of procedures that needed to be learned. Many of the students regarded the exclusive act of practising procedures as inconsistent with the development of a broader, conceptual understanding.

In contrast, while the students in Grape and Orange school used the same, or similar, textbooks as students in the other four schools they did not work through the exercises producing answers that were supported or invalidated by the teacher. Instead they were asked to discuss the different questions, and consider the meaning of possible solutions with each other. This act of negotiation and interpretation meant that mathematics did not appear to the students to be an abstract, closed and procedural domain, but rather was seen as a field of inquiry that they could discuss and explore. Thus the students developed very different views about the nature of mathematics and learning:

> M: I don’t know, it just seems like math is more important. In my English class, I can just kind of flow, and whatever’s going on, write an essay about whatever, it’s not a lot, well, in my case, it’s not a lot of deep thinking. Not a lot under the surface.
>
> JB: Is there in math – deep thinking?
>
> M: Yeah. Yeah because the thing, being conceptual, and that’s a lot harder than just like memorizing formulas, definitely. (Melissa, Grape school)

When students were encouraged to discuss the meaning of the procedures they encountered in mathematics, they appeared to develop profoundly different perceptions about the nature of mathematics, and a greater propensity to strive towards conceptual understanding. The students’ enjoyment of mathematics was largely related to the extent to which they identified as a mathematics learner (Boaler, 1999); their perceptions of the subject were strongly linked to these.

Enjoyment and Identification.

Most students in the US schools, despite being relatively successful mathematics learners, reported disliking mathematics, not because the procedural nature denied them access to understanding, although that was important, but because their perceptions of the subject as abstract, absolute and procedural conflicted with their notions of self, of who they wanted to be. For example:

> Well it's not that I don't understand it, when I understand concepts I like doing it because it's fun. I'm more of a language/history person, kind of and sometimes the things he explains I find really hard to understand. And later, even when they try to ask for help, I get so confused so I don't really like that aspect of it and also there's only one right answer and you can, it's not subject to your own interpretation or anything. (Susan, Cherry school)
>
> I'm more of a visual art kind of person, so I always like stuff that was more logic, rather than straight math. Oh, yeah, my dad did this thing back in elementary school, family math, there was a night where it was like parents could come with their kids to the library at school, it was more like little games, little puzzle-type things, but it was fun. I thought it was fun. Back in 4th grade. I enjoyed that. (Amy, Lime school)
>
> JB: Do you like math?
>
> V: No, I hate it.
>
> JB: Why do you hate it?
>
> V: It's just too, I'm into the history, English (…) It's like too logical for me, it always has to be one answer, you can't get anything else BUT that answer. (Vicky, Lime school)
>
> I used to love math, but now I think, it's like I'm going to make sure that I don't major in math or anything because it's starting to be like too much competition, it's so weird. When it came to calculus and precalculus, I just kind of lost interest. It's like I'm going to do this for the points, I don't really care. I care more about science and English, stuff that makes sense to me where I think I'm learning morals and lessons from this, where I can apply it to something. (Betsy, Apple school)
>
> JB: Why wouldn’t you major in math?
>
> C: I think I'm a more creative person, I can do it and I can understand it but it's not something I could do for the rest of my life and I think if I had a job I'd like one that let me be a little more creative.
>
> JB: Math isn’t creative…?
>
> C: No. (Cathy, Lemon school)
>
> I think women, being that they're more emotional, are more emotionally involved and math is more like concrete, it's so "it's that and that's it." Women are more, they want to explore stuff and that's life kind of like and I think that's why I like English and science, I'm more interested in like phenomena and nature and animals and I'm just not interested in just you give me a formula, I'm supposed to memorize the answer, apply it and that's it. (Kristina, Apple school)
>
> T: There’s definitely a certain type of person who’s better at math. Generally, if you’re better at English they seem to be more social. And the math people. I don’t know, they’re just as social, but in a different way. They express themselves differently, they like to see things in

black and white. They don't see the colors and greys between. With English people they like things that don't necessarily have an answer. They like to explore that. (Tom, Lemon school)

It seems to us significant that so many of the students related their rejection of mathematics to the type of person they believed themselves to be. The students above variously described themselves as a 'language/history, visual arts, history/English, creative, emotional or social' *person*. They did not discuss their choice of subject in cognitive terms, detached from broader notions of identity – yet such notions have pervaded theories of learning and discussions of subject choice. The students' comments suggest that procedural presentations of mathematics do not only make the subject less enjoyable, or preclude understanding for some, they a represent a potential life-path that is uninviting for most students.

These attitudes did not come through so strongly from the UK students. Of course, as might be expected, there were many students who disliked mathematics—some with real intensity (see Boaler, Wiliam & Brown, 2000). On the other hand, there were many students who *did* like mathematics but very few of the students who liked the mathematics identified *with* the mathematics—their reasons for liking mathematics were primarily related to their perceptions of being good at it, or because it would lead to a desired further stage of education or employment. The following quotation is typical:

DW: Do you ever work hard on something just because you are interested in it?

C: Yeah, but not in maths. (Colin, Redwood school)

Mathematics was seen as a necessary price to pay for educational or vocational progress, and this was more or less burdensome depending on how easy one found the mathematics. However, even these successful learners did not see the 'ideal' student that their teachers seemed to have in mind as in any way relevant to their own developing identities.

S: They expect us to be like, just doing it straight away.

M: Like we're robots. (Simon & Mitch, Alder school)

He explains it as if we're maths teachers. He explains it like really complex kind of thing, and I don't get most of the stuff. (Paul, Redwood school)

Yeah, I don't know when we use algebra, I don't know when that comes in. I just think it's to see how our brain works, that's all, our knowledge. It never comes in to anything though (Alwyn, Willow school)

Discussion

It is our contention that any explanation of what happens in mathematics classrooms will be incomplete if it ignores the essentially social nature of schooling. The students who are learning mathematics in secondary schools are also trying to negotiate conflicting constraints in developing their identities as sons or daughters, as males or females, as members of various friendship groups and of course, as learners. Most students want to be successful at school, not least to avoid conflict with parents, but they also want to negotiate a way of being successful that does not alienate them from groups with whom they feel affinity. In some cases, the playing out of these social process will lead students *towards* particular individuals or groups, while in others, it will be influenced by a desire *not* to be like an individual or a group (see, for example, Griffiths, 1995). The extracts from interviews described above, and the many more that we could have selected, show clearly that mathematics classrooms in the United States and the United Kingdom present to the apprentice an unambiguous vision of what it means to be successful at mathematics, and of what it means to be a mathematician. However, it is also clear that this vision is one with which many, if not most, students find it hard or impossible to identify. They want to be successful at mathematics (so that they can get on to the next phase of education, or into a job they want), they may even like some parts of the mathematics they do, but they don't want to be successful *as mathematicians*. 'Becoming a mathematician' seems to play no part in their plans. From a psychological perspective this might well be cast as a problem of the 'ability' of the students. However, we believe that more useful insights into the nature of mathematics education, particularly of the 'able' students who are qualified to study mathematics further but choose not to do so, would be gained by looking at this as an issue not of 'ability' but of 'belonging'.

Changing the emphasis from 'ability' to 'belonging' also demythologises the special status of mathematics. The idea of 'belonging' immediately raises the question of 'belonging to what?', allowing the possibility of multiple communities of practice, rather than a single monolithic edifice. This will have particular importance for those practitioners who are keen to develop perspectives on mathematics that are consistent with a view of knowing as 'connected' to human existence, in contrast to the prevailing view of mathematics as 'separate', abstract, remote and 'alien' (Boaler, in press).

Adopting such multiple perspectives would also suggest a redefinition in the way we look at 'success' and 'failure' in mathematics classrooms—the kinds of strategies adopted by teachers in the face of a student's 'failure to belong' would be very different from those suggested by a 'failure of ability'. It would also suggest a move away from a view that the 'problem' lies with those who cannot identify with mathematics as presented in school, and instead to a concern with why anyone would be, and would want to be, successful at, something as abstract and dehumanised as the traditional diet of secondary school mathematics.

Notes

[1] All names of schools and students are, of course, pseudonyms. The US schools are named after fruit, and the UK schools are named after trees.

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Mathematics teacher development and learner failure: challenges for teacher education

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This paper presents a case-study of a mathematics teacher who has substantively developed her mediational strategies and her conceptions of mathematical knowledge as a result of her participation in an in-service programme. Her new teaching practices raise serious dilemmas for her in relation to her teaching context, in particular, the limited knowledge of her pupils. Using Vygotsky's notion of mediation in the zone of proximal development, it is argued that her successes, dilemmas and constraints raise important questions for mathematics teacher development programmes in such contexts. In particular the position of contextually grounded pedagogical content knowledge is considered.

Introduction

> "I can't just go to the next lesson, I can't...I can't just go. You have seen maybe two, three, four students they understand what they are doing but I cannot just go to the next lesson because two, three students understand ... if I don't finish I'll be sorry but I think it will be better if the things that I have done they understand, unlike to finish the syllabus only to find that they...they know nothing of the things I've done. ... if I don't finish the syllabus of course I will be worried but it will be better if they know one thing that I have treated with them."

The above quote expresses the dilemma of a teacher who wants to work with pupils' meanings but also needs to ensure that pupils learn the knowledge of the curriculum. Jaworski (1994) and Edwards and Mercer (1987) have documented how teachers in England manage this tension. The teacher who said the above words is a secondary mathematics teacher in a rural, over-crowded, under-resourced South African school. For her the dilemma is acute, because so many of her pupils fail mathematics[1]. Her context introduces a second dilemma, implicit in the above quote. Does she focus on building the mathematical knowledge of the majority of pupils or does she focus on the few who might pass? Focussing on the majority would require working at a much lower level, she would not cover much of the Grade 10 syllabus, and would therefore disadvantage the few who are at Grade 10 level and might be able to pass. If she focuses on the few relatively more successful[2] pupils, she will disadvantage the majority. Similar dilemmas are experienced by all teachers. However, the pervasiveness of failure in this teacher's context foregrounds the dilemmas and makes them more acute. In this paper I will argue that such teacher dilemmas present particular challenges for research and teacher education in such contexts.

[1] I am noting this failure as a reality which needs to be addressed. I do not wish to set up deficit explanations, and I particularly do not think it is attributable to either the teacher or the learners.
[2] These pupils are successful relative to the pupils who fail badly in this school. In reality their mathematical knowledge is also severely limited and in schools with better results, they would be considered weak.

To do this, I will present some of the story of the teacher whose dilemmas I have outlined above. She was a student on an in-service programme[3] and a participant in a research project which followed a sample of teachers who enrolled in the programme in 1996[4]. She was a successful student on the course, both in our terms and in hers. She feels that through the course she has changed from being an angry, frustrated teacher, to someone who is able to understand pupils' difficulties, both in and outside of the classroom. The research observed that she was able to appropriate and use in her classroom much of what she had learned on the course, in particular with respect to pedagogy. She elicits and hears her pupils' meanings and attempts to work with them in order to develop new knowledge. In Vygotsky's terms she attempts to mediate in the zone of proximal development (Vygotsky, 1978). We (Brodie, Davis and Lelliott, 1999) have characterised her as a teacher who has substantively taken up learner-centred mediational strategies from the programme. However, the interactions in her classroom and her subsequent dilemmas raise new questions in this context.

What happens when such a teacher adopts learner-centred practices? This paper will describe classroom incidents, her reflections on them, and the dilemmas that arise for her. On the basis of these dilemmas, it will address the question as to what the notion of mediation means when pupils' knowledge base does not enable access to new concepts. Thus the paper will raise important questions about learner-centred mathematics teaching in South Africa, and contribute to arguments that the notion of learner-centred pedagogy and its theoretical underpinnings may need to be tempered with local realities, particularly, but not only in developing countries (Tatto, 1999; Clark, 1999).

The teacher and her context

Ms Nhlapo[5] teaches mathematics and science in a rural secondary school in the Northern Province. She originally trained as a primary school teacher, but moved to help found her current school when the community decided that a secondary school was necessary in the area. She has been teaching Grade 8-10 mathematics since 1995, and feels that the FDE has given her the confidence to move into Grades 11 and 12 (in 1999 she is teaching grade 11). Ms Nhlapo's knowledge of mathematics is strong in relation to the rest of our sample, although there are certainly limitations. She did well in her mathematics courses in the FDE

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[3] The programme provides a Further Diploma in Education, and is known as the FDE.

[4] The aims, methods and first two years of analysis from this project have been written up in Adler et al, 1997, 1998. This paper is based on the data of one teacher over the three years of the project. This teacher was observed, and her data analysed over the three years by Kgethi Setati, Jill Adler and myself.

[5] This is a pseudonym

and in her previous studies. Although she is primary-trained, she is clearly competent and confident in the topics that she teaches up to grade 10 level and her conceptual difficulties in class originate in very subtle mathematical issues.

Ms Nhlapo's school is very poor. There are eleven classrooms for 705 pupils and 18 staff members. Seven out of the eleven classrooms are not complete, with windows, doors, floors and roofs missing. There are not enough desks and chairs, and many that are there are broken and hardly functional. The classrooms have small portable chalkboards which are not always visible from all points in the classroom. The school receives limited numbers of textbooks. The number of pupils in a class ranges from 40 to 120, and the classrooms are built for 40. There are high rates of absenteeism among pupils, with 10 or more pupils absent from most of Ms Nhlapo's lessons. There is also much teacher absenteeism.

Many pupils in the school are overage and some are refugees from Mozambique. In order to get a sense of what pupils know and can do in mathematics we examined their books and tested the pupils[6]. A detailed examination of the books of Ms Nhlapo's Grade 10 class suggests that much of the work in the syllabus is not covered, and of those sections that are covered, only the easier concepts and examples are dealt with. As the quote at the beginning of the paper shows, Ms Nhlapo is acutely aware of how much of the syllabus she has not covered, and she is extremely concerned about it.

The pupils' test results in the table below tell a depressing story. The April and June tests were set by Ms Nhlapo based on what she had taught. The research tests were developed by the research team to try to access pupils' conceptual and procedural knowledge. Ms Nhlapo's Grade 10 pupils were given the Grade 9 test and were able to complete only the very basic questions. Questions requiring slightly more complex mathematics, or a combination of skills were not done successfully.

	No. in class	**No. wrote**	**pass**	**fail**	**below 20%**
April	44	42	9	33	19
June	44	36	8	28	15
Research (Gr 9)	44	34	1	33	28

Poor test results were not unique to Ms Nhlapo's class. Similar results were pervasive in our research, they get worse in the higher grades, and culminate in an overwhelming matric

[6] The purpose of the tests was to see what testing could tell us about pupils' knowledge. In fact they told us very little beyond the fact that pupils know very little. In order to get at the complexities of pupils' knowledge we would need more sophisticated instruments.

failure rate in South Africa[7]. Pervasive failure as a consequence of an under-resourced context limits possibilities for effective mediation in the zone of proximal development. Given her lack of material resources, Ms Nhlapo views herself and her pupils as her primary resources and tries to work extensively with pupils' knowledge in relation to mathematical knowledge. She tries to probe pupils' meanings and work with their ideas through a questioning strategy which breaks with conventional classroom discourse. However, her new style of questioning brings her dilemmas to the fore, makes them more acute and raises difficulties with the kinds of mathematical knowledge she is trying to teach.

The "Teacher's Dilemma"

Ms Nhlapo describes the aim of her questioning strategy as follows:

> "where I use question and answer ... I like them to tell me <u>why</u> they say this ... I don't just take the answers I do follow up to ask <u>why</u> they say that, whether it's correct or wrong I like to follow up because they might be guessing or something like that or to be able to help other students." (my emphasis)

Consistent with her aim of working with pupils' meanings and misconceptions, Ms Nhlapo often probes for a range of answers and then asks pupils to explain why they have answered in a particular way, before settling on the right answer. Thus she does not evaluate immediately, but rather waits for a number of ideas to be in the public domain before doing so. In doing this, she breaks substantially from the conventional I-R-F exchange structure with a strong evaluative component, where the teacher goes for the correct answer and ignores wrong answers (Edwards and Mercer, 1987). This practice was evident to a limited extent in Ms Nhlapo's teaching in 1996 and she used it more frequently in 1997 and 1998. For example:

T: *How many terms [writes 4x3x2]*
P1: *3 terms*
P2: *0 terms*
P3: *1 term*
P4: *0 terms*
T: *Anyone with a different answer?*
P5: *2 terms*
T: *Can you count your 2 terms*
P5: *Its 1 term*
T: *(to pupil 5) Why now 1 term?*
(silence)
T: *Can you tell us why 3 terms?*
P1: *4 x 3 x 2*
T: *Meaning you have counted the numbers 4,3 and 2*
Why are you saying 1 term? Its correct to understand 1 term but <u>why</u> do you say it?

[7] It is crucial to note here that the above results cannot serve as an indication of whether Ms Nhlapo is a successful teacher or not. We did not investigate whether there was any improvement in her pupils' mathematical knowledge and thinking as a result of her improved teaching. The above table serves to underline one of the major constraints that Ms Nhlapo works with, her pupils' weak subject knowledge.

P: *(inaudible)*
T: *You multiply it and get 1 number so its one term.*
Now who said 0 terms, Confidence[8]*?*

Here, Ms Nhlapo allows all possible answers to be tabled before asking for reasons and evaluating them. Her tone of voice in her questions: “why 1 term?” and “why 3 terms?” is the same and does not suggest which one she favours. Even when a correct explanation has been achieved she continues to ask for explanations for wrong answers. This is a considerable break with conventional participation and evaluation patterns in the classroom. The difficulty in developing and sustaining such an approach cannot be underestimated, particularly when there are pressures of time and limited pupil knowledge. That Ms Nhlapo continues to use it after three years suggests that she finds it a useful teaching strategy. Her rationale for this teaching strategy, expressed in the quote above, is firstly that she does not have to do all the explaining, pupils who understand can do it for her, which may help the others, and secondly it enables her to identify and work with pupils' misconceptions and mistakes.
There are however a number of limitations to the way this mode of interaction is played out in Ms Nhlapo's classrooms. Firstly, although Ms Nhlapo is careful not to favour particular answers and explanations, in the end she must emphasise the correct answer, because this is what pupils need to know. Thus Ms Nhlapo finds herself firmly in the teacher's dilemma (Jaworski, 1994; Edwards and Mercer, 1987). The teacher wants to hear, acknowledge and affirm a range of pupil ideas. However, ultimately she must approve the right answer.

The teacher's dilemma is one in which all teachers find themselves and can be understood from a Vygotskian analysis of teaching. Teaching is constituted as mediation by a knowledgeable other in the zone of proximal development. Teachers mediate between what pupils know and can do, and what they must learn, the knowledge and actions (or scientific concepts) of society (Vygotsky, 1978; 1986; Wertsch, 1984). The “gap” of the zpd is necessary for fruitful interaction and learning to take place. Good teaching involves negotiating this gap and the tensions that arise from working in it. However, what happens when the zone is too large?

A second example from Ms Nhlapo's classroom illustrates just how large the zpd can be. Ms Nhlapo had written the expression 3i x 3b x 3a ÷ 6z on the board, and asked the question: “how many terms?”. One pupil answered that there are four terms in the expression. Ms Nhlapo asked why, and the pupil did not answer. Another pupil clicked his hand to answer and Ms Nhlapo reprimanded him saying: “give him time to think”. When the (first) pupil still did not answer, Ms Nhlapo tried to scaffold his thinking, asking “what are terms separated by?”. The pupil answered: “addition and subtraction”. She then asked: “what are the terms in the expression?”. He did not answer and she called him up to the board asking him to write an addition sign. He wrote a subtraction sign and she asked the class if he's correct. The class

[8] This extract has been reconstructed from field notes, not transcribed from a video tape so it may not reflect the exact words of the pupils in some cases.

laughed and she reprimanded them. She then wrote an addition sign and explained in XiTsonga to the pupil.

In reflecting on this incident in her interview, Ms Nhlapo said:

> "I called him because I could see that Mm, Mm...there is more problem in him than just a minor problem that I was thinking, because ...I asked him whether he seen a negative or addition sign there, which makes him say four terms, while it was multiplication, and he said addition sign. Then I said maybe it's my handwriting because multiplication and addition are...the same sort of, if you don't put them nicely and my handwriting is not good. So I thought maybe he might think my multiplication sign is addition. I thought that way when I called him to come and write the addition sign. I was amazed when he wrote subtraction...So I said oh my God then it means it's a serious problem, meaning I should just sit with him down and try to help him more. I will try to find some ways, I don't know how but I could see that he's having a serious problem not just a minor problem. It's very serious and you can see he's older. ..... I was shocked, I didn't even know what to say, really I didn't know... I was worried, I was worried. I didn't know what to think quickly or what to say even now I'm still worried..."

I shared Ms Nhlapo's shock when the incident occurred. It is untenable that a pupil in Grade 8 draws a subtraction sign when asked to draw an addition sign, even if he may be feeling panic that he has not so far managed to answer the teacher's questions and on the spot in front of the class. The above incident is significant in that Ms Nhlapo tried to follow through the thinking of an individual pupil in order to deal with his difficulty. However, she was unable to do so in a manner which shielded him from ridicule, which helped his learning of mathematics, and which helped the class to progress with their work. The incident shows the level of pupil difficulty that Ms Nhlapo must deal with among her many pupils, and the difficulty of doing this through probing misconceptions and difficulties in class. Thus although she views her pupils' knowledge as a resource, it can be a severely constraining factor in her teaching. Further, in developing a strategy which is helpful for some pupils in some cases, ie: probing pupils' knowledge, Ms Nhlapo creates new challenges for herself, which she is yet to develop ways of dealing with.

New challenges for the teacher

The first of Ms Nhlapo's new challenges was mentioned in the introduction. Her new questioning strategies reveal her pupils' knowledge to her and so she is more aware of its limitations. Her learner-centred approach does not allow her to ignore these, she must work with them. However, her pupils' limited knowledge base means that her initial attempts might create zpd's that are too large. Learner-centred pedagogy would then require that she creates appropriately smaller zones which support the learners' progress towards the ultimate goal. This is what she tries to do, but given where many of her pupils are, she does not often achieve her teaching goals, and this disadvantages the stronger pupils. Ms Nhlapo makes a choice in this regard, she chooses to work with the knowledge of the majority. However, given the enormity of this task, the implications of her choice are that most of her pupils' achievements in mathematics are not improving[9] **and** she is not managing to cover enough of the syllabus. The zone is too wide for her to negotiate in ways in which she is trying to at the moment. We could ask whether she manages to create a zpd for most pupils.

A second, related challenge concerns the mathematics that Ms Nhlapo is trying to teach. She characterises her mathematical goals as:

> "I would like my pupils to learn the...the content itself, the language in mathematics using correct language... My aim is to teach my students to..to use these concepts, mathematical concepts correctly and the calculations itself to calculate properly"

Ms Nhlapo emphasises developing the language and concepts of mathematics with her pupils. This is something that she has learned about from the FDE programme. She integrates aspects of the programme, by using pedagogical strategies that she learned about in one part of the programme, with teaching mathematical language that she learned about in another.

Unfortunately, it may be the case that the strategies she chooses are not the most appropriate to teach what she intends to. In the above extracts, Ms Nhlapo is working on the definition of the notion "term" in an algebraic expression. Probing pupils' responses in these cases may help her in identifying misconceptions. However, in both extracts above, the range of possible answers that pupils might give is limited, and ultimately there is only one correct answer, because of the definitional nature of the task. Almost all of the pupils' meanings that Ms Nhlapo elicits in her lessons are definitional or procedural (when she works with calculations, another key aspect of her teaching).

It may be the case that many pupils cannot read the ground rules (Edwards and Mercer, 1987) of her new form of classroom discourse. They may not understand why she keeps asking "why", or that her questioning of ideas does not necessarily mean they are incorrect. Moreover, because the tasks ultimately only allow for one correct answer, the pupils might become confused when a range of possibilities are put up and not strongly evaluated, particularly given their difficulties with mathematics. They may not be able to identify which are the correct answers. Ms Nhlapo could address this by trying to make her discourse rules explicit. Ms Nhlapo might also try to refine her use of the question "why". When she asks "why" she is asking for different kinds of responses at different times, including clarification, justification and explanation.

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[9] We did not set out to investigate in a textured way what pupils were learning as their teacher changed her practice, and thus we cannot identify even slight, improvements in their mathematical thinking which may result from her increased attention to their existing knowledge. These may be there, and may make the substantial effort she is putting into changing her practice worthwhile. However, what is clear is that Ms Nhlapo's pupils' achievement and knowledge remain extremely poor. There are no obvious indications of improved learning.

The question arises as to whether the strategies Ms Nhlapo has worked so hard to achieve might be more appropriate if she set more exploratory tasks, or worked more conceptually. This aspect of her practice is one that Ms Nhlapo has not yet developed and so we do not have the empirical means to answer the question in this case. However, remembering the size of the zone between where pupils are and where they need to be, we might predict that Ms Nhlapo would face similar problems and we might be challenged to think about how to help her face them.

New challenges for teacher development

What can we learn from Ms Nhlapo's teaching which can help us to think more appropriately about learner-centred teaching in contexts characterised by pervasive and acute failure and limited pupil knowledge? I have argued that Ms Nhlapo struggles to mediate between existing and new knowledge because of the large distance between these. She makes choices to close the gap and work at lower levels, and still she does not succeed. I have also argued that there are mismatches between her pedagogical strategies and the mathematics that she is trying to teach.

An analysis of the FDE programme shows that Ms Nhlapo's achievements in her classroom come from thoughtful study and participation in the programme. The mathematics teaching course in the programme emphasises pedagogical strategies and is complemented in this respect by education courses. The mathematics content courses have helped her to develop a broader view of what she might aim to do in the classroom and have increased her confidence and competence in teaching mathematics. However, these two aspects have not yet been brought together to help her to think about pedagogical content knowledge. By this I mean, how might this teacher bring together her knowledge of pedagogical strategies and her knowledge of her pupils' mathematical thinking to help her think about the most appropriate strategies to create and mediate within appropriately sized zones of proximal development.

In order to achieve this, two key research and development projects are necessary in South Africa. First, we need to develop more accurate pictures of what South African mathematics learners know and can do at various levels in the education system, ie we need to find out about learners' actual levels of development. Researchers and teacher educators need to focus on learners if we are to enable teachers to do so. This is a substantial research project, and requires theoretical frames and research instruments which can capture what pupils really do know, in context. Second, we then need to develop, together with teachers, more realistic possibilities for teachers to work with pupils' knowledge. In “zpd” language, we need to consider how to create successive zpd's which move from where learners are to where they need to be. One example would be to refine possible teaching strategies. For example, Ms Nhlapo could benefit from reflection on her different purposes in asking “why”, and what mathematical and pedagogical function her questions serve in the zone of proximal development.

What I am suggesting here is that contextually grounded pedagogical content knowledge should be a key aspect of teacher development programmes. This is the case in all contexts. In contexts characterised by pervasive failure, particular pedagogical content knowledge is required which enables teachers to mediate large or non-existent zones of proximal development. Dilemmas experienced by teachers such as Ms Nhlapo should be foregrounded in our thinking about mediation and made the object of discussion with teachers. Hopefully in this way resolutions might be found in relation to particular mathematical knowledge in particular circumstances.

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Numeracy, numeracy, numeracy and ideology, ideology, ideology

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In this paper I examine recent changes in policy in mathematics education in England and Wales. In particular, I attempt to give an account of the educational ideology that lies behind the introduction of the National Numeracy Strategy into primary schools. I describe this ideology as a form of managerialism. I explore the hegemonic aspects of this ideology but also point out contradictions within it and within the National Numeracy Strategy, which give the possibility of a more critical practice.

"I can't emphasise enough however, how important we think this is. We have a situation where 11 year olds, many of them, do not have the required skills in the very basics they need in later life."

Tony Blair, U.K. Prime Minister, introducing the National Numeracy Strategy training video (DFEE 1999b)

On election to government Tony Blair announced his three priorities would be 'education, education, education'. In the context of mathematics education I believe the government has prioritised numeracy, numeracy, numeracy.

The introduction of the National Numeracy Strategy (NNS) in primary schools and other aspects of government education policy with respect to numeracy in England and Wales raise important questions. It represents a very direct and focussed political intervention by government into both the *content* of the mathematics curriculum and *prescription of teaching methods* and therefore helps to make transparent a number of aspects of the politics of mathematics education that are sometimes obscured.

Aims of the paper

To some extent this paper was prompted by the question 'Why is the Numeracy Strategy so politically important that the Prime Minister introduces a training video?' I attempt to address how the Strategy came into existence and what is the explicit and implicit message it gives about how children learn mathematics and what the aims of mathematics education should be.

In addition the paper has a number of theoretical aims. The development of theories of social and cultural reproduction that move beyond the reductive and economistic has been much explored (Apple 1979, Apple 1995, Giroux 1979). These have drawn on the ideas of Gramsci to understand the complexity of reproduction at the level of the school. However, in seeking to avoid 'reductionism' there is a risk of not recognising that there are occasions when the economic priorities of governments and states do directly shape educational policy. I believe that the introduction of the National Numeracy Strategy (NNS) is one such occasion. This is important because in debating educational policy we may find ourselves more directly engaged in a debate about what sort of society is economically and environmentally sustainable and can meet the needs of all people. A second aim is to foreground the concept of 'hegemony' as essential to understand current curriculum change. Thirdly, to highlight the importance of understanding contradictions in the curriculum as arising from economic and social contradictions and not simply out of the inclusion of different viewpoints. I believe, and I have only the space to assert this, that these points of contradiction are potentially the most productive sites for critical action.

The limits of the paper

I feel it is worth stating some of the limits of this paper. Firstly, it focuses on government policy and the curriculum; it does not address the important and complex issues of how the National Numeracy Strategy has actually been implemented. Secondly, it is based upon analysis of NNS documentation, video material and newspaper articles. I attempt to give an account of the origins of the Strategy without having access to reports by actual participants in the process. The third limit is the theoretical framework of historical materialism. I believe this is essential to understanding some of the reasons for curriculum change, for example their connection to the prevailing economic and social context. However, it may not be as helpful in giving direct insight into certain aspects of the political process surrounding such changes, for example the importance of parental demands on the education system, or the way the need to legitimise such changes affects policy.

The concept of ideology is central to the paper. Here I generally use it in a limited sense of a set of ideas about society and, in this context, education, held by an individual or group that are more or less coherent and more or less contradictory. A key feature of the concept is that ideologies do not exist separate from and must be understood in dialectical relationship with the groups that hold them and importantly the interests of these groups. Gramsci argued “material forces are the content and ideologies the form, though this distinction between form and content has purely didactic value, since the material forces would be inconceivable historically without form and ideologies would be individual fancies without the material forces” (1971/1929-35, p377). The importance of education is due to it being, in the language of classical Marxism, both part of the base and the superstructure of society, a ‘material force’ and a site of ideological reproduction. This in part explains why the Prime Minister takes the trouble to introduce a teacher training video.

In discussing the origins of a new hegemony in education, I touch on a more extensive concept which understands ideology as “something people *inhabit* in everyday material ways” (Lather 1991 page 2, original emphasis), “a mode of consciousness” (Giroux 1981 page 22) or how “hegemony acts to ‘saturate’ our very consciousness” (Apple 1979 page 5). Understanding ideology in this sense would be essential to analysis of how the Numeracy Strategy is actually being implemented. A further limit arises from the unavoidable fact that I ‘inhabit’ a particular ideology. It is difficult to summarise even those aspects of my ideological position that I am critically aware of, not least because it is developing and changing, let alone those aspects that I am more dimly familiar with. It does have at its centre the belief that people should be the subject of social processes and not the objects of them. I am not simply opposed to the Numeracy Strategy as means of teaching mathematics but also because it is a part of an attempt to maintain capitalism. Perhaps a small part but not an unimportant one for those directly affected by it.

The National Numeracy strategy

The National Numeracy Strategy, alongside the parallel introduction of the Literacy Strategy, is at the centre of government educational policy. It consists of a detailed description of the content of the primary curriculum in terms of the mathematics children should be expected to know at particular ages. Secondly it provides a framework for teaching both in terms of lesson structure and teaching style. Lessons are to last up to one hour daily. This in itself represents a considerable increase in the time devoted to the teaching of mathematics. There is a prescribed three-part lesson structure. Firstly, 5-10 minutes for oral and mental work. This is teacher led with an emphasis on learning and recalling facts and procedures. Secondly, the main part of the lesson that consists of teacher directed learning. Lastly there is a whole class plenary in which the work of the lesson is discussed.

The origins of the Numeracy Strategy

In 1996 the Conservative Government established the National Project for Numeracy and Literacy. The debate about numeracy (and to a lesser extent literacy) at this time centred on an implied relationship between numeracy skills and industrial competitiveness. Comparisons were made between the rates of growth in the UK and those in countries that out performed the UK in TIMMS (Keys et al 1996). A further concern was the widespread and alleged unreflective use of commercial schemes in Primary classrooms (DFEE 1999a). One of the first acts of the New Labour government was to establish a Numeracy Task Force, sponsored by National Power, a privatised electricity generating company and by KPMG, a major national accountancy group.

The National Numeracy Task Force was constituted in a very different way to the 1988 National Curriculum Working Group. At this time there were a number of competing voices as to which countries' teaching methods the UK should seek to emulate. Analysis of articles from the Times Education Supplement, the main weekly teaching journal, shows that the most frequently mentioned countries were Switzerland and Taiwan. In addition there existed an interest in Eastern Europe, most notably Hungary. A number of small-scale projects were established to explore aspects of these different methods in the UK context. Some of the different individuals that went on to form the Numeracy Task Force were associated with favouring different countries. It seems reasonable to suppose that the favoured country reflected the ideological leanings of the individuals involved. Different ideological positions can be identified with different countries' methods or at least the perceptions of those countries' methods, perceptions affected by Eurocentrism. There is a risk of simplifying the process by which the different individuals involved arrived at the detail of the Strategy, of seeing them simply as ideological representatives. It is not clear exactly how much debate took place or how important the representatives of the government in the task force were to the process. However, in comparison with the level of conflict during the introduction of the National Curriculum, the NNS has been developed with a marked lack of public dispute and has been done very quickly. This is good evidence of a new hegemony. I would argue that the task force's purpose has been to technically implement a project, which already had a clear ideological direction. The NNS can be seen as a classic, if crude managerial solution to a perceived problem: examine competitors' production methods and copy them.

Educational ideologies in the UK

In thinking about educational ideology, particularly in the UK, an important theoretical starting point is the one developed by Paul Ernest (1991). Ernest's model is of five distinct ideologies: the Industrial Trainer, the Technological Pragmatist, the Old Humanist, the Progressive Educator and the Public educator. This model was developed to understand the ideological fight between different groups within and outside the education community during the introduction of the National Curriculum in 1988/89. Using this model Ernest analysed the contestation chiefly between the Progressive Educators and Industrial Trainers with the Old Humanist and Technological Pragmatist groups playing a less important role. The outcome was a mathematics curriculum that was an ideological compromise.

Ernest's model is rooted in its identification of particular ideologies with particular social groups. The Industrial Trainers represent those industrialists concerned primarily with the production of a compliant and willing workforce. The Technological Pragmatists are those capitalists concerned with the need for a more highly skilled and flexible workforce. The Old Humanists can be characterised as the establishment elite concerned with the cultural aspects of social reproduction. Ernest develops a model that identifies how these ideologies differ over key features of what mathematics education should be, such as the view of mathematics, the implicit and the theory of teaching and learning (for details see Ernest 1991 and 1998).

Ernest recognises that there are weaknesses in the model. In particular that it is mono-dimensional ignoring the possibility of overlaps between different ideologies (Ernest 1998). The model does then allow an account of contradictory consciousness within educational ideologies to be developed. In considering the recent new curriculum for Primary Initial Teacher Education, Ernest concludes that it represents the dominance of Industrial Trainer ideology with Technological Pragmatist and Old Humanist ideologies having less influence. Ernest also points out the underlying managerialism and market metaphor in the curriculum documentation (Ernest 1998 pages 164/165).

One strength of the model is the way it explains the events leading to the introduction of the National Curriculum. Ernest's view is supported by and explains various accounts, including those of some of participants in the process (Noss 1990, Dowling 1990, Graham and Tytler 1993, Brown and Johnson 1996). The model is also supported by more general explanations of the ideology of the Thatcher government of the time as a coalition of various groups with differing ideologies (Chitty 1989, Gamble 1988). The model is intended as an abstraction. However, I wish to contend that whilst it is one that fitted well the ideologies prevalent at a specific time, the outcome of the contestation between the ideologies was a new hegemony that has features of the three dominant ideologies but is different from each. This is in part a reflection of a wider new hegemony in society that in terms of policy is best characterised by the managerialism that Ernest identifies.

Hegemony and contradiction

Hegemony is central to the Gramscian theory of ideological contestation. Hegemony for Gramsci is not simply about cultural dominance, he states that "though hegemony is ethico-political, it must be economic, must necessarily be based on the decisive function exercised by the leading group in the decisive nucleus of economic activity" (Gramsci 1971/1929-35 page 161). There is a danger in crudely applying a theory developed to understand the contest between classes and organisations and ideologies based on class relations to changes in political policy. However, if we survey political developments in the U.K. over the last twenty years there has been a dramatic change in the terms of political debate. I discuss later the nature of the current ideology of the New Labour Government.

I wish to highlight two important features of how hegemony arises. Firstly, the process of gaining hegemony leads to developments in ideologies. This is partly due to the dialectic of contestation between ideologies and partly due to the need to gain legitimacy, for the hegemonic ideology to be accepted and believed. Thus the call for 'back to basics', that Ernest identifies with the Industrial Trainer ideology, features in current political discourse not simply because it is a call to meet the perceived needs of a powerful group of capitalists but also because it appeals to parental concern about the quality of their children's education. It should not be a surprise that we find that the current hegemonic ideology in education has features of more than one of the dominant ideologies that Ernest identifies as it is in a sense a synthesis of them.

The second feature of understanding the development of hegemony is the importance of the relationship between 'doing' and 'thinking' in the creation of ideology. In the context of recent educational changes this points to the importance of how structural changes, for example the National Curriculum, the impact of league tables, the inspection regime and increased testing, lead to changes in what teachers *do* and therefore mean they are more open to accepting the hegemonic ideology as what Gramsci terms 'commonsense'. The National Numeracy Strategy is, in a sense, a mirror of what is already happening, of what is already believed by many about the teaching and learning of mathematics and not simply a new imposition.

It is also important to recognise that consciousness and ideology can be 'contradictory' and are likely to be so given the complex patterns of relationship to structures of various sorts and the position individuals and social groups have within them. In particular educational ideology in capitalism is likely to be contradictory in terms of the ideologies held by dominant groups that arise out of the relationship between education and social reproduction. There are two inter-related aspects of social reproduction through education: firstly, cultural reproduction and secondly the skilling of future labour. Put crudely, the needs of cultural reproduction may require children to learn to obey but there may be an economic need for labour that can work independently and creatively.

The ideology of New Labour

New Labour is a new political formation that is still evolving and is as yet under analysed. Its ideology has been formed in response to the hegemony that the ideas of the New Right gained in the eighties, that is economic liberalism, preference for market solutions, reduction of the welfare state and conservative social policy. New Labour represents the reconstruction of social democracy in the UK in acceptance of these ideas but, as such a reconstruction, it is not simply a continuation of the New Right.

Tony Blair has labelled his ideology 'the third way'. The key feature of the third way in the context of education is the importance given to human capital as the key to economic competitiveness rather than machinery or plant (White 1999). Even those aspects of policy, which reflect the legacy of New Labour's reformist past such as the aim of reducing social exclusion, prioritise economic needs over those of the individual. For example by compelling the unemployed, disabled and single parents to take up low-paid employment or training schemes under the slogan 'welfare into work'. Generally, moralistic pronouncements and a form of opportunistic populism mark social policy, for example in attacking refugees' rights or the parents of school truants.

The Education Minister has given a succinct statement of New Labour's rationale for educational policy:

" With increasing globalisation, the best way of getting and keeping a job will be to have the skills needed by employers, the flexibility to adapt and above all the confidence that comes from these attributes.... [T]he United Kingdom will need a myriad of skills underpinned by rigorous grounding in numeracy, literacy and communications." (Blunkett 1999)

Here we see the key features of New Labour's policies - firstly a focus on the needs of the economy, the concern for 'basics' and the need to compete internationally. These translate into more detailed educational policy as an ideology that is best described as managerialism. It takes the methods of modern industrial management and applies them to the classroom. Key features of this ideology are a high level of control over the production (learning) methods, production objectives in the form of target setting, the teacher as a manager directing the learning process, the aim of learning as the acquisition of skills that are transmitted and a concept of learning as being hard work.

However, the legacy of New Labour's reformist past means that it also emphasises the need for social inclusion, although how far the attacks on teachers' 'poverty of aspiration' for working class children is cynical rhetoric is an open question. What is clear is that there are differences between the managerialism of the NNS with its emphasis on whole class teaching and the managerialism that has been identified in for example the U.S. with an emphasis on individualised learning (Apple 1995). Speculatively, this may be linked to the differences in the political traditions within the two countries.

The ideology of the Numeracy Strategy

Here I illustrate New Labour’s managerial ideology with reference to the Numeracy Strategy. An important aspect of this ideology is an emphasis on control. The NNS not only prescribes *what* is to be learnt but *how* it is to be learnt. Prescription of teaching methods to this degree is a new feature of education policy within the UK. Schools are provided with planning grids which detail what topics should be covered in each week of the school year. Three weeks per year are unallocated but teachers are advised to use these for assessment. In order to ensure that the NNS is implemented, 300 Numeracy Consultants have been employed. Notice the term ‘consultant’, a term borrowed from management discourse. These Numeracy Consultants themselves are given detailed descriptions of exactly what they are to do, including what appear to be sales scripts for marketing the strategy in schools (DFEE 1999b).

There is an emphasis on target setting. Overall there is a national target for improvement in national test scores. This is translated into targets for local education authorities and schools. In turn the class teacher is advised to set targets for individual children. The suggestion is that two days per half term should be used for assessment tasks. The lesson structure itself is based on discrete teaching objectives. The training materials for the NNS propose and exemplify teacher language such as “today I will show you how to...” or “today I will teach you...” (DFEE 1999b).

The teaching strategy is based on teacher directed learning with the teacher as the manager of the class and the key knowledge holder. The elements of “good direct teaching” are said to be (in this order) directing, instructing, demonstrating, explaining and illustrating, questioning and discussing, consolidating, evaluating pupil’s responses, summarising (DFEE 1998 page 11/12).

Mathematics is seen as a hierarchical body of knowledge. The importance of learning facts and being able to recall them quickly is emphasised. The name Numeracy Strategy is partly misleading as it contains all the aspects of the primary mathematics curriculum. However, there has been a marked shift in the balance of the curriculum. The Framework has five strands, each with a number of sub-strands (in brackets); numbers and the number system (5), calculations (6), solving problems (3), measures, shape and space (2) and handling data (1). The narrowness of the curriculum is, if anything, worse than these figures suggest, as solving problems consists of “Making decisions: deciding which operation and method of calculation to use; reasoning about numbers or shapes; solving problems in context; ‘real life’, money, measures” (DFEE 1998 Page 39). Of the 17 sub-strands of the Framework only one does not mention number or is not related to number.

Above all, learning (and teaching) mathematics in the view of the NNS is hard work. Primary children will now spend up to two hours a day (taking into account the literacy hour as well) in a strictly controlled learning environment. The government has produced guidelines for the amount of time children, including primary children have to spend on homework. This is against a background of discussion about the lengthening of the school year, the introduction of after-school classes and summer schools. Even reception children (age 4 years) are not exempt from the strategy as a supplement has been included detailing what they "should know".

Contradictions

The managerialist ideology and the National Numeracy Strategy are rooted in the contradictory needs of capitalism with respect to education. This leads to contradictions in the Strategy which do allow some scope for a more critical approach within its structure. There is space only to touch on this here.

New Labour's aim is to manage capitalism in the light of technological change by emphasising the importance of human capital but this leads to contradictory objectives. As Richard Noss (1998) points out, the definition of numeracy used in the National Numeracy Strategy is narrower than that used in 1959 and does not serve as a basis for the sort of numerical and mathematical understanding that those new technologies require. It is ironic that both the strategy and the draft new National Curriculum (QCA 1999) minimise the importance of data handling. Yet in devising Numeracy tests for trainee teachers one of the specific 'numeracy' skills teachers are said to need is the ability to interpret statistical information produced by the government about educational standards.

The need for a different sort of mathematics to be learned and for the power of alternative approaches is implicit in some parts of the Strategy documentation. The plenary session, for example, can be used to "ask pupils to present and explain their work" or to "discuss and compare", "discuss the problems that can be solved" and "make links to other work" (DFEE 1999a page 15). The approach to calculation itself, which has an emphasis on mental approaches and non-standard written methods, potentially requires a high level of discussion not only about algorithms but also about the structure of the number system as well.

In addition to the contradictions within the Strategy itself there is also the complicated question of how it will actually be implemented in schools. Ideology is not simply transmitted within educational structures or through them. As Henry Giroux, has put it we need to focus on "the dynamic nature of the antagonistic relationships that actually transpire at the day-to-day level of schooling" and explore the "concept of resistance" (Giroux 1981 page 29). However, given that the ideology behind the strategy is hegemonic we should not expect, and have not witnessed, open defiance. Teachers have an unusual role in being workers whose work is, from the point of view of capitalists, to produce workers. The education system is based on compulsion and children are the objects of the process. At the same time teachers themselves are oppressed as workers, they too are the objects of compulsion and their work is commodified. The transparency of this commodification is increasing. This contradiction can lead to different sorts of action. At best teachers may defend themselves and children from the worst aspects of capitalist policies in education, for example the teachers' boycott of national testing arrangements in the UK in the early nineties. At worst it can lead to the scapegoating and pathologizing of ‘bad children’, for example at the Ridings School and similar cases where teachers have taken strike action or threatened it to have children excluded from school (TES 1996).

The ideology that informs New Labour’s education policy is hostile to both individual and social emancipation. But if it is to be resisted or subverted it is important to recognise that it *is* hegemonic and therefore powerful but also that it is contradictory. These contradictions arise from the synthesis of the different interests that have influenced its formation, the need to legitimise the changes in policy and practice but also as a reflection of the contradictory forces in global capitalism. I contend that the National Numeracy Strategy is irrational with respect to its implicit and explicit aims and this irrationality arises from the irrationality of capitalism itself. These contradictions allow the possibility of resistance to its worst aspects but also to the potential legitimisation of the uncritical teaching of mathematics that has been prevalent in many UK Primary schools. Pessimistically, there is anecdotal evidence that it is leading to an increase in setting and grouping by ability for younger children. However, the managerialism of New Labour does mean that it is explicit about its aims and goals, its ‘mission statements’. As a result, as the National Numeracy Strategy is implemented, there is the possibility of a more open debate, than was perhaps possible in the past, with teachers, parents and students about the purposes of mathematics education.

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Mathematical thinking: bringing together alternative perspectives

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This symposium arises from the work of the project 'Teaching and Learning Mathematical Thinking'[1]. This project aims to bring together a range of theoretical perspectives, including socio-cultural, semiotic and sociological theories and methods, in order to illuminate school mathematical activity. The overarching research question is: What characterises mathematical thinking? Within this, we are focusing on three themes: the recognition and legitimation of mathematical thinking; conceptualising relationships between mathematics in school and in other contexts; the role of affect in development of mathematical thinking.

In the two sessions of the symposium, the members of the research team will report on the ongoing work of the project and illustrate some of the ways in which we have worked to bring our various perspectives to bear together on the research themes. We will invite participants to join our conversations.

Session 1: After an introduction to the project and an overview of the theoretical perspectives from which we approach our research questions, we will illustrate one of the ways by which we bring together and synthesise our various perspectives. This illustration will take the form of a 'conversation' about assessment.

Session 2: Another way we have worked is through bringing together alternative approaches to and analyses of specific situations of mathematical problem solving. Participants will be invited to join with us in analysing and interpreting a transcript of such a situation, focusing on the themes of affect and relationships between different contexts (including what is sometimes called 'transfer'). The transcript will be provided in advance of the session.

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THE RELATIONSHIP OF CHILDREN TO MATHEMATICAL KNOWLEDGE AND ITS ALIENATION THROUGH AND BY THE SCHOOLING EVALUATION PRACTICES

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Abstract

Adopting a relational perspective on learning, knowledge and education, and interpreting preliminary evidence derived from an ongoing research, it is claimed in this paper that children's relationship to mathematical knowledge is initially constructed as a concrete "use-value" in daily life pragmatic activities, but through, and by the school evaluation practices it is gradually transformed into a relationship of an abstract 'exchange-value' for current and future educational use. Such a relationship of children to mathematical knowledge bears the fundamental characteristics of an alienated relation which in turn shapes conformably their mathematics learning activity.

Background Orientations

This paper stems from an ongoing research on children's learning of mathematics within the context of formal schooling practices both conceived from a relational perspective. Main lines of theoretical background orientations of this study, as well as an interpretation of preliminary findings are briefly presented. The reported account, however, is inevitably fragmentary and incomplete, not only because of the space prerequisites for a conference paper, but mostly because, trying to overcome a subjectless sociological and an unsocial psychological approach to the subject matter, many theoretical and methodological components of this venture are repeatedly reconsidered and revised as, in the course of the empirical research, they are being put to test

The point of departure is a tangible necessity for approaching learning as an individual and, at the same time, as a social, activity which as such, incorporates and, at the same time, is incorporated into a cluster of determined social relations. It is anticipated, that analysing learning from such a perspective wiil make it possible to arrive at an explanatory theoretical basis for developing actual practice in mathematics education.

A Relational Perspective

It is an inherent feature of the modern Western mode of academic thinking, and as a consequence it is well established as a legitimate and appropriate practice in the theory and research of education, to operate with various kinds of separation in mind, fragmenting and slicing the concerns and topics of any particular inquiry. Mainstream psychology for example, which fosters to a large extent educational theory and research, treats cognition, perception, emotion, learning, motivation, memory, etc. separately as distinguishable from one another and discernible units of study; the same holds true for sociology and many other disciplines which permeate educational theory and research.

The manifold ways in which these isolated and differentiated units are linked, either as mutually dependent wholes, or as components in some larger whole, are afterwards attempted to be shown. Any such attempt at linkage, however, remains ultimately grounded within a realm of separation. It endeavours to link and articulate what are still conceived of as conceptually separate and distinguishable features of reality. Mathematics education, we are particularly concerned, is not an exception to this prevailing mode of thought, which may also be seen as a central feature, not only of particular processes of 'differentiation and specialisation,' but also of broader issues of human alienation.

This realm of separation in mind has been addressed by a philosophy of “internal relations” which is developed in Marx's writings and is employed to express a conception of things and their inter-relations. Interrelations which are not, however, fixed but changing. A version of this philosophy elaborated by Ollman (1971) has been adopted as a general founding framework of the mentioned research project.

The founding thesis of Ollman's treatment of internal relations, as a prelude to a discussion of children’s relations to mathematical knowledge and their alienation through and by particular schooling practices, may be epitomised in the following way. Ollman argues, that for Marx, concepts are many-faceted and their sense depends on the relations which may be considered to exist between their different components. Thus any subject-matter for inquiry must be conceived of 'relationally.' In Marx’s writings, for instance, capital, labour, property, value, commodity, etc. are all grasped as relations, so that capital contains labour within it as an internally related component and vice-versa. Relations are considered to be internal to each 'factor', and when a relation alters, the factor itself alters. According to Marx, for example, if wage-labour disappeared, that is if the workers' relation to capital radically changed, capital as such would no longer exist. In such a framework, each factor (e.g., property, labour, product) has the potential to take the names of others (e.g., commodity, production, value) when it operates as they do within a relational whole. In other words, each factor is, according to Marx, internally related to its own past and future forms, as well as to the past and future forms of surrounding factors. By the way, it must be emphasised that although the use of terms such as 'factor' or 'inter-relation' may call to mind the vocabulary of functionalism, such a similarity in glossary conceals fundamental differences, as functionalism, on principle, approaches reality in terms of discrete, isolatable entities, and consequently conceives categories in a confinable, tangible way which in practice under-emphasise time, change and movement in the world leading to an emphasis on system stability rather than social change.

Subsumed in such a relational perspective is the importance of context (for this factor, in this context, this is the influence most worth noting), history (each social factor is related to its own past and future forms), and dialectic (things may be viewed as moments in their development in, with and through other things) emphasised by Marx at many instances.

Learning, Knowledge and Education

In the relational perspective adopted in the mentioned research venture, human learning is conceived of as a reflexive process involving the individual psyche in relationship with the surrounding natural and social world. Thus, learning is primarily approached as a subjective process which demands direct and successful experience of purposeful activity upon the real world, and in this sense, it is self-determined, in contrast to the neural activity which characterises animal adaptive learning. At the same time, human learning incorporates and is incorporated into social relations, so as to produce, via symbol-referent language, the capacity to imagine a range of alternative purposeful actions.

Knowledge is understood as the social organisation of individual learning into objectified systems and principles. Thus, its characteristics vary with the historical character of social relations at any given time, and its structure is itself a feature of those relations. Knowledge is, therefore, not originally conceived as an object, but as a set of relationships among human beings and between human beings and the natural and symbolic universe; relationships which become objectified as knowledge in the conventional sense. This is understood by an appeal, in principle, to Marx's notion of 'objectification of the subject' through the co-operative subjugation of nature involved in original human productive activity. The psychological effect of such activity is that man in subjugating nature by a co-ordinated effort 'changes his own nature'(Marx, Capital, I, p. 177-178). This premise provides a material basis for human learning as well as for its objectification as knowledge.

Education, in the form of schooling, is considered as the social organisation of the subjective learning process in a defined historical situation and in response to a specific mode of social relations. In contemporary society, education is an aspect of the division of labour preparing individuals for the dominant class relations. Since it is a social process, education is also constituted of defined social relations, and these relations may change to some extent independently of the general social relations. The appreciation of the material as opposed to the abstract character of education, requires the awareness of the basic involvement of social relations in human learning and, therefore, of the inextricable connection between the social-psychic process, learning, and its social organisation, education. It is precisely this conjunction between the material nature of learning and its social organisation as education, which, in fact, needs an elucidation.

As the social organisation of a subjective learning process, education can be seen to form part of the established general social relations and, at the same time, to consist of specific social relations. Human beings learn from other human beings, directly or indirectly - either by having their learning systematised, or simply by using the social invention of language to construct and reconstruct concepts of the physical and social universe. In education, learning is systematised, given a social organisation which reflects epistemologically the social relations of the society, but it is also made subject to specific, personalised social relations. The social organisation of learning represents the social relations *of* education, which under capitalist social conditions are exploitative, appropriating the mental activity of the many to the purposes of the few. The specific, reflexive relations are those generic to learning and they are the definitive social relations *in* learning. These are obviously related to the general social relations, but they are not indispensably determined by them. Under certain circumstances, the dominant purposes of capital are deflected, and conflicting purposes are incorporated into social relations within education. Such an analysis of education (Holly, 1977) is presumably indispensable to explain the fact of historical development itself.

Learning - which in a sense is always social learning - either reproduces existing social relations or projects new ones. Its systematic organisation, as education, contains always the possibility of conflicting versions of the world, the success of one or other of which rests upon the dialectic interaction between specific learning relations and general social ones. As a result of this outcome, consciousness is either changed or maintained, but, in any case, the purposive activity of individuals is involved, though some such purposes will be more influential than others.

Since education, the social organisation of learning, is based on one or another specific theory of knowledge, arising out of one or another specific mode of social relations, it is important to grasp some such general or basic conception of human knowledge and its material derivation; otherwise we are in danger of either assuming one particular historical practice to be fundamental (as is done by many traditionalistic philosophies of education) or floundering in a morass of subjective relativism (as is done by many phenomenological approaches to knowledge).

A viable approach to knowledge needs, on the one hand, to take into account the historical character of objectified knowledge and, on the other, the nature of the social relations temporarily determining a given social stratification. Thus, it is necessary to posit, a priory, a general basis for all human knowledge, one which can be seen to be independent of the particular interests of a particular human group. In this account, a material basis is intended as implied by Marx, that is one which takes the species-being of man as founded on a humanising productive activity of men and women in social co-operation. In other words, it is proposed that knowledge is the objectified learning relationships of human beings to the real world and to one another in the subjugation of that world to their purposes. If this is considered to be the species-nature of human knowledge, its historical character, as temporarily determined by the present system of social relations, is different. Under capitalist social conditions, and as a consequence of the elevation of the division of labour under capitalism into an all-embracing principle of social organisation, knowledge appears as alienated from the subjective learning of individuals altogether and as existing in the form of an independent entity. (Marx, Capital, p. 354). It is precisely in the character of alienated knowledge, and particularly of 'scientific' knowledge in its modern sense, which is alleged to be neutral of social interests, that we must look for the alienative aspects of the present social organisation of learning.

Having presumed the basically relational nature of human learning, it is asserted that the social organisation of these learning relations, as education, is subject to the same conditions as the social organisation of human labour. In fact, it is asserted that learning can become alienated through, and by, formal educational processes. In so doing, the question of how this state of affairs can change, and to what extent, independently or not of changes in the general social structure is indirectly arisen.

Children's Relationships to Mathematics: Research Preliminary Evidence

As already mentioned, this paper capitalises on preliminary evidence derived from an ongoing research that aims to explore the formation and development of children's relations to mathematical knowledge during their primary schooling, under the influence of predominant schooling practices. The inquiry is in progress in the context of formal mathematics classrooms in two primary schools of Thessaloniki, Greece, employing an essentially ethnographic perspective. Additional research approaches include, according to the case and the purpose in hand, action research interventions and case studies.

About twenty children in each grade of each primary school (total 250 children) are the subjects of the study, having been followed for two consecutive school-years up to this day. Most of their parents are also occasionally involved contributing required data. Their teachers (18 men and women) are actively participating in the project observing, questioning and registering the relevant data according to pre-established protocols.

A clinical technique of inquiry is applied to probe children's relations to mathematical activities, and to the personal knowledge produced by these activities. It is presumed that aspects of these relationships are being revealed to a sufficient extent by verbal statements that children express, as well as by non-verbal modes of behaviour that they manifest, during reflective discussions on, or in the course of their engagement in, mathematics classroom tasks and homework.

Two interrelated initial findings from the inquiries carried out up to the present are utilised for the present form of analysis.

The first, derives from a retrospectively composed history of children's relation to mathematical knowledge as a product of their mathematical learning activity; it concerns the transformation of value ascribed by children themselves to their mathematical knowledge, already constructed, under construction or in prospect. As "value" is considered here a specific relation between the child and his/her activity, product and other children (Marx, 1965). The children under inquiry, when in lower primary school grades, are found to appreciate their mathematical knowledge mostly as a concrete 'use-value' in daily life pragmatic activities, such as, for instance, playing, shopping, cooking or travelling. By "use-value" is meant a power of mathematical knowledge to satisfy some human need. As children, however, move on to upper primary school grades, they are inclined to conceive mathematical knowledge mostly as an abstract 'exchange-value' in their current and future educational or professional career. By "exchange-value" is meant a power of mathematical knowledge to relate to other products or services on a measurable basis, i.e., a trading power.

The second finding is yielded from a qualitative analysis of children's behaviour instances being evidenced, and their related thoughts being verbally expressed, during mathematical activities that they were carrying out in their classrooms, as well as in their homework. The relationship of children to their mathematical activities seems to be initially shaped by their personal knowing interests and needs, but gradually, as children move up school grades, it is apparently being more and more regulated, and in the end it is essentially transformed, by the influence of the (indirect and direct) school evaluation demands and requirements.

For children attending upper primary school grades, school evaluation standards have been singled out as the most prominent factor dominating on their comments, and most frequently being referred to as a criterion, whenever they are asked to give reasons for particular choices and decisions made in regard to characteristics of their mathematical learning activities carried out, not only in their mathematics classroom, but also in their mathematics homework.

Parents also invoked mostly school evaluation standards and implied credits, when they were interviewed on their stance towards facets of their children's mathematical learning activities either in classrooms or at home.

On the contrary, children attending lower primary school grades provided comments on, and explications for, their relevant modes of behaviour mostly referring to personal criteria, practical or emotional, such as, for example, working convenience, individual pleasure, curiosity, self-confirmation, etc.

Learning, Knowledge and Evaluation: The alienation of children's relationship to their intellectual activity and its conceptual products

The elaborated perspective to learning, knowledge and education involves emphasising particular influences in particular contexts, their history and their interdependence. For the purposes in hand, in this context, against the background of the particular research evidence, 'evaluation' has been singled out as the schooling practice which predominantly shapes the relationship of children to their mathematical activity, the conceptual products of this activity and other people engaged in this activity, inscribing to it the fundamental characteristics of an alienated relationship.

It is through the workings of the evaluation practices that the mathematical work of children in school may be represented as a feature of commodity production. It is suggested, that it is possible to see teachers and pupils, producing or reproducing mathematical knowledge through their work in school mathematics. This object, knowledge, embodies the subjectivity of both of them, but through commodity fetishisation the relations of subject and object are ultimately inverted. Children, as all human beings, inevitably objectify themselves through forms of their work. By submitting the product of their mathematical work to an evaluation which deals with it by treating it as a unit of production to be ranged alongside and compared with the work of others, the children learn to see this work in terms of exchange-value. Their work thereby loses its individual purposiveness. Its relation to their needs and intentions is submerged by being given a 'natural price' in relation to the work of others through the intermediary mechanisms of grades, marks, etc. It can be here hinted at Marx's more general sense of the alienation of labour, which he considers as "giving up the use-value of one's productive activity; the most important of all human functions is put under the control of another" (Ollman, 1971). The object is taken away in the sense that mathematical knowledge is seen as other than the children's product; it is dependent on others, selected by curriculum designers, registered by text-books writers, evaluated by school examiners.

However, the alienation manifests itself prior to the actual offering of the conceptual product by the children to the school evaluators. Alienation resides in the children beginning to work with the notion of evaluation, and thus the notion of exchange, in mind. It is not that the children are necessarily working for themselves when doing their mathematics tasks and then they become alienated when they present their product to be exchanged in the market of record-cards or mark-sheets. Rather, they are alienated in beginning work with the perception in mind of treating their school work as a commodity. School mathematics work is truncated to exchange-value. Parts of the being of children are split off and undergo their own transformation. This means that only certain aspects of themselves are emphasised and developed at the cost of their wholeness. Thus children turn their school mathematics work into a commodity. They treat it as a form of private property to be disposed of through exchange.

That the conceptual product of mental work, knowledge, also becomes a commodity means that it is treated not as an object created through children's (and teachers') social, creative activity, but as something abstract and possessing an existence independent of them. Or, in other terms, the curriculum is seen not as practice, the outcome of human production, but as 'fact' (Young, 1975). Again, in Marx's words, 'the social character of man's labour appears to them as an objective character stamped upon the product of that labour'. In this context, the school evaluation is ordinarily considered 'useful' as it is through it that everything that is ours becomes saleable. Abstract, objective, universal, standardised procedures are used to return to the human producers the value which their products has. In this process of fetishisation the original producing subject is reduced to the level of an object to be bought and sold in the market like any other commodity. The object, knowledge, that children and their teachers produced, becomes the abstract subject. Knowledge thus functions as if it were a subject, and its producers become the object. The creative potential inherent in individuals is neglected.

Given this view of children's school work and knowledge production as a commodity, the children as pupils come naturally to be regarded as passive recipients of 'banked' knowledge, as it has been, in many ways, evidenced by teachers and parents participating in our research. Production produces not only the object, it creates a specific manner of consumption; it creates the ability to consume as a need. Into this knowledge they are initiated by someone who 'knows' and literally possesses the 'subject'. Teaching and evaluation may then become matters of efficient technique and calculation. As with the calculation of wages as exchange values in the labour market, the 'value' of children's work is seen to be calculable according to technical laws and rules of procedure. The social relations of evaluation are mystified through the generation of models of child development, learning theories and evaluation schemes. These are used to explain how the exchange process was carried out and the basis on which marks or grade-value are attached to the work of children as pupils. Elaborated theories are used to address the different abilities and capacities which children bring to bear upon their school work. Complex forms of tests and measurements are devised to make the evaluation of this work finer, more discrete and fairer in relation to the ages, aptitudes and abilities of children. And all of this within a context of reified forms of knowledge which are seen as providing the epistemological grounds on which children's school work can be evaluated and given its 'natural price'.

In this account it is possible to see the internal relationships between learning, knowledge and schooling practices as constituent parts of a whole, incorporating and at the same time being incorporated in the prevailing mode of social relations, and to trace the translations of constituent dimensions of the one into constituent dimensions of the other. School evaluation, for example, does not merely have connections with, relate to, or lead to the social division of labour; it is internally related to the social division of labour, in the sense that it contains it. This is not to argue some kind of mechanistic 'correlation' between school evaluation and location within the occupational structure, but rather that such an evaluation practice is predicated upon the social division of labour. Alienation appears whenever the division of labour is the operative principle of economic organisation. If we accept this supposition and view education as a mode of production, children's activity in schools can then be seen as a relation and expression of activity in society.

The division of labour is part of a complex which includes private property, exchange, social class divisions. A brief argument on the division of labour in education, on how knowledge is conceived of as private property, and on the relation of knowledge to class divisions is indispensable. It has been suggested by several writers that in the educational context 'capital' can be thought of as cultural capital, and that the prevailing view of knowledge in our society is as if it was private property (Bernstein, 1971, Bourdieu, 1973). The transformation of the relationship to knowledge into a property relation, however, is not the realisation of human personality but its negation. The knowledge that is related to economic rewards - high status knowledge as it is considered to be mathematics - is kept in such a way that it entails non-possession by others. Children working privately at their desks, putting their arms round their work to hide what they read and write was a stereotyped evidence of our research. Access to mathematics is limited to a selected few, and, even where access to it is given, pupils are usually evaluated in such a way that they are presented with a selection of mathematical knowledge which gives them only restricted access to the subject matter. An abolition of the relations of private property will mean the abolition of the conditions which produce and reproduce the private proprietor.

As it has been argued, the prevailing conception of knowledge inverts the relations between the human subject and the world of object, and thus transforms knowledge from an object of the will into a master. To say this is to say that children become, ultimately, a predicate of knowledge. In schools, therefore, it is the conceptual product, knowledge, that determines what the pupil does. The power of any of the worker's products over the worker, Marx wrote, always reflects the power of the people who dominate it and use it as an instrument. Thus, it should not be surprising that children come to dissociate knowledge from themselves, and that some of them see themselves and teachers as opposed to one another. This split is analogously an expression of reified social relations.

Concluding Comments

The purpose of this paper was to report on a developing framework for approaching children's learning of mathematics within the concrete context of formal schooling practices both conceived from a relational perspective.

Although in view of mathematics education, it may appear to have nothing more done than providing a critique to school evaluation. In one sense, it may be so, but it has been attempted to approach the multiple relationships between learning, knowledge and education conceived of also as relations, within which mathematics education is embedded and constitutes a fundamental component. In this account, it has been attempted to locate a crucial aspect of education, the school evaluation practices, within the forms of learning relations which are moulded by these practices and to some extent within the forms of social relations which ground their application.

Mathematics by its distinctive self-referenciality camouflages the value-laden and socially biased character of many schooling practices; it thus has been unquestionably established, more than any other school subject, as a privileged domain for the application of school evaluation. Mathematics education, as a result, constitutes a field where many aspects of alienation of the children's relationship to their conceptual activity, its products and to other people, become first and foremost evident.

It is clear that a number of questions which are of central importance have not been considered in this paper. By employing a philosophy of internal relations as an organising principle and thus treating learning, knowledge, etc. as forms of each other, as expressions of a common totality little space have been left to deal with other, and perhaps even more important aspects, such as for example the relationship between the arguments set out and the material base of the mode of production .By focusing on alienation it has not sufficiently stressed the importance, for example, of economic contradictions, and the materialistic conception of history.

In conclusion, it is claimed in this paper that a relational perspective is a necessary part of comprehending the current reality of mathematics education, in an effort not merely to grasp, record and analyse its problems and contradictions, but to challenge prevailing conceptions of that reality.

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Title:

School maths in 'themes': moving beyond 'traditional' and 'progressive' pedagogies?

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Abstract: *Recently there has been quite a lot of attention on renewing teaching styles and lesson organisation in maths classrooms. In this realm, the use of 'themes' in varied forms is often associated with a child-centred ideology and has become largely unpopular with a back-to-basics agenda. As such, it tends to loose grounds in teaching practice. This is not unusual in current school settings where maths teachers are in low supply, teachers are under a continuous pressure for accountability and change and argue '..that they have no time for doing interesting mathematics' (see Lerman, 1998, p.1/18). This paper discusses the issue of employing a thematic approach to mathematics teaching by using the contrasting cases of traditionally and progressively oriented teachers. The constructs of 'traditional' and 'progressive' have been widely used to denote diverse teaching styles and pedagogies. This can create polarity when it serves to 'label' teachers and to pathologise their practice. In this context, 'traditional' and 'progressive' serve as analytic units for exploring contrasting cases of practice. As such they can enable analysis of embedded meanings and reflection on the limits of these espoused pedagogical orientations. Findings in this study support the argument that teachers' espoused pedagogy influences the math lesson organisation around a 'theme', but leaves ineffective the ways they address mathematical content.*

Introduction: school maths and 'themes'

The beneficial role of theme-based mathematics is well documented. For example, Boaler (1997) has recently shown that a whole school culture for project work is a major factor for all pupils' meaningful experience of mathematics (and especially female and lower social class). And a number of research studies have focused in identifying and describing the teaching and learning processes entailed in the classroom implementation of such contexts (e.g. Treffers (1987) on 'realistic mathematics', Bussi (1998) and Boero (1992) on the use of 'common sense activities', Jaworski (1994) on investigations, D' Ambrosio (1985) and Gerdes (1996) on ethnomathematics). The focus in these studies has been on charting and unravelling the complexity of dynamic classroom interactions and on characterising the nature and quality of strategies. These often exemplify patterns of negotiation of mathematical meanings and argumentation as well as sophisticated monitoring of pupils' learning. Differences exist in the above projects, not only in terms of analytical and methodological frameworks but also in epistemic conceptions of what should be the goals and the potential of mathematical activity. For example, the nature of tasks is often seen as a regulative procedure carefully designed to enable pupils to 'read' the mathematics in the offered thematic context and to organise and systematise their thinking towards mathematising (Treffers, 1987). Whilst others claim that understanding the significance of mathematics should come through understanding the features, structure and significance of the task itself (Christiansen, 1997), and that the 'theme' in the task should provide opportunities for a critical reflection on mathematics itself (Skovsmose, 1994).

However, participation in certain institutionalised mathematical practices, such as lessons, can create stories of not so much of an effective practice as some projects describe. Research which takes a sociological view into account recounts a different story and seems to point out that by and large, such outcomes do not map the reality of typical classrooms. For example, Paechter (1995) in a study of teachers' use and assessment of interdisciplinary work, identifies that teachers are not always able to integrate meaningfully a subject area which differs from their teaching. This needs to be seen not only as a difficulty for conceptualising the relevance of 'themes' as linkages across subjects, but also as a matter of organising workload and collegial dynamics amongst subject departments within the school, often referred to as the 'micropolitics' of curriculum innovation (see Ball, 1987). Further, Dowling (1998) based on a sociological analysis of textbooks, has pointed out that 'themes' can easily serve either for a superficial exploration of both mathematics and theme (e.g. cooking as a referent for mathematics), or for mythologising mathematics itself as a universal and powerful language that can be easily and unproblematically used to describe and explain almost every aspect of our lives (see Dowling, 1998: myths of reference, participation, emancipation: pp. 4-17). Overall, the meaningful incorporation of a 'thematic' approach in the school maths curriculum is a complex matter and its success depends on several factors such as; the teachers' experience and motivation in integrating knowledge of a thematic area with subject knowledge; flexibility within the school organisation that would allow co-operation amongst different subject teachers (and school departments); the choice of 'themes' that are relevant to teachers' and pupils' interests and lives; the representation of 'themes' in mathematical practices (e.g. teaching materials, teaching) in ways that do not mythologise the embedded mathematics.

The study: questions, teachers, analysis

As mentioned above, although manifestations of 'good practice' are important to be charted and reported, it seems equally important to know about how 'typical' teachers cope in their everyday reality and what are the constraints and conflicts which they face. Thus, instead of creating a specific plan for teachers' intervention (e.g. as in teaching experiments) or specifying a collaborative agenda that reflects a particular philosophy in using theme-based mathematics, the present study focused on the uninterrupted observation of teachers' teaching. The questions that drove this study were kept purposely open: How do teachers appreciate, value and incorporate a 'thematic' approach in their maths teaching? How do they use the 'theme' in their lessons? How do they address mathematics within the theme? Are there differences amongst teachers in the ways they use and/or transform 'themes' in their maths lessons?

A set of activities (based on the theme of art and geared towards teaching geometric transformations) were offered to a small number of experienced teachers working in two rural comprehensive schools, for use with their 11-12 year old pupils. The teachers were each followed for almost half a year[1], observing their teaching not only in lessons where the art-theme was used but also in lessons of their ordinary teaching. Tasks in the activities provided a range of experiences and teachers were asked to use them as they wanted. Data were collected through participation in their classrooms and interviewing teachers immediately after lessons as well as on the basis of videotaped lesson transcripts (see Chronaki, 1997). The choice of focusing the analysis on 'typical' teachers deserves further clarification. It is reasonable to ask what might a 'typical' teacher look like and it may also be reasonable to accept hundreds of valid answers or no answer at all. However, in this study the choice of 'typical' has actually found expression on the contrasting cases of 'traditionally' and 'progressively' oriented teachers for reasons that will be explained below.

Firstly, although there was no purposeful sampling, 'traditional' and 'progressive' teachers emerged as distinctive categories of teaching styles in two comprehensive schools of the same area where the study took place. Moreover, Peter and Simon, the teachers whose practice is analysed here, were aware of their espoused pedagogies and had explicitly characterised their teaching as such. Their accounts were possible to be verified through observations in their lessons as will be described in subsequent sections. Peter was broadly oriented towards a free explorative style using project based work and investigations whilst Simon favoured a 'duty' approach towards pupils (i.e. disciplining and organising their time in lessons), seeing himself responsible to cover the curriculum prescribed content. Is it then reasonable to assume that 'progressive' and 'traditional' pedagogies are oftenly

[1] The present paper on the practices of two teachers consists a meta-analysis of case-studies (of a larger sample) presented in Chronaki (1997) and Chronaki (1998).

met in schools and thus represent typical cases of teaching practice? It was certainly so in the context of this study.

Secondly, an underlying assumption in this study is that teaching is not value-free, but instead embedded in participants' ideologies about the values and purposes of their practices. According to Ernest (1991) 'ideologies' in mathematics education can be realised in varied settings and can be expressed by different social groups (e.g. mathematicians, maths teachers, educators). He has identified five educational ideologies (i.e. industrial trainer, technological pragmatist, old humanist, progressive educator, public educator) which represent a far more complex and complete picture as compared to the two categories of 'traditional' and 'progressive'. Why then these two cases can be useful analytical units?

In fact, the polar traditional/progressive represents a dichotomy, heavily value-laden and with stressful consequences for teachers in schools. A few years ago to be 'traditional' was a curse and 'progressive' (or for a few 'constructivist') was viewed as the desired teaching style, steamed from a 'child-centred' orientation. As Walkerdine (1998) comments, these 'common-sense' categories do not really exist, but are in fact socially constructed and serve the politics of the time. This is certainly true for 'traditional' and 'progressive' and thus seems more important to pay attention on the type of 'use' one makes of them than on the categories themselves. For example, the current educational discourse (mainly on a politics level) often favours the return of 'traditional' teaching and is constructed in varied 'discursive practices' (e.g. media) as the best practice (see Hardy, 1999, p.203 for a Foucaultian analysis of a BBC video on maths teaching). Moreover, 'traditional' teaching is used as a term to describe mainstream practices in developing countries (see, Kaahwa, 1999). This polar is used here not as a way for 'labelling' teachers, but with the aim to deconstruct its hidden meanings (and possibly hidden power) for classroom practice and in particular for the use of a thematic approach in mathematics curriculum. By means of comparing the practices of these two teachers some further questions can then be asked: How far such diverse pedagogical positioning produces equally diverse mathematical practices? In other words, how far teachers' espoused pedagogical orientation influences the incorporation of a 'thematic approach' in their maths teaching?

The analysis of data took place in two main stages; looking *at* and looking *through* teachers' practice (reflecting and interpreting issues). Looking *at* their practice, it was possible to describe the features that characterised teachers pedagogical orientation and the ways the 'theme' was used in maths lessons in each case (i.e. describing teaching and verifying teachers' accounts). Looking *through* their practice it was noticeable that teacher's espoused pedagogical orientation tend to influence the interpretation and the placing of the thematic context in lessons but not the ways in which mathematical content was addressed in close interactions with pupils. Below, these issues will be analysed and discussed.

Looking *at* pedagogies as views and practice

These two teachers' ways of rationalising their teaching in terms of its style and aims for pupils' learning reveal two diverse orientations. Peter's main aim is to get the pupils to feel responsible about their learning, and believes that learners need to be confident in addressing their weaknesses. He claims that: "If the pupil says -'I don't get it', then you don't have much to work with. But if she says, I have drawn these pictures, but I don't think that the shape is right or it doesn't look right, there is something to work with. I can point to the co-ordinates and have him check the co-ordinates. So, at least showing what they understand" (Peter, 20/09/93). In contrary, Simon wants pupils to work on a task, to be productive and confident about the final result, or in Simon words: *"getting the job done"*. He wants them to arrive somewhere, to grasp the content. He feels responsible for delivering knowledge to his pupils. Peter expresses a 'child centred' pedagogy and Simon locates the rationalisation of his actions in a duty oriented view of teaching where his main responsibility towards pupils is covering the mathematical content as represented in the curriculum and as required for exams. In short, Peter thinks the responsibility for learning rests with the pupils' own active endeavours, whilst Simon believes that the teacher is mainly responsible for pupils' learning. Peter, believes that pupils can learn by themselves and Simon sees himself obliged for knowledge transmission during lessons. Two very different stances on 'what is learning?' which result in providing different lesson experiences to their pupils.

Peter's progressiveness

Peter's lesson organisation was quite loose. Pupils worked in small groups and lots of noise and chatter was going on during lessons. As Peter explained, he did not intend on having pupils working together at the same task or having regular whole class discussions and presentations. Most of the time, Peter would start a lesson with a short introduction, and then pupils would continue working individually on their tasks. Pupils could move around in the classroom and when they had problems, they would approach Peter for questions. He said that he is happy when: "...they (pupils) will come up and they will say I think I've got this wrong, rather than 'Have I got this wrong?' If I come and find the group has started working in my absence, I think I am doing well with the group. Because they don't need me to get them started. They are interested enough. They've got enough maturity" (Peter, 20/09/93).

Peter, tries not to interact much with pupils in the sense of structuring overtly their work in whole class discussions correcting them or making sure that they progress with their tasks. Peter's style in managing the class gives the impression of a free, unstructured environment where learning was mainly based on individual work and on pupils' confidence to seek help and ask questions. In discussing this with Peter, he explained that he wanted his pupils to become confident, independent and responsible for their own learning. A view that may be interpreted either as a genuine expression of his wish to develop pupils towards this direction or as a 'naïve' belief on pupils' readiness in being confident learners and equal partners in the classroom setting. He explains: "I have, hopefully, a broader knowledge than they do. That's may be why I am teaching them. But I don't think it is right for me to pretend -it is like this, like this. Pupils can make discoveries for themselves. So, I don't have a monopoly of knowledge." (Peter, 20/09/93).

Simon's 'didactic' approach

Simon employs what he calls a 'formal' or 'didactic' style of teaching. His notion of didactic teaching is captured in the following: "... I still do the didactic teaching now and then. I must say, I am not, I am not against doing didactic teaching. I mean, I do sometimes stand up the front and explain work to kids. And I think it's important to explain work to kids. And it depends on what they are doing, it depends on what stage they are at" (Simon, 7/12/93).

In contrast to Peter, Simon's lessons appeared to have a distinct routine consisting of regular whole class introduction and recapitulation, and even though pupils were working in groups, the level of noise was low. Talking about his style of teaching, he speaks through an example: "..Actually, about the bearing, I probably ask them to copy some work on their books, and have an example. And I don't see any harm in that. I think that's quite good. To discuss it first, to get information from the kids, to actually do an example on the board for them, to get them to copy that in their books. Then to get them to do their own examples (...) I mean, that's what I call 'didactic' teaching" (Simon, 7/12/93). The core of concern for Simon is to deliver the curriculum in the sense of covering the necessary content for all pupils. Due to this, he argues that using 'practical work' cannot be his primary choice in teaching. Simon recounts the varied constraints and pressures such as; syllabus framework, the national curriculum, the assessment, as well as the difficulty in having available materials (resources, worksheets, visual aids) that can match all topics and levels of pupils' learning abilities. Added to this, is the time and pressure required to deliver the content of a lesson and his own confidence in using new materials effectively.

Looking *through*: placing the 'theme' in maths lessons: periphery or core?

Both teachers were very motivated in using the theme based activities. They said that by using them, pupils would be able to investigate symmetry in the patterns, enhance their vocabulary in geometrical terms and enrich their spatial experiences. They justified this by explaining that the grid provides a plurality of transformations for them to explore; *'..the Running Pelta pattern gives a variety of transformations that are possible at any time. So, it helps to see one transformation within the world of any transformation possible'* (Peter, 2/12/93) and they particularly liked the tasks in *'..looking at things that change and things that stay the same'* (Simon, 7/12/93). In particular, Peter commented that the art pattern *'...is an underlying idea, very important'*. They commented on the beauty of the art patterns during the slides' projection in the first lesson and they tried to initiate some discussion with pupils. However, the artistic part (i.e. viewing slides of artistic work, drawing patterns) as a theme was placed in the periphery of Simon's lessons whilst in Peter's it was a central part of his teaching. It can be said, that the art context became an expression of Peter's 'child centred' pedagogy, whilst it was used by Simon at its minimum. The two teachers used very different ways of timing and spacing lessons and their teaching through using the context, as will be seen below.

As previously said, the 'theme' was placed *in the periphery* of Simon's lessons. Simon, soon after the slides projection, shifted away from the content of art patterns and focused pupils' attention on the completion of tasks. Mellin-Olsen (cited in Christiansen, 1996, p. 24) refers to such activity as the 'exercise discourse' where the mathematical activity or the communication between teacher and pupils is along the lines of the exercises with the exam or the next grade level as the goal. Simon, being concerned about pupils' involvement in completing their work in tasks did not want to spend time in emphasising pupils' involvement with artistic work. Pupils of course did the tasks in making patterns, but this was done instrumentally, in the sense of completing the tasks within time limits. None of his class discussions were geared to addressing pupils' experiences on this activity or discussing the features of the theme (e.g. possible pupils' enjoyments, difficulties, ways of progressing in making art patterns), and pupils were not encouraged to express their views or enthusiasm in doing quality work in the artistic part. Artistic work was largely seen as an interesting starting point that should be quickly abandoned and therefore not really relevant for the mathematics at hand. For example, Simon said in one of the first lessons: "…this task is to create your own designs, using the pelta. Now we aren't expecting you to draw this absolutely (…) perfectly. You don't need to get (…) You can do a free hand drawing on that, and if you feel that you need tracing paper—to help you recreate one shape—in other places of your page, or if you want to use tracing paper, maybe to turn the shapes, to see where they would go. I've got tracing paper available for you." (Simon, lesson2, 14/1/94, my underline).

On the other hand, the 'theme' became *the core* of teaching for Peter, who**,** whilst introducing the activities, talked extensively and expressively about the nature of patterns in the varied slides projection of artefacts. Peter's aim was to have pupils appreciating the plurality of patterns in the artefacts and enjoy the beauty and mystery entailed. He focused mainly on making pupils aware that mathematical regularity can be experienced in cultural artefacts, such as the pictures in slides. Concerning the slides projection he commented after viewing one of his videotaped lessons: "From seeing the video I seemed to stress mathematics in the context of culture. I was talking about Islam and Christianity and use of geometry and attitudes to that, which I wasn't expecting to. I didn't realise I knew that much about them all. It's just as it came out. I was interested in repetition, pattern and in colour and in maths as an art form or geometry as an art form anyway. Those are the main things to stress on that occasion and getting them to be aware that you have repetition in pattern and that if you describe displacement, movement from one part of the pattern to another, which is where the transformations come in. So just to make them aware that transformations exist, without going into how you describe them" (Peter, 7/12/93). Reference to 'mathematical information' embedded in the artefacts was implicit. Peter, did not aim to alert pupils to explicit descriptions of geometric transformations. Due to this, he did not make any explicit suggestions to pupils about a desired mode of work and he did not attempt to clarify any focusing purpose for the activities. In fact, since his main aim was for pupils to appreciate and enjoy the viewing, drawing and constructing of art patterns, making explicit any predetermined purposes and objectives would seem contradictory.

The art theme remained the core of Peter's teaching in all lessons. He used two extra lessons (one based on hand drawing and the other on using the computer) on making art patterns and towards the end of a series of lessons he arranged a show of pupils' own art patterns. Pupils were asked by Peter to come in front of the class in groups of three. They all had quite detailed, creative and complicated patterns and they used interesting colouring. Peter talked about the aesthetic appeal of the patterns, commenting on their visual impact, sensitivity in colour and design and expressed his appreciation of them. For example some of his comments included: "These patterns strike a lot more from a distance. Can you see this bit here? It's got a nice Chinese style (...) Yes quite nice shapes here! But you don't see that until you get back from a distance. Then, it stands out. And using just the two colours is a good combination // Yours is very delicate. Yours is very symmetrical, isn't it? With the original tile you use (...) Because the original tile is symmetrical, any transformation won't actually change it visually. Because when you transform it, it still looks the same // Yes. It is a very nice pattern, but you can have lots of different transformations there, and it looks as though you haven't done it, because of the original tile. So any transformation is lost, because it looks the same when you lay it in different ways. This has a mystical effect, hasn't it? It makes it quite mysterious and quite three dimensional as well" (Peter, lesson 4, 22/11/93).

Looking *through* mathematical discussions: 'covert' or 'overt' explicitness?

As seen above, teachers place the 'theme' in their lessons in radically different ways, which accord with their interpretation of its relevance to pupils' learning and their teaching style (i.e. traditional/ periphery, progressive/core) . Around this, they organised time and space in lessons for pupils to use particular tasks. They made choices about what tasks pupils would use, for how long and why and they provided not only instructions but also encouragement and enthusiasm for pupils' activity. Despite teachers' different pedagogical views and ways of placing the art theme in their lessons, it was striking to identify commonalities when delving deeper into Peter's and Simon's teaching and in particular through analysing their ways of interacting in subject related discussions[2] in lessons. These very different pedagogically oriented and positioned teachers, from first glance, appeared to have very similar ways of addressing mathematical conversations with their pupils. Interacting with pupils[3] enabled teachers to identify their difficulties which often became the starting points for discussions – a main starting point for teacher-pupil interactions as also observed by Bauersfeld (1988). It was by looking deeper into such episodes of lessons that similarities could be discerned concerning teachers' ways of talking about mathematics with pupils, that could not be noticed by means of observing the flow of lessons.

Peter: covert explicitness (and covert power)

Looking through Peter's talk with pupils an absence of delving into the pupils' own images of mathematical concepts was noticeable. Inquiring pupils' own views and building on them in classroom settings has been explored systematically and regarded by Bauersfeld and Cobb (1995) as the core for developing meaningful mathematical learning. It can also be seen as the initiation of a dialogue about subject knowledge and pedagogy. Given, that the introduced mathematical concepts (as well as their visual and textual representation through the theme context) may be new for the pupils, it would be reasonable to expect that the teacher would make room for them to express ideas in their own words. For example, by means of asking them to provide another example or to involve them in a discussion where their actions and observations could be reflected upon. Instead, Peter provided overt explanations in a 'telling' form, with minor efforts for checking whether his 'telling' has been meaningfully understood by pupils.

[2] The subject related discussions are also called 'thematic patterns'. Thematic patterns have been described by Voigt (1998) as the patterns of teacher-pupil(s) interaction over a mathematical theme and which strive for some thematic coherence. He says: *'In these processes, the students and the teacher achieve a thematic coherence in their discourse. Interactively, they constitute a mathematical theme that, on the one hand depends on the participants' contributions, whereas on the other hand cannot be sufficiently explained by the thoughts and intentions of any one person alone*' (p.164). Elsewhere, he provides a clarification of the mathematical theme: '*In the course of negotiation, the teacher and the students (or the students amongst themselves) accomplish relationships of mathematical meanings that are taken as shared. From the observer's point of view, I call these relationships of meanings a mathematical theme'* (ibid, p.174).

[3] Looking at pupils' activity in worksheets and in lesson episodes, pupils' problems included; using formal vocabulary; producing an articulate and complete verbal description of the features of geometric transformations; making accurate constructions and using measuring instruments (see Chronaki, 1997).

Moreover, Peter was friendly, polite (and his voice soft), showing willingness for co-operation in an almost equal partnership status. As Peter mentioned, his aim was to cultivate pupils' confidence and independent work at his immediate supervision. Thus the main feature of his interactions was the creation of a supportive and positive working atmosphere, the projection of the teacher as a non-authoritative figure and of the pupils 'in charge' of their own activities. This type of relating with pupils, a relaxation of power relations, has been seen as typical of 'progressivist' classrooms, and could be described as a 'weak classification' of 'power relations' between teachers and pupils in the classroom, in Bernstein's (1990) terms[4]. It is this climate, through which Peter was 'explicit' when addressing mathematical content, which can be described as a covert explicitness concealed through a friendly discourse.

Simon: overt explicitness (and overt power)

Simon used to provide instructions for pupils to attend. His focus was on suggesting the necessary 'techniques' to be employed for particular tasks (e.g. repeating, listening carefully, responding to questions, trying out his suggestions). Although he addressed pupils with questions, it was Simon who constructed the meaning and relevance of questions and answers. Anticipating what might not be understood by pupils, no much room made for them to think and reflect on alternatives.

Simon was also explicit in his interaction with pupils and this explicitness was realised in every aspect of his teaching, and as such it could be described as 'overt'. Giving instructions at the beginning of each lesson and recapitulating main points was the natural format of his teaching. He also spent time during lessons in explaining extensively how pupils should organise their work and how to spend their time in specific tasks. His interaction with pupils was a manifestation of a strong 'classification' and 'framing' pattern (see Bernstein, 1990, pp. 187-197). Classification, as a way to define the 'power relations' between teachers and pupils in the classroom, describes 'who' is to be taught and 'what' is to be learned. In Simon's case these roles were formatted with the teacher having the clear authority. By framing, Bernstein refers to the ways in which these power roles (of teacher and of pupils) are negotiated in practice during the course of pedagogic communications and refer to the 'how' some particular content is to be taught. This form of regulating classroom interactions was also strongly defined in Simon's teaching. For example, he was identifying materials for the pupils to use such as protractors, mirrors and tracing paper, not allowing much room for pupils to consider what might be done in tasks or to invent their own ways for tackling questions. At the beginning of one of the lessons, Simon addresses the whole class: "You will need a protractor for the work involved here. I hope you realise why. I hope you realise why we need a protractor... Because we're talking about turning! We're talking about turning, rotation this week. Rotation involves turning. And we use a protractor to measure an amount of turn, don't we? If you want to borrow one, just put your hand up..." (lesson 4, 4/2/94)

Conclusions

This study started with an interest to explore the employment of a thematic approach to school maths curriculum in the practices of 'typical' teachers. Teachers who espouse 'traditional' or 'progressive' pedagogies were conceived here as 'typical' cases. Analysis in the ways these teachers use a thematic approach in their maths lessons (as reported in above sections) has provided insights in the ways these teachers employ, value and talk about 'themes' and 'maths in themes'. Comparison across these contrasting cases has also enabled to appreciate embedded meanings and limits presented by these pedagogical orientations.

The limits of 'traditional' and 'progressive' pedagogies

[4] 'Classification' is a concept through which Bernstein (1990, p.197) defines the regulative function of pedagogic discourse and refers to the flexible or rigid boundaries between the two main social categories in the classroom (teacher and pupils).

Although the two teachers espouse contrasting pedagogies, ‘progressive’ and ‘traditional’, they have similar styles of talking about mathematics with their pupils. This may come as a surprise since both teachers, as seen earlier, had very different styles in organising their lessons and ways for employing the art theme in their teaching. Inconsistencies between teachers’ espoused views and their actions in classroom practice have been observed by Lerman (1992: beliefs and beliefs-in-practice) and others interested in characterising the disparity between espoused and enacted beliefs (Raymont, 1997). However, this study highlights that teachers’ espoused pedagogies are consistent with the ways teachers plan and organise their lessons (e.g. the placing of ‘theme’ in lessons) but not in the ways they talk about maths with their pupils (e.g. the ways they provide help when pupils are faced with conceptual difficulties).

For Simon, it was important that pupils arrive at some sort of ‘right’ and ‘correct’ answer and he tried to correct pupils’ work and to ensure that pupils did not leave their tasks with errors. For Peter, it was important to let pupils enjoy without stressing what they should do in particular tasks, assuming that pupils would eventually discover the mathematical content for themselves. But, in both cases, mathematical knowledge was seen as a commodity or an object to be either transmitted by the teacher or discovered by pupils in the course of the activity. This underlying belief of ‘what is mathematics’ was embodied in the ways teachers talked with pupils about their mathematical activity. The views of both the progressive and the traditional teachers arrive at a very similar point: that is to ‘tell’ explicitly what is conventionally taken as ‘mathematically correct’. Next to their ‘explicit’ manner for communicating, lie subtle differences due to their diverse pedagogical rootings. Peter’s ‘progressive’ style disguises ‘explicit telling’ in a friendly and polite discourse with pupils (e.g. provides straight answers, tries not to correct them so as to make them feel that they make mistakes, holds a naïve view about ‘equal’ relations in classroom). Simon’s explicitness is covered by a ‘funelling’ type of questioning and instructing where he guides pupils to guess and produce his own interpretation of the activity.

Recent discussions about desired styles of teaching in maths classrooms have stressed the notion of an ‘inquiry’ type of classroom discourse that fosters mathematical discussion, argumentation and justification (Bauersfeld and Cobb, 1995). This was not the case for our two ordinary teachers whose classroom talk did not exemplify a form of ‘classroom negotiations’. Seen through this perspective, both practices (and pedagogies-in-practice) can be characterised as ineffective for stimulating pupils’ active engagement. In fact, teachers, were providing mathematical information to their pupils in a direct, explicit style described as ‘direct mathematisation’ by Voigt (1995), following either an ‘overt’ or a ‘covert’ form, and their main concern was the provision of ‘right’ answers, or answers which ‘fitted’ pupils’ questions.

Looking into lesson episodes, one common feature was that teachers’ efforts were geared towards accustoming pupils to a culture of ‘procedures’, ‘skills’ and ‘techniques’ acquisition. Teachers, in a sense, tried to make available to their pupils the ‘tools’ and ‘rituals’ of what a ‘school maths culture’ might be. They were ‘passing on’ culture in a way similar to Lave’s description of learning as enculturating participants in a practice (see Lave and Wegner, 1991), but without making room for discussing the significance and the emergence of these tools as cultural and social artefacts. Through the ‘eyes’ of Lave, it can be claimed that both of our teachers are engaged in forming ‘communities of practice’ in their lessons and that both try to enable pupils entering a ‘culture’ of school maths by providing them with the necessary ‘tools’ (i.e. of a conceptual, organisational and/or technical nature) for them to cope with maths in school (e.g. curriculum cover, lessons, exams). However, one needs to ask how far this availability of ‘tools’ (by the forms of telling) is being transformed into tools for thinking, action and critique in the hands and minds of pupils themselves (i.e. how far do they promote democratic educative relations), and whether this concern is primary one for teachers. Issues that certainly deserve further exploration.

The placing of ‘themes’ in maths lessons: traditional or progressive?

Concerning the placing of 'theme' in maths lessons it was noticed that although both teachers in this study perceived the theme of artistic work as a valuable underlying motive for pupils' engagement in learning, it was really their pedagogic orientation (e.g. preferred style of teaching and aims about pupils' learning) which acted as the driving force for the placing of the theme in their lessons. Teachers' espoused pedagogy determines the ways the theme will be used and its integration in lessons and influences teachers' decisions and judgements about what 'fits' best into their lesson planning, as well as the overall framing concerning goals for pupils' activity in lessons. Specifically, Peter's progressive style favours a rich use of the theme's features and places it at the core of his teaching. Pupils in his lessons spend quite a lot of time in viewing, drawing and making art patterns. Simon's formal and traditional style locates artistic work at the periphery of his lessons and pupils are merely offered an introductory flavour of the theme.

Overall, these two teachers have constructed very different experiences for their pupils around the use of the theme-based activity. Their teaching conveys different meanings about the status and significance of the theme as well as its relations with mathematics and maths teaching. Avenues for pupils to experience mathematics (and to construct images for the subject) are relayed not only in the communicative acts that take place in thematic interactions with the teacher, but also in the broader structuring and orchestration of the activities in lessons and in classroom culture. Both the placing of the theme in lessons and teachers' ways for guiding and framing mathematical conversations with pupils entail cues about 'what a maths culture might be' and opportunities for pupils experiencing the mathematics in it. Boaler's (1997) work shows clearly the significance for using theme-based projects for pupils' learning, motivation and meaningful activity. She has also discussed the importance of a broader 'progressive' school culture that promotes such curriculum work. My study agrees that a 'progressive' construct of classroom culture is the necessary background for undertaking theme-based activities in the classroom. Making room in lessons for exploring the theme and the mathematics through the context of a theme necessitates a 'progressive' pedagogical orientation as in Peter's case. Whilst a 'traditional' one, such as Simon's, tends to overemphasise mathematical content in a manner that dislocates mathematics from the features of the theme.

However, I would argue that the 'progressive' metaphor also needs unpacking and problematising since the term may embrace a conservative flavour when a closer look in classroom talk is undertaken. Teacher-pupil interaction (and other social interactions in the classroom such as whole class and group discussions) entail opportunities for talking about mathematics within a theme and for constructing images, attitudes and knowing. Making a good use of these opportunities seems to be not simply a matter of pedagogical orientation, but also a matter of knowing how to manage ongoing discourse. Towards this direction, Evan's (in press) work seems to offer a promising construct for theorising the merging between the spaces of 'theme' and 'mathematics'. He claims that the 'building of bridges' across contexts and practices is possible and the identifying of 'bridging' strategies (e.g. context-sensing questions) is rooted in a careful analysis of the developing discourse between participants. This view provides an optimistic direction for identifying ways in which both 'theme' and 'mathematics' can be seen as parts of a 'building bridges' process that would enable us to construct meaningful contexts for learning about mathematics and theme.

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TRANSDISCIPLINARITY AND CURRICULUM ORGANISATION

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ABSTRACT

The school organisation in closed and separated disciplines is not compatible with the complexity of the existing knowledge and the problems that society has to face nowadays. One of the ways to adapt school to the existing needs and demands could be a curricular organisation in which transdisciplinary project work would be placed in a central position. In this paper, we argue that school mathematics has much to gain from participating in such projects.

INTRODUCTION

We have been noticing that many students do not understand mathematics as a human construction that provides them with tools to better interpret and interact with their world. Instead, they look at mathematics as a game of symbols, generally unfruitful, that they do not understand and in which they cannot or do not want to participate. From our point of view, that fact has much to do with the way teaching is organised: Knowledge is arranged in closed disciplines, with the back to each other; teachers are organised in groups by disciplines; and time and space in school are strictly split in tight cells. Besides, mathematics teaching has been over-stressing symbolic manipulation techniques, neglecting, in a great extent, other issues like problem solving, mathematics applications, modelling and investigations.

We firmly believe that, to have all students learning mathematics, we must organise teaching in activities that fully provide meaning to mathematical ideas and objects, in order to promote mathematical thinking. Learning mathematics is learning how to think mathematically. In order to think mathematically it might be necessary to know some of the mathematical language and some of the techniques of mathematics, but that is not enough. The most important is to have something-worthwhile thinking about. However, something-worthwhile thinking about is not necessarily the same for all students. That’s why we believe that interdisciplinary or transdisciplinary project work can be a potentially meaningful context to the learning of mathematics. Its open character stimulates students to get involved in the development of the activities and its global nature may provide rich experiences.

Those concerns led us to question the way knowledge has been organised in society and in school, and the appropriateness of that organisation in our current world.

In this paper, which we view just like an excuse to provoke a debate, we start presenting some ideas related with the organisation of knowledge and its implications to education. We will also try to clarify the meaning we give to the words *interdisciplinarity* and *transdisciplinarity*. In its second part we present some results and some questions following an analysis that fell upon curricular orientation documents and official documents that define Portuguese curricula. In the third part we briefly describe a project in which one of us has participated and that we think it is a good example of transdisciplinary project work. We believe that this example of school practice, and the reflection that it allows, can help to clarify the understanding of learning mathematics in a transdisciplinary project work context.

Knowledge organisation and education

The role of school in reproduction and maintenance of knowledge organisation has important consequences in social terms. Pombo, Guimarães & Levy (1994) stress some of them:

> It seems that, as specialisation goes forward, it becomes more and more difficult to find comprehensible answers, not only to the new and complex questions that we face nowadays, but also to the more concrete and (apparently) more elementary questions that human kind has always raised. (…) as far as the specialised knowledge of some grows, we feel more and more ignorant and set apart. (p.25)

D'Ambrosio (1997) argues that the way knowledge is organised in disciplines has been granting power to the specialists and to the societies usually called first world's, and has consequently been giving place to unequal distribution of power and wealth within the several societies and cultures.

Vithal, Christiansen & Skovsmose (1995), citing Knud Illeris, claim that disciplines "have been developed through tradition, the basis of which is far in the past and dependent upon societal conditions which long since have vanished" (p. 211).

The illusion of modernity, that science would bring us the full understanding of reality and the power to manipulate that reality, has fallen down. The "logic-positivistic program that intended to construct the unity of science and a formal grammar to account for that unity" (Pombo et al, 1994, p.26) has failed. At present, the great complexity of the physical, social and cultural world we live in is recognised, and there has been emerging perception that the understanding of that complexity cannot be reached only through partitioned knowledge, broken into disciplines.

Without denying the value of disciplinary knowledge "for deep research and for practical applications that became possible because of this type of knowledge" (p.26), we must reconsider the organisation of knowledge, space and time in school, specially if we want school to form autonomous, critical and intervening citizens.

Vithal et al (1995) discuss project work in university mathematics education, based on the concepts of problem-centered studies and interdisciplinarity. The way work was organised revealed to be successful in many aspects. Although this paper refers to university, we are mentioning it because we think this kind of approach can be even more adequate to school before university, for all students.

The ideas of interdisciplinarity and transdisciplinarity

The Portuguese words for *interdisciplinarity* and *transdisciplinarity* are used within school practices and in literature as response to the questions we have raised before. However, as Pombo et al (1994) note, there is "no consensus about the meaning of the concept of interdisciplinarity, neither between teachers nor in specialised literature" (p.4). For these authors there are no defined boundaries between the meanings of *multidisciplinarity*, *interdisciplinarity* and *transdisciplinarity*. These words mean different levels of interaction and integration of disciplinary knowledges.

For Ubiratan D'Ambrosio (1997), the word *transdisciplinarity* means a proposal that goes far beyond the school:

> Transdisciplinarity (…) lays in an attitude of recognition that there are no privileged cultural space and time that allow us to judge and categorise hierarchically – as if they were more correct or true – complexes of explanations and forms of living together with the reality around us. (p.9)

In this work, our main concern is the school and the curricular organisation. The concepts of problem-centered studies and interdisciplinarity proposed by Vithal et al (1995) are the key concepts for the idea of work project in curriculum:

> The central feature of problem-centered instruction is that it does not originate in the subjects themselves – (…) - but in currently relevant problems which are addressed using knowledge, methods, and theories from different disciplines to the extent they are relevant to the problems (Illeris, 1974, cited by Vithal et al)

> Interdisciplinarity implies drawing on different disciplines to the extent to which it is useful for the treatment of a specific problem. (Vithal et al, p. 211)

These two concepts together are very close to our idea of *transdisciplinarity*. Our proposal of terminology, which we use along this paper, namely in the analysis of several documents, is the following:

<u>Interdisciplinarity</u>: Knowledge is organised in a set of disciplines. An interdisciplinary perspective takes into account the relations and intersections of those disciplines, in a movement of *synthesis* of knowledge. The

interdisciplinary school is still organised by disciplines that collaborate and co-operate with each other in order to contribute, from the confluence of different points of view, to a better understanding of the same object of study.

Transdisciplinarity: Knowledge has a holistic and complex character, which crosses and goes beyond every discipline in which it is organised because of the educational, teaching, work division, and specialisation necessities within each society or culture. In a transdisciplinary perspective, those disciplines are always viewed as part of a whole context which is responsible for their making sense. The transdisciplinary school is organised around the unified, global and complex knowledge. In a first analytic movement, it looks at the objects of study through the several disciplines, to return to the unity and growing complexity of the knowledge.

INTER AND TRANSDISCIPLINARY PERSPECTIVES IN PORTUGUESE CURRICULA

The work that gave rise to this paper was our participation in a research project, in the context of a master degree in Mathematics Education[1]. Within this project, our goal was to understand what are the positions of the Portuguese curricula with respect to knowledge integration. We started by analysing some international curriculum orientation documents that, from our point of view, have had a direct influence in Portuguese mathematics' curricula:

Mathematics counts (Cockcroft report) – U.K. 1982
Everybody counts – U.S.A. 1989
Reshaping School Mathematics – U.S.A. 1990
N.C.T.M. Standards – U.S.A. 1989/98

We have also analysed a Portuguese curricular orientation document for mathematics, which is the result of a seminar promoted by the Association of Teachers of Mathematics (A.P.M.[2], 1988) and recent documents of global curricula orientation, for grades 5-9 (M.E. DEB[3], 1996 and 1997) and grades10-12 (C.N.E.[4], 1998).

Concerning the present Portuguese curricula, we have analysed the acts and documents that define them in general (M.E. 1991a and 1991b), as well as the mathematics syllabuses for the grades mentioned before.

At last, two working documents of the Portuguese Ministry of Education were studied. They are the current orientations of the educational policies that are being implemented in the same school grades (M.E. DEB, 1998a and M.E. DES[5], 1997).

In the analysis of each document, we looked for information that could clarify the way each one considers, explicitly or implicitly, the following issues:

1. Perspectives about knowledge integration

2. Relationships between mathematics and other disciplines
3. Applications of mathematics
4. Teaching methodologies for knowledge integration
5. Curriculum organisation

The different nature of the mentioned documents is reflected in the different ways they approach the questions we posed. Some of the questions have more to do with the general curriculum and others with the mathematics curriculum.

1. Perspectives about knowledge integration

   There are some references that can be interpreted as transdisciplinary conceptions of knowledge, in the international documents of mathematics curriculum orientation, except in the NCTM's. However, those conceptions do not have strong consequences in those documents' recommendations. We identified also the idea that the integration of knowledge would be important during the first grades of schooling and that the curriculum should aim at disciplinary specialisation and interdisciplinary work at more advanced grades (Cockcroft, 1982, for example).

   That same idea is also present in the Portuguese acts that establish the general curricula (Act 46/86 and D.L. 286/89). However, signs that knowledge integration would be a principal concern were not found in the Portuguese mathematics syllabuses.

   In the recent recommendations (C.N.E., 1998, and M.E, 1998b) and in the recent working documents of the Ministry of Education, it is explicitly assumed the need of a global reconsideration of the curricula, according to transdisciplinary perspectives of knowledge, in all school grades. Therefore, it seems that there is a growing awareness that, in school, the knowledge organisation in closed disciplines is not sufficient to face the complexity of the problems that society and citizens have to deal with, nowadays.

2. Relationships between mathematics and other disciplines

   Naturally, it was in the documents concerning the mathematics curriculum that we found some indications about the way that relationships between mathematics and other disciplines are viewed. There are, in every one of them, strong references to the skills and abilities that mathematics promotes and that cross all knowledge – communication (Cockcroft), mathematical thinking, (N.R.C., 1989), problem solving, mathematical power (N.C.T.M.), etc. In Portuguese syllabuses, those are represented in the formulation of the general objectives of abilities and attitudes that cross the several curricular components. However, both the international mathematics' curriculum orientation documents and the Portuguese mathematics' syllabuses fail in regarding the integration of mathematical ideas in a global knowledge, except for the applications of mathematics which have strong importance in all documents.

3. Applications of mathematics

Although all documents of orientation or definition of mathematics curriculum place a high value on applications of mathematics, there are differences in the ways that the several documents view them in the curriculum. In general, the applications of mathematics are viewed as an objective in the earlier documents. They consider that students should learn mathematics in order to apply it afterwards, in a job, in college or even in everyday life. In more recent documents it is more and more clear that it is important to consider applications of mathematics as a curricular content or methodology.

In Portuguese syllabuses for grades 5-9, the applications of mathematics are present in some topics, but with the perspective of first learning the mathematics to apply them afterwards. For grades 10-12, mathematical modelling is a topic that crosses all the other subjects, although there are much more suggestions of modelling activities in the subject *Functions*. The applications of mathematics appear sometimes as a pretext to introduce mathematical concepts in these grades.

4. Teaching methodologies for knowledge integration

In the earlier documents, it is barely found methodological recommendations addressed to the integration of knowledge. Surprisingly, the A.P.M. document from 1988 stresses interdisciplinary project work as a more suitable methodology to that purpose.

The Portuguese global curricula, created in 1989, included a component of interdisciplinary or multidisciplinary project work, but several reasons contributed to its failure and to its disappearance with the time, in most schools. Some of the reasons were: there was not any syllabus for that project work, nor any kind of evaluation was considered; there was not any teacher responsible for it; the syllabuses of the disciplines, including mathematics, did not refer to that project work; and there was not a specific time, in the students timetable, to work on the project.

In the more recent Portuguese documents, it is very clear the idea that project work is the methodology adequate to the integration of knowledge.

5. Curriculum organisation

It is very recent, and not yet generalised, the awareness of the need to implement methodologies with the purpose of integrating knowledge. However, it is even more recent the need to organise the curricula with that same purpose. That idea is emphasised in the recommendations of the C.N.E.

document from 1998, in which the need to reorganise school is stated. That reorganisation should consider less exclusively disciplinary criteria, but more inter and transdisciplinary ones.

The documents that point out the basis for next changes in curricula foresee one or more transdisciplinary components, side by side with the disciplines. These components will have their own space and time, teachers and syllabuses, so it seems they might gain a place in the curriculum. This constitutes an innovative situation.

In short, both a holistic view of the knowledge or an interdisciplinary perspective, or even just a perspective of collaboration between disciplines, fade away as far as curriculum becomes specific. It seems to us that it is possible to identify large gaps amongst the intentions that appear in the more general documents, and the way those intentions are made operational in the Portuguese mathematics syllabuses and the practices in schools. That led us to the question: what can be done to diminish these gaps?

To tackle these difficulties, new documents pointing out ways of organising curricula, which are different from the existing disciplinary logic, have appeared. They also propose different ways of managing space and time in school and different teaching methodologies such as project work.

THE GEODESIC PROJECT

Social and cultural context

The experience we are about to describe involved one of us in her school. It is an Art school, for grades 10-12, in which it is possible to identify particular cultural issues, such as the high value placed on creativity, on being different, on the aesthetics of things, and a somehow low value placed in other aspects such as logical and abstract thinking, usually identified with mathematics. In a general way, students come to this school because they chose it and because it responds to their projects for the future.

The students who participated in the experience were 12^{th}-formers, enrolled in the course of “Equipment Design”, and they did not attend, at that time, a course in mathematics. They had failed the discipline before and gave it up, once it was optional. All of them stated that they did not know and did not like mathematics.

In the Equipment Design course, the curricular component ‘Project and Technologies’ usually has three teachers, with different backgrounds, who work in three different spaces – a drawing room, a workshop for wood, metals, and other materials, and a room equipped with computers to work mainly with

AutoCAD. Students develop their projects in small groups, moving freely on the three spaces and interacting with the three teachers and other students.

The project

As a schoolwork for 'Project and Technologies', students intended to elaborate a project of a 'giant and hollow sphere', and execute it afterwards. Naturally, one of the teachers, a designer, led them to study geodesic structures and looked for information about them in specialised literature.

Both the teacher and the students were convinced that most geodesic structures were composed by equilateral triangles, all congruent, but did not succeed in understanding how to arrange the triangles to build a net for a geodesic structure. As the language used in the book (Marcolli, 1978) was mainly mathematical, they turned for aid to a mathematics teacher.

Several work sessions were carried out with the design teacher, the students and the mathematics teacher, in order for students to understand the geometry involved and, together with the teachers, to reach a solution to their project. For those sessions, the mathematics teacher brought some manipulative materials for them to construct some polyhedra and to understand that is impossible to construct regular polyhedra with more than 20 triangles. She discussed with them the processes of mathematical construction of a geodesic figure starting from an icosahedron, as she had studied in the design teacher's book (Marcolli, 1978). She also used the descriptive geometry methods, more familiar to those students in those practices, and dynamic geometry software to determine the lengths of the edges of the structure.

The processes used were adequate to conclude successfully the project. The materials chosen were inspired in the geometric models brought by the mathematics teacher, and led to the formulation and discussion of new mathematics problems.

The structure was built with a diameter of three meters, and was set up at the front of the school. Every teacher and student of the school saw it, appreciated it and commented on it.

Some comments on the project

We consider this project a transdisciplinary one because it did not come from the disciplines, neither was it drawn to serve them. Instead, it came from a student's idea, and went to the disciplines to better support its development.

This project settled a suitable context for the learning of mathematical ideas and procedures, because it provided them with meanings within the students' social and cultural practices. Besides, students learned to give value to mathematics and to the role it can play in their activities, although they kept considering themselves unable to work with mathematics.

The access of the mathematics teacher and of mathematics to students' practices was an important issue to the success of this project, which is not usual in that school. Usually, school mathematics practices do not have anything in common with the practices of 'Project and Technologies'. In the latter, students are engaged participants, sharing the space, the artifacts and the meanings of what they learn, because their goal is to become a designer, just like the teacher. The group mathematics and the mathematics teacher to come in because of a specific and prompt need. However, it was the teacher's engagement in their practices, her interest and involvement in the activities of the project, formulating and discussing diverse problems, mathematical or not, that made mathematics to become part of their activities too.

FINAL REMARKS

School organisation has to change in order to respond appropriately to the needs and demands of the existing society. Both the most recent recommendations and the official documents seem to tend to a reformulation of schooling, in which there is a place to transdisciplinary knowledge. Project work seems to be an appropriate methodology and curricula are about to be changed, following these perspectives. However, changing curricula is not enough – school is much more than a curriculum and there are other questions to be raised that can be useful to help to reconsider school in a holistic perspective of knowledge. We present here a few:

Generally, teachers' initial preparation falls upon only one discipline. Should it be reconsidered, accounting to knowledge integration?

School organisation (teaching groups, representation in management teams, etc) is ruled by a disciplinary logic. Are there other ways to organise educational agents in order to promote a more global teaching?

The educational system tradition values highly the knowledge divided into disciplines. It places them in a hierarchy, depending on the importance they are supposed to have in society. How can we integrate, in school, physical, scientific, literary, artistic, etc., activities, without valuing some more than the others?

Students' evaluation always falls upon disciplinary knowledge. Are there any other processes of assessment to replace the examinations we have nowadays? Is it really necessary that 12^{th} grade students pass through a selection process that influences all school practices in these last grades? Evaluation being a valued process in our system, would it be possible to use it with the goal of better understanding the learning processes of students, instead of selecting them?

The teams who elaborate syllabuses are usually chosen within only one discipline. Curriculum development processes should also be reconsidered, and taken over by multidisciplinary teams.

The present possible reflection about the geodesic project consolidates our conviction that within the existing legal system it is possible to implement knowledge integrating practices in schools. It is possible, nowadays, to promote the learning of meaningful mathematics in context of transdisciplinary project work. Once we stand for truly democratic practices in schools, is it reasonable to claim for the transformation of school as a consequence of changing official curriculum documents?

NOTES

1 We are referring to the project 'Aprendizagem e Tecnologia em Educação Matemática' (Learning and Technology in Mathematics Education), from the Faculty of Sciences of the University of Lisbon, sponsored by IIE (Institute of Educational Innovation).

2 A.P.M. is the Portuguese Association of Teachers of Mathematics.

3 DEB is the department of the Ministry of Education (M.E.) which is in charge of the basic education, that is, grades K–9. In Portugal, compulsory education corresponds to grades 1–9.

4 C.N.E is the Portuguese National Education Council, which is a consultative committee with representatives from several social, cultural and economic groups.

5 DES is the department of the Ministry of Education (M.E.) which is responsible for the secondary school, that is, grades 10–12.

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WHAT IS A CRITICALMATHEMATICS LITERACY FOR THE WORKING CLASS????????????

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At a 1994 conference designed to develop a program for ensuring that all adults in the USA become numerate, the opening presenter argued that shop-floor workers need to be numerate because mid-level managers are being fired and workers need to assume responsibilities that involve reading complicated graphs and charts related to the production line. Instead, a criticalmathematics literacy for the working class argues that workers need to be numerate so they can read the graphs and charts that show how much profit management extracts from their labor, and so they can fight for new ways of allocating the resources and results of production.

CRITICALMATHEMATICS LITERACY FOR WORKERS' (INTELLECTUAL) EMPOWERMENT

The following annotated examples illustrate various categories of mathematics problem-solving that a criticalmathematics perspective views as important for workers' knowledge: respecting workers' knowledge; understanding the mathematics of workers' conditions; understanding the politics of the mathematics of 'official' descriptions of workers conditions; and, understanding the politics of mathematical knowledge as officially presented in the class. These examples raise issues that go beyond most workers' knowledge, opening up questions about the responsibilities of educator-activists to build upon and to challenge students' ethnomathematical knowledge, developing a criticalethnomathematics.

Respecting Workers' Knowledge There are various ethnomathematical studies that analyze workers' mathematical knowledge: Scribner (1984) found that dairy workers invent their own units (full and partial cases) to solve problems of product assembly on the job. Carraher, Carraher and Schliemann (1985) analyze the mental calculation algorithms developed by children working in their parents' marketplaces. These various knowledges can be analyzed to deepen all workers' mathematical knowledge. Further, considering the general daily problem-solving occurring in workers' lives presents a challenge to the split between mental and manual labor, helping workers connect that daily knowledge to their studies in school.

EXAMPLE Students read and discuss this excerpt from a literacy primer prepared with Paulo Freire's help for Sao Tome and Principe (Friere & Macedo, 1987, pp. 76-7).

> The Act of Studying I. It had rained all night. There were enormous pools of water in the lowest parts of the land. In certain places, the earth was so soaked that it had turned into mud. At times, one's feet slid on it. At times, rather than sliding, one's feet became stuck in the mud up to the ankles. It was difficult to walk. Pedro and Antonio were transporting baskets full of cocoa beans in a truck to the place when they were to be dried. At a certain point the truck could not cross a mudhole in front of them. They stopped. They got out of the truck. They looked at the mudhole; it was a problem for them. They crossed two metres of mud, protected by their high-legged boots. They felt the thickness of the mud. They thought about it. They discussed how to resolve the problem. Then, with the help of some rocks and dry tree branches, they established the minimal consistency in the dirt for the wheels of the truck to pass over it without getting stuck.
>
> Pedro and Antonio studied. They tried to understand the problem they had to resolve and, immediately, they found an answer. One does not study only in school. Pedro and Antonio studied while they worked. To study is to assume a serious and curious attitude in the face of a problem.

The Act of Studying II. This curious and serious attitude in the search to understand things and facts characterizes the act of studying. It doesn't matter that study is done at the time and in the place of our work, as in the case of Pedro and Antonio, which we just saw. It doesn't matter that study is done in another place and another time, like the study that we did in the Culture Circle. Study always demands a serious and curious attitude in the search to understand the things and facts we observe.

A text to be read is a text to be studied. A text to be studied is a text to be interpreted. We cannot interpret a text if we read it without paying attention, without curiosity; if we stop reading at the first difficulty. What would have become of the crop of cocoa beans on that farm if Pedro and Antonio had stopped carrying on the work because of a mudhole? If a text is difficult, you insist on understanding it. You work with it as Antonio and Pedro did in relation to the problem of the mudhole. To study demands discipline.

> To study is not easy, because to study is to create and re-create and not to repeat what others say. To study is a revolutionary duty!

Notes *Studying the above passages leads to a discussion about the falseness of dichotomizing mental and manual labor. When workers are convinced that they already know methods for studying problems, they are motivated to develop new methods for studying new, academic problems.

*Analyzing this further leads to a discussion of intellectual diversity. One of the most significant contributions of Paulo Freire (1982) to the development of a critical literacy is the idea that "our task is not to teach students to think--they can already think, but to exchange our ways of thinking with each other and look together for better ways of approaching the decodification of an object." This idea is critically important because it implies a fundamentally different set of assumptions about people, pedagogy and knowledge-creation. Because some people in the USA, for example, need to learn to write in 'standard' English, it does not follow that they cannot express very complex analyses of social, political, economic, ethical and other issues. And many people with an excellent grasp of reading, writing and mathematics skills need to learn about the world, about philosophy, about psychology, about justice and many other areas in order to deepen their understandings. In a non-trivial way we can learn a great deal from intellectual diversity. Most of the burning social, political, economic and ethical questions of our time remain unanswered. In the USA we live in a society of enormous wealth and we have significant hunger and homelessness; although we have engaged in medical and scientific research for scores of years, we are not any closer to changing the prognosis for most cancers. Certainly we can learn from the perspectives and philosophies of people whose knowledge has developed in a variety of intellectual and experiential conditions. Currently "the intellectual activity of those without power is always labeled non-intellectual"(Friere & Macado, 1987, p.122). When we see this as a political situation, as part of our "regime of truth", we can realize that all people have knowledge, all people are continually creating knowledge, and all of us have a lot to learn.

*On the other hand, we discuss the need to avoid what Youngman (1986, p.179) calls Freire's tendency toward an "uncritical faith in the 'people' [which] makes him ambivalent about saying outright that educators can have a theoretical understandings superior to that of the learners." While it is vitally important that, as teachers, we listen to students' themes, it is equally important that we organize those themes using our critical theoretical frameworks, and re-present those themes to students as problems that may challenge their previous perceptions. Also, we suggest new themes, themes we judge are important to shattering the commonly held myths about the structure of society and knowledge, myths that interfere with the development of critical consciousness.

Understanding the Mathematics of Workers' Conditions In spite of the mathematical knowledge workers' have learned and adapted in their own situations, many workers have been mathematically disempowered through their schooling experiences. This usually results in their 'avoiding' numbers, and not regarding numerical data as a part of understanding economic, political and social issues. Learning math in the real-life context that clarifies the institutional structures of a capitalist economic system teaches the kinds of mathematical information and analysis crucial for workers' 'reading the world.'

EXAMPLE Students are asked to discuss what numerical understandings they need in order to decipher the following chart. These figures were compiled in time-and-motion studies, conducted by General Electric, and published in a 1960 handbook to provide office managers with standards by which clerical labor should be organized. (Braverman,1974, p. 321)

Open and close	Minutes
Open side drawer of standard desk	0.014
Open center drawer	0.026
Close side drawer	0.015
Close center drawer	0.027
Chair Activity	Minutes
Get up from chair	0.039
Sit down in chair	0.033
Turn in swivel chair	0.009

Notes * Students see that an understanding of how very small these decimal fractions are, so small that watches cannot even measure those units of time, illuminates the viciousness of time-motion studies in capitalist 'scientific management' strategies. As MES2 reviewer Madelena Santos pointed out, this does not mean that students instantly develop a deep understanding of worker exploitation under a capitalist economic system. Occasionally, in my classes of adult students, there is someone who actually remembers being 'time-motion studied' on the job, and this clearly helps strengthen our analysis. However, due to the time constraints of learning under capitalism, where my students often need a few jobs to support their families and their public education costs $5000 a year, their study and classroom time is limited. In this situation, the focus of the criticalmathematics literacy curriculum is on how math can help workers to understand and fight to change conditions of exploitation.

* To more deeply understand how math underpins this kind of worker control, we read about William Henry Leffingwell who:

> calculated that the placement of water fountains so that each clerk walked, on the average, a mere hundred feet for a drink would cause the clerical workers in one office to walk and aggregate of fifty thousand miles each year just to drink an adequate amount of water, with a corresponding loss of time for the employer. (This represents the walking time of a thousand clerks, each of whom walked only a few hundred yards a day.)… All motions or energies not directed to the increase of capital are of course 'wasted' or 'misspent.' That every individual needs a variety of movements and changes of routine in order to maintain a state of physical health and mental freshness, and that from this point of view such motion is not wasted, does not enter into the case. The solicitude that brings everything to the workers' hand is a piece with the fattening arrangements of a cattle feed-lot or poultry plant, in that the end sought is the same in each case: the fattening of the corporate balance sheet. (Braverman, pp. 310-11)

* To look at this outrage even more deeply, we can discuss the theory behind the 'scientific management' of workers. The idea is to conceive of the worker as a general-purpose machine operated by management, displacing laborers as the subjective element of the labor process and transforming them into objects. Braverman (p.190-2) states:

> This mechanical exercise of human faculties according to motion types which are studied independently of the particular kind of work being done, brings to life the Marxist conception of 'abstract labor.'… The capitalist sees 'labor not as a total human endeavor, but [abstracts it] from all its concrete qualities in order to comprehend it as universal and endlessly repeated motions, the sum of which, when merged with the other things that capital buys--machines, materials, etc.--results in the production of a larger sum of capital than that which was 'invested' at the outset of the process. Labor in the form of standardized motion patterns is labor used as an interchangeable part...'

* Finally, as Madelena Santos pointed out, conditions of worker exploitation are less visible today than in the 1960's, so it is more difficult to see how these conditions of control are exercised. In the future, I will add information about computer "numerical control" of work to address this issue.

Understanding the Politics of the Mathematics of 'Official' Descriptions of Workers' Conditions To critically 'read the world,' it is also important to analyze the 'official' information through which most people are reading the world, to know the questions to ask to better understand that information, to discover which questions have not been asked, to find out who has been asked, and so on.

EXAMPLE Students are told that in the USA the unemployment rate is defined as: the number of people unemployed divided by the number of people in the labor force. Then they are given figures from December 1994 (in thousands, rounded to nearest hundred thousand) of various groups of workers, and asked to discuss who should be counted as unemployed, who should be considered part of the labor force and why.

(1)	101,400	Employed full-time
(2)	19,000	Employed part-time, want part -time work
(3)	4,000	Employed part-time, want full-time work
(4)	5,600	Not employed, looked for work in last month, not on temporary layoff
(5)	1,100	Not employed, on temporary layoff
(6)	400	Not employed, want a job now, looked for work in last year, stopped looking because discouraged about prospects of finding work
(7)	1,400	Not employed, want a job now, looked for work in last year, stopped looking for other reasons
(8)	3,800	Not employed, want a job now, have not looked for work

in the last year

(9) 60,700 Not employed, don't want a job now (adults)

Notes *****Discussion brings out that there is political struggle involved in deciding who counts as unemployed (and once that is decided, changing the fraction to a decimal to a percent does not involve political struggle, but instead is based on the way the numbers work). The USA official definition counts (4) and (5) as unemployed and (1) through (5) as part of the labor force, giving an unemployment rate of 5.1%. If instead we count (4) through (8) plus half of (3) as unemployed, the rate would be 9.3%. And there are other groups we could count, such as the 2.5 million people who worked full-time, year round in 1994 and earned below the official poverty line. (Sklar,1995) Further, in 1994 the Bureau of Labor Statistics stopped issuing its U-7 rate, a measure which included categories (2), (3), and (6)-(8), so now researchers will not be able to determine 'alternative' unemployment rates. (Saunders,1994)

*****More extended discussion brings up the issue of the unpaid labor done largely by women at home, and how this free labor is an important factor in increasing business profits—wages would have to be much greater if men had to pay people to cook, clean, care for children, and so on. In spite of the increasing participation of USA women in the paid labor force, the amount of time women spend working in the home is still substantial, and much greater than that of their husbands. (see Frankenstein, 1989, pp.66-7)

*****More extended discussion also brings up issues such as the effects of racism in employment data. In 1986, for example, 14.8% of black workers and 10.6% of Hispanic workers were unemployed, compared to 6.8% of white workers. Even with a college degree, blacks had higher unemployment rates than whites (in1986,13.2% to 5.3%); blacks with a college degree had higher unemployment than whites with only a high school diploma (13.2% to 10.1%). "When people of color bear a large share of the burden of unemployment, they buffer whites against the ups and downs of the business cycle." Further, when blacks find jobs, those jobs usually pay less than those of their white counterparts. "One way to measure the combined effect is to multiply the median earnings of the different groups by the percentage of the labor force of that group that is employed. This provides an estimate of the typical earnings of a member of the labor force….[For example] individual black men working full-time in 1983 earned 75% of what white men earned, but a typical black man in the labor force earned only 52% of what a typical white man earned--because the black man was far more likely to be unemployed." (Folbre,1987, charts 4.7, 4.8, 4.12)

***** Discussions of the even broader picture of black-white income inequality requires taking into account the role that the free labor of slaves had in building the fortunes of capitalists. "By 1820, half of America's exports were slave-picked cotton. By 1860, American slaves picked two-thirds of all cotton grown on earth." Although hard to calculate fully, since slave labor benefited white workers as well as capitalists (cotton, for example, was the raw material for factory workers in the USA and Europe, which in turn generated jobs in transportation, handicrafts, trade and so on), economists in *The Wealth of Races* estimate that the face value of American slaves in 1860, in 1983 dollars, was $17 billion.

> Assuming that the money slaves produced for others from 1790 to 1860 grew in the economy, economists, using conservative compound interest rates, calculate that the value of such incomes by 1983 was between $1.4 trillion to $4.7 trillion…. [At the end of the Civil War] white confederate landowners got their land back while African-Americans got virtually nothing...[Racism manifesting itself in] government-approved housing segregation, property redlining, and job discrimination…cost African Americans at least $1.6 trillion in lost wages from 1929-69…. even if one considered all social programs from 1953-83 as 'reparations' [for slavery] the black-white wealth gap was still $500 billion. And by 1983, of course, such programs were under attack by…Reagan…the goal of reparations should be to 'restore the black community to the economic position it would have had if it had not been subjected to slavery and discrimination.' [Reparations] would be America's recognition that white wealth is based, to a horrendous extent, on black credit.(Jackson,1997)

***** Finally, discussions about unemployment that consider the broader theoretical context, examine why 100% employment cannot exist in a capitalist economic system, because that would create a situation in which the workers would have too much power to change the conditions of their exploitation.

Understanding the Politics of Mathematical Knowledge as 'Officially' Presented in the Class At a math conference on problem solving, a presenter who was illustrating Polya's (1957) ideas on problem solving, stated that in the problem, 'if three people take ten hours to dig a hole, how many hours would it take ten people to dig that same size hole?' it is obviously equivalent whether people dig a hole or machines dig the hole. From a critical mathematics perspective, whether people or machines dig holes involves crucial issues of concern to our society, issues such as automation, unemployment, and quality of life. To study mathematical problems solving without engaging these issues is to become functionally, as opposed to critically, literate. At its worst, this way of learning mathematical knowledge can result in the kind of blind pursuit of 'neutral' knowledge which produces, for example, nuclear weapons without awareness or questioning of the interests behind the science and the consequences ahead. As Marcuse argued:

> in this society, the rational rather than the irrational becomes the most effective vehicle of mystification...for example, the scientific approach to the vexing problem of mutual annihilation--the mathematics and calculations of kill and over-kill, the measurement of spreading or no quite-so-spreading fallout…is mystifying to the extent to which it promotes (and even demands) behavior which accepts the insanity. It thus counteracts a truly rational behavior--namely, the refusal to go along, and the effort to do away with the conditions which produce the insanity. (1964, pp.189-90)

To learn mathematics in a way that avoids this insanity, to learn mathematics in a way that fosters a critical 'reading of the world,' it is important, therefore, to analyze and challenge the 'hidden messages' which are presented in math learning situations.

EXAMPLE Students are asked to critically reflect on the messages encoded in their mathematics textbook.

Notes * We focus our analysis on the content of word problems, but we discuss that even traditional math courses which provide no real-life data and only consist of the symbolic language of mathematics, carry the non-neutral hidden message that learning math is separate from helping people understand and control the world. We further discuss that texts contain other kinds of hidden messages--might, for example, a very boring text serve to prepare the student for very boring work?

* We discuss how even trivial math applications, like finding the total from a grocery bill, carry the non-neutral hidden message that it's natural to distribute food according to individual payment. We look at Gill's (1998) analysis of two problems from an English math text that involve figuring out average wages:

> (1) In an office, 5 people earn a wage of £36 each, 3 earn a wage of £40 each and 2 earn a wage of £42 each. Calculate the average wage.
>
> (2) In three weeks a man earns £60, £50 and £58. His average weekly earnings for four weeks is £54. How much did he earn in the fourth week?

Gill argues that "The first example describes, and because it does not comment, appears to legitimize the hierarchical organization of labour in offices, with the majority of workers on lower pay than the minority... [The second} can be seen to legitimise systems of employment in which people's weekly income is uncertain and variable…"(p.124). She also analyzes typical exercises dealing with shopkeepers and car dealers making profit or loss, where "profit" is defined as "the difference between the selling price and the cost price." She contrasts this "antiseptic" definition with Marx's concept of "profit as ultimately unpaid labour"(p. 123).

* We also look at the treatment of workers in other texts. Anyon (1979) does this for economic and labor history between the Civil War and World War II in seventeen widely used USA high school history texts. She found a striking absence of working class history:

> The average length of the section…on labor history is 6 pages. Most strikes are not even mentioned, and although there were more than 30,000 during this period, the texts only describe a few of them. Fourteen of the 17 books chose from among the same three strikes, ones that were especially violent and were failures from labor's point of view…(p.373)

Further, there is an absence even of the concept of, or a label for 'the working class.' There is a clear underlying theme that:

> …the methods appropriate for solving economic and labor problems and the view of consensual and orderly social change inherent in them are actions that maintain the balance of power in society; confrontation between contending groups which could increase the likelihood of changes in the power structure are not implied. (p.383)

CRITICALMATHEMATICS LITERACY FOR WORKERS' EMPOWERMENT?

So, will all this respect and understandings lead to an actual change in the conditions of workers' lives? Freire's writings on the details of how critical consciousness leads to radical change (e.g.,"This pedagogy makes oppression and its causes objects of engagement in the struggle for their liberation" (1970, p.33)) leaves him open to Mackie's critique that by ignoring "the political economy of revolution in favor of an emphasis on its cultural dimension [Freire's] talk of revolution tends to become utopian and idealized."(1981, p. 106) Carby (1990, p.85) highlights this issue in her remarks on the changes in the literary canon at universities, where African-American women have become subjects on the syllabus, but the material conditions of most African-Americans, including women in the academy, are still ignored. She challenges us to think through issues of real power: "are the politics of difference effective in making visible women of color while rendering invisible the politics of exploitation?" As Jim Hightower (1989) says:

> To clean up this mess, we've got to go to work. We have got to create a new people's politics. My Aunt Beulah told me, "you won't ever clear up the water till you get the hogs out of the creek." You don't get a hog out of the creek by saying, "Here hog, here hog, pretty please." You get a hog out of the creek by putting your shoulder to it and shoving it out of the creek. That's what we've got to do.

And we need all the shoulders we can get! Before we can push together, we need to understand what divides us, and we need to not divide our own understandings.

Understanding the Politics of Division The historical development of various structural divisions in the United States helps explain why these splits have been so effective in clouding class consciousness, solidarity and struggle (see Hogan,1982). For example, we can trace the structural division of racism to the beginnings of the black workforce in slavery. After "emancipation" racism forced blacks into dead-end, low-wage employment, or unemployment. White workers benefited as a group, certainly in the short-run, by getting the (however slightly) better jobs. These conditions made it easy for employers to use blacks as strike-breakers, fueling the white strikers' racism and diverting their struggle against capital. As blacks organized during the Civil Rights and Black Power movements, the competitive, individualistic ethic of capitalism combined with racism to exacerbate white workers' fears of losing their relatively privileged positions (see Boggs,1970). Furthermore, racism, again reinforced with the 'dog-eat-dog/survival of the fittest' view of human society promoted by the capitalist class, impedes class solidarity by providing psychological benefits to poor and working class whites. These white workers can feel better about their own exploited situation by comparison to, and in numerous cases, by active participation in, others' oppression (Reich,1978, p.387). An illustration is found in an ethnographic study of two male teenage peer groups in a low-income housing project—one white, one black. MacLeod (1987) found that the white kinds were aware that "the opportunity structure is not open" and that there are "external obstacles to their social advancement." But "in the absence of any systematic critique of capitalism," they turn to racism and explain their own lack of success by blaming blacks and affirmative action. MacLeod shows how the white teenagers explanations muddle class and race issues, so that the kinds "spare themselves blame, but then the social order is also spared any serious scrutiny"(pp.121-2). Further, the more recent history of capitalism in the United States has added new conflicts among the working class. For example, the international division of labor, with run-away shops and super-exploitation of workers in poor countries diverts the struggle of USA workers when they blame their job losses on cheap labor in other countries (Hymer,1978, pp 497-9).

EXAMPLE Students are asked to reflect on how the ways in which statistical data is reported reinforces divisions. In one case, the USA. government rarely disaggregates health data by social class. In 1986, when it did this for heart and cerebrovascular disease, it found enormous gaps:

> The death rate from heart disease, for example, was 2.3 times higher among unskilled blue-collar operators than among managers and professionals. By contrast, the mortality rate from heart disease in 1986 for blacks was 1.3 times higher than for whites…the way in which statistics are kept does not help to make white and black workers aware of the commonality of their predicament. (Navarro, 1991, p.436)

Notes *****We discuss other statistical analyses that support breaking down the divisions, but that don't get reported. For example, some white workers may believe that racial discrimination against blacks benefits whites because of reduced competition for jobs. Reich (1978) uses correlation coefficients between various measures of racism and white incomes to show that racism results in lower wages for white as well as black workers, and higher profits for the capitalist class.

***** We look more broadly at how the mass media atomizes the divisions in the working class by bombarding us with images of how 'everybody's really the same,' so all you need is motivation and persistence to make it to the top (see DeMott,1991).

*We also examine how schools obscure and strengthen the structural oppression of class society through their ostensible organization according to merit. Students who don't do well in school internalize blame, taking individual responsibility for their 'failure,' lowering their aspirations. They "accept their eventual placement in low status jobs as the natural outcome of their own shortcomings."(MacLeod,1987, p. 113) The reality, as Bowles and Gintis (1976) documented, is that socioeconomic background is the major predictor of educational attainment (p.31) and that, regardless of educational attainment, socioeconomic background is the major predictor of future economic success (p.142). We look at what happens in schools (see Anyon,1980 and Weis,1990) to make

> the structure of society appear as permanent, or beyond human control…what human beings have created comes to seem immutable, 'natural,' transformation becomes individualized…American society is characterized by an appearance of permanence as a system, but by a reality of permeability by individuals…(Sennet and Cobb,1972, p.271)

Understanding the Politics of (Intellectual) Division/Fragmentation

An anecdote in the *New York Times* sums up the failure of class politics in the United States. Sharon Duke, 22-year old white, single mother, explained her vehement support for white racist candidate David Duke (no relation to her) all in terms of race. "They just have those babies and go on welfare," she complained to a reporter. She herself, though, was unemployed and on welfare. "Yes" she replied "but the blacks get more."(Frank,1992) In the absence in the USA of a unifying liberation struggle, small victories become isolated instances that may even support, rather than challenge, the status quo. One way to contribute to counteracting this and to helping build a unified/unifying struggle, is to develop an overarching theory, which the individual events illuminate, or challenge, and which unclouds the organizing structures of society and clarifies that people control those structures. As Marcuse (1964) argues

> The trouble is that the statistics, measurements, and field studies of empirical society and political…science are not rational enough. They become mystifying to the extent to which they are isolated from the truly concrete context which makes the facts and determines their functions. This context is larger and older than that of the plants and shops investigated, of the towns and cities studied, of the areas and groups whose public opinion is polled or whose chance of survival is calculated…. This real context in which the particular subjects obtain their real significance is definable only within a theory of society. (p.190)

And theory matters. Nteta (1987, p.55) argues that "revolutionary self-consciousness [is] an objective force within the process of liberation." He shows how the aim of Steve Biko's theories, and of the Black Consciousness Movement in South Africa, was "to demystify power relations so that blacks would come to view their status as neither natural, inevitable nor part of the eternal social order…[creating] conditions that have irreversibly transfigured South Africa's political landscape." (pp.60-1)

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Markets, Marx, Modernity and Mathematics Education. A Response to Michael Apple

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I very much welcomed the invitation to respond to Michael Apple, because he is one of those people who was once one of my gurus. That is not to say that he is no longer a guru, its just that I no longer have gurus. If I did, he would probably still be one. His writing usually gives me that feel good factor. You know when you read something and subliminally - or out loud - find yourself saying "Yes!" and feeling that you wished you had said that. This paper is no different for me. It is a paper representing a position that just has to be said, and said louder. However here I am going to respond by taking a slightly different slant to focus my torch onto a corner of the room Michael has chosen not to bring to the fore. In this pre-conference paper, I will sketch out the argument I will be making in my response to the conference.

In his paper, Michael has identified quite clearly the current dangers and processes of the growth of the new right ideologies in the development of subjectivity, society and education and I want to take some of his arguments and thrusts slightly further. What we need to help us understand these tendencies is the development of a theory of structure and agency in advanced capitalist societies in which most of us are located, and in the developing capitalist economies in which the rest of us are located.

I want to enlarge upon what I see as a significant tendency in the development of market discourses - that of neo-liberalism. Liberalism has a long history in arguing for support for the individual freedom. Neo liberalism takes this into a new arena and elevates oxymoron to an art form. Neo-liberalism as portrayed by Margaret Thatcher, values the freedom of the individual, the free market, parental choice. Such a gradual development of discursive chains has changed the orientation of popular discourses, and in so doing, has managed to deflect attention away from mechanisms of dominant and inequity and forms of authoritarianism.

How is this related to our meeting in a maths education conference? I will use the argument that a neo-liberal political economy will create lower educational standards (Lauder 1991, p 417). Furthermore, it favours individual freedom over democratic participation and in so doing, in favouring the pursuit of self-interest, produces socially undesirable outcomes (i.e. undesirable in my terms).

Standards, quality in education and universal advancement are not part of the essential programme of the market economy. Quite the contrary – the development of capitalist relations of production (which I see as equivalent to the "*Market Economy*" or a "*low wage, low technology economy*") requires a low skill economy, where managers can keep costs down by reducing wage costs, and thereby fight to increase surplus value. This is part of the ongoing drive to counter what Karl Marx identified as the tendency for the rate of profit to fall. In such an economy, profits can be made out of cheap labour. Such a *low wage, low technology economy* can be contrasted with a *high wage, **high technology economy*** requiring high levels of skill and autonomy. In such an economy, would be needed a range of skills at all levels requiring a strong sense of equality of opportunity. In addition, a ***democratic economy*** requires a universally high level of skills so that all individuals may contribute and participate in a democratic society. In addition, a strong sense of equality and equity is required in order to ensure that previously and traditionally silenced voices may be heard in order to participate fully in society.

So, what are we going for? I don't believe we can challenge the rhetoric of the market economy without challenging the legitimacy and desirability of the market economy. I put up my stall in the democratic socialist economy and all the implications that has.

You will probably have noticed by now that I am likely to have little truck with argument that we live in an age of postmodernity, and maintain that we need to quite urgently pull back from the postmodern abyss. There are arguments that in this age of global markets, mass communications, changing employment practices and rising living standards, we live in an age where capitalism as described and conceptualised by Marxists has fundamentally obliterated itself. For Francis Fukuyama we have reached the "*end of history*" with the triumph of liberal democracy (Fukuyama 1992, p 338). What I do need to do, since I have been arguing for a political meta-narrative that postmodernism would deny, is to give my rationale for rejecting arguments for postmodernity.

Alex Callinicos describes how, because the characteristic structures of capitalism have not undergone any fundamental transformation, postmodernity, seen as a perspective which eagerly embraces the present as the beginning of a new era of unprecedented fluidity, social mobility, and individual choice is "*historically dubious*" (Callinicos 1999, p 260). Michael, whose work is located within critical education, holds a similar position as he has written elsewhere.

> Capitalism may be being transformed, but it still exists as a massive structuring force. Many people may not think and act in ways predicted by class essentializing theories, but this does *not* mean the racial, sexual and class divisions of paid and unpaid labor have disappeared nor does it mean that relations of production (both economic *and* cultural since how we think about these two may be different) can be ignored if we do it in non essentializing ways.
>
> (Apple 1997, p 599)

I do feel passionately that there is something we can do and I reject arguments that we are in a post-modern era. The imperative here is the need to distinguish as clearly and coherently as we can those aspects of the debate over postmodernism that are supportive of social justice, emancipation and democracy from those which are more individualising and fragmentary, marginalising or even rejecting the struggle for equality and freedom. My position is to base a theoretical framework on a model of social organisation that takes the underlying relations of production as a central force. This means not assuming that individuals are fragmented, constituted by discourses, but rather are embedded in a stratified society and consequently reflect this social structure in their cognitive structures and interpersonal relations. In doing this though, we can adopt the position that the construction of one's individual social frameworks are likely to be somewhat fragmented due to the complex nature of the society we are bought up in.

What neo-liberalism does cleverly is to draw on liberalism's Achilles heel – the valuing of individual liberty – another oxymoron. There is some assumption that not only is individual liberty possible as a characteristic, but that it is strategically possible. Liberalism achieves this through the assumption of the universality of shared values, such as equality, equity, caring, sharing. These are not value free values, but are tied closely to underlying social assumptions. I would argue that they are the values of a particular tendency and tradition – that of the left. Alternatively, we have values of struggle, competition, rational self-interest, freedom. Which are the views of the right. I ought to apologise to those of you who find such binary assertions painful, quaint, naïve or politically or intellectually ignorant. It is I feel a failure to recognize such dialectical tendencies, which result in a lack of clarity about how we move forward to construct what Michael calls,

> *defensible, articulate and fully fleshed out alternative progressive policies and practices in curriculum, teaching and evaluation.*

Arthur Halsey has shown that in the UK, differences in success in the education system (particularly in our divisive separation between state and private schools) can largely be explained by the differences in parents' social class background. We can extend this further, the root causes of failure in mathematics classrooms is not fundamentally the teaching sequence, misconceptions, imagery, mental representation and other constructivist concerns, but it is poverty, social disadvantage, low wages, poor housing, social exclusion, and so on. But of course, this is counter-hegemonic. It is almost a heresy. Its not even common sense!

> *Neo-liberals and conservatives have shown how important changes in commonsense are in the struggle for education.*

We will not change common sense by working within the commonsense boundaries of current conservative discourses. What we need to be now is to be counter-hegemonic. We need to flip the coin and create and exploit the dialectical space between inevitable tensions. I will offer three examples.

Hegemonic position	**Counter-hegemonic position**
Homework is good, helps to reinforce work done in school, consequently raises standards and improves learning opportunities.	Homework is bad for children.
Working class parents don't involve themselves enough in their children's education.	Parents from disadvantaged background need to distance themselves from official schooling. They have nothing to gain form it in it present configuration and need to organise to change rather than sallow themselves to be implicated.
The National Numeracy Strategy in the UK is about raising standards.	The national numeracy strategy is about reducing the gap between pupils of different social classes. It is about favouring the poor at the expense of the rich.

Where we start from is an articulation of our own ideological orientation. This short paper is an attempt partially to achieve that. Erna Yackel has argued for a similar elaboration.

> *In our view, Apple is right when he calls for us to clarify the ideological, social and political dimensions of our efforts to initiate reform in mathematics education. Only then can we guard against the possibility that we will unknowingly foster even greater inequities.*
>
> *(Yackel and Cobb 1994, p 32)*

Where can we start. Well for me there are some precursors, basic assumptions that need establishing and clarifying.

- That society is a conflict between differing;
- That the economic structure, the mode of production, is a fundamental determinant of social life;
- That we need to consider the interconnectedness of the whole social system rather than explore in isolation locations of social activity;
- That life is essentially social;
- That educational research should be critical and emancipatory, through analysing power relations.

Now that's a radical strategy. It is not one that is going to be easy, because it requires us to accept that we are all ideological, but that ideological orientations are related to fundamental drives deriving from our social imagery and dispositions. It also requires us to begin to conceptualise fundamental theoretical orientations.

Other maths education conferences have only interpreted the world in various ways; the point of this one, however, is to change it. As Michael Apple says

> *There is political and practical work that needs to be done. If we do not do it, who will?*

Our task here is to begin that theoretical and organisational process for change.

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Researching multicultural classes: a collaborative approach

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In this paper we discuss the adequacy of using a collaborative model as a research approach for studying the dynamics of a multicultural mathematics classroom[1]*. The goal of the research is the analysis of the immigrant students' processes of adapting to a new cultural context, from their home and school culture to the school culture that hosts them, understanding the construct 'culture' in its broadest sense. Our study has shown the importance of involving in-practice teachers in researching crucial issues related to their day-to-day teaching.*

The research within its context

The research we present is framed within a wider one that clearly has an intention of change in the politics of the educational system in Catalonia, an autonomous region in northern Spain (capital Barcelona). Since 1997, the authors are working in a project[2] commissioned by the Ministry of Education in Catalonia, concerned with finding more appropriate ways to teach mathematics in schools with large numbers of immigrant students.

In recent years, there has been increasing immigration into Catalonia that has led to significant changes in the school population. The immigrant population in Catalonia is about 1,6% of the whole population and, according to the register of inhabitants in 1998 in Barcelona, the area of the project, it reached 2,3%. Even if this number may seem not to be significant quantitatively, it is certainly significant qualitatively, because the reality of a multicultural classroom raises many questions related to issues concerning equity and

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[1] The political and social contexts of the research, as well as its global aims, were already presented in a contirbution to MEAS1, 1st International Conference on Mathematics Education and Society, held at Nottingham on September 1998, under the title *'Starting a research project with immigrant students: constraints, possibilities, observations and challenges'* by N. Gorgorió.

[2] The project is funded by a Catalan private foundation devoted to education, Fundació Propedagògic.

justice (Keitel et al., 1989). At present, most of the immigrant pupils come from North Africa (Magreb), but others come from countries in other parts of Africa, North and South America, Asia and Eastern Europe.

Even if the project was initially the result of a request from the administration, the team's understanding of the multicultural situation in the school goes far beyond that of the educational administration. It has been a long process of discussion with the administration to negotiate the goals of the research. The team has negotiated strongly, and continues to do so, to change what initially was a policy-driven 'research' project into a research project with no inverted commas. However, it is hard to change the views of the people in the educational administration about mathematics being a cultural product rather than a universal matter, to have them abandon the simple idea of the mathematical utilitarian culture, and especially to have them compromise about the social and political implications of our research.

Probably the most difficult arguments with bureaucrats and politicians have been about mathematics being a cultural product, about learning and teaching mathematics being linked to values, beliefs and expectations, and about the fact that these emotional aspects can explain many of difficulties immigrant students experience when learning mathematics. We have also spent part of our time trying to change the educational bureaucrats' idea of a 'cognitive or cultural deficit' to be 'compensated', by emphasising with them the importance and possibility of building on the potentialities every student has.

The authors consider that explaining the difficulties immigrant children have in school in terms of cognitive deficit is too simplistic and questionable (Ginsburg & Allardine, 1984; Nunes, Schliemman and Carraher, 1993; Rasekoala, 1997). Moreover, this interpretation has social implications, because it projects particular expectations onto concrete cultural groups, which confirms our opposition to it because of our belief in the importance of students' personal values and experiences. To explain the difficulties that many immigrant students face in their mathematics learning it is not enough to consider them in terms of the students' cognitive abilities, in terms of the quality of the teaching they receive, or in terms of the adequacy of the communication tools. Mathematics education, to be understood, should be embedded in the comprehension of the social and cultural reality where it takes place (Oliveras, 1996).

The mathematics classroom is a situation where many social and cultural conflicts arise that should be incorporated into the research models in mathematics education. In particular, we understand that the difficulties immigrant students experience when learning mathematics are often linked to the ‘distance’ between their own social and cultural frames of reference and the implicit ones within school. Our starting point is the consideration of the cultural contribution of ethnic minorities and of different social groups as a source of richness to be maintained and shared. The team does not see cultural differences, and the cultural conflicts arising from them, as a ‘problem to be solved’ nor as a ‘diversity to be treated’, but as a potentiality.

Regarding the analysis of the social dynamics of the mathematics classroom and the cultural conflicts arising there, and as a result of the different meanings attached by the participants to its different aspects, our research has three goals:

- to document, analyse and reflect on how mathematics teachers, as agents of change, understand the cultural and social conflicts and disruptions arising in their mathematics classrooms,
- to identify and interpret different external manifestations of cultural conflicts related to the different meanings attached to the norms that regulate the social dynamics of the mathematics classroom and the norms that regulate classroom mathematical practice,
- to identify, experiment with, and analyse different teaching strategies and social dynamics that could contribute to making visible the cultural conflict and to co-construct meanings in order to facilitate the learning process of immigrant students.

The research model from a theoretical perspective

The lack of relationship between research and practice has been fully documented, both in education in general and in mathematics education in particular. It is clear however from various research summaries (Bishop et al, 1996; Grouws, 1992) that preserving the dichotomy between research and practice is no longer acceptable, and that there are increasing moves to involve teachers much more in research. However, often in research that has the classroom as the object of its study, even if the teachers are participants, their roles consist only in developing the researchers’ proposals, most of the time without even knowing the grounds for their actions (Cook & Reichardt, 1986).

Therefore, the need exists for knowing more about teachers' perspectives on practical issues which researchers could seriously address, and for counting on their expertise and knowledge to find ways to research them and to interpret the results. As Bishop (1996) states: 'As mathematics education is a practitioner-dependent activity, the research process should therefore be practitioner-focussed. This means that it should address practitioners' issues and problems, and that practitioners should be involved at all stages of a research project, particularly at the start. It does not mean that there is no place for reflection or for theory, but only that the practitioner-orientation should predominate.' (Bishop, op. cit., p. 4)

The main goal of our project being to 'promote changes' in the educational context, we therefore considered, as do Kemmis & Taggart (1988), that the best approach would be action-research; that is, research done by people on their own work, following an essentially critical approach to schooling, and with the explicit aim of improvement. As Schon (1983) points out, action-research should enable us to build bridges from what is going on in educational research to the intuitive educational acts. Action-research was originally introduced into the field of education with the intention of creating real improvement in educational practice, because it was considered that the educational research developed until then did not bring with it any real improvement in practice. More than any other approach, action-research takes 'change' as its focus, and encourages practitioners at different levels to research collaboratively their shared problems, and as these authors point out, 'the approach is only action research when it is collaborative' (Kemmis &Taggart, op.cit., p. 5).

Insisting on the ideas of teachers reflecting on their actions, rather than just being technicians, and on researchers becoming more aware of classroom limitations and constraints, should be beneficial for both practice and research. We consider, as do Cohen and Manion (1990), that action-research is an alternative both to external research which has little connection with classroom realities, and also to subjective practice, which omits the guarantees of external observers that ensure triangulation of the data and their interpretation. However, the development, if not the survival, of action-research will not be possible without the contribution of the different educational administrations. They need to facilitate it by such means as: giving economic support, reducing the teachers' school schedules, facilitating their attendance at conferences, promoting in-service working groups, reforming rigid working structures, and restructuring the power hierarchies (Baumann, 1996).

There is no doubt that balancing research and practice is not an easy task. Wong (1995) remarks that there are two kinds of conflict between the role of teacher and the role of researcher, those of purpose and of conduct. Wong refers to the tension emerging between the purposes of the two perspectives: as a researcher observation has to take place without intervention, without guidance, while teaching requires the leading of the learning process, even if this act alters the observable situation. However, the use of an interpretative paradigm, and the use of qualitative methods reduces the conflicts stated by Wong (1995). Wilson (1995) criticises Wong's perspective on conflicts, arguing that it is due to the use of a quantitative paradigm, where the researcher is a subject alien to the studied phenomena. In contrast, according to this author, viewed from an interpretative approach, conflicts can become a kind of compatibility.

The contributions of different research projects dealing with the socio-cultural analysis of learning situations, some of them integrating the complex interaction among affective, cognitive and cultural aspects, together with our personal experiences in previous studies convinced the team that the most adequate research approach, given our goals, was a collaborative one under a qualitative and interpretative paradigm.

Reflections from the perpsective of our project

Doing research within a collaborative model with in-service teachers, and having teachers participate in action-research projects implies a move that, at least in our context, still needs to be justified within both the university community and the school systems. Our university system, both academically and administratively, is still reluctant to accept in-service teachers as full members of research teams. Academics at the university do not consider that practitioners have enough knowledge and expertise in the field of research. Neither does its administrative system yet consider practitioners as members of research groups for the provision of grants. Within the school system, teachers at mathematics departments, principals and inspectors and, more globally, the educational administration, find it difficult to accept and justify practitioners devoting part of their time to research. In particular, as Elliott (1989) states, from different instances related to educational research, action-research is reluctantly accepted or considered because of the complexities of the many variables that this kind of research involves. This argument conveniently forgets that this approach facilitates, more than any others, changes in the practice and the interiorising of the results of the researchers by those that are assumed to implement them.

The reasons for deciding to develop our research collaboratively are several. At present, the research agendas are still dominated by the researchers' questions and aims and not by those of the practitioners. However, we are addressing a crucial issue related strongly to social demands within a particular context. Change being the final goal of our research project, we are convinced that the teachers' contributions on stating the points that should be addressed and on how to address them are crucial.

The issues addressed in our research are actual teachers' problems and have to do with actual teaching constraints and limitations when teaching mathematics in multicultural settings. For instance, in our particular context, teachers feel that one of the biggest problems that they have to face in their classes is communication, because most of their immigrant students have difficulty with the language of the teaching. On one side, the lack of a common language makes it difficult for the teachers to come to know about their students' thinking processes, and for the students to achieve the construction of mathematical meanings[3]. In particular, very often students react to this lack of communication through the 'suspension of sense making' (Schoenfeld, 1994). On the other hand, language being a social tool for communication, the lack of a common framework of reference is at the origin of different interpretations of the social norm, of the norm of the mathematical practice and of the socio-mathematical norm and, therefore, at the origin of many cultural conflicts[4]. Therefore, one of our research goals, related to one of the 'limitations felt by the teachers', namely the difficulty of communication, is to find teaching strategies and methods to minimize the lack of communication and the cultural conflicts.

Concerning the search for teaching strategies that could make visible cultural conflicts and minimize them, it is teachers who can conform more to practitioners' criteria and methods. Teacher's experience and knowledge about the real possibilities for change and their implementation has been crucial in the search for teaching styles and classroom organizations that could modify the social dynamics of the mathematics classroom. And it is

[3] See Gorgorió & Planas (forthcoming): 'Teaching mathematics to immigrant students: A multilingual situation'. *Educational Studies in Mathematics.*

[4] See Gorgorió, N., Planas, N., & Vilella, X. (forthcoming): 'Immigrant children learning mathematics in mainstream schools'. In G. Abreu, A. Bishop & N. Presmeg (eds), *Transitions between Contexts for Mathematics Learning,* Dordrecht, Netherlands: Kluwer Ac. Pub.

also teachers, with their day-to-day life with their students, who have brought to the group the necessary knowledge of the students' real interests and motivations that allow us to create classroom activities that create meaning for the students.

Moreover, teachers have a better knowledge of the particular research context than researchers from university have. This is especially true in our project because, at least in our context, university researchers still have very little knowledge of multicultural classrooms. For instance, concerning methodological approaches, the teachers' knowledge of their students was the only way of establishing links between what was desirable from a theoretical point of view and what was really possible within the classroom. Some methods, like videotaping a classroom or interviewing students, that are crucial for a socio-cultural approach and that could be considered 'easy' by a university researcher, were reluctantly accepted by the students, because of their personal values. It was only through the negotiation of the teachers within the groups, and on the basis of the emotional relationship that the students established with them, that the students agreed to cooperate.

On the other hand, teachers' questions and explanations derive from a knowledge domain that is distinct from, and complementary, to that of isolated researchers from university. Working collaboratively facilitates communication between the different domains, overcoming the mutual exclusion of practice and research. The research focus being on teachers and teaching, we needed to contact and work with practitioners. The presence of teachers as full members of the research group, legitimates and facilitates the contact and the communicating process with other teachers, and also helps with finding ways to disseminate the research findings and the innovation proposals. Moreover, their participation in the research process, benefits themselves as teachers and their own teaching practices. It can also benefit other teachers in other ways, particularly by sharing research approaches with them that any teachers can use. Besides that, the presence of teachers on the research group strongly enriches the process of interpreting the situation and the triangulation of the data, illustrating the complementary points of view of university researchers and practitioners.

The collaborative work allows us to take into consideration not only the factors that condition practice, but also the connections with published theory. Both of these play an important role in shaping the research, by establishing the possibilities, limitations and constraints of the context, and also by offering the dimensions of generality that gives sense to the research.

Working collaboratively facilitates the researchers' contribution to the development of practice by not only contextualising the research within the classroom realities, but also by establishing the whole study in terms of practitioners' needs and schemes of knowledge. The study thereby becomes both an analysis of practice and a search for explanations towards a development of theory.

At this moment, when mathematics teaching is facing many tensions with the implementation of the new educational system in Spain, we consider that collaborative research and action-research can help the educational community to commit itself to the changes in the educational practices. By 'educational community' we mean agents, for example researchers and teachers, and also structures, the educational administration. On the other hand, given that the aims of the research were under negotiation, essentially with bureaucrats and politicians who in general believe that university research is far away from the school real needs, the idea of collaborative research contributed greatly to that process of negotiation.

After two years working, we are convinced from the perspective of our project, that the understanding of the situation and the outcomes we could get through a collaborative and action-research approach, under a qualitative and interpretative paradigm[5], were worth all the constraints and conflicts we were facing in getting involved in action-research.

The teachers as researchers in our project

Having received the request to address an issue that is mainly connected with schools, the first author, being the commissioned researcher and a university lecturer, argued for the necessity of working in a team which included different members of the educational community. The negotiation with the educational administration resulted in a collaborative team, whose other members are in-service secondary and primary mathematics teachers, with a deep experience in multicultural settings, and already linked with research at university, and who also have a partial release of their teaching hours to devote time to the project.

[5] See Gorgorió, N., *'Starting a research project with immigrant students: constraints, possibilities, observations and challenges'*, on the proceedings of the MEAS1, 1st International Conference on Mathematics Education and Society, to know more about the research procedures used.

In the previous sections we have argued, both from a theoretical perspective and from the perspective of our project the importance of having teachers as full members of the research team. In the following section, we will present, through some particular examples, how we establish our collaboration, taking always into account the expertise of the teachers, the published theoretical frameworks, the issues to be investigated, the actions to be developed and the results of our project.

The first questions we usually begin with concern particular cases that arise in day-to-day practice. From the joint discussion of the case we situate it within a theoretical framework to explain it. For instance, during a mathematics lesson, Kamrum, a Pakistani boy, left the classroom because he refused to work in the small group where he was assigned. We understand this case as being one where the teacher and the student attach a different meaning to the classroom organization and to the teacher's role. The student was interviewed by his teacher in the next session. Kamrum, on the one hand, did not accept to work with the girls on the group and, on the other hand, he considered that it was his teacher's duty, according to his previous cultural framework, to solve the problem on the blackboard. Through his explanations, what initially could have been thought of as a disruption, was just an adjustment between the meanings he and his teacher were attaching to the same situation.

Through the discussion of this case in the light of the existing theory, we identified other similar cases, which concerned not only Kamrum, but also Saima, Sajid, Aftab..., and many others. It was not anymore about living in the classroom, but about the need to pay explicit attention to the norms that regulate the social dynamics of the mathematics classroom. Similar processes led us to the need to take into account also the norms of the mathematical practice and the socio-mathematical norms.

From the discussion of the particular cases we decide that some actions should be taken in order to create classroom situations that allowed us to elicit the conflicts arising from different understandings of the norms and to 'positivise' them. Some proposals are made concerning implementing particular classroom organizations, learning activities and teacher interventions that could be helpful to our purpose. The different proposals are developed in their classrooms, the teacher keeps a diary, the lessons are videotaped, and one of the other members of the group acts as an external observer.

In our weekly meetings, we jointly discuss the data coming from the lessons. We analyse them, from the perspective of our goals on the basis of the transcription of the videos, the teacher's diary and the notes of the external observer. All the process is constantly under feedback, in every meeting new ideas and new points of view are discussed that modify the proposals for new actions to be taken next. The group also keeps a diary of the meetings, which is very useful for analyzing our own process and how our initial assumptions have evolved. We want also to note that the fact of publishing and presenting our research, both in academic contexts and in in-service teacher programs, has also forced us to go deeper in the reflections.

However, collaborative research and action-research also have their limitations, both practical and methodological. On the one hand, the teachers involved must continue to do their jobs within the schools, and this means within the constraints of the administration, including in particular the time they are allowed to devote to the project. On the other hand, even if following this approach makes it easier for teachers to participate in research studies than other approaches, some difficulties and tensions inevitably arise from the fact of teachers researching their own teaching. One of the needs we were aware of was to how to ensure a distance between the two simultaneous roles of being a teacher and a researcher.

In particular, we had to face the tensions between the teachers' responsibility to the students and to the research. In particular, there were tensions regarding issues of students feeling reluctant to participate in the research, by for instance not wanting to attend a class if it was going to be videotaped. Our response to this kind of tension was this: the research team explicitly agreed that we had a responsibility as teachers that was over and above that which we had as researchers, even if that could mean a 'loss' for the study. We understand this kind of limitation as part of our work with people having their own system of beliefs and values.

We had also to face the risk of bias when interpreting the data obtained from a classroom where the role of teacher and researcher were played by the same person. Analyzing the data obtained in a study developed in one's own class requires important control actions (Robinson, 1998). Discussing and contrasting the different points of view within the research team, having an observer in the classroom who is different from the teacher, documenting and analyzing the developing of the lessons through the video recording and the teacher's diary have all helped to control the biases.

After three years of working on the project, we are aware that there are still plenty of unresolved issues, which are of concern not just to us, but we suspect to anyone involved in such kind of research. For example, up to what point can/does the research on mathematics education reflect the real needs of the society where it takes place? How are these needs established? Who in charge is to decide which changes should be implemented in the mathematics curriculum in order to guarantee mathematics learning for ALL? Why, so often, does the educational administration pay so little attention to what is being done in the research field? Is it the fault of the researchers themselves because of failing to communicate their results in such a way that they can be useful for the classroom? Or is it because the research questions addressed are far away from the reality of the classroom?

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The Process of Teacher Learning within an INSET practice stimulated by curriculum change: emerging methodological and epistemological issues

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"You know before I always used to introduce myself as the music teacher, now I introduce myself a maths teacher" BL in interview about INSET course – 20/07/99 – Heerengrach school

The research which I am presenting is part of a broader doctoral research plan in which I investigate mathematics teacher learning as it relates to implementing >new= aspects of mathematics education as emphasised within South Africa's new national learner centred outcomes based curriculum. What is presented here is work in progress with some discussion around some preliminary findings. In particular I raise methodological and epistemological issues of the study, for discussion within the MES context.

In this paper I will briefly outline:

C the social and political context within which the study takes place
C the broader research study within which the paper is embedded
C teachers' initial understandings about the new curriculum
C the demands of the curriculum change process on mathematics teacher=s fashioning of new teacher roles and related identities (Lave and Wenger, 1991)
C some emerging elements of teacher practice in relation to implementing (adopting and acting) 'new' roles within the context of the INSET practice
C emerging methodological and epistemological issues for further discussion

Background on current South African curriculum change

South Africa is currently embarking on radical curriculum change. The curriculum change has been stimulated by the major political changes which occurred in the country during the 90's and which brought about the abolition of apartheid and the production of a democratic South Africa. Throughout the nineties education debates were raging as to how to develop an education system which ensures: greater accessibility for all to education, redresses inequalities and enables and encourages democratic citizenship and which enables articulation between vocational and formal education.

Dr Chabani Manganyani, the DDG of Education, in the forward to 'Curriculum 2005 Lifelong Learning for the 21st century. A users guide' writes: Athe curriculum is at the heart of the education process. In the past it has perpetuated race, class, gender and ethnic divisions and has emphasised separateness, rather than common citizenship and nationhood. It is therefore imperative that the curriculum be restructured to reflect the values and principles of our new democratic society." (National Department of Education, 1997)

The curriculum change aims to implement a learner centred, outcomes approach to education. In mathematical terms the curriculum shifts its focus from developing abstract mathematical knowledge with a focus on mastering mathematical skills and algorithms to developing mathematical meaning. Mathematical learning, instead, is to be relational, shifting from its current production of inert knowledge, to knowledge which is flexible, transferable and applicable to everyday life and other learning areas (Adler, Pournara and Graven 2000).

Resonating with the political aims of preparing learners for participation in a democratic society, the curriculum places a significant emphasis on the contextualisation of mathematics, socially, politically, economically and historically. It also places a significant emphasis on particular mathematical processes such as mathematics communication, interpretation and justification.

Recognition of the social and cultural influences on learning and the construction of knowledge has largely been informed by developments in Psychology and Social Anthropology. Recent cognitive and anthropological research in the field of mathematics learning (e.g. Carraher, Carraher and Schliemann, 1985; Lave 1988; Saxe 1990) provide support for the thesis that:

1. mathematics learning is a human creation evolving within social and cultural contexts
2. learners actively construct mathematical knowledge through interaction with the social and cultural environment.

These emphases are inscribed within the new curriculum definition and within the ten specific mathematics outcomes which were designed by the National Education Department to inform all of mathematics teaching from grade 1 up to grade 9[1] (see appendix A). The definition of mathematics as outlined in current National Curriculum documents is:

"Mathematics is the construction of knowledge that deals with qualitative and quantitative relationships of space and time. It is a human activity that deals with patterns, problem-solving, logical thinking etc., in an attempt to understand the world and make use of that understanding. This understanding is expressed, developed and contested through language, symbols and social interaction." [National Department of Education, 1997]

In pedagogical terms the new curriculum requires teachers to use a range of learner centred methodologies, to use co-operative methods of learning which facilitate conceptual and relational understanding rather that rote or procedural knowledge.

However, while the changes demanded by the new mathematics curriculum reflect many of the developments in mathematics curricula around the world, a specificity of this curriculum is that it makes the socio-political aspects of mathematics education explicit and makes explicit the role of mathematics in preparing learners for critical democratic citizenship. Other factors particular to the South African context which influence the process of teachers' learning include the wider political changes taking place and the legacy of apartheid, and the relatively weak knowledge base and the difficult working conditions of many teachers. These specificities are a focal point of the broader research study.

[1] The new curriculum is broken up into phases. Grades 1-3 make up the foundation phase, grades 4-6 make up the intermediate phase, grades 7–9 make up the senior phase and grades 10-12 make up the further education and training phase (FET).

About the INSET Programme

Previous research conducted by Graven (1997, 1998) indicated that teachers in many schools around Johannesburg (the largest city in South Africa) were relying on traditional approaches to teaching, in which the teachers showed step-by-step procedures and rules to follow (often rotely) when solving mathematics problems. Analysis of teacher questionnaires indicated that teachers identified school mathematics as a Abag of facts, rules and skills@ and that its importance for learners was predominantly in terms of providing access into further education and training (FET). This research also indicated that very little information about curriculum development was being disseminated to schools. It is within this context that the Programme for Leader Educators in Secondary Mathematics Education (PLESME) was developed.

PLESME began workshops with 16 teachers from schools in Soweto and Eldorado Park (both urban 'non white' townships outside Johannesburg) in January 1999. The programme works intensively with teachers over a 2 year period. The primary goal of this programme is to:

* create leader teachers in mathematics with the capacity to interpret, critique and implement current curriculum innovations in mathematics education in South Africa and to support other teachers to do the same.

A central aim of these workshops is to support teachers in understanding the specific mathematics outcomes, especially those that were not included in the previous curriculum. These include those which contextualise mathematics within historic, social, political and economic contexts and those mathematical processes such as communication, interpretation, conjecturing and justification.

Why the study focuses on teacher learning

A theory of learning which has developed out of research on socio-constructivist perspectives of learning and which are being increasingly drawn on by mathematics educators to explain student and teacher learning, is that of situated cognition and legitimate participation in communities of practice (Lave&Wenger 1991; Chaiklan and Lave 1993). According to their model, learning is:

C located in the process of co-participation and not in the heads of individuals
C not located in the acquisition of structure but in the increased access of learners to participation
C an interactive process in which the learner simultaneously performs several roles

Watson (1998) edits a collection of articles in a book titled: *Situated Cognition and the Learning of Mathematics.* In this book, educators from several countries, report on the way in which theories of situated cognition and the learning of mathematics, within communities of practice, can be useful in describing mathematics learning. Watson (1998) notes however, that situated cognition and the learning of mathematics has not yet developed into a full blown theory, but it is being usefully applied as a tool for mathematics educators to think with. Adler (1998a) notes that while Lave=s model does not easily apply to student learning in mathematics classrooms, it does apply to teacher learning and therefore has application to understanding teacher development. Previous research conducted by myself (Graven 1997) indicates that teacher education should involve incorporating teachers into a supportive community of practice in which reflection-in-action (Schon 1983) is encouraged. Lave=s model of learning supports this conclusion and it is expected to provide some useful insights for analysing the research data of this study.

As yet there has been no research done on teacher learning in relation to understanding the new mathematics curriculum at the senior phase level in South Africa. Furthermore, teacher voices have been largely absent from curriculum discussions and there has been an absence of sustained debate among both teachers and educators (Jansen, 1997). There is a need for qualitative long term research which 'puts the teacher at centre stage' (Nelson, 1997), and analyses, from the inside of schools, the processes and the dilemmas which teachers go through as they are expected to learn (fashion) new mathematical beliefs, knowledge and teaching practices which >fit= more closely with the new curriculum.

Within the above framework, the study conducts rich qualitative school based research which can explore in depth teacher learning in relation to the questions:

1. How do teachers interpret and enact the new aspects of mathematics education over time?
2. To what extent and how do teachers participate in and make use of a community of practice, stimulated by INSET within the context of curriculum change?

A review of the literature on teacher development indicates a focus on teacher <u>change</u>. The focus of this study attempts to reconceptualise teacher change in terms of teacher <u>learning</u>. Thus the question will not be whether teachers have changed, or how they have changed, but rather what is the process of learning which teachers have experienced through involvement in PLESME at a time of curriculum change. The term 'teacher change' is particularly problematic in the South African context where curriculum support materials set up dichotomies between the 'old' practice and the 'new' practice and quite blatantly refer to all 'old' practice as bad and all 'new' practice as good. These documents call for radical 'teacher change' where old practice is completely replaced by new practice. Once this has happened the learning process is complete. What happened to the idea of learning as a life long process? Such a view of teacher change is clearly disempowering for teachers and furthermore is not educationally sound.

My assumption is that the implementation of the new curriculum does not simply involve following a set of curriculum instructions or replacing 'old' practice with 'new' practice. Rather implementing the new curriculum is a process of fashioning the curriculum in such a way that it becomes part of the teacher's 'way of being' (Lave and Wenger, 1991). In fashioning the curriculum in this way, teachers will 'change' themselves and modify the curriculum. My assumption is that this learning (fashioning of a new way of being) will take place within the context of participation within the INSET practice, which includes practice within schools.

These assumptions were not evident to me at the start of the research study but rather developed over time through observing teachers make sense of the new curriculum and reflect on their learning process. In interviews with teachers about their learning within the context of PLESME, it became evident that teachers themselves, saw their learning as a process of fashioning a 'way of being.' Here are just a few quotes, from these teacher interviews, which support this statement:

"You know before I always used to introduce myself as the music teacher, now I introduce myself the maths teacher" BL– 20 July 1999

“ Its more than that, the programme develops you as a person and you can take it to all areas of your life” - KC – 17th June 1999

“It has broadened my horizons very much…For myself, if I open a newspaper I think what can I use in my class, or think this is another way of drawing a graph, I saw one (a graph in a newspaper) that started at 17. Like the example we (PLESME group in a workshop) did on holiday, I start to realise how much they (advertisements) are bluffing you. I use it in everyday life (maths), like you told us to look for the per gram price in shops….” EN – 22nd June 1999

Two key notions, which I will draw on, will be teacher roles (often referred to in national curriculum documents) and teacher identities. The object of the broader study will be to elaborate on what these mean. For Lave (1991) learning is a way of being in the social world, not a way of coming to know about it. She writes, “…learning and a sense of identity are inseparable: They are aspects of the same phenomenon.” (p115) The meaning of identity and teacher roles in this paper are tenuous and unformed. The study will need to fashion a meaning for these concepts within the context of mathematics teacher learning within the INSET practice within the context of broader curriculum change.

Methodology of the study

The study uses qualitative ethnography as its research methodology. In the study, I work as a >participant= observer with teachers in various schools over an eighteen month period. In this sense I am the main research instrument. The schools were selected with the help of the Gauteng Department of Education. The schools have been historically disadvantaged but are running relatively smoothly compared with other schools in the area. At the present time in South Africa, for the purposes of INSET or for the purposes of research, it is not conceivable to randomly select schools. The criteria which were discussed with the District Departments for the selection of these schools were that participating schools needed to be traditionally >disadvantaged= and that the schools needed to be operating >relatively= smoothly. In this sense I have chosen both a purposive and an opportunity sample for my study. Vithal (1988) highlights the importance of working with opportunity samples for research in mathematics education in South Africa. She writes: “Rather than telling it like it is, the challenge is to tell it as it may become.” (pg 480)

Within this framework I use a wide range of techniques such as classroom observations, focus group interviews, individual interview, teachers writings, field notes, journal entries and group reflection sessions in order to provide multiple entries into accessing teacher learning. I am hoping that these tools will provide rich qualitative data, which will enable access to teacher learning as participation in practice and also access to the fashioning of individual teacher roles and identities. These tools are shaped by the study and new tools are developed based on data which is collected. For example a brief analysis of teacher interviews led to the design of a tool which required teachers to give a diagrammatic representation of who they engage with about mathematics and mathematics teaching. It is expected that further tools will emerge as the study progresses.

Initial understandings of the PLESME teachers about the new curriculum

Baseline interviews were conducted with all sixteen of the PLESME teachers prior to the programme beginning. These interviews were recorded and later transcribed. Here is a summary of some of the findings. In terms of teacher feelings about the change responses could be categorised as positive, negative, insecure and ambivalent. The most frequesnt response was that of insecurity. The positive and negative responses were almost equal and many teachers expressed a mixture of these responses.

Only half of the teachers had attended a once off workshop on the new curriculum which was run by the department and which was not mathematics specific. The other half had heard about it from friends, colleagues and the media. Thus teachers were very poorly informed and none of them had received any curriculum documentation.

Teachers gave a range of responses when asked what they understood the new curriculum to be about. The most common response (from 6 teachers) was that it was different from the 'old' curriculum. Many other responses could be classified as understanding it as being learner centred. Teachers used phrases such as 'individual attention', 'learners work at their own pace' 'it takes learner background into account' 'learners are active, enjoy it, are interested'. In terms of teaching methods some teachers saw the curriculum encouraging more discussion, facilitation and group work. Furthermore maths would be more practical and relevant and problem solving would be important. Some teachers expressed concern that the curriculum would be mainly for stronger learners and that assessment of learners would become difficult.

When asked what teachers understood by the mathematics specific outcomes (see appendix A) most teachers were unable to find meanings for specific outcomes 4, 8 and 10. These outcomes included terms such as 'social, political, economic, natural forms, cultural products' and the teachers were unable to find a connection between such ideas and mathematics. While the outcomes resonated with teacher's political ideologies the mathematical content of these outcomes was opaque. Here is one teacher's response which illustrates support for these outcomes while struggling to make mathematical meaning of them.

"I think in a way it is attempting to make maths real to children in having to analyse relationships using social and economics I would say political, I am not sure if they could relate to that yet. Historical development and cultural context I think they are objects which culturally they can link up with their shape, geometric sense or mathematical sense. I think it is good in that sense" IT January 1999

In summary, while teacher spontaneous responses to the new curriculum were uneven and diverse it was common for teachers to struggle to move beyond the rhetoric of the outcomes.

What fashioning of teacher roles is demanded by the curriculum change process?

What is emerging from working with teachers in workshops and in mathematics classrooms is that the practice of implementing the new curriculum requires teachers to shift between three different orientations of mathematics. These three different orientations can be identified as:

- C Maths is a useful subject for everyone, it is both relevant and practical and is applicable to everyday life. Embedded in this orientation is critical analysis of the mathematics in socio-political relations and mathematics for democratic citizenship.
- C Maths is a subject which inducts learners into what it means to be a mathematician (or to think mathematically), it has its own beauty and can be explored for its own sake. (Mathematical investigation is emphasised).
- C Maths as a subject within which conventions (skills and algorithms) are learnt which enable access to further education and training

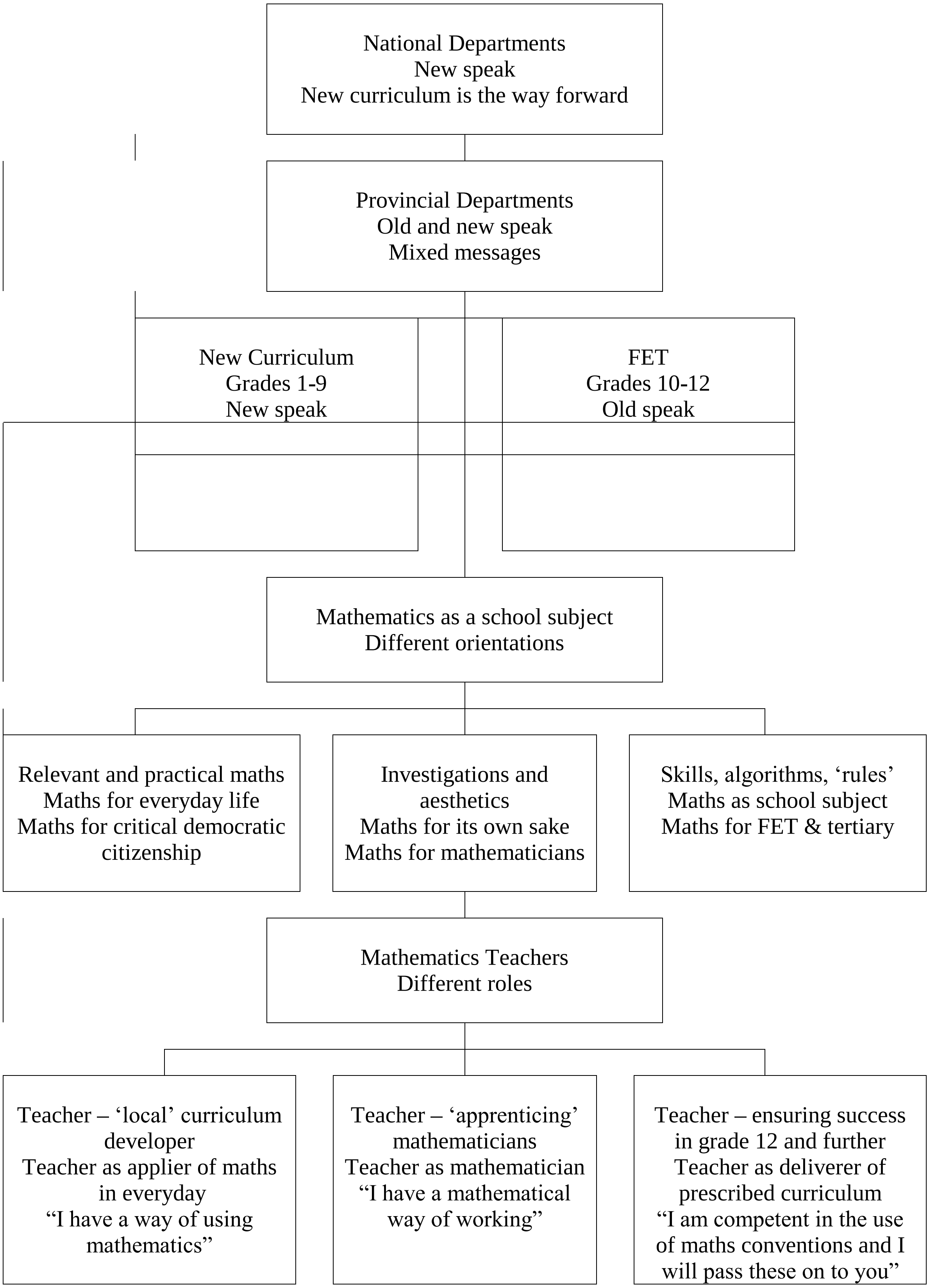
Aspects of all three of the above orientations of school mathematics can be found in the new national curriculum. However, implementation support provided to teachers (in the form of pamphlets or district workshops) tend to emphasise the first orientation as the correct one and the third orientation (most familiar to teachers) as 'old' and wrong. The orientation involving mathematical investigation tends to be overlooked. Thus the district workers, in providing 'support' for teachers are sending out confusing and mixed messages. While they emphasise the integration of mathematics across learning areas they say nothing of the importance of integration between the various orientations of mathematics.

Thus a process of pendulum swinging is taking place in which it is communicated to teachers that it is time to swing form their 'old' and bad practice to the 'new' and good practice which in their view is exclusively the first orientation which involves mathematical relevance to everyday life and other learning areas. An intended outcome for PLESME which has developed out of the experience of working with teachers in implementing the new curriculum has been to assist teachers in working with all three orientations of mathematics in an integrated way.

An understanding of school mathematics in terms of these various orientations demand that mathematics teachers develop multiple 'roles' (including both existing roles and new roles) in relation to their practice as a mathematics teacher. The skills, rules and algorithms approach resonates with a teacher role as a deliverer of a prescribed curriculum and enabling the success of learners in further education. The Investigations and aesthetics approach resonates with a teacher role of themselves being a mathematician and apprenticing learners as future mathematicians. The relevant and practical approach resonates with a teacher role as a local curriculum developer and as themselves being an applier of maths in their everyday life. The diagram below provides a summary of the above.

The position of the education departments district workers is indeed very difficult. There is very little capacity to run workshops with teachers and most of their time is taken up by administrative work. Furthermore, no changes have been made to the curriculum for the further education and training grades (FET). This FET curriculum is still firmly based on >traditional= approaches to teaching and approaches maths as a subject whose orientation is exclusively about the mastery of skills, algorithms and rules which are necessary in order to access further studies. In summary there are disjunctures within the national curriculum between the compulsory education phase and the FET phase. This creates dilemmas for teachers when their schools are ultimately still judged on the results of the final FET examination and especially for teachers who teach mathematics across these phases. Furthermore the district departments tend to spend much of their energy focusing on the improvement of the FET results. For example, the PLESME teachers in Soweto have been made to write common assessments (set by schools in white areas) which do not reflect any of the current curriculum changes and are based entirely on the previous curriculum. The explanation given for this is that these schools tend to perform much better in the final FET examinations.

HYBRID ORIENTATIONS AND MIXED MESSAGES



Where to from here?

The research study is still in its early stages, and further data will be collected over the next six months. Thereafter a thorough analysis of the data collected over the eighteen month period will begin. However, it seems that teacher participation within PLESME, which enables teacher access to a range of resources, the most important being access to participation within a community of practice which encourages debate and critical reflection on practice, the teachers are fashioning new 'identities' or 'ways of being' a mathematics teacher within the context of PLESME. This clearly needs further exploration and analysis of data.

Two critical issues for further discussion invite critical comment.

A concern of this study is finding and developing appropriate methodological tools for accessing teacher learning as a 'fashioning of identity' within the PLESME 'community of practice'. I have said a bit about this in this paper but realise that there is still a long way to go.

In the paper I argue that the new curriculum requires mathematics teachers to work with three different orientations to mathematics and to move between three related teacher 'roles'. Is this an appropriate epistemological model of what it means to work mathematically in schools? Is this an appropriate model for the different roles which teachers need to take on when teaching mathematics? Is it appropriate to expect teachers to work with this model in an eclectic way or are there inherent contradictions or difficulties in working in this way?

I invite your critical comment and ideas for taking this study further.

Acknowledgements

I would like to acknowledge the critical comments made by my supervisor, Professor Jill Adler. Her critical comments and engagement with the paper have been most useful

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Appendix A

1. Demonstrate understanding about ways of working with numbers.
2. Manipulate number patterns in different ways.
3. Demonstrate understanding of the historical development of mathematics in various social and cultural contexts.
4. Critically analyse how mathematical relationships are used in social, political and economic relations.
5. Measure with competence and confidence in a variety of ways.
6. Use data from various contexts to make informed judgements.
7. Describe and represent experiences with shape, space, time and motion, using all available senses
8. Analyse natural forms, cultural products and processes as representations of shape, space and time.
9. Use mathematical language to communicate mathematical ideas, concepts, generalisations and thought processes.
10. Use various logical processes to formulate, test and justify conjectures. [Department of Education, 1997]

An experiment in using video to scaffold method with pedagogy

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We outline our actions during the design, development and implementation of a primary mathematics method subject over a period of several years. This has culminated in the production of a video series which it is hoped will be used to more effectively address the broader pedagogy that must necessarily scaffold method. Our initial experiments are discussed in the paper and the conference session will be used to first view and then talk about the exemplar episode that is discussed in the paper. It is hoped that this might be used as a basis for inviting comment and discussion from conference participants. A framework for such discussion is outlined in the paper and the conference session, therefore, will need to proceed on the premise that participants will have had the opportunity to read the full copy of the paper that has been placed on the conference web site

Introduction

> "Teaching is an uncertain and complex enterprise and if teachers are to cope with the inherent uncertainty and complexity of their profession, teacher education must adopt an <u>enquiry</u> perspective which emphasises the cultivation of research skills about teaching and the multi-dimensional contexts of teaching. This perspective differs from other paradigms of teacher education, the most powerful of which is the <u>behaviouristic</u> perspective. Within this perspective teacher education focuses on learning the knowledge, skills and competencies that are thought to be most relevant to good teaching."
>
> (McLeod, 1998, p344)

I am a researcher and a teacher of twenty-three years with experience in the primary, secondary and tertiary sectors. I write about our journey, a journey that we've been on for the past seven years, one with many stopovers but one that has yet to arrive at a final destination. I write about significant turning points, where we have been and where we intend going.

The context

Our journey commenced seven years ago when I was first approached about the possibility of coordinating the teaching of the mathematics method subject in our four-year undergraduate primary teaching degree course. The aim of the subject was to teach pre-service teachers appropriate "method" for teaching primary school children number and operations involving number. I inherited eighty students, a recommended text, a weekly two-hour lecture slot and four one-hour tutorial slots. Following tradition I gleaned what I considered to be important method from various sources, wrote weekly lectures around such method, included the odd bit of research here and there, some occasional analysis and presented this material to our students in the form of a lecture.

I felt assured that this approach to the teaching of the subject was both appropriate and effective. There appeared to be ample anecdotal evidence to support this belief. How did we conduct the tutorials? The usual discussion questions, any questions the students had were addressed, with sometimes a little time being spent adding to or revising what was covered in the lecture. How was the assessment structured? We stayed with tradition and included an essay type assignment (40%) and an examination (60%). The examination contained typical method questions, for example, "Explain the build to ten strategy for addition" or "Discuss the first stage of counting that a child normally goes through".

The changes

Over the next few years I steered through a number of changes within the subject. The very first concerned the duration of the lecture – two hours of lecture becoming one hour. Students readily embraced this change and appeared relieved when I expressed my sympathy for anyone who had to listen to me for more that an hour. We changed the assignment weighting from 40% to 20%. Later on I recommended dropping the assignment altogether and replacing it with a mid-semester examination that was allocated a 20% weighting. I personally found this refreshing because my classes were no longer half-empty at assignment [due date] time! Initially students reacted negatively to such a change, but later they became more accepting of the idea especially after we explained that the format and style of the examination would be identical to the final examination. We also gave a commitment that the first examination would be marked and returned so that students would then have an indication of their own strengths and weaknesses and some indication of how we [academics] tend to "mark" an examination script. As promised

I took responsibility for collating collective misconceptions and I transcribed these onto a central computer database for the use of students.

A two-hundred-page workbook emerged over a period of years and students identified this as being one of the most desirable features of the subject. Initially tutorial attendance was non-compulsory, then we decided to make it compulsory and when that didn't work either, we returned it to being non-compulsory. I don't need to say which of these options was favoured more by students. Independent one-hour small group workshops then became a feature with activities written into the workbook and complemented by numeration kits [boxes] that could be borrowed through the educational materials library on campus. This inevitably lead to certain changes in tutorial activity – tutors became more of a guide when students began using the tutorial as a forum for discussing the outcomes of their independent small-group workshop activity. Tutorials began to evolve into more of an informal seminar with tutors taking on the role of chairperson.

Students used the session to present mini-seminars on the results of their workshop deliberations for a particular week. It soon became apparent to me that the lectures, for which I was responsible, were fast becoming obsolete as more and more students were researching the required information without my direct assistance. This eventually led to our decision to scrap the lecture and put in its place a large group meeting, which I decided to label an "exposition". This was in order to distinguish it from the more traditional idea of a lecture. Unlike the now defunct lecture the exposition was not a class that preceded all other classes at the beginning of the week. As coordinator of the subject I used it as an opportunity to meet with all students at the conclusion of the week in order to reaffirm their progress [or lack thereof]. I was able to do this very successfully using a series of demonstrations that involved modelling of the method within a semi-pedagogical context.

The present

This subject now has an enrolment of two hundred and fifty students. We have a comprehensive workbook containing sequenced independent workshop activities that are based upon the use of concrete materials; these are, as highlighted earlier, available in kits through the library-borrowing scheme. Upon completion of an independent workshop, students come to the exposition; this is their opportunity to tie up any loose ends in their understanding. Since students bring with them their completed workbooks they rarely find it necessary to take comprehensive notes during the exposition – just the occasional addition and/or clarification here and there. The exposition serves as a polishing exercise; it prepares students even further for their seminar sessions. The final seminars give staff and students the opportunity to confirm that the goals of the subject are being met.

This was the point in our journey four months ago.

A new challenge

It may come as a surprise to many that four months ago I also decided to shelve much of the above [good] work and start afresh! I have a number of reasons for doing so. A visiting academic once reminded me that when everything appears as though it could not be any better then this is usually a good indication that things need changing – at the time I thought he was being cynical.

Two years ago our faculty began the process of developing a new four year undergraduate teaching degree and one of the key criteria for this new degree was that it must be able to be completed entirely off-campus. There was a time when I was presumptuous enough to believe that I was an essential part of the teaching and learning process; now it is much less so but I still had difficulty seeing how I could effectively teach my subject from a distance. Also, our institution has a campus four hundred kilometres away and I have been informed that as of the beginning of next year my subject will need to be offered in distance mode. As much as I have resisted the thought I have slowly come to terms with the reality that students in the future could be and will be more attracted by courses that afford them maximum flexibility in terms of how, when and where they study.

The most compelling reason, however, was my ever growing belief that specific method and broader pedagogy are inextricably linked and when studied in isolation, the method will invariably lack conviction and be far removed from reality. The subject that I was responsible for <u>was</u> a method subject – pedagogy was not a required part of it because there were other subjects within the course that dealt with this.

I wondered if the answer lay in <u>the use of video as a tool for bridging method with pedagogy</u>. The challenge facing me was the redesign of the subject in such a way that it addressed all of the above issues. Ultimately we would have a subject that not only bridged method with pedagogy but it would also be a subject that could be completed on-campus, or off-campus or even using a combination of both modes of study.

The next step

This has been a full-time six-month project. With the cooperation of a group of our education students and some of our schools, we have developed a video series containing a total of twenty-one teaching segments. The duration of each video segment ranges from between 3 minutes for the shortest and 8 minutes for the longest segment. The series was produced in the television studios of our media services division here at the University of Southern Queensland; it is they who hold the marketing rights. Each segment vividly illustrate a separate element of method and for me the great attraction is the potential that this series has for being used as a tool of enquiry.

This paper commenced with a quote that said in part “teacher education must adopt an enquiry perspective, which emphasises the cultivation of research skills about teaching and the multi-dimensional contexts of teaching”. It is our view that the video series provides us with such an opportunity because of the manner in which the method is scaffolded within a pedagogical framework. In other words it is now possible to give students the opportunity to vary, modify and/or adapt method in an effort to complement pedagogy and equally, vary, modify and/or adapt pedagogy to complement method.

In each episode we see two people assisting each other, sometimes with varying degrees of success, to elicit understanding in the third person, a child. One of the two people involved is a novice teacher while the other is myself, a teacher-educator. I would like to emphasise that this video does not necessarily set out to illustrate examples of best practice, although, at times this can be an unintentional outcome of the discourse that occurs within some of the episodes. I must also add that I myself have learned much about my own teaching from the production of this video, and, as always, I am continuing to learn new ways to enhance my effectiveness as a practitioner.

Using the video

Using two of my education students I also experimented with some ideas for using the video segments. The segment I experimented with was titled “Interpreting Number (part a) ” and featured student teacher Nicole and child Tiffany. At the conference presentation in Portugal I will use this five-minute segment as a basis for further discussion.

I didn’t think it was good idea for the person viewing the video to attempt to interpret and/or make value-judgements about the teaching practices therein. Discussions with John Mason (UK) who has done considerable work in this area helped to reaffirm this view. I felt it was more important to encourage a student to adapt, discuss and refine particular teaching methods as a way of developing both his/her competence in the method and skill in the pedagogy. While developing competence in the specific method I was hoping that the student would also learn how to adapt and refine his/her pedagogy to match variations in the method.

I asked two of my students, Peter and Jane (not their real names) to view this segment, uninterrupted. When they had done so I instructed them to spend a few minutes mentally replaying the video segment – this I knew to be a difficult task and I encouraged them both and emphasised the need to be persistent. I now asked that they write down some of the more salient and/or striking moments of the video segment. It was important that this be done in as value-free and non-judgmental a way as possible. I asked them to describe in a "brief but vivid" way what they saw. I suggested that it might be any one of the following, for example: voice tones, timing, pauses, and gesture, head and/or shoulder posture, body actions eg. steadiness/tentativeness, who holds the pen, things said, etc. The very first suggestion Jane made was: Nicole might have used better timing when she asked the question “How many ones are there in this?” I responded that this was unacceptable since it included a value judgement. With a little more effort they were both able to get to a stage where they were able to offer me a list consisting of three or four points. I will focus on Jane’s list:

- When Nicole asked: “How many are there in each square there?”,
- When John asked: “How many tens are there in this number?”,
- When Tiffany answered “A hundred” and changed it to “fifty”.

I encouraged Peter and Jane to share their chosen moments with each other; I allowed them about five minutes to do this. I then asked them to, again stressing the requirement that it must involve <u>as little</u> judgement and/or analysis as possible, to explain to each other <u>why</u> the particular moments so captured their attention. Jane again, as enthusiastic as she was, had the tendency to include a little too much interpretation when she said to Peter: the question, “How many are there in each square there?” was unnecessary. With a little more prompting, however, she soon had the idea and I was able to transcribe the following condensed version of Jane’s contribution to the conversation.

- The part when Nicole asked: “How many are there in each square there?” struck me afterwards when I mentally replayed the segment because I wondered about the purpose of the question,

- The part when John asked: “How many tens are there in the number, 405?” struck me because I initially thought it was a trick question but later I wasn’t sure,
- When Tiffany answered “A hundred” and changed it to “fifty”, I began thinking if the question that was posed to her could be asked differently.

Both Peter and Jane had little difficulty in identifying the significant numerical concept that “came to light” in the video segment. At that stage I decided to leave them for an hour or so while they studied a couple of readings that I gave them. I asked them to make particular note of the different methods that are recommended for teaching this and related concept/s.

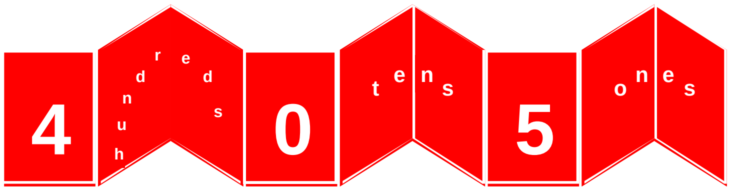
Upon my return I replayed the video segment, asking Peter and Jane to select a small component containing either a questioning sequence, or a modelling sequence, or a verbal explanation, or a teacher response, or any other interesting aspect that focussed on pedagogy. I suggested that they try describing an alternative way in which that section might be conducted. Such a description might include detail such as alternative questions, alternative materials, alternative teacher responses and so on. I stressed that their description should include not just changes or adaptations to the method but also any refinements they might make to the pedagogy. I returned two hours later … Jane was still working so I studied Peter’s response. Interestingly, Peter had decided to reword a questioning component that Jane had identified earlier. Jane had said:

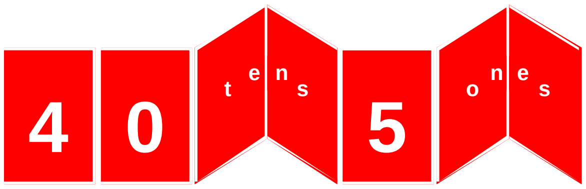
- The part when John asked: “How many tens are there in the number, 405?” struck me because I initially thought it was a trick question but later I wasn’t sure.

Peter had written the following alternative for this question:

- “Does the fact that there are no tens in the tens place of the number mean that there are no tens in the hundreds place?”.

Jane’s response was somewhat more comprehensive. She had decided to use an alternative method to elicit understanding of the same concept. Her readings talked about a device called a number expander and she had found this interesting. She was intrigued by the idea and felt that for her the explanation would make more sense if such a device were to be used. She made one and excitedly demonstrated how one might use such a device in their explanation (see diagram).





Conclusion

At the beginning of this article I remarked that I write about a journey, one that we are several years into and one that still continues today. Like many before, this has been yet another of those stopovers. I will now continue on with the journey as I endeavour to implement some of these new ideas. In the meantime I continue to remind myself that one way of encouraging students to "adopt an enquiry perspective which emphasises the cultivation of research skills about teaching and the multi-dimensional contexts of teaching" is by actively engaging in such behaviour in one's own teaching.

It is with this in mind that I have asked a colleague to help me to make some suggestions for the running of the conference session. I invite comment and discussion along the following themes. This does not necessarily preclude other possible themes that might arise on the day itself:

- Video episodes such as the one aired at the conference session could also be used to make explicit and illustrate certain components of practical theories of teaching, to invite reflection on these and to encourage their incorporation, where appropriate, into personal practical theories. For example, the episodes might allow discussion of such components of practical theories as strategies, principles, teacher attributes, values, beliefs, goals and contextual variables and the part each plays in approaches to teaching. Making these components of practical theories explicit could well assist pre-service teachers to develop frameworks for thinking about and articulating their own approaches to teaching primary mathematics.

- Another characteristic of this unit is its flexibility. Although the unit and its resources have been designed primarily for pre-service teachers, they could also find a ready use in programs of professional development for inservice teachers. In fact, the materials designed for use in this unit are in a form which could be easily adapted for sale commercially. This is a possible outcome which warrants serious attention and so should be kept in mind in the planning and conduct of a systematic evaluation of the unit and the unit materials.

(With acknowledgment from Perc Marland, PhD).

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PROBLEMATISING CULTURE AND DISCOURSE FOR MATHS EDUCATION RESEARCH : tools for research.

Tansy Hardy (presenter) and Tony Cotton

There is much talk of visible problems in teaching and learning mathematics in the UK today. Tony and I argue that, in order to find ways of addressing these problems, Maths Education research must acknowledge that its work is deeply contextualised and cultural in every sense and find valid ways to work with social and cultural theory. We discuss what this might mean for our researches and try out particular theoretical 'toolkits' in an attempt to reveal the cultural practices of school mathematics education and how these practices become licensed. We also start to consider the effects these practices have on both learners and teachers.

INTRODUCTION

Much national and international mathematics education research has moved to appeal to notions of the 'social' and the 'cultural' to address increasingly visible problems in teaching and learning mathematics and to account for the failure of mathematics education to produce a much demanded mathematically literate citizenship. Tony and I argue that if we are serious about addressing this failure then those of us working in Maths Education research need to acknowledge that our work is deeply contextualised and cultural in every sense. We must also think through what requirements such an acknowledgement may place on the forms of our research and, in particular, on the theoretical frames to which we appeal. This paper intends to discuss the nature of these requirements and to explore how we might work validly and critically with social and cultural theory.

The fields of social and cultural studies do, indeed, offer ways of theorising the functioning of any endeavour of human society. This clearly embraces the endeavour that is learning and teaching mathematics. However, we are concerned that the complexity of working within social and cultural contexts in mathematics education is often not fully acknowledged. There is an inevitable privileging and denial in any theory that we might use in our work: particular aspects of practices, particular actors will be highlighted whilst others are ignored or camouflaged. However, superficial applications of social theories give 'flat' accounts of the classroom - free from power and affection, which we view as key elements in the functioning of mathematics education. Such applications repeatedly fail to account for a persistent failure to help certain groups of learners.

This superficiality can result from singular or isolated conceptions of the interrelations and culture practices of teachers and learners and produces accounts that repeatedly ignore significant groups and hide particular effects of our practices in teaching and learning mathematics. We are particularly concerned about the privileging effect of any theorising in Maths education research (this concern also applies to our own researches) and the implications this has for issues of exclusion and injustice. We urgently need to identify ways of working that will offer richer, more valuable accounts of the effects of our education practices that give us some indication of why, as teachers and as researchers, we are not helping many groups of learners to learn maths (this includes the groups of learners that have been repeatedly identified and targeted in UK government education policies over the last 20 or more years).

There are researchers in Mathematics Education who have shown concern over this privileging effect and are striving to identify more productive theoretical frames. The divisive nature of such a privileging effect can be seen when Mathematics education works with the idea of 'individual difference'. This has exclusive and normalising effects and produces a deficiency model - using terms such as 'average (levels of attainment)' and 'raising standards' defines 'normal' and generates the 'abnormal' ('below average' and 'failing to meet the required standard'). A shift from talking of individual difference to foreground a view of diversity can be seen in the work of Valerie Walkerdine (1988, 1994) who works within critical psychology where she sees the individual in the intersection of 'overlapping language games'.

Mairead Dunne (1999) provides another example. She also perceives that there is a paucity of research adequately considering social or cultural aspects in mathematics education literature and asserts the inadequacy of applications of some theories of learning in revealing the cultural nature of the maths education . She moves to foreground the political nature of everyday activity in mathematics education;

> *'Within recent work, including social constructivism, the notion of classroom culture carefully circumscribes its concern as within the confines of the classroom, sometimes as if disconnected from any external influences, for example, what the students bring with them into the classroom. This construction of 'culture' is evidently too limiting for the development of mathematics education research with a social justice concern. Such work necessarily connects the micro- to the macro- level; local practices to policy; individuals to communities.'* Dunne 1999 p117

Together with a growing body of maths educators (e.g. Lerman (1998), Zevenbergen (1996), Dowling (1998)) we are striving to think differently about Maths Education Research; the nature of the objects of its study, its concepts and its methods of study. It is our view that this requires a shift from talking of individual difference to foreground a view of diversity in our work.

To this end, my specific project here is to draw out particular theoretical notions from the ideas of contemporary cultural theorists and from these, identify 'toolkits'[1] with which I can research teachers' and children's practices. Using this I will try to expose the ways in which both are involved in culture creation through classroom practices; and the way particular cultural practices become validated.

In this paper, I use extracts from a journal and transcribed interviews where children and teachers describe their practices, together with the theorised 'toolkit'; drawn from of Michel Foucault's conceptions of power and the production of knowledge. From this I give a 'reading' which considers the multipositioning of both teachers and pupils. This may give some indication of why, as teachers and researchers, we are not helping many groups of learners to learn maths. I also use this strategy to help me identify to whom this reading may offer a productive account and to consider the validity of this way of working as a Maths Education researcher

In previous writing[2] Tony and I have use two theorised 'toolboxes'; one drawing from notions of social justice and the other from the conceptions of power/knowledge. The two 'readings' are juxtaposed to consider the irresolvable nature of mathematics teaching and learning. We acknowledge that offering these two readings raises the question of the comparative validity of these differing accounts. We wish to discuss in our presentation, strategies we might use as researchers to avoid the unproductive effects of championing one particular account.

From our introductory discussion it can be seen that we want to argue that, if we are to tackle issues of mathematical literacy, equity and exclusion, this will mean engaging with critical approaches to educational discourse. Together with my fellow researchers, I must identify ways in which I can become aware of the effects of My practices as teacher and researcher. Paul Dowling (1991 p.2) takes my attention to this concern when he refers to Foucault's work:

> *People know what they do; they frequently know why they do what they do; but what they don't know is what what they do does. (*Foucault and Deleuze 1972 p208)

It has been suggested (eg Bourdieu & Passeron 1977) that schools serve to reproduce the existing injustices in society through practices seen as common sense in school, but which are based on the class structure present in society. It is this trick of power to masquerade as 'common sense' that leaves us unaware of the effects of our practices. Similarly researchers in Mathematics Education must become aware of the effects of their practices, of the effects of how they describe classrooms, children, teachers and mathematics and of how and where they discuss their work - of 'what what they do does'.

CULTURE AND DISCURSIVE PRACTICES

Culture is an extremely difficult term to pin down. Learners in our schools are at a point of cultural transition, and in many ways operate at the intersection of several cultures, or indeed, move between cultures depending on the context in which they find themselves. Learners in multi-ethnic schools illustrate this cultural transition with great clarity .

We are arguing for the need for a complex view of culture. Some views will perpetuate the 'naturalness' of particular constructs and will hide the effects of educational practices. The structural metaphor that we choose effects what takes our attention in our examination - stressing some aspects and ignoring others. This in turn will influence our interpretation of that site. For example, a view of 'dominant' culture with satellite cultures, offers only one way to gain access to the dominant culture: by a complete shift from the subculture to take on all values of the dominant culture. This model is one of exclusivity, dominance, and disconnectedness. Any structural metaphor outlines a view of identity and culture. Pupils and teachers in schools are engaged in culture creation and through this process identity creation. Tony and I see this as continually shifting and changing - we are seeking metaphors which reflect this shifting nature.

The central term of Discourse in its broadest sense means anything written or said or communicated using signs (including actions and interactions in the classroom, resources used, and arrangements of the furniture). Foucault (1972a) extends this to consider how knowledge is actually produced and induces power through what he calls 'discursive practices' in society. The modifier 'discursive' stresses the ways in which all practices are bound up in systems of knowledge. Institutional sites can be studied in terms of these rule bound sign systems that infuse everyday activities. Foucault argues that these discursive practices differentiate people in relation to cultural norms that constitute self regulatory ways of knowing.

Foregrounding the discursive nature of mathematics education practices marks a shift away from seeing culturally accepted norms and knowledge as formed and perpetuated through traditions and towards seeing knowledge produced through a process of describing and ordering things in particular ways. For example, the categorisation embedded in the constitution of the National (School) Curriculum for Mathematics in the UK (DFE 1995) has become a cultural norm, regulated through descriptions that have come to be taken as natural and obvious.

POWER/KNOWLEDGE: Useful notions from Foucault's 'toolbox'

In this section I will discuss Foucault's conception of power and what I believe it can offer mathematics education research.

> *Early attempts to do this (inject power, as the energy behind social, economic, and cultural movement, into theory) within educational theory often conceived of power as a scare commodity, like money or cultural capital, which people 'have' in relative amounts.*
> Appelbaum 1995 p37

Foucault attempted to rethink the nature of modern power; rejecting totalising schemes that anchored power in ruling or dominant classes and that saw power's effects as entirely repressive He developed perspectives that interpret power as dispersed, productive and dynamic. In this he abandons individualistic ways of viewing power and offers an alternative conception of power- understanding it as a property of relationships - that is, not invested in one individual to exert over another. He writes:

> *What characterises the power we are analysing is that it brings into play relations between individuals (or between groups)....* (Foucault in Dreyfus and Rabinow, 1983, p. 217 *)*

> *...The exercise of power consists in guiding the possibility of conduct and putting in order the possible outcome)....* (Foucault in Dreyfus and Rabinow, 1983, p. 221)

The exercising of power produces what is held to be knowledge; what is the right interpretation; the true meaning; the valued act or utterance within that practice. From this Foucault collapses the distinction between power and knowledge, that is, to begin with a single category of power/knowledge.

More complexly, he connects this single category inextricably to a notion of the production of a sense of self which he refers to as 'the production of subjectivity'. For Foucault's analysis these are inseparable facets of the modern human condition . Others in mathematics education have found the theories of Michel Foucault valuable in providing new understandings on 'the production of subjectivity' in education and written about this for mathematics teachers and learners. Valerie Walkerdine's (1988) major analysis of 'the developing child' shows the ways in which psychology has produced this 'subject' as its object for scientific investigation. She describes how the practices of mathematics education become sites for the production of 'the self-regulated child'. Paul Dowling (1998) has also written about the production of mathematics and the learner of mathematics within mathematics teaching texts and, for example, Mary Klein (1998) considers the production of the mathematics teacher's sense of self through investigatory teaching approaches. When these authors write about subjectivity, they inevitably engage with the circulation of power and the production of knowledge.

It is the indeterminable nature of Foucault's conception of power and the inseparable meshing of the people, their actions, their relations, their subjectivity, their institutions that I argue can valuably be applied to the realms of mathematics education and can offer a fresh view of my professional contexts.

MY THEORETICAL TOOLS: Power is productive.

For Foucault power is internally contradictory. Organised forms of knowledge, working together with their associated institutions, have significant effects on people and their possible actions, repressing and enabling.

> *If power were never anything but repressive, if it never did anything but to say no, do you really think one would be brought to obey it? What makes power hold good, what makes it accepted, is simply that fact that it doesn't only weigh on us as a force that says no, but that it traverses and produces things, it induces pleasures, forms, knowledge; it produces discourse. It needs to be considered as a productive network which runs through the whole social body, much more than as a negative instance whose function is repressive*. (Foucault in Rabinow 1986 p61)

When power circulates it determines, to some extent, possible ways of acting and limits in some ways what can be done but it also the mechanism that enables one to act. My analysis will need to attend to the discursive nature of professional practices and the simultaneity of how human beings (teachers and children) are defined by discourse's use (that is in fact defined by human beings) whilst at the same time the discourses describe them.

I have experienced this constititve nature of discourse when, as a researcher observing in a classroom, a child has tried to elicit ‘my help’. For this child a characteristic of a 'teacher' might be 'one who helps'. I will define myself as 'teacher', or not, by my response to her. At the same time I contribute, through that response, to the definition of what a teacher is and does.

I am working predominately with Foucault's accounts of the operation of power within groups and institutions. He describes particular 'techniques of power' and invites us to look at how power relations function at the micro level within institutions. Jennifer Gore (1999) did this for pedagogic sites as varied as a physical education classroom and a feminist reading group practice and claims that 'techniques of power' could be seen to appear in both. I have used these 'techniques of power' as tools to give me a way of looking at how children and teachers talk about how they how they view their practices. Through this approach, I aim to establish whether these techniques of power are readily recognisable in pedagogic discourse in mathematics classrooms .

Foucault uses the word technology in a similar sense to the early Greek word 'techne'- to mean an 'art or applied science'. It refers to the physical or mental acts of constructing reality. This is a clear form of knowledge-making. When Foucault talks about technologies of power, he recognises that for every technology deployed there is a simultaneously constituted domain of knowledge. This is the inseparability of Power/Knowledge. In using the term technology Foucault gives support to the notion that human beings are essentially constructed by the seemingly nondiscursive background practices into which they are thrown

Normalisation and Surveillance

Any mathematics education discourse positions and categorises children (and teachers) in particular ways. For example, children are often portrayed as the ones who 'have difficulties' or 'misconceptions'. There are two particular techniques of power that can bring about this pathologising:- 'normalisation', and 'surveillance'. Much of Foucault's work is focused on the construct of normalisation.

The process of normalisation is the mechanism that categorises people into normal and abnormal . Linking the notions of normalisation and power as a productive network allows us to see the process that determines what is considered to be valid knowledge in the classroom and how that knowledge can be expressed and by whom.

It is the process of normalisation that determines who is included and who is excluded in this discourse - who 'has the difficulties' and who does not.

Foucault (1977, p.184) also claims that examination '... is a normalising gaze, a surveillance that makes it possible to qualify, to classify, and to punish. It establishes over individuals a visibility through which one differentiates them and judges them.' The specific forms of normalisation: - individualisation and totalisation are interesting. For totalisation a group or collective specification is given, asserting a collective character. This forms a readily recognisable element of pedagogic activity where 'we' or a class name is used in addressing whole groups of participants. For example, 'Well done, 3W. I'm pleased with the way that you moved back to your desks'. Individuals and their behaviour is ignored or erased by this statement. It permits regulation of the group behaviour and assertions of the group's characteristics and subjectivities. A child claiming an individual voice could find themselves excluded from the group and from the classroom culture. Individualisation is the technique of giving individual character to oneself and may be an attempt to resist unwelcome totalisation. However it can also be a way of drawing attention to a child's deviance from the classroom norms and establishing their abnormality - again a common classroom practice.

TRYING OUT THE TOOLKIT:

I will work with a Foucauldian conception of power as productive and look for patterns in children's and teachers' descriptions of their work where techniques of power, particularly forms of Normalisation and Surveillance involving examining, classifying and a play of totalisation and individualisation, might be recognised.

This section is made up of extracts from Tony's journal (see Cotton 1999). This contains significant (to Tony) sections of group discussion, which he has transcribed. He has also included some of his commentary on these discussions from his journal. This is followed by my Foucauldian reading of the text.

Tony's Journal:

> 22/8/95 ... yesterday we had a training day. One of the aims was to try and move away from standardised tests or traditional beginning of term 'find out what they know' tests to assessment based more precisely on the needs of the kids or the information required by the teacher to help them plan.
>
> I offered many exemplars of these types of materials, which seemed to be enthusiastically received by my colleagues and agreements were reached to trial certain materials. However today some teachers have immediately fallen back on their 'traditional' methods. Do they see training days as detached from their' real lives' of teaching and so forget very quickly?
>
> I notice also how embedded these testing practices are in the learners' image of schooling and learning. The children have to find ways of making sense of these practices within a wider context - here that is a school which professes an ethos of learner centred teacher behaviour.

22/8/95 (Reflections on my observation of ‘school tests’).

In the classroom I observed the kids do not seem threatened by the tests however - some move themselves to sit next to someone doing the same test so that they could work together, despite the teacher deliberately alternating mathematics and language reviews to stop 'copying'. This didn't disturb the teacher in the slightest - in fact the teacher was quite amused by it.

So although the school ethos of 'learner-centredness' remains, an assessment system which could not be described as learner-centred begins to embed itself in both learners' and teachers' common sense. This is only exposed when alternatives are offered to both teachers and learners. This can be seen in the ways that these teachers do not develop learner-centred assessment as part of their everyday practices and in the ways pupils resist the implementation of these testing practices.

At a focus group meeting with the children (11-year-olds) we discussed the process of SATs (National Standard testing). This is an extract from the discussion:

Tony: What did you think about the SATs.

Group voices : Err - quite easy

Tony: Somebody said easy - why were they easy?

Lucy: I dunno - we've like done em before haven't we?

Mehnaz: We haven't done 'em before, that was revision.

Rupa: We did that Science test revision thing.

Tony: Right.

Lucy: Science Test B was easiest because we had just done that work really recently.

There followed an excited discussion about what had been easy and what had been hard. Kenny explained that he was trying to forget all about the tests as quickly as possible. There was general agreement about how useful it had been to have covered the topics on Science test B, electricity and evaporation just before the test. So the immediate reaction appeared to be one of enthusiasm to discuss the process and the questions on the test rather than resistance. Even Kenny and Lucy, previously the most resistant to ideas of testing joined in enthusiastically. I also noted how at this early stage the practice of doing past papers as revision had already been accepted by the group as 'good practice' and the thing which made the tests easy. The discussion moved on,

Tony: Do you think the SATs are a good way of testing?

Whole group (in unison): Yeah.

Tony: Why do you think they are a good way of testing? Kenny?

Kenny: Because it will give you a fair idea of what you’re, like, gonna get, when you are in senior school. When you come to do your exams.

Tony: Right. Other reasons why they are - go on.

Lucy: Because, they like see what sort of level you are on, instead of just saying, oh you did a level 2 in your infants so you're all on level 2, you've got to get like a different level.

Tony: Right so you know where you are. Sairah.

Sairah: It helps you practice for bigger exams in the future.

Group (several) : Yeah

Tony: Right, any other reasons? They are reasons why they are a good thing to do maybe. Because it helps you practice, it tells you where you are, I've forgotten what you said Kenny.

Kenny: It will give you a fair idea of what exams will be like.

Tony:	Right, so now you will all be given a level. Do you think the level you will be given is true, is a fair reflection of what you actually are?
Imran:	No.
Tony:	Let's go round. Imran, you said no. Why is it not?
Imran:	Because on the day your nose might be blocked or you might not be feeling well.
Group;	(giggles)
Tony:	Right. So you might not do as well as you could do.
Imran:	Yeah.
Sairah:	Because the test might not be on the things you know, it might be completely opposite to the things you know and there might be some of it on another sheet that you know all about and on another sheet that you don't know anything about so you might get lower marks on one and higher marks on another.
Mehnaz:	Yeah, it’s like er if you haven't learnt something and like it just like new and your like erm what am I going to do and everything if you haven't like learnt it before.
Tony:	What about you Rupa?
Rupa:	Don't know.
Tony:	If it's not a fair way, what would be a fairer way of finding out what level you are?

The discussion that followed showed the group seeing the school curriculum defined by the test. They suggested that you should be tested on what you had just learnt and that the teachers should make sure they covered everything that would be on the tests each year. The group was all clear what level they should be - 'we should all be at level 4' although Imran described this as boring. "It's boring being level 4 because that's what everyone is apart from some clever people." Kenny went on to say that he had been estimated at level 2 by his teacher, when asked how he felt about this he said, "If I come out as level 2 I will kill myself." Rupa had also been estimated at level 2 by the teacher, she chose not to describe her feelings on finding this out.

9/5/96 (Parmjit's journal)

> I'm just a little bit nerves about the S.A.T.S. because I think I might get a really bad score or level. I hope I get at least level 4 because I think it is my ability but I will be very happy if I get a level 5.

Parmjit is clear that the 'expected' level is level 4. A fact that was supported by the focus group interview. The teachers in Y6 had not pushed the fact that the Government's expected level at the end of Keystage 2 is level 4 but it was clear in many of the pupils' minds.

A Foucauldian reading from Tansy

I offer here a 'reading' of Tony's journal extracts where I look for techniques of power: surveillance, examining, classifying, a play of totalisation and individualisation, and normalisation.

At the start, Tony talks about his 'training day';

> 'Do they see training days as detached from their' real lives' of teaching and so forget very quickly?'

I recognise here the workings of discursive practice(s)- putting in order the meanings that teachers make and how these are related to patterns of classroom practice. The training appeals to teachers' concern for children as individuals - a learner-centred discourse. This leads to enthusiasm about alternative methods (the rich information about children's individual needs will be valuable to teachers' future planning) but does not dislodge the need for 'find out what they know' tests. It has not disrupted their knowledge that there is a need to view the class as a whole and talk about 'what they know'. The contradiction that Tony sees does not disable these teachers - their everyday practices fudge over the disjuncture. Tony might be aware of the way that children become classified and then included or excluded from future opportunities by the regulating effect of this 'whole class' gaze; he searches for alternative, richer assessment strategies. The teachers he describes are caught up with the common sense of 'traditional methods' and can seesaw between individualisation (concerned with particular children) and totalisation (treating the class as a whole) - valuing one while doing the other.

> ‘The kids do not seem threatened by the tests however - … … This did not disturb (the teacher in the slightest - in fact the teacher was quite amused by it.’

This is a story of regulation through written tests. There seem several colliding techniques operating that bring about normalisation of the learners and the teacher. Through this, aspects of their identities are constituted and they come to know the truth of who they are in this educational site. The final written test script will determine who and what the learners are. We can erase any information about what each child understands and can do - all there is their script. Does the teacher recognise this 'surveillance' and attempt to disperse its point of action? The gaze can shift to the class as a whole and possibly to the teacher her/himself and avoid exclusion of any one child. This illustrates well the foregrounding of texts and language in a discursive analysis and the effective erasure of the subject (as an individual person) that can result.

The most significant use of the National Standard Assessment Test results is to examine the school and its teachers. Whilst the ‘tests’ referred to are school based tests the results will to some extent still reflect on the teacher and the school as a whole. Does this account for the 'not disturbed' response from the teacher: - what individual children do does not directly matter to the school? At the same time, written tests are a surveillance technique that can bring about the regulation of individual children. It is in the collision of this totalising and the individualising that the students can effect their resistance to the implementation of this examination - a transgression of the ordering of the classroom space. Interestingly, it does not reclaim their voice but it exploits their invisibility. Does the teacher's pleasure arise from this contradiction?

In the focus group discussion a group voice is identifiable. It asserts 'The tests are easy and fair. We find them easy; we aren't crushed by them'. The pupils are also aware of the individualisation that can be effected through the scripts. This attributes a level to each child('s name). The technique of normalisation can be clearly traced out in the effects of the SATs. A 'normal' child will achieve level 4 at Key Stage 2. If a child is given a lower result then they are not up to the required standard. They become 'abnormal'. These primary school children are acutely aware of this defining effect, of how it determines how their efforts, past and future, are interpreted, and of ways in which this determines their future position. This is so, despite what teachers or parents might do to reassure them that these test results would not determine their future. Their results may put one of them outside e.g. Level 2. Abnormal. But the tests seem to make a false offer of redemption -'if you work hard enough, practice, revise, then maybe you will be successful in the next test, and be back 'inside''. They can see specific cases where the tests are not fair - but the only way back in (from the outside) that they talk of is a retiming of the tests. If I talk differently i.e. not in terms of levels or SATs then I am mad. To deny SATs is to be totally silenced.

It is interesting to note that again an attempt is made at reclaiming of part of their sense of self from this process. Success at the test is to do with revision and familiarity (not directly, for example, their ability, their understandings). At the very least they give the pupil's a fair idea of what they are like. Did I work well? Did I practice enough? This is surveillance: the anonymous SATs writer might be both the examiner and arbiter. This attempt at distancing themselves from the process of normalisation is not (completely) successful. I will come to know my place/my position in the order, whether I am normal. There is an erasure of self through the levelling process. I will become what I do on that day. Is it melodramatic to echo Kenny - If that proves to be 'abnormal' you might as well be dead?

CONCLUSION: What we have done, what we must do, what we will lose.

Every education system is a political means of maintaining or modifying the appropriateness of discourses along with the power/ knowledge they bring with them. I have said that there is a need for Maths Education research to rethink its concepts and objects of study in order to become aware of why the process of schooling, and of learning school mathematics in particular, has the effects it does on groups of our children. And to think about how this process and the possibility of alternatives can remain hidden.

For example, a model of the social where the individual is separated from the 'outside', and where that outside is seen as forming the individual may produce disjunctures in research analyses which is often ignored in the desire to make generalised claims about the causes of our failure to help children to learn mathematics. Tony and I have argued that mathematics education research must struggle to find more productive ways of examining teachers, learners (and researchers) practices and that to do so we must acknowledge the discursive nature of the field in which we operate.

As a start I have tried to trace some elements of the process of schooling through discussion of tools or strategies I related to Tony's transcript. I have discussed how these strategies can reveal the discursive practice(s) of mathematics education and the ways this can position people within the classroom, effecting their actions and determining what each has to say to be heard. More importantly, I have begun to identify some tasks to engage in as part of that struggle. My commentary on the transcript points to important, though not easy, steps that can be taken to reproblematise aspects such as assessment practices.

I have not aimed to order or interpret the 'reality' of mathematics education, indeed I do not believe that there is such a thing as 'reality' of experience within our schools - rather a multiplicity of 'realities'. Neither have I tried to understand the lived experiences of pupils and teachers; rather I have searched for ways of producing knowledge about maths classrooms that reveal their constructed nature and through which we can think about alternatives. This form of knowledge involves a very different attitude towards meanings, so the aim is not to understand per se but to criticise and transform. Clearly results take on a different meaning here.

I have worked with Foucault's vision of normative nature of assumption; that, for the vast majority of learners, school is an 'unreal' world - where they are not the 'normal'. This has involved trying to know better the 'conditions of education' as our society have established them and the representations and practices that have given rise to our current maths education practices. A rethinking of our curriculum and our practices, an undermining of common sense ideas about children, teachers and mathematics will lead to some insight into why many groups of learners are not being helped to learn mathematics.

In taking up the challenge to examine how we live our professional lives we have to find ways and terms that acknowledge the multiple positioning of the actors in the mathematics classroom and see and work with diversity. We also need to abandon normalising assumptions and ask the hard questions of why we act the way we do, why we construct the subjectivities we do. I believe I have started to address this as a teacher. I need also to take a reflexive stance to my research practices. As researchers, we have to be wary of hidden assumptions about looking for generalisable knowledge and seek alternative ways of validating our work.

So what might such a hard, but interesting question might be for mathematics education? I may wish, for example, to reproblematise aspects of my role as a teacher. It is a challenge to articulate this in practical terms. I might be approaching such an articulation in the reflection I offer below:

> I have been aware of the persistence of a perception of ‘maths as right or wrong’ over the 25 years of my professional experience. In particular I have a strong sense of recognition from pre-service students of the predominance and effects of this (perceived) characteristic of mathematics. In order to explore what it is that I do that contributes to this perception, I might critically examine the strategies I use which I intend to dislodge this notion of maths as (always) right or wrong. One such strategy is in my selection of particular teaching styles, for example: investigatory, discussion based, and my rejection of others. However, my current research task as I see it should move me beyond this to consider how power continues to work throughout changes in constructions of school mathematics curriculum and teaching and learning approaches so that this construct of ‘right or wrong maths’ remains. Teaching style per se may not be an important focus. I should ask whether all pedagogic contexts are liable to this construction of ‘right or wrong’ knowledge? And what examples there are where this has been successfully

resisted? And why it is particularly difficult to resist this in the mathematics classroom? Also are there specific instances of resistance from learners to the normalising effects of ‘getting maths right or wrong’?

Tony’s and my work here offers a lens through which we can search for the perpetuation of exclusion and injustice and for domains within which we may have the power to intervene. It also suggests that there is much that can be done within schools: by examining the curriculum offered to learners, by being critical about the practices adopted, and by being aware of the social and political structure and contexts within which teaching and learning operate. Also by thinking about alternatives, by exploiting the lack of stability of many professional notions, we might open up spaces from which we can counter pathologising, ill-posed problems and look for sites of resistance. So the challenge for educational researchers interested in redefining the experience and effects of learning is to locate the ways in which the curriculum and its assessment, pedagogy and social and cultural environment can be reconstructed. Not an easy task to describe or take on - but we suggest an exciting task.

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[1] Foucault in discussing the role of theory (Foucault & Deleuze 1972b) made the invitation to use his theoretical notions ‘as a toolbox’. Also he role of theoretical tools in practitioner research is more fully discussed in Hardy 1996.

[2] This paper draws from a chapter under review for *'The Social Dimension of Mathematics Education: Theoretical, Methodological, and Practical Issues*', Eds. Jo Boaler, & Paola Valero.

Some words about
"Re-searching mathematics education
from a critical perspective"

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I would like to begin my talk by thanking the Organising Committee of MES2 for the invitation to discuss the paper of Renuka Vithal. I feel honored at this opportunity which has challenged me to think about questions that, in a way, have also been the subject of my reflections as mathematics educator. As a professor in a teacher training course, researching the field of Ethnomathematics, I have dedicated myself to thinking about the central theme discussed by Renuka in her paper. This concerns the theoretical-methodological implications resulting from the very act of researching, when the latter is associated with a social, cultural and political approach, and the empirical subject to be analyzed is pedagogical work with student teachers. Possibly all of us present here, educators committed to a critical approach to Mathematics Education, fully agree with Renuka when she says that "critical paradigm is significantly under-explored and under-represented in mathematics education research". In fact, I consider that most of what has been produced in Mathematics Education is still closely connected with very conservative ways of seeing Mathematics: a science embedded in neutrality, asepsis, in which, as Brian Rotman pointed out, "assertions proved stay proved forever (and must somehow always have been true), where all the questions are determinate, and all the answers totally certain" (Rotman, 1980:129). The mainstream Mathematics Education, as a recontextualizing field, in Berstein's words, is clearly connected to that vision of Mathematics. As opposed to this, the critical approach, in its different modalities, in recent years is achieving greater visibility, even knowing that it is still situated on the edge of Mathematics Education research. And it is not surprising that it is exactly in the so called peripheral countries that this edge approach of Mathematics Education has a wider tradition. If critical approaches in Education are relevant in countries where most of the population is wealthy, they are even more forceful in realities where the majority is poor, as in South Africa, Renuka's country and in Brazil, from where I come. Renuka clearly emphasizes this aspect in the section of her paper entitled "Context, Change and Instability".

Our experience in dealing with social inequalities has given us great pain, and a strong feeling of impotence. At the same time, this experience has enabled us to constantly seek new ways of dealing with the relationship between research and practice, which will allow us to contribute, very modestly, to diminish the suffering of many of those who live in peripheral countries. Thus, our harsh experience in dealing with the underprivileged places us in a position which I would call, to a certain extent privileged, to provide us with a profound understanding of their effects in constructing social identities.

Paulo Freire's original thought must be understood in this scene. With his African experience as educator and his work in Brazil with the poor, he learned about poverty and education as a political tool. In the early sixties, he argued that Education had a political dimension. Later on, he went further in his argument, saying that Education is political. Looking back at Paulo Freire's intellectual trajectory, authors like Tomaz Tadeu da Silva (1999:62) argue that "if he in a way anticipated the cultural definition of curriculum which would later characterize the influence of Cultural Studies on curricular studies, it can also be said that he initiates what could be called (…) a postcolonial approach to the curriculum" (...), an approach which seeks to problematize the power relations between countries which, in the previous situation were colonizers and those that were colonized (…), that "seeks to privilege the epistemologicla perspective of the dominated peoples". The contemporariness of postcolonial studies is increasingly strengthened at a time of excluding globalization which produces profound social and economic inequalities around the world.

The ideas developed by Paulo Freire and by the critical pedagogy, in consonance with modern thinking, greatly inspired mathematics educators committed to social change. Today, we are moving toward another landscape in which the contributions of postmodernism, poststructuralism , postcolonialism , feminist theorizations and in a broad sense, the Cultural Studies are being incorporated.

Based on these most recent contributions we can rethink what we have been doing in the field of Mathematics Education and construct new perspectives for this field of knowledge. Renuka's paper analyzes many of these new perspectives, "examining differentes facets of the research process and forms of participation". Since it is dense text, with many well thought-out ideas and profound arguments which discusses a complex range of facets of the subject studied, I chose to discuss a facet which permeates the whole paper.

This concerns the role which we educators play as intellectuals involved in teacher training. Renuka, supported by her qualified empirical research presents this point very clearly when she problematizes the "relation between the researcher and the research participants" (p.11). What is our role as intellectuals in this power relation? What are its effects in the construction of social identities?

To start the discussion, I would like to clarify the meaning I am lending to the expression "intellectual". Following Thomas S. Popkewitz (1991:218), I argue that it refers to the

> (...) institutional position and social relations of those who produce knowledge, more than to a normative criterion as to who has the knowledge and discernment.The intellectual is therefore, a class connected to historical formations and social positions of the occupations of producers of knowledge.

This statement allows us to consider educators as intellectuals, and their teaching activities to be seen as intellectual work. Thus, agreeing with this approach, my comments about Renuka's paper are connected to the examination of the intellectual work from the perspective of its function in the social world of which it is part, which is the equivalent to thinking about it as a social practice. As Renuka showed, her social practice was developed and articulated in two dimensions: the first of them is connected to the scientific field, " the world of academia", in which she has a position as a researcher, and the other is linked to the activity developed with the student teachers and the practitioners.

Pierre Bourdieu attempts to denaturalize academic practices, examining not only their internal determinations, the properly scientific ones, but also their social determinations, showing how the intrinsic and extrinsic interests are necessarily connected.

I would like to emphasize that the demystification of academic space allows the intellectual to examine also her/his own scientific practices from the sociological standpoint, looking at them as determining and determined by a field of forces, in which they act as agents, trying to maintain or increase cultural or social capital. Renuka's discussion about different paradigms of research (p. 7) may be analysed in this perspective. In effect, choices of problems, themes, methods or scientific procedures — considered more relevant inside academic communities, at different historical periods — can elucidate the indissociable connection between scientific and social strategies used by intellectuals who are attempting to satisfy their interests.

In our case, especially, who are committed to critical approaches in Mathematics Education, it is important to consider that also from the standpoint of the internal dynamics of the scientific field, the researcher who deals with some theme considered socially relevant faces a rather peculiar situation. What would appear to simplify her/his investigations, namely, the fact that the relevance of the theme studied might spare a more careful construction of the objects of research, effectively constitutes an element which renders work more difficult, to the extent that it might be involved in the illusion of obtaining easy profits. It is in this sense that one may interpret the observation by Bourdieu regarding " women's studies, black studies, gay studies", which can also be extended to investigations connected with critical approaches. According to the author, those studies:

> (...) are, certainly, so much the less protected against the 'naïveté' of 'good feelings', which does not necessarily exclude the well-conceived interest in gains associated to "good causes", the less they have to justify their existence and the more they confer on those who take them over an efective monopoly (frequently demanded as a right), but leading them to enclose themselves in a sort of scientific ghetto (Bourdieu, 1990:28).

In brief, the above considerations have attempted to show that, if on the one hand the involvement of educators in themes directly linked to social approaches, shows their commitment to relevant social problems, on the other it also indicates another kind of profit which, often, goes unnoticed by them. That unveiling operation allows academic practices to be examined with less idealization, the myth of science for its own sake to be problematized, and ensures that the analysis that the intellectual her/himself performs of her/his role in the social world contain a larger dose of self-reflexiveness and criticism, qualifying the discussion on relations between knowledge, power, intellectuals, teaching and research.

This qualification assumes that the elements involved in these relations — knowledge, power, intellectuals, teaching and research — be thought of in a historical and geographic situation, connected to specific contexts, based on which one does not attempt to produce extrapolations. From this standpoint, Popkewitz (1991) presents an important contribution to the theme when emphasizing historicity in the formation of social patterns and their connections to a specific power juncture, establishing an inseparable connection between knowledge and power.

The author's approach deals with the idea that power is articulated in two conceptual dimensions. The first, associated with a more traditional view, is situated in a structural perspective, linking power to the global processes of dominance and subordination of social space. Without frontally opposing this focus on power, Popkewitz, however, problematizes it, indicating the effects which may be produced when one dichotomizes the social world between oppressors and oppressed: to see each of these groups as completely homogeneous and monolithic tends to cover up the actions and practices of individuals through which power also operates. Renuka clearly showed her attention to avoiding this dichotomy in research and the need to take into account the "multiple identities of the researcher and the research participants" (p.16).

The second dimension of power incorporated by Popkewitz in his theoretical formulation is strongly influenced by the ideas of Michel Foucault, in which, as opposed to the ideal of an oppressive, centralized power, from a single source, the productive dimension of power is emphasized, a power distributed by capilarity throughout the social tissue without occupying fixed places. It is from this standpoint that Popkewitz (1991, p.223) considers that power

> is intrinsically connected to rules, patterns and styles of reasoning, though which individuals speak, think and act in producing their daily world. Power is relational and regional.

In introducing the idea of regionalization to the concept of power, following Foucault, the author attempts to stress the "multiplicity of social forms and power relations which occur in specific historic places" (Popkewitz, 1991:221). This approach allows intellectuals to be thought about not in generic, universal terms, but as connected to specific social struggles.

The theoretical approach of Popkewitz, articulating and looking for complementarity between these two conceptual dimensions of power — the first macroscopic and the second microscopic — is fertile ground for the analysis of the role of an educator as a researcher. In some way, Renuka deals with these two conceptual dimensions of power in her paper. Moreover, she pointed out the key importance of analysing power relations produced in the interactions of the researcher and the researched. This kind of issue provides us with a more profound understanding of the role of the intellectual in our times. I will argue that this role is moving from a Gramscian perspective to a Foucautian one.

In fact, from the Gramscian perspective, the role of the intellectual who is organically linked to the subaltern classes is privileged as the one who will perform the process which would lead these classes to "higher levels of culture", one of the conditions to build up a counter hegemony. This position, when transferred to the present times and situated in the specific contexts of social movements, points toward two questions.

The first of them concerns the possibility that the fragmented consciousness of a group — to Gramsci that of the "simple ones" — might become a "higher" conscience, unified, homogeneous, a concept which has been appropriately problematized by postmodern thought. The second question concerns the outstanding political position given to the intellectual. The privileged position given to the intellectual may not be seen as something "natural", inevitable. Foucault points out the problem of this position very appropriately. In his view, the more traditional view of an intellectual was conceived as " the one who told the truth to those who did not yet see it, and in the name of those who could not say it: conscience and eloquence". In this sense, the intellectuals placed themselves " 'a bit ahead, or a bit to the side' to say everyone's mute truth". (Foucault, 1989:71). According to the French thinker

> what the intellectuals recently discovered is that the masses do not need them to know, they know ,perfectly well, clearly, much better than the intellectuals; and they say it very well. But there is a power system which bars, forbids, invalidates this discourse and this knowledge. A power which is not found only in the higher instances of censorship, but which penetrates very deeply, very

> subtly, into the tissue of society. The intellectuals themselves are part of this system of power, the idea that they are agents of 'conscience' and of discourse is also part of this system (Foucault, 1989:71).

One of the deductions which may be obtained from Foucaultian thought is the relevance of intellectuals placing permanently as the object of their concerns the need for self-reflection, so that their own discursive practices may be analysed and interpreted as participants and producers of a power system. Because, as said Foucault (1989:71), the role of intellectuals is "primarily to fight against the forms of power exactly where it is at the same time object and instrument: in the order of knowing, of 'truth', of 'conscience', of discourse".

More and more the space for "universal "intellectuals, who, with their powerful narratives will act as " awareness makers" and leaders of the "masses" diminishes. About the replacement of the character of the "universal" intellectual by the "specific" intellectual, Foucault (1989: 8-9) says:

> For a long time the so-called 'left' intellectual talked and saw acknowledged his right to talk as the only person who knew truth and justice. People would listen to him, or he intended to be heard as representative of what is universal. To be an intellectual was a bit to be everyone's conscience(...). It has been many years since anyone has asked the intellectual to play this role . A new mode of 'connection between theory and practice' has been established. The intellectuals have become used to working not at the 'universal' , the 'exemplary' , the 'fair-and-true-for-all', but in given sectors at precise points in which, they were placed by their own living or working conditions(...). Certainly, in this way they gained a much more concrete and immediate awareness of the struggles.

Authors such as Popkewitz (1991) have called attention to the point that the great proximity of a theoretical analysis of reality to the problem of immediate change privileges the researcher as an agent of this change, bestowing on her/him a heightened authority in this process. It is in this sense that Popkewitz argues that the intellectual needs permanently to practice her/his capacity for self-reflection.

As can be deduced from Renuka's study, she has had a special concern in exercising, locally, the self-reflection of which that author speaks. Renuka is aware that the privileged position from which she spoke to those student teachers could not be avoided. This leads her to discuss throughout the paper the problem of imposition, as in page 5, when she says: “While it must be acknowledged that interventions can never escape the problem of imposition, the difficulty or contradiction that occurs in this research, however, is related to how to deal with this impositional issue with reference to a particular perspective in theory, methodology and participation which argues against it in the educational setting.”

Precisely this kind of concern can lead each of us to consider her/himself as a ”specific intellectual“, in the Foucaultian sense of the term. In this case each of us is seeing her/himself as an educator who performs

> a much more modest role, much less universal and much more local (...) a much more symmetrical role in relation to the other participants in the social struggles in which she is involved, in the sense that her knowing, her vision and her discourse owe as much to the interests of power as to those of any other participant (Silva, 1994:251).

As far as I can deduce from Renuka’s arguments, although it is not mentioned, she considers herself as a “specific intellectual”. In agreement with this position, she ends her paper problematizing the notions of emancipation and empowerment in research, saying that “rather than to speak of research carrying emancipatory intent, it may be useful speak of research as carrying possibilities and hope, an idea also put forward in the theoretical educational landscape.” Renuka ends her paper referring to possibilities and hope in the educational landscape, and I will end my talk following Robin Usher and Richard Edwards (1994: 31), when they say:

> More than ever, education needs a critical scepticism and a suitable degree of uncertainty whilst close attention must be paid to the need of a careful deconstruction of the theorisations and discourses within which educational practice is located.

I hope that these thoughts on “Re-searching mathematics education from a critical perspective” that I have presented here will be a small contribution to the discussions which may provide elements for that deconstruction.

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The Unseen Social and Cultural Substance Of Written Responses in Mathematics

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Students' written responses to open-ended (extended constructed response) tasks show that student performance differs qualitatively based on such social and cultural factors as knowledge of valued forms of communication in mathematics, language and reading proficiency, knowledge of task-designers' implicit perspective, and type of logic. Real-world contexts also introduce greater opportunities for divergent, yet reasonable responses from students. Extended constructed response tasks are a viable means for meeting some of the challenges of equity.

Introduction

Knowing how to participate in the socially constructed forms of mathematics and science communication is an expectation of the current standards-based reform in mathematics and science (Champagne & Kouba, 1997a). Morgan (1998) argues that this expectation is a hidden, but very present aspect of assessment. She states that in many assessment situations the learner "not only needs to 'understand' a particular piece of school mathematics but also needs to know the forms of behaviour that will lead to recognition of this and how (and when) to display these forms of behaviour" (p. 4). The difficulty is not just that students do not heed directions to explain and justify, as Dossey, Mullis, and Jones (1993) state, but that students do not know what is implied by such directions. Morgan (1998) frames this issue as one of equity:

> There is a fundamental equity issue here that is often ignored. Those students with the linguistic awareness and skills that are generally associated with advantaged, literate backgrounds are more likely to 'pick up' the unspoken distinctions and display the valued behaviour in the appropriate situations. Others, from less advantaged backgrounds, are less likely to come to school with these skills; they must, therefore, rely

> on their teachers to provide them with the necessary awareness of the forms of behaviour that will be valued. The naïve guidance (such as 'draw a diagram') commonly provided by teachers is not adequate for such a purpose. An important task for teachers and researchers who are concerned with equity in assessment, therefore, must be to investigate assessment practices at a level of detail that can identify which aspects of students' behaviour are likely to be recognised as mathematical and valued as signs of mathematical understanding. Such investigations would also need to develop a language to describe these valued behaviours – a language that teachers and students can use both to help students to display the behaviours that will lead to success in the assessment process and critically to interrogate the assessment practices themselves. (p. 6)

Our current research on mathematics and science literacy, as part of the National Center for Research on English Learning and Achievement, is an investigation of students' written explanations in mathematics and science. One of our goals is to systematically examine student responses for clues on how culture and language may be part of "disadvantaged" students' performance, especially in terms of mathematical behaviors valued within the larger mathematics community and culture.

As we examine student responses, we also must keep in mind Tate's (1996) admonition that any delineation of expected behaviors, especially in testing situations, ought to be done from a multicultural and social reconstructionist philosophy that allows students to "solve problems from their lived reality" (p. 195).

History of the Task

One of the most difficult 1996 NAEP extended-response items for eighth-grade students was:

> This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> of your work.
>
> Julie wants to fence in an area in her yard for her dog. After paying for the materials to build her doghouse, she can afford to

> buy only 36 feet of fencing.
> She is considering various different shapes for the enclosed area. However, she wants all of her shapes to have 4 sides that are whole number lengths and contain 4 right angles. All 4 sides are to have fencing.
> What is the largest area that Julie can enclose with 36 feet of fencing?
> Support your answer by showing work that would convince Julie that your area is the largest. (NCES, 1999)

Less than 1 percent of the 1,615 eighth-grade students in the NAEP sample provided an extended or satisfactory response, 29 percent gave a partial response, 4 percent gave a minimal response, nearly 40 percent gave an incorrect response, and nearly one-third omitted the item (NCES, 1999; Kenney & Lindquist, in press).

Eighth-grade students' responses in the NAEP sample were scored as follows:
Extended -- Correct response (Justification need not include table, but should account for all possible length-width combinations or demonstrate evidence that all combinations have been considered in the formulation of the explanation.)
Satisfactory -- Response that indicates that 9 x 9 square has maximum area (81 sq. ft.) or that another rectangle has maximum area but accompanying work contains a minor error OR Response contains all work for nine rectangles (widths 1 through 9), but maximum area is not indicated (or response indicates that a rectangle other than the 9 x 9 has the maximum area).
Partial -- Response shows at least 3 different rectangles (dimensions and areas); may indicate that one of those rectangles has maximum area OR any response with no work that indicates that the 9 x 9 square has maximum area.
Minimal -- Response demonstrates a minimal understanding that area and perimeter formulas for rectangles are needed in the solution and may show a beginning attempt to organize the data and information. For example, this might be illustrated by generating one specific value for the length and width that fits the given information.
Incorrect – any other incorrect answer. (NCES, 1999)

Kenney and Lindquist (in press) suggest that the low performance on this item could have been a result of a limited scoring guide. They felt that the NAEP scoring guide did not account for students who had worked on similar items and "just knew" that a square yields the maximum area for a quadrilateral with a fixed perimeter. Based on our research, we also believe that narrowly interpreted scoring guides on high-stakes tests may depress reports of performance (Kouba, 1999). However, our results suggested that the low performance in our sample seemed more a societal or cultural result of students' lack of knowledge about the expected forms of literacy (see Champagne and Kouba, 1997b for more on forms and levels of literacy in science and mathematics).

Study and Results

Expected Forms of Responses

We were curious about what a detailed examination of a large sample of students' responses to the dog yard item would reveal about students' prior knowledge and understandings of expected forms of response. We administered the dog yard item to 315 eighth-grade students across three middle schools in an urban school district that is racially, linguistically, and economically diverse. The students in our sample took the item as the last one of their year-end science exam (a situation outside the environment of the mathematics classroom, much as the NAEP assessments were administered outside of the usual mathematics classroom routine).

We did not use the NAEP scoring guide, except to look at the extended and satisfactory responses. The students in our sample faired somewhat better than the NAEP group, but the results still were low with 2 percent scoring in the extended or satisfactory level. On the encouraging side, we did have a couple of explanations where students used the expected justification structure of showing all possible areas of rectangles and concluding that the square had the largest area. We also had responses that showed an ability to argue from a more abstract perspective. For example, two students gave the following responses:

1

"No matter how many 4 sides, 4 right angled figures you try, you will find that a perfect square [a 9 x 9 square] will have the highest area. Look at [a 2 x 16 rectangle] for example, it has 32 square feet. [A 10 x 8 rectangle] has 80 square feet. [A 7 x 11 rectangle] has 77 square feet. In fact, the farther you get from having both sides equal, the more your area will reduce. So to get the most for your money, go with the square"

2

"This [a 9 x 9 square, A = 81] would be the largest area she could have because any other one would have one side bigger than the other which will end up causing it to have a smaller area than the square. It's [a 6 x 12 rectangle, A = 72] still smaller than the square because the sides aren't all even."

Both of these responses have the structure of a justification, a stated conclusion with a warrant (because…), and both demonstrate an understanding of the task and pattern of change in areas as length and width are altered. Both also rely on a linguistic rather than a symbolic or diagramatic explanation.

But, what of the students who did not garner a rating of Extended or Satisfactory? Was poor performance more a lack of mathematical understanding and mathematical background, or a lack of understanding the societally determined expectations for presenting a convincing argument?

Kenney and Linquist (in press) suggested that middle school students have prior experience with determining the maximum area for figures with fixed perimeters, and thus know the mathematics necessary to solve the item. For our sample of students, we checked with the schools and found that most of the eighth-grade students had done problems that looked at maximum areas possible with fixed perimeters. We also have evidence in the students' responses of prior knowledge and experience with such items. The latter of the two responses displayed above seems more a reiteration of an established conclusion than an explanation of a relationship discovered as a result of doing this particular dog yard item. We also had students who wrote,

> "She will have to enclose near a barn or house OR make the fence circular";
> "Using metal fencing and enclose the yard & fence in a big circle";
> "She could make it so each side has 9 feet of fencing! The shape would be a box and that would make 4 cornors [sic]. She could also make a circle that runs around 36 ft!"

The mention of a barn or house, and the suggestions of a circle also seem indicative of prior experience with similar types of items. Thus, students in our sample who answered:

"Square because it covers more area"; or

"9 ft by 9 ft is your best bet, it has the largest amount of area and you use up all the fencing,"

may have clearly understood the mathematical relationships within the item, but not the testing expectations in terms of providing a convincing justification. Based on our qualitative analysis of the students' responses, at least a third of the eighth-grade students had the requisite mathematical understanding. Thus, we strongly agree with Morgan's (1998) recommendation that the expectations for form of response must become part of the daily instruction in mathematics. Equitable instruction and assessment require that students get systematic instruction in the valued forms of mathematical explanations.

Linguistic Concerns Related to Equity

We also found evidence of linguistic or mathematical reading comprehension difficulties such as reading "four sides" as "four equal sides" or reading "whole number lengths" as meaning "even lengths":

"She can have 9 feet on each side. Ok you have 36 feet and 4 sides all sides equal you divide 36 by 4."

"all right angles, all even lengths, fencing on all sides, …"

Some students also thought of right angles as "squared" angles, thus requiring Julie to make "a squared fenced in area for her dog." These linguistic difficulties led students to a correct shape and area, but an incorrect justification. Other students simply wrote that they could not make sense of some of the words in this task or did not understand what the task was about.

Although language poses difficulties for all students who struggle with reading, it is a particular concern for students for whom English is not their first language. In order to be a good problem solver, one must be proficient in the language in general as well as the technical and symbolic languages of mathematics. Therefore, the limited English proficient (LEP) child may be at a disadvantage, not because he or she does not possess the necessary skills to solve the problem but because of a lack of "accessibility" in the second language (Mestre, 1981). Failure to master formal discourse styles may interfere with students' understanding of word problems (Cummins, 1991). Mathematics has its own specific forms of discourse. Therefore LEP students must master this language as well. Students are required to combine their linguistic, cognitive, and meta-cognitive development to successfully comprehend the reading. Thus, in addition to the mathematics skills that they need to solve the problem, students must simultaneously develop the requisite comprehension skills while encountering text that is culturally biased. (Fencing in dogs is not a universal cultural activity.)

A second linguistic dimension to the interpretation of the dog-yard task emerged from our consideration of the students' responses. Six students suggested making the dog yard some shape other than a square or rectangle (e.g., circle, hexagon, pentagon, or irregular shape using the sides of the doghouse or of Julie's house). As we discussed these responses, we kept trying to make sense of these students' written comments to Julie that their shapes were "what would work best." Were the students just answering the last question, "What is the largest area that Julie can enclose with 36 feet of fencing?" Or were they thinking in some other way? English education members of our research group suggested thinking of inflection as a factor in students' reading comprehension. Julie may <u>want</u> to make a rectangular shape and use fence for all sides of the enclosure, but the <u>best</u> solution is clearly something else. In other words, the students may have interpreted the task as one of convincing Julie that her wants were not the best solution.

Social and Cultural Concerns Related to Equity

About three percent of the eighth-grade students (11 students) gave answers that indicated a divergent, yet reasonable interpretation of the context. Two students said that the problem could not be solved because they didn't know the shape of Julie's yard, i.e., "…I really don't know how her yard looks and how her yard is measured;" or "[maybe] she can't do it because her yard is too small." These responses support the concerns that Tate (1996) raises; that is, that some items are constructed from a cultural or economic perspective quite different from that of many of the students (e.g., a White, middle class, or Eurocentric perspective). The dog yard task seems designed from the perspective of having a relatively large yard. This is contrary to the "lived reality" of many of the students in our study. The lived reality for many of the students in our sample, especially those from the lower socioeconomic groups, is that they live in urban apartments or brownstones which have tiny (4-foot by 4-foot) front yards and no backyards. And those urban buildings that do have backyards often have oddly shaped ones that would not allow for a 9-foot by 9-foot square dog yard. Thus, students who placed themselves entirely within the context from their perspective may have dealt in a literal and self-situated way with the direction to find the maximum area for Julie's dog yard.

An equitable approach to preparing students to respond in reflective ways to items such as the dog yard task might be that teachers help students to identify and solve from multiple perspectives. Tate (1996) suggests that teachers employ multicultural and social reconstructionist approaches where students ultimately are expected to solve the same question from the perspective of different members of the class, school or society. This necessitates teaching students the scientific habit of mind of always considering alternative assumptions and always suggesting solutions from alternate assumptions.

Logic, Reality, and Equity

Some divergent responses initially might be perceived as just idiosyncratic differences in people. However, we view these as indicative of child rather than adult logic, (as in Piaget's argument that the intellectual structures of children are not the same as those for adults). Children's thoughts, references, and logic are embedded in the details of the reality of the context. Once the mathematics has been embedded in a context, children are less able than adults to see the context as just a vehicle for understanding the mathematics. Children are less able to extract the mathematics from the context. For some students, this leads to an inability to respond as expected, because they cannot "get past" a contextual detail that the adult may have never considered. For example, some students in our sample were concerned about having a gate so that "the dog would be able to get out" and suggesting fencing around the dog house, but keeping enough pieces of fence to make a gate. Another student, who seemed to have prior knowledge about appropriate shapes of pens for dogs said a 6 by 12 rectangle should be used because that was the largest area that we could have while at the same time giving the dog room to run. One student thought it couldn't be done with only 36 feet of fence and indicated some relationship between the height of the fence and the amount of fencing. As we discussed this response, we realized that the student seemed to be thinking that Julie had 36 board-feet of fence. Perhaps this student had experience buying lumber by the board-foot.

Although some of these types of attention to detail can be avoided by careful pilot testing and construction of tasks, there is no way to make a context free from alternative interpretations. We are brought again to the need to help students learn to provide multiple solutions from multiple assumptions (e.g., if we ignore the need for a gate, the answer is x; if we take into consideration the need for a gate, the answer is y).

Conclusion

Our work has brought us to the conclusion that the use of extended constructed response items in assessment offers a far better prognosis for equitable assessment than a return to multiple choice testing. The use of extended constructed responses opens the possibility to let students reveal their logic and argue from their lived realities (as well as from the realities of others). We see the requirement of written explanations or well constructed convincing arguments as a means to allow students to demonstrate what they know. We also see it as a means to change from a staid objective view of mathematics to what Tate (1996) argues for, mathematics "as a tool to guide social decision-making…influenced by the values of those who use it in human affairs" (p. 187). The mathematics education community is well into understanding and grappling with the challenges of using extended-constructed tasks in testing and the concomitant cultural, social and political implications.

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A TEACHING METHOD / RESEARCH TO STUDY OVERCOUNTING

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Abstract:

This article presents a teaching method/research for the study of overcounting, with students from kindergarten (5 to 7 years old). It shows that overcounting in adding problems depends on the memorization of the natural numerical sequence, from a certain number different from 1. The method proposed allowed a good understanding of the evolving learning processes of the students as well as the role of certain social interactions in the classroom. Besides that, it enabled classes to follow their original course and that the activities be adapted to the cognitive conditions of each student. We reached a good level of control of the pertinent variables of the research, without overlooking the complexity of the classroom situation.

1. Introduction

Based on ERMEL (1991), in the 20th century, we may take into account two remarkable periods in the teaching of numbers, which have influenced the current period:

- From 1945 to 1970, the teacher presents the numbers to the students, in a repetitive scenario, whose main stages are: global introduction, formation and decomposition of the number. These numbers are introduced following their sequence: a week is dedicated to each of the first ten numbers, during which the number of the week is approached, as being the number of the previous week, added by one unit. For example, 6 = 5 + 1. The number is then named, written down in letters, decomposed in the sum of two terms in all possible ways. From number 11 on, the number is introduced with reference to number 10. Global introduction of the number means showing the quantity of objects corresponding to the numbers studied, besides its denomination, with several examples. Thus, 5 is just another way of designating ✈✈✈✈✈ or /////. The formation of a number means its concrete representation from the preceding number (in the previous example presented with 6, 5 + 1 or 22222 1). The decompositions serve as references for the formation of the number (13 = 10 + 3; 17 = 10 + 7) and also as a basis for automatisms related to sums (or subtractions). Therefore, the teaching concept is based on the sensual empirism, through which knowledge is formed from the experiment and observation, from simple to complex. The student learns by observing, imitating and repeating. In general, the sequence of the numbers greatly determines the progression of teaching. We must observe that, within this context, the *social knowledge* of the students is disregarded.

- From 1970 to the 80’s, the concepts are constructed by the students and serve as a frame for the progression: starting up from the work with sets of objects, aiming at the so-called prenumerical concepts (classification, ordering, designation), a concept of number (the cardinal of a class of equipollent sets) is developed. Thus, there is emphasis on the one-to-one correspondence and in the comparisons of the quantity of elements of sets, which serve both for the development of the number concept and for the numerical designations and comparisons. The decompositions are important, both for the understanding of the numbers and the sum. This teaching concept is largely supported by the Piagetian theory and by mathematical theories as well. As opposed to the previous period, tasks such as the verbalization of the numerical sequence are considered repetitive; its memorization is devalued. It is important to remark that the *social knowledge* of the students is not actually taken into account: there is resistance in utilizing the numbers before the construction of the concept of number.

In short, before the 70’s, we have an authoritarian teaching framework; between the 70’s and the 80’s, the teaching becomes democratic and focuses on the active role of *the students’ experiment*, on the importance of the mathematization process, in which the student has an essential role. However, the students’ social knowledge is not actually taken into account, and certain practices, such as the memorization of the natural numerical sequence, are disregarded.

The Referencial Curricular Nacional para a Educação Infantil (National Reference Curriculum for Children’s Education) (1998), considers the most adopted pedagogical practices in Children’s Education in Brazil, regarding the teaching of numbers, which are similar to the previous ones. It introduces its current approach, with 3 basic axes to orient the teaching of numbers and numbering systems: counting, notation and numerical writings; operations. From the analysis of this document, we remark that the construction of the number concept as the cardinal of a class of equipollent sets is outdated. A synthesis of the cardinal and ordinal approaches of the number is suggested, since the cardinal is largely emphasized between the 70’s and the 80’s, disregarding the ordinal. The practices of counting objects and memorizing numerical sequences are recovered, with a new approach: *this time the social knowledge of the students and their hypotheses about the numerical formation are taken into account.*

From our experience, an important aspect of the numerical practices in the kindergarten is the one related to the overcounting: counting from a certain number, different from 1. The overcounting can be used for the resolution of adding problems. Several previous researches, with 5- to 7-year-old-children, show that the memorization of the natural numerical sequence is necessary for the counting of objects, because the student makes the following one-to-one correspondence: he says the number 1 of the natural numerical sequence and catches one object; he says the following number, catches another object and so on. A good synthesis about these researches (which do not refer to overcounting, but to counting) can be found in Nunes and Bryant (1997). It is therefore obvious that the *overcounting in adding problems may depend on the memorization of the numerical sequence*, *from a certain number different from 1.* In this study, we wanted to know whether kindergarten students: a) can overcount when reciting the natural numerical sequence; b) can overcount in the solution of adding problems. Besides that, we wanted to know if the overcounting (a), when reciting the natural numerical sequence, is required and enough for the overcounting (b) in the solution of adding problems.

2. Theoretical Framework

In this study, we looked for problems in which the students could *initiate* a research procedure and *validate* the answers, as per the theoretical concepts of Mathematics' Didactics, developed by Douady. According to this researcher, certain concepts of the students are developed by means of the old-new-dialectics[1], according to certain phases of the tool-object-dialectics[2] and by means of the inter-domain-interaction[3]. The old-new-dialectics intends to formulate problems for the students, in such a way to enable them the formation of new knowledge, while using the old one. We foresee that the students will be able to solve them, at least partially, although their knowledge will not be enough for the whole solution. In order to achieve that, the students will have to use knowledge from at least two domains. Douady (1984) considers as domains: the geometrical, the numerical, the physical (physical actions over objects) and the representational (drawings, codes, signs or symbols, in general). We want to emphasize the three latter ones. Those domains are chosen in such a way that one serves as a reference to the other, so as to allow the use of adequate knowledge to the solution of each problem and to the validation of what is produced as *new knowledge* by the action of the students themselves. Then, a formulation and validation phase is promoted, and some errors or contradictions can be overcome by the confrontation of ideas. Because of that, this phase is also a learning source and therefore a cognitive development phase as well.

[1] dialectique ancien-nouveau, in French
[2] dialectique outil-objet, in French
[3] jeux des cadres, in French

This theoretical framework allows us to hand the students the initiatives about the working methodology. According to Maranhão (1999), from the students' productions, the researcher chooses the domains, related to the problems proposed, that is, to what the research wants to analyze. Old knowledge is identified (through the procedures or means they utilize to solve the problems proposed) and, thus, we can conduct its progression, taking into account their cultural background (supplied by school or other sources). This framework also allows us to have ideas on pertinent issues along the research, which we might have overlooked at the start. That means that we can formulate new questions or hypotheses from the production of the students.

When we elect the domains used for the solution of the problems, we identify the tools (knowledge) available for the students to solve the problems proposed and, at the same time, we recognize certain knowledge required for their progression. In this research, we also want to choose the domains: ordinal (the overcounting by reciting a numerical sequence) and cardinal (the overcounting in the solution of adding problems). The knowledge is focused on the chosen domains. In new problems, we analyze the interactions between these domains, produced by the students themselves, that is, the use of knowledge about one domain for the evolution of knowledge in another one. As we can see, this theoretical framework enables us to observe concepts in evolution.

In this process, we may identify some procedures which are considered as non-pertinent and then, by means of validation, we may lead the students to the choice of a new one. The teacher will evaluate the adequacy to the individual potential of each student. In this phase, group discussions among the students may be encouraged and we can formulate questions. The teacher/researcher has an essential role, either mediating the discussion or formulating new questions. Supplying certain clarification, always respecting the students' freedom, without providing answers to the problems proposed. The students' progression can also be analyzed, due to this phase, and certain mediations can be evaluated, according to their efficacy and coherence with the theoretical framework.

It is important to remark that the theoretical concepts by Douady, selected for this research, enable us to articulate the teaching and researching activities, as a case study, according to Nisbet and Watt (1978). Such articulation provides us the necessary flexibility, because our main purpose is the understanding not only of a *singular instance* but also the *concepts in evolution*.

According to Nisbet and Watt (1978), we can characterize the development of the case study in three phases: open or exploratory; systematic; analysis and systematic interpretation of data in the elaboration of the report. During the open phase, the critical issues emerge from a speculation based on the personal experience of the members of the group of researchers. During the systematic phase, we work on information gathering, using more or less structured instruments. The third phase includes interpretative elements, besides the descriptive ones.

These phases are not completed in a linear sequence, but in a dialectical one. Therefore, they can be articulated with the phases that we had chosen, from the tool-object dialectics, originally conceived for the formulation of a teaching sequence and that contemplates a researching activity in the solution of problems. This way, beyond a singular instance, we can understand an evolving learning process.

According to Lüdke and André (1986), the case study, among other aspects, aims at: discovery; revelation of the vicarious[4] experience; representation of the different elements and, sometimes, conflicting points of view which are present in a social situation; emphasis of the interpretation of context; use of a more accessible language and way, as compared to other reports from the research. As a restriction, we have the fact that only naturalistic generalizations are allowed, that is, instead of asking "What is this case representative of?" the reader should ask: "What can (or can't) I apply from this case to my situation?"

In accordance with these theoretical frameworks, the *teaching/researching method*, proposed herein, aims at the discovery of procedures utilized by the students in the solution of problems, the revelation of domains made available for the conceptual evolution. It also foresees discussions among the students and between the students and the teacher/researcher, revealing points of view sometimes conflicting, which are present in the classroom situation. This teaching/researching method also aims at adapting the researching sessions to the reality of the classroom and the teacher, continuously elaborating and evaluating the working strategies together with him/her. Besides that, there are the main questions derived from the professional background of the group involved in the research.

[4] "one who represents another person" – DICMAX – Michaelis Português – Moderno Dicionário da Língua Portuguesa/1999

3. Methodology

The study was carried out at a public school in São Bernardo do Campo – São Paulo, Brazil, in a kindergarten classroom with 32 students.

3.1 Open Phase

In order to obtain a smooth flow of the teaching/researching method, we promoted various discussions with the teacher, under the form of open interviews, before the application phase of the research, in the classroom. The purpose was to know her working methodology with the students, the mathematical concepts previously taught, the teaching/learning method, the individual behavior of the students and possible difficulties specific of each one of them, according to the point of view of the teacher.

We studied, together with the teacher and two observers, some theoretical elements: the objectives and main questions of the research; the old-new-dialectics, formulation and validation phases; the concept of inter-domain-interaction, in the functioning of these phases; the teaching/learning concept proposed by Douady, and the articulation of this concept with the teaching/researching method proposed in this study.

We also discussed, in meetings held during the application phase of the research, in the classroom, the activities performed in class, their objectives, what we intended to know from either the production of the students or the discussions. The activities were conceived according the vision of the teacher about the previous knowledge of her students and according to the objectives of the research. We foresaw possible and adequate interventions from the teacher and the researcher. We also discussed some interventions that would not be adequate to the teaching/researching method.

3.2 Systematic Phase

Two sessions were conducted by the teacher or the researcher. Each session had more than one activity. We used a camera, in certain activities, and in others we used a tape recorder in order to obtain accurate data from the procedures in classroom. Besides those data, we obtained others, form the notes taken by the observers. During the first session, the first part aimed at getting the students acquainted with the people and equipment and the second one aimed at identifying the counting procedures known by the students. This second part had two activities. In the first activity we tried to observe the counting of objects and, in the second one, we tried to identify the available knowledge of the students about overcounting, when reciting a numerical sequence. In the second session, we aimed at the revelation of the knowledge applied to the resolution of adding problems (research and validation).

3.3 Description of the sessions

First session: During the first part of the first session, we worked with groups of 4, to follow the regular display of group working, in this classroom. The students collected the material that would be used in the next activity. We chose used, empty matchboxes, because they had this object in the school and, in general, the students knew it. The teacher oriented the students about the objectives of the research. We did not gather data from this part.

In the second part of the first session, we used a game circuit, so that all students were active along the session, in a similar way to what happened at regular classes. By using the circuit we intended to control the data gathering of the research, following the teaching method. We then had two circuits, which we named A and B, each of them with two activities. While 4 groups were in circuit A, 4 groups were in circuit B. The groups were defined by the teacher. In circuit A, the students did activity 1 first and then activity 2. In circuit B, the students did activity 2 first, and then activity 1. The teacher explained the task of activity 1 to the students and delegated its sequence to the researcher. In this activity, the students were invited, one by one, to get 4 matchboxes from the teacher's desk, so that their group could wrap them. Each group would wrap 16 matchboxes. An observer took notes on the counting procedures of the 4 objects. In activity 2, the teacher told each student a number, different from 1: 6, 7 or 8. They should recite the natural numerical sequence, from that number on. The same activity was repeated, starting from a lower number, any time a student made a mistake or showed indecision. An observer filmed this activity.

Second session: In the second session, we formed new groups of 4 students, according to the knowledge analyzed from the previous session. This time we conducted the box game. The teacher showed an empty blue box, in which several matchboxes previously prepared in session 1, would fit. In the first round, she put 2 matchboxes inside and then 3 more and closed the box. She asked the class how many matchboxes were there inside the blue box. Each group had a paper and pen, to take notes during the game. The observers took notes on the procedures used by students of all groups and, besides that, all students were questioned by the teacher about the solution and how they obtained it. This way, the productions of the students were validated, through explanations on how and why they did what they did. Afterwards, the teacher would open the box so that the students could confirm or review their answers, whenever she thought it convenient. This session was filmed by the researcher. In the second round, we used loose sheets of paper with a drawing indicating the situation of the box game, with the amounts 2 and 7, for all students. While the teacher questioned some students, the other ones were developing the second round. Some students received proposals for new rounds: the ones who did not provide us with enough data about the overcounting in the two first rounds. The extra rounds were similar to the second one and the numbers used were: 10 and 5, 17 and 8, 19 and 9, 10 and 16, 20 and 32, 31 and 51, 120 and 130.

4. Results

Transcriptions representative of the productions by some students.

Activity 1, first session: Dialog 1

R–Get materials for your group.

S – (took 4 boxes, counting one by one.)

The previous example illustrates the production of 28 students, who counted correctly, one by one. Fernanda refused to do the activity. William split it into two groups, of two matchboxes each. Bruno and Pedro each got the 4 matchboxes directly.

Dialog 2

Activity 2, first session: *R – Patrícia, 7*

S – 8,9,10,11,12,13,14...

The previous example illustrates the production of 27 students, who answered at once and correctly, upon the teacher's talk. One student, Vitor Augusto made two attempts in the activity: he did not answer when the teacher said number 7 and he answered correctly when she said number 4. It took Wesley, Luana and Joyce some seconds to answer correctly. Fernanda made a mistake in the sequence from number, and a group of students corrected her. The activity was redone with number 4 and this happened again.

Second session: Dialog 3

R – How did you do it, Érica?

E – I put 2 in my finger then 3 more and counted

R – How did you count? Show me.

*S – Like this: 3,4,5. Then I got 5. (*touched the fingers to indicate the spoken counting).

The previous procedure illustrates that 15 students used their fingers and 5 used drawings of little balls or sticks to answer. Six students, Pedro, Bruno, Guilherme, Jean, Henrique and Letícia calculated correctly. The observers noticed that three of them said the correct result (immediately after the teacher's activity) before writing a sum. Three of them did just the opposite. When questioned, they said they did it "by heart". Other rounds were proposed to these students, increasing the numbers. We could observe that when questioned about the result of 120 + 10, they changed the procedure, because they did: 120, 121, ... up to 130, using the ten fingers to count, from 121 on. Wesley did not solve the problem correctly. After being questioned, he wrote 1+1 as the answer. Fernanda, Joyce, Luana, Denis and João did not solve the problem. These students seemed blocked during the questioning. During the validation phase, with the box open, except for Fernanda, all the students answered correctly.

5. Conclusions

We observed that 31 out of 32 counted 4 objects correctly, showing that they memorize the natural mathematical sequence up to number 4 and they applied this knowledge to count objects. We confirmed, then, the previous researches made about this.

We also observed that 31 out of 32 students said the numerical sequence out loud, from a certain number on, different from 1. Of these, 3 needed some seconds to recite it. We understand that these 3 students might have used these seconds for mental counting, from a number lower than the one told by the teacher. Therefore, we can affirm that 28 overcounted from the specific number given by the teacher. For two students out of these 28, a repetition was required, lowering the original number given.

Of the 28 students that overcounted, when reciting the natural sequence, 26 used the overcounting in the adding problems. All students who used overcounting in the adding problems had overcounted when reciting the natural numerical sequence, from the number spoken by the teacher.

Therefore, we concluded that kindergarten students: a) can overcount when reciting the natural numerical sequence; b) can overcount in the solution of adding problems. Besides that, the memorization of the natural numerical sequence, from a certain number on, different from 1, is necessary, but is not enough for the overcounting in the resolution of adding problems.

It is important to observe that 15 students used the physical domain (overcounting of fingers), for overcounting when solving adding problems which involved numbers 2 and 3, or 2 and 7. Five students used the representation domain (overcounting of drawings), to solve the same problems of numerical domain. Six students used the ordinal domain (overcounting when reciting a numerical sequence) to solve the problem formulated in the cardinal domain. They did that only when the numbers involved in the problem were 120 and 10. That indicated to us that the problems with numbers under 100 did not promote a researching procedure on the part of the students and, therefore, were not a learning source. Six students did not solve the problems involving numbers 2 and 3 or 2 and 7. The mediations by the teacher, using the physical domain (counting of matchboxes), during the validation phase, were enough for the correct solution of the problem, for 5 of these students.

This teaching/researching method provided us a good understanding of the evolving learning processes and the role of the interventions (social interactions) in the classroom, in the activities proposed. These interventions could be evaluated and, them the activities were adapted to the cognitive conditions of each student. This way, we could promote learning for 31 out of 32 students. Our methodological choice provided *flexibility*, which we consider essential for the study of teaching procedures which promote cognitive evolution. We could ensure a high level of control of the pertinent variables in this research, without overlooking the complexity of the classroom situation, because even adapting the activities to the conditions of each student, the classes took their course as close as possible to their regular routine.

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Diversity Acknowledged and Ignored: Achieving Equity in School Mathematics

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There are many pedagogical approaches that genuinely strive to foster equity in mathematics education. In this paper we first analyze why, although these approaches do acknowledge diversity, equity is not really accomplished. We then describe our research-based TAP (Together-And-Apart) approach that has been implemented in two projects in very different contexts, ISTAP in Israel and MALATI in South Africa. We describe how TAP achieves equity by both acknowledging diversity and ignoring diversity thus disarming school-mathematics of its traditional role as the gatekeeper of students' future. Finally we focus on a research site and one teacher's struggles and achievements in his attempt to accomplish TAP's goals.

> "Mathematics Education is a key discipline in the politics of education. Mathematics qualifications remain an accepted gatekeeper to employment…Mathematics education also tends to contribute to the regeneration of an inequitable society through undemocratic and exclusive pedagogical practices..." (MEAS1 Proceedings, page 3).

The above paragraph makes two main claims: (1) that mathematics is a "gatekeeper" for managing students' futures; (2) that mathematics instruction, being unable or not inclined to respond to the diversity in the learners, perpetuates inequity. One way to deal with this issue is to cease using mathematics as a 'discriminating' gatekeeper, in the same way that sex, race, religion, etc do not pose discriminating gatekeepers in students' progress. Similarly, as in most places mastery of history or art are not criteria for learners' acceptance or rejection to future enterprises, unless directly related to the specific enterprise involved. Nevertheless, most mathematics educators accept, and even justify, using mathematics as a filtering device. They suggest that inequity be dealt with via the employment of miscellaneous methods of instruction and class organization thus, ostensibly, securing maximum success for maximum students.

Studies have shown that the most widespread approach for dealing with inequity and students' diversity is ability-grouping, either by setting up 'same-ability' groups within the same class or by placing students with different abilities in separate classes. Research indicates that teachers view ability-grouping as the best way of improving the scholastic achievements of all students and as the only 'fair' way for dealing with students of different ability-levels (e.g. Oakes, 1985). *"The question of how early some form of instructional grouping of students should occur...My response would be...as soon as the teaching and learning of mathematics occurs..."* (Dialogues, November 1998).

Recent research, however, has clearly shown that the tracking systems contribute to the regeneration of an inequitable society. Studies of this sort have concluded that the placement of students in ability groups, in and of itself, increases the gap between students beyond what would be expected on the basis of the initial differences between them (e.g. Linchevski & Kutscher, 1998; Slavin, 1990).

Other attempts have tried to support equity by designing learning environments that permit and encourage different levels of mathematical knowledge and sophistication within the same community of learners. They suggest that the way to cope with within-class inequity is by developing learning environments that are sufficiently flexible to allow all students to show what they know and can do (MSEB, 1993).

In our view, however, the latter attempts and its practices deal with only one aspect of the equity principle. What they actually do is legitimize different levels of mathematics without taking into account the gatekeeper-effect of mathematics.

What actually happens in most of the above-described systems is that diverse levels of mathematics are legitimized in the early stages of students' mathematics education. However, at a certain point in time (which may be different in different systems) certain specific mathematical knowledge is required in order for the student to be accepted into a prestigious learning trajectory, for instance allowing the student to study in a mathematics class leading to an 'accepted' high-school diploma. This filtering process occurs more than once during the students' learning career. The students who have learned in the lower tracks in the tracking system or in alternative 'tracks' in the heterogeneous system find themselves unprepared for these critical moments. The system did not check, repeatedly and continuously, whether the educational system was equipping the students with tools to ready them for these crossroads.

We believe that accepting the current situation and not exploring and exhausting all educational means to keep the gates open to as many students for as long as possible, contradicts the espoused goal of equity. We believe that the goal of ensuring more success for all students is not only desirable but also achievable. This is based on our conviction that to a great extent the high failure rate in school mathematics is linked to the nature of the school system and to the ways mathematics is taught in too many schools and classes. If we really mean it, we must move beyond rhetoric and build on research about learning, teaching and curriculum.

Can such an educational approach be developed and implemented?

In this paper we describe our research-based TAP (Together-And-Apart) approach that was developed in Israel and has been implemented in two projects in very different contexts, ISTAP in Israel and MALATI in South Africa. We first describe the major assumptions and guidelines of TAP. We describe how this approach genuinely supports equity not only through appropriate learning environments, but also by providing learning interventions that prepare students for their mathematical crossroads. We then focus on one teacher's struggles and achievements in his attempt to accomplish TAP's goals.

TAP's major assumptions

The two major assumptions of TAP are:

- Tracking systems violate equity. We believe that equity in school mathematics can be achieved only when all learners are members of a fruitful, diverse mathematical community where there are many opportunities for rich mathematical experiences. We believe that equity in school mathematics can be achieved in a learning environment that features the positive aspects of higher-track mathematics classes. We build on theoretical approaches that describe learning as an individual process nourished by interpersonal interaction (e.g. Voigt, 1994). For these theorists the study group is not a mere administrative division, but a crucial component of the learning environment. We realize, however, that a rich learning environment in and of itself cannot guarantee each member genuine school-mathematics. We also know that such a community can be mathematically productive and endure to the satisfaction of all its members only if on the one hand its members have sufficient shared mathematical knowledge to make meaningful interaction possible, and on the other hand there is enough space for all members to express their mathematical diversity and to experience success.

- Certain essential mathematical knowledge (henceforth called ‘Indispensable Mathematical Knowledge’ or IMK) should be owned by all students notwithstanding the acceptance of diversity in other parts of their mathematical knowledge. Indispensable Mathematical Knowledge is that part of genuine school-mathematics that enables the heterogeneous mathematical community fruitful interaction to the satisfaction of all its members, culminating in open doors to higher education. If we want to give learners a fair chance to succeed in school-mathematics in the long term, and not only in the short term, we have to be able to discriminate between cases in which legitimizing a wide range of “different levels of mathematical knowledge and sophistication” (MSEB, 1993, p. 92) is the right approach and cases in which it is, eventually, at the expense of a fair chance to cope with future activities in mathematics and with society requirements. Equipping each student with IMK supports equity by enabling all students to be full partners in the heterogeneous mathematical community. In our view, it is the teachers’ duty to identify IMK as well as to identify students whose IMK is insufficient, and to take responsibility for providing these students with repeated opportunities for acquiring it.

It is clear that the mathematics curriculum and thus IMK might vary among different educational systems. It is also clear that the choice of curriculum is one of the mechanisms certain systems use as a filtering device. We believe that systems’ decisions, regarding their choice of curriculum-derived IMK, in and of itself, may promote or violate equity. This issue deserves a separate analysis and will not be dealt with in this paper. For the purpose of this paper we assume that the curriculum-derived IMK is given, feasible and justifiable.

<u>TAP’s main guidelines</u>

In our view the above-introduced requirements can be realized only if the learning environment is designed to concurrently ‘acknowledge diversity’ and to ‘ignore diversity’. By acknowledging diversity, we mean in TAP that we recognize diversity in students’ ‘entry’ points and allow and encourage all students to fulfill their mathematical needs, abilities and preferences. Thus, acknowledging diversity should lead to the construction of a learning environment that accommodates differences in the ways learners think about, construct and display mathematical knowledge and understanding. It should lead to the design of a teaching model that responds to students’ diversity.

However, the above-introduced requirements also imply that at certain carefully defined points in the learning process, TAP sometimes ‘ignores’ diversity: In these cases TAP “does not accept” diversity in students’ exit points. Ignoring diversity means that IMK should be owned by all students. Thus, ignoring diversity should lead to the design of a learning environment that guarantees students’ acquisition of IMK.

Acknowledging diversity while ignoring it, two ostensibly contradictory goals in our perception of equity, is achieved in our teaching model by alternating between two basic types of learning groups: heterogeneous groups and homogeneous groups. The various heterogeneous groups are generally engaged in the same activities (Together), while the homogeneous groups are generally engaged in different activities (Apart). (For more details see Linchevski & Kutscher, 1996 &1998.) The evaluation model is designed to accommodate, evaluate and reward equally the diverse thinking processes that different students display, as well as the diverse activities in which the different students are involved. The evaluation model is also designed to guarantee that IMK is followed up.

The research site: Stonehill High

As previously mentioned, TAP has been implemented in two different countries, ISTAP in Israel and MALATI in South Africa. A report and description of TAP's success in accomplishing equity in ISTAP as measured by students' mathematical achievements may be found in Linchevski & Kutscher (1998). In the current paper we report on the implementation of TAP by MALATI in Stonehill High School, South Africa.[1]

Stonehill High is one of seven schools participating in the MALATI Project in South Africa. This school is situated in a traditional black township and is, in many ways, typical of schools in disadvantaged areas in South Africa.

The class-size at Stonehill High ranges from 40 to 50 students per class where students frequently have to share desks and seats. A considerable portion of teaching time at Stonehill is lost due to administrative reasons. For example, students' registration and time-tabling is finalized only at the beginning of the school year. At the beginning of the 1998 school year ten school days were used for the latter purposes. Teaching-time at Stonehill is disrupted on a regular basis mainly due to administrative issues and school events. Stonehill has developed its own method for dealing with these regular time-consuming disruptions. Each class will still be conducted but the length of each period will be considerably shortened, from 45 or 50 to 20 minute periods, resulting in nearly impossible teaching situations. More learning time is lost during the weeks that are devoted solely to examinations, one to two weeks at the end of each of the four school quarters. Many students do not return to school for the week following the examinations, but begin their vacation early. During teacher strikes of protest about service conditions, wage increases and the retrenchment of teachers, the school day is also shortened. All time lost is not compensated for.

Classroom practice at Stonehill prior to working with MALATI was typical of that in South Africa and elsewhere: Lessons were teacher-centred with whole class teaching the norm and dominated by low level questions and the mastery of procedural skills (Taylor and Vinjevold, 1999). Teaching was authoritarian with very little room for analysis or critique. Students at Stonehill were not accustomed to working in groups – they did not listen to one another and struggled to communicate orally or in writing. There was also little culture of doing homework at the school.

Assessment at this school was typical of the wider practice in South Africa. It was exam-driven, with "control tests" occurring at the end of a section or school term, and was used for reporting purposes (Niewoudt, 1998; Taylor and Vinjevold, 1999). The curriculum is divided into sections so that at the end of each quarter examinations were administered for

[1] The name of the school has been changed.

each grade separately culminating in final, end-of-year examinations that assessed all the material learned throughout the school year.

Promotion of students from grade to grade is not automatic, but based on a final examination, together with a cumulative mark, obtained primarily from tests and examinations held during the school year. A considerable number of students in each grade at Stonehill are failed each year. These students, nicknamed “repeats”, are required to repeat the entire year of schooling and formally are pupils of the same grade learning with younger students although in effect they are quite isolated in the class. The pass rate for mathematics in the final year (grade 12) at this school is very low.

Prior to the MALATI intervention, after an examination teachers had typically addressed any problems arising from the assessment with the whole class, usually by re-solving some items during the period that follows the marking process. The teacher then moved on to the next section in the syllabus. No IMK identification or consolidation[2] took place. No diversification[3] between students was carried out. There was no attempt to follow up students’ difficulties nor to take responsibility for bridging essential gaps. No analysis about what was and was not crucial for understanding subsequent topics was done. The teachers seldom used other forms of assessment such as projects or oral assessment.

Compounding these difficulties is the political background from which the Stonehill teachers stem. Complicated problems that are the product of the recent emergence of equality due to political changes in South Africa must all effect the teachers’ grapple with MALATI’s concept of education and, especially, equity. As members of a society previously discriminated in South Africa, these teachers had been part of the struggle for equality and democracy. Despite this, in their role of mathematics teachers they unwittingly continued to practice in their schools all the elements of undemocratic pedagogical practices, analyzed and discussed in this paper. And if in the past few years there has been more awareness (fostered through constant exposure to the state’s new philosophy of a learner-centered, outcomes-based curriculum) that the school’s pedagogical practice regenerates inequity within the school (and thus, eventually, jeopardizes its students’ future) through its undemocratic methods, the teachers usually blamed outside forces for these problems, and expected external interventions to assist them in solving their problems.

The state’s attempt to redress past inequalities in the distribution of human and physical resources has resulted in uncertainty that has led to an exodus of teachers from the profession and low-morale amongst those that remain. Unlike many South African schools, Stonehill has a mathematics department that has changed little during recent staffing changes. The six mathematics teachers at Stonehill have taught at the school for at least 7 years. All these teachers have at least three years of professional training (at university), with two having studied up to Masters level. Prior to MALATI, mathematics departmental meetings mainly dealt with administrative issues.

[2] Process of guaranteeing IMK to students in need, after assessment indicated that their IMK was not yet acquired.

[3] In this article diversifying means organizing the class in homogeneous groups in order to cater for the differential needs of the students

The decision to form a partnership with MALATI was taken by the whole school mathematics staff. This decision was facilitated by the fact that TAP is in line with the state's new philosophy of teaching and learning. The mathematics teachers seemed to be open to innovation and change, and participated enthusiastically in the MALATI project, forming a cohesive unit from its introduction.

MALATI supported the Stonehill teachers by providing learning materials, by its counselors' frequent visits to their mathematics classes and by weekly workshops where the teachers discussed appropriate strategies for cooperative learning, assessment and class organization in their heterogeneous mathematics classes according to TAP's principles.

In the context of the above factors and difficulties we now present a case study of Mr L and his attempts to implement TAP over a two-and-a half- year period. We will report in more detail on his first two years until his major break-through.

<u>Mr L:</u>
Mr L was a competent teacher with full command of his class. He felt most comfortable in his role as "center-star" in his teacher-centered classroom. But, as we will soon see, this quality interfered with his success in adopting and implementing more learner-centered environments. At this stage of MALATI, Mr L's perception of teaching mathematics meant demonstrating the solution process of an exercise and thereafter practicing it for a predetermined period of time. At the time he believed that: *"The answer is more important than the process"; "when a pupil can use a mathematical procedure correctly he understands it"; "if students methods are inefficient they* (these methods) *should not be encouraged"; "mathematics tasks can be solved only in one way".* His main source for exercises was the textbook, usually inspired by the type of exercises these students would solve in their matriculation examinations - given they would reach this stage. This practice was dominant regardless of the students' grades, knowledge or success. The fact that many students failed, and that most students did not reach 'matric', did not trigger any process of reflection in Mr L other than devoting more preparation periods before end-of-term and end-of-year exams. Mr L did not believe in group-work. His guiding philosophy was: *"pupils cannot solve mathematical problems effectively unless they have been shown how to do them"*. He believed that assessments should take place at prearranged times that were decided on at the beginning of the year; it was not necessary to consider *"when pupils or teachers feel that the pupil is prepared".*
<u>1997: First half year</u>: MALATI started the interaction with Stonehill in July 1997[4]. In the first half year of interaction almost no change was observed in Mr L's practice. Most of the workshops and class visits were devoted to getting the teachers acquainted with the TAP rationale and MALATI materials. There were discussions of changes but in effect none were implemented.
<u>1998: 10th January, First year:</u> Mr L's attempts at TAP started by organizing the class for group work. The children were grouped randomly because *"I feel I don't know the them well enough yet.*" Despite the groups, the learning was whole-class and teacher -centered.
<u>Six weeks later</u>: Finally school seemed to be starting on time: *"I find the changing times very frustrating as I cannot plan my lessons".* Mr L was trying to initiate improvements in the school. He had submitted a number of proposals for ways of stabilizing the timetable

[4]The school year starts in January.

but to no avail. He was frustrated by teacher work ethics and lack of discipline both in the staff and in the students. He was attempting to inculcate his own students with better learning habits but the school culture worked against him. In the TAP spirit, after the first evaluation he reorganized his class into heterogeneous groups based on the test results: *"In each group we have a person that performed well, average, below average and I grouped them according to that"*. Although he went through the motions of fine-tuning the group composition as if readying pupils for 'real' group-work, he was unable to relinquish his role as 'center-star': The IMK consolidation was done in a whole-class setting.

9th March: Mr L reported that when marking the exam papers he noticed names of pupils he didn't recognize from class (even though they obviously attended his classes). And two of the latter students had outperformed his others pupils! He strode into this class, sought out these 'unknown' students, and then proceeded to lecture his class on their lack of motivation and hard work. The pupils had been introduced to the topic of "Probabilities with Dice" and since the MALATI tasks were inquiry-oriented and not procedure- and-drill, the students were not taking them seriously. Mr L commented to the pupils that some of them were complaining that the (mathematics) work they were doing was just a game. He assured them that this was still mathematics, with a new approach. Mr L's views of mathematics-learning were apparently starting to change.

Three months into the school year: Important TAP changes could be observed in Mr L's class. He was reviewing their control test. After giving them a pep talk on the importance of mathematics for future employment, he divided them into two groups[5] according to the information derived from this test's profile. Those students who needed IMK consolidation were divided into homogeneous pairs; the rest formed a small group at the back of the class. The students moved willingly and quickly. He found teaching in this learning environment *"a scary process".* While he was doing IMK consolidation, the other group continued with other activities and *"continued and worked very quickly, and then I was split again and I still have difficulty handling the different levels that the learners are at. And sometimes it's difficult also within that classroom situation to cope, but I did have them working on their own also….I find that some people can finish their activity quite quickly, and then they have a negative input, or they become playful..."*

His practice had undergone a major change: the tasks were not procedure-and-drill. However:*" They* (the children) *don't like it because it's 'easy' and 'different'. The kids are so used to struggling with maths that they don't know how to handle it".* As was apparent on the 9th of March, we observe here too that Mr L had to contend not only with his own difficulties in the process of his changing views of mathematics learning, but also his students' difficulties, all having come from a completely authoritarian culture of learning in general, and steeped in a mathematics culture of procedure-and-drill in particular.

Two weeks later: The children were still sitting in their homogeneous pairs designed previously for IMK consolidation, although they had started a new topic – geometry. He handed out an activity and gave them five minutes to tackle it. Most of the children struggled (not having stronger students to confer with in their pairs) with the activity. Just when they got going, they were told time was up and Mr L initiated a whole-class discussion. The nature of his class's discourse had undergone a metamorphosis. He

[5] This means that there are two different, concurrent learning-plans so that every small group is involved only in on of the learning-plans.

encouraged them to think, suggested and legitimized different answers, getting them to clarify what they meant, and specifically encouraged the use of mathematics terminology when appropriate. This was the classroom culture he fostered until the end of the term examinations – one month's duration. But, he still seemed to need to be "center-star", not allowing the students to grapple with their problem without his leadership.

30th August: Slowly he became aware of his shortcomings and tried to be more conscious of the time devoted to group-work. He explicitly encouraged the students to work more independently in their groups. He was beginning to trust the students' abilities of learning: "*Many times I will leave them, but I will leave them with that doubt that I am not happy, so they will see where the problem is, if any, but I don't like guiding them in that direction."*

10th October: Mr L appeared quite comfortable with group-work. After assessment he again diversified the class for IMK consolidation. He prepared extra activities for those who performed well while he himself interacted with the others. Pupils got down to work quickly and continued so most of the lesson. When asked how he felt, he said: *"I think it helped the 'front'* (consolidation) *pupils. But I think I will have to assess them again to be more sure."* He was becoming more convinced of the benefits of group-work and of diversifying: *"In the smaller groups I find that I can be more attentive to them whereas the others who I feel don't need that much attention can go ahead."* He was gaining confidence in the ability of children to work on their own.

3rd November: Mr L was very frustrated by his lack of success in having the students do their homework and he constantly expressed his disappointment. He related most of this failure both to the lack of school culture in this area and to the pupils' own laziness: *"They were going to 'drop'* (stay down a class)*, and the reason being that they are lazy...".* He maintained group-work and some diversification but started to express dissatisfaction: *"I might be neglecting the stronger pupils and I need to work on this".* His beliefs and attitudes seemed to have undergone major changes. He was aware that he was battling on three fronts: 1) the children's views of what mathematics is; 2) school and department regulations; 3) his old practices and beliefs.

1999: First week, January, third year: The school was still not organized for scheduled learning due to administrative reasons. Mr L arrived to attend his second lesson but an unplanned administrative session that morning took more than an hour so all classes were shortened. He said that he had proposed to the principal that periods should be no less than 50 minutes but the principal had some objections. Mr L felt he could handle the class in the TAP spirit and would need assistance only after the first assessment.

11th February: Once again learning time was wasted on administrative purposes thus the lessons were very short. The students were sitting in rows while Mr L was conducting a traditional teacher-centered class. As the lesson progressed he gradually encouraged the "quickers" to pair off or form groups which they did quite readily. He was concerned that *"the learners who work very quickly will get frustrated when working with slower learners".* The counselor realized that Mr L was implementing mainly homogeneous groups. He suggested that the learners work in heterogeneous groups, at least for core activities, so that all could benefit from the interaction. Mr L seemed determined to work on it (and on himself).

15th February: During the workshop Mr L spoke up against end-of-term exams since *"learners only work for these examinations".* He suggested a system whereby "*tests be spread throughout the year, be non-standardized and learners be given the opportunity to* *be reassessed."* TAP was slowly taking effect: he had started questioning the system. It looked like the mathematics department would adopt the "mini-exams" system but they were unwilling to give up the weeks set aside solely for examination preparation. Mr L seemed very frustrated by this.

9th March: Following the exams Mr L maintained homogeneous groups even after IMK consolidation for a topic was completed. He indicated that the 'repeats' had benefited from IMK consolidation. The counselor urged him to implement heterogeneous groups especially for the core material.

One month later: Mr L indicated that the 'repeats' had given up on mathematics. So he decided to try to integrate some of them with the rest of the class[6]. He said that he had been inspired by the movie "Patch Adams".

16th April: A general staff meeting was held for 50 minutes in the middle of the school day. Once again the timetable was disrupted. Later that day during a scheduled workshop the MALATI counselor emphasized again the importance of working in heterogeneous groups. Mr L shared with his colleagues how he had expanded his mixed-ability groups also to incorporate the 'repeats' as a way to motivate them.

22nd April: It seems that Mr L had made himself the commitment of adopting heterogeneous groups as his dominant class-practice. The class was organized in mixed-ability groups and Mr L moved from group to group struggling to get the learners to compare their answers. During class discussion the different groups had to report back. He tried to show his class how one could infer from the group-members' responses whether the group was cooperating well. He had adopted the practice of sitting with a group for an extended period, regardless of whether the learners were seated in heterogeneous or homogeneous groups. This personal interaction with the students may explain why this year he knew not only the strategies used by the different groups but also by the different students, whereas at approximately the same time the previous year he barely knew his students' names.

From this point on Mr L's practice was focussed on fostering cooperative group-work in mixed-ability groups within a mathematical culture of inquiry and discourse. It was clear that Mr L was able to and indeed did successfully implement most of TAP's principles. He no longer saw himself as 'center-star': *"They can learn from one another, that is – is what I have learnt…they can also learn from me."* There is no better way to describe the change than using Mr L's own words: L: *"Perhaps I am scared because it worked…";* (Mr L and the counselor both laugh) C: *"Why are you scared?"* L: *"Because they don't need me".* However, it was also evident that he was still oscillating between his old beliefs and his new experiences: He was not yet a full partner of TAP. He consistently needed proofs that TAP strategies were truly beneficial. At times he provided different groups with different activities but once the students were involved in these activities he began to have second thoughts, focussing on the negative aspects on what each group missed by not doing the others' activities instead of realizing what they had gained having had their specific needs addressed. At other times he would see a positive aspect and then seem to draw back as if to reinforce his original beliefs. He would declare that weak learners could benefit from working in mixed-ability groups: *"Uhm, what I'm finding is that in many groups certain people adopt those people that are not performing well…"* and in the same

[6] The traditional practice at Stonehill was to sit the repeats at the back of the class. They were usually physically much bigger than the 'regular' students in their class.

breath he could say that he was not sure that the weak students benefited from learning in mixed-ability classes. It was clear to him that mixed-ability group-work benefited the "strong" learners: *"I find that when they* (the "strong") *communicate in the* (mixed-ability) *group they also learn some other skill – of speaking mathematics, which is of great help for them"* - echoing Vygotsky (1986). And again a need to retract: *"I need proof that the strong learners would benefit from working in mixed-ability groups".* If previously Mr L was concerned that he *"might be neglecting the strong pupils"* when they learned independently in the homogeneous groups, he now believed that *"within the group there is over enough intelligence to actually run through the activities."* But he still had a problem of *"a difficulty of the letting of one group go ahead."* This last difficulty was not only one of class-management and logistics that he was still experiencing. These expressions of contradictory beliefs were characteristic and representative of the way he expressed and exposed his inner conflicts with TAP's principles and practice.

When summing up, we can see that even under the objective difficulties – school culture, facilities, students' learning culture and the like – Mr L's practice underwent a remarkable change in terms of TAP. But from the many discussions and interviews, it was apparent that his beliefs did not undergo the same change. Why would a teacher with so much evidence, even hard data (*"looking at the results of last year versus the results that they obtained thus far… out of a class of 48 only five people have not improved on their mark of last year"*) and with a successful record of implementation, still cling to his old beliefs? Along with all the commonly recognized factors that affect beliefs, such as the change-agent's role, beliefs lagging behind practice, personality etc, one cannot ignore the social-context factor in which Mr L's change occurred. Most of the aspects in Mr L's old practice, such as teacher-centered lessons, end-of-term 'control' tests, 'failing' students becoming 'repeats' etc, had been shared also by previously privileged S.A. – and it seemed to work for them! Thus from this standpoint it might be reasonable to believe that with inequalities redressed and improved resources, most of the problems that the school experienced would disappear. Taking this perspective, it might be very difficult to be convinced that it was the old practice that posed the problem. Only honest reflection on the old practice will allow change where beliefs and practice go hand in hand.

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THE EPISTEMOLOGY OF JPFs:
WHEN IS RESEARCH IN MATHEMATICS EDUCATION VALID, FOR WHOM, AND UNDER WHAT CIRCUMSTANCES?

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ABSTRACT

The term *research* has been appropriated by people with a vested interest in their own formalised methods and expertise. Yet formal research has had very little impact on the practice of individuals, certainly in relation to the amount of money spent. What is needed is an approach to *re-search* which enables ordinary practitioners to make sensible decisions about their own practice, informed where possible by indications of possibilities arising from the work of colleagues.

An approach which meets these needs can be constructed based on the natural acts of *just plain folks*, by addressing the questions of the title. The approach draws upon traditions in which assertions are considered to be made for particular people at a particular time in a particular place, rather than having some objective validity, and in which assertions are merely signals or stimuli to make distinctions previously not made. These can be systematised into a research practice for *just plain practitioners.* Furthermore, validity is a matter of testing things out in your own past, present, and future experience, not one of accepting what someone else says because they claim to have 'proved' it in some other context.

INTRODUCTION

Colleagues all over the world find themselves in research situations which are in conflict with their immediate concerns: social-instability makes sampling impossible; subjects given a pre-test or interview are dispersed or unavailable for follow-up study; social and personal factors such as insufficient food to concentrate strongly influence any attempts to validate or develop general theories about teaching or learning; and social responsibilities beyond the school take up available time and energy. Such factors render the controlled conditions of rationalist cause-and-effect-based enquiry as at best inappropriate.

In this paper I suggest that there is an alternative approach to research, located through closely interrogating experience in the following way.

> By attending to and describing how most of us learn most of the time, we can then refine that description to produce a systematic method founded on an epistemology which reflects the everyday approach of 'just plain folks' like ourselves. This is the epistemology which actually lies at the heart of research paradigms as well as of everyday practices.

> Those practices bring a form of research within the grasp of unusual circumstances by allowing the researcher to investigate their own lived experience.
>
> The real product of research is the enhanced and refined sensitivity of the researcher to notice aspects and subtleties previously overlooked.

The term *just plain folks*, or JPFs for short, was introduced by Lave (1988) to refer to ordinary 'everyday' people, and their normal 'everyday' practices, untainted by academic analysis. My intention is to describe and then extend the epistemology and methods of JPFs, that is, of all of us most of the time and most of us all of the time. This even includes researchers who pride themselves in some specific epistemological position and in strict methodical practices, and yet only use these when performing in a professional context.

Despite elaborate articulations of method and epistemology, I suggest that there is a great deal in common in how people approach making sense of the world, finding out what seems to be the case, and validating what they find. Whether they say they believe there is a truth to be found, a 'something that is the case', or whether they espouse a highly relativist position, they all actually act in the same way. This 'way', when made precise and when employed systematically, constitutes an approach to *re-search* which empowers ordinary practitioner JPF's.

I begin by elaborating on the issue of validity. Because of a shortage of space I cannot do more than offer a very attenuated summary of the reasoning which constitutes and justifies the epistemology of JPFs. Elaboration and justification can be found in an expanded version of this paper, available from the author.

VALIDITY: FOR WHOM, AND UNDER WHAT CIRCUMSTANCES?

TRADITIONAL RATIONALIST APPROACHES

Western rationalist philosophy can be read as a struggle to find a method which guarantees finding the truth about situations by locating 'what is the case'. The problem is to eliminate the possibility of being swayed by false or incomplete reasoning, by charismatic appeal to emotion, by social conditioning etc.. Of course, no such method has ever been discovered. Indeed, current philosophical positions would argue that no such method is possible, by observing that no method of ascertaining knowledge can be value or culture free, and so all methods are circumscribed and constituted by the language in which they are expressed, and the values which they manifest.

Aristotle, Euler, Leibniz, Descartes, and Boole were typical explorers in the long search for universal validity. They were all insightful mathematicians who expressed a desire to formalise philosophical argument in mathematical terms so that there could be no dispute about validity. But ordinary philosophy resists the rigour of mathematical argument, for axiomatisation is rarely possible. Axiomatisation is certainly inappropriate in mathematics education, where the 'objects' under discussion are often ephemeral and invisible, and have to do with human beings, not relationships between mentally imagined concepts.

Post-modernism tries to convince us that there is no global story, no objectivity, no truth to be found. Yet for many people, particularly those who have engaged in mathematical research, this flies in the face of experience. Some mathematical situations are not in rapid flux, and so some assertions are more appropriate, more valid than others; some 'truths' have at least a sense of permanence. In mathematics education too, there are invariances amidst change, even if they are, in the long term, relative. For example, the children in a classroom I observe do not score very highly on the tests that they sit; research students tend to collect a lot of data, and only then seriously address the question of what they are going to do with it. Neither are universal, but both have a validity at a certain time and in a certain place, for me.

Post-modernism is in part a reaction to Popper's Objectivism (Popper 1972), but there are ways to counter Popper other than by denying a sense of truth altogether. Popper wanted to escape the flurry of un-testable, un-attackable theories which were popular in post-war Europe (as they are again, for example creationism, linguistic hegemonism, and constructivism in its various forms), and so he developed the notion that theories have to be falsifiable, in the sense that one has to be able to imagine how the theory could be proved wrong in order for it to be considered part of a scientific contribution to knowledge.

If educational research could be formalised mathematically so that assertions were either true or false in Popper's sense, then we would live in a totally mechanised, robotic world with no place for the creative, complex, organic responses of human beings to hazard. We would long ago have found the 'best way to teach mathematics' so earnestly sought by politicians. For any reasonable assertion in mathematics education, it is possible to construct an alternative interpretation in which it is false, or to imagine a circumstance in which its negation is also true. For example, "practice makes perfect" is only valid when the practice draws attention away from that which is practiced.

Traditional Humanist Approach

Eastern philosophical and psychological enquiry has much in common with humanistic phenomenological enquiries into 'lived experience,' into 'what it is like to be or to experience' something (van Maanen 1990). Instead of demanding that a theory must be falsifiable, there is a long tradition of localism: what is said is taken to be at best valid at that time in that place for those people. It is the responsibility of individuals to test out assertions in their own experience (that it makes sense of or fits with the past and present, and that it informs the future). Furthermore, assertions made are only contributions to the development of sensitivities to notice, and to the development of awareness of aspects and subtleties not previously noticed. For example, work on gender issues which heightens teacher sensitivities to their own behaviour is much more valuable than assertions about the presence of gender bias in classrooms in general. Neither are fruitful unless alternative behaviours are available in the moment when needed.

Validity for JPFs

Validity for most people in most situations, that is, for JPFs, is similarly temporary and situated, even though it may be experienced in the moment as universal. For example, when I find myself noticing a particular behaviour during some observations in a classroom, my awareness is dominated by that behaviour; I experience it as universal, as 'being the case', and I tend to see it everywhere. But later I may 'get it in perspective'. I may recognise that what I am observing is revealing something about my sensitivity, as much as it is about those observed.

Observation is usually accompanied by ontological commitment, signalled by that sense of universality. Goethe (1810) suggested that "In every attentive look on nature we already theorise", while Stoppard (1988) rephrased it as "The act of observing determines reality . . .; You get what you interrogate for". For example, when I am caught up in looking for some particular behaviour in a group of subjects I am researching, I am inclined to see 'it', and to assert that 'it' is present (e.g. beliefs, attitudes, abilities, etc.). But the 'it' may be my own ontological construction in order to account for what I observe. Furthermore, when I show data to others, they may see other things, and have alternative interpretations. The 'truth' about data is that it serves as a mirror to reveal the observer's sensitivities to make certain distinctions, or as Storm (1985) put it in the context of North American Indians, "The world is a mirror for the people".

As researchers we want to see 'what is there', but this implies a commitment to something being there to be seen. An alternative, which corresponds more closely to experience of data collection and analysis, is that events consist of the multiple stories that are woven about observations, including by those who only have access to later versions of the stories. For example, Piaget and Freudenthal both made observations about their children or grandchildren, but their accounts are immediately recognisable even by those of us who never met their 'subjects'. They resonate with our own experience. The stories they tell are augmented by our own stories, creating a rich fabric of meaning which is taken as shared in a growing community.

The fact that something ‘feels’ true does not make it true. That is why it is vital that we check assertions not just against our own experience, but against the experience of others.

The purpose of checking out an assertion is not, as is commonly assumed, to validate it, but rather to develop the sensitivity in ourselves to notice whatever it is the assertion draws to attention, so that we can see if it informs past, resent, or future. When we check out with others, we are checking whether they can develop a similar sensitivity, or at least whether they can report on what they notice using the same language! Asking a direct question about what people notice or are aware of is not usually very effective, since they are most often embedded in and caught up *in* that awareness, but not aware *of* it. A more effective way is to construct and refine task-exercises through which others reveal what they notice, and through which they can be sensitised.

The age-old concern is that we might convince colleagues through charismatic intensity. But there is no antidote, no guarantee of neutrality and validity. Each of us is responsible for checking that what is offered makes sense, or challenges constructively, the stories we have already woven. Each of us is responsible for seeing whether it fits with present experience, and whether it informs future actions.

Of course each of these three domains are circumscribed by practices of the communities in which we are embedded, and the pre-judices we have developed from the stories we have heard and woven in and about the past. Our desire to be accepted within a community, which sometimes includes desire to disagree, may result in stressing some features through activating certain awarenesses, while ignoring others. The only recourse we have is to remain in question, to keep as much conjectural as possible, continually checking against our own experience, and seeking new people with whom to test out conjectures.

If something fails to check out, fails to make sense of the past or present, fails to inform the future, then it may be that there is something inappropriate in the assertion-awareness, but it may also be that it is not appropriate at that time in that place for that person. When there is a strong reaction against something proposed as worthy of noticing, it is more likely to be a signal of an imminent significant shift of attention, than when there is a warm acceptance. Disturbance is more creative than flow, as long as the disturbance is not excessive.

For example, most teachers are caught up in the exigencies of the immediate teaching and administration, and so find the thought of working-on-their-practice an unnecessary additional burden. It is only when they experience a dissatisfaction, a disturbance in the flow of lessons or in the responses of students, that they begin to question themselves and the system. Only then are they likely to make sense of suggestions which they have heard in the past about alternative practices or perspectives. Teachers caught up in a fluid socio-political situation may not be in a position to attend to subtleties in approaches to task design. They may appear not to heed what they are offered, just as children may appear not to heed grammatical corrections or other comments made by an adult, yet later they suddenly incorporate what they were told. When your 'message' is not heeded in the present, it may be useful to trust in the complex organic nature of human beings that they will later be in a position to respond to and develop sensitivities which are currently inappropriate.

I suggest that sensitivity-based localism is how JPFs actually operate. By elucidating and refining this epistemology, we can arrive at an approach to knowledge-development which can be as systematic and methodical as one likes, and which may be applicable in situations where traditional research methods are inappropriate. The result is an approach to *re-search* which fits with much of modern theorising, which admits a relative objectivity in time, place, and person, and which supports the education of awareness, that is, the strengthening, broadening, and refining of sensitivity to notice. Details of such an approach can be found in Mason (1994, 1996, 1997).

In summary then, research in mathematics education results in heightened and precised sensitivities to notice, in the researcher. The most valuable products of that research are devices such as task-exercises and assertions, which stimulate others to notice similarly. Disturbance is a central mechanism. Validity is the responsibility of the individual, within the practices of the community, to test out in their past, present, and future experience, and in the experience of others. Validity is person, place, and time dependent, and concerns the sensitivity to notice in emergent situations, not facts about situations before they happen.

THE EPISTEMOLOGY OF JPFS

Due to lack of space, this section consists of a very brief summary of some of the reasoning which justifies my claim to be describing the epistemology of JPFs. I have chosen those aspects which seem most relevant to conditions in which traditional research methods appear to be inappropriate.

Disturbance triggers attention, awakening-to-detail, and hence awareness;

> We spend most of our time caught up and immersed in activity. Expertise requires the growth of an inner monitor separate from the action. E.g. when you suddenly become aware while interviewing; 'My questions are shaping this person's responses", your monitor may be waking up.

Noticing (awakening, awareness) is based on making distinctions.

Noticing literally means ‘to make a distinction’. For example, you suddenly find yourself distinguishing different postures or gestures that children or teachers are using, or different ways they respond to each other.

Resonance is an essential and on-going need for human beings

As human beings we seek the affirmation of colleagues as part of our on-going re-need for acknowledgement and hence reassurance of our existence and worth (our reality). We join social groupings (such as conferences) because we expect to get strokes from folks we respect and with whom we share ideas and ideals. We seek resonance (and in extreme cases, dissonance) from those we respect, but we tend not to respect those from whom we obtain resonance too easily! There is a delicate balance between criticism and acceptance.

To make a distinction requires resonance with past experience

To make a distinction requires some features to be stressed or brought to the fore-ground while others are ignored or back-grounded. The identification of ‘thing’ which is stressed, is based on prior experience, since something that is unrelated to anything experienced in the past cannot actually be seen.

Noticing can lead to action, (including apparent inaction), and activity.

Non-habitual acts require that a possible act should come to mind in the moment just before a habitual act is triggered. I need to notice that I am about to ask a typical question, to repeat back what a pupil has said, to raise my voice, etc., just before I actually do it. I also need an alternative to employ. It is not sufficient that alternatives have simply been ‘seen in the past’. They have to come to mind, as part of awakening to the situation in the moment. Only then can noticing lead to action (including a choice not to act). The point is to experience a moment of choice, a moment of liberation, not be subject to habitual reactions and automatic behaviours built up over many years.

Humans have natural powers of thought and action which include ways of testing assertions

They do this by testing against both specific and generalised past experience, expectation and ‘history’, against current experience, and against whether future actions are informed. Only when these are satisfied can we say that we have ‘learned something’.

TOWARDS METHOD THROUGH REFINEMENT OF THE EPISTEMOLOGY

The assertions so far refer to JPFs and their epistemology. But when these are refined into actions and made more systematic and methodical, a research-method emerges. One such has been elaborated elsewhere as the Discipline of Noticing (Mason 1994, 1997, 1998) so it is inappropriate to repeat it here.

All research involves

* Recognising and formulating a question; formulation takes place within a discourse and a practice, and so privileges associated methods;

Questions concerned with personal change or development find resonance within the discipline of noticing, since trying to change others is doomed to trivialisation, superficiality, and failure in the long run. But all research has an element of sensitisation to notice.

* Articulating a method whereby investigation can be systematic and replicable (in the sense that others can investigate similarly);

The Discipline of Noticing provides such a systematic method. It depends on clarifying the phenomenon (through brief-but-vivid accounts of incidents that others can recognise), collecting alternative actions to employ, and working to employ those actions in the future. Distinguishing the phenomenon (what is noticed) from explanations and valuations, is vital.

* Bringing data into existence by 'collecting' it and nominating it as such;

Data in noticing consists of brief-but-vivid accounts and distinctions that appear fruitful; it also includes responses from colleagues who are invited to engage in task-exercises etc..

* Analysing data;

Data can be analysed through seeking patterns or threads, which form the basis for more general claims and

* Crystallising, abstracting and generalising through seeing data as generic;

Noticing is most effectively disseminated in the form of task-exercises accompanied by attention drawn to distinctions, frameworks, or challenging assertions.

* Validating assertions through some form of logic (rational argument, statistical reasoning, or experiential evocation);

Distinctions are validated by the user as described in the earlier sections. Other paradigms present other forms of products and use other forms of validation.

Sensitivities can be intentionally developed, and are influenced by social and psychological forces, but sensitising yourself to notice is a great deal harder than appears at first sight. Intentional 'work' is hard to maintain without a supportive community because the mechanicality of habit is very strong. Sensitivities do not reside purely in the individual. They are more appropriately seen as co-emergent mutualities between individual, socio-cultural context, and situation.

FINAL REMARKS

As JPFs we try to make sense of what strikes us. We make sense of much of what impacts us by ignoring, since it does not conflict with or disturb our equilibrium. What we experience internally *is* our version of events. The fragments of experience are woven into stories which account for, that is, make sense of, and hence become, that experience.

As JPFs we make sense by employing natural powers such as

> Responding to generalities by particularising (seeking examples) in our own experience, and generalising the particulars of our own experience to see if it makes sense of, fits with, or challenges in some way, that experience;
>
> Recalling the past (as stories woven from fragments of experience) and imagining the future (acting in non-habitual ways);
>
> Expressing the past and future by weaving stories, some of which appear as propensities to act (to respond or react), and some of which appear as analysis or theory. These can be verbal, visual, aural, and physical or with symbolic as well as iconic and enactive layers of personal meaning.

The social and the psychological are deeply interwoven in complex ways, for resonance with others is a major driving force for the psyche (in relationships, in self-image, in professional and social activity). Resonance comes about through concensual coordinations of actions (carrying out practices in an outwardly consistent manner) and, as Maturana (1988) has pointed out, through concensual coordinations of those coordinations, in and by means of language. The form in which resonance is recognised is itself an interpretation of physical manifestations (postures, gestures, words, etc.) mediated by experience of community practices.

Why might it be worthwhile to attend to the epistemology of JPFs? In the first instance, it is possible to fool ourselves into believing that what we think we know is more firmly founded than in fact is the case, because an espoused method may be more of a smokescreen than a magnifying lens. Secondly, attempting to describe how most people enquire enables us to refine what we do, to make it more precise, and to employ it more systematically. Thirdly, it provides the basis for a research paradigm which lies at the heart of most other explicit paradigms. The result is an approach for practitioners seeking to develop their practices so as to empower their students to employ their powers fully and effectively.

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Dilemmas in assessment — Dilemmas in mathematics education

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Assessment is a difficult subject for me. As part of my professional life I have to assess students and teachers, and, for several years, I have taught a small course on assessment to prospective teachers of mathematics. Still, it remains a difficult subject for me. As a teacher, I feel it is difficult both because of the conflicts it occasionally raises, and because I know that every aspect of my evaluation procedures (my informal comments, my grades, my global evaluation) are ultimately (radically) subjective. They are the end product both of "objective" pieces of evidence as written essays, and feelings that come out either of observation of students' classes or of minor details of their manners, personality, interaction with their peers, etc. I am aware of the necessity of this subjectivity and, paradoxically, I try to overcome it as much as I can and came up with a final evaluation that I believe to be fare. But the feeling of arbitrariness about the whole process remains, and it is composed with the knowledge of the consequences of assessment in students' future lives. In the end it is a mystery for me that only a small number of students disagree with my evaluative comments and my grades. As a teacher, I would be happy if we could devalue this portion of our professional life.

In her paper Discourses of assessment — Discourses of mathematics, Candia Morgan (2000) takes this feeling further, turning it into a research issue and questions the very fundamentals of our assessing practices. The paper questions assessment in two areas. The first is the notion that we must pay attention to the reasons for assessment, the *why* to assess, in Candia Morgan's terms. The second area is the reflection upon the consequences of assessing students, especially when this assessment is accomplished under the "naïve" assumption of trying to address students' needs.

Commenting on her paper, I will take the opportunity to raise two issues I believe important for mathematics educators. The first deals with the issue of the use of power in our educational practice and the second with our answers to the very existence of assessment. Essentially, I will try to raise questions, more than provide answers, as I believe our community does not have clear solutions, and, moreover, we are still struggling to posing the appropriate questions themselves.

The use of educational power

Issues debating the use of power are overwhelmingly present in Candia Morgan's paper. Teachers are "coerced" into specific practices, the use of assessment to implement curriculum reforms is "ultimately disempowering to teachers", assessment has a "regulative" role, to mention just a few instances. Adequately, it is stressed that assessment involves the exertion of power of some participants in the educational enterprise over others:

- teachers over students, in the psychological discourse,
- reformers over teachers, in the curriculum reform discourse,
- educational officials, politicians, and the overall society over teachers and schools, in the standards discourse.

Teachers of prospective teachers, as most of us are, still have another kind of power over their students, as they must ensure the correct acquisition of evaluative skills by their students. Sometimes they extend this incumbency to the teachers themselves.

These kinds of power can be put to very damaging effects in so far as the mathematical education of many students, the conditioning of innovative teachers' practices in schools, or the reproduction of social and cultural inequalities are concerned, as Candia Morgan's discussion so clearly shows. But there are arguments supporting the idea that power can be put to good use. Alan Bishop (1988), for example, proposes that learning mathematics can be thought as an intentional process of mathematical enculturation. Others (Melin-Olsen, 1987; Skovsmose, 1994), more or less implicitly, argued for the viability of similar educational endeavours. Many participants in this conference have vivid (and successful) experiences showing the ways in which they used their power to build "empowering" educational environments.

The issue, however, still demands a reflection from us: *is there a way to legitimately use our educational power*[1] *to contribute to the empowerment of students*? Note that I am stressing the *legitimacy* and not the *effectiveness* of power. The former is linked to the realm of values and has consequences for a political perspective of mathematics education, whereas the later focus on the outcomes answered by a technocratic perspective. Alan Bishop (1988) proposes some principles under which

[1] Bishop's formal, sapiential and achieved power will not be discussed here, and I will use the umbrella term educational power.

this educational power can be used (legitimacy of the use of power, a constructive and collaborative engagement, a facilitative influence) which I believe are very good starting points for a discussion on this issue.

The viability of assessment

In her paper, Candia Morgan strongly questions the use of assessment to address the perceived needs of students. She convincingly argues that 1) it is not possible to know objectively the contents of students' minds; 2) even if it was possible, assessing to determine students needs and teaching accordingly works against the same students that concerns the liberal/progressive perspective. Candia Morgan finishes her paper pointing to a split between our role as researchers, understanding the impossibility of the assessment task, and as teachers, looking for ways to improve our assessing procedures. I will try, here, to shift Candia Morgan's arguments and take them as a challenge to the community of mathematics educators, specially those concerned with the role of diverse social and cultural backgrounds on students' learning of mathematics. I will argue that this community must take an active role in the search for (at least) partial ways out of the split researchers/teachers.

Assessment will not discover truth

The conviction that, even if there is a reality "out there," we will not be able to know it objectively has dominated mathematics educators' discourse for some time (Kilpatrick, 1987). More recently the perspective that the mind is a social entity (Gergen, 1985), and that teaching and learning mathematics is inseparable from their contexts, composed with the belief that mathematics educators have to consider the cultural, the social and the political dimensions of their actions (Bishop, 1988; Skovsmose, 1994), has influenced the work of many mathematics educators. Both perspectives have shattered the common sense belief in the existence of a knowable unique objective reality with which we could compare our conceptions and decide about their truthfulness (Matos, 1991). But, at the same time, both perspectives did not paralyze teaching or research, and fruitfully have forced teachers to reconsider their teaching practices and investigators to shift research paradigms. What were initially statements about impossibilities ("we cannot know") became the source of new ideas.

Similarly, I propose that we must look for ways to live with these impossibilities in our assessment practices, especially in formative evaluation[2], searching for “viable” models of students’ minds and devising consensual teaching (and assessment) practices. Maybe we will find that we will not be able to overcome

> “passing total judgements, armed with the unconscious criteria of social perception on total persons, whose moral and intellectual qualities are grasped trough the infinitesimals of style and manners, accent or elocution, posture or mimicry, even clothing and cosmetics” (Bourdieu & Passeron, 1990, as quoted by Morgan, 2000).

Maybe we will only devise tinker-toy procedures for use in classrooms (and maybe these will be valid only in *our* classrooms) that will compound with our pedagogical knowledge. But formative evaluation is a key instrument in adjusting teaching practices and we must look for ways to use it adequately. Exposing limitations in assessment, questioning its underlying values, and stressing that it will not discover true ideas in individual students’ minds, as Candia Morgan did in her paper, is a very important first step on the way to improve assessment procedures.

Formative evaluation is necessarily biased

In the conclusions of her paper, Candia Morgan proposes a dichotomy between our role as teachers (seeking to perfect assessment procedures) and as researchers (knowing the futility of that effort and aware of their potential perverse consequences). Although I side with her analyses, I question her conclusions.

Mathematics has profound effects in today’s society. It permeates (formats) many aspects of our societies (Skovsmose, 1994) and Western mathematics, in particular, has a pervasive (and secret) ideological influence in changing the cultures of the entire world (Bishop, 1990). I believe this situation endows us with special responsibilities either as teachers or as researchers. In either role, we must seek ways to answer: “do students know mathematics?” This question comes from different directions. It is posed internally by our reflection about our teaching practices and by our research on teaching and learning mathematics, and it

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[2] The term *formative evaluation* is commonly used in Portugal and corresponds to the assessment to address pereceived needs in Candia Morgan’s paper.

is posed externally by other teachers, educators, social forces, politicians, etc. Among those that question us are our own students who want to know whether they are successful in their efforts, and, among these, are the "low achievers", precisely the group of students Candia Morgan shows are at the weakest position as far as assessment is concerned. If they are to overcome their situation, they must know the quality of their mathematical knowledge, and they are turning to their teachers, who then turn to the researchers, looking for answers. I do not claim that we are the only ones that are supposed to be looking for answers to this question. Other educational researchers and many social forces must also contribute. But we are certainly among the ones that should be searching earnestly. Postulating the impossibility of an answer renders the teachers powerless and prone to using unacceptable forms of assessment. At the same time, it limits the scope of researchers' action to a distant reflection over educational practices. In summary, I believe Candia Morgan's analysis of the consequences of using assessment to address students needs shows that this form of assessment has many problematic consequences. It is our responsibility as teachers and researchers to look for alternatives.

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Some reflections on democracy with regard to curricula and vice versa

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Abstract

In this paper it is argued that Mathematics Education can and should contribute to de development of a democratic competence, that is, the competence to effectively cultivate democracy in a democratic State. It is stressed the need of such competence and arguments are offered to support it.

Finally it is presented a way of working on mathematics topics -- using some examples taken from an educational project -- as a form of unveiling aspects of reality and as a means to arouse and cultivate a critical consciousness.

"We do not say that a man who is not interested in politics is a man who is attending to his life; we say that he has no place here at all"

Pericles

"In being born a citizen of a free State and a member of the sovereign power, however weak the influence of my voice may be in public activities it is enough for me to have the right to vote in them to impose upon myself the duty to inform myself about them".

Rousseau, in *The Social Contract.*

1. We were at war with Yugoslavia for 77 days. Without our consent, without any consultation on the part of our representatives (if indeed our members of parliament do represent us, but this is a question I shall address later).

We live, so they say, in a democracy. Is this the *kratos* of the *demos*?

2. In the European Parliamentary elections abstention won, with around 60% of the votes.

So the *demos* don't want the *kratos*?

What is it that leads the Portuguese to prefer roasting themselves in the sun to using their share of power? Political analysts offer various explanations. For now, I will offer just two: 1) one cannot disregard the will of the people on some occasions and call on them to participate on others; 2) the lack of information available to the people is not compensated by electoral campaigns which are based more on creating noise than explaining ideas. *Most people still do not think with ideas, but with words: thus «politician», in the parlance of our times,*[1] *is the man who calls himself «politician», or who others call by this name, and not merely what he is* (...) (Sérgio, 1980).

3. Yet this phenomenon of abstention does not only affect Portugal, it also reached significant proportions in 10 out of 15 of the member States.

If man, as Aristotle says (I, 2, 1253a) *is by nature a political animal*, how do we explain this indifference, this apathy on the part of the citizen? Is it the rejection of the idea of a European Nation, or the dissociation of the people from the politicians? In the latter case, does this come from a weariness of empty promises or is it the fruit of parliamentarianism?

Whilst limiting myself to the country whose political practices I am familiar with, that is, my own country, I would argue that the Republican Assembly has not been a deliberative assembly with a common interest – the general good, the good of the Nation – but rather it has functioned as a congress of groups with different, often conflicting interests.

Could it be otherwise?

4. By definition, in a democracy, sovereignty resides in the people. As sovereignty consists essentially in *general will* (Rousseau, 1974), it cannot be represented. *Who can represent me?* asks D. H. Lawrence (cited in Arblaster, 1988) – *I am my own self.*

A representative is someone who, in a determined situation, acts in the same way as would the person he/she is supposed to represent. What do the MP's of the party we vote for know of our values, our interests, and our realities?

In voting for a particular party, what we are doing is giving a mandate to this party – whether it is in power or in opposition – to put its electoral manifesto into practice. In voting for this particular party, what we are really doing is abdicating our will.

Where representation begins, democracy ends (Harrison 1995).

5. Even if we reduce the participation of the people to the choice of governors[2] (as Shumpeter, 1987 argues), this choice cannot be blind, it cannot be based on

affections or sympathies generated in walkabouts, it cannot rest on the charisma[3] of a particular candidate.

But it is not enough simply to vote; civil society must, during the government's term of office, *scrutinise and react to political power* (which Sérgio, 1980, considers to be the essence of parliamentarianism) and use the vote in future elections either to reward, or to punish those who could not[4] or would not[5] carry out their promises.

6. *To live is not to breathe, but to act* (Rousseau 1966).[6]

The idea of democracy can be applied to a whole society, it can be a way of life. For this to happen, all citizens have to believe in themselves as transforming agents, to organise themselves into neighbourhood associations, to assert themselves as interventionists in the workplace and in the instances of power in which they have some say.

7. And yet *praxis is not blind action. It is action and reflection* (Freire, 1974). It is **sustained action** – a form of action based on information about, and knowledge of, society and the world we live in, supported by critical reflection on this reality and our way of being in it and sustained by the consciousness that the situation in which we find ourselves is not inevitable but may be transformed by our actions (Moreira and Carreira, 1998).

8. However, in a highly technological society, the information and the reasoning necessary for decision taking may be beyond the reach of the ordinary citizen. The question can therefore be raised as to whether it is in fact possible to preserve citizen participation in such a highly technological society.

9. Determinist theories, which appeared in the late 1960's and 1970's, underlined the effect of the reproductive role of schools in relation to the social system. The fatalist character of these theories pointed to the perpetuation of the status quo and took away any room for manoeuvre from educators, themselves the products of the educational system. Education is thus *a process of social conformism* (Gramsci, cited in Morrow and Torres, 1997).

But, given that the true vocation of man is that of *transforming reality* (Freire, 1974), the objective of education has to be one of change towards a fairer society in terms of distribution of power, wealth and opportunities (Giroux, 1988).

10. If mathematics is the logical support, if mathematics constitutes the basis of technological society, then mathematical literacy becomes a pre-requisite for social and cultural emancipation; mathematical education is the individual's passport to citizenship (Skovsmose, 1992).

11. But what is mathematical education?

Nowadays new roles are being constructed for mathematical education, including the development of aptitude for: a) the critical evaluation of mathematical models and modelling processes; b) the discovering of materialised mathematical models which are part of our day-to-day lives; and c) the questioning of the use and abuse of mathematical models in present-day society.

According to Skovsmose (1992), mathematical literacy has to be rooted in a critical spirit and therefore a further aim of mathematical education is the development of reflective knowledge. Reflective knowledge implies various levels. In the first two levels we apply mathematical tools – Have I chosen the appropriate algorithm/procedure? Have I used it correctly? Are there other procedures I could have used? In the third and fourth levels, the relationship between the tool and the task is reflected upon – Is the result adjusted to the context? Could this solution have been reached without using mathematics or, at least without using formulaic procedures? In the fifth level, *we leave the classroom definitively*, searching for broader consequence of the use of specific techniques and reflecting on the *formatting* power of mathematics – How does the application of this algorithm affect our way of seeing the world? In the sixth and final level, we reflect on the way in which we thought about all of the previous questions, we reflect on the use of mathematics (Skovsmose, 1994).

The ideas of both Keitel and Skovsmose reflect the concept of liberating education, as defended by Freire (1975). According to Freire, education should aim towards a constant demystification of reality and the student, in the face of each problem, needs *to grasp the particularities of the whole problem, which are understood as being units in interaction by the reflective act of his/her consciousness which is becoming critical.*

12. In the academic year 1992/1993, after eleven years without any direct contact with students, I began teaching Mathematics again, at the *Escola Superior de Gestão, Hotelaria e Turismo* (School of Business Studies, Hotel Management and Tourism), University of the Algarve. I was faced with a strong rejection of Mathematics as a subject and consequently a high rate of failure – a situation, which is common enough in other schools.

Over the next three years, I came to understand what sort of mathematics the students needed and what kind of challenges had to be met by these future managers, as well as what the students' expectations of and attitude towards Mathematics actually was.

My work with the students then became organised around three major lines of attack: the applications and solving of real problems were put first and foremost; co-operative work was to be used as *modus faciendi*, and finally I stressed the importance of communication. These options were based on three types of questions: 1) utilitarian – what will these students need when they enter the professional marketplace? 2) psychological – how can the effectiveness of the teaching/learning process be improved? 3) affective – how can the liking for mathematics be developed? (for further details, see Moreira, 1996, and Moreira and Carreira, 1998).

During the academic year 1995/96, informal interviews conducted with my students led me to conclude that they recognise:

a) that the "practical" applications of mathematics on the one hand let them (finally) see the importance and the usefulness of mathematics, and on the other hand "made them think", in other words these applications work as a means of putting abstract concepts into practice, thus making learning more effective;

b) the possibilities of co-operative work, valuing this as being a means of brainstorming, a hotbed for ideas and a fountain of resources as well as a stronghold of solidarity and the practising of tolerance;

c) the importance of the classroom presentation of group work, for besides being good training for real-world tasks, it means "going into greater depth" as each group has to present its findings and be able to respond to questions put to them by the rest of the class.

13. As teachers, by force of circumstance we know better than anyone the young people often called the 'shallow' generation. Taking courses in areas for which they don't feel any vocation, threatened by the spectre of unemployment, living in a society which values comfort, consumerism and ease; their power of vindication does not go much beyond the question of university fees.

We all know how well informed they are on matters that really interest them, but we also see the wide information gap and lack of understanding about facts and phenomena which affect our (and, obviously, their) lives. The inability to give an opinion or present an argument about a real problem is obvious each time someone (including the media) calls upon them to do so. Mobilisation for a cause is limited to certain, restricted groups.

Yet we will never have true democracy – political, economic, and social – *if the majority of the (republican)*[7] *youth do not study the social problems of their* *times and do not want to intervene in civic debates and raise them to the height of their own ideals.* (Sérgio, 1980).

14. During the academic years 1996/97 and 1997/98, I endeavoured to structure the Mathematics class as a place and a time of *conscientization* so as to form a basis for **sustained action**. To this end, I gave preference to mathematical applications in areas of economic, political and social impact.

We studied models of the evolution of the number of AIDS sufferers – in the short and mid-term – using national and international data, as the harsh reality of the figures could well provoke a dissuasive effect regarding certain risk practices. We also saw how a knowledge of the numbers involved and the consequent construction of the model can lead to conclusions about the nature of the disease and therefore about the measures which can be taken to diminish its spreading.

From a purely mathematical point of view, this was merely an opportunity to study different models of population evolution. And, as the students are future managers (or police officers or bartenders, depending on the opportunities given by the job market or unemployment), we also studied the models which show the evolution of the number of consumers, according to the publicity of the product is done via the media or from consumer to consumer.

We used the concept of elasticity (an instantaneous rate of relative variation and therefore an extension/application of the derivative concept), not only to draw conclusions about the effect of an increase in price on goods or services but above all to have more data to **sustain** a viewpoint on the issue of decriminalisation of soft drugs.

When we studied models of biological population evolution it was not only to find the zeros of the function in question or its extreme values, but, armed with this knowledge, to come to some conclusions about the propriety, or impropriety, of taking measures such as the banning of whale hunting or, from the perspective of maximising profit, to reach a conclusion about the best fishing policies without endangering populations.

The Gini index is twice the area between the straight-line $y=x$ and any Lorenz curve (cumulative distribution curve of the income of a population). It is therefore a measure of deviation relative to the equal distribution of income but it is also, above all, a tool for: 1) startling us into an awareness of the sharpening of this deviation throughout the world over the last four decades despite the prodigious progress of technology; 2) making us conscious that Portugal tops the tables of inequality in the EU; 3) shocking us with the contrast between the 6% of total net income which is divided between the poorest 20% of households and the 46% earned by the richest 20%;[8] 4) revolting us, in unison, with the realisation that a third of Portuguese families live below the poverty line.

Figures released by the Intergovernmental Panel for Climatic Change[9] were used as the basis for a study of the greenhouse effect – what it is, what consequences it has, what can upset the balance of this natural process. Data on the concentration of carbon dioxide in the atmosphere (over six five-year periods) permitted, through regression analysis, the construction of a model and the identification of the circumstances, which limit its powers of prediction. By knowing the function which allows us to determine average air temperature through the concentration of carbon dioxide, it was possible, by composition of functions, to arrive at another model which can be used to estimate the average temperature of any particular year and, by the same process, arrive at a further model which permits the calculation of the rise in sea level.

Last but not least, in the face of the evidence about climatic alterations, it was recognised that by reducing environmental aggression we can live in a healthier way and that the growth of developing countries can be ensured without their being obliged to use processes or products which are harmful to the environment. We began to question the lack of political will, on the part of the most polluting countries,[10] to implement measures to combat pollution. We condemned the fact that sectarian interests prevailed against the general interest to the extent that international agreements are broken.

News reports on the Chernobyl nuclear disaster and on the Russian nuclear cemeteries were the starting point for the study of radioactive decline and consequently an in-depth study of the exponential function. This led to a gathering of further information on the pros and cons of adopting nuclear power and therefore to a constant act of *demystification* (Freire 1975) which enables one, if and when necessary, to take a **sustained** position.

I could go on giving more examples, but what I want to draw out here is my attempt, through this type of work, to ensure that the students *begin to understand, critically, how they stand in the world in which they find themselves* (Freire, 1975).

15. In the above-mentioned examples, I used the first person plural. This was not a question of style, but a natural consequence of the work we undertook and the attitude I assumed with the students.

To applaud democracy and not to use it in the classroom is a farce.

To commend dialogue and then to shut out the contributions of others, or deny our own, is a lie.

To cite equality and not share the power is mendacious.

To call for solidarity and then to stimulate competitiveness is illogical.

From a purely mathematical point of view, the programme content was non-negotiable,[11] but all the rest – methodology, organisation of time, means of assessment – was negotiated with the students.

If I suggested activities, they suggested topics; if I handed out a bibliography, they searched for additional material (via the Internet, for example). I often asked for their opinions without previously giving my own. In the debate, which followed I took the role of chairperson first of all, but soon handed over this role to the students. Nonetheless, I made sure I asked questions to get the students to clarify or develop an idea. I also never refused to give my own opinion as soon as the students realised they were allowed to ask for it as we were involved in a genuine debate and not merely in a classroom situation set up for the sole purpose of assessment.

But, as Meirieu (1997) points out, the more the contract is negotiated, and the more imaginative it is in terms of articulating previous experiences of the students, present motivations and the teacher's objectives, it always puts the two parties in a doubly asymmetrical position where each one takes on a certain anteriority regarding the other: the goal of teaching precedes the learning situation, the teacher anticipates what is "good" for the student; the act of learning precedes what is learnt.

Consequently, that "we" is a result of two previous subjects who, *whilst mediated by the world nonetheless reflect upon it in a way which is more and more critical in spirit, and which is inseparable from ever more critical action* (Freire, 1974).

16. No education is neutral. The activities I chose, the news reports I selected, the position I took were what those and not others. I assume the politics of my practice (to paraphrase Freire, 1993), but I reject the accusation of indoctrination – I value the rights of the citizen as highly as freedom of opinion and thought. I presented facts to the students, real facts, and I asked for their opinions: **sustained** opinions.

17. The art of being a teacher is not an easy one – it involves *doing everything, whilst doing nothing* (Rousseau, 1966).

NOTES

[1] The times were different, but behaviour... (This note is, obviously, the responsibility of the author).

[2] Then democracy would be merely the process of choosing governors.

[3] Charismatic personalities often turn out to be heavyweight dictators – we need only think of Hitler.
[4] Due to incompetence.
[5] Because of simple demagogy or because of lobbying from pressure groups.
[6] It may be more appropriate to the times to say: to live is not to consume...
[7] The parenthesis is my own.
[8] Figures released by Eurostat, 1993.
[9] Report to be discussed at the Quioto Convention in Japan.
[10] For example, the Americans, who account for 4% of the worlds' population, produce more than 20% of the gases responsible for the greenhouse effect.
[11] I could have taken the position that the mathematical content arises according to necessities and according to specific projects – I have done so before, in another context, some years ago. The pros and cons of this option cannot be discussed in the space I have in this issue but to be honest it was not due to the 'cons' that this option was not presented to the students (nor was it suggested by them), but rather so as not to put the eventual success of the work into question because of not adhering strictly to the programme, or nor choosing content more suitable to the work developed.

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DISCOURSES OF ASSESSMENT – DISCOURSES OF MATHEMATICS

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MAINSTREAM DISCOURSES OF ASSESSMENT IN MATHEMATICS EDUCATION

Current debates about assessment in mathematics education have focused on the idea of 'authenticity' of assessment tasks and on the influence that various forms of assessment may have – for good or for bad – on the mathematical experiences and learning of students (see, e.g. Leder, 1992; Niss, 1993; Romberg, 1995). The big question has been *how* to assess in order to fulfil various functions rather than *why* to assess at all. For most of those involved in education and educational research, assessment appears to be an essential and natural part of educational processes. Without some form of assessment, how could we teach and how could we know about learning? Although in some circumstances particular forms of assessment may be seen to be inappropriate or even harmful, there is a strong consensus that, in principle, assessment is necessary and even beneficial to teaching and learning. We have, however, seen changes over time and differences between countries and between groups of educators, researchers, and policy makers in the forms of assessment that are valued and the types of knowledge sought through assessment processes.

In this paper, I intend to examine the discourses that dominate thinking about assessment in mathematics education – that is, to analyse the sets of constructs, assumptions and values that underpin research, curriculum development and teacher education in relation to assessment. Such an analysis necessarily lays these constructs, assumptions and values open to question by identifying their contingent, historically and socially situated nature. It also identifies tensions between competing discourses associated with current practices. The dominant discourses within mathematics education obscure the social functions that assessment fulfils within the classroom and in the broader society. I shall argue that, if we are concerned with social issues within mathematics education, we must challenge these dominant discourses and the practices associated with them.

Psychological discourses

Until fairly recently, research in most aspects of mathematics education has been heavily dominated by constructs and methods located within explicitly psychological discourses. This has been particularly true of assessment. The main aims of researchers in this area have been the development, use and validation of improved assessment instruments to characterise the attributes of individual students or to construct models of the general characteristics of knowledge and understanding in a given area of mathematical activity. The types of attributes and aspects of mathematics involved include both ‘traditional’ areas of study, such as ‘geometry’, and areas associated with current curriculum reform movements, such as ‘problem solving’. Some of the studies reported appear to be ‘pure’ research while others are explicit in their intention to provide tools for teachers to use or to influence teachers’ practice. Some have adopted a broadly Piagetian framework, assessing the stage that children have reached; others, more recently, work within a Vygotskian framework, developing the idea of dynamic assessment. While there may be substantial differences in the aims, content and theoretical framing of such studies, they all share two fundamental assumptions. Firstly, it is assumed that individuals possess attributes (such as knowledge, understanding, skill, ability, etc.) that are discoverable and measurable. Secondly, the primary purpose of assessment is seen to be to discover and measure these attributes.

It is not only research that has been dominated by this psychological discourse. It has also had a strong influence on policy and practice in schools. I shall illustrate this influence by looking at some extracts from documents issued recently by the UK government Teacher Training Agency, describing what trainee teachers in England and Wales must learn and be able to do before being accredited as qualified teachers. The first extract shows clearly that assessment is presented as a straightforward means of determining the characteristics of students’ understanding. Teachers are expected to know:

> how to use formative, diagnostic and summative methods of assessing pupils’ progress in mathematics, including:
>
> (i) identifying from pupils’ oral and written work and from observation of their practical mathematical skills, the basis of their understanding of mathematics; …

(iii) preparing oral and written questions and setting up activities and tests which check for:

- misconceptions and errors in mathematical knowledge and understanding, to identify specific mathematical issues which need further attention;
- understanding of mathematical ideas and the connections between different mathematical ideas (DfEE, 1998a)

Teachers are to be experts, not only in using instruments devised by others, but also in preparing their own instruments to assess pupils' understanding. The actions the teachers are to perform – *identifying* and *checking* – suggest a world in which observation provides absolute knowledge of the character of the object observed. Interpretation of the information appears not to be an issue.

There have, of course, been changes over time in the types of mathematical knowledge, skills or understanding to be assessed, the methods used to do this and the theories of learning underpinning the assessment (see Table 1). As Gipps (1996) points out, the original development of psychometric testing was based on a notion of uni-dimensional intelligence, and what we now perceive as 'traditional' multiple choice tests and examinations of knowledge and skills were based on behaviourist principles. The mathematics assessed by such tests tended to be restricted to knowledge of facts, skills and standard procedures. More recent developments, with their emphasis on 'authentic' assessment (e.g. Romberg, 1995) have been more or less explicitly grounded on constructivist theories of learning and views of the nature of mathematical knowledge. Even in the document we have just seen (produced by an agency not well known for its progressive views), the teachers' assessment will not only find errors in the mathematical *texts* produced by the pupils – it will find errors in their *understanding* and, going even more deeply into the cognition of individual pupils, the *basis* of their understanding. Mathematics and the object of mathematical education, therefore, are not just composed of facts and skills but also involve individual conceptions and connected ideas.

The assumptions of the psychological discourses of assessment are all rooted in a strongly positivist tradition. That is, they are predicated on the belief that there is an underlying truth to be assessed/discovered and that it is theoretically possible to get as close as you might wish to this underlying truth. This positivist tradition is perhaps even stronger in mathematics than in other subject areas: there are only right or wrong answers; you either know the right answer or you don't. Uncertainty and non-excluded middles in mathematical contexts are deeply uncomfortable for many people, even for those who might find them less surprising in other disciplines. Interestingly, currently fashionable constructivist theories of learning challenge both the idea that there is some absolute 'truth' about students' understanding of mathematics and the idea that any instrument could observe and measure such a state. Yet such epistemological concerns have had little impact on thinking about assessment (Galbraith, 1993). The authentic tasks associated with constructivist-inspired curriculum reform still seek for 'authentic' knowledge of student understanding.

Table 1: Models of assessment[1]

assessment instruments	**theories of knowledge and learning**	**assessment discovers …**	**nature of mathematical knowledge**
psychometric testing	uni-dimensional intelligence	absolute measure of intelligence	irrelevant (because dependent on general intelligence)
traditional tests and examinations	behaviourism	skills attained	facts, skills and standard procedures
'authentic' tasks	constructivism	nature of personal understanding	personal and contextualised

The traditional psychological discourse of assessment was concerned only with the cognitive attributes and development of students. In considering the ways in which assessment might support teaching and learning, knowledge of these attributes was the only factor considered. More recent developments take a broader view of the student and of ways in which assessment may affect learning. Particularly influential, both in the United States and internationally, has been the reform agenda of the National Council of Teachers of Mathematics, based on a more flexible view of mathematics and mathematical learning, encompassing student creativity, processes and attitudes as well as traditional content (NCTM, 1989). Accompanying its recommendations about mathematics and about classroom teaching and learning processes, the reform has also addressed assessment issues. In doing so, it has broadened the role of assessment and enhanced its importance within mathematics education. No longer is assessment just a neutral means of measuring students' attributes – its neutrality guaranteed by statistical standardisation and elimination of bias. It is now explicitly seen as contributing to teaching and learning in complex ways and, in doing so, promoting the values embodied in the intended curriculum.

> In order to develop mathematical power in *all* students, assessment needs to support the continued mathematical learning of *each* student. This is the central goal of assessment in school mathematics. In our view, assessment occurs at the intersection of important mathematics content, teaching practices, and student learning. Assessment that embodies the vision of the six standards presented here will be a dynamic process that informs teachers, students, and others and supports each student's continuing growth in mathematical power. (NCTM, 1995, p. 6, original italics)

The idea that it should embody a vision brings assessment explicitly into the realm of values, while the notion of mathematical power is one that is closely identified with the accompanying curriculum reform. The formative aspects of assessment are to the fore here – it is envisaged as dynamic and as supporting learning rather than simply as providing a measure.

The officially beneficial nature of assessment is also apparent in the UK reforms. Teachers are to use it to improve their teaching and to intervene 'purposefully' in pupils' learning:

> Those to be awarded Qualified Teacher Status must, when assessed, demonstrate that they: … assess and record each pupil's progress systematically, including through focused observation, questioning, testing and marking, and use records to: …
>
> - monitor strengths and weaknesses and use the information gained as a basis for purposeful intervention in pupils' learning;
> - inform planning (DfEE, 1998b)

By characterising students' understanding, teachers are to be able to adapt their teaching to make it more effective. (Of course, any underlying theory of how learning might progress, given a particular state of understanding, is absent.)

But gaining information about students is not the only function of assessment. Information about cognitive attributes may even take a back seat, as in this extract from a recent book about assessment of ‘significant achievement’ in mathematics addressed to primary school teachers:

> The purpose of the assessment process is to make explicit children’s achievements, celebrate their achievements with them, then help them to move forward to the next goal. Without children’s involvement in the assessment process, assessment becomes a judgmental activity, resulting in a one-way view of a child’s achievement. Information gathered in this way has minimal use. When shared with the child, assessment information is more likely to result in a raising of standards, because the child is more focused, motivated and aware of his or her own capabilities and potential. Good assessment practice enables children to be able to fulfil their learning potential and raises self esteem and self-confidence. (Clarke & Atkinson, 1996, p. 9)

The underlying theory of learning here emphasises the role of affective factors such as motivation. The outcomes of assessment thus contribute to teachers’ planning of interventions not only to influence students’ cognition directly but also to influence their “self esteem and self-confidence”. As well as participating in the individualised psychological discourse of enabling children to “fulfil their learning potential”, the author here also makes use of the idea of using assessment for “raising standards” – a component of the curriculum reform discourses that I shall turn to next.

Curriculum reform discourses

In recent years, educators and governments around the world have been engaging in debates about the mathematics curriculum and have instigated major curriculum reforms. These curriculum reforms have, in many cases, been associated with and accompanied by reform of assessment. We have seen increasing interest in the role of assessment in the context of curriculum reform among researchers as well as among curriculum developers (the two groups are, of course, not distinct) and this has been marked by a move away from a strictly psychological discourse. Within what I am calling curriculum reform discourses of assessment there are two strands, focused on the practical problem of curriculum implementation and on “raising standards” – on the regulation of the system.

Implementation

Assessment is clearly used for more than just to inform teachers’ planning and teaching. It is widely recognised that assessment emphases and structures have a strong influence on the curriculum experienced by teachers and students. This is especially the case where tests and assessment tasks and norms are imposed and designed by an authority at a level higher than the individual teacher (whether at school level, local, state, national, or even international level). This has led to calls for assessment to be deliberately designed to lead curriculum reforms, modelling the values and principles of the intended curriculum in “beautiful” (Burkhardt, 1988) or “balanced” (Ridgway & Schoenfeld, 1994) assessment schemes. From this perspective, assessment methods are not only expected to match the values of the curriculum reform but are also to be used to coerce teachers into teaching in ways consistent with the curriculum objectives. Although *coerce* is a word that is not acceptable within this discourse (teachers are to be *encouraged* and *supported*), I am using it to highlight the relationship between teachers and those with the power to instigate curriculum and assessment reform. Such coercion may be successful in changing teachers’ practices to enable more students to match the expectations of the assessment tasks. This is not necessarily equally effective in making teaching practices match curriculum aims, particularly where assessment values such as reliability and objectivity are in tension with reform curriculum values such as creativity and collaborative working. (See, for example, Morgan, 1997 on the distorting effects of institutionalisation by assessment on the ideals of investigative mathematics.)

Even where the idea that assessment drives the curriculum is not so explicit, contestation over the nature of the curriculum often manifests itself in debates about the nature of assessment tasks and systems. Some examples from the UK context:

1. Contrast the unquestioned authority of a question appearing in a national examination paper in 1985 (in the context of the Falklands/Malvinas War between Britain and Argentina):

   > A pilot flying an aeroplane in a straight line at a constant speed of 196m/s and at a constant height of 2000m, drops a bomb on a stationary ship in the vertical plane through the line of flight of the aeroplane. Assuming that the bomb falls freely under gravity, calculate, (a) the time which elapses after release before the bomb hits the ship, (b) the horizontal distance between the aeroplane and the ship at the time of release of the bomb, and (c) the speed of the bomb just before it hits the ship.

with the fuss made by a government minister about another examination question, this time labelled “unacceptable”, comparing military spending with the resources needed to address human needs:

> The money required to provide adequate food, water, health and housing for everyone in the world has been estimated at £11,500 million. How many weeks of NATO plus Warsaw Pact military spending would be enough to pay for this?[2]

Should the mathematics curriculum be neutral (i.e. reflect the dominant ideology of the current rulers) or may it address issues of values?

2. Consider the attack by the Secretary of State for Education on the “elaborate nonsense” of assessment tasks devised for the first national assessment of 14-year-olds in 1991. The contract for developing these tests was subsequently cancelled (Broadfoot & Gipps, 1996). Should the mathematics curriculum engage students in extended and open problem solving or should it concentrate on disseminating facts and procedures?

The power of assessment to influence the curriculum is a double-edged sword. It is necessary to ask who is controlling the reform and in whose interests they act. In recent years in the United Kingdom, we have seen a change in the relationships between teachers, curriculum reforms and assessment practices. In the 1970s and early 1980s, reformers who wished to see greater diversity in the curriculum and opportunities for wider groups of students to participate in mathematics made use of innovative assessment methods to encourage the teaching of problem solving and the use of mathematical investigation in the classroom (see, for example, Love, 1981). Many of those actively involved in setting the agenda for such reforms were themselves classroom teachers. In 1988, with the introduction of a new national system of examination for England and Wales, some of these practices were officially endorsed and, eventually, made compulsory. This use of assessment to instigate universal reform actually acted to distort and impoverish the types of rich mathematical activity it was apparently intended to encourage[3] (Morgan, 1997). Since the late 1980s, assessment has increasingly been used as a tool in the move towards centralised control of the curriculum. Teachers have lost most of their opportunities to innovate and to have their innovations validated through the official assessment system. Both the content and the method of teaching have been deliberately engineered through the introduction and shaping of national tests for political as much as educational purposes. As Galbraith argues, the now generally accepted idea that external assessment requirements should be used to influence the curriculum is “ultimately disempowering to teachers in impeding the growth of full professional responsibility, and to students in making their choices and interests irrelevant.” (Galbraith, 1993, p.82).

Standards

A second discourse of curriculum reform that is currently powerful within the United Kingdom and elsewhere is the discourse of *standards* (using a meaning for standards rather different from that of the NCTM) and target setting. Here, rather than directing the reform effort at changing the processes of teaching, it is directed at the outcomes. The kinds of educational experiences offered to students are irrelevant except in so far as they lead to high scores when the students are assessed. Rather than focusing on the learning needs of individuals, this discourse focuses on the outcomes of education, usually at a higher level in the education system. Thus targets are set for individual pupils based not only on assessment of their personal cognitive state but on ‘benchmarks’ for attainment set at a national level. The same document that demands that teachers should use information gained through assessment to “intervene purposefully” in students’ learning also expects them to

> know how to use national, local, comparative and school data ... to set clear targets for pupils’ achievement . (DfEE, 1998b)

Targets are also set for schools and teachers in terms of the examination results their pupils should achieve. A natural consequence of this is that schools and teachers focus their attention and efforts on meeting the targets by whatever means are available. For example, secondary schools are compared by reference to the proportion of their students gaining grades A-C in national examinations. There is plenty of evidence to suggest that some schools and teachers pay extra attention (including better resourcing and extra teaching time) to those students on the borderline for achieving these grades rather than distributing resources according to the learning needs of the individual students concerned. As Gillborn and Youdell (1999) point out, those excluded from this special attention because they are considered unlikely to reach the crucial ‘C’ threshold include “a disproportionately high number of working-class children; pupils with special educational needs; and African Caribbean young people.”

The discourse of standards is more or less explicit about its regulative function. International competitiveness and the needs of industry are appealed to as justification for raising standards – though the link between achievement on international comparative tests and the economic well-being of the country is less than proven. (And the correlation between shrinking employment opportunities for young people and government policies for the expansion of further and higher education tends not to be mentioned.) At the same time, however, the term *standards* is used as a transcendental signifier, an unquestionably good thing that does not need definition. In debates in the UK about the curriculum for 16-19 year-olds those who wish to conserve the traditional academic elitist structure and those who wish to introduce a broader reformed structure giving equal value to academic and vocational studies both appeal to the goal of maintaining or raising standards.

Summary of mainstream discourse

The main features of the various mainstream discourses of assessment that I have discussed above are summarised in Table 2. It would be very easy at this point to make value judgements about the aims and values of each of these discourses and to say “this way of thinking about assessment is good” and “this way is bad”. In particular, for many of us who are concerned with the ways in which individuals and groups of individuals are disadvantaged and oppressed by educational practices and systems, the psychological discourse with its concern for individual needs and the pedagogic role that it constructs for teachers seems most congenial. Moreover, the use of assessment systems to coerce teachers to adopt imposed practices and sets of values offends liberal sensitivities. While we may reject the overt regulative aims of the curriculum implementation and standards discourses, I would argue that we must also recognise the regulative role played by “assessment to support learning” as championed within the psychological discourse.

Table 2: Summary of mainstream assessment discourses

	Psychological	**Curriculum Implementation**	**Curriculum Standards**
focus	individual learner	system-wide curriculum	system-wide outcomes
aims	to produce valid knowledge about individual students	to effect reform	to produce higher achievement
assessment should be	‘authentic’ in the sense that it identifies real mathematical understanding	‘authentic’ in the sense that it matches the values of the desired curriculum	normative and challenging
individual students will benefit because	teaching will be matched to learning needs	teaching methods will match curriculum aims	the national economy will improve, leading to better individual opportunities
teacher’s role	to know students and support their learning	to (be coerced to) implement changes in curriculum and teaching methods	to (be coerced to) adopt strategies that will lead to higher outcomes
student’s role	learner	receiver of curriculum	future worker

THE REGULATIVE FUNCTIONS OF ASSESSMENT

We are all familiar with the explicitly regulative functions of assessment in the selection of students. We know that mathematics qualifications serve in many societies around the world as a means of discriminating between individuals when allocating educational and occupational opportunities, even where knowledge of mathematics itself may be irrelevant to the future performance of the individual. As Noss claimed in his critique of the UK National Curriculum, the purpose of assessing ability to perform long division is to “divide and rule” (Noss, 1990). But surely, you say, this is the function of those bad, summative forms of assessment arising within the discourse of standards. Surely we need to engage in some form of assessment in order to match our teaching to the needs of our students? It is easy to assume that ‘assessment to support learning’ can only have beneficial effects. I will outline two challenges to this assumption.

Firstly, does assessment really identify students' 'needs'? Secondly, what are the consequences of attempting to address these needs?

A challenge to the assumption that assessment is about discovering truth

Although teaching is no longer seen as simple transmission of knowledge and there is a general recognition among mathematics educators that students interpret what teachers say in multiple ways, this insight into the contingent nature of meaning making is not usually extended to how teachers interpret what students say or write. Mainstream thinking about assessment is still based on a naive transmission view of the nature of communication in which meaning resides within the text, independent of the reader, carrying the author's intentions exactly. The teacher/assessor's role is thus to 'extract the meaning' from the text produced by the student. Obvious failures to communicate – where different modes of communication (for example, a written test and a teacher observation of a child working) provide different messages about the 'same' student competence or where the teacher/assessor is unable to make sense of a written or spoken text produced by a student – are usually seen to be a 'language problem' for the student. But on what basis do we assume that, when teachers and other assessors do succeed in making sense of a student's text, they then know what the student intended to communicate? A more consistent epistemology would suggest that there is no necessary simple correspondence between a piece of text and the meanings its various readers construct. Rather, the meanings constructed will depend on the resources brought to bear on the text by individual readers. These resources will vary according to the discourse within which the text is read and the positions adopted by a particular reader within that discourse as well as the reader's previous experience (Kress, 1989). There can never be a guarantee that the interpretations made by the assessor are exactly those intended by the student. Indeed, studies of teacher/assessors demonstrate how different assessors can construct entirely different interpretations from the same text (Morgan, 1996; Watson & Morgan, 2000).

Moreover, even if teacher/assessors do succeed in reaching an interpretation of a student's text that is close to the meanings intended by the student, how can we assume that they then have a valid basis for making inferences about the nature of the student's mathematical understanding? Unless the student has a complete grasp of the ground rules (Edwards & Mercer, 1987) of the classroom and the assessment genre[4], they may not attempt to communicate the particular aspects of their mathematical understanding that are anticipated by the teacher/assessor. This mismatch has been demonstrated in cases where mathematics assessment tasks are presented in 'context' (see, for example, Cooper, 1998). Kearns' (1998) interviews with students working on such tasks revealed that some students made deliberate and conscious choices between using mathematical knowledge or everyday knowledge for their solutions – choices that in some cases did not coincide with those their mathematics teachers would expect. Making the 'wrong' choice in these circumstances would be likely to lead to an assessment that the student's mathematical understanding was faulty, even though the student may have considered and deliberately decided to reject a solution that would have demonstrated 'correct' understanding.

Assessment practices that justify themselves in terms of a psychological discourse, therefore, discriminate between students not solely on the basis of their mathematical understanding but also on the basis of the extent to which they share the more general resources and expectations of their teachers, schools and assessment regimes. This results in disadvantage for students from non-dominant social groups – and the disadvantage is likely to be greatest where the ground rules for formulating acceptable responses are least explicit.

> Class bias is strongest in those tests which throw the examiner onto the implicit diffuse criteria of the traditional art of grading, such as the dissertation or the oral, an occasion for passing total judgements, armed with the unconscious criteria of social perception on total persons, whose moral and intellectual qualities are grasped through the infinitesimals of style or manners, accent or elocution, posture or mimicry, even clothing and cosmetics. (Bourdieu & Passeron, 1990, p. 162)

The challenge for the student, then, is not to acquire knowledge and understanding of mathematics but to acquire knowledge of the characteristics of the forms of behaviour that will allow her to be seen to know and understand, together with the skills necessary to display the appropriate behaviour. In Bernstein's terms, she needs to acquire the recognition rules that "regulate what meanings are relevant" and the realisation rules that "regulate how the meanings are to be put together to create the legitimate text" (Bernstein, 1996, p. 32). The ideals of ‘reform’ mathematics curricula, unfortunately, increase this challenge for the student. By weakening the framing of the pedagogic discourse - valuing creativity rather than industry, student empowerment rather than rule following - the criteria by which students are to be evaluated become increasingly implicit and invisible. This does not mean that assessment criteria are any less determinate, merely that it more difficult to determine what they are.

Lerman and Tsatsaroni (1998) have argued that, just as traditional (strongly framed) forms of pedagogic discourse are inaccessible to working class students, these same students may be further disadvantaged by the discourse of ‘reform’ curricula and evaluation practices. Cooper and Dunne (Cooper, 1998; Cooper & Dunne, 1998) show that working class children, already achieving at a lower level overall, were even less successful on ‘realistic’ questions. They argue that, whereas the rules for answering traditional ‘esoteric’ mathematics questions are clear-cut, in order to answer such contextualised questions successfully, students have to judge very finely exactly how much everyday ‘realistic’ knowledge to use. The relatively poor performance of working class children on such contextualised tasks appears to be related to their use of inappropriate ‘everyday’ modes of response when they would need to draw on more formal mathematical methods in order to achieve the answers expected by the test setters. The implicit evaluation ‘rules’ applied within ‘reform’ curricula, valuing ‘authentic’ means of assessment, are likely to be most accessible to those groups of students whose cultural and linguistic background is closest to that of the school.

What are the consequences of addressing perceived needs?

I do not intend to go in detail into the obviously regulative uses made of summative assessment results at points of transition in students’ educational careers. Rather, I shall consider briefly the consequences of assessments that teachers make in their day-to-day interactions with students. As Watson (1999) has argued, the judgements a teacher makes about an individual student affect the ways the teacher interacts with that student in the future. In particular, this will affect the tasks provided for the student and hence their opportunities for learning. If assessments are partial, inaccurate or biased (as I have argued they must be) there are obvious implications for (in)equity of opportunity (see Watson & Morgan, 2000).

But let us suppose for a moment that assessment is successful in identifying different levels or different kinds of understanding. As I showed earlier, according to mainstream psychological discourses, individual students will benefit from this assessment because it will facilitate teaching that will be matched to their learning needs. Differentiation of the mathematics curriculum on the basis of perceived differences between the 'needs' of individual pupils or groups of pupils is portrayed as desirable in current curriculum documents. This is consistent with constructivist views which stress the individual nature of knowledge and learning. It is important to consider, however, the nature of the differentiated curriculum offered to different groups of students and the longer term consequences of such differentiation. For example, the latest version of the Mathematics National Curriculum for England and Wales (DfEE, 1999), due to start in September 2000, provides two different curricula for students in the final two years of compulsory schooling (15-16 year-olds), describing the 'Foundation' level curriculum as being intended to meet the needs of 'disaffected' students because of its focus on 'everyday' applications of mathematics that the students have already met in earlier years. There are a number of interesting issues that arise from this: the conflation of low attainment with disaffection; the idea that 'everyday' mathematics is more motivating and/or easier; the assumption that this group of students needs to continue to work on material they have already met rather than moving on to more advanced mathematics.

Given what has already been said about the differential outcomes of assessment processes for students from different social groups it seems that working-class students and those from other non-dominant groups are likely to be over-represented among those directed into the 'foundation' curriculum. Dowling's (Dowling, 1991) analysis of differentiated texts suggests that the 'everyday' mathematics provided those students assessed to be lower achievers constructs these students as engaged in manual rather than intellectual labour, hence reproducing existing class distinctions through the curriculum. Cooper (Cooper, 1994) provides a useful historical overview of differentiation, highlighting the issues for equity involved in providing a curriculum intended to meet the ‘needs’ of those identified as low achievers, and indicating the way in which constructing differences between groups of students serves the purpose of preparing students to take up different positions within society.

Regulation of teachers – tensions between discourses

It is not only students who are regulated by assessment. As I have already indicated, the curriculum discourses of implementation and standards focus on regulation at the level of the education system itself. Teachers are placed in an intermediary position as agents of the system. Official pronouncements on assessment addressed to teachers by governments, trainers and advisers assume that focusing on individual students and their learning needs is completely compatible with a simultaneous focus on system-wide standards (see, for example, TGAT, 1987). Teachers have to operate in curriculum and assessment frameworks that make use of both psychological and curriculum discourses.

During my research into the discourse of mathematical investigation in schools in the UK (Morgan, 1995; 1998), I interviewed teachers as they engaged in the task of assessing students’ reports of their investigative work. It emerged that they were often predominantly positioned within a psychological discourse. Thus they aimed towards the idea that the assessment ought to seek for a true representation of the student’s mathematical understanding and used the evidence of this understanding in a student’s text in order to suggest ways of supporting that student’s future learning. However, they also exhibited tensions within this discourse and occasionally shifted out of it – painting an altogether different picture of the assessment process and of their positions within it. This occurred especially when the text they were assessing appeared unusual to them – a situation in which they were unable to rely on set routines and were therefore apparently prompted to reflect on and justify their judgements, often referring to past experience or common practices. For example, Dan highlighted the difference between what students know and can do and the requirements of the examination system.

> I had to pin people down and say I really can’t give you the marks you deserve on this [….] they knew exactly what they were doing but they had to go back and rework that piece of work. (Dan)

Here the purpose of the assessment is not simply to measure what the student knows or can do. Dan appears to be working with two forms of measure: what the student deserves (presumably some absolute measure of his or her knowledge or capability) and the marks that can be allocated for the particular piece of work – the concrete text produced. The two measures cannot coincide until the student presents work in the form required by the examination. This focus on the concrete outcome is a feature of the discourse of standards. Dan's claimed inability to give the student "the marks you deserve" sets up a conflict between his own apparently preferred values (those of a psychological discourse, focusing on the characteristics of the individual student) and the values of the official examination system within which he is working (focusing on normative standards). At the same time he positions himself as powerless within the system.

It is not only students whose behaviour must conform to the expectations of an external authority; teachers also must abide by and impose the rules, even when these do not coincide with their own values and beliefs about the curriculum:

> We're actually marking by the criteria laid down by the exam board and so we rank them [the students] according to their [the exam board's] criteria perhaps rather than according to the criteria that we might use here. (Andy)

Andy's use of *we* here suggests that he is locating his own preferred criteria within a more widely accepted curriculum reform/implementation agenda. (It may also suggest that he assumes his interlocutor shares this agenda.) But he is unable to implement his preferred curriculum values because they do not coincide with those embodied in the official assessment system.

These teachers were working in a context in which their assessment activity was explicitly regulated by an external agency. Their assessment of their own students was subject to moderation and possible alteration by external assessors with high-stakes consequences both for their students (in terms of future educational and occupational opportunities) and for themselves (in terms of possible loss of face and professional standing). It is thus not surprising that a discourse of regulation emerged as they engaged in the assessment process: a discourse marked by the modality of compulsion seen in Dan's description of his own and his students' actions and by Andy's subordination of his own preferred criteria to those laid down by the examination board.

The explicit face of assessment as regulation emerges here where the assumptions of the psychological discourse and the curriculum implementation discourse break down as they come into tension with the standards discourse. Assessment cannot be about discovering and measuring the attributes of students if what the teacher knows to be the true state of a student's capability cannot be acknowledged because it is expressed in the wrong form. Assessment cannot reflect the values of the curriculum if there is a mismatch between the criteria arising from shared curriculum values and those imposed by an external authority.

CONCLUSIONS

Attempts to reform curriculum and assessment in accordance with constructivist or liberal/progressive principles seem doomed to come into conflict with the needs of the system to regulate the supply of future workers. Assessment is a major tool in this regulative process whether it is explicit, as in the case of traditional examination systems and the discourse of standards, or whether it is implicit, effected through the differential reading of texts produced by students with different degrees of cultural capital and through the differentiated curriculum provided to meet the 'needs' of these students. As well as acting to differentiate between students, assessment plays a major role in regulating the curriculum and the extent to which teachers can act autonomously (though here too the regulation may be implicit or explicit).

Many of us here are teachers and are involved with curriculum development and teacher education as well as research. When we are positioned as teachers, as curriculum developers, as teacher educators, there is a tendency to engage with attempts to find 'better' ways of assessing. I certainly see this tendency in myself as I work with student teachers who find themselves in schools, expected to assess their students and required to fulfil the government prescribed standards in relation to assessment that I have quoted from earlier. (Indeed, I am required to assess how well they assess their students and to devise 'good' means of doing so.) When we position ourselves as researchers at a conference about Mathematics Education and Society, however, I would suggest that the search for better assessment is not an appropriate aim. Rather, we must aim to understand how assessment works in mathematics classrooms and more broadly in education systems, and to understand what its consequences are for individuals and for groups within society.

The mainstream discourses of assessment that I have identified serve to naturalise the regulative functions of assessment acts. Within these discourses it makes good sense to see assessment as essentially benign, bringing benefits to all students both as individual learners and as citizens of a prosperous society. I have argued that this 'good sense' can and should be challenged.

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[1] The 'traditional' and 'authentic' types of assessment instrument are strongly allied with the Type 1 (traditional) and Type 2 (liberal/progressive) pedagogic practices classified by Lerman and Tsatsaroni (1998).

[2] The source of these examples is a cartoon in *Mathematics Teaching* 116 (1986, p. 29) based on letters from Richard Noss and David Pimm.

[3] A more cynical reader might suggest that the intention was to harness and hence control and modify the teacher-led innovations.

[4] And I would agree with Cooper & Dunne (1998) when they suggest that specifying all the rules is an impossible task.

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In Reaction of “Standards, Markets, and Inequality” by Michael Apple
Fernando Nunes
Portuguese Schoolteacher within this World

Trying to make it clear

I was asked to “reagir”, using the English language, towards an original essay written in English by Michael Apple. My obviously limited knowledge of that language drove a first concern to look for an equivalent word in English, and its semantics. Being satisfied with “to react”, as it has the same Latin original root, the word *reagere*, I have found the following meanings: “**1**. To act in return, or in turn, *upon* some agent or influence. **b**. *spec*. in *Chem*. Of the action of reagents. **2**. To act, or display some form of energy, in response to a stimulus; to undergo a change under some influence. **3**. To act in opposition to some force. **4**. To move or tend in a reverse direction; to return towards a previous condition.”

Among these alternatives I didn’t hesitate. I feel comfortable with “To act in return”, allowing “to undergo a change under” the influence of Michael Apple’s essay.

The second issue that I want to clarify relates to the inescapable limitation of a written text. I know there’s more to it than a simple collection of letters trying to make sense. Moreover, I don’t know the author from Adam and I have never read a single one of his many -I do know now- works. So, my comments and questions arise only from my reading of the text, and are shaped by my opinions, beliefs and knowledge.

General Impressions

The paper is a call for action, as it tries to “*deconstruct the policies of conservative modernization in education*”, against a general movement existing in the Anglo-Saxon world (USA, England, and Australia are the only named, alongside with “*other countries*”). It offers several arguments, considerations and information to document and base the author’s reasoning. This variety allowed me to find a rich text, even the pleasure to read some delicious writing, and overall I do believe there are several questions that need further discussion. I'll try to contribute to this task.

Two or Three Questions (no hierarchy intended)

• Inequality is a central concern of the text, and I wasn't able to find out the concept behind it. **Which kind of equality does the text aims at?** We can view inequality from a number of perspectives: female/male; special needs/"average"; through social layers; race and ethnic groups; etc. We can ask if is it possible to have a school that fosters equality in a world/society well known for its intrinsic inequality. This is really a complex issue. Moreover, the data we can gather can be misleading. Let me quote a Portuguese example: in the students entering the university, women are the majority. Can we assume that the Portuguese school favors girls and neglects boys? I don't think so.

• We can feel also in Portugal the trends of neo-conservative and liberal policies, in what regards education, albeit there are different details – after decades of deep normative national curricula, a more flexible approach is presented in official texts, allowing each school, public or private, to organize its own curricula, although with several constraints. But national testing, a novelty in Portugal at grades 4, 6, and 9 – Mathematics along with Portuguese are the first subjects to suffer these tests - as well as the need to set up standards and a social pressure to classify schools are evident. **How can research help to disavow these trends?** I must confess my skepticism. I can see that, maybe, here or there we can use research results to argument against the "*conservative modernization*", but I look forward to hear a more detailed and supportive reasoning to broaden my views.

Where are the teachers? I feel a "central committee" view of education throughout the text, as if the issues could be solved with ideological disputes between intellectuals. The teachers are at a gray backdrop, and the fact that they are losing power and status is no news: that is happening for some time now. Teachers are a heterogeneous group, where it can be found a wide variety of beliefs, political views, social status etc, and some of us are also responsible for that status devaluation, so it is not easy to appeal to them. Anyway, I do believe that it is not possible to change schools without their active or passive collaboration. This important and crucial fact should not be overlooked and any analysis has to take it into account.

I am convinced that the modern power has a name: economics. It’s by and with economic arguments that competing states fight each other, and it’s using economics that the state has our lives controlled. Increased marketization seems to be a feature of economics, with all the consequences so rightly presented in the paper. School, as an institution, is part of this world ruled by economics. Is it possible for the school to escape? **Can we have a kind of purified environment in our schools, allowing our students to grow and appreciate values different from the economical claims? And how can we achieve that?**

Commentary
Discourses of Assessment – Discourses of Mathematics

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I'm a Mathematics' teacher at a middle school in Lisbon and my pupils are ten years to sixteen years old. I tell you this so that you would take that situation into account as you listen to my comments.

This commentary reflects what I think as a Mathematics' teacher. I would like to remark that during seven years I coordinated the "Directores de turma", which are the teachers that solve problem that arise between students of a given class, their parents, teachers, etc. This responsability was very important to shape my opinion about evaluation in general and assessment in particular.

I will begin by expressing some ideas about the Portuguese assessment culture. Assessment creates tensions to Portuguese teachers. The 1989 curricular reform brought great tensions and dilemmas to teachers and to the Portuguese society in general, and this is one of the most difficult problems for teachers at the moment.

This reform introduced a true paradigm shift in assessment. Before the reform, the purpose of assessment was to select pupils and this process was instrumental in helping teacher to coerse students to learn. Nowadays assessment has a regulative function, and teachers and society in general have difficulties to cope with this. In Portuguese secondary school, however, the situation did not change. At this level, schooling is not compulsory (as it is in middle school), and so assessment is associated with selectivity.

Well, let´s go into the conference "Discourses of Assessment – Discourses of Mathematics" and comment it since it is the reason of my presence here.

Candia Morgan analyses, mainly, UK perspectives. My view comes from the context of Portuguese schools. Here, in many cases, even in compulsory schooling, the practice of assessment is far from the idea of authenticity. However the mathematics education research, in Portugal, is very similar, in conceptual terms, to the one that is discussed in the paper.

The authoress tell us the history of assessment in mathematics education framing it within several conceptions of learning that arose during last years. Her paper intends to examine the dominant discourses of thinking about assessment in mathematics education – "that is, to analyse the sets of constructs, assumptions and values that underpin research, curriculum development and teacher education in relation to assessment" (p.2).

The authoress argues that research in mathematics education has been dominated by psychological discourses, and these discourses had a strong influence in policy and school practices, independently of the implicit theoretical framework. Even discourses about curriculum reform or about assessment reform, underly psychological discourses.

The "curriculum reform discourses of assessment", as Candia Morgan says, develop in two strands: the curriculum implementation and the "raising standards" and both contain a regulation of the system.

Assessment has been used to lead curriculum reforms and to coerse in changing teachers' practices, in spite of not being posed as such. Candia Morgan identifies tensions between discourses and practices and proposes that dominant discourses should change in order to introduce social issues in mathematics education. Her discourse focus on the question: why to assess?

Discourse of beliefs and Discourse of subjectivity

I organized my comments based on three ideas, one statement and two questions: "Assessment to support learning" has beneficial effects, What do you mean by truth? and Why am I ceptic about curricular assessment?.

Assessment to support learning has beneficial effects

One thing that I felt compelled to react to Candia Morgan's analysis is her perspective of assessment to support learning. In education new ideas usually give rise to contradictions. Although this kind of assessment has a negative side, as Candia points, it involves beneficial effects. Teachers' work is between the desirable and the possible, that is, teacher must know how to negociate the essential and to yield the accessory. Teachers' practical knowledge is constructed in a permanent conflict between what must be, what is expected and what the day-to-day creates and this last aspect decides their opinions about the assessment system. Formative assessment helps me to adequately live within this negotiation.

Those practicing formative assessment have to face (literally) students' diversity and contrast it with scholar failure. This leads to feelings of guilt, but at the same time, they become aware of the absurd of some contents, aims and proposals of the curriculum.

The teacher's practice is always based on beliefs about what is the child, how child learns, what is teaching, what is the reality and what we want to make with it. Teacher's work includes always more or less explicit ways of assessment. Besides, I can only see my work if it incorporates this kind of assessment and I believe that this has influence in the self-esteem and in the self-confidence of my students. I am however aware that a strict formative assessment would create a great stress that, instead of being beneficial, would restrain a friendly classroom environment.

What do you mean by truth?

The challenge Candia Morgan poses to the assumption that assessment is about discovering truth makes me wonder: what do you mean by truth? Truths don´t exist independently of the author. Truths in education are always charged with contradictions and imprecisions, and these can be important to the teacher´s reflexion about practice. If the intentions are to help students to develop mathematics reflexive thinking so as to become democratic citizens, practicing formative assessment means to run risks, and to criticize and to contradict interests as we confront external "truths".

It seems reasonable to me to admit that the one who practices formative assessment is not looking for the absolute truth, but has some conviction about what she desires for her pupils. That kind of assessment helps not only to guide own work, but also helps pupils to think, and to understand why they make certain things and not others.

The teacher must make inferences about the nature of students mathematical understanding, if not, how would she regulate her actions? They are based on possible interpretations, eventually correct, that guide her pedagogic practice. But teacher´s practical knowledge is confronted with a diversity of situations that allow for changing inadequate inferences about students' understanding. The same cannot be said about an external assessor that corrects pupils' paper and pencil tests, since that "crossed reading" can only be made by teacher in classroom situations.

Why am I ceptic about curricular assessment?

In Portugal, arising from a necessity prompted by the reform, assessment of the 4^{th}, 6^{th}, and 9^{th} grades pupils, using paper and pencil tests, in Mathematics and Portuguese

Language is being implemented. The reasons intended by the legislator are well-intentioned. The “Provas Aferidas” (which are similar to criterion referenced test) emerge as a good idea to measure the level of accomplishment of minimum goals, to control the quality of the educational system, to help making decisions in order to its improvement and also to increase the social confidence on the educational system. It looks solelly for the curricular outcomes and adopted procedures as stated in official documents. It has no effects on pupils scholar progression.

In our schools, teachers started, already, to prepare pupils to perform well in “Provas Aferidas”. This already happens with examinations in secondary school, that make contingent formative assessment. But the answers to the tests only provide a partial look about pupil´s global learning, no matter how well they are constructed.

Another justification for “Provas Aferidas” is that this assessment procedures will help to guide teachers on their practices. But I think that this is a fallacious way to place the question. The teachers are not fulfiling the expectation of the new curriculum and, “Provas Aferidas” can show to the teachers what they must be emphasize. I see on this position naiveté and cynicisme.“Provas Aferidas” are paper and pencil tests, and in them it is impossible to make visible certain amount of capacities, attitudes and values expressed on the mathematical curriculum. They are done in “objective” conditions that do not correspond to the usual classroom situations.

How teachers can be asked to work with the pupils to develop knowledge, capacities and attitudes, and at the same time to implement a curricular assessment procedures based on the pupils achievement with paper and pencil tests?

I can see this has an aim: to show to the society whether the pupils “are learning or not”. But in the most favorable situation we will be told that nowadays pupils “know much less”, reinforcing in this way what some opinion-makers are saying.

The official expectation is that teachers will be encouraged to change their practices according to ”Provas Aferidas”. But in Portugal, political power has an ambiguous position. They started by stating that “Avaliação Aferida” intended to assess the educational system. Later, they restricted this purpose and said it was only ment as a device to improve each school’ project and each teacher’s practices. The original intention of assessing the educational system was lost. However, the feasability of this new “formative” role of the curricular assessment can be questioned. The will earnings be bigger than the costs? What can a curricular assessment (paper and pencil test) modify in terms of curricular practices? To think that the teachers will change their educational practice only because curricular assessment exists is to consider teachers as “receivers” in which any ideas can be placed.

At this moment, I argue that this form of assessment is a cynical way to transmit an idea that only intends to mask another. They choose part of curriculum, a set of items transform it into and send them back to the teachers and after expect the teachers to understand, reproducing now equal things or similar to the ones that appear on the tests.

The *media* will say that the pupils are not well prepared and teachers do not know how to teach, but some schools are better than others because their pupils’ outcomes are better.

During the last years in Portugal a lot of assessment work has been done encompassing various educational areas, and the conclusion was that almost always everything is was wrong. I don’t believe that the perverse effects of the curriculum assessment will be emphasized over other possible positive effects. Who wants to teach “bad pupils” that do not allow teacher “to shine”? Who will innovate if what would be validated is what will be questioned in “Provas aferidas”?

Concluding remarks

We cannot discuss nor to think about curricular assessment in mathematics education without understanding other social and political phenomena.
The common sense discourse and the scientific and the political discourses have at their bases the well-being . The well-being is the base of education. I think that is necessary to question schools from an educative and not from an economical perspective that has underlie the neo-liberal politics. When we emphasize formative assessment we are paying attention to the pupil as a person, when we are emphasizing the curricular assessment we are overestimating society's interest. At this point we must ask: but what society are we talking about? Today, who owns the power? Who asks for schools to become competitive? Who wants to transform the pupils in competitive agents?
I'm aware that the relation between these two perspectives about assessment is dual, and it cannot be dichotomous. Curriculum is a game that incorporates a permanent conflit that involves multiple compromises and where parents can also be called to participate.
The assessment that arises connected to the curricular reform has a goal to calm down society and parents. And also, perhaps, who knows?, to calm down teachers who are feeling confused with the instability of the current curricular policy.
I argue that a political compromise must be established between what must be and what schools can do. But can never be a "compromise" imposed by the administration. The curriculum and the curricular assessment is a conflicting field, but it can also be a field of consensus, that results from reconstructions, depending on where we go.
In Portugal social discourses about assessment influenced by social pressures over schools, have prevailed over psychological discourses. This has prevented a lot of teachers to use formative assessment.
Although I am aware of the costs devaluing "Avaliação Aferida", I emphasize the formative assessment. I propose myself to be ceptic about curricular assessment become of the perverse effects that it carries. The closeness of the curriculum to the diversity either of students' background or of innovative teaching practices.

As I said, my statement is situated on the Portuguese context that has a different story than the UK, of course. Thus is why I felt the need to talk a little about my reality so as to allow to our distinct discourses to be understood and intertwined.

The will to mathematics: minds, morals and numbers

Wenda Bauchspies, Pennsylvania State University and Sal restivo, Rensselaer Polytechnic Institute

The 1990s have been The Decade of Sociology in mathematics education. The sociology of mathematics has become a core ingredient of discourse in mathematics education and the philosophy of mathematics and mathematics education. Unresolved questions and uncertainties have emerged out of this discourse that hinge on the key concept of social construction. More generally, what is at issue is the very idea of "the social." Within the framework of the general problem of "the social," we want to open a discussion of boundaries and margins in mathematics and mathematics education. By theorizing the divisions of purity and danger, we will be able to better understand the intersection of logic, mathematics, and thinking with gender, race, and class, and morals, ethics, and values in the classroom. The process of transforming the sociology of mathematics and the sociology of mind into pedogogical tools for mathematics educators and philosophers of education has already begun. One of the tasks before us is the development of a more profound and at the same time more practical grasp of "the social." Our objective in this symposium is to move ourselves and symposium participants in the direction of just such a grasp of the social.
In the past, we have lectured on these topics to audiences like this one. The results have not always been clear. Therefore, instead of adopting a standard posture of the expert voice, we would like to serve as facilitators in a discussion of key concepts, theories, and ideas in the sociology of mind and mathematics. We will prepare a paper that will be available prior to the conference. This paper will summarize the reigning ideas in the sociology of mind and mathematics, and provide a starting point for discussion. Having made the paper available, we will make brief opening statements on the first day of the symposium. At the conclusion of our opening statements, we will offer a key question coupled with a quotation taken from our sociological theory tool kit as a stimulus for discussion. We will then join the participants in an open discussion. At the end of the first day's symposium, we will offer one or two key questions coupled with a quotation or two. We will select questions and quotations that will help push the group's grasp as well as our own in more profound and practical directions. We assume that the structure we propose is open to change in accordance with the will of the group, and the directions and dynamics of the larger setting we are part of. We also hope that participants who regularly interact with students in classroom settings will provide us and the group with

challenging questions and problems. In this way, we may make some additional progress in using the sociology of mathematics and mind to facilitate intellectual and political change in the mathematics classroom.

PEDAGOGICAL TRADITIONS IN MATHEMATICS TEACHING: ENGLISH ARITHMETIC BOOKS FROM 1780 TO 1850.

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Abstract

A distinction between the sociology and social history of mathematics is drawn, and an approach to textual analysis via a social epistemology of mathematics is proposed. Examples of texts are introduced to exemplify some different epistemological beliefs, and some reflections on past and present day conditions are considered.

1. Introduction

In an earlier paper (Rogers 1998) I described the broad social and ideological background which determined the kind of mathematics taught to different groups of people in England in the period from 1750 to 1900. In that sense I was endeavouring to provide the reader with an insight into the social and economic context in which the changes in mathematics teaching were taking place. This paper is an attempt to examine more closely some of the evidence for the 'divide' that I referred to, by looking at the changes in some of the arithmetic textbooks of the period from 1780 to 1850.[1]

Traditional interpretation of text books is often at a superficial level, and misrepresents the nature of the knowledge enterprises of the time. For example, texts like Yeldham (1926, 1936) claim that they are about the teaching of mathematics. However, what they do is to portray technique, and give a 'progressive' view of the skills which were developed over a period of time. On the other hand, one book which does, by its very title, suggest an important social context is Swetz's (1987) "Capitalism and Arithmetic" which discusses the place of the first printed arithmetic in Italian (and European) economic life, and Schubring's (1987) critical paper on Lacroix is a good example of the problems we face.

2. Sociology and Social History of Mathematics

Struik (1942), was one of the early writers to bring the problems of the context of mathematics to our attention in claiming that ,

> "The sociology of mathematics concerns itself with the influence of forms of social organisation on the origin and growth of mathematical conceptions and methods, and the role of mathematics as part of the social and economic structure of a period." (p.58)

For Struik, a classical Marxist[2] interpretation sees history of science (and in particular mathematics) as related to productive forces and modes of production. For example, the emergence of *descriptive geometry* may be seen as the organisation of what was fundamentally elementary mathematical knowledge in reaction to the needs of the training of French engineering and gunnery officers in the Ecole Polytechnique, while *projective geometry* then became the formal mathematical analysis of that practical knowledge. We can extend this argument and show that even the beginnings of mathematics were subject to 'productive forces' - namely the practical problems of everyday life.

This kind of interpretation is better called the social history of mathematics, for sociology and social history are different. The sociologist generally regards historiographical work as the raw data which is used to exemplify abstract theoretical ideas,[3] while on the other hand, the historian's aim is to try to reconstruct a web of events (sometimes based on minimal empirical evidence) and to attempt to provide an acceptable interpretation by describing the activities and motives of particular individuals in the history of mathematics.

I have argued elsewhere (Rogers 1995, 1998) that not only is a rational reconstruction of mathematics in history impossible, but that the very nature of the historian's enterprise is one of interpretation, and this interpretation has an ideological background resting in a set of beliefs which are necessarily contemporaneous to the writer, and which determine the kinds of questions to be asked, and the ways in which they are asked. Therefore historiography has a narrative character, while in contrast, the mathematician's view of the history of his subject is generally anti-sociological. A series of events in mathematics is not seen as a 'story' which could have been told in a different way, and the whole is viewed as a rational reconstruction of the past, governed by the laws of mathematics itself. Furthermore, 'Mathematician' is a problematic term which has often been restricted to the person producing original mathematical knowledge (or at least to assume that this is who we mean). However, this is clearly inconsistent with historiographical practice: many examples range from the 'mathematicians' of pre-classical antiquity through the 'mathematical practitioners' of Taylor (1954, 1966) to the actuaries, statisticians and modellers of today. All of these categories of people produce mathematical knowledge of some kind, and many see it as their duty to pass on this knowledge to others. Where our concern is Mathematics Education (by which I mean both didactics and pedagogy), the duty of the possessor of the knowledge is seen to be the handing on of that knowledge in an organised form, and so most periods of history of mathematics pay some attention at some level to this process. The fact that these 'handing on' processes have not been studied very much, is due to the rational-internalist views of what constitutes legitimate mathematics and hence what is deemed worthy of study.

The study of what is handed on and how it is done looks at the perceived needs, motives, social and economic contexts, preferences and priorities of the particular period. But not only this, I claim that there are sets of underlying epistemological beliefs that guide the kind of transmission practices we may be able to discover by looking at the texts themselves.

3. Social Epistemology of Mathematics

Schubring (1987) discusses this point in his analysis of Lacroix's texts[4]

> "...it is necessary to enlarge the interpretation of a text in order to reconstruct its meaning: a first basic rule for such an endeavour is that a text can only be interpreted adequately together with its *context.* And, as an approximation to a reconstruction in its proper conceptual field, one should analyse its *contemporaneous context.* .....One has to reconstitute the whole context of the debates and the conceptions of the contemporaneous authors together with their embeddings in the cultural structures of the time." (p.44)

Often the interpretation of textbooks rests on the assumption that the function of knowledge in society is merely its intended use writ large; the texts produced, when read in the 'right' way afford human beings a greater understanding of their world. For example, it might be assumed that if one book on Newtonian mechanics succeeds at representing the physical universe to one person, then many books in the hands of many people will have the same effect. But as we well know, the author's intention and epistemological stance may not be what the reader interprets from the text. Fauvel et. al. (1988) clearly show the other agendas and sources of knowledge and inspiration underlying the production of Newton's original text. What Laplace and Voltaire may have made of it is another matter entirely.

While identifying the primary source of knowledge to be transmitted - in our case the mathematics textbook - is easy enough, specifying the exact social function served by the textbook is much more difficult. The problem with the 'one-many' assumption above is that something is being taken as a property of a 'whole' (a community of readers) simply because it is a property of a 'part' (one author), and so the exact outcome of this situation (many books in the hands of many readers), namely, how the particular distribution of texts ends up affecting the whole social order, is far from clear.

Denotative and connotative problems abound (Barthes 1977) and the whole process of translation, interpretation and representation of the text and the concepts it promotes, intervenes so that the knowledge distribution is locally constrained and structured, and not systematically controlled, even though the implicit aim of the author may be to produce a codified system of knowledge. This lack of centralisation can result in a cluster of different communities, each with it's overt and covert agendas, and each with it's 'understanding' of the 'master text'. Where control is attempted in this situation, it is in the institutionalisation of knowledge within an education system.

4. Different Communities

During the period in question, educational institutions were being formed - not so much as we may regard them today - but as communities each with their own sets of aims and beliefs, set in the social and economic contexts of the time. It is these emerging institutions that I aim to describe through the samples of the textbooks below. The reader may argue that I am being selective; in a short paper this is inevitable, and I acknowledge that this deserves a longer and more careful study. However, from about 1750 we see the emergence of a 'school book industry' which is answering the various educational needs of the industrial revolution. Typical among these is Thomas Dilworth's

The *Schoolmasters Assistant. Being a Compendium Both Practical and Theoretical.* (1780)[5]

The content is much the same as other arithmetics at this time; however Dilworth presents each introduction to new ideas in a 'question and answer' mode, and then goes on to give the reader a series of operational rules and a variety of problems to practice.

> *"I have shown the Subject of the following Pages into a* Catechetical Form, *that they may be the more instructive; for* Children *can better judge of the Force of an* Answer than *follow* Reason *thro' a Chain of* Consequences." (pp.v-vi)

So on the first page we have:

"The INTRODUCTION.[6]

Of Arithmetic in general

> Q. *What is* Arithmetic?
>
> A. *Arithmetic* is the Art or Science of computing by Numbers, either Whole or in Fractions.
>
> Q. *What is* Number?
>
> A. *Number* is one or more Quantities, answering to the Question, *How many?* ....
>
> Q. *What is* Theoretical Arithmetic?
>
> A. *Theoretical Arithmetic* considers the Nature and Quality of Numbers, and demonstrates the Reason of Practical Operations. And in this Sense Arithmetic is a *Science.*
>
> Q. *What is* Practical Arithmetic?
>
> A. *Practical Arithmetic* is that which shews that Method of working by Numbers, so as may be most useful and expeditious for Business. And in this sense Arithmetic is an *Art.* "

It can be seen that the text is not only instructing us in the nature of arithmetic and its operations, but is also using a pedagogical device where key concepts are signalled by typographical changes. For example:

"*Of the* Single Rule *of* THREE.

Q. *How many parts are there in the Rule of* Three?
A. Two: *Single* or *Simple*, and *Double* or *Compound.*
Q. *By what is the* Single Rule *of* Three *known?*
A. *By three Terms,* which are always given in the Question to find a *fourth*.
Q. *Are any of the Terms given to be reduced from one* Denomination to another?
A. If any of the given *Terms* be of several *Denominations,* they must be reduced into the lowest *Denomination* mentioned.
Q. *What do you observe concerning the* first *and* third Terms?
A. They must be of the same Name and Kind
Q. *What do you observe concerning the* fourth Term?
A. It must be of the same Name and Kind with the Second.
Q. *What do you observe of the three given* Terms *taken together?*
A. That the two first are a *Supposition,* the last is a *Demand.*
Q. *How is the third* Term *known?*
A. It is known by these, or the like Words, *What cost? How many? How much?*

He then goes on to describe direct proportion and gives fifty examples of the rule of three and direct proportion applied in a wide variety of trades
This is quite different from the 'dialogic' tradition of Robert Record, (Fauvel 1989, Rogers 1998) where much attention is given to explanations of principle and background concepts, as well as procedures. Dilworth's book was intended as an instructional manual as the dedication in the preface admits: "*...the following* Pages *which are intended as an* Help *towards the more speedy* Improvement *of your scholars in* Numbers..." (p.i).

The large number of examples from different trades suggest that Dilworth is intending to show the universal applicability of the arithmetical process. However, in his context, arithmetic is applied almost piecemeal in a series of separate and very complex systems of weights and measures in common use. While it is possible to conceive that an individual may be able to operate with simple calculations within their own trade, it appears that people are unable to transfer the technique to another area because the skills had been set up by teachers as contextually, and therefore conceptually, distinct. However, it must be pointed out that while we may contrast the pedagogical approaches of Recorde and Dilworth, their texts were produced for very different purposes, and different audiences. On the one hand Recorde was, as far as we know, the first English textbook writer.[7] He had a few precedents. In the fifteenth and sixteenth centuries the Hindu-Arabic numerals were transmitted through Europe into the Netherlands and Kool (1999) shows how the development of arithmetic books in this period was related to the tradition of the 'Abacus Books' and Yeldham (1926) lists a number of English workers on arithmetic before the advent of printing. While Recorde was writing mainly for those wishing to learn mathematics on their own, in Dilworth's time there was already a series of texts to copy, and teaching was already becoming institutionalised, so he was writing for an already established market. This is clear from the 'advertisements and recommendations' printed in the front of the book.

Some fifty years later, we find a very different approach in the work of Augustus De Morgan. Much has been written about De Morgan as a mathematician and a pedagogue,[8] and he was one of the founders of the 'Society for the Diffusion of Useful Knowledge' which was supported by liberal politicians and others with a humanist philosophy to bring 'useful knowledge' to the population at large.[9] The SDUK produced serials like the 'Penny Cyclopaedia' in which the useful knowledge was marketed in convenient packages.[10] As a member of the 'Analytical Society' De Morgan was clearly promoting new ideas in mathematics and became involved in the translation of the 1806 edition of Lacroix's Calculus.[11] From Schubring's (1987) account it would seem that De Morgan probably learnt something about a pedagogical style of writing from Lacroix.

The first chapter of De Morgan's 'Study and Difficulties of Mathematics' (1831) is a discourse on the 'Nature and Objects of Mathematics' where he expounds a Platonist philosophy, describing its logical structure, but also its excitement, interest, and practical applications. In the previous year, he had published 'The Elements of Arithmetic' (1830) and later, in the preface of the 1850 edition he says,

> "At the time when this work was first published, the importance of establishing arithmetic in the young mind upon reason and demonstration, was not admitted by many. The case is now altered: schools exist in which rational arithmetic is taught, and mere rules are made to do no more than their proper duty. There is no necessity to advocate a change which is actually in progress, as the works which are published every day sufficiently show. And my principal reason for alluding to the subject here, is merely to warn those who want nothing but routine, that this is not the book for their purpose."

In contrast to earlier arithmetics we find a careful setting out, much attention to explanation, and no typographical devices. In the first part of the book, ('The Principles of Arithmetic') the various direct and inverse proportion rules are summarised in a single statement of generalised algebraic proportion, a:b::c:d (pp100 - 106). In the second part, ('Commercial Arithmetic') the principles of proportion are applied and illustrated in examples including the 'Rule of Three' (p.144). The approach here is to produce a text that promotes the structural unity of mathematics (in this case arithmetic) and enables he user to apply general procedures. It is also interesting to note that the text itself appears quite sophisticated in its discursive style, and quite demanding intellectually. This book seems to be aimed at a particular section of the population.

Bearing in mind the quotation from De Morgan above, it is interesting to see that the belief that mathematics teaching had ‘improved’ was not necessarily born out by the evidence and about 1850, the title page of a book published by Crossley and Martin proclaims,

> “Every Boys Arithmetic. The Intellectual Calculator, or Manual of Practical Arithmetic. Compendium with all the usual rules, a much larger number of business questions on each elementary rule, than has been ever before published. And a Complete Course of Mental Arithmetic reduced for the first time to a system: embracing all the arithmetical requisites of the school, the counting house and the shop.”

While the authors on the one hand aim (p.7):

> “... to render the work in every way complete, and to make it what its name imports, an intellectual book, in which the rationale of the science should be demonstrated in a manner calculated to draw forth the *thinking* powers of the *child.* ”

Gone are the exhortations, recommendations and the detailed explanations and examples of setting out; much is condensed into lists and tables, the text is crowded, typographical devices are used, and the book contains a “complete course in mental arithmetic’ reduced to a ‘regular system’ which “..renders familiar many of the general principles and much of the subsequent operations of arithmetic.” (p.110) which the student was supposed to be able to apply to any situation.

The ‘Rule of Three’ re-emerges, albeit in a more simplified form in a section on Simple Proportion (p.58), but no real explanation is given.

> “RULE.-Put in the THIRD place that term which is of the same kind as the answer. If the answer is to be greater than the third term, place the greatest of the remaining terms in the

> SECOND place. Put the remaining term in the FIRST place. Multiply the second and third terms together, and divide by the first."(p.58)

This text both in content and approach is clearly produced for a different market. There are seventeen advertisements on the endpapers for a variety of books by these and other authors intended for both school and home use. It is almost as though the changes that De Morgan referred to had never occurred.

5. Some Reflections

The period I have chosen here is one of enormous change in many aspects of society. It is one thing to identify the mathematics taught in this period, but quite another to specify the role it plays in the greater scheme of things. Mathematics was regarded not only as an intellectual enterprise, but as a tool for industry and commerce and a means of personal advancement. While more applications of different areas of mathematics were being found, the principal content of arithmetic at this level remained much the same. However, in the hands of different teachers, it was conceived differently and used for different purposes. The use of the same book does not necessarily reflect a uniformity of epistemological approach. Clearly, this brief study needs extending, there is much more here than at first meets the eye. The gradual institutionalisation of mathematics teaching by these different communities gave rise to a situation which often reinforced fundamental divisions in society. How we regard mathematics is at least, in part, not only a function of our cultural contexts, but also of the particular institutional embedding which we can still detect today.

Notes

1. My choice of arithmetic books may be politically and topically motivated: I admit I have some ideological problems with the National Numeracy Project which is now being extended into Secondary schools in England.
2. For a discussion of Sociological approaches see Restivo (1985)

3. For a contemporary discussion of the Philosophy of Social Science, see Fay (1996)
4. De Morgan was involved in the English translation of Lacroix's *Differential and Integral Calculus* of 1812.
5. My personal copy is of 1780, but there were various editions of this book from about 1740 onwards.
6. Typographical accuracy is difficult to follow in this and the following quotations: for example there are large and small capitals used in the titles of the sections and apart from the endings of words, the old English ' long s' (which looks like the letter 'f') is used.
7. By 'textbook' I mean a printed book, covering a range of applications and produced specifically for the purpose of use in instruction either by a teacher, of by oneself.
8. For a recent survey see Rice and Wilson (1998)
9. While the aims of the society were thus declared, it soon became clear that only the 'middle classes' were really able to profit from its publications.
10. De Morgan's *Differential and Integral Calculus*, published as a book in 1842, first appeared as 25 parts (1836 to 1842) in the 'Library of Useful Knowledge', another SDUK publication.
11. Babbage and Peacock were principally responsible for this translation which appeared in English in 1814.

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A Numerical Archeology

by Jose Antonio Segarra

Harvard University 2000.

A Genealogy of a Process

> To follow the complex course of descent is to
> identify the accidents, the errors, the false appraisals,
> and the faulty calculations that gave birth to those things
> that continue to exist and have value for us; it is to
> discover that truth or being do not lie at the root of what
> we are, but the exteriority of accidents.
>
> -Michel Foucault, *The Archeology of Knowledge.*

Just as important as coming to a place in one's own understanding from where one begins the laborious struggle to create a work, lies the trail of ". . . errors, false appraisals and faulty calculations . . ." (Foucalut, 1972, p.150) that brought us to where we are. It is my goal to try as critically and as clearly as possible to revisit my own,". . . complex course of descent . . ." (p.150) in my learning process. The purpose of retracing the "genealogy" of my work to this point in this pedagogic journey lies in its usefulness to my present and future work, insofar as it affords me: first, the opportunity to create a critical blueprint or cartographic space from past locations from which new paths may emerge in my work, secondly, it assists to begin approximating the limitations and the realistic parameters within which risk and movement can occur within and beyond those limitations. Such a process of identifying and describing this non-linear, often times tangled and seemingly contradictory accumulation of raw information, serves as a heuristic compass pointing, albeit at times tentatively, to possible directions for future areas of engagement and transformation through research and practice.

Through a thorough process of self-interrogation and the "disentanglment" of my previous actions, I would like to begin illuminating the set of assumptions that I have made relative to the topics of interest that were generated by such processes. Ultimately there will be an analysis of the subsequent rationales used to justify the creations of proposed research and action. The reason for this is to create as accurate a re-construction of the routes taken by those modes of inquiry, their limitations, their contributions to furthering other roads and inspiring other paths in my thinking towards a more organic vision for my continued inquiries and investigations.

The Origins of an Inquiry

In the search to make mathematics meaningful to the lives of students, I worked within the community center called El Puente, in Brooklyn, where I tutored high school students in mathematics. My students were poor and from working class backgrounds who were predominantly Dominican and Puerto Rican, with other students who were African-American or from the West Indies. This is the community where I grew up and lived, Williamsburg, and was therefore familiarized, and even had personal relationships (friendships) with some of my students and their families. Their forms of resistance, that I had initially understood as 'apathy', was further fueled by their 'anxiety' surrounding the content of the mathematics materials from their school work and the way in which they had been conditioned to 'do math'. I knew I had to try something different and relevant to each of us in order to make it pertinent, meaningful, and fun.

We began the process by putting out a questionnaire to see what things the students saw as mathematical in their community. Our task as facilitators, utilizing Paulo Freire's problem posing pedagogy, was then to try to problematize and operationalize their observations into their mathematics material. At the beginning many students said that they could not see any mathematics in their community and in their lives whatsoever. Through further discussions we challenged their sense of 'mathematizing' the world around them, and in the process they gave us clues that guided us inasmuch as they provided insights into what stimulated their curiosities to want to know more about issues in their community, as well as outside of it. What we finally developed with the students was something referred to as the "Math Pack."

The "Math Pack" was a product of the collaborative work done by the tutors with the students. The 'pack' was representative of three things:

- that the students work was not only valuable to us the tutors, but would be shared by the whole group as a meaningful expression of their abilities to generate mathematical ideas;

- that mathematics was not only pertinent to them, but it existed in every facet of their lives, as it influenced many of the ideas and the decisions that they made;

- that this community organization could meet the criteria set forth by the Department of Employment for their funding by using an alternative assessment tool developed by the students, in community, in the area of mathematics.

From Dominoes to Dominology

Of all the projects developed by the students during that year at this community center, the one that stands out the most for me was the concept developed by one of the students, Hector Arias, who was 15 years old at the time and attending Eastern District High School in Brooklyn. He developed a concept called *Dominology,* or the study of dominoes. In this study of dominoes he talked about how people in the community would socialize themselves around dominoes, as well as tell stories and find strategies to beat their opponents in the game. In sum, he was creating a type of ethnographic study of the world within which dominoes played, while describing the mathematical complexities of the game within the context of his community. Using Hector's work and ideas, we created a tournament of dominoes as well as doing fractions using the domino pieces, that we Xeroxed, and made copies for everyone.

Relearning Mathematics: Number, Power, and Being Counted

> The kind of mathematical literacy needed to clarify issues, to understand the structure of society, and to support or refute opinions is more than the ability to calculate. It is the ability to understand what numbers mean-how the number system is constructed to describe aspects of the world . . . in order to demystify learning processes, and to emphasize the empowering purposes of learning, so that you can build on your experience learning mathematics to learn anything you need in the struggle for personal and collective liberation.
>
> - *Marilyn Frankenstein, Relearning Mathematics*

As I began a more focused inquiry into ways in that mathematics learning and teaching could be made more meaningful in my work with students as well as raising consciencia of the racial, class, cultural and gender biases and assumptions that are created by the media and in many ways sustained by each of us. Graduate studies afforded me the luxury and the privilege to pursue these ideas. Many of the assumptions that are made with regards to mathematics are: boys are better than girls in mathematics, or that, if I can't do math I must be stupid, and most of the students thought they could not do math. Many of my students assumed that they must be stupid because they could not 'do' math. The logic sequence that they developed I understood as the following:

1. If I can do mathematics I am smart;
2. If I cannot do mathematics I am not smart;
3. Since I fail my tests in mathematics that means,
   I cannot do mathematics;
4. Because this happens therefore I am not smart.

I found that they also applied this same formula with all their other subject disciplines being taught in the public schools. This formula had a few corollaries attached to it as well, such as:

> If I am smart, I will go far, get a good job, be able to go to college and 'make it';
>
> If I am not smart, I will not go far, I won't be able to get a good job, or go to college and I will not 'make it.'

I interpreted these things as meaning that attached to performing well in school, particularly in mathematics, one needed to be smart, therefore one was capable of doing well and of becoming successful. What both my students and myself had not realized was that 'making it' in conventional terms had very little to do with academic success or failure, but more with whom your parents were, how much they owned and had endowed you with, and what elite schools were preparing you for further elite settings. What my students had no access to was the world of those who had 'made it.' They could not see was that docility, obedience, complacency, mediocrity, and a Machiavellian work ethos were what Pierre Bourdieu calls the “rules of the game” of these privileged settings. The equation that they had been sold, and conditioned to believe was not true at all, in fact it was one of the many logical equations that were falsehoods, because they were grounded or premised in false assumptions. Some of these assumptions were:

- that mathematics is a value-free, neutral, and exact science;
- if you are mathematically literate by the standards set forth by the Board of Education (or any other Institution that legitimizes the kind of knowledge deemed necessary to obtain competency or proficiency in an area of study) , you are intelligent, therefore useful, and you will be a successful and productive citizen In our society;
- the reason many of them could not "do math" was because they weren't' smart or capable enough to do it.

All of this work with students inspired me to investigate the history of mathematics of Puerto Ricans, and my own research interests.

An archeology of research

> Colonization carried forward by the armies of war
> is vastly more costly than that carried forward by
> the armies of peace, whose outposts and garrisons
> are the public schools of the advancing nation.
>
> (In Walsh taken from the United States Bureau of Education,1902, p.257).

Initially, the working title for the first draft of a proposed graduate research project was: *The Dialectics of Puerto Rican Numeration*. In this project my objective was to:

> . . . contextualize the different mathematical systems
> which Puerto Ricans developed from the
> Pre-Colombian peoples and societies, the
> African societies, and of the emergent
> creole enclaves on the island. This project is meant to
> look at how mathematics systems developed
> in places where working class Puerto Ricans resided
> on the island of Puerto Rico whether it be in
> the cafetales, (the coffee plantations) the canaverales
> (costal sugar cane communities) or the surplus
> labor force that was forced to migrate en masse to the
> United States.
>
> (Segarra, 1995)

The assumptions guiding the research project were that there may be some primary sources that would be available that would help further the mathematics investigation. A thorough investigation proved this hypothesis to be an overly ambitious one. That there had not been anything written about mathematics as it pertained to the Puerto Rican people, motivated a change in the direction and the subsequent momentum of the research.

The next phase of the research focused on finding materials around the working title *Puerto Rican Mathematics: A History of Mathematics on the Island of Puerto Rico*. Initially the intent of this research focus was to do an overview of the literature regarding mathematics education on the island from the end of the 18th Century through to the 20th Century. The idea for doing such work was to trace the historical trajectory from the Spanish colonial education system through the American colonial education system. As I became aware of the vast scope of this process I realized that there was a need to focus the research even further. I also intuitively knew that if mathematics education is value-laden it must therefore have a set of ideological apparatuses that fuel and inform this knowledge. As important was finding out in what ways the Puerto Rican people had developed their own numerical meaning making systems.

After taking some time to reflect on the this facet of my work, I realized that I was approaching mathematics as regards the Puerto Rican people from two distinct positions, with each position having a particular set of assumptions. They were:

- Puerto Ricans as creators/ producers of mathematical knowledge,
- Puerto Ricans as consumers of mathematics as a discipline of study

Through a detailed analysis of the role numbers and counting played in the lives of Puerto Ricans, I had hoped to garner a clearer understanding as to how mathematics developed as a historically, socially, and culturally constructed organizing modality. Furthermore, I assumed that:

a) knowledge is not value neutral;

b) the knowledge for popular consumption vis-à-vis forms of facilitated imposition for the colonized is determined by the colonizer to meet the imperatives of the colonizer;

c) it takes power and resources to determine how knowledge is created, legitimated, and replicated on a large scale.

I made a second set of assumptions that focused on the Puerto Rican people as consumers of mathematics in the form of disciplines of study. In particular I decided to focus on the progression of mathematics study as Puerto Rican children experienced it on the island after U.S. occupation in 1898. I began this search interestingly enough through United States Congressional Records, not by way of the Department of Education, but by way of the Department of War that was in charge of cataloguing things that were deemed necessary or important on the island, including the inhabitants of the island and its schools. Immediately what became evident from these records was the need to focus in on:

1) functions of a colonial school system on the island,

2) understand what an American Public School education meant as a vehicle for Colonization

3) investigate what were the first texts that were developed for such a purpose, (i.e. teachers manuals, curricular material, mathematics books)with a particular focus on mathematics education in the primary grades.

The beginnings of such a search afforded me the opportunity to see that arithmetic and number work were a profoundly important subjects of study for the newly created 'American public schools' curricula on the island. The amount of time allotted per day to arithmetic and number work, the expenditure of resources to facilitate its ease of instruction, as well as the importance that as many children become as numerate as possible lead me to further hone the area of investigation. What became increasingly evident was that arithmetic was seen as a foundation, used as an organizing principle if you will, for the numeracy of Puerto Rican children on the island. As Tobias Dantzig states in his *Number the Language of Science*

> Arithmetic is the foundation of all mathematics, pure or applied. It is the most useful of all sciences, and there is, probably, no other branch of human knowledge that is more widely spread among the masses.

What is truly fascinating is the idea that arithmetic as a non-neutral, value-filled area of study, contained within it the logic of colonialism, and the structure by which popular numeracy could be facilitated as a means of colonizing students to become Puerto Rican students numerate towards the needs of the American colonizers. Hence the working title for the future work: *The Arithmetic of Colonization: How we Counted and were Counted.*

I have conducted research to investigate the availability of primary research texts as regards arithmetic, education, teacher training manuals, as well as any original historical sources that will shed light on my theme of the arithmetic of colonialism in the University of Puerto Rico, that was founded in the beginning of the 20th century by one of the presidentially appointed U. S. Commissioners of Education, as the first teacher training center for the island. As a researcher and as a teacher, I have had the opportunity to see that historically the explicit and implicit roles of mathematics pedagogy in a colonial context, has been the maintenance of the intellectual, cultural, and educational state of affairs. In many ways the texts and the materials contained within them for example, "legitimate" and make "official" (Apple, 1991) the necessary knowledge for competency in an area of study, such as arithmetic. As a teacher, I am working to have students generate their own texts, driven or inspired by their own contexts, and then have them problematize and develop possible solutions/ strategies to their problematizations. This process of learning and teaching, begins with a respect for the learners knowledge allows them the freedom to generate, imagine, and problematize the world around them. Colonial projects generate, imagine, and problematize upon people, their land, and their minds imposing on the colonized, and the learners, values, ideas, contexts, histories that are foreign to themselves. Let us call this process where students mimic or duplicate and perform material that is not grown or born of their own experiences, intellectual and cultural alienation. Therefore to begin to counter such a process of pacification, an anti-colonial teacher allows the process of learning to become a place of spontaneity, generativity, and engagement. In my research I am searching for and exploring how mathematics education was used in Puerto Rico during Spanish colonization to enculturate students vis-s-vis textvooks, so as to valorize, and naturalize particular values, beliefs, and ways of knowing. In my classroom, for example, I use mathematics as an instrument with my students, who are in their majority workers or from working-class backgrounds, to dismantle and formulate refutations to arguments leveled against them, allowing them the freedom to develop material borne and reflective of their experiences, and to create a learning environment where solidarity and collective collaborative work, not individual achievement alone, constitute the rigor, the values, and the organizing principle behind their intellectual development in their mathematic educational work.

From Archeology to Digging in to do Work

As you may have observed from this process, it has not been a linear progression of thinking or of actions that have lead me to many of these transitory conclusions about my theory and my practice. But I am attempting to bridge my research and my pedagogy through the creation of a unique adult numeracy curriculum.

The overall movement of my organizing project is contained in the ¡VAMOS PA'LANTE ! learning strategy. ¡VAMOS PA'LANTE ! , means "Let's Go Forward!" in Spanish . The acronym stands for Voicing and Articulating Mathemetics Operating Strategies from a Progressive Applied Language And Numeracy Transfomational Ethos. This pedagogy is grounded in providing the learners the opportunity to come to construct thier own mathematics work and find the medium(s) through which to operationalize their knowledge. “Pa’lante Siempre Pa’lante!” is the phrase that Puerto Rican activists have used in their struggles in the United States to represent the sense of mission and of agency that are needed for all of us to actively address, challenge, and transform the structures of domination that systematically effect all working-class, poor, and people of color in the U.S.. In this case mathematics is the instrument to help move us Pa’lante.

As an instructor in the College for Public and Community Service, at the University of Boston my goal is to create a pedagogy that opens up new terrain's of possibilities. I envision a pedagogy that treats both the student/learner and the teacher/learner as historical and cultural beings, as co-creators in learning , and as language and mathematics as critical producers and consumers. A numeracy ethos, within this context, deals with students ability to extrapolate meaning from numerical data, (i.e., by reading and learning to interpret statistical data), to learn to en-code and de-code the words, the numbers, the world, toward substantive reflection and meaningful action. Numeracy in this context would be the numerical mirror and complement to a literacy project. By putting the power to create and re-create knowledge by allowing the actualization of the transformative potentials of these tools in the hands of those who work and study everyday, gives them another tool in their struggles.

As a pedagogical ethos, it allows for the creation and configuration of alternative modalities, spaces, and places from which to de-code and interpret their world. The role of this pedagogy is as a means of dealing concretely and tangibly with mechanisms and paradigms that devalue, demean, and suffocate. The objective of such a philosophy would be to further develop projects that would be articulated and trans-created both on the individual, and the collective levels to counteract structures that aren't necessarily meant or intended to generate pedagogical possibilities for concrete and grounded work.

As and educator who has attempted to uncover and chronicle the assumptions aas well as the “exteriority of accidents” in his research and practice, it is my hope that by stimulating students to engage in their own archeologies, they too will discover their own truths and as yet undiscovered interiorities, and ultimately learn that the fundamental measure of number lies in the simple and most elusive measure of being human.

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CLASSROOM-BASED RESEARCH: FROM *WITH OR ON* TEACHERS TO *WITH AND ON* TEACHERS

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1. INTRODUCTION

There are inescapable political and ethical issues for researchers as they engage in classroom-based research. Political issues are about power. They include questions such as, whose project/research agenda is it? Who participates? How do they participate? Who benefits? And how do they benefit? Ethical issues on the other hand are about responsibility, and they include questions such as: who is the researcher responsible to? Who is the researcher accountable to? What is the researcher accountable for?

Questions about *how* the researcher gains access to schools and then classrooms and about the involvement of teachers in the whole process are crucial ones to ask. Recently, Vithal (1998) raised the issue of the relationship between the researcher and the participants in the research process in the South African context and in particular the attitudes of the participants to the researcher and the research. In many instances researchers are viewed as people who use teachers and students for their own benefit. These concerns, inevitably, come from a history of research tradition in South Africa where ethical issues have consistently been marginalised (Vithal, 1998).

During my presentation at a recent research conference, a representative from one of the teachers' unions challenged me. He wanted to know "where are the teachers you are talking about?" and "What did they benefit from the process?" It is important to note here that the representative did not ask the same question to a presenter before me who had mentioned that he bought gifts for the teachers as a thank you. While I had bought teaching aids as gifts for the schools and teachers who were involved in the research to use with their colleagues, I did not mention this during the presentation. I do not think that buying gifts either for the teachers or for the schools answers the crucial question about how and what teachers benefit by being involved in a research project. The common sense assumptions about what teachers can benefit from involvement in a research project is that the benefits should be in the form of material gifts. Today many researchers buy gifts or even pay teachers or schools to get involved in a research project. This for me raises questions about what will happen if teachers and schools expect or get used to being paid for involvement in a research project? Who will be privileged/advantaged or disadvantaged by this expectation? In this case ethical and political issues slide into each other and are not easy to disentangle. Is it ethical to pay teachers to get involved in a research project? If it is ethical to pay teachers for participation in a research project, then this raises political issues about disadvantaging and advantaging researchers. Is this good for research?

Within educational research, the political and ethical issues between research and teachers have pivoted on notions of working *with* as opposed to *on* teachers. In this paper I am going to take issue with the dichotomising between researching on and researching with teachers because it posits power as being unidirectional. In my research I have struggled with the questions: am I working enough with the teachers or am I abusing my access and my power as a researcher. I think there are ethical ways of working with teachers that understand classroom-based research as working with and on rather than with or on teachers. From my experience, I do not think the researcher is all-powerful and the teacher powerless in the hands of the researcher.

I am going to take issue with this kind of dichotomising and challenge the common sense view that when researchers move into a school they do that to do research on teachers unless it is a mutually constituted, collaborative project from the start. We cannot wish away the power relations between the researcher and the teacher, however, if research is done in an ethical way there is bound to be mutuality about it. My argument will be based on a research project where I came in with an agenda and my learning through that project is that when teachers agree to participate in a project they too come in with agendas. Therefore I want to move the debate from a dichotomy between *with or on* to understanding research in schools and classrooms as a process of *with and on* teachers. I am going to talk about this as a reciprocal power relationship.

I am going to describe what I did in my project, the issues around negotiating access and working with teachers. I will highlight some of the problems and complications involved in doing classroom-based research in South Africa, in so doing I will point to the political and ethical issues that are inevitably part of any research process.

2. THE RESEARCH PROCESS

The main aim of my research was to investigate and explore ways in which teachers' language practices affect the learners' communication of mathematical ideas and concepts in multilingual intermediate[1] classrooms in South Africa. As is typical in a qualitative study, I needed to work in depth with a small number of teachers, therefore I selected six Grade 4 teachers together with their learners to observe and interview during the first phase of the study. Out of these I further selected three who I observed and interviewed for the second phase. These three teachers have been involved in the study for two years now. It was important therefore that I worked with teachers who were really interested and also that I found a way of working with them that would keep them interested. As a black researcher and also given the fact that I could not afford to pay these teachers or schools for being involved in my project I wanted to make sure that teachers do not feel abused. I wanted them to adopt my project as theirs, to feel that as much as this is my project it is also theirs, to have ownership of the project.

3. NEGOTIATING ACCESS

3.1 To Schools

Schools are systemic entities; they are linked-in with a whole set of communal and social relations. In South Africa, schools are located in communities who in some instances contributed towards the building of the school. The School Governing Bodies (SGB) which consist of the parents, teachers and learners run them and principals are in charge of the day to day administration and management. The schools' first line of responsibility to the government is through the district offices. Within the schools there are hierarchies and therefore a Power structure that controls them (here I use Power with a capital 'P' to show hierarchical power). The Power relations in a school are at multiple levels: the government department in the form of the district office, the community, the principal, the teachers and the learners. So, when a researcher wants to do research in a school, with whom and how should they negotiate access/entry? One of the important questions to consider is: whom does the school belong to? This is important because it gives direction as to who the researcher needs to negotiate with to gain access to the school and the classrooms. There are at least four possible answers to this question:

[1] This refers to Grade 4 to 6 classrooms which mainly have learners aged between 9 and 14.

1. To the government department: if this is the case, then the researcher needs to negotiate with the nearest district office. From my experience, research is not a priority in district offices. It is in fact seen as benefiting the researcher and not the district nor the schools/teachers. This perception makes the negotiations for access into schools very difficult. In my case, this negotiation involved committing myself to doing maths development workshops with the schools/teachers in the study. I was able to do this because the organisation that I worked for had an agreement and funding to work with the schools in the area and therefore my development work became part of that initiative. For some researchers it might not be possible to make the kind of commitment I made due to financial and time constraints.

2. To the community in which the school is located: if this is the case, then negotiations should be held with the community structure that works with the schools in the area. In the area where I did my research, for instance, I was told that the Community Empowerment Forum co-ordinates all development programmes in the area (including the schools). While I did not negotiate with the forum to gain access to the schools, I informed their chairperson of my plans to do research in the area. Community structures like these may be seen as less important. However, in South Africa they have the power to disrupt any research project if they are not consulted and therefore they should not be ignored.

3. To the school community (principal, teachers and learners): One of the most difficult factors when negotiating with the schools relates to perceptions that exist in schools about researchers and their agendas. Many schools see researchers as people who use them to get more qualifications or money and therefore are ambivalent about participating in research projects. Of course, many schools have been victims of researchers who drove in, collected data, drove out and never came back to share their findings, and therefore some schools are very sceptical of anyone wanting to do research in their school. The other factor is that choosing a particular school for a research project raises questions like: why our school? Is there something wrong we are doing? This is particularly the case in black schools where during the apartheid era 'authority' always seemed to find fault with the school or the teacher. For my research it was important to get the principal's agreement first before negotiating with the teachers because I did not know the names of the specific teachers I wanted to work with. Permission to involve learners in the project was done through the teachers rather than the parents. In South Africa, there are practical constraints of requesting permission to involve learners in a research project. These involve the fact that many parents cannot read or write. It is important however that as these issues get more and more attention in discussions about research, more suitable ways of seeking permission from parents can be devised.

4. To the School Governing Body[2] (SGB): Handling requests to do research in a school could be one of the functions of the SGB, particularly because it has representation from parents, teachers and in secondary schools learners as well. There are, however, problems around SGBs in many schools in South Africa since this is a relatively new structure. Training for SGBs, particularly in the black schools only started in 1998 and in my view, some of them are still not sure of their roles yet.

Whichever approach one uses to gain access to schools, the point is that as a researcher one should be ethical and this includes recognising and showing respect to everyone involved in the research project. Researchers need to respect teachers as fellow human beings who are

[2] Policy around School Governing Bodies was instituted in 1997 in the South African Schools' Act.

entitled to dignity and privacy. Researchers also need to show respect for educational research in order to enhance the image of research (Bassey, 1999: 74). Respecting teachers as fellow human beings and showing respect for educational research can often be in tension. Holding this tension is important. It is therefore crucial that each of the above mentioned structures is not ignored when negotiating access to schools because they can actually threaten the whole research process.

The other factor here is that there are other stakeholders within a school who may not have the Power but can use their power to deny the researcher access into the school (here I use power with a small letter ‘p’ to show individual agency or the ability to do or act). These may include ordinary teachers, parents or learners in the school who may refuse to participate or picket at the gate on the day of the researcher’s visit or withhold information. Unlike Power, power is never unidirectional; it shifts and is about ownership.

To gain access to the schools in my project I approached the nearest education district office for assistance to identify possible teachers. Even though I needed six teachers for my study, I asked the district to identify eight teachers. This was done so that if any of the teachers dropped out of the project I would still remain with at least six teachers. A discussion on how I gained access to the teachers follows in the next section. I then held meetings with each of the principals to share my research and also offered them development workshops after data collection. A letter that also required the principals to give me the permission in writing to do research in their schools followed each of the meetings. One out of the eight principals declined to participate. The principals did not have to state the reasons why they do not want their schools to participate. This for me was important because as I mentioned earlier, it was important that I get schools that were interested in the project.

Political issues, therefore, in negotiating access to schools are about Power and power. As indicated earlier, Power in a school situation is about hierarchy, it is also about managerial and administrative authority and superiority, while power is mainly about ownership and social influence and does not reside in any one person or a group of people.

As a researcher, my main aim was to do research in these schools. I have learned, from my own experience as a teacher involved in a research project, that everyone who gets involved in any research project benefits in one way or another. However, as to what or how an individual benefits depends on what they want to benefit and their kind of involvement. The kind of involvement that teachers can have in a research project is to a large extent defined or determined by the researcher. As part of my research agenda, I needed to observe and interview one grade 4 teacher and a few learners at each of the schools. While I had made an offer to the schools for development, I did not have a defined agenda for it. The schools therefore were given the space to define an agenda for development in their schools. In this case when I talk about the school I am referring to the principal, all the teachers, learners and also parents.

The nature of the offer I made the schools meant that individual schools could ask for different things at different times. It also meant that the schools could work together on a common programme and then present it to me. Only two out of the six schools involved in the project took me up on this offer. The demands from these two schools were also different. In one school, I was requested to run only one workshop throughout the year while in the other school, I was given a timetable of dates for workshops every two months and lesson demonstrations in some of the classrooms. While all the six schools were involved in the same way in the project in terms of the research, benefits would inevitably accrue in different ways. This is mainly because each of the schools had to decide how they would benefit from the presence of a researcher in their school. Even the two schools that took my offer for development work must have had different benefits because each of them made a decision or choice regarding the type, duration, frequency and content of the development work.

So, what we are seeing here is that I had two roles in the schools, at one level I was a researcher and at another level a teacher educator. The nature of my relationship with teachers as a teacher educator was hierarchical and therefore I had the Power. While as a researcher my relationship with the teachers was reciprocal and therefore had shared power with the teachers.

While in a school, a researcher engaged in classroom-based research, collects data in classrooms. In many schools access to classrooms is not negotiated with the principal but with the teachers.

3.2 To Teachers

After gaining access to the school, I also needed to negotiate access to the classrooms with the teachers I wanted to work with. The problem here was that I did not have specific names but criteria, which I shared with the principal. In addition to teaching mathematics in grade 4, needed to be multilingual, qualified (posses an M+3 qualification[3]) and experienced (at least three years teaching experience). Only one teacher per school was needed. In schools where there was only one grade 4 teacher, the teacher was asked to participate and in schools where there was more than one, they were all called and after explanation of the research project a volunteer was requested. A letter was then written to each of the teachers who volunteered to show in writing their agreement to participate in the project.

One out of the seven teachers in the schools that agreed to be involved declined to participate. Initially this teacher was willing to get involved on condition that a video recorder was not used in her class. One of the factors impacting on negotiating access to classrooms with the teachers is the lack of trust that teachers have about anyone wanting to observe their lessons. There are also questions such as ‘why me?’ and suspicions that the researcher is being used by those with Power (e.g. government department) to give information about the teacher’s competencies for the purposes of redeployment or retrenchment. In fact, the teacher who declined to participate raised these issues with me and it was not possible to convince her that the district office would not have access to the video recording of the lessons. Even after agreeing with her not to use a video recorder for data collection in her class, she was still suspicious and therefore we (myself and her) agreed to exclude her from the study. What this shows is the shift of power from the principal and the researcher to the teacher when it comes to the classroom.

It is important to note here that, agreement to participate was not just individual but also a contextual matter. The research took place during a period of transformation and restructuring in South Africa and so for the teachers, this was both a time of threat and opportunity. A time of threat because the department of education had just announced numbers of teachers in each school that would either have to be redeployed or retrenched. The teachers thus could not be

[3] An M+3 qualification means that a teacher passed grade 12 and completed a three year teaching diploma either at a training college or university. Given the number of unqualified and underqualified teachers in South Africa, it was important to specify these qualifications and teaching experience for my research to diminish the possibility that the teachers’ language practices were due to lack of teaching experience or recognised qualification.

sure about how their involvement in the project was going to factor into whether they got redeployed or retrenched. The six teachers, who agreed to participate, therefore came into the project with no job security. This was also a time of opportunity because, with the transformation there were a lot of new senior positions either within the schools or in the district office that were being advertised that teachers could apply for. Here again, teachers could not be sure about how their involvement in the project would enhance their possibilities of being appointed to the positions they apply for. These are issues that researchers will come across when doing research in an unstable environment.

One of the issues that some teachers raised was why they were not getting paid for being involved in the research project. This question was raised because some of them had been involved in research projects in which they were paid. While I had included in my research budget ‘gifts for schools/teachers’, I did not want to tell teachers about these because I do not think this should be the reason for teachers to get involved in a project. I therefore made them aware that what they benefit will depend to a large extent on them. I of course made them aware of the offer to do development work in the schools. The teachers, however, had the responsibility to initiate and co-ordinate these development workshops.

From my experience, there is not an easy way into the classrooms or an easy way out of these issues - whichever option the researcher chooses it is possible that there will be problems. Another fact is that having been given permission, for instance, by the district office (or any government office) does not mean that the researcher will be accepted with open arms in the schools and classrooms. In fact, in certain instances, the fact that a researcher has obtained permission from the district office (or any government office) creates problems at schools. This usually happens if a conflict exists between the schools and the particular government office (e.g. district office).

4. GIVING FEEDBACK

The important questions here are: who is the feedback for? When do you give feedback? Why do you give feedback and how do you give feedback? All these questions are about ethical issues of responsibility and trust. As a researcher, I am firstly responsible to the research community and so the feedback is for this community. This feedback can be given in a written publication or verbally at conferences and it is mainly for the growth of the community and growth of knowledge. Secondly I am responsible to the teachers in whose classes I collected data and therefore should give feedback to them. Feedback to the teachers can also be in writing or verbally, however it may be different in form and even content to the one given to the research community. The differences are due to the differences between the practices of teaching and researching. For instance, in giving feedback to teachers the focus may be on teaching and learning while giving feedback to the research community also include issues about researching. Thirdly, I am responsible to the principals and the district office particularly if access to the school was done through them.

In most of the schools that participated in my project, the principals asked for feedback at the end of the week of data collection. This can put pressure on any researcher because principals seem to expect an evaluative feedback from the researcher. In my case I insisted on a feedback meeting with all the maths teachers and the principal or whole staff where possible and this was done for the purposes of transparency. In these meetings I just gave descriptive comments about the lessons I observed and promised to give detailed feedback after the analysis.

I believe that the teachers who were directly involved in the study should be given first priority in terms of getting feedback after the analysis. In this way they can challenge the researcher on any interpretations and claims made without evidence. In my project, each of the teachers was given a copy of the sections in which their lessons were analysed and asked for comments. They then gave their comments and corrections, which were then incorporated into the report. During these feedback sessions with the teachers, there were teachers who raised disagreements either about what they said during the interview or some of the practices that I observed in their classrooms, like chanting. In these cases, I had a conversation with the teacher about what they thought they really said or why they thought chanting did not happen during the lessons observed. I further used recorded interviews or a videocassette of the lesson to convince the teacher that what was written in the report was accurate. These conversations with the teachers can be used as data for further analysis. The kind of changes that I had to make related mainly to the details about their qualifications and experience which I had recorded inappropriately in the case of one teacher. These feedback sessions were also opportunities for other teachers to learn about new concepts like ‘discourse’. Going back to the teachers for feedback is thus not for “respondent validity” (Silverman, 1993: 159), but it is about responsibility, respect for the teachers and trust.

In ethical research, responsibility has to do with trustworthiness. When teachers allow a researcher into their classrooms, they do it out of or with trust. I believe that in ethical reciprocal research, teachers should be given feedback as soon as data analysis has been completed in order to further this relationship of trust.

I have so far dealt with the political and ethical issues in South Africa that relate to access and giving feedback. It is true that these issues can occur in any research context, however, in South African where education was politicised during the apartheid era that lead to a lot of resistance by the oppressed masses, these issues cannot be escaped. In the next section I will draw from experiences in my research project to discuss the nature and extent of the teachers’ involvement. This will lead to a discussion on reciprocity, which is the main substance of the paper.

5. TEACHERS’ INVOLVEMENT IN THE PROJECT

As mentioned earlier, I went into the schools with a well-defined research agenda. I formulated the questions, decided on the research design and methodology and time frames. I also had criteria to guide who could be involved in the project. While I wanted teachers to be involved in this project as partners and also recognised the fact that as a researcher I am not an omnipotent possessor of knowledge, there was no space for teachers to change my research agenda (or should I say I did not make any space for teachers to change/shape my research agenda). This non-negotiable agenda was shared with the teachers before they committed themselves to being involved in the project. The challenge here was and remains creating a way for the teachers to own the agenda with me.

While I had a clear research agenda, I am a teacher educator and at the time of the research had done some teacher development work in that area in the past. What this means is that I had/have strong views about what a teacher development programme in that area should involve given the recent political and education policy changes in South Africa. The other thing is that three of the six teachers knew my background and had been in some of the programmes I conducted in the area. It became clear as the project progressed that for some of the teachers the motivation to volunteering to being involved in the project was due to their knowledge of the researcher as a teacher educator.

> *In my case, it was not for the first time I saw you, so at least I had attended about three of your workshops, three if not four. So I knew your stuff. I was excited and thought if she comes here it means we are going to learn more, a lot, even though some of my colleagues were scared. (Reflective group interview: 1999: 2)*

Even though I had explained that I am coming in as a researcher, these teachers continued to see me as a teacher educator. This of course impacts on how they participate and their expectations of what they will benefit from being involved. Given their multiple identities in terms of culture, gender, age, histories and future plans, these teachers who had an experience of working with me as a teacher educator participated in different ways and had different expectations. However compared to the other three teachers who did not know me, these teachers had a history with the researcher that shaped (consciously or unconsciously) their expectations and participation.

Irrespective of whether teachers viewed me as a researcher or teacher educator, the power relations operated in my favour. Teachers saw me as an expert and this was evident in how they responded to me during and after lesson observations. Almost all of them, particularly those I have a history with, asked me after observing a lesson, "How was the lesson?" For this question, the teachers expected a response from me, which would characterise them as either 'good' or 'bad'. This was very difficult because whilst I did not want to judge them, I am human and there are lessons that I generally prefer and those that I do not like for a variety of reasons. On the other hand I did not want to communicate my lesson likes or dislikes to teachers or declare them 'good' or 'bad', however, whatever I said teachers heard what they wanted to hear. Even a simple or vague response such as "interesting" was interpreted as meaning 'good'. I then decided not to give any comment at all. Whenever teachers asked, "How was the lesson?" I responded by explaining that after I have done an analysis of all the lessons I will give them detailed feedback on their lessons. It also seemed that the teachers wanted to please me either as a researcher or teacher educator.

> *Kgethi: Did you feel during my observations that you had to meet my expectations, which you did not know?*
> *Lindi: Yes, it happened. Like I said that I knew you before you came for the research, so I know what you have been facilitating all about, when you were talking about OBE and so on. So sometimes I wanted to be up to standard, then at the end of the day I would say but I don't know. Let me do my thing. But there is an element of saying hey, I hope I am doing the right thing.*
> *Kuki: I also had the very same feeling, because from time to time I would go to Kgethi and ask: "Am I on the right track? Am I doing the right thing?" This was an indication of uncertainty. I was not sure whether I am doing the right thing or the wrong thing. You know I was a bit worried. Mmh to be honest (Reflective group interview: 199: 3).*

As a researcher I cannot ignore the fact that my presence in the classrooms is not benign. This is captured in the above extract and of course raises questions about the kind of data produced in these classrooms. During the reflective group interview, which happened 15 months after the start of the project, all the teachers indicated that they were more free, more relaxed and had an idea of what I expected from them in the second year of the study. This interview was with the three teachers who I continued to work with for the second year. In the extract that follows the teachers talked about some of the things which changed due to my presence.

> *Gugu: Actually switching is not allowed in my school. So, when you came for the first time, last year, I did not switch at all, you see, but when you are not*

there, to speak the truth, we usually switch. But when you came, I did not know what you expected, what you wanted from me. I thought that maybe we are not supposed to speak Zulu at all, so they did not switch. I told them uguthi (that) 'no switching' you see. But when I started to see uguthi (that), okay Kgethi, I mean she does not discourage this switching, I told my learners to be free. We started to be free the way we use it, the way we usually learn some other days when you are not around. But the first time they did not switch at all because I told them not to switch.
Lindi: Oh, you wanted to abide by the school policy, language policy, thinking that maybe Kgethi was spying.
Gugu: Of course. (Reflective group interview: 1999: 8)

Given what Gugu is saying in the above extract about her teaching in the first year of the project, what does it mean for the data that I collected in that period? And what does it mean for the analysis?

This brings us to another important but related issue concerning why teachers get involved in a research project (especially if they do not know the researcher)? One of the teachers (Kuki) pointed out during a group reflective interview that she got involved because she wanted her work to be recognised. So, teachers also have agendas for getting involved in a research project. Teachers have a choice to make their agendas explicit or keep them implicit. Kuki made her agenda clear: she had been teaching for many years and felt that she was due for promotion. At certain points during the research she requested me to write letters to either the principal or district officials about her involvement in the research project.

One of the undertakings I made the teachers when I started with the project was that their real names would not be used in any reports on the research. I also promised that permission will always be sought if videos of their lessons were going to be used for any purpose and that if I wrote any paper on their teaching they would be given an opportunity to read and comment on it. While not using the teachers' real names in reports is ethical and the teachers had agreed to it, Kuki backed down on this agreement. When she was given a paper based on her lessons to read in which pseudonym had been used, she insisted that she would like her real name to be used in future. This is mainly because Kuki is a very confident teacher who believes that she is doing good work and so wanted her work to be in the public domain. Of course the other teachers were also confident, however, they had different agendas and therefore whether their real names or pseudonyms were used they did not have a problem. The agreement about the use of names in reports was therefore changed.

As indicated earlier, there were instances when I needed to use the videos of these teachers' lessons for other purposes and each time I would request permission to do so. It was interesting, however, to see their excitement each time I asked them for permission. When I enquired why the excitement, Lindi indicated that if their videos are used for workshops with other teachers then it means there is something to learn from them and that made her feel good about it. The teachers also mentioned that I do not need to get permission from them before I used the videos.

What the above discussion highlights is that reciprocity between the researcher and teachers in a research project enables teachers to pursue their own agendas while fulfilling the researcher's agenda. In this kind of relationship research is inevitably with and on in both directions. Teachers will come in with agendas and these will work on and with the researcher; the researcher also has agendas that will work on and with teachers. The point here is how do we strengthen this reciprocal relationship?

Working *with* is about partnership with the teachers and an important part of partnership is agreement. In this case then research becomes a mutually constituted, collaborative project with the teachers. The researcher and teachers are equal partners: they both belong and also participate equally. Working *on* is about subjugation. In this case teachers are just subjects of the research and they may or may not benefit from participation. Working *with and on* is about reciprocity and it includes negotiation and choice. In this case both the teachers and the researcher benefit from the relationship. This is different from partnership in the sense that while both the researcher and the teachers receive or benefit from participation in the project, they are not equal partners, the researcher has Power. Both partnership and reciprocity involve choice and are ethical. In a relationship of subjugation teachers usually do not choose whether they would like to participate in a research project or not. For instance, if a school is selected as a pilot school by the government department and researchers have to evaluate that pilot project, both principals and teachers do not have a choice about whether they want to participate or not. However, they are still not powerless in the research process, they can still choose how they participate. For instance, if a teacher does not want to be observed, he/she can play sick during the periods when they are supposed to be observed. Therefore any research relationship involves a certain amount of choice.

Looking at the relationship between the researcher and teachers raises questions about the attack that I described earlier from the teacher union representative at a recent conference. If research is conducted ethically, then teachers will have choices and to constitute them as being used is actually to undermine them. While the attack is important for pushing researchers to reflect critically on the politics and ethics of their work with teachers, it does not understand political and ethical dimensions of research. To constitute teachers as powerless, undiscerning participants in research is actually to contradict the motivation for attack.

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FORMATTING POWER OF MATHEMATICS - A CASE STUDY AND QUESTIONS FOR MATHEMATICS EDUCATION

by
Ole Skovsmose and Keiko Yasukawa

Abstract: In this paper, we will argue the thesis that mathematics, as a constituent part of an apparatus of reason, has socio-technological agency. Mathematical agency is manifested by what we call its 'formatting power' - that is, the ways in which mathematics can both constrain and generate socio-technological imaginations. Taking a case study for our thesis, we illustrate the germ of a methodology for a 'sociological' investigation of mathematics, and analyse our findings against a theoretical framework of the formatting power of mathematics. The test case which we discuss concerns the mathematics of encryption.

1 Introduction

Tests as descriptors of intellectual ability, performance indicators in work-places, identification of people by their 'digital persona', and GNP as an international measure of 'progress' - these are only some of the measures used to position individuals and social groups in relation to the wider social structures.[1] If we consider the social implications of such examples, the notion of the *formatting power of mathematics,* the idea that mathematics can influence, generate or limit social actions, becomes a believable, seductive concept. However, it is a concept which also needs to be scrutinised and teased out.[2]

We seek to examine in particular the notion that mathematics, embedded or 'packaged' in technological systems, has formatting power. We will 'excavate' the mathematical make-up of a particular information technology used for electronic mail (e-mail) security - PGP, an abbreviation for Pretty Good Privacy. As part of our concluding remarks, we raise some questions about the implications of the formatting power of mathematics for mathematics education.

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[1] See, for instance, discussions about social indicators in Eckersley (1998), and Dorling and Simpson (eds.)(1999).

[2] For a discussion of the formatting power of mathematics, see, for instance, Skovsmose (1994, 1998, 1999) and Yasukawa (1998).

2 Mathematics and the Apparatus of Reason

The descriptive, predictive and prescriptive functions of applied mathematics have been recognised, for example by Davis and Hersh (1988: 115-121). However, such a framework for thinking about mathematics could lead one to believe that there are specific and predicated objects of description, prediction and prescription. What we posit here is that in appropriating mathematics as a way of making meaning of a social or technological phenomenon, mathematics can itself become part of and inseparable from the phenomenon. This in turn, we would argue, serves to both limit and generate new ways in which social realities unfold.

By the *apparatus of reason*, we refer to a *resource for carrying out technological actions and decision-making concerning technology and for constructing new technologies*. We choose a broad interpretation of technology, including not only artifactual technologies such as computers and automobiles, but also abstract systems such as forms of economy and communication management. Engineering science has been a resource for carrying out 'classical' technological actions (for example the design and control of physical processes and environments), but technology is now developing from a broader base, and is both influenced by and influencing social and abstract processes and environments. Generally speaking, any action is informed and directed by a set of resources. By the apparatus of reason we refer to one such resource which is relevant for interpreting a range of large scale socio-technological actions. We argue that mathematics constitutes and important part of this apparatus of reason.

3 PGP - a cryptographic technology

To illustrate how mathematics constitutes an apparatus of reason, we will examine the role that mathematics plays in a specific piece of 'intellectual' technology - cryptography.

Cryptography, the process of encoding messages to achieve confidentiality, is a technique which could most readily be associated with warfare - coding of secret strategic information about enemy movements, dissemination of false information to fool enemy intelligence, leakage of intelligence information, and so on. Interest in cryptography has now expanded far beyond the military sphere into the wider social spheres of commerce, and to personal communication.

Suppose we call the original message or the *plaintext* P. Then the secret message or the *ciphertext* C is produced by a transformation of P by an *encryption* function, say E. The relationship between P and C is:

$$C = E(P)$$

Decryption is the process of recovering the plaintext from the ciphertext through some function D:

$$D(C) = P$$

The function D is the inverse function of E: the decryption function reverses the action of the encryption function.

Modern cryptography refers to encryption and decryption functions which take the form of mathematically based computer algorithms. In talking about modern encryption, we are assuming a computerised process where the original message in natural language has already been converted into a suitable machine readable representation. The process of providing a machine readable representation of a message is, however, not essential to the problem of encryption.

Encryption (and decryption) algorithms, except possibly those used in military and intelligence work), are public algorithms. PGP is a software package which was developed and in 1991 released by an individual Phil Zimmerman (rather than a commercial software firm) in the USA to provide electronic mail and file storage security.[3] One can purchase (or in the case of PGP freely download) these encryption packages. It may appear that making algorithms public would make encrypted communication less secure. The consensus, however, has been on the contrary; the more widely exposed algorithms are to potential hackers, the more confidence one can have in ones that survive. Because the algorithms themselves are public, the confidentiality is established by a 'key'. In cryptography, a key is needed for encryption (locking) as well as for decryption (unlocking).

Modern cryptography can be classified into two distinct approaches: (1) *conventional*, *single-key* or *symmetric cryptography*, and (2) *public-key* or *asymmetric cryptography*. In *symmetric* encryption, the same key is used by the sender to encrypt, and the receiver to decrypt the message. The big step forward in cryptography was to avoid the need to use the same, or closely related, keys for encryption and the decryption. Using the same key for encryption and decryption requires a system of key distribution - somehow the key which has to be kept secret between the two communicating bodies must be safely distributed from whoever generates the key to the other. Any capture of this key by a third party during the process of distribution compromises the confidentiality of the communication.

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[3] See Hoffman (1996).

The public key system, on the other hand, relies on an *asymmetric*, or two key system where each party has a *public key* which is known and shared by both the source and the destination (and could be known to the wider public without compromising the security system), and a *private key*, unique and known only to the each of the parties. Hence, a message is encrypted by a sender *A* using the public key of the receiver *B*. This message can only be decrypted by the intended receiver *B* using the private key which even the sender does not know. The strength of the public key schemes is based on the difficulty of determining the private key from the knowledge of the public key and the ciphertext alone, while the generation of the public and private keys with additional information known only to the holder of the private key is computationally simple.

4 What is in the package?

At the surface where PGP is implemented and used, PGP's mathematical artefacts are invisible. People implementing PGP would typically download from the Internet or purchase from a vendor, a whole *package* within which the various public-key and symmetric algorithms reside. Whilst in some cases, a version of PGP may offer options in features such as key sizes, the user would not look beyond the *technical specifications* of the product to determine whether or not it is the appropriate (meaning reliable), secure, user-friendly, machine compatible product to buy. The package would have a user-friendly *interface*. Thus at the level at which PGP is implemented and used, the 'formatting power' of mathematics is elusive.

When we ask 'what mathematics makes up the package?', we are effectively asking what mathematics underpins each of the different algorithms which make up the package. In particular, we are asking how functions such as encryption, decryption, and secure key exchange are facilitated by mathematics.

For public key systems, implementation involves the generation of the public and private keys, the encryption of confidential data into the corresponding ciphertext using the recipient's public key, and the decryption by the recipients using their private key. Compromising the system involves determining the private key from knowledge of the public key and possibly some ciphertexts. The class of mathematical functions upon which the designers of public-key systems have employed to achieve the requirements of the systems is what is known as *trap-door one-way functions*. These are functions where the function values are easy to compute but where, given a function value, the inverse (the value(s) which the function acted upon) is or are difficult to compute without additional information. An example of this is a function which takes two prime numbers and computes their product; multiplying two prime numbers is simple, but determining the prime factors of an arbitrary number, especially a large number, is difficult. This prime factorisation problem is in fact the mathematical basis of the algorithm underpinning the PGP secure key exchange service.

There exist infinitely many prime numbers; a proof of this was constructed by Euclid, and the statement of this result is known as Euclid's Theorem. The implication of this result on the analysis of encryption schemes is two fold: a hacker trying to determine the private key would have to go through the process of searching for a pair of prime numbers over an infinitely large set, while those generating the key pairs have the benefit of an infinite set from which to choose a suitable pair of prime numbers.

The process of encrypting P is the following: P is raised to a certain power e, but only the remainder of dividing that number by a pre-determined natural number n is kept. This remainder, C, is then the encrypted version of P. Thus the encryption is expressed by the equation

$$C = P^e \mod n$$

The decryption of the encrypted key is achieved by the application of the equation

$$P = C^d \mod n$$

The second equation is of the same form as that for the encryption process: a number is raised to a power d (d being the decryption exponent) and the remainder from dividing by the number n will be the desired number, in this case the ciphertext C. [4] As mentioned earlier, these encryption and decryption functions are public - anyone who cares to know how the RSA algorithm works will be able to find out this much. That it is possible to choose e, d and n relating C and P as expressed by the encryption and the decryption equations relies on number theoretical results.[5]

The actual implementation of this method presupposes that the parameters of the algorithm, namely the encryption and decryption exponents e and d, and the modulus n have been pre-determined in some way, and communicated to the appropriate parties. In

[4] In modular arithmetic, each number is an element of an equivalence class of all numbers which share the same remainder when divided by the modulus. Hence, in mod 3 arithmetic, the numbers 2, 5, 8, 11, ... all belong to the same equivalence class (when divided by 3, there is a remainder of 2 in each case). In this text, when a statement such as $P = C^d \mod n$ is made, it signals an assignment of the smallest positive element of the equivalence class of $C^d \mod n$ to P .

[5] See also Kobliz (1994), Loshin (1998), Massey (1988), Schneider (1996), Stallings (1999), and van der Lubbe (1998).

particular, the receiver of the ciphertext message must know, and be the *only* one to know the private key (d, n), and the sender must know the public key (e, n).

Providing a system which is secure means firstly, ensuring that the private key cannot be captured by an attacker. Putting aside a direct capture of the private key, and in particular the decryption exponent d, security of the private key means preventing its easy derivation from the public encryption exponent e and the modulus n. A number theoretical clarification will show that determining the decryption exponent d from the encrypting exponent e and the modulus n, depends on the factorisation of n.

A systematic 'hacking', would therefore depend on the availability of a reliable and computationally efficient algorithm for factoring n. Although factoring is a simple process to describe mathematically, it is a time consuming task when we are working with large numbers: 'At present, a 200-diget number that is the product of two 100-digit numbers cannot be factorised in any reasonable time ... In fact, not so long ago, the most efficient factoring algorithms on a very fast computer were estimated to take 40 trillion years, or 2000 times the present age of the universe.' (Schroeder, 1997: 131)[6]

5 New significance of mathematical formulae

If we study classics in Number Theory, like *Introduction to the Theory of Numbers* by Hardy and Wright, then it is clear that no kind of applications to anything external to pure mathematics are considered.[7] The study stays well within the walls guarding the purity of mathematics from the contamination of the real world. However, the significance of a mathematical theorem is relative to the context where it is applied. When a theorem is presented in relation to the derivation of another theory or in a textbook, it might appear 'pure' and insignificant as far as its impact on lives on most of the rest of us; but when it appears in an application package, such as PGP, its significance may be completely different. In fact, this change of significance is an essential new departure point for the philosophy of mathematics. In cryptography, classical and fundamental results in Number Theory, like Euclid's Theorem, the Prime Number Theorem, Euler's Theorem, and Fermat's Little Theorem, have come to acquire significance well beyond the walls of pure mathematics.

Thus, the effectiveness of the RSA algorithm (a public key algorithm used in PGP), and in particular, its key generation algorithm, rests heavily on the ability to find suitable

[6] In a previous version of the book Schroeder wrote: 'At present (1983), a 100-diget number that is the product of two 50-digit numbers cannot be factorized in any reasonable time ...'

[7] See also Landau (1958) and Baker (1984).

pairs of primes p and q for forming the encryption modulus n. To this end, one of what Hardy calls a 'real' mathematical theorem (Hardy, 1967: 91) is central - Euclid's theorem that there is an infinite number of prime numbers. '... Euclid's theorem is vital for the whole structure of arithmetic. The primes are the raw material out of which we have to build arithmetic, and Euclid's theorem assures us that we have plenty of material for the task'(Hardy, 1967: 99). Thus not only is Euclid's Theorem a resource for mathematicians but it is also a vital resource for cryptographers by ensuring an infinite pool of prime numbers from which to choose suitable pairs. Hardy, however, assumes that as important as this theorem is in building the theories of arithmetic, it has little practical relevance: 'There is no doubt at all, then of the 'seriousness' of either [Euclid's or Pythagoras's] theorem. It is therefore the better worth remarking that neither theorem has the slightest 'practical' importance. In practical applications we are concerned only with comparatively small numbers ... I do not know what is the highest degree of accuracy which is ever useful to an engineer - we shall be very generous if we say ten significant figures .... The number of primes less than 1,000,000,000,000 is 50,847,478: that is enough for an engineer, and he can be perfectly happy without the rest' (Hardy, 1967: 101-102). Hardy not only predicts incorrectly the uselessness of Euclid's Theorem, but also the critical importance on the size of numbers for some engineering applications. It is recommended that the size of the prime numbers for RSA encryption is in the order of 75-150 digits (van der Lubbe, 1998: 143; Stallings, 1999: 181; Schneier, 1996: 467).

6 The formatting power of mathematics

Mathematics can 'materialise' out of its abstract existence as symbols and theorems and emerge as a functional entity which drives a computer package. This package becomes a functional entity, and it provides a specific meaning to the more general claim, made, for instance, by Bell (1980), that knowledge and information become intellectual technology. The package can be installed and implemented, and its implementation 'makes a difference' on a number of fronts - social, political and technological. A package, underpinned by mathematics, functions in sharp contrast to any 'scientific theory'. It materialises into both the physical world and people's economic and social reality. Mathematics gets a new significance being part of a package.

Each society exhibits technological propensities, that is, each society has capacities for building and realising new technological innovations, some of which come into fruition. This propensity for technological actions is represented by the apparatus of reason. And mathematics is involved:

(1) By means of mathematics new technological alternatives are presented - alternatives which are not possible to grasp and identify without mathematics as a tool for

analysis and construction. However, mathematics also limits the set of hypothetical situations which are presented, as mathematical construction is only one way of expressing a sociological imagination. (2) By means of mathematics we can investigate particular details of situations not yet realised. A particular strength of mathematics is to enable hypothetical reasoning, which refers to reasoning about technological details of a not yet realised technological construction. However, mathematics may also produce blind spots concerning the effects of such a not yet realised construction. Those effects cannot be foreseen until they emerge when the technology has been implemented. (3) When choices are made and the technological ideas are translated into new technological and, hence, social realities, mathematics simultaneously 'enters' this reality in a concrete form. Mathematics assumes an essential functional role within technological packages, and once that happens, the mathematical influences on this reality becomes inseparable from the other social realities in which it is acting. Mathematics will have become 'socialised'.

We take these three aspects as a summary of what can be meant by the formatting power of mathematics. In the case of PGP, the technology arose as a response to a perceived threat on people's privacy by a move towards government controlled cryptographic systems in the USA - systems which allow government bodies to hold the encryption keys, hence disabling truly confidential communication between parties. Both the technology which presented the threat, and PGP which was developed to counter the threat relies on mathematical results and reasoning. The mathematical basis of these algorithms allow each of the competing bodies to hypothesise the levels of threats or counterthreats, with reference to the mathematical effort needed in 'cracking' each other's algorithms. In this way, mathematics becomes part of a fundamental social struggle over the right to privacy.

7 Conclusion

The apparatus of reason does not in itself provide resources to establish new social forms and relations, but it represents society's overall dispositions for socio-technological action. It produces new spaces for technological actions (this is the constructive element of the apparatus of reason) and helps to clarify particular aspects of certain elements of this space (this is the analytic element of the apparatus of reason).

The PGP example, and encryption more generally, are technological approaches to confronting perceived problems in the structure of trust in society. From perceived needs for these technologies, and the interactions (sometimes competitive) between these technologies, we see the formation of new types of social relations which are defined by these technologies. Even the 'purest' mathematics can be included in packages.

Mathematics represents a powerful resource for the ongoing, rapid and unpredictable development of the apparatus of reason: New technological options are generated. Technological innovations are supported by mathematics, because mathematics often helps to establish hypothetical situations and analyse particular aspects of (some of) these situations. In this was mathematics provides a resource of a technological imagination. Eventually, mathematics becomes part of the social reality in which the technological actions are finally carried out.

This analysis of the formatting power of mathematics, while limited to only one case study raises some questions about implications for mathematics education. If mathematics has formatting power in many more ways than encryption, then what new imperatives does this present about the teaching and learning of mathematics? Can, and should mathematics be taught without explicit reference to the part it plays in the apparatus of reason? Is it possible to contemplate a thorough social inquiry without mathematics being an dimension of that inquiry? And if as educators, we want our students to question and challenge the social conventions around them, then wouldn't an appreciation of the formatting power of mathematics have to be a critical part of their education? We are not prepared to answer these questions at this point, but we feel that these questions need to be on the agenda, not only for mathematics educators, but educators more generally.

Note

This paper is a shortened version of Skovsmose and Yasukawa (1999). The inspiration for tracking down the formatting power of mathematics through an 'excavation' of PGP emerged from our conversations at the conference Mathematics Education and Society in Nottingham in September 1999.

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Policy, Labour Markets and School ‘Pathways’: school mathematics and social justice

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This paper argues that school mathematics education, of all the curriculum areas, is a key mechanism in the production and reproduction of social inequality. The proposition is based on a combination of what interests school education policy-makers, strong signals from the global labour market and illustrations from some Australian Year 12 ‘pathways’ studies. Paradoxically, these very conditions provide the opportunity to fulfill some of the social justice aspirations that radical mathematics educators espouse, if there are changed conceptions of how mathematics education fits schooling, society and economy.

PREAMBLE

Zygmunt Bauman tells a salutory story. Until the late 1980s, intellectuals always considered culture to be their special sphere of influence and authority. They strenuously defended their right to define culture against the encroachments of the state. When the interest of the state in culture waned, intellectuals saw themselves as beneficiaries. However, culture was opened up to market-led consumption as entertainment rather than to the free reign of intellectuals. Bauman’s point is that intellectuals accused the new cultural rulers (record companies, publishers, web site developers, managers of mass communications) of imposing a homogenized standard on what was now presented as a previously rich and diversified cultural field. ‘Cultural uniformity’, Bauman (1992: 17-18) says, ‘lost its attractiveness when someone else --- forces beyond the intellectuals’ control – was to set its standards and preside over its implementation’.

Current discussions about curriculum development and its implementation seem be a parallel conversation to what Bauman describes. Over the last decade or so, the clash between liberal-humanist education and what might be called neo-liberal and vocational views of education have dominated curriculum policy discourse. Education professionals are worried about the intrinsic worth of education for the individual person, and with the Good Life rather than with making a ‘good living’. They tend to condemn new regulatory and accountability mechanisms as ‘managerial’. Above all, there is strong resistance from educational professionals to any real or perceived erosion of their rights and privileges to define what counts as necessary knowledge, competence and essential practice.

On their part, governments and their education policy-makers view education as an infrastructural input into social and economic planning. All major civic institutions are conceptualised as contributing to social capital, including schools, voluntary and charitable associations, firms and businesses. There is new interest in identifying and doing something about exclusionist institutions and generating tolerance among individuals. Citizenship, wellbeing, employability, life chances and educational success are central to this policy agenda.

Governments and some intellectuals believe that in a period of speed and connectivity, the role of education is too important to be left to educators. Performance criteria like demographic analyses of educational outcomes that point to massive inequalities and international comparisons of school system indicators put pressure on the professional judgements and practices of the professional education establishment. Its voice is often heard as a defence of flawed institutions. Governments are not interested in pumping more resources into the same basic structure that is unlikely to provide the changes that are needed for newer agendas.

Thus, everywhere there is pressure on school curricula to change and to assume new roles. ‘The purpose of education systems is to prepare young people in appropriate ways for the challenges and responsibilities they will face throughout their lives’ Bentley (1998: 38) says, ‘and if society is changing, so should the way in which we introduce young people to it.’ Mathematics education is hardly an exception.

Consonant with this temper, I argue that school mathematics education, of all the curriculum areas, is the essential mechanism in the production and reproduction of social inequality. I base this proposition on a combination of what interests school education policy-makers, strong signals from the global labour market and illustrations from some Australian Year 12 ‘pathways’ studies. Paradoxically, these very conditions provide the opportunity to fulfill some of the social justice aspirations that radical mathematics educators espouse, if there are changed conceptions of how mathematics education fits schooling, society and economy.

SOME THEMES IN SCHOOL MATHEMATICS EDUCATION

Several themes in the mathematics education literature catch my attention. First, there are those propositions about current patterns of school mathematics teaching that reinforce a curriculum of algorithms and calculation. At the classroom level, it is reported, many students continue to experience a traditional mathematics curriculum of memorization and procedures. Teachers talk and students listen and practice whatever they practise. These patterns are deeply sedimented in school systems, schools, teacher cultures, teacher education, and society. A second theme is that teachers' personal theories about content and their pedagogical content knowledge influence what they teach and how they teach and evaluate curriculum content. Some mathematics teachers like teachers of history or English or whatever are fixated by the power, internal structure and elegance of mathematics as a discipline. They are wont to think about theory and abstraction as the most highly valued centre of their discipline. The beauty and elegance of mathematics rather than learning is the main focus of mathematics education for such people. Still others take different tacks and there are many ways in which mathematics teachers think about their discipline. Nevertheless, together these two themes generate and sustain a certain insularity in mathematics education which is of particular concern for this group of mathematics educators.

A third theme is that waves of mathematics education reforms, like many other kinds, often appear little more than cosmetic and anyway, they soon pass. The fundamental curriculum, or what students are expected to learn, remains largely unaltered. The most used pedagogies, or how students are expected to learn mathematics, seem timeless. In this respect, mathematics education is unremarkable. Overall, pedagogic innovation is the less developed aspect of innovation in education and training.

Finally, within these broad parameters, there is a strong, emergent concern with the social consequences of mathematics education. There is recognition that students are systematically excluded by class, race, gender, and location and so on from learning and using mathematics. It is hardly surprising then that mathematics educators, like others in curriculum studies domains, generate even more research about how people learn mathematics and how barriers to learning might be removed. This work often carries strong emancipatory intentions as people struggle to overcome culturally dominant and hegemonic curriculum and teaching practices. These are crucially important and noble concerns for someone with a cultural theory background and who has been involved in curriculum policy formation as well as teacher education.

At the risk of oversimplifying the existing work of mathematics educators, my purpose is to focus attention on policy matters that bear directly on school mathematics curricula and pedagogical outcomes. These outcomes are fundamental to emergent patterns of social and economic wellbeing for individuals and society. They challenge the purposes of school mathematics education. My purpose then is to broaden the context of the discussion about how we might think about school mathematics today. Let me elaborate.

EDUCATION, SOCIETY AND ECONOMY: POLICY CHALLENGES FOR MATHEMATICS EDUCATORS

One of the most important signals for school education is the work of the OECD, thus:

> Across OECD countries, governments are seeking policies that make education more effective while searching for additional resources to meet the increasing demand for education. The OECD education indicators enable countries to see themselves in the light of other countries' performance…indicators presented represent the *consensus of professional thinking* on how to measure the current state of education internationally. They reflect both on the human and financial resources invested in education and on the returns on these investments (OECD, 1997b, emphasis added).

Notice the emphasis here on ‘countries’, ‘governments’, ‘resources’, ‘demand’ and the will to judge outcomes. For many school and university-based educators, there is much to complain about in such an agenda. The discourse signals supply-side policy directed at social investment in education and infrastructure projects. Such ideas seem especially distasteful to some when put against the prevailing socio-institutional and educational practices, pedagogic arrangements and the modus operandi of schools and teacher education. They clash with a set of values, dispositions, attitudes and expectations that maintain boundaries between ‘education in itself’ and education as an instrument or input into the economy (Cullen, 1998). They are a direct challenge to the relative autonomy of the educational sphere and educators whose professional ethos defines accountability and monitoring as matters internal to the professional field.

I am of course just as capable as others of railing against the encroachment of political and economic agendas on professional domains, with two caveats. First, in dealing with these new agendas, educators are forced to define themselves in relation to others rather than by their own peculiar combination of ideas in the professional field. This seems like a good idea to me. That is, attempts to transform an education system include ‘whole-of-government’ strategies that link economic strategy, family services, community planning and so on. If failure to achieve at school has multiple causes, then tackling it requires strategies that bring together agencies that more usually work in isolation, if not antagonism. Policies and programs that appear to have little to do with education may therefore contribute to the raising of achievement in schools, especially government schools given their social location and roles. This more rounded approach to the transformation of education is needed to counter the cynicism and lowering of morale that have followed unrealistic and poorly grounded reform programs based on intervening in the education sector alone in the past.

Second, I see few reasons to defend school systems that unremittingly produce too many students with no formal qualifications and whose lack of basic competence drastically reduces their social and economic life-chances. This is a greater threat to the basic solidarity or cohesion of societies and liberty than global markets and attempts to overhaul education systems as part of a strategy to create sustainable conditions of economic improvement (Dahrendorf, 1999: 25). Moreover, OECD policy formation is grounded in assumptions about the efficacy of education and its effects on individuals, societies and economies. It is assumed that education is of value to individuals, social solidarity and the economy simultaneously. In turn, education policy is aimed at inclusiveness as a social and economic imperative because exclusion means the development of an underclass in today’s world. Thus, I am a future-oriented optimist interested in educational reform, despite the risks of political and economic excursions into education. I think risk and flexibility are opportunities, as well as threats (Beck, 1999).

I have laboured these points because I think there are significant professional and social opportunities for mathematics educators in what appears to some as a bleak landscape. School mathematics holds a special place in education pathways that lead to higher education and training and successful jobs and careers. These pathways are now recognised as fundamental elements of government policy. Moreover, mathematical knowledge and skills are now seen as essential for all students if they are to have life chances in the kinds of trends that I now highlight.

LABOUR MARKET TRENDS

Before proceeding, it is important to disassociate school mathematics from the general claim that future jobs will require ‘advanced technological skills’, the vision of a high-technology economic future. The November 1997 issue of the USA journal **Monthly Labor Review** for example, forecasts that in the period 1996 to 2006, ‘technicians’ account for a mere 3.7% of the US labour force. After adding more ‘technical’ categories, the MLR concludes that technological skills still account for only a quarter of all jobs. There is a growing awareness that people in jobs of the future will need to have some technical skills, such as using computers, but it is not technology as such that will define their jobs.

Carnevale and Rose’s (1997) report undermines the image of an America splintering into lucrative high-technology jobs and low-skilled, low-paying jobs. They based their analysis on what workers actually do instead of the industry in which they work. Jobs are classified as ‘elite’, ‘good’, and less-skilled. In place of the factory-floor jobs that are typically counted in census surveys, the data show that in the USA, between 1979 and 1995, ‘Office’ work accounts for 59% of all new jobs. This is a much higher proportion than at any time in the past. Office jobs include accountants, technicians, salespeople and a thousand other specialised positions developed to deal with a service economy. They have names that demonstrate their newness -- facilitators, co-ordinators, consultants -- because they did not exist a generation ago.

Five ‘office’ categories account for about 70% of all jobs: executive (10.5%), professional specialist (15.2%), marketing (17.1%), clerical (8.7%), and service (16.7%). Of the 53 million office workers in the USA, about 1.5 million are science-based professionals, such as engineers, architects, and chemists, who also work in an office. In 1995, 44% of Office jobs were ‘Elite’ and ‘Good’ (business managers and professionals). When the 20% of low-skilled service sector is added to these categories, there is not a lot left for the high-tech science/math jobs that are often proposed as the trend for the future.

The US Department of Labor's Dictionary of Occupational Titles categorises jobs by the levels of skill they demand in language, mathematics, and reasoning. The authors point out that the jobs being lost are almost all classified as ‘low skilled’. But, paradoxically, 71% of new jobs will require only low or moderate levels of skill in language, 67%

will require only low or moderate levels of skill in reasoning, and fully 84% will require only low or moderate levels of skill in mathematics (D'Amico, 1997: 31-32).

Applying Carnevale and Rose’s analysis to the Australian economy indicates that jobs in the ‘Office’ now represent 43% of the Australian workforce - that's 3.3 million jobs in a country of 18 million people. Salaries and educational qualifications tend to be higher in this sector and over 60% of what Carnevale and Rose refer to as ‘Elite’ and over half of the ‘Good’ jobs are found in this sector in Australia (ANTA, 1999).

These job characteristics are part of what Leabeater (in OECD, 1997a: 5) describes as two ‘very powerful trends.’

> First, a growing share of what we produce and consume is “immaterial”: information, judgement, analysis, service, entertainment, advice. The assets we use to produce these immaterial goods are increasingly immaterial as well. We increasingly rely on information technology, software, design and personal skills…
>
> Second, the generation, application, orchestration and exploitation of knowledge is becoming critical to how companies, regions and economies develop and sustain competitive advantage.

In the knowledge society, school and subsequent education underpins future security, wellbeing and opportunity in at least four ways (OECD, 1997). First, individuals who complete Year 12 are likely to be more employable than those who fail to do so, even when that person subsequently completes a vocational education and training (VET) qualification. Again, year 12 leavers who are employed, tend to earn more than those who do not complete year 12. Third, there are links between qualification levels attained by a population and economic performance. Finally, education contributes to a wide range of social benefits including greater social cohesion, lower crime and better health (Giddens, 1999).

Accordingly, the OECD (1998: 93), in a recent examination of the policy implications of differences in human capital measures, including qualification profiles between countries, pointed to two ‘key shortfalls that should concern any country’. They are the number of adults who have not attained upper secondary [ISCED 3-7] qualifications or the measure of the adult capacity to extend knowledge and skills. The second is the proportion of the population that does not display the level of literacy and other skills required to tackle the demands of 21st century life and work. This is the capacity for using human capital.

Now, it seems to me that these are very important indicators for school education outcomes. Carnevale and Rose show that while a higher proportion of workers than before are doing well, middle and low income workers in comparison with elite workers, have fared significantly worse since 1979. Their report points to the emergence of two tiers of American workers also reflected in Australia and I guess elsewhere. Carnevale and Rose provide a telling portrait of how education levels and their consequences divide a society.

The payoff from post-year 12 education comes, Carnevale and Rose contend, not so much from the technical skills acquired. Instead, the pay-off comes from the fact that such an education is *necessary* to break into the categories of jobs that sustain more than basic existence. The new Office economy will reward the people who know how to deploy technology, integrate technology, and market technology, but better technology alone does not win. Staying at school then and achieving a year 12 + education in a *qualification pathway* matters very much indeed.

The kind of policy work undertaken by the OECD and the study by Carnevale and Rose raise questions about the *type* of education that best fits emergent social and economic conditions. The Office economy may require communication skills, social ease, and basic reasoning abilities as much as, if not more than, technological expertise. However, acquiring such knowledge and skill today however general they seem may only be possible through higher education. In higher education, students are exposed to a sophisticated culture, a variety of experiences, and varying disciplines that require analysis of facts and concepts (Madrick, 1998). In this sense, education has become a basic meal ticket. Inadequate curriculum design and pedagogy offered in inappropriately organised schools will (is!) consigning millions to reduced life chances and permanently low incomes. School mathematics education is deeply implicated in this chain of processes which brings me to the policy issue of school 'pathways'.

SCHOOL EDUCATION PATHWAYS

Schools differentially convey symbolic social power on their students. They do this in a dynamic process that is partly internal to school education and learning and partly shaped by socio-economic and political factors. The dynamic cannot be modified at will yet nor can it be thought of as completely determined by economic or ideological circumstances. I now briefly survey some Australian evidence of how the dynamic works at the end of high school in order to show that mathematics education is centrally implicated in the accomplishment of social inequality (Long, Carpenter, Hayden, 1999).

The Australian evidence shows that girls are more likely than males to have completed year 12 and participated in higher education by age 19. Moreover, young people from higher parental status, better parental education levels, and greater family wealth are more likely to have completed Year 12 and to have participated in higher education by age 19. They are less likely to have participated in vocational training courses than other students. Success in university-entry level mathematics is central to the profile.

Similarly, students from elite independent (private) schools are more likely to have completed year 12 and participated in higher education directly from school. Moreover, students with higher achievement scores based on reading and mathematics (standardised multiple choice reading and mathematics tests) were far more likely to have completed Year 12 and participated in higher education by age 19. To identify an ideal type, high SES, non-English speaking females who attended private schools in urban areas and who had high previous achievement in English and mathematics are more likely to participate in higher education. Success in university entrance level mathematics is a defining feature.

While the detail may vary in different places, the international research literature over several decades indicates that such links between the distribution of school outcomes and the reproduction of social order are endemic.

The gross relationships however hide the mechanisms, especially the effects of Year 12 subject choices and combinations on future education and work experiences. As post-compulsory education has become mass-education oriented, there are general pressures on school systems and schools to provide a more diverse curriculum that is inclusive of differences in motivations, abilities and learning styles. The main policy issue for these systems is how to maximise the value of the extra years of post-compulsory education for individuals and for social purposes. In short, there is a challenge to ensure that different subjects and combinations of subjects offered to all students in the post-compulsory years are adequate pathways to something worthwhile educationally and to future life chances.

Once again, the Australian experience is that high SES young people from private schools, and non-English speaking backgrounds who are high achievers, select combinations of subjects that lead to higher education and the professions (Lamb & Ball, 1999: v). These include mathematics and science subjects such as advanced mathematics, physics and chemistry a few other pathways. Students of low SES background tend to participate in academic strands that are less demanding and contain subjects such as ‘general mathematics’, biology, business studies and computing.

There are clearly discernible patterns in subject combinations and and their effects. Again, this is the expected pattern and can be interpreted as evidence that the organisation of the curriculum in the senior years reinforces social, cultural and academic differences among young people. It does this by making available school knowledge and school credentials in differential ways so that different groups of students are attached to formalised pathways that lead to differentiated opportunities and ultimately Carnevale and Rose’s divided society.

In addition, the Australian data suggest that, over and above the type of school a student attends, social origin, gender, ethnicity, and previous achievement, *participation in particular combinations of subjects* matters for access to future life chances. I imagine this is true elsewhere as well. In short, the curriculum also has *independent effects* on the social distribution of school success. Some combinations of subjects increase the likelihood of participating or not participating in higher education, and vocational education and training, or neither. Crucially, all of the ‘Elite’ and ‘Good’ job-bound combinations contain mathematics beyond the elementary level.

It is important to register the contribution of previous ‘achievement’ on Year 12 subject choice and subsequent pathways. Low achievement is not surprisingly associated with low SES so that the hierarchy of subjects becomes a social hierarchy (Teese & Charlton, 1999). Because there ‘can be no doubt that the achievement of students in the early years of secondary school has an impact on school trajectories’ (Lamb & Ball, 1999: 19), achieving satisfactory Year 12 outcomes becomes a P-12 policy matter. Low achievers are likely to leave school early because of the deterioration in their school experiences linked to failure. Low achievers who stay on, are likely to select subject combinations that lead nowhere compared to those high achievers who select maths and physical science type combinations. Their minds are constructed according to the very order of things (Bourdieu & Wacquant, 1992: 168). Differences in early school achievement lead to segmentation and increasing homogenisation of the student body. In turn, particular kinds of strands at the end of high school are 'residualised' as high and low achievers end up in different places. This is not to say that there are no differences in capacity amongst students but to point out that advanced mathematics, physics and chemistry subjects are overwhelmingly taken by high SES rather than low SES students. How is it that the independent effect of the curriculum has such an impact?

To summarise, there are three propositions that bear directly on the work of school mathematics educators. First, curriculum and pathways patterns demonstrate that mathematics is the 'new Latin' in so far as it acts as the *gatekeeper to most other worthwhile post-school education, training and vocational aspirations* for the young.

Second, mathematical knowledge and skill are important elements in the social capital required for participation in the emergent 'knowledge economy'. Such knowledge and skill has to be *transferable and applicable* while remaining, nevertheless, *mathematical*.

Third, the mathematics curriculum, expressed as subject choices in Year 12, has a decisive *influence on post-school outcomes.* School mathematics educators, more than in any other curriculum area, are strategically placed to make a difference in the lives of young people by curriculum and pedagogical intervention.

In short, the mathematics education profession has a special responsibility to ensure that school mathematics is a passport, or transfer, that enables students to proceed rather than being a maze in which students are gridlocked from the earliest years. The primary rationale for improving school mathematics is not competitiveness, but equity: There is no justification for approaches to mathematics education that filter out those with greatest need and equip only the high SES students for productive high-income careers.

INNOVATIONS

Mathematics is now important in many areas where it has not previously played much of a role including the social sciences. Mathematically based techniques are increasingly used in the workplace and have transformed the way decisions are made and business is done. In the interests of social justice, there is a need to improve the mathematical foundations laid at school level for all students, whether or not they are likely to proceed to higher education (The London Mathematical Society, 1995).

There is no room for ‘dumbing down’ the curriculum further. The common curricular alternative of vocational or consumer mathematics provides a narrow range of skills limited to middle school topics and is devoid of the conceptual understanding required in a knowledge society (Forman, 1999). If mathematics defines valued pathways, we need to discover how to open them up for all.

It appears to me then that there are two high priority research and development issues in school mathematics education that go to the core of the matters I have discussed. The first is that of *identifying appropriate pedagogies* for teaching school mathematics. Here I refer to the concept of pedagogic practices, especially the invisible embedded in visible pedagogies that act selectively on students of different social backgrounds (Bernstein, 1990). As Gardner (1999: 126, 130) suggests, the art of teaching for contemporary conditions inheres in ‘aiding students to acquire the moves and insights of major disciplinary fields.’ If it proves too difficult to convey a topic’s interest and relevance, then one ‘should probably seek another entry point’ (Gardner, 1999: 130).

The second then is that of the *packaging and presentation* of school mathematics. The use of IT to change the modality, pace and sequence of mathematics teaching need to be explored as ways of creating ‘generative topics’ designed for understanding (Gardner, 1999: 130). The challenge for school mathematics education is to provide school leavers with the ability to use mathematics-based tools without requiring that they prepare for mathematics-based careers.

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A ‘democratic classroom’, but who speaks loudest?
Research with basic mathematics students[1]

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Abstract *This paper presents five ‘snapshots’ of data collected by adult basic maths students taking part as co-researchers in a project to investigate the discourse of adult basic maths classrooms. The work is presented against a background of the Freirean influence on adult literacy and numeracy; the notions of ‘empowerment’ and ‘dialogue’ in the Freirean model are challenged. I argue that these samples present evidence that some of our usual ways of gathering data are flawed, and further that a research culture in the classroom fosters shifts in both classroom and research discourses.*

When I started work as a part-time tutor on a research project into discourse in adult basic maths classrooms, I invited the students to take part as co-researchers. In this paper I use snapshots from our joint work to raise questions about ‘empowerment’ and research. Four course groups, each between six and ten students, were involved over a period of two years (with some overlap when students continued for a second year), meeting for two hours a week. Most of the students had difficulties with reading and/or writing and spelling, and also attended literacy or English courses. Some have become actively involved in designing the project and see themselves as co-researchers; others were less consistently involved but took on specific pieces of work, including data collection and/or analysis. Here I look at five examples of our work, which I will argue illustrate approaches to teaching and learning maths but which also present challenges to some of our usual ways of gathering data.

1. Maths life histories

Four students in one group drew line graphs of their maths histories. This work was done as part of an introduction to line graphs, and came after about 15 meetings of the group. In that time, we had already discussed students’ experiences of maths in some detail, and all had done some writing about it (some dictated it, to enable them to get round spelling problems) which was read in the group.

Joyce drew a graph which started high, with a written comment ‘Handling money, but you didn’t count that as maths’, then slid down: ‘decided no good at maths when I started adult classes’. Emma said hers would be flat: ‘I need the paddles, like in *ER*!’ *ER* is a US hospital drama; Emma’s idea of graphs uses the image of a cardiograph, with a flat graph indicating a stopped heart. In fact her completed graph for her adult years was highest when she had paid work and consistently

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fell when she was unemployed; paid employment and confidence in maths seem for her to go together. One student recorded on her graph that she had been to special school and had had speech difficulties; one of the dips in her school maths had the comment ‘Went down in maths when doing graphs etc’, thus telling us that the particular section of course we were starting reminded her of one of the worst times at school. Carol’s was flat but *high*; she said she had always enjoyed maths, even though she had not had a successful school experience. These graphs were shared with another group, where Sandra said, ‘Oh, I hate graphs, I hate that paper!’ She then drew her own map of her maths experience, on plain paper.

We haven’t space here to go into everything the graphs showed. We discussed them at length in the two groups, and in the discussion the writers added extra comments. We noticed that all the graphs had a y-axis which represented confidence (rather than, say, skills in maths, or time spent studying or using maths) and all had an x-axis which represented age. So the students’ measurement of ‘up’ or ‘down’ in maths revolves around confidence rather than technical skills, and the graphs are a new way of showing autobiography.

This seems, then, for some people to be a useful way of representing maths histories and an alternative to some of our more usual approaches. The students were perhaps creating a new genre, using visual images, numbers and words combined. The graphs get round the need for good technical reading and writing skills, and offer a vehicle for group discussion and analysis. However, they also raise an issue about how students and tutors get to know each other. We had already worked on personal maths histories, and indeed my notes of the discussions and the students’ writing form part of the data for my research. The graphs revealed omissions in the earlier work. I hadn’t known that adult maths education knocked Joyce’s confidence down; that Emma’s maths was tied to employment; that one student had a history of speech difficulties; that two people dreaded or hated work on graphs; or, indeed, that one was entirely cheerful about maths. The point here is not that graphs are a magical way forward (some students, for example, chose not to do them at all), but that we should be more aware of how much of their experience students may choose to withhold from the group or the tutor. The liberal discourse of adult basic education holds that tutors ‘build on students’ experiences’; we need to be more sceptical about how much we understand of those experiences.

2. An interview: ‘the course is a bit wishy-washy’

We use interviews as a way of trying to understand what our students think, or what their experiences are. But one piece of work shows the limits of what we find out.

Pat and Cathy interviewed each other about writing their maths diaries. I gave them a tape recorder and some questions, with one open-ended question at the end, about whether there was any advice for students or tutors. I meant the ‘advice’ to be about using maths diaries. This is part of the ‘advice to tutors’:

Cathy	I have enjoyed the course, but sometimes I think it is a bit wishy-washy. You get told that you have done very well because you're *almost* right, or on the right tracks. But in maths I think you're either right or you're wrong. I wish you were told you had got it right, or you had not got it right. It's kidding yourself.
Pat	But you are gaining more than you thought you would. I think the confidence I do have is because of the teacher. Being adults, and having children of our own, and feeling inadequate when our kids come home and we're not able to help them - having the right teacher and being in the right atmosphere and company, it does help.
Cathy	Yes, and maybe that's why the teacher never says, 'You've got that all wrong'. What would be the point? You probably wouldn't come back. And it's only Basic Maths, perhaps at this stage it's not all that important.

The students seized a bit of time that was less directly controlled by the tutor to debate and work on issues that I had understood quite differently. This interview gave me a view of the course that would usually be closed to me. I thought I was teaching a course that might help people to see themselves as creators of maths; to understand maths is not always right or wrong; and to understand maths is socially constructed. Pat and Cathy saw me as *kind* to them; I confirmed for them that they are only 'basic'. This is a revelation about a relationship of patronage.

The interview was supposedly about diaries; Cathy and Pat used the one more open question to tell me much more important criticisms. Pat played the tape for the rest of the group; it was not meant to be a private word in the teacher's ear. They knew I would be listening to the tape, so this is not in any way a 'window' into their 'true' ideas about the class. It does however suggest that the more usual interviews, in which the tutor/researcher interviews the students, may be very restrictive.

3. The students' meeting: 'Tutors are sort of loudly spoken'

A group of eight students, from two centres, worked together to organise a meeting for students about maths. The meeting, held in a community centre in South London, was attended by about forty students, who came from four different organisations (a Further Education college, two community centres and a local authority education service). One issue was the dominance of tutors who had come to support their students. These are some comments from student organisers, made in discussion after the conference:

> Tutors, I thought there was no tutors! At the back please, keep quiet, just take notes, I felt like saying. The whole idea was for the teachers to listen, and take notes or whatever, and for us to do everything. [A tutor] kept on

> asking questions, she was directly looking at me, and do you know what I mean? I felt really intimidated. (Shazia)
>
> I think the tutor should have took a back seat and let the students interview the students, and just listen to what's being said. (Jeremy)
>
> Tutors are sort of loudly spoken. (Lorraine)

I should stress here that I (a tutor) did not see any evidence of domination by tutors, other than the anger and anxiety generated in the students. Our rôle as tutors, with all the influence and authority it carries, makes us intimidating, however democratic our intentions. The message is that to discuss how maths education should be organised, students need space without tutors there.

The students used most of the time at the conference to discuss the curriculum and compare experiences of South London maths education. These are some quotes about the curriculum:

> The woman [a tutor] was saying you need maths for measuring and all of that thing what you is doing, and I thought, how can she ask those questions? Why can't she just go? (Antoinette)
>
> The [question] that got me, is the one with, which is the best thing, going through text books or doing ... news articles. And I said both, I said both. (Shazia)
>
> Many different ways to do maths. (Notes from a small group discussion)

There is not one curriculum. People want choices, and they don't want tutors telling them what they need. The conference was a success, and led to ...

4. A magazine of students' writing

Global Maths, a 52-page magazine, was written, edited and produced by students, and includes maths life histories and reports of the discussions at the students' meeting. It represents a written report, addressed to both students and tutors, of some of the students' research in this project, and is thus a challenge to the public discourse of adult basic education, where it is taken for granted that students don't do research: aside from their 'lack of basic skills', they 'lack confidence'.

I analysed data from the magazine and the conference, and found the students' reasons for studying maths fell into four main areas (with much overlap): for everyday needs, including shopping, measurement, and employment; to recover from previous failure; to help children; and for enjoyment and intellectual challenge.

The problem is - so what? This does not distinguish basic maths students from research mathematicians. This leads me to a difficulty about generalising about students. Generalisations we can make, like this one, are useless because they are not specific - they cover the population of the world. Other generalisations are rejected by students, as we saw in their discussion of the curriculum.

5. Classroom observation

One group decided to observe their own class. I assumed one or two would do the observing, and I took in six sample observation schedules for the group to choose from, or amend. In fact the group of six students decided they would *all* observe the class, each using a different schedule. We did it for two lessons, and then four of the students together collated the results.

Tutor's questions to ...	**group**	**women**	**men**
Genuine question - wanting to know the answer	1	8	6
A question to find out if the student knows something.		2	11
A question to help the student work something out.		3	6
Other sorts of questions	3		3
Totals	4	13	26

Here I reproduce the collated results of just two of the observations: 'Tutor's questions', and 'Who does the talking?' (The others were students' questions, timed observation of class activity, teacher/student interaction and students' responses to the class.)

This first table shows the collated results of the 'Tutor's questions' observation schedule. In it we see that I asked the two men twice as many questions as I asked the four women.

Names ⑦	Andy	Alison (tutor)	Priya	Trevor	Theresa	Carol	Joyce
Talking to whole class		3		1	1	1	
Talking in a small group or a pair	6			2	9	11	7
(Students) Talking to the tutor	4	6	6	6	2		1
TOTALS	10	9	6	9	12	12	7

The next table shows *'Who does the talking?'*

Theresa, Carol and Joyce all have a higher score for 'talking in a small group'; in the group discussion about the observation results they said they enjoy working together and helping each other.

This data is inaccurate, of course. Every observer was also observed, and meanwhile we were trying to continue 'ordinary' classes. We all had to remind each other to keep observing, and we ended up spending most of both the observed classes laughing at each other. I was listed as talking to myself six times, so we can see some of the tallying was inaccurate (and this was also shown by a tape transcript of one of the classes). However, that's not the point. The observation exercises, and discussion of the data (both the transcript and the tally sheets) changed the discourse of the classroom. For example, my lesson plan for the first observed class included individual work with two students (who happened to be men), and I knew I would balance it the following week - but that plan was mine, not the students', and what was apparent to the group was simply the teacher spending time on the men. After the observations and discussions, one of the women was able to interrupt one of the men talking to me: 'You're a man! consider your teacher!' Students' own preferred ways of working, including whether they like to be quiet (like Priya) or discuss things with each other, are up for debate. Students' evaluations of the term (including comments from two women about how much the men talked) were more critical than usual. We became more self-aware and could talk about group processes more easily.

Students as researchers

These five pieces of work suggest that data I had previously collected was flawed. We say we build on students' experiences - but we need to question how we come about our knowledge of that experience. Students' discussed and written maths histories, material from interviews, any discussion with a tutor present, my own generalisations about students' aims, and class observation have all been questioned. I am not suggesting that these new forms of data are more 'true', but that as researchers we need to be more aware than I was of the transforming potential of students' participating as active researchers. This is a reflexive process: the students' work has changed my own ways of working.

The 'teacher/student interaction' observation (another of the six class observation schedules) identified the tutor as 'dominant'. When we discussed this in the group, the students said they thought it correct that a tutor should be dominant; that was part of my job. So the processes of joint research have not in any way made tutor and students 'equal', but they have, I argue, opened up for group discussion the question of what empowerment or democracy might mean in an adult basic education context.

We commonly say in adult basic education, in both literacy and maths, that we are aiming to 'empower' our students. The term is used differently by different writers and in different traditions; here I want only to point to three strands which have influenced the research reported here.

Adult basic education has been strongly influenced by Freire's work, which originated in work on adult literacy (e.g. Freire, 1972); though, not surprisingly, it is used more in adult literacy work, some mathematics educators also cite his work (e.g. D'Ambrosio, 1997; Fasheh, 1991; Frankenstein & Powell, 1994). Diana Coben gives a very useful critical overview of Freire's legacy for adults learning mathematics (Coben, 1997) and adult education more generally (Coben, 1998).

Within literacy work there is an established practice of using students' own language (spoken, dictated, written, taped ...) as learning material, and of publishing students' writing (see Mace (1995) for more detailed discussions, based in British experience). A parallel development in mathematics education, growing from studies in ethnomathematics, may be using students' own problems, methods, algorithms and mathematical traditions - seeking to acknowledge and value students' existing knowledge and strategies and build on them (e.g. Gerdes, 1997). Knijnik (1997) describes work with landless people working to develop both their traditional and 'standard' methods for land measurement so that they can choose and *use* mathematics so far as possible to their own advantage. (Elsewhere I discuss the use of students' own questions in both literacy and basic maths work (Tomlin, 1998).)

There are further influences from political theory and organisation in the women's liberation movement ('second wave', 1970s). 'Consciousness-raising groups' sought to build their own theory out of examination of their and other women's own experience. In common with those political groups, the mathematics work I discuss here relies on group solidarity and students' willingness to learn from each other; it is organised in small groups which sometimes meet together to share experience; it assumes that 'failure' in mathematics has, by and large, a socio-economic rather than individual origin; and it is optimistic.

Our ideas of 'building on students' experience' depend on students having space to express that experience. In the 'student-centred' approaches we seek to use in adult basic education, there is a risk that the focus on the individual's relationship with the tutor sidelines the students' relationships with each other; as tutors we may be inadvertently isolating students. When the tutor is pushed slightly to one side and students are seen as active researchers, some of the constraints of the usual discourse are shifted and students find or make contexts that may allow them to work more openly and to share critiques of their classes.

The Freirean concept of 'dialogue' as a route to 'empowerment' is effectively critiqued by Ellsworth. She found that recognising 'the students' and professor's asymmetrical positions of difference and privilege' (Ellsworth, 1994: 314) required recognising also that

> Acting as if our classroom were a safe space in which democratic dialogue was possible and happening did not make it so ... we needed classroom practices that confronted the power dynamics inside and outside of our classroom that made democratic dialogue impossible (ibid., 315).

The students' work illustrated here shows ways not of 'solving the problem' (of empowerment, or the curriculum, or teaching strategies) but of opening up issues with students themselves so that the explanations, solutions and compromises are not all those of the tutor (or government). The students are theorising about their own education. The research includes more formal research processes: members of the group who organised the students' meeting and magazine have presented a workshop for practitioners (Gray et al., 1999). As part of the project some students have also read others' research: for example, students have discussed Knijnik's (1997) work and tried out two Brazilian methods of area measurement; tried addition and subtractions approaches from Netherlands colleagues (Beishuizen & Anghileri, 1998); and discussed nurses' use of averages at work (Hoyles, 1999).

A research culture in the classroom, in which it is assumed that the students and tutor are learning together using both human and written study materials and that there is no ready-made answer, fosters shifts in both classroom and research discourses. This project lies broadly within the fields of participant action research (e.g. Merrifeld, 1997) and teacher research. Cochran-Smith & Lytle (1993) argue that

> What is missing [from current research] ... are the voices of teachers themselves, the questions that teachers ask, and the interpretive frames that teachers use to understand and to improve their own classroom practices. (op. cit. p. 7)
>
> Teacher researchers are both users and generators of theory. (ibid., p. 17)

I argue that this is true for student researchers too. Cochran-Smith & Lytle claim that

> because teacher researchers often inquire with their students, students themselves are also empowered as knowers. (ibid., p. 43)

The research described here is into the students' and tutors' own discourse, rather than some outside issue; the tutor is as much the 'object' of research as the student, and as we have seen, the students challenge the idea that tutors, however well-intentioned, 'empower' their students. Instead, it seems we may silence students: tutors are 'loudly spoken'; we should more often 'keep quiet, just take notes'.

Group work and discussion of the meanings of maths and of students' own maths histories can be used to generate reading materials which include a range of student voices, and which are based on students' own accounts of their experiences (whether of schooling, work, bringing up children, using maths or the classroom itself) rather than on the myth of the typical student. In turn the group's own work can be used for that group and others to read and discuss. For example, an edited version of the tape transcript of Pat's and Cathy's discussion (quoted above) was read aloud as a dialogue in other groups, as a starter for discussion about the conduct of their courses. This form of dissemination and analysis (by students) of research data sidelines the tutor/researcher's voice and places students' experiences at the centre of the classroom discourse. Tutors remain tutors; the dominance of standard mathematics as a gatekeeper means students will often want formal maths qualifications and hence a restricted curriculum.

However, inviting students to see themselves as researchers unsettles these discourse structures.

I wrote above that we cannot make many useful generalisations about students' views of maths, pedagogy and ways of learning because students themselves reject such generalisations. One central insight does emerge from the work discussed here, however: they want to be actively involved in the planning of their courses, including both curricular and pedagogical issues. The students represented here are *organising*: coming together to develop ideas alongside joint strategies for change. They are engaged in praxis, the term for the union of theory and action used by Freire and others (e.g. Fasheh, 1991). We don't always have to agree with them; as with any other writer, politician, learner, teacher or mathematician we can engage in debate (though as the students point out, echoing Ellsworth, we should beware that in such a debate tutors start from a dominant position). The organisational and teaching practices discussed here support students working towards this openness, solidarity and individual presence:

> The students in this project are researchers, and have a free hand to organise everything. We feel more relaxed. When tutors are there, we are more intimidated. Students want to work more on an equal basis. Sometimes the tutor is a friend; but in some situations the tutor is an authority. At the students' conference and at RaPAL [Research and Practice in Adult Literacy conference] we organised how the group discussed things, and what questions we asked. For example, one older woman at the students' conference was quiet and we wanted to be able to learn from her, so we asked her some questions, and she talked and talked. The room went silent - it was intense listening. We learned so much from her. We become experts listening to students from other centres, and students relate more easily to each other than to a tutor. We are in the same situation. (Gray et al., 1999: 18)

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Working with pre-service primary schools' teachers: project works in mathematics and society

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Abstract:

The mathematical formation of students who are being trained to be primary schools' teachers is a study field that admits different answers, but it is far from having found its complete definition and resolution. We are thinking about the necessity to keep in mind, in this training, the position of mathematics in society as an element of power, differentiating and classifier. If we begin with the non-neutrality of mathematics in social problems and its role in the transmission of values, we wonder which should be the way of approaching a didactics of mathematics lecture for pre-service teachers. The project work is considered as a way of paying attention to these socialization and enculturación processes within education.

Introduction.

The teachers' role in the construction of a democratic society is a matter which has not been paid enough attention lately (Carr, 1999). As a general rule, a lack of appreciation of the degree in which a democratic society depends on the educational body exists. The teachers' function is generally perceived by the society like that of some professionals who have the duty of communicating some knowledge to the student during a period of time. We have two conceptions here: the way of acting (the teachers') and the object of their performance (the subject).

It is admitted in many sectors that the transmission of the knowledge should be carried out as if it were "to fill" the minds of the students, accepting the "banking" conception so criticised by Paulo Freire. On the other hand, the object to transmit, the knowledge, is thought it should be classified in some of the typical subjects: language, history, sciences, art, mathematics. If we think like this, it is obvious that education has very few possibilities to exert influence in society and to contribute to the construction of a democratic society.

However, although what was previously mentioned previously is most of the population's feeling, the laws which regulate the educational system have a slightly different vision of the role that teachers and education should complete in general.

In Spain, the current Law of General Regulation of the Educational System, LOGSE, accepts in its preamble that *"education's first and fundamental objective is to provide a full formation... directed to the development of their capacity to exert, in a critical way and in a plural society, the freedom, the tolerance and the solidarity."* This, in practice, only remains as a "pretty" or "beautiful" text, the means that the Administration provides don't allow, at the moment, to make big changes; everything is in hands of the teachers' good will and love for their profession. Although values and attitudes are mentioned in the official regulations, the teachers have all the opportunities to keep on doing the same thing they did ten, or even fifty years ago, with the only difference that the text book has changed a lot its appearance but not much its content, the subject's syllabus.

In this whole situation, the teachers' training represents a very important part, those of primary school as much as those of secondary and high school. The LOGSE practically leaves total freedom to universities, which are in charge of the initial training of all the pre-service teachers. If only we, the teachers, who have the responsibility of this pre-service, take into account the role that education represents in a democratic society, education will be able to become a tool for the construction of a fairer and more equitable society.

At this point I think it is not necessary to argue the role mathematics represents in education and society. Many references to the non-neutral role of mathematics in society and to the way they are used to empower and control the information, can be found in the bibliography (Bishop, 1988; D'Ambrosio, 1990, 1994; Frankenstein, 1997; Powell and Frankenstein, 1997; Skovsmose, 1994). From all this we can deduce its importance in the basic formation of all citizens.

To what I will refer from now on, is to the necessity that the conscience of the omnipresence and of power of mathematics reaches the teachers' training process, specially the primary teachers' training.

The context.

In Spain, the teachers of Primary School have to study three courses at university, in which the subjects cover the general areas of pedagogy, psychology, sociology and some specific ones as language, mathematics, history, geography, science, literature, art, physical education and artistic expression. In the specific areas, the teaching includes, in general, so much the teaching of the subject, as of its didactics. Also, the students will carry out some practices (250 hours) in schools, distributed in two years. The situation we have is that teachers of primary school receive a general training that tries to cover many areas and in short time.

Regarding mathematics, in the Corunna University, the Primary School pre-service teachers have two compulsory subjects of didactics of mathematics, with a total duration of 90 hours. It is here, therefore, where we start off with the idea that our work can to influence the education of the future citizens of a fairer and more equitable society.

A theoretical conception.

The mathematical formation of the students who are being trained to be primary teachers is a study field that admits different answers, but it is far from having found its complete definition and resolution. From here we present the necessity to keep in mind the position of the mathematics in the society as an element of power, differentiating and classifier. If we begin with the non-neutrality of mathematics in the social problems and its important role in the transmission of values, which should be the way of approaching a didactics of mathematics lecture for pre-service teachers?

When one wants to transmit some didactic knowledge regarding mathematics to students who are being trained to be teachers, the habit is to think only of thatsubject, mathematics, and so, what is discussed are the teaching methods, the problems and difficulties caused by the teaching of mathematics in the different educational levels. It is it what our students want: how to teach children each of the mathematical topics they have to learn. And, if we do this, we will omit several questions: why do children have to learn such mathematical topic?, why do they have to learn it that way?, why do they have to learn mathematics?, what are mathematics?, what is learning in mathematics?, what is learning?, why and for what goal does education exist?

I assert that the general questions referring to education, its aims, its methods, its reason for being, are not oblivious to the area of mathematics and therefore, they are matters which we deal with in our classrooms for pre-service primary teachers. If we didn't do this way, university teachers would be transformed into suppliers of techniques and "tools" to work with the mathematical contents in the primary classroom and, although it didn't become explicit, the objective of this whole educational process would only be the acquisition of mathematical knowledge. After all, the way we adopt to work in the classroom has a lot to do with our philosophy of education, with our vision of the role of mathematics in the society and with our conception of citizenship.

However, if we have to consider the problems of mathematical education in the primary level, I believe that one of the main obstacles which define the failure in the mathematics classroom is the conception the primary pupils, as much as their teachers, and our students, have of the subject. For the latter and for the teachers, mathematics is the most important subject that has to be imparted in the school curriculum, along with language. Maybe this importance is due, in their opinion, to its utility and necessity in other subjects, to its prestige (mostly the prestige obtained by those who stand out in mathematics) and, mainly to the fact that it is imparted in all the courses and the students will have mathematics in every school year. The understanding students have of mathematics, is similar in a certain way, but seasoned with a strong dose of objectism, control and mystery in detriment of the values of rationalism, progress and opening (Bishop, 1988, chap. 3).

However, if we deepen a little in the reasons which are located behind this point of view of the subject, we can understand that everybody perceives the mathematics studied in the classroom in a very different way from the mathematics which are used in daily life, in society. We can consider, then, that there is a difficulty in finding that relationship, and it is a difficulty because it isn't thought that it should exist or because it isn't know what to do to find it.

An answer.

We believe that the root of the problem is mostly the fear or the anxiety forwards the subject and for this reason the teacher hides behind the text book and 'what has always been made', which means safety. We should also consider that in most of the classrooms the teacher's role is accepted as the possessor of the information, the possessor of the knowledge and the solution. This role is assumed by the teacher and the students, who also presuppose the accuracy and the infallibility of mathematics.

Another fear that is usually found among teachers is the ‘fear of students ', not only the one that comes from the scarce knowledge of who they are, of how they behave, of which are its necessities, etc., in which we cannot really intervene; but also the one that comes from their behaviour towards the subject and which should this behaviour be.

I believe the answer that should be given to face these two fears is the following one: the teacher builds his knowledge along with his students; the teacher, in his performance in the classroom, should make clear to the students that he can only help them in their learning and understanding. The answer is easy and utopian, and as such we should accept it, but not underestimate it, because what is really being looked for is that the teacher feels full of confidence in front of the class, because he realises his students know the possibilities and the limitations of mathematics, because his students feel the teacher is also a partner in the search of knowledge and because the students feel that the subject, mathematics, is not oblivious to the world where they live.

Within the previous answer it is contained, as I said, an utopia, but I am firmly convinced that it is only looking at that utopia how education can be improve.

It is an utopia that is very linked to Ubiratan D'Ambrosio's thinking when, in the foreword of the A. B. Powell and M. Frankenstein's book, *Ethnomathematics. Challenging eurocentrism in mathematics education*, he says:

“Hence, education is an act of love. Mathematics education is not different. Why should it be odd to discuss these items, to bring love into our reflections, when we are talking about mathematics?

Couldn't we, as mathematicians, see ourselves as heralds of a new era for humankind. Why not?” (D'Ambrosio, in Powell and Frankenstein, 1997, pp. xvii and xx).

All these convictions and feelings should be taken to the classroom of pre-service primary teachers. If we believe what we comment in the previous paragraphs should happen in the primary school, all this has to be communicated and shared in university courses of teachers' training. It is something as important as the didactic contents, in fact, they are a fundamental part of the didactic contents we should impart. And the only way to carry this out is to do it in our own university classrooms. We should ask ourselves, which is the aim of the mathematics education? The subject itself or Education and, as a last resort, the near future society? I think the second answer is the one we should give and it takes us to reconsider the mathematics didactics classes.

In our classrooms we must confront several variables, among them: the scarce knowledge our students have of mathematics, the role of mathematics in society and the educational goals in primary school. We should consider which must be the way to handle these variables maintaining at the same time the vision of the mathematics teacher described previously. I assert that an answer comes from work through projects.

If we admit the thesis that mathematics is a social construction and that they becomes a tool for the student empowering him/her and allowing him/her to adopt a critical posture and to exert an influence in society, the work through projects is contemplated as the best way, or maybe the only one, to make the student, who will later be a teacher, a person who can pass on to his/her future students that idea or feeling about mathematics.

I understand the work through projects just as Ole Skovsmose describes it:

"The teaching-learning process should be oriented towards the goal of providing students with opportunities to develop their critical competence in the form of qualifications necessary for their participation in further democratisation processes in society." (Skovsmose, 1994, p. 61).

A ‘project ' is a work undertaken by the class or by a group, with the teacher as another member, with the aim to study, to investigate or to develop a topic that all the group considers interesting. The topic has to be interesting by itself, because it is something located in the group's life, in their culture, and it is not merely a part of the curriculum. Concepts, techniques and procedures appear in this study, which the student should know and to use.

A project is characterised by the personification of teaching, the interdisciplinarity and the critical analysis. To carry out this last characteristic the subsequent reflection to the realisation of the project is fundamental. This reflection will try to analyse the relationships between knowledge and teaching with the characteristics of the world of the students and the processes of social and cultural reproduction. It should all be useful to take our first steps in the language of criticism and possibility.

The project work arises from the idea that it cannot be somebody external to the student the one who gives or proposes all the ways of learning. It arises from the understanding that the teacher is a motivating and revitalising element in learning. But if what the learning looks for is to achieve education, the teacher can only provide it by means of the teaching situations that he/she proposes or allows.

In the project works education and socialisation walk hand in hand; the student, inside the group, links the school subjects with the society in which he/she lives, and the learning becomes useful. It is useful because it allows him/her to interpret the reality in which he/she moves or the one which he/she has contact with. It is also useful and productive because it allows him/her to handle that reality, either to adapt it to his/her environment, to take advantage of it and use it, or to criticise it and modify it. Through the projects, it is also wanted that the "subjects" they learn should be useful and tools for life. But we are not talking about justifying the teaching of all the contents for their utility; on the contrary, discovering the "utility" which can be found in all knowledge.

Some examples.

During this year, my students on pre-service primary teachers course are involved in carrying out several project works. Some of these don't fit in the project conception I exposed before. The reason can probably be found in the freedom they have in the election of the topic. I am concerned about providing the rules so they can find a topic they assume and consider as something of their own, looking for it in their particular situations, hobbies, interests or concerns, which will make them feel a certain responsibility in its realisation. Because of this, not all the students feel involved in the vision of mathematics I try to present, and what they mainly look for in the realisation of a project is "teaching mathematics ".

Fortunately, other groups have chosen topics which will help to understand the global role of mathematics in society. Among them we can give the following examples:

"Study and design of the distribution of cars in a parking of the Faculty". One of the parkings of the Faculty is an enclosure with an irregular ground in which the parking lines have not been drawn yet, and therefore the cars are parked in a chaotic way. What the students have to carry out is a study of the land, looking for the design of the parking that allows parking cars in the best way. There are implicit several questions that the students have to define: the appropriate size for the parking of a generic car, the necessary space to manoeuvre for parking, the necessity or not for establishing two types of parking squares, the number of reserved squares for bicycles or motorbikes.

"Analysis and criticism of the change in the parking modality in a street". The way of parking has been changed in some streets of the city recently, from parking in a single line to parking in front/rear to the curb. The council authorities reason was the better use of the space, both for parking and for traffic. An analysis of what happens now in those streets is that the traffic has got worse: where there were two lanes for the cars before, there is only one now; the lines that have been drawn for parking are not even appropriate for small cars. A slight variation in the angle which the marks should form with the line of the sidewalk could be the cause. However, among the aims of the project there is something more than to analyse the causes and to describe what happens; students are wanted to investigate who take benefits of the change and who loses out.

"Decrease of the speed limit in the city". In Spain, there is a speed limit of 50 km/h in urban streets. Some people consider this excessive and you can frequently consider it the cause of accidents. It is sought to make a detailed study of times and distances in different journeys in the city, trying to discover how they would be affected if the maximum speed decreased to 30 km/h. In view of the fact that in the different journeys we have traffic lights or zebra crossings, and the traffic is quite dense, it is expected that there won't be a remarkable decrease of the time used in each journey, being able to increase the security considerably. The work could also be completed with an analysis of consumption of fuel, trying to discover who is the biggest beneficiary in the increase or reduction of the speed limit.

Finally, I want to mention the aspect of the evaluation. The fact of carrying out a teaching focused on the work through projects, necessarily implies that the evaluation changes. The considerations made on the global process of the education in the different educational environments and on the mathematics role in the current, technological and mass media society, will also imply that we have to consider not only the accomplished work, but also the individual and group conditions, the attitudes, the motivation, the capacity to establish relationships, the participation, etc., in the projects' evaluation. Here, I want to raise a doubt which is the following: can we demand our students to have an interest for the teaching of mathematics that goes beyond the goal of obtaining a good mark in the final evaluation? How could they be convinced to have that interest?

At the Congress I hope to be able to expose the results of the work with my students and the conclusions I have obtained.

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Dilemmas of Social/political /cultural Research in Mathematics Education

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Based on our experience in two different research projects that explicitly adopt a social/political/cultural approach to research in mathematics education, we explore four different dilemmas that we have faced in our endeavor. The dilemmas of mathematical specificity, scope, scientific distance and relevance refer to four types of critical situations where researchers have to make decisions that are not documented and discussed enough in the community of research in mathematics education. Our intention with the paper is to start a discussion about ~~what~~ these dilemmas ~~are about~~ and the reasons why they emerge in this kind of endeavor.

Introduction

During the last decade research in mathematics education has given an increasing attention to the social, cultural and political aspects of mathematics teaching and learning. This kind of research has slowly been driving mathematics educators to engage in understanding phenomena for which the psychological tradition of research in the discipline does not offer possibilities of tackling in serious ways. In the search for a scientific identity, a kind of independence of the discipline has been achieved through the construction of own specific frameworks to give an account of learning and teaching processes of mathematics. These frameworks have emphasized both a mathematical point of view in order to give an account of the complexity of the knowledge involved in teaching-learning processes, and a mainly psychological approach to learning. Despite ~~of the fact~~ that mathematics education has been ~~several times~~ defined as a discipline at the intersection of several others –what would appeal to a strong interdisciplinary nature of mathematics education–, in reality researchers have not taken seriously and to its latest consequences the perspectives of social sciences like anthropology, sociology or political science. Adopting these points of view as central in giving an account of what the practices of mathematics education are is certainly a recent and marginal endeavor. Being as such, there is not much discussion not only about the results of this type of research, but also about the process itself of carrying out a study that stands on a non-psychological, non-mathematical perspective.

Our intention with this paper is to ~~open~~ start a discussion around some of the difficulties ~~or dilemmas~~ that a researcher in mathematics education faces when adopting a social, political or cultural approach. ~~We will discuss,~~ B~~b~~ased on the description of two ~~independent~~ research projects in which we have been involved, we will present ~~what we see as a social-political and a cultural approach in research in mathematics education, and we will present~~ four dilemmas that we have encountered ~~in this endeavor~~when undertaking research from these perspectives. We conclude with a set of questions ~~for discussion~~ about the dilemmas themselves, and the reasons why they emerge so strongly in these~~is~~ types of approaches.

Two Research Projects

In this section we present two ~~different~~ research projects that served as a base for our collective reflection for this paper. For the last three years, Paola has been conducting the first study as her Ph.D. project. And João Filipe has been the leader of the research team who conducted the second project during the last three years. ~~in which each one of us has been involved in the last years. Although the projects are not directly related and we have not cooperated in their development, we found that both offer us a base to discuss critical situations in which we have had to make key decisions that have shaper the development of our research.~~ The following descriptions intend to illustrate the research dilemmas that will be discussed later.

Reform, Democracy and Mathematics Education in Secondary School (RDME)

This research follows a recent tendency in mathematics education, which considers the effect of the school organization in the way teachers actually develop their teaching and implement current reform ideas (Krainer, 1999; Pence & Becker, 1996; Secada et al., 1995). It is also based on previous research experiences, which lead to the formulation of the notion of Institutional System of Mathematics

Education –ISME (Perry et al., 1998). The ISME is the set of relevant elements and relationships among them, more directly associated with the teaching of mathematics inside the school organization. There are at least three groups of elements that give an account of the functioning of school mathematics: (a) The *administrators* in their leadership hold power to strengthen the mathematics teachers' professional culture by means of entrusting responsibilities about the performance and decisions of teachers in their professional activity. (b) The *group of mathematics teachers* constitutes a professional culture, where teachers as a community share a reference framework for their interaction. (c) And *teachers as individuals*, with their beliefs, professional knowledge and reflective capacities, engage in the teaching practices of mathematics in the classroom.

The core assumption of approaching secondary school mathematics through the ISME is that, in order to understand school mathematics functioning and its possibilities for change and reform, one needs to consider mathematics teaching as a part of a network of connected school practices ~~that are immersed~~ embedded in the school organizational structure. The analysis of this complex network of practices offers a deeper view of how mathematics education works in the framework of a school.

Adopting this position is relevant because, on the one hand, recent reform or change proposals poses challenges on teachers that exceed the boundaries of the individual practices in the classroom. They require that teachers get involved in processes that happen in larger structures inside the social organization of the school –e.g., the idea of mathematics teachers as curriculum designers and developers brings teachers to operate inside a professional community that might not have a very strong role before. And, on the other hand, the emphasis of these recent proposals on the association between mathematics education and democracy introduces the necessity of investigating the political dimension of school mathematics practices, and discussing ideology, power and conflict as some of its characteristics. Therefore, opening the focus of attention from the classroom or the individuals to the whole ISME is necessary to gain a deeper understanding of possibilities for change –or stagnation– in school mathematics teaching and learning.

This project is under development. It aims at proposing a theoretical model of the Institutional System of Mathematics Education, through a process of cyclic ~~*hermeneutical*~~*hermeneutic* *reasoning* that combines both theoretical reflections and interpretations of empirical data. This empirical data, used as a resource for theorization, involves the examination of three cases in three schools in different countries (Denmark, South Africa and Colombia). In these case~~-studies~~s different data collection methods were used: semi-participant observation, observation-based interviews and discussions with the members of the school, and the collection of written materials. These cases have been used to produce *vivid images* of critical issues that might be discussed and included in a comprehensive theory about the ISME.

This research is considered to adopt a socio-political approach. It is social because it views secondary mathematics teaching and learning as practices that are constituted in the complex network of interactions among different actors inside the school organization. And it is political because it also considers the value-laden, ideological nature of those practices and the power conflict that emerge in actors' interactions and practices around the teaching and learning of mathematics in secondary school. At large, the researcher also takes a social and political stance towards the implications of the research process and its results.

Culture, Mathematics and Cognition (CMC)

School mathematics learning has been the object of many research projects everywhere for a number of years. Mathematics education researchers believe that if it is possible understand better how people learn, and if the notion of what it means to know mathematics can be refined, then education in mathematics could be improved in terms of the quality and of the power of its aims. There are quite different lines of work within this field reflecting different concerns and possibilities.

The project "Culture, Mathematics and Cognition. Reflecting on Learning in Portugal and Cabo Verde" aims at identifying, describing and analyzing (a) forms of mathematical thinking that adolescents use in institutionalized spaces –the school and other informal but socially organized settings– and (b) connections between those forms in different social practices. The overall research question addressed is how to characterize school mathematical practices. Drawing on the theoretical framework proposed by Lave and Wenger (1991), one of the relevant issues became understanding the idea of "learning as an integral part of generative social practice in the lived-in world" (p.35). The project takes for discussion and uses as a research tool the idea of "learning as increasing participation in communities of practice" (p.49).~~ and t~~The analysis of these ideas has led us to try to understand the meaning of participation in a social practice (and therefore in a community of practice). The practice of the *ardinas*[1]~~–~~in Cabo Verde Islands, in Africa, constituted an important part of the empirical setting for the project and turned out possible to develop an understanding of how practices relate to school mathematics knowledge. Our goal was to look into the ways in which (mathematics) learning relates to forms of participation in social practices where mathematics is present, but where the characteristics of the school environment are not.

We believe that culture is an unavoidable fact that shapes our way of seeing and analyzing the world. That is why we decided to look into a culturally distinct practice, that constituted a really foreign domain for us: the practice of the *ardinas* in Cabo Verde. The need to find a practice to observe, that actually allowed the presence of the researchers, had a strong influence on our choice of the observation setting. Previous contacts were made with several local practices – e.g., young people washing cars in the street, boys helping carrying luggage at the airport entrance. But the presence of favorable circumstances determined the choice made. So, for about five months, one of the members of the research team collected data observing the practice of the *ardinas* in the streets, interviewing them and video recording their activity and interaction with the researcher. This data started to be analyzed taking into account the theoretical framework and seeking answers for the research questions defined (Santos & Matos, forthcoming).

Research Dilemmas

~~Our experience in~~In these two projects ~~has~~ we have faced ~~brought us to~~ critical moments in ~~the development of the research where~~which we have ~~had to make~~ made decisions about the ~~research~~ investigation process itself. These critical moments have posed *dilemmas* with which we had ~~had to deal~~dealt in different ways. Here are our reflections and questionings about those situations.

[1] *Ardina* is the name given to the young boys who sell newspapers in the street.

The Dilemma of the Mathematical Specificity

This dilemma emerges when a question like "Interesting, but, can it be considered research in mathematics education?" is raised. The dilemma builds on the different conceptions that different researchers hold about what research in mathematics education is and about the legitimacy of certain research questions, theories and methods in the area. Commonly, mathematics education is defined as the discipline studying "the practice of mathematics teaching and learning at all levels in (and outside) the educational system in which it is embedded" (Sierpinska & Kilpatrick, 1998, p. 29). In this field, "[…] mathematics and its specificities are inherent in the research questions from the outset. One is looking at mathematics learning and one cannot ask these questions outside of mathematics."(p. 26). This definition focuses research on the *didactic triad*, that is, the relationship between teacher and learners, taking place mainly in the context of a classroom, and having mathematical content as its main constitutive element. There is strong emphasis given to this triad, and specially to mathematics in it.

When ~~social-political~~socio-political and cultural approaches are taken, the didactic triad opens up and mathematics education starts considering research objects and questions that are located outside of the triad. Then, as one goes deeply into these approaches, the key role of school mathematics and of a mathematical approach in the research tend to vanish or to be questioned. Instead, other phenomena and relationships receive more. The dilemma of the mathematical specificity is represented, then, by the tension between the high or low priority and importance that a mathematical point of view is given in the research, and the high or low priority and relevance given to other aspects such as social, cultural and political settings and relations in which the learning and teaching of mathematics are embedded. And this gives room to the discussion of the nature of mathematics itself.

In the CMC project, it was clear that the mathematics used by the *ardinas* was embodied in the practice, and could not be seen as an isolated, objectified entity. The way the ~~children~~ adolescents addressed "mathematical" problems immersed in their practice used the frame of the school or the frame of the *ardinas*' practice itself. The mathematical aspects of the social practice under analysis were in fact not seen as a priority by ~~children~~the adolescents. And at the same time the research questions were not formulated within a mathematical frame but within a social and cultural one.

In a similar way, the project RDME did not make a choice for studying in detail each one of the individual teachers and their knowledge, beliefs and practices in the classroom as a means to give an account of the way individuals put in practice ideas of reform. Instead, there is an emphasis on bringing to the surface the social structures of the "professional community" of mathematics education in the school. In so doing, it was possible to highlight the factors that, associated to the functioning of such a community, empower or obstruct possibilities of a collective change in the teaching and learning of mathematics in the school. These structures offer a plausible explanation to change reluctance in mathematics education, but are not the kind of explanation that current research in the area would overtly support.

The Dilemma of the Scope

One very frequent comment to hear about these projects is: "Good plan, but, don't you think that you should shape your research questions better? Since cultural and ~~social-political~~socio-political approaches in mathematics education research open up and shift the focus of attention, it is expected that they enlarge the research questions that one poses. The dilemma here appears in whether it is desirable (and coherent with the adopted position) to have a very narrow question or whether it is imperative to have a research which tackles a broader scope.

For us, this dilemma is closely connected with the way questions, theory, choice of analysis units, methodology and analysis get mutually constituted in the research process. In most of the psychology-oriented research in mathematics education, the unity of analysis tends to be small and the methodologies and theories focussed. In contrast, ~~social-political~~socio-political and cultural approaches open to more inclusive theories, less focussed methodologies and more complex units of analysis. In the RDME project the focus on structures inside the school as a whole does not permit the researcher to consider exclusively individual teachers' practices in a classroom. There is a need to look at the teachers,~~;~~ at the collective activities happening among the teachers, and ~~to~~ at the connections between the school administrators and mathematics teachers.~~; and the group of teachers, or the individual teachers~~. Choosing the classroom as the unity of analysis is contradictory with the basic assumptions of the research.

The dilemma of the scope also touches on the problem of context. In the CMC project, when the ~~children~~ adolescents-in-action in the practice of selling newspapers was chosen as the unit of analysis, it was evident that we could not forget that this practice takes place in a certain city in a certain country with constraints and opportunities. The context is important and should not be dismissed, but at the same time should not be looked as the “cause” providing explanations for the hypothetical results. Instead, it is our job as researchers to try to construct meanings in the light of the relevant aspects of the context. It is a fact inherent to research that we as researchers are certainly making a recontextualization of our observations to our readers.

The Dilemma of the Scientific Distance

A very deep assumption of research quality is the preservation of a distance between the “researcher” and the “researched” in order to assure quality. Even in qualitative research, the motivations and the reasons why a researcher deliberately privileges some aspects and makes questions about them are not made visible to those involved. In fact, interview techniques have been developed in order to hide the intentions of the researcher so that evident answers are not possible and, then~~,~~ better, supposedly “unpolluted” data can be collected.

In our view this dilemma could be seen from two perspectives. From the point of view of the community of practice of researchers in mathematics education, the researcher ~~must~~ should supposedly preserve reliability –the consistency in a coding process when carried out in different occasions and/or by different researchers– and validity –the confirmation of the extent to which the research instruments are in fact measuring what the researcher thinks they are capturing. ~~. This can be achieved through achieving consistency in a coding process when carried out on different occasions and/or by different researchers, and can be seen to be a measure of the extent to which the researcher is measuring what they think are measuring.~~ But we see that in research from a socio-political or cultural approach the expected quality criteria are problematic in two ways. ~~In fact these criteria raise ethical problems and consequently raise at least two~~

~~issues. The~~First of all,~~ issue of~~ the complexity of the relations between participation and observation is more than a technical matter. This complexity is associated with the fact that these two enable one another. In a concrete research situation researchers obtain information from specific moments of interaction with the participants, that can only be interpreted and perceived by the researcher –an the people involved in that particular situation–, and where there are no possible replication or objective measures to grasp the revealing character of the information collected. Furthermore, the expectation of keeping a “scientific distance”~~ is the issue of the~~ can easily lead to the objectification of the people linked to the research. This objectification refers to the actual status given to participants ~~Some critics of this supposed objectified relations between the researcher and researched argue that the people being observed can be seen as~~ who are considered as ~~mere~~ puppets ~~that are~~ under the scrutiny of the researcher and who are not allowed to have a real say in the research findings. ~~of the research~~. ~~F~~ And from the point of view of the participants or those involved we could ask: What does it mean to participate in a research project? What role are they expected to have –or be given by the researcher– in order to get them living in a significant activity and not as mere “objects”?

The dilemma of the scientific distance emerges critically when one is doing research in mathematics education f~~o~~rom a ~~social-political~~socio-political and cultural perspective. It questions the extent to which the “scientific distance” can be kept and at the expense of what. It shakes deeply the ethical basis of research. In most cases, research in mathematics education is done to satisfy the interests –of knowledge, of attaining a certain academic degree, of pleasing funding agencies– of the researchers and very seldom allows seriously the worries, interests and expectations of the “researched”. This poses the problem of *responsibility* and *responsiveness*. Responsibility is the researcher’s capacity of being openly accountable to those involved in the research, about the observations, interpretations and results. And responsiveness is the capacity to reply and react~~s~~ readily to the influence received from the research interaction. In other words, in contrast to the classical situation in which researchers take information out of a social situation and use it outside ~~that situation~~ of it for knowledge production purposes, the responsiveness and responsibility principles require a constant and open “giving back” to the people or situation involved in the research.

This does not necessarily mean that the researcher has to get involved in solving the actual problems of the people and context in focus. But this at least points out to the necessity of establishing a de-objectifying dialogue between researcher and participants, so that they can get acquainted with the research purposes and intentions, and can also benefit intellectually from it.

This whole dilemma invites the creation of different research quality criteria that can better represent the complexity of the relations between observation and participation and between researcher and participants, in order to legitimate the process of data collection, analysis and report.

The Dilemma of Relevance

From the point of view of those who, as a consequence of their induction in their field of mathematics education, see themselves as members of the dominant mathematics educator community, mathematics assumes an overshadowing emphasis within research projects. Research in mathematics education has as an intrinsic assumption the idea that mathematics is important for people. Research exploring the justifications of mathematics education (Niss, 1996) shows that historically the community of mathematicians and mathematics educators has~~ve~~ always had a good reason to sustain the teaching and learning of mathematics in school as a relevant subject. Besides, there seems to be a tacit social agreement on that, to the extent that few would dare to imagine an educational system without the explicit teaching of mathematics.

But what could happen if we take away, for one moment, the ~~glasses of~~ mathematician or mathematics educator' s glasses, and allow the view of "lay people" to emerge? We will find students who perceive school mathematics as an integrated part of their school experience and for whom mathematics education has nothing special. It does not contribute in any particular and relevant way to their lives, and it certainly does not improve their performance in the outside world in any significant way. In the project RDME, an effort was made to listen to the overall appreciation of students about their school experience and their mathematical experience in school in a ~~low~~ working class, secondary, Colombian school. Two students' opinions were particularly strong and questioning. The following fragment of an episode in the school reveals the students' concerns:

> The talk started and they were curious to know my [Paola's] intentions, my motivations to be in their schools. They wanted to know about my life, where I lived, where I have studied, why I was living outside Colombia. They couldn't understand why I was there in that "poor" school, talking to poor people if:
>
> Andrés: We can see in your face that you have never suffered. You've got it easy.
>
> I was sincere with them. I have not suffered, that is true. But that did not mean that I had got it easy. I studied hard to have the chance of doing what I was doing. My intention was to tell them that there were reasons to study and to be interested in the school and in that mathematics. But they could not see it in their lives:
>
> Juan: The only class I would like to pay attention to is English because I want to get out of this fucking place, and go to the U.S. Still, I don't even manage to say "Good morning" (Valero, 2000, image C1).

Even more, teachers' intentions of making mathematics education a powerful experience attracts very few because ~~the expectations of~~ students' expectations and what they can see as their future possibilities have~~s~~ nothing to do with being good at mathematics. This point of view could be easily shared by the majority of students in the world —especially those in poverty— who will not pursue a university education and for whom school mathematics offers only a boring experience.

Taking a social/cultural/political frame, we can argue that it is relevant to see mathematics included in the school activity provided that mathematics is treated as a social/cultural/political tool. This goes directly into the question of the aims of schooling under which the role of mathematics learning in education it is defined –in an explicit or implicit way.

Discussion

There could be two possible lines of discussion about the dilemmas that we have outlined. One line could explore the dilemmas themselves. Then, some relevant questions could be:

- Are these dilemmas specific to social/political/ cultural research in mathematics education? If not, how do they differ from possible similar dilemmas in other kinds of research in the discipline?

Or we could also dig into the reasons why these dilemmas emerge and their implications on defining the task of doing research in mathematics education from a social/cultural/political perspective. We could then ~~formulate~~ think about questions like:

- Is~~Does~~ the recognition and particular management of these dilemmas a matter of (not) belonging to a defined "community of practice" in mathematics education research? And, therefore, is there an agreement about the legitimacy of research that, when questioned, makes these dilemmas emerge?

We leave these issues open for discussion in an attempt to get a clearer understanding of the task of ~~carrying out research~~ investigating in mathematics education from alternative perspectives.

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MATHEMATICS EDUCATION AND SOCIETY REVISITED

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Abstract

How can a sociologist contribute to the understanding of mathematics education processes? Apparently, there isn't any possibility of interception between these two sets of knowledge - mathematics and sociology.

However, the sociological perspective aims precisely to question the social processes, although in a rather different way from that which the individuals themselves do during their everyday life. In fact, the sociological perspective intends to reach wider forms of understanding, namely by discovering other levels of significance hidden in such processes.

Both Mathematics and Mathematics Education are, undoubtedly, social processes and, therefore, potential sociological objects. In this paper, we will try to think of a possible example of a sociological research on mathematics education. We will reflect upon the underachievement in mathematics as a social problem and the conditions for transforming it into a sociological problem.

Re-searching mathematics education from a critical perspective

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Abstract: To undertake such research raised at least two methodological problems, which are discussed in this paper. The first is that practices and theoretical concerns associated with a what may be referred to as social, cultural, political approach which integrates a critical perspective in mathematics education, are still at the margins of the realities of mathematics classrooms. That is, they are not widespread in the current mathematics education system and readily available for investigation. Hence, the methodological concern is that of producing a research design and process that brings these theoretical ideas into the realm of practice. This problem is addressed by allowing ourselves (researchers and practitioners) to imagine a hypothetical situation inspired by a theoretical landscape, and to create an arranged situation for research, by intervening in some existing, current actual situation. The second concern is to develop an appropriate methodology in which the research processes and practices are consonant with the educational processes and practices. That is the theoretical underpinnings and commitments of a critical approach to mathematics education need to be maintained within the research approach. To this end a critical approach to research in mathematics education is explored through a series of key issues in the third part of this paper, which were identified in undertaking such research.

Introduction

The idea that mathematics and mathematics education could or should somehow be quite directly and explicitly connected to issues in society is now well established. It is possible to sketch a wide theoretical landscape of a social, cultural, political approach to teaching and learning mathematics. This landscape, comprises at least four developments in mathematics education: a critical mathematics education for and by all; and a critical ethnomathematics education; concerns about dimensions of diversity including race/culture, gender, class, and language; and people's mathematics in people's education for people's power from the Apartheid era. Taken together these may be interpreted as offering a critical perspective in mathematics education. Arguably, these ideas that could be said to constitute a social, cultural, political approach to a mathematics curriculum are far more developed in theory than in practice. Hard evidence to support (or refute) theoretical propositions about empowerment, emancipation, democracy, social justice, equity and so on through mathematics education are still rather thin. Moreover, a commitment to these kinds of theoretical ideas also needs to include a concern for making such ideas more widely available to practitioners for interpretation and critique. This has created an imperative to begin to consider the means by which to investigate such an approach to mathematics education.

Throughout the paper I illustrate and give meaning to the ideas I develop with reference to my research[1]. The research question I was interested in was: what happens in a mathematics classroom when student teachers who have been introduced to a social, cultural, political approach to the teaching and learning of mathematics, which integrates a critical perspective, attempt to realise such an approach. Student teachers are typically introduced to new ideas in teacher education curricula with the implicit assumption (or hope) that they will somehow integrate these ideas into their teaching. A co-operative collaboration with them offered an opportunity to take a closer look inside classrooms, to examine what meaning is given to these theoretical ideas in practice. In this process the student teachers would learn to teach by teaching in a new and different way. This opportunity created the means for myself, as the teacher educator, and the student teachers (and the class teacher) to reflect on both the theoretical ideas introduced in their coursework and the forms they take in practice. In this paper I do not focus on the findings of the study but rather seek to open discussion on methodological concerns that arose in conceptualising and undertaking the study.

In essence this was a study in theory-practice relations in mathematics education but here the focus is on that relation in research. I use the term theory rather broadly to refer to a network of ideas and concepts in a theoretical landscape, which, in particular, explains an approach to the school mathematics curriculum that focuses on social, cultural and political aspects and integrates a critical perspective. What are the means for one to say anything – critical or supportive - to this approach, its theoretical basis, and to any associated practices, especially when these are not readily available? Once produced, what are the sources for developing any theory and associated practices further?

Actual, hypothetical and arranged situations

Distinguishing these three "situations" assists in describing and clarifying a process of researching innovative theoretical ideas and practices in mathematics education from a critical perspective[2]. I take these situations to offer theoretical tools for thinking and talking about researching a theory-practice relation when a particular theoretical landscape and related practices are deliberately introduced into a context because these are not dominant in the mainstream educational setting.

The actual situation

This is the situation that actually exists: in a class, a school, a teacher education institution or even the educational system as a whole. In a significant amount of research in mathematics education, some aspect of the current, existing, actual situation is researched: learners, teachers, texts, curricula and so on. That is, researching *what is.*

There are at least two ways in which the theory-practice relation in this research could have been investigated by referring to the actual situation. First, I could have searched for an actual situation in which, for example, teachers are working with the curriculum approach I am interested in studying. This would be quite a challenge given the dominant existing modes of mathematics teaching and learning in South African classrooms. A description of the actual situation, currently in South Africa can be characterised as one in which authoritarian modes of interaction and transmission teaching dominates, with rigid adherence to the mathematics syllabus strongly tied to tests and examinations and all of which largely occur in under-resourced large classes (see for e.g. Naidoo 1999). Although not all schools or classrooms fit this description, what has been described as a social, cultural, political approach is not a feature of the actual situation in the vast majority of South African classrooms. A second approach may be to study and interpret the actual situation as it occurs, through the lens or theoretical framework of a critical perspective in mathematics education (see for example Cotton, 1998). In this latter approach to the research I could consider issues of democracy, equity, social justice, etc. in actual situations as they are currently played out in mathematics classrooms. By studying *what is*, we may produce ideas for *what could be* or *what ought to be*.

In my research, the actual situation is not considered directly. It is important to the extent that I together with other participants could intervene in an actual situation and arrange a situation for research. Knowing the actual situation is, however, important for the analysis and theorising later to explain what occurs in the arranged situation. My research interest lies in making a concerted effort to introduce prospective teachers to a particular theoretical landscape and its associated practices and then to examine its recontextulisation when facing the reality of classrooms. The focus is not on the existing actual situation per se but rather on some new and different situation that is organised and created with ideas from a particular theoretical landscape.

The hypothetical situation

To posit a hypothetical situation assumes that the link between a theoretical landscape and practices associated with it, is not direct. It is mediated by what we allow ourselves to imagine could be. The hypothetical situation represents an ideal situation being thought of by the researcher or teachers who are engaging the theoretical landscape. It contains hypothetical ideas, concepts and also related examples of practice which are selected and reinterpreted from the theoretical landscape according to their understanding of the context in which the practice will take a particular form. It is constituted by what is imagined by the various research participants. We could also think of the hypothetical situation as the "recontextualising field" (Bernstein 1996) through which there is a de-location from the theoretical landscape and a re-location into practice. It is distinguished from a theory or theoretical landscape, which contains a certain network of ideas and concepts developed with reference to and offering explanations about what happens in a variety of contexts. These theories vary in distance from the context under consideration in the research being undertaken. For example student teachers are studying theories and practices developed in Denmark or the USA which must be re-conceptualised and re-interpreted for the general South African context and for a particular setting such as urban or rural. It is precisely for this reason that the hypothetical situation is important since it offers a space for reforming or transforming elements from the theoretical landscape. Perhaps it could also be called the "situation of hope" or the "hoped-for situation" because it offers inspiration for envisaging changes in the actual situation. No doubt the hypothetical situation has its source in the theoretical landscape that the researcher wants to investigate but also includes other ideas arising from the context. It is the situation imagined by the researcher and informed by the ideas and concepts in the theoretical landscape. A researcher constantly interprets and reinterprets a theory and its fundamental ideas and concepts throughout the research process according to the actual situation she is confronted with.

In my research since the student teachers were seen as co-researchers, their interpretations also belong in the hypothetical situation. The hypothetical situation in this study constitutes my understanding and that of the student teachers, of the theoretical landscape that sketches a critical social, cultural, political approach to mathematics education. This also includes our ideas of related educational practices such as project work. The hypothetical understandings and interpretations of the different participants in the research are not in any sense equal as they have different vested interests in the theoretical landscape and in the associated practices, and therefore also play out differently according to the differing power relations of the researcher and research participants in the situation being created. This means that all is not harmonious in the hypothetical situation, which could contain conflicts and contradictions. For this reason hypothetical situations are essential for investigating a critical perspective in mathematics education because they also assist in mediating the imposition of an intervention and allow for critique and dissent from all participants.

The hypothetical situation is also not static as it is likely to be constantly developing and changing through what happens in the school and classroom contexts. The student teachers' understanding of a social, cultural, political approach to mathematics education (see Vithal 1997) gives some indication of the student teachers' hypothetical thinking and reasoning before entering the school or classroom and to some extent the theoretical landscape that I have constructed does the same for me as a researcher. This hypothetical situation is also dynamic in that it changes as the proximity to the classroom increases. For instance, shifts in student teachers hypothetical thinking and reasoning are discernible as they approach the situation to be arranged and as they begin to negotiate with the class teacher for the opportunity to try a new or different approach in the mathematics lessons. That is, the hypothetical situation for the student teachers during their coursework when the possibility to realise the approach appeared remote is different from the period when they are closer to actually preparing to do something in a school and classroom.

The arranged situation

The arranged situation is a reorganised actual situation, which is created and constituted by the researcher and research participants. It is developed with reference to ideas and inspiration from the hypothetical situation. The arranged situation may be negotiated by the researcher, but it nevertheless represents an imposition on the actual situation. This is the case even if the intervention is considered progressive and represents democratic practices and emancipatory pedagogy. Furthermore, the arranged situation as it is developed with reference to the hypothetical situation, creates the opportunity for the researcher to see how events unfold in relation to the theoretical landscape. The arranged situation represents a temporary situation in itself, but is likely to have some lasting consequences for the actual situation which may or may not be significant.

The arranged situation in this research is one in which student teachers negotiated and realised an opportunity to 'try out' a social, cultural political approach which integrates a critical perspective, and in which the nature of interactions and the way in which mathematics is taught and learned were fundamentally transformed. The educational practice employed in this particular arranged situation was that of project work which was selected by the student teachers from the range of practices made available in this theoretical landscape. The arranged situation here could be thought of as a mathematics "curriculum development laboratory". In appropriating this image of a laboratory, a place for exploring and experimenting with a 'new' or different curriculum approach is suggested, in which all the actors and their actions are being considered. The mathematics classroom as a whole constitutes this "curriculum development laboratory" offering an opportunity for theory and practice to confront each other precisely in the place in which both must be given meaning. Through negotiation, a novel arrangement is set up in the mathematics classroom, which acknowledges and is aware of the scrutiny of a research process for a predetermined period. In the curriculum laboratory, it is possible to study the curriculum as it develops and unfolds in the classroom.

Descriptions of the arranged situation must allow the researcher to reflect on the hypothetical situation. The researched situation (which may be considered as a fourth situation) is located inside the arranged situation since the researcher may focus on some features of the arranged situation and may miss others or not be aware of the total impact of the intervention in a specific context. It could be that the researcher might choose to focus on a specific aspect of the arranged situation. Thus, only a part of the arranged situation becomes data, which is analysed, interpreted and explained, selected in relation to the hypothetical situation and the research question.

Arranged situations are needed in countries like South Africa which are attempting fundamental curriculum changes based on theoretical and hypothetical speculations through curriculum policies whose consequences are not known in the diversity of classrooms in the schooling system. Creating and studying arranged situations are also important in contexts where curriculum importation is undertaken because they open to scrutiny theoretical assumptions underpinning such curricula and the viability of their related practices, which may have remained masked in other places.

Reflections on actual, hypothetical and arranged situations

The construction of these actual, arranged and hypothetical situations offer tools for understanding and explaining the research process in investigating a theory-practice relation, in particular, one that embeds a critical perspective. While research in actual situations is closer to researching something typical or ordinary - studying *what is,* research in the arranged situation allow a researcher to study *what ought to be* or *what could be*. Researching the actual situation could also lead to studying *could be* if the researcher is able to find a situation that is in a sense ideal/exceptional or close to the ideas in the hypothetical situation. The arranged situation makes it possible to study what does not currently exist or exits only as remote theory and practices. Arranging a situation for investigating a critical perspective in mathematics education involves choice, negotiation and reciprocity. The reasons for participating need to be open and to some extent shared.

An important observation is that as a result of organising an arranged situation for research, the actual situation may never return to its original form. *What could be* can become a part of *what is*, even if only partially and incompletely so. In other words, the arranged situation could become an actual situation, but different from the one the research started with. The main point here is that no classroom intervention can be made and withdrawn without leaving some impact, however small or large, however visible or invisible. The implication is that arranged situations, created by temporarily significantly transforming actual situations, could begin to produce 'new' or changed actual situations. The act of arranging a situation for research is not without consequence for all the participants involved. Once the research has ended, the classroom does not revert to the exact same actual situation that existed before. A 'new' actual situation, produced in the process of constructing and living in an arranged situation, through the interaction of the hypothetical situation and the actual situation that existed prior to the intervention, resembles neither the original actual situation nor the arranged situation. This means that each of these hypothetical, actual and arranged situations are dynamic in themselves, constantly changing as the unfolding events in the arranged situation force shifts in the hypothetical situation and bring changes in the actual situation.

Distinguishing the hypothetical, actual and arranged situations offer a means for thinking about how to establish and research innovative practices associated with emerging theories. In itself this is not really a special problem and there are many studies that implement and investigate interventions in mathematics education. A main concern in this study is that of the imposition that occurs when an actual situation is made into an arranged situation by taking a particular hypothetical situation into account, that is, an approach which embeds a critical perspective. While it must be acknowledged that interventions can never escape the problem of imposition, the difficulty or contradiction that occurs in this research, however, is related to how to deal with this impositional issue with reference to a particular perspective in theory, methodology and participation which argues against it in the educational setting. The problem of imposition occurs in all three situations, hypothetical, actual and arranged.

A serious contradiction arises in exploring a theory that attempts to introduce a critical democratic perspective in an education setting without it being an imposition. Yet, an imposition has to be made, in the first instance, in the hypothetical situation precisely in order to make such ideas more widely available and to understand what they can mean in reality. Participants such as teachers and students cannot be coerced into accepting the importance of such a curriculum approach or to be critical. Moreover, nor can the form, content and direction of their critique be pre-determined. In the hypothetical situation researchers and teachers select, reject and interpret ideas and concepts from a given theoretical landscape according to their understanding and interpretation of the context of the actual situation.

A second problem is a methodological one of imposing a critical perspective in the arranged situation. It may be argued that to study an innovation, it must first be carefully developed, and then implemented in a classroom or school. The very use of a term such as implementation includes an implicit implication of having to do something with little or no choice. There are several questions that have to be considered when introducing a critical approach to a curriculum. To what extent, and in what form can such a curriculum approach be developed before entering the educational setting? What are the means for bringing it into a classroom so that it becomes a shared responsibility? And who should have the main responsibility for developing and shaping this curriculum approach and be accountable for what happens within the classroom? A curriculum approach that seeks to value the intentions, participation and actions of learners in the arranged learning situation, needs to value and invite the teachers in similar ways into learning about such a curriculum approach.

A third level of contradiction resides in the actual situation. The attempt to introduce a critical pedagogy may fundamentally and significantly contradict the prevailing culture and ethos of the existing learning context. The issue here is about how much and in what ways the teachers' goals and strategies for teaching and learning mathematics in the actual situation diverge from those built into a curriculum approach that embeds a critical perspective. The established norms and traditions of the actual situation in a mathematics classroom which rest on assumptions of what teachers believe and know or do not know, and according to which they act, can be seriously challenged to differing degrees. For example, giving learners choice about their learning in mathematics can significantly conflict with teachers' (and pupils') beliefs, goals and ways of working. The issue of what constitutes the hidden curriculum of a critical pedagogy needs to be considered in terms of how it could give learners conflicting and contradictory messages in contrast to the hidden curriculum of traditional pedagogy.

One approach to the problem of imposition is that choice, negotiation, reciprocity and reflexivity must be key features, theoretically and methodologically, in a critical perspective. But this can mean that the distance between the hypothetical and arranged situation increases, as negotiation with the actual situation always involves compromises. For instance, the researcher may find it difficult to recognise key features of the new curriculum approach in the arranged situation according to her hypothetical understanding and reasoning.

These theoretical and methodological concerns provide a means for discussing the potentiality of situations, to describe and create a narrative about a future situation. It allows us to describe a future context for acting, for teaching and learning mathematics. However, the descriptions of potentiality themselves are in tension with the actuality of situations. This is because the theoretical concepts and ideas in the hypothetical situation are not rooted in actual situations but in the idea of potentiality. This potentiality is derived from theoretical and practical considerations in the theoretical landscape but given life through interactions with reality in the arranged situation. Moreover, many of these considerations in the theoretical landscape are from contexts which are different from the actual situations in which the study is located. However, theoretical concepts and ideas need not only be imported, they may also grow from arranged situations. It is precisely the opportunity to imagine a potential situation that this research attempts to offer and is one of the most important reasons for creating an arranged situation. Possibilities imagined in both theory and practice arise from descriptions of arranged situations. A constructive confrontation between concepts and criteria from theory (or the hypothetical situations) and concepts and criteria from practice (the arranged situation) can bring advancements in theory and in practice (or in new actual situations). From this potentiality of situations, emerge inspirations for new hypothetical, actual and arranged situations.

In summary, my main concern so far has been to primarily address the question of finding a methodology that allows one to investigate an approach to the school mathematics curriculum that by and large is not found in the current system. To this end, I have constructed theoretical tools for explaining a methodology for investigating a social, cultural, political approach to the school mathematics curriculum. Through these theoretical methodological tools it is also possible to engage the inherent contradictions of intervention and imposition of a critical perspective in mathematics education. By setting up a meeting between a theoretical landscape and practice, the opportunity to bring new meanings to a curriculum approach that integrates a critical perspective could be realised. This in turn has the potential to improve both ideas for theory and practice. The question still to be addressed is what should be the fit between a theoretical landscape which integrates a critical perspective and a research methodology that seeks to investigate the landscape itself? This second question will now be discussed.

A critical approach to mathematics education versus a critical approach to research

In this part of the paper I raise issues that I consider to be necessary (but by no means sufficient) in seeking a methodology for investigating an approach to mathematics education that integrates a critical perspective. I do this by using three broad well-known categories: positivist, interpretivist and critical paradigms in research which serve as a map for the rest of the discussion and for locating my research concerns. I then discuss what may be considered a serious difficulty in researching a critical perspective in mathematics education – that of a researcher trying to find consonance between her research approach and her educational approach. The search for a research methodology for mathematics education from a critical perspective takes two routes – one into mathematics education and the other outside it – which is reified through the discussion of some key aspects in the relationships between the researcher, the research participants and the research process.

A distinction quite commonly made between different research approaches in the literature is: a) the empirical-analytical, logical positivist or behaviourist paradigm; b) the interpretive, hermeneutic, phenomenological or symbolic paradigm; and c) the critical paradigm. These categories have been imported into research discussions in mathematics education in various ways by writers such as Romberg 1992; Nickson 1992 and Kilpatrick 1988[3]. This classification is one of many, and not in any way exhaustive, we need only refer to the growing research debates related to postmodernism and feminism. Nevertheless, the first paradigm has dominated mathematics education research as a glance at journals reporting research in mathematics education show. In recent years, with the strong emergence of constructivsm, the second paradigm has also gained much ground. However, if the research journals and the recent handbooks published in mathematics education (see Grouws 1992; Bishop et al 1996; Sierpinska and Kilpatrick 1998) are taken as indicating the state of the art in research in mathematics education then it is reasonable to conclude that the critical paradigm is significantly under-explored and under-represented in mathematics education research. In positioning my research it seems quite logical, even natural, that a critical perspective in mathematics education must surely reside in a critical paradigm. But what exactly is a critical research paradigm? And what relation, if any, could it have to a critical perspective in mathematics education?

A main purpose of this study is to explore the relation between a particular theoretical educational approach and its recontextualisation into practice by student teachers. The educational approach has been described as a social, cultural, political one, which integrates a critical perspective, and I constructed a particular research process through which I explored its realisation in a mathematics classroom. A question that arises from the description of the research process is, what is the underlying theoretical base supporting the empirical work and methodology? The question may be extended to: what is the relation between the theoretical assumptions upon which the research process is based, and the educational theory that is being examined in its interpretation into practice? Indeed, what could or should be the nature of the relation between the educational theory being investigated and the "research theory" – the theoretical underpinnings that inform the construction and enactment of the range of research practices within an empirical study?

Typically, the theoretical framework set out in a study provides the theoretical tools by which the data will be analysed. Is it possible to explore a deeper, more broader link between the theory underpinning educational practice and the research process itself in all its facets: such as in the nature of the question asked, the relation between the researcher and the researched, the involvement of the research participants in the activity of data generation, and the criteria for verification and evaluating the study. The assumption being made here is that just as there is no neutral and value-free mathematics education, the research enterprise is neither neutral nor value-free. The problem then is to not only understand the assumptions which (dis)connect theory and practice in education but also the theoretical assumptions that underpin the research methodology through which that link is explored and understood – the theory and practice of the research itself.

In the way in which the above questions are framed, one could posit a separation within the theoretical considerations in an empirical study. That is, a possible disjuncture between the educational theory and its practices and processes on the one hand, and the research theory and its practices and processes on the other. The research paradigms distinguished above can assist in making visible the theoretical assumptions in the research process and the fundamental ideological differences in how research is understood, engaged and its goals achieved. The theory-practice relation in the different research paradigms, as Carr and Kemis (1986) state, is understood differently. In positivist research "theory is regarded as a source of disinterested principles which...may be taken to prescribe for action". In interpretative approaches to research, interpretations do not prescribe action but "merely informs teachers about the nature, consequences and contexts of past actions, and require that practitioners use their own practical judgement in deciding how to act". From a critical perspective, the relation between theory and practice "is seen to require the active participation of the practitioners in collaborative articulation and formulation of the theories imminent in their practices and the development of these theories through continuing action and reflection" (ibid.: 152)

To explore further the issue of theoretical considerations in research methodology and its link or disjuncture from theoretical considerations in education consider the following from my research. Perhaps an approach to the research could have been to develop a set of criteria or prescriptions from the theory to guide the student teachers, to follow this with classroom observations and interviews with them, and then to analyse the data against a predetermined set of indicators of this critical perspective to mathematics education. The idea that a set of criteria or indicators can be found and applied in the research process comes into a serious and significant contradiction with the theoretical positions within a critical mathematics education and conflicts with the educational process. That this is observed as a conflict, of course, depends on how a critical perspective in mathematics education is understood. Taking a critical perspective in mathematics education cannot be equated to, as Skovsmose and Nielsen (1996) point out, "a sort of methodological principle". Critical mathematics education does not refer to a particular form of mathematics education but rather to a perspective in an educational landscape which involves mathematics. As such, it cannot be outlined as a set of rules for action and content and then followed in order to realise a 'critical mathematics education' (Skovsmose and Nielsen 1996). The problem can be concretised more sharply. In the educational theory, a particular educational relationship is argued for between teachers and pupils, for instance, pupils cannot be 'forced' to learn or become critical. The question to be considered is then similarly, what should the research relationship be between researcher and teacher, for instance, teachers too cannot be 'forced' to take a critical perspective in mathematics education. Hence, the difficulty is that whilst the educational theory is located in a critical paradigm, the theoretical underpinnings in the research could become lodged in a positivist paradigm. My experience in trying to investigate a critical perspective in mathematics education is that this conflict arises quite easily if the theoretical assumptions on which the research is based is not explicitly considered by the researcher in the research process, and its connection to the educational theory is not explored and maintained. What needs to be understood is how and what mediates the way in which a researcher understands the theory-practice relation and chooses to act in particular ways as a researcher, and in constructing research relationships. In this research it has to do with my views about what constitutes a critical perspective in mathematics education versus the research paradigm in which I locate myself as a researcher.

One difficulty in grappling with these problems is that the literature on critical mathematics pedagogy seldom makes its research methodology explicit. In the review of research and literature on ethnomathematics Gerdes admits that "Ethnomathematical - educational research, including the study of possible educational implications of ethnomathematical research, is still in its infancy" (1996: 927). Much of the attention, as can be seen in the work of Frankenstein on a criticalmathematics literacy, is focused on advocating a critical approach to mathematics education, and developing a theoretical base for the approach and related educational practices. Skovsmose (1994) develops his theory of a critical mathematics education by referring to teachers' descriptions and not his observations. More recently critical mathematics education is used to refer to both "educational practices as well as to research on this practice" (Skovsmose and Nielsen 1996: 1260) and it is suggested that research in critical mathematics education can be largely identified with ethnography and action research. This is not to say that descriptions of research processes do not exist. What has still not been adequately developed are a set of reflections at a meta-level in research that begin to put forward a coherent and comprehensive theoretical framework for doing research in this mathematics educational landscape. In the next section I attempt to initiate such a discussion by tentatively marking out some means for making these reflections and identifying some issues for consideration, particularly as they arose in my research.

Sources for developing a research methodology for a critical perspective in mathematics education

There appears to be at least two sources for developing a research methodology for a critical perspective in mathematics education - one from inside mathematics education and one from outside.

The search inside mathematics education

A source inside mathematics education for developing a methodological base for researching a critical perspective in mathematics education is to consider the corresponding educational landscape itself i.e. to draw on theoretical formulations within mathematics education that elaborate a critical perspective which could inform the research process. These may be sought in the work of Skovsmose (e.g. 1994), Frankenstein (e.g. 1987), D'Ambrosio (e.g. 1990) and others who write to develop a theoretical base for a critical perspective in mathematics education. For instance, Skovsmose and Nielsen identify several "concerns" of a critical mathematics education such as "Critical mathematics education concentrates on life in the classroom to the extent that the communications between teacher and student can reflect power relations" (1996: 1257). This could also become a concern in a critical mathematics education research methodology in the relation between the researcher and the research participants (as I will elaborate later). Several other concepts such as 'reflective knowing'/'knowing as an open concept' may have a parallel interpretation in research as reflexivity; 'intentionality' in learning may give insights into understanding the interests of the research participants in the research process; and the notion of 'exemplarity' could provide alternative meaning to the issue of generalisation in this study. What may be observed here is how concerns and concepts in the educational landscape could be recontextualised in the research landscape. This means that the hypothetical situation serves both the researcher and the practioner in recontextualing ideas from theory both for practice and for research.

A counter to this proposition may be that the task, goals and discourse of education must be distinguished from those of research - there may be overlap but they are essentially different human activities. The dilemma, however, is that in preserving this necessary separation, the researcher runs the risk of seriously contradicting the democratic project at the heart of her study. This correspondence between research and educational theory is not only desirable but is essential if a mathematics education theory that is concerned with questions such as, "Does mathematics education reproduce inequalities ... (that) are reinforced by educational practice?" (Skovsmose and Nielsen 1996: 1261) is not to become implicated in reproducing or reinforcing forms of inequality in the research processes and methodology employed to study those theories and related practices. The main thesis here is that any study that puts issues of democracy in the centre of an educational theory must equally be concerned with issues of democracy within the research process itself. A theory, which draws attention to the politics of mathematical knowledge as an integral part of mathematics education, must concern itself with the politics of knowledge production within the research enterprise that seeks to investigate such ideas.

The search outside mathematics education

A second source comes from the progress that has been made in developing and using such research approaches outside mathematics education, in response to similar kinds of questions. A critical mathematics education research methodology could draw on the advances made in methodological issues from outside mathematics education because critical perspectives in mathematics education are inspired by and rooted in a critical paradigm, and draw on the work of those same theorists/theories outside mathematics education. Ethnomathematics, feminist and critical mathematics educators draw on the work of theorists such as Freire, Giroux and others who are proponents of critical theory and perspectives. This means that further elaboration can be found by examining the relation between research and educational theory as it has developed outside mathematics education. There seems to be agreement that (educational) theory and practices that locate themselves within in a critical paradigm must be investigated through methodologies which are themselves located in a critical paradigm. I draw on the writing of Robert Young and Patti Lather who deal with this dilemma.

In *A Critical Theory of Education*, Young identifies the need to "re-theorise or reconstruct general methodological understandings in educational research" and the problem of employing research methodology informed by positivist ideas in studies to investigate critical education: “The existing literature theorises the activity of researchers in epistemological terms and not as social agents. That is, for the most part, educational researchers are theorised as privileged epistemological actors within a theoretical model which is conceptually quite distinct from the theory in which the behaviour of teachers Twenty-two points, plus triple-word-score, plus fifty points for using all my letters. Game's over. I'm outta here. and pupils is theorised.” (1990: 138-9). The clear implication is that there should be some kind of harmony between the educational relationships advocated in a particular theory and the research relationships constructed in the research process. But what does such a research methodology look like and how does one create a correspondence between the theoretical or epistemological base of the educational and research processes and practices?

Going outside education, this issue is perhaps most directly and succinctly discussed by Lather in her paper “Research as Praxis” (see also Lather 1991) in which she explores “the methodological implications of the search for an emancipatory social science”. She writes “The essence of my argument, then, is that we who do empirical research in the name of emancipatory politics must discover ways to connect our research methodology to our theoretical concerns and commitments. At its simplest, this is a call for critical enquirers to practice in their empirical endeavours what they preach in their theoretical formulations” (1986: 258). A key aspect of what is described as an “emancipatory approach to research”, an approach that is “explicitly committed to critiquing the status quo and building a more just society” (ibid.: 258), is the relation between the researcher and the research participants and the form in which this relation gets express through the research process in the generation of the research question, the data, the analysis and theory and even the writing of the research report. The main point to be gleaned from this, for the discussion at hand, is that whatever the understanding of democracy, creating a research process that is characterised by democratic concerns is essential in a study that itself puts issues of democracy in the centre of educational theory and practice. Researchers who involve the research participants in a democratised process of inquiry, according to Lather, engage in research that features negotiation, reciprocity and empowerment. In this way she argues, empowering approaches may be developed to the generation of knowledge. However, as she also points out there are few clear strategies for linking critical theory and empirical research. One approach is to build in opportunities for reflexivity into a critical enquiry at all levels in the research process not only for the researcher but also for research participants.

It is possible to observe, across writers, that to take a critical perspective in theory and practice, means also to take a critical perspective in research. They argue that ways must be found to make this connection and refer to the growing phenomenon of critical research (see for e.g. Cherryholmes 1991; Carspecken and Apple 1992). Two examples of methodologies linked to critical research are action research (see for e.g. Carr and Kemis 1986) and more recently critical ethnography (see for e.g. Quantz 1992, Kinchloe and McLaren 1994)

Toward a critical research methodology: some key issues

The main challenge I set out for researching a critical perspective in mathematics education earlier, I now illustrate with respect my own study - in what ways, and to what extent have I dealt with integrating a critical perspective in my research? I do so by examining different facets of the research process and forms of participation. Several key aspects are identified and discussed in an attempt to connect concerns of practice and theory in research on the one hand, and in mathematics education on the other; and for the different research participants - students teachers, class teacher and pupils - and the researcher.

1. Choice

The choice to participate in the research recognises the aspect of agency, the freedom and the capacity to decide to act, which must be considered, given its central role in a critical education setting. In the research, student teachers were invited to participate as co-researchers to investigate what for them was an innovative, even radical approach to teaching and learning mathematics. Sample selection was based on voluntary participation involving student teachers interested in the opportunities the research provided; who were familiar with the approach and had some understanding of it; and who were to some extent committed to the ideas of the curriculum approach under scrutiny. Several preparation session were held prior to the students entry into schools which provided students with information about the study, so that the choice for their involvement could be informed and based on their willingness, interest and commitment to the educational ideas and the research process. Just as I created an opportunity for joint ownership of the research question, some student teachers created similar opportunities for their learners to own their project problems. So learner interest, a key idea in a critical approach to education, paralleled the student teachers' interest in the research. Notwithstanding, interests themselves differ and are vested in different ways in different parts of the process.

The choice to participate is followed by the choice to shape that participation. This is observed in the choice excercised in the interpretation of the approach within the arranged situation. Although at the beginning of these preparation sessions I had left open the educational tasks and ideas for implementation (for e.g. critical literacy tasks involving newspapers - see Vithal 1997), in the discussions that followed and in school they focused on project work. That is, although the research focussed on investigating a social, cultural, political approach to the mathematics curriculum, the student teachers reshaped that focus to project work as its main interpretation in practice. Within project work, further choices were made, for e.g. in handling the teaching and leaning of mathematics. The dilemma was should it be taught first and then ask learners to apply it or could it be learned as a part of the process of working in the project. In dealing with such dilemmas, my relationship to the student teachers reflected the role of a teacher-supervisor in project work that they were attempting to model in their projects. The student teachers, for instance, give the learners a relevant chapter from a textbook to deal with the mathematical issues that arose in the projects. Such a radical deviation from the conventional approach was negotiated and tried. But was it also in some sense imposed?

There are also the choices that researchers make throughout the research. By the time teaching practice ended I was not able to be present in the classes for three of the projects. This had serious consequences in that these projects were somewhat marginalised in my study. Even though I explained my lack of equal physical presence in the projects to the student teachers and the way in which I was making the choices, this was, however, not without consequences especially for my relationship with the student teachers and the projects not visited regularly, and for those student teachers' lack of participation in later activities. Within the research process itself student teachers chose to collect data and participate in an initial analysis in order to produce a paper for a conference. Drawing on Skovsmose's (1994) notion of intentionality and learning as action, it may be argued that learners learn when they own the reasons and goals to learn. Similarly, student teachers' engagement in the research process was also mediated by their own interest to become researchers and/or practitioners. Student teachers who expressed a strong interest in doing research, collected far more classroom data than other student teachers. After teaching practice, it was these student teachers who drove the process for writing and presenting the paper which came to be a powerful means to see themselves as intellectuals and generators of knowledge.

Choice is an important element in researching a critical educational perspective and essential for participants in the research process because it constructs them as free agents in that should their experience of the research process become in anyway exploitative they could withdraw or change the nature of their participation. This freedom to choose, however, is bounded since it is exercised within other constraints, such as the requirements of their teaching practice course for their degree and the commitment to the school. Choice assists the researcher to not fall foul of practices that are contradictory to the theory on which the research rests. It serves to counter, in part, imposition that any negotiation might lead to especially when the researcher is in an inherently more powerful position than the participants because of the unequal balance of knowledge and skills specific to the situation and to research itself. Choice is also essential if participants are to maximise their participation in the research especially in terms of the effort and commitment that any successful research project requires. The postulating of a hypothetical situation, which offers a space for mediating between theory and practice, supports the element of choice in that the recontextualisation of theoretical ideas need not be uniform or consistent within or across research sites, though they may be negotiated. Choices are shaped by hypothetical reasoning to anticipate actions in the arranged situation.

2. Negotiation

Once the choice to participate is made, there are different kinds of negotiations related to different aspects of the research endeavour that need to be negotiated. First there is the negotiation of research relationships and identity among the different participants: student teachers, learners, teacher and researcher. These must be managed both with reference to the research situation and the educational setting which comprise the arranged situation. My relationship to the student teachers is foregrounded in the study and for each of the students in the study it is different, not least because our histories are both similar and different given our multiple identities in terms of race, class, gender, age, etc. I was aware through their reflections and evaluations that they identified with me in different ways at different times as a teacher educator, a fellow teacher, a researcher, a woman, and as a friend. The research process allowed greater closeness than usual during normal supervision in teaching practice. This relationship underwent further change during the course of the study with a closer relationship developing with those students in whose project I was present a great deal more whilst a distance emerged between those students for whom I was not there. Six students who did not participate in writing the paper, were almost all students whose projects were on the margin. From the outset of the research project I emphasised a relationship of colleagues jointly interested in the same research question.

Once the choice to participate is made, participation has to be negotiated given the different vested interests and identities of the researcher and research participants in the educational and research endeavour. Although I construed the student teachers as co-researchers this belies the unequal knowledge, skills, interest and participation in the research. I declared my dual role of researcher/teacher educator and debated how these could be managed. The student teachers and I shared an interest to know the outcomes in real classrooms of these new theoretical ideas and practices developed in other countries discussed in their teacher education course. However, as their teaching practice supervisor I was required to give an assessment during the research process. Hence, assessments were opened to negotiation. Some student teachers showed extraordinary commitment to the research project. They were very enthusiastic and constantly talked to each other and to me about their ideas, especially those who showed a strong interest in being involved in research themselves. Other students were sceptical because the ideas were considered to be radical, but they were curious about its possibilities and therefore interested to participate. The student teachers in turn, entered negotiations with the teacher and with learners in school to realise a different approach. The preparation sessions held prior to meeting the teachers and learners, and continuous post lesson reflections during the research prepared student teachers for their negotiations and confrontation with the actual situation and to assist them in arranging a situation in a school during teaching practice. These also gave an indication of the student teachers' and my hypothetical understandings and interpretations of the theory and practice under investigation, and a space for a continuous process of negotiation. Negotiation of participation involves also a negotiation of practice. Despite drawing student teachers attention to other forms of practice, they focussed on project work. However the project work itself took different forms and evolved through negotiations in the arranged situation relating to both the actual and hypothetical situations and for both educational and research considerations.

Negotiation is the key to creating the possibility for change. Negotiation allows a situation in which, whatever the idea that is put forward by the researcher and by the participants, it has the status that it can be challenged, critiqued, discarded, reformed or transformed. This means that the quality of reasoning both in the practice and in the theory may be improved. Negotiation is essential and central to the relationships between the hypothetical, actual and arranged situation. Firstly, between the ideals of the hypothetical and the reality of the actual situation, the researcher and research participants need to negotiate their creative pedagogical imagination to develop possibilities for action in practice. Secondly, for a workable and realistic interpretation of the hypothetical situation into the arranged situation, a collaborative transformation of the actual into the arranged is needed. Thirdly, negotiation enables theorising to occur from the ground in the arranged situation, back into the hypothetical situation, and to the a priori theoretical landscape. Throughout these relations, negotiations serve to enhance the quality of the participation of the research participants and therefore of the research. However, negotiation is not without its problems. Given the inherently unequal power in research relations, negotiation itself can dilute different perspectives and contradictions in seeking consensus to act in a particular situation.

3. Reciprocity

Reciprocity ensures that the goals and outcomes of the research process will meet the needs and interests of both the researcher and the research participants. Given the availability of choice and negotiation, reciprocity keeps at bay the possibility for the research process to collapse by helping to secure the commitment and participation of the research participants in the arranged situation. It assists in bringing equity to the research partnerships since both are seen as needing something the other can offer which in turn contributes to effort and commitment. All involved participants should have a clear idea about what is in it for them in the research process. Through reciprocal partnerships, participants are made accountable to each other even if that accountability lies in different domains and interests. It is in reciprocity that the ethics and politics of research are reified and the aspect of rewards and reasons for participation need to be dealt with. Unequal power and differing vested interests make reciprocity crucial in critical approaches to research. In my research, rewards were offered that the participants could decide on in negotiation with me. Of the several suggestions discussed, student teachers chose to jointly write a paper about their experience in the project and participate or present at a conference. Nevertheless, I still recognised that the power relations operated in my favour by virtue of my status as researcher and teacher educator, and nor was I able to apply these concerns in equal measure to the teachers in the school and especially to the pupils in study. Even offering rewards are not without their difficulties since, in themselves, they constitute a power-induced intervention mediated between the researcher who has the authority and power to reward, and those who receive that reward.

Despite having employed many of the strategies similar to those described by critical researchers, I was aware of the inherent hierarchical nature of the relation with the student teachers. In both these settings of researcher and teacher educator I was often construed as the one who should know. Having discussed at length various alternatives for a particular classroom situation in the project work, student teachers still asked for my opinion as an "expert" – "What do you think? Am I doing the right thing?" Thus, in the teaching and research process, where student teachers are learning to become teachers of a critical approach to mathematics education and are also participating as novice researchers, the power relations operate in favour of the teacher educator and researcher. However, there are spaces where this is reversed or equalised, for instance, during discussions of practice related to knowledge about the learners and classroom organisation issues (such as how to arrange the groups in a particular class or what to do with a specific group or student). These shifting power relations are at work throughout interactions between the researcher and the research participants. It is important to recognise this because it influences what data are produced and how these are analysed, especially when the student teachers are also involved in the analysis and writing.

The question is to what extent were my suggestions, in fact experienced by student teachers as impositions. There appears to be an inherent paradox in critical research described by Lather as follows: "The potential to create reciprocal, dialogic research designs is rooted in the intersection between people's self understandings and the researcher's efforts to provide a change enhancing context" (1986: 269). The problem is that whilst the researcher and the research process seek to avoid being impositions (but inevitably are), at the same time the research participants need to be empowered to think and act in new and transformed ways. For the student teachers it was a constant struggle between traditional ways of thinking, acting and being mathematics teachers, and their new role as supervisors or facilitators in project work. And this struggle played out within the constraints of how the school views and organises mathematics teaching, learning, assessments etc., as well as their role as student teachers in subject matter areas other than mathematics.

Throughout the research process I sought to respect the views of the research participants no matter how much they differed from my own as opportunities were created for the ideas in the theoretical landscape to find expression. But this also meant being confronted with racist and sexist views and views that condone corporal punishment. The research process required staying in dialogue with the student teachers, challenging them to be innovative, supporting and building on their own ideas, yet also developing their capacity to critique not only what was happening in the classroom but also their own deeply held beliefs and values. One of the most important considerations in designing the research was to construct a partnership that did not exploit the participants in the research process. This reciprocity concern, however, was not extended in equal ways to teachers and learners in the research. In return, I remained open to the possibility that my deeply held beliefs and the heart of my research concern that mathematics education has a role to play in building a democratic and just society and that cultural, social, political issues can and should be discussed in mathematics classrooms, may be seriously challenged and completely shattered when faced with the reality of mathematics classrooms and schools. Choice, negotiation and reciprocity are important features not only in methodology but also in criteria for evaluating such research, such as in democratic participatory validity, which requires the researcher to make visible the extent to which the research participants participate in the research (see Vithal 2000).

4. Reflexivity

Reflexivity opens for constant critique in research and in the educational setting. Opportunities for reflecting on the arranged situation were made available through: a) various interviews with student teachers, both in the project and outside, and the class teacher; b) data generated by the pupils in the class which included their diaries, work done on projects, charts produced and any other written material; c) through presentations made by the student teachers to a PhD reference group and a faculty research seminar; and d) the production and presentation of a joint paper at a national conference for mathematics education (see Vithal et al 1997). Reflections on the arranged situation are important because they give us the means by which to connect back to the hypothetical and actual situations and to seek shifts in these. Involving multiple levels and points for reflections and involving different participants both in the centre and at the margins in the research and educational processes gives rise to a reflexivity which brings both an insider-outsider perspective, and opens opportunity for self-critique. Student teachers responded differently to critique from me, the class teacher and their peers who were assisting in the project work. The student teachers' involvement in these research activities demonstrate how spaces for reflexivity may be created in the arranged situation and as they leave it to return to the actual situation. Moreover they have methodological implications because they contributed to an early and initial analysis of the data. The issues they identified in these reflections were debated and discussed and through these processes became part of my analysis in the research. They showed how hypothetical situations were changing and therefore also how they would act in an actual situation and in a newly created arrange situation.

Reflexivity is needed for managing the multiple identities of the researcher and research participants. Throughout, I was acutely aware of my shifting framework for observing and interacting as I grappled with and tried to understand what it meant to be both the researcher and the teacher educator. For instance, when standing at the back of the classroom with the video camera, I was to all intents and purposes a non-participating observer. That immediately changed when the student teacher asked my opinion about something or if the pupils drew me in with a question. A significant difficulty was in trying to simultaneously reflect and understand what was going on as a researcher while at the same time acting as a participant in the process as a teacher or teacher educator. The position I adopted as far as the classroom interactions were concerned was to participate, if I was invited by the student teacher or the pupils. The researcher/teacher educator dilemma also emerged in the post lesson discussions as I presented alternative solutions and strategies from the ones the student teachers were considering which were usually informed by a traditional mathematics pedagogy. Reflexivity serves to flatten the hierachy of relationships in research and in the educational environment because all reflections are considered and valued.

Reflexivity makes possible more equitable theory-practice relations and allows for the development of both theory and practice through shared reflection and critique. The post lesson discussions provided strong opportunities for reflexivity and joint analysis, on a regular basis, of what was going on in the classroom. It was during this time that alternatives were discussed and decisions negotiated for the day to day running of the project. In writing their projects, began a preliminary data analysis process and overview of the project for each student teacher. The opportunity and process of writing the paper provided a space to reflect and more closely resembled what researchers do – having collected data, doing an analysis and writing. It was during these times that students could reflect on what had happened in the different projects across contexts, which led to alternative explanation for events observed in their own projects and affirmed or showed gaps in the theory.

This brings us to another important, but related concern about how the relation between theory and practice is constructed in the research situation and the role of the researcher and the research participants in the critique and generation of ideas for both theory and practice. Two questions must be raised here: first, how should the critique and development of theory and practice be managed in research; and second who should be involved in this process? Typically theory is priviledged and this priviledge is extended to the researcher - theory speaks to practice. Reflexivity in critical research gives practice the opportunity to speak back to theory. Both theory and practice become matters for negotiation between the researcher and the research participants. It is also for this reason that the recognition of a hypothetical situation, which mediates how theory and practice are recontextualised during the study, is important. Moreover, a critical perspective positions the researcher not only as someone who seeks to understand the research situation, but legitimates the researcher's active involvement in, and on the research situation. This means that while the researcher has an important role in sharing reflections on practice, student teachers have equally a space to reflect on their experience with respect to the theory.

Reflexivity is also essential for contexts in which there is theory and practice importation. The main focus of my research was to give meaning to a particular theoretical landscape in a context vastly different from that in which it was conceived and then to see how the process could yield new insights both for theory and practice. Thus, while the theoretical landscape led to a particular practice which generated specific data, the data in turn would come to inform the theory. Lather (1986) brings this issue to the fore by examining theory building versus theory imposition as dialectic. In this sense the dialectic of theory imposition versus theory building played itself out in my study as theory imposition creating opportunities for theory building by both the researcher and the student teachers. This point is important in that seeing theory and practice as dialectical (see Roman and Apple 1990) is not the only consideration in a critical approach to research, but also recognising that the practitioners or the research participants have an active role to play in theory building. This is however, easier said than done because what needs to be considered is the research participants' (lower) interest in theory building. For most of the student teachers, their main interest was in improving practice rather than building theory.

5. Subjectivity-Objectivity

The relationship of the researcher to the research process takes different forms in each of the positivist, interpretivist and critical approaches. In positivist educational research the researcher is the "instrument by which research is undertaken" as an objective observer. The interpretivist researcher reconstructs and interprets events for greater understanding which "become part of the language of their time and influence(s) the decisions made by others". However, in a critical approach to educational research the researchers' "participation in the development of knowledge is comprehended as social and political action which must be understood and justified as such" (Carr and Kemmis 1986). This does not mean that these are discrete relationships. Although the relationship of the researcher to the research process may be driven by one paradigm, and in this study the critical paradigm, there are instances during which the other relationships do manifest. That is, in the practice of research, there are times when the researcher might be positioned as the 'objective observer'. However, the overall research process must itself be guided and grounded in methodology that corresponds with the theoretical commitments of the researcher.

The relationship of the researcher to the research process and the research participants has often been discussed through the debate about objectivity and subjectivity in research methodology. A rather simplified tracing of the history of this debate seems to suggest a shift from a preoccupation with and concern about establishing objectivity in research in the positivistic paradigm to a situation in which subjectivity is recognised as important and understood in a multiplicity of ways in research. There might be what is called "disciplined subjectivity", as one example in the interpretivist paradigm. This debate however rages on (see for e.g. Eisner and Peshkin 1990). Roman and Apple argue that subjectivity and objectivity should not be "treated as a binary opposition in which the absence of one is seen as the presence of the other" but rather what needs to be acknowledged is "the reciprocal determinacy that "subjectivity" and "objectivity" - the conflicting sets of historically specific power relations and material interests - have upon one another" (1990: 39). Research that is "openly ideological" (Lather 1986) or is constructed or seen explicitly as "an ethical and political act" (Roman and Apple 1990), and which attempts to address concerns for inequalities and injustices, has forced researchers to question and re-examine the "Subject-Object dualism" in new and different ways. An assumption here, is that objectivity and subjectivity are separate but in dialogue. It is possible to posit yet another approach, and that is one in which objectivity is interpreted as inter-subjective agreement - giving up any search for objectivity and settling instead for inter-subjectivity. There is no 'truth' to be found through research, but only multiple truths depending on the position taken or occupied in the research setting. But even such a position is not unproblematic since these do not all have equal status given that "meaning is jointly constructed between researchers and the research subjects in the context of interests that are formed out of contradictory power relations" (Roman and Apple 1990: 40). How researchers think about and resolve the objectivity and subjectivity positions in their individual research is important because it deeply affects how research relationships are created, the research process and procedures set up, what is accepted as data, how and who participates in the analysis and production of theory.

6. Context, Change and Instability

Perhaps what may be considered a silence or gap in discussions about critical research methodology is the question of: how is critical research context related? Does matter if the methodology is considered in Denmark or South Africa? What does it mean to do critical research concerned with issues of equity and social justice, in societies marked by rapid change, deep and structural inequalities, violence and poverty? The powerful contestations and resultant disruptions which produce instability, the heart of a critical research agenda, threaten the very existence of the research situation. What distinguishes countries like South Africa from other countries such as the USA, is the scale of the disruption and instabilty, and its location at the centre of society, involving mainstream concerns, rather than at the margins. There is an assumption about stability in research, including critical research that assumes a research situation and its participants are constantly available to the researcher.

The description of the research methodology for my research could be described as a relatively normal, steady process. There is little indication of the history of the research process, the material conditions in which it was located, or the transformations that were taking place in the context in which the research was happening. Yet the research was marked by severe and consistent disruptions and instability with strikes and protest action. Early in the study, while working with student teachers, the university closed down and the preparation sessions (which were also disrupted) replaced substantive negotiations with teachers in schools, which resulted in a more marginal participation by class teachers in the project work. Later, in schools, during teaching practice, teachers went on strike which impacted directly on how the project was realised, for instance, one project was completely abandoned. Disruptions to carefully conceived research designs are the norm rather than the exception in educational research in South Africa. Why disruptions are produced in research, how they come to feature in the research process, and what is/can be done with disruptive data has been discussed elsewhere (Vithal 1998). It may be argued that the stability assumption built into the research situation, in research methods and methodologies, are largely imported from the "north", from the more "developed" contexts, and often applied in "developing" countries in the "south" which is characterised by rapid and huge transformations in virtually all its institutions (see Valero and Vithal 1998). Such an assertion attempts to bring an analysis of power relations engaged within theory and practice, and within research and education, also to the inequalities in the production, importation, ownership and legitimation of the *means* by which knowledge is produced-the research methodologies themselves in mathematics education. Critical research has emerged in particular contexts, in response to a particular set of concerns and conditions, and is itself implicated within the larger global inequalities. The potential for more equitable dialogue is considerable, if for instance, the disruption and instability thrown in such sharp relief in the South African context is seen to bring greater focus on similar situations in other contexts where such concerns may appear marginal or marginalised. The challenge for critical researchers is to take seriously instability in research situations, both methodologically and theoretically, and to consider what it means to focus on unsanitised "disruptive data" not only as a procedural matter but substantively, both practically and theoretically. The politics of knowledge production within critical research itself may be problematised through a focus on context and the concern to theorise disruption and instability in methodology.

7. Emancipation, Empowerment and Hope

In this concluding aspect I return to a central idea of any critical education and research. What distinguishes a critical approach from other approaches to research, Carspecken and Apple (1992) write, is that the critical researcher is deeply concerned about "inequality and the relationship of human activity, culture, and social and political structures." These concerns guide the questions that are posed and the nature of the inquiry in which the critical researcher acts on the world with others "in democratic ways so that this world may change". Critical researchers make explicit that their research agenda is not only about understanding the inequalities and injustices but also includes an openly transformatory and emancipatory research agenda which means that they seek to bring some change to the participants' lives' and their contexts. Although this intention can be read in my broad research focus in the choice to select and study a particular approach to mathematics education that is concerned with empowerment and emancipation, this was not a direct or explicit goal in the research.

Nevertheless, I must address the notions of empowerment and emancipation, which have an important position in the hypothetical situation, but raise several difficulties within the arranged situation in the research. The first is that whilst a research project may claim an emancipatory intent or purpose, this cannot be predetermined. The researcher cannot know the direction, nature or content of an empowerment or emancipation nor its impact on the whole life of the participants. Empowerment in some aspect brings disempowerment in others. Second, is the inescapability of the "imposition of emancipation". Someone, usually a researcher, as insider or outsider, selects and decides to involve participants in a research process. The focus on selection based on disadvantage and oppression in critical inquiry leads to the situation of someone in an inevitable position of power defining a group as such and beginning a research process. The third is the need for a deeper understanding of the nature of the participation of the researched in all aspects of the research process. The participants do not have the researcher's skills and knowledge and an inherent unequal situation cannot be avoided between the researcher and researched even though the researcher may act to reduce that in specific ways. This is the problem of the researcher knowing best in the research situation. A fourth problem is that of a "once and for all transformation". A main focus of critical research is the notion that the transformation that is hoped for, occurs in the research process, but what of its sustainability or transient nature. A fifth difficulty lies in seeing the research process itself as constituting the transformation. That is, the processes and practices for studying the change do not necessarily coincide with the processes and practices for making the change. The problem is that of being involved in transforming a situation and simultaneously studying it. Finally whilst no methodology is neutral, no methodology is inherently emancipatory or positivist. Rather it depends on the theoretical assumptions built into it, the way in which it is given meaning in its use within a research design and the researcher's theoretical leaning[4]. Problematising the aspects of empowerment and emancipation is not intended to discard such notions but rather to see them as located in the hypothetical situation which inspires and gives reasons and goals for thinking and acting in the arranged situation.

Rather than to speak of research carrying emancipatory intent, it may be useful speak of research as carrying possibilities and hope, an idea also put forward in the theoretical educational landscape. Although this may be interpreted as weakening a critical approach, it serves equally to address some of the difficulties mentioned above. But the question is what can the "principle of hope" found in a critical mathematics education (Skovsmose 1994) mean in a critical approach to research? I was aware that student teachers would be inducted into their first major teaching experience through a radically different pedagogy. The research situation that I arranged with them and through them created the possibility and potential for change at a number of levels and in different areas - their role as student teacher, teacher of mathematics with a new approach and so on. Whatever the intent and my direct intervention, changes are difficult to anticipate in contested unstable contexts. But still, it is the hope for change, which inspires and drives initiative and effort to arrange situations. The research situation itself revealed what possibilities (if any) there were for change and what may in fact change, even if temporarily. Distinguishing an actual, arranged and a hypothetical situation offers a means for talking about a critical approach to research that focuses on possibilities and hope, on potentiality and actuality. Even though critical researchers may enter into the investigation with their epistemological assumptions and political agenda admitted upfront (Kincheloe and McLaren 1994), sustaining these concerns throughout the research may be far more difficult.

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[1] This paper is an edited version of a section from my doctoral dissertation (see Vithal 1999)

[2] These ideas first proposed by Ole Skovsmose have been developed in research seminars in South Africa, Denmark and Brazil and become part of my PhD project through the supervision discussion process.

[3] The latter two writers refer to the last category as action research rather than as a critical paradigm. Although action research is often elaborated with respect to the critical paradigm (see for example Carr and Kemmis 1986) not all action research fits into this category.

[4] Survey research, for instance, can be used for purposes of empowerment because even though it assumes a particular relationship between researcher and researched, it can be subverted toward more egalitarian ends (Singh and Vithal 1999).

TEACHER-ASSESSMENT AND EQUITY

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This paper presents a critique of teachers' assessment practices from a social justice standpoint. It is based on two studies of different aspects of the established system of teacher assessment in the UK. Each study found that teachers' own perspectives, resources and interpretations led to the construction of views of students' mathematics which differed from those constructed by other teachers or researchers. The authors conclude that professional critique of assessment decisions made about individuals needs to be developed, and raise further research questions about equity in assessment practices.

Recent research and curriculum development related to assessment in mathematics education has been associated with changes in conceptions of the nature of mathematical learning. There have been some powerful critiques of the poverty of the information provided by some traditional methods of assessment and of the inequity inherent in them (e.g., Burton, 1994). Assessment methods associated with "reform" curricula have also begun to be critiqued from an equity perspective (e.g., Cooper & Dunne, 1998). In rejecting crude positivism, however, most reformers and researchers have still made the assumption that, with "improved" methods, it is possible to achieve valid insight into students' thinking. We wish to problematise this assumption.

In a number of countries, the wish to broaden the repertoire of strategies for assessing students in mathematics has led to an increase in the responsibility devolved to teachers for assessing their own students. Among the potential problems, Clarke (1996) identifies teachers' lack of expertise in "devising, employing and interpreting assessment tasks" (p. 333). This formulation assumes that it is possible to have "expertise" in interpreting students' responses and that teachers can be supported in gaining this. We shall argue, both theoretically and through our empirical investigations of the practices of teachers engaged in such interpretation, that this is not a simple issue. The two independently conceived but complementary research programmes that we shall describe (Morgan, 1998; Watson, 1997; 1998) both address concerns with the nature of teachers' "expertise" as assessors.

Assessment is contextualised and interpretative

Every act of assessment takes place within a specific context, consisting not only of the current circumstances but also of the resources brought to bear by the assessor (including personal knowledge of mathematics and of the curriculum, experience and expectations of a particular child and of children in general, beliefs about assessment, experience of the classroom, etc.). These "reader resources" (Fairclough, 1989) arise both from the assessor's personal, social and cultural history and from their current positioning within a particular discourse. The professional enculturation of teachers seems likely to ensure a certain degree of common resource, as may be seen in the success of training programmes and joint moderation meetings in achieving consensus about ranks, levels and grades in relation to individual pieces of work and portfolios (see, for example, Wiliam, 1994). However, teacher-assessors will also make use of more individual resources, including their past and present relationships with individual students or with groups of students and the extent of their authority and autonomy as professionals. A range of positions are available to teachers in relation to their students, other teachers and external authorities (Morgan, 1994). Different positionings are likely to give rise to the use of different sets of resources and hence to different actions and judgements by different teachers or by a single teacher at different times in different circumstances (cf. Evans & Tsatsaroni, 1998; Walkerdine, 1988) .

The two studies of teacher-assessment that we discuss below are located within interpretative paradigms. In both studies attention was focused on possible ways of interpreting texts, where "text" is taken to encompass physical actions, facial or body language as well as spoken or written language. The teacher "reads" any text produced by a student in an interpretative and contextualised way, constructing meaning in terms of the student's mathematical attainment. A new orthodoxy of mathematical activity and interactive classroom practice, rooted in various versions of constructivism, is fairly well-established in mathematics education, based on recognition of the interpretative and contingent nature of students' construction of knowledge. Less consideration has been given to how teachers base pedagogic decisions on their constructions of individual students' mathematics. Though Simon (1995), for example, writes that "the teacher can compare his understanding of a particular concept to his construction of the students' understandings, not to the students' 'actual' understandings" (p.135), the cumulative effect of such interpretations is not discussed, and any doubts about the validity of summative statements of student achievement tend to be expressed in terms of the quality of the sampling of student behaviour (due to situatedness, temporal specificity, assessment style etc.) rather than in terms of its interpretation.

The impressions and expectations of their students that teachers develop through their readings of the evidence available to them are incorporated into the curriculum and into future interactions with the student. Hence expectations tend, unsurprisingly, to be fulfilled (Nash, 1976). Early impressions are therefore crucially important and may have such a strong effect that subsequent events are noticed and interpreted only insofar as they support the original impression (Nisbett and Ross, 1980). Moreover, teachers' expectations about students' mathematical learning may not be based on mathematical evidence, but on the evidence of other behaviour, social skills, and social class background (McIntyre et al, 1966; Walkerdine, 1984). What appears salient to one teacher may not to another; this is a particular issue within mathematics with its wide range of possible modes of communication.

The research studies

The two studies we will discuss originated within the assessment system in UK state schools, where teachers' assessments of students' mathematics have been a statutory part of national assessment procedures for over ten years. All teachers have been trained to incorporate assessment into their teaching, and to use assessment criteria. While the particular national context necessarily affects the detail of the teacher-assessor practices that we will describe, our analyses illuminate broader issues which have to be seen as features of an established system.

Study A: Teacher's constructions of views of students' mathematics

The initial aim of study A was to develop a description of ways in which teachers interpret and accumulate their experience of students in mathematics classrooms, not only during formal assessment situations but also during normal day-to-day activity (Watson, 1995; 1999). Interviews with teachers revealed a number of problematic aspects of practice. When asked about individual students, teachers talked more of their learning behaviour in general than specific achievements. For example, it was common to say that students who were regarded as strong mathematicians were "well-organised and self-motivated" or "quick" rather than to describe particular mathematical features of their work. Although teachers thought it very important to take into account students' oral work, they tended to do this unsystematically. A combination of records of written work and personal recollection formed the basis for most teachers' assessments. Teachers spoke of "getting to know" students as a frame for interpreting their work, rather than as an outcome of informal assessment. Summative assessment was done partly by personal recollection – some teachers, knowing the flaws of testing, regarded this as much fairer than testing.

These issues were explored further in a study of two teachers "getting to know" new classes (Watson, 1997). The researcher observed ten students in one lesson a week with each teacher during the first term with the new class. All public verbal utterances and some one-to-one interactions by the target students were noted, all written work produced by each student during the term was scrutinised, behaviour and actions were observed and noted using a combination of systematic observation punctuated with records of complete incidents, as interpreted by the researcher. Considerably more observed information about behaviour and actions of the target students was available to the researcher than to the teacher, though they had similar oral data and the same written data; the researcher also had more time to consider the data. Both the researcher and the teacher formed views about the target students' mathematics and discussed them regularly with each other, but the differences in the data to which they had access contributed to substantially different interpretations. The following example illustrates typical differences.

Sandra (aged 11) took part enthusiastically in the routine start of each lesson when the class marked their own answers to mental arithmetic questions done at home. She sometimes called out answers and often put her hand up energetically, waving it around to attract attention. Nearly all her enthusiastic contributions to class arose from work done at home or, very occasionally, from discussions the teacher had with her. When her answers were wrong Sandra made comments like "but my father said ...." It was as if she knew that to get approval from the teacher you had to show right answers publicly, and she was skilled in getting those right answers not from her own head but from her father or from the teacher. Other evidence to support an interpretation that Sandra was weak in number work was her use of fingers, particularly for subtraction, in situations where a competent arithmetician would have known number bonds or have developed some patterning strategies.

The teacher, while being aware that Sandra sometimes altered answers as she marked them, was not aware of the extent of the alterations and had a view that Sandra was mainly good at mental arithmetic, changing answers in order to boost her confidence. The teacher's estimation of her arithmetic, initially low as a result of a test, had risen to a relatively high level as a result of her oral contributions to class. The researcher's estimation was that she lacked skills, lacked useful internalisation of arithmetical facts, and had previously relied on performance of algorithms which she failed to remember. It was as if the teacher had a neutral view of her until something outstanding happened, in this case her enthusiastic contribution to homework feedback sessions. Once there had been an event which allowed him to differentiate her mathematics from that of other students, the view he formed remained strongly with him so that subsequent events were interpreted in that light. Hence her alteration of answers became under-represented in his picture of her and the pattern of her responses was not obvious to him.

In contrast, the teacher felt that Sandra was relatively weak in her ability to think mathematically while using and applying mathematics or when tackling new ideas. However, several instances of mathematical thinking were inferred by the researcher from Sandra's verbal comments and from her actions. In these instances she appeared able to devise strategies, adapt strategies which have been effective in the past, describe patterns and make conjectures resulting from patterns.

Relative to the researcher, therefore, the teacher *appeared* to overestimate skills in the area of mathematics in which Sandra wanted him to be interested, and to underestimate her skills of reasoning. The teacher, seeing her work always in the context of what the rest of the class did and what his own expectations were, makes a judgement which is *comparative* to what she has done before and to the rest of the class. But "what she has done before" includes creating an impression in his mind, therefore judgements are relative to the picture already formed. Of course, the researcher was able to see a pattern to Sandra's oral contributions that was not visible to the teacher. It was the pattern and circumstances of the contributions, rather than their quantity or nature, which indicated the contrast between her arithmetical abilities and her desire to be good with calculations. The teacher has seen this analysis and accepts it as a description of aspects of Sandra's work and the problems of informal judgement of which he was not, *and could not* be aware. The competing demands on the teacher's attention in a classroom make detailed observation impossible.

Analysis of data about the ten students in this study led to the identification of a number of issues (see Watson (1997) for a fuller discussion):

- teachers may see only part of the whole story;
- teachers may see, or fail to see, patterns in responses and behaviour;
- some behaviour may be over- or under-represented in the teacher's mental picture;
- teachers may be strongly influenced by students' strong or weak social skills;
- teachers interpret work in the light of existing impressions;
- time constraints on the teacher prevent full exploration of mathematics;
- perceptions of external purposes affect assessment;
- teachers are unable to see and use all the details which occur in classrooms.

In no sense are we suggesting that there is a true view to be achieved, nor that the researcher is correct and the teacher wrong. On the contrary, the study suggests that teachers' informal assessments, made in classroom contexts, are *inevitably* influenced by a variety of unavoidable factors which may have little to do with mathematical achievement. In the UK system, these informal assessments contribute to formal, summative, high-stakes assessments as the teacher brings her perceptions of the student to bear on subsequent interpretations of mathematical performance. It is of concern that this takes place within a system in which teachers have been trained to use detailed, tested, criteria, and where assessments contribute to high-stakes decisions.

Study B: Teacher assessment of written mathematics in a high-stakes context

The second study was set in the context of the high-stakes GCSE examination for students aged 16+ in England and Wales. The coursework component of this examination, which is assessed by teachers, most commonly takes the form of reports of one or more extended investigative tasks. These reports are intended to include evidence of the mathematical processes that students have gone through (for example: systematising, observing, conjecturing, generalising, justifying) as well as the results of their investigation. The original concern of this study was to investigate the forms of writing that students produced in their coursework and to consider the match or mismatch between student writing and the forms of mathematical writing valued by teacher assessors (Morgan, 1998). Analysis of interviews with 11 experienced teachers reading and evaluating students' coursework texts explored the teachers' assessment practices, the features of the texts that the teachers attended to, and the values that came into play as they formed judgements about the texts and about the student-writers. The issue emerging from the results of these analyses that we wish to consider here is the diversity that was discovered, both in the ways different teachers approached the task of reading and assessing student texts and in the meanings and evaluations they constructed from the same texts (Morgan, 1996; 1998).

All the teachers had been trained in the use of common sets of criteria, were experienced in applying these criteria to their own students' work, and had participated in moderation processes. Nevertheless, their relationships to the criteria and their methods of approaching the task of applying them were in some cases very different. The following example illustrates the ways in which teachers reading with different resources can arrive at very different judgements about the same student.

One student, Steven, had written a report on his work on a task called 'Topples' that involved investigating piles built up of rods of increasing lengths, seeking a relationship between the length of the rod at the bottom of the pile and the length of the rod that would first make the pile topple over. Steven found a formula expressing this relationship, $(A+A)+\left(\frac{A}{2}\right)=b$, and showed that he could use it to find the length of the 'topple rod' for piles starting with rods longer than those he had available to build with. He then presented an alternative method for finding results for piles starting with very long rods by scaling up his results for piles starting with short rods:

> *An alternative way to do this would be to take the result of a pile starting at 10 and multiply it by 10*
>
> $$(10+10)=20 \quad \left(\frac{10}{2}\right)=5$$
>
> $$20+5=25$$
>
> $$25\times 10=250$$

No derivation or further justification of this method was provided in the text. In order to evaluate Steven's work, each teacher-reader constructed his or her own understanding, not only of the method itself but also of the means by which Steven might have derived it and of Steven's level of mathematical achievement. The following extracts from interviews with two teachers reading this section of Steven's work illustrate how different these understandings can be.

> <u>Teacher 1: Charles</u> *Um ok so I mean he's found the rule and he's quite successfully used it from what I can see to make predictions about what's going to happen for things that he obviously can't set up. So that shows that he understands the formula which he's come up with quite well, I think. There's also found some sort of linearity in the results whereby he can just multiply up numbers which again shows quite a good understanding of the problem I think.*

Charles recognises the mathematical validity of the alternative method, relating it to the linearity of the relationship between the variables. He takes this as a sign that the student has "come up with" the formula as a result of understanding the linearity of the situation. This results in a positive evaluation of Steven's mathematical understanding.

> <u>Teacher 2: Grant</u> *It's interesting that the next part works, I don't know if it works for everything or it just works for this but he's spotted it and again he hasn't really looked into it any further. He's done it for one case but whether it would work for any other case is er I don't know, he hasn't*

> *looked into it […] And it's just a knowledge of number that's got him there I think intuition whatever. He may have guessed at a few and found one that works for it.*

Grant appears less confident with the mathematical validity of the alternative formula, expressing some uncertainty about whether the method would work in general. Perhaps because of this uncertainty, his narrative explaining how Steven might have arrived at the method devalues the student's achievement, suggesting that the processes involved were not really mathematical: "spotting" the method, not looking into it properly, guessing, using "just a knowledge of number" or "intuition".

In this case, the teachers' different interpretations of Steven's level of understanding and different hypotheses about the methods he might have used to achieve his results seem to be connected to their personal mathematical resources. It is Charles, expressing a clear understanding of the relationship of the alternative method to the linearity of the situation, who makes the most positive evaluation of Steven's understanding, while Grant , apparently uncertain of the general validity of the method, constructs a picture of the student working in relatively unstructured or experimental ways. Other differences in interpretation and evaluation of Steven's results and his means of achieving them were also evident in other teachers' readings of his text.

In order to make sense of a text and to use it to evaluate the student-writer's achievement, each teacher must compose an explanatory narrative, drawing on the resources available to them. These resources include common expectations of the general nature of investigation and investigative reports. Where students had produced 'standard' work and expressed it using the conventions of the investigation report genre it was found that teachers' evaluations coincided closely. However, if a student text diverges from the 'usual' to the extent that it is not covered by the established common expectations, each teacher must resort to their more personal resources, thus creating the possibility of divergence in the narratives they compose. Major differences between teachers in their interpretations and evaluations of students' texts occurred primarily where the student's text diverged from the 'norm' in some way – either in its mathematical content or in the form in which it was expressed.

Other ways in which teachers' interpretations and approaches to evaluation were found to differ included:

- different hypotheses about work that the student might have done in the classroom that was not indicated in the written text – and different attitudes towards valuing such unrecorded achievement;
- different judgements about factual aspects of the text, for example, whether the wording of the problem given by the student had been copied from the original source or had been paraphrased in the student's own words;
- different approaches to the task of assessing a piece of student work: some teachers appeared to focus on building up an overall picture of the characteristics

of the student, some were interested in making mathematical sense of what the student had done, while others focussed solely on finding evidence in the text to meet specific criteria;

- different ways of resolving tensions between the teachers' own value systems and their perceptions of the demands of externally imposed sets of assessment criteria.

Conclusions

The two studies illustrate the variations that are possible in judgements teachers make about students' mathematical achievements and suggest some of the sources of these variations. The first study suggests that teachers form views of students' mathematical strengths and weaknesses based on information which is inevitably partial, due to the impossibility of seeing and hearing everything, the need to stress some aspects of a student's performance and decide which others are unimportant. The second study shows that judgements about written work in mathematics are intimately connected with the values and experiences that teachers bring to their interpretations of student text. It seems likely that such values and previous experiences are also significant in influencing the aspects of student behaviour that teachers notice and attach importance to in the classroom. While the evidence teachers are able to attend to in the classroom is inevitably partial and will vary between different observers, even when exactly the same evidence is available to all observers (as it is in the case of written texts) the two studies show that different teachers can interpret the same or similar student texts of all kinds in very different ways, attending to different salient features and placing different values on similar features. These differences can occur both in informal classroom assessment and in formal high-stakes situations.

In both studies the teachers were experienced and had been trained in assessment methods. They were also aware that they were involved in research about assessment. They may thus be assumed to have been making judgements in the most professional ways they knew about. Differences are unlikely, therefore, to be attributable to a lack of skill or a lack of professionalism. In both situations the judgements could influence what happens next in the student's mathematical career. Judgements made in the everyday classroom influence the teacher's future interactions with the student and the mathematical teaching provided, while judgements in more formally summative contexts have obvious 'high-stakes' consequences for progression to further education and employment opportunities. Yet, in both situations the positions and priorities, values and experience of the teacher influence the judgement.

While it may be possible to resolve such differences in summative assessments where there is time available and teachers can meet and discuss decisions (Clarke, 1996; NCTM, 1995; SEAC, 1991), we would argue that, because of the interpretative nature of any act of assessment, it is not possible to avoid differences altogether. This raises important issues for equity in education both in summative and formative situations. In particular, those students who do not share the social, cultural and linguistic background of the teacher-assessor are less likely to have access to the resources that would enable them to produce texts that will have the features that the teacher will attend to and value highly. Moreover, the consequences of even informal teacher judgements are not merely fleeting but have lasting influence on the educational opportunities available to students.

We are not suggesting that teachers should not make judgements; people have to make subjective judgements about others in order to communicate. Moreover, we are not arguing that teachers are "bad" at assessing and need to be trained in "correct" methods. Rather we would argue for further professionalisation of the ways teachers read and evaluate students' mathematical texts, and the ways they make use of their informal evaluations. This might involve the development of self-critical approaches to making assessments about others, including awareness of the relative, partial, and interpretative nature of assessments, of the prejudices which may influence them and of the ways in which assessment can deny students access to equal opportunities. A critical professional environment in which colleagues are expected to question and justify decisions could support such development.

Further questions

In considering the implications for future research related to teacher assessment in mathematics education, we are concerned both with the quality and nature of the judgements made by teachers and with the issues of equity raised by questions about students' access to the means of expression that are likely to lead to high evaluation of their mathematical competence. Questions that need to be addressed include:

- What are the characteristics of student behaviour that will lead to a student being seen as a competent mathematician, and that will lead to high evaluations?
- To what extent are students belonging to various social and cultural groups aware of their teachers' values and expectations and able to demonstrate them?
- To what extent are teachers, teacher educators and assessment designers aware of the ways in which assessments may be influenced by various teachers' values and expectations?
- How can teachers raise students' awareness of their assessors' values and expectations and enable students to behave in (mathematical, linguistic and social) ways that will lead to high evaluations?
- Do the differences between and within teachers' judgements act structurally to disadvantage certain social groups? How do any such inequities relate to those known to be inherent in formal written tests?

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Meanings and consequences of research in mathematics education

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Introduction

Renuka Vithal's paper '*Re-searching mathematics education from a critical perspective*' engages with the two methodological issues that arise in undertaking such work. The first is concerned with what is researched, and specifically, whether we research existing practice or research instead 'what might be'. Traditionally the emphasis has been on the former, but more recently, there has been much more emphasis on 'design experiments' designed to transform the settings they study, but little attention has been paid to the problems inherent in such studies. The second issue is concerned with how the research is conducted, and in particular how the framing of the research reconciles the conflicting priorities of the production of research findings that transcend the immediate context of the research while also being conducted in ways that are consonant with the precepts of critical mathematics education.

Although it is traditional, and perhaps even expected, in a response to such a paper to engage in a critique of the paper, I find myself so wholly in agreement with Vithal's paper that to attempt a critique would be an artificial and sterile exercise. Instead I shall simply engage with the issues raised by Vithal and attempt to outline a theoretical framework which I believe might be helpful in developing these debates further.

What is and what might be

Vithal's paper illustrates very clearly the associated difficulties of *what* should be researched and *how* this should be done. Although these two are at first sight, different questions, a cursory look at the history of research in mathematics education (and in education more broadly) shows very clearly that issues about how research should be carried out have a great influence on (indeed, some would say has largely determined) what gets researched.

In her paper, Vithal distinguishes between various different settings for research. One could research the actual situation—that which we find around us right now—but as Michael Apple also points out in his paper at this conference, the analysis of *what is* has led to a neglect of *what might be*. In some cases, no doubt, this stems from a carefully thought-out epistemological position that it is better to start with where we are now, and understand what is going on, before we try to change things. However, in my view it is also the case that the researching of what is rather than what might be is regarded as easier, more straightforward, or more likely to generate research outputs that are acceptable to the academy. Vithal's engagement with these two questions is therefore timely.

As Vithal points out, any attempt to move from what is to what might be is theoretically driven. It is our theories that tell us what might be an improvement on the current practice. In some cases, these theories are explicit, so that what would count as an improvement is reasonably obvious, if not completely predetermined. In other cases, while there are no explicit theories, the choices about what kinds of changes would be desired are made within a discourse that shapes the possibilities for our thoughts. In this sense, we can think of Vithal's *hypothetical situation* as providing a kind of *telos* for our interventions—in other words a direction for our trajectories of change. However, as well as providing a telos, the hypothetical situation also provides a calibration point in this direction. In other words, as well as showing the direction in which change is sought, the hypothetical situation provides a vision of what it would be like actually to reach the desired point.

Because the hypothetical situation provides both a direction for change (ie a telos) and a marker for how much change is desired (by envisaging the end-point), it becomes important to consider whether, in determining the hypothetical situation, the actual situation is taken into account. Vithal's answer on this point is clear:

> In my research, the actual situation is not considered directly. It is important to the extent that I together with other participants could intervene in an actual situation and arrange a situation for research. Knowing the actual situation is, however, important for the analysis and theorising later to explain what occurs in the arranged situation. My research interest lies in making a concerted effort to introduce prospective teachers to a particular theoretical landscape and its associated practices and then to examine its recontextualisation when facing the reality of classrooms. The focus is not on the existing actual situation per se but rather on some new and different situation that is organised and created with ideas from a particular theoretical landscape.

In other words, where people are starting from is important for analysis of what goes on, but should not be taken into account in determining the goal. In this sense, therefore, it seems to me that the hypothetical situation functions more as a telos than a desired endpoint.

So, we know where we are by observing the actual situation, and the hypothetical situation (driven by our theories) tells us the direction in which we think it would be appropriate to move[1]. The *arranged situation* emerges then as an interaction between the current situation and the telos provided by the hypothetical situation, but mediated by the agency of those involved and the structures in which they operate. In this sense, as Vithal implies, both the hypothetical and the arranged stituations are emergent properties of the interactions between researcher and researched.

Now, of course, it may be that the arranged situation approximates the hypothetical situation, but in most cases the arranged situation differs in important respects from the hypothetical situation. Traditionally, this has been regarded as a 'problem' of implementation. In other words, the researchers have a clear idea of what needs to be done, if only the teachers would do as they are told, much as medical researchers regard the failure of patients to follow pharmaceutical regimes as a 'problem'[2].

Such notions are at the heart of 'centre-periphery' models of dissemination of practice in which it is assumed the knowledge is created at the 'centre' (ie by researchers) and then is disseminated to those at the periphery who put the research into practice. Such models may work reasonably well in those situations in which the knowledge being disseminated is explicit, but as is being increasingly realised in ‘high-technology’ industries, such models are simply ineffective for the dissemination of implicit knowledge. Indeed, as Nonaka and Tageuchi (1995) have pointed out, the transfer of implicit knowledge is a quite different process from the transfer of explicit knowledge[3].

Involving teachers as co-researchers in the research process is therefore not just good public relations, or a way of assuaging guilt about the power relations involved in the research process. In complex domains such as education, where knowledge cannot easily be transferred explicitly, involving teachers is essential because the researchers have left something out. The broad principles of, to take the current example, critical mathematics education, provide some insight into the hypothetical situation, but do not provide a model for what to do. And however experienced the researcher is as a teacher—even if the researcher is an exemplary practitioner of critical mathematics education—she or he does not know what it is like to try and ‘do’ critical mathematics education with this class, in this school, at this time. This might be compared to the relationship between science and technology. While a scientist might understand well the principles of the internal combustion engine, and may even be able to produce a working model of such an engine on a laboratory bench, it requires the very different skills of a technologist to produce an engine that is small enough to fit into a car and which will run for over 30 million cycles before needing to be serviced. In the same way, while the researcher can provide strong guiding principles, turning this into an operational pedagogy that functions reliably and effectively in the context of a particular classroom requires considerable skill and development[4].

How then, can we deal with these very real dilemmas posed by Vithal. How do we research racist and sexist teachers? To what extent should we involve the researched in the research process? How do we deal with the very real differences in power between the researcher and the researched, especially in situations where the researcher is placed in a dual role of teacher and evaluator. A possible way of engaging with these tensions in the research process lies in addressing the question of what is to count as *evidence* in research.

Inquiry systems

What is to count as evidence in research was used by Churchman (1971) as the basis of a typology of methods of inquiry. He distinguished five kinds of inquiry systems, each of which he labelled with the name of a philosopher:

Inquiry system	primary source of evidence
Leibniz	rationality, reason
Locke	empirical observation
Kant	representations
Hegel	dialectic
Singer	ethics, morality, values

In a *Leibnizian* inquiry system, the primary source of data is the logical relation between the elements. Critical mathematics education would be supported or opposed simply in terms of the logic of the situation. For example, if we had a curriculum that required students to be critical of the use of mathematics then it might be regarded as 'obvious' (at least to some) that a teaching approach based on critical mathematics education must be better than one that was not.

In a *Lockean* enquiry system, the principle source of evidence is empirical data. Either a theoretical prediction is made, and validated by reference to empirical data, or empirical data is collected, and a theory is built to account for it. The effects of an attempt to introduce critical mathematics education could, for example, be investigated by defining aims for critical mathematics education, and then evaluating the extent to which the intervention achieved its intended aims. In order to further strengthen the warrant for the claims being made, we might set up a control group, and compare the outcomes for those who had been introduced to critical mathematics education and those who did not.

The major difficulty with a Lockean approach is that, because observations are regarded as evidence, it is necessary for all observers to agree on what they have observed. However, what we observe is dependent on the theories we hold, and in areas which are under-theorised (certainly the case with most educational issues), different theories lead to different results. This is true even in the physical sciences where Werner von Heisenberg observed that "What we learn about is not nature itself, but nature exposed to our methods of questioning" (quoted in Johnson, 1996 p. 147).

Because those with different theories will observe different things in the same setting, what gets observed cannot be regarded as 'out there' in any non-trivial sense, but are the result of the interaction between the brute physical world and the theories held by the observers[5]. This is recognised in a *Kantian* inquiry system, which involves the deliberate framing of multiple alternative perspectives, on both theory and data (thus subsuming Leibnizian and Lockean systems). This can be done by building different theories on the basis of the same set of data or by building two theories related to the problem, and then for each theory, generating appropriate data (it might well be that different kinds of data were collected for the two theories). Therefore, what counts in a Kantian enquiry system is the coherence of each of the multiple representations—there is no requirement for the representations themselves to be coherent with each other. Indeed, it is often the case that the representations are not even commensurable—in other words there is no way (without further theory building) of saying whether the representations are saying the same thing or not. However, often this work of investigating the connections and links between representations can lead to fruitful theory building.

In our introduction of critical mathematics education, one researcher might investigate the effects on achievement, whether defined in terms of the traditional mathematics curriculum, or in terms of some more radical aims, such as the ability to think critically in a given classroom situation. Another researcher might evaluate the initiative in terms of the extent to which students feel empowered to mathematize in 'real-life' situations outside the mathematics classroom, while a third might evaluate the introduction of critical mathematics education in terms of participation in classroom dialogue. None of these perspectives, with their associated theoretical perspectives and data, can be regarded as 'better' than the others. Each provides only a partial account of what is going on.

In an effort to prompt further theory-building, however, we can attempt to reconcile two (or more) rival theories through the development of antithetical and mutually inconsistent theories, which is the defining feature of a *Hegelian* inquiry system. Not content with building plausible theories, the Hegelian inquirer takes the most plausible theory, and then investigates what would have to be different about the world for the *exact opposite* of the most plausible theory itself to be plausible. The tension produced by confrontation between conflicting theories forces the assumptions of each theory to be questioned, thus possibly creating a synthesis of the rival theories at a higher level of abstraction.

For example, an Hegelian enquiry into our introduction of critical mathematics education in a particular setting might begin by building a coherent account of what was going on from some theoretical perspective, but would then investigate the extent to which the same data could support exactly the opposite conclusion. In this particular instance, we might have collected data that we believe shows that our intervention has been successful, so we would then attempt to use the same data to show that the intervention had not been successful. This could be sharpened by asking what would be the minimum change in our data that would be necessary to provide data consistent with the opposite view that the intervention had been ineffective (or even counterproductive). If only trivial changes in our data were to allow the opposite to be concluded, then this suggests that our interpretations are unlikely to be well-founded.

The differences between Lockean, Kantian and Hegelian inquiry systems were summed up as follows by Churchman:

> The Lockean inquirer displays the 'fundamental' data that all experts agree are accurate and relevant, and then builds a consistent story out of these. The Kantian inquirer displays the same story from different points of view, emphasising thereby that what is put into the story by the internal mode of representation is not given from the outside. But the Hegelian inquirer, using the same data, tells two stories, one supporting the most prominent policy on one side, the other supporting the most promising story on the other side. (Churchman, 1971 p177)

However, Churchman's typology also recognises that we can inquire about inquiry systems, questioning the values and ethical assumptions that each inquiry system embodies. Such an inquiry into inquiry systems is itself, of course, an inquiry system, termed *Singerian* by Churchman after the philosopher E A Singer which entails a constant questioning of the assumptions of inquiry systems. Tenets, no matter how fundamental they appear to be, are themselves to be challenged in order to cast a new light on the situation under investigation. This leads directly and naturally onto examination of the values and ethical considerations inherent in theory building.

In a Singerian inquiry, there is no solid foundation. Instead, everything is provisional; instead of asking what 'is', we ask what are the implications and consequences of different assumptions about what 'is taken to be':

> The 'is taken to be' is a self-imposed imperative of the community. Taken in the context of the whole Singerian theory of inquiry and progress, the imperative has the status of an ethical judgment. That is, the community judges that to accept its instruction is to bring about a suitable tactic or strategy [...]. The acceptance may lead to social actions outside of inquiry, or to new kinds of inquiry, or whatever. Part of the community's judgement is concerned with the *appropriateness of these actions from an ethical point of view*. Hence the linguistic puzzle which bothered some empiricists—how the inquiring system can pass linguistically from "is" statements to "ought" statements— is no puzzle at all in the Singerian inquirer: the inquiring system speaks exclusively in the "ought," the "is" being only a convenient *façon de parler* when one wants to block out the uncertainty in the discourse. (Churchman, 1971 p. 202; my emphasis in fourth sentence).

The important point about adopting a Singerian perspective is that with such an inquiry system, one can never absolve oneself from the consequences of one's research. Educational research is a process of *modelling* educational processes, and the models are never right or wrong, merely more or less appropriate, more or less defensible, for a particular purpose.

I suggest that a Singerian enquiry system provides an appropriate framework for the further investigation of the complex issues raised in Vithal's paper. For example, Vithal raises the issue of democracy in the research process. What does it mean to make (and what are the consequences of making) the 'researched' equal participants in the research process? In a Singerian inquiry, the researcher must defend the choice to the research community. In some cases, it will certainly be the case that making the researched equal participants in the process is appropriate (as judged by the community), but in other cases it might not. For example, Colin Lacey's influential study of 'Hightown Grammar' (Lacey, 1970) attended little to the needs of the researched individuals in that school, but arguably its effect on the system of education in the United Kingdom was great enough to compensate for this weakness. In my own research, I am involved in a variety of projects that examine the impact of ability grouping practices on students' learning and identity (see Boaler, Wiliam and Brown, 2000, and the paper by Boaler, Wiliam and Zevenbergen at this conference). In these projects, neither the students nor the teachers could be regarded as co-participants in the sense envisaged by Vithal. Is this defensible? The search for transcendent meanings that can serve to take forward a political project aimed at reducing the use of ability grouping in mathematics classrooms in England and Wales, can, I believe, justify the lack of impact in those schools in which the research is carried out, although, within a Singerian framework, this would be a matter for the research community.

In other research work, however, (see, for example, Wiliam, 2000) my focus is different. I and other colleagues are working with mathematics and science teachers to develop the role of formative assessment in classrooms. In this project, the requirements of producing data that could in some sense be regarded as generalisable has been given less emphasis than the need to allow the teachers to develop their practice in whatever ways they feel is appropriate. In this context, it is interesting to note that the original research design (perhaps somewhat naively) called for teachers to develop these skills in one or two of their classes, so that the progress of students in these classes could be compared with that of other students taught by the same teacher. As might have been predicted, because the teachers are finding the skills that they are developing lead to more effective learning, they are inevitably (and in some cases unconsciously) using these skills with all their classes. The meanings of the research data are compromised, but this is compensated by the consequences for these teachers' practice (though again whether the compensation is enough to justify this course of action would need to be subjected to the ethical and moral scrutiny of the community).

The framework of Singerian enquiry also illuminates the politically charged question of whether (and if so, how) to research racist and sexist teachers. Vithal describes how she maintained a dialogue with these teachers in which she challenged their views. The purpose of these challenges was not, of course, to help them develop more coherent defences of their racist and sexist positions (which is always a danger in such situations) but rather to produce change. Implicit in such an approach is the assumption that the existing positions and views held by these teachers are unacceptable and need changing. In other words, the researcher seeks to impose her or his views on the researched. Within a Singerian framework, such an approach is neither right or wrong, but simply more or less justifiable to the community.

In the same vein, the question of the 'emancipatory possibilities' of the research is an inevitable element for the Singerian enquirer which arises naturally. In research with the oppressed, traditional methodologies may require a consideration of the extent to which the research is emancipatory, but this is generally left to the whims of the researcher. For a Singerian enquirer, such questions are always present, even though, in a particular study, the emancipatory potential may be limited, because of the pressure from competing priorities.

Conclusion

Most research in mathematics education can be characterised as a process of assembling evidence that a particular chosen interpretation of data is warranted on the basis of the available evidence and the primary concern is with establishing the extent to which such interpretations can be generalised beyond the immediate context in which the data arose. However, in addition we should also be concerned to investigate plausible rival interpretations of the data and be prepared to marshal evidence that our preferred interpretation is more justifiable. Stopping at this point entails, in effect, adopting a rationalist position on the nature of research in mathematics education (which includes both positivist and interpretivist paradigms). It is also generally held, within such a perspective, that critiques should not question what is researched, but only how well it has been carried out.

However the issues raised by Vithal in her paper cannot be effectively addressed within such a rationalist programme. The decision to engage in critical mathematics education arises from a set of values that cannot be separated from the research process itself, which entails, in effect, adopting a Singerian mode of inquiry.

In such a mode, we are held accountable for all the decisions we make, both as to what we research and how we research it. Most people who carry out research from their jobs in universities, whether on fixed-term or permanent contracts, are privileged, given the luxury of time and facilities to carry out research. Our decisions about what to research, as much as our resolutions of the questions around negotiation, reciprocity, reflexivity, objectivity, instability and emancipation raised by Vithal, are, at root value-based decisions which we should expect to have to defend.

Notes

1 Of course, it is also important to note that the theory-dependent nature of observation means that what we observe about the actual situation is also driven by our theories.

2 Originally, medical researchers described the match between instructions given to participants in (say) drug trials as an issue of ‘compliance’, and more recently as one of ‘adherence’. However, the realisation that both these metaphors canonise a particular ‘correct’ method of behaviour has led to describing the question of the match between the actions of the participants and the wishes of the researchers, as one of ‘concordance’. That this is a real issue is illustrated by Collins and Pinch (1998) who cite the example of individuals involved in trials of drugs for the treatment of HIV/AIDS. The trials are double blind randomised controlled trials, so that neither those administering, nor those receiving the medication know whether they are receiving a drug believed to be effective in delaying the onset of AIDS or a placebo. Because being in the placebo group is effectively a death sentence, individuals with HIV/AIDS seek out others in the same trial and share medication, to increase the probability of getting some of the drug believed to be effective.

3 The importance of rapid dissemination of findings is demonstrated by studies of product development reported by Gleick (1999) which show that projects which deliver products 50% over budget, but on time, generate greater profits than those that deliver products on budget but six months late (p48).

4 The use of ‘technologist’ as a metaphorical counterpart for the role of the teacher in the teacher-researcher relation should not be taken as endorsing a view of teaching as a ‘technical’ profession. On the contrary, it seems to me that the fact that technologists have to make substantial use of implicit knowledge makes their work *more* professional than that of scientists, who are able to depend more on explicit knowledge. Accessing high-level explicit knowledge may require intellectual skills that are not widely-shared, but does not seem to me require the application of professional judgement to the same extent as is required in responding effectively to relatively unstructured situations.

5 Too often ontological discussions become polarised into whether the objects of discourse really exist, or are created. In almost all cases, the answer is both. For example, many languages do not distinguish between green and blue. This does not, of course mean that the speakers of this language see grass as the same colour as the sky, but rather that the need to distinguish these colours with different labels has not arisen. Conversely, it appears that Newton distinguished between indigo and violet simply because it was felt to be more auspicious to have 7 colours in the spectrum than 6.

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Pathways: Possibilities for Reform and Social Justice

A Reaction Paper to Smith

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From Smith's paper, I understand him to be raising a number of points. First, that the pathways that students take through mathematics should be meaningful and lead to productive lives in the new labour markets. He suggests that the old forms of mathematics teaching are not as effective as they should be and that there is a need for reconceptualising the teaching of mathematics if it is to be made accessible to all rather than the hegemonic practice that we are, at this conference at least, critically aware. In posing his challenge to mathematics educators that there is a need to develop more effective pedagogical forms that will enhance equity outcomes, three key considerations come to mind. In the first instance, the hegemony of mathematics education as a traditional pedagogy informed by transmission models of teaching must be considered. The discourses informing mathematics education are dominant and very resilient to change. Many teachers in the primary sector tend to be uncomfortable with mathematics and rely on traditional pedagogical forms when teaching this subject more so than with other subject areas. Effecting change in such contexts is difficult. The second issue deals with the current push in education based on economic rationalism where the discourses are based on accountability and the need to ensure that the employing bodies and other interested parties are receiving value and return for their expenditure. Current estimations would indicate that the return from the equity dollar is perceived to be low and that social justice money is not being effective. The third and final issue deals with the notion of pathways. This is central to Smith's paper, but it is not extended in a way that is possible given his "optimistic futures" perspective. I would propose that contemporary education is in a critical phase due to the substantial changes occurring within schools and the wider society. The schooling system was designed for the modern society, but it is evident that there is significant change and that the needs and learning patterns of contemporary students are very different from previous generations. Schools and mathematics education need to be changed significantly in order that we cope with the demands of the new millenium. The notion of pathways provides a means through which this is possible, but Smith's notion needs to be extended.

Pathways and Mathematics Education

The role of mathematics in subject choices and life trajectories has been understood for some time. It had its birth with the reforms in the gender equity. The movements in gender reform highlighted the importance of more girls undertaking the study of mathematics. What became clear was that not only were girls not participating in mathematics, that where they were participating, they were not taking the "right" mathematics. Their subject choices could lead to them into better careers where there were greater social and economic rewards if they took the "right" forms of mathematics. For example, to gain entrance into medicine or veterinary science, girls needed to take the more complex forms of mathematics rather than general forms mathematics. As the focus moves away from gender, it is now recognised that other groups in society are equally marginalised through their lack of participation and success in the "right" forms of mathematics. This approach to pathways seems to be central to Smith's conceptualisation of the same.

There have been a number of studies that track the effects of non-participation in mathematics for social justice target groups (such as working-class students, girls, and racially-marginalised students). Not only must participation be considered, but also success and retention. Such studies have shown the low participation, low success and/or low retention rates of target students. Within dominant discourses in mathematics education, the focus has been on individual characteristics such as motivation, self esteem, attributions of success and failure, and so forth. Such approaches engender a victim-blaming mentality that fails to account for the structural exclusion of students. One of the themes of this conference is to bring to the fore, discussions that critically appraise the forces that help to exclude students from mathematics. Smith poses a challenge to the mathematics education community to examine mathematics pedagogy more critically for the ways in which invisible pedagogy (Bernstein, 1990) acts "selectively on students from different backgrounds" (Smith, 2000, p. 12). His call to make mathematics pedagogy transparent for all students and to prepare them for the real world is a very realistic argument and one which we should consider seriously.

Mathematics Education and Resistance to Change

I would find it difficult to believe that there is a mathematics educator, particularly at this conference, who believes that mathematics pedagogy is not in need of renewal in some form or another. While social justice outcomes may not be the focus for all mathematics educators, it is certainly a key focus for this conference. Indeed there are ample studies that highlight the need to for innovative practice and a plethora of studies that show how students learn a range of mathematical concepts and processes. Our counterpart conferences are imbued with such studies, yet they have made little inroad into reconceptualising mathematics pedagogy. By and large, the teaching of mathematics has remained a conservative practice that is still dominated by traditional practices. This is in spite of the large number of studies (and money) that has been spent on identifying quality practice and attempts at improving practice in order to improve social justice outcomes. This begs askance of why this is so.

Anyone associated with initial teacher education will attest the resilience of traditional teaching methods. In providing preservice teachers with the knowledge of new approaches to teaching mathematics, two forces appear that impeded reformed teaching methods. In the first instance, primary preservice teachers have generally avoided the study of mathematics so have resistance to the subject and very poor perceptions of it. As a consequence, they often rely on their history to position their thinking and practice of mathematics education. In the second instance, they place a high priority on the value of practice so their school practice experiences are seen to be invaluable in their formation of ideas about effective teaching and learning. Yet in most cases, they report that these experiences are the antithesis of what they experience at the university level. Over a period of three years, I have surveyed my third year students (in a four year Bachelor of Education degree) and sought to identify the types of experiences they have had in their practicums insofar as the teaching of primary mathematics. The response has been anywhere between 86% to 95% (depending on the cohort) that they have had traditional teaching experiences across their last three practicums. Sullivan and Mousley (1993) have noted that there needs to be some caution in using interpretations of teachers' practice due to the subjective framework within which such interpretations are made. However, the qualitative descriptions provided by the students indicate that their experiences are significantly traditional forms of pedagogy when it comes to teaching mathematics. As such, the practices of the past - "chalk and talk", "teacher directed maths", rote and drill, are very evident in the students' experiences. What is disconcerting is the high number of students who claimed to have observed only traditional forms of mathematics education thus suggesting that the practice is very common.

Furthermore, there were numerous comments that the supervising teachers did not give university studies much credence within the school experience. Many students recanted that their teachers would comment that university studies had little to do with what happens in the reality of the classroom. Together, these comments suggest that the reality for preservice teachers and the students in mathematics classrooms are ones where there is a resistance to change. Some of the comments offered by the students included:

> S1: My teacher said that the stuff we learnt at Uni had nothing to do with what happens in the classroom. All the stuff like constructivism, group work, and that didn't work and that it was best just to forget what we learnt there and do like they do in schools. At least we know it works.

> S2: When I was on prac my teacher said that the uni lecturers really have no idea what is happening in the classroom. She said that you really can't do all that fancy teaching that we learn at Uni. It doesn't work and that we are better to forget it and just get on with teaching. She said it was fine to have all that theory, but it really doesn't help much in the classroom.

> S3: My last teacher was very old fashioned. She said that everything just goes around in circles. She has seen it all before and it really doesn't make any difference. She reckons that once you come out of uni you have too many things to do and that the only way to survive is to do it the traditional way. It has always worked so why bother with the other stuff. In the end it will come back again [to traditional teaching].

> S4: My last teacher had only been teaching for a few years so I was surprised by her comments about how to teach maths. She used the textbooks a lot. She said that she never felt confident teaching maths and she did not do so well at uni. She reckons that the textbooks are really good as they have books for the teachers and they [the books] give you ideas on how to teach the content to the kids. She says she tries something new from time to time, but she only uses a little bit of the problem solving stuff that she learnt at uni. She said her lecturer at uni was down on textbooks but she has found that they work really well.

The comments selected above indicate teachers' resilience to change. Equally, they indicate the almost futility of change within preservice education. Where practicum experiences assume a very high profile, the denigration of new ways of teaching (based on research) allows many preservice teachers the opportunity to legitimately reject the contemporary approaches to teaching mathematics. In so doing, it makes implementing change quite difficult. Similar comments were made in relation to assessment. Again, the research that has shown that traditional pencil-and-paper testing is not an effective means of assessing student understanding but that this has not filtered down into classroom practice.

> S5: My teacher did the Friday tests. That way she can work out who has learnt what they were taught during the week and who hadn't. She said that the other ways of assessing kids' work is all fair and good, but it takes too much time and as a teacher, there is too much going on to be doing all the other things.

These comments are representative of the majority of comments offered by the students. There were some students who were fortunate enough to experience more innovative practices, but they were in the minority.

When these experiences are coupled with the past experiences of the students and their fear of mathematics, there is a strong tendency to rely on past methods rather than embracing new reforms. The teachers' comments help to reinforce the values held by the students and as such legitimate past practices. Given that prac is seen as such a valuable component of initial teacher preparation, the comments offered by students and the results of these surveys suggest that the capacity to effect change in the teaching of mathematics is restricted. As such reproduction of old values inherent in past methods remain dominant.

Considerable research has been undertaken by members of the mathematics education research community that notes the need and value of reform in mathematics teaching. The wide scale reform proposed by the National Council of Teachers of Mathematics (1991) has not been implemented as effectively as would have been hoped. For example, Frykholm (1996) noted the difficulty in translating the standards into practice.

Bourdieu's (1992) notion of field is most useful in trying to understand this phenomenon. For Bourdieu, field is an arena where relationships of power are negotiated and lived out. Within the field of education, of which mathematics education and schools are subsets, there are certain practices that hold more sway than others at particular points in time. These are not fixed but can be quite transitory in nature. For example, in mathematics education, the discourse of constructivism was quite powerful in the past decade. Those who purported to support the discourse gained more power through such links. This power was achieved through the symbolic power conveyed through the field which, in this case, could be attained through objects such as publications (journal articles, books, etc), conference keynotes which were then convertible to other forms of capital, including economic capital, through forms such as promotions, royalities and so on. For teachers entering the school system, power is conveyed through the discourses that dominate school education. For preservice teachers, this means that they need to demonstrate the qualities desired by the teaching profession in order that they gain entry into the profession. For such students, this may be in the form of professional interviews where the prospective employee must demonstrate the qualities demanded by the profession. Those prospective teachers who do not show the characteristics desired by the employing body are less likely to gain employment than their peers who show such characteristics. Accordingly, the preservice teacher must comply with the unspoken rules of the field and assume the qualities of a good teacher. In this case, it may well be that the employing school sees quality mathematics teaching within traditional discourses so that the prospective employee needs to demonstrate these qualities in order to gain employment with this school.

However, the previous discussion revolved around the primary sector and questions need to be asked as to the effect in the secondary sector. In the past, teachers entering secondary school classrooms could be considered to have a reasonably strong grasp of mathematics. However, this is not necessarily the case in contemporary schooling. In their study of mathematical knowledge of secondary teachers, Kanes and Nisbet (1994) found that many teachers did not have a strong knowledge base and indeed, many teachers were being given mathematics classes to fill their teaching quota even though they were not qualified in the discipline. This finding suggests there is a strong synergy between the primary and secondary sectors than would have been expected in the past. This situation is likely to worsen with less people seeking teaching as a career and even less undertaking the study of mathematics. It is highly likely that many teachers facing junior secondary classes may be very similar in profile and attitudes towards mathematics as their primary school peers.

Accountability and Teacher Change

In this section, I take up the issue raised by Smith insofar as the moves towards accountability in education. One of the ethical dilemmas in researching educational failure is the effect on teachers and their standing in the community. One must be very careful so as not to engender teacher bashing as teachers and schools are often blamed for all manner of social ills. This is hardly fair. However, as Smith also notes, the issue of accountability is a key to educational change. Whereas other professions are held accountable for their actions, traditionally teachers have not. In a large-scale study of American teachers, it was found that newer teachers were more effective than teachers of 20 years in the field, producing better learning outcomes than their more experienced peers (The Australian, Jan 29, 2000). Using test results across a number of years, researchers were able to identify individual teachers and schools that were more or less effective than others. They claim that there were teachers who are not improving (or in some cases detracting from) student learning, and such teachers should be held accountable for their teaching. This is a relatively new phenomenon and has met with considerable debate. Where the results are being used to control teachers, there is a need for criticism but where the scores are being used to identify consistently poor teachers, there may be some merit in the process.

Current moves in education have focused on accountability through wide scale measuring schemes. The rhetoric behind such schemes has been to identify and monitor student progress, and where necessary, implement necessary educational programs to enhance student learning. However, as is often the case, such rhetoric can appear to be empty and a thin veil for controlling education. There is a need for healthy cynicism for such programs. However, if the programs were designed for social justice purposes and indeed, if there were systemic problems noted and redressed, then there is some value in having such programs. For example, in the Australian context, indigenous students consistently scored significantly lower than non-indigenous students . Similar studies have shown the same results in the United States (Reyes & Stanic, 1988; Secada, 1992). Such systemic failure must be addressed. The problem has been long standing. Whereas the gender reform initiatives have had a considerable impact on educational outcomes, there has been significantly less impact for students from socially disadvantaged backgrounds or students from particular racial backgrounds.

If the results of such tests continue to show poor performance, then there is a need for some intervention. It is unjust that disadvantaged students can continue to be excluded from curriculum studies. However, where the programs are used in contexts for pegging salaries or closing schools, then they [the programs] should be called into question. As has occurred in the UK, schools in disadvantaged areas have performed poorly on testing scales and were closed due to poor performance. Little consideration had been given to the demographics and special needs of the students, but rather, the schools have been closed for political reasons and the schools have been used as scapegoats for wider systemic problems.

One of the difficulties in mathematics education has been the dominance of particular forms of knowledge and pedagogy even though research has shown such practices to be inequitable. In part, this is due to the values (and fears) that some teachers hold towards mathematics. One of the dangers of being critical of the teaching of mathematics is that it can engender teacher bashing. Hence many studies of mathematics teaching are couched in terms of effective teaching and seek to identify effective teachers of numeracy or mathematics. In their large scale study, Askew et al (199?) identified a number of features of good teachers of mathematics. In so doing, it implicitly suggests that those teachers who do not have the characteristics identified in the study may not be as effective.

One of the characteristics noted by Askew et al (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997) is the involvement in professional development. Such characteristics include undertaking regular, sustained and focused professional development. That is, professional development that improves the education and deep learning of the teacher as opposed to the hands-on, half day, inservices often taken by teachers. The latter forms of professional development do little to improve teachers' understanding of teaching and learning, but rather give access to quick lessons that can be implemented with little understanding of the links between other aspects of the mathematics curriculum – a further quality of good teachers identified in the Askew et al (1997) study. Indeed, the resistance to change has been borne in studies where teachers theorised their work and it was found that how they talked about their work could be correlated with when they undertook their initial teacher training (Andrews, 1997). Thus is would appear that teachers must undertake sustained study in order that they move with the times and gain a currency in their work. By invoking some forms of accountability, teachers may need to undertake the forms of professional development advocated by Askew et al (1997) in order to ensure that they are up to date with current research and approaches to effective teaching and learning.

Many education authorities are seeking accountability in some aspects of teachers' work – namely student performance, yet if they are serious about teacher improvement and moving teacher perceptions beyond the comments offered in section one, then the work by Askew et al suggests that there is some rationale for teachers to undertake effective forms of professional development. Furthermore, if this is indeed the case that good teachers of mathematics are those who undertake such forms of professional improvement then employers need to recognise teachers who are undertaking forms of professional development that enhance learning outcomes for students, and in so doing improve the status of schools, education and society.

Pathways and Social Reform

Smith has noted that the pathways undertaken by students in their study of mathematics are critical to their life chances. Not only does he advocate undertaking the right mathematics, but that the mathematics that is undertaken should be meaningful and purposeful in the reformed labour markets. This is a very valid observation and one that has been noted by a number of key researchers. Changes to the nature of work mean that Marxist conceptions no longer hold the sway that was evident in modern society. The labour market trends that he notes bear testimony to the changed workplace and the need for graduates for these positions. In concert with the changed workplace, is the role of high retention rates. Using Giddens (1999), Smith argues that education to Year 12 improves job prospects, earning capacity and social gains including greater social cohesion, lower crime and better health. Lamb [REF NEEDED] has shown similar findings in his longitudinal work of Australian school leavers. Lamb's analysis of school subject choices has shown the role of mathematics in the student subject profile to be a critical factor in subsequent life successes. These studies show categorically that students need to complete 12 years of schooling in order to maintain a reasonable to high quality of life. Thus, it is in a society's and the individual's best interest to keep students in the education system as long as possible. However, some caution is needed. Not only is completion of Year 12 an indicator of improved life chances, the choices of subjects within the suite of year 12 subjects must be considered.

Lamb's comprehensive analysis of subject choice alerts us to the potential for a ghetto-ising effect of choices. Clearly the choice of mathematics is critical to improving life chances, but as Lamb (1997) and Teese (Teese, Davies, Charlton, & Polesel, 1995) both argue, it is not only mathematics that is important, but the type of mathematics. Based on subject selection across various social, cultural and gender groups, the patterns of subject selection are noticeable. It is more likely that working-class (or low SES) students are likely to select the soft options – general maths and biology – whereas middle-class (or high SES) students are more likely to select the "hard options" maths B and C (which contain topics such calculus, trigonometry etc) physics and chemistry.

However, these pathways represent traditional forms of knowledge packaged within traditional modes of pedagogy. Clearly subject choice needs to be more informed, but it assumes a particular form of rationality in subject choice. This is not always the case, and maybe subjects are chosen for personal or subjective reasons, some of which will bear significantly on life chances, but not all students chose for the seemingly logical choices. Subjects may be chosen because the student likes a particular person or that it is the only subject of any interest in a particular block of subjects within the timetable. However, the old structures of schooling remain intact. What may be necessary is a total reconstruction of curriculum and modes of delivery. The debate about boys and education has brought to the fore many of the issues about the suitability of contemporary practices for the post modern students. Many students would consider taking on more employment during school hours so that there is a need for more flexible school hours. Students may be able to undertake practical studies in their nominated curriculum areas. It may be useful for students considering engineering as a career to undertake field work and see how the mathematics is used within the workplace and see the relevance of the mathematics in situ and also to observe how engineers undertake the calculations necessary within particular contexts. Often teachers do not have this hands-on knowledge so the mathematics becomes deconstextualised. Examples used in classrooms are often ones that are used repeatedly so students do not gain the knowledge necessary for the applications to the field.

Where students are struggling with mathematics and school in general, there maybe some usefulness in allowing them greater flexibility within their study programs. It is not uncommon for early school leavers to resume study at vocationally-orientated colleges. These colleges have developed curricula more suited to the needs of the less-academically orientated students, but students gain significantly more from these learning environments than their school experiences. In part, it is the pedagogical approaches used by the teachers, but it is equally the case that many students see the colleges as a second chance to improve their employment opportunities. Different pathways through school may be able to encapsulate these flexible forms of delivery. Students maybe able to engage in part time work in the very casualised workplace and come to realise that this type of employment is not desirable. In so doing, they may be able to remain in school study, in a reduced capacity, but undertake study within the school context that supports their employment while enhancing their job opportunities. For example, the student working in the casualised retail industry may undertake work on the shop floor or cash register while remaining at school but undertaking studies in retail management. Subjects, often perhaps in a modularised form, support and extend the activities of the workplace and in so doing, give the students a chance to see the relevance and applicability of the school studies.

Towards a Socially-Just Mathematics

Smith (2000, p. 12) advocates the need to provide "school leavers with the ability to use mathematics-based tools without requiring that they prepare from mathematics-based careers". This would seem to be a reasonable enough claim as most students do not pursue tertiary mathematics as a career. Rather, as mathematics educators in this sector bemoan, fewer students are undertaking sustained studies in mathematics. Rather, tertiary mathematics is of the form where students drop in and out of subjects as they need to fill in quotas for other courses. Smith appears to be advocating for a mathematics curriculum that enables students to think mathematically rather than being able to recite meaningless formulae. By providing all students with the capacity to think mathematically, they should be competing on a more equal playing field than has been done in the past. This is the dream of most mathematics educators, but it is far from being realised. We are acutely aware of the need for more students to have access to mathematics as it is a powerful tool for contemporary society – in terms of employability as well as quality of life. We know from situated cognition studies that people undertake tasks in a variety of ways, most of which are not like those that are taught in formal school mathematics. Yet, equally we are acutely aware that the power of thinking mathematically about such tasks is often more efficient, effective and accurate than the methods developed *in situ*. For example, Lave et al's (Lave, Murtaugh, & de la Rocha, 1984) shoppers who needed to check the price of cheese could have calculated the price of the cheese using ratio and proportion, but had to rely on a seek and search technique to find other balls of cheese the same mass and then compare prices. The ability to think mathematically in these situations has the potential to be far more empowering than the methods used. However, it is not always the case, and invented algorithms can be just as effective when they have meaning to the inventors such as the street vendors in Carraher, et al's work (Carraher, 1988; Carraher, Carraher, & Schliemann, 1985).

We are acutely aware also of the routinised ways in which mathematics frequently is taught to students. Students are taught particularised steps in solving mathematical tasks. As Bourdieu would argue, such practices come to be embodied by the students within a mathematical habitus. This habitus, in turn, provides the lens through which students come to see and interpret the mathematical world. For many of them, mathematics is little more than lock-step processes which they need to follow in order to solve tasks. This process does little to develop deep understanding of the mathematics. As such, practices such as these tend to be mathematically disempowering for students.

In conclusion, I support Smith's claim that there is an urgent need for reform in mathematics that will make it accessible for more students and that such mathematics will meet the needs of the reformed market place. In order that we achieve these changes, there are obstacles to be addressed. I have highlighted some of these. What is essential is that the changes that do need to occur and that will move mathematics pedagogy away from the exclusory practices of the past, must be supportive. Teachers live in very demanding contexts. As Michael Apple's (Apple, 1995; Apple & Jungck, 1992) work attests, teachers' work is intensifying and they are at risk of having many of their professional responsibilities removed, so it is critical that many of the professional decisions that teachers make in relation to student performance and progress are protected. Teachers need to be supported in the current climate where there are increasing demands if they are to be able to adopt and adapt to new forms of mathematics pedagogy, particularly those highlighted in this session where the aim is to be more inclusive.

The simplistic (and outmoded) model of teacher education that I proposed in relation to my students' experiences during practicum experiences highlights the weakness in such models of teacher education. Perhaps there is potential in breaking down the school/university dualism, and that the professional development of preservice teachers, school teachers and university teachers is undertaken as a learning team, where each learns from the other. This may support teachers in developing new forms of socially-just pedagogy; may provide university staff with insights and understandings of contemporary classrooms along with increased understandings of the effectiveness of these proposed reforms; and to break down the dichotomy and tensions that preservice teachers experience during their initial training. This may give legitimacy to both practice and theory and in so doing bring about informed practice for all participants.

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