Mathematics Education and Society
Proceedings of the 4th International Mathematics Education and Society Conference

Edited by Merrilyn Goos, Clive Kanes and Raymond Brown

Centre for Learning Research, Griffith University

Griffith UNIVERSITY

Proceedings of the Fourth International Mathematics Education and Society Conference
Edited by Merrilyn Goos, Clive Kanes and Raymond Brown
First published 2005
Published by the Centre for Learning Research, Griffith University

ISBN 1 920952 35 7

All papers in this publication have been peer reviewed anonymously to comply with the verification requirements of the Department of Education, Science and Training for conference publications — full written paper refereed.

For further information please refer to the 2003 Higher Education Research Data Collection: Specifications for Preparing Returns for the Collection of Data, Higher Education Division, Department of Education, Science and Training, Canberra.
Available online:
www.dest.gov.au/sectors/research_sector/online_forms_services/higher_education_research_data_collection.htm

The Conference Organising Committee accepts no responsibility for any errors in or omissions from this publication.

Dick Roebuck
Conference Manager

MES4 Organising Committee

International:
Tamara Bibby, Karin Brodie, Dimitris Chassapis, Tony Cotton, Peter Gates, Joao Filipe Matos, Candia Morgan, Paola Valero, Renuka Vithal, Robyn Zevenbergen

Local:
Bill Atweh, Raymond Brown, Carmel Diezmann, Merrilyn Goos, Romina Jamieson-Proctor, Clive Kanes, Elizabeth Warren, Robyn Zevenbergen

MES4 Secretariat

Dick Roebuck, Centre for Learning Research, Griffith University

TABLE OF CONTENTS

CONFERENCE INTRODUCTION AND PLENARY PAPERS

INTRODUCTION

This is the fourth international meeting of the Mathematics Education and Society group—the first meeting to be held outside Europe. The first conference took place in Nottingham, Great Britain, in September 1998. The second conference was held in Montechoro, Portugal, in March 2000. The third conference was held in Helsingor in April 2002. On these occasions, people from around the world had the opportunity of sharing their ideas, perspectives and reflections concerning the social, political, cultural and ethical dimensions of mathematics education and mathematics education research in present world societies.

As a result of the success of the first three meetings, it was decided to have a fourth conference in the southern hemisphere. The Fourth International Conference on Mathematics Education and Society was held at the Gold Coast, Australia, in July 2005.

The conference has been promoted by the Centre for Learning Research, Griffith University and is a co-operation between the Universities in the Brisbane Corridor – Griffith University, University of Queensland, Queensland University of Technology and the Australian Catholic University.

Aims of MES 4

Education is becoming more and more politicised throughout the world. Mathematics education is a key discipline in the politics of education. Mathematics qualifications remain an accepted gatekeeper to employment. Thus, managing success in mathematics becomes a way of controlling the employment market. Mathematics education also tends to contribute to the regeneration of an inequitable society through undemocratic and exclusive pedagogical practices that portray mathematics and mathematics education as absolute, authoritarian disciplines. There is a need for discussing widely the social, cultural and political dimensions of mathematics education; for disseminating research that explores those dimensions; for addressing methodological issues of that type of research; for planning international co-operation in the area; and for developing a strong research community interested in this view of mathematics education.

The MES 4 Conference aims to bring together mathematics educators from around the world to provide such a forum, as well as to offer a platform on which to build future collaborative activity.

As a result of an evaluation of the thematic organisation of the previous conferences, MES 4 has a central discussion theme on the relationship between theory and practice in mathematics education research from a social/political/cultural/ethical perspective.

Conference Program

The conference has been organised having in mind the importance of generating a permanent dialogue and reflection among the participants concerning the central discussion theme. There is a range of activities directed towards the aim of generating this constant discussion.

Opening session and conference introduction

The opening session included an Indigenous "welcome to country" and a presentation on the origins, history, and aims of MES.

Plenary addresses and reactions

Three invited keynote speakers were asked to address the conference's central discussion theme. Each keynote presentation was followed by reactions from two mathematics educators.

Working groups

Groups, formed at the beginning of the conference, discussed the plenary lecture and the reactions. Each working group produced a brief report detailing key questions or issues to be addressed by the speaker and reactors in a plenary response session.

Plenary response session

In these sessions there was an opportunity to bring back to the whole audience the questions and issues raised by each working group, and to have further comment from the plenary speaker and reactors.

Symposia

Two symposium proposals were accepted after review of the local organising committee. The proposals are published in the conference Proceedings. Following acceptance, presenters were asked to prepare a short paper outlining their contribution to the symposium. The papers are available on the conference website. Both symposia included full papers that had already been accepted for presentation and publication in the Proceedings via the peer review process outlined below.

Paper discussion sessions

Each of the research papers submitted was peer reviewed before publication, without the author or reviewers being known to each other, by two experienced mathematics education researchers. For each paper at least one reviewer was a person who had participated in a previous MES conference or had volunteered for this task via the MES email list. Strict guidelines were followed; in particular, reviewers were asked to assess the significance of the paper in the light of the aims of MES4. However every effort was made to be as inclusive as possible of the diverse research interests and backgrounds of members of, and newcomers to, the MES community. The Co-Editors of the Proceedings together read all reviews. Where reviewers disagreed about the acceptability of a paper the Co-Editors made a decision based on their reading of the reviewers' reports and the original paper.

After peer review of all paper submissions, 30 papers were accepted for presentation and discussion during the conference. The full text of accepted papers was posted to the conference website and published in the Proceedings.

Agora

There are two sessions dedicated to an open, informal exchange of ideas on topics to be proposed by participants and the future of MES.

Networking

Within the program there are slots dedicated to discussing possible co-operation among participants.

Concluding panel

There is an increasing concern on the over-representation of dominant cultures and classrooms pervading mathematics education literature. The plenary panel, comprising a small group of international MES participants, will focus on issues related to under-represented cultures and regions of the world and the impact of non-participation or low quality education of the majority of students in the world on mathematics teaching and learning.

Participants

We welcome participants representing the international community of mathematics education researchers from Australia, Brazil, New Zealand, South Africa and the United Kingdom.

Merrilyn Goos, Clive Kanes and Raymond Brown
June 2005

POWER AND ACCESS IN MULTILINGUAL MATHEMATICS CLASSROOMS

Mamokgethi Setati
University of Witwatersrand
<setatim@educ.wits.ac.za>

This paper explores how teachers and learners position themselves in relation to use of language(s) in multilingual mathematics classrooms. It draws from two studies in multilingual mathematics classrooms in South Africa. The analysis presented shows that teachers and learners who position themselves in relation to English are concerned with access to social goods and positioned by the social and economic power of English. They do not focus on epistemological access but argue for English as the language of learning and teaching. In contrast, learners who position themselves in relation to mathematics and so epistemological access, reflect more contradictory discourses, including support for the use of the their home languages as languages of learning and teaching.

INTRODUCTION

> Classroom conversations that include the use of […] the [bilingual] students' first language as legitimate resources can support students in learning to communicate mathematically (Moschkovich, 2002, p. 208).
>
> If we changed our [mathematics] textbooks into Setswana and set our exams in Setswana[i], then my school will be empty because our parents now believe in English (Lindi, a Grade 4 mathematics teacher).

It is widely accepted that language is important for learning and thinking and that the ability to communicate mathematically is central to learning and teaching school mathematics. What is still under constant debate and investigation is which language is most appropriate for learning a subject such as mathematics especially in a multilingual contexts. The quotes above capture the essence and complexity of the debate. Research argues that the learners' main languages are a resource in the teaching and learning of mathematics while teachers argue for the use of English. Herein lies the heart of the problem explored in this paper. These arguments are equally compelling as they are about access to mathematics and social goods. The main aim of this paper is to give substance to the debate by exploring how multilingual mathematics teachers and learners position themselves in this debate and what might this mean for research and practice.

The data used in the paper is drawn from two research projects in multilingual mathematics classrooms in South Africa. Using data from South Africa is convenient but also appropriate: South Africa is an extraordinarily complex multilingual country. While the multilingual nature of South African mathematics classrooms may seem exaggerated, they are not atypical. In South Africa, there is a general view that most parents want their children to be educated in English and that most learners would like to be taught in English. While there is no systematic research evidence, it is also widely held that many schools with an African student body choose to use English as a language of learning and teaching (LoLT) from the first year of schooling (Taylor & Vinjevold, 1999). The TIMSS results in South

Africa were very poor. Studies that have emerged from this argue that the solution to improving African learners' performance in mathematics is to develop their English language proficiency (e.g., Howie, 2002). What does this recommendation mean for mathematics learning? The question explored in this paper is how the power dynamics of language play out in the mathematics classroom context, and in whose or what interests? Issues of power and access are by no means straightforward and so it is important that they be problematised.

The work on the politics of language is complex, not well developed in mathematics education and often misrepresented. To put this debate in perspective it is important to provide a brief overview on the political role of language.

THE POLITICAL ROLE OF LANGUAGE AND ITS USE IN MULTILINGUAL MATHEMATICS CONTEXTS

Language, like multilingualism, is always political (Hartshone, 1987; Reagan & Ntshoe 1987; Mda, 1997; Friedman, 1997; Heugh, 1997; Granville; Janks; Mphahlele; Reed; Watson; Joseph, & Ramani, 1998; Gee, 1999). It is one of the characteristics that are used in society to determine power (Gutiérrez, 2002). In South Africa the issue of language has always been interwoven with the politics of domination and separation, resistance and affirmation. During apartheid, the language of learning issue became a dominating factor in opposition to the system of Bantu Education. Though not unmindful or ashamed of African traditions per se, the mainstream African nationalists have generally viewed cultural assimilation as a means by which Africans could be released from a subordinate position in a common, unified society (Reagan & Ntshoe, 1987). They therefore fought against the use of African languages as languages of learning and teaching because they saw it as a device to ensure that Africans remain oppressed. Lindi's views above that the parents of learners in her school believe in English are thus not surprising.

The political nature of language is not only at the macro-level of structures but also at the micro-level of classroom interactions. Language can be used to exclude or include people in conversations and decision-making processes. Zentella (1997) through her work with Puerto Rican children in El Bario, New York shows how language can bring people together or separate them. Language is one way in which one can define one's adherence to group values. Therefore decisions about which language to use in multilingual mathematics classrooms, how, and for what purposes, are not only pedagogic but also political (Author, 2003). Most research on mathematics education in multilingual classrooms has argued for the use of the learners' home languages as resources for learning and teaching mathematics (e.g., Addendorff, 1993; Adler, 2001; Arthur, 1994; Khisty, 1995; Merrit, Cleghorn, Abagi, & Bunyi, 1992; Moschkovich, 1999, 2002; Ncedo, Peires, & Morar, 2002; Author, 1998; Author, 2001; Author, 2002). They have argued for the use of the learners' home languages in learning and teaching mathematics, as a support needed while learners continue to develop proficiency in the language of learning and teaching (e.g., English) at the same time as learning mathematics. While research in general education on language and minority learners is strongly rooted in the socio-political context of learning (Cummins, 2000), most research on multilingualism in mathematics education has been framed by a limited conception of language as a tool for thinking and communication. To ignore the political role of language in mathematics education research and practice would assume that power relationships do not exist in society.

In this paper, I use the work of Gee (1996, 1999) to take the work on multilingualism in mathematics education further by explaining the language choices of teachers and learners in multilingual mathematics classrooms beyond the pedagogic and cognitive. Gee's work is relevant because he considers language as always political (1996, 1999). He argues that when people speak or write they create a political perspective; they use language to project themselves as certain kinds of people engaged in certain kinds of activity. Language is thus never just a vehicle to express ideas (a cultural tool), but also a political tool that we use to enact (i.e. to be recognized as) a particular 'who' (identity) engaged in a particular 'what' (situated activity).

Gee uses the theoretical construct of cultural models to explore the identities and activities that people are enacting. Cultural models are shared, conventional ideas about how the world works, which individuals learn by talking and acting with their fellows. They help us explain why people do things in the way that they do and provide a framework for organizing and reconstructing memories of experience (Holland & Quinn, 1987). Cultural models do not reside in people's heads, but they are embedded in words, in people's practices and in the context in which they live. The question that is relevant for this paper is what cultural models do teachers and learners in multilingual mathematics classrooms enact in relation to language and mathematics? In what follows I use the notion of cultural models to explore why teachers and learners prefer the language(s) that they choose for learning and teaching mathematics. Thereafter I will look at the implications of such language choices for research and practice.

TEACHERS' LANGUAGE CHOICES

The data that I draw on in this section comes from a study that involved six primary school mathematics teachers in multilingual classrooms in South Africa. Data was collected through teacher individual interviews, focus group interviews and classroom observations. During the pre-observation interview teachers were asked, "Which language do you prefer to teach mathematics in? Why?" Over and above all else, *English is international* emerged as a dominant cultural model that shaped the teachers' language choices. All the six teachers stated ideological and pragmatic reasons for their preference to teach mathematics in English. As the extracts below show, these reasons ranged from the belief that English is an international language to the fact that textbooks, examinations and higher education are all in English.

Vusi: I prefer to teach in English because it is a ***universal language***.

Kuki: I think all the languages must be equal although English as the ***international language***, it has to still be emphasised and mother tongue I think it's high time that the kids ***learn mother tongue and be proud of it***.

Lindi: … it is said that [English] is an ***international language*** … I encourage them to use English … The ***textbooks are written in English***, the ***question papers are in English***, so you find that the child doesn't understand what is written there. (my emphasis)

These teachers are aware of the linguistic capital of English and the symbolic power it bestows on those who can communicate in it. They see English as international and universal and thus 'bigger than' themselves. The way Kuki and Lindi express themselves in the above extracts also suggests that they do not want to take responsibility for the status of English. The status of English is what it is and they cannot change it. Kuki uses the phrase "I think …", while Lindi uses "It is said …," suggesting that they see themselves as being caught up in the dominance of English. They do not have any control over the international nature of English. All they can do is to prepare their learners for participation in the international world, and teaching mathematics in English is an important part of this preparation. It is thus not surprising that all the teachers saw English as the natural choice for use in mathematics teaching. All their lives they have lived in an environment that values English more than any other language. Furthermore, as Lindi points out, the mathematics textbooks and examinations are in English. Over the years, no mathematics textbook in South Africa was written in an African language. During the time when 'mother tongue' instruction was enforced in primary schools, the mathematics textbooks at this level were translated from English or Afrikaans into the African languages. The secondary school mathematics textbooks have never been published in African languages in South Africa. Therefore, for many African teachers and learners, mathematics is associated with the English language because it is the language of mathematics textbooks. As a result, English has become the natural choice for teaching and learning mathematics. What is interesting is that none of the teachers challenged the power of English or the fact that textbooks and examinations are in English while learners are not fluent in it.

While the other three teachers did not explicitly highlight the international nature of English, they also indicated that they encourage their learners to use English and their reasons focused on the social goods that learners can access through English.

Gugu: I think English, ***it empowers*** them [the learners], you understand. At this stage of eight, nine years, they can be able to ***speak English unlike us***. ***We never did English in primary*** and at college we were supposed to answer in English in lectures. So we had a problem with this language, ***so at any early age they just become used to it***.

Mpule: I ***encourage them to use English*** because if they do not learn the language how will they be able to ***cope in higher classes***, they will not cope.

Rosina: I encourage them to use ***English always***... So that they can ***learn the language*** (my emphasis).

Gugu wants to make English accessible to her learners early in their schooling. In her view, making English accessible will assist in undoing the wrongs of the past, which she experienced as a learner. Gugu's view of making English accessible is similar to Granville et al.'s (1998). While Granville, et al. insist that all South African learners must learn at least one African language they argue that all school learners should have access to English, which is "the language of power" at present. The argument is that if everyone had access to English; it would no longer be an elitist language. In this way English could come to be seen as a resource, not as a problem" (Granville, et al., 1998). The challenge now is that even the learners who do not have access to English are learning mathematics in English. Gugu's view is that the mathematics classroom to be another opportunity for learners to gain access to English. An important question to ask here is, what is the cost of focusing on making English accessible to the learners during mathematics teaching?

Mpule highlights the fact that English is the language of higher education. Higher education in South Africa is only available in English and Afrikaans. As a primary school teacher, she feels responsible for ensuring that the learners are ready for higher classes and the ability to speak English is an important part of preparation for that. What is interesting is that Mpule, like all the other teachers in the study does not highlight the importance of ensuring that learners are mathematically competent for higher classes. While this absence of a concern for mathematical competence may not be deliberate, it is important to note. What is in the foreground in the teachers' cultural models above is English. Explanations for their preferred language(s) for mathematics teaching focus on English and not mathematics. These teachers position themselves in relation to English (and so socio-economic access) and not mathematics (i.e., epistemological access).

Of all the teachers, Kuki is the only one who indicated some awareness of the fact that all the official languages in South Africa are equal. What is interesting is that even with this recognition, Kuki still maintains that English has to be emphasised. As the above extracts show, Kuki is working with conflicting cultural models of wanting to honour the African languages on the one hand, and on the other hand ensuring that the learners have access to English. During the focus group interview both Gugu and Lindi also displayed the same kind of conflicting cultural models.

Gugu: To me those different languages must be respected, we must never look down upon different people speaking different languages. I think to me they are all important. Much as we are respecting English as an international language but I think it is high time that we realise that we need to interact with other languages.

While Gugu wants to respect and honour the African languages, she still feels pressured by the international nature of English. During the pre-observation interview, she was emphatic about the need to focus on English and in the focus group interview, she emphasises the need to respect and interact with other languages.

> Lindi: …*t**he past has already killed our nation***, the only language that has been respected the most is English. ***If you don't know English you look like a fool, and you are considered as not intelligent***. If someone knows English it means that person is intelligent, it seems as if they are associating this knowledge of English as having a good intellect. (my emphasis)

Lindi's reference to the lack of respect for African languages as a brutal act that 'killed the nation' is typical of the emotive language that these teachers used at different times in the study. Gugu also used this language of 'killing' during the pre-observation interview. Gugu, however, talked about 'killing the children' by not exposing them to English, while Lindi is talking about 'killing the nation' by not allowing them to use and therefore develop their own languages. Lindi also shows her anger about the past and the status of English - the fact that to be respected one has to be fluent in English. Despite her anger, Lindi does not want her school to use an African language in teaching mathematics because her "school will be empty".

The analysis presented above highlights the teachers' preference for English as the language of learning and teaching mathematics and the cultural models that inform these preferences. The discussion also shows the conflicting cultural models which teachers work with. A glaring absence in the teachers' cultural models is any reference to how learning and teaching in English as they prefer, would create epistemological access for the learners. This absence suggests that the teachers position themselves in relation to English and not mathematics. What is more prevalent in the reasons for preference of English are: economic, political and ideological factors. The section that follows explores the learners' language preferences and how they relate to those of teachers.

LEARNERS' LANGUAGE CHOICES

The data used here is drawn from a wider study still in process which involves secondary school learners. I analyse individual interviews with five Grade 11 (16-year-old) learners from Soweto, the largest and most multilingual African township in South Africa with a population of about 3 million people. All of these learners are multilingual (they speak four or more languages) and learn mathematics in English, which is not their home language. They chose their preferred language for the interview. With the exception of one (Basani), all their schooling has been in Soweto. They all made a choice to do mathematics and indicated that they like doing mathematics. Three indicated that they prefer to be taught mathematics in English while the other two felt that it really does not matter what language mathematics is learned in.

For the learners who preferred to be taught English (Tumi, Sipho and Nhlanhla) the cultural model of *English as an international language*, which positions

English as the route to success, emerged as dominant in their discourse. Their preference for English is because of the social goods that come with the ability to communicate in English.

> Tumi: English is an international language, just imagine a class doing maths with Setswana for example, I don't think it's good.
>
> Researcher: Why?
>
> Tumi: I don't think it is a good idea. Let's say she taught us in Setswana, when we meet other students from other schools and we discuss a sum for instance and she is a white person. I only know division in Setswana, so I must divide this by this and don't know English, then he I going to have problem. So I think we should talk English. English is okay.

Tumi sees English as an obvious language for learning and teaching mathematics. It is unimaginable to him for mathematics to be taught in an African language like Setswana. The use of English as a language of learning and teaching mathematics is common sense to him; he cannot imagine mathematics without English. This resonates with the teachers' cultural models above, which are exacerbated by the fact that mathematics texts and examinations are in English. Another factor that emerges Tumi's views above is the fact that he wants to be taught mathematics in English so that he can be able to talk about mathematics in English with white people.

> Sipho: I prefer that ba rute ka English gore ke tlo ithuta ho bua English. If you can't speak English, there will be no job you can get. In an interview, o thola hore lekgowa ha le kgone ho bua Sesotho or IsiZulu, ha o sa tsebe English o tlo luza job. *(I prefer that they teach us in English so that I can learn English. If you can't speak English, there will be no job you can get. In an interview you will find a white person not able to speak Sesotho or IsiZulu, you will loose the job because you don't know English.)*

Sipho's preference for English is because he sees it as a language that gives access to employment. Sipho also connects employment with white people by arguing that during the interview one must be able to express oneself in English because white people conduct interviews. This connection of jobs to white people and English is as a result of the socio-political history of South Africa in which the economy was and still continues to be in the hands of white people with English as the language of commerce, hence Sipho's expectation that a job interview will be conducted by a white person in English. Like Gugu, Tumi and Sipho see the mathematics class as an opportunity for them to gain access to English—the language of power.

Unlike Tumi and Sipho, Nhlanhla, who also indicated a preference for English, positioned herself in relation to mathematics. Nhlanhla, however, had conflicting cultural models.

> Nhlanhla: …is the way it is supposed to be because English is the standardized and international language.
>
> Researcher: Okay, if you had a choice what language would you choose to learn maths in.

Nhlanhla: For the sake of understanding it, I would choose my language. But I wouldn't like that [English as language of learning and teaching] to be changed because somewhere somehow you would not understand what the word 'transpose' mean, ukhithi uchinchela ngale *(that you change to the other side)*, some people wont understand. They would not understand what it means to change the sign and change the whole equation.

While Nhlanhla recognises the value of learning maths in a language that she understands better, she does not want English as LoLT to change because English is international and the African languages do not have a well-developed mathematics register. There are conflicting cultural models at play here: one that values the use of African languages for mathematical understanding and another that values English because of its international nature.

Researcher: What if there are students who want to learn mathematics in Zulu, what would say to them?
Nhlanhla: I would say its okay to have it but you have to minimize it because these days everything is done in English especially maths, physics and biology.
Researcher: Why does maths, physics and biology have to be done in English?
Nhlanhla: I don't know, think that's the way it is.

Nhlanhla's conflicting cultural models are evident in the above extract. They are indicative of the multiple identities that she is enacting. As a multilingual learner who is not fully proficient in English, she does not want to loose the social goods that come with English. As a mathematics learner it is important for her that she has a good understanding of mathematics and using her language, as she says, facilitates understanding. While the teachers (Kuki, Lindi and Gugu) also experienced conflicting cultural models, theirs were about access to social goods and not to mathematics.

Basani and Lehlohonolo are the two learners who felt that it really does not matter what language is used for mathematics. As indicated earlier, Basani is new in the school. Before coming to the school in Soweto, he was a student at a suburban school, which was formerly for whites only. At the time of the study, it was his second year at the Soweto school, which he came to because his mother could no longer afford the fees at the former white school. Basani's level of English fluency was clearly above all the other learners interviewed. During the interview, he explained that he was doing Grade 11 for the second time because he failed IsiZulu and Mathematics the previous year. He however insisted that he has no problem with mathematics and that he failed mathematics because he was not as focused as he should have been.

Basani: Maths is also a language on its own, it doesn't matter what language you teaching it. It depends if the person is willing to do it.
Researcher: What would you say to learners who want to be taught maths in their African languages?
Basani: I would not have problem. If that's the way they wanna do it , well its their choice. I have a friend here at school he is Sotho, I help him with Maths.

Sometimes when I explain in Sesotho he doesn't understand and when I explain it in English he understands.

Researcher: Why is that?

Basani: I don't know that's something I cannot answer because, how should I know, I never had a problem with maths before.

As the above extract shows, Basani believes that mathematics is a language and thus it does not make any difference what language it is taught and learned in. Basani is very confident about his mathematical knowledge and seems to be working with a cultural model that says, the key to mathematics learning is the willingness to do it. Lehlohonolo, who is also very confident about his mathematical knowledge, also felt that it does not matter what language is used for mathematics. The class teacher explained that he is the best performing learner in mathematics in his class. Another interesting thing is that when I gave them the information letters and consent forms to participate in the study, Lehlohonolo immediately indicated that I should use his real name because he wants to be famous. During the interview, Lehlohonolo focused more on mathematics rather than language.

Researcher: Does it matter which language you do maths in?

Lehlohonolo: To me it doesn't matter just as long as I am able to think in all languages and I can speak and write in those languages then I can do maths in those languages.

Lehlohonolo is connecting language to learning in very sophisticated ways. For him fluency in a language (ability to read, speak, write and think) facilitates ability to learn in the language. As he explains below, fluency in a language is not sufficient to make a learner successful in mathematics.

Lehlohonolo: What I have realized is students that are I go with in class fail maths but they do well in English, I don't think English is the cause of why they failing maths. Some of them they chose maths because of their friends, some of them are in the wrong class. From my past experience they are not good in maths so they shouldn't have gone with maths. Even if you do it in IsiZulu, things will be the same, the problem is not with the language. They don't want to think, they don't want to be active; they don't interact with the teacher. If the teacher does the exercise and ask them if they are okay with this, they just agree, but when it comes to writing they don't understand.

For Lehlohonolo, language cannot be blamed for failure or given credit for success in mathematics. He sees the important factor in succeeding in mathematics as being the learners themselves and the choices they make about how they participate in the mathematics class. The above extract suggests that Lehlohonolo enacts a cultural model that mathematics should be taken only by those who are good at it and being good at mathematics is not connected to language.

Researcher: So if you had a group of students who want to do maths in Zulu, what would you say to them?

Lehlohonolo: That's their own problem because if they out of high school, they cannot expect to find an Indian lecturer teaching maths in Zulu. English is the simplest language that everyone can speak so they will have to get used to English whilst they are still here.

While Lehlohonolo does not connect failure or success in mathematics to language, in the above extract he seems to be suggesting that learners should choose to learn in English because in higher education no lecturer will be able to teach in their languages. This is an emergence of a conflicting cultural model for Lehlohonolo, which says even if there is no causal link between success in mathematics and the language used for learning and teaching, English cannot be ignored.

The above discussion shows that the learners' positioning and cultural models are not as clear as those of teachers are. What we can see is that the learners who prefer to be taught in English position themselves in relation to English. Nhlanhla is the only one who preferred English but she also positioned herself in relation to mathematics. Tumi and Sipho are more concerned with gaining fluency in English so that they can access social goods such as jobs and higher education. They enact the same cultural model as teachers that *English is international*. This cultural model emphasises the belief that the acquisition of the English language constitutes the major content of schooling. This is inconsistent with the content of schooling, which is about giving epistemological access and to research and the Language in Education Policy (LiEP) in South Africa, which promotes multilingualism and encourages use of the learners' home language. The assumption embedded in this policy is that mathematics teachers and learners in multilingual classrooms together with their parents are somehow free of economic, political and ideological constraints and pressures when they apparently freely opt for English as LoLT. The LiEP seems to be taking a structuralist and positivist view of language, one that suggests that all languages can be free of cultural and political influences.

As indicated earlier, the learners who position themselves in relation to the mathematics seem to be working with conflicting cultural models—one that is about mathematical understanding and the other that is about English fluency. While teachers also worked with conflicting cultural models, they did not position themselves in relation to the mathematics.

WHAT DOES THIS MEAN FOR RESEARCH AND PRACTICE?

Research argues that to facilitate multilingual learners' participation and success in mathematics teachers should recognise their home languages as legitimate languages of mathematical communication (Khisty, 1995; Moschkovich, 1999, 2002; Author & Adler, 2002). As alluded to earlier in the paper, all the studies that recommend the use of the learners home languages have been framed by a conception of mediated learning, where language is seen as a tool for thinking and communicating. The studies foreground the mathematics but do not consider the political role of language. The analysis presented in this paper shows that the language choices of teachers and learners who prefer English are informed by the political nature of language. The challenge is in bringing the two together. Research shows that in bringing the two together, English dominates.

A recent detailed analysis of a lesson taught by Kuki suggested a relationship between the language(s) used, mathematics discourses and cultural models that emerged (Author, in press). During the lesson, Kuki switched between English and Setswana. However, her use of English tended to produce procedural discourse while her use of Setswana tended to produce conceptual discourse. The same observations were made in Lindi's class, while procedural discourse was dominant in Gugu's class who used only English during her teaching (Author, 1998, 2002). While it can be argued that the observations made in Kuki, Lindi and Gugu's classrooms cannot be generalised to all the teachers in multilingual classrooms, they give us an idea of what the dominance of English in multilingual mathematics classrooms can produce.

Recent research in South Africa points to the fact that procedural teaching is dominant in most multilingual classrooms (Taylor & Vinjevold, 1999). In most cases, this dominance of procedural teaching is seen as being a function of the teachers' lack of or limited knowledge of mathematics. What the above discussion suggests is that the problem is much more complex.

CONCLUSION

The analysis presented in this paper shows that teachers and learners who position themselves in relation to English are concerned with access to social goods and positioned by the social and economic power of English. They argue for English as LoLT. Issues of epistemological access are absent in their discourse. In contrast, learners who position themselves in relation to mathematics and so epistemological access, reflect more contradictory discourses, including support for the use of the learners' home languages as LoLT. The work presented in this paper provides an important contribution in dealing with the complex issues related to teaching and learning in multilingual classrooms. Much remains to be done.

ACKNOWLEDGEMENTS

This paper is based on work supported by the National Research Foundation under Grant Number, GUN 2053954. Any ideas expressed are, however, those of the author and therefore the NRF does not accept any liability. I appreciate critical comments from Jill Adler, Thabiso Nyabanyaba and Godfrey Sethole on earlier drafts of this paper.

[i] Setswana is one of the 11 official languages in South Africa. The other official languages are: IsiZulu, IsiXhosa, TshiVenda, Xitsonga, Sesotho, Isindebele, Siswati, Sepedi, Afrikaans and English.

References

Adendorff, R. (1993). Code-switching amongst Zulu-speaking teachers and their pupils. *Language and Education*, *7*(3), 141–162.

Adler, J. (2001). *Teaching mathematics in multilingual classrooms.* Dordrecht: Kluwer Academic Publishers.

Arthur, J. (1994). English in Botswana primary classrooms: Functions and constraints. In C. M. Rubagumya (Ed.), *Teaching and researching language in African classrooms* (pp. 63–78) Clevedon: Multilingual Matters Ltd.

Cummins, J. (2000). *Language, power and pedagogy: Bilingual children in the crossfire.* Clevedon: Multilingual Matters.

Friedman, S. (1997, 3 March 1997). Democracy stifled by language barrier. *Business Day*, p. 7.

Gee, J. P. (1996). S*ocial linguistics and literacies: Ideology in discourses.* London: Falmer Press, Taylor and Francis group.

Gee, J. P. (1999). *An introduction to discourse analysis: Theory and method.* London: Routledge.

Granville, S., Janks, H., Joseph, M., Mphahlele, M., Ramani, E., Reed, Y., & Watson, P. (1998). English without g(u)ilt: A position paper on language in education policy for South Africa. *Language in Education, 12*(4), 254–272.

Gutiérrez, R. (2002). Enabling the practice of mathematics teachers in context: Toward a new equity research agenda. *Mathematical Thinking and Learning, 4*(2 & 3), 145–187.

Harsthorne, K. B. (1987). Language policy in Africa education in South Africa, 1910–1985, with particular reference to the issue of medium of instruction. In D. Young (Ed.), *Bridging the gap: Essays in honour of L. W. Lanham* (pp. 62–81). Pretoria: Maskew Miller Longman.

Heugh, K. (Ed.). (1997). *Multilingual education for South Africa.* Johannesburg: Heinemann.

Holland, D., & Quinn, N. (Eds.). (1987). *Cultural models in language and thought.* Cambridge: Cambridge University Press.

Howie, S. (2002) *English language proficiency and contextual factors influencing mathematics achievement of secondary school pupils in South Africa.* Den Haag: CIP- Gegevens Koninklijke Bibliotheek.

Khisty, L. L. (1995). Making inequality: Issues of language and meaning in mathematics teaching with Hispanic students. In W. G. Secada, E. Fennema & L. B. Abajian (Eds.), *New directions for equity in mathematics education.* (pp. 279–297). Cambridge: Cambridge University Press.

Mda, T. (1997, November 1997). Learning best in home language. *The Teacher*, p. 6.

Merritt, M., Cleghorn, A., Abagi, J. O., & Bunyi, G. (1992). Socialising multilingualism: Determinants of code-switching in Kenyan primary classrooms. *Journal of Multilingual and Multicultural Development*, *13*(1 & 2), 103–121.

Moschkovich, J. (1999). Supporting the participation of English language learners in mathematical discussions. *For the Learning of Mathematics, 19*(1), 11–19.

Moschkovich, J. (2002). A situated and sociocultural perspective on bilingual mathematics learners. *Mathematical Thinking and Learning. 4*(2 & 3), 189–212.

Ncedo, N., Peires, M. & Morar, T. (2002). Code-switching revisited: The use of languages in primary school science and mathematics classrooms. *Tenth Annual Conference of the Southern African Association for Research in Mathematics, Science and Technology Education* (pp. 308–312). Durban: University of Natal.

Reagan, T. G., & Ntshoe, I. (1987). Language policy and black education in South Africa. *Journal of Research and Development in Education*, *20*(2), 1–8.

Taylor, N., & Vinjevold, P. (1999). *Getting learning right.* Johannesburg: Joint Education Trust.

Zentella, A. C. (1997). *Growing up bilingual: Puerto Rican children in New York*. Malden, MA: Blackwell.

IS MATHEMATICS EDUCATION A PRACTICE? MATHEMATICS TEACHING?

Stephen Kemmis
Charles Sturt University
<skemmis@csu.edu.au>

Practice is a rich and complex notion whose nuances remain elusive for many practitioners, researchers, policy-makers and administrators. As this paper shows, the question of whether teaching is a practice is contested. The theoretical density of practice is frequently underestimated by researchers who too frequently view it from narrow and limited perspectives. The paper presents a framework that aims to illuminate the richness of practice in particular cases and in research on practice. It is also argued that the practice of education is corrupted and impoverished when education is viewed in mechanical and instrumental terms, as has happened in recent years as states have acted to administer, regulate and assess teaching more closely.

In 2002, the *Journal of Philosophy of Education* initiated a debate about whether teaching is a practice. The source of the debate was this statement from Alasdair MacIntyre (MacIntyre & Dunne, 2002) the philosopher whose work on practice has been central for many in education and other fields:

> … teaching itself is not a practice, but a set of skills and habits put to the service of a variety of practices. The teacher should think of her or himself as a mathematician, a reader of poetry, an historian or whatever, engaged in communicating craft and knowledge to apprentices. It follows that you cannot train teachers well, until they have been educated into whatever discipline it is they are to transmit… (p. 5)

In the dialogue with MacIntyre reported in the article, Joseph Dunne responds to this challenge by showing how teaching appears to involve a number of features central to MacIntyre's conception of practice, first elaborated in his (1982) *After Virtue*—quoting MacIntyre against MacIntyre (pp. 7–8) to argue that teaching

- is 'a complex form of socially established cooperative human activity',
- has its own 'standards of excellence', and through teaching 'human powers to achieve excellence are systematically extended',
- has characteristic 'internal goods' (realised in students' learning and development) which are in tension with 'external goods' (like money, power, status, which accrue both to teachers and those they teach),
- that "teaching is 'the good of a certain kind of life'" (p.7), which is to say that it is a principal good in the life of a teacher when that life is understood as a whole,
- that teaching, like other practices, has a history of its own furnishing it with "a wider tradition of exemplary figures and indeed of fundamental debate, with its proper ends being defined and redefined in canonical writings" (p. 8) and that
- "the dialectic between 'practice' and 'institution' seems to be faithfully reflected in the case of teaching and the school" in which "'the ideals and creativity' as well as the 'cooperative care for common goods' of teaching are

always 'vulnerable to the competitiveness of the institution' of school and its 'corrupting power'" (p. 8).

To this, MacIntyre responds

> It's not clear to me how far we disagree. You say that teaching is itself a practice. I say that teachers are involved in a variety of practices and that teaching is an ingredient in every practice. And perhaps the two claims amount to very much the same thing; but perhaps not. For it is part of my claim that teaching is never more than a means, that it has no point and purpose except for the point and purpose of the activities to which it introduces students. All teaching is for the sake of something else and so teaching does not have its own goods. The life of a teacher is therefore not a specific kind of life. The life of a teacher of mathematics, whose goods are the goods of mathematics, is one thing; the life of the teacher of music whose goods are the goods of music is another (p. 9).

So: on MacIntyre's view, it appears that mathematics teaching, for example, is not worthwhile in and of itself, but only as a means to the practice of mathematics which is worthwhile in and of itself.

Now this is not just a matter of status—in the sense of the relative worth or reputational standing of mathematics (or other practices like medicine or law or chess or farming) and teaching. What hangs on this is whether teaching is the kind of activity which, among other things, deserves to be taught in universities, whether it is or has its own distinctive disciplinary knowledge, for example. Early in their dialogue, Dunne had put this proposition to MacIntyre:

> … you have developed the notions of a 'practice', the 'narrative unity of a life' and 'tradition' as together providing the conceptual matrix for a reconstruction of 'virtue'. I'm wondering whether you would see these three ideas as *also* offering resources for a fruitful reconceptualisation of education (p. 3).

And now it appears that MacIntyre has ruled out teaching being a practice, so the conceptual matrix seems unlikely to provide a platform for building a new conceptualisation of education.

In the dialogue that follows, it seems to me that not enough is made of the conceptual differences between 'teaching' and 'education' and 'schooling'. Had these distinctions been made, perhaps other conclusions might have been drawn by the protagonists.

A year or so after the publication of the MacIntyre-Dunne dialogue, in a special issue of the *Journal of Philosophy of Education* edited by Joseph Dunne and Pádraig Hogan (*37*(2), 2003), several commentators on and respondents to the dialogue nudged the argument a few steps further. Nel Noddings, for example, who initially felt that not much was to be gained through debating the terms, came to the conclusion that teaching is a practice of a special kind which she called a 'relational practice'. Like others, she also believes that it is important to the standing of teaching as a profession that its work be describable as a practice. David Carr takes issue with MacIntyre's conceptions of practice and (especially) virtue from the "more mainstream Aristotelian virtue-ethics concepts of moral character and agency" (p. 253). He proceeds to show that teaching does indeed involve moral character and agency, and that it cannot be reduced to technical-managerial expertise. He also distinguishes more carefully between 'education' and 'schooling' (as the institutionalised form of education), to make a strong case that many (perhaps all) people—like parents—teach even if they are not teachers by profession or occupation.

To my mind, more might have been gained by distinguishing more clearly between 'education' and 'teaching'. MacIntyre's argument that *teaching* (especially if in the even more limited sense of 'instruction') is a means to the end of some other practice seems good to me—indeed we *do* teach things in the interests of having our students be able to do or be the things we teach (e.g., to program a computer or to be morally responsible for their actions). But it seems to me that the same cannot be said about *education*. Properly understood, teaching is one among a number of means to education—the education of the learner, or even more, 'the educated person'. Something more is meant by the education of learners than simply that they have learned this or that which we either taught or did not teach them. We may say of Sally "I taught her quadratic equations", but not "I educated her quadratic equations" or even "I educated her in quadratic equations". Because, to the extent it could be true, one might only say (immodestly and wrongly) "I educated *Sally*". The nub of it is this: the point of Sally's education is, on the one hand, *her self-development as a person and her learned capacity to continue her self-development*, and, on the other, *the development of the society and world in which she lives through her educated, civil and capable participation in it*.

These two interrelated aims—contributing to the self-development of learners as persons and contributing to the development of the good society through the participation of educated persons—are the distinctive aims of education, associated with distinctive goods—the notions of the good person and of the good society. Though other people, practitioners and professionals might value these aims and goods or contribute to them in the conduct of their own distinctive work, no other practice or profession has, as its central aim and good the double task of contributing to the self-development of learners as persons and to the development of the good society through the participation of educated persons. This double aim seems to me to parallel what is distinctive about farming which MacIntyre rightly regards as a profession—in my view, it is parallel with the interrelated aims and goods of farming in contributing to the sustenance of people and the sustenance (sustainability) of the land being farmed. Such a view also shows affinities with the character of other practices like medicine which concerns itself with people's health through action against ill-health (as against their self-development in the case of education), or the practice of nursing which concerns itself with providing care in the interests of the well-being of those cared for.

On this view, *teachers*, insofar as they regard themselves as *educators*, may indeed live a distinctive kind of life—one committed to the service of those ends and goods, and against which they may offer themselves to be judged. In other contexts, I have remarked on my lack of success as an educator, since, through my work in teacher education and research on education I have not apparently 'produced' a sufficient number of teachers/educators committed to these goods through whose work better persons and a better society might thrive. Indeed, it seems to me, there is much to bewail about our contemporary society, and I would agree with MacIntyre about many aspects of his diagnosis of what our society has become in the absence of a more educated public, to the extent that the notion of an educated public remains viable in our contemporary, compartmentalised, fragmented and fractious world—a world in which technical, functionalist and managerial reason everywhere ignores and attempts to displace moral and practical reasoning, and to empty politics of its moral content in favour of policies that can be 'sold' as slogans to a harried, disbelieving, and often cynical public that seems no longer to expect governments to be legitimate or act legitimately. In short, one of the yardsticks by which I measure the *technical* success of my work as a educator is whether or not I appear to have contributed to the 'production' of more educated persons and a better, more civil society, but this is overridden by the *moral* imperative which gives my work meaning and significance to me as an educator. I may take a little comfort in the thought that things might be even worse without my efforts and those of others committed to these educational aims and goods, but my moral task as a teacher seems to me to have been badly disfigured—not so much by other teachers or educators who hold other views, but by governments and state agencies which have acted contrary to these profound goods that give my life meaning as a "certain kind of life", as MacIntyre puts it—that is, the life of an educator committed to ends and goods which are distinctive to the practice of education. Commitment to these ends and goods in turn gives rise to a distinctive conception of what counts as a virtuous life for an educator, including, for example, modelling commitment to the self-development of others and enacting civility in relationships with students, their families, colleagues and others.

An audience of mathematics educators will no doubt be interested that MacIntyre—and Dunne and Noddings and Carr—uses the example of mathematics education. MacIntyre says teaching is not distinctive as a practice because it is only a means to the practice of mathematics; the others resist this view. In my view, mathematics education is but one subspecies of the distinctive practice of education, and mathematics teaching is but one means to the distinctive and enduring ends and goods of education (namely, the self-development of learners and the societies in which they live)—something that can be attained only through the education of citizens, including education in fields and practices as important as mathematics.

As a preliminary answer the questions in the title to my paper, I thus conclude that, yes, mathematics education is a practice (in the sense that it is a subspecies of the more general practice of education), and, no, that mathematics teaching is not in itself a practice, being only one of a variety of means to (a) the distinctive ends of education (participation in the practice of education as the self-development of learners and the societies in which they live), and (b) the practice of mathematics (though only some learners will go on to participate in mathematics as a practice—they will simply use mathematics in other pursuits and practices). And, in both cases, one other means to those ends, and to the conduct of those practices, is learning in the absence of explicit or implicit teaching (instruction or self-study of instructional texts, for example).

A More Encompassing View of Practice

MacIntyre's view of practice is rich in implications for practice. But there are other views of practice which also offer insights and implications for the conduct and development of practices, including the practice of education. I would like to use this opportunity to elaborate on a view of practice described in a paper I presented in Umeä last year (Kemmis, 2004). In that paper, I argued that practice is not best understood in terms of 'professional practice knowledge', as, for example, in the view presented by Higgs, Titchen and Neville (2001) who suggest that professional practice knowledge can be described in terms of

> (1) propositional, theoretical or scientific knowledge – e.g., knowledge of pathology;
>
> (2) professional craft knowledge or knowing how to do something;
>
> (3) personal knowledge about oneself as a person and in relationship with others (p. 5).

Against the view that practice can best be understood from the perspective of practitioners' knowledge—that is, what is in the heads of individual practitioners—I argued that practice has a number of *extra-individual features*, and that neither practice itself nor the process of changing practice can be adequately understood without reference to these extra-individual features. I drew on a variety of theorists of practice (like MacIntyre, Bourdieu, Foucault and Habermas) to show that, beyond the individual person of the practitioner, practice is also socially-, discursively-, culturally- and historically-formed.

One reason for making this argument was to address the educators of professionals: to argue that we should not limit our teaching to instilling professional practice knowledge in the form of technical, craft and personal knowledge, but rather to insist that neophyte and developing professionals should understand how practices are constructed in the social and other dimensions just listed. If I might put it this way, understanding and changing practice requires work outside the heads of practitioners as well as inside them. I argued for opening communicative spaces—public spheres constituted for public discourse—in which both communities of practice and practitioners and their clients could thematise and explore problems and issues of practice, and the effects and longer-term consequences of particular kinds of practice.

A second reason for arguing for a more encompassing view of practice is addressed to researchers studying practice in different fields. In our chapters for the second and third editions of *The Handbook of Qualitative Research* (Kemmis & McTaggart, 2000, 2005), Robin McTaggart and I have argued that research on practice has frequently proceeded with impoverished views of practice as an object of study, and that to understand practice in a more multi-dimensional way it must be studied using multi-disciplinary, multi-method approaches which allow it to be viewed from at least the five perspectives sketched in Figure 1, in part because they characteristically rely on the kinds of research methods and techniques sketched in Figure 2.

Focus: **Perspective:**	***The individual***	***The social***	***Both: Reflexive-dialectical*** view of individual-social relations and connections
Objective	**(1)** Practice as individual behaviour, seen in terms of performances, events & effects: behaviourist and most cognitivist approaches in psychology	**(2)** Practice as social interaction - e.g., ritual, system-structured: structure-functionalist and social systems approaches	
Subjective	**(3)** Practice as intentional action, shaped by meaning and values: psychological *verstehen* (empathetic understanding) and most constructivist approaches	**(4)** Practice as socially-structured, shaped by discourses, tradition: interpretive, æsthetic-historical *verstehen* & post-structuralist approaches	
Both: Reflexive-dialectical view of subjective-objective relations and connections			**(5)** Practice as socially- and historically-constituted, and as reconstituted by human agency and social action: critical theory, critical social science

Figure 1. Relationships between different traditions in the study of practice.

Focus: **Perspective:**	***The individual***	***The social***	***Both: Reflexive-dialectical*** view of individual-social relations and connections
Objective	**(1)** *Practice as individual behaviour:* Quantitative, correlational-experimental methods. Psychometric and observational techniques, tests, interaction schedules.	**(2)** *Practice as social and systems behaviour:* Quantitative, correlational-experimental methods. Observational techniques, sociometrics, systems analysis, social ecology.	
Subjective	**(3)** *Practice as intentional action:* Qualitative, interpretive methods. Clinical analysis, interview, questionnaire, diaries, journals, self-report, introspection	**(4)** *Practice as socially-structured, shaped by discourses and tradition:* Qualitative, interpretive, historical methods. Discourse analysis, document analysis.	
Both: Reflexive-dialectical view of subjective-objective relations and connections			**(5)** *Practice as socially- and historically-constituted, and as reconstituted by human agency and social action:* Critical methods. Dialectical analysis (multiple methods).

Figure 2. Methods and techniques characteristic of different approaches to the study of practice.

These characteristic approaches to the study of practice mean that the practice one researcher 'sees' is likely to be very different from what is 'seen' by a researcher from a different tradition. These differences betray profound disagreements about what *research* is, which in turn give rise to disagreements about what *practice* is—whether practice in general, or in the case of particular professions or occupations, or in the case of particular practitioners.

In our chapter for the second edition, we therefore argued for 'symposium research'—drawing on different disciplines and employing multiple methods—in the study of practice. I hope that some researchers studying mathematics teaching as a means to mathematics education and education in general will explore the possibility of multi-disciplinary, multi-method 'symposium research' of this kind.

In Figure 3, I identify a range of different features of practice all of which seem to me significant in adequately understanding a practice. I would like to claim that there are no other interesting categories to consider about practice than the ones pointed to in my summary—but no doubt I have missed aspects of practice just as important as the ones identified here, or have inadequately expressed some of the ideas intended. Repairing some such omissions, the Figure also includes aspects of practice not explicitly discussed in my Umeä paper—particularly (in column 2 of the Figure) the material-technical aspects of practice as behaviour assumed in that paper. I hope the key words listed in each cell provide sufficient pointers to the work of other thinkers and theorists of practice; clearly, there is not time or space here to provide a comprehensive justification of all of the elements included—that task would require a book.

Before I proceed to my list of key features, it might be useful as a thought-experiment if you could think of

(1) a case of successful practice which you know well (e.g., a successful case of mathematics education practice), and

(2) an example of an excellent research study examining practice in some field (e.g., an excellent study of mathematics education practice).

I invite you to consider (1) the extent to which my list captures what you regard as the most significant aspects of your example of successful practice, and (2) the extent to which my list of key features provides a framework for identifying presences and absences in the example of research into practice you have in mind, thus suggesting a framework for critique of particular studies of practice and identifying how other studies might complement and strengthen the research undertaken in the example you have chosen. I hope my list (1) touches on aspects of your practice that you will agree are significant—and that it does not leave out the things you believe are most significant, and (2) indicates how different research studies of practice illuminate particular aspects of practice even though they may overlook others.

These features are summarised in Figure 3 on the pages that follow.

		Extra-individual features of practice		
	(1) Individual features of practice	(2) Material-technical features	(3) Social features	(4) Cultural features
A Meaning and purpose	Practice is not just activity: it involves ***meaning and intention***, and draws on ***professional practice knowledge*** (including technical, craft and personal knowledge)	Practice involves ***action and interaction in and on the world*** (with others and objects) to address identified ***needs*** or ***problems*** in pursuit of characteristic ***goals and ends***	Practice involves and expresses ***values*** (value-laden), ***social norms*** (guided by moral and ethical concerns) and ***virtues***.	Practice is always ***'theoretical'*** – it refers to theory that informs it (of which practitioners may or may not be aware)
B Structured	Practice is always ***experientially-formed*** – it realises and is realised in the ***identity*** of the practitioner as a practitioner	Practice involves the use of ***learned skills and techniques*** (that have themselves developed and evolved over time) in ***structured systems of relationships*** between people (e.g., practitioners-clients) and people and things (e.g., practitioners-instruments)	Practice is always ***socially formed and socially structured*** – it realises and is realised in social interactions and relationships (incl. characteristic role relationships like practitioner-client, teacher-student, nurse-patient) (cf. Bourdieu's ***social capital***)	Practice is always ***culturally and discursively formed and structured*** – it realises and is realised in language, words, ideas, specialist discourses and theories (cf. Bourdieu's ***cultural capital***)
C Situated	Practice is always ***embodied*** – it is what particular people do, in a particular place and time – inevitably involving ***identity work*** and ***emotional work*** (e.g., as a painful consequence of caring)	Practice involves ***action on the material world in the material here-and-now*** (particular times, places, objects)	Practice realises and is realised in characteristic forms of ***social integration*** (e.g., care via nursing, education via teaching, sustenance of people and land via farming)	Practice is grounded in the agreements and debates that form the discursive histories and social relations of relevant ***communities of practice***
D Systemic	Practices are frequently preserved, maintained and developed through the development of the ***professional role***	Practices involve ***material and economic interactions, exchanges and transactions*** (e.g., role-	Practices are frequently preserved, maintained, developed and regulated in ***institutions and***	Practices are frequently subject to ***accreditation and regulation*** through law and policy and

	of the practitioner	related functions, payment for services, professional status relations) (cf. Bourdieu's ***economic capital***)	***organisations***, and the cooperative work of ***professions***	professional standards and guidelines
E Temporally-located	Practice is always ***dramaturgical*** in character – it unfolds in human and social action, against the narrative background of individuals' lives (biographies)	Practice occurs in/over ***time***, through processes (***transformation*** of raw materials into end products via labour), against a technical background of education, training and development	Practice is always ***historically formed and structured*** – it is always the product of a local history (in this situation, among these people) and history in the wider sense	Practice is always culturally-located against backgrounds of symbolic and discursive ***tradition*** characteristic of particular groups, societies and histories
		Extra-individual features of practice		
	(1) Individual features of practice	(2) Material-technical features	(3) Social features	(4) Cultural features
F Forms of reasoning	Practice always involves ***practical reasoning***, using knowledge in the face of uncertainty, and understanding that action is always a kind of exploration of what might be done (***exploratory action*** in the face of ***the dialectic of the actual and the possible***) – guided by a ***practical*** knowledge-constitutive interest in acting wisely and prudently in given circumstances	Practice always involves ***technical reasoning*** about the (most appropriate and efficient) use of means for given ends in particular material contexts, and ***functional reasoning*** about organisational capacities to achieve organisational goals – guided by a ***technical*** knowledge-constitutive interest in achieving particular ends using appropriate means	Practice always invites ***critical reasoning*** in which participants collaboratively explore the nature and consequences of what they do against the criteria of comprehensibility, truth (in the sense of accuracy), truthfulness (sincerity) and moral appropriateness – guided by an ***emancipatory*** knowledge-constitutive interest in identifying and overcoming incomprehensibility, irrationality, deception (including false consciousness or self-deception) and injustice (including suffering, domination and oppression)	

G Reflexivity and transformation	Practice is always ***reflexive***, in the sense that the act of doing the practice invites practitioners to bring to mind and treat as problematic the relationship between themselves as knowing subjects, the (new) situation of their practice in the here-and-now, and their practice as *praxis* (informed committed action)	Practice is ***materially or strategically reflexive***, in the sense that it makes transformations of the world (e.g., raw materials to end products), and in the sense that the operations and interactions involved can themselves be transformed (redesigned, re-engineered, improved)	Practice always invites ***opening communicative space*** (and the creation of ***public spheres***) in which practitioners, and practitioners and clients and others interested or affected, explore issues and themes of common concern or interest	Practice always invites ***communicative action*** (i.e., collaborative action oriented towards mutual understanding, intersubjective agreement and consensus about what to do)

Figure 3. Key features of practice.

I look forward to hearing discussion of the adequacy of my list of key features of practices, and to discovering whether the table helps to identify the features of practice observed in different kinds of research on practice.

If the list does provide a provisional framework for understanding and researching practice, it shows how rich and complex practice is, stretching out from the here-and-now of particular episodes of behaviour and action in time and physical, semantic, social and cultural space. It suggests what *lies behind* or *may* lie behind particular acts, in the minds of those participating in them. It suggests what cannot be 'seen' by research that limits its purview just to the actions seen by an observer, or in the perspectives of particular participants. It suggests a kind of illimitability of practice, for example in the dimension of history and tradition, even though it points towards a genealogy of connections between these people and acts and others long gone and far distant. And this illimitability of *practice* makes a mockery of most 'measures' of practice that observe only particular behaviours or acts.

It is possible, however, to explore at least the nearby regions of the illimitable space occupied by a practice, for example, by considering the relationship between practitioners' and clients' perspectives on practice—a topic to which I shall now turn.

PRACTITIONERS' AND CLIENTS' PERSPECTIVES ON PRACTICE

In the time that remains, I would like to invite you to consider some further questions about practice which I believe are central to understanding them as practices—namely, that they involve social interactions in which 'clients' are not merely 'objects' operated on or influenced by practitioners, but persons-in-themselves who are, to a greater or lesser degree, *knowing subjects* who are co-participants in practice. Thus, for example, learners are not merely 'objects' on which teachers 'operate', but persons-in-themselves who are co-participants in the joint activity better described as 'learning and teaching' than merely 'teaching' (which directs our attention to just one of the players in the game of learning and teaching).

To a greater or lesser degree, 'clients' of different practices—patients or students for example—are knowledgeable about the practices and know something about how they are to participate in them. Even an acute hospital patient meeting, say, an occupational therapist for the first time knows something about how to interact with this person—for example, that they are to get some kind of help through some kind of 'therapy', that the conversation between them will probably be conducted in a 'professional' manner, and that this is a service somehow linked to an institution (like a hospital) and a profession with relevant professional bodies and a distinctive specialist discourse (sometimes perceived as jargon). The acute patient meeting the occupational therapist for the first time thus begins learning how the particular 'game' of occupational therapy is played, in terms of the languages and discourses appropriate to it, the kind of activities and work processes involved, and the social relations and organisational and institutional goals, roles and rules that apply to their interactions.

I want to suggest that one might explore the client's perspective on practice using the table of key features of practice presented in Figure 3. Indeed, I would like to suggest that 'learning the game' of the practice involves the client (patient, student) in aligning their perspective on the practice with the perspective implied in the words and actions and social relationship offered by the practitioner. Sometimes practitioners must re-align their presuppositions about the conduct of their practice to connect with those of their clients, and almost assuredly clients will need to re-align their presuppositions to connect with those of the practitioner.

Without the detail of Figure 3, Figure 4 below is intended to portray the juxtaposition of practitioners' and clients' perspectives, though inadequately demonstrating that both have some ideas and experience related to all or most of the cells in the matrix.

		Extra-individual features of practice		
	(1) Individual features	(2) Material-technical	(3) Social	(4) Cultural
A Meaning & purpose				
B Structured				
C Situated				
D Systemic				
E Temporally-located				
F Forms of reasoning				
G Reflexivity & transformation				

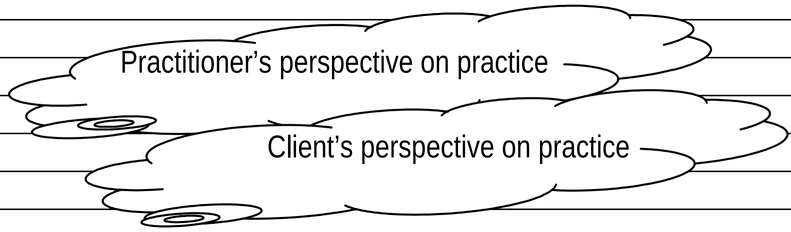


Figure 4. Clients' and practitioners' perspectives on practice in relation to the key features of practice.

Of course neither practitioners nor clients exist in a social vacuum. On the one side, from the perspective of a professional practice, we might readily point to the community of practice of the practitioner—the professional bodies and institutions, frequently including universities, carrying the knowledge and traditions of the practice of that profession, and may be responsible for accreditation and regulation of members of profession. On the other side, from the perspective of the client, we may also point to those social groups, including family, community and other kinds of affiliations and connections that furnish a background of meanings, purposes, values and the rest brought by the client to the practice situation. And it should be noted that the practitioner also has a background of family, community and other connections that she or he brings to the situation. These backgrounds are roughly portrayed in Figure 5.

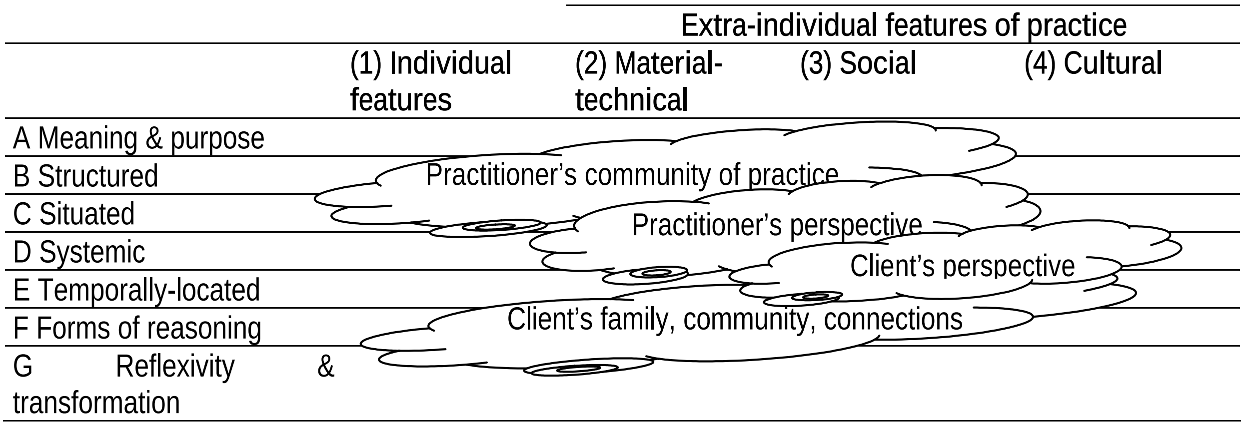


Figure 5. The widening social context of clients' and practitioners' perspectives on practice.

As suggested in relation to practitioners' and clients' perspectives, the presuppositions and perspectives of communities of practice and the social groups to which clients belong may also be considered against the framework of key features of practice listed in Figures 3 and 5. Here, this is simply at the level of a thought-experiment. Perhaps the example of successful professional practice you considered earlier allows you to speculate about the relationships between practitioners' and clients' perspectives in that case (cf. Figure 4); the task becomes far more demanding in relation to the variety of perspectives depicted in Figure 5.

In my view, pursuing an analysis of the kind suggested by Figure 5 takes us, reflexively, back into the key features of practice presented in Figure 3. It begins to show, at greater depth, what the columns referring to the social and cultural features of practice refer to, and what the rows referring to forms of reasoning and reflexivity refer to. Perhaps it is to suggest something about the 'forms of life' practices represent, as a Wittgensteinian analysis of practice might begin to show. Some steps towards such analyses have been taken in some recent writing on practice (for example, Shotter, 1996).

Applied to the case of mathematics education, we might think of the relationships depicted in Figure 5 in terms of the perspectives of a mathematics teacher, her or his students, any community or communities of practice with which the teacher is involved, and the families, communities and other connections of the students. Clearly, the social networks brought into contact at the point of learning and teaching stretch far beyond the teacher and students in physical and social space, in time, and in terms of discursive resources and relationships. As Shotter suggests, in the poetics of conversations like those between students and teachers, *worlds* of meaning connect or collide, occasionally re-orienting both students and teachers as they glimpse aspects of each others' realities through the windows of their words in the here and now, sometimes yielding surprising insights into how each construes their apparently-shared world. Of course, this refers in one way to the "aha!" experience that teachers revel in whenever they see it, and to the idea of "the teachable moment" that teachers aim to construct or respond to when they find it. But it also refers to the "aha!" of the teacher who makes sense of the nature of a student's misunderstanding, or is surprised by facts about a student's family life or background that explain why there have been difficulties 'connecting' with John or Jane.

What I hope to do by juxtaposing the practitioner's and client's perspectives on practice with the framework of features of practice sketched in Figure 3, however, is to say more than that the world of the mathematics teacher and student are *different*—I hope also to suggest some of the *ways* they differ. If we think about a case of practice like the example of successful practice you considered earlier, involving some practitioner—perhaps a mathematics teacher or teacher educator—and a 'client' or 'clients—perhaps that teacher's students—we can explore the richness of the space of practice by considering each of the rows and columns in the framework presented in Figure 3. We can begin this task here—just pointing to topics referred to in the framework, but I think it will also show, in practice, how practice richly understood is illimitable.

INTENTION AND MEANING

Reading the labels of the rows in Figure 3, teachers and students may have different *intentions* and draw on different resources of *meaning*—as individuals, and, reaching out from their encounter as people, in their modes of interaction with the material world, the social world, and the discursive and symbolic resources of culture.

STRUCTURE

In terms of the *structure* of their reciprocal participation in the practice of mathematics learning and teaching, clearly they are also quite differently oriented in and to the practice by personal experience, by familiarity with and expertise in relevant skills, the social relationships characteristic of the practice, and the nature and history of the discursive and cultural forms relevant to the practice (which is experienced not only as mathematics learning and teaching, but also, for example, learning or enacting rules of participation in this class or learning setting, behaving civilly, learning or enacting the value of persistence and "getting it right", and many other things).

SITUATEDNESS

In terms of the *situatedness* of practice, clearly student and teacher live in their own bodies, with their own identities, doing emotional work of different kinds in their encounter. They act on different aspects of the material world—for example, the teacher at the whiteboard and the students at their desks. They probably have different views about and responsibilities for the work of social integration in the classroom, and for the exercise of care in the conduct of their activities as part of the practice of mathematics education. And of course, teacher and students bring different backgrounds of *situated* discourses, and different chronologies and experience, to their encounter. One wonders about the extent to which these dimensions are made explicit in the life of most classrooms in schools, colleges and universities, and in workplaces in which education occurs—about how social class and gender and cultural differences are recognised and handled in the caring relationship of the practice of mathematics education.

SYSTEMIC

In terms of the *systemic* character of the participants and their relationship, clearly teacher and students occupy different and reputedly reciprocal roles—ones very ill-captured by the notion of the student as a client receiving services or a consumer exchanging money for goods, though these ways of framing learning and teaching have become increasingly prevalent in the discourses of educational administration and policy in recent decades. Teacher and students are also reciprocally enmeshed in material exchanges of work for grades, for example, as part of larger institutional systems of schooling, educational administration, teacher professional development and educational research and evaluation, with different perspectives on the nature of schooling (at all levels) as an institution and education as the practice schooling is intended to promote and nurture. And both students and teachers find themselves enmeshed in institutional processes of evaluation, assessment, accreditation and regulation as part of the social system they jointly inhabit, with characteristically different, sometimes cooperative and sometimes mutually-resistant perspectives on what it means to be enmeshed (or entrapped) together in these systems.

TEMPORALLY-LOCATED

Clearly, too, in terms of the *temporal location* of practices, teachers and students have characteristically different perspectives on the unfolding drama of education offered and received, through all its episodes, and at the different stages in the lives of each—and the careers of each. It is composed on multiple timescales—the 'period', the unit of work, the term or semester, the year, the stage or level, and so on. Is the teacher just teaching third grade, or this particular subject in the vocational education and training trade certificate course, or this particular subject in the bachelor's degree, or is each teaching a person with their own narrative understanding of the unity of their life and career, and their own personal goals and ideas of the good for humankind? And is the student experiencing *only* Mr Jones the mathematics teacher, or *also* Mr Jones the person, with his own character, background and view of life. Student and teacher also have very different views of how the present class, the present episode of practice can be viewed against the background of history. Is Mr Jones merely old-fashioned, or does he believe that his social constructivist view of mathematics education has roots reaching back as far as, and perhaps beyond John Dewey, and so he teaches in a way some regard as 'progressive' but that he regards as justified because it is necessary to draw on students' experience to make explicit the relationship between the students' knowledge and experiences and the topic now before them? Of course the students also bring a history to the class—a history of success and failures in schooling, of interests inflamed and extinguished, of expectations raised or lowered in a history of attainments in schooling and outside it. Some, of course, experience their greatest educational successes in educational episodes outside the school—in workplaces, social clubs, family life, and the adventures of adolescent peer group activities that raise the ire or eyebrows of adults. And each draws in different ways on the historically-given store of meanings in words, discourses and theories available to them, and each draws on these resources in different ways, for different purposes which may, in the end, converge in something like the practice of mathematics or ideas about the good life or the good society—but which may not converge, and will probably diverge as students go on to live their lives by other lights than the ones that guide their teachers.

Forms of Reasoning, and Reflexivity and Transformation

I will not say more about the differences in the use, or opportunity to use, different forms of reasoning available to students and teachers in the mathematics classroom—though differences there are—nor about the different views of reflexivity or transformation available and accessible to students and teachers in those roles, especially in the context of compulsory schooling. You can read the opportunities and differences off the table, and give them shape and substance in relation to examples of practice you have in your mind. These are important topics, however, and they go to the heart of what it means to teach or to learn, especially in formal education, and they shape quite different views of pedagogy, by which I mean not merely the science or art of teaching, but the *transformative* point and purpose and goods of education, for individuals and for societies.

Reading Versus Measuring Practice

These purposes and goods, in the end, are what *education* is for, and they are the things to which *teaching* may be *one* means. They are things to which mathematics teaching may be one means. They are the things which give a more profound measure of the quality of mathematics education and mathematics teaching than can ever be given than by results of students or classes or schools and colleges on examinations or standardised tests. And they give a more profound reading of practice than can be gained by assessing the 'quality' of teaching against performance measures created by proponents of 'authentic' or 'productive pedagogies', no matter how well-researched, 'scientific' or well-intentioned they may be. We may hope such measures point towards those unmeasurable aspects of quality, but they cannot capture the quality of practice in the more encompassing sense outlined in Figure 3—*nor can they be expected to do so*. Making some assessment of the outcomes of learning and the conduct of teaching may be technically-necessary if one is to have an idea of whether one is achieving one's aims as a student or teacher, but that is pretty much as far as they go. The quality of learning and teaching in the richer sense of participation in the practice of *education* is simply unmeasurable.

One can make a reading of an act, an episode, or a life of learning and teaching, against a framework of features of practice like the one I have offered here, and make one's own judgement—which may disagree with the judgements of others—of the quality of education 'given' or 'received', but such a reading is not a measure or an assessment, it is an elucidation of the way in which the act or episode or life holds up as a consistent, developing effort to realise moral and educational goods in one's own life, in the lives of others with whom one works in education (not only students), and in a society. And it is *one's own* elucidation of the 'facts' of the act or episode or life with which one is presented. The judgement tells as much about the judge as what is judged. This is what makes the ruthless and reckless drive to performance measurement throughout schooling the more appalling—it aims to make judgements about quality of education through technical measures which cannot grasp its materiality, its practicality, its morality, or its actual or likely contribution to the self-development of individuals and the development of the societies in which they live. Indeed, this is what makes the drive to performance measurement *anti-educational* and *anti-intellectual*. It mistakes form for substance, the measurement for the thing measured. At best, it may provide technical assistance; at worst, it subordinates the practice of education to the imperatives of administration (itself no longer a worthwhile practice of public administration conducted by a civil or public service, but a technical tool for policy-makers and states).

Against the objectification of the practitioner and the practice constructed through the instruments of performance measurement, consider the figure of the educator as a moral agent, a person. These are persons bound to model and enact the values and virtues for which they stand, evolving over a career or a lifetime. And they are bound to do so despite this commitment being merely assumed or noted in passing by institutions and the state, and in general being irrelevant to institutional decision making of almost any kind—though what makes a person admirable or genuinely morally worthwhile may be noticed in decisions about promotion or new appointments, and more frequently in interviews than in reading a *curriculum vitae*. Against the performance measures, remember the person, their history, their commitments, their aspirations. And consider the extent to which any of these are essential to the job specification statement, the functions described in association with a role in an organisation. The latter are generally expressed in so-called 'neutral' and 'objective' language, and they have their purpose—assigning duties, enabling incumbents to be held to account. They have institutional administrative purposes that intend discreetly and dispassionately to overlook the personhood of the person to see them only in terms of functionality—capability to function in the role. It is the amplification of this administrative gaze through our institutions, corporations, state and society that empties them of moral content and significance. And this, in turn, endangers practice, as Joseph Dunne so aptly put it in the title of his paper to the (2004) Umeä *Participant Knowledge and Knowing Practice* conference.

At the end of *After Virtue* MacIntyre wrote a wonderful passage comparing the 1980s to the fall of the Roman Empire:

> A crucial turning point in that earlier history occurred when men and women of good will turned aside from the task of shoring up the Roman *imperium* and ceased to identify the continuation of civility and moral community with the maintenance of that *imperium*. What they set themselves to achieve instead—often not fully recognising what they were doing—was the construction of new forms of community within which the moral life could be sustained so that both morality and civility might survive the coming ages of barbarism and darkness. If my account of our moral condition is correct, we ought also to conclude that for some time now we too have reached this turning point. What matters at this stage is the construction of local forms of community within which civility and the intellectual and moral life can be sustained through the new dark ages that are already upon us. And if the tradition of the virtues was able to survive the horrors of the last dark ages, we are not entirely without grounds for hope. This time however the barbarians are not waiting beyond the frontiers; they have already been governing us for quite some time. And it is our lack of consciousness of this that constitutes part of our predicament (p. 263).

What horrors more of our "new dark ages" have been revealed in the twenty-five years or so since he wrote those words—Iraq, September 11, and so many more. And with what inadequate moral resources most of our governments address them—calculations of national interest, profit, and political polling.

MacIntyre spoke of "new forms of community within which the moral life could be sustained so that both morality and civility might survive", and he has famously addressed the possibility that universities might be such places (1988, 1990). We all know that universities, too, are creatures much domesticated by the state, indeed, creatures of the state in many countries. And schools and schooling are even more so creatures of the state or, all too frequently, of other sectional interest groups.

I put it to you that the task of education and for educators remains the same as it has always been—harder now if we forget the resources of our educational traditions and yield ourselves to the imperatives of the state, to keep alive awareness of the enduring tension between education and schooling, or, one might rather say, the contradiction of education versus schooling. This is not just a call to heroic resistance by teachers—most of us state employees or of state-regulated institutions—but a call to teachers to do their best to enact the practice of education, not just schooling, if necessary in the interstices of state-sanctioned curricula. Most classroom educators—not just teachers—have been conducting this resistance for a long time. Indeed, many classroom educators today believe that teaching is what they do 'on the lines' of their day-to-day work, and that education is what they do 'between the lines'. And it is a call on behalf of students and our society—to work in the interests of the self-development of every single student, and the development of our societies as moral and civil communities—to enact *education* in our classes, not just the *teaching* of mathematics or music or science or studies of society or literacy or literature. For us, as educators, teaching these are merely means by which persons may become educated in the knowledge, morality and civility necessary for our societies to emerge from "the new dark ages that are already upon us".

Responding to this call requires conviction and courage, as it always has. And it requires preparedness for the inevitability of disappointment created by circumstances outside the educator's control—for example, by government policies that reduce the resources available for education, or public housing policies that ghettoise poverty with the effect that particular schools must deal with a greater proportion of students in troubled circumstances. Such disappointments may be inevitable, but the practice of education also offers a more-than-compensating sense of exhilaration when the teacher *does* connect with students and others in the interests of their self-development and the common good.

In my view, the task of sustaining and defending the practice of education is urgent, and our circumstances dire. I do not expect that modern states, societies or the mass of public opinion will emerge from this crisis in my lifetime. All the more reason, it seems to me, for us to take more seriously, and embrace more fervently, the ends and goods that our traditions of education have bestowed upon us, to keep them alive in and through education *and teaching*. In other times, educators have taken education into the hedgerows and farmhouses to resist the schooling offered by their states. I take it as part of our *moral* task to keep the aims and goods of education alive for a time when the folly of the current ideology of expertise, efficiency and technical reason in the service of interests behind the machinery of the state, is past. I mean to point to no conspiracy here—though of course I refer to the interests of global capital—but rather to suggest that, despite the contrary intentions of many, we ourselves are all too frequently the servants of those self-same interests, when we extend the technical rationality or effectiveness or efficiency of schooling *at the expense of* education and our students as persons and our societies as civil forms of community.

Our Prime Minister, John Howard, famously remarked last year that public schools in Australia do not teach—or was it have?—values. Teachers in those schools were outraged; many felt deeply betrayed. It seems to me that they *do* have values, and the values of education are among them. Perhaps, seen through the Prime Minister's eyes, these are not values but irrelevancies. I fear, however, that the truth is at once more sinister and more explicable: that he believes the values of public education embrace aims that threaten to inoculate people against domestication and submission to the state and the interests of the powerful, and that valuing public education is against the principle of providing every possible service through a market to generate private profit. These are views, I believe, against which every educator must stand opposed, even those in private education. For they are values which undermine the sustainability of morality, civility and the state itself. If we follow the Prime Minister's logic, we are sawing off the branch we are sitting on, messing our own nest, eating our own children. Dear visitors to Australia for this conference, please believe that our Prime Minister intends to be the friend of education even while his government is its nemesis.

References

Carr, D. (2003). Rival conceptions of practice in education and teaching. *Journal of Philosophy of Education*, *37*(2), 253–266.

Dunne, J. (2004). Endangered practices. Invited address to the conference *Participant Knowledge and Knowing Practice*, Umeä University, Sweden, March 26–27. (Forthcoming in *Pedagogy, Culture and Society*.)

Higgs, J., Titchen, A., & Neville, V. (2001). Professional practice and knowledge. In J. Higgs & A. Titchen (Eds.), *Practice knowledge and expertise in the health professions*. Oxford: Butterworth-Heinemann.

Kemmis, S. (2004). Knowing practice: Searching for saliences. Invited address to the conference *Participant Knowledge and Knowing Practice*, Umeä University, Sweden, March 26–27. (Forthcoming in *Pedagogy, Culture and Society*.)

Kemmis, S., & McTaggart, R. (2000). Participatory action research. In N. Denzin & Y. Lincoln (Eds.), *Handbook of qualitative research* (2nd ed.). Thousand Oaks, California: Sage.

Kemmis, S., & McTaggart, R. (2005). Participatory action research: Communicative action and the public sphere. In N. Denzin & Y. Lincoln (Eds.), *Handbook of qualitative research* (3rd ed.). Thousand Oaks, California: Sage.

MacIntyre, A. (1990). *Three rival versions of moral theory: Encyclopaedia, genealogy and tradition*. London: Duckworth.

MacIntyre, A. (1988). *Whose justice? Which rationality?* London: Duckworth.

MacIntyre, A. (1982). *After virtue*. London: Duckworth.

MacIntyre, A., & Dunne, J. (2002). Alasdair MacIntyre on education. *Journal of Philosophy of Education*, *36*(1), 1–19.

Noddings, N. (2003). Is teaching a practice? *Journal of Philosophy of Education*, *37*(2), 241–251.

Shotter, J. (1996). Living in a Wittgensteinian world: Beyond theory to a poetics of practices. *Journal for the Theory of Social Behaviour*, *26*(3), 293–311.

SYMPOSIA

MATHEMATICS EDUCATION (RESEARCH) AND/AS PERFORMANCE

Tony Brown
Manchester Metropolitan University
<a.m.brown@mmu.ac.uk>

Tony Cotton
Nottingham Trent University
<tony.cotton@ntu.ac.uk>

Tansy Hardy
Sheffield Hallam University
<t.hardy@shu.ac.uk>

Paramjit Oberoi
Derby University

Helen Toft
Catalyst 17
<helen.toft@ntu.ac.uk>

> *Mathematics Education tends to contribute to the regeneration of an inequitable society through undemocratic and exclusive pedagogical practices which portray mathematics and mathematics education as absolute, authoritarian disciplines. (Aims of MES)*

This aim would suggest that one challenge for members of MES is to explore research processes and research which offers a view of Mathematics and Mathematics Education as democratising and inclusive. Such research would develop a view of learning and researching mathematics which is tentative, multi-faceted and participatory. This is the task the organisers of this symposium set themselves.

This symposium draws together four researchers engaged in work exploring the connections between mathematics education, mathematics education research and elements of performance and art. The symposium will bring together research papers and expect participation to allow for the exploration of the outcomes of such research and to develop a methodological approach to this work through participation. The expectation is that the members of the symposium will engage in research work though performance which will be shared with other members of the conference.

THE ART OF MATHEMATICS: BEDDING DOWN FOR A NEW ERA

Professor Tony Brown, *Manchester Metropolitan University*

What analogies might we productively draw between mathematics and art education? How might we see the promotion of aesthetic appreciation as a motivating factor in mathematics? How might we define the relation between mathematical and artistic objects and human subjects? These questions led to more general concerns with how humans relate to mathematics, and, in stepping back from that, to how we might understand the notion of "relation" in this context. Ultimately, it addresses the question of how we might understand the shifting borders defining the space that houses mathematical thinking and learning as we begin a new century where "mathematical" and "pedagogy" become increasingly contentious terms.

WHAT IS IT LIKE TO BE HERE? A METHODOLOGY FOR 'VOICE'

Dr Tony Cotton, *Nottingham Trent University*

This paper will develop a methodology which takes as its starting point the 'voice' of those engaged in they research. It suggests that the exploration of educational settings should be a collaborative activity engaging those who live and work in the settings as well as the researcher. This gives a much deeper understanding of the current context within the setting and offers areas for intervention and action both by all engaged in the research. Using case studies the paper will illustrate the use of photography, drama and other creative research methods as a way to work with 'voice'. Working with Helen Toft the group will be invited to engage with these methods to explore the use of participants 'voice' in a reflection on 'what it is like to be here' in terms of Mathematics Education research on the international stage.

LEARNING AND TEACHING MATHEMATICS: A CONFIDENCE TRICK?

Dr Tansy Hardy, *Sheffield Hallam University*

This paper is both an exploration of theorisations of 'identity' and of what it means to be confident in learning and teaching maths. This involves a discussion of the notion of 'subjectivity' and what this offers to understandings of the experience of many learners and teachers of mathematics. I present these explorations in an experimental form, mixing textual commentary and a patchwork of vignettes from my teaching and research experience in mathematics education. I intend that these are brought together and held in juxtaposition within this paper. This will use 'what is to hand' to create something new. This is intended to evoke connections and parallels that might be concealed by more traditional modes and to offer a more authentic glimpse of how a re-examination of practices and descriptions operates and new meanings are formed for me as a mathematics education researcher. I connect this form of presentation to the term from art and literature bricolage (see for example Levi-Strauss, 1966 and Heriot Watt University webpage, 2005). Bricolage refers to the process of adapting and juxtaposing old and new texts, images, ideas or narratives to produce whole new meanings. There is an opportunistic and perhaps playful process of selection. One will borrow, appropriate from what is to hand and re-present to generate new senses. It offers the possibility of challenging habitual ways of understanding.

The task I outline is to work in such a way that it will shake up some of my and the reader's constructs of what it means to be a 'good learner' or a 'good teacher' of mathematics. Later in the paper I will consider what has been created/generated for me and offer a prompt to consider what has stood out for you through this experiment. I hope to use this to extend my understanding of how some learners become marginalised through their attempts to learn mathematics.

BRITISH ASIAN WOMEN AT WORK: RESEARCH AS PERFORMANCE

Paramjit Oberoi, *Derby University*
Helen Toft, *Catalyst 17*

This paper explores the use of innovative research methodologies developed in school settings to explore the lived experience of British Asian Women in work. The women range in age from school age young women to women with long experience in work. The research process used methods associated with performance to develop written and performed reflections on the life experiences of British Asian women in and out of educational settings. The paper reflects on the process, reports on the outcomes and offers a methodology for research though performance as well as research as performance.

The paper will use this study as an exemplar for the methodology of research as performance and invite participants to engage in a process allowing the group to explore research as performance in terms of mathematics education and its international context. The paper will be predominantly interactive and expect engagement and participation.

SYMPOSIUM 2 PROPOSAL

PAPERS

MATHEMATICS THROUGH/FOR UNDERSTANDING SOCIAL LIFE: PRODUCTIVE PEDAGOGIES MEETS CRITICAL MATHEMATICS

Bill Atweh
Queensland University of Technology
<b.atweh@qut.edu.au>

Derek Bland
Queensland University of Technology
<d.bland@qut.edu.au>

This paper discusses how the involvement of young people in "real" research activities can be an effective pedagogy for learning for mathematics as well other life skills. However, such collaboration with young people presents dilemmas to their teachers. The concepts of productive pedagogy developed by one school reform movement in Australia are used to reflect on the SARUA project that works for students from underrepresented backgrounds in higher education.

In many countries, mathematics enjoys a special role in school curricula and is seen by many teachers and parents as particularly important for the education of their children. In many curriculum documents, mathematics is seen as essential for the economic well-being of the nation based on its contribution to science and technology. Students often grow to believe that studying mathematics is important for their future lives and that it opens the door to better jobs. However, many students fail to see any relevance of the specific content studied. In spite of being seen as highly important, and probably partly because of it, mathematics is the cause of considerable levels of anxiety for many students who struggle to make sense of it and for many school teachers who have to teach it. Further, as many international comparative studies such as TIMSS and PISA have demonstrated, mathematics achievement and participation remain inaccessible to students based on their gender, cultural and ethnic background, and country of origin.

During the past 50 years, there have been many reforms in mathematics education as well as an escalating body of research on its teaching and learning. Atweh (2004) commented that the effect of research and reform programs in changing actual school practice is still open to debate. Perhaps, the limitation of research and reform to affect classroom practice can be attributed to three causes. First, the gaps amongst research, classroom practice and policy—gaps in time (generate knowledge now and apply it later), in personnel (demarcation between academics, bureaucrats and teachers) and in dissemination (academic vs. teacher journals and conferences)—limit the interaction between these three areas in the discipline. Second, some educational reforms may lead to a demoralising and disempowering of teachers (Hargreaves & Evans, 1997) and may be seen by teachers as external demands on them, hence they are resisted (Sprinthall, Reiman, & Thies-Sprinthall, 1996). Third, in a book with the provocative title of *The Predictable Failure of Educational Reform* (Seymore, 1990 cited in Hargreaves, 1994), the author identifies the piecemeal approach that many of these reforms take as contributing to their failure; e.g., separate agendas for reforms for the curriculum, assessment, teacher professional development, school structures and organisations, and so on. The first aim of this paper is to discuss one reform in

Australia that avoids some of the pitfalls of earlier reforms affecting mathematics education pointed to in the above comments. One corner stone of this reform is what is called "the New Basics" including the Productive Pedagogies.

Here we argue that reforms in schools should also include young people themselves who have been referred to as "the missing voice" in educational research (Cook-Sather, 2002, p. 5). In the fast-changing climate of the early twenty-first century, Cook-Sather said, "students must be included among those with the authority to participate both in the critique and in the reform of education" (p. 3). There are a few instances of projects involving students as key participants and researchers in educational reform processes, particularly in the United Kingdom (Cook-Sather, 2002; Fielding, 2001; Kirshner Thomas, 2000). Kirshner and O'Donoghue (2001) noted, "while great advances have been made in theorizing researcher-practitioner partnerships, research collaborations with youth remain under-theorized and under-utilized" (p. 4). Using late modernity theorisation according to writers such as Habermas and Kemmis, and other writers within the action research literature, Bland and Atweh (2004) theorised the concept of young people as researchers. They discussed potential benefits and limitations of such involvement and identified some issues that need to be considered in planning and reflecting on collaboration with young people as researchers, such as voice, i.e., insider/outsider, expert/novice, and the question of empowerment. The second aim of this paper is to briefly discuss one such project where high school students from underrepresented backgrounds in higher education have been involved in action research studies that led to the development of mathematics knowledge in a real world context. Finally, this paper attempts to demonstrate how students' research can parallel the principles of Productive Pedagogies elaborated in the Australian reform noted above.

PRODUCTIVE PEDAGOGIES

One reform movement in the state of Queensland, Australia, called the New Basics, that went on trial in 2000, attempts to provide an integrated approach to public school reform based on a) an examination of directions that education should take to prepare students for an ever changing society, b) our knowledge of effectiveness of teaching methods, and c) associated assessment practices. The three basic components of the reform are illustrated in Figure 1 below.

While not discarding traditional subject areas in the curriculum, New Basics presents new ways of coordinating, focusing and integrating teaching programs in schools. It is a reform that is centred around the teacher as a professional in that it "provides teachers and schools with ways of renewing knowledge of fields in light of dynamic changes and blending of disciplinary knowledge that have occurred since their initial training" (Education Queensland, 2000; p. 37).

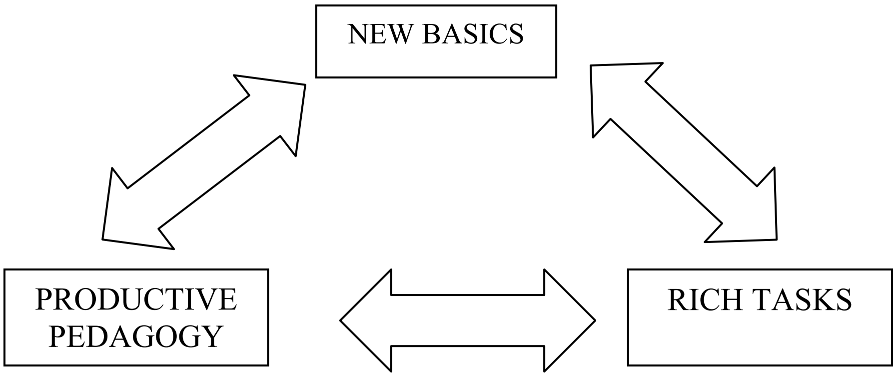


Figure 1. The components of New Basics.

New Basics refers to four basic themes and practices that are seen as essential for students' present and future work and life. They do not represent new topics or content but rather organisers for all content areas studied. The four organisers are: i) *Who am I and where am I going?* Life pathways and social futures; ii) *How do I make sense of and communicate with the world?* Multiliteracies and communications media; iii) *What are my rights and responsibilities in communities, cultures and economies?* Active citizenship; iv) *How do I describe, analyse and shape the world around me?* Environments and technologies.

The Rich Tasks, on the other hand, are interdisciplinary assessment points, covering Years 1–3; 4–6; 7–9, that "legitimate and underscore the New Basics and Productive Pedagogies by making available assessable activities that are intellectually challenging and have real-world value, two characteristics which research identifies as necessary for improved student performance". Typically they are big projects on which students collaborate for several months during the assessment year.

Finally, Productive Pedagogies are classroom principles that teachers can use to critique their teaching methods to improve educational outcomes. Productive pedagogies are critical in nature, empowering students to create their own history and to become agents for democratic, social change (Zyngier, 2003). By moving away from notions of education as preparation for a possibly non-existent world of work, schools can enable students to connect to their own realities. The productive pedagogies concept, as the term implies, is pluralistic and does not propose any single model of classroom practice. There are 20 Productive Pedagogies in the New Basics Framework, grouped under four categories: intellectual quality, supportive classroom environment, connectedness, and recognition of difference.

Intellectual quality: This dimension includes higher-order thinking, deep knowledge, and deep understanding. It includes "substantive conversation", or "talk leading to sustained conversational dialogue between students, and between teachers and students, to create or negotiate understanding of subject matter"

(Education Queensland, 2001, p. 6). There is evidence that high expectations of intellectual quality benefit all students and reduce equity gaps (Education Queensland, 2004). An example of how higher order thinking might be experienced in a mathematics classroom is provided in the "Classroom Reflection Manual" (Education Queensland, 2004) in which Year 2 students grouped and regrouped objects according to criteria they determined themselves. The students had to articulate reasons for their classifications and justify placing some in overlapping sets.

Supportive classroom environment: A supportive classroom environment is an essential component of productive pedagogies, especially for students from educationally disadvantaged backgrounds (Education Queensland, 2004). This includes opportunities for students determining their activities in the lesson. The Classroom Reflection Manual (Education Queensland, 2004) provides a cross-disciplinary example in which Year 8 students discussed what they wanted to learn about themselves and the world. These questions formed the basis of their curriculum for that year with the students involved in determining curriculum content and activities. Again, high expectations of students play an important role in establishing a supportive social environment, in which it is possible to take risks and attempt challenging work. Academic engagement in such an environment can be assessed through student self-regulation, enthusiasm and contributing to group activities.

Connectedness: The concept of connectedness includes linking new knowledge with students' background knowledge as well as connectedness to the world outside the classroom through a focus on identifying and solving intellectual and/or real-world problems (Education Queensland, 2004) thus allowing learning to occur more easily and meaningfully (Moulds, 1998). Creating connections may present a particular problem for mathematics teachers where the applications can be complex to the level of mathematics available, but integrated, thematic, and interdisciplinary approaches can provide creative possibilities to enhance learning and transcend subject matter bounds (Lonning, DeFranco, & Weinland, 1998).

Recognition of difference: The valuing of non-dominant cultural knowledges is a key aspect of recognition of difference which would include deliberate attempts to increase the participation of the diversity of students. This enhances the building of a sense of community and identity and encourages active citizenship within the classroom (Education Queensland, 2004) and avoids the disengagement of those from otherwise unvalued backgrounds and cultures. In an example of classroom practice provided in the Classroom Reflection Manual (Education Queensland, 2004), year 7 students gathered comparative statistics on global issues relating to poverty. This study led to the students creating a library display and making recommendations for the school to become involved with human rights agencies.

THE SARUA PROJECT

The Student Action Research for University Access (SARUA) project (Atweh, 2003) consists of groups of senior high school students, working in collaboration with their teachers and staff from the university to a) conduct research activities on the barriers to higher education for students from social backgrounds underrepresented at universities, and b) plan, implement, and evaluate school-based projects to overcome the problems identified. The project was conceived as an equity and access project rather than as a pedagogical project to develop school-subject learning. SARUA is committed to promoting "students' knowledge and interest about university at the same time as they are developing some of the skills required at tertiary level" (Atweh & Dornan, 1999, p. 7). Examples of student-produced research through the SARUA project for their high schools include:

- An inquiry into the low tertiary entrance rate of students from the school, leading to the development of a homework centre, a tertiary shadowing program, a school-university buddy system and positive publicity about the school through local media and school publications (Bajar, Brennan, Deen, James, Nguyen, Nguyen, Owens, Peace, Rice, Rilatt, Strachan, & Tran, 1993)
- An investigation into tertiary aspirations of years 8–12 students leading to the implementation of school-based projects on self-esteem and year 10 assistance in subject choice (Bevan, Fawke, Gladman, Tuigamala, & Fidow, 1996).
- An inquiry into factors affecting the participation of Aboriginal and Torres Strait Islander females in secondary and tertiary education, which noted a strong desire but lack of role models and information (Allberry, Borey, Morris, Cobb, & Jarrett, 1996).

In a typical year, students are invited to participate by their teachers based on a combination of criteria including their motivation to participate, their social and ethnic background, and academic achievement. Students receive two days of training on social issues, project management and introduction to research methods. The training session concludes with plans for projects for the rest of the year. Students and their teachers work on a weekly basis on their projects at the school. At times, this may be possible during the school timetable –mostly, however, students work on the project in their own free time. Close to the end of the year, they return to the university for at least two days to analyse their data and write their reports. All through the year, staff from the university provide assistance, advice and specialised training as requested by the school.

STUDENTS' RESEARCH AS A PRODUCTIVE PEDAGOGY

While the SARUA project described above is not conceived as an activity to teach mathematics directly, students in the project have utilised a significant amount of mathematical content such as percentages, decimals, fractions, and graphs. In this project, these were used implicitly in meaningful real world contexts. This is in keeping with critical literacy and critical mathematics. The mathematics used in writing up the student reports was arguably already known to the majority of the students. However, an argument can be raised that similar contexts could be used with lower age students that may be more useful in developing these concepts and skills. Using contexts such as real research to develop the mathematics not only provides a way of giving meaning to these concepts, but also allows for the development of higher order thinking strategies not possible while using meaningless numbers. For example, in making decisions on the most appropriate graphs to represent the data, students engaged in elaborated conversations with each other about the advantages and limitations of each type of graph in conveying the specific message that they want to communicate. Hence, through students' involvement in authentic research activities, mathematics can be developed *through* attempts to understand the social reality of the students.

Further, developing the mathematics within this real context allowed students to reflect on the real world and reasons for their disadvantage. Mathematics was shown to be a powerful tool to understand their social reality and to change it (Mellin Olsen, 1987). Very rarely do students have the opportunity to develop meaningful data from their classroom activities in mathematics, and even less frequently would they act on the knowledge to improve aspects of their world. Hence through students' involvement in authentic research activities, mathematics can be developed for understanding their social reality and empowering them to act on it. This development of mathematics for and through understanding the social reality of the students establishes essential connections with the world outside mathematics. It also connects mathematics with literacies developed in other school subjects.

Mathematics teaching has been critiqued for being detached from the interest of the learner and society (Frankenstein, 1994). Often, the applications of mathematics to real world problems that are used are taken from the natural world and, at times, from business. Often, these are taken as non-problematic, perhaps reflecting the widespread belief that mathematics is value free (Bishop, Seah, & Chin, 2003). Frankenstein (1994) proposes that "[c]riticalmathematics literacy … involves the ability to ask basic statistical questions in order to deepen one's appreciation of particular issues, and the ability to present data to change people's perceptions of those issues. A critical understanding … prompts one to question 'taken-for-granted' assumptions about how a society is structured and enabling us to act from a more informed position on societal structures and processes" (p. 23). Kellermeier asserts that "a criticalmathematics curriculum would then weave a discussion of social issues into the learning of functional and mechanical mathematics thus preparing students to better participate as global citizens" (1996, p. 9).

Atweh (2003) point out how the students involved in the SARUA project demonstrated considerable "research sense" and a critical appreciation of the research process itself. This was clearly illustrated in the research reports they produced. For example, they were able to identify the strengths of using questionnaires for data collection in order to "question a large anonymous audience, within a minimal amount of time" (Borowicz et al., 1993, p. 2). They also identified that the attitude of the data collector towards the respondents was a major factor in obtaining valid information. They concluded "one must commit oneself to the task, taking a professional outlook and reflecting this image toward the respondents" (p. 3). Similarly, they were not afraid to go beyond the data and raise hypotheses about its causes. For example, in noting that 71% of the young men and 29% of the young women surveyed have university aspirations in spite of the fact that girls indicated that they enjoy school more than boys, the young researchers were able to offer the explanation that: "Possibly this may be due to a lack of female role models who have completed university other than teachers, as well as early motherhood which is common in [this suburb], rather than women concentrating on careers" (p. 21).

Here, we have demonstrated that when students are involved in "real" research activities not only do they have a chance to develop learning of high intellectual quality, but that learning is necessarily interdisciplinary and connected to their real world concerns. However, from our involvement in SARUA we also have learnt the importance and, we should add, the dilemmas, of providing students with support and recognise issues related to their differences.

Atweh (2003) argues how through students' engagement in authentic research activities, they are developing collaborative learning skills in a supportive and trusting environment. Working with students in this mode is not without its problems (Atweh, Cobb, & Dornan, 1997) and requires continual self critique and reflection. It challenges the normal demarcations of power between teachers and students. It also opens the door for challenges and new opportunities to work in productive ways. Successful collaboration between students and researchers demonstrates a parity of esteem (Grundy, 1998), whereby the participants work to develop a reciprocal sense of trust and respect, and a common commitment towards the content of research shared by all parties involved – students, teachers, and university staff.

All the university participants approached researching with school students with a great sense of ethical responsibility. As much as possible, we dealt with the students as equal partners and dealt with them with the same respect that we did each other. We respected and attempted to promote students' freedom in their decision-making. However, the students were also aware of the "duty of care" responsibilities. We were in a more privileged position as we had more experience in the planning and conduct of research as well as our knowledge of theoretical issues. The boundary lines between the authority that we had and the freedom that we advocated for the students were sometimes confusing to them as well as to us.

At times, students and their schoolteachers were hesitant to proceed on a decision without checking if it was what university staff wanted them to do. However, these requests for "permission" became less frequent as the project progressed each year.

The university researchers learnt two means to deal with these confusions about our roles. First was the process of open negotiation with the students and teachers about each partner's roles. This negotiation started when the university staff were explaining the project to the schools, the volunteering teachers and the students themselves. The advertising material sent to the schools about the project specifically outlined lists of responsibilities of the various partners. Further, this negotiation was continuous throughout the life of the project. Whenever possible, decisions that we made were explained to the students. Likewise, the students were invited to evaluate the sessions and the processes of the project.

Further, at various times in the deliberations with students, it became clear that the students need some assistance in considering the options of what is possible before they can decide on an appropriate action. For example, in helping them decide what type of data collection tool to use, we discussed with them various data collection methods with their advantages and disadvantages. Naturally, in choosing the list of methods discussed, we selected methods based on our assessment of what was appropriate and what was achievable by the students. From the discussed options, the students had to make their own decisions on which instrument they used.

At the initial stages of the project, students worked in homogenous groups based on gender and cultural background. This was done in response to demands from certain schools themselves. Atweh, Cobb and Dornan (1997) identified several benefits of organizing the groups this way. These included,

- an increase in the participation of students from certain backgrounds who were hesitant to join the project as a minority group working within mixed groups;
- an opportunity to consider aspects of their culture that may not have arisen in culturally mixed groups;
- an opportunity to address issues of race and prejudice in their discussion and research;
- the avoidance of the possible tension that can arise between students from both genders about equal participation; and
- the development of leadership potential within the various groups.

However, the grouping of students in homogeneous cultural backgrounds and gender was not without its dilemmas. One of the indirect aims of the project design was for the participants to become aware of social disadvantage and oppression as widespread phenomena that affect different people according to their gender, race, socio-economic or other source of disadvantage. The university team believed that such awareness is best achieved in groups of students from different backgrounds working collaboratively where they have a chance to develop mutual respect and understanding. On the other hand, the project was also founded on the belief that research into the factors affecting underrepresentation should be contextualized in terms of the various factors of disadvantage. The experience of disadvantage varies in different communities. Such contextualization could be best achieved in homogeneous gender and cultural groups. This presented a dilemma for the project.

To satisfy both these conflicting considerations, the project in 1996 was planned so that students with similar backgrounds worked in homogenous groups yet shared their plans and results with other groups. For example, during the training workshop at the university in 1996, eight school groups were represented: four consisted of Aboriginal students and Torres Strait Islander students, one was an all Pacific Islander group and three were mixed gender students from low socio-economic backgrounds (including non-English speaking backgrounds, Aboriginal students and Pacific Islander students). During the training sessions the students participated in joint sessions and worked in their school groups. This meant that there were regular opportunities for sharing the issues discussed in the small groups with the whole project. During the year, attempts were made to issue a regular newsletter informing the groups of the activities at other schools. These arrangements gave all students a sense of belonging to a local group while, at the same time, functioning within a larger project that included students from diverse backgrounds.

The second dilemma encountered was the match between the schoolteachers' backgrounds and those of the students. The selection of the school liaison people is a crucial component in the success of such projects with students (Atweh, Christensen, & Dornan, 1998). The university team believed that the deeper the understanding the liaison teacher had of the culture of the student the more successful the project would be in achieving its aims. Arguably, this was important with respect to both gender and cultural background. Not only would a person from the "inside" be more able to understand the issues faced by the students, but they would also be able to provide a better role model to the students. This has not always been possible. In many cases there were no Aboriginal, Torres Strait Islander or Pacific Islander teachers in the respective schools. Further, in some cases where there were teachers within the school from the targeted backgrounds they were already overburdened by their heavy involvement in a variety of other school activities and projects.

REFERENCES

Allberry, M., Borey, K., Morris, J., Cobb, A., & Jarrett, L. (1996). *Bayside young women's project* (A report of the SARUA project for Cleveland District State High School). Brisbane: Queensland University of Technology.

Atweh, B. (2003). On PAR with young people: Learnings from the SAURA project. *Educational Action Research, 11*(1)*,* 23–40.

Atweh, B. (2004). Understanding for changing and changing for understanding: Praxis between practice and theory through action research in mathematics education. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education* (pp. 187–205). Boston: Kluwer Academic Publishers.

Atweh, B., Christensen, C., & Dornan, L. (1998). Students as action researchers: Partnerships for social justice. In B. Atweh, S. Kemmis & P. Weeks (Eds.), *Action research in practice: Partnerships for social justice in education* (pp. 114–138). London: Routledge.

Atweh, B, Cobb, A., & Dornan, D. (1997). Students as partners: Potentials and dilemmas in action research. Paper presented at the annual conference of the Australian Association of Research in Education. Brisbane: Australian Association of Research in Education.

Atweh, B., & Dornan, L. (1999). *SARUA: Training and resource manual* (3rd ed.). Brisbane: Queensland University of Technology.

Bajar, R., Brennan, K., Deen, A., James, S., Nguyen, J., Nguyen, T., et al. (1993). *Bridging the gap: A university access project* (A report of the SARUA project for Inala State High School). Brisbane: Queensland University of Technology.

Bevan, T., Fawke, J., Gladman, R., Tuigamala, J., & Fidow, C. (1996). *The next step: Planning for university* (Report of the SARUA Project for Deception Bay State High School). Brisbane: Queensland University of Technology.

Bishop, A.; Seah, W., & Chin, C. (2003). Values in mathematics teaching: The hidden persuaders? In A. Bishop et al., *The second handbook in mathematics education* (pp. 717–765). Netherlands: Kluwer.

Bland, D., & Atweh, B. (2004). A critical approach to collaborating with students as researchers. In E. McWilliam, S. Dandy & J. Knight (Eds.), *Performing educational research: Theories, methods and practices* (pp. 331–344). Brisbane: Post Pressed.

Borowicz, B., Davis, N., James, S., Le, T., Nguyen, T., Owens, K., et al. (1993). *Finding out* (Report for the SARUA Project). Brisbane: Queensland University of Technology.

Cook-Sather, A. (2002). Authorizing students' perspectives: Toward trust, dialogue, and change in education. *Educational Researcher, 31*(4), 3–14.

Education Queensland. (2000). *The New Basics Project* [website]. State of Queensland (Department of Education). Retrieved 13 February, 2004, from http://education.qld.gov.au/corporate/newbasics/

Education Queensland. (2001). *The Queensland school reform longitudinal study* (Vol. iii, Teachers' Summary). Brisbane: The State of Queensland (Department of Education).

Education Queensland. (2004). *Productive pedagogies: Classroom reflection manual* [website]. State of Queensland (Department of Education). Retrieved 13 February, 2004, from http://education.qld.gov.au/public-media/reports/curriculum-framework/productive-pedagogies/html

Fielding, M. (2001). Students as radical agents of change. *Journal of Educational Change, 2*(2), 123–141.

Frankenstein, M. (1994). Understanding the politics of mathematical knowledge as an integral part of becoming critically numerate. *Radical Statistics*. V56.

Grundy, S. (1998). Research partnerships: Principles and possibilities. In B. Atweh, S. Kemmis & P. Weeks (Eds.), *Action research in practice: Partnerships for social justice in education* (pp. 37–46). London: Routledge.

Hargreaves, A. (1994). *Changing teachers, changing times: Teachers' work and culture in the postmodern age.* London: Cassell.

Hargreaves, A., & Evans, R. (Eds.) (1997). *Beyond educational reform.* Buckingham, UK: Open University Press.

Kellermeier, J. (1996). Statistics in social context: Using women in development issues for a criticalmathematics literacy curriculum. *Hunger TeachNet*, *6*(4), 9–11.

Kirshner, B., & O'Donoghue, J. (2001, April). *Youth-adult research collaborations: Bringing youth voice and development to the research process.* Paper presented at the Annual Meeting of the American Educational Research Association, Seattle.

Lonning, R., DeFranco, T., & Weinland (1998). Development of theme-based, interdisciplinary, integrated curriculum: A theoretical model. *School Science and Mathematics, 98*(6), 312–318.

Mellin-Olsen, S. (1987). *The politics of mathematics education*. Dordrecht: D. Reidel Publishing Company.

Moulds, P. (1998). The teaching and learning of complex reasoning: A perspective from the literature. In B. Baker, M. Tucker & C. Ng (Eds.), *Education's new timespace: Visions from the present* (pp. 49–55). Flaxton: Post Pressed.

Sprinthall, N., Reiman, A., & Thies-Sprinthall, L. (1996). Teacher professional development. In J. Sikula (Ed.), *Handbook of research on teacher education* (pp. 666–703). N.Y.: Macmillan.

Thomas, L. (2000). Staffordshire universities' maths summer school (SUMSS) evaluation strategy. *Widening Particpation and Lifelong Learning, 2*(3), 41–44.

Zyngier, D. (2003). Connectedness: Isn't it time that education came out from behind the classroom door and rediscovered social justice. *Social Alternatives, 22*(3), 41–49.

"OUTSIDERS" IN A COLLABORATIVE LEARNING CLASSROOM

Mary Barnes
University of Melbourne
<marysb@iprimus.com.au>

During a study of collaborative learning in senior mathematics classrooms, it was observed that some students attempted to participate in small-group discussion, but were frequently interrupted or ignored by other group members. They were treated at times as though they did not have the same right as the others to contribute to the discussion; as though they were not full members of the group. I present case studies of two such "Outsiders", analysing their patterns of participation and non-participation. These are discussed in terms of the constructs of marginal and peripheral participation developed by Etienne Wenger (1998). I conclude by presenting some suggestions for ways of developing greater inclusiveness within groups engaged in collaborative activity.

Much recent research in mathematics education is based on sociocultural theory (see, e.g., Lerman, 2001) asserting that learning mathematics is inherently a social and communicative activity involving the internalisation of processes developed in interaction with others, mediated by signs and cultural tools. From this standpoint, teaching mathematics involves establishing a community of practice (Wenger, 1998) in which "ways of thinking, modes of inquiry, communicative conventions, values and beliefs characteristic of the wider community of mathematicians can be progressively enacted and appropriated" (Goos, Renshaw, & Galbraith, 1997, p. 1).

These ideas are supported by mathematics education reform documents that stress the importance of fostering communication skills and encouraging mathematical dialogue (e.g., Australian Association of Mathematics Teachers, 2002). Some teachers seeking to implement these ideas adopt a collaborative learning approach, in which students work in small groups on challenging, unfamiliar tasks. The aim is to construct new concepts by recalling prior knowledge and combining and applying it in new ways. Later, in whole-class discussions following the group work, students explain solutions, ask questions, and share insights, and attempt to reach a consensus.

Cohen (1994) identified conditions for effective collaboration. They include reciprocal interdependence—it must be necessary for everyone to contribute for the group to achieve its goal. Cohen describes a good group task as challenging and rewarding, requiring a variety of skills and procedures, having more than one answer or solution pathway, and unable to be completed more efficiently by a single person. Given reciprocal interdependence and a good group task, learning gains depended on the amount of task-related interaction. Students who participated less, learned less.

Recent research on collaborative learning has studied the interactions within groups, focussing mainly on cognitive and metacognitive aspects (e.g., Forster & Taylor, 1999; Goos, Galbraith, & Renshaw, 2002; Williams, 2000). I believe, however, that more attention needs to be given to social aspects of interactions, because poor social relationships and communication within a group can limit the range of approaches considered, or result in a group failing to engage fully with the task. This was a major motivation for my study.

THE STUDY

The research reported here is part of a larger study of student-student interaction in coeducational classrooms in which experienced teachers were implementing collaborative learning methods (Barnes, 2003). In a multi-site case study, three senior classes were chosen to incorporate as much variety as possible. They came from both city and country, and included government and independent schools, male and female teachers, small and large classes, and varied ethnic and social class backgrounds.

To facilitate a detailed study of student-student interactions during collaborative learning, lessons were videotaped, making it possible to return to the video as often as necessary to check interpretations. During small-group discussions the camera focused on one group and a desk microphone captured their speech. Additional data included interviews with teachers and selected students, field notes, worksheets, and student written work. Each class was observed for two periods of about three weeks, with a gap of a few weeks in between to allow for reflection and preliminary analysis.

POSITIONING THEORY

Positioning theory provided a theoretical framework for the analysis. Harré and his colleagues (e.g., Davies & Harré, 1990; Harré & van Langenhove, 1999) argue that in conversational interactions, people can be thought of as presenting themselves and others as actors in a drama, with different parts or 'positions' assigned to the various participants. Positions are not fixed, but fluid, and may change from moment to moment. People may actively try to adopt a position, or it may be assigned to them by others. If a position is assigned, they may acquiesce, contest the assignment or try to subvert it. Being positioned in a particular way carries obligations or expectations about how to behave, constraints on what may be meaningfully said or done, and rights, such as the right to be heard. Thus positioning theory can help to illuminate issues of power.

Linehan and McCarthy (2000, p. 442) claim that "both students and teachers have a degree of agency in how they position themselves in interactions but this agency is interlaced with the expectations and history of the community" In a mathematics class using a collaborative learning approach, the expectations and history will include behavioural norms for small-group work that the teacher has negotiated with the class, such as a duty to listen attentively to what other group members have to say, and an obligation to justify any assertions made.

Analysis of the video data was undertaken in stages, beginning with an in-depth study of a single lesson. I noted who introduced new ideas; how others responded; who controlled the discussion by initiating a topic or deciding when to move on;

who helped to keep the group on-task; and who attempted to distract them from it. This revealed that the adoption of an idea had less to do with its usefulness or correctness than with the status of the person who proposed or supported it and their positioning within the group. Students who took up positions as 'Manager' or 'Expert' exerted a disproportionate influence on the discussion. Others regularly positioned themselves as 'Entertainer' by attempting to amuse or distract the group. A few were positioned at times as 'Outsider' and their contributions ignored.

The subsequent stages of the analysis concentrated on identifying and describing the range of positions occupied by students during small-group discussions. After a scan of the entire database, eight additional lessons were selected for detailed study, because they gave evidence either of positions not found so far, or of key incidents which could provide additional insight into positions already described. This resulted in a list of positions, with a description of empirically-observed behaviours for each. These descriptions were then applied to all remaining lessons, and the positioning of each student in the focus group identified. This was done directly from the video record, so that body language, facial expressions and other contextual cues could be taken into account, as well as what was said. No new positions were identified at this stage. Given the large cultural and social differences between the three classes, this suggests that the list was fairly comprehensive, although it is likely that so-far undescribed positions might emerge in unusual situations. Validation procedures for the analysis included comparison with an independently-generated categorisation of student behaviour, and discussion with colleagues about points of uncertainty.

The positions identified included Manager, Expert, Spokesperson, Facilitator, Critic, Collaborator, Helper, In Need of Help, Entertainer, Audience, Networker and Outsider. These are described in detail elsewhere (Barnes, 2003, 2004). This paper focuses on the position of *Outsider*.

STUDENTS POSITIONED AS OUTSIDERS

Students were described as being positioned as *Outsider* at times when they did not fully participate in the activities of their group. From my observation, this could happen in one of two ways. Students *assumed* a position as Outsider when they made little attempt to join in group activities or discussions, and gave no sign of attending to what others in the group were saying or doing. Often a student would take up this position for a short time, and then begin again to pay attention and participate in the activities—sometimes in response to a comment or question from someone else in the group. More rarely, a student took up this position on a regular basis. Some students, on the other hand, were *assigned* the position of Outsider by other group members. These students made attempts to participate and contribute to the discussion, but frequently either an attempt to speak was interrupted, or what they said was ignored by the rest of the group. It seemed as though their right to join in the work of the group was not fully recognised, and their right to be heard was not acknowledged.

Goos, Renshaw and Galbraith (1997) discussed students who "resisted, rejected or subverted" (p. 5) attempts to include them in a classroom community of inquiry. That is, these students *assumed* the position of Outsider. While this positioning appeared to be largely their own choice, the authors noted the influence of the actions of other students and the teacher. This paper deals with students *assigned* to the Outsider position by others in their group, and the effect of this on the learning outcomes for those students and the groups they were in. I present two case studies to illustrate this.

CASE 1: SELENA

Selena was a newcomer to her school, having arrived only a few weeks before the research began. Before moving with her family to Australia, she had lived in Hong Kong, where she had her primary schooling. She had then spent three and a half years at a secondary school in another state before coming to this school. She had not been at the school long enough to have made any close friends in the mathematics class.

Selena was in the research focus group for four lessons and was interviewed three times. Although the groups contained only three or four students, Selena found it difficult to get the other students to attend to what she had to contribute to their discussions. When she tried to speak, she was often interrupted—other students talked over her. She often had to make many attempts before anyone would listen to her, and when she did manage to complete what she wanted to say, her suggestions were often ignored. On many occasions, ideas initially put forward by Selena were accepted only after another member of the group made the same suggestion.

In one lesson, Vic, Zoe and Selena were trying to make sense of a question that involved folding a sheet of paper to make an open box:

Vic: You // can't fold /
Selena: // No you just /
Zoë: / But you can. [responding to Vic, ignoring Selena]
(For an explanation of the symbols used in transcripts, see Note 1 at the end of the paper.)

No-one ever heard what Selena was about to suggest. She did not complete what she started to say because Zoe cut her off. A single instance like this may not seem very significant, but when such events are repeated again and again they convey a clear message that Selena's input was of less value than that of others in the group.

On another occasion, Selena, Mike and Jacqui had agreed on an (incorrect) answer. Mike then began an off-task conversation with a friend in a nearby group about a favourite television program. Meanwhile Selena reflected on their answer, realised that something was wrong, and said "But then the graph would not be right". No-one paid any attention to this. Mike continued to talk about television until Jacqui called the group back to work. They then moved on to the next part of the task without further discussion of the error.

Another time, Sally, Charles and Selena were working on a problem that required them to find where a cubic graph crossed the *x*-axis. Sally took the lead in the discussion, and tended to be dismissive of Selena's ideas. For example:

Selena:	Can you do this? [shows Sally what she has written]
Sally:	Let me see. Expand it, you mean?
Selena:	Mm, yeah.
Sally:	I dunno. [dismissively] You just do it, and see, if it works.

A little later, Selena decided that it would *not* work.

Selena:	There's no point in expanding this thing. [Begins erasing what she has written.]

Neither Charles not Sally paid any attention to this, or made any reply. They went on working independently until, about two minutes later, Sally announced "Oh that's pointless, multiplying it out." They then began to discuss alternative approaches.

Selena often spoke hesitantly, sometimes pausing mid-sentence. For example, when she wanted to suggest that the solution of a problem might involve limits, she said "S-so like the, lim-it". That is, she hesitated and stammered slightly on "so", paused for a moment after "the" and then hesitated again on "limit", separating the two syllables. Occasionally she used gestures instead of words to complete a sentence. For example, when it dawned on her that her group had misunderstood a problem they were working on, and that it was about volume rather than surface area, she said, "Oh:h. Oh, so we were just thinking that …" and gestured to indicate a flat surface. At other times, she expressed suggestions in the form of questions, as when she said "Do we do a derivative in that?" These ways of expressing herself are less forceful than the ways in which many other students spoke, and may have given the impression that she was less confident.

Another of aspect of Selena's speech may have made it easier for others to ignore or interrupt her: If someone else began to speak while Selena was speaking, she nearly always broke off at once, yielding the floor to the other student (as happened in the first transcript above with Vic and Zoe). She would then wait for a suitable pause before speaking again. Selena's hesitant manner meant that her utterances carried less conviction and authority than those of other students. At the same time, by pausing and breaking off, she made it easier for them to interrupt her. Thus her speech patterns contributed to her Outsider status.

Another relevant factor was that, because she was new, she had not yet established an academic reputation. Other students in the class were unaware of her mathematical capabilities, but they did know that in her previous school she had not learned about certain topics which they had studied earlier in the year. It is likely that many of them generalised from this and placed a low value on anything she had to contribute.

Although Selena was positioned as an Outsider during every lesson in which I observed her, she did not occupy this position all the time. She clearly wanted to be accepted by other students, and made obvious efforts to fit in with whatever group she was assigned to, engaging eagerly and thoughtfully with all their activities. For example, she complied willingly and capably with requests to carry out routine calculations, thus acquiescing in being positioned as Helper. Whenever she saw an opportunity, she attempted to take up a position as Collaborator, often joining in the group's activity by speaking in chorus with another student or by completing their utterances. She also frequently tried to take up a position as Critic by pointing out errors, suggesting a better method, or asking for clarification, but these attempts were often contested. Requests to others to explain what they were doing were interpreted as indicating a lack of understanding on Selena's part rather than a lack of clarity or a need for justification by the others, and she was positioned as In Need of Help.

I noticed, however, that Selena was able to increase her participation gradually as time progressed. As students got to know her better, they accepted her more readily. The teacher assisted in this by drawing attention to and praising good ideas or good solutions that Selena produced. The first time she was asked to present her group's work to the class, another member of the group immediately volunteered to do it instead (possibly uncertain how well she would be able to explain). The teacher replied firmly "No, Selena's fine." Selena then gave a very clear and full presentation of her group's solution. When one boy began to fool around while she was speaking, the boy who had volunteered to present in her place silenced him. When she finished, this boy smiled and gestured approval. Thus the teacher's interventions gradually increased recognition by the class that Selena had something of value to contribute.

It is worth noting that Selena was the only student of Asian background in the class, although the school did have a number of other Asian students. I wondered if Selena's ethnicity might be contributing to her positioning as an Outsider, and looked for indications of racial prejudice. After reviewing both the lesson videotapes and interviews with Selena and others, I could find no evidence of this. Indeed, Selena's report to the class was given a rather better hearing than reports by other girls. I concluded that the problem was her newcomer status rather than her 'race'.

CASE 2: CHARLES

Charles, in the same class as Selena, was also frequently positioned as an Outsider, but presents a contrast in personality and behaviour. Charles was in the research focus group on two occasions. He had not been selected as a key informant at the start of the research, so was never interviewed. In part, this was because I realised how shy and inarticulate he was, and doubted whether interviewing him would generate useful data. It was only after data collection had been completed that his positioning as Outsider was identified as of particular interest.

Awkward, shy and diffident in manner, Charles appeared to be a 'loner' with no close friends in the class. Although he regularly sat with two other boys, Robert and John, he was not close friend of either. I did not ask students their opinions of their classmates, but Robert volunteered that Charles "knows what he's talking about":

> … I don't really like, I don't um I don't really like Charles as a person, but he's very very intelligent, he's very *very* intelligent, and that just comes out when he's working because he like sees obvious things and stuff, and … yeah, him and John are very very smart.

Comments made earlier by the teacher supported what Robert said about Charles' ability. She described him as "very bright, a critic" but added that he had poor communication and social skills. His reasoning and problem-solving capabilities may not have been obvious to other students in the class, however. He had a tendency to stammer, and sometimes expressed himself incoherently when answering questions in class or reporting on behalf of a group. He tended to omit steps in a solution or fail to explain in full what he was doing. Evidently, as Robert suggested, it was all obvious to him, and explanations were unnecessary. In addition, Charles' inadequate written expression tended to lower his marks in assessment tasks, so he may not have been perceived by most other students to be 'good at maths'.

Some students responded negatively to Charles. For example, he arrived late to class one day, and was told to join a group (Zoe, Vic and Selena) that had already begun work. When Zoe heard that Charles was to be in their group, she muttered "Beautiful!" in an ironic tone of voice. A few seconds later, when Charles joined them, no-one greeted him or acknowledged his arrival by any sign that was visible to the camera or audible by the microphone.

Observing Charles' participation in group activities, I noted that he spent much of the time sitting in silence, staring at the table in front of him. He appeared to think things through carefully before speaking aloud. While he often proposed useful ideas, he tended to express them clumsily, and other students did not understand what he was saying. Like Selena, he was frequently interrupted or ignored, but unlike her, he could be very persistent in putting forward an idea that he thought was important. Even after being interrupted, he still persisted in trying to make his point.

For example, when working with Zoe, Vic and Selena, Charles had an idea about to how to tackle the problem they were working on, but it took him several turns before he was able to complete what he wanted to say:

Zoë:	We need a formula that ends up like, you know that … [pauses to scratch her head]
Chas:	Well, if we say /
Vic:	/It's like Year Twelve maths /
Zoë:	/I know /
Chas:	/if we say that's X, well then we'll work out, // a formula

Zoë: //X is this
Chas: find out what that equals. [Points to the diagram]
Zoë: Not a bad idea. All right.

In spite of having interrupted him twice, Zoe did eventually accept his suggestion.

Later, Charles three times proposed graphing the function they were investigating and using the graph to find the maximum value. Twice Zoe and Vic dismissed or ignored this suggestion, but on the third occasion Vic partially grasped what he was trying to say, and he and the rest of the group then adopted the idea with enthusiasm.

Charles was not rejected by everyone in the class, however. Robert and his friend John appreciated his mathematical insight, as did at least one other student, Sally. When Charles was in the same group as Sally, she frequently positioned him as an Expert and asked for his opinion on what she was doing. When Charles made a rather cryptic observation "It's zero" without saying what "it" referred to, Sally asked him to explain. Later, she explicitly asked "Hey, Charles, what do you reckon about this?" At other times, she glanced toward him when speaking to see how he would react, or listened to what he had to say and indicated her support. Sally's reaction to Charles contrasts strongly with the way she treated Selena in the same lesson (see above).

Unlike Selena, who was very aware of how other people responded to her, and made an effort to adapt to their ways of working, Charles appeared to be uninterested in interacting with other people. When he spoke, he did not make eye contact with the people he was speaking to. He did, however, appear to be wary of Vic, the dominant male in one group. On one occasion, after making a suggestion, I saw him glance sideways in Vic's direction, as though anxious about how he would react.

Charles seemed to be more interested in doing mathematics than in building relationships with other students. He became engaged in a task when he saw it as a challenge, but his interest was limited to solving the problem as he understood it. When he thought he had found an answer, he lost interest and disengaged again from the discussion. He was not very concerned about explaining or justifying the answer, and he showed little interest in looking for alternative ways of solving the problem or ways of extending the task beyond what had been explicitly asked.

DISCUSSION

The work of Cohen and her colleagues (Cohen, 1994; Cohen, & Lotan, 1997) casts light on possible causes of unequal participation in group activities, such as that observed in the two cases described. Cohen describes how status hierarchies can develop within collaborating groups, resulting in some students being more active and influential than others. Status may depend on the student's perceived expertise in important subjects (academic status), or their popularity and social standing among classmates (peer status). Academic and peer status have been found to be closely related, and in combination create expectations for competence on classroom tasks. In some contexts, factors like gender, ethnicity and social class also create expectations of competence and contribute to the status hierarchy.

As a newcomer, Selena was an unknown quantity. She had not yet established an academic status, because she had not undertaken any tests or other assessments. All the other students knew was that she had not studied some topics that they had done. She had also not been in the school long enough to acquire a social status among her peers. Thus she may have been assigned low status by default. Charles, on the other hand, had low peer status, demonstrated by his lack of popularity among the class. His academic status was variable—a few students, like Robert and Sally, recognised his mathematical ability, but others clearly did not. Thus expectations based on status help to explain how Selena and Charles came to be positioned as Outsiders.

The work of Wenger (1998) casts light on the contrasts between the two cases. Wenger distinguished two different forms of incomplete participation, which he called *peripherality* and *marginality*. Peripheral participation is described as an enabling form of non-participation that provides opportunities for learning and leads to more complete participation in the future. Peripheral participants are "on an inbound trajectory" (Wenger, 1988, p. 166). Marginal participation, conversely, is described as a restricted form of participation that prevents full participation. "Long-standing members [of a community of practice] can be kept in a marginal position, and the very maintenance of that position may have become so integrated in the practice that it closes the future" (Wenger, 1998, p. 166).

Selena paid attention to other group members, and interacted collaboratively with them whenever she could. Her eager involvement, empathetic behaviour, and complaisant manner suggest that she was doing whatever she could to increase her acceptance by the class. In Wenger's terms, she was a *peripheral participant*, on a trajectory that would clearly lead to fuller participation in the future.

Charles, by contrast, had been in the class much longer than Selena, but in spite of this was frequently positioned as an Outsider. His own behaviour contributed to this, including his hesitant, often slightly incoherent speech and his rather anxious manner. These made it easier for more confident students to contest his attempts position himself as Expert, Critic or Spokesperson. By concentrating only on the mathematical problem, and withdrawing into silence when he thought he had solved it, he cut himself off from the social interaction among the students in the group. There may have been a historical explanation for the lack of friendly interaction between Charles and other students—he may, for example, have been reacting to previous rebuffs—but I have no knowledge of this. But it is clear that Charles' own way of participating in group discussions, and other students' perceptions of him, combined to restrict his opportunities to take part. In Wenger's terms, he was a *marginal participant*, on a trajectory that was likely to maintain his marginal position into the future.

CONCLUSIONS

The significance of these observations lies in the detrimental effects of a student's positioning as an Outsider on the learning outcomes of what was intended to be a collaborating group. If this happens only rarely there may be few serious consequences, but not if it happens frequently or for long periods. Students positioned as Outsiders have fewer chances to articulate, explain and justify their own ideas, or to question or challenge ideas and interpretations put forward by others. As Cohen (1994) pointed out, students who participate less, learn less. Not only does this deny them learning opportunities, but it may seriously affect their confidence and motivation. In addition, others in the group lose the opportunity to learn from the student who has been positioned in this way. In the case of students like Selena and Charles, who both showed considerable mathematical insight, this constitutes a serious loss to the group.

These studies have implications for teachers seeking to implement collaborative activities in their classrooms. While students are working in groups, an important task for the teacher must be to monitor groups and observe the positioning work that is taking place. In particular, they need to look out for students being positioned as Outsiders, who have little power, and can do little to change the situation themselves. With time, peripheral participants like Selena are likely to move gradually towards full participation, but this is not the case for marginal participants like Charles.

The priority must be to establish a classroom culture that supports collaboration. Norms for collaborative work should be discussed and negotiated with the class, but need to include, as a minimum, an expectation that everyone will contribute, that they will be treated with respect, and that others will listen carefully and courteously to what they have to say. It is not sufficient to discuss behavioural norms only when first introducing collaborative work—regular re-introduction and reinforcement are needed. Teachers in my study had a number of strategies for doing this, including asking for written feedback from students after a period of group work and using this as a starting point to stimulate class discussion about collaboration, and what they could do to make it more effective. One teacher demonstrated a particular skill to a student who was less confident and assertive, and encouraged her to teach it to the rest of her group. Another took time to remind her class about the benefits of collaboration and to explain that she formed groups by putting together students with different abilities and skills and different ways of thinking. Strategies such as these can help to change students' expectations about other students and what they might be able to contribute to the work of the group.

Note 1

Key to symbols used in transcripts:

//	marks the beginning of overlapping speech.
/	no noticeable pause between turns, along with indications that the first turn was incomplete.
:	indicates a lengthened sound.

REFERENCES

Australian Association of Mathematics Teachers. (2002). *Standards for excellence in teaching mathematics in Australian Schools*. Adelaide: AAMT. Retrieved 10 January, 2005, from http://www.aamt.edu.au/standards/.

Barnes, M. S. (2003). *Collaborative learning in senior mathematics classrooms: Issues of gender and power in student-student interactions.* Unpublished PhD thesis, The University of Melbourne, Melbourne.

Barnes, M. S. (2004, November–December). The use of positioning theory in studying student participation in collaborative learning activities. Paper presented at the Annual Meeting of the Australian Association for Research in Education, Melbourne.

Cohen, E. G. (1994). Restructuring the classroom: Conditions for productive small groups. *Review of Educational Research, 64*, 1–35.

Cohen, E. G., & Lotan, R. A. (1997). *Working for equity in heterogeneous classrooms: Sociological theory in practice*. New York: Teachers College Press.

Davies, B., & Harré, R. (1990). Positioning: The discursive production of selves. *Journal for the Theory of Social Behaviour, 20*, 43–63.

Forster, P., & Taylor, P. C. (1999, November–December). Learning mathematics through conversation and utilizing technology. Paper presented at the annual conference of the Australian Association for Research in Education, Melbourne.

Goos, M., Renshaw, P., & Galbraith, P. (1997, December). Resisting interaction and collaboration in secondary mathematics classrooms. Paper presented at the annual conference of the Australian Association for Research in Education, Brisbane.

Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: Creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics, 49*, 193–223.

Harré, R., & van Langenhove, L. (Eds.). (1999). *Positioning theory: Moral contexts of intentional action*. Oxford: Blackwell.

Lerman, S. (2001). Cultural, discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics, 46*, 87–113.

Linehan, C., & McCarthy, J. (2000). Positioning in practice: Understanding participation in the social world. *Journal for the Theory of Social Behaviour, 30*, 435–453.

Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge: Cambridge University Press.

Williams, G. (2000). Collaborative problem solving and discovered complexity. In J. Bana (Ed.), *Mathematics education beyond 2000: Proceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia, Fremantle, WA* (pp. 656–663). Sydney: MERGA.

TOP SET IDENTITIES AND THE MARGINALISATION OF GIRLS

Hannah Bartholomew
University of Auckland
<h.bartholomew@auckland.ac.nz>

This paper draws on research carried out in a London Secondary school to explore the ways in which the dynamics of a 'top set' mathematics class serve to marginalise girls. Drawing on the notion of a 'figured world' and the work of feminist post-structuralist theorists in mathematics education, the dynamics of the group are examined from the perspective of the identity-work being done by students engaged in mathematics in this class.

In seeking to promote their subject, mathematics educators have understandably regarded a major aspect of their role as that of maximising participation and attainment among students of mathematics. But such concerns tend to sideline questions about what exactly it is that is being promoted, and whose interests are being served in the process (Apple, 1992). The increasing focus on learning as a process of becoming, and as such as being inextricably bound up with an individual's identity (Boaler, Wiliam, & Zevenbergen, 2000), brings these questions to the fore; if being successful at mathematics conflicts with other aspects of who a student wants to be, then they are likely to opt out of the subject. And when certain groups within society opt out disproportionately, it becomes clear that those groups are not well served by mathematics education as it is currently constituted. In this paper I discuss one top set mathematics class, and explore the different relationships that girls and boys form with the practices of this class. In the process I hope to illustrate some of the ways in which discursive construction of mathematics as abstract and asocial is both misleading and unhelpful.

THEORETICAL UNDERPINNINGS

There has been a discernible shift in recent decades away from theories of learning which attend to individual cognitive processes, towards those which stress the ways in which knowledge is socially constructed and situated (Boaler, 2000; Lerman, 2000). Rather than regard learning as something that takes place inside people's heads, situated cognition theories regard learning as social practice (Wenger, 1998); it is less about acquiring concepts and abstract structures, and more about becoming a participant in a community. Using an apprenticeship model of legitimate peripheral participation within communities of practice, Jean Lave and Etienne Wenger (Lave & Wenger, 1991) have developed a theory of learning which locates knowledge "in the evolving relationships between people and the settings in which they conduct their activities" (Agre, 1997, p. 73).

These ideas have proved an effective tool for demonstrating how classroom norms affect the ways in which meaning is negotiated by students and teacher (Cobb, 2000), and for understanding the difficulties experienced by many students in applying school learnt mathematics to 'real world' problems (Boaler, 1997). They also place issues of identity at the heart of what it means to learn, making it possible to move beyond the idea that ability is the sole determinant of success (Boaler et al., 2000). However, they have been criticised on a number of counts. The narrow focus on the classroom (or other context in which learning takes place) does not permit an analysis of the wider socio-cultural factors that have a bearing on the ways students experience school mathematics. Lave and Wenger's model offers only one trajectory, and differences in the behaviour of individuals is not easy to account for; "there appears to be a goal for the learning which are characteristic of the practice, and apprenticeship into it is monolithic in its application" (Lerman, 2000, p. 27). Similarly, emotional responses, power relations, and the social and historical location of subject and practice are not adequately accounted for (Walkerdine, 1997).

Two perspectives which retain a focus on identity, but which attend to these criticisms are Holland, Lachicotte, Skinner and Cain's notion of a figured world, and the work of feminist post-structuralist theorists within maths education. Figured worlds have some things in common with communities of practice. Like Lave, Holland et al are concerned with the ways person and setting 'engage dialectically' (Agre, 1997), but they take a much broader view, both on the range of factors which shape figured worlds, and on the range of perspectives towards them. Figured worlds are:

> [Landscapes] of objectified (materially and perceptibly expressed) meanings, joint activities and structures of privilege and influence—all partly contingent upon and partly independent of other figured worlds, the interconnections among figured worlds and larger societal and trans-societal forces (Holland, Lachicotte, Skinner, & Cain, 1998, p. 60)

In developing a theory of identity, Holland et al point to the limitations both of analyses which emphasise cultural norms and those which emphasise social positioning, arguing that both approaches make too little of human agency. They seek to account for both the structural factors which constrain action, and the improvisational nature of individual responses:

> Though an overemphasis on social constraints leaves little room for human agency, many accounts make the opposite error: they neglect the cultural and social contexts that inform the "playing field" to which human action is directed and by which it is shaped (Holland et al., 1998 p. 275).

The other strand of work that I have found helpful is that of feminist post-structuralist work within mathematics education. Post-structuralist theory has many manifestations, but researchers working with these ideas share a recognition of the irreducibility of social phenomena, eschew essentialist accounts of difference, and seek to uncover the taken-for-granted assumptions underpinning the ways we regard the world, thereby exposing the ways politics and power play out in localised settings.

Like the model of a figured world, this work engages with the relationship between the individual and the social, and the ways in which they are constitutive of one another. A central concept within post-structuralist thinking is that of discourse, that is "a way of speaking, thinking or writing that presents particular relationships as self-evidently true" (Paechter, 2001, p. 41). Discourses structure the way we can think or speak about things, "operating within regimes of truth, not because of their power to describe reality but because of their power to produce it" (Mendick, 2003b). Individuals are positioned within and by discourses, and whether particular dominant discourses are accepted or rejected, they cannot be ignored. A post-structuralist perspective thus offers an alternative to the essentialism that has dominated much research in gender in mathematics education: "feminist post-structuralist theory recognises the ways in which gender is contested and reconstructed daily through the multiple discursive practices in which individuals participate" (Barnes, 2000, pp. 146–7).

THE STUDY

This paper draws on research carried out as part of a larger study of two schools. I embarked on this research with an interest in the ways students become positioned as particular kinds of learners in mathematics lessons. In order to research this issue I worked in the mathematics departments of two London schools over a period of three years.

I report here on data collected over two years in the top set maths class at a school I have named Clyde. Three quarters of the students in this class were from professional backgrounds (compared to under 30% in the school as a whole), and of the 27 students in the class only 7 were female. At the time of the study, the students were in years 10 and 11 (the final two years of compulsory schooling in England) and were preparing for their General Certificate of Secondary Education (GCSE) examination in mathematics. The data collected include observations of approximately 15 lessons, interviews with 10 students, progress reports written by the teacher for parents and data on students' attainment at various points over the two years.

In focusing on the top set I am discussing the experiences of a group of almost exclusively middle class boys and girls. This is not to downplay the (much more dramatic) differences between groups, and in particular the very different opportunities available to the predominantly working class students in low sets and the predominantly middle class students in high sets (for a discussion of these issues see Bartholomew, forthcoming), but my focus here is the gender issues that emerged from my studies of set 1 class.

INTRODUCING CLYDE SCHOOL

Clyde is a popular mixed secondary school that has been a successful player in its local area in establishing itself as a "high profile, elite, cosmopolitan, maintained school" (Gewirtz, Ball, & Bowe, 1995). The school is concerned with image management and markets itself assertively. It is over-subscribed recruiting students from a wide area. During the course of the study an entrance exam was introduced as the basis for selecting a proportion of each new intake. Students' performance in National exams is consistently above local and national averages, and in recent years, partly as a consequence of the change in admissions policy, this advantage has increased to the point that Clyde is now an extremely high performing state school.

A notable element of the school's marketing strategy is its emphasis on its 'most able' students. For example, the school prospectus contains a section outlining the provisions made for "more able pupils", including early identification of potential Oxbridge candidates, and support for these students. Students at Clyde are taught in ability groups throughout the school—in banded form groups for the years 7–9, and in sets for individual subjects in years 10 and 11—leading to considerable differences between classes. These points are expanded further in (Bartholomew, forthcoming).

It is against this backdrop that students in the top mathematics set come to regard themselves. These students are in a highly privileged position within the school, and have a high profile. The majority of the students in this group have been in 'top' groups throughout their time at Clyde, and are acutely aware of this position. This awareness of pecking orders leads to intense competition in some classes. Anna (all names are pseudonyms), a band one student who is in set two for maths, describes this climate as follows:

HB: So would you say there was quite a lot of competition in your class?

A: In our school. We've been, like all through the years it's been like—you have to be the best at everything you do.

HB: Why do you think that is?

A: Because we're banded.

HB: What do you mean because you're banded? What difference does that make?

A: Well, I don't know what happens to the other classes, but definitely in band 1 classes which [I was] in we were always told 'you're the best so you can't have this attitude towards this because you're a band 1 class. You should be above everyone else.

…

HB: And what—so how does that affect the class, do you think?

A: Well, that's made us all competitive I think, because we're like, then there's like, sort of 'who wants to be best' and it's all very, like—I mean, I've never heard any teachers say 'Oh, so and so did much better than you there' or something, but you sort of have that underlying feeling.

Anna, female student, set 2

THE FIGURED WORLD OF THE TOP SET

Set one maths lessons have a distinctive atmosphere. In comparison with other maths groups at Clyde, there is a relaxed and relatively informal approach to lessons which seems to stem from the assumption that all students in this class are keen to succeed and will knuckle down and get on with their work when necessary. The thinking seems to be that these are able students who have a serious attitude to school, and their teacher, Rob Sharpe, can afford to relax with them in lessons. Something of the culture of the class is captured in Rob's response when I asked if I could audio record one of his lessons. He summed up what I would be likely to hear on the tape as follows:

> There are two or three pupils who I kind of banter with all lesson—I'll say something, then quick as a flash, they'll be back with a response, and this goes on all lesson.
>
> Rob Sharpe, set 1 teacher

Despite the light heartedness of these lessons, they left me feeling uneasy, as it was clear that the figured world of the top mathematics set at Clyde was rather exclusive. While some students evidently thrived in this environment, others, notably the girls in the class, were marginalised. This figured world was created by the interaction of a range of factors including: the ethos of the school, and its attitude to academic achievement; the discursive construction of mathematics as "the ultimate form of rational thought and so a proof of high intelligence" (Mendick, 2003b, p. 5); and the composition of the class itself—both in terms of the gender and social background of its members, and in terms of their individual personalities, histories and ways of interacting. Hence while being indicative of the particular relationship that Rob Sharpe had with this group of students, it did not spring from nowhere.

An important element of the figured world of the top mathematics set at Clyde was that students able to sustain high achievement for (apparently) minimal effort attained a high status within the group. This can be related to the fact that these students are almost exclusively middle class; Walkerdine found that while 'hard working' is a positive trait in predominantly working class schools, it is viewed negatively in predominantly working class schools, where high attainment is expected of all and those who have to work hard to achieve it are seen to be lacking in ability (Walkerdine, 1998). It also draws on discourses which see mathematics as being associated with clear-cut right answers, and mathematical ability as something that a select few possess. An edition of the Clyde school newsletter reported that three students who had won prizes in a recent mathematics competition must have "found it rather easy"—it is hard to imagine this form of words in relation to an essay writing competition. These values feed into the competitiveness of the class, and make for a highly visible, if simplistic, means of ranking students:

R:	The boys in my class are really competitive about maths … they're always trying to get one up on each other as far as maths is concerned.
HB:	Why do you think that is?
R:	I don't know. … I think boys are generally more competitive [than girls], but I don't know what it is about maths. … It's just one of the few subjects

> where—for some reason in our class being good at maths is really important. And that everyone else sees it.
>
> Rhiannon, female student, set 1

Current concerns about boys' 'laddish' behaviour highlight the fact that working hard at school can be socially problematic for boys (Lucey & Walkerdine, 1999); for those who could pull it off, attaining high marks in mathematics without appearing to try very hard conferred considerable prestige, both academically and socially. Within the discourses around ability, mathematics attainment and masculinity which predominate in the top set, being good at mathematics is all but synonymous with 'mucking about' in lessons, to the point that Ben, who attained the second lowest mark in the exam taken at the end of year 10 (18%) was widely regarded as very clever. Indeed, when asked whom he would choose to work with in mathematics, Ian—who was himself chosen by five of the twelve boys who nominated another student as being the best at maths in the class—chose Ben on the grounds that "he is the only one with a greater intellect than myself".

GIRLS, BOYS AND BELONGING

Turning now to focus on gender issues, it is worth reflecting again on Rhiannon's comments, quoted above. She is comparing the behaviour of boys in the class with that of girls, and goes on to say:

> Most of the girls I know want to get a good mark in maths, but it's not important to them that everyone sees that they're the best in maths, and they get the answers first. I think they are more happy to make sure they've got all the notes down and they're all prepared when it comes up to the exam.
>
> Rhiannon, female student, set 1

However, when asked specifically about her own competitiveness, she talks first about the fact that, unlike most girls, she contributes a lot in lessons. This slippage between *competing* and *contributing* is interesting, suggesting that entering into the competition is the only form of contribution acknowledged. She then refers to her relationship with Jo, another girl in the class:

> Jo and I picked all the same GCSEs, and she's a deeply, *deeply* competitive person, and I think it is just a bit of friendly rivalry.
>
> Rhiannon, female student, set 1

It is striking that, in casting her own competitiveness with Jo as "a bit of friendly rivalry", she takes care to distance herself from the aggressive competition of the boys in the class. She and Jo are certainly the two girls who contribute most and are regarded as most successful, and this evidently raises tensions for her. Walkerdine, Lucey and Melody (1999) argue that middle class girls must negotiate a delicate balance between attaining high levels of academic success whilst maintaining a feminine identity, and Mendick's analysis that "doing maths is doing masculinity" (Mendick, 2003a, 2003b) suggests that this issue is likely to be brought into sharper focus in mathematics lessons. Certainly in this class, it was those students who behaved most 'like a boy' who were regarded as most successful. Thus while the attainment of the 7 girls in this class was similar to that of the boys, this was neither widely acknowledged nor entirely comfortable for them. Rhiannon is careful to place herself on the margins of the group while talking to me (by denying her participation in the competition); Jo, when given the opportunity to take GCES maths a year early at the end of year 10, declined; of the remaining 5 girls, 2 opted to enter GCSE at a lower level than the rest of the class, though other students with lower marks stayed at the high level and the other and the remaining 3 all expressed their discomfort at being in the class to me, indicating that they would rather move down to a lower group.

By contrast no boys that I spoke to expressed such doubts to me, and whatever private anxieties they may have had, were keen to present themselves as core members of the group. A notable example is Prashanth. He began year 10 in set 4, but was promoted to set 1 on the basis of his performance at the end of the year. His account of being in set 4 is infused with a sense of not belonging there:

> I was in Ms Floyd's class and I didn't feel that I was getting the best maths work. I knew I could do better. And … most of my friends were actually in higher classes, so I didn't feel I had a lot in common with the work I was doing and with the people around me.
>
> Prashanth, male student, set 1

In relation to set 1 he spoke enthusiastically about the atmosphere in the class. In claiming to speak for the whole group he is placing himself at its heart:

> Yeah, well, everyone loves [Mr Sharpe's] humour. He sometimes talks about what was on last night, but everyone loves it. I'm sure everyone does. But, I mean, even though there is humour, there is a serious side as well. He does tell us … things about exams, and he tells us we need to be ready. He does pressurise us too.
>
> Prashanth, male student, set 1

DISCUSSION

Mendick's argument that "what [students] enjoy when doing maths is the identity work they do through it" (Mendick, 2004, p. 1) speaks to the issues raised in this paper. Although I did not interview all of the boys in the class, those I did speak to were enthusiastic about their mathematics lessons. Like Prashanth, they liked being in set 1, with the status this conferred, and they enjoyed sparring with one another, and proving their intellectual prowess. While their markers of success drew on discourses that place mathematical attainment on a pedestal of abstract rationality, attributing their enthusiasm straightforwardly to their pleasure in engaging in an abstract and asocial practice appears to me to be missing the point. Indeed, my observations of their lessons suggest that these boys were primarily concerned with working quickly and succeeding visibly. It was the status that mathematics afforded them that they enjoyed, and it was in terms of this status that their success was defined.

In the case of the girls in the class, much of their discomfort in lessons appeared to be associated with the tensions that contributing in lessons presented for their ongoing gender identity work. While, on a superficial level, many stereotypes about anxious, hardworking and co-operative girls appeared to be played out in these lessons, it would be misleading to attribute this behaviour to a distaste of, or difficulty with, engaging with mathematics *per se*. Many of the girls in the class spoke with enthusiasm about doing mathematics in certain situations, and their attainment attests to their capability to succeed in the subject.

In neither case is it possible to separate the objective, rational and abstract from the subjective, emotional and relational (Mendick, 2004), still less to attribute one side of this dichotomy to the boys (and to mathematics) and the other to the girls in any essential way. My data suggest that the identity work in which students are engaging, and the associated emotional factors, are implicated at all levels, not as a background which may facilitate or hinder mathematical achievement, but as an inevitable part of what it means to do mathematics and regard oneself as mathematical.

REFERENCES

Agre, P. E. (1997). Living math: Lave and Walkerdine on the meaning of everyday arithmetic. In D. Krishner & J. A. Whitson (Eds.), *Situated cognition: Social, semiotic and psychological perspectives* (pp. 71–82). Mahwah, NJ: Lawrence Erlbaum Associates.

Apple, M. W. (1992). Do the standards go far enough? Power, policy, and practice in mathematics education. *Journal for Research in Mathematics Education, 23*(5), 412–431.

Barnes, M. (2000). Effects of dominant and subordinate masculinities on interactions in a collaborative learning classroom. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning*. Westport, CT: Ablex Publishing.

Bartholomew, H. (forthcoming). The market, social class and the construction of ability in secondary mathematics.

Boaler, J. (1997). *Experiencing school mathematics: Teaching styles, sex and setting*. Buckingham, UK: Open University Press.

Boaler, J. (2000). Exploring situtated insights into research and learning. *Journal for Research in Mathematical Education, 31*(1), 113–119.

Boaler, J., Wiliam, D., & Zevenbergen, R. (2000). The construction of identity in secondary mathematics education. Paper presented at the Mathematics Education and Society conference, Montechoro, Portugal.

Cobb, P. (2000). The importance of a situated view of learning. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 45–82). Westport, CT: Ablex Publishing.

Gewirtz, S., Ball, S. J., & Bowe, R. (1995). *Markets, choice and equity in education*. Buckingham, UK: Open University Press.

Holland, D., Lachicotte, W. J., Skinner, D., & Cain, C. (1998). *Identity and agency in cultural worlds*. Cambridge, MA: Harvard University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, UK: Cambridge University Press.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning*. Westport, CT: Ablex Publishing.

Lucey, H., & Walkerdine, V. (1999). Boys' underachievement: Social class and changing masculinities. In T. Cox (Ed.), *Combating educational disadvantage* (pp. 37–52). London, UK: Falmer Press.

Mendick, H. (2003a). Choosing maths/doing gender: A look at why there are more boys than girls in advanced mathematics classes in England. In L. Burton (Ed.), *Which way social justice for mathematics education?* (pp. 169–187). Westport, CT: Praeger.

Mendick, H. (2003b). Mathematical stories: Why do more boys than girls choose to study mathematics at A level? Paper presented at the British Educational Research Association conference, Edinburgh, UK.

Mendick, H. (2004, 4–11 July). Objective subjectivities, subjective objectivities and guilty pleasures: Exploring the possibilities of deconstructing the separated/connected opposition for thinking about gender and mathematics. Paper presented at the 10th International Congress on Mathematical Education, Copenhagen, Denmark.

Paechter, C. (2001). Using poststructuralist ideas in gender theory and research. In B. Francis & C. Skelton (Eds.), *Investigating gender: Contemporary perspectives in education* (pp. 41–50). Buckingham, UK: Open University Press.

Walkerdine, V. (1997). Redefining the subject. In D. Kirshner & J. A. Whitson (Eds.), *Situated cognition: Social, semiotic and psychological perspectives* (pp. 57–70). Mahwah, NJ: Lawrence Erlbaum.

Walkerdine, V. (1998). *Counting girls out* (2 ed. Vol. 8). London, UK: Falmer Press.

Walkerdine, V., Lucey, H., & Melody, J. (1999). Class, attainment and sexuality in late twentieth-century Britain. In C. Zmroczek & P. Mahony (Eds.), *Women and social class—International Feminist perspectives* (pp. 51–68). London, UK: University College London Press.

Wenger, E. (1998). *Communities of practice: Learning, meaning, and identity*. Cambridge, UK: Cambridge University Press.

DODGING THE DRAGON: STRATEGIES FOR MATHEMATICS PROFESSIONAL DEVELOPMENT IN LOW SOCIO-ECONOMIC AREAS

Bill Barton
The University of Auckland
<b.barton@auckland.ac.nz>

Judy Paterson
The University of Auckland
<paterson@math.auckland.ac.nz>

Barbara Kensington-Miller
The University of Auckland
<bmiller@math.auckland.ac.nz>

Hannah Bartholomew
The University of Auckland
<h.bartholomew@auckland.ac.nz>

This paper theorises strategies for the delivery of effective professional development to mathematics teachers in schools in low socio-economic areas. A project working in eight such schools in the Auckland region of New Zealand has enabled the identification of some particular features of such a situation, described in an earlier paper. This paper follows on by proposing professional development approaches that are likely to be effective. Schools in low socio-economic areas are characterised by higher teacher stress and turnover, relatively under-prepared teachers, fewer opportunities for successful outcomes, and teachers who are isolated in their practice. We confirm the work of others in identifying confidence as a key feature, and use the metaphor of a dragon to explore the requirements of an effective programme. The project has confirmed the importance of a supportive community with a focus on reflective practice and an emphasis on mathematics. In addition to the usual features, (for example long-term commitment), a stance of professional self-actualisation is hypothesised to be important.

Following on from an earlier paper in which the characteristics of mathematics professional development in a low socio-economic situation were examined (Bartholomew, Barton, Kensington-Miller, & Paterson, 2005), this paper describes a metaphor for conceptualising one of the resulting key problems for developing effective professional development. It then lays out some of the features necessary for a programme to be successful. Evidence from the professional development component of a project working with senior mathematics teachers in Auckland, New Zealand, is presented in support of these conclusions. The whole project was described at MES-3 in Copenhagen (Barton, Autagavaia, Poleki, & Alangui, 2002).

BACKGROUND

The Mathematics Enhancement Project (MEP) aims to enhance the participation and achievement of senior secondary students (ages 16–17 years) in mathematics in eight schools that are situated in a low socio-economic area. The project is multi-faceted, working with students, teachers, and the community in a development and research mode. The project is a long-term commitment that explores ways of operating on a long-term basis without large inputs of one-off funding. The Needs Analysis was done in 1999–2000, the Pilot Phase ran from 2001 to 2003, and a 4-year programme began in 2004. It is hoped to find an effective development programme that can be used in other similar schools on a sustainable basis (Barton, Autagavaia, Poleki, & Alangui, 2002).

The schools in the study are in the Manukau region, just south of Auckland city in New Zealand. The students are predominantly of Pacific Island (Polynesian) descent, although most have lived in New Zealand for a considerable time. There are also Maori students, and new immigrants mostly from Asia and the Middle East.

The teacher professional development component of the project consists of a team of six researchers from the Mathematics Education Unit of the University of Auckland working with the three or four teachers of senior classes from each school. There is a total of about 25 teachers in any year. With respect to research on the professional development process, each of the university researchers are gathering data on their specific area of interest in a variety of ways, including recording group discussions, interviews, research journals, questionnaires, and written feedback records. The university researchers have shared their individually gathered data for this paper.

The teacher development is organised in two overlapping structures. In the first a variety of opportunities are provided for mathematics teachers to experience development activities. These include regular teacher meetings, mentoring between teachers, mentoring by university mathematics educators, mathematics lectures, and the provision of written material. The second structure involves each teacher within a research team led by one of the university researchers. For example, teams have been formed that are examining: classroom didactic contracts; the use of mathematics lectures as a stimulus for reflecting on ones own learning and teaching; developing identities of mathematics learners; language issues in the mathematics classroom; the changing conceptions of calculus; and the benefits of mentoring.

THEORETICAL & PHILOSOPHICAL POSITIONS

As reported in Barton, Autagavaia, Poleki, and Alangui (2002), the review of research conducted as the project developed indicated that, with respect to teacher development:

- socio-economic factors are critical, but solutions are likely to be systemic;
- constructive rather than remedial interventions are preferable;
- an effective programme will be long-term, will involve teacher-initiated classroom reflection, will be school-based, will recognise the complexity of teacher's lives, and should involve mathematical development.

A research study in the early years of the programme confirmed the systemic nature of the issues (Kensington-Miller, 2004), and this paper reports on evidence that confirms the third point above.

The project was established under a set of principles that includes the following.

- Teachers are the means to improvement, they are not the 'problem'.
- The project must work within a national mathematics curriculum, an existing staffing situation, pre-determined student intakes, and parental expectations.
- Teachers are assumed to be professionals within a community of practice in which they work as practitioner, as researcher, and as mathematician.

The philosophy behind the project is one in which teachers are regarded as professionals who take responsibility for ongoing improvements as a normal part of their practice. The project is based on a conception of professional development as reflective practice that is less about instructing teachers in best practice and more about opportunities for teachers to step back from the realities of teaching to look at what they do. Therefore a key aim is the development of a community of practice in which teachers are empowered as professionals through a shared focus on the processes of teaching and learning (Hiebert, Gallimore, & Stigler, 2003). The resulting model of development has the following characteristics.

- It includes the good from the past, and implies that it is always possible (and necessary) to improve (as opposed to rejecting the past).
- It is a gradual implementation tailored to the resources available (as opposed to requiring new infrastructure and complete retraining).
- It is dependent on the active, voluntary participation of those involved (as opposed to being imposed, unresponsive, and insensitive to their needs).
- It is education theory and research driven (as opposed to being driven by a political ideology).

The various political issues were discussed noted in Barton, Autagavaia, Poleki, and Alangui (2002). In particular, it was noted that, despite the need to operate within an existing educational and political environment, the project does seek to address issues of positioning. The current paper is partly a response to the need to "bring about an awareness of ethnic and socio-economic aspects of the habitual behaviour of all participants, students, teachers, researchers, the school, and the community alike" (p. 47).

CHARACTERISTICS OF LOW SOCIO-ECONOMIC SCHOOLS

An earlier paper details the characteristics of mathematics education in this low socio-economic situation, and its impact on professional development (Bartholomew, Barton, Kensington-Miller, & Paterson, 2005). In summary, in comparison with schools in higher socio-economic areas, these schools exhibit: poorly resourced classrooms; higher absentee rates; a more transient student population; less emphasis and time allocated to mathematics; higher proportions of students with English language difficulties; lower achievement on national examinations; a more pyramidal student demographic and lower retention rates in mathematics; and students with poorer entry achievement standards. Suitably trained relief staff are more difficult to obtain, teachers of senior classes work in isolation, mathematics teachers are less likely to have been educated in the environment in which they teach, and mathematics teachers are, on average, less well qualified. Teacher stress is higher and staff turnover is faster. There are fewer personal and institutional resources available to cope with the situation.

Two positive characteristics are that student successes that do occur are more highly rewarding to all involved, and there is a strong identification between teachers in these schools of succeeding in an environment in which many other teachers would probably not cope.

The consequences of the above for professional development relative to higher socio-economic mathematics education environments were noted as: more difficult to sustain a long-term and supportive community of practice; fewer opportunities to share and reflect; more difficult to organise professional development activities; more difficult to experiment with pedagogical practices (particularly active ones); and greater pressures to focus on examinations, content rather than process, and skills rather than applications.

The MEP professional development component has had some success in overcoming these negative consequences. Teacher feedback mentions the long-term commitment, and the emphasis on mathematics and research as well as classroom practice as contributing factors. The importance of the formation of a community of teachers with common experiences and with increasing input into the shape of the programme is also clear.

Reaching this point and sustaining continual development, are not without problems. There has been a tension between the professional development philosophy and principles adopted for the project (see above), and the professional development activity expectations of many of the teachers. This includes a tension between teachers asking for resources and teaching exemplars which can be directly used or emulated, and the desire of the project organisers to include new mathematics learning and research activities that do not have immediate or direct application to the classroom. Achieving a suitable balance is the subject of ongoing debate within the community, with the development of self-actualising responsibility remaining the ultimate aim.

The subject of this paper, however, is that of teacher confidence, conceived in a broad manner. This includes both positive aspects, such as knowledge of successfully surviving in difficult conditions, and negative ones, such as awareness of inadequate mathematical content knowledge. Lack of confidence has been both observed by researchers and reported by teachers in this project as contributing to non-participation in professional development activities. Similarly, strong confidence in classroom performance has been observed and reported by teachers as contributing to willingness to have other teachers or researchers in their classroom.

Graven (2004) has recently highlighted the need for a concept of confidence in a social practice account of teacher development, arguing both from evidence gathered within a professional development project and through an analysis of Wenger's (1998) theory. We find ourselves in agreement with her conclusion that confidence enables teachers to become active members of a professional community of practice. Evidence from the MEP project provides further support. Thus we see a development in the understanding of teacher development that can directly contribute to the design of effective programmes.

THE DRAGON

There is a request at the end of Graven's paper for more research and theorising to be undertaken in this area. Our research is still in process, and will be reported on fully at the end of 2005. However the data gathered so far has led us to trying to conceptualise confidence in a way that will allow us to improve the professional development programme within the project. We acknowledge the systemic difficulties of our teachers, and the positive components of their confidence, however observed and reported lacks in confidence also need to be addressed.

Observing that insecurities are real obstacles for teachers, the challenge becomes that of finding ways of working with them in a sensitive and constructive way. We have found it helpful to conceptualise these fears as a dragon. The dragon appears to sit in its cave blocking the way to teacher development, and possibly to other types of change in mathematics education. Our work tells us that, for teachers in low socio-economic areas, this dragon is larger and more fearsome than in other areas.

What can be done about such a dragon? In seeking strategies to implement development or initiate change, this metaphor encourages us to note that it is possible either to confront the dragon in its cave with all the attendant dangers, or to walk around the dragon on a strategy of avoidance. Confronting the dragon requires considerable courage: the teacher might get hurt, or might be defeated. Not everyone has the courage to face dragons: we are not all St George. There is a commonsense logic that there is no point in upsetting the dragon and making it roar even loader. If the teacher has a survival strategy, then why move from that zone of relative comfort?

In our context this is interpreted as the risk that a teacher might expose something far bigger than they are ready to deal with within the professional development programme. Perhaps this feeling is part explanation for non-attendance at professional development meetings. At best, exposing oneself to professional fears would make the professional development experience ever more threatening. At worst it could precipitate the end of a teaching career. Such possibilities are not likely to be engaged with unless the chances of success in the confrontation are known in advance to be high. No programme of professional development can guarantee that.

Note that confronting and surviving the dragon (but not getting past it) can also have its problems. The concept of defended subjects (Hollway & Jefferson, 2000) makes us realise that teachers who see themselves as successful may position themselves and others in ways that limit development options. For example, in positioning themselves as 'teachers who can cope', their students are simultaneously positioned as 'students who must be coped with', and university researchers are likely to be positioned as 'people who don't really understand what it is like'. This was discussed further in our previous paper (Bartholomew, Barton, Kensington-Miller & Paterson, 2005, pp. 4–5).

Walking around the dragon seems to imply skirting round the real issues for the teacher and thus restricting the possibility for anything beyond superficial change. This is exemplified by professional development programmes that "have the answer" and ask teachers to adopt a new method without consideration for their past experiences. Research literature has long indicated the inadequacy of such design and sought to construct other models (see, for example, Leder, 1989; Loucks-Horsley & Matsumoto, 1999; Robb, 2000; and the Key Group model of Robinson, 1989).

However, walking around the dragon can be done in two ways: in the knowledge and cognisance of the dragon; or pretending that it does not exist. Pretending that our fears do not exist will not take us very far in the long term. It may allow us to continue with a practice that is adequate, or even to make some surface changes or try out well-supported new ideas. However, when ultimately left on our own, the fears re-emerge to prevent substantial development.

On the other hand, our attitude is that, whatever the size of the dragon, it is not very dangerous and it is tethered to the back of its cave. It does, nevertheless, have a fairly loud roar. If you walk around the dragon and pretend that it is not there, then when it roars you become paralysed and speechless. If however, you are aware of its presence, accept that it will make its presence felt now and again, but decide not to feed it, moving forward is possible. That is, our fears can be acknowledged without being confronted, in such a way that they do not obstruct development. Indeed, this stance, it is suggested, is exactly the stance of a professional.

Walking around the dragon in cognisance of it means that we can learn to talk about the dragon, enquire as to its origins, and learn to recognise its different roars and what they mean. That is to say, this stance includes the important ability for teachers to come to know their fears, and to understand them. This may include recognising their own role in creating the dragon in the first place—to some extent it may be a paper monster.

Graven (2004, p. 207) can be interpreted in this metaphor. Having fears, insecurities, or areas for improvement, and learning to acknowledge them without being disempowered by them, is a major characteristic of what we wish to call professional confidence, that attitude of confidence that is necessary for the profession of mathematics teaching. It is exhibited below by a teacher in the MEP as he recognised the role of fear in learning for students and acknowledged it also in himself when preparing material:

> Coming to these sessions in groups, it's made me look at what my students feel when they're in the classroom. And I think initially, … when somebody comes up with something, you're taken aback a bit. And I think initially the feeling is fear. … I think that for me learning here and for them learning from me as well. Because anything that's unknown is scary. It doesn't matter what you've got in life, if you don't know what's round the corner there's a bit of nerves, there's a bit of tingling sensation, and there's a bit of fear in that, and if [as a teacher] you're trying to learn something new which you feel like, 'I don't know if I'm going to know this, but they're going to expect me to know that', I think that's fear. (Teacher, MEP Project)

Returning to the metaphor, perhaps the aim of professional confidence can be seen as from moving from a Western conception of a dragon (characterised as a danger, something to be feared and overcome) to a more Eastern conception (still a symbol of strength, but not necessarily bad or needing to be destroyed). Being descended from a dragon is a positive attribute, the dragon is what gives you power to be yourself.

MOVING BEYOND DEFENDED PRACTICE

Here, then, is an idea that can be used to design professional development: set up professional development so that teachers come to acknowledge their fears without being disempowered by them, understand their fears comfortably, recognise that self-construction is a part of their origin, and learn to operate with their presence. It is argued that this is a particularly important design feature for teachers in low socio-economic schools because the dragon is that much more fierce.

First it should be noted that the important features of professional development indicated in the literature are supportive of this idea: long-term commitment and teacher-control (Loucks-Horsley & Matsumoto, 1999; Putnam & Borko, 2000). As the community of practice develops within the MEP, we have seen increasing evidence that it is becoming easier for all of us to acknowledge our insecurities and our anxieties, and the environment feels a safe one in which to do so. Simply being together for a long time, and knowing that the relationship will continue, is at least correlated with more insightful acknowledgement of self-identified areas of concern.

Teacher control obviously enables acknowledgement to take place at an appropriate time in an appropriate way. However there is a problem here: total teacher control might lead to avoidance. What is indicated, therefore, might be teacher control within some professional parameters. More work needs to be done to clarify this area.

But it is the development of the community itself that requires attention. It often seems to be assumed that putting professionals together will usually result automatically in a community of practice. Our experience in the MEP is that this is not the case. Most of the teachers in the project had met each other before and been part of many professional development meetings. However, at the beginning of the project, they did not exhibit participation in a community despite avowing a desire for it (Kensington-Miller, 2004). The community is now slowly developing. What is contributing to this? How can it be consciously generated? We do not know all the answers to these questions, but the single most important message coming from our work is that, particularly in a low socio-economic setting, time and attention must be devoted to construct the community of practice.

Two parts of the MEP seem to be particularly important in this respect. These are the re-introduction of mathematics and of research into the role of the teacher-as-professional in addition to the existing educator/practitioner role that is the usual focus. The MEP teachers are enrolling in high numbers in university courses, some to study more mathematics and some to study mathematics education as a research field. They report their appreciation of these opportunities, and have voluntarily invested valuable weekday evenings to these activities. These opportunities were always available and advertised, and are not a compulsory part of MEP participation. We can only conclude that the teachers are finding the engagement in new mathematical ideas and being part of a research team as rewarding in themselves—that is, such activity enhances their professional confidence.

These two components of the project also provide opportunities to acknowledge fears. The tapes recording discussions following presentations of new mathematical ideas contain many examples of teachers admitting being at a loss during the lecture, of not understanding ideas, and of asking questions of each other. This appears to be a safe context for mathematics teachers to expose themselves. Similarly, formal research observation in classrooms inevitably reveals the inability of all of us, teachers and researchers alike, to explain all that is seen—and forces discussion about it.

Teachers in these situations do not seem to pretend that they are not afraid of the dragon. They speak about it, and there is some evidence (see, for example, the extended quote above) that they come to deeper understanding of these feelings.

In this respect there is a small, practical matter: professional development must be frequent, regular, and planned. We note that it has been unusual for the MEP teachers to have eight planned sessions in a year, and they were initially concerned about the call on their time or disruption to classes. They now look forward to the next one, and many are willing to give up several days of their holiday for this. There is no doubt that the professional community is an increasingly enjoyable place to be—it provides their opportunity to talk about their work and the dragon.

Finally, we believe that there is one other important factor about professional development programmes that contributes to the development of professional confidence: the philosophical stance of professional self-actualisation. As noted above, this philosophy continues to be in tension with some teachers' desire to be given prepared practical classroom resources or be told 'how-to-do' some aspect of their work (particularly if there are new curricular requirements).

In practice, a stance of professional self-actualisation means that sessions are designed around providing generic learning experiences, and teachers are given opportunities to make of them what they will. For example, there may be a university lecturer talking about some new ideas in mathematics (not ones that can necessarily be used in a secondary classroom, but ideas that will be of interest to teachers), and then question time is extended into discussion groups. Another example is to provide a time when teachers of the same level classes get together without outside input. A further example is the formation of mini research teams that look at a topic like whole class discussion. Teachers are free to choose whether or not they extend this activity into their own classrooms and practice.

We do not have evidence that will link professional self-actualisation to professional confidence. Some teachers have reported that the MEP sessions have been more satisfying than some other recent Ministry of Education sponsored sessions that follow a provision model. Therefore we present this feature of professional development design as a hypothesis, and will be directing some attention to gathering evidence to test it.

CONCLUSIONS

This paper supports the emergence in the literature of confidence as a key component of professional development for teachers. It builds on previous work by the same authors to argue that this component is especially important for mathematics teachers working in a low socio-economic environment (and one in which the culture of the students is different from that of the teachers). In it we discuss the construct of professional confidence, an important part of which is the self-acknowledgement of teacher needs and fears without disempowerment.

Such a construct leads to the question of how professional confidence is developed. The most critical conclusion is that it emerges within a community of practice, but that such a community must be deliberately developed over a period of time, again, especially for teachers in the low socio-economic context. It will not necessarily naturally result from teacher gatherings.

Some emerging evidence from the MEP indicates that important strategies in the design of professional development, include commitment, some level of teacher control, attention to the mathematician and research roles of the teaching profession as well as the practitioner one, and pursuing self-actualisation within teacher-development.

ACKNOWLEDGEMENTS

The professional development component of the MEP research has been funded by the New Zealand Ministry of Education Teaching and Learning Research Initiative. The wider project is funded by the Woolf Fisher Research Centre, The University of Auckland, and Manukau City.

REFERENCES

Ball, D. L. (1996). Teacher learning and the mathematics reforms: What we think we know and what we need to learn. *Phi Delta Kappa, 77*(7), 500–508.

Bartholomew, H., Barton, B., Kensington-Miller, B., & Paterson, J. (2005). Mathematics teacher development in low socio-economic areas. Accepted for presentation at the *15th ICMI Study on The Professional Education and Development of Teachers of Mathematics*, Brazil, May, 2005.

Barton, B., Autagavaia, J., Poleki, A., & Alangui, W. (2002). The Mathematics Enhancement Project: A theoretical approach to research and development. In P. Valero & O. Skovsmose (Eds). *Proceedings of the Third International Mathematics Education and Society Conference* (pp. 144–153). Copenhagen: Centre for Research in Learning Mathematics, Roskilde University.

Graven, M. (2004). Investigating mathematics teacher learning within an in-service community of practice: The centrality of confidence. *Educational Studies in Mathematics, 57*(2), 177–211.

Hiebert, J., Gallimore, R., & Stigler, J. W. (2003) The new heroes of teaching. *Education Week, 23*(10), 42–56.

Hollway, W., & Jefferson, T. (2000). *Doing qualitative research differently: Free association, narrative and the interview method.* London: Sage.

Kensington-Miller, B. (2004). Professional development of mathematics teachers in low socio-economic secondary schools in New Zealand. In I. Putt, R. Faragher & M. Maclean (Eds.), *Mathematics education for the third millennium: Towards 2010. Proceedings of the 27th Annual Conference of the Mathematics Education Research Group Australasia* (pp. 320–327). Melbourne: Mathematics Education Research Group of Australasia.

Leder, G. (1989). Teachers as agents of change in mathematics education, in N. F. Ellerton & M. A. Clements (Eds.), *School mathematics: The challenge to change* (pp. 217–222). Geelong: Deakin University Press.

Loucks-Horsley, S., & Matsumoto, C. (1999). Research on professional development for teachers of mathematics and science: The state of the scene. *School Science and Mathematics, 99*(5), 258–271.

Putnam, R., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher*, *29*(1), 4–15.

Robinson, I. (1989). The empowerment paradigm for the professional development of teachers of mathematics, in N. Ellerton & K. Clements (Eds.), *School mathematics: The challenge to change* (pp. 269–283). Geelong: Deakin University Press.

Robb, L. (2000). *Redefining staff development: A collaborative model for teachers and administrators.* London: Heinemann.

Wenger, E. (1988). *Communities of practice: Learning, meaning, and identity*. New York: Cambridge University Press.

VALUES IN MATHEMATICS AND SCIENCE EDUCATION: RESEARCHERS' AND TEACHERS' VIEWS ON THE SIMILARITIES AND DIFFERENCES

Alan J. Bishop
Monash University
<alan.bishop@education.monash.edu.au>

The paper describes the genesis, background and some data from a recently completed research project on values in mathematics and science education. Values are theorised as being the deep affective qualities of the subjects which are revealed through the educational process. This project involved two mathematics and two science educators and the research was carried out with teachers and their students in both primary and secondary schools. Here we show that although there are some strong similarities between the values ascribed in the research literature to the two subject areas, there are some important differences perceived by the educators in those fields. From the other data we can see that differences are perceived by the teachers, although not always the same differences.

INTRODUCTION AND BACKGROUND

In the modern knowledge economy, societies are demanding greater mathematical and scientific literacy and expertise from their citizens than ever before. At the heart of such demands is the need for greater engagement by students with school mathematics and science. As the OECD/PISA definition of numeracy puts it:

> Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen (OECD, 2003).

Values are an inherent part of the educational process at all levels, from the systemic, institutional macro-level, through the meso-level of curriculum development and management, to the micro-level of classroom interactions (Le Métais, 1997) where they play a major role in establishing a sense of personal and social identity for the student.

The notion of 'values' is not new in anthropology (e.g., Kluckhohn, 1962), in psychology (e.g., Kohlberg, 1981; Krathwohl, Bloom, & Masia, 1964; Rokeach, 1973), or in general education (e.g., Halstead, 1996; Nixon, 1995; Raths, Harmin, & Simon, 1987). However the notion of studying values in mathematics education is a relatively recent phenomenon (Bishop, 1999) and even in science education the study of values in classrooms is not a major focus of research. Nevertheless, in mathematics and science education values are crucial components of classrooms' affective environments, and thus have a crucial influence on the ways students choose to engage (or not engage) with mathematics and science. Clearly the extent and direction of this influence will depend on the teachers' awareness of, respectively, values ascribed to the particular discipline, the values carried by their selection from the available pedagogical repertoire, and their consciousness or otherwise of imposing their own personal values (Pritchard & Buckland, 1986). Data from a previous research project, Values and Mathematics Project (VAMP) has shown that teachers of mathematics are rarely aware of the values associated with teaching mathematics (FitzSimons, Seah, Bishop, & Clarkson, 2000). Furthermore, any values 'teaching' which does occur during mathematics classes happens implicitly rather than explicitly (Bishop, 2002).This paper will report on ideas developed during a more recent research project concerned with values in both mathematics and science education. There were three basic questions on which the research focused:

1. What values are (a) implicit and (b) explicit in the intended mathematics and science curriculum and assessment documents, as well as in textual and other resources utilised in the classrooms under study?
2. What values do teachers (a) espouse socio-historically, epistemologically, and pedagogically and (b) actually portray in their classrooms? How do each of these sets of values relate to those in 1?
3. What values do students in these classrooms hold, and how do these relate to the values in each of 1 and 2? How, if at all, are they influenced by pedagogical interventions within the timeframe of the school year?

This project therefore differed from the earlier one in two significant ways. Firstly it involved a comparison between values in mathematics and science. This was done because of both the similarities between the two subjects and their differences. The second difference with VAMP was that this one would also involve collecting data from students, and would attempt to explore their values and how these are related to any the teachers might hold.

VALUES IN MATHEMATICS AND SCIENCE

Regarding their similarities, both mathematics and science are taken as ways of understanding that are embedded in rational logic - focusing on universal knowledge statements. Both are seen by society in general as essential components of schooling, rivalled only by literacy. Hence, teachers of each face substantial political and social pressures from outside the school (e.g., system-wide assessments of student performance, purposes for teaching seen as being directly related to technological development, etc.). In their teaching, both involve following routines, although not exclusively. Both involve modelling, albeit with different emphases. Similarly each is incorporated into the other's applications but in an asymmetrical relationship.

On the other hand, science curricula/texts commonly contain a section on "The Nature of Science" while mathematics rarely contains the equivalent. While the values embedded in mathematics teaching are almost always implicit, in science teaching some are quite explicit. For example, curriculum movements such as

Science-Technology-Society make some values explicit and central to the intended learning outcomes; laboratory work seeks to make explicit such values as 'open mindedness,' 'objectivity,' etc.; and content described as *The Nature of Science*, for example, also makes some values explicit (see also UNESCO, 1991).

Among the general public, although the concept of 'a science industry' or 'scientific industries' is widely recognised, this is not the case for mathematics. In the popular media (e.g., magazines, newspapers, books, radio, television), science receives much more attention than mathematics, despite there having been a few recent movies featuring mathematical prodigies. Even when it is present, mathematics is generally subsumed under science. In the case of the popular pursuit of gambling, where mathematical thinking might be considered to play an important role, this is generally not the case as 'luck' seems to be considered a critical factor for many people.

Yet, mathematics plays a much more prominent role as a gatekeeper in society than does science. For example, it is often used as a selection device for entry to higher education or employment, even when the skills being tested are unrelated to the ultimate purpose. In broad terms (e.g., modelling or simulations which reduce costs and/or danger), mathematics is considered to be publicly important; at the very same time as it is considered to be personally irrelevant (Niss, 1994), apart from the obvious examples of cooking, shopping and home maintenance. Politically, mathematics has been ascribed a *formatting role* in society (Skovsmose, 1994).

DIFFERENCES IN VALUES BETWEEN MATHEMATICS AND SCIENCE

This project was built on the work of the earlier VAMP project, and it used as the basic conceptual framework the six values component model developed by the author (Bishop, 1988) through analysis of the activities of mathematicians throughout Western history. In this model six sets of value clusters are structured as three complementary pairs, as shown in Figure 1.

The 3 dimensions are based on the original work of White (1959), a renowned culturologist, who proposed 4 components to explain cultural growth. White proposed these as technological, ideological, sentimental or attitudinal, and sociological, with the first being the driver of the others. The author argued that mathematics can be considered as a symbolic technology, representing White's technological component of culture, with the other three being considered as the values components.

The project involved two mathematics educators and two science educators, and in the first part of the project there was considerable discussion and analysis of this initial framework, particularly in relation to whether the same structure could hold for science (see Corrigan, Gunstone, Bishop, & Clarke, 2004). As a result of this analysis, a comparison of values between the mathematics and science educators was achieved, as shown in Figure 2.

1. Epistemology of the Knowledge (Ideological values)

1a Rationalism Reason Explanations Hypothetical Reasoning Abstractions Logical thinking Theories *1b Objectivism* Atomism Objectivising Materialism Concretising Determinism Symbolising Analogical thinking **2. How individuals relate to the knowledge (Sentimental or attitudinal values)** *2a Control* Prediction Mastery over environment Knowing Rules Security Power *2b Progress* Growth Questioning Alternativism Cumulative development of knowledge Generalisation **3. Knowledge and Society (Sociological values)** *3a Openness* Facts Universality Articulation Individual liberty Demonstration Sharing Verification 3b *Mystery* Abstractness Wonder Unclear origins Mystique Dehumanised knowledge

Figure 1. Values of western mathematical knowledge (Bishop, 1988).

Mathematics	Science
Rationalism Reason Explanations Hypothetical reasoning Abstractions Logical thinking Theories	***Rationalism*** Reason Explanations Hypothetical reasoning Abstractions Logical thinking Theories
Empiricism Atomism Objectivising Materialism Concretising Determinism Symbolising Analogical thinking	***Empiricism*** Atomism Objective Materialisation Symbolising Analogical thinking Precise Measurable Accuracy Coherence Fruitfulness Parsimony Identifying problems
Control Prediction Mastery over environment Knowing Rules Security Power	***Control*** Prediction Mastery over problems Knowing Rules Paradigms Circumstance of activity
Progress Growth Questioning Cumulative Development of knowledge Generalisation Alternativism	***Progress*** Growth Cumulative development of knowledge Generalisation Deepened understanding Plausible alternatives
Openness Facts Universality Articulation Individual liberty Demonstration Sharing Verification	***Openness*** Articulation Sharing Credibility Individual liberty Human construction
Mystery Abstractness Wonder Unclear origins Mystique Dehumanised knowledge Intuition	***Mystery*** Intuition Guesses Daydreams Curiosity Fascination

Figure 2: Comparison between values associated with mathematics and science.

As can be seen there is a considerable amount of agreement, but there are some important differences. As far as the Ideological dimension is concerned there are both similarities and differences. In the cluster of Rationalism there is much agreement, as both subjects require the use of all the logic skills available and thus emphasise the range of values associated with those skills. With the value cluster of Objectism, which became recast as 'Empiricism' in order to accommodate the scientists' approach, there is also some agreement, but the highly empirical nature of science means that it has many more value aspects there than does mathematics. The experimental and observational activities of science bring other values into play than we can find in doing mathematics.

For the Sentimental dimension, with the complementary pairing of Control and Progress, there was once again some agreement between the mathematics and science educators about the Control value cluster, with its emphasis on prediction, mastery, and procedural rules. However the circumstances of the activity and different paradigms are significant in science but have little meaning in mathematics. Likewise with Progress, the idea of the cumulative development of knowledge is clearly similar, but the role of science teaching in continuing to deepen learners' understanding of a phenomenon again has no parallel in mathematics development.

Some other differences appear with the Sociological dimension, that is the way society relates to the knowledge of the subjects. In relation to the Openness value cluster, the emphasis of science on credibility and human construction are significant, compared with the idea of 'facts' in mathematics and the value of verification, sometimes via proof. With Mystery, which itself is a rather mysterious category, the dehumanised nature of mathematics with its abstractness and unclear notions of the origins of ideas contrasts strongly with the intuition, daydreaming, and empirically-based guesses of the scientists.

When considering these contrasts it is important to remember that this framework involves pairs of clustered values along the three dimensions. So the two clusters should not be considered as dichotomous, but rather as complementarities of each other. For example, Openness is the complement of Mystery, and therefore both clusters are present to some extent in that value dimension. Furthermore, what the model suggests is not that science and mathematics are markedly different but that there are strong similarities in their values, as befits their common heritage. There are however some interesting and, in terms of education, revealing different values represented also.

TEACHERS' VIEWS OF MATHEMATICS AND SCIENCE VALUES

We now turn to some of the data collected from primary and secondary teachers by means of specially constructed questionnaires. They are based on the three complementary pairs, Rationalism and Empiricism, Control and Progress, Openness and Mystery, discussed above. For the purposes of this paper only part of the questionnaires will be considered. These are the questions which concern the relative emphasis given by teachers to the values described above (see Appendix 1). The statements in these questions are the same for mathematics and science, and 13 primary and 17 secondary teachers volunteered to answer these questionnaires. Primary teachers in the state system in Australia teach both subjects to their classes, and we selected secondary teachers who also taught both subjects to the same classes. However one primary teacher responded only to the mathematics questionnaire and one secondary teacher responded only to the science questionnaire.

The structure of the questions 3 and 4 is as follows. Each question contains 6 statements to be ranked by the teachers. Each statement relates to one of the values clusters, for example, the statement "It develops creativity, basing alternative and new ideas on established ones" relates to the value of Progress. The other statements follow closely the other value descriptors in Figure 2 above although their order is different in the two questions. Note also that the teachers were not made aware of the value structure underlying the two questions and each of the six statements. Tables 1 and 2 show the results from the two groups of teachers in terms of their rankings and the means of the rankings for the six values clusters.

For Question 3, "Maths/Science is valued in the school curriculum because…" the primary teachers showed considerable similarity between the orders for maths and science with Empiricism and Rationalism being the most important values for both. Control was seen as by far the least important value, which is surprising given the findings about Control in the textbook study of Seah (1999). For the secondary teachers we can see an important and perhaps predictable difference between the rankings for mathematics and science between Rationalism and Empiricism. Once again Control is a distinct last choice for both. There are also interesting, and once again predictable, differences between the rankings of Mystery for mathematics and science for both groups of teachers.

Table 1
Rank orders & mean ranks for Question 3

(a) Primary

Value	*Rationalism*	*Empiricism*	*Control*	*Progress*	*Openness*	*Mystery*
Maths rank (mean rank)	2 (2.30)	1 (1.46)	6 (5.23)	4 (3.15)	3 (3.53)	5 (3.61)
Science rank (mean rank)	2 (2.75)	1 (1.41)	6 (4.91)	4 (3.41)	5 (3.66)	3 (3.00)

(b) Secondary

Value	*Rationalism*	*Empiricism*	*Control*	*Progress*	*Openness*	*Mystery*
Maths rank (mean rank)	1 (1.94)	2 (2.05)	6 (4.52)	4 (3.88)	3 (3.35)	5 (4.29)
Science rank (mean rank)	4 (3.18)	1 (1.25)	6 (5.87)	4 (3.18)	3 (3.06)	2 (2.81)

For Question 4 "Mathematics/Science is valuable knowledge because…", once again the primary teachers put Empiricism firmly at the top of the list for both subjects, but their second choices are interestingly different. For mathematics they favoured Progress while for science they favoured Mystery. Their last choices are also markedly different, with Mystery being given that place for mathematics and Control for science. For the secondary teachers, Rationalism and Empiricism stand out as the top values for mathematics, while Empiricism stands very much alone at the top for science. At the bottom, the pattern is the same as for the primary teachers, with Mystery occupying that place for mathematics and Control for science.

Table 2
Rank orders and mean ranks for Question 4

(a) Primary

Value	*Rationalism*	*Empiricism*	*Control*	*Progress*	*Openness*	*Mystery*
Maths rank (mean rank)	3 (3.66)	1 (1.33)	5 (3.75)	2 (3.00)	3 (3.66)	6 (3.83)
Science rank (mean rank)	4 (3.41)	1 (1.41)	6 (4.75)	3 (3.33)	5 (3.83)	2 (2.58)

(b) Secondary

Value	*Rationalism*	*Empiricism*	*Control*	*Progress*	*Openness*	*Mystery*
Maths rank (mean rank)	1 (1.70)	2 (1.82)	3 (3.44)	4 (4.00)	4 (4.00)	6 (4.47)
Science rank (mean rank)	3 (3.12)	1 (1.25)	6 (4.12)	2 (3.00)	5 (4.06)	4 (3.33)

CONCLUSIONS AND IMPLICATIONS

The comparison of the values between the science and mathematics educators in the project has revealed perceptions of some important differences between the two subjects. It has also helped to clarify the values structure underlying the current project. In particular, regarding the Ideological dimension, there was evidence that mathematics favours the cluster of Rationalism while science emphasises Empiricism.

With the Sentimental dimension, while both subjects favour Control, the values of Progress differ, with science education seeking to deepen understanding of relationships rather than constructing new knowledge as in mathematics. Concerning the Sociological dimension, there are important differences in both the Openness and Mystery clusters with science seeming to relate more to the humanising aspects of knowledge compared with mathematics.

The comparisons between the values in mathematics and science for the teachers also show interesting differences, reflecting their concerns with the curriculum and teaching at their respective levels. At the primary level the teachers favour Empiricism over Rationalism for both science and mathematics, though both are important, and this contrasts with the findings above. At the primary level of course much mathematical work is empirical in nature. For the Sentimental dimension, Control is much less favoured than Progress also for both. The main difference between the subjects appears in the Sociological dimension where Openness and Mystery reverse their positions with the two subjects, the first being more favoured than the second in mathematics and the reverse in science. This difference shown by the primary teachers reflects the educational implications of the educators' views above.

For the secondary teachers, the Ideological dimension reflects the educators' views, with mathematics favouring Rationalism and science favouring Empiricism, disagreeing with the primary teachers. For the Sentimental dimension, the secondary teachers largely agree with their primary colleagues and for the Sociological dimension, they again agree with their primary colleagues favouring Openness for mathematics compared with Mystery, and reversing these for science. Indeed mystery for science is ranked 2 and 4 by the secondary teachers and ranked 2 and 3 by the primary teachers, showing how significant they consider that aspect to be.

In general, the conceptualisation put forward for this project has begun to show interesting and interpretable results. Discussions with the teachers have revealed an interest in the issues of values teaching in all subjects, but also a lack of vocabulary, and conceptual tools to enable them to develop explicitly the values underlying mathematics education. One of the goals of this project is by contrasting mathematics and science, to help teachers develop those conceptual tools further. As we have seen, and as has been shown above, the contrasts between these two closely related forms of knowledge are provocative, and already reveal worthwhile challenges for mathematics teaching to pursue.

For example, the difference between the emphasis on Empiricism at primary level and on Rationalism at secondary level implies some important challenges for explicit values development in the teaching of mathematics at those two levels. How should that values development be smoothed across the primary/secondary divide?

The differences in the views on Progress are also revealing, with the development of understanding in science contrasting with the construction of new knowledge in mathematics. How can we reconstruct our views of the mathematics curriculum so that progress through that curriculum is not just a matter of acquiring new knowledge but of ensuring that it also deepens learners' understanding of what has been taught before?

Finally could the dehumanised, highly abstract and mystique-laden value of Mystery of mathematics which appears to be such an obstacle to mathematics learners be made more explicit so that it could be challenged by the more humanised and personal intuitive nature of that value which science appears to enjoy?

ACKNOWLEDGEMENT

Thanks are due to my colleagues Debbie Corrigan, Barbara Clarke, and Dick Gunstone for their contributions to this project and to this paper.

REFERENCES

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht, the Netherlands: Kluwer Academic Publishers.

Bishop A. J. (1999). Mathematics teaching and values education: An intersection in need of research. *Zentralblatt fur Didaktik der Mathematik*, *31*(1), 1–4

Bishop, A. J. (2002, April). Research policy and practice: the case of values. Paper presented to the Third conference of the Mathematics Education and Society Group, Helsingor, Denmark.

Corrigan, D. J., Gunstone, R.F., Bishop, A. J., & Clarke, B. (2004). Values in science and mathematics education: mapping the relationships between pedagogical practices and student outcomes. Paper presented at Summer School of the European Science Educational Research Association, Mulheim, Germany, August, 2004.

FitzSimons, G. E., Seah, W. T., Bishop, A. J. & Clarkson, P. C. (2000). What might be learned from researching values in mathematics education? In T. Nakahara & M. Koyama (Eds.), *Proceedings of the 24th conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, p. 153). Hiroshima: Hiroshima University.

Halstead, M. (1996). Values and values education in schools. In J. M. Halstead & M. J. Taylor (Eds.), *Values in education and education in values* (pp. 3–14). London: Falmer.

Kluckholm, C. (1962). *Culture and behaviour*. New York: Macmillan.

Kohlberg, L. (1981). *The philosophy of moral development: Moral stages and the idea of justice* (1st ed.). San Francisco: Harper & Row.

Krathwohl, D. R., Bloom, B. S., & Masia, B. B. (1964). *Taxonomy of educational objectives: The classification of educational goals* (Handbook II: Affective domain). New York: David McKay.

Le Métais, J. (1997). *Values and aims underlying curriculum and assessment* (International Review of Curriculum and Assessment Frameworks Paper 1). London: School Curriculum and Assessment Authority.

Niss, M. (1994). Mathematics in society. In R. Biehler, R. W. Scholz, R. Strässer & B. Winkelmann (Eds.), *Didactics of mathematics as a scientific discipline* (pp. 367–378). Dordrecht: Kluwer Academic Publishers.

Nixon, J. (1995). Teaching as a profession of values. In J. Smyth (Ed.), *Critical discourses on teacher development* (pp. 215–224). London: Cassell.

OECD. (2003). *Assessment framework – mathematics, reading, science and problem solving: knowledge and skills*. Paris: OECD.

Pritchard, A. J., & Buckland, D. J. (1986). *Leisure, values and biology teaching.* (Science and Technology Education, Document Series No. 22). Paris: UNESCO.

Raths, L. E., Harmin, M., & Simon, S. B. (1987). Selections from 'values and teaching'. In J. P. F. Carbone (Ed.), *Value theory and education* (pp. 198–214). Malabar, FL: Robert E. Krieger.

Rokeach, M. (1973). *The nature of human values*. New York: The Free Press.

Seah, W.T. (1999). The portrayal and relative emphasis of mathematics and mathematics educational values in Victoria and Singapore lower secondary school textbooks: A preliminary study. Unpublished Master of Education thesis, Monash University, Melbourne, Australia.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer Academic Publishers.

UNESCO (1991). *Values and ethics and the science and technology curriculum*. Bangkok: Asia and the Pacific Programme of Educational Innovation for Development, UNESCO-729.

White, L. A. (1959). *The evolution of culture.* New York: McGraw-Hill.

APPENDIX 1

QUESTIONS 3 AND 4 FROM THE MATHEMATICS QUESTIONNAIRE FOR TEACHERS

For the next two items please rank the six statements accordingly in the accompanying boxes, where '1' indicates your first choice, '2' your second choice, '3' your third choice, etc. Note that the same ranking value can be given to more than one statement. Please rank each statement.

3. "For me, Mathematics is valued in the school curriculum because…."

	Ranking
It develops creativity, basing alternative and new ideas on established ones	☐
It develops rational thinking and logical argument	☐
It develops articulation, explanation and criticism of ideas	☐
It provides an understanding of the world around us	☐
It is a secure subject, dealing with routine procedures and established rules	☐
It emphasises the wonder, fascination and mystique of surprising ideas	☐

4. "For me, Mathematics is valuable knowledge because…"

	Ranking
It emphasises argument, reasoning and logical analysis	☐
It deals with situations and ideas that come from the real world	☐
It emphasises the control of situations through its applications	☐
New knowledge is created from already established structures	☐
Its ideas and methods are testable and verifiable	☐
It is full of fascinating ideas which seem to exist independently of human actions	☐

TEACHING MATHEMATICS FOR EQUITY: LEARNER CONTRIBUTIONS AND LESSON STRUCTURE

Karin Brodie
Wits University
<brodiek@educ.wits.ac.za>

This paper engages with the debate as to whether reform pedagogies promote or inhibit equity in mathematics learning, particularly in under-resourced countries such as South Africa. A key issue in the debate is whether teachers can simultaneously engage with learners' current knowledge and experience, as well as ensure mathematical progression and development. Using data from two classrooms in South Africa, one well resourced and one poorly resourced, I show how this can be done. I use two coding systems, one for learner contributions and one for lesson structure to show how the teachers manage this tension in their lessons.

INTRODUCTION

This paper engages with the relationships between reform pedagogy and equity. It is well known that mathematics achievement is inequitably distributed both among countries (as shown by international comparison studies such as TIMMS) and within countries, along the dimensions of race and class. Curriculum developments (reform mathematics) in many countries over the past 20 years have as a goal the reduction of inequalities in mathematics achievement and learning. In South Africa a new curriculum has been in place since 1998, which promotes teachers' listening to and taking learners' ideas seriously, and using learners' ideas and experiences to enable their mathematics learning. An important critique has been raised of the new curriculum, which is that it will widen, rather than narrow, the gap between advantaged and disadvantaged learners and schools, because working with learners' ideas may impede mathematical progress and development. In this paper I use data from two schools in South Africa, one extremely well resourced and serving predominantly wealthy learners, and the other relatively poorly resourced and serving extremely poor learners, to argue that it is possible for teachers in low SES (socio-economic status) schools with weaker learners to structure lessons in ways that both take account of learners' ideas and enable them to make mathematical progress. To do this, I use two sets of codes to analyse the data. The first set describes learners' contributions in the lessons and the second describes lesson structure. So a second contribution of this paper is the development of methods of analysis for mathematics classroom interaction.

WHO IS SERVED BY REFORMS?

The key question that this paper addresses is whether reform pedagogies will promote or inhibit equity in mathematics learning. A number of researchers have addressed this question. In a small study in her own classroom, Lubienski (2000) observed that lower and higher SES students responded differently to the open, contextualized nature of the problems and to the less directive nature of her pedagogy. She argued that lower SES students demanded more guidance and direction from the teacher. In a study of about 600 students, Cooper and Dunne (2000) found that middle-class students did better than working class students on test items that relate to everyday life. They argued that with any realistic item, only some aspects of the 'real' context are drawn on to solve the problem, while others need to be bracketed out. Choosing which aspects to bracket out is a difficult task. Middle-class children traversed more easily the boundaries between what is appropriate or not to include or leave out in their thinking about the problems. Working class children were more likely to focus on the real constraints of the task and in so doing masked their mathematical competence. While they could do the mathematics that the task required, they chose not to, because they interpreted the situation, sensibly, in a way that did not require the particular mathematics that was being tested. In relation to race and reform pedagogy in English teaching in the United States, Delpit (1986) argues very powerfully that while communicative and meaning making approaches are important, black students also need to be taught the codes of power, which include the technical and grammatical skills necessary for reading and writing, and the ways of speaking and communicating that are dominant in society. She argues that society is structured according to certain codes, which are defined by those in power who are predominantly white. Children who are members of advantaged communities have access to these codes through participation in their everyday culture. The role of equity-producing schools should be to transmit these codes to black students who do not have access to them through their homes or other resources.

In South Africa, the debate is complicated by two conditions. First, there is a severe shortage of qualified mathematics teachers (Department of Education, 2001) and so qualified mathematics teachers can generally be attracted by the better conditions at higher SES schools. Second, many teachers who teach in lower SES schools are themselves the products of such schools and therefore themselves have weaker mathematical knowledge. The legacy of poor teaching in apartheid teacher training colleges means that those who did specialize in mathematics often did not develop much depth in mathematics beyond the school curriculum. So teachers in lower SES schools are often not confident or qualified for traditional mathematics teaching, let alone reforms which are much more demanding. A review of the new curriculum suggested that while most teachers are extremely supportive of it and try to teach in the ways it suggests, they are not usually successful, and teacher-centred practices remain prevalent (Chisholm et al., 2000). This is confirmed by other research studies (Brodie, Lelliott, & Davis, 2002a; Jansen, 1999; Taylor & Vinjevold, 1999). What these research studies also suggest is that teachers interpret the new curriculum as requiring particular forms, such as group work or increased learner talk, rather than paying attention to the substantive content of what learners are saying (Brodie, Lelliott, & Davis, 2002b). When teachers, particularly, but not only, those with weaker mathematical knowledge, struggle with the dilemma of validating learners' thinking and developing it at the same time, they usually resort to one or the other. They might ignore the learners' contributions in order to focus on the mathematics that needs to be learned, reverting to teacher-centred teaching. On the other hand, they might focus on the learners' contributions at the expense of the mathematics. In the second case, classrooms have been observed where there is a lot of learner talk on task, but very little mathematical development through the lessons because the teacher does not engage with learner ideas (Brodie, 1999; Brodie et al., 2002b; Taylor & Vinjevold, 1999).

In response to this situation, some South African scholars predict that if teachers try to facilitate learner participation rather than teach the mathematics directly, learners who are least resourced will suffer most. They argue that low SES, black learners with already weak mathematical knowledge will be most disadvantaged when the close control and framing that are features of the traditional curriculum are lost (Chisholm et al., 2000; Taylor, Muller, & Vinjevold, 2003; Taylor & Vinjevold, 1999). Similarly to Delpit (1986), they argue that if schools are to produce equity they must make the knowledge of power accessible to all. If this knowledge is implicit or less available in the classroom, those who have other means of obtaining it will do so, whereas those who do not will be left disempowered. One response to such arguments is to maintain traditional pedagogy, and try to strengthen it, for example by increasing teachers' conceptual knowledge (Taylor & Vinjevold, 1999).

Boaler (2002) argues cogently that maintaining traditional pedagogy will lead to even more inequities, because the benefits of reform pedagogy will be available only to those who are already advantaged, and poorer learners will continue to be under-served by pedagogies of poverty (Haberman, 1991). Boaler (2002) argues for the strengthening of reform pedagogies by explicitly teaching learning practices that are important in reform classrooms, for example: how to interpret mathematical and real-world problems, how to find ways to begin exploring open-ended problems and how to explain and justify ideas.

There is some evidence, from the United States and England, that teaching aligned with visions of reform produces better and more equitable achievement and learning in mathematics (Boaler, 1997, 2003; Hickey, Moore, & Pelligrino, 2001; Schoenfeld, 2002). There is little similar evidence from South African schools, mainly because of the difficulties of deciding what counts as "reform" practice. Since we have a national curriculum, which most teachers support, all teachers are ostensibly working with a reform curriculum. However, the research quoted above suggests that very few actually are. This paper looks at two teachers, one in a low SES school and one in a high SES school who developed a set of tasks to elicit and engage with learners mathematical reasoning and ideas. In describing their practices, I show how both teachers manage to both work with learners' ideas as well as make mathematical progress in the lesson.

THE TEACHERS, THEIR CONTEXTS AND THE DATA

Mr. Peters teaches in a government school that has only black learners who come from low SES homes, some of which are extremely poor. I used school fees and teachers' reports of parent occupations as an indication of SES. Mr. Peters' school charges R400 per year and most parents are unskilled workers, with many unemployed. The Grade 10 class in which the research took place had 45 learners. Mr. Peters' classroom had an "old style" desk with adjoined chair for each learner. There was a chalkboard and chalk, no overhead projector and screen and no electricity, so on rainy days the classroom was dark. Some windows were broken and the paint was peeling off. The school is located in an area where there is gang activity and some of this spills into and involves learners at the school.

Ms. King teaches in a private school, where most learners are from extremely wealthy homes, with the exception of the teachers' children and a few learners on scholarships. Almost all learners are white. The school fees are R40 000 per year and many parents are professionals or company executives. The Grade 10 class in which the research took place had 27 learners and was the second highest of seven classes in a tracked grade. Ms. King's classroom is part of a newly built wing of the school, is carpeted and has air-conditioning. There is a big table and chair for each learner. There are whiteboards and pens, cupboards and tables for storing paper and worksheets, an overhead projector and screen, and a television set which can be used for presentations from a computer.

Mr. Peters had 13 years mathematics teaching experience and Ms. King had 11. Both have extremely good mathematical content knowledge and pedagogical content knowledge. They were both students on an in-service mathematics education course and were thus better informed than most teachers about the new curriculum. As part of the research study, they worked together to develop a set of tasks that would elicit mathematical reasoning from their learners and planned how they would go about working with learners' thinking. I observed and videotaped the lessons where they used these tasks. I also interviewed both teachers a number of times and conducted task-based interviews with the learners. On the basis of the learner interviews and my observations it was clear that Mr. Peters' learners were operating way below Grade 10 level, while many of Ms. King's learners were advanced for Grade 10.

This paper focuses on my analysis of the lessons. The lessons were videotaped, transcribed and coded on the transcript while watching the videotapes. Data from the interviews were drawn upon where relevant. To analyse the lesson data, I developed two coding schemes, one for learner contributions in the lesson and one for lesson structure and progression in the lesson over time. I coded three lessons for each teacher[i].

LEARNER CONTRIBUTIONS

Learner contributions are indicators of a number of important aspects of the teaching and learning that is happening in the classroom. First, they show the extent to which learners are participating in and grappling with important ideas in the lesson. Second, they give indications of how the teacher is enabling learner participation. Third, they are indicators of the strength of learners' knowledge and confidence in the subject. I developed the following set of categories for learner contributions: Basic Errors, Appropriate Errors, Missing Information, Partial Insights, Complete and Correct and Beyond Task.

A first important distinction is between contributions that could count as complete and correct responses to a task, and those that were partial responses in some way, along either of two dimensions: completeness or correctness. Complete, Correct contributions are those that provide an adequate answer to a particular task or question. For example, in response to the task: *Can $x^2 + 1 = 0$ for x a Real number?*, the response: "No, because for $x^2 + 1$ to equal zero x^2 would need to be –1 and x^2 cannot be negative", is a Complete and Correct contribution.

Partial contributions are those that are either incomplete or incorrect in some way. There are three kinds of partial contributions, one of which is incorrect, one of which is incomplete, and one of which is both. An Appropriate Error is an incorrect contribution that mathematics teachers would expect at the particular grade level in relation to the task. For example, a claim that –x represents a negative number, coming from a Grade 10 learner would be classified an Appropriate Error. Appropriate Errors are distinguished from Basic Errors, which will be discussed below. A contribution that is Missing Information is correct but incomplete, when a learner presents some of the information required by the task, but not all of it. For example if a learner says that x^2 is always greater than zero, she is missing the information that x can be zero and therefore x^2 is always greater than or equal to zero. A Partial Insight contribution is one where a learner is grappling with an important idea, which is not quite complete, nor correct. An example of a Partial Insight would be when a learner argued that as she substituted lower numbers, the value of $x^2 + 1$ decreased. Therefore, if she tried a negative number for x, she would obtain zero for $x^2 + 1$.

A second distinction that is important is between Appropriate Errors and Basic Errors. Basic Errors are errors that one would not expect at the particular grade level, for example, errors in multiplying 1/2 by 1/2 in a Grade 10 classroom. Basic errors are in a different relation to the task, because they indicate that the learner is not struggling with the concepts that the task is intended to develop, but rather with other mathematical concepts that are necessary for completing the task, and have been taught in previous years.

Finally, contributions that go Beyond Task requirements are contributions that are related to the task or topic of the lesson but go beyond the immediate task and/or make some interesting links between ideas. For example, in response to the task: can $x^2 + 1$ be 0 if x is a real number, the response: If x is $\sqrt{-1}$, then x^2 can be 0, goes beyond the task because it brings in complex numbers.

These codes could account for all of the learner contributions in the lessons except for a small number that were so unclear that I could not make out the learner's meaning. This classification scheme is useful in a number of ways. First, it provides a way to characterize what happens in lessons. A lesson with many Complete and Correct contributions will look very different from one with many partial contributions, and both of these will look different from a lesson with many Basic Errors. Second, the kinds of contributions suggest the level of learner participation in the lesson. Third, they indicate how learner knowledge is implicated in lesson participation. Table 1 shows the distribution of learner contributions across the two classrooms.

Table 1
Distribution of learner contributions (percents of total)

	Mr. Peters	Ms. King
Basic Errors	21	1
Appropriate Errors	19	17
Missing Information	11	10
Partial Insights	8	3
Complete and Correct	35	59
Beyond Task	3	7
Unclear	4	2

The biggest differences across the two classrooms are in the Basic Errors and Complete and Correct contributions. This can be accounted for by the differential knowledge of the learners, but, as I will show in the next section, can also be directly linked to the teachers' different pedagogies. Also of interest are the small differences between Partial Insights and Beyond Task contributions. The Beyond Task contributions can be accounted for by the stronger knowledge of Ms. King's learners and by their access to other resources, since half of these came from a discussion of $\sqrt{-1}$ solving the equation $x^2 + 1 = 0$, which a group of learners had discussed with another teacher the previous day. The fact that Mr. Peters' learners showed more Partial Insights is particularly interesting, given their weaker knowledge, and can be directly linked to his pedagogy. Finally, the fact that Appropriate Errors and Missing Information contributions are similar across the classrooms suggests that these are not directly related to learner knowledge or the teachers' pedagogy. In the next section, I will discuss how the teachers' pedagogies account for the differences in Basic Errors, Partial Insights, Complete and Correct, and Beyond Task contributions.

TEACHING FOR EQUITY

In order to describe teaching over time, I developed a set of categories for lesson structure. In Ms. King's lessons, I identified two kinds of lesson structure: *parallel* and *hierarchical*, while Mr. Peters' lessons could be characterised as *hierarchical with detours*.

In a *parallel structure*, the ideas are linked only in that they are all responses to the same question. Neither the teacher nor the learners explicitly link the ideas to each other, or to other mathematical ideas. Each idea is discussed for some time and then the conversation moves to the next idea. In response to the task "Can $x^2 + 1 = 0$?", Ms. King put up four learner responses, which they had generated the previous day in their groups and which she had read. Two of these were Complete and Correct and two were Appropriate Errors. Ms. King asked for comment on each solution and encouraged learners to discuss them. Thus each idea was given equal status as an idea worthy of discussion. Through the conversation, Ms. King worked to correct the Appropriate Errors. However, she did not make links across the different solutions. For example, the two Appropriate Errors were: a learner arguing that x^2 can be represented as $–(1)^2$; and a learner arguing that $–x$ represents a negative number. Both of these suggest difficulties with the representation of negative numbers algebraically, which Ms. King might have chosen to work on in more detail, bringing the two responses together.

Parallel structures can be associated with reform teaching, and occurred most often in report-backs from group work. They give learners opportunities to express their thinking and to hear and discuss the thinking of others. However, they may not take the learners beyond their current ways of seeing, except to correct mistakes.

In a *hierarchical structure*, the mathematical ideas build on each other. Usually the teacher initiates ideas and builds the progression, but sometimes learners do. Table 2 below shows how Ms. King's questions built hierarchically to the point she wanted to make. There is a more obvious, linear progression of mathematical ideas.

Table 2
Hierarchical structure

Teacher Question	Learner Contribution	Code
If I want to represent an even number, how would I do it in algebra?	2x	complete, correct
And an odd number?	2x-1 or 2x+ 1	complete, correct
Now, we've thrown this x in, what does x mean?	x can be any real number	appropriate error
x can be what?	x can be any real number	appropriate error
Uh, can it? Let's talk about even numbers. For even numbers, what can x be? Can x be minus a half?	No, Yes	appropriate error/ complete correct
Can x be .3?	No	complete, correct
What can x be?	Natural numbers	complete, correct
What are natural numbers?	1,2,3	complete, correct

Hierarchical structures are usually associated with traditional teaching and look very similar to the IRE structure identified by Mehan (1979). They enable progression in the lessons but do not leave much space for extended learner contributions and grappling with ideas. The example above shows how this structure worked to constrain learners' contributions and how most of them are Complete and Correct. Ms. King's teaching moved between parallel and hierarchical structures, suggesting that she worked with both reform and traditional pedagogies in ways in which she thought appropriate. This gave space for both extended learner contributions and for her to ensure mathematical progression.

Mr. Peters' pedagogy illuminated another way of balancing learner contributions with mathematical progression, what I have called a *hierarchical structure with detours*. Similarly to Ms King, Mr. Peters had read the responses to the task generated by the groups the previous day. He chose three contributions to put on the board for discussion. The first was an Appropriate Error and was made by the vast majority of groups in the class. The second was a Partial Insight made by three groups and the third was Complete and Correct which only one group had. So Mr. Peters chose contributions to illuminate different ways of seeing the task, both correct and incorrect, as did Ms. King. In addition, he had a very clear sense of a hierarchy among the responses. He structured his lesson using this hierarchy and planned to work through the responses in the above order, identifying the strengths and weaknesses of each solution and showing how each was an improvement on the previous one.

As each response was discussed, it raised a number of Basic Errors. For example, the first contribution (an Appropriate Error) was that $x^2 + 1$ could not equal 0 because $x^2 + 1$ could not be simplified and had to remain $x^2 + 1$. As Mr. Peters opened this up for discussion, some learners claimed that $x^2 + 1 = 2x^2$ and others claimed that $x^2 = 1$ because "there's a 1 in front of the x". Mr. Peters detoured from the main discussion to deal with these Basic Errors and then came back to the contribution under discussion. He dealt with the Basic Errors relatively quickly, in a way that led the class to a Correct and Complete contribution in relation to the Basic Error.

Once he had spent considerable time on the first contribution, he moved on to the second, a Partial Insight, where learners had substituted various values for x to test whether $x^2 + 1$ could be zero. Mr. Peters used this contribution to engage with the error in the first contribution, trying to get the learners to see that $x^2 + 1$ took a range of values as x took a range of values. Again, as this discussion progressed, many Basic Errors surfaced, for example learners saying that $1/2 \times 1/2 = 1$. Mr. Peters detoured to deal with these and then came back to his main point, which was how to test the conjecture that $x^2 + 1$ could not be zero.

The learners with the Partial Insight had tested the expression with positive numbers and zero, but had not gone beyond zero. Only the group who got the Complete Correct response had done this. Through discussion, Mr. Peters got other learners to see this as a natural progression of testing the expression and the conversation moved smoothly into a discussion of the Complete Correct contribution, which was that for $x^2 + 1$ to equal zero x^2 would need to be -1 and x^2 cannot be negative.

As Mr. Peters moved through the three contributions, learners generated some new Partial Insights (in addition to the Basic Errors discussed already discussed). For example, some learners began to express the idea, somewhat inarticulately, that since x^2 already gave you a "value" and you were adding 1, the result could not be zero. Mr. Peters challenged them to grapple with the nature of the "value" that x^2 could be.

So Mr. Peters worked hierarchically with a clear sense of where he was going over the lessons as a whole, and at the same time was able to diverge from his carefully planned structure to open up and take into account learners' ideas. However, he never diverged totally; he always came back to his agenda of discussing the task and more importantly he linked the different contributions together as increasingly appropriate ways of approaching the task. He built a progression through the contributions for the learners. His pedagogy accounts for the interesting mix of Basic and Appropriate Errors and Complete Correct contributions, together with a small number of Partial Insights as shown in Table 1. This suggests that while learners made many mistakes and Mr. Peters did deal with them, they were at the same time beginning to grapple with important mathematical ideas in a way that Ms. King's stronger learners were not.

CONCLUSION

The analysis above shows both that and how a teacher of low SES, poorly achieving learners can use their contributions to structure a lesson that both works with their contributions and makes mathematical progress. Mr. Peters worked with a range of learner contributions in different ways. He accepted the many Basic Errors that his learners made and moved quickly to correct them. He structured discussion around Appropriate Errors and Partial Insights, which enabled learners to grapple with some important mathematical ideas. The many errors that his learners made presented a demanding challenge for Mr. Peters, but he was able to work with these in interesting ways.

Ms King, a teacher of high SES, high achieving learners was able to use the resources that she and her learners had to enable them to go Beyond the Task requirements and to think about further mathematics in more depth. The percentages in both classrooms of Partial Insights and Beyond Task contributions, together with the qualitative analysis, suggest that both teachers were able to work with their learners' current knowledge and take this knowledge further.

These two cases contribute to the argument that reform pedagogies can be used appropriately in lower SES and lower achieving classrooms and that they can contribute to more equitable mathematics learning[ii]. However, it may be that reform pedagogies look different in different classrooms. Ms. King moved between parallel and hierarchical structures while Mr Peters built a hierarchical structure with detours for learner contributions.

It should be noted that both these teachers were highly skilled and have very good mathematics and pedagogical content knowledge. Whether less skilled and knowledgeable teachers can also manage the tension between learner contributions and mathematical progression will be the subject of a subsequent research project. The codes for learner contributions and lesson structure will enable the analysis of a range of different classrooms from an equity perspective.

---

[i] There were four teachers in the study although this paper focuses only on two. The codes were developed for and apply across all the teachers.

[ii] I use the word learning and base this claim on the Learner Contributions data. I do not have achievement data.

REFERENCES

Boaler, J. (1997). *Experiencing school mathematics: Teaching styles, sex and setting*. Buckingham: Open University Press.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33*(4), 239–258.

Boaler, J. (2003). *Equitable teaching practices: The case of Railside.* Paper presented at the Annual conference of the California Mathematics Council, Asilomar, California, December 2003.

Brodie, K. (1999). Working with pupils' meanings: Changing practices among teachers enrolled on an in-service course in South Africa. In O. Zaslavsky (Ed.), *Proceedings of the 23rd conference of the International Group for the Psychology of Mathematics Education* (Vol. 2, pp. 145–152). Haifa: Israel Institute of Technology.

Brodie, K., Lelliott, T., & Davis, H. (2002a). Developing learner-centred practices through the FDE programme. In J. Adler & Y. Reed (Eds.), *Challenges of teacher development: An investigation of take-up in South Africa.* (pp. 94–117). Pretoria: Van Schaik.

Brodie, K., Lelliott, T., & Davis, H. (2002b). Forms and substance in learner-centred teaching: Teachers' take-up from an in-service programme in South Africa. *Teaching and Teacher Education, 18*, 541–559.

Chisholm, L., Volmink, J., Ndhlovu, T., Potenza, E., Mahomed, H., Muller, J., et al. (2000). *A South African curriculum for the twenty first century. Report of the review committee on Curriculum 2005.* Pretoria: Department of Education.

Cooper, B., & Dunne, M. (2000). *Assessing children's mathematical knowledge: Social class, sex and problem solving*. Buckingham: Open University Press.

Delpit, L. D. (1986). Skills and other dilemmas of a progressive black educator. *Harvard Educational Review, 56*(4), 379–385.

Department of Education. (2001). *National Strategy for Mathematics, Science and Technology in General and Further Education and Training.* Pretoria: National Department of Education.

Haberman, M. (1991). The pedagogy of poverty versus good teaching. *Phi Delta Kappan, 73*, 290–294.

Hickey, D. T., Moore, A. L., & Pelligrino, J. W. (2001). The motivational and academic consequences of elementary mathematics environments: Do constructivist innovations and reforms make a difference? *American Educational Research Journal, 38*(3), 611–652.

Jansen, J. (1999). A very noisy OBE: the implementation of OBE in Grade 1 classrooms. In J. Jansen & P. Christie (Eds.), *Changing curriculum: Studies on outcomes-based education in South Africa.* (pp. 203–217). Cape Town: Juta.

Lubienski, S. T. (2000). Problem solving as a means towards mathematics for all: An exploratory look through a class lens. *Journal for research in mathematics education, 31*(4), 454–482.

Mehan, H. (1979). *Learning lessons: Social organisation in the classroom*. Cambridge, MA: Harvard University Press.

Schoenfeld, A. H. (2002). Making mathematics work for all children: Issues of standards, testing, and equity. *Educational Researcher, 31*(1), 13–25.

Taylor, N., Muller, J., & Vinjevold, P. (2003). *Getting schools working: Research and systemic school reform in South Africa*. Cape Town: Pearson Education.

Taylor, N., & Vinjevold, P. (Eds.). (1999). *Getting learning right: Report of the President's Education Initiative Research Project.* Johannesburg: Joint Education Trust.

THE ART OF MATHEMATICS: BEDDING DOWN FOR A NEW ERA

Tony Brown
Institute of Education, Manchester Metropolitan University
<a.m.brown@mmu.ac.uk>

What analogies might we productively draw between mathematics and art education? How might we see the promotion of aesthetic appreciation as a motivating factor in mathematics? How might we define the relation between mathematical and artistic objects and human subjects? These questions led to more general concerns with how humans relate to mathematics, and, in stepping back from that, to how we might understand the notion of "relation" in this context. Ultimately, it addresses the question of how we might understand the shifting borders defining the space that houses mathematical thinking and learning as we begin a new century where "mathematical" and "pedagogy" become increasingly contentious terms.

This paper was inspired by a session I attended at a Psychology of Mathematics Education conference. Nathalie Sinclair (2003) was presenting some of her work on how we might see mathematics in terms of its aesthetic qualities. Various contributors at the session commented on the qualities of different mathematical proofs using such criteria as "economic", "neat" and "elegant". I found the session especially intriguing and engaging as it was targeted at examining how such qualities might become more prominent in our teaching with children.

Nevertheless, I found myself adopting a fairly guarded attitude, as so often, when mathematics is presented as aesthetically pleasing, it draws attention to qualities missed by so many people. As a result, I found myself wrong footed in that my main motive as a mathematical educator is to find ways of the subject reaching a broader audience and in an environment where mathematics' prominent position on the school curriculum is underwritten by its perceived utility. Sinclair's session appeared more like an art appreciation class and in art education rather different sorts of motivations are offered in defending its more marginal position on the curriculum. As a result I started to explore the ways in which mathematics and art educators sell their wares differently to children in schools and to their sponsors. And in doing this I began to question how a notion of aesthetic might function as a motivation in creating and evaluating mathematical or artistic objects. I commence by considering the human subject directs their thinking and float some options as to how this subject might be understood. By considering how the thinking might be understood in relation to an art object I prepare the ground for drawing analogies for the perception of mathematical objects. Art, however, is not about the simple production of objects. Nor is it just about aesthetics. Rather its performative aspects draw in the response that it activates to become part of its own domain. Thus we consider how mathematics and its learning might be seen as elements of performance shaped by societal conceptions of the task it serves. After considering some examples of mathematical performance I conclude with identifying some immediate implications for interpreting our present practice in mathematics teaching and a longer term prognosis for how mathematics might evolve in tandem with newer conceptions of the human subject and how they learn.

PHYSICAL OBJECTS, HUMAN SUBJECTS

In the voluminous works of Sigmund Freud a central concept was that of *ego*. Nevertheless, Freud's work developed over some fifty years and the way in which Freud deployed such key terms evolved through successive meanings. The notion of ego has been the basis of some especially contentious debate. Without doubt Freud was ambivalent on this issue and some of his later work left it unresolved. In his earlier work (e.g., Freud, 1923) Freud understood the ego as a biological entity and his paper established a cartographic representation of the human mind comprising ego, id and super ego. In this conception of the ego, psychoanalytical treatment was understood in terms of developing the ego to increasingly occupy the territory governed by the id. This was announced by the slogan: *Where the id was the ego shall be*. This version of the ego was embraced by the US ego psychology school and has gained an image of seeing psychoanalytic therapy in terms of calming the ego to be more conformist. At various other points, including some of his very latest work, the ego was understood very differently. It was understood as a relational entity produced through the subject's identification with other people and the world around. It was this version of the ego that has been developed by Freud's belligerent disciple Jacques Lacan. For Lacan the ego is an inauthentic agency derived from a delusional stance in which the human subject has an image of his or her self. As such the ego is something to be challenged. Therapy is targeted instead at locating the truth of the patient's desire. That is, treatment is seen in terms of understanding how the unconscious functions in conditioning the patient's expressed demands. Lacan suggests that when the analysand says "I", the analyst should be mistrustful. That is, the image of self portrayed needs to be inspected to discover how it is a distortion of the desires being activated. Grosz (1989) offers a helpful summary of the two egos.

In the first model we would find a fairly familiar depiction of an individual human subject existing as a standalone biological entity who casts his or her attention over mathematical or artistic objects that also standalone. The psychological basis of so much mathematics education research would fall under this banner as encapsulated in the tradition of the Psychology of Mathematics Education organisation. The alternative is not a sociological model as may be supposed. Or rather, it is not sociological in that we work from a holistic conception of society that shapes individuals within it. Lacan's relational ego is a result of fantasy. This fantasy does not have negative connotations. Rather, our understanding of reality is seen as being structured through such fantasies. Fantasies might be seen as the filters through which we inspect reality, a reality that in a sense cannot be perceived directly, and in some other senses is not there at all except through its manifestation in the fantasies of individuals. In this cartography we would not have a standalone biological human confronting an independent object. For analytical purposes the space would be carved up differently. There would be no overarching perspective from an independent arbiter. The analysis would be centred on the human subject's supposed relation to the object and the world he or she crafts around it.

Let us take some examples from art to illustrate potential understanding of this relationally produced subject. In his book, *Post-Modernism, or the Cultural Logic of Late Capitalism*, Jameson (1971, pp. 6–10) made an interesting distinction between Modernism and Post-Modernism. He associates hermeneutical depth interpretation with the former and the textuality of post-structuralism with the latter. He illustrates this by contrasting two paintings described briefly here. I have discussed this more elsewhere (Brown, 2001).

"A Pair of Boots" by van Gogh

Jameson suggests that van Gogh's painting of a pair of peasant's boots gives rise to the possibility of various interpretations. He offers the magnificence of the bucolic landscapes we might normally associate with the paintings of van Gogh or alternatively, the stark peasant lifestyle suggested by such clothing. Either view can be developed as a fairly full account of what the painting might be seen as evoking.

"Diamond Dust Shoes" by Warhol

Warhol's effort, a dark, sparse, shadowy affair that may have been produced with the help of an X-Ray machine or a photocopier seems to defy any such generation of stories. It seems to be *all in the surface* - it begins and ends with the painting.

These two works, a century and a half a century old respectively, were both challenges to their respective publics. They were not about beauty. They redefined draughtsmanship or aesthetics in any previously known sense. They were both seemingly targeted at activating something in the viewer beyond mere appreciation. But one might suggest that we have moved on from here and the domain of art criticism has entered very different territory. Let us take two newsworthy British examples from the last five years.

Tracy Emin's Bed

Tracy Emin caused one of the most notorious controversies in British public life with her entry for the prestigious Turner Prize. The exhibit comprised her bed. The bed, allegedly, had been transported in to the gallery "as it was" with unmade sheets and various items unmentionable in polite society.

Holiday in Spain

A group of art students from Leeds were given a grant to enable them to produce their degree show. They took some photos purportedly to depict how they had spent the money. The photos showed the group in Spain enjoying a holiday and they organized a reception committee to greet them as they arrived at Leeds-Bradford airport. Soon after the British tabloid press had a field day berating lazy students for wasting taxpayer's money, while the more serious papers published articles discussing the meaning of art. The students then revealed that the whole thing had been a carefully staged event and that the photos were in fact a fabrication taken at a beach a few miles away in cold conditions. They had not spent any of the money. Their degree show comprised the "original" photos, the newspaper headlines, a performance at the airport and an account of how they had produced the fabrication.

Both works presumably were designed to be provocative. In some sense these two works take post-modernism one step further than Jameson's account. That is, the art is not so much in the surface, as outside of it. The work of art is to some extent defined by the reaction to it. For Emin's piece the question seemed to be why is this seen as art? How come it is being considered for a major award? What can we say about the way in which this work activated public reaction? How does it transgress public conceptions of the personal? This very transgressive attitude, asserting its own awkwardness of fit into what counts as art, activated the response that became part of the art object's specific evolution in public conception and of art generally in its now expanded form. Emin's piece became a British cultural icon of the nineties and she has established herself as a major artist, part of a re-defined British art establishment. As I write the Weekly Telegraph reports that the Tate Gallery has bought a selection of her major works to be put in a room dedicated to her work. The *Holiday in Spain*, as a degree show, had its public response and media feedback built into its very physical presence. Public or, at least, media reaction was something that the work sought to alert us to in a new way. Here, the relational space between human and object is not usefully seen as a space between an object and biologically defined ego. The relational ego is rather more helpful in coming to terms with such art in that the works are about the way in which you relate to them and what that assumed relationship tells you about yourself and your connection to the world of which you are part. They were extremely successful in activating a multi-layered debate with wide participation on the question of what is art? Arguably, they achieved this more than any other British artist apart from Emin. Would the staging of such a large-scale public event be possible to address the question of what is mathematics?

MATHEMATICS (EDUCATION) AS PERFORMANCE

Mathematics must surely join art in seeing its patch as transcending singular notions of beauty. Mathematics, like art, can teach us about ourselves, but not necessarily through didactic means. That is, the student may be allowed to learn their own lesson rather than the one supposed by their teacher or some other authority. Further, Deborah Britzman (2003) has shown us how the lessons of school can take many years to settle and take us by surprise when we least expect. The content as defined by the teacher has a very different cartography to the assimilation of it made by students. Elsewhere, I have argued that we have to see the boundaries of mathematical activity more broadly (Brown & McNamara, 2004). This point will be developed a little in the final section. What follows are brief accounts of four mathematical events; a public lecture on advanced mathematics, the publication of a popular book discussing mathematical physics, a public mathematics examination for aspiring primary teachers and a controversy surrounding the publication of some school examination statistics. These accounts each situate mathematics in a public domain and are chosen to exemplify cultural manifestations of mathematics that perhaps offer some opportunity to better understand the public image of mathematics. This provides a route towards understanding how mathematics, inspected more locally, might enable us to learn a little about ourselves through the way in which mathematics is used and understood. The descriptions will be followed by an evaluation of them in terms of how they predicate conceptions of mathematics and people responding to mathematics as a cultural phenomenon.

CAROLINE SERIES LECTURE

The mathematician Caroline Series recently gave a number of lectures in New Zealand. The publicity for the events featured intricate and colourful images reminiscent, to a novice at least, of the widely known Mandelbroit images of fractals. See Mumford, Series and Wright (2002). The lecture in Hamilton was widely attended. It would be unusual for many mathematicians to command such attention. The mathematics presented was probably inaccessible to many of the people present including myself, yet the images were sufficient for me to warm to the associated symbols. It was as if the beauty of the images invested the symbols with a meaning that could not be deduced through the symbols themselves. To what extent might we see this oscillating between the depth of van Gogh insofar as the meaning of his painting is understood as being a profound association between the symbols and the pictures? Yet for so many, certainly for me, it appeared to be in the surface, as with Warhol's shoes, attractive images that stand in the way of access to any depth behind. In some sense this is familiar territory as regards assumed links between mathematics and art. Geometric images such as Islamic patterns are common in classrooms as learning (e.g., Sinclair, 2001; Sharp, 2001). Here art and mathematics are seen as intersecting in images or objects for the physically independent human to behold. This appears to be in the same genre as Sinclair's work with children. It certainly had the same attraction for me except that I understood the mathematics in Sinclair's session and that I could envisage the possibilities for children across a broader spectrum. More recently, Sinclair (2004) has supplied some intriguing hints at how such aesthetic concerns can link to affective experience of mathematics.

STEPHEN HAWKING'S "THE BRIEF HISTORY OF TIME"

Stephen Hawking's book was a best seller (Hawking, 1988). The contemporary successor to Lacan, the cultural theorist Slavoj Zizek (1997, p. 173) offers a somewhat unsympathetic reading of this success. He asks "Would his ruminations about the fate of the universe, his endeavour to 'read the mind of God', remain so attractive to the public if it were not for the fact that they emanate from the crippled, paralysed body, communicating with the world only through the feeble movement of one finger and speaking with a machine generated impersonal voice?" Zizek contends that Hawking's iconic status results from this very disability in that Hawking stands in for the general state of subjectivity today (p. 135). Can one imagine any other mathematician standing in for his performance on Radiohead's OK Computer, often voted the most popular rock album of recent years? As a phenomenon it is not the meaning of what Hawking says but the mode of performance and how the performative aspects of his delivery speak for mathematics and physics and the relationship they are perceived as having with the world by a broader public. Zizek (2001, p. 213) argues that Hawking is an exponent of what Brockman (1996) has called *The Third Culture*. Here, a "new type of public intellectual … who, in the eyes of the wider public, stands more and more for the one 'supposed to know', trusted to reveal the keys to the great secrets which concern us all." Zizek (2001, p. 215–216) goes on to suggest "as they are clarifying the ultimate enigmas … (they) silently pass over the burning questions which actually occupy centre stage in current politico-ideological debates". Thus such a public image of mathematics is an image problem for those enticing young students in to the area. Furthermore, such singular conceptions of beauty and totality do not only support a Third Culture, they have also been known to lead to the Third Reich.

THE BRITISH NUMERACY SKILLS TEST FOR PRIMARY TEACHERS

The image of mathematics being a private activity of a lone student grappling with symbols has surely been disrupted in certain contexts such as the UK where mathematics has become a rule governed activity externally defined by government agencies that check that prospective teachers share the same understanding. My colleague Olwen McNamara describes the following scene, reported in Brown and McNamara (2004), set at the Institute of Education, Manchester Metropolitan University (one of the largest Initial Teacher Trainingproviders in the UK).

The cast: 830 trainee teachers.

The audience: (absent but "overwhelmingly" supportive) the Great British public who "never forget a good teacher" (Teachers Training Agency, 1999).

The stage: 34 rooms across the Campus. The largest held 120, the smallest 16. There were special rooms set aside for dyslexic students, non-native English speakers and latecomers.

The script: included oral/mental and written components and was devised by the United Kingdom Teachers Training Agency and naturally remained undisclosed to the cast until the performance; but was known to focus upon their "wider context of their professional role as a teacher".

The rehearsals: practice scripts were available in abundance and managed centrally through "web-based resources", although hard copies and help lines were also available.

The directors: worked solidly for days prior to the performance enlisting back stage support, planning, producing room lists, counting out scripts (lack of sufficient spares available made contingency arrangements exceedingly tricky); preparing individualised instruction packs for the stage managers/runners; and (to reduce commotion and disruption) stopping builders from building, gardeners from cutting lawns, and beer lorries from delivering.

The stage-managers: 40 invigilators and 20 runners (provided with mobile phones due to the size of the campus) were drawn from amongst the academic and administrative staff.

The stagehands: a House Services team worked tirelessly for days setting out the requisite amount of chairs and tables in the 34 rooms.

The props: audio equipment was provided for each room to deliver the mental/oral test, pre-recorded on audiotape. Above and beyond what was already available, this alone cost £1000. Backup calculators, pens, rulers, paper cups and water were also supplied in great numbers.

The pre-performance briefing: planned, according to the director, with "military precision" took place in Lecture Theatre A at 10.15am.

The performance: almost faultless – the stage managers reported only one audiocassette to be mal-functional. Only 2 of the 830 cast were late and a further one reported with a slight malaise at the beginning of the performance (most probably a case of stage fright). Less impressive, however, was that one in ten of the cast forgot their registration number and/or their photographic identification.

This highly staged event was very much about mathematics in the eyes of the students and of the government setting the test. The event tells us a lot about the country, its mode of governance, its sense of how things get done. Underlying this social nexus is a very specific interpretation of mathematics linked to some specific assumptions about the mathematical knowledge teachers must demonstrate before they begin earning a salary as a primary teacher.

THE (IM)POSSIBILITY OF PERFECTION IN CHILDREN'S ACHIEVEMENT

Recent education policy for secondary pupils in New Zealand has been based around an assessment regime designed to include all pupils. There is a rhetoric that everyone can achieve. Nevertheless, recent controversy led to the dismissal of a prominent school principal after 100% success had been declared for her pupils. It transpired that children had been assigned a whole variety of spurious tasks to enable them to make this perfect score an achievable and defensible reality. Others, however, felt that the principal's tactics had been a blatant attempt to massage life to create statistics.

The story was headline news in New Zealand for a number of weeks, focusing on the assessment strategies used, the dispute that led to the principal's dismissal and the subsequent student protests. This public performance of statistical arguments and their underlying premises exposed how they are used and abused to shape our world. The mathematical dimension could not have been extracted to become independent of this highly interpretive environment.

CONCLUDING REMARKS

If we were pursuing Freud's first conception of the ego we might wonder how we would now operate on the ego towards transforming it, perhaps strengthening it, through some sort of educative process. The mathematics education research community centre this understanding around the work of Piaget. The principal departure from this has been towards the earlier work of Vygotsky, who offered a more socially oriented conception of the learner. The relational ego of Freud, however, displays certain delusional aspects that would prevent an easy alignment with Vygotsky. It is not something that one would wish to strengthen, in oneself or others. Rather, one is more concerned with exploring the ego to see how it functions, how it reveals the person's desires. We would not be interested in getting the ego to mirror some model of perfection in an external object. Rather, the task is to learn about oneself through your understanding of your relationship to the stimulus offered.

Art education itself has been shaped around somewhat outmoded or, at least, limited conceptions of art (Atkinson, 2002). Contemporary art has long since moved on from notions of art objects being admired by independent observers. Similarly, practitioners in mathematics education, insofar as they see themselves sharing some of art education's aspirations, (such as attending to aesthetic qualities, self expression, learning about oneself) are potentially locked into a similar time warp as regards how we should understand mathematical objects. In mathematics education there is much benefit to be gained from understanding the discipline's aesthetic qualities and in finding ways to enable our students to share these pleasures. This paper was motivated by the need for this whilst trying to understand these qualities in a broader way. There are risks however in underwriting those pleasures as though they provide access to a more beautiful world beyond. The works of mathematicians and physicists like Caroline Series and Stephen Hawking paint a picture of mathematics that offers few markers for school children looking for ways to tackle real life problems. No more than classic art providing anchorage to contemporary manifestations of art. The fringe artist, the struggling student and the Sunday painter are potentially all just as helpful. The aspiration to a grand theory anchored around world leaders can neglect more immediate tasks and map out the territory in ways that might seem distorted within certain readings. It is unhelpful, I believe, to demand that school mathematics needs to be underwritten by its supposed connections to such advanced mathematics. Mathematics, seen more broadly and more locally, provides models of varying scales that enable us to inspect the world, for its beauty and for its deformities. School mathematics needs to have a brief that enables more possibilities in enabling children to learn about themselves and learning to express themselves. The children need to understand how the mathematics they learn is shaped around life and how life is shaped around mathematics.

The description of the Numeracy Skills Test paints a picture of a country in which former aspirations that mathematics be created in classrooms by teachers and children are replaced by a notion of mathematics externally defined by the government seeking to ensure that its ideological take on mathematics is embraced by all. This seems to be about mass ego management predicated on Freud's first conception of ego. The policy is seemingly designed to have a mechanical effect on the brain of each individual through an instrumentally defined route. Perhaps the social effects (Who am I? How do I fit in?) are served better by the second conception of ego. This issue is discussed at length by Atkinson, Brown and England (forthcoming). By seeing the boundaries of mathematics extending into its social performance we enable more possibilities but to support this we need a conception of human subjects that accepts this intrinsic relationality in their very formation.

Finally, for the New Zealand principal, her very attempts to deliver the ideal held up by the government as supposedly achievable, made the system itself, to which she was subscribing, all too faithfully, appear to lack credibility. Zizek (e.g., 1989) argues that regimes require a cynical distance among their populace for their ideologies to work. Statistics no longer remains a measure of life if life is forced to fit the required image. The mathematical measure demonstrating perfection or the guidelines advising how this could be achieved, showed that the conception of perfection was not altogether perfect. Models crack and the world cracks but we can learn from these events but not necessarily by holding them against notions of perfection or universality that generally have had a poor track record.

Such issues have immediate consequences for the interpretation of our everyday teaching in schools. Pedagogical strategies and the demonstrable skills of children are often seen as subordinate to the mathematical conceptions they seek to engender. Perhaps, however, such a view should be contested since the teaching devices of school mathematics need also to be understood as constructed and implicit components of the mathematical ideas we wish our students to encounter (see Brown & McNamara, 2004). The commodifications of mathematics are not all bad. The performative aspects of mathematical activity need to be understood as well as the mathematics supposedly underlying this. Elsewhere I provide an example of children attending to these aspects (Brown, 2001, pp. 101–102). This does not preclude the possibility that the mathematical thinking can be "sufficiently abstracted to be removable from its practice" (Barton, 2004, p. 23). Yet the social and linguistic conditioning of mathematics is necessarily a crucial aspect of the discipline being addressed in school and vocational courses. Proficiency with concretisations is integral to the broader proficiency of moving between concrete and abstract domains, as are the social dynamics that surrounds it, proficiencies that lies at the heart of mathematical endeavours (at least in schools). Indeed, one might suggest that for many students and many teachers proficiency in specific concretisations forms the core and key motivation of activity pursued within the classroom.

Meanwhile we can also be attentive to longer-term trends. Art has traditionally provided apparatus for (wo)mankind to inspect itself. Mathematics also has reflective and reflexive functions worthy of further development and attention, that help humans to affirm images of themselves and also to disrupt these images for further growth. These are encountered through objects, rituals and other events. Learning mathematics is intricately tied up with the architecture of emotional and intellectual space. But societal relations define the very contents of individual brains and perhaps individuals cannot see themselves outside of those parameters (Althusser, 1971; Butler, 1997). Yet those are the parameters that govern and explain the individual's actions. Mathematics can be part of the kit bag of resources that might enable us to better understand how those parameters work.

REFERENCES

Althusser, L. (1971). Ideology and ideological state apparatuses. In *Lenin and philosophy and other essays*. London: New Left Books.

Atkinson, D. (2002). *Art in education: Identity and practice*. Dordrecht: Kluwer.

Atkinson, D, Brown, T., & England, J. (forthcoming). *Regulative discourses in education: A Lacanian perspective*. Bern: Peter Lang.

Barton, B. (2004). Mathematics and mathematical practices: WHERE to draw the line. *For the Learning of Mathematics*, *24*(1), 22–24.

Britzman, D. (2003). *After education.* Albany: State University of New York Press.

Brockman, J. (1996). *The third culture.* New York: Simon and Shuster.

Brown, T. (2001). *Mathematics education and language: Interpreting hermeneutics and post-structuralism*. Dordrecht: Kluwer.

Brown, T., & McNamara, O. (2004). *New teacher identity and regulative government*: *The discursive formation of primary mathematics teacher education*. Norwell, MA: Springer.

Butler, J. (1997). *The psychic life of power*. Stanford: Stanford University Press.

Freud, S. (1923). The ego and the id. In S. Freud (1991) *The essentials of psycho-analysis*. London: Penguin.

Grosz, E. (1989). *Jacques Lacan: a feminist introduction*. London: Routledge.

Hawking, S. (1988). *A brief history of time.* New York: Bantam Books.

Jameson, F. (1971). *Postmodernism or the cultural logic of late capitalism.* Durham, NC: Duke University Press.

Mumford, D., Series, C., & Wright, D. (2002). *Indra's pearls: The vision of Felix Klein.* Cambridge: Cambridge University Press.

Sharp, J. (2002). Pictures inspired by Theo van Doesburg. *For the Learning of Mathematics*, *22*(3), 18–19.

Sinclair, N. (2002). Re-constructing a painting with geometric eyes. *For the Learning of Mathematics*, *22*(3), 19–22.

Sinclair, N. (2003). Aesthetic values in mathematics: A value oriented epistemology. In N. Pateman, B. Dougherty & J. Zillox (Eds.), *Proceedings of the Joint Meeting of PME and PMENA*. Honolulu: CRDG College of Education, University of Hawai'i.

Sinclair, N. (2004). The roles of the aesthetic in mathematical inquiry. *Mathematics Thinking and Learning, 6(*3), 261–284.

Teachers Training Agency. (1999). *National skills tests: A guide for trainee teachers*. London: Teachers Training Agency.

Zizek, S (1989). *The sublime object of ideology.* London: Verso.

Zizek, S. (1997). *The plague of fantasies.* London: Verso.

Zizek, S. (2001). Did somebody say totalitarianism? London: Verso.

EXPLORING SPACES OF COLLABORATIVE LEARNING WITHIN A COLLECTIVE ARGUMENTATION CLASSROOM

Raymond Brown
Centre for Applied Language, Literacy and Communication Studies, Griffith University, Australia
<ray.brown@griffith.edu.au>

Collective Argumentation is a collaborative form of teaching and learning designed to involve children in negotiating the development of conceptual knowledge through the use of a key word structure. This key word structure utilises the strategies of representing, comparing, explaining, justifying, agreeing and validating to guide children's activity at the small-group and whole-class level and to scaffold the construction of spaces of collaborative learning. This paper explores the nature of small-group interaction within a Year 7 classroom that employs Collective Argumentation as a tool of learning. Particular attention is paid to the spaces of collaborative learning that children construct around the shared practices of the classroom and to the interaction of these spaces with individual children's levels of learning and development.

Collective Argumentation (Brown & Renshaw, 2000) is a format of classroom organisation designed to enable children to negotiate understandings of key concepts embedded within the school curriculum. It is derived from a sociocultural theory of learning (Vygotsky, 1981) that advocates that a child's learning is a personal activity that may be transformed through engagement in social interaction. According to this theory, changes in the level of sophistication of a child's understanding can only be understood when we view learning as being part of dialogue, of collaboration, of communication, of a zone of proximal development – that is, those spaces in a teaching learning relationship that promote learning.

THE NATURE OF PARTICIPATION WITHIN A ZONE OF PROXIMAL DEVELOPMENT (ZPD)

Participation in a ZPD is revealed in the social patterns of engagement and influence that different individuals and groups achieve within an activity setting such as a classroom. These local patterns of influence and privilege are not regarded as merely random and incidental, but as revealing social, cultural and historical distributions of power. Hence, participation in a classroom activity is an extension of past-participation in classroom events - an extension that is directed towards accomplishing personal, social, and cultural goals that have not yet been accomplished (Rogoff, 1995). The recognition that a student's present-participation in the classroom is a dynamic process mutually constituted through the interaction of past experience, on-going involvement, and yet to be accomplished goals, requires that participation within the ZPD be conceptualised as being both *product* and *process* - entities that do not merely alternate, but actually generate each other. Participation within the ZPD, therefore, may be seen both as a product—the momentary embodiment of the evolving relationship between learner and sociocultural context (e.g., what is said, what is represented, the role that is assumed)—and as a process—sense making through negotiating actions and privileging certain ways of knowing and doing over others (e.g., the saying, the representing, the enacting of a role).

The aim of this paper is to explore the nature of the ZPDs constructed by students as they engage in a collaborative form of classroom learning—Collective Argumentation. To help this exploration the interactions of two Year 7 children, Allan and Annie, are tracked as they consider what it means to write a mathematical word problem in a classroom that employs Collective Argumentation on a regular basis.

COLLECTIVE ARGUMENTATION

The social processes of Collective Argumentation are scaffolded by a 'key' word structure (*represent*, *compare*, *explain*, *justify*, *agree* and *validate*) that focuses participants' interactions within key social situations that range from everyday interaction to communal validation. In simple terms, collective argumentation involves the teacher and students in small group work (2 to 5 students per group) where students are required, initially, to individually 'represent' a task by using pictures, diagrams, drawings, graphs, algorithms, numbers, etc. Students are then required to 'compare' their representations with those of other group members. This phase of individual representation and comparison provides the potential for differences in understanding of curriculum content to be exposed and examined. Subsequent talk by the students regarding the appraisal and systematisation of representations is guided by the keywords—'explain', 'justify', 'agree'. Finally, moving from the small group to the classroom collective, the thinking within each group is validated for its consistency and appropriateness as it is presented to the whole class for discussion and validation.

When students enter the Collective Argumentation classroom they bring with them their learning histories reflecting the interaction patterns and products of their functioning in everyday and institutional settings. For example, Allan and Annie's participation in previous mathematics classrooms has been limited to individually completing textbook tasks, answering teacher directed questions and listening to adult explanations. One result of such participation is Allan's and Annie's view that mathematics is about remembering and reproducing sets of unrelated facts rather than being an inquiry-type activity directed toward enhancing "our understanding of our world and the quality of our participation in society" (Australian Education Council, 1991, p. 5).

CONTEXT OF THE COLLECTIVE ARGUMENTATION SESSION

The Year 7 classroom referred to is situated in a metropolitan primary school located near the centre of Brisbane. The population of this class of 11 to 12 year-old children is comprised of one teacher, and 15 female and 11 male students drawn from middle and working class sections of the community. The class has been the focus of a year-long, intensive research study (See Brown, 2001). In this paper we focus specifically on two students, Allan and Annie. Allan and Annie are fraternal twin siblings who usually chose to work with other students during Collective Argumentation sessions, but were required, for research purposes, to work with each other once a fortnight over the course of the study. Audio and video-recordings of Allan and Annie's group interactions and whole-class interactions were made during these fortnightly sessions.

EXPLORING ZPDS WITHIN A CLASSROOM CULTURE OF COLLECTIVE ARGUMENTATION

A COLLABORATIVE SPACE WHERE CHOICE OF TASK IS NEGOTIATED

The children had been set the task of choosing one of three activities to do:

(1) Write a word problem that can be expressed in the form of a ratio and solve it for the class.

(2) Write a word problem that can be expressed in the form of a number sentence and solve it for the class.

(3) Write a word problem that can be expressed in the form of an equation and solve it for the class.

These activities were unconventional in terms of primary school mathematics as they required children to write a word problem, a task usually associated with the role of the teacher. Allan and Annie were negotiating which task to do (See Table 1).

Table 1
Allan and Annie's negotiation of a classroom task

Turn/Speaker	***Text***	
01 Allan	I want to go with . . .	
02 Annie	With (problem) one!	
03 Allan	Why?	
04 Annie	With question one Allan. It's a bit of	a challenge.
05 Allan	Okay we'll do number one.	
06 Annie	Do you agree with it, because if you (problem one).	don't agree with it we won't go with it
07 Allan	Okay.	

In this sequence Annie convinces Allan to do question one, offering as her argument that the question offers the group a challenge. Annie's desire to ensure that Allan agrees on which task to do illustrates that in the Collective Argumentation classroom all children are required to share in the responsibility for task management and to assume the obligations associated with task completion. The children's interactions also illustrate an important 'ground rule' (Mercer, 1995) of Collective Argumentation, namely, that the motivation for choosing a task should be based on a desire to advance learning through attempting tasks which offer a 'perceived' challenge rather than on a desire to display acquired knowledge through the quick and easy completion of familiar tasks. Perceived challenge is a defining feature of the ZPD as the ZPD is about meeting potential goals in learning (Goos, Galbraith, & Renshaw, 2002). If the function of the ZPD is to expand potential learning by providing unpredictable outcomes (Kinginger, 2002), it must be constructed around activities that lead learners to reflect on their own goals as they negotiate meaning. Perceived challenge, thus, provides learners with the motivation to collaborate around a task in order to achieve common solutions that fulfil personal goals. For one student, collaboration may facilitate the construction of new knowledge for another it may facilitate social acceptance.

A SPACE WHERE PERSONAL POINTS OF VIEW ARE EXPRESSED

Allan and Annie then commence their first attempt at representing their ideas about the task as illustrated in Table 2.

Table 2
Considering Allan's idea

Turn/Speaker	***Text***
08 Allan	(To Teacher) I'm doing question one and it goes, to "write a word problem that can be expressed in the form of a ratio and solve it for the class". Well, I've gone, to make a block they need seven shovelfuls of cement and three shovelfuls of gravel.
09 Teacher	Ah, hum.
10 Annie	(To Allan) And then what do you do with that?
11 Allan	And then you put seven shovelfuls of cement is to three shovelfuls of gravel (looks to teacher).
12 Annie	Yes, but Allan you have to say, write this in a ratio form. If you just go like that . . .
13 Teacher	(To Allan) What's the question? Every word problem has to have a question. What's the question you're going to ask at the end of the statement? What's your's (word problem) Annie?

Here the teacher and Annie engage critically but constructively with Allan's ideas as they ask questions and offer statements, suggestions and counter-challenges for joint consideration. Within the social space supported by Allan's willingness to share his idea, the interactions of the participants give rise to differing views of the meaning of the task. Allan views the task in terms of simply stating the relationship between two quantities (cement and gravel). Annie views the task as requiring people to state the relationship between two quantities in terms of the signs and symbols associated with ratio. In other words, that it is the form of expression rather than the substance of the mathematics that is important. The teacher, through his statements, attempts to extend the children's thinking beyond the surface form of the task to consider the mathematics that the task may invite the class to engage in. In other words, that it is the substance of the mathematics behind the task being attempted that is important.

In the above text we see Allan, Annie and the teacher 'contextualising' the activity. 'Contextualising' is a term coined by Van Oers (1998) that captures the dynamic nature of the ZPD where what is being learnt is dynamically constructed and re-constructed on the basis of evolving interactions between participants and the sociocultural setting of the classroom. In other words, all three participants add new meaning to the given situation (considering the relationship between empirical quantities; stating the relationship in terms of the signs and symbols of mathematics; and viewing the relationship in terms of mathematical concepts) in order to characterise this situation in terms of what *could* (making concrete blocks) or *should* (engaging in a school task) be done.

A SOCIAL SPACE WHERE A POINT OF VIEW CAN BE RECONSIDERED

In the above text, all three participants have provided collectively valid statements related to the nature of the task. As such, Allan's and Annie's ideas remain active within the group. However, the stage has been set in this social space for the expansion of Allan's and Annie's thinking to include the substantive idea that school-based word problems ask challenging questions which require the application of mathematical knowledge. This is illustrated in Table 4.

Table 4
Reconsidering Annie's idea

Turn/Speaker	***Text***
14 Annie	(To Teacher) To make a cake a chef must put to every four cups of flour, three cups of egg-white. Write this in ratio form.
15 Teacher	Is that a problem or is it just asking you to state the words of the problem in a ratio? Is there a problem there? Is there something that people have to work out?
16 Allan	(To Teacher) Yes, write a problem in the form of ratio.
17 Teacher	Write a word problem that can be expressed in the form of a ratio.
18 Annie	(To teacher) Well this isn't . . . (referring to her work).
19 Teacher	It's not a problem, it's a statement isn't it?
20 Annie	Yes.

The teacher, Annie and Allan engage in quality talk as both children attempt to counter the teacher's argument that their representations are statements rather than word problems. By doing this, the children's knowledge of what constitutes a word problem is made more public and accountable and, therefore, more open to being influenced by the conventional understanding of what constitutes the genre of a mathematical word problem. Allan and Annie argue that the aim of the task is to express relationships between quantities in the form of a ratio. The teacher, in an attempt to deny Allan and Annie's contention, expands on the conventional understanding of a word problem going from an abstract utterance, 'problem', to a more concrete interpretation, "Is there something that people have to work out?" Annie eventually accepts that her point of view is inadequate, thereby allowing the more conventional point of view to gain prominence in the discussion.

The inability of Annie's and Allan's statements to keep their point of view on the table creates a space within the dialogue where the teacher's previous statement that *"every word problem has to have a question"* is reconsidered by Annie *(Well* *this isn't . . .)* as being an important aspect of task completion. As such, the teacher's statements in this sequence, although similar to utterances spoken previously, are more powerful as the context of the ZPD expands and the children commence to seriously consider the 'voice' of the local mathematical community.

> The notion that the ZPD can and should extend beyond the walls of the mathematics classroom is an idea well supported in the literature (cf., Lampert,1990). However, the relationships between the mathematics of the local classroom and the mathematical practices of an advanced community of mathematicians are complex. For one thing, the breadth and depth of knowledge of the local class community is limited by comparison. The challenge for the teacher and the students at the local level is to instantiate in some authentic way, selected aspects of broader mathematics communities. Annie's reconsideration of her point of view provides such an instantiation.

A SOCIAL SPACE WHERE THE ACTUAL MEETS THE POTENTIAL

Annie's reconsideration illustrates an important 'ground rule' of Collective Argumentation that shares a resonance with an important norm of a broader mathematical community, namely, 'wise restraint' (See Lampert, 1990), that is, Annie's agreement with the teacher is born out of an inability to defend her idea in light of the teacher's statements and out of an 'openness' to reconsider another point of view rather than out a desire to please the teacher. Allan's mode of arrival at this point of 'reconsideration' differs to that of Annie's as illustrated in Table 5.

Table 5
Allan and Annie's collaborative solution to the task

Turn/Speaker	***Text***
21 Allan	Well what's wrong with this (referring to his representation)?
22 Teacher	Yes, well, what's your question? (Reads from Allan's representation) "To make a cement block you need seven shovelfuls of cement to three shovelfuls of gravel." Well, where's your question? Where's your problem?
23 Allan	Problem?
24 Teacher	How many shovelfuls of cement would they need if they were going to make?
25 Allan	Three shovelfuls? Cement?
26 Teacher	You're just guessing now. You have to stop, think about it, reflect.
27 Annie	(To Allan) What I was trying to say is (points to Allan's word problem) . . .
28 Allan	Look, shhhh!
29 Annie	No, just wait. You've got to make a cement block. You need seven shovelfuls of cement and three shovelfuls of gravel. You have to then say, write it in a ratio. But now you need a problem. Something like this, to make a cake a chef must put for every four cups of flour, three cups of egg-white.
30 Allan	Yes.
31 Annie	How much flour and egg-white will you need to make three cakes? Write your answer in ratio form.
32 Allan	Aaaaah!
33 Annie	Is that a problem? That is called a problem!

Here the teacher and Allan engage in talk that helps Allan to focus on the inadequacy of his response. Allan's contention, that his statement relating one quantity (cement) to another quantity (gravel) is a word problem, has weakened, but, unlike Annie, he attempts to guess the answer to the teacher's question. The teacher directs Allan back to the task of individual representation and encourages him to think about his word problem and to reflect on how it might be turned into a mathematical word problem. However, personal reflection is insufficient for Allan to progress in his understanding. He requires more than individual representation to come to the 'common understanding' of the task as shared by the teacher and Annie. Allan requires and receives social support from Annie.

At first, Allan rejects Annie's support *("Look, shhhh!")* showing that his intellectual resources are unable to cope with this challenge. However, Annie's 'persistence' forces Allan to become socially responsive, with him listening to Annie as she expands on his representation of a ratio word problem to include the notion that problems ask questions. In the process, Annie offers an alternative representation which helps Allan to attain the group's understanding *("Aaaaah!")* of what a word problem is - an understanding that shares congruence with Allan's initial idea and with the conventional 'voice' of mathematics. During this social interaction, Annie not only expands Allan's understanding, but also clarifies her own thinking as to what a 'word problem' is, answering her own question, *"Is that a problem?"* with the emphatic statement, *"That is called a word problem!"*

Annie's interactions in this sequence illustrate an important 'ground rule' of Collective Argumentation, namely, that students share with the teacher in the responsibility for the establishment of mutual understanding. It must be remembered that the teacher is not only working with Allan and Annie's group, but also with eight to ten similar groups in the classroom. As such the teacher's participation in any one group is constrained by time restrictions, syllabus requirements and the needs of different groups of children. With Allan and Annie, the teacher is directly involved in focusing the children on the mathematical challenge of the task and in keeping this challenge before them. However, it is Annie who is able to stay working within Allan's zone of proximal development, scaffolding his efforts as he moves from his personal representation of what a ratio word problem is, to the more abstract notion that ratio word problems offer a mathematical challenge to students that can be met through using the language of ratio.

A Social Space Where the Student Becomes the Teacher

The social space created by the teacher, Allan and Annie has changed. The flexibility of roles occupied by participants in this teaching and learning process denotes the ZPD as being a generative intellectual space. That is, a social space created and re-created in the moment as a result of the interactions of specific participants (Zack & Graves, 2001). In this space, the teacher's role of focusing and directing attention has now receded and Annie has assumed the role of teacher within the group. Annie's role as teacher continues in the next sequence as the children come to a consensus on an approach to the task that they can both articulate to the class (See Table 6).

Table 6
Allan and Annie reach a common understanding of the task solution

Turn/Speaker	***Text***
34 Annie	Well, let's go with your idea.
35 Allan	Why not?
36 Annie	No, look. I like your's because . . . it just sounds good, except …
37 Allan	What about yours? You've got a cake, it's sweet.
38 Annie	Yeah, I'm . . . Okay, this is mine. To make a cake a chef must put to every four cups of flour three cups of egg-white. How much flour and egg-white would you need for three cakes? Write your answer in ratio . . .
39 Allan	That's good. I can understand that.
40 Annie	And then you times . . .
41 Allan	I can't understand mine.
42 Annie	And then you times four by three which will give you twelve and three times three which will give you nine. And your answer would be twelve is to nine.
43 Allan	Yeah, well we don't work that (the answer) out.
44 Annie	Yeah, you have to work it (the answer) out.
45 Allan	Why?
46 Annie	It's exactly the same as yours.
47 Allan	Well, we'll go with yours.
48 Annie	So you want to do mine?
49 Allan	Yes, yes.
50 Annie	Alright, we'll go up and do ours.
51 Allan	But, no, no, wait (laughs). Just before you leave, what's that? (Points to the calculations in Annie's math pad.)
52 Annie	That's how I worked it out. I timesed four by three and three by three and that gave me the answer. You see (points to the word problem), I said three cakes. You know?
53 Allan	Yeah.
54 Annie	I want to make three cakes. So you multiply by three. Do you understand?
55 Allan	Yeah.

Here Annie and Allan come to a consensus on which 'word problem' to present to the class. The consensus is not based on one child dominating or submitting to the other, but on Allan's desire to go with a 'word problem' that he can understand. However, Allan views the application of mathematics to the word problem as being irrelevant to the completion of the task. Annie redirects his thinking by stating that *"you have to work it out"*, thereby focusing Allan's attention on the teacher's earlier point of view that it is the substance of the mathematics behind the question being asked that is important. Allan's recognition of this point of view is reflected in his actions as he calls Annie back to the task and requests an explanation to clarify his understanding of calculations of 'equivalence' and 'proportion' that Annie has applied to the representation.

Annie's explanation and her querying as to whether Allan understands is evidence that she now considers the 'word problem' as belonging to both her and Allan and that his understanding is essential to the completion of the task. This recognition of co-authorship is succinctly reflected in Annie's utterance, *"we'll go* *up and do ours".* The children's interactions in this sequence illustrate a basic 'ground rule' of Collective Argumentation, namely, that each member of the group be able to articulate a common understanding of the group's response to the task. More importantly, the children's interactions embody the nature of the ZPD that scaffolds their participation – a space where the essence of the product of their endeavours ("our" word problem) reflects the process of the participants' collective endeavours (individual representation, co-operative comparing, explaining & justifying, and consensual agreement). In this way, the ZPD can be seen as a process of personal growth and as a product of collective action - a characterisation of the ZPD as defined by Zack and Graves (2001). As such, it can be stated that both Allan and Annie have gone beyond their individual needs to know and are now pursuing a collaborative goal directed toward full participation in their classroom community of mathematicians.

CONCLUSION

This paper set out to explore the nature of the ZPDs constructed by Allan and Annie as they engaged in a collaborative form of classroom learning - Collective Argumentation. The above analysis of student-student and teacher-student interaction suggests that within a collective argumentation classroom zones of proximal development emerge around shared practice. That is, spaces where on-going processes for adding meaning to a given situation such as representing, comparing, explaining, etc., are available and students' representations, ideas and points of view have the potential to become unique products of the moment. Participants' interactions imply that within these zones of proximal development practices may be negotiated that have the potential to promote the co-construction of knowledge and an awareness of the 'self' as operating with mediational means. Within a collaborative classroom, therefore, zones of proximal development are constructed that not only scaffold the knowing, but also the doing.

The ZPDs that Allan, Annie and the teacher co-constructed in coming to know and write a mathematical word problem provided multiple opportunities to reflect upon and manage the task, to relate task elements (as expressed through personal points of view) to a collaborative goal, to reconsider personal viewpoints in the light of constructive criticism, to progress from personal to conventional understandings, and to adopt the voice of the teacher to further participation. In other words, Allan and Annie's ideas came from ZPDs scaffolded by the social processes and products of Collective Argumentation.

The interactions between the teacher, Allan and Annie, made it possible not only for Allan and Annie to understand the substance of a mathematical word problem, but also to participate positively in the collaborative activity of the classroom. As such, it may be said that zones of proximal development that emerge around shared activity promote not only the appropriation by students of the knowledge associated with a subject discipline but also facilitate students' moves toward full participation in the sociocultural practices of their classroom community.

This paper has provided evidence also that students within a Collective Argumentation classroom construct differently the zones of proximal development that emerge around shared activity and that this construction is an on-going process of adding meaning to a given situation through characterising the situation in terms of what *should* or *could* be done when engaged in shared practice. For Annie, the zones of proximal activity that she negotiated afforded her the opportunities to (1) influence Allan to do the challenging work of the classroom *("With question one Allan. It's a bit of a challenge");* (2) link the purposefulness of his actions with the conventions of mathematical word problems *("You have to then say, write it in a ratio. But now you need a problem. Something like this…");* (3) assist Allan to connect his thinking with the practices of the classroom *("Yeah, you have to work it out")*; and to (4) influence Allan to participate, 'positively', in the collective work of the classroom community *("Alright, we'll go up and do ours").* For Allan, his participation within these zones of proximal development assisted him to (i) generalise his thinking about a task so that he, Annie and the teacher could work as a group to compare and reflect on ideas *("Well, I've gone, to make a block they need seven shovelfuls of cement and three shovelfuls of gravel")*; (ii) objectify his ideas so that the group could work co-operatively to accept, reject or modify ideas on the basis of reasoned argument rather than convenience *("Well what's wrong with this [referring to his representation]?")*; (iii) expand his ideas so that he and Annie could work collaboratively with the teacher to make the expression of such ideas consistent with the 'voice' of mathematics *("Aaaaah!")*; (iv) share his thinking about the task so that he and Annie could assist each other to establish a consensus based on shared understanding *("That's good. I can understand that")*; and to (v) access a classroom culture based on joint practice and communal validation *("Well, we'll go with yours.").*

The emergence of zones of proximal development within this classroom did not happen by chance. The collective processes and products in this class have been scaffolded by the teacher over a period of time as he introduced and supported the students in producing explanations and convincing arguments. The physical classroom setting that facilitated the emergence of these spaces was organised around the negotiated decisions of the students and teacher. This scaffolding and classroom organization permitted students to encounter mathematics within spaces that assisted them to compare, reflect upon, reject, modify, and expand their ideas through accessing shared resources and to place themselves in the positions of mathematicians so as to see and to know something of what a mathematician could experience.

REFERENCES

Australian Education Council. (1991). *A national statement on mathematics for Australian schools.* Melbourne: Curriculum Corporation.

Brown, R. A. J. (2001). A sociocultural study of the emergence of a classroom community of practice. Unpublished Ph.D. Thesis, University of Queensland, Brisbane.

Brown, R. A. J., & Renshaw, P. D. (2000). Collective argumentation: A sociocultural approach to reframing classroom teaching and learning. In H. Cowie & G. van der Aalsvoort (Eds.), *Social interaction in learning and instruction: The meaning of discourse for the construction of knowledge* (pp. 52–66). Amsterdam: Pergamon Press.

Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: Creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics, 49,* 193–223.

Kinginger, C. (2002). Defining the zone of proximal development in US foreign language education. *Applied Linguistics, 23*(2), 240–261.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29–63.

Mercer, N. (1995). *The guided construction of knowledge: Talk amongst teachers and learners.* Clevedon: Multilingual Matters Ltd.

Rogoff, B. (1995). Observing sociocultural activity on three planes: Participatory appropriation, guided participation, and apprenticeship. In J. V. Wertsch, P. del Rio & A. Alvarez (Eds.), *Sociocultural studies of mind* (pp. 139–164). New York: Cambridge University Press.

Van Oers, B. (1998). From context to contextualizing. *Learning and Instruction, 8*(6), 473–488.

Vygotsky, L. S. (1981). The genesis of higher mental functions. In J. V. Wertsch (Ed.), *The concept to activity in Soviet psychology* (pp. 144–188). Armonk, NY: M.E. Sharpe.

Zack, V., & Graves, B. (2002). Making mathematical meaning through dialogue: "Once you think of it, the z minus three seems pretty weird". *Educational Studies in Mathematics, 46,* 229–271.

MATHEMATICS EDUCATION AND POWER RELATIONS

Valéria de Carvalho
HIFEM – FE – Unicamp
<val.carvalho@uol.com.br>

Sandra A. Santos
DMA – IMECC – Unicamp
<sandra@ime.unicamp.br>

Antonio Miguel
HIFEM – FE – Unicamp
<miguel@unicamp.br>

This twofold aimed paper is an inquiry into some regimes of truth empowered by mathematics, and argues that the dominant conception of mathematics teaching has been used to support many discourses and actions that maintain asymmetric power relations inside and outside school. Excerpts from college students' investigative reports sustain the inquiry.

INTRODUCTION

As mathematics educators, we usually ask ourselves: what world are we "preparing" our students for if we do not offer them adequate mathematical literacy for dealing with their everyday situations in a critical way? We will not argue in favor or against a pragmatic conception of mathematics and mathematics education. Instead of focusing on this philosophical discussion, we would rather concentrate on another discussion that is equally philosophical, and has an immediate repercussion in the educational field: the ethical-political and ideological development of the social uses of mathematics, and the nature of the values that are promoted and spread by the mathematical culture that is transmitted in school or in any other space, institutionalized or not. In this sense, we highlight the works of D'Ambrosio (1993), Borba and Skovsmose (2001), Skovsmose and Valero (2002a, 2002b), Zevenbergen (2002), among others.

Our purpose in this paper is to pursue an inquiry of certain types of asymmetric power relations that are produced or reinforced by means of particular social uses of mathematics in different social practices. Besides, we seek to emphasize how mathematical concepts, mathematical language and the status that surrounds this area of knowledge have sustained and facilitated the decision-making process that interweaves the economic and political features of society. To reach these goals we have pursued to exemplify and question particular ideological uses of some values associated with mathematics, such as rationality, objectivity and neutrality. This inquiry rests mostly upon the concept of *ideology of certainty* developed by Borba and Skovsmose (2001) and in the notions of *regime of truth* and *power-knowledge* from Foucault (2000, 2003). Selected statements from college students in the context of an investigative work on the theme "critical citizenship" sustain our reasoning.

AN OUTLOOK ON THE STUDENTS' INVESTIGATIVE WORK

The first author of this paper developed an investigative work with students enrolled in the second year of the course Social Communication, within the discipline of Statistics I, at a Brazilian private university, in Campinas (São Paulo). To compose the proposal, the teacher got inspiration in the report by Dimenstein

(1999) about the difficulties that people in general face in understanding the newspapers, a vehicle filled with concepts such as inflation, social debt, progressive tax, GDP, among many other concepts. As Dimenstein says, "without understanding the meaning of such words, it is impossible to know the meaning of citizenship" (1999, p. 9).

The teacher instigated the process of investigation by surveying, with the students, a collection of concepts, words and abbreviations that were commonly used in the press, and whose meaning they suspected were unknown by most of the population. One of the beliefs that motivated the teacher to launch this investigative project was that the notions of *ideology of certainty* and *illusion of numbers*—which will be discussed below—in association with the non-acquaintance of some concepts, words and abbreviations, are some of the forces that restrict a plentiful, critical and aware exercise of citizenship for the majority of the Brazilian population.

The students, divided in teams, interviewed people from a general public. The project consisted of preparing an opinion poll involving selected themes with social, political and mathematical dimensions, together with bibliographical research, gathering of hypotheses, preparation and application of a questionnaire, data analysis and production of a printed publicized material, giving meaning to the chosen themes and putting them in context. In the end, the students produced a collective report including the research, the questionnaire with tabulated and analyzed data, the publicized material with written defense, and individual reflections.

The majority of the Social Communication students enrolled in this study are uncommitted with the basic disciplines of their formation, which encompass Statistics. They fit into the medium-high class, and in general have deficient cultural and educational basis. The following statements show changing of attitudes of some students as far as the statistical knowledge is concerned, before and after their involvement in the project:

> Before starting the project, I felt indifferent to the subject under consideration. I did not care about the meaning of the technical terms of the current social, political, and economic systems that drive our world. The accompanying numbers had no meaning to me; it was as if they did not at all make part of the text (L. A.).
>
> Every time I listen to the theme **minimum wage**, I am sure that the plot formed in my mind will be totally different than the one I had before acquiring such knowledge. To me, as for many people in Brazil, such topic does not attract much interest (J. F.).
>
> (...) Applying the theory from the Statistical classes in practice was the best way to be prepared for future marketing research, besides developing the knowledge acquired during the semester (M. S.).

In the preparation of the poll, the teacher advised the teams to choose and focus on at least three concepts, words or abbreviations from a selected collection. The following passage, selected from the report of one of the teams, shows the conceptual option of its members and the question they prepared to guide their investigation:

> In our research, we have also elected the theme **minimum wage**. According to the Brazilian Constitution of 1988, the minimum wage must fulfill the basic vital needs of the employee and his/her family, on housing, food, education, health, leisure, clothing, hygiene, transportation, and social welfare, with periodical adjustments to correct its purchasing power. However, the minimum wage in Brazil – around US$90.00 monthly for 8h of work per day – certainly does not cover the basic needs. In March of 2004, the Inter-union Department of Statistics and Social-Economic Studies estimated the value of around US$485.00 for the minimum wage to maintain a four-people family (two adults and two children), almost six times the current minimum wage value. Therefore, we decided to include in our poll a question to investigate how much should the minimum wage be to maintain a five-people family (a couple and three children), with unique income. 42% of the interviewed people estimated that the minimum wage of this family should amount to at least US$345.00. Inquired about the fundamental item to be covered by the minimum wage, food was chosen by 66% (L. creations).

The pedagogical proposal has promoted discussion and creation of material about themes that favored interaction and dialogue among students, between teachers and students, among teachers of distinct disciplines, and also between students and the interviewed people. Motivational factors to the teacher were the possibilities of integrating the discipline of Statistics with the knowledge of other disciplines of the course, as well as seeking to exploit the social and political dimensions of her task as a mathematics teacher. The aim was that the discipline should provide the students the opportunity of producing statistical knowledge in an investigative way, so that the ideological uses of this knowledge in some social practices might be evident.

In the inquiry of the projects developed by the several teams, the teacher sought to highlight the connection between the selected themes and the current Brazilian moment, the motivations to prepare the questionnaire, in addition to the interpretation and analysis of the constituted data, going much further than the work with statistical exercises that are usually present in textbooks. The theme *minimum wage*[i] was elected by the majority of the teams and has social relevance in our country.

SOME ASPECTS OF THE BRAZILIAN REALITY

In this section we comment on some Brazilian data, by means of the mathematical language present everyday in the communication vehicles: the economic indicators.

The accumulated inflation, from June 1994 to May 2004, was 142.8% according to the IPC (Consumer Price Index). The variation recorded by the IPCA (Consumer Price Broad Index), on the other hand, was 167.21%, whereas the one registered by the IGP-DI (General Price Index – Internal Availability) was 296.46%. These indicators are not the only ones used in Brazil, but the large discrepancy of the figures shows, at least, that different methods of computing inflation will give rise to different results. The inflation indices are used to systematically correct prices, contracts and public tariffs, silently exerting influence in the budgets and, many times, turning future plans unfeasible. According to the economist Carlos Thadeu de Freitas, "the population is replacing the buying of goods by the payment of tariffs[ii]" (Folha de São Paulo, Caderno Dinheiro B1, 10/07/04).

Along the ten years of Brazilian *Plano Real*,[iii] it was verified that as the minimum wage had an effective increase of 25%, the profitability of the financial funds was 399%, whereas the public tariffs were corrected in 255%. The profit of the ten largest banks increased in 1039%. As the official inflation of the period 1994–2004 was of 143%, the citizens pay 255% more in their taxes.[iv] The governmental representatives, at different levels, use certain indices to update public tariffs and prices controlled by govern, that imply in gathering.[v] However, such indices are out of the question when it comes to correcting, in the same period, the income tax or the income loss. According to IBGE, 85% of the Brazilian families, including those who earn more than the minimum wage, are out of money before the end of the month.

ESTABLISHING NEXUS BETWEEN POWER-KNOWLEDGE, REGIMES OF TRUTH, IDEOLOGY OF CERTAINTY AND MATHEMATICS EDUCATION

> The language of education is not simply theoretical or practical, it is also contextual and should be compromised in its genesis and development as part of a wider net of historical and contemporary traditions, so that we can be self-conscious of the principles and social practices that give meaning to it (Giroux, 1997, p. vii).[vi]

Certainly, many occurrences in society have been marking our actions as teachers and human beings. We are not teachers independently of the context in which we live – but how can we ensure that the teaching of mathematics performs the role that we expect from it in our epoch, in the present history (which, somehow, we are responsible for)? Which connections would we like to establish between mathematical language and society? Which transformations of forces that define the asymmetric power relations in every level and context of the human actions could these connections accomplish? Which meanings should be given to the connections that we would not like to establish?

The questions above indicate the need to focus our attention on the social and ideological roles that mathematics and mathematics education have been playing, beyond the way they are transmitted by scholar circumstances and textbooks. We should also ask ourselves: which processes of social elaboration, synthesis and re-elaboration of knowledge would we have been privileging, voluntarily or not? Which groups and interests would they have been promoting and benefiting? In every context and level (institutionalized or not) in which human relations are established, intentions, interests, knowledge and several values are produced, usually diffuse and conflicted. That is the reason why such human productive relations are, always and simultaneously, power relations[vii] and resistance relations. According to Rios: "School is always *positioned* in the range of correlation of forces of the society in which it is inserted, and thus, it is always serving the forces that struggle to maintain and/or to transform society" (2003, p. 43).

In this sense, the current official mathematics curricula are also expressions of this tense historical process, reinforcing practices and values, many times contradictory and with perverse social effects, but which appear as neutral, natural and ethically elevated in the eyes of students, teachers and parents. Therefore, it is necessary to look more carefully into the personal and social developments of such practices and values. D'Ambrosio, after reflecting about the question "why is mathematics taught with such intensity and universality?", find answers "in a multiplicity of reasons associated to five values: (1) utility; (2) cultural; (3) formative; (4) sociological; (5) esthetic" (1993, p. 19). He also emphasizes that we should think the curriculum as a strategy of pedagogical action. In our view, besides being a strategy of pedagogical action, the curriculum is also a strategy of power (not in a negative sense, but in Foucault's sense for the expression of correlation of forces) actioned by certain social groups, which somehow and for some (not always consonant) reasons value the scholar space.

Social groups with different intentions, interests and political projects have different expectations about what school in general and the teaching of mathematics in particular should supply to society as a whole. These groups, by means of mass media and other vehicles diffuse ideas, practices and values, make claims, and take decisions and actions in certain directions that affect public or private entities. The pressures from these groups gain some visibility and invite responses that can lead to various repercussions for the groups whenever they materialize in symbolic forms such as in official curricular proposals; on the agenda or headlines in various communication channels; in our discourses, concerns and claims as teachers; or in the political discourses of candidates for elected positions.

In recent years, the assessment and the inquiry of the social practices of mathematics which take place in schools started to consider and to value other social practices of a mathematical nature which take place in other contexts that are not scholarly or academic. We highlight that the mass media vehicles might have a singular importance in the mathematics education of both people who attend and do not attend school. On one hand, they release images, messages and information of our everyday lives at an increasing rate, thus suggesting a set of criteria to distinguish right from wrong, good from bad, necessary from superfluous. Equally, the work performed by these various mass media vehicles may inform us about social, political and economic indices, and also about results of statistical research related to different social practices and thematic areas, thus allowing their audience to access a diversity of knowledge from which they can build questions, debates, hypotheses, and establish connections between mathematics and society. This latter role can thus give strength to promoting an active citizenship.

It is worth mentioning that not only the social and economic indices but also the results of statistical research which is always made up based on numbers, while expressed in different types of geometric-visual languages, are constituted and interpreted in terms of certain methods, and are divulged, in form and content, to serve certain interests which, in general, are not properly made explicit. It is clear that these indices and results interfere, directly or indirectly, in people's lives (e.g., in survival expenses) as well as in their future actions and realizations. How is this possible? The social, political and economic decisions together with the definition of priorities, made at the different levels of public power (municipal, state and federal) and by several kinds of private enterprises, are generally (although not exclusively) legitimated by mathematical arguments. Consequently, conceptions of mathematics education, citizenship, democracy, social and economic justice are related by means of regimes of truth. According to Foucault,

> The truth has a round connection with systems of power, that produce and support the truth, and with effects of power that reproduce the truth and are induced by it (...) Each society has its regime of truth, its 'general policy' of truth: that is, the types of discourse that accepts and makes work as true; the mechanisms and instances that allow to distinguish true sentences from false ones, the means from which each sentence is sanctioned; the techniques and procedures valued in acquiring the truth; the status from the ones charged with telling what counts as true (citation from Gore, 1995, p. 10).

The essential question, as regards the production of indices, data collection for statistical treatment or elaboration of mathematical models applied to pursue an approach to economic problems, is how to realize if it is possible to build a new regime of truth. The difficulty does not reside in changing the awareness of people or what they have in mind, but in discussing the political, economic and institutional regime of truth production. Foucault also asserts that

> (...) it is exactly in the discourse that power and knowledge are articulated (...) it should not be assumed a world of discourse divided between the admitted and the excluded one, or between the dominant and the dominated discourse; on the contrary, as a multiplicity of discursive elements that may enter into different strategies (...). The discourses, as the silence, are neither submitted to power nor opposed to it. It is necessary to admit a complex and unstable game in which the discourse may be, simultaneously, instrument and effect of power, and also obstacle, support, point of resistance and starting point of an opposed strategy. The discourse releases and produces power, reinforces but also undermines it, exposes, debilitates and allows bringing it down (citation from Gore, 1995, p.14–15).

We have a huge challenge in mathematics education, together with professionals of other knowledge areas: to create strategies for producing a nexus between the historical-cultural context and statistical data and/or social-economic indices. Using and releasing quantitative information disconnected from their historical-cultural contexts is equivalent to conceiving a society as static, and the individuals as passive.

The contextual isolation of the historical series[viii], in most of the cases, is inappropriate to depict a social-economic scenario, since the expectations and basic needs of society do not remain the same. By hiding the intentions of those who produce and manipulate these data, one might obscure the historical features of the data. Furthermore, the social-political consequences of their use might give strength to inequalities and maintain or increase the asymmetry of power relations.

In the mathematics education field, Borba and Skovsmose (2001) discuss the political dimension of mathematics by means of *the ideology of certainty*. To the authors, the ideology of certainty is a general and fundamental structure of interpretation that contributes to the political control of an increasing number of questions, that turns mathematics into *a language of power*. We should stress the intentions present in the collection of data, in the quantification of information, in the creation of models[ix] and indices, once the fundamental aims of the creation of models based on quantitative data are not only to understand the object under analysis, but also to support decision taking that will influence thousands, even millions of people.

Therefore, it is essential to be aware of our posture as mathematics educators so that our reasoning, or the lack of it, does not give more strength to the ideology of certainty. Naive and non-critical postures always contribute to maintain the *status quo*. If schools keep a radical separation between traditional scholarly and non-scholarly practices, performed and promoted in institutions and in other contexts, and, moreover, if they neither encourage in students an inquiring attitude towards television programs nor discuss with them headlines from the news, interpretation of poll results and indices divulged by the mass media—for what kind of world are they educating children and young people? Which regimes of truth are they promoting and/or reinforcing?

Broadly speaking, the scholar culture does not make any effort to interact with non-scholar social practices, which adds difficulties to the development of new proficiencies for both teachers and students. Should not the school be in charge of helping the new generations to interpret, analyze and question a large number of social practices, especially those connected to decisions that influence the majority of a population, from a city, a country, or even from the whole world?

The research developed by the students in the aforementioned project reinforces our point:

> (...) we had chosen themes usually present in the mass media (...) Along the development of the poll, I realized the most aware of the subject were those used to reading, and not those with greater degree of instruction. As a result, it can be noticed that knowledge, in general, does not depend on an academic formation, but on the concern about understanding the opportunities and the fragility of the nation (D. B.).
>
> I realized the enormous lack of knowledge about the analyzed concepts (...). Some people commented that they had read texts with such words and, many times, had faced interpretation difficulties, without understanding exactly their meaning. In my opinion, the scholar level did not act in favor or against the interviewed people; the turning point to ensure a plentiful exercise of citizenship was the access to information (C.V.).

In our opinion, as far as mathematics education is concerned, discussions should be started to open up our social-historical understanding of economic, financial, and administrative social practices, such as the management of inflation, of cost of life, of internal and external public debt, of GDP, of minimum wage, of social debt, of public taxation and gathering, of misappropriation, of corruption, of *per capita* income, among others, as the citizen does not live isolated from the political, economic, financial, fiscal, and social contexts. With the end of the Brazilian military dictatorship, we started to find, inserted into teaching proposals, and also into official curricula of mathematics, indications of the intention nurture and integrate into society citizens who are reflexive and critical. However, how can we integrate citizens that are not being prepared to critically perform the reading of the political, economic, financial, fiscal, legal, scientific, and technological practices that take place in our country? What kind of citizenship is produced by a mathematics education that does not assume the responsibility of pursuing inquiries in the social practices that, somehow, involve mathematics?

This perspective should be taken into account to seek the inquest of a scholarly mathematics education aimed at the citizen formation. This is a task that part of the society still delegates to school, although it is not an exclusive responsibility of the schools. It is a historical process that we teachers might make easier if, in partnership with others, we build the knowledge about how mathematics is produced and/or appropriated, and how it receives new meanings through different social practices that take place within the contemporary world. The building of this knowledge, if possible, should rest upon the understanding of historical, social, political, ethnic, educational and economic inequalities that might exist. To us, everyday relations and information are shaped by ideologies, that is, by practices, discursive or not, that seek to justify and legitimate inequalities and asymmetric power relations. On the other hand, we should encourage our students and pursue a mobilization to contribute, as best as possible, to this cyclic and continuous process of knowledge that is produced in effective social practices in which the power relations are, all the time, supporting this production and being supported by it. Citizenship goes beyond ensuring legally established rights and duties.[x] We must question the rights and privileges, what should be valued and what must be transformed, with special attention to the several forms of injustices that generate inequalities. The next excerpt reveals a student's perception in this sense:

> Due to this work we have started to face the real problem of the Brazilian people: the lack of information. (...) The more informed and aware the members of our population are, the less prone we are to be manipulated (E.B.).

OBJECTIVITY AND NEUTRALITY MYTHS OF MATHEMATICS

> (...) Mathematics cannot be seen as the 'Queen of Sciences' nor be asleep in the limbo of asocial, amoral and apolitical neutrality. It cannot be conceived independently of the people that have created it, and use it in a historical and social process – cannot be separated from the values, intentions and interests of such people (Martin, 1997), nor can be detached from the context of social analysis where it has grown, or from the historical and social structures which gave power to it (Skovsmose & Valero, 2002).

The myth of the objectivity of the mathematical data and, therefore, of the "exclusive paths" that are attributed to an "almost straightforward" consequence of mathematical data and models, and their results, deserve more study and research. In this sense, we stress that not only a mathematical argument is not, by itself, superior to other arguments, but also mathematics gives us one perspective of the reality, not necessarily the best, the most correct, the most neutral, or the most important.

We believe that mathematics education can contribute both to the promotion of life and political, economic and social democracy, and to the maintenance of inequalities, injustices, social submission and asymmetric power relations in the extent of human relations. In other words, our postures, as mathematics educators, may serve as instruments of power-knowledge. We should be aware of the ideology of certainty spread all around society and oppose it. The ideology of certainty is supported, in part, by ideological discourses that rest upon "truths" produced by different social practices. In these social practices, the mathematical discourse is shown, somehow, involved in a positive, productive and effective sense, independently of the ethical orientation of the aims of such practices. As teachers, it is crucial to call the students' attention to the "super-powers" (cf. Borba & Skovsmose, 2001, p. 132) attributed to the mathematical discourse, by questioning it and, whenever necessary, demolishing its mythical nature.

The mathematical discourse, when reinserted critically into the social practices that constitute it or give it new meanings, simultaneously becomes effect and instrument of power-knowledge, revealing another facet of the power, that is, the resistance that allows the constitution of new discourses. Moreover, the mathematical models produced to explain, reproduce, correct, simulate or forecast circumstances from natural or social world allow one to "project" just a small part of reality, because models are always selective and thus non-neutral approximations in which some variables or factors are chosen to be included and others to be excluded. We refer to the aforementioned example of the different methods for computing inflation indices in Brazil.

In the classroom, to mobilize our students and to value our discipline, it is common that we discuss the importance of mathematics for the development of society in many different areas. Such discourse helps to diffuse, and to make absolute and natural, in the scholarly sphere, the questionable belief that using mathematics in any situation is a good, legitimate and intelligent way to proceed and to have something done well. On the other hand, this partial and generous view of what is done with or by means of mathematics hides its "perverted" uses. Mathematics, together with other areas of knowledge, has contributed to the development of powerful weapons, able to kill, with more precision, a larger amount of people. Mathematics is the tool that "neutrally" justifies the economic, political and social exclusion of worldwide populations. It is also an essential tool for the finances and for professionals of the economic area,[xi] both in public and private spheres.

FINAL REMARKS

In this paper we have defended that the building of bases for the social, political and economic dimensions of mathematics should indeed be integrated into the curriculum of the scholar mathematics education. These bases should generate discussions and effective pedagogical actions so that they are part of a multiplicity of discourses that should compose the regime of truth of scholarly mathematics education. We realized that the neutrality discourse of scholarly mathematics education has been excluding political discourses and, consequently, more political practices. We agree with Matos when he states that:

> Knowledge is not permanently fixed into the abstract properties of mathematical objects. Acquiring and producing knowledge are two moments of the same cycle. This idea involves the notion that knowledge is a product from human consciousness and reality. (...) The role of the teacher cannot be limited to simply teach mathematics. It is essential to recognize the social, ethic and political dimensions in the teaching of mathematics and not assume any neutrality in this teaching (2003).

There are teachers, like us, who question what should be done, even with a possibly "conservative" formation. Our response is to join other people and study. Freire, considering the action of study, states that "it is indeed a hard work" and "a critical and systematic posture cannot be gained unless we practice it" (1982, p. 9). About the difficulty of changing things, the warning of Chomsky applies: "Nobody reaches anything alone besides complaints. Joining other people might help reaching some changes" (1997, p. 134). The statement below, from a student that joined the aforementioned project, reinforces this point of view:

> True citizens fight for their aims and for social ones, integrating society and its movements, not accepting to be another piece of information in the statistics. The "paper citizens", on the other hand, are alienated and believe that they will never be able to change reality: "This is the way things are. There is nothing to be done." The moment of reflection might empower many people (C. P.).

ACKNOWLEDGEMENTS

The authors are indebted to the students that joined the investigative work as well as to the opinion poll participants. We are also grateful to an anonymous referee who improved the initial version of this paper.

---

[i] As reported by the IBGE (Brazilian Institute of Geography and Statistics), in 2002, among the employees, 31.8% received the minimum wage or less. This amounts to 21.6 million of the Brazilian working force (www.ibge.gov.br).

[ii] In Brazil the large amount of taxation contrasts with the low social security provided by the government.

[iii] Economic plan started in 1994, aimed to reduce current inflation of 70% per month to a single figure per year, based on high interest rates, increasing of taxation, reducing of income and cutting of public investments.

[iv] Data from the journal Folha de São Paulo (Caderno Dinheiro), 07/27/04, article *Dez Anos de Plano Real*.

[v] This category encompasses water and sewerage system, electric power, telephone, transportation, health plans, etc.

[vi] All the original quotations are in Portuguese, and were translated by the authors.

[vii] According to Foucault, the power is not necessarily repressive, since it incites, induces, seduces, makes it easier or harder, increases or limits, makes it more or less probable. Moreover, it is exercised or practiced instead of possessed and, thus circulates, passing through any force related to it. Power and knowledge are related (cit. by Gore, 1995, p. 11).

[viii] The historical series may represent a plot that signs the rate at which certain events are occurring or not.

[ix] Skovsmose (2004) discusses the mathematical models as part of the social reality.

[x] If citizenship were only concerned about rights and duties, without questioning and transforming them, slavery might be legally accepted up to the current days.

[xi] More (neo) liberal professionals of this area find strong motivation in mathematical models to support their decisions.

REFERENCES

Borba, M. C., Skovsmose, O. (2001). Ideologia da certeza em Educação Matemática. In O. Skovsmose (Ed.), *Educação matemática crítica: A questão da democracia* (pp. 127–148). Campinas: Papirus.

Chomsky, N. (1997). *Segredos, mentiras e democracia*. Brasília: Editora Universidade de Brasília.

D´Ambrosio, U. (1993). *Etnomatemática: Arte ou técnica de explicar e conhecer*. São Paulo: Ática.

Dimenstein, G. (1999). *O Cidadão de Papel: A infância, a adolescência e os Direitos Humanos no Brasil*. São Paulo: Ática.

Foucault, M. (2000). *Microfísica do poder* (15th ed.). Rio de Janeiro: Edições Graal.

Foucault, M. (2003). *A verdade e as formas jurídicas* (3rd ed.). Rio de Janeiro: Nau Editora.

Freire, P. (1982). *Ação cultural para a liberdade* (8th ed.). Rio de Janeiro: Paz e Terra.

Giroux, H. A. (1997). *Os professores como intelectuais: Rumo a uma pedagogia crítica da aprendizagem*. Porto Alegre: Artes Médicas.

Gore, J. M. (1994). Foucault e Educação: Fascinantes Desafios. In T. T. Silva (Org.), *O sujeito da educação: Estudos Foucaultianos* (5th ed., pp. 9–20). Petrópolis: Vozes.

Matos, J. F. (2003). A educação matemática como fenômeno emergente: Desafios e perspectivas possíveis. *XI Conferência Interamericana de Educação Matemática*. CD-Rom, 12p.

Rios, T. A. (2003). *Ética e Competência* (13th ed., vol. 16). São Paulo: Cortez.

Skovsmose, O. (2004). Mathematics in action. *Philosophy of Mathematics Education Journal, 18*(October). Retrieved 1 January, 2005, from http://www.ex.ac.uk/~PErnest/pome18/ contents.htm.

Skovsmose, O., & Valero, P. (2002a). Quebrando a neutralidade política: O compromisso crítico entre a educação matemática e a democracia. *Quadrante, 11*(1), 7–28.

Skovsmose, O., & Valero, P. (2002b). Democratic access to powerful mathematical ideas. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 383–407). Mahwah, NJ: Lawrence Erlbaum Associates.

Zevenbergen, R. (2002). Citizenship and numeracy: Implications for youth, employment and life beyond the schoolyard. *Quadrante, 11*(1), 29–39.

THE IMPACT OF PRIMARY STUDENTS' INFORMAL EXPERIENCES ON THEIR DIAGRAMMATIC KNOWLEDGE

Carmel M Diezmann
Queensland University of Technology, Australia
<c.diezmann@qut.edu.au>

The Matrix, Network, and Hierarchy are three general purpose diagrams that are useful for representing various mathematical relationships. This paper examines how informal experiences impact on Grade 3 and Grade 5 students' knowledge of these diagrams. The results revealed that primary students associated a variety of informal experiences (e.g., Art, Sport) with one or more of these diagrams. The results also revealed that there were substantial differences between the quantity and type of associations made by Grade 3 and Grade 5 students. These differences are more likely to be due to students' informal experiences than their formal schooling because diagrammatic instruction was not part of the students' mathematics curricula.

DIAGRAMS IN MATHEMATICS

All citizens need to be able to create and interpret a variety of diagrams that represent mathematical relationships. Diagrams facilitate the conceptualisation of problem structure, which is fundamental to problem solving (van Essen & Hamaker, 1990). Additionally, diagrams support visual (wholistically-oriented) reasoning, which complements linguistic (sequentially-oriented) reasoning (Barwise & Etchemendy, 1991; Mayer & Gallini, 1990). Moreover, diagrams support knowledge generation (Karmiloff-Smith, 1990) because they are an-inference-making knowledge representation system (Lindsay, 1995). For example, the use of diagrammatic representation resulted in the Pythagorean discovery of irrational numbers (Simon, 1995). Although there are a myriad of types of diagrams, general purpose diagrams assume a particularly important role in mathematics because they are applicable to a range of problem structures. General purpose diagrams comprise Matrices, Networks, and Hierarchies and a range of diagrams that exhibit Part-whole characteristics (Novick & Francis, 1993). The former three diagrams are spatially-oriented and their unique spatial structures are used to represent specific mathematical relationships (Novick, 2001; Novick & Hurley, 2001). For example, the Matrix has a row and column structure and is useful for representing combinatorial and deductive problem situations. In contrast, Part-whole diagrams have no unique spatial structure.

Despite the importance of diagrammatic knowledge in mathematics, students of all ages are reluctant to use diagrams and have difficulty using diagrams effectively (Veloo & Lopez-Real, 1994; Yancey, Thompson, & Yancey, 1989). Hence, instruction plays a crucial role in the achievement of a diagrammatically literate populace. Although instruction in diagram use has long been advocated (e.g., Diezmann & English, 2001; Yancey, 1981; Yancey et al., 1989), there is a need for research and theory development to guide instructional practice (e.g., Shigematsu & Sowder, 1994). If teachers are to understand students' mathematical thinking, they need to appreciate how students connect new ideas to existing ideas (Carpenter, Fennema, & Franke, 1996). Hence, knowing the types of experiences that impact on students' diagrammatic knowledge contributes to an effective curriculum. Because students encounter diagrams in their primary years and there is generally limited formal instruction about diagrams in the mathematics curriculum, this paper focuses on how primary students' informal experiences impact on their diagrammatic knowledge.

INFORMAL EXPERIENCES AND LEARNING

There has been an increasing awareness of the role of informal experiences in learning throughout life. These informal experiences include visits to museums, historic houses, art galleries, sculpture walks, science centres, zoos, aquaria, botanical gardens, and national parks (Henwood, 2002). Other informal experiences that provide learning opportunities are workplaces, hobbies, peer and family life, the media, information communication technologies and consumption (e.g., goods, services, entertainment) (Aittola, 1999).

A primary advantage of many informal environments is that they provide opportunities for individuals to learn through observation of and participation in authentic contexts, such as community life (Rogoff, Paradise, Arauz, Correa-Caevez, & Angelillo, 2003). In such environments, children learn from adults and other children and through their own contributions within a context. Informal environments that have an educative purpose (e.g., museums) support learning in three distinctive ways. First, they try to meet the needs of different types of learners (Henwood, 2002): "the learning compulsives, who want to know everything about something, and the *learning grasshoppers*, who want to know everything about everything" (p. 1, emphasis in original). Second, they adopt a multi-sensory approach that includes the use of objects, tactile experiences, physical space, labels, written information and visual effects (Henwood, 2002). Finally, they provide opportunities for interactions between visitors and more knowledgeable individuals, such as guides or education officers (Henwood, 2002). These forms of support in educationally-oriented informal environments accommodate different learning styles, facilitate self-paced learning, and allow visitors to determine the time spent at particular exhibits (Melber & Abraham, 1999).

There are three main disadvantages of informal learning opportunities. First, in informal environments that do not have an explicit educative purpose (e.g., entertainment) there is a lack of cohesion in what is learnt and a lack of support for learning (Aittola, 1999): "The learning based on different spheres of everyday life is usually unplanned, unsystematic and fragmentary" (p. 7). Second, in informal environments with an educative purpose, exhibits that are designed for wide appeal, may not appeal to individuals of specific ages or abilities (Lederman & Niess, 1998). Third, in informal settings with an educative purpose, particular cultural values are transmitted through "what is selected, how it is arranged, what prominence it is given, the language used to describe it, and, most importantly what is left out" (Henwood, 2002, p. 7). Thus, an important and challenging part of the visitor's task in learning from these environments is to explicate the embedded cultural messages. This may include interpreting a variety of visual representations.

Notwithstanding the disadvantages of informal learning environments, they are ubiquitous and have advantages that are typically not found in formal educational settings. Teachers can capitalise on learning opportunities in informal environments and address their shortcomings in two key ways. First, teachers can support students to make personal meaning from informal environments. This might involve being familiar with the resources available in that environment (Melber & Abraham, 1999) and being an active participant in the experiences in which the students' engage in informal environments (Lederman & Neiss, 1998). Second, teachers can support students to make connections between their existing knowledge and new knowledge developed within the environment (e.g., Carpenter et al., 1996). The effectiveness of each of these forms of support is dependent on teachers' awareness of the type of knowledge that students derive from informal experiences irrespective of whether these experiences have an explicit educational purpose or not.

DESIGN AND METHODS

The development of primary school students' knowledge of diagrams is being monitored using an accelerated longitudinal design (Willett, Singer, & Martin, 1998) in which two differently-aged cohorts are being studied for a 3-year period. The 69 students in Cohort 1 and the 68 students in Cohort 2 were aged approximately 8–9 year olds (Grade 3) and approximately 10–11 year olds (Grade 5) when the study commenced. The student sample comprises all students in the three Grade 3 classes and three Grade 5 classes from the same school whose parents or guardians gave them permission to participate. The school is in an outer suburban school of a large city.

The aspect of the study which is investigated in this paper is the out-of-school experiences which students reported have influenced their knowledge of spatially-oriented diagrams (i.e., Matrix, Network, Hierarchy). In-school-experiences were also explored but these are not presented here. Students' out-of-school (and in-school) experiences that have contributed to their knowledge of diagrams were investigated in an individual interview: "How did you learn about diagrams?" The students were then asked to describe any informal activities outside of school that contributed to their knowledge about diagrams. They were also asked to draw any diagrams that they identified (e.g., a tennis competition draw). The students' explanations were analysed for themes associated with the Matrix, Network or Hierarchy and the frequencies of these themes were then calculated. Students' explanations and diagrams provide exemplars of these themes. Cross-sectional data allowed comparisons of the themes and frequencies of themes across Grade level cohorts. Follow-up interviews in two subsequent years using the same approach will provide longitudinal data.

RESULTS AND DISCUSSION

A total of sixteen categories of responses were derived from the Grade 3 (*N*=69) and Grade 5 (*N*=68) students' comments and drawings about their out-of-school experiences related to the Matrix, Network and Hierarchy. These categories were Advertisement, Art, Chart, Checklist, Computer game, Family Tree, Friendships, Food Pyramid, Graph/Grid, Map, Music, Puzzle/Game, Sport, Timetable or Other, which is an amalgam of miscellaneous responses. While some out-of-school experiences (e.g., Family Tree), were specifically associated with a particular diagram (i.e., Hierarchy), other experiences (e.g., Sport) were linked to two or more diagrams (i.e., Matrix, Network. Hierarchy) (see Figure 1).

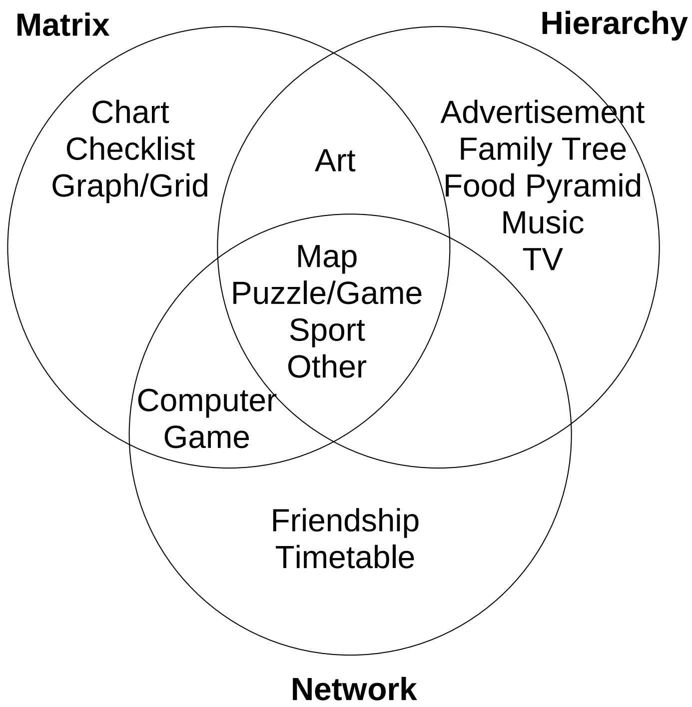


Figure 1. The associations between informal experiences and the Matrix, Network and Hierarchy.

The following sections present (1) an overview of students' responses for the Matrix, Network and Hierarchy, and (2) the proportion of students' responses about everyday experiences that they associated with these spatially-oriented diagrams.

STUDENTS' RESPONSES FOR THE MATRIX, NETWORK AND HIERARCHY

The calculation of Grade 3 (*N*=69) or Grade 5 (*N*=68) students' responses to the Matrix or Network or Hierarchy in this section is based on the percentage of students in each grade who made a particular response.

INFORMAL EXPERIENCES AND THE MATRIX

The students' responses to the Matrix were associated with Art, Chart, Checklist, Computer Game, Graph/Grid, Map, Puzzle/Game, Sport, and Other (see Figure 2). For example, Celia's (Grade 3) response and diagram shows a practical application in which a Matrix being was used as a Checklist (see Figure 3). Three types of responses (with the exception of "Other") were reported for the Matrix by more than 10% of Grade 3 (*N*=69) and/or Grade 5 (*N*=68) students (see Figure 2). A similar percentage of students related the Matrix to a Chart in Grade 3 (13%) and Grade 5 (14.8%). Hence, there appears to be negligible age/grade effect for this type of response. In contrast, the students' responses about the Graph/grid more than doubled from Grade 3 (11.6%) to Grade 5 and their responses about the Checklist were more than eight times greater from Grade 3 (2.9%) to Grade 5 (24.1%).

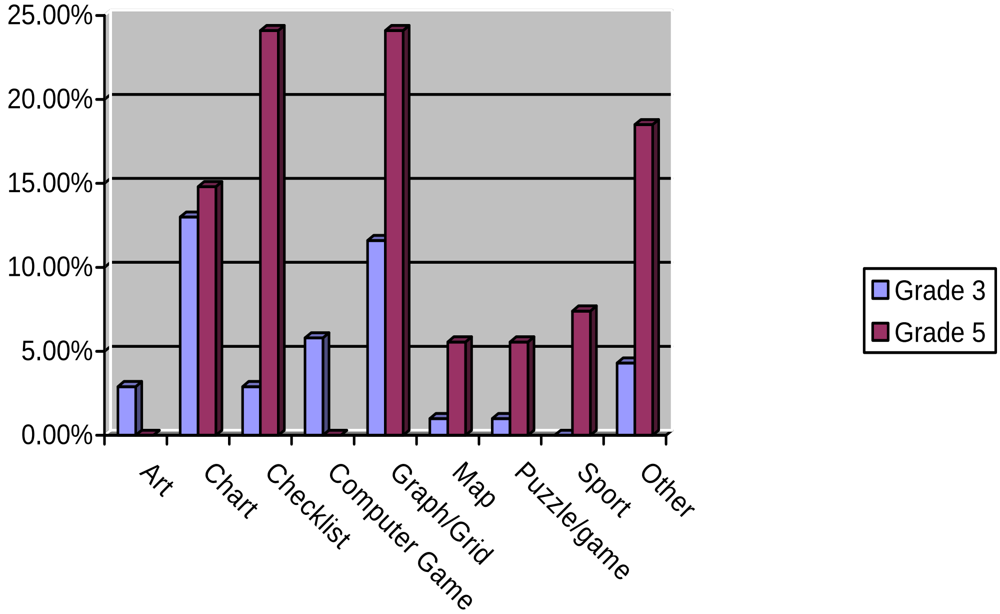


Figure 2. Percentages of students who associated informal experiences with the Matrix.

Celia [C] (Grade 3); Interviewer [I]

I: Okay can you think of a time when you used that [matrix]…?

C: When all my friends came over and we had to try and figure out who wanted what from McDonalds and stuff like that….

I: …can you tell me how you used this [matrix] one for your birthday party?

C: We just drew up a rectangle and line down on the places and we put up the top of each area we put like CB stands for cheeseburger and N stands for nuggets and S stands for soft drinks and M stands for milk shakes and we asked everybody what they wanted and wrote it down in the box next to it and then in the first box we put their initials in it.

I: And how was that helpful to you or to your Mum?

C: Instead of Mum remembering everything and getting everything confused, she made up a graph instead to tell her what everybody wanted?

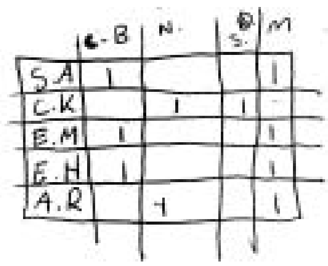


Figure 3. An example of a Matrix response.

INFORMAL EXPERIENCES AND THE NETWORK

The students' responses for the Network related to Computer Games, Friendships, Map, Puzzle/Game, Sport, Timetable, and Other (see Figure 4). For example, Kate's reference to a Network related to a Map of the physical layout at the school (see Figure 5). As for the Matrix, three types of responses (with the exception of "Other") were reported for the Network by more than 10% of Grade 3 and/or Grade 5 students. A similar percentage of students linked the Network to a Map in Grade 3 (15.9%) and Grade 5 (16.2%). As a Network is commonly used for a Map (e.g., a public transport map), the lack of a substantial increase in Grade 5 students' responses is a concern. In contrast, there was a substantial increase in Grade 5 students' references to Sport (10.3%) whereas no Grade 3 students' gave this response. There was also a marked decrease in the percentage of Grade 3 students (11.4%) compared to Grade 5 students (2.9%) who associated a Network with a Puzzle/Game.

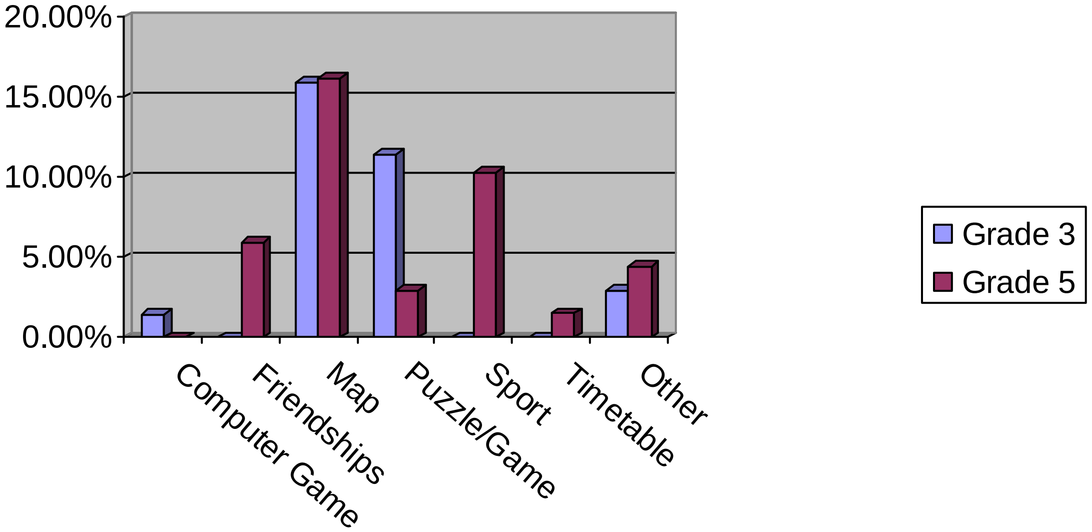
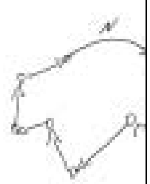


Figure 4. Percentages of students who associated informal experiences with the Network.

Kate [K] (Grade 3); Interviewer [I]

I: Now what about the network? You said you could sometimes, you'd seen the network. How does that one work?

K: Well, because, like, we use the whole oval…here…this area's for the cooking and stuff, before we put the food out, he'll draw a map, a big map, showing, like, the first half of the school and the other half of the school.



4.

Figure 5. An example of a Network response.

INFORMAL EXPERIENCES AND THE HIERARCHY

Students' responses for the Hierarchy related to Advertisements, Art, Family Tree, Food Pyramid, Map, Music, Puzzle/Game, Sport, TV, and Other (see Figure 6). For example, Brent's response and diagram show a real-life application of a Hierarchy to Sports (see Figure 7). None of the responses from Grade 3 students relating to the Hierarchy reached 10%. More than 10% of Grade 5 students associated the Hierarchy with Sport (14.8%) and Art (11.5%). These Grade 5 responses represented a substantial increase from the Grade 3 responses in these categories of 1.4% and 0% respectively. These increases are promising over a 2-year period.

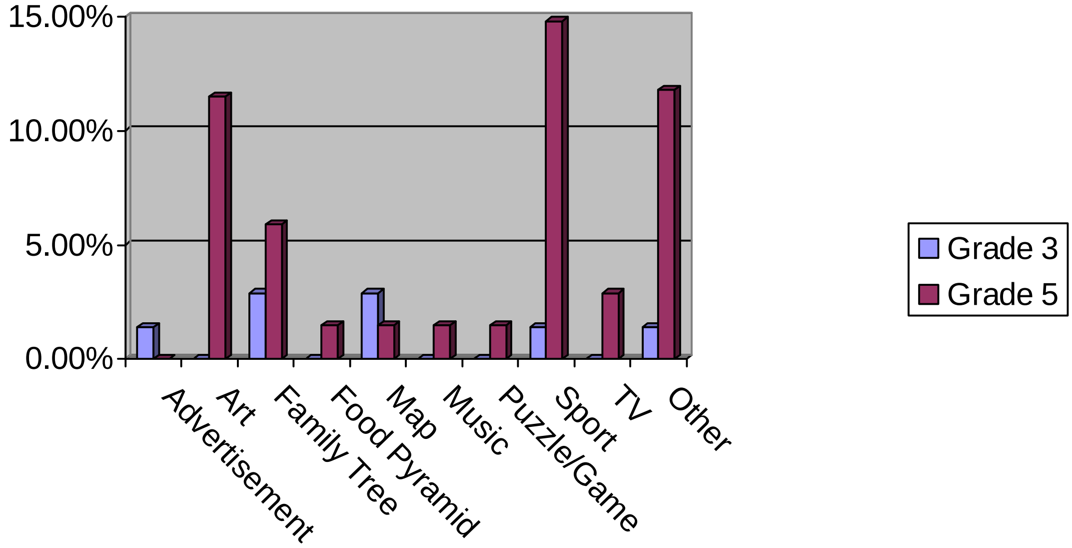


Figure 6. Percentages of students who associated informal experiences with the Hierarchy.

Brent (Grade 5)
B: My brother did a handball competition and he had a hierarchy diagram showing the contestants down the bottom and the winner up the top.

Figure 7. An example of a Hierarchy response.

Proportion of Students' Responses about Informal Experiences

The calculation of Grade 3 (*N*=69) or Grade 5 (*N*=68) students' responses to the Matrix or Network or Hierarchy in this section is based on the proportion of students in each grade who made a particular type of response (see Table 1). The number of responses for Grade 3 and Grade 5 were respectively: 30 and 54 for the Matrix; 16 and 32 for the Network; and 7 and 29 for the Hierarchy. Thus, substantially more Grade 5 students than Grade 3 students identified links between each of these diagrams and out-of-school experiences.

The results indicate that students associated specific diagrams with particular out-of-school experiences. More than 20% of responding Grade 3 students associated the Matrix with a Chart (30%) or a Graph/Grid (26.7%); the Network with a Map (68.8%); and the Hierarchy with a Chart (28.6%) or a Family Tree (28.6%) or a Map (28.6%). More than 20% of responding Grade 5 students made links between the Matrix and a Checklist (24%) or a Graph/Grid (24%); the

Network and a Map (34.4%) or Sport (21.9%); and the Hierarchy and Sport (34.5%).

Table 1
Proportions of Informal Experiences associated with the Matrix, Hierarchy or Network

	Matrix		Network		Hierarchy	
Types of Responses	Grade 3 (*n*=30)	Grade 5 (*n*=54)	Grade 3 (*n*=16)	Grade 5 (*n*=32)	Grade 3 (*n*=7)	Grade 5 (*n*=29)
Advertisement	0%	0%	0%	0%	14.3%	0%
Art	6.7%	0%	0%	0%	0%	3.4%
Chart	30%	14.8%	0%	0%	28.6%	0%
Checklist	6.7%	24%	0%	0%	0%	0%
Computer Game	13.3%	0%	6.3%	0%	0%	0%
Family Tree	0%	0%	0%	0%	28.6%	13.8%
Friendships	0%	0%	0%	12.5%	0%	0%
Food Pyramid	0%	0%	0%	0%	0%	3.4%
Graph/Grid	26.7%	24%	0%	0%	0%	0%
Map	3%	5.5%	68.8%	34.4%	28.6%	3.4%
Music	0%	0%	0%	0%	0%	3.4%
Puzzle/Game	3%	5.5%	6.3%	6.3%	0%	3.4%
Sport	0%	7.4%	0%	21.9%	14.3%	34.5%
Timetable	0%	0%	0%	3.1%	0%	0%
TV	0%	0%	0%	0%	0%	6.9%
Other	10%	18.5%	18.8%	21.9%	14.3%	27.6%

NB. Totals may not add to 100% exactly due to rounding of percentages in each category.

There also appear to be links between particular out-of-school experiences and specific diagrams in Grade 3 and Grade 5. In Grade 3, the Map appeared to be the most influential or memorable experience with more than 28% of students identifying it for the Network (68.8%) and Hierarchy (28.6%). However, in Grade 5, the Map was identified by fewer students for both the Network (34.4%) and Hierarchy (3.4%). The lower proportion of responses to the Network can be explained by Grade 5 students making links to a broader range of experiences, such as Sport (21.9%) and Friendship (12.5%). Students' association of a Network and Friendship suggests that their associations extend beyond recall of a diagram in an everyday situation because it seems unlikely that they would have seen a diagram representing a network of friends. The substantial proportional drop in responses in the association of a Hierarchy and a Map from Grade 3 (28.6%) to Grade 5 (3.4%) can be explained by a reduction in students' links to an inappropriate representation. Although some associations between diagrams and everyday experiences are appropriate, other associations may be inappropriate. For example, although a Family Tree is an exemplar of a Hierarchy due to its branching structure and levels, these structural features of a Hierarchy are generally not consistent with a Map. Although similarities between representations can result in positive transfer, they can also be misleading (Baker, Corbett, & Koedinger, 2001). The most commonly identified referent in Grade 5 was Sport with 21.9% of students identifying it for the Matrix and 34.5% of students identifying it for the Hierarchy. While there is a clear association between Sport and the Hierarchy in the representation of knock-out sporting competitions (see Figure 7), the link between Sport and a Network though less apparent is also conceivable (see Figure 5).

CONCLUDING COMMENTS

This exploration revealed that informal experiences appear to influence primary students' diagrammatic knowledge. Grade 3 and Grade 5 students identified a variety of everyday experiences that they associated with a Matrix, Network, and/or Hierarchy. Thus, these informal experiences provided students with authentic opportunities to develop their knowledge of diagrams. The majority of experiences that students associated with a Matrix, Network or Hierarchy could have been recalled (e.g., Sport). However, some students also made associations between an abstract structure, such as Friendship, and a visual structure, such as the Network. Thus, some students are capable of sophisticated thinking about structural relationships.

The differences between the percentages of Grade 3 and Grade 5 students who were able to identify everyday experiences associated with diagrams and the change in proportions of particular types of responses suggest that there can be a substantial change in students' diagrammatic knowledge over a 2-year period. These differences can largely be attributed to the knowledge that students have recalled or constructed from their informal experiences because instruction in diagram use was not part of these students' mathematics curriculum and they reported scant formal experiences with diagrams.

The results suggest two avenues for further diagrammatic research. First, which of the experiences that students associated with particular diagrams enhance their understanding of these diagrams and which experiences are misleading? Instruction in effective diagram use involves fostering appropriate associations between experiences and diagrams and addressing inappropriate relationships. Second, why were some students able to recall diagrams from their everyday experiences, whereas other students who presumably had similar experiences were seemingly unaware of these diagrams?

At a broader level, the results of this study have revealed that daily life is an important informal learning environment. Hence, conceptions of informal learning environments need to extend well beyond particular sites that are characterised by exhibits and visitor guides.

ACKNOWLEDGEMENTS

This project is supported by funding from the Australian Research Council. Special thanks to the Research Assistants who are working on this project.

REFERENCES

Aittola, T. (1999). Possibilities of learning inside and outside the school. In P. Alheit, J. Beck, E. Kammler, R. Taylor & H. S. Olesen (Eds.), *Lifelong learning inside and outside schools: Collected papers of the 2nd European Conference on Lifelong Learning* (Bremen, Germany, Febraury 25-27, 1999). [ERIC Document Reproduction Service No. ED 469 193]

Baker, R. S., Corbett, A. T., & Koedinger, K. R. (2001). Toward a model of learning data representations. In *Proceedings of the 23rd Annual Conference of the Cognitive Science Society*, (pp. 45–50). Mahwah, NJ: Erlbaum.

Barwise, J., & Etchemendy, J. (1995). Heterogeneous logic: Reasoning with diagrammatic representations. In J. Glasgow, N. H. Narayanan & B. C. Karan (Eds.), *Diagrammatic reasoning* (pp. 211–234). Menlo Park, CA: AAI Press.

Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal, 97*(1), 3–20.

Diezmann, C. M., & English, L. D. (2001). Promoting the use of diagrams as tools for thinking. In A. A. Cuoco (Ed.), *2001 National Council of Teachers of Mathematics Yearbook: The role of representation in school mathematics* (pp. 77–89). Reston, VA: National Council of Teachers of Mathematics.

Henwood, J. (2002). Worlds still to discover: Museums and other informal learning environments. *Gifted,* December, 6–8.

Karmiloff-Smith, A. (1990). Constraints on representational change: Evidence from children's drawing. *Cognition, 34,* 57–83.

Lederman, N. G., & Niess, M. L. (1998). How informed are informal educators? *School Science and Mathematics, 98*(1), 1–3.

Lindsay, R. K. (1995). Images and inferences. In J. Glasgow, N. H. Narayanan, & B. C. Karan (Eds.), *Diagrammatic reasoning* (pp. 111–135). Menlo Park, CA: AAI Press.

Mayer, R. E., & Gallini, J. K. (1990). When is an illustration worth ten thousand words? *Journal of Educational Psychology*, *82*(4), 715–726.

Melber, L. M., & Abraham, L. M. (1999). *Science Activities, 36*(1), 3–4.

Novick, L. R. (2001). Spatial diagrams: Key instruments in the toolbox for thought. In D. L. Merlin (Ed.), *The psychology of learning and motivation, 40,* 279–325.

Novick, L. R., & Francis, M. (1993, November). Assessing students' knowledge and use of symbolic representations in problem solving. Paper presented at the 34th annual meeting of the Psychonomic Society, Washington.

Novick, L. R., & Hurley, S. M. (2001). To matrix, network or hierarchy, that is the question. *Cognitive Psychology, 42*(2), 158–216.

Rogoff, B., Paradise, R., Aruaz, M. R., Correa-Chavez, M., & Angelillo, C. (2003). Firsthand learning though intent participation. *Annual Review of Psychology,* 175–204.

Shigematsu, K., & Sowder, L. (1994). Drawings for story problems: Practices in Japan and the United States. *Arithmetic Teacher, 41*(9), 544–547.

Simon, H. (1995). Foreword. In J. Glasgow, N. H. Narayanan & B. C. Karan (Eds.), *Diagrammatic reasoning* (pp. i–iii). Menlo Park, CA: AAI Press.

van Essen, G., & Hamaker, C. (1990). Using self-generated drawings to solve arithmetic word problems. *Journal of Educational Research*, *83*(6), 301–312.

Veloo, P. K., & Lopez-Real, F. (1994). An analysis of diagrams used by secondary school pupils in solving mathematical problems. In J. P. da Ponte & J. F. Matos (Eds.), *Proceedings of the 18th Psychology of Mathematics Education Conference* (Vol. 1, pp. 80). Lisbon, Portugal.

Willett, J. B., Singer, J. D., & Martin, N. C. (1998). The design and analysis of longitudinal studies of development and psychopathology in context: Statistics models and methodological recommendations. *Development and Psychopathology, 10*(2), 395–426.

Yancey, A. V. (1981). *Pupil generated diagrams as a strategy for solving word problems in elementary mathematics.* Unpublished dissertation for the degree of Specialist in Education, University of Louisville. [ERIC Document Reproduction Service No. ED 260 922]

Yancey, A. V., Thompson, C. S., & Yancey, J. S. (1989). Children must learn to draw diagrams. *Arithmetic Teacher*, *36*(7), 15–23.

CAN ADULT NUMERACY BE TAUGHT? A BERNSTEINIAN ANALYSIS

Gail E. FitzSimons
Monash University
<gail.fitzsimons@education.monash.edu.au >

This paper draws on theoretical foundations elaborated by Basil Bernstein in order to interrogate the teaching of numeracy to adult learners. It distinguishes between the vertical discourse of mathematics and the horizontal discourse of numeracy, discussing curricular and pedagogical implications for face-to-face encounters as well as for flexible delivery via new learning technologies.

In relation to mathematics or numeracy, adults return to study sometimes of their own volition seeking to participate in courses which reinforce and extend their previous formal and informal learning—to complete "unfinished business", to prove something to themselves and others, often justified in the name of helping family members on educational and/or business grounds. Sometimes adults are required by social security or employers to undertake further study. In both cases self-esteem and interpersonal relations can be threatened or enhanced. There are also opportunity costs of time and money—particularly under neoliberal "user pays" regimes.

DISTINCTIONS BETWEEN NUMERACY AND MATHEMATICS

Over recent years, the term *numeracy* has gained popularity in English-speaking countries such as Australia and England—both for school children and for adult education. Other terms gaining currency include *mathematical literacy* in relation to international surveys of adults and school students (Gal et al., 1999; OECD/PISA, 2002) and *techno-mathematical literacies* in relation to workplace activity (Kent et al., 2004). The use of the term numeracy may be an attempt to popularise the discipline of mathematics, but carries certain dangers for adult education in particular, especially when it implies a reduction in content to the four basic processes with rational numbers and perhaps simple measurement skills (FitzSimons, 2002). Research into adults using mathematics in the workplace and in everyday life for a variety of purposes reveals a far greater breadth of mathematical activities and depth in certain understandings, especially when practice demands it (FitzSimons, in preparation, in press; FitzSimons & Mlcek, in preparation).

Bernstein, like Popkewitz (2002), recognises that just as the school subject of woodwork cannot reflect the activity of carpentry, neither can the school subject of mathematics reflect or represent the activities of professional mathematicians. In FitzSimons (2004) I drew on the work of Basil Bernstein (2000) to distinguish between mathematics and numeracy. The following is drawn in part from that 2004 paper where I argue that Bernstein's distinction between *vertical discourse* and *horizontal discourse* may be fruitfully applied to distinguish between mathematics and numeracy, respectively.

Vertical discourse and horizontal discourse will result in different forms of knowledge construction. Bernstein (2000) describes vertical discourse as having a coherent, explicit, and systematically principled structure. This may be in a hierarchically organised form, as in the sciences (*vertical knowledge structures*), or in the form of a series of specialised languages, as in the social sciences and humanities (*horizontal knowledge structures*). Mathematics is included under the latter—as a horizontal knowledge structure, due to the specialised languages of its many sub-disciplines—within the vertical discourse structure.

Bernstein (2000) claims that these two fundamental forms of discourse have tended to be seen as oppositional rather than complementary. In the educational field they are known as: school(ed) vs everyday common sense knowledge, or 'official' vs 'local' knowledge. Common sense knowledge is likely to be: "oral, local, context dependent and specific, tacit, multi-layered, and contradictory across but not within contexts" (p. 157). Crucially it is segmentally organised—that is, "the realisation of this discourse varies with the way the culture segments and specialises activities and practices" (p. 157). Accordingly, Bernstein defines horizontal discourse as entailing "a set of strategies which are local, segmentally organised, context specific and dependent, for maximising encounters with persons and habitats" (p. 157). Discussing the inter-relations between horizontal discourse and the structuring of social relations, he notes that the latter "generates the forms of discourse but the discourse in turn is structuring a form of consciousness, its contextual mode of orientation and realisation, and motivates forms of social solidarity" (p. 158). He adds that "these 'knowledges' are related not by integration of their meanings by some co-ordinating principle, but through the functional relations of segments or contexts to the everyday life" (pp. 158–159). Thus, the knowledges of horizontal discourses are

> contextually specific and *context dependent*, embedded in on-going practices, usually with strong affective loading, and directed towards specific, immediate goals, highly relevant to the acquirer in the context of his/her life. … Although competences/literacies are localised, they do not necessarily give rise to highly coded inflexible practices. Indeed, any one individual may build up an extensive *repertoire* of strategies which can be varied according to the contingencies of the context or segment. … From the point of view of any one individual operating within *Horizontal discourse*, there is not necessarily one and only one correct strategy relevant to a particular context (pp. 159–160).

The research into workplace numeracy by myself and others (FitzSimons, in preparation, in press; FitzSimons & Mlcek, in preparation), supports understanding numeracy in this way—as a horizontal discourse. The crucial features of numeracy, then, are the contextuality, the immediacy of extra-mathematical goals, relevance, and the flexibility and segmentation of strategic practice; other people and the habitat (or environment) can also play a significant role, explicitly or tacitly. This is in direct contrast to common understandings of (school) mathematics as decontextualised, abstract, aimed at furthering mathematical understanding, teleologically directed towards successful completion of assessment tasks, generally lacking in personal relevance to the learner, broadly applicable and generalisable, with predetermined solutions generally known in advance by teachers and/or examiners; the individual is in focus and the broader social and environmental context is generally absent, although sociality within the learning setting is encouraged under socio-constructivist teaching approaches. These have fundamental implications for curricular content and pedagogy, in particular for courses purporting to offer *Adult Numeracy*.

According to Bernstein (2000, p. 159), the pedagogy of horizontal discourse "is usually carried out in face-to-face relations with a strong affective loading." It may be tacitly transmitted by modelling, by showing or by explicit means. Either the pedagogy is exhausted in the context of its enactment, or it is repeated until the particular competence is acquired—for example, counting change. "The segmental pedagogies of the peer group may well depend strongly on modelling/showing" (p. 159). Bernstein notes that "in *Horizontal discourse* there are distributive rules regulating the circulation of knowledge, behaviour and expectations according to status/position" (p. 157). The acquisition of horizontal discourse is marked by competence.

By contrast, vertical discourses such as mathematics consist "not of culturally specialised segments, but of specialised symbolic structures of explicit knowledge" (p. 160). There are strong distributive rules regulating access, transmission and evaluation. Knowledge circulation is accomplished through explicit recontextualisation and evaluation.

> The procedures of *Vertical discourse* are then linked, not by contexts, horizontally, but the procedures are linked to other procedures hierarchically. The institutional or official pedagogy of *Vertical discourse* is not consumed at the point of its contextual delivery, but is an on-going process in extended time. (p. 160)

That is, the pedagogical focus is on recontextualising institutional knowledge developed elsewhere, with the individual being assessed on graded performance. Bernstein reminds us that "both discourses, *Vertical* and *Horizontal*, have an arbitrary pedagogic base" (p. 159).

CURRICULAR AND PEDAGOGICAL IMPLICATIONS FOR ADULT NUMERACY

From the above, it follows that numeracy in practice relies on a combination of mathematical and extra-mathematical knowledges and skills, developed through formal and informal processes, inside and outside of recognised educational institutions and other workplace or community settings where official knowledges are taught (or *delivered*, to use common neoliberal parlance). Numeracy practice, in the workplace at least, operates in association with a range of so-called *key* *competencies*, such as: (a) collecting, analysing and organising information, (b) communicating ideas and information, (c) planning and organising activities, (d) working with others and in teams, (e) using mathematical ideas and techniques, (f) solving problems, and (g) using technology (Mayer, 1992)—described by Bernstein (2000) as *generic modes* of knowledge structure. (For further discussion of generic modes in relation to vocational mathematics education, see FitzSimons, 2002, chapter 3.) Unlike typical school mathematics activities, numeracy is rarely if ever undertaken in isolation from communication with other people, quite possibly utilising some form of new technology, such as a calculator or computer, as a tool to think with and as a source of information (FitzSimons & Mlcek, in preparation; Kent et al., 2004). Further, in situations where people and the environment are potentially at risk, an ethical component underlies (or should underlie) decision-making. In commercial and private enterprises, there are likely to be time and/or money constraints and many occasions when a timely decision is imperative. But what does this mean for content and pedagogy of adult numeracy courses, face-to-face and via distance education means?

Supporting findings from the literature, FitzSimons (in press) notes that the range of Bishop's (1988) pan-cultural mathematical activities (counting, locating, measuring, designing, explaining, playing) were present to a greater or lesser extent in the workplaces she visited. This suggests the need to keep general preparatory courses as broad as possible in content and numerous recommendations for adult numeracy curricula have been made internationally (see FitzSimons, in preparation). However, as noted above, the competency of numeracy is inevitably linked in practice to other key competencies in order to achieve particular extra-mathematical objects or goals rather than the pursuit of technical skills and/or deeper mathematical understandings that are the goals of formal mathematics education. In other words, preparation for the activity of numeracy requires a considerable depth and breadth of mathematical understanding even if the topics are thought to be "elementary" in terms of lists of learning outcomes for school children. Meaningful context is essential, together with the interplay of other relevant social and cognitive competencies.

Internationally, adult numeracy texts tend to resemble school mathematics texts in that they are topic driven, following the arbitrary nature of school mathematics curricula (Bernstein, 2000; Ernest, 1991) in addressing topics such as number, measurement, geometry, statistics, and so forth. That is, they follow (probably for good political reasons) neoliberal principles of atomising the curricula—as is the case for competency-based training in Australia and elsewhere (FitzSimons, 2002). However, they do make serious attempts at adult contextualisations such as shopping, personal finance, home handywork, utilising local demographics, and so forth. In order to develop the vertical discourse of mathematics to support numeracy activities, there is a need for knowledge and skills derived from deep learning, with a focus on meaning making—and this is apparent in many texts, both printed and electronic (See FitzSimons, 2002, chapter 4, for discussion of a counter-exemplary text.).

Although there is an abundance of recommendations for *functional* mathematics curricula in relation to school students and adult learners (e.g., Forman & Steen, 1999; Hoyles et al., 2002; Lindenskov & Wedege, 2002), there are few which address *critical* mathematics in relation to adult learners (e.g., Frankenstein, 1996; Johnston, 1994; Knijnik, 2000). As a complement to functional mathematics, Alrø and Skovsmose (2002) propose that:

> Critical mathematics education is concerned with how mathematics in general influences our cultural, technological and political environment, and the functions mathematical competence may serve. For this reason, it not only pays attention to how students most efficiently get to know and understand the concepts of, say, fraction, function and exponential growth. Critical mathematics education is also concerned with matters such as how the learning of mathematics may support the development of citizenship and how the individual can be empowered through mathematics (p. 9).

The challenge for adult mathematics educators is to negotiate a way to address these issues even though they are extremely unlikely to form part of the prescribed curriculum of officially designated competencies or learning outcomes in neoliberal economies. It may also be that they are outside of the expectations of adults returning to study, and sensitive negotiation needs to take place.

While there is a substantial body of literature from policy-makers, professional associations and practitioners, recommending the use of learner-centred, hands-on, concrete materials, and everyday contexts, Alison Tomlin (2002) cautions that these may not always meet the actual needs of adult learners, especially when their desire is to focus on mathematics from an academic perspective. Once again, adult numeracy teachers should not presume to make choices on behalf of their students, without consultation or dialogue—even when cognisant of recommendations in policy documents. Tomlin also observed, first-hand, that the most efficient solution mathematically may not be the most appropriate in practical terms to meet adults' real-life numeracy problems.

Following Bernstein's discussion of the pedagogy of horizontal discourse, the acquisition of numeracy competence could be compared to the development of tradespersons' knowledges, skills, and techniques, developed via an apprenticeship model. This model blends theory and practice, formal and informal learning, and the physical presence of an experienced practitioner is essential. Eraut (2004) asserts that most workplace learning occurs on the job rather than off the job.

INFORMAL LEARNING

In relation to learning on the job, Eraut (2004) notes that informal learning:

- is in contrast to formal learning, suggesting "greater flexibility or freedom for learners"

- recognises the social significance of learning from others but implies greater scope for individual agency than socialization
- attends to "learning that takes place in spheres surrounding activities with a more formal overt purpose"
- takes place in a wide variety of settings
- can be considered as complementary to learning from experience, which is more personal than interpersonal (p. 247).

Also consistent with Bernstein's horizontal discourse, but adopting an activity theoretical perspective, Griffiths and Guile (2003) offer some ideas about learning associated with work, albeit from a work experience perspective. They note that current and historical contexts are important, as are mediating artefacts in the form of tools, equipment, conversations, manuals, and records. New workers must learn to transform knowledge gained in school and vocational education communities of practice, via social participation, into their workplace community of practice. At the same time, it is recognised that new knowledge is being created as continually evolving problems arise in the workplace. These observations were supported by FitzSimons and Mlcek (in preparation).

Drawing from the extensive range of exemplars in Eraut's (2004) eight-category typology of aspects of informal learning in the workplace, FitzSimons and Mlcek (in press) identify outstanding impressions of integral components of successful workplace numeracy learning in chemical spraying and handling as including: (a) having an awareness and understanding of the problems and risks; (b) having the confidence and knowing when to seek and gain information and confirmation from other workers, manuals, package labels, historical records, and even the internet; (c) being able to cope with the complexity of information potentially available; (d) having the ability to learn from experience; and (e) developing the teamwork skills of joint planning and problem solving. The research showed that supervisors allowed novices restricted parameters for decision making, always under guidance, until they had a proven record of safe practice. In this way serious mistakes could be avoided, yet opportunities for reflection on misjudgements could be provided as learning experiences. This is in striking contrast to the individual focus typical of formal mathematics education where mistakes are commonplace but without any serious consequences. However, increasingly new learning technologies are coming to play an important role in post-compulsory mathematics and adult numeracy education.

NEW LEARNING TECHNOLOGIES AND ADULT NUMERACY EDUCATION

New learning technologies have been promoted as offering possibilities for extending educational opportunities for students across time and space and for ostensibly doing more for less cost to institutions. Although their uptake is certain to proliferate in coming years, and many claims have been made for their success in remediation of cognitive deficits among students commencing university mathematics courses, care must be taken in their adoption for adults returning to study in mathematics/numeracy courses. However, Schapper and Mayson (2004) observe that in the context of falling levels of government support for higher education (in Australia at least) and the need to maintain competitive advantage in global education markets,

> universities face contradictory tendencies: they must market and deliver their educational services across the globe while simultaneously accommodating the diverse and localised and decentred needs of specific student groups. … Education becomes a commodity … delivered to "customers" in rationalised and economic ways, with only lip service paid to learning outcomes or educational objectives of diverse student groups (p. 192).

Oftentimes adults, particularly women, welcome the opportunity for social interaction in supportive institutional or community settings. However, face-to-face classes may have been reduced or abolished as educational authorities attempt to save money and adult numeracy students are required to go online or work with CD-ROM packages for some or all of their studies. This is not to say that socialisation cannot take place—rather that it might take different forms. Flexible delivery which utilises distance education methods is useful for people who are socially or geographically isolated, for those whose family and work commitments, paid or unpaid, do not allow them the luxury of attending regular classes at convenient locations. As well as being a vector for the delivery of educational materials, new learning technologies also provide tools such as calculators of various kinds, computer software packages such as spreadsheets, graph plotters, geometric explorers, statistical data manipulation and display, simulation packages, and so forth, in order to assist learning and calculation processes.

DESIGN AND EVALUATION PERSPECTIVES

Discussing technology and didactics, Nordkvelle (2004) draws on the 1998 work of Wallin who called for:

> a 'reflective approach to educational technology.' He elaborated this as a technology that questions the relations between ends and means and illuminates the contextual and situated nature of teaching. This 'new' technology of teaching should be able to operate according to the rules for action within a strategic, not a rule-directed, framework. He specified that a 'reflective educational technology' should explicitly declare its value premises and its relation to the social context, demonstrate consistency within the knowledge base of the technology—implying the justification of ends and means—and help learners analyse the relationship between the values and knowledge of the technology and the consciousness of the learner (p. 438).

Nordkvelle concludes that "didactics is a technology, in a broader politicized sense, a technology that should be used in ethically and socially responsible ways. … new didactical artefacts, such as ICT in education, are not automatically tested and proven" (p. 440). Both critical and creative didactic approaches are necessary.

Hedberg (2004) proposes that new design discourses are necessary if interactive learning resources are to effectively combine the skills of all members of the development team. The discourses emphasise the importance on interactive design of the learner's role as an active participant in the learning environment if the resultant product is to be engaging and meaningful. A representational balance is needed between subject matter display, visual and sound design, software engineering design, and learning strategy.

Drawing on Aldrich *et al*. (1998, cited in Hedberg, 2004, p. 247) four forms of interaction (visibility and accessibility, manipulability and annotatability, creativity and combinability, experimenting and testing) "represent the assumption that the learner will have an active engagement with the learning task and ensure that its execution and representation is in accord with constructivist learning ideas." He continues that "interactive multimedia task design often coalesces the task into the method of its assessment (p. 248). Ideally, learning tasks require the tacit use of the concepts being learned to achieve the final task completion. Hedberg (p. 248–249) states that "The choice of tools … often dictates the form and interactions that are possible. … web tools in themselves do not enable many of the interactions previously included in CD-ROM packages. … The advent of learning objects, however, provides the opportunity to change the discourse to integrate software tools. In the CD-ROM-based interactive multimedia products he notes that there could be a notebook and a PDA [personal digital assistant] to explore "relationships between inputs and consequences. Both of these objects could be developed as stand alone objects that could collect and manipulate data from multiple sources (p. 249).

Boud and Prosser (2002, cited in Hedberg, 2004) have sought to specify the characteristics of high quality learning outcomes from the learners' perspective. They ask:

- How do learning activities support learner engagement? Reasons for the learner becoming involved and the way the tasks require them to reflect or employ their previous interests and understandings.
- How do learning activities acknowledge the learning context? Does assessment employ real world skills? Does it support the transfer between learning context and professional practice?
- How do learning activities seek to challenge learners? Novices need supportive structures, experts require information for an existing knowledge structure, too much ambiguity can turn a novice away, too little and they become bored. Students might need support to extend the information provided as part of a problem-solving scenario.
- How do learning activities provide practice? Matches between assessment, learning tasks and the transfer tasks might align and model performance. Feedback must support the ongoing development of the learning (p. 251).

Hedberg (p. 252) observes that "often environments employing open-ended learning strategies throw students on their own management resources and many students fend poorly in the high cognitive complexity of the learning environment." He recommends cognitive support tools (e.g., context sensitive, visual and aural help) and the explicit acknowledgement of the double agenda of metacognitive self-management and learning.

Hedberg concludes by recommending the establishment of a continuity between learning task, interaction and visual representation, in order to create the balance desired: "Communication between designer and learner is inherent in the effective development of online applications" (p. 253).

Clearly CD-ROMs cannot offer the communication between designer and learner sometimes available to online production when there is a commitment to maximising learners' achievements rather than corporate profits. Certainly online courses have the potential to foster interaction and communication among fellow learners and tutor, to provide opportunities for reflection, to illustrate complicated calculation techniques through software demonstrations, and to provide simulations as well as video clips of actual numeracy practice. However, online or offline, it seems that numeracy-related project work—utilising the best technology available to learners, whatever their circumstances—comes closest to encapsulating the principles of teaching numeracy as a horizontal discourse.

CONCLUSION

In this paper I have drawn on Basil Bernstein's theoretical distinctions between vertical discourse and horizontal discourse in order to interrogate the teaching of numeracy to adult learners. In discussing curricular and pedagogical implications for face-to-face encounters as well as for flexible delivery via new learning technologies, I have asserted that a pedagogy for adult numeracy should be supported by mathematics knowledge, but needs to draw on characteristics of informal learning typical of the workplace. Recognising that coursework often needs to address official knowledge requirements, as well as those of the learners, one solution might rest with the implementation of reality-based, technologically-supported, project work which recognises that mathematical skills and techniques are but part of numeracy practice which is historically, socially, and culturally located. From a Bernsteinian perspective, teaching mathematics and teaching numeracy are two different activities, with different goals and objects. As a horizontal discourse, teaching numeracy to adults presents challenges to conventional wisdom regarding teaching mathematics—even social constructivist approaches.

ACKNOWLEDGEMENT

This paper is based upon work funded by the Australian Research Council Discovery Project, DP0345726.

REFERENCES

Alrø, H. & Skovsmose, O. (2002). *Dialogue and learning in mathematics education: Intention, reflection, critique*. Dordrecht: Kluwer Academic Publishers.

Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research, critique* (Rev. ed.). Lanham, MD: Rowman & Littlefield.

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education.* Dordrecht: Kluwer Academic Publishers.

Coben, D. (with Colwell, D., Macrae, S., Boaler, J., Brown, M., & Rhodes, V.). (2003). *Adult numeracy: Review of research and related literature*. London: National Research and Development Centre for Adult Literacy and Numeracy. Available as PDF from www.nrdc.org.uk

Eraut, M. (2004). Informal learning in the workplace. *Studies in Continuing Education, 26*(2), 247–273.

Ernest, P. (1991). *The philosophy of mathematics education*. Hampshire, UK: Falmer Press.

FitzSimons, G. E. (2002). *What counts as mathematics? Technologies of power in adult and vocational education.* Dordrecht: Kluwer Academic Publishers.

FitzSimons, G. E. (2004). *An overview of adult and lifelong mathematics education*. Keynote presentation at Topic Study Group 6, 10th International Congress on Mathematics Education [ICME-10]. Retrieved 18 January, 2005, from http://www.icme-10.dk

FitzSimons, G. E. (in preparation). *Adult numeracy and workplace learning.* Adelaide: National Centre for Vocational Education Research. [under review]

FitzSimons, G. E. & Mlcek, S. (in preparation). *Chemical spraying and handling*: *Teaching and learning numeracy on the job.* Final Report. Adelaide: National Centre for Vocational Education Research.

FitzSimons, G. E. (in press). Numeracy and Australian workplaces: Findings and Implications. *Australian Senior Mathematics Journal, 19*(1).

Forman, S. L., & Steen, L. A. (1999). *Beyond eighth grade: Functional mathematics for life and work.* (Research Report MDS-1241). Berkeley, CA: University of California at Berkeley, National Center for Research in Vocational Education.

Frankenstein, M. (1996). Criticalmathematical literacy: Teaching through real real-life math world problems. In T. Kjærgård, A. Kvamme & N. Lindén (Eds.), *Numeracy, race, gender, and class: Proceedings of the third international conference on the Political Dimensions of Mathematics Education (PDME III)* (pp. 59–76). Landås, Norway: Caspar Forlag.

Gal, I., Tout, D., van Groenestijn, M., Schmitt, M. J., & Manly, M. (1999). *Numeracy* [working draft]. Retrieved 14 January, 2004, from http://www.ets.org/

Griffiths, T., & Guile, D. (2003). A connective model of learning: The implications for work process knowledge. *European Educational Research Journal, 2*(1), 56–73.

Hedberg, J. (2004). Designing multimedia: Seven discourses. *Cambridge Journal of Education, 34*(2), 241–256.

Hoyles, C., Wolf, A., Molyneux-Hodson, S., & Kent, P. (2002). *Mathematical skills in the workplace. Final report to the Science, Technology and Mathematics Council. Foreword and executive summary.* London: Institute of Education, University of London: Science, Technology and Mathematics Council. Retrieved January 27, 2004, from http://www.basic-skills-observatory.co.uk/uploads/doc_uploads/522.pdf

Johnston, B. (1994) Critical numeracy. *Fine Print*, *16*(4), 32–36.

Kent, P., Hoyles, C., Noss, R., & Guile, D. (2004). *Techno-mathematical literacies in workplace activity.* Paper presented at International Seminar on Learning and Technology at Work, Institute of London, March, 2004. Retrieved November 24, 2004, from http://www.ioe.ac.uk/tlrp/technomaths.

Knijnik, G. (2000). Ethnomathematics and political struggles. In D. Coben, J. O'Donoghue & G. E. FitzSimons (Eds.), *Perspectives on adults learning mathematics: Research and practice* (pp. 119–133). Dordrecht: Kluwer Academic Publishers.

Lindenskov, L., & Wedege, T. (2001). *Numeracy as an analytical tool in mathematics education and research.* Roskilde: Roskilde University, Centre for Research in Learning Mathematics.

Mayer, E. (Chair). (1992). *Report of the Committee to advise the Australian Education Council and Ministers of Vocational Education, Employment and Training on employment-related Key Competencies for post-compulsory education and training*. Melbourne: Australian Education Council and Ministers of Vocational Education, Employment and Training.

Nordkvelle, Y. (2004). Technology and didactics: Historical mediations of a relation. *Journal of Curriculum Studies, 36*(4), 427–444.

OECD/PISA. (2002). *Programme for International Student Assessment*. Retrieved April 15, 2002, from http://www.pisa.oecd.org/math/def.htm

Popkewitz, T. S. (2002). Whose heaven and whose redemption? The alchemy of the mathematics curriculum … . In P. Valero & O. Skovsmose (Eds.), *Proceedings of the Third International Mathematics Education and Society conference (Part 1)* (pp. 34–57). Copenhagen: Centre for Research in Learning Mathematics.

Schapper, J. M., & Mayson, S. E. (2004). Internationalisation of curricula: An alternative to the Taylorisation of academic work. *Journal of Higher Education and Management, 26*(2), 189–205.

Tomlin, A. (2002). 'Real life' in everyday and academic maths. In P. Valero & O. Skovsmose (Eds.), *Mathematics education and society. Proceedings of the Third International Mathematics Education and Society conference (Part 2)* (pp. 524–532). Copenhagen: Centre for Research in Learning Mathematics.

WHY STUDY GRADE 11 MATHEMATICS: WHAT HAVE COMPUTERS GOT TO DO WITH IT?

Helen J. Forgasz
Monash University
<helen.forgasz@education.monash.edu.au>

Students in grade 11 were surveyed twice, two years apart (2001 and 2003), about a range of issues related to the use of computers for the learning of mathematics. In this paper, the focus is on whether prior computer use for secondary mathematics learning had influenced students' decisions to study mathematics at grade 11, and whether gender differences were evident. The students' reasons for persisting with grade 11 mathematics, even though it is no longer a compulsory study at that level in Victoria, and their future intentions for mathematics studies beyond schooling were also examined. The results and their implications are presented in this paper.

BACKGROUND TO THE STUDY

In Australia, as elsewhere, there is growing concern about declining enrolments in the enabling sciences (Dobson & Calderon, 1999). The persistent gendered patterns of enrolments in some high school level mathematics and science subjects are also of concern. Despite higher female retention rates to grade 12 in Australia—78.5% compared to 66.4% for males in 1999 (Collins, Kenway, & McLeod, 2000)—higher proportions of males than females enrol in the most demanding grade 12 mathematics courses offered (Dekkers & Malone, 2000); the male to female ratio is approximately 1.5:1.

In computing, the patterns of male and female participation rates are more strongly gendered than for mathematics and science subjects. For the two computing subjects offered at grade 12 in Victoria (Australia) in 2002, the male to female ratio was 1.5:1 for Information Processing and Management, and 7.6:1 for Information Systems (Statistical Information, VCE Assessment Program 2002, 2003). With respect to participation rates, both computing and the challenging subjects in mathematics can be considered male-dominated. A similar trend is evident in the workforce. In the popular press, it was reported that women comprise only:

> 29 per cent of jobs in the top 250 ICT companies in Australia…. The relatively small number of women is closely related to the number of women studying ICT at university (Yelland, 2003, p.6).

In the past, Australian males consistently outperformed females in mathematics. In recent years the performance gender gap appeared to have closed; more recently, however, it appears to have widened again. The population 2 (13 year-olds) results in the *Third International Mathematics and Science Study* [TIMSS] revealed that Australia was one of only a few countries with no statistically significant gender differences in performance (Lokan, Ford, & Greenwood, 1996). However, among grade 8 students in the 2003 results of the *Trends in International Mathematics and Science Study* [also known as TIMSS], gender differences in favour of males appear were apparent (Ruddock et al., n.d.). With respect to computing, the few females enrolled in Information Systems in Victoria appear to be doing as well if not better than the males (Statistical Information, VCE Assessment Program 2002, 2003).

Historically, the disciplines of mathematics (Fennema & Leder, 1993), computing (Forgasz, 2002) and the physical sciences (Kelly, 1987) have been considered *male domains*, that is, considered more suitable for males than for females. Contemporary researchers concerned with monitoring and understanding this phenomenon include gender as a variable in their studies of computer use for mathematics learning (e.g., Forgasz, Leder, & Vale, 2000). In the study described in this paper, gender was of particular interest and was included in the research design.

Affective factors including beliefs and attitudes towards mathematics and towards oneself as a learner of mathematics have been strongly implicated in gender differences in mathematics learning outcomes including achievement levels and future participation in mathematics and related disciplines (see Fennema & Leder, 1993; Leder, 1992). Affective dimensions were also a central component of the present study. Beliefs related to mathematics learning, computer use in education generally, and computer use in mathematics classrooms in particular, were sought, although not all findings are reported in this paper.

Technology use in schools is strongly endorsed by federal and state governments in Australia (e.g., Victorian Curriculum and Assessment Authority [VCAA], 2001, website) and elsewhere (e.g., National Council of Teachers of Mathematics, 2000). Rhetoric about the positive effects that technology (calculators and computers) will have on student learning outcomes is rife. However, there appears to be a paucity of research evidence to support such claims. Computers and hand-held technologies are now widely used in secondary mathematics classrooms. While there has been considerable work on students' learning outcomes with respect to specific computer applications and/or with respect to specific mathematical content (see, for example, various chapters in English, 2002), rarely have affective dimensions or gender been included in the research designs.

THE PRESENT STUDY

Exploring whether the use of computers for the learning of secondary mathematics might challenge or re-inforce the extent of gender-stereotyping of mathematics issue was one of the aims of the three-year study discussed in this paper. Another aim was to determine if computer use may be a contributing factor in students' decisions to persist with higher level studies in mathematics. Whether there were gender differences in students' responses to these issues were also of interest.

Data were collected from two large cohorts of grade 11 students from the same schools in 2001, and again in 2003. In this paper, some of the findings from the survey responses of the two cohorts of grade 11 students are presented and the implications discussed.

SAMPLE AND METHODS

Students from a representative sample of co-educational schools across the three Australian educational sectors—Government, Catholic, and Independent (non-Catholic, non-Government)—participated. The schools were representative of the Australian socio-economic profile, and metropolitan and non-metropolitan schools were included. Virtually identical forms of the survey questionnaire were administered to the two grade 11 cohorts in 2001 and 2003. Open and closed response formats were used. A range of information was gathered including: biographical data (e.g., gender, socio-economic status, language background, number of siblings etc.), self-rating of mathematics achievement at grade 10, computer ownership information, subjects being studied, previous use of computers for learning mathematics, career intentions, and others. The samples of grade 11 students who participated in the study are summarised in Table 1.

Table 1
Sample of participating grade 11 students in the 2001 and 2003 cohorts

			Gender			Language background		Student socio-economic status			
	Schools	**All**	**F**	**M**	**?**	**NESB[1]**	**ESB**	**High[2]**	**Medium**	**Low**	**?**
2001	23	519	237	281	1	153	365	88	328	93	10
			46%	54%	.2%	30%	70%	17%	63%	18%	2%
2003	22	376	166	210	-	94	282	81	223	66	6
			44%	56%		25%	75%	22%	59%	18%	2%

1 NESB = Non-English speaking background
2 The Australian Bureau of Statistics [ABS] provides an index of socio-economic categories – high, medium, and low – based on postcodes/zipcodes. Students' home postcodes were gathered.

It should be noted that of the 23 schools from which data were gathered in 2001, 22 were again involved in 2003, with only one school declining the invitation for the follow-up participation.

For this paper, two sets of questions included in the survey questionnaire are of interest. The first set involves a series of questions about the students' current and future involvements with studies in mathematics:

a. Are you studying mathematics this year (grade 11) ? **Yes / No**
b. Do you intend studying mathematics in grade 12? **Yes / No**
c. Do you intend continuing with postsecondary studies? **Yes / No**
d. Is mathematics likely to be included in your postsecondary studies? **Yes / No**

The second set of questions is associated with the use of computers for the study of mathematics in high school:

1. Have you used computers in any mathematics classes in high school? **Yes / No**
2. Do you believe that using computers for learning mathematics helped ***you*** understand mathematics better? **Yes / No / Unsure**
3. Did using computers made learning mathematics more enjoyable? **Yes / No**
4. Do you think your experiences with computers influenced your decision to study (or not to study) mathematics in Year 11? **Yes / No**

   Why do you say this? [Open-ended follow-up question]

Questions 2 and 3 above were only answered by those who answered **Yes** to Question 1. Thus, the statistical analyses associated with responses to these questions refer only to the 378 students (73% of entire cohort) in 2001 and 285 (76%) students in 2003 who answered "yes" to Question 1. For Question 4, the responses considered in this paper are from students who answered "yes" to Question 1 (they had used computers for high school mathematics) and only for those who indicated that they were studying mathematics in grade 11. In 2001, this involved 367 (71% of whole cohort) students; in 2003 this came to 279 (74%). Interestingly, in 2001 there were only seven students who were not studying mathematics but had used computers earlier in high school; in 2003 there were only two such students.

DATA ANALYSES

Data for the Yes/No responses to the first set of questions, Questions a-d, and to Questions 2 and 3 described above, were analysed for differences in frequency distributions using χ^2 (chi-square) tests with $SPSS_{PC}$. The quasi-longitudinal research design allowed for responses to be analysed by year of data collection. For each year data were gathered, the responses were also analysed separately by gender.

RESULTS

STATISTICAL DATA

The categorical responses to Questions a-d and to Questions 2 and 3 above were analysed by the year in which data were gathered (2001 and 2003). Chi-square analyses enabled differences in the frequency distributions of the responses to be examined. The results for Questions a-d are shown in Table 2.

The data in Table 2 reveal that

- most of the students were studying mathematics at the grade 11 level;
- a large majority intended studying mathematics at the grade 12 level;
- a large majority of the students intended continuing with postsecondary studies (a larger proportion in 2003 than in 2001); and
- over 50% of the students believed it likely that mathematics would be included in their postsecondary studies, with a larger proportion in 2003 than in 2001.

The responses to each question (a-d) for each year of data gathering (2001 and 2003) were analysed separately by gender. The results are summarised in Table 3.

Table 2

Results of χ^2 analyses for responses to questions a-d by year of data collection

	a. Studying Maths in Gr.11		**b. Intend studying Maths in Gr.12**		**c. Continue with postsecondary studies**		**d. Maths likely in postsecondary studies**	
	2001*	**2003**	**2001**	**2003**	**2001**	**2003**	**2001**	**2003**
Yes	506	370	433	328	429	334	234	207
	98%	98%	84%	88%	84%	90%	59%	65%
No	10	6	80	47	83	36	166	112
	2%	2%	16%	13%	16%	10%	42%*	35%
***p*-level**	ns**		ns		$p<.01$		$p<.05$	

* Valid percentages provided ** ns = not statistically significant

Table 3

Results of χ^2 analyses for responses to Questions a-d by gender for each year of data collection

	a. Studying maths in Gr.11		**b. Intend studying maths in Gr.12**		**c. Continue with postsecondary studies**		**d. Maths likely in postsecondary studies***	
	2001	**2003**	**2001**	**2003**	**2001**	**2003**	**2001**	**2003**
	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Female 2001: 237 2003: 166	227 97%	162 98%	192 82%	143 86%	202 86%	154 94%	87 46%	78 54%
Male 2001: 281 2003: 210	278 99%	208 99%	240 86%	185 89%	226 82%	180 87%	146 70%	129 74%
***p*-level**	$p<.05$	ns	ns	ns	ns	$p<.05$	$p<.001$	$p<.001$

* For both years, there was a high proportion of missing data for this item

The educationally significant gender differences evident from the analyses shown on Table 3 were related to the likely inclusion of mathematics in students' postsecondary studies. It was clear that in 2001 and in 2003 there was a much higher proportion of males than females who believed that mathematics would be included in their postsecondary courses. This finding is consistent with Australian postsecondary enrolment data in many mathematics, science and Information technology subjects in which males outnumber females (Dobson & Calderon, 1999).

The results for students' responses to Questions 2 (Do you believe that using computers for learning mathematics helped ***you*** understand mathematics better?) and 3 (Did using computers made learning mathematics more enjoyable?) are shown in Table 4. The data in Table 4 reveal that there were no statistically significant differences in the response patterns to the two questions over the two-year period covered by this study. About a third (36%) of the students who had used computers for mathematics learning believed that the computers had helped their understanding of mathematics (Question 2); about two thirds (62%) of those who had used computers agreed that computers had made mathematics more enjoyable. It seems that computers may enhance enjoyment of mathematics for many students. Fewer students, however, felt that their mathematical understandings had been improved.

Table 4
Results of χ^2 analyses for responses to Questions 2 and 3 by year of data collection

Q2. Computers help students understand mathematics			Q3. Using computers made mathematics more enjoyable		
	2001 ***N*=371**	**2003** ***N*=281**		**2001** ***N*=371**	**2003** ***N*=279**
Yes	121 (36%)	99 (35%)	**Yes**	229 (62%)	172 (62%)
No	156 (42%)	113 (40%)	**No**	142 (38%)	107 (38%)
Unsure	94 (25%)	69 (25%)	***p*-level**	ns	
***p*-level**	ns				

Of those who had used computers in high school and who were studying mathematics in 2001 and 2003, the numbers of students reporting that computer use had affected their decisions to study mathematics in grade 11 (Question 4) were: 2001 – 58 students (ie. 17%); and 2003 – 36 (14%). There were no statistically significant differences by year of data collection. Thus, for most of the students, computer use had not influenced their decisions to study grade 11 mathematics.

For the 2001 and 2003 cohorts, the responses to Questions 2-4 were also examined separately by gender. The results are shown in Table 5.

Table 5
Results of χ^2 analyses for responses to Questions 2-4 by gender

	Q2. Believe computers helped their maths understanding		Q3. Computers made maths more enjoyable		Q4. Computer use influenced decision to choose Gr.11 maths	
	F	M	F	M	F	M
2001	*N*=178	*N*=203	*N*=180	*N*=205	*N*=185	*N*=208
Yes	30%	34%	59%	62%	14%	21%
No	49%	37%	41%	38%	87%	79%
Unsure	21%	30%				
p-level	p<.05		ns		p<.05	
2003	N=134	N=161	N=135	N=160	N=138	N=166
Yes	26%	43%	50%	70%	11%	17%
No	47%	34%	50%	30%	89%	83%
Unsure	27%	24%				
p-level	p<.01		p<.01		ns	

The data in Table 5 reveal that compared to females:

- a smaller proportion of males believed computers had not helped them understand mathematics better (2001 and 2003);
- a higher proportion of males believed that computers made mathematics learning more enjoyable (2003 only); and
- a higher proportion of males agreed that their experiences with computers had influenced their decisions to study mathematics at grade 11 (2001 only).

OPEN-ENDED RESPONSES TO QUESTION 4

The reasons students gave for choosing to study mathematics subjects in grade 11, whether or not computer use had influenced their decisions, were very informative. A random selection of about 20% of the surveys completed by the grade 11 students was selected from those completed in 2001 and a similar number of completed surveys was randomly selected from the 2003 surveys. Approximately equal representation of males and females from each school was desirable, resulting in 110 surveys from 2001 and 114 from 2003 being selected for analysis.

Of the 110 2001 student surveys analysed, 10 students responded "yes", that is, that computers had influenced their choice of mathematics for grade 11 with only six students providing reasons; 69 answered "no" with 54 providing reasons; and 31 gave no response. Of the 114 surveys from 2003, there were 19 "yes" responses, 15 with reasons; 74 "no" responses, 61 with reasons; and 21 students did not respond. Since the categorical responses to Question 4 showed no significant difference over the two years of data collection, the open-ended responses were pooled for the analysis. The responses were grouped following a "grounded" approach (Charmaz, 2000) and are summarised in Table 6.

The data in Table 6 reveal some interesting patterns:

YES, prior computer use did influence choice of grade 11 mathematics:

- the most frequent reason given by both females and males was because computer use had helped their understanding of mathematics; and
- the next most frequent reason by both females and males was that computers made mathematics more enjoyable or more interesting

NO, prior computer use did NOT influence choice of grade 11 mathematics

- the most frequent reason provided was that computers had not been used much in prior high school mathematics learning; interestingly, a higher proportion of females than males thought this to be the case;
- the next most frequent reason was that mathematics was needed for future careers or was a pre-requisite for future courses; a higher proportion of males than females provided this response (consistent with a higher proportion of males believing it likely that mathematics would be included in their post-secondary studies – see above);
- that computers were irrelevant in the decision-making and that computers had made no difference to mathematics learning were the

next most frequent reasons put forward; in both cases a slightly higher proportion of females than males gave these reasons; and

- several students wrote that their decisions were related to their positive reactions to mathematics or that they would have studied mathematics irrespective of computer use.

Table 6

Categories and frequencies of open-ended responses to Question 4 by gender (in order of most to least frequent response category)

YES, prior computer use in high school influenced decision of grade 11 maths			**NO**, prior computer use in high school influenced decision of grade 11 maths		
CATEGORY	FREQUENCY		CATEGORY	FREQUENCY	
	Female	Male		Female	Male
Computers helped understanding	4 44%*	3 25%	Computers were not used much	20 31%	6 11%
Computers made maths more enjoyable or more interesting	2	4	Maths is needed for career/prerequisite	6 9%	11 20%
VCE maths doesn't need computers	1	1	Computers were irrelevant in decision	9 14%	4 7%
Computers are the way of the future	1	1	Computers made no difference to learning	8 13%	5 9%
Computers provide new perspectives on maths	0	2	Other	2	11
Other	1	1	Like maths/good at it/ maths is interesting	4 6%	7 13%
			Would have done maths anyway	5 8%	5 9%
			Computers are not needed in VCE maths	1 2%	3 5%
			Computers are just a different way of learning	1	2
			Maths is not needed for the future	2	0
			Grade 11 maths is a school requirement	2	0
Total	9	12		64	54

* Percentages have been calculated for 'larger' numbers for comparative purposes

CONCLUSIONS, IMPLICATIONS AND FUTURE RESEARCH

In a world in which the technology today is obsolete tomorrow, the study's quasi-longitudinal research design provided a unique opportunity to examine snapshots, two years apart, of grade 11 students' views about the effects of computer use for the learning of mathematics. Over the two year period between administrations of the questionnaire, the statistical analyses revealed little change in students' views with respect to the questions analysed and reported in this paper. Due to the likelihood of increased exposure to computing capabilities, it was expected that students' beliefs may have changed over the two-year period, although the direction would have been unpredictable.

What can be said is that the findings of this study do not provide evidence to support the rhetoric about the beneficial outcomes of using computing technologies in mathematics classrooms. Only about a third of the grade 11 students believed that computer use for mathematics had helped in their understanding of mathematics, although almost all had used computers in mathematics some time during high school, although evidence from the open-ended responses suggests that computer use was fairly infrequent for many students. Interestingly, the open-ended responses analysed also suggest that if computers do have the effect of aiding students' understanding of mathematics, this can impact on their decisions regarding the pursuit of higher level mathematics study. Clearly more research is needed to support or refute the findings reported here and to identify what aspects of computer use are viewed by students as being beneficial to their mathematics learning.

When the students' responses were examined by gender for the 2001 and for the 2003 cohorts, it was found that in both years males were less likely than females to believe that computers had not helped their mathematical understanding. Only in 2003 were gender differences favouring males found with respect to beliefs that computers made mathematics more enjoyable. In 2001 only, although the proportions of students were generally small, males were more likely than females to indicate that computer use had been a contributing factor in their decisions to study grade 11 mathematics. In both years, males were much more likely than females to believe that mathematics would be included in their postsecondary studies. It would appear that computers serve as stronger motivators for males' than for females' enjoyment of mathematics and in persisting with mathematics at higher levels of schooling and beyond. These trends need careful monitoring as they appear to suggest that a consequence of computer use in mathematics classrooms may involve inequitable learning outcomes with respect to females' future participation in mathematics-related fields, thus perpetuating gendered enrolment patterns in mathematics and related courses and careers.

There are many fruitful paths for future research based on the findings reported here. Replicating the measures obtained in similar Australian settings and in other international contexts, as well as employing other data gathering techniques, might provide further insights into the issue of the benefits of computer use, both cognitive and affective, for mathematics learning.

ACKNOWLEDGMENT

This study was supported by an Australian Research Council [ARC] Large Grant.

REFERENCES

Charmaz, K. (2000). Grounded theory. Objectivist and constructivist methods. In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (2nd ed., pp. 509–535). Thousand oaks, CA: Sage Publications, Inc.

Collins, C., Kenway, J., & McLeod, J. (2000). *Factors influencing the educational performance of males and females in school and their initial destinations after leaving school*. Canberra: Commonwealth Department of Education, Training and Youth Affairs.

Dekkers, J., & Malone, J. A. (2000). Mathematics enrolments in Australian upper secondary schools (1980–1999): Trends and implications. *Australian Senior Mathematics Journal, 14*(2), 49–57.

Dobson, I. R., & Calderon, A. J. (1999). *Trends in science education: Learning, teaching and outcomes 1989–1997*. Melbourne: Australian Council of Deans of Science.

English, L. (Ed.). (2002). *Handbook of international research in mathematics education.* Mahwah, NJ: Lawrence Erlbaum Associates.

Fennema, E., & Leder, G. C. (Eds.). (1993). *Mathematics and gender*. St. Lucia, Queensland: Queensland University Press.

Forgasz, H. J. (2002). Computers for learning mathematics: Gendered beliefs. In A. D. Cockburn & E Narda (Eds.), *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education, Vol.2* (pp. 369–375). Norwich, UK: University of East Anglia.

Forgasz, H. J., Leder, G. C., & Vale, C. (2000). Gender and mathematics: Changing perspectives. In K. Owens & J. Mousley (Eds.), *Research in mathematics education in Australasia 1996–1999* (pp. 305–340). Turramurra, NSW: Mathematics Education Research Group of Australasia Inc.

Groves, S. (1997). Developing number sense in a calculator rich environment – Implications for classroom practice. In H. Neill & A. Ralston (Eds.), *A zero-based curriculum: Proceedings of a Working Group at the Eighth International Congress on Mathematical Education, Seville, July 1996* (pp. 73–89). Nottingham: Shell Centre for Mathematical Education, University of Nottingham.

Kelly, A. (Ed.). (1987). *Science for girls*. Milton Keyes: Open University Press.

Leder, G. C. (1992). Mathematics and gender: Changing perspectives. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 597–622). New York: Macmillan.

Lokan, J., Ford, P., & Greenwood, L. (1996). *Maths & science on the line: Australian junior secondary students' performance in the Third International Mathematics and Science Study*. Melbourne: Australian Council for Educational Research.

National Council of Teachers of Mathematics [NCTM]. (2000). *Principles and standards for school mathematics*. Retrieved 11 December 2002, from http://standards.nctm.org/document/chapter2/index.htm

Ruddock, G., Sturman, L., Styles, B., Gnaldi, M., & Vappula, H. (n.d.). Where England stands in the *Trends in International Mathematics and Science Study* (TIMSS) 2003. Retrieved 22 January 2005, from www.nfer.ac.uk/research/timms/Section6.pdf

Statistical Information, Victorian Certificate of Education Assessment Program 2002, (2003, April). Retrieved 25 January 2005, from http://www.vcaa.vic.edu.au/vce/statistics/2002/statinfo2002.html

Victorian Curriculum and Assessment Authority. (2001). *Curriculum and Standards Framework II [CSFII] – online, Overview*. Retrieved 8 April 2004, from http://csf.vcaa.vic.edu.au/ov/ov.htm

Yelland, P. (2003, 16 Dec.). The glass cyberceiling. *The Age*, p. (Next) 5.

HOME, SCHOOL AND COMMUNITY PARTNERSHIPS FOR NUMERACY EDUCATION IN A REMOTE INDIGENOUS COMMUNITY

Merrilyn Goos
The University of Queensland
<m.goos@uq.edu.au>

Lesley Jolly
The University of Queensland
<l.jolly@uq.edu.au>

Alex Kostogriz
Monash University
<alex.kostogriz@education.monash.edu.au>

This paper explores the nature of partnerships between families, schools, and communities to support children's numeracy education in contexts of diversity and disadvantage. We draw on the findings of a large Australian research project that investigated the nature of effective partnerships and the extent to which the needs of educationally disadvantaged children were being met. These issues are illustrated with data from a case study of an innovative approach to improving children's access to pre-school education in geographically remote Indigenous communities.

This paper explores issues arising from research on educational partnerships between families, schools and communities in contexts where diversity and disadvantage impact on children's numeracy learning and achievement. It is widely recognised that parents and families are the primary educators of children and are responsible for laying down the social and intellectual foundations for their learning and development. This assertion is also grounded in the education research literature, conveying the clear message that parental and community support benefits children's learning, including their numeracy development (Cairney, 2000; Epstein, 2001; Horne, 1998). Numeracy education has become a high priority in Australia, and the government policies and strategies that have been formulated to address this area typically capitalise on the need to build partnerships with homes and communities (e.g., DETYA, 2000). Yet there are discrepancies between the rhetoric of policy documents and the practice of family and community involvement in education, as current partnership models disregard how families' material and cultural conditions and feelings about schooling differ across social groups (deCarvalho, 2001). These were some of the issues we addressed in a national research project that investigated home, school and community partnerships in children's numeracy education. We analysed features of effective partnerships in the primary school and pre-primary years, with particular emphasis on the extent to which the needs of educationally disadvantaged children were being met. In this paper we draw on one of our case studies to discuss characteristics of successful numeracy education partnerships in remote Indigenous communities in Australia's Northern Territory.

THE ROLES OF "HOME" AND "COMMUNITY" IN PARTNERSHIPS

Epstein (1995) defines home, school and community partnerships as exemplifying a relationship between "three major contexts in which students live and grow" (p. 702) and in which shared interests in and responsibilities for children are recognised. However, while recent shifts in educational policies are partly based on the recognition that good relationships between parents and schools benefit students, consensus has not been reached about how these effective relationships should be achieved, who holds responsibility for what, and where power and control should reside in making educational decisions. Cutler's (2000) historical study of connections between home and school in American education demonstrates that recognition of parental influence in children's education in practice has been often blended with the construction of parents as adversaries who are either uninvolved and irresponsible or overly demanding and intrusive. This idea echoes with Sarason's (1995) view that the present governance structures of schools define, and indeed limit, the nature and scope of parental involvement. In particular, low-income parents often feel and are treated as "less" than the professionals in schools (Fine, 1993).

Communities are powerful learning environments for children, creating potential for their development as they engage in social practices with others. Drawing on communities' funds of knowledge can capitalise on children's culturally diverse home environments. Interestingly, community partnerships focusing on numeracy issues do not usually do so exclusively, and Hexter (1990) notes that community-based programs deemed exemplary for their interventions in support of educational access are often based on more than numeracy and take a more holistic approach. As Kahne (1999) points out, the most important aspect of community programs is the development of long-term relationships in support of positive social change.

Questions of power, control, and access emerged in many of the case studies we conducted as part of our research on partnerships that support numeracy education. In the remainder of the paper we consider how these questions were addressed in one such case, an innovative program that brought mainstream pre-school education to Indigenous children living in remote communities. We begin by giving an overview of the research project as a whole and our case study methodology. This is followed by our observations of the Mobile Pre-school Pilot Program. We then analyse the context and history of the partnership and discuss its significance in the light of the theoretical issues raised above.

RESEARCH OVERVIEW AND CASE STUDY METHODOLOGY

The research project as a whole involved conducting seven case studies of exemplary, sustained numeracy education programs in sites around Australia. Visits to each case study site lasted 3-6 days and involved: observation of classrooms, school staffrooms, teacher-parent interactions, and families in their homes; interviews with teachers, school administrators and support staff, and parents; and analysis of teaching materials, policy documents, and evaluation reports. Cases were selected from analysis of an Australia wide questionnaire survey of education organisations, parent and community groups, and primary school Principals, and of interviews conducted with key personnel in government, Catholic and Independent school sectors in every State and Territory. We selected cases to sample a range of partnership initiation strategies, stakeholder perspectives, and target groups of students. These are the dimensions that framed our analysis of cases.

The first dimension of the analytical framework attends to relations between educational systems, schools, families and communities in terms of how partnerships are *initiated and funded*. Here we distinguished between partnerships that were *top-down* or *top-supported* (i.e., those initiated and sponsored by an education system, and implemented with varying degrees of uniformity in terms of program goals and processes across schools), *school-generated* (initiated by a school independently of an education system), and *home or community-generated*. Clearly, partnership initiation strategies and funding regimes are bound up with issues of power and authority in stakeholder relations.

The second dimension of the framework recognises the different *perspectives of stakeholders* on what constitutes partnerships and what their roles might be. We classified these as school-centred, family-centred, or community-centred. For *school-centred* perspectives we drew on Epstein's (1995) well known work on home-school partnerships to describe how schools understand the roles of families and communities: parenting, communicating, volunteering, learning at home, decision-making, collaborating with the community. Less attention has been given to the ways in which families and communities might see their connections with schools and with each other, and this in itself is suggestive of power relationships between these groups. We drew on available literature in this field (James, Jurich, & Estes, 2001; Jordan, Ozorco, & Averett, 2001; Katz, 2000; Keith, 1999) to identify a range of family-centred perspectives on partnerships and roles, such as acknowledging home practices that support numeracy education, and community-centred perspectives, such as utilising community resources for school reform and curricular enrichment.

The third dimension of the framework looks at ways of responding to diversity and educational disadvantage by identifying the *groups of students targeted* by the program. These include students from Indigenous (i.e., Aboriginal or Torres Strait Islander), non-English speaking, and low socio-economic backgrounds; students in geographically isolated locations; and low achieving students deemed to be at risk of failing to meet State mandated benchmarks for numeracy performance.

THE MOBILE PRE-SCHOOL PILOT PROGRAM

One of our seven case studies was of the Mobile Pre-school Pilot Program, which develops pre-school programs and materials to distribute to Indigenous children aged 3–5 years in remote locations in Australia's Northern Territory. Previously there was no access to pre-school education because of the small numbers of children in each community. (The government's funding formula for staffing schools required enrolment of at least 12 children in any one centre in order for a qualified teacher to be employed. However, most remote communities are too small to satisfy this requirement.) This is an example of a top-supported partnership in that it is government funded but without the requirement for uniform implementation across all sites in the Northern Territory. Our investigation of the history of the program also revealed that many elements were originally, and continue to be, community-generated, thus increasing family and community participation in making educational decisions. Although the partnership does feature school-centred perspectives on the roles of families and communities, its derives its strength from community-centred perspectives, especially the role of local communities in deciding whether and on what terms to accept the program and in deriving financial and social benefits from their participation.

The aim of the program is to increase enrolment, attendance, and participation of Indigenous children in remote areas and prepare them for formal schooling through pre-literacy and pre-numeracy activities. Materials consist of a variety of play activities and items that can be packed into large (90 cm × 50 cm) plastic containers, such as painting materials, puzzles, counting, colour and shape matching games, picture story books, play dough and block construction as well as larger equipment like tricycles, prams and dolls, climbing and sand play equipment. Materials are developed and organised by trained early childhood teachers who prepare and store the materials in their home bases and transport them to surrounding areas by light aircraft or off-road vehicles. The play-packs are often compiled around themes such as transport, communication, colours, and insects, and are rotated between sites weekly or fortnightly, depending on the contingencies of visiting the site.

Teachers travel with the play-pack and introduce the materials to the local teaching support officer (TSO). The TSO in most cases is an Indigenous person chosen by their community to take on the role of organising and running the pre-school sessions in their area. When teachers visit individual sites they introduce the materials in the play-pack to the TSO, explaining how each item might be used. The TSO bases his or her work in the ensuing week or fortnight on the new activities provided in the current play-pack. Teachers circulate between locations, which are grouped into clusters for organisational and planning purposes. This paper deals with our case study observations in the Arnhem and Katherine regions. In the Yolngu/Arnhem Cluster we visited Yirrkala and Dhalinybuy, and in the Katherine Cluster we observed operations with the Bulman Indigenous community.

OBSERVATIONS OF THE PARTNERSHIP IN OPERATION

Pre-school is usually held in the morning three to five days per week, for about two hours, with a morning tea break half-way through. TSOs lead sessions with the aid of parents who follow the TSO's lead in helping the children to use the materials. Older siblings may also attend and help, and younger siblings, if present, take part in the pre-literacy and pre-numeracy activities. Food for morning tea is provided by the teacher on her visiting days and shared with others from around the community.

In Yirrkala the pre-school is run in combination with the child care centre on their premises. This was an organisation of convenience as the child care centre had lost numbers, and the Principal of the local school, when faced with a similar issue, had decided to relinquish his pre-school teacher. When the Mobile Pre-school was established, it joined forces with the child care centre for their mutual benefit. Dhalinybuy has a one-teacher school in which classes are taught by a qualified Indigenous teacher. The Dhalinybuy pre-school was conducted outdoors on a large woven mat under a shady tree. The Bulman pre-school worked in conjunction with the primary school and used one of its rooms, and an open covered area.

At all the sessions we were able to observe, the visiting teacher was present and set the agenda, with support and help from the local TSO. Where the pre-school was closely associated with a school, the teacher there also played a significant role. Parents were present in a fairly liminal fashion but community support was clearly crucial, especially in deciding whether the program was to operate in their community or not. For instance, when the TSO at Dhalinbuy, who is also chair of the local school council, appealed to his community for someone to take on the TSO role he was told "No, you be the teacher". The clear implication was that community people senior to the TSO made this decision. Overall, the visiting teachers are cast in the role of experts in the field who make suggestions to the local TSOs and encourage them to adapt the program for the week to immediate circumstances. Without direct observation of days when the visiting teacher is not there it is impossible for us to say how roles are negotiated in that event. Good personal relationships between the visiting teachers and the community members seem to allow for a certain equity in the partnership, thus reinforcing the trust between participants that seems to be crucial for the success of the program.

There is a dynamic interchange of activity and communication among people at all levels of local community and in the organisation of the pre-school program. Both teachers are very familiar with the Northern Territory and have known the people in the communities and in the educational and child care organisations for many years. This has established a level of trust among participants that is fundamental to the success of the program. Communication occurs predominantly by word of mouth and the play-pack is a means of providing materials that people in local areas may use and adapt in their own ways. In Yolngu/Arnhem territory, the teacher relies on the TSO and other parents for translation between Yolngu and English. In the Katherine cluster, more English is spoken, though Kriol is the home language. Since the TSOs and even the teachers are intimately involved in everyday affairs in small communities there is a transparency between the program and the community that facilitates communication about routine details and individuals. Communication among teachers and TSOs is maintained not only by weekly (or at least regular) personal visits but by bi-semester or bi-term workshops in the central location (Yirrkala or Katherine) where the program is assessed and future plans are made. Participants at this level can also phone each other regularly. This is only possible because the main actors all share a long history of commitment to early childhood education and the welfare of the communities concerned.

NUMERACY PRACTICES

The pre-numeracy activities we observed are typical of those conducted in mainstream Australian pre-schools, and aimed to develop number, measurement, space, and chance and data concepts (Curriculum Corporation, 1994). Active play with puzzles and toys such as cars required shape and colour matching as well as sequencing and counting. Songs and stories provided reinforcement of the language used to make comparison, describe size, shape and sequence and discuss ideas about chance and uncertainty. Games such as "Follow the Leader" addressed sequencing, following instructions and counting. However, it is tempting to argue that many of the toys and activities provided may not have been meaningful for children whose everyday experience was living on Aboriginal homelands. Several puzzles made use of cars, trucks, traffic lights and all the accoutrements of city-based transport. It is not that the children are totally unfamiliar with such things, but there is not a close fit between red double-decker buses represented in the puzzles and the minivan that serves as a bus in their local community. Of course this lack of "relevance" would also be an issue for other Australian children whose life experiences are not represented by the play activities and materials provided to them. Nevertheless, as we discuss below, local people insisted that children needed to become familiar with the world beyond their own communities.

CONTEXT AND HISTORY OF THE PARTNERSHIP

INDIGENOUS EDUCATION

Indigenous education in Australia has been complicated by the history of colonisation. Many studies have documented the damaging effects of attempts to transplant an education system that embodies quite different epistemologies, attitudes and normal behaviours into Aboriginal communities (Folds, 1987). In recent years "two-way education", taking something from both western and Indigenous culture, and adapting it to local conditions and aspirations, has become a popular catch-cry (Harris, 1990; Malcolm, 1999). However the pressures of mainstream culture are hard to resist, and the task is further complicated by the fact that every Aboriginal area, indeed every community, has its own history and hence its own needs and aspirations. The sites we looked at here are informative in this regard.

The Yolngu of Arnhem Land were among the last Aboriginal groups to have been directly affected by colonisation. These were largely confined to the operations of a small Christian Mission until, in the late 1960s, bauxite began to be mined on the Gove Peninsula. This was sanctioned by government without consultation with local Yolngu landowners. Legal disputes over this issue led ultimately to the emergence of land rights and native title as political issues in Australia, in part because of the tenacious engagement of Yolngu elders who realised that their culture was threatened by such incursions of the State and that in order to battle them they had to find ways to speak across the cultural abyss between white and Aboriginal people (Williams, 1986). In order to do this they had to understand non-Indigenous epistemologies, law and politics. As a result western-style education has long been considered necessary to Yolngu people in pursuit of their own cultural agendas. However, acceptance of the education has been regulated according to local culture. Like Aboriginal people elsewhere in Australia, the Yolngu want to hear what this education has to say, but they want to decide for themselves how to use it. This includes a determination to maintain local languages and the patterns of life associated with residence in small communities on homelands, while having regular schooling in English, as it is in mainstream schools.

Aboriginal people in the Katherine area have by no means suffered the degree of dislocation and disruption as people in other parts of Australia but they have a different history from the Yolngu. Bulman is a case in point. Here relationships had been built up over generations between cattle ranchers and local groups. While these were certainly not entirely voluntary or favourable to Aboriginal people, they allowed continued contact with country and a compromise way of life that came to be seen as valuable for many. In the late 1960s or 1970s the local cattlemen made it impossible for their Indigenous staff to remain on the property and these people walked off and set up Bulman near to one of their sacred sites. The group was mixed, including Rembarrnga speakers, Dalabon speakers and others. Because of the long history of living alongside other Aboriginal groups but being forced to conduct much daily business with English speakers, the local languages are not much spoken now. Instead, the home language is Kriol, a new Indigenous language with an English lexical base and an Indigenous grammatical structure. The aspirations of this kind of community are commonly more like those of the mainstream. While ownership of their own land and the right to make decisions regarding it will always be important, a good life is seen to include settled employment in jobs that require mainstream education. While there are intermittent programs attending to the original languages of the Bulman students, there are none that take account of the fact that the home language is Kriol. While this community appears to value and desire mainstream education, their relationship to it is very different from that of the Yolngu.

PLANNING/GETTING STARTED

Although the Mobile Pre-school program has been running for a relatively short time in its present form, it is based on nearly a decade's work by teachers and communities, and its success is intimately tied up with this long lead time and strength of personal commitment and relationships between participants.

In the latter half of the 1990s discussion started on the desirability of providing early childhood education to all Aboriginal children, especially those in remote locations. In general, these discussions were initiated by teachers but in all cases proceeded through lengthy and careful negotiations with communities. With collaboration between staff in education and other government departments, and with the active help of community teachers and women's centre staff in some communities, the concept of pre-school in a box slowly evolved. It seems that all of the central participants, from the program officer in Darwin to the mobile pre-school teachers in the regions, as well as some of the community members, have been involved in the program from very early planning days. Before that they all enjoyed positive and longstanding relationships with communities and this depth of history undoubtedly is important to the success of the program.

What appears to have been a fairly informal arrangement at first could be expanded only with substantial funding. This was lobbied for and gained throughout 2000-2001. The difference between this program and previous ones seems to lie in its greater flexibility. One previous model was to equip a vehicle with all the necessary pre-school equipment and a teacher, who then toured remote communities. This meant that each community was seldom visited had little opportunity to have input to the program or ownership over it. The present scheme provides extensive support to communities who are substantially left to run the daily activities of the pre-school.

In the case of individual communities, there appear to be several ways in which they became participants. In some cases they heard about MPPP visiting another community and asked about joining in. Sometimes influential people (usually women) in the community instigated discussion of pre-school as a good thing and urged the council or other community body to explore options. In others, the suggestion came from teachers, although communities had ways of electing not to participate. Once a community decided to participate, workshops were held to inform community members about what was involved and seek interested people to act as TSOs. The existing relationships between teaching staff and communities were felt to be crucial in this process as they are in the continuing mentoring relationships between regional pre-school teachers and TSOs.

SIGNIFICANCE OF THE PARTNERSHIP

Successful government intervention in Aboriginal communities, whether in matters of health, education, social order or employment, is always likely to be fraught. A common criticism is that such services are simply ways in which the State continues to colonise and oppress Aboriginal people by imposing cultural values and behaviours on them that are unwanted and inimical to social and cultural health. For this reason, special care has been taken by those organising this program to take account of the sensibilities of the participating communities. There was no evidence that communities or families were unwilling partners in the program.

This partnership is significant primarily for the success of its articulation of school, home and community sectors in pursuit of better educational outcomes for children. This program demonstrates the truth of arguments in the literature that success depends on sustained mutual collaboration, support and participation of school staffs and families at home and at school. Although this program is relatively short, it depends on exactly those sorts of relationships built up over many years. It demonstrates that such essential relationships cannot be mandated from outside nor built up overnight, but depend on trust and mutual respect which can only be gained over time.

The literature also suggests that community characteristics influencing students' success include social coherence and neighbourhood stability. Our case study cautions us against interpreting this observation simplistically. In the Yolngu case social coherence is strong, but the program works equally well in Bulman with a more mixed and mobile population. The school teacher there observed that while stability of residence was helpful in providing continuity in education, stability in the household and the relative status of that household in the community were likely to be more closely linked to success. In other words, "stability" is another term that must be interpreted according to context. Here the program itself provides a measure of stability thanks to the longstanding relationships between participants and their responsiveness to local circumstances.

CONCLUSION

Our analysis of this case study shows that cultural difference does not necessarily translate to different educational goals. The participants we talked to were unanimous in wanting their children to succeed in school "the whitefella way". On the other hand, this cannot be taken to mean a wholesale adoption of mainstream epistemologies, values and attitudes. Those planning and implementing interventions in any community need to be very careful to understand exactly what people mean when they express their goals in the words provided for them by the planners. Only constant and open communication between partners can ensure that enthusiastic outsiders (including teachers) do not run ahead of their brief.

The Mobile Pre-school Pilot Program has had positive outcomes for schools, teachers and communities consistent with the benefits of community-centred education programs identified by Kahne (1999). For example, the local communities benefited because of the new jobs created by the program. This is a direct financial benefit but also a social benefit in that community members are given positions of trust and responsibility, their opinions are listened to (in fact eagerly solicited) and they thus provide a role model and exemplar of one kind of success and one kind of use for education for others in the community.

Readers may have formed the opinion that MPPP is not exclusively about numeracy education – and that is precisely the point we wish to make. This partnership, together with several others identified in our case studies, was not initiated as a numeracy program but instead took a holistic approach (cf Hexter, 1990). Our research indicates that building strong home-school-community partnerships around children's learning in general can lay the groundwork for numeracy-specific learning. In culturally diverse communities we would suggest that partnership building is of paramount importance, and should precede—or at least accompany—the introduction of educational programs that seek to initiate children into numeracy practices that are valued but different from those of their home culture.

ACKNOWLEDGEMENT

This research was commissioned by the Australian Government through the Department of Education, Science and Training. The project was funded under the National Strand of the Numeracy Research and Development Initiative.

REFERENCES

Cairney, T. (2000). Beyond the classroom walls: The rediscovery of the family and community as partners in education. *Educational Review*, *52*(2), 163–174.

Curriculum Corporation. (1994). *Mathematics – A curriculum profile for Australian schools*. Melbourne: Curriculum Corporation.

Cutler, W. (2000). *Parents and schools: The 150-year struggle for control in American education.* Chicago, IL: University of Chicago Press.

deCarvalho, M. (2001). *Rethinking family-school relations: A critique of parental involvement in schooling*. Mahwah, NJ: Erlbaum.

Department of Education, Training and Youth Affairs (DETYA). (2000). *Numeracy, a priority for all: Challenges for Australian schools.* Canberra: Commonwealth Government of Australia.

Epstein, J. (1995). School/family/community partnerships: Caring for the children we share. *Phi Delta Kappan*, *76*(9), 701–712.

Epstein, J. (2001). *School, family, and community partnerships: Preparing educators and improving schools.* Boulder, CO: Westview Press.

Fine, M. (1993). [Ap]parent involvement: Reflections on parents, power, and urban public schools. *Teachers College Record*, *94*(4), 682–729.

Folds, R. (1987). *Whitefella school: Education and Aboriginal resistance.* Sydney: Allen & Unwin.

Harris, S. (1990). *Two-way Aboriginal schooling: Education and cultural survival.* Canberra: Aboriginal Studies Press for the Australian Institute of Aboriginal and Torres Strait Islander Studies.

Hexter, H. (1990). A description of federal information and outreach programs and selected state, institutional and community models. *Symposium on information resources, services and programs. Background paper number three*. Washington DC: Advisory Committee on Student Financial Assistance.

Horne, M. (1998). Linking parents and school mathematics. In N. Ellerton (Ed.), *Issues in mathematics education: A contemporary perspective* (pp. 115–135). Perth: Mathematics, Science and Technology Education Centre, Edith Cowan University.

James, D., Jurich, S., & Estes, S. (2001). *Raising minority academic achievement: A compendium of education programs and practices. Report*. Washington, DC: American Youth Policy Forum. Retrieved January 21, 2005, from http://www.aypf.org/publicatons/index.html

Jordan, C., Ozorco, E., & Averett, A. (2001). *Emerging issues in school, family and community connections. Report*. Austin, TX: Southwest Educational Development Laboratory.

Kahne, J. (1999). Personalized philanthropy: Can it support youth and build civic commitments? *Youth and Society*, *30*(3), 367–387.

Katz, Y. (2000). The parent-school partnership: Shared responsibility for the education of children. *Curriculum and Teaching*, *15*(2), 95–102.

Keith, N. (1999). Whose community schools? New discourses, old patterns. *Theory into Practice*, *38*(4), 225–234.

Malcolm, I. (Ed.). (1999). *Two-way English: Towards more user-friendly education for speakers of Aboriginal English.* East Perth, WA: Education Dept.

Sarason, S. (1995). *Parental involvement and the political principle: Why the existing governance structure of schools should be abolished*. San Francisco, CA: Jossey-Bass.

Williams, N. M. (1986). *The Yolngu and their land: A system of land tenure and the fight for its recognition.* Canberra: Australian Institute of Aboriginal Studies.

LEARNING AND TEACHING MATHEMATICS: A CONFIDENCE TRICK?

Tansy Hardy
Sheffield Hallam University
<t.hardy@shu.ac.uk>

This paper is both an exploration of theorisations of what is often named 'identity' and an exploration of what it might mean to be confident in learning and teaching maths. This involves a discussion of the notion of 'subjectivity' and what this can offer our understandings of the experience of many learners and teachers of mathematics. The form in which I present these explorations are experimental to me in a form of *bricolage*. Vignettes from my teaching and research experience in mathematics education are brought together and held in juxtaposition within this paper. This will use 'what is to hand' to create something new. The task I set myself to do this in such a way that it will shake up some of my and the reader's constructs of what it means to be a 'good learner' of mathematics and extend my understanding of how some learners become marginalised through their attempts to learn mathematics.

This paper is both an exploration of theorisations of what is often named 'identity' and an exploration of what it might mean to be confident in learning and teaching maths. I have experienced a limitation in the understandings of the experience of many learners and teachers of mathematics that can be produced using some models of self image and individuality. I will discuss the notion of 'subjectivity' and consider in what ways this might offer a better analytical frame.

I present these explorations in an experimental form, mixing textual commentary and a patchwork of vignettes from my teaching and research experience in mathematics education. I intend that these are brought together and held in juxtaposition within this paper. This will use 'what is to hand' to create something new. This will mean that the experience of reading this paper is likely to be of inevitable fragmentation and possibly somewhat jarring. I hope that it will nonetheless evoke connections and parallels that might be concealed by more traditional modes. It does offer a more authentic glimpse of how a re-examination of practices and descriptions operates and new meanings are formed for me as a mathematics education researcher. I connect this form of presentation to the term from art and literature *bricolage (*see for example, Levi-Strauss, 1966 and Heriot Watt University webpage, 2005). Bricolage refers to the process of adapting and juxtaposing old and new texts, images, ideas or narratives to produce whole new meanings. There is an opportunistic and perhaps playful process of selection. One will borrow, appropriate from what is to hand and re-present to generate new senses. It offers the possibility of challenging habitual ways of understanding.

The task I outline is to work in such a way that it will shake up some of my and the reader's constructs of what it means to be a 'good learner' or a 'good teacher' of mathematics. Later in the paper I will consider what has been created/generated for me and offer a prompt to consider what has stood out for you through this experiment.

CONFIDENT LEARNERS OF MATHS

I have chosen to explore in particular how ideas of confidence are inscribed in teachers' and learners' images of themselves and each other in mathematics education. I have been struck by the repeated occurrence of *confident* as a addend to descriptors of learners of mathematics. As an attribute, confidence brings with it reference to many aspects of the social practices that make up the experience of individual learners of mathematics.

Engaging with this brings into question the nature of 'individuality' and 'the social', and in particular the assumptions about the relationship between the individual subjects and the social domain implicit in our researches in mathematics education. This is a question that has engaged me in my previous work (see e.g., Cotton & Hardy, 2004). I have found helpful theorisations that do not talk separately of the social and the individual, that do not place these as complementary. This theme was furthered in the work of a conference discussion group in 2003 (Gates et al., 2003). In this we considered how the disciplinary paradigms of psychology and sociology can complement each other in enhancing our understanding of the particular contribution that mathematics education plays in bringing about social exclusion. As all theoretical frames promote and hide aspects of the landscape/field we looked for conflicts inherent in their different interests and assumptions.

It was a session during this group's activity that highlighted the problematic process through which learners are inscribed within a discourse of confidence. Starting with my offering of a classroom anecdote linked to the didactic tension (Mason, 1988) of confidence v challenge, Andy Noyes showed an extract from two children's 'video diaries'. He used the intimacy of a big armchair in a private room with which children could record a video diary. This strategy elicited contributions from the children that would bring together the fields of family, school and youth culture within this school location. The contributions evolved into articulations of their maths experiences and broader relations to school, friendships, home and their growing up. Our discussions focussed on the ways in which the two children could be seen as *confident* learners in maths. This paid attention to bodily gestures, positionings, eye contact with camera.

Pupil 1 Girl	*Pupil 2 Boy*
Deep in the chair, wrapping the edge of her jumper round and round her hands, rather worn jumper too, talking of her and her friend's difficulties in maths lessons, of her mum and her family and their relation to learning. She looks into the camera and appeals directly for the help she needs with her maths.	He adopts a full frontal position, uses an assertive language form: 'I am good at…' 'It's easy …' but he seldom looks at the camera. He gave examples from (hard?) astronomy and talked of collecting coins as 'maths out of school'. He talked of his father who wanted him to good well in maths.

Initial discussion described the second pupil as confident that he was 'good at maths' and that first pupil's body gestures displayed nervousness and lack of confidence. But when looking again there was speculation that in fact it could be argued that the reverse was true. This could go like this - For the girl she describes a strong and supportive background where her happiness matters. She can acknowledge she needs help and where she can get it from. Her appeal to the camera is a direct link to a teacher who will help. For the boy, he appeals to examples from scientific world to show how he engages in maths out of school but his descriptions do not included any mathematical references. He tries to portray a coherent and firm stance that say's I'm OK, but averts his gaze. We can ask what he is trying to disguise and who is he trying to satisfy? His absent father?

SUBJECTIVITY

Subjectivity comes from frames that work with notions of 'discourse' and 'practice', with attention on how the subject, 'the learner', 'the teacher', 'the subject mathematics' is inscribed within the language and actions of an education discourse .

The following report illustrates how the subject is both constructed within a similar discursive practice (of coaching and women) and simultaneously contributes to understandings of what a coach can and cannot be.

> A female PE teacher who was refused a coaching license by the Football Association because it claimed women are 'too emotional' was awarded more than £1600 in damages. Ms Hardwick twice failed to win an advanced license to coach despite scoring higher assessment marks than some male coaches who are former Premiership players. 'I'm not normally this aggressive," said Ms Hardwick, who now plans to coach in the USA. "But sometimes you just have to stand up for yourself" (Reported in the Times Educational Supplement 1999).

The subject contributes and colludes in the possibilities and limitations of being a coach. In her case high assessment results were not evidence that she could 'be a coach'. Her supported challenge will have contributed to a shift in the possible constitution of 'coaches'. However her 'aggression' may not shift the claim that women as 'too emotional'; it may not be seen as assertive and controlled, it could compound women's responses as always being emotionally excessive.

> (This formation of subjectivity) allows self awareness… but understands that subjects are dynamic and multiple always positioned in relation to particular discourse and practice and produced by these (Henriques et al., 1984, p. 3).

Some theories of the individual talk about a variety of selves and multiple identities. However these multiplicities are seen as contributing to a resultant coherent and rational individual, reducing the social to the intersubjective. This locates responsibility for contradictions or discontinuities with particular individuals, constituting these as failings and (schizophrenic?) disorders of the individual.

This formation of subjectivity produces a subject existing as a set of multiple and contradictory positionings or subjectivities. But how are such fragments held together? What accounts for the continuity of the subject, and the subjective experience of identity? However, this frame does not imply that people are simply and mechanically positioned in discourses. This would offer no explication of either the possibilities for change or individuals' resistances to change. But can it also contribute to an account for experiences of the predictability of people's actions, as they repeatedly position themselves within particular discourses? (from Henriques et al., 1998, p. 204).

> In contrast, psychoanalysis gives space to our fundamental irrationality: the extent to which will or agency is constantly subverted to desire, and the extent to which we behave and experience ourselves in ways which are often contradictory… (but) development of individuals and their implications are neither entirely predictable nor reproducible, nor are they controlled from within (Henriques et al., 1984, pp. 205–206.)

From setting out the analytical frame that I bring to the context of this exploration I recognise a concern to identify ways in which learners, teachers and researchers are driven to project an imaged coherent identity. This is a drive has also been described as trying to complete the picture of yourself. (Jones & Brown, 1999).

MORE BRICOLAGE

The extracts below are drawn from data and reflections from particular research projects that I have undertaken. Clearly these were 'to hand'. The first of these is an analysis of video guidance material (DfEE/SEU, 1999) to explore the discursive practices of teachers and children in exemplar 'National Numeracy Strategy' lessons. (see Hardy, 2004). In a second project "Exploring whole class interactive teaching: its meanings and its effects I worked with both practicing teachers and pre-service student teachers. I have also referred to the research of others with which I make connections and have borrowed incidents that were related to me in interviews with teachers. Several of the terms and expression I use in my commentary have been appropriated from others' accounts of their research.

> **Confidence**
> • **noun 1** the belief that one can have faith in or rely on someone or something. **2** self-assurance arising from an appreciation of one's abilities. **3** the telling of private matters or secrets with mutual trust. (Oxford Compact Dictionary)
> • **noun 2** the quality or state of being certain (Merriam-Webster 2003 online dictionary)
> **origin** *con-* completeness, *fidere-* to trust (Chambers Twentieth Century Dictionary)
>
> **Confidence trick** (N. Amer. also confidence game)
> • **noun** an act of cheating someone by gaining their trust. (Oxford Compact Dictionary)

FAST, PACEY AND NO SWEAT

Preservice primary maths student teachers said that **for children in school confident learners of maths…**

Put their hands up for nearly every question; Are eager to give answers and might shout out; Speak out more; Can explain how they got their answer; Will have a go at things; Are fast workers and complete their work quickly.

For their peers in maths sessions confident learners…

Can get straight on with trying something out; Go too fast for others to follow; Contribute without thinking about the 'end result'; Can explain why something works.

In the classroom scene: children are 'warming up their maths brains' by working out the doubles of the numbers to 12 as fast as they can.

Teacher: We are going to practice and practice and practice our doubles, see if we can be faster.

Teacher: Brilliant, very well done. That's excellent. I can't believe how fast you did that. Easy peasy!

Teacher comment: We started off by sitting on the carpet. I like that. Because we are more focussed when we sit together, and I think I can keep them as a group better. Children really see maths lessons as fun. They really get a chance to really join in and participate.

Video Transcript: Year 3 Teacher, National Numeracy Project/Hamilton Maths Project, 1998

Recurring themes in the research of Jo Boaler (1997) and Hannah Bartholomew (2000) include girls' reluctance in using the term confidence in their descriptions of themselves in relation to their maths teaching. The girls included in their research talked of their discomfort in their maths classes, of not having enough time to understand, that learning maths was hard work, of being uncertain about their likely test results. This includes comments on the discourse used by those who are seen as good at maths. These girls may sound unconfident as they don't use 'its easy' or portray their work as 'no effort required'.

> We go though the topics very quickly, without having enough time on one. A lot of the people in the class are naturally very clever, and it is embarrassing to get something wrong in front of them. Tania, set 1, Willow (in Bartholomew, 2000).

Yet when prompted these girls are very clear that they are as good at the subject as the boys.

What are these girls talking of when they describe their discomfort and uncertainties, and with what intent? Verbalising self doubt might be a symptom of their recognition that to take on the challenge of learning maths that they need to be prepared to live with uncertainty. That they recognise that their understanding is incomplete, and want to work on this further, not just produce outward signs of 'can do'.

Many women of the preservice students interviewed refer to positive feedback and praise having an important effect on learning. In (some) classrooms girls receive less attention in comparison to boys. Boys demand more interaction and will receive feedback from teachers. Praise is often used as a management strategy (this may often be undue praise). Does this give boys more opportunity to develop confidence in the ease of 'getting it right'? (Research Journal Entry)

RISKING GETTING IT WRONG

Pre-service primary maths students said **of themselves that they were confident in maths** & will have a go when… I know the subject very well; I know I'm right; When I'm 99% sure If I have some basic understanding I've done something similar Someone else got it wrong first No-one else will see the consequences I've got time to try things out; If I can have a go on my own; if no one is watching if I get it wrong; When the group will laugh with me not at me	A teacher's comment: The first part of the lesson we start with a mental warm up to try to get children's confidence up, that they know the answer encourage them to have a go at the answer, even if it's wrong it doesn't matter." Another teacher: A few children don't put their hands up. They try to hide but that's the idea, there is no hiding place. You encourage them all as long as you give them quality feedback even if they get it wrong they are not scared to give an answer. NNP/HMP 1988 Video transcript
Pre-service primary maths students said **Of their peers that confident learners…** Answer quickly even if they are not right; Will share ideas even when wrong and work on them until the correct solution is achieved; Less likely to be embarrassed; Speak out, even if they don't fully understand; Less confident learners stay quiet if they think they might say the wrong thing. **Of children that confident learners** Don't ask to check answers before writing them down; Less confident children copy from others; Will have a go at things.	A teacher researcher describes how he came to realise that his students reported enjoying the more open, active and more discussion and group work based way of working he had developed with them and most talked of feeling more confident and empowered with their work particularly calculations in maths. Despite this, for a few students, maths had become a less certain realm and so reported feeling less confident about their ability to tackle the work in maths. Kevin Thompson's Masters' dissertation (Thompson, 2003)

They're having a go, they're risking things and you don't gain anything unless you have a few risks. (Teacher comment from NNP/HMP 1998)

Trust: Wrong doesn't matter, Willingness to try is what matters in learning maths
The Trick: Trying is not enough, you'll have to learn to be inured to the embarrassment

Preservice primary maths students said that **teachers help learners be confident** when they: Are not negative about wrong answers; Consider all ideas offered; Always give positive feedback; Praise children for any contribution; when they are right; where it's due; Give help and advice to help overcome difficulties, are re-assuring; Don't put them on the spot, don't make a show of them by 'individualising'; Encourage them to take risks even if wrong

I notice the persistence of a perception of 'maths as right or wrong' amongst pre-service students with whom I work. To explore what I do that contributes to this perception, I critically examine the strategies and teaching styles I intend to dislodge this notion of maths as (always) right or wrong. However, I need to go beyond this to consider how power continues to work regardless of changes in school mathematics curriculum and teaching approaches so that this construct of 'right or wrong maths' remains. Questions to ask: Is it particularly difficult to resist this in mathematics? Are there specific instances of resistance by learners to the normalising effects of 'getting maths right or wrong'. What desires and images does maintaining this construction of maths serve?   From research journal (see Hardy, 2000)

Contributing to a classroom or whole group session is to open yourself up to be judged by peers and by teachers. The discourse practice works to produce valid contributions that are fast and slick made with ease. Only contribute when you are sure and you understand. There is no time to work on ideas, to clarify and evaluate. There is fracture and an erasure here between pre-service student teachers' description of the conditions where they feel confident to contribute, and their descriptions of the the acts of their peers and of children to whom they attribute confidence. How does this contradiction arise? Only one student said that it was possible to be confident and not want to contribute.

> We have not only illustrated the motivational dynamics through which individuals are positioned in discourses, but also opened the possibility that those processes which position us are also those which produce the desires for which we strive" (Henriques et al., 1984, p. 205).

PARTICIPATION, PERFORMANCE AND STAGE FRIGHT

Classroom scene: a 'times table' challenge. The audience of children and teacher survey a child sitting on a chair alone at the front of the classroom. The teacher asks the other children to support him 'be thinking the answers for him'. 'Give him a big clap' 'It wasn't easy in front of all these people, good boy!' NNP/HMP 1998	The whiteboard tale… I had already recorded the repeated behaviour of one boy whose hand always goes up quickly, so often that it is unlikely that the teacher will actually ask him to answer the question. I'm interviewing a teacher who is undertaking a classroom research project on using individual whiteboards in his whole class teaching to engage more of the students and access their ideas. He reports one child who has learnt to hold his board up flat, in such a way that the contents are obscured from the teacher's view.
For children in school confident learners of maths…. Offer answers, speak out more; are able to explain their own thinking Myhill (2002) finds a relationship between participation and under-achievement where learners who do not actively participate are less likely to achieve well on assessment tasks.	

<table>
<tr><td colspan="2">'The act of learning can be regarded as a political act; the learner has to grant his or her assent to learning.' (Evans quoted in Myhill, 2002)</td></tr>
<tr><td colspan="2">Classroom video scene. A child is asked to come out to the front to place a coin card in the correct position on a line. The child moves slowly and thoughtfully towards the front.
Teacher: Quick, quick.

It's as if the child's silent careful thinking is undesirable, even cheating somehow. When another child comes out to the front thinking out loud as he approaches, this 'speaking subject' is praised for by the teacher.</td></tr>
<tr><td>Preservice primary maths students said for their peers confident learners of maths….
Can get straight on with trying something out; Contribute without thinking about the 'end result' and give clear explanations;
Not afraid to have a go in front of others;
Will go to the front to use the board;
Volunteer; Contribute to whole class discussion regularly and doesn't sit back;
Always the same people answering questions:
Will chat with peers, not always about subject or the task.</td><td>Preservice primary maths students said for themselves that they feel confident in contributing in maths sessions if…

Others will listen to me; and they aren't overpowering & insist they're right;
If the group is not too big;
If there is support from the group, if I know they won't judge me.</td></tr>
</table>

To be attributed with *confidence* you must act as confident learners of maths do. Fluency in the discourse practices of the mathematics classroom is crucial, particularly in how to make a contribution that will be seen as valid and be seen making it. Participation and performing with and in front of others is necessary too.

CONFIDENCE; IS IT ALL A TRICK?

This discussion and attribution of confidence is a tactic that reveals the differences between the inside and outside for both the watching attributer and the experiences of the actor. Feeling confident inside but not performing fluently with the normalising language of the classroom would result in this confidence and associated ability remaining unacknowledged. However using theoretical frames drawing on the workings of the discursive practice and its effects on the subjects/subjectivities (learners, teachers, mathematics as a subject, confident learners) reveals better the instability and contradiction of notions like learners' abilities and confidence.

Here lies a possible confidence trick—where learners inscribe themselves in the performed symptoms, the noticed actions that have been seen to construct one actor as *confident*, they put their trust in the teacher's assertion that participating and trying is what matters. But there is no guarantee that this process will work for them, that they will become *confident*. The descriptions of confident learners above include those who know how to start a problem, extend their work and who can say why something works. These are more difficult characteristics to replicate. Perhaps retaining a limited view of maths as about right or wrong answers permits students to sustain a more complete picture of themselves as learners in relation to maths as a subject. It's easier to completely trust the maths is it's about getting the right answers.

However this does not fully address how is the performative element is able to hold sway in teachers' descriptions? What is there to gain for them by focussing on this aspect of maths practices? Certainly the more that is spoken out loud and acted out in the classroom, the better they can observe and judge and contribute to the regulation of these learners of maths.

From the point of view of maths education as a subject, how does the inclusion of confidence as an important part of capability in maths offer? In what way does it bring coherence to the image of maths education? It does offer a broadening to include an affective element; confidence to express yourself becomes a fundamental attribute of a good maths learner. This may give a more accessible feel, and make educators feel less anxious about an enduring concern with end points and right answers. Although here I have no evidence that this has succeeded in loosening this concern.

Further, whose interests does a focus on performative aspects fulfil? For some learners it offers them something they can at least strive to do, to work towards (if you aren't given time to think things through and clarify your explanations). For others it allows them to sit back and keep quiet, "I've never been any good at maths, anyway". For others it alienates, does not confirm their subjectivity, their self image. Their desire for understanding, for time to work it out, to enjoy challenges and to sweat, work on something hard are unfulfilled. These desires are excluded and these learners may be placed on the boundary - achieving well enough but really not acting or participating in the right way, so not confident learners of maths.

I have given some indication what new understandings have been created for me and questions that have been generated for my further engagement. I conclude by offering a prompt for any readers to consider what has stood out for them through following this experiment.

REFERENCES

Bartholomew, H. (2000). 'Top set' identities and the marginalisation of girls. Paper presented at King's College London School of Education.

Boaler, J. (1997). When even the winners are losers: Evaluating the experiences of 'top set' students. *Journal of Curriculum Studies, 29*(2), 165–182.

Chambers Twentieth Century Dictionary. (1974). A. M. MacDonald (Ed.). Edinburgh: W&R Chambers

Cotton, T., & Hardy, T. (2004). What does it mean to research social and cultural contexts in mathematics education? In R. Zevenbergen & P. Valero (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 85–103). Dordrecht: Kluwer Academic Press.

DfEE/SEU. (1999). *National numeracy strategy: Framework for teaching mathematics.* London: CUP for DfEE

Gates, P., Lerman, S., Zevenbergen, R., Hardy, T., & Askew, M., (2003). The role of mathematics education in social exclusion: Reviewing the interface between psychological and sociological research paradigms. At the 27th conference of the international group for the Psychology of Mathematics Education, July 2003, Honolulu. Discussion group proposal in proceedings and full report in Newsletter.

Hardy, T. (2000). Tracing Foucault's power in the mathematics classroom. In C. Morgan & T. Rowland (Eds.), *Research in mathematics education Vol 2.* (The Second British Society for Research into the Learning of Mathematics Yearbook), (pp. 207–224). Bedford Way London: BSRLM.

Hardy, T. (2004). There's no hiding place: Using Foucault's notion of normalisation to examine the effects of teachers' and children's practices in a mathematics lesson. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 103–119). Greenwich, CT: Information Age.

Henriques, J., Hollway, W,. Urwin, C., Venn, C., & Walkerdine, V. (1984/1998). *Changing the Subject: Psychology, social regulation and subjectivity*. London: Routledge

Heriot Watt University 2005 Department of Business Organisation – Semiotics. Retrieved 2 January, 2005 from http://omni.bus.ed.ac.uk/opsman/quality/SEM_black_run_7.htm

Jones, L., & Brown, T. (1999). Tales of disturbance and unsettlement: Incorporating and enacting deconstruction with the purpose of challenging aspects of pedagogy in the nursery classroom. *Teachers and Teaching; Theory and Practice, 5*(2), 187–202.

Levi Strauss, C. (1966) *The savage mind*. London: Weidenfeld and Nicolson.

Mason, J. (1988). Tensions. In D. Pimm (Ed.), *Mathematics, teachers and children* (pp. 164–169). Milton Keynes: The Open University.

Merriam-Webster online dictionary. Retrieved 2 January, 2005 from http://www.m-w.com/dictionary.htm

Myhill, D. (2002). Bad boys, good girls? Patterns of interaction and response in whole class teaching. *British Educational Research Journal, 28*(3), 339–353.

National Numeracy Project/Hamilton Maths Project. (1998). *Numeracy in action: Effective strategies for teaching numeracy*, Video material.

Oxford Compact Dictionary. (2005). Retrieved 2 January, 2005 from http://www.askoxford.com/

Thompson, K. (2003). *Effective teaching of numeracy in a secondary school*. Unpublished Master's dissertation, Sheffield Hallam University.

THEORISING THE PRACTICES OF ALIGNING ASSESSMENT STRATEGIES WITH OTHER COMPONENTS OF THE SECONDARY MATHEMATICS CURRICULUM

Clive Kanes
Centre for Learning Research, Griffith University
<c.kanes@griffith.edu.au>

The paper indicates the need for new work in the areas of practice and theory around the secondary mathematics curriculum. It offers a plan for proposed research into the topic of aligning assessment strategies with other components of the secondary mathematics curriculum. The proposed research is to be conducted over three years, each year a cycle framed by Engeström's (1999) theory of expansive visibilisation. In building a plan for this research three key aims are identified and discussed in relation to the research literature. The first of these explores a model that visibilises components of the secondary mathematics curriculum and identifies the matter of alignment among these. The second aim explores how alignment between assessment and other curriculum components is obtained in practice. The third aim is to offer a theorisation of the practices of alignment and seeks to generate new practices of alignment. The paper ends with an outline of methodological issues that draw on cultural-historical activity theory, and frame the proposed study.

BACKGROUND

If the last 20 years has seen major changes to our understanding of learning and pedagogy in mathematics, much of this organised around the theme Lerman (2000) has identified "the social turn", then assessment has largely been left behind. This is not to suggest, however, that mathematics *teachers* have left assessment behind – the wisdom of mathematics teaching practice has for a long time regarded the matter of what is taught and what is assessed as closely related. But such alignments tend to be motivated by what Habermas has famously characterised as an instrumental knowledge interest, which is a concern for a means-end calculus (Carr & Kemmis, 1986). Thus, what is left out in Habermas' terms is a study of these curriculum relationships from a critical/emancipatory perspective. Such an analysis moves from the instrumentalism of traditional practice and leads to a broad consideration of the practices of assessment, their structure and social and political status. For instance, in exploring mathematics assessment, few studies have systematically explored the relationships among assessment and learning, pedagogy, the discourse of syllabus construction, numeracy, and the technologies mediating these components and their interactions. As a result, the pre-emption of mathematics pedagogies in received practices of assessment has gone mostly unchallenged, and this has the effect of sidelining the "social turn", and producing reproductive pedagogies where transformatory practices were intended.

In this paper there are two main goals. The first is to motivate the need for a new understanding of what methods can help to achieve the alignment of assessment strategies with other components of the secondary mathematics curriculum. Such a need, if met, would have the practical consequence of countering the pre-emption of pedagogy figured above, thus removing a significant obstacle to the more general implementation of a broader range of pedagogies in secondary mathematics curriculum. The second main goal of this paper is to outline methodological plans for conducting the proposed exploration. It is hoped that critical feedback and discussion relating to these lead to a refinement and improvement of the proposal.

A plan for the current paper is as follows. In the first section three aims for the proposed study are identified and their evolution from the research literature is traced. In the second section brief comments summarising the significance of the proposed research are made. In the penultimate section cultural-historical activity theory is identified as the overarching theoretical framework for the methods to be used in the proposed study. The paper ends with a short conclusion.

AIMS FOR THE PROPOSED STUDY

Aims for the proposed study are divided into three key points.

AIM 1: TO ARTICULATE A MODEL VISIBILISING COMPONENTS OF THE SECONDARY SCHOOL MATHEMATICS CURRICULUM AND IDENTIFY THE PROBLEM OF MAL-ALIGNMENT AMONG THESE COMPONENTS

By curriculum in this proposal is meant what Pinar (2004) and others have referred to as both the physical and psychological artefacts of teaching and learning as well as the processes associated with implementation and acquittal of curriculum outcomes such as assessment, disciplinary knowledge, syllabus and reporting methods. This broad definition of curriculum suggests that components of curriculum practices may be identified in the following way (Fowler, 2004):

- Mathematics disciplinary knowledge and mathematics syllabus
- Mathematics learning goals, outcomes or competencies
- Mathematics teaching methods and technologies
- Mathematics assessment methods and technologies
- Mathematics reports and statements of achievement.

In Figure 1, following, is shown a hypothetical model of how these components may interact within the context of mathematics curriculum. Also shown (in the arrows) are the spaces of what is referred to as 'practices of alignment'. Now little attention in the literature is directed towards ascertaining how curriculum components interact in practice and, in particular, how the practices of alignment are structured and how they mediate interactions. Largely, in fact, these practices are invisible. Notwithstanding this the question of alignment lies at the heart of the legitimacy of curriculum practices, their validity and reliability. Indeed, drawing on anecdotal evidence, standard curriculum practice does not generally explicitly recognise the need to conceptualise the practices of alignment and theorise their mediations. Consistent with this practice the literature also glosses over discussion and analysis of practices of alignment. For instance, Anderson, Brown & Lopez-Ferrao (2003) note

> The comprehensive assessment programs to be developed and used must address the concerns of the public and the practitioners. Different kinds of assessments are required and their appropriate roles and use must be specified with a much stronger emphasis on the use of results at the district, school, and classroom levels (p. 625).

Thus, on their view, curriculum processes are certainly found to be multilayered, however it seems to be assumed that the processes by which these layers are in dialogue are either known and well understood, or not known and considered unproblematic. Such assumptions, however, are not well grounded in research or practice relating to mathematics. Indeed, looking at the literature, it is clear that practices of alignment remain deeply embedded within assessment cultures for mathematics.

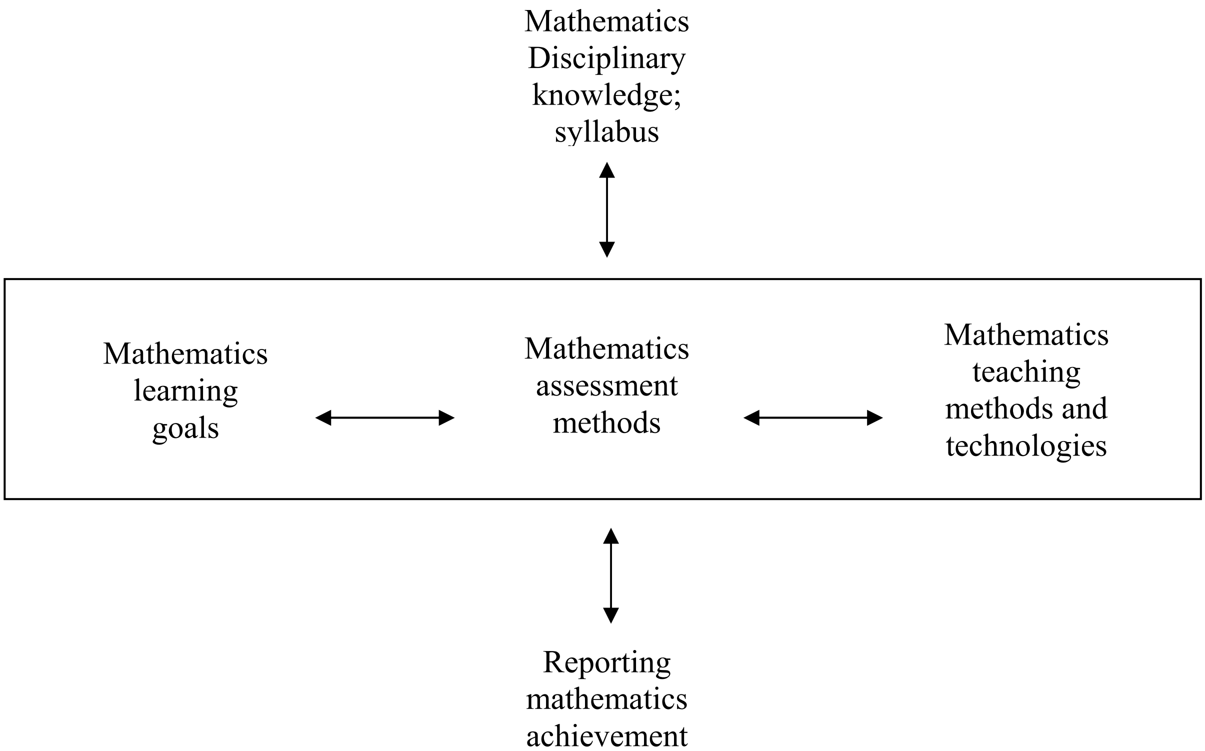


Figure 1. Model of components of mathematics curriculum in which the double-headed arrows indicate the need for alignment – the work of what I have called 'practices of alignment'.

Of course, a pragmatic view would be that this apparent omission does not signify a serious problem. Such a view might be made to depend on the observation that even though the typical practices of alignment are for the most part unreflective, no serious problems, in practice, arise. However, this to is not the case. For instance, in a pair of superb studies of mathematics teachers' positions and practices in "discourses of assessment", Morgan (1998) and Morgan, Tsatsaroni and Lerman (2002), identify "tensions between liberal progressive and traditional modes of pedagogic discourse" (p. 458). Thus we see that in these studies tensions or mal-alignments are identified within everyday mathematics curriculum practices. Moreover these tensions are expressed explicitly, in part at least, as mal-alignment among the discourses of assessment and those of pedagogy.

In another study, de Vijer and Phalet (2004) have studied the problems of assessment within the context of a multicultural student body. Their findings indicate the critical importance of implementing methods that are able to assess the degree of acculturation of a migrant student *prior* to routine assessment strategies. They find that "this information is essential in determining a testee's or client's *testability*. Without such information it is difficult to know whether or to what extent norms for mainstreamers can be applied." (italics added, p. 230). In other words, in the case of migrant students, the practices of curriculum alignment need to be identified, closely observed, and modified where necessary. Moreover, if the finding of de Vijer and Phalet is correct, then the need to produce significations of culture in mathematics assessment methods is pressing. Such a development of assessment in a multicultural society with a significant population of indigenous people, such as is, for instance, Australia in modern times, would therefore be made urgent.

Thus, Aim 1 of the proposed research is to articulate a model visibilising components of the secondary school mathematics curriculum and identify the problem of mal-alignment among these components.

AIM 2: TO EXAMINE HOW THE PRACTICES OF THE ALIGNMENT OF ASSESSMENT TO OTHER COMPONENTS OPERATE

Now, are argued above, key curriculum components in mathematics are numerous and possibly interacting. This suggests that any proposed ethnography of assessment (see the methods section below) would be a complex and highly nuanced enquiry. In response to this, this study proposes to concentrate on the curriculum component of assessment. This is chosen because mathematics assessment, though well provided with professional 'how to do' kits – all at the level of technical knowledge - nevertheless lacks a comprehensive theoretical and practical understanding of how it combines with disciplinary knowledge, teaching methods and learning. Another way of making this point is to suggest that mathematics teachers may often become submerged with technical questions relating to assessment and as a result have insufficient scope for constructing critical responses to mathematics assessment practices.

Fowler and Poetter (2004) in their detailed analysis of contemporary French curriculum practice find that the "skilful use of formative assessment to guide teaching" (p. 304) is one of four reasons advanced to explain why the French mathematics school results have scored highly in studies such as the Third International Mathematics and Science Study. The reasons advanced by these authors for this success is found to relate to the close alignment of teaching and assessment. In another study, Harlen and Winter (2004) argue that a focus on formative assessment around the rubric of "assessment for learning", will "take us, as teachers, closer to the learning of learners and make us think more clearly about the purposes of classroom assessment and how it can be made the 'partner' of learning rather than, as we can sometimes feel, the driver of what we do." (p. 406). Thus, in these studies, we see one practical way that learning and assessment can be aligned, and this is by means of overlap between assessment and learning. A problem with this view however is that it seems to assume, to some extent at least, that learning and assessment are essentially similar – whereas, in practice, these are strongly separated by students, teachers, the general public, and other principal stakeholders.

Thus, Aim 2 of this proposed research is to examine how the practices of the alignment of assessment to other components operate.

AIM 3: TO OFFER A THEORISATION OF THE PRACTICES OF ALIGNMENT OF ASSESSMENT WITH OTHER COMPONENTS OF THE SECONDARY MATHEMATICS CURRICULUM AND GENERATE NEW PRACTICES OF ALIGNMENT BASED ON THIS THEORY

Over the last 15 years mathematics educators and curriculum specialists have generated a range of new methods of assessing knowledge in mathematics. The intention of these pioneers has been to create a diverse range of mathematics assessment techniques. The following typical list (taken from the Mathematics Years 1 to 10 Syllabus, Queensland Studies Authority, 2004) illustrates many of these new style assessment methods as produced by numerous authors both nationally and internationally (e.g., Akinson, 1997; Beyer, 1993; Clarke, 1997; Henricus, Scott, Jennings, Hatton, & Oates, 1997; Perso, 1998, 1999; Shafer & Romberg, 1999; Sullivan, 1997; Sullivan & Lilburn, 1997):

Anecdotal records	Observation notes
Annotated work samples	Reflection sheets, diaries, scrapbooks
Audio and visual recordings	Reports of test results
Checklists	Self- and peer- assessment sheets
Feedback sheets	Reflective journals
Folios	Student/teacher journals
Learning logs	Worksheets

Now each of these presents a challenge to the question of how assessment is aligned to other components of the mathematics curriculum, and for reasons advanced above, these challenges need to be met if the application of such a repertoire is to be considered valid and reliable. The proposed study is a response to this challenge. The proposal is to develop a theorisation of the alignment of assessment and other components of the mathematics curriculum in order to negotiate a systematic, research-and-teaching-practice-rich, and thus expanded practice of alignment. In order to generate such an expansion, at least two candidate models are to be explored, as follows.

In the first, the problem of inter component alignment is conceptualised as the problem of *truth-telling*—in the sense of the question: Is such and such an assessment strategy "true" to learning goals, teaching methods, and other curriculum components, etc? Now, the advantage of this move is that philosophy provides three main theories of truth-telling, and a combination of these may be of utility in theorising and constructing an expanded practice of curriculum/assessment alignment. These theories are: the correspondence theory of truth (something is true to the extent that it *corresponds* with a fact); the coherence theory of truth (something is true to the extent that it can be *conceptually absorbed* into the possibility of the other); and the pragmatist theory of truth (something is truth to the extent it *affords* desired/appropriate outcomes). Of these, the coherence theory we have already seen above—in the examples relating to formative assessment. In the proposed empirical study, however, it is conjectured that instances of all forms of truth telling are to be identified within observed practices of assessment. The challenge, however, is to identify these instances and see what they recommend as procedures for 'truth-telling' in curriculum practice.

In the second alternative candidate for theorising the practices of mathematics assessment curriculum alignment, it is suggested that much can be made of the triple set of relationships in mathematics curriculum created by Boaler (2002). In this theory disciplinary knowledge is directly related to pedagogic practice and the formation of mathematically rich identities by students as shown in Figure 2.

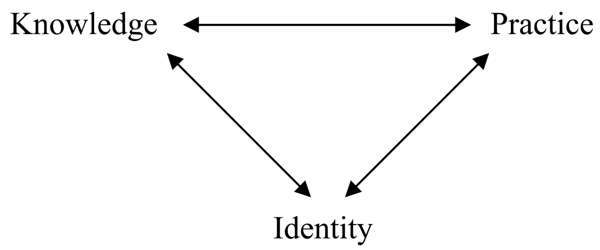


Figure 2. Boaler (2003), a possible map of secondary school mathematics curriculum.

In a nutshell, this model is taken to recommend that the practices of assessment alignment can be formed around relationships among disciplinary knowledge, pedagogic practice and identity formation. Many important questions arise here, for example:

- Is Boaler's model robust or should the various elements be revised? For instance, what is meant by 'knowledge', one of the most problematic terms around education matters?
- Should 'identity' in Boaler's model really be 'subjectivity'? It is observed that discourses of identity tend to have the frequently unintended consequence of spilling over into liberal/libertarian views and are often played out in discourses of the individual. Nonetheless, the significance of identity

formation in learning discourses, is frequently judged critical, see Ecclestone & Pryor (2003).

- Can/need other components be introduced to the model, for instance, technology?

Thus, if a Boaler-like model were to be engaged, much theoretical and empirical work would be required.

Now, the advantage of the former truth-telling approach is its relative simplicity – a range of criteria for alignment could be fashioned out of truth telling conditions, and these incorporated into formal criteria for assessment alignment. A disadvantage, however, is that sociological and psychological dimensions known to be salient to assessment practices (and by extension the practices of assessment alignment) would not be explicitly addressed. In contrast, an advantage of the latter approach is the direct inclusion of political and cultural variables – but, unfortunately, at the possible price of considerably more complexity and difficulty. In the proposed study these alternative possibilities are investigated in order to build a theory adequate to the assessment tasks specified. The theory generated would then be used to frame a practice of curriculum alignment adequate to the task of validating curriculum alignments in practice.

Thus, Aim 3 of this research is to offer a theorisation of the practices of alignment of assessment with other components of the secondary mathematics curriculum and generate new practices of alignment based on this theory.

SIGNIFICANCE OF THE PROPOSED RESEARCH

Drawing together the previous material, a summary of the points of significance of the proposed study may be stated as follows.

First, as argued above, the current practices of alignment around assessment in secondary mathematics curriculum are underdeveloped, and this works to side-line the "social turn" in secondary curriculum teaching practice. The proposed study addresses this problem by enriching the repertoire of practices of alignment, thus opening up to teachers a broader range of secondary mathematics pedagogies.

Second, the proposed study visibilises the micro practices of alignment among all components of the curriculum relating to assessment. This is the first step in the practice of critical social science hinted above. Subsequent studies adopting socially critical perspectives could build on this work.

Third, the proposed study offers a rich theorisation of the practices of alignment among components of the curriculum relating to assessment and this allows for a broader political and cultural evaluation of assessment within the mathematics curriculum. This also is a further step in the process of critically social science.

Fourth, to the technical-rational question of: How can we be sure that there is validity and reliability in our mathematics curriculum/assessment profiles?, the study offers a new theoretical framework. Referred to above, this framework helps curriculum workers identify gaps in alignment against a coherent theoretical framework, and suggests strategies for redress.

Fifth, purposefully enriching practices of alignment, and mapping these enriched practices against mathematics pedagogy, can lead to more effective teaching and learning.

Sixth, the proposed research may also have practical implications for the way mathematics curriculum components can be specified and thereby promote curriculum coherence and transparency.

In the next section I present a summary of the methods planned to be implemented in the proposed study.

RESEARCH APPROACH AND METHODOLOGY

The approach adopted in the proposed research is ethnographical in spirit. This means that data is gathered and analysed in order to come to terms with the variety and richness of experience of those (eg students, teachers, school authorities, parents, etc) involved in mathematics curriculum and assessment work. As stated previously, the theoretical framework used in structuring data sets and organising the analysis of data is cultural-historical activity theory (CHAT). This theory originated in the work of Vygotsky (1978) as built on by Leontiev (1981), Luria, Cole and Scribner and more recently Engeström (1987). Central to this theory is the notion that human activities are driven by motives that have an historical dimension. Such activities are in turn populated by goal directed actions and operations. Another key idea in CHAT is that activities, actions, and operations are mediated by artefacts. These may include, for instance, tools (physical and psychological), rules and norms, community and value, division of labour. It follows from this description that CHAT suggests important steps in filling out the proposed ethnography are to identify motives and activities, goals and actions, operations, outcomes and artefacts that mediate these components of activity. Data organisation is consequentially constructed around these CHAT structures.

Now in CHAT activities, goals and operations are mediated by artefacts, however they also draw participants into situations of conflict, tension, stress, double-bind situations, and so on, all linked to artefacts in the life of the activity system. However, in CHAT, the presence of conflict and contradiction is not a matter for *ad hoc* remedy; it is an opportunity for the activity system to engage in historical development. As the activity engages with conflict, artefacts are reorganised and repopulate the activity system in new ways. This leads to the potential 'expansion' of the activity system. Expanded activity systems may appear to resolve/remediate old conflicts, but in doing so are likely to introduce new ones —and thus the cycle of expansion and renewed tension potentially leads to yet more historical development.

The cyclic process described above is called by Engeström (1999a) 'expansive visibilisation'. The methods of collecting and analysing data in the proposed study are organised around this concept. Particular stages in the cycle are listed as follows (note that in these descriptors I quote verbatim from Engeström (1999a, pp. 68–84):

Visibilisation 1 (V1): Mirroring and analysing troublesome matters, making components of the activity system visible, identifying contradictions
Visibilisation 2 (V2): Modelling activity systems
Visibilisation 3 (V3): Designing and implementing new actions
Visibilisation 4 (V4): Following and revising.

Expansive visibilisation is chosen because, firstly, this theoretical tool affords comprehensive methods for collecting, organising, analysing data, and theorising and prefiguring the transformation of practice. Secondly, it provides powerful and useful tools for planning, implementing and critically reviewing organisational change. It is noted that CHAT also affords a range of alternative tools for organisational change, such as: *Knotworking* (Engeström, Engeström, & Vähäaho, 1999), *Change Laboratory* (Engeström, 1996), *Boundary Crossing Laboratory* (Engeström, 2000), and so on. The merits and utility of these tools is to be discussed in the proposed study and may inform further research.

Principal data for the study consists of transcripts associated with interactions among a set of mathematics teachers formulating assessment practices, implementing these practices, transforming and re-engaging with these practices in line with the expansive visibilisation model. The group of teachers is to be known as the Assessment Innovation Group (AIG). Teachers are to be chosen to belong to the AIG on the basis of their willingness to engage in transformatory collaborations around assessment – membership of the AIG is in the first instance restricted to teachers currently engaged in teaching Year 8 classes (chosen as these students, in Queensland, are in their first year of secondary school).

Methods of data collection relating to the AIG are to include: semi-structured interviews and questionnaires gathering information about students' and teachers' knowledge, beliefs and experiences of assessment and the alignment of assessment and pedagogy (protocols to be shaped by CHAT and the ecological validity of meanings); focus group analysis of data indicating practices of alignment, technologies, contradictions (in this using as a stimulus video-taped assessment practice, tools associated with practices of alignment, other stimulus material etc); and other data acknowledged within the CHAT paradigm and deemed useful in the development work of the AIG. Analyses of data are to make use of critical episode, constant comparison, and additional concepts utilised from grounded theory approaches.

Sites for data collection are to be the mathematics departments in a small number of high schools in South East Queensland though this may be broadened depending on resources). The exact number is to depend on the progress of the research and, in particular, on the kinds of alignment practices identified and the scope and willingness of teachers for transformatory practice. Schools with specifically low socio-economic variables are chosen as the need for transformatory practices around assessment may be most pressing for this group. It is foreseen that each school will have its own AIG, and that expansive visibilisation processes are, in the first instance, focussed within each school. Cross germination of expansive visibilisation cultures is also important, this will be accommodated in later stages of the study.

It is intended that the proposed study is to have three annual expansive visibilisation cycles. The aims for these are as follows.

- <u>Cycle 2006 aim</u>: To articulate a model visibilising components of the secondary high school mathematics curriculum and, in particular relation to assessment, identify problem of the alignment among these components
- <u>Cycle 2007 aim</u>: To examine in practice how the alignment of assessment to other curriculum components is obtained, and identify areas of mal-alignment within the junior secondary level curriculum
- <u>Cycle 2008 aim</u>: To offer a theorisation of the alignment of assessment with other components of the junior high school curriculum and generate a profile of new assessment strategies appropriately aligned with components of the curriculum

Within each of these cycles the intention is to advance the research through the four-step process of visibilisation and theory/practice building as indicated above. Thus Cycle 2006 is to consist of four stages, V1-V2-V3-V4, and V4 in 2006 is to lead into V1 in 2007, and so on.

CONCLUSION

In this paper I have made a case for exploring secondary mathematics assessment processes around the concept and theory of the 'practices of alignment'. I have suggested that these particular practices are largely invisible within school contexts, and this has serious limiting consequences for the revolution of mathematics pedagogies build around the "social turn". As a result I have indicated that research addressing this situation may be required. In addressing this prospect I have proposed a research alternative structured around CHAT. In the first two parts of this proposed research it is planned to visiblise practices of alignment within the secondary mathematics curriculum, and identify practices of alignments relating to assessment. In the third part, a proposal is made to engage the powerful change tools of CHAT to create a theory of alignment in assessment, and generate new practices of alignment.

REFERENCES

Anderson, B., Brown, C., & Lopez-Ferrao, J. (2003). Systemic reform: Good educational practice with positive impacts and unresolved problems and issues. *Review of Policy Research*, *20*(4), 617–627.

Atkinson, S. (1997). Assessment and record keeping. In *Developing a scheme of work for primary mathematics*. London: Hodder & Stoughton.

Beyer, A. (1993). Assessing students' performance using observations, reflections, and other methods. In N. Webb (Ed.), *Assessment in the mathematics classroom: 1993 Yearbook*. Reston: V: NCTM.

Boaler, J. (2002). The development of disciplinary relationships: Knowledge, practice and identity in mathematics classrooms, *For the Learning of Mathematics*, *22*(1), 42–47.

Carr, W., & Kemmis, S. (1986). *Becoming critical: Knowing through action research*. Victoria: Deakin University Press.

Clarke, D. (1997). *Constructive assessment in mathematics: Practical steps for classroom teachers*. Berkeley, CA: Key Curriculum Press.

De Vijer & Phalet (2004). Assessment in multicultural groups: The role of acculturation, *Applied Psychology: An international review*, *53*(2), 215–236.

Ecclestone, K., & Pryor, J. (2003). 'Learning careers' or 'assessment careers'? The impact of assessment systems on learning. *British Educational Research Journal*, *29*(4), 471–488.

Engeström, Y, J., Virkkunen, M., Helle, J., Pihlaja, & Poikela, R. (1996). The change laboratory as a tool for transforming work. *Lifelong Learning in Europe*, *2*, 10–17.

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Helsinki: Orientat-Konsultit.

Engeström, Y. (1999a). Expansive visibilization of work: An activity-theoretical perspective. *Computer Supported Cooperative Work*, *8*, 63–93.

Engeström, Y., Engeström, R., & Vähäaho, T. (1999b). When the centre does not hold: The importance of knotworking. In S. Chaiklin, M. Hegegaard & U. Jensen (Eds.), *Activity theory and social practice: Cultural-historical approaches*. Aarhus: Aarhus University Press.

Engeström, Y. (2000), Activity theory as a framework for analyzing and redesigning work. *Ergonomics*, *43*(7), 960–974.

Fowler, F., & Poetter, T. (2004). Framing French success in elementary mathematics: Policy, curriculum and pedagogy. *Curriculum Inquiry*, *34*(3), 283–312.

Harlen, W., & Winter, J. (2004). The development of assessment for learning: Learning from the case of science and mathematics. *Language Testing*, *21*(3), 390–408.

Henricus, J., Scott, S., Jennings, W., Hatton, L., & Oates, M. (1997). Rich assessment tasks: Their use as projects in the classroom. In D. Clarke, P. Carlson, D. Gronn, M. Horne, L. Lowe, M. McKinlay & A. McDonough (Eds.), *Imagining the possibilities*. Melbourne: MAV.

Leont'ev, A. N. (1981), *Problems of the development of the mind,* Moscow: Progress.

Lerman S. (2000). The social turn in mathematics education research. In Boaler J (Ed.), *Multiple perspectives on mathematics teaching and learning*. Westport, CT: Ablex.

Morgan, C. (1998). *Writing mathematically: The discourse of investigation.* London: Falmer.

Morgan, C., Tsatsaroni, A., & Lerman. S. (2002). Mathematics teachers' positions and practices in discourses of assessment. *British Journal of Sociology of Education*, *23*(3), 445–461.

Perso, T. (1999). Assessment in the mathematics classroom. *Australian Mathematics Teacher, 55*(4), 17–20.

Perso, T. (1998). What's in a test? *Australian Mathematics Teacher, 54*(1), 40–41.

Pinar, W. (2004). *What is curriculum theory?* Mahwah, NJ: Lawrence Erlbaum.

Queensland Studies Authority. (2004). *Mathematics Years 1 to 10 Syllabus*. Brisbane: Queensland Studies Authority.

Shafer, M. & T. Romberg (1999). Assessment in classrooms that promote understanding. In E. Fennema & T. Romberg (Eds.), *Mathematics classrooms that promote understanding*. Mahwah, NJ: Lawrence Erlbaum Associates.

Sullivan, P., & Lilburn, P. (1997). Using 'good' questions in your classroom. In *Open-ended maths activities: Using 'good' questions to enhance learning*. Melbourne: Oxford University Press.

Sullivan, P. (1997). Rich tasks: Making assessment better and simpler. In D. Clarke, P. Carlson, D. Gronn, M. Horne, L. Lowe, M. McKinlay & A. McDonough (Eds.), *Imagining the possibilities*. Melbourne: MAV.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes.* (M. Cole, V. John-Steiner, S. Scribner & E. Souberman (Eds. & Trs). Harvard University Press.

HOW THE 'MADNESS' OF METHOD AFFECTS ENGAGEMENT AND EQUITY: A POSTSTRUCTURALIST ANALYSIS

Mary Klein
James Cook University
<mary.klein@jcu.edu.au>

In this paper I attempt to show how humanist assumptions about 'relevance' and learner 'autonomy' in mathematics methods can result in teaching and learning interactions that preclude high levels of intellectual engagement and equitable participation. As well, teachers' and researchers' a-critical acceptance of method can lead to maintenance of the status quo, as attention is diverted away from quality and equity in teaching and learning interactions, towards defining and redesigning instructional protocols. A poststructuralist analysis does not allow a mind/body dualism or split in learning; whenever and however the mind is engaged so too is the body, and a pressing problem in mathematics education is that far too many bodies are walking away (Devlin, 2000; Forman & Steen, 2000).

Recently there has been a strong emphasis in mathematics education and research on epistemologies that emphasise the active role of the learner in knowledge construction; as well, sociocultural (Vygotsky, 1978) and sociocognitive (Lakoff & Johnson, 2000) perspectives encourage educators to move beyond a sole focus on the individual to incorporate social context and collaboration as important elements in learning. In keeping with these relatively new perspectives on what knowledge is, and with interpolations regarding how it is learned and taught, 'constructivist', 'open-ended' and 'problem-solving' methods have been promulgated to supposedly support a rich and rigorous knowledge construction process. These methods commonly emphasise less direct instruction from the teacher, 'relevant' content, collaboration and engagement in processes of exploration, conjecture and generalisation. However, many students continue to leave school disaffected, with very low levels of mathematical understanding. Kilpatrick & Silver (2000, p. 224), for example, paint a dismal picture indeed:

> Students aren't learning mathematics well enough; they leave school hating it. Teachers don't know enough mathematics and don't know how to teach it effectively. The school mathematics curriculum is superficial, boring and repetitious. It fails to prepare students to use mathematics in their lives outside school.

These well renowned mathematics educators and researchers (Kilpatrick & Silver, 2000) reproduce 'deficit' readings of the problems of school mathematics that resonate with readings reported in the popular press and the community generally; students aren't learning well enough, teachers don't know enough and the curriculum is boring. They 'psychologise' failure, that is, they defer to the psychological to find explanations of why students are 'turned off' mathematics and why they "dislike both the mathematics and the learning of it" (Willoughby, 2000, p. 8). However, readings such as this are dangerous where engagement and equity are of concern. First, they encourage educators and policy makers to try to 'fix' the problem (students, teachers and curricula) by finding new and even more efficient teaching methods. Methods, though, can not take account of and properly cater for the multitudes of intersecting socio-cultural, political and identity issues that support or suppress student engagement in learning mathematics. They launder the learning process of "contradiction, contestation and ambiguity" (McLaren, 2003) which, if recognised and addressed, can, I later suggest, lead to more purposeful engagement and equitable outcomes for students. Second, such readings are very safe and self serving for those not currently engaged in classroom teaching. In attributing some sort of 'deficit' to teachers and students they deflect attention from alternative readings of, and careful research into, how this state of affairs has developed over time. As McLaren (2003, p. 236) warns "Psychologising school failure indicts the student [and one could add teacher] while simultaneously protecting the social environment from sustained criticism".

So, while most commentators look at classrooms and ask "why" this state of affairs has arisen (and usually find someone or something to blame), a poststructuralist analysis asks "how". How can it be, in the twenty-first century, with all the research that has been done on the teaching of mathematics, and the millions of teachers who continue to do their very best for those they teach, that so many students find learning mathematics to be an alienating experience? A useful contribution might come from poststructuralist thought that suggests that learning mathematics is not just about intellectual aspects; also important are the ways in which students are positioned as learners. For example, are their histories and lives affirmed and legitimated in the pedagogical practices, are they able to initiate communicative acts and questions and make sense of the social experience in ways that reflect those in the world outside school? Are they actually taught the structural patterns and relationships of mathematics? To what extent are students able to establish themselves as legitimate participants with/in the social practices of school mathematics? All of these are important, as they are constitutive of the numerate citizens of the future. However, at the moment the picture in school mathematics education could be seen to be more or less as Smith (1987, p. 32) paints:

> It is like a game in which there are more presences than players. Some are engaged in tossing a ball between them; others are consigned to the role of audience and supporter, who pick up the ball if it is dropped and pass it back to the players. They support, facilitate, encourage but their action does not become part of the play.

There are students in the classroom, who because of the operation of the discursive practices in the various teaching methods, never really get their hands on the ball other than to pass it over to those who have established themselves as legitimate players – as has been the case for so long in transmission methods of teaching.

In this paper I examine how assumptions about what is 'relevant' to learners, and 'autonomous' learners, lead inexorably to classroom practices that, in not reflecting lived experience, may seem a little 'mad' and alienating to many learners. As well, from a poststructuralist perspective, one could argue the 'madness' of continuing with methods that focus on teaching strategies, rather than the quality of teaching/learning interactions that form, or are constitutive of, the mathematicians of the future.

PEDAGOGIC WORK IS IDENTITY WORK

Within poststructuralist thought, all pedagogic work is identity work. School mathematics, whether based on transmission, 'constructivist' or 'poststructuralist' epistemological views, operates as a discourse and its discursive practices position the various participants as able or not to act in powerful ways. The social context of the classroom is not separate from the individual as thought to be the case in theories of socialisation, but is in or constitutive of the learner (and teacher). Davies (1994, p. 44) states: "To the extent that discursive practices shape or make real certain ways of being, they are constitutive of the persons who take them up as their ways of speaking the world and themselves into existence". For example, in the mathematics classroom, the act of construction of mathematical ideas and relationships can not be separated from the constitutive force of the teaching interaction. The quality of the teaching interactions influence the extent to which learners can establish themselves as legitimate players in the "game of truth" (Foucault, 1984a) that is school mathematics. Conceptually the notion of individual ability or aptitude is displaced with a focus instead on the ways in which students are produced (as able or not) within the discursive play(s)—recognising of course that there are many other domestic and civic discourses that also influence identity formation.

Thus the subject of poststructuralism is not the rational, coherent, unitary, non-contradictory self of times past, when humanist thought prevailed. Rather, the subject itself "is the effect of a production, caught in the mutually constitutive web of social practices, discourses and subjectivity; its reality is the tissue of social relations (Henriques et al., in Davies, 1991, p. 42). In poststructuralist thought the perfectly knowable teacher and student have vacated the classroom and been replaced by contradictory, fragmented and changeable identities renewed and reshaped by (re)positioning in the available discourses. (See Davies, 1991, for a comprehensive analysis of the concept of person in humanist and poststructuralist theories). These are not essentially rational nor autonomous persons, and their actions in the classroom (and beyond) reflect their constituted *knowing* (Lather, 1991) about themselves and their world. What students know and learn, from a poststructuralist perspective, has an intellectual and an ontological dimension. As students engage in the discursive events of the classroom, as they attempt to learn the mathematics, they come to know themselves as legitimate participants (or not) in the social practice of mathematics education. This coming to know is not a cognitive, conscious event but rather an unconscious sense of whether or not their voice and presence in the discourses are valued. Walkerdine (1990, p. 5) reminds us that "An individual can become powerful or powerless depending on the terms in which her/his subjectivity is constituted". This is where positioning becomes important.

In the mathematics classroom many subject positions are available; the teacher usually takes up the subject position of 'good' at mathematics, as someone able to speak its 'truths' with certainty, as do some of the students. However, in institutionalised discourses such as this, all students need to be able to recognise themselves as active players in Smith's (1987) ball game, not supporters. The discursive practices of school mathematics need to operate in ways that, even as students are at basic levels of intellectual knowledge construction, make spaces for them to participate as rightful members of the social practice (discourse), not as sideline groupies. Quality teaching from a poststructuralist perspective is a social practice; it comprises intellectual and social relationships, these are constitutive of all students who come to know themselves as learners who can and should speak and be heard within and beyond the established discursive practices. As I attempt to demonstrate in the following section of the paper, new conceptions of learners and learning are needed to ground new practice—essentialised notions of the rational, autonomous learner will not suffice.

THE MADNESS *IN* OUR METHODS

In the following section of the paper I use the poststructuralist concepts of discourse, identity and positioning (power) to analyse how humanist assumptions currently framing practice blind teachers to what is actually going on in the various teaching methods. Teachers strive to make learning as engaging as they can for students, they choose 'real world' problems, they question them to get them involved and they include group work and other collaborative activities to have students actively engaged. However, humanist assumptions about identity and engagement prejudice quality learning and send teachers (and students, policy makers) relentlessly back to readings of deficit, disadvantage and dysfunction when outcomes are not what had been anticipated. In the following couple of classroom excerpts I want to show how assumptions about

- essentialised identity, what is 'relevant', and
- autonomous engagement, in processes of problem solving and inquiry

allow the discursive practices of the classroom to go on unchecked and unaltered, as they cement existing relationships of power and powerlessness in the classroom.

ASSUMPTIONS OF 'RELEVANCE'

Zolkower (1996) provides an interesting example of a teacher in East Harlem, New York City going through a problem solving lesson with her young students. Looking at what the teacher does, one observes she is doing everything right according to the basic tenets of such lessons, yet the students plough through an experience that one could read as stultifying and painful. First the teacher sets what she imagines to be a 'relevant' problem:

Suppose you had 50cents on Monday night. You took a job for a week that doubled your money each day. For example, Tuesday night you had $1.00. How much money did you have the next Monday night?" (Zolkower, 1996, p. 68).

As well, she invites collaboration (assuming autonomous engagement):

Could someone give me some ideas on how we are going to solve this? Let's see if we can work on this together.

To ensure they are fully and actively engaged, she leads them step by step through the problem, engaging in triadic dialogue that she imagines will produce the knowledge they need to know. When they get too noisy she turns off the light, which brings them back on task. She recognises and affirms the presence of those who answer questions by writing the answers on the blackboard (and she adds her corrections) (Zolkower, 1996).

Though the teacher's aim is to engage students in solving a supposedly 'relevant' problem, so that they will engage in mathematical thinking and construct important mathematical relationships and ideas, her assumptions about learners and learning render this outcome unlikely at best. The teacher here, like most classroom teachers, assumes she is teaching students with essentialised identities. She takes for granted that a problem based on a job situation, with an intriguing twist where the pay doubles every day, will be interesting to them all. As well, there is the assumption of autonomous engagement, that all students will be eager to participate. However, she does not realise that the students already know lots about jobs, and what they have come to know about them and the working situation is not reflected here. Many of the students' parents do not have jobs, and those that do, do not work over the weekend, nor does their pay double every day. Second, the ontological experience of being in a job may not be a life aspiration for many of the students, and may render this 'problem solving' exploration tedious at best. As well, the teacher takes over all authority and the authorship of all sense-making action as she engages the students in triadic dialogue. Lemke (1990, p. 168) states:

Triadic dialogue is an activity structure whose greatest virtue is that it gives the teachers almost total control of the classroom dialogue and social interaction. It leads to brief answers from students and lack of student initiative in using scientific language. It is a form that is overused in most classrooms because of a mistaken belief that it encourages maximum student participation. The level of participation it achieves is illusory, high in quantity, low in quality.

From a poststructuralist perspective, not only do the students not construct much in the way of important intellectual knowledge, but also the discursive practices, centered on the teacher, position the learners as 'extras' in this game of truth (Foucault, 1984a). This is a problem as the students are not able to find the discursive spaces to establish themselves as 'problem solvers'; rather they are relegated to marginal positions of dependence on the teacher.

ASSUMPTIONS OF 'AUTONOMY'

Of great important in mathematics education is that students engage in mathematical thinking process in the construction of mathematical ideas, patterns and relationships. It is accepted that students should estimate, generalise, communicate with each other, represent mathematical ideas and so on so that they build up strong and robust conceptual structures. The process of knowledge construction is critical, and in an effort to have students engage fully in this process, teachers use methods and approaches thought to appeal to young learners to get them involved. The use of open ended tasks is one such approach.

Sullivan, Zevenbergen and Mousley (2003, p. 113) provide an example of an 'open ended' task chosen by one teacher:

> The mean height of three people in the room is about 155 cm. You are one of those people. Who might be the other two?

Sullivan et al (2003, p. 113) give details of the teacher's thinking behind this choice of task:

> The task was 'open ended' and so allowed a range of possible approaches, procedures, and answers for any student. It was intended that the openness of the task provide an environment for the learning of mathematics that emphasised the possibility of multiple responses, making explicit to the students that it was their own exploration that was required, and both valuing and learning from the range of responses produced.

Here there is the assumption that students will want to engage with the task because it is 'open ended', meaning that there could be many correct answers, they could author their own path towards a solution and they could learn from others in the group. It is assumed that because students are given a task that is 'open ended' some sort of social energy will be released that will cause them to suddenly find ways of being a learner of mathematics that are pleasurable and fulfilling, focused on finding out for themselves, exploring and working with peers. However, from a poststructuralist perspective, the setting of a task does not 'provide an environment' while the intersection of identities, power relations (positioning) and knowledge does. I shall raise some issues on behalf of each of these in turn.

The first issue has to do with identity and recognises that most students will engage at some level in the activity. Regardless of the fact that they may not really know what the 'mean' means, or that they have come to know themselves to be far too short or too tall for their age, they will get themselves involved in the set task. This is because they want to be recognised by others and themselves as legitimate with/in the social practices of school mathematics. Students' identities, though shifting and changing day by day, depend on their being seen as able to cope adequately with the discursive practices and positionings that engulf them. Zolkower (1996, p. 60) speaks of how students do get themselves involved in even the most meaningless tasks:

> We may be surprised to find that seven-to-eleven-year-old students often want to learn even the most apparently obsolete chunks of knowledge presented to them on a daily basis by their math teachers, and some of them enjoy such trivial activities as math challenges in which exhibiting perfect mastery over the multiplication tables is the only condition for winning the class struggle.

One result of this, though, is that teaching-mathematics-as-usual goes on unchallenged and unchanged.

A second issue is that there is an assumption that there are no power relations involved, or if there are, they are positive in making it possible for the students to engage in this learning on their own terms. However, an Indigenous Australian (Sullivan et al., 2003, p. 117) makes it clear that there are always power relations, and in this case they operate in ways that make her feel (position her as) uncomfortable:

> [The teacher] is not a neutral person standing there. It's a woman who looks a certain way. There's no difference from her stance and that of a lawyer or police officer and that would be very threatening to a lot of students, especially if they've had at high school previous contact with authority figures. There's no chance of anything spontaneous happening in there and no chance to talk with the other students. Her stance and body language almost sends a message that if you do something outside of what I want from you there'll be repercussions. Now that may not be the case, but as an Aboriginal person, I'd find her way of teaching quite offensive.

This then leads to the final issue of knowledge; what students know and have learned about what mathematics is and how it is done. Poststructuralists recognise as important not only the intellectual knowledge constructed in mathematics classrooms, but also the ontological ways-of-being a learner that have been, and are being, constituted. Student may not know a lot of mathematics, but they do know lots about how it is learned and about themselves as learners. What they have learned is that if you wait long enough the teacher tells the answer or scaffolds so well that you don't have to do any thinking (Forman & Steen, 2000), that the tasks really have no purpose in the present but may be useful in the future and that teachers and texts know, and students don't. This constituted *knowing* (Lather, 1991) may compromise their full engagement in mathematical thinking processes no matter how interesting or 'open ended' the set activities are intended to be.

Here again, essentialising practices, such as those based on the assumption of autonomous engagement by all students in activities that are problem based, investigative or open-ended "fix relations of power and powerlessness" (Davies, 1994, p. 28). They assume that the intellectual can somehow be divorced from the social, that engagement in thinking processes can somehow be divorced from constituted knowing that one is not meant to think, that one has no useful knowledge and that one should listen and learn. While the rhetoric is of active engagement, the practice is one of constituted passivity and alienation for more students than is commonly thought to be the case. I refer here to the fact that even white, middle class students tend not to find the learning of mathematics a pleasurable experience. Methods come and go, yet students continue to turn away from it as it is currently taught (Devlin, 2000; Willoughby, 2000).

THE MADNESS *OF* METHOD

All methods are useful to some extent in that they work in some way for some students some of the time; however, they are dangerous if used carelessly because of what they fail to notice, fail to address and leave out. Surprisingly, perhaps, too often it is the learner who is sidelined, left out; her/his experiences, life, knowledge and need to express them are not acknowledged in the 'teaching method equals learning' equation. Ladson-Billings (1994, p. 31), for example, talks about how teachers try not to notice racial difference, ignoring all that makes children special, and suggest that "by claiming not to notice, the teacher is saying that she is dismissing one of the most salient features of the child's identity and that she does not account for it in her curricular planning and instruction".

Learning, however, is a social process which is about the mathematics, and also the operation of the discourse (positioning). It is a process wherein all students should be encouraged to establish themselves as valued and respected participants. First, it goes without saying that it is important to teach the mathematics, for students can not establish themselves as powerful in any discourse if they do not have access to the discursive 'truths'. Though it is crucial on whose terms the mathematics is taught. Too often the content to be learned is mystified, and students are not given the conceptual tools they need to be able to engage fully in quality learning processes. Second, the 'tone' or climate of the interactions is important. Students should be treated as valued contributors to the knowledge production that is going on in the classroom; they should be able to engage in dialogue and initiate questions and ways of making sense of the mathematics, speaking from their own experiences about what they know and can do. This authoring of speech acts and ways of making sense is very important, as it has to do with positioning and identity formation; as McLaren (2003, p. 245) states: "The author's voice is a constitutive force, and the author defines him/herself as an active participant in the world". Too often, in the various methods, too many students are sidelined, and seem to be active only in supporting those who have established themselves as authoritative and legitimate contenders in the game.

Learners historically have always been placed on the passive side of educational binaries; teacher/student, knower/known, adult/child. Teachers attempt to make learning more seductive and choose teaching activities and methods that promise to engage students by connecting them somehow to the material to be taught. They try to make the learning tasks 'relevant' and engaging. However, all students come to the mathematics classroom with rich experiences, backgrounds and knowledge that should be acknowledged and used to energise the learning process and make it relevant. For example, McLaren (2003, p. 250) states that learning processes should "confirm and legitimate the knowledges and experiences through which students give meaning to their everyday lives" (emphasis in original). Relevance and engagement are to be found in learning processes that students sense to be purposeful and productive; relevance and engagement are produced in interaction, they are not something that is external to, nor can they be added or artificially painted on to, the learning interactions. However, in the assumption that they can, the status quo is inexorably maintained.

CONCLUSION

Mathematics education is a social practice; it is like Smith's (1987) ball game wherein knowledge, identities and discourses mix, and from which far too many are currently marked out of the game. Taken-for-granted methods and practices rub against the sole (soul?) like ill-fitting football boots, leaving blisters and scars that can cripple for life. Perhaps it is time to take seriously the words of Foucault (1984b, p. 343) that "everything" in mathematics education, "is dangerous". It is not that everything is bad, just dangerous. And as Foucault (1984b, p. 343) so tellingly adds: "If everything is dangerous, then we always have something to do". In mathematics education, in the name of engagement and equity, we certainly have plenty to do.

REFERENCES

Davies, B. (1991). The concept of agency. *Social Analysis*, *30*, 42–53.

Davies, B. (1994). *Poststructuralist theory and classroom practice*. Geelong: Deakin University Press.

Devlin, K. (2000). The four faces of mathematics. In M. Burke & F. Curcio (Eds.), *Learning mathematics for a new century* (pp. 16–27). Reston, VA: National Council of Teachers of Mathematics.

Forman, S., & Steen, L. (2000). Beyond eighth grade: Functional mathematics for life and work. In M. Burke & F. Curcio (Eds.), *Learning mathematics for a new century* (pp. 127–157). Reston, VA: National Council of Teachers of Mathematics.

Foucault, M. (1984a). What is enlightenment? In P. Rabinow (Ed.), *The Foucault reader* (pp. 32–50). London: Penguin.

Foucault, M. (1984b). On the genealogy of ethics. In P. Rabinow (Ed.), *The Foucault reader* (pp. 340–372). New York: Pantheon.

Kilpatrick, J., & Silver, A. E. (2000). Unfinished business: Challenges for mathematics educators in the next decade. In M. Burke & F. Curcio (Eds.), *Learning mathematics for a new century* (pp. 223–236). Reston, VA: National Council of Teachers of Mathematics.

Ladson-Billings, G. (1994). *The dreamkeepers: Successful teachers of African American children*. San Francisco: Jossey Bass.

Lather, P. (1991). *Getting smart: Feminist research and pedagogy with/in the postmodern*. New York: Routledge.

Lakoff, G., & Johnson, M. (2000). *Philosophy in the flesh: The embodied mind and its challenge to western thought*. New York: Basic Books.

Lemke, J. (1990). *Talking science: Language, learning and values*. Norwood: Ablex.

Malloy, C. (1997). Including African American students in the mathematics community. In J. Trentacosta & M. Kenny (Eds.), *Multicultural and gender equity in the mathematics classroom* (pp. 23–33). Reston, VA: National Council of Teachers of Mathematics.

McLaren, P. (2003). *Life in schools: An introduction to critical pedagogy in the foundations of education* (4th ed.). Boston: Pearson Education.

Smith, D. (1987). *The everyday world as problematic*. Boston: Northeastern University Press.

Sullivan, P., Zevenbergen, R., & Mousley, J. (2003). The contexts of mathematics tasks and the context of the classroom: Are we including all students? *Mathematics Education Research Journal*, *15*(2), 107–121.

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processe*s. Cambridge, MA: Harvard University Press.

Walkerdine, V. (1990). *Schoolgirl fictions*. London: Verso.

Willoughby, S. (2000). Perspectives on mathematics education. In M. Burke & F. Curcio (Eds.), *Learning mathematics for a new century* (pp. 1–15). Reston, VA: National Council of Teachers of Mathematics.

Zolkower, B. (1996). Math fictions. In S. Aronowitz, B. Martinsons, M. Menser & J. Rich (Eds.), *Technoscience and cyberculture* (pp. 57–96). New York: Routledge.

QUALITY LEARNING FOR PRESERVICE TEACHERS OF MATHEMATICS: PROBLEMS AND PROCESSES FROM A POSTSTRUCTURALIST PERSPECTIVE

Mary Klein
James Cook University
<mary.klein@jcu.edu.au>

In this paper I consider what quality learning for preservice teachers might look like, given the poststructuralist notion of the discursive construction of knowledge. First, I attempt to tease out the notion of what it means 'to learn' to teach mathematics, when what one knows has both epistemological and ontological dimensions, influencing one's professional identity and practice. Second, I contemplate the effects the pedagogies of school and university mathematics have (had) on preservice teachers' ability to teach in ways that get beyond readings of individual 'deficit' and disadvantage. I then contemplate a small window of opportunity for teacher education; it may be possible to engage preservice teachers in teaching/learning interactions that celebrate uncertainty regarding identities 'set in stone' and teaching as an unalienable good. This climate of inquiry would be, to some extent at least, constitutive of the teachers of the future, and may encourage them to think differently about what they do in the name of quality teaching and equity.

As times have changed, so too have notions of quality learning. Once about receiving and remembering lots of facts, skills and procedures, new social and economic imperatives have over time enforced, and depended on, a series of redefinitions. Currently, quality learning of mathematics is commonly equated with high levels of understanding: "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM, 2000, p. 20). The reason understanding is so important is that it enables students "to solve the new kinds of problems they will inevitably face in the future" (p. 21). In teacher education, too, it is taken as given that the preservice teachers should understand basic mathematical ideas and structures, so that they can teach mathematics competently. Similarly, the Department of Education, Science and Training (DEST) (2003, p. 145) sees an easy alliance between understanding and competent action; it states that teacher education must ensure that all preservice teachers "improve their broad understanding of the forces of change in Australian society and the importance of science, mathematics and technology in underpinning the knowledge economy and society" (DEST, 2003, p. 145).

While understanding is important, the assumed relation between it and purposeful action is problematic. Especially in teacher education, where students learn theories about how mathematics is learned and how it should be taught, there is often scant evidence of these theories being put into practice in the classroom. Indeed Schon (1987, in Johnston, 1996) demonstrates that there are many instances in which teaching practice is inconsistent with the theories a teacher can articulate. The problem (and possibilities), from a poststructuralist perspective, is not with the understanding, but with the processes within which it was, and is being, constructed (or not, for some students). In the processes of mathematical knowledge construction, whether through 'constructivist', transmission or a combination of these means, professional identities are also being produced, as preservice teachers struggle to establish themselves as competent and proactive in the mathematics education community(ies). A recognition of the productive force of all pedagogical interventions in teacher education opens up new avenues of research and practice in/for the future.

QUALITY LEARNING FROM A POSTSTRUCTURALIST PERSPECTIVE

As previously mentioned, in the past, and currently to a considerable extent, learning to teach mathematics has been about the construction of mathematical and pedagogical ideas and their application in classroom contexts (DEST, 2003). Issues of class, gender and race are seen to be important, and included in subject readings and discussions in teacher education. It is assumed that students who know the mathematics, and are well informed about and understand socio-cultural issues, will be well equipped for future responsible and responsive classroom practice. However, these assumptions are predicated on the preservice teacher as a rational and reflective humanist individual, one who constructs knowledge about what must/should be done, and then follows through appropriately in the classroom. St Pierre (2000, p. 500) describes the individual of humanism as "a conscious, knowing, autonomous, and ahistoric individual who is endowed with a will, a freedom, an intentionality which is then subsequently expressed in language, in action, in the public domain".

Fortunately for teacher education, at a conceptual level at least, the humanist individual has vacated the physical and cyber-space of instruction. The preservice teachers I teach, like all individuals, are complex and contradictory beings, one day on top of the world about their teaching successes, and the next day down in the dumps. From day to day the discourses that engulf them, within which they attempt to establish themselves as competent and resourceful professionals in the making, pull them in one way, then another, making the learning to teach process rugged and dangerous terrain (see, for example, Annie's experiences in Youngblood Jackson, 2001). However, the poststructuralist notion of professional identities as malleable and changing presents an opportunity for teacher education; it means that preservice teachers are always in the process of becoming something else, of learning and knowing more, of beginning again. If our students' identities were stable and set in stone, each student essentially good/bad, motivated/unmotivated, bright/dull, there would be little for us to do other than provide the curriculum and stand back. On the other hand, if professional identities are growing and changing, the onus is on teacher education to get the discursive practices operating in ways that engage students purposefully and critically in the learning-to-teach-mathematics process.

Teaching, from a poststructuralist perspective, is less about strategies and instructional practices, and more about ensuring that students are engaged in quality teaching and learning interactions and relationships that position them as legitimate teaching professionals *in process*. In mathematics education it is the intellectual and social quality of interaction that matters. It is in the moment of interaction that learning occurs, when relationships of power, knowledge and identity intersect and coalesce to position participants differentially in the discourse. Strangely enough, the teacher may be oblivious to much of the learning, or *coming to know* (Lather, 1991) that transpires; in poststructuralist thought teaching and learning come together and teaching/learning is a social process wherein subjectivities are (re)constituted as content knowledge is constructed. Recognising that "all pedagogical work is always and everywhere *identity work* of some kind" (Chappell, Rhodes, Solomon, Tennant, & Yates, 2003, p. 4) I examine some of the problems and possibilities of/for teaching/learning interactions in teacher education. I begin with what I see as contemporary problems before I move to consider some possible directions for the future.

ANOTHER FACE OF PEDAGOGY

One problem of contemporary pedagogies is that only intellectual knowledge is acknowledged; constituted *knowing* (Lather, 1991), although it has an important influence on future practice, is ignored. The humanist learner and teacher reign supreme because preservice teachers and teacher educators have been constituted through normalising practices (discursive practices) in school and university that essentialise and categorise according to humanist interpretations of ability, gender and socio-cultural status. They have come to know that there are those who can do mathematics, and those who can not. For example, psychological discourses that inform classroom practice take for granted that there are motivated/unmotivated learners and management discourses speak of the engaged/disengaged students. It is as if learners have essential qualities that define them, that are unchangeable and indicative of their 'proper' positioning on the positive or negative side of the binary. This constituted *knowing* (Lather, 1991) about the nature of learners, and the interactional protocols appropriate to learners positioned on either side of the binary, anonymously influence the preservice teachers' practice. Ultimately, if they come to know the children they teach in school are essentially good or bad, motivated or not, there is little need to vary their instructional routines; the good, motivated students will learn, while the 'others' will not.

Added to this is the problem that in teacher education programs the preservice teachers know themselves as learners in essentialist terms (Sumsion, 2003). They are subjected to normalising assumptions based on the teacher/student, expert/novice binaries. The preservice teachers struggle to achieve themselves as some essentialised notion of the 'good' teacher of mathematics, supposedly present in the classrooms they visit on practicum rounds. Youngblood Jackson (2001, p. 387) states:

> The normative discourse holds that those who have the most experience possess the most power and knowledge, and those who tout this discourse expect novice teachers to conform and fluidly take up an identity similar to that of their mentor, who is the master teacher.

University based teacher educators, too, have investments in having prospective teachers see teaching in the ways they do, and can, often unknowingly, demand deference to their ideas through formative assessments and exams. McCotter (2002), a teacher educator, describes how she fell into habits of naming and blaming her own students and the teachers in schools, at the same moment as she derided these habits in the preservice teachers. However, the fact that "our histories come with us daily to our professional practice" (Cole & Knowles, 2000, p. 27) need not be seen as a liability; where this is recognised, all manner of possibilities for mathematics education present themselves.

A second commonly unrecognised aspect of school and university teaching is that the preservice teachers have come to know teaching as a linear process; something done for or to students for their own good. In school, the authority, or authorship of ways of making sense of the discursive practices of the classroom, belonged to the teacher. The pedagogic practices through which preservice teachers were taught mathematics were primarily about making the student an observer and participant only on terms decided by the teacher and text. These students were born to passivity; they were not given the opportunity to speak from their life experiences, to author or initiate questions, to come to know mathematics on their own terms. As they sat through worksheets, pages of textbooks, 'tables' tests and so on, engagement was probably minimal, somehow divorced from the person who would one day triumph if only s/he knew the mathematics. Learning was about waiting to be told and shown, and teaching was mostly about helping, showing, scaffolding and nurturing. This constituted knowing about teaching and learning informs classroom practice, and is, I would argue, extremely problematic for hopes of quality in mathematics education in the future.

Normalising discourses (student/teacher; expert/novice) frame teacher education too, and valorise 'experience' as if "learning to teach is a linear process in which a novice student becomes a teacher through the function of unproblematic experience" (Youngblood Jackson, 2001, p. 386). However, 'experience', whether on campus, or in schools, is never unproblematic and can have positive or negative effects on developing professional identities. To the extent that teacher education "remains a bastion of traditional pedagogical practices" (Luke, Luke, & Mayer, 2000, p. 10), out-dated authority relations prevail. Preservice teachers depend on their lecturers, school-based teacher educators, booklets of readings and texts to make available to them the selective skills and knowledge said to be needed to make them recognisable as teachers. In schools they are often expected (Youngblood Jackson, 2001) to 'model' themselves on the school based teacher educator, establishing themselves as apprentice to the knowledgeable and 'experienced' teacher. However, as Luke et al (2000, p. 9) make clear, such practices "are geared not so much toward the creation of a 'generative' teacher for new ecologies and technologies, but more towards the representation and reproduction of particular historical models of 'good teaching', as culturally generalisable and as universally practical".

Although teacher educators and researchers often lament the lack of knowledge preservice teachers have when they enter mathematics education subjects, a poststructuralist analysis makes visible the ontological *ways of being* a learner and teacher that have been constituted and influence their teaching of mathematics (and learning to teach mathematics in teacher education). I would argue that these constituted knowledges, of essentialised identity and teaching as an undeniable 'good', are dangerous; they are dangerous because they render invisible the productive power (the constitutive effects, which can be positive/problematic) of all teaching/learning interactions and relationships in mathematics education. However, the question arises as to what teacher education might be able to achieve in terms of interrupting this constituted knowing about how teaching and learning are done. It is important to recognise, of course, that whatever is done is constitutive of the mathematics teachers of the future, although many other discourses also vie for expression in teachers' classroom practice.

POSSIBILITIES IN TEACHER EDUCATION

Teacher educators have always endeavoured to ensure quality learning for preservice teachers, especially in this case for those who are going to teach mathematics. In the past, and currently in *Australia's Teachers: Australia's Future* (Department of Education, Science and Training, 2003) quality is defined in terms of the mathematical and pedagogical knowledge constructed. The assumption seems to be that the more teachers know and understand, the better they will teach. However, the simple logic of this assumption is deceptive; it is based on positivist notions of knowledge, it says nothing about the power relationships in the contexts in which students learn to teach mathematics and it oversimplifies the ways in which that learning might be linked to educational change. From a poststructuralist perspective, I want to suggest that quality in learning should involve also a constituted sense that what one knows is never innocent, that how one has come to know it is not innocent - though each critically influences how one interacts with students in the classroom. A small window of opportunity opens for teacher education to engage students in a 'border pedagogy' (Davies, 2000) of learning to teach that, in celebrating uncertainty about learners and learning mathematics, interrupts taken for granted assumptions that currently frame practice. After all, preservice teachers are expected to interact with students in investigative ways that encourage the construction of mathematical ideas, patterns and relationships. The trick, from a poststructuralist perspective, is to ensure that inquiry (or investigation) for preservice teachers is not merely an intellectual, regulated task, but a constituted *way of being* in teaching/learning interactions that render both knowledge and learning tentative and uncertain.

Teachers, says McLaren (2003, p. 296) "need to become warriors against certainty". One could add here teacher educators, and researchers on whom the onus falls to find ways of encouraging preservice teachers to look beyond the commonsense of mathematics education, to find new and more inclusive ways of working with students. If preservice teachers are to achieve themselves as lifelong learners, as inclusive and generative facilitators of learning in others, then this sense of the professional self has to constituted *in* pedagogic interactions in teacher education and related sites; it can not be taken for granted to *follow from* the acquisition of disciplinary and pedagogical knowledges alone, as is too often assumed. The task for teacher education though is not an easy one; preservice teachers, especially those teaching at the primary and early childhood levels, come into teacher education programs with little mathematical knowledge and understanding, and, as previously discussed, commonly defer to readings of deficit to theorise student misunderstanding or lack of motivation. However, as Luke et al. (2000, p. 11) suggest "Remaking the teacher and the school and redesigning teacher education for new times go hand in hand". In the following section of the paper I contemplate what redesigning teacher education might look like from a poststructuralist perspective. The ball is clearly in the teacher educators' court; and the constitutive force of the discursive practices of teacher education indicates a tentative place to begin.

Change, from this novel perspective, goes beyond a concern for understanding and reflection, to a concern for the operation of discourses, through which preservice teachers have been, and are being, formed. Discourses comprise "historically, socially and institutionally specific structures of statements, terms, categories and beliefs" (Scott, cited in Adams St Pierre, 2000, p. 485) and organise ways of knowing into ways of acting in the world. A new discourse of teacher education, with new 'truths' and ways of interacting with learners is needed to bump into, to confront and interrupt the operation of existing discourses in the interests of redefining notions of what it means to learn and teach mathematics in the twenty-first century. New discourses can supposedly rewrite the world, as novice teachers are configured at the intersection of multiple intersecting discourses, living/acting in and between them finding comfortable spaces and investments (not necessarily conscious) in discourses that enact new truths and ways of operating. Change is accomplished as a result of contradictory positioning, due to the co-existence of the old and the new; every relation and every practice to some extent articulates such contradictions and therefore is a site of potential change as much as it is a site of reproduction (Hollway, 1984, p. 260). A 'border pedagogy' (Davies, 2000) of teacher education might incorporate two distinct but co-requisite elements: it might operate in new ways, constituting novice teachers to new purpose and participation through new relationships of power, and in making visible the productive force of pedagogy, it might imbue new 'truths' about knowledge and identity that renew and revitalise teaching/learning partnerships and interactions.

CONSTITUTING PURPOSE AND PARTICIPATION

Learning to teach mathematics is a social process; it is about becoming recognisable, and being able to recognise oneself, as a legitimate participant, and teacher, in the intersecting discourses of mathematics education. To this end, the preservice teachers will need to be able to speak the language of mathematics and current pedagogies and they will need a repertoire of skills, knowledge and pedagogical practices that make them recognisable as teachers. In the past teacher educators have tried to make all this knowledge available in textbooks and readings where "The selective traditions of our course readings and textbooks, seminars and practica are bids to shape student repertoires of skill and competence, knowledge and discourse" (Luke et al., 2000, p. 9). However, teacher educators should be wary of on whose terms the mathematics and theory are taught and learned. As teacher educators we need to get beyond certainty ourselves, to look more closely not only at what is learned, but at how—how preservice teachers are positioned in teaching/learning processes and how this is affecting their professional identity and view of mathematics as a worthwhile field of study. Too often, we (Klein, 2002) identify with ways-of-being a teacher that blind us to how our instructional (discursive) practices position the students; we can get carried away with methods that promise much in the way of knowledge construction, and forget how the teaching/learning processes are forming professional identities. It may be that what we are teaching is completely negated by feelings of alienation and inadequacy generated in students through the instructional processes we choose. However, there is an alternative. Rather than taking the knowledge first, and trying to 'fit' it to the student, we might take the student and make spaces for self initiated processes of learning to teach. Rather than giving out readings and tasks that must be completed, preservice teachers could be encouraged to author their learning journeys, with lecturers giving navigation help and advice.

The critical difference here would be the positioning of the preservice teacher as respected and valued participant in dialogue and action in the mathematics education community; such positioning would be framed by notions of all participants (including teacher educators) as teachers in process, and knowledge as similarly evolving and growing. Since it is futile trying to cover all that preservice teachers might need to know in the short time span available, teacher educators could make broad content choices, allowing the preservice teachers to determine learning processes. Here, I am suggesting that purpose and participation are constituted in pedagogical interactions and relationships, and that these are not individual attributes nor attitudes as has sometimes been thought to be the case.

RECOGNISING THE PRODUCTIVE FORCE OF PEDAGOGY

An additional issue is that preservice teachers should recognise both the intellectual and social (identity) knowledges produced in pedagogical interactions. They learn that each person's identity is an invention, a "social negotiation among discourses" (Phelan, 1996, p. 344) produced in schooling and community sites. This is an equity issue, because in recognising the often contradictory discourses through which persons come to know themselves and the world as they do, they may move away from seeing learners in humanist terms, as essentially motivated/not motivated, clever/dull or Anglo/Indigenous. It may be that teachers who recognise the discursive constitution of identity could do something to revise pedagogical practices and relations that they perceive to operate oppressively.

In her teaching of preservice teachers Phelan (1996) uses approaches such as "mapping the self" (p. 344), where they recognise and analyse their constitution through multiple discourses and think about which ones appear convincing and difficult to reject…and what this might mean for how they will interact with students in the classroom. Interestingly, this approach includes analyses of the intersecting and contradictory discourses of teacher education, within which the preservice teachers struggle to establish themselves as competent teachers of mathematics. In enacting discursive practices framed by notions of teachers and learners as constituted, and all learning contexts as socially and politically compromised, the hope would be to build up in prospective and new teachers a sense that 'things could be different' and that "nothing is ever settled completely" (Phelan, 1996, p. 344). Phelan (1996) sees this as a positive, though it may not sit well with all preservice teachers nor teacher educators who may feel more comfortable with order and a clearly defined direction in their teaching.

CONCLUSION

This paper is written to continue a conversation; a conversation about how preservice teachers can be better prepared to teach mathematics in the new millennium. Taken aback by the number of preservice teachers who demonstrate little allegiance to, or knowledge of, mathematics as a field of inquiry, and students continuing to leave school disengaged from, and disenchanted with, their experiences of school mathematics, I attempt to interrupt and open up what seems to be considered ordinary, 'natural' and commonsense practices in teacher education, to other readings and possibilities (St Pierre, 2000). Clearly my writing muddies the waters of teacher learning and change, though it must be conceded that new life may spring from muddy waters.

While uncertainty may not sit well with old models of teaching or experienced teachers, Davies' (1994, p. 35) words sound a warning: "While consistency and total coherence are pleasurable and satisfying, they involve a large degree of selective perception and ignorance."

REFERENCES

Chappell, C., Rhodes, C., Solomon, N., Tennant, M., & Yates, L. (2003). *Reconstructing the lifelong learner*. London: RoutledgeFalmer.

Cole, A. L., & Knowles, J. G. (2000). *Researching teaching: Exploring teacher development through reflexive inquiry*. Needham, MA: Allyn & Bacon.

Davies, B. (2000). Eclipsing the constitutive power of discourse: The writing of Janet Turner Hospital. In E. A. Pierre, & W. S. Pillow (Eds.), *Feminist poststructural theory and methods in education* (pp. 179–198). London: Routledge.

Department of Education, Science and Training. (2003). *Australia's teachers*: *Australia's future*. Canberra: DEST.

Hollway, W. (1984). Gender difference and the production of subjectivity. In J. Henriques, W. Hollway, C. Urwin, C. Venn & V. Walkerdine (Eds.), *Changing the subject* (pp. 227–263). London: Methuen.

Johnston, S. (1996). Questioning the concept of 'dissemination' in the process of university teaching innovation. *Teaching in Higher Education*, *1*(3), 295–304.

Klein, M. (2002). A poststructuralist perspective on the productive power of process. In B Barton, K. Irwin, M. Pfannkuch & M. Thomas (Eds.) Mathematics education in the South Pacific, *Proceedings of the 25th annual conference of the Mathematics Education Research Group of Australasia, Auckland* (pp. 390–397). Sydney: MERGA.

Lather, P. (1991). *Getting smart*. New York: Routledge.

Luke, A., Luke, C., & Mayer, D. (2000). Redesigning teacher education. *Teaching Education*, *11*(1), 5–11.

McCotter, S. (2002). If only we had better students: Re-learning how to be a good teacher. *Teaching Education*, *13*(3), 329–340.

McLaren, p. (2003). *Life in schools* (4th ed.). Boston: Pearson Education.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Phelan, A. (1996). Strange pilgrims: Disillusionment and nostalgia in teacher education reform. *Interchange*, *27*(3&4), 331–348.

St Pierre, E. (2000). Poststructural feminism in education: An overview. *Qualitative Studies in Education*, *13*(5), 477–515.

Sumsion, J. (2003). Rereading metaphors as cultural texts: A case study of early childhood teacher attrition. *The Australian Educational Researcher*, *30*(3), 67–87.

Youngblood Jackson, A. (2001). Multiple Annies: Feminist poststructural theory and the making of a teacher. *Journal of Teacher Education*, *52*(5), 386–397.

POLICY AND PRACTICE IN MATHEMATICS EDUCATION RESEARCH

Stephen Lerman
London South Bank University, UK
<lermans@lsbu.ac.uk>

Anna Tsatsaroni
University of the Peloponnese, GREECE
<tsatsaro@uop.gr>

In this paper we discuss the ways in which relations between the research community and official policy makers affect the productions of research and the identities of researchers. We note that the current state of mathematics education research indicates a relatively active and vibrant community, apparently able to produce research in spite of direct regulatory mechanisms of the state or indirect marketisation processes, although of course circumstances vary across the world. Recently there have been a number of articles and research teams looking at the field itself and its research productions, a sign, we argue, of a relatively confident intellectual community; but, perhaps, also of tendencies towards changes in the strength of boundaries structuring relations between the domains within the pedagogical field, with implications for the community members' identity.

> "Educational research is located in a knowledge-producing *community*." This claim "is significant because it is to recognise that it, like all research, is a social practice" (Usher, 1996, p. 34).

In this paper we are concerned with looking at the social practices in which we, as researchers, are engaged. We pose the questions: "What is the state of the mathematics education research community? To what extent is its state a factor of the relations between the official field of education decision-making in each country and the research community?" We begin by some general comments about the activities of mathematics education researchers and we then review recent studies on the research productions in the community, including our own research project. Subsequently we will sketch some strategies for examining these questions, illustrated with an account of the history of mathematics education research in one country, namely the UK.

A ROUGH PICTURE OF THE FIELD

If we look at the state of publishing of research then we can see a rapidly expanding field of activity. The two major international journals are now 35 years old in the case of the *Journal for Research in Mathematics Education* (JRME) and 36 years in the case of *Educational Studies in Mathematics* (ESM). One 'specialist' journal, the *Journal of Mathematics Teacher Education*, is now seven years old. A more general journal, the *International Journal of Science and Mathematics Education* (IJSME), published by Kluwer, is now in its second year and has a very healthy flow of high standard articles on mathematics education research being submitted. This is a journal where only half of the articles would be of interest to the vast majority of the international mathematics education research community, as until now there has been very little which could be said to be of common interest to the two research communities constituting the constituency of the journal. Nevertheless, there is, one can assume, such pressure on researchers to publish that in spite of both ESM and JRME increasing the number of issues published each year there is still a perceived need for further outlets; hence the success already, at this early very stage, of IJSME. There are a number of other journals that have appeared in recent years too, which have been very successful, including the journal *Mathematical Thinking and Learning* and the *International Journal of Computers for Mathematics Learning* to take just two examples.

Regarding conferences, the International Congress on Mathematical Education (ICME) held its first meeting in Lyon in 1969. The International Group for the Psychology of Mathematics Education (PME) was founded in 1976 at the third meeting of ICME and will hold its 29th meeting in 2005. At about the same time the British Society for the Psychology of Mathematics Education, which later changed its name to the British Society for Research into Learning Mathematics (BSRLM), was founded with largely the same aims as PME. The group *Mathematics, Education and Society* (MES) is now holding its fourth international meeting, the first having been held seven years ago.

We can say, then, that whilst mathematics education research is young when compared to psychology as an intellectual field of research, for example, it is nevertheless now more than 35 years old and well established. There are disputes, debates and diversification, all, we would argue, signs of a dynamic and productive research community.

However, to read the debates on 'quality' of education institutions at all levels and particularly Higher Education institutions, and academic research, especially educational research, makes one reflect again on our description above of developments within mathematics education research. For instance, looking at the debate over the *No Child Left Behind* bill of President George Bush and the denigration of the quality and usefulness of the output of the education research community from that administration and by some within the education community itself (see the journal *Educational Researcher* for evidence of these debates) one wonders what the future will bring for the mathematics education research community, in particular because the bill has brought with it a clear demand for large-scale random experiments to evaluate the effectiveness of educational innovations and an implication of withdrawal of funding from research of other kinds. Hence we consider it timely to examine the relations between mathematics education research and researchers and the field of policy. To give one more indication of how important it is to open up this rather complex issue, we merely note at this stage that, like the word 'quality', the word 'learning' featured above and used by mathematics education research community perhaps to indicate a (change) of its interests, is, today, also part of what Readings (1996) has called 'de-referentialised discourse', i.e. words empty of meanings increasingly mobilised for 'image-building' purposes.

A REVIEW OF REVIEWS

Recent years have seen a number of studies that look inwards to the mathematics education research community, its orientations and interests, and its research productions. In today's audit culture, academic researchers are supposed to, mainly, look 'outwards', towards the field of application of their knowledge in the 'real world' of practice. They are 'encouraged' to look 'inwards' only in order to reflect upon their practice and to develop procedures of self- evaluation. Depending on the discourse, this is in order to improve their practice, to minimise abuses in the exercise of their power upon the less powerful (e.g., teachers), and often to 'assist' the work of external evaluators of their practice. However, looking 'inwards' is certainly not something new, and it hasn't always been instrumentally put to the purposes just mentioned above. Instead it has been a pivotal characteristic of the life of professionals and academics, when the latter had a different relation to knowledge, and when a different conception of knowledge was current. Furthermore, one aspect of this inward movement was 'reflexivity', i.e. a constant reflection on how one does things, crucially how one governs (Rose, 2004, p. 173–174). In view of the important observation that "[k]nowledge, after nearly a thousand years, is divorced from inwardness and literally dehumanised" (Bernstein, 2000, p. 86), and that today a different notion of knowledge prevails, it is likely that research discussed in the text are open to different interpretations, and risk being subject to ideological and political manipulations.

We should look back first to 10 years ago, when Kieran's (1995) retrospective look at mathematics education research on learning presented interviews with two leading researchers looking back over that period, followed by an analysis of articles published in JRME in its first 25 years building on the remarks of the researchers in the first section. She argued that there has been a shift towards integrating learning with understanding and studying them together, as well as an increasing orientation towards interactionist studies drawing on Vygotskian ideas. The other studies have all taken place in the last 5 years (Niss, 2000; Lubienski & Bowen, 2000; Chassapis, 2002; Hanna & Sidoli, 2002; see Tsatsaroni, Lerman, & Xu, 2003 for a more detailed description).

The concerns of these studies vary from being mainly pedagogical to sociological and they also attempt to capture qualities to do with the research activity characteristic of the field. These studies contribute to sketching a rough picture of the field, pointing to some of its developments over time. Therefore, in terms of their approach these studies intend to find out 'what' or 'what kind' of developments took place in the field, and to remind the research community itself of issues to do with teaching or equal treatment that potentially are of interest to the research community.

For the 2004 meeting, the tenth, of ICME in Denmark, two survey teams carried out surveys of aspects of the work of the community. Survey Team 1 on "The Relations Between Research and Practice in Mathematics Education", whose

Chair was Anna Sfard, asked researchers across the world a series of questions: "How would you describe your work in mathematics education over the last 5 years or so; during this period, to what extent was your work influenced by the current state of mathematics education; do you think that your work had, or is going to have, an actual impact on the practice of mathematics education?" 74 people responded from across all continents with responses from North America being amongst the smallest, although Europeans constituted the largest group. The evidence was presented in a narrative style, as a dialogue between a composite researcher voice and the team voice. Sfard (forthcoming), in her conclusions, notes the shift to a focus on the teacher, calls for acceptance of diversity of discourses amongst researchers, notes a qualitative trend, and suggests that research affects practice in teaching more than most researchers believe.

Survey Team 3 on "The Professional Development of Mathematics Teachers", whose Chair was Jill Adler, carried out a survey of the research publications of the community in the field of mathematics teacher education from 1999 to 2003. Their systematic study examined a wide range of journals, conference proceedings and the Second International Handbook of Mathematics education for articles on mathematics teacher education between 1998 and 2003. The team presented alternative voices as reactions to the findings, which included: small-scale qualitative research predominates; most teacher education research is conducted by teacher educators studying the teachers with whom they are working; research in countries where English is the national language dominates the literature; and some questions have been studied, not exhaustively, but extensively, while other important questions remain unexamined.

We can note here that the two surveys are somehow different from the group of studies already reviewed, in that they can be read not simply as a response to internally developed concerns of an 'autonomous' group of researchers interested in exploring possibilities of growth, but also as a 'reaction' to outside (perceived or real) pressures, as when one needs to defend oneself by giving an account of his/her actions. The methodological 'trick' of 'voice' could be variously interpreted and justified here, but in fact it is not one voice. Rather the discourse is 'multivocal', as one can hear not only the voice of the researcher narrating her work, and not only the teacher struggling to change conditions that make her silent, but also the state official's voice. However, in the sociological discourse the notion of 'voice' has recently given rise to an interesting debate, which is not possible to recapitulate here (but see Moore & Muller, 1999; Young, 2000; Arnot & Reay, 2004). The broad aim of our research project (Tsatsaroni, Lerman and Xu, 2003) was to analyse the processes whereby mathematics educational 'theories' are produced and the circumstances whereby they become current in the mathematics education research field, as well as the changes over the years. We aimed to construct a representation of the field of mathematics education research through which to explore the reproduction of identities, of researchers and teacher educators in the field. Our method of working to create this picture was to look at specialised texts of the research field, namely a representative sample from 12 years of the papers in the Proceedings of PME, and of articles in the two journals ESM and JRME. Whilst the choice of years of publications to analyse was to some extent arbitrary, it was based on two factors: we wanted to bring the analysis up to the period in which the research was carried out; and we were most interested in the years since the entry of more social theories into the field (see Lerman, 2000). But what we are still interested in is to understand such apparent openings of this small sub-field to influences coming from the wider intellectual field, and by tracing any changes in the pattern of influences to analyse their consequences for how knowledge is defined; the latter taken to constitute the basis for identity formation.

We developed a tool of recording and analysing the specialised texts of the research community by drawing broadly on Basil Bernstein's work. The tool changed as we interrogated more articles and found our categories inadequate or requiring modification. A key factor has been the development of justifications for judgements, what Bernstein (2000) calls recognition and realisation rules, for what made us place an aspect of an article in one category or another in an explicit manner. We were concerned that our project should be an empirical, study, and at the same time will draw on a theory, the 'grammar' of which could assist description and facilitate understanding (Moore & Muller, 2002).

One of our findings was that the main addressee (almost three quarters of all articles) of research appears to be the category 'researchers and teachers' in spite of the fact that few teachers attend PME or read ESM or JRME. We see here a clear tendency to a positioning which looks outwards, beyond researchers, to teachers but also to teacher educators. Our interpretation is that the most dominant positioning is of the *researcher as teacher educator* and this claim can be further supported by our finding that most papers (more than 80%) explicitly construct a pedagogical model. It is worth noting here that this model is of the *individual learner-knower*, and it is consistent with the interest in problem solving as the focus of research—with almost all the features of liberal-progressive pedagogy being valued overall.

Whatever the research questions posed in these studies, and whatever the conclusions reached, we can say that the mathematics education research community is very active and very productive. Indeed one can argue that a community that examines, classifies and criticises its research orientations is a maturing community. However, it is also important to note that, especially when research practice itself becomes an object of investigation, the insistence on inquiring into the capacity of theory to contextualise, re-conceptualise and re-define the object of study, that is the insistence on holding on to a fundamental criterion and value of our research tradition is also a sign of the degree of autonomy of that field. In the remainder of this paper we attempt to approach this question of the field's autonomy by discussing how relations between the research field and the official field might affect research activity.

MODELLING RELATIONS BETWEEN AGENTS AND AGENCIES

The description that follows is taken and adapted from Morgan, Tsatsaroni and Lerman (2002), and draws largely on Bernstein's theories to characterise the relations between communities concerned with mathematics education.

Official discourses are produced by agents operating in the *Official Pedagogic Recontextualising Field* (OPRF) (Bernstein, 1996), for example, the examination boards, government departments and agencies. To produce these discourses official agents draw on a set of discourses and practices available within the sub-field of recontextualisation, and have subsumed them under their own aims and purposes. Among such discourses are those produced in the field of production of knowledge by the activities and practices of the mathematics education research community and circulated within the *Unofficial Pedagogic Recontextualising Field* (UPRF) (Bernstein, 1996). Elements of these can be appropriated by official agents, often constituting central elements of the official discourse. Elements of discourses produced by other educational communities and circulated within the UPRF, such as discourses on school management, school effectiveness, etc., might also become elements of the official discourse. However, discourses produced by the mathematics education research and other communities might equally remain outside the official pedagogic discourse, forming *unofficial, oppositional educational discourses or simply remaining silent*. Whether research in the community is appropriated or remains outside is a function of the degree of *boundary maintenance* (Bernstein, 2000) that regulates the relation between official agents on the one hand, and other agents (such as educational advisers, researchers, mathematics teacher educators) and teachers on the other.

There are a number of issues regarding the field of mathematics education research and its relationship to the official discourse, notably its status and its internal organisation. To start with its status: mathematics education research comprises a sub-field within the general field of educational studies in the field of intellectual production. Though small, this sub-field is continually growing nationally and internationally and, relative to other sub-fields, enjoys some significance in the field of educational studies. It is the largest special interest group in the American Educational Research Association (AERA), for example. On the other hand, within the general field of intellectual production, educational studies is a most vulnerable area, subject to interventions by the state, mainly because its purpose is always assumed to be the education of teachers. As a consequence of current economic, cultural, technological and social changes, such state interventions are in fact reducing its significance, as teachers' 'education' is redefined in instrumental terms as work-based 'training', fact which de-legitimises certain forms of knowledge.

The second issue is the way its knowledge is organised. To think about its organisational form we find informative to mention two kinds of descriptions that Bernstein provides, one referring to the intellectual fields themselves, the other on their recontextualisation in Higher Education curricula through which members are socialised into a field of knowledge. Because of the limited space available, it is not possible to refer to the second kind, except to note that the terms in which curricula are constructed are increasingly shaped by external forces (Bernstein, 2000; Beck & Young, 2005). According to the first, the internal structure of the education research field comprises sub-fields of activity—mathematics education research being one such sub-field—with *horizontal knowledge structures* (Bernstein, 1999). This means that, unlike hierarchical knowledge structures, exemplified by the natural sciences, each education research sub-field consists of a series of specialised languages with specialised modes of interrogation and criteria for the construction and circulation of texts. Developments take the form of the addition of a new language, an additional segment, rather than greater generality and integrative potential. One crucial point here is that horizontal knowledge structures form a kind of collection code, often with a *weak grammar*, i.e. with a conceptual syntax not capable of generating unambiguous empirical descriptions. Interestingly, the discipline of mathematics is also a horizontal knowledge structure but with a strong grammar. It has been conjectured (Adler *et al*, forthcoming) that the mismatch of the strong grammar of mathematics and the weak grammar of education makes the process of mathematics teacher education problematic.

Overall, the status and organisation of knowledge in the field of mathematics education research points to the shifting and uncertain nature of the field. This affects voice constitution and power relations and makes positioning precarious since there is no stable single specialised language, therefore no clear distinction between official and oppositional discourses. Instead there are complex relationships between the official voice and other voices.

We move now, finally, to examine these arguments in relation to one national community, that of the UK.

CASE STUDY – THE UK

Following the analysis above, we will discuss, first, status and internal organisation of mathematics education as research and as academic subject. We will then present a brief narrative of mathematics education's history, in order to be able to comment on the boundary between the OPRF and UPRF in the UK as it affects the mathematics education research community. We should point out that this account is based on experience, participation and perception. We recognise the need for a systematic and careful study of this history through document and journal analysis for our observations to have validity.

As a sub-field within the general field of educational studies mathematics education research can be seen as quite strong, in that BSRLM is one of the most active research associations in curriculum areas of schooling. There is a special interest group within the British Educational Research Association (BERA) for mathematics education research, which will be co-ordinated by BSRLM, although

BERA itself is quite small when compared with the US equivalent, AERA. Educational research as a whole has to compete with economic and social science research for Government research funding and our perception is that education receives quite a small proportion of those funds, although there are charitable sources of funding. It would of course be very interesting to have access to information on the projects funded and those not funded over a period of years to evaluate trends and to identify possible influences on the direction of educational research, and mathematics education research in particular. There have been moves by Government to control educational research funding, given that schooling is so tightly regulated, both in content and in teaching style, as is teacher education. At the time of writing there is no sign that this will happen, at least overtly. At the same time, all research in Universities is subject to periodic peer evaluation (the Research Assessment Exercise, RAE) that controls direct funding for research within the overall budget given to Universities. It has been suggested that the panel which makes decisions on educational research has been particularly harsh in previous RAEs. It produced far fewer top-rated Departments than most other subject panels. A case of 'shooting ourselves in the foot'. If, as a community, we do not rate our own research very highly who can blame others for the same? Certainly it has had a profound effect on research communities and researchers.

Regarding internal organisation, the UK mathematics education research community exhibits the same horizontal knowledge structure with a weak grammar as the international community, although the relative strengths of different discourses, and indeed the strength of the interactions between those discourses, exhibit some differences. We might conjecture that there is greater interest in sociological approaches, given that Bernstein was British and there is a general interest in Bourdieu's work in the UK. We would conjecture that the radical constructivist orientation that has dominated the US community for some decades is not so strong in the UK. We repeat that these conjectures would benefit from a systematic analysis; – some of the data produced in the research project reported earlier, as well as within the Survey Team 3 can certainly be revisited to create a better picture.

Before 1988 there was no National Curriculum (NC) for schools in the UK, the curriculum being organised around the various options for national examinations at age 16. The UPRF enjoyed a degree of relative autonomy but the relations between the OPRF and the UPRF were not oppositional. On the contrary, the Cockcroft report (1981) drew on, and indeed commissioned research in order to prepare the report for Government. The NC initially incorporated some of the suggestions from that report. Subsequent iterations of the NC, and in particular the ambitious large-scale reform initiative, the National Numeracy Strategy, were very partially informed by research, but not in interaction with the research community. Indeed one of the authors of this paper was present at a meeting of representatives of the mathematics education research community with an official of the Strategy who, when asked if the community could produce research that would inform further iterations, said that changes would be informed by information gathered by inspections of implementation in schools, not by research. Indeed it appeared that 'research' was being re-defined as that resulting from inspection reports.

As we mentioned above, schooling in the UK, and mathematics teaching and learning in particular, is highly regulated by Government, perhaps amongst the strongest regulating regimes in the world, given the content, teaching styles, inspection, national tests, public reporting of outcomes of tests by each school, and teacher education mechanisms of regulation. At the same time the mathematics education research community is active, producing more research as the years go on, and producing more doctoral dissertations too. Our project indicated that the research community does not, in general, engage with policy, but given the 'cold shoulder' exhibited by the policy makers this is perhaps not surprising. There are some notable exceptions, the group at King's College London in particular.

Our project also indicated that there is little research explicitly focused on the character and impact of school and teacher training policy, and furthermore how this affects mathematics education as a field of research and as an academic subject.

Finally then, we might conjecture (based only on an impressionistic narrative of the UK situation) that there are considerable shifts affecting relations within and between the field of mathematics education. Education policy, through current initiatives on school mathematics and on the training of teachers, attempts to side-step mathematics education research community. Where exactly boundaries are weakening and where becoming stronger, and the implications for the place of mathematics education in Higher Education and in relation to schools and to the Policy field needs now to be more systematically addressed.

REFERENCES

Adler, J., Ball, D., Krainer, K., Lin, F.-L., & Novotna, J. (forthcoming). Mirror images of an emerging field: Researching mathematics teacher education.

Arnot, M., & Reay, D. (2004). Voice research, learner identities and pedagogic encounters. Paper presented to The Third International Basil Bernstein Symposium, Clare College, Cambridge, U.K.

Beck, J., & Young, M. F. D. (2005). The assault on the professions and the restructuring of academic and professional identities: A Bernsteinian analysis. *British Journal of Sociology of Education, 26*(2), 183–197.

Bernstein, B. (1996). *Pedagogy, symbolic control and identity: Theory, research, critique*. London: Taylor & Francis.

Bernstein, B. (1999). Vertical and horizontal discourse: An essay. *British Journal of Sociology of Education*, *20*(2), 157–173.

Bernstein, B. (2000). *Pedagogy, symbolic control and identity: Theory, research, critique* (2nd ed.). New York: Rowman & Littlefield.

Chassapis, D. (2002). Social groups in mathematics education research: An investigation into mathematics education-related research articles published from 1971 to 2000. In: P. Valero & O. Skovsmose (Eds.), *Proceedings of Third International Mathematics Education and Society Conference, 1* (pp. 273–281). Danish University of Education: Centre for Research in Learning Mathematics.

Hanna, G., & Sidoli, N. (2002). The story of ESM. *Educational Studies in Mathematics, 50*(2), 123–156.

Kieran, C. (1994). Doing and seeing things differently: A 25-year retrospective of mathematics education research on learning, *Journal for Research in Mathematics Education, 25*(6), 583–607.

Lubienski, S. T., & Bowen, A. (2000). Who's counting? A survey of mathematics education research 1982–1998. *Journal for Research in Mathematics Education, 31*(5), 626–633.

Moore R., & Muller, J. (1999). The discourse of 'voice' and the problem of knowledge and identity in the sociology of education. *British Journal of Sociology of Education, 20*(2), 189–206.

Moore, R., & Muller, J. (2002). The growth of knowledge and the discursive gap. *British Journal of Sociology of Education* (Special Issue: Basil Bernstein's theory of social class, educational codes and social control), *23*(4), 627–637.

Morgan, C., Tsatsaroni, A., & Lerman, S. (2002). Mathematics teachers' positions and practices in discourses of assessment. *British Journal of Sociology of Education*, *23*(3), 445–461.

Niss, M. (2000). Key issues and trends in research on mathematical education, Plenary address to Ninth International Congress on Mathematical Education, Makuhari, Japan.

Readings, B. (1996). *The University in ruins*. Cambridge, MA: Harvard University Press.

Rose, N. (2004). Nikolas Rose: Governing the social. In N. Gane (Ed.), *the future of social theory* (Ch. 9), London and New York: Continuum.

Sfard, A. (forthcoming). *What could be more practical than good research? On mutual relations between research and practice of mathematics education*.

Tsatsaroni, A. Lerman, S., & Xu, G. (2003). A sociological description of changes in the intellectual field of mathematics education research: Implications for the identities of academics. Paper presented at annual meeting of the American Educational Research Association, Chicago. ERIC# ED482512.

Usher, R. (1996). Textuality and reflexivity in educational research. In D. Scott & R. Usher (Eds.), *Understanding educational research* (pp. 33–51). London: Routledge.

Young, M. F. D. (2000). Rescuing the sociology of educational knowledge from the extremes of voice discourse: Towards a new theoretical base for the sociology of the curriculum, *British Journal of Sociology of Education, 21*(4), 523–536.

Millennials and Numeracy in New Contexts, Cultures and Spaces

Tom Lowrie
Charles Sturt University
<tlowrie@csu.edu.au>

Robyn Zevenbergen
Charles Sturt University
<rzevenbergen@csu.edu.au>

Increasingly researchers and educators recognise that changing social conditions impact on learners. This is particularly the case in post-industrial times where technology has shaped young people's interests and dispositions. This paper explores the ways in which technologies have influenced the learning contexts and entertainment spaces young students inhabit in a technology age. It is argued that young people, or Millennials, have very different orientations to thinking and working mathematically from their older counterparts and that this has serious implications for school mathematics.

Within social theories and among literacy educators, there is a strong recognition that technology has impacted significantly on how young people go about their work and their thinking. Social commentators (Howe & Strauss, 2000; Mackay, 1997) have noted the intergenerational differences between Baby Boomers, Generation X and Millennials. Although there are differences in how these generations are circumscribed temporally, there is consensus that social conditions create different opportunities for the participants within a generational group. The focus on post-World War II generations has shown 3 distinct generations within Western cultures. The most recent of these has a number of names including Millennials, Generation Y, Generation Why, and Nexters (Zemke, 2001). Collectively, this generation is different from previous ones in that this is the generation that has grown up in a world immersed with technology. This is the generation for whom technology is an integral part of their social world. One only has to consider how the mobile phone is an extension of the Millennials' way of being in the social world. Yet, the mobile phone is multifunctional – a phone, a text machine, a camera, a calculator, a GPS navigation system, and a time device. Having grown up in a technology-rich environment, Millennials have been immersed in an environment where there is almost instant gratification, where they can play games (on phones, X Box or in arcades) where their actions are rewarded by some form of technology. They are multi-skilled and are used to processing multiple forms of information. Moreover, they have begun to develop a range of multiliteracies in order to make sense of literacy and numeracy demands (Lowrie & Clancy, 2003). One only has to consider the Millennial playing a video game, listening to a walkman, interacting with a real person, and holding a phone conversation. This is not unusual for this generation. While literacy educators have been working with the ideas that surround the impact of technologies on literacy learning and representation, less of this has been undertaken in numeracy education (Lowrie, 2002, 2004; (Zevenbergen, 2004).

THE INFLUENCE OF GAMES ON CULTURE AND SPACE

Within this literature, there is a growing recognition of the impact of games technology on how young people begin to read texts (such as those represented through the games) as well as how they come to see and be seen in the world. Generally scorned by educators, games such as Prince of Persia, Grand Tarismo and Tony Hawkes are part of the popular culture of Millennials. Where this generation is seen to have low attention spans (which impacts on how they learn in school settings) and in need of instant (and often gratuitous) feedback, exploring games environments offers insights into the habitus of Millennials. Gee (2003), for example, has been exploring the impact of video games on participants' understandings of self as well as the impact on cognition as related to literacy and other aspects of dominant school learning. These games include those played on hand-held technologies (Gameboys), relayed through television (such as Xbox and Nintendo) and through arcade games (Rap Dancer). In contrast to the public perception that young people have low attention spans, games environments can see them engaged for substantial periods of time. As such, questions need to be asked about the elements of games technologies that are counter to those in schools. Researchers (including (Gee, 2002); Clancy & Lowrie, 2002, 2003) have been seeking to identify these elements for literacy learning but there has been little research on this in terms of numeracy learning (with the exception of the Lowrie's 2003; 2004 work). As Gee notes, the games environments are very rich in problem solving. He maintains that in such environments, the player needs to go through worlds where he/she needs to keep scores. This would suggest that there is a high degree of numeracy needed however there is a paucity of literature to support this. While there is some literature in mathematics education that explores the impact of various technologies on mathematical understanding such as Groves' work with hand-held calculators (1995); Geiger (1998) and Vincent (2003) with graphing calculators; and Van Rijswik (1999) work with spreadsheets, there is little research that explores the potential of digital formats used in games technologies on potential new numeracies as suggested by Gee. However, we would contend that there may be considerably more potential in this environment for identifying numeracy learning opportunities. If, as has been found by literacy educators, games environments have considerable potential for literacy educators, then consideration needs to be made of their potential for numeracy education.

BARRICADES BETWEEN IN AND OUT-OF-SCHOOL NUMERACY SPACES

It could be argued that Millennials will find it increasingly difficult to find connections between the activities they undertake at school and the problems they solve out of school—particularly in light of the fact that these young people have access to more than any generation before them. De Corte, Verschaffel and Greer (2000) maintained that in order for students to make meaningful connections between problem solving and real-life contexts they need to be immersed in innovative learning environments that are radically different from traditional classroom practices. They proposed that tasks should be well structured, diverse and authentic. Since skills and knowledge are best acquired within realistic contexts (Grabinger, 1996), authentic tasks should be aligned between the context in which learning is represented and the real-life setting in which that knowledge will be called upon (Bennett, Harper, & Hedberg, 2002). Consequently, the problem solver should be able to engage within the problem context from both sense making and process perspectives. Ideally, students should be encouraged to extend, adapt, revise and adopt mathematical ideas to a context that they can place themselves within.

Lowrie (2004) maintained that young students were using such skills and processes in game environments. He suggested that the authenticity of new technology games provided opportunities for the game player to become immersed in the game's surroundings and personalise the game-playing experience in ways that school curricula did not seem to offer. The seemingly "realness" of the learning space magnified the game experience to an extent where players felt that they were in control of their learning and consequently remained highly motivated to solve problems as part of the game context. Similarly, in a study that traced the out-of-school mathematical environments to which children were exposed, de Silva, Masingila, Sellmeyer and King (1997) observed middle school children a) using several mathematical concepts within a single activity, (b) making decisions that were based on optimising goals, and c) communicating their ideas in order to make sense of complex relationships. Importantly, they concluded that these children were exposed to potentially rich mathematical contexts.

WHERE NEW LITERACIES, NUMERACIES AND TECHNOLOGIES COLLIDE

In the subsequent sections of this paper, we draw on a case study of one student from a much larger project where the focus was on the phenomenon of young students using Pokemon within the Games Boy environment. The investigation examined the way in which a 7-year-old boy accessed a range of literacy and numeracy skills when playing an electronic game in a naturalistic setting. The Pokemon phenomenon—which is used as an example of out-of-school mathematical engagement—consists of a range of different synergistic texts such as movies, videos, books, internet cheat sites, card games, computer games, board games and hand held Game Boys. One of the key aspects of playing the Pokemon Game Boy is the notion of journey. Each game involves having a mission that involves going on a Pokemon journey in order to collect different species of Pokemon. These journeys require the players to move across a range of landscapes. Over time the game continues to evolve and so players constantly need to seek out information from the different forms of text.

The Pokenav (see Figure 1) provides access to important information about the location of cities and pathways (Routes) that are recommended for travel from one city to another. Morgan (the 7 year old case study participant) accessed additional information about specific pathways from the Pokemon books. In these magazines Morgan encountered different graphical representations of cities—including maps with different scale, orientation and perspective.

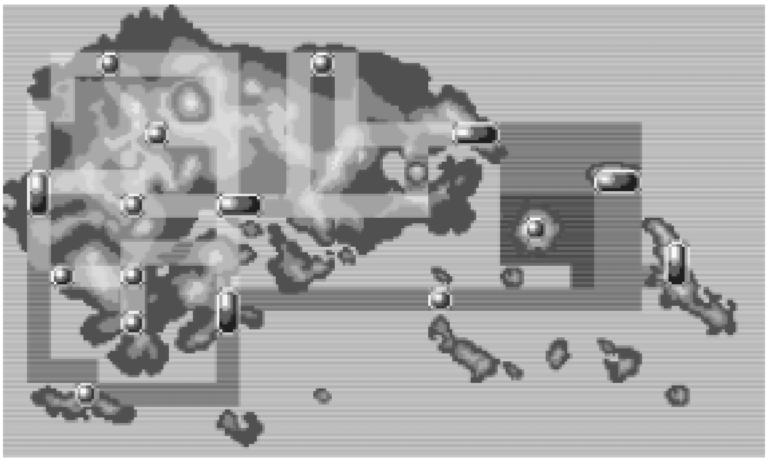


Figure 1. A visual representation of the map illustrated in the Pokenav.

The Pokemon screen is very small so students rely on memory, artefacts (such as magazines and computer downloads) and visualisation skills to navigate around the worlds. As such, they rely on more than the immediately available visual screen. Thus, for us, this creates a novel environment to explore the ways in which they think and use spatial concepts and processes not traditionally found in the school curriculum. For us, we were interested in how they resolved their pathways through seen and unseen worlds as this was a very different way of working with traditional school mapping formats. In the following sections, we discuss a number of strategies used by the participant.

ACCESSING AND USING MAPS IN OUT-OF-SCHOOL CONTEXTS

In order to play the game more efficiently, Morgan accessed and utilised various artefacts that involved interpreting maps. In fact, Morgan demonstrated the capacity to access important information from the Pokemon world by analyzing maps in different representations and scaled forms—including graphical information from magazines. These maps included "full" maps that represented the entire Poke-world (see Figure 1) and more detailed "zoom" maps that allow the player to navigate his way through towns (see Figure 2), cities, and various natural environments (including caves, mud slides and waterfalls) between these cities and towns. In addition, less detailed positional maps (see Figure 3) were regularly analysed in order for Morgan to determine where he was positioned in relation to significant landmarks.

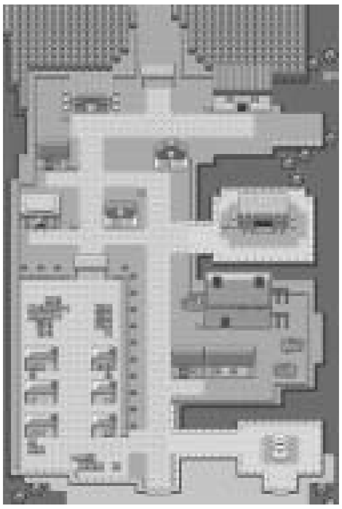


Figure 2. A town (Slateport City) displayed within the Gameboy.

Figure 3. A positional map within the Gameboy.

Essentially, the maps were utilised to locate information that was necessary to find (or catch) Pokemon. Morgan's capacity to reason visually and locate information in a relatively sophisticated manner was required in order to solve both routine and open-ended problems within the game context. As Morgan commented:

> The Mountain Falls [is] the closest city you can go to. Once you go from Mauville City, that's where I am [showing the location on the screen], you go up there to there [pointing to another location on the map], then you go across here and follow that thing [a pathway], you end up in Mountain Falls. That's where the Magna Team are. You need to battle the leader two times…And this is Everyday City right over here [pointing]. That's the whole thing. I need to go over there, that's the Pokemon Centre right over there (Morgan is referring to the Pokenav that shows the whole Houen area map and the individual cities that are colour coded to represent different buildings).

The Pokenav (see Figure 1) provides access to important information about the location of cities and pathways (routes) that are recommended for travel from one city to another. Morgan accessed additional information about specific pathways from the Pokemon books. In these magazines Morgan encountered different graphical representations of cities—including maps with different scale, orientation and perspective.

Although the magazine maps were more detailed (and in a larger scale) than the corresponding graphical representations in the Gameboy, Morgan found it advantageous to cross reference information whilst playing the game. The magazines became an important reference point for travel between cities because these maps provided more information within a single frame—not only was the scale easier to interpret, more information was represented within the given space. The Gameboy screen is relatively small (7 cm x 4 cm) and as a consequence the player would need to use scroll buttons (across eight compass-point directions) to view the information that could be represented in the magazine maps. Within the game-play context the player is able to navigate through space in both "full" and "zoom" modes (represented in Figures 1 and 3). The zoom mode displays information in a more detailed manner (possibly magnified ten fold) than the map that represents the Houen City. Morgan simultaneously moved between these two perspectives while regularly referring to the maps in the magazine.

> [There are] two maps…The little map that shows you half of the town. You have to move over and it shows you the other half…You flip backwards and forwards between the cities to get more information [and] to see where you are going.

Morgan had developed an awareness of scale and proportion. Moreover, he appreciated the fact that you could only see part of the map in the zoom function mode and realised that one part of the map was connected to the other even though both parts of the map were not visible on the single screen.

The ways in which Morgan switched between the various representations – magazine, normal screen and zoom screens – offered insights into the multiplicity of representations needed (digital and paper) to solve the task. Morgan also drew on his knowledge of the different cities and obstacles needed to overcome in order to reach the goal of this first city. Thus, unlike the paper mapping activities of school mathematics classrooms, the Pokemon format drew on a range of resources, including the paper map, for mapping. Morgan displayed a competency in working through these various cities as well as a metalanguage for talking through the ways to solve the task.

In terms of coming to understand the ways in which young students make sense of their experiences, what was observed was Morgan's capacity and willingness to switch between multiple forms of representation in almost simultaneous modes – that is, he switched very quickly from one format to another with ease. We contend that such a capacity is a very different skill than that used in most contemporary classrooms. As such, we propose that this experience, at such an earlier age, is a skill that has not been readily available in previous generations and offers very different potentialities for learners and learning. As has been identified with Millenials in their capacity to multiskill and multitask and from our observations of

Morgan (and his peers), the switching between the various formats noted here may provide a formative arena for developing such skills and dispositions.

TRANSFERRING SKILLS WITHIN AND ACROSS DIFFERENT CONTEXTS

Impressively, Morgan not only remembered the directional sequences when revisiting cities, he was able to explain why it was important to go back to these destinations. His conversations referenced the Gameboy, the magazines and the Pokedex within the Gameboy simultaneously. Moreover, Morgan was able to effortlessly move between several graphical representations when describing his movements "inside" and "outside" the game context.

> …the big [map] shows you just where the towns are. Like if they are going longways, tallways, diagonals or circles. It shows you the world, like here the world and here are the little towns and here's the other one and there are little ones here (referring to the maps on his Pokenav).
>
> [With the zoom function] There's actually three maps… Because when you go on like if you click anywhere and go small, so you just see one more part and if you click say again, it just shows the whole world and then the little one. So it shows you one little square, it can show you the whole world and it can show you how cities are.

He was able to appreciate that his (the trainer's) location within a city or town could be represented in different ways on the same screen (as in Figure 3). Although he had not encountered notions of scale, proportion or perspective (in a school context) he was able to conceptualise the relationship between landmarks in different spaces as a series of routes. Furthermore, he was able to integrate these routes into networks of landmarks in ways that allowed him to make approximations of relative distances, and thus constitute a form of scale (Lehrer & Pritchard, 2002).

As a 7-year old, it was certainly the case that Morgan had not encountered notions of scale, proportion or perspective at school and yet he was interpreting maps and applying knowledge to situations with a level of sophistication that would probably not be expected of him at school for another three or four years. While his language was not as rich as would be expected within school, conceptually, his comments indicated that he had an understanding of these complex ideas. What this suggests to us is that the forms of mathematics that Morgan encountered in the Pokemon format was contextualised within a social framework that was both meaningful and personally authentic. Embedding mathematics within activities that engage students in ways that make sense and connections appear to foster potential to develop deep understandings of complex ideas.

CONCLUDING COMMENTS AND IMPLICATIONS

Within the confines of a conference paper, we are unable to draw out the richness of the data that has been yielded through the exploration of one tool (Game Boy) and in one game (Pokemon). However, what we have sought to show are a number of key elements that we see as critical to reconsiderations of what it is to be a young student in a technology-rich world. For Morgan (and his peers), the Pokemon game offered potentialities that were not available to previous generations. Morgan's capacity to switch between different textual representations (in this case, paper and digital) where spatial concepts were central but contained other mathematical constructs (such as size, proportion, direction) were evident. What was apparent to us in the observations and interviews with Morgan was a disposition to switch easily between contexts (paper and digital) with fluency. The young age of Morgan would suggest that as he ages and is exposed to other multiple literacy formats, that the dispositions he shows here will become more complex and as such develop into generational characteristics – characteristics that epitomise the Millennial generation.

The complexity of mathematical concepts that were evident in Morgan's understandings of mapping through the use of the different scales of the maps suggests that authenticity in activities supports learning in rich ways. For Morgan, his understandings of scale and proportion were well developed for his age (as defined by curriculum conventions within the Australian context). What is emerging from these data is that authenticity supports learning. While it may be argued that Pokemon is hardly an "authentic" learning environment, we contend that for this age group, the Pokemon phenomenon is authentic since the game is part of a much wider social phenomenon. Cartoons and movies as well as magazines are a part of this age group's social world. As such, the Pokemon game is authentic to the age grouping. Being seen as a consumer group by the producers of Pokemon, young children are targetted for particular items. These groups can be as young as 3 years old (such as the Wiggles market) through to much older consumers (such as adolescents). Unlike any other generation, Millennials are large consumers (Zemke, 2001) and as is evident from Morgan's use of Pokemon and its multiple forms of consumerism (Games and magazines), young children are targeted by producers.

When Morgan's use of Pokemon is seen within a larger context, the authenticity of his activity becomes apparent. The maps, cities and objects that are integral to the game are represented in other formats (cartoons and movies) so that as he moves through the various worlds, he is able to make connections with these experiences so that, for him, the Pokemon adventure is very authentic.

REFERENCES

Australian Association of Mathematics Teachers. (1997). *Numeracy = Everyone's business.* Report of the Numeracy Education Strategy Development Conference. Adelaide: AAMT.

Bennett, S., Harper, B., & Hedberg, J. (2002). Designing real life cases to support authentic activities. Australian Journal of Educational Technology, 18(1), 1–12.

Clancy, S., & Lowrie, T. (2003). Maps that come alive: Numeracy engagement across multimodal texts. In L. Bragg, C. Campbell, G. Herbert & J. Mousley (Eds.), *Mathematics education research: Innovation, networking, opportunity* (pp. 215–221). Referred Proceedings of the Annual Mathematics Education Research Group of Australasia Conference, Geelong.

De Corte, E., Verschaffel, L., & Greer, B. (2000). Connecting mathematics problem solving to the real world. *Proceedings of the International Conference on Mathematics Education into the 21st Century: Mathematics for Living* (pp. 66–73). Amman, Jordan: The National Center for Human Resource Development.

de Silva, R., Masingila, J., Sellmeyer, N., & King, K. J. (April, 1997). Exploring multiple mathematical concepts within a single context. Paper presented at the *RAC/SIG-RME Research Presession of the Annual Meeting of the National Council of Teachers of Mathematics*, Minneapolis, Minnesota.

Gee, J. P. (2002). New times and new literacies: Themes for a changing world. In B. Cope & M. Kalantzis (Eds.), Learning for the future: Proceedings of the learning conference (pp. 3–20). Spetses, Greece: Common Ground Publishers.

Gee, J. P. (2003). *What video games have to teach us about literacy and learning.* New York: Plagave MacMillan.

Geiger, V. (1998). Students' perspectives on using computers and graphing calculators during mathematical collaborative processes. In C. Kanes, M. Goos & E. Warren (Eds.), *Teaching mathematics in new times. Proceeedings of the 26th annual conference of the Mathematics Educaiton Research Group of Australasia.* (Vol. 1, pp. 217–224). Gold Coast: MERGA.

Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2000). Perspectives on technology mediated learning in secondary school mathematics classrooms. *Mathematics Education Research Journal, 12*(3), 303–320

Groves, S. (1995). The impact of calculator use on young children's development of number concepts. In R. P. Hunting, G. E. Fitzsimons, P. C. Clarkson & A. J. Bishop (Eds.), *Regional collaboration in mathematics education* (pp. 301–310). Melbourne: Monash University.

Howe, N., & Strauss, W. (2000). *Millennials rising: The next great generation.* New York: Vintage Books.

Lankshear, C., Synder, I., & Green, B. (2001). *Teachers and technoliteracy: Managing literacy, technology and learning in schools.* St Leonards: Allen & Unwin.

Lehrer, R., & Pritchard, C. (2002). Symbolizing space into being. In K. Gravemeijer, R. Lehrer, B. van Oers & L. Verschaffel (Eds.), *Symbolizing, modeling and tool use in mathematics education* (pp. 59–86). The Netherlands: Kluwer Academic Publications.

Lowrie, T. (2002a). The influence of visual and spatial reasoning in interpreting simulated 3D worlds. *International Journal of Computers in Mathematical Learning, 7*(3), 301–318.

Lowrie, T. (2002b). Young children posing problems: The influence of teacher intervention on the type of problems children pose. *Mathematics Education Research Journal, 14*(2), 87–98.

Lowrie, T. & Clancy, S. (2003). Narrative constructions from multimodal texts. *International Journal of Learning, 10* (article 93).

Lowrie, T. (2004). Problem solving in out-of-school settings: Children "playing" in ICT contexts. In J. Cia. & J. Mamona Downs (Eds.), *Problem solving in mathematics education.* Refereed Proceedings of the Problem Solving Topic Study Group (TSG, 18) of the International Congress of Mathematics Education, Copenhagen, Denmark. Available online from http://www.icme-10.dk/.

Mackay, H. (1997). *Generations: Baby boomers, their parents and their children.* Sydney: Macmillan.

Noss, R. (1987). Children's learning of geometrical concepts through LOGO. *Journal for Research in Mathematics Education, 18*(5), 343–362.

Stake, E. A. (2000). *Case studies.* In N. K. Denzin & Y. S. Lincoln (Eds.), *Handbook of qualitative research* (2nd ed., pp. 435–454). Thousand Oaks, CA: Sage.

Steen, L. A. (1999). *Numeracy: The new literacy for a data-drenched society.* Educational Leadership, October, 8–13.

Van Rijswik, R., & Hoey, G. (1999). Using Microsoft Excel and Powerpoint in your mathematics program. *Prime Number, 14*(1), 6–9.

Vincent, J. (2003). Year 8 students' reasoning in a Cabri environment. In L. Bragg, C. Campbell, G. Herbert & J. Mousley (Eds.), *Mathematics education research: Innovation, networking and opportunity. Proceeedings of the 26th Annual Conference of the Mathematics Educaiton Research Group of Australasia.* (Vol. 2, pp. 696–705). Geelong: MERGA.

Zemke, R. (2001). Here come the millennials. *Training, 38*(7), 44–49

Zevenbergen, R. (2004). Technologising numeracy: Intergenerational differences in working mathematically in new times. *Educational Studies in Mathematics, 56*(1), 97–117.

BETTER BUY

Tamsin Meaney
University of Otago
<tamsin.meaney@stonebow.otago.ac.nz>

This paper reports on the oral responses given by primary school students to a mathematical assessment task, Better Buy. It is from a larger study which documented the typical language that Year 4 and Year 8 students used when giving mathematical explanations and justifications to four tasks. The responses to this task were particularly interesting as students were surprisingly consistent in the structure of their explanations, when providing an accurate response. However, students depending on their age, gender and socio-economic background gave different sorts of responses. It seemed that Year 8 boys from high decile schools were most likely to give explicit, extended responses which were accurate whilst Year 4 girls at low decile schools were most likely to give simple and inaccurate responses. The consistency of responses plus the distinct differences between groups of students was not as great in any of the other tasks which were studied. This paper reports on the text structures used in responses to Better Buy and investigates the features of the task itself which may have contributed to the consistency in text structures and the distinctions between groups.

INTRODUCTION

With the recent introduction of the National Certificate of Educational Achievement into New Zealand, there has been interest in the use of explanations and justifications within mathematics assessments (Meaney, 2002). Yet Bicknell (1999) showed in earlier research that there was little understanding of what constitutes good explanations and justifications, with both teachers and students suggesting that there was a need for help in writing these. It is also known that some groups of students 'tend to generate misfitting response patterns in large-scale tests' (Lamprianou & Boyle, 2004, p. 239). One of the suggested reasons given for these response patterns is 'language deficiencies' (Lamprianou & Boyle, 2004, p. 240). Certainly, sociolinguistic research has suggested that students with different demographic characteristics such as age, gender, socio-economic background will use language in different ways (Meaney, 2005). It is also known that teachers make judgements about students' ability, based on how they speak (Haig & Oliver, 2003). As a result, our original intention in undertaking this research was to document the language that primary school children used in providing mathematical explanations and justifications (Meaney & Irwin, 2005; Meaney, 2005). We anticipated that this would enable us to see whether different language use related to accuracy of responses.

However, what we found was that there was significant variety in the ways that students structured their responses to different assessment tasks (Meaney & Irwin, 2005). Although some of this variety was related to the accuracy of responses, this was not the only determiner. Using the work of Krummheuer (1995), Yackel (2001) and Forman, Larreamendy-Joerns, Stein and Browns (1998) found that in jointly constructing arguments, students working in groups and with the teacher used combination of components such as claims, grounds, warrants and backings.

Although it was possible to recognise the linguistic embodiment of these in some students' responses, it was not always necessary to use all these features in order to provide a clear and accurate response (Meaney & Irwin, 2005). This finding supported Ellerton and Clarkson's (1996) conjecture that there was no simple relationship between mathematics and language. However, it was apparent that students in deciding how to respond linguistically were influenced by the requirements of the task itself.

The responses to the Better Buy task produced the most consistent set of text structures, with a clear relationship between text structure and accuracy of the response. It also seemed that the gender, age and socioeconomic background of students affected the text structures used and the likelihood of the responses being considered accurate. This paper first describes the task, before providing information on the distribution of students who gave accurate responses and the text structures they used. Finally, it discusses how the task requirements may have affected students' perceptions of the appropriateness of linguistic choices.

THE ASSESSMENT TASK

Each year the National Educational Monitoring Project (NEMP) assesses around 3000 randomly selected New Zealand primary school children in a range of different subject areas, half in Year 4 and half in Year 8. Each subject in the curriculum is assessed every four years, with mathematics being assessed in 1997 and 2001 (Flockton & Crooks, 1997; Crooks & Flockton, 2001). Assessments are done in a number of different formats including a videoed interview between an individual student and a specially trained teacher administrator. The Better Buy task came from the 1997 administration and asked students to indicate which of two boxes of Pebbles was better value for money.

Of the 72 students whose responses were analysed, half were in Year 4 and the other half in Year 8. Half were boys and half were girls. One third of students came from high decile, one third from middle decile and the remaining third from low decile schools. In New Zealand, it is generally accepted that the decile level is related to socioeconomic background. This is because a school's decile level is determined by the Ministry of Education based on a series of factors such as parental occupation and ethnicity of the school population (Bicknell, 1999; Flockton & Crooks, 1997).

Place the 100g and 50 g boxes of *Pebbles* in front of the student.

In this activity you will be using some boxes of *Pebbles.* The big box holds 100 grams of *Pebbles* and costs $1.30. The smaller box holds 50 grams of *Pebbles* and costs 60 cents.

1. Which one is better value for money?
   Prompt: Which box would give you more Pebbles for the money?
2. Why is that box better value for money?
3. How do you know that?

Figure 1. Instructions for the Better Buy question.

The task requirements are given in Figure 1. The instruction to the teacher administrator is given in bold whilst the instructions that they read to the students are in normal font.

ACCURACY OF STUDENTS' RESPONSES

Although in responding to other tasks, some students opted not to give a verbal response, for this task all students chose one of the boxes as their answer and then gave some information when prompted for an explanation. Table 1 shows the distribution of students on their accuracy of responses.

Table 1
Use of text structures by different groups

Text Structures	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Medium	High	
Accurate	13	20	4	29	6	12	15	33
Discussed price and mass but no comparison	7	4	9	2	6	3	2	11
Discussed only one aspect	16	12	23	5	12	9	7	28

Although it could be expected that year level would affect the likelihood of a student giving an accurate response, gender and decile level of school attended are also significant indicators. For gender, $\chi^2 = 14.166$, df = 2, $p < 0.001$. For year level, $\chi^2 = 52.66$, df = 2, $p < 0.0005$ and for decile level of school attended, $\chi^2 = 19$, df = 4, $p < 0.001$. These groups could be considered, therefore, as providing misfitting response patterns such as those described by Lamprianou and Boyle (2004). It would seem that boys were more likely to give an accurate answer and students attending low decile schools were more likely to give an inaccurate answer. As a result, it was worth considering how different groups constructed their responses.

THE TEXT STRUCTURES OF STUDENTS' RESPONSES

Text structure is the combination of linguistic components which are used regularly in a particular situation or 'contextual configuration'. Hasan (Halliday & Hasan, 1985) stated that the contextual configuration 'can predict the OBLIGATORY and the OPTIONAL elements of a text's structure as well as their SEQUENCE *vis-á-vis* each other and the possibility of their ITERATION' (p. 56, capitals and italics in original). However, as Hasan pointed out, the relationship between language and situation is bi-directional with some elements of the text structure helping to construct the situation. For example, when a student provided a minimal response and the teacher administrator kept prompting, sometimes this prompting became more about teaching the student than about assessing their current understanding.

If these further questions become an obligatory element of the situation, then the situation changed from one of assessment to one of teaching.

It was clear from the data that every student's explanation contained one or more or the following three features. These were *Premise*, *Consequence* and *Conclusion*. A premise is a statement of ideas upon which a student's reasoning is built. The students in responding to this task used two types of Premises. One was the repetition of a fact that was given in the question, such as '[i]t's fifty grams and that's 100 grams'. These were labelled as factual Premises. The other Premise was when a hypothetical situation was mooted such as '[i]f I buy two of them …'. Descriptions of the ideas built on these Premises were labelled as Consequences. For example:

Because if you buy two of these boxes, it's going to equal a hundred grams and only cost a dollar twenty.

Premise Consequence implicit Conclusion

The final feature of these explanations was a Conclusion. This is where the student referred to better value. Only nine students used an explicit Conclusion in their response. However, 25 other students used words such as 'more', 'only', 'but' to cue the listener to the fact that a comparison had been made. These were labelled as implicit Conclusions. Given that these were oral explanations where the context was shared between the student and the teacher administrator, it is to be expected that the listener would supply some unspoken background information (Halliday, 1985). It, perhaps, is more surprising that some students chose to be so explicit. If the Conclusion came before the Premise (and the Consequence), then the student was most likely pre-empting the question asking for their reasoning when they responded to the question about which box was better value.

The 72 students used one of ten different combinations in giving their reasoning. Table 2 provides examples of each of these combinations and the number of students who used the different types of Premises in these combinations. In the examples, Qs stand for a question or prompt from the teacher administrator.

When responding to the 'why is that box better value for money?', every student provided a Premise and this would be the obligatory feature. Optional elements were Consequences and Conclusions as not every student included these elements. In these responses, Premises always came before Consequences but they were also found after or before Conclusions. Consequences do not occur in students' explanations unless preceded by a Premise. As can be seen in Table 2, Consequences were more likely to occur after a hypothetical Premise. In regard to iteration, Premises and Consequences occurred repeatedly within a student's explanation so there may have been Premise – Premise – Consequence – Premise – Consequence, or Premise – Premise, or Premise – Consequence – Premise – Consequence. Conclusions, whether explicit or implicit, only occurred once in any child's explanation except for two students who both began and ended their explanations with a Conclusion.

Table 2
Text structures

Text structures	Examples	No. of students using hypothetical premise	No. of students using factual premise
premise (1, 2, …) – consequence (1, 2, …) – conclusion	Because if you, um, if you put two of them together it will only cost, um, if you buy two of these it will cost a dollar twenty, um, and fifty times two is a hundred, and that one's, ah, ten cents more, than if you buy two of these.	5	3
premise (1, 2, …) – consequence (1, 2, …) – implicit conclusion	Because if you do fifty, if you do it's sixty cents so then you do two times sixty, and it equals one twenty and that's one thirty, and it should be one twenty.	10	5
premise (1, 2, …) – conclusion	Because, that there try, that's half the size of this one, and they charge ten cents more.	0	2
premise (1, 2, …) – implicit conclusion	Well, there's lots of pebbles in it and it costs only sixty cents.	1	8
conclusion – premise (1, 2, …) - consequence – implicit conclusion	Umm, this one's better value because if you bought two of these you'd have a hundred grams and it would only costs a dollar twenty.	3	0
implicit conclusion – premise (1, 2, …) – implicit conclusion	Just buy two of those. Q Because those are sixty, and that's a hundred, and you get ten cents off.	0	1
conclusion – premise (1, 2, …)	Probably this one here because you don't have to pay as much. Q But this one here would be the best to buy cos it has the most.	0	3
implicit conclusion – premise (1, 2, …)	Because it's only sixty cents and that's one dollar thirty.	1	2
premise (1, 2 ,…) – consequence (1, 2, ...)	Because, when you add it, sixty and sixty together which equals that it's a dollar twenty.	4	7
premise (1, 2 ,…)	Because it's a fifty gram and not a hundred gram, oh, a hundred gram and the hundred gram is a dollar thirty and the fifty gram is only sixty cents.	0	17

Bills (2002) in research on the linguistic features that students used in responding to questions about how they did some mental calculations found that use of personal pronouns ('you' and 'I'), present tense and logical connectives such as 'because', 'so' and 'if' related to accuracy. In responses to the Better Buy task, students usually used 'you' or 'I' in a hypothetical Premise. In all cases, 'you' could have been replaced by the more formal 'one' as it was not used to refer to the teacher administrator but to a generalised person. Rowland (1995) commented on a similar use of 'you' in his research and suggested that it pointed to an expression of a generalisation. In responding to this task, the students seemed to use it more to provide a description of the conditions under which the comparison of the two boxes would be true. 'If you got two of those it will be the same as that but it would be ten cents less' enables the cost and mass of both boxes to be made equivalent, thus allowing a comparison of cost, which is a necessary for determining which box is better value. This suggests that 'you' was used in a very specific part, the Premise. If it is not used in the Premise, it very rarely appeared in other elements of the text structure. However, if it was used in the Premise, it was also likely to be continued to be used in the other elements found in that response.

As can be seen in Table 3, there are some clear differences in which groups use which text structures. On the whole, boys in Year 8 from high decile schools were most likely to use a Premise – Consequence – Conclusion structure. On the other hand, Year 4 girls were most likely to just provide Premises or a Premise – Consequence combination. These text combinations were also more likely to be used by students attending low- decile schools. Boys were much more likely to use explicit Conclusions in their explanations than girls (13:4) but an equivalent number of boys and girls used implicit Conclusions (12:15). Year 8 students were much more likely to include a Conclusion (implicit or explicit) in their text structures than Year 4 students. However, if Year 4 students did use a Conclusion, it was more likely to be implicit than explicit. This suggests that as students get older they are more inclined to complete an explanation with a rounding off statement which links directly back to the original question. However, it would seem that decile level of school attended and gender affected a student's likelihood of giving a Conclusion.

Table 3
Use of text structures by different groups

Text Structures	Gender		Year Level		School Decile Level			Total
	Girls	Boys	Year 4	Year 8	Low	Medium	High	
premise – consequence – conclusion	1	7	2	6	0	3	5	8
premise – consequence – implicit conclusion	10	5	2	13	3	7	5	15
premise – conclusion	1	1	0	2	2	0	0	2
premise – implicit conclusion	6	3	7	2	2	4	3	9
conclusion – premise - consequence – conclusion	1	2	1	2	0	1	2	3
implicit conclusion – premise – implicit conclusion	0	1	0	1	0	1	0	1
conclusion – premise	0	3	2	1	0	1	2	3
implicit conclusion – premise	0	3	1	2	1	1	1	3
premise – consequence	6	5	5	6	6	4	1	11
premise	11	6	16	1	10	2	5	17

TEXT STRUCTURES AND ACCURACY OF RESPONSE

Table 4 sets out the most common text structures and how they related to accuracy of response. It is clear that if a student gave an accurate response they were most likely to be using a Premise – Consequence – Conclusion (implicit or explicit) combination. If they gave an inaccurate response, they were more likely to just give a Premise.

Table 4

Text structure and accuracy of response

Text structures	Premise	Premise – Consequence	Premise – Consequence – Conclusion/ implicit Conclusion	Premise – Conclusion/ implicit Conclusion	Other
Accurate	0	4	18	4	7
Discussed price and mass but no comparison	2	3	3	3	0
Discussed only one aspect	15	4	1	6	2

Would students who knew about an expected text structure be able to use it to their advantage in helping them solve the problem? Bills (2002) suggested that as students could use linguistic features in non-mathematical explanations, their use or non-use in mathematical explanations reflected their thinking. Our results certainly suggest that on the whole Year 4 students were unable to determine a successful strategy to solve this problem and providing them with a text structure for their answers may not be useful. However, some Year 8 students by knowing about other text structures may be able to use them to help them solve the task appropriately. For example, students, who used Premises in inaccurate responses, may be supported to give clearer, more accurate responses by describing a hypothetical situation with a generic 'you' as the doer of the action. Although some students mostly those from high decile schools already do this, it may need to be taught explicitly to other students who are not familiar with such a text structure. Further research is needed to see whether such an intervention is beneficial.

As well as differences in the linguistic choices made by students from different socio-economic backgrounds, the responses to the Better Buy task suggested that gender affected the explicitness of the Conclusion given. What made boys chose to be more explicit than girls? This was a task in which there was a lot of shared experience between the teacher administrator and the student, yet these boys chose to be very explicit in their reasoning. We can consider the text structures that were used, as being on a continuum from requiring little of the listener to requiring them to bring a large amount of background knowledge to what they were hearing. Students who only gave a Premise in their response require the listener to back-fill in most of the necessary information in order for the reasoning to be considered acceptable. As has been suggested elsewhere (Meaney, 2002), students' perceptions of who their audience is will have an impact on the information they provide in their responses. It may well be that the boys who provided very explicit responses were aware that an assessment situation requires them to presume that the listener has no prior knowledge. It may be that making girls aware of the needs of assessors could support them to provide necessary detail, especially when they move into high school and write their responses in formal examination situations.

The responses to this task were interesting in that there appeared to be more consistency in who gave accurate responses and the text structures used with these responses than to other tasks. It would certainly seem that the use of particular text structures was closely related to whether the responses were accurate or not. It may be that the language used by certain groups results in these groups being perceived as giving misfitting response patterns. However, given that the text structures used in accurate and inaccurate responses in other tasks were more varied, the task itself may also have influenced some students' language choices.

TASK REQUIREMENTS

For students to determine an appropriate response, they needed to understand that better value required them to work out an equivalent mass so that they could compare prices. Consequently, there was only one way to achieve a correct solution. In other tasks, we investigated this was not the case. Often there might be more than one way to arrive at a solution or more than one solution which could be considered correct. If students did not know about better value and many students did not appear to, as they gave, for example, shopping stories for their responses, they were unlikely to be funnelled into considering how to compare mass and price. Students at high decile schools and in Year 8 more often realised that they needed to compare price and mass. Once this realisation had been made, then a text structure of Premise – Consequence – Conclusion becomes the most appropriate for effectively giving information on how this comparison could be achieved.

The results initially suggested that students' language choices related to their ability to provide accurate responses. However, the essential feature which influenced both accuracy and text structure was knowledge of a better buy. If they knew this, students were able to choose to use the most efficient text structure for their solution. It is, therefore, essential that consideration of language differences are done in tandem with consideration of context.

References

Bicknell, B. (1999). The writing of explanations and justifications in mathematics: Differences and dilemmas. Making the difference: *Proceedings of the 22nd Annual Conference of the Mathematics Education Research Group of Australasia Incorporated* (pp. 75–83). Adelaide: MERGA.

Crooks, T., & Flockton, L. (2001). *Mathematics: Assessment results* Dunedin: Educational Assessment Research Unit.

Ellerton, N., & Clarkson, P. (1996). Language factors in mathematics teaching and learning. In A. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick & C. Laborde (Eds.), *International Handbook of Mathematics Education* (pp. 987–1023) Dordrecht: Kluwer Academic Publishers.

Flockton, L., & Crooks, T. (1997). *Mathematics: Assessment results* Dunedin: Educational Assessment Research Unit.

Forman, E., Larreamendy-Joerns, J., Stein, M. K., & Browns, C. A. (1998). 'You're going to want to find out which and prove it': Collective argumentation in a mathematics classroom. *Learning and Instruction, 8*(6), 527–548.

Haig, Y., & Oliver, R. (2003). Language variation and education: Teachers' perceptions. *Language and Education, 17*(4), 266–280.

Halliday, M. A. K. (1985). *Spoken and written language*. Geelong: Deakin University Press.

Halliday, M., & Hasan, R. (1985). *Language, context and text: Aspects of language in a social-semiotic perspective*. Geelong: Deakin University Press.

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *The emergence of mathematical meaning: Interaction in classroom cultures* (pp. 229–270) Hillsdale, NJ: Lawrence Erlbaum.

Meaney, T. (2002). Aspects of written performance in mathematics learning. In K. Irwin, B. Barton, M. Pfannkuch & M. Thomas (Eds.) *Mathematics education in the South Pacific* (Proceeding of the 25th Annual Conference of the Mathematics Education Research Group of Australasia). Auckland: MERGA.

Meaney, T. (2005). Ways and means to understand the linguistic choices of children's mathematical explanations. In *Turning the kaleidoscope: Proceedings of the New Zealand Association of Research in Education Conference*. Conference held 24–26 November, 2004, Westpac Stadium, Wellington.

Meaney, T. (2005, forthcoming). Mathematics as text. In A. Chronaki & I. M. Christiansen (Eds), *Challenging ways of viewing maths classroom communication*. Westport, CT: Information Age.

Meaney, T., & Irwin, K. C. (2005) *Language used by students in mathematics for quantitative and numerical comparisons.* NEMP Probe Study Report. Dunedin: EARU, University of Otago.

Yackel, E. (2001) Explanation, justification and argumentation in mathematics classrooms. In A Cockburn & E. Nardi (Eds.), *Proceedings of the 26th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 9–25). Norwich: PME.

MAKING SENSE OF CURRICULUM INNOVATION: A CASE STUDY

Candia Morgan
Institute of Education, University of London, United Kingdom
<c.morgan@ioe.ac.uk>

The rhetoric greeting the first few years of the introduction of the National Strategy into lower secondary schools in England has been optimistic and some aspects of the curriculum development have been widely welcomed by mathematics teachers and implemented in classrooms. The nature of that implementation, however, is observed to be varied. As a first step towards understanding this variation, I present an analysis of the official discourse of the Strategy characterising the ways in which it constructs mathematics teaching. In concluding, I discuss how this discourse may be understood by teachers drawing on various alternative discourses.

INTRODUCTION

What is teaching? In much curriculum discourse, teaching and learning are presented as two sides of the same coin: good or effective teaching is that which leads to good learning. Curricula tend to be defined in terms of what pupils should learn or what they should be taught – these two being presented as essentially the same – or in some cases the kinds of experiences they should have. For example, the National Curriculum for England and Wales contains "Programmes of Study" consisting of lists of what "Pupils should be taught" to do, while the "Attainment Targets" against which achievement is measured consist of parallel lists expressed in terms of what pupils themselves have learnt to do (DfE, 1995). *How* to teach has generally been considered a matter of professional judgement, to be debated among teachers and others with a professional concern with education but not to be explicitly prescribed. In England[i], however, teaching is now regulated not only by specification of content and assessment regimes but also by increasingly detailed descriptions of teaching methods. In this paper, I begin to examine the ways in which the nature of teaching is constructed within this official discourse and to consider how teachers may make sense of this and incorporate it into their practice.

THE CONTEXT OF CURRICULUM INNOVATION

Since the late 1980s, the United Kingdom curriculum in general, and the curriculum for state maintained schools in England in particular, have been subject to increasing degrees of state regulation through specification of what must be taught, imposition of mandatory testing regimes with high stakes for pupils, teachers and schools, regular and frequent inspection of individual teachers and schools and, more recently, the introduction of performance management schemes to control teachers' careers. Until relatively recently, however, what actually happened in individual classrooms was regulated only indirectly, especially through the design of assessment instruments intended to encourage teachers to adopt specific teaching practices in order to prepare their pupils for the tests and examinations they would have to take. The introduction of the statutory National Curriculum in 1988 was accompanied by Non-Statutory Guidance about approaches to teaching but this faded quietly into the background as teachers and schools concentrated on ensuring that they fulfilled their statutory obligations and achieved the highest possible results in national tests.

While schools are still legally bound only by the specification of the content they must teach, increasing pressure has been exerted on both primary and secondary schools and on teachers in England to organise the 'delivery' of the curriculum and to teach in officially approved ways. What started as a curriculum development project to improve standards of literacy and numeracy at primary level, especially in schools considered to be under-achieving, has expanded to become a 'National Strategy' addressing an ever widening range of aspects of schooling from discussion of mental calculation strategies, through approaches to formative assessment, to behaviour management.[ii] For mathematics teachers in secondary schools, the *National Framework for Teaching Mathematics*, a key document of the National Strategy, describes and exemplifies an approach to organising and teaching the content of the National Curriculum for pupils in Years 7, 8 and 9 (aged 11–14). Though schools are not required to use the *Framework*, they "are expected to … be able to justify not doing so by reference to what they are doing" (DfES, 2001, p. 2). Individual teachers are not required to teach in the ways described but are obviously subject to pressure from managers within their schools to comply. Moreover, support materials, resources and training, whether provided by government agencies or independent sources, including teachers' professional associations, increasingly assume compliance with the model of teaching presented in the *Framework*.

UNITY AND DIVERSITY

Evaluations of the implementation of the National Strategy have indicated that it has had some influence in the vast majority of classrooms in primary and secondary schools in England (Earl et al., 2003; Ofsted, 2003). However, they also report that the nature of the changes implemented has not always matched the intentions of the strategy (Ofsted, 2002, 2003; Stoll et al., 2003) and that many teachers have been "'tweaking' rather than radically changing practice" (Stoll et al., 2003, p. 1)—a finding supported by studies of the practices of primary teachers that have identified qualitative differences in the nature of activities implemented (Askew et al., 2000) and persistence of "traditional" forms of classroom interaction (Hardman et al., 2002).

Part of my job as a teacher educator involves visiting a variety of schools in the Greater London area, observing trainee mathematics teachers in the classroom and talking about teaching with them and with the teachers who mentor them. With the introduction of the National Strategy, my own work with trainees has been shaped by its discourse and regulated by a curriculum for Initial Teacher Training that demands that new teachers should demonstrate that they "use the relevant frameworks, methods and expectations set out in the National Strategy" (DfES, 2002, p. 12). Thus, however critical our discussions may be, we certainly make use of the *Framework* and part of our joint work is to find ways of demonstrating that we are doing so. A feature of this experience that has struck me as both interesting and puzzling has been the extent to which the *Framework* has been accepted as a guide to practice by mathematics departments and teachers. While some teachers may object to the slight to their professionalism offered by official recommendations about teaching methods, few challenge the validity of the methods themselves and those who claim not to pay much attention to the document often justify themselves by saying "I'm doing that already", thus claiming compliance by default. Organisational aspects of the National Strategy, including planning and record keeping formats and the idea that a lesson should have a three-part structure, have been adopted in broadly similar ways that show direct relationship to the official guidance. Yet the teaching practices observed in schools appear as diverse as they did before the introduction of the National Strategy. Even where teachers are explicitly implementing what is referred to in the *Framework* as the "oral and mental starter" component of a lesson, there is considerable variation in the objectives, the type of activity and the extent and form of interaction between teacher and pupils. This variation persists, even into what is now the fourth full year of implementation, suggesting that the recommendations have been understood and implemented in substantially different ways by different teachers and schools.

Explanations of problems in implementation of curriculum development tend to focus on teachers' resistance to or transformation of new curricula (e.g., Fullan & Hargreaves, 1992), while evaluations of the National Strategy have identified "teacher capacity" as a concern (Earl et al., 2003; Ofsted, 2003). Such identification of teacher deficit as a major barrier to successful development focuses attention on intervention at the level of training and support structures for teachers but fails to take into account other factors that may affect the success of curriculum development, including those related to the form of the curriculum development itself. The sets of concepts and values expressed in the dissemination of a curriculum development may not constitute straightforward guidance for practice. For example, Brown et al. (2000) identify ambiguities in the discourse of the primary National Strategy, allowing alternative interpretations of recommended pedagogy. Similarly, Jones and Tanner (2002) report that secondary mathematics teachers involved in a development and research project, implementing "whole class interactive teaching" as recommended by the National Strategy, differed in their practices, in spite of training, support and apparent consensus and commitment to the overall values of the programme.

My starting question is, therefore, how can it be that such widespread consensus about the legitimacy of the teaching methods recommended by the National Strategy and claimed compliance with its recommendations can coexist with continued diversity in classroom practices?

A DISCOURSE ANALYTIC APPROACH

In a previous study of teachers implementing curriculum development in high stakes assessment (Morgan, 1998), I identified different practices and different ways in which teachers were positioned in relation to their task of assessing pupils' texts. A systematisation of these findings by Morgan, Tsatsaroni and Lerman (2002), drawing on Bernstein's theory of pedagogic discourse (Bernstein, 1996), related the positions to a variety of educational and everyday discourses on which teachers were able to draw as they recontextualised the official discourse of the curriculum. Moreover, the official discourse itself could be seen to be a recontextualisation of other discourses drawn from various fields, resulting in tensions among the various concepts and values of the curriculum development. In order to understand the ways in which teachers may be making sense of and implementing the National Strategy, I have chosen to start by analysing the official discourse of the curriculum development itself, aiming in particular to identify the nature of teaching and the concepts and values associated with teaching as they are presented within the texts available to teachers.

An initial problem in attempting to study the discourse of a curriculum innovation such as the National Strategy is the wealth of sources available, including: documents produced both by government agencies and by commercial publishers; videos; internet based resources; training sessions. In order to make the initial task manageable I have chosen to focus on limited selections from one key document, the *National Framework for Teaching Mathematics* (DfES, 2001). This is probably the most widely distributed and well known of the official materials, provided free of charge to all serving and trainee secondary mathematics teachers. It contains detailed guidance for mathematics teachers as well as definitive lists of learning objectives for pupils in each year of the lower secondary school and extensive exemplification of how these might be interpreted in practice. While the guidance contained in this document has been supplemented and revised by more recent publications, the *Framework* itself is still regarded as core to the National Strategy.

The "Guide to the Framework" is divided into sections:

1. Introduction, which outlines the policy level context of the document;
2. Mathematics at Key Stage 3, which has sub-sections related to each of the components of the mathematics curriculum and to cross-curricular links and ICT;
3. Teaching strategies;
4. Inclusion and differentiation;
5. Assessment and target setting;
6. Planning.

Like many curricular documents, the *Framework* is addressed to multiple audiences, including school managers and heads of departments as well as teachers themselves. I have chosen to analyse section 3 *Teaching strategies* initially, as this most explicitly addresses teachers themselves about the nature of teaching.

The analytic approach is that of Critical Discourse Analysis (Chouliaraki & Fairclough, 1999; Fairclough, 1995), drawing on the grammatical tools of Halliday's functional systemic linguistics (Halliday, 1985) and interpreting the features thus identified in relation to the social context in which the texts are used and other discourses commonly available to mathematics teachers. As my interest is in how the nature of teaching and of teachers is constructed, I have focused primarily on those grammatical features that perform ideational (or experiential) functions, identifying in particular the ways in which teachers and teaching are involved in the text as actors and the processes in which they are presented as agents. In the analysis that follows, I shall also comment on some interpersonal aspects, which affect the ways in which its teacher-readers may be positioned in relation to the text.

TEACHING AND TEACHERS IN THE *FRAMEWORK*

The section entitled 'Teaching strategies' is marked by a distinct lack of any room for readers to question or debate its description of teaching. Thus the section starts by stating:

> The recommended approach to teaching is based on ensuring:
> - sufficient regular teaching time for mathematics, including extra support for pupils who need it to keep in step with the majority of their year group;
> - a high proportion of direct, interactive teaching;
> - engagement by all pupils in tasks and activities which, even when differentiated, relate to a common theme;
> - regular opportunities to develop oral, mental and visualisation skills.

The nominal phrase "recommended approach to teaching" succeeds in obscuring who is doing the recommending, thus preventing the teacher-reader from questioning their authority or the validity of their recommendation. The modality, as throughout the document, is high. In particular, the frequent use of unqualified intensive relational statements (of the form "A is b"), characteristic of scientific writing (Halliday, 1998), constructs the text as a straightforward description of the way things are.

The authoritative presentation is reinforced by addressing the teacher-reader with imperative instructions:

> **Aim** to spend a high proportion of each lesson in direct teaching, often of the whole class, but also of groups and of individuals. (*my emphasis*)

and by the use of modifiers that strengthen the value judgements implicit in the text:

> **High-quality** direct teaching is oral, interactive and lively, and **will not** be achieved by lecturing the class, or by **always** expecting pupils to teach themselves indirectly from books. (*my emphasis*)

The only type of teaching identified in this "recommended approach" or, indeed, named in this section of the document as a whole is "direct interactive teaching" (valued as good), defined by contrasting it with "lecturing" or expecting pupils to teach themselves (bad). While only a "high proportion" of each lesson is recommended to be occupied by this type of teaching, no mention is made of any other approved kind of teaching that might happen in any other part of the lesson. The value attached to "direct interactive teaching" is heightened by being modified by qualifiers "high-quality" and "good" at several points in the text – the possibility that there might be bad examples (or even indifferent ones) is never raised.

A list of "teaching strategies" is then presented:

> Good direct teaching is achieved by balancing different teaching strategies:
>
> - Directing and telling …
> - Demonstrating and modelling …
> - Explaining and illustrating …
> - Questioning and discussing …
> - Exploring and investigating …
> - Consolidating and embedding …
> - Reflecting and evaluating …
> - Summarising and reminding …[iii]

The lack of explicit agency expressed by the passive *is achieved* and the authoritative presentation of the list of nominalised strategies again make it hard to question, but also introduce some ambiguity about who is doing these things. This ambiguity is compounded by the mixture of types of processes included in the list. Although all are presented as aspects of teaching, some, especially those involving non-verbal processes, are glossed in ways that suggest the pupils are the actors. (Each point in the list is expanded by a further, more elaborated list, expressed in similar nominalised form.) For example, the point headed *Exploring and investigating* continues:

> asking pupils to pose problems or suggest a line of enquiry, to investigate whether particular cases can be generalised, to seek counterexamples or identify exceptional cases; encouraging them to consider alternative ways of representing problems and solutions, in algebraic, graphical or diagrammatic form, and to move from one form to another to gain a different perspective on the problem…

Leaving aside the question of whether the activities listed may be considered to constitute exploring and investigating, it is clear that the teacher's role in this is to *ask* and *encourage* pupils rather than to explore or investigate themselves. This pattern is apparent in considering the types of processes ascribed to teachers and to pupils throughout this section of the *Framework* (summarised in Table 1). The proportion of verbal processes in the table under-represents the emphasis on "telling" as a significant number of the material processes also involve giving information, for example, "using blackboard instruments to demonstrate a geometric construction, using a thermometer to model the use of negative numbers".

Table 1
Processes involving teachers and pupils: "Teaching Strategies", National Framework for Teaching Mathematics

	Teachers		Pupils	
Type of process	*n*	%	*n*	%
Behavioural	1	2	2	6
Material	27 (of which 8 are related to communication)	52 (15)	16 (of which 3 are related to communication)	46 (9)
Mental	3	6	10	29
Verbal	21	40	7	20
All	52	100	35	101*

* does not sum to 100 due to rounding

In spite of the statement that "High-quality direct teaching … will not be achieved by lecturing the class", a very high proportion of the processes in this list of its components involve talking by the teacher. Even *Reflecting and evaluating* involves "using [pupils' errors] as positive teaching points by talking about them" and "giving [pupils] oral feedback on their written work". The description of teaching thus comprises contradictory messages; the contradiction is even incorporated into its name—it is both *direct* and *interactive*. On the one hand, it is not lecturing. Lecturing is not itself defined but belongs to a discourse that contrasts the pair teaching-learning with the pair lecturing-listening. The lecturer (according to this contrast) delivers a lecture without interaction with the audience or concern for their understanding; the teacher ensures learning. On the other hand, teaching is "direct", involving telling, demonstrating, explaining, etcetera.

The recommended approach to teaching is thus constructed as:

- *unquestionable* – There is no author to debate with and the qualities of good teaching are presented as scientific facts, though without any reasoning that would allow space for disagreement or debate.
- *verbal and teacher-centred*
- *unitary* – Only one type of teaching is named. This is contrasted and opposed to "lecturing" (which is not teaching) and to pupils teaching themselves (which again is not teaching).
- *all encompassing* – The headings of the list of teaching strategies succeed in incorporating aspects such as exploring, investigating, discussing, consolidating, which might be thought to belong to a more pupil-centred philosophy associated, for example, with the recommendations of the Cockcroft Report (DES, 1982). On closer examination it may be seen that the terms are transformed here to accommodate them to a teacher-centred approach.

For the teacher who was previously most comfortable with a teacher-centred approach, the *Framework* offers few major challenges. Although other materials provided by the National Strategy suggest that a traditional "chalk-and-talk" approach may not be compatible with the intentions of at least some of the agents behind the innovation, it is certainly possible to read the description of teaching offered in this key document in ways that provide justification for the claim "I am doing it already". At the same time, for the teacher comfortable with pupil-centred approaches, aspects of compatible discourses are also present in the *Framework* text, allowing readings that involve valuing approaches to teaching and learning such as investigation, exploration and problem solving.

CONCLUSION

The anonymous authors of the Framework have managed to perform the difficult task of presenting a picture of mathematics teaching that is simultaneously:

- authoritative, consistent with the assumption of current government and media discourse that teachers need to be forced to change their practice and that debate is a feature of sixties libertarianism and the airy-fairy theorising of the 'educational establishment';
- new, satisfying the political demand for reform; and
- familiar enough that a majority of teachers is likely to be able to identify with it in some degree, ensuring that most teachers and schools will be positioned as compliant and that hence a success may be claimed for the National Strategy.

This analysis suggests a possible explanation for the widespread acceptance of the National Strategy and its diverse manifestation in classrooms. I have argued that teachers can position themselves as compliant by relating their practice to that described, interpreting the components of teaching specified in the *Framework* by drawing on other discourses of teaching. The question of whether and to what extent the implementation of the National Strategy has led to genuine changes in pedagogy remains open.

---

[i] The various countries making up the United Kingdom have varying degrees of autonomy in determining their education policies. The legislation for the National Curriculum for England and Wales was introduced before the devolution of substantial autonomous powers to Wales in 1997. Since that date, the central government has had direct control over education only in England. The National Strategy discussed in this paper thus applies only to schools in England.

[ii] The panoptic nature of the National Numeracy Strategy, the fore-runner of the National Strategy, was identified at an early stage by Tansy Hardy (Hardy, 2000). The extent of surveillance of both teachers and pupils is now even greater.

[iii] It is interesting to note how the *Framework*'s characterisation of recommended teaching contrasts with the corresponding section of the Cockcroft Report, previously the UK's best known and most influential recommendations about teaching approaches.

> 242 … We are aware that there are some teachers who would wish us to indicate a definitive style for the teaching of mathematics, but we do not believe that this is either desirable or

possible. Approaches to the teaching of a particular piece of mathematics need to be related to the topic itself and to the abilities and experience of both teachers and pupils. Because of differences of personality and circumstance, methods which may be extremely successful with one teacher and one group of pupils will not necessarily be suitable for use by another teacher or with another group of pupils. Nevertheless, we believe that there are certain elements which need to be present in successful mathematics teaching to pupils of all ages.

243 Mathematics teaching at all levels should include opportunities for

- exposition by the teacher;
- discussion between teacher and pupils and between pupils themselves;
- appropriate practical work;
- consolidation and practice of fundamental skills and routines;
- problem solving, including the application of mathematics to everyday situations;
- investigational work.

(DES, 1982, p.71).

Here, the modality in the first paragraph is low, presenting teaching methods as contingent on mathematics, teachers and pupils. The recommendations are presented as the beliefs of the authors rather than as scientific fact. The elements of teaching mentioned in the first paragraph of the extract is transformed in paragraph 243 into a list of "opportunities" to be included. It thus does not focus, as the *Framework* does, on the actions of teachers but more generally on what might happen in the classroom.

REFERENCES

Askew, M., Denvir, H., Rhodes, V., & Brown, M. (2000). Numeracy practices in primary schools: Towards a theoretical framework. In T. Rowland & C. Morgan (Eds.), *Research in mathematics education volume 2: Papers of the British Society for Research into Learning Mathematics* (pp. 63–76). London: British Society for Research into Learning Mathematics.

Bernstein, B. (1996). *Pedagogy, symbolic control and identity: Theory, research, critique*. London: Taylor & Francis.

Brown, M., Millett, A., Bibby, T., & Johnson, D. C. (2000). Turning our attention from the what to the how: The National Numeracy Strategy. *British Educational Research Journal, 26*(4), 457–472.

Chouliaraki, L., & Fairclough, N. (1999). *Discourse in late Modernity: Rethinking critical discourse analysis*. Edinburgh: Edinburgh University Press.

DES. (1982). *Mathematics Counts: Report of the committee of inquiry into the teaching of mathematics in schools under the chairmanship of Dr W. H. Cockcroft*. London: HMSO.

DfE. (1995). *Mathematics in the national curriculum*. London: HMSO.

DfES. (2001). *Key Stage 3 National Strategy – Framework for teaching mathematics: Years 7, 8 and 9*. London: Department for Education and Skills.

DfES. (2002). *Qualifying to teach: Professional standards for qualified teacher status and requirements for initial teacher training*. London: Teacher Training Agency.

Earl, L., Watson, N., Levin, B., Leithwood, K., Fullan, M., & Torrance, N. (2003). *Watching and Learning 3: Final report of the external evaluation of England's National Literacy and Numeracy Strategies*. Toronto: Ontario Institute for Studies in Education, University of Toronto.

Fairclough, N. (1995). *Critical discourse analysis: The critical study of language*. Harlow: Longman.
Fullan, M., & Hargreaves, A. (1992). Teacher development and educational change. In M. Fullan & A. Hargreaves (Eds.), *Teacher development and educational change*. London: Falmer Press.
Halliday, M. A. K. (1985). *An introduction to functional grammar*. London: Edward Arnold.
Halliday, M. A. K. (1998). Things and relations: Regrammaticising experience as technical knowledge. In J. R. Martin & R. Veel (Eds.), *Reading science: Critical and functional perspectives on discourse of science* (pp. 186–235). London: Routledge.
Hardman, F., Smith, F., Bramald, R., & Mroz, M. (2002). *Whole class teaching in the literacy and numeracy hours* (No. Final report ESRC grant no. R000 239213). Newcastle: University of Newcastle.
Hardy, T. (2000). Tracing Foucault's power in the mathematics classroom. In T. Rowland & C. Morgan (Eds.), *Research in mathematics education volume 2: Papers of the British Society for Research into Learning Mathematics* (pp. 207–224). London: British Society for Research into Learning Mathematics.
Jones, S., & Tanner, H. (2002). Teachers' interpretations of effective whole-class interactive teaching in secondary mathematics classrooms. *Educational Studies, 28*(3), 265–274.
Morgan, C. (1998). *Writing mathematically: The discourse of investigation*. London: Falmer.
Morgan, C., Tsatsaroni, A., & Lerman, S. (2002). Mathematics teachers' positions and practices in discourses of assessment. *British Journal of Sociology of Education, 23*(3), 445–461.
Ofsted. (2002). *The Key Stage 3 Strategy: Evaluation of the first year of the pilot*.
Ofsted. (2003). *The Key Stage 3 Strategy: Evaluation of the second year*: Office for Standards in Education.
Stoll, L., Stobart, G., Martin, S., Freeman, S., Freedman, E., Sammons, P., et al. (2003). *Preparing for change: Evaluation of the implementation of the Key Stage 3 Strategy pilot – Executive Summary*. Nottingham: Department for Education and Skills.

MATHEMATICS FOR TEACHING AND COMPETENCE PEDAGOGIES IN FORMALISED IN-SERVICE MATHEMATICS TEACHER EDUCATION IN SOUTH AFRICAN UNIVERSITIES

Diane Parker	Zain Davis	Jill Adler
University of KwaZulu-Natal	*University of Cape Town*	*University of the Witwatersrand*
<parkerdc@ukzn.ac.za>	<zd@humanities.uct.ac.za>	<adlerj@educ.wits.ac.za>

In this paper we examine two instances of the use of competence models in formalised in-service teacher education courses, finding that they prioritise the use of the visual as a central resource for the modelling of teaching mathematics and of the teaching and learning of mathematics. The way in which the visual is used in competence models produces an emphasis on the sensible that at the same time seems to disrupt the intelligible and so principled reproduction of mathematics teaching and of school mathematics. These instances of teacher education practice raise challenging questions about the selections from mathematics and teaching in mathematics teacher education.

INTRODUCTION

A central concern of the QUANTUM Research Project is that of answering the question: *what* is constituted as mathematics knowledge for teaching in formalised in-service teacher education in South Africa and *how* it so constituted? The discussion elaborated here is part of the attempt to answer that question. Previous and forthcoming work towards answering the question are reported on in Adler (2002), Adler and Davis (2003; Forthcoming), Adler et al. (Forthcoming), Long (2003). Embedded in the question is an understanding that, in practice, selections into mathematics teacher education are varyingly drawn from the domains of both mathematics and teaching. In this paper we present part of an emerging and challenging theme in our study, of complex hybrids of competence and performance models of curriculum and pedagogy in mathematics teacher education. We draw mainly on the work of Basil Bernstein who proposes that pedagogies and curricula might be broadly described in terms of two general models—competence and performance models—which he develops from his sociological analysis of the notion of *competence* (Bernstein, 1996). His analysis reveals a range of features which we would argue are hegemonic in curriculum and pedagogy reform discourses in general, and in post-apartheid education in South Africa.

Bernstein uses the term *social logic* to refer to "the implicit model of the social, the implicit model of communication, of interaction and of the subject which inheres in this concept" (1996, pp. 55–56). His analysis of the social logic of competence reveals key features that, briefly, include: an announcement of a universal democracy of acquisition; all are inherently competent with no deficits, only differences; the learner is active and creative in the construction of a valid world of meanings and practice; an emphasis on the learner as self-regulating with development or expansion not advanced by formal instruction; a critical, sceptical view of hierarchical relations, and a conception of teaching as facilitation, accommodation and context management. In contrast, again briefly, performance models emphasise 'absences', and so what the learner is to acquire and the outputs s/he is expected to produce.

An examination of official pedagogic discourse over the past decade and the Revised National Curriculum Statements (RNCS), the first of which appeared in 2002 (for Grades R to 9), shows a strong resonance with Bernstein's description of the social logic of *competence*. Since 1994 in South Africa the distance between official pedagogic discourse and the discourse circulating in higher education teacher training has diminished, suggesting a general convergence in the education arena towards the privileging of competence models.

In their analysis of curriculum and pedagogy in systemic school reform in post-apartheid South Africa, Taylor, Muller and Vinjevold (2003, pp. 4–5) argue that teacher education providers reveal a strong ideological commitment to competence models of pedagogy, and (with Bernstein) that the analytic distinction between performance and competence models does not necessarily mean these models are mutually exclusive in practice. They go on to propose a 'rapprochement' of features across the two models for effective practice. Our study of formalised in-service mathematics teacher education appears to confirm the non-exclusivity of these models, but suggests that there are varying hybrid forms. Interestingly, this hybridity was initially obscured by what we now consider to be a dominant *ethos* of competence. Teaching practices we are studying suggest the co-existence of interesting elements of both models, with varying apparent effects on learning.

The hedging above is a function of this still being work-in-progress, and also of the difficulty of further elaboration within the space constraints of this paper. We have chosen to focus here on selected instances of practice where competence models are clearly at work. The forms these take, and particularly how mathematics for teaching comes to be constituted, are challenging and troubling. They present provocative situations for critical reflection. We come to this through a focus on what Bernstein recognises as the central feature of competence models, that of the structuring of education along the lines of so-called *similar to* relations.

> In the case of competence models there is a focus on procedural commonalities shared within a group. In the cases we have analysed the group is children but procedural commonalities may well be shared with other categories, e.g. ethnic communities, social class groups. From this point of view competence models are predicated on fundamental 'similar to' relations (Bernstein, 1996, pp. 64–65).

In other words, the central organising principle of competence models emphasises the self-recognition of the pedagogic subject in others and in knowledge. Metaphorically, it is a principle encouraging an apparent mirroring back to the pedagogic subject of him/herself. Here we will discuss the apparent effects of competence models on the production of mathematics for teaching with special reference to two cases, taken from two different teacher education sites where teachers were enrolled in in-service upgrading programmes specialising in a fourth and final year of accredited mathematics teacher education.[i]

The question explored in this paper is, then: *what seem to be the effects of the deployment of competence models in teacher education on the production of mathematics knowledge for teaching?*

SOME METHODOLOGICAL COMMENT

After an initial review of programmes across South African universities we selected three sites of focus because of the continuum they offer with respect to the integration of mathematics and teaching (content and method) within courses. From across those three sites, the two cases that have been chosen for discussion here are from programs at either end of the continuum. The first case discussed is drawn from a program where courses integrate content and methods, and specifically from a course on the teaching of algebra at the level of grades 7 to 9. The second case discussed is drawn from a program that includes but separates post secondary level mathematics courses and mathematics education courses; and specifically from a (non grade specific) course on professional practice in the teaching of mathematics. In each of the two cases we discuss here, a teaching sequence from the particular course was selected for illustrative purposes. The teaching sequences have been chosen to illuminate a particular production of mathematics for teaching in the context of each course and its apparent competence model at work. Neither of these do justice to the courses in general, as there are elements in each where hybridity is at work. The scope of this paper does not allow for such a full and nuanced discussion. We are instead using instances that typify competence models at work and that provoke critical reflection on apparent effects of particular forms of mathematics teacher education practice.

For each of the Cases we will start with a general description in terms of Bernstein's work discussed earlier, followed by the production of analytic statements supported by illustrations from the selected teaching sequences. The unit of analysis is referred to as an *evaluative event*, that is, a teaching-learning sequence focused on the acquisition of some or other content. Each of the Cases discussed here refer to course lectures that were chunked into a succession of evaluative events over the period of a complete course. Following our discussion of each of the Cases we will then move on to a more general discussion of the implications of the use of competence models for the production of mathematics knowledge for teaching.

CASE 1: THE TEACHING OF ALGEBRA

In Case 1, the practice to be acquired is a particular pedagogy that is modelled by the lecturer who presents the activity as a specific practical accomplishment. This is clearly recognised in and across the course sessions. The lecturer also states on a number of occasions: "I am not teaching you content, that you must do on your own. … I am teaching you how to teach [algebra]". In other words, teachers on the course are to (re)learn how to teach Gr 7 – 9 algebra[ii]. A number of important consequences flow from this central feature of Case 1. First, the principles structuring the activity are to be tacitly acquired since the particular pedagogy is not an explicit object of study; the teachers, through their pedagogic experience are required to emulate the activity of the lecturer. In other words, at the level of immediacy, the privileged texts to be produced are oriented towards the (re)production of an iconic similarity. Second, because the principles of the activity remain tacit, those principles need to be recognised by the teachers in the form of something which stands in their place. That which stands in place of the principles can then be (a) an assemblage of pedagogic procedures and (b) localised in the form of the teaching/learning experiences of the teachers and experienced as instances of the activity to be acquired. Third, and what follows, is that the production of the meaning of the activity will privilege the sensible (in the strict sense of that term) over the discursive (or the intelligible). Fourth, while not a necessary consequence, the third does however predispose both the lecturer and teachers to an orientation towards mathematics which privileges the sensible. It is this feature of a competence model at work that we find provocative.

In order to reveal how a particular teaching/learning content progresses in each of the courses, we examine the appeals that are made to some or other ground in order to fix signification. In this particular case, we find the distribution of appeals shown in Table 1. Since the activity is that of teacher education, elements of teaching are always present, even if they are merely implicit. The distinction drawn between Mathematics and Teaching in Table 1 indicates what type of object was the explicit object of intended acquisition. So, in Case 1, we see that only four of thirty-six events explicitly appealed to teaching; three of those appeals were to the localised experiences of the teachers and one to the official curriculum. No appeals were made to the arena of mathematics education. This observation supports the point made earlier that the teaching of mathematics is presented as a practical accomplishment where its principles are to be tacitly acquired.

Table 1
Distribution of appeals in Case 1

	Mathematics	Mathematics Education	Metaphorical	Experience of either adept or neophyte	Curriculum	Authority of the adept
Mathematics	15	0	25	1	0	0
Proportion of appeals (*N*=41)	36,6%	0%	61%	2,4%	0%	0%
Teaching	0	0	0	3	1	0
Proportion of appeals (*N*= 4)	0%	0%	0%	75%	25%	0%
Mathematics & Teaching	15	0	25	4	1	0
Proportion of appeals (*N*=45)	33,3%	0%	55,6%	8,9%	2,2%	0%
Proportion of events (*N*=36)	41,7%	0%	69,4%	11,1%	2,8%	0%

We also note from Table 1 that the meaning of mathematics was strongly grounded in metaphor. This, interestingly, reflects Shulman's (1986) identification of appropriate metaphors as an important element of teachers' pedagogic knowledge. Here, for purposes of greater generality across Cases we have not disaggregated the metaphor types used. However, in Case 1 the lecturer frequently employed everyday and visual metaphors, sometimes combining them. For example, the distribution of tools and chicken feed are used to establish the meaning of the distribute law:

> Now the distributive law. What's that about? I've got all my tools packed up in the factory and then I distribute them. I take them out to where we are going to sell them. So your distributive law takes whatever is in front and multiplies them by what ever is inside the brackets. I don't know if any of you remember that Farmer Brown chicken advert. What was it? They look so good because they eat so good! Something like that. Now I want you to think of this fellow here as Farmer Brown, okay, and here he has got all his chickens. Now Farmer Brown is feeding each chicken in turn (draws arrows) – each term in the bracket he feeds. So if there are three terms in the brackets he feeds each chicken. Do you understand that? (Case 1 transcript)

Figure 1 shows what was written down as the distributive law was explained. Note the stick drawings of Farmer Brown and his chickens. Later, when discussing the product of binomials shown at the bottom of Figure 1, the metaphor was extended to include Farmer Brown's assistant, standing in place of the second term of a binomial, feeding the chickens their pudding. Figure 2 shows the use of a visual metaphor in which the areas of squares and rectangles are used to establish some sensibility for the distributive law. The appeals to Mathematics in Case 1 where the focus was on learning to teach some of the rules of algebra were, for the most part, of the form of using numbers to test and assert the validity of mathematical statements, or, of actually asserting a procedure or rule (as with the distributive law), which was then redescribed metaphorically.

A second focus in the course was on generalising number patterns and producing algebraic statements expressing relationships between sets of numbers. There are instances within this mathematical focus, where appeals are made to visual descriptions that are general (i.e. hold in all cases). More often, the production of mathematical statements was achieved through the use of the inductive treatment of regularities in sequences of numbers, accompanied by some or other visual support (like arrangements of matchsticks, for example). In these instances, it appears that mathematics is to be treated as an inductive practice, the statements of which are validated through empirical testing. Here, the intelligibility of mathematics is transmuted into a sensibility produced through metaphorical redescription and empirical testing of rules and procedures.

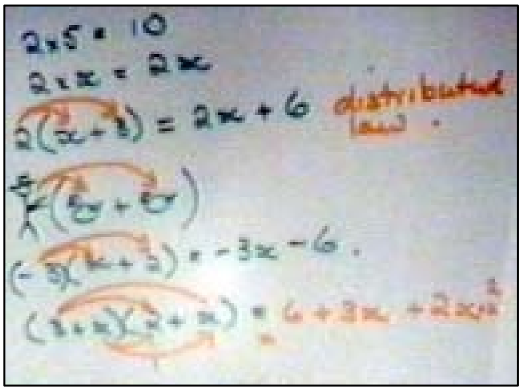


Figure 1. Farmer Brown & the distributive law

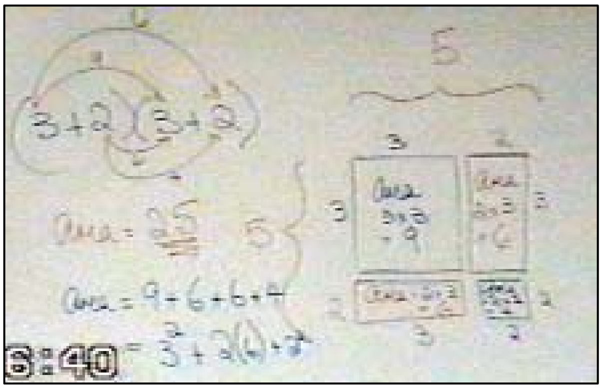


Figure 2. Area & the distributive law

CASE 2: PROFESSIONAL PRACTICE FOR MATHEMATICS TEACHING

There is also a practice to be acquired in this course viz. reflection (conscious examination and systematisation of one's own practice). The course sits within a multi-modal program, delivered through a combination of written materials and face-to-face contact sessions. Specific post secondary level mathematics courses run alongside the mathematics education course in focus in this paper. All the elements of the description of the social logic of competence detailed above (Bernstein, 1996) are visible in Case 2. In the materials for the text and in the contact sessions the lecturer explicitly positions teachers as experienced and knowledgeable. In the course notes it is suggested that teachers will acquire the 'tools and the space' to think about and improve their teaching through action research—it will help them to 'systematise what they already do', viz., reflect on their practice to improve mathematics teaching and learning. The course is thus predicated on the principle of 'similar to' relations both with respect to knowledge and with respect to others, i.e., there is no alienation and no deficits. The principles that are to be made visible by engaging with the course content are presumed to always-already inhere in the learner (teacher). The course is about making explicit the expertise already held in order to further enhance that expertise, hence the focus on self-reflection and action research. Teachers, as self-regulating autonomous subjects, are expected to use their existing mathematical and professional competence to engage independently at home with the course materials so as to produce resources from their own practice for reflection and elaboration in contact sessions.

This presumed mathematical competence for teaching is, however, imaginary. Major obstacles appear when it turns out that the presumed competence is absent. In response, the lecturer has to attempt to insert the absent competences. In this case, she does so by modelling the 'expert practice' required. The principles of the practice that she herself uses are backgrounded. It appears that the logic of competence prevents her from making visible the principles that she is using in the contact sessions.

It is an interesting feature of the course that the textual materials for the course do carry evaluative principles for the legitimate text, but they are probably only recognisable to those students that already have access to these. The logic of competence operates in the text through a curious device. The recognition and realisation rules for the production of legitimate texts are elaborated but they are always accompanied by an additional statement which suggests that teachers have the freedom to choose what to do; for example:

> In the reader for this unit, you will find a worksheet with a number of activities/questions meant to guide learners through realising a number of things relevant to the conversions of decimals to fractions and *vice versa*. It is not given here as a prescription for how to make activities or construct activities. It is only one out of many possible ways of engaging learners with this topic. (Case 2, course notes, Unit 5, pp. 3–4)

The teacher can therefore follow the activities relevant to conversions (i.e., the privileged text) or rely on his/her local knowledge and experience. From the perspective of the teachers, as self-regulating subjects, they should be able to produce a text that exhibits at least some of the features of the privileged text, so that these can then be worked with and 'systematised'. Their freedom to choose is a forced choice. In this Case, the majority of students do not follow the expected practice (suggestions), with the result that the resources required in the contact sessions for enabling progress in the module are absent. Since the students do not bring the resources required for engagement in the expected practice, progress is thwarted. The lecturer tries to overcome the problem through a pedagogy that involves modelling (an example) of the required expert practice. There appear to be two texts that are interrogated through this modelled practice: a professional practice (including bureaucratic aspects and mathematics for teaching) and a mathematical practice (focussed on mathematical reasoning), both of which attempt to engage learners in a particular orientation to knowledge. The lecturer draws on principled knowledge to produce the example she uses. As noted earlier, the principles that structure her activity are backgrounded and so remain tacit.

The example that follows illustrates a typical instance of such modelling. In the third contact session the teachers had been given elaborate instructions about designing a 'Hypothetical Learning Trajectory' (HLT), a model for planning a sequence of student work for learning selected mathematical knowledge, based on Simon (1995). They were required to design a HLT for one of their own classes, a teaching sequence focussed on a particular mathematical topic in the curriculum that would become the basis of their action research project. They were expected to assess their students' readiness for following this trajectory by designing questions that would elicit responses which could be analysed to assess their prior knowledge and readiness for the topic chosen. They were expected to bring their students responses to these questions for discussion in the following contact session. The whole session depended on the teachers producing the required student work for analysis during the session. Only two of the 25 teachers do so. In the face of the absence of the expected resource, the lecturer was forced to produce a text of her own to illustrate the points she had intended would be revealed to the teachers though reflecting on their own practice. She produced the text through choosing a particular example and modelling the kind of thinking she had expected them to engage with.

L: I'm going to ask you to do a little something here. (writes 2^3 on the board). […] Now my question […] is not what the answer is, my question is to you: How many different questions can you ask about this? How many different questions? There is no need to do a lot of group-work […] I think you can just start spitting out questions. You should be able to ask about 25 different questions – nice questions. What is a question you could ask about this?

S: Ask your learners?

L: Yes, ask your learners. (Case 2 transcript)

The students respond by providing possible questions and the lecturer prompts them when they get stuck, and thus modelling an orientation to asking student questions, and a practice for generating questions, that she hopes they will adopt. She writes their answers on the board as she goes along.

L: Okay. Why is the answer not six? That's a good question. Okay. What tells you how many two's to write? What did we get .. one, two , three, four, five, six, seven, eight, nine, ten eleven, twelve thirteen, fourteen, fifteen different questions. We could probably come up with a few more. If you wanted to … But the point of this is such a simple thing – we often tend to just want the answer. Once we have explained one time, we may ask for the extended form. We might ask two or three questions. But if you look at how much information is hidden in such a short notation doing this gives us an idea of how many problems the learners could run into when you just quickly say write on your papers this problem: the base is 2 the exponent is 7 – what's the answer? Do we allow for all these possible misperceptions …
(Case 2 transcript)

Table 2 summarises the appeals made for grounding (legitimating) the texts within this practice. The main text and focus of this module is clearly the modelling of professional practice: 33 of 36 events. The overall pattern reveals that the legitimating appeals are located in the student's experiences and the authority of the lecturer, based on her 'expert' knowledge of the professional practice she models. There are also some appeals made to mathematics and to mathematics education. The three cases where the mathematical text is the focus of the event were diversions from the main teaching text. All three relate to a particular worksheet, intended to be an example (model) of mathematical activity focussed on a specific section in the curriculum—decimal fraction/ common fraction conversions—that was to be analysed to reveal the desired orientation to mathematical knowledge and pedagogy. It became necessary to focus on the mathematics referenced in the worksheet, in place of engaging with the worksheet itself, since students did not engage with it independently in preparation for the session. In these three episodes the appeals were made almost entirely to mathematical principles.

Table 2
Distribution of appeals in Case 2

	Mathematics	Mathematics Education	Metaphorical	Experience of either adept or neophyte	Curriculum	Authority of the adept
Mathematics	3	0	1	0	0	1
proportion of appeals (*N*=5)	60%	0%	20%	0%	0%	20%
Teaching	3	10	0	28	5	23
proportion of appeals (*N*= 69)	4,4%	14,5%	0%	40,6%	7,3%	33,3%
Mathematics & Teaching	6	10	1	28	5	24
proportion of appeals (*N*=74)	8,1%	13,5%	1,4%	37,8%	6.8%	32,4%
proportion of events (*N*=36)	15,4%	25,6%	2,6%	71,8%	12,8%	61,5%

CONCLUDING DISCUSSION

From our analyses of Cases 1 and 2, and notwithstanding their differences (in terms of levels, focus, mode of delivery and intended integration of the domains of mathematics and teaching), it would appear that the structuring of mathematics teacher education by *similar to* relations produces forms of pedagogy that might well work against principled elaboration of both mathematics and mathematics teaching. It would seem that mathematics for teaching within a competence model exhibits features of an empirical activity: inductive procedures supported by empirical testing. A crucial additional feature is the endemic deployment of the visual, or the image, in various forms.

First, the visual inheres in the form of the modelling of practice to the learner who is required to mirror the activity of the adept (lecturer). An important difference between Case 1 and Case 2 is the emphasis of what is modelled. The former models grade specific teaching practice. The latter models an expert professional practice with respect to both mathematics and teaching. Second, the visual recurs in the extensive use of metaphor to explain contents, constructing everyday and pictorial images as place holders for contents, as was seen in Case 1. By the term images we are recognising both pictorial as well as linguistic image; for example, narrative is linguistic imagery. Third, the visual is personalised in the recruitment of the experiences of learners, and often of the adept, as images of that which is to be acquired, as we saw in Case 2. Fourth, more generally, and this is the central point we wish to make, the visual prioritises sensibility, which is experiential. Hence our interest in these practices, and the challenges they present to mathematics teacher education practice. Sensibility is an important feature of the teaching and learning of school mathematics, where some meaning in mathematics remains absent for many learners. But this cannot be at the expense of intelligibility. Specialised knowledges, including mathematics and mathematics for teaching, in part aim at rendering the world intelligible, that is, providing us with the means to grasp in a consistent and coherent fashion that which cannot be directly experienced. Consistency and coherence, however, require principled structuring of knowledge.

In the context of mathematics teacher education in South Africa, access to privileged forms of knowledge by those previously disadvantaged by apartheid is an imperative for overcoming the inequitable distribution of high status knowledge, and so life chances, for the majority of the population. Competence models are attractive because of the apparent democratising of education and knowledge, with a promise of universal access and non-alienation. However, our analysis suggests that competence models produce a pedagogic practice that backgrounds principled features of specialised knowledge. Why is this so? Why is the sensible so prevalent? What then are consequences for acquisition (by whom and of what)? In a context of historical educational neglect and inequality, how do we confront the current contradictory social logic at work, where evaluative rules are invisible to many learners (and so too teacher-learners), and practices produce localised knowledge? What pedagogic practice(s) in mathematics teacher education enable, for example, a principled study of metaphors for both sense and intelligibility of mathematics? Perhaps it is in the mutual working of these oppositional orientations to knowledge that we find the kernel of mathematical knowledge for teaching.

ACKNOWLEDGEMENTS

This paper is based on work supported by the National Research Foundation under Grant number 20535225. Any opinion, findings and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Research Foundation.

This paper forms part of a wider research project on Mathematical knowledge for Teaching, directed by Professor Jill Adler, at the University of the Witwatersrand.

---

[i] In South Africa teachers are required to obtain a four year post-school qualification in education to practice. Those teachers who obtained only three (or fewer) year qualifications under previous dispensations are now required to enrol for further study on in-service programmes to upgrade their teaching qualifications.

[ii] Most of the teachers on this program were initially primary trained and upgrading a 3 year qualification, and level of teaching. An intention built into this course was that by learning to teach algebra they would themselves have opportunities to (re) learn algebra.

REFERENCES

Adler, J. (2002). INSET and Mathematics teachers' conceptual knowledge-in-practice. In C. Malcolm & C. Lubisi (Eds.), *Proceedings of the 10th Annual SAARMSE Conference* (pp. 1–8), Durban, Kwazulu-Natal January 22–26.

Adler, J., & Davis, Z. (2003). An analysis of the structuring of evaluative tasks: A focus on level 6 mathematics INSET. In Jaffer, S. (Ed.), *Proceedings of the 9th National Congress of the Association of Mathematics Education of South Africa, University of Cape Town, July 2003.*

Adler, J., & Davis, Z. (Forthcoming). Opening another black box: Researching mathematics for teaching in mathematics teacher education. Mimeo.

Adler, J., Davis, Z., Kazima, M., Parker, D., & Webb, L. (Forthcoming). Working with students' mathematical productions: Elaborating a key element of mathematical knowledge for teaching. Mimeo.

African National Congress. (1994). *A policy framework for education and training*. Johannesburg: ANC.

Bernstein, B. (1996). Pedagogizing knowledge. In *Pedagogy, symbolic control and identity: Theory, research, critique.* London: Taylor & Francis.

Bernstein, B. (1999). Vertical and horizontal discourse: An essay. *British Journal of the Sociology of Education*, *20*(2), 157–73.

Department of Education. (2002). *Government Gazette No.: 23406, Volume 443, May 2002: Revised National Curriculum Statement Grades R–9 (Schools), Policy: Mathematics*. Pretoria: Department of Education.

Long, C. (2003). Mathematics knowledge for teaching: How do we recognise this? In S. Jaffer (Ed.), *Proceedings of the 9th National Congress of the Association of Mathematics Education of South Africa, University of Cape Town, July 2003*.

Simon, M. (1995). Reconstructing mathematics Pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114–45.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4–14.

Taylor, N., Muller, J., & Vinjevold, P. (2003). *Getting schools working.* Cape Town. Pearson Education South Africa.

IMMIGRANT MATHEMATICS TEACHERS' NEGOTIATION OF DIFFERENCES IN NORMS: THE ROLE OF VALUES

Wee Tiong Seah
Monash University, Australia
<weetiong.seah@education.monash.edu.au>

This paper reports on an essentially qualitative study exploring the socialising experiences of immigrant mathematics teachers in Victoria, Australia. Culturally different socio-mathematical norms and norms of mathematical practice have been documented as potential sources of marginalisation and disempowerment amongst these teachers. By interpreting associated feelings of dissonance in terms of how underlying values are seen to be emphasised differently in the mathematics education workplace, this paper offers a lens through which 'exotic' norms associated with the teaching and learning of school mathematics may be further understood. The potential for proactive professional support from this value perspective is identified.

INTRODUCTION

In 2004, Australia admitted one of its highest number of immigrants, an exercise co-ordinated by a government which only eight years prior had regarded the annual migrant intake to be too large (Shaw, 2005). This follows a trend of increasing intake over the past few years. Most of these modern-day settlers have qualified skills that are needed to fill up positions in vocations such as healthcare, accountancy and education. In Australia and in many other developed countries around the world, skilled migration is the answer to current and emerging shortage of professionals as the baby-boomers begin to reach retirement age, and as the population in these countries begin to 'grey'.

In education, the median age of Australian teachers has increased from 34 to 43 years old between 1986 and 2001 (Australian Government Department of Education Science and Training, 2003). Look at it from another way, in 2001, 44% of Australian teachers were more than 45 years old, up from 17% at the beginning of the same period. A shortfall of up to 30,000 teachers in the years leading up to 2010 is projected in Australia (Australian Ministerial Council on Education, Employment, Training and Youth Affairs, 2003), especially in subjects such as mathematics. There are further serious implications for the quality of mathematics education in schools, as tertiary mathematics enrolment continues to plunge and where mathematics is not the highest qualification for about 12% of Australia's secondary mathematics teachers (Australian Ministerial Council on Education, Employment, Training and Youth Affairs, 2003).

In this context, it is perhaps not surprising that the employment of immigrant teachers of mathematics can be an efficient solution to address the shortfall of mathematics teachers. These teachers have already received pre-service teacher training and have classroom mathematics teaching experience in their respective home countries. Immigrant mathematics teachers might have thus responded directly to advertisements for job vacancies in mathematics classrooms in

Australia, or might have been recruited indirectly as their families migrated across geopolitical borders.

A survey conducted in Victoria, Australia in late 2000 had identified 110 immigrant mathematics teachers from 34 different countries in 159 secondary schools, representing 6.6% of all mathematics teachers in these schools (Seah, in press). These immigrant teachers risk becoming an overlooked group of professionals, partly made invisible by the fact that immigrants from 'Western' countries look local anyway, and also partly hidden by the group of second- or third-generation immigrants who were teacher-trained in Australia and who are generally integrated into the Australian culture and ways of life.

In fact, the study on which this paper is based had documented several examples of immigrant mathematics teachers experiencing marginalisation or disempowerment. An example of the former was shared by Carla, an immigrant teacher from Romania. She recalled her weekly summon into the principal's office while teaching in a rural secondary school in Victoria, when the principal would remind her to teach mathematics the 'Australian way',

> by isolating me and my teaching manner from the rest. By remainding [sic] me all the time that I have a different background (like I wouldn't know that). (Carla, November, 2000)

Such actions reflect a deficit model mentality by the dominant, host culture, threatening to marginalise the professional practice and knowledge of mathematics teachers from other cultures.

> They [that is, colleagues in Australia] don't try to talk to you as a person coming from somewhere, to see whether you know more or less, or better, or change ideas. This is what I sometimes don't understand …. They know that the [mathematics educational] system I come from is very good and very strong. They know that, but they don't want to find out about that. It's like, you know, let's follow the Americans, because they're Americans, rather than use your own resources …. I mean, it's not that I think our system is perfect, or where I came from is perfect. It's not perfect. There are weaknesses in the system as well. Why not share what's good? Why not? And usually, they are not interested in how you think as a person. (Carla, May, 2001)

On the other hand, Karim, arriving in Australia from Lebanon, is an example of an immigrant teacher who was experiencing professional disempowerment in his Australian workplace:

> When I have something to say, normally not many people [are] willing to listen …. I remember years ago when I started to talk about culture with teachers. Nobody, you know, [was] willing to listen, and now, they start to change something in the system, referring to what we said before. But they don't say, "you said that," because this is like normal, this is natural. Always, you know, you try to practise ideas before you accept it, before you adopt it. So many times, we talk about changes in the exams system, in the assessment system, but no real change, no people willing to listen. Now they start to change it. (Karim, July, 2001)

The phrase which Li Kang (an immigrant mathematics teacher from Malaysia) often used in discussions with him reflects a similar feeling:

> Have to soldier on! (Li Kang, August, 2001)

One of the immigrant teachers from Fiji, Manoj, also conveyed this sense of disempowerment:

> You don't want to go to the next [class] generally feeling upset, go home and can't sleep and so on. Well, you turn yourself off. I just turn myself off, I go and watch TV, do what I like, fine …. I've done what I think I should have done. My conscience is clear. Like, you can take them to the well, but you can't make them drink unless they want to drink. So I mean, you can only do so much. (Manoj, November, 2000)

INTERPRETING THE PROBLEM WITH A SOCIO-CULTURAL PERSPECTIVE

These examples are cause for concern, especially if immigrant mathematics teachers had embarked on their respective journeys to Australia subscribing to the widely-held view that mathematical activities are purely cognitive exercises. Such a view would position the immigrant teachers to regard school mathematics teaching as being a culture-free activity that is easily transportable across cultural boundaries. As Carla said,

> I strongly believe that Maths is Maths in any culture. I teach Maths my own way, with a great passion and commitment to the students I teach. (Carla, May, 2001)

The fact that mathematics is socially-constructed knowledge (Bishop, 1990) which has been developed over time in response to human needs appears to have been hidden by general observations that the same factorisation and Pythagoras' Theorem are being taught in mathematics classrooms everywhere. Li Kang talked about protecting this perceived pan-cultural nature of school mathematics education:

> Maths is universal. Worded questions should not be culturally biased, for example, questions involving cricket, football (Aussie rules) may be biased in favour of Australians or students who play in the sport. How in questions like probability, involving playing cards, if assumed that all students are expected to know what a 'pack of cards' are. (Li Kang, June, 2001)

More importantly, how this mathematical knowledge is taught in different cultures is also socio-culturally constructed. After all, "school mathematics is mathematics as it is conceptualized, represented, structured, and sequenced to share with the next generation through the formal schooling experience" (Schmidt, McKnight, Valverde, Houang, & Wiley, 1997). Furthermore, it is hard to imagine mathematics teaching merely as transfer of cold, hard mathematical knowledge when (school) mathematics teaching is really an "encounter between contextualised, historically grounded human beings and their activity in particular settings and spaces that are socially structured" (Valero, 2004). As a result, Orton (1992) noted how the 'meticulous translations' of 'Western' mathematics teaching styles by some developing countries had led to disastrous outcomes in these countries, highlighting the perils of regarding mathematics pedagogy as one-size-fits-all.

The concern, then, is how best can immigrant teachers of mathematics be supported professionally, so that attrition rates in a profession that is already known to 'eat its young' may be controlled, and so that the immigrant teachers' 'cultural funds of knowledge' (Moll, 1994) may interact with local pedagogical traditions to facilitate even more effective mathematics education programs in schools.

In Victoria, Australia, immigrant teachers of mathematics need only be registered with the relevant authorities to practise in local schools almost immediately. The registration process authenticates applicants' overseas teaching qualifications and their respective residency status. Incidentally, under the Trans Tasman Mutual Recognition Act 1997, mathematics teachers from New Zealand are automatically registered for practice in Australian schools. There is thus no formal initiation program for immigrant mathematics teachers in Victoria, many of them new-arrivals in Australia. At the same time, no in-service professional development course is known to have been organised in Victoria to help immigrant teachers of mathematics socialise into the profession. There are indications that this situation is similar in other educational systems elsewhere as well.

(Mathematics) educational research related to immigrant students/teachers and to ethnicity generally suffers from under-representation. Lubienski's (1999) survey found that only 0.2% and 3.7% respectively of 3,011 mathematics education research articles which had been published in 48 authoritative education research journals between 1982 and 1998 belonged to these categories. Zeichner and Gore's (1990) review reported that "the socialization of minority teachers … has been totally neglected in the literature to date" (p. 335), a point not lost in the research of Su, Goldstein, Suzuki and Kim (1997).

TRANSITIONS, NORMS, AND VALUES

Unlike those minority teachers who might have grown up in − and naturally socialised into − the dominant culture of the host country, minority teachers who are first-generation immigrants have to live through a transition process as they socialise into the local education culture. Interpreting and understanding the professional lives of immigrant teachers with a socio-cultural stance has meant the acknowledgement of the complexity related to the ways in which these teachers construct meaning, think and reason in their day-to-day professional interactions in the classroom micro-culture and in wider social settings such as the school, both of which involve communicating with students and colleagues who generally do not share their respective cultural heritage.

In attempting to understand how immigrant (mathematics) teachers interpret and make sense of cultural differences in their socially-conceptualised sites of practice, it has been useful to consider the classroom interactions manifesting as what Gorgorió and Planas (2005) called the socio-mathematical norms and norms of mathematical practice. They had adapted similar definitions proposed by Erna Yackel and Paul Cobb in the late 1990s, to regard

> socio-mathematical norms … [as being] shaped by representations and valorisations of mathematical knowledge and its ownership. They regulate and legitimise interactions and communication processes of mathematical practice. Norms of mathematical practice, as representations of what mathematics in schools is/should be about, regulate the content of practice as legitimised within the classroom (p. 96).

Just as the immigrant students in the research conducted by Núria Gorgorió and Núria Planas mentioned above encountered divergences in the way these norms were interpreted by them and their teachers/peers, the eight immigrant mathematics teachers participating in the study on which this paper is based had also observed similar differences in the Australian mathematics classroom. Indeed, the common theme that runs through educational research on the professional lives of immigrant, expatriate and ethnic minority teachers appears to be that of cultural differences in norms and values (see Bascia, 1996; Horowitz, 1986; Kamler, Reid, & Santoro, 1997; Meacham, 2000; Su et al., 1997).

The construct of (continued) participation is as relevant here as it is emphasised in the theory of socialisation (Danziger, 1971) and Rogoff's conceptions (1995). This notion of participation is all the more demanding of immigrant teachers because unlike (immigrant) students, teachers cannot withdraw participation in the lived moments of classroom interactions. They do not have the 'luxury' of tuning out of the lesson in progress, looking out of the classroom window or sending off a SMS text message. In all of the cases in Gorgorió and Planas' (2005) study,

> discrepancies in the understanding of norms, both of the mathematical practice and socio-mathematical, were not dealt with by negotiation of meanings. When the students felt that they themselves or their practices were valued negatively, the lack of negotiation caused obstacles to communication that led them to abandon their participation (p. 101).

Indeed, Bishop's (2002) observation that "all mathematics education is a process of acculturation, and that every learner experiences cultural conflict in that process" (p. 195) can reasonably be extended to include all participants in the school mathematics teaching and learning activity.

While immigrant teachers who had abandoned their participation would have left the teaching profession, the experiences of the eight practising immigrant teachers in this study suggest a stronger desire by teachers (by virtue of their authority in the classroom, or due to the reality of livelihood?) to negotiate differences in norms and to mediate what underlie such differences. The question then, is how may this negotiation process be understood? What constructs would be useful in guiding our thinking about ways in which cultural differences of norms are reconciled by immigrant mathematics teachers?

The perspective adopted here is one that regards the interpretation, reasoning and reconstruction of socio-mathematical norms and norms of mathematical practice as being guided by the individual's own value schema. For example, a teacher introduced into a mathematics classroom where students accept what teachers say as absolute truth may not associate herself with such a socio-mathematical norm. This teacher's meaning-making, thinking and reasoning in response are then likely to be guided by the extent to which qualities such as *mystery* (Bishop, 1988), *student-centredness*, *participation* and/or *authority* are valued by the teacher herself.

RESEARCHING IMMIGRANT TEACHERS' NEGOTIATION OF PERCEIVED VALUE DIFFERENCES

As such, making explicit the nature and the negotiation of such value differences as they are perceived by the immigrant teachers affords us a look into ways through which these teachers are or are not able to interpret and reconstruct both the socio-mathematical norms and norms of mathematical practice that are constituted in their respective Australian workplace. In the study on which this paper is based, an essentially qualitative research with eight immigrant mathematics teachers identified through purposive sampling in Victoria, Australia had allowed for the collection and analysis of data from a questionnaire, lesson observations, semi-structured interviews, and teacher assessment of student work. In particular, field data was collected from each teacher participant involving three lesson observations of critical incidents (Tripp, 1993), followed by semi-structured interviews focussing on teacher sharing of interpretation and responsive action relating to each of these critical incidents. In particular, the research aimed to explore the nature of value differences perceived by the immigrant teachers, how these were negotiated as part of the teachers' socialisation process, and how socio-cultural factors mediate such negotiation of perceived value differences.

EXAMPLES OF PERCEIVED VALUE DIFFERENCES

This paper will now provide two examples of the 34 critical incidents documented during the fieldwork study, highlighting each in terms of a disruption in what is considered to be expected socio-mathematical norms and norms of mathematical practice, as different values are activated and portrayed through the discourse and interaction of all participants in the mathematics classroom.

PORTRAYAL OF ETHNIC DIVERSITY

Deanne is an immigrant mathematics teacher from Canada. Over the six years she has been practising in Australia, Deanne has encountered a number of differences in the way the socio-mathematical norms and norms of mathematical practice in the Australian and Canadian classrooms play out. These norms are expressed through the different aspects of the curriculum, an example from the implemented curriculum being a perceived lack of emphasis on ethnic *diversity* in the textual discourse of textbooks and assessment items in Australia. As such,

> you often see people of different nationalities [in Canadian textbooks] … whereas in the Australian books, there is very strong 'John', 'Tim' and 'Sarah'. You will come across all sorts of names and things [in Canadian textbooks], which I think is a very good thing which we are lacking here …. (Took out a copy of a Canadian textbook, pointing to an exercise question within) And here we have 'Takzan has three times as much as Paul'. We are not going to see that in an Australian book …. You think there is a very strong Aboriginal community in Australia, and yet you don't see a lot of Aboriginal names up in our textbooks, which is a shame. (Deanne, November, 2001)

This relative lack of ethnic representation in Australian textbooks was also reported by McKimmie (2002) in a comparative study of three Victorian mathematics textbooks. This 'phenomenon' was obvious to Deanne because it did not fit in with her understanding of the need for textbook discourse to reflect ethnic *diversity* in the society. It was a dissonance that also reminded Deanne of her role as an educator, not just as a teacher of mathematics. In the community Deanne was practising in,

> you might be having a conversation with kids particularly given what happens [sic] in the States [i.e., the September 11 tragedy at New York], and with the refugee crisis [ie the plight of asylum seekers at detention centres in various parts of Australia]. Some of the attitudes are quite shocking at times, part of that is because they are kids, and part of that is maybe they are attitudes they get from older parents. But they are not quite as embracing difference as I like them to be. (Deanne, November, 2001)

From her point of view, Deanne was also concerned that students in the more multi-ethnic city (Melbourne) were reading a similar message from their textbooks, for

> the textbooks that we use [here in country Victoria] will be the same as the one they'll use there [in metropolitan Melbourne]. And still we're seeing the same sort of names [in the textbooks]. (Deanne, November, 2001)

Despite Canada and Australia being multicultural societies, Deanne's socialisation experience in the Australian mathematics classroom had highlighted to her how differences in textual discourses in the two countries might lead to a different representation and production of cultures, in the ways *diversity* and ethnic *participation* in doing mathematics are portrayed.

PURPOSE OF TEACHER-POSED QUESTIONS

The next example of a teacher-perceived difference in norms relates to professional teaching experience in Lebanon and Australia of immigrant teacher Khaliq. Amongst the several culturally different ways of valuing mathematics and mathematics teaching was one concerned with teacher-posed questions to students in the mathematics classroom. While teacher posing of questions is a pedagogical strategy used in both Australia and Lebanon to assess student understanding of concepts taught, the formality of this assessment can differ between the two educational systems. In Lebanon,

> sometimes you know [when] the teacher in Lebanon asks the students to the board to solve the problem [posed], it's part of the assessment …. So, "okay, he can do it, give him that [particular] mark. He can't do it, so lower mark," it depends. (Khaliq, immigrant teacher from Lebanon, July, 2001)

whereas in the Australian classroom,

> the idea behind … [getting] one of the students to the board to do the work is not just you and him, you together and the other students do something else. No, everybody should share with him the work, try to help, try to get, you know, ideas. (Khaliq, July, 2001)

thus implying different roles expected of peers as well.

Yet, despite classroom questions serving broader pedagogical purposes in the Australian mathematics classroom, Khaliq (and several other immigrant teacher participants of the study) found students in Australia to be generally more self-conscious of volunteering their opinions and answers in class:

> Over there [in Lebanon], you asked the students to come to the board and do and try to solve exercise. [In Australia] we won't do [this], you don't ask the kids to come to the board, "excuse me, whatever student, you know, come to the board and try to solve this question for me." We won't do this. Why? Because this can be a bit embarassing [for the nominated student] …. Overseas, everybody has to go to the board. Even if you cannot do it, so, bad luck! (Khaliq, July, 2001)

Teacher question-posing in the mathematics classroom is often such an integral part of the pedagogical process that this perceived cultural difference in the nature of its purpose and of student participation can stimulate dissonance in the practice of an immigrant teacher everytime a lesson is conducted in Australia, as it did to Khaliq. Such a difference in the socio-mathematical norms accentuates differential cultural valuing of *assessment*, (student) *confidence*, *participation*, and even values related to the nature of mathematics in terms of whose voice and whose ideas are most emphasised.

CONCLUSION

These and the other differences of socio-mathematical norms and norms of mathematical practice being perceived in the Australian mathematics classroom by the immigrant teachers suggest that the teaching of a supposedly culture-free subject such as mathematics across geo-political borders is anything but culture-free. Not only is mathematics value-laden, so are the pedagogical aspects of the subject, the expected role of teacher as value educator, and systemic expectations of the professional function of the mathematics teacher.

Obviously, an immigrant teacher can—and does—respond to such cultural differences in her own mathematics classroom in different ways as she intuitively attempts to restore harmony and equilibrium to the experiencing of dissonance (Festinger, 1957) in this process. An expectation for immigrant teachers to either enculturate or assimilate to the dominant, host culture is likely to be as insensitive and too simplistic a 'solution' as one that only celebrates diversity in the profession and entertains pedagogical variety in a student's mathematics learning process. Adopting the values framework, this study has found that not only were a range of responsive approaches adopted by the eight immigrant teacher participants to negotiate about the perceived value differences inherent in the different sites of mathematics teaching and learning, more significant was the observation that each immigrant teacher responded to the differences using different approaches. In fact, the different socio-cultural factors operating at the different times when the same value difference is experienced have been found to introduce different contextual values in such a way that an immigrant teacher may respond to this same value difference differently. What this means is that for each immigrant teacher, professional experience in the host culture does not necessarily eliminate value difference (and thus, perceived difference in norms). Changing socio-cultural contexts ensures that the process in which immigrant teachers look at and look through (Chronaki, 2004) each of the perceived value difference situations is a repetitive and iterative one.

Indeed, in negotiating the difference in norms, these differences were not always eliminated from the immigrant teachers' professional practice. That the majority of the teachers' responsive approaches were amalgamation and appropriation (Seah, 2003), that each and every immigrant teacher participant of the study was engaged in them, reinforce Bishop's (2002) inclusive partitioning view of conflict and consensus co-existing in dynamic equilibrium in the cultural interaction process.

This paper is aimed at highlighting how immigrant mathematics teachers may be left marginalised or disempowered if their attempts at negotiating perceived differences of socio-mathematical norms and norms of mathematical practice are not recognised and professionally supported. While the immigrant teachers may be predisposed to negotiate difference, not all teachers have been successful, and not all negotiation processes have been straightforward. If the cultural funds of knowledge of immigrant teachers of mathematics and their potential for enriching the 'Australian' mathematics pedagogical culture are to be valued, if the professional and affective lives of immigrant mathematics teachers are to be supported for the reasons discussed earlier in this paper, then proactive in-service professional development programs should be devised by relevant organisations such as mathematics teacher associations to empower this yet small but increasingly significant group of mathematics teachers, so as to optimise the mathematics learning experiences of all students in our schools. The study on which this paper is based has suggested that an approach from the values framework can be promising in this regard, at the same time that it acknowledges the active role of all participants (that is, including the immigrant teachers themselves) in the mathematics classroom in interpreting and reconstructing the 'Australian' mathematics education culture.

REFERENCES

Australian Government Department of Education Science and Training. (2003). *Australia's teachers, Australia's future: Background data and analysis* (Committee review). Canberra, Australia: Australian Government Department of Education, Science and Training.

Australian Ministerial Council on Education Employment Training and Youth Affairs. (2003). *Demand and supply of primary and secondary school teachers in Australia*. Victoria, Australia.

Bascia, N. (1996). Inside and outside: Minority immigrant teachers in Canadian schools. *Qualitative Studies in Education, 9*(2), 151–165.

Bishop, A. J. (1988). *Mathematical enculturation: A cultural perspective on mathematics education*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Bishop, A. J. (1990). Western mathematics: The secret weapon of cultural imperialism. *Race and Class, 32*(2), 51–65.

Bishop, A. J. (2002). Mathematical acculturation, cultural conflicts, and transition. In G. d. Abreu, A. J. Bishop & N. C. Presmeg (Eds.), *Transitions between contexts of mathematical practices* (pp. 193–212). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Chronaki, A. (2004). Researching the school mathematics culture of 'others'. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 145–165). Dordrecht, The Netherlands: Kluwer.

Danziger, K. (1971). *Socialization*. Baltimore: Penguin.

Festinger, L. (1957). *A theory of cognitive dissonance*. Stanford, CA: Stanford University Press.

Gorgorió, N., & Planas, N. (2005). Social representations as mediators of mathematics learning in multiethnic classrooms. *European Journal of Psychology of Education, 20*(1), 91–104.

Horowitz, T. R. (1986). Attitudes of Soviet and American immigrant teachers in Israel. *Education and Society, 3*(2), 90–94.

Kamler, B., Reid, J.-A., & Santoro, N. (1997). *Who's asking the questions? An attempt to research race, ethnicity and teachers. Paper presented at the 1997 AARE Annual Conference, Brisbane*. Retrieved July 4, 2000, from http://www.aare.edu.au/97pap/kamlb218.htm

Lubienski, S. T. (1999, April 19–23). What's hot? What's not? A survey of mathematics education research 1982–1998. Paper presented at the 80th Annual Meeting of the American Educational Research Association, Montreal, Canada.

McKimmie, T. (2002). Gender in textbooks. *Vinculum, 39*(4), 18–23.

Meacham, S. J. (2000). Black self-love, language, and the teacher education dilemma: The cultural denial and cultural limbo of African American preservice teachers. *Urban Education, 34*(5), 571–596.

Moll, L. (1994). Literacy research in community and classrooms: A sociocultural approach. In R. Ruddell, M. Ruddell & H. Singer (Eds.), *Theoretical models and processes of reading* (pp. 179–207). Newark, DE: International Reading Association.

Orton, A. (1992). *Learning mathematics: Issues, theory and classroom practice*. (2nd ed.). London: Cassell.

Rogoff, B. (1995). Observing sociocultural activity on three planes: Participatory appropriation, guided participation, and apprenticeships. In J. V. Wertsch, P. D. Rio & A. Alvarez (Eds.), *Sociocultural studies of mind*. Cambridge, UK: Cambridge University Press.

Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E. (1997). *Many visions, many aims* (Vol. 1). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Seah, W. T. (2003). Teachers in transition and the negotiation of value differences in the classroom: Perspectives of immigrant teachers of mathematics in Australia. *Proceedings of the 2003 NZARE/AARE Conference*, from http://www.aare.edu.au/03pap/sea03594.pdf

Seah, W. T. (in press). *The negotiation of perceived value differences by immigrant teachers of mathematics in Australia.* Unpublished PhD dissertation, Monash University, Victoria, Australia.

Shaw, M. (2005, January 10). Migrants, refugees in new peak. *The Age*.

Su, Z., Goldstein, S., Suzuki, G., & Kim, T. (1997). Socialization of Asian Americans in human service professional schools. *Urban Education, 32*(2), 279–303.

Tripp, D. (1993). *Critical incidents in teaching: Developing professional judgement*. London: Routledge.

Valero, P. (2004). Socio-political perspectives on mathematics education. In P. Valero & R. Zevenbergen (Eds.), *Researching the socio-political dimensions of mathematics education: Issues of power in theory and methodology* (pp. 5–23). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Zeichner, K. M., & Gore, J. M. (1990). Teacher socialization. In W. R. Houston (Ed.), *Handbook of research on teacher education* (pp. 329–348). New York: Macmillan.

DIALOGICAL ENGAGEMENT BETWEEN THEORY AND OBSERVATIONS WITH REFERENCE TO WEAKLY CLASSIFIED ACTIVITIES

Godfrey Sethole
Tshwane University of Technology
<setholegi@tut.ac.za>

This paper provides a theoretical tour I took in a quest to develop a language of description for mathematical tasks which incorporated the everyday. The substance of the argument is that I continuously had to abandon or modify my initial theoretical constructs as dialogue between data and theory ensued. I illustrate this point by retracing how the construct, weak classification, became limited in providing a more accurate description of the type of activities initially categorized as weakly classified. Summoning Dowling's language of description for texts for this purpose, I also illustrate how I had to modify it in order to provide a better explanation of my study's purpose. I argue that the 'lens' metaphor for theory may be misleading.

THEORY AS A LENS?

In 2001, I embarked on a doctoral study whose focus was the learners' perspectives on the inclusion of the everyday in mathematics. The empirical data for the study was relatively clear to me. In that regard, I had a better picture of the type of empirical data I needed for the study and how I was going to collect it. Less clear, though, was a theoretical framework I needed and whether I needed one in the first place. This lack of clarity over the role of a theoretical framework was shared by a number of doctoral students during a South African doctoral students' session organized by the National Research Foundation (NRF) in 2003.

Much of the literature, claims Usher (1998, p. 134), views theory as a platform for mapping experience. It helps researchers not "to go out into the world with completely vacant minds" (Martin, 1997, p. 24). In other words, 'theory is like a lens' through which one views practice (Olivier, 1992, p. 193). However, the metaphor of a lens for theory is problematic and to some extent misleading. Inherent in its use is an acknowledgement that theory enables the observations that a researcher sees and does not see. However, it also backgrounds the role that these observations can play in shaping up theory, since it is only through the lens that an object may be viewed and not the other way round. In this paper I outline the way in which data for my study spoke back to my initial theoretical framework based on Bernstein's concept of classification. Though it provided a focus, it became apparent that the concept of classification was inadequate to help me engage the data I had collected. I will outline this experience by discussing

1. The way in which Bernstein's concept was appropriated for the study
2. The way in which data forced a modification of my initial theoretical framework
3. The significance of dialogical engagement between theory and data.

1. SUMMONING BERNSTEIN'S CONSTRUCT – CLASSIFICATION

My study took place within a context of a new South African education system which was in what may be referred to as a boundary-blurring or non-segregationist mode (DoE*, 1997). Not only was there a political intention to blur the boundaries between different races, sexes and classes; there was also an educational intention to blur the boundaries between different school subjects and between each school subject and the everyday. My study hinged on (1) an explication of what the everyday is and (2) what the incorporation of the everyday in mathematics entailed. Bernstein's constructs helped me conceptualise these two aspects. Firstly, I viewed the everyday as one entity, any common observable phenomena. Common, as Bernstein (2000, p. 157) outlines, "…because all potentially or actually have access to it". Secondly, I recruited Bernstein's (2000) construct of classification to describe the incorporation of the everyday in mathematics. In espousing this construct, Bernstein draws a distinction between weak classification and strong classification. "We can distinguish between strong and weak classifications according to the degree of insulation between categories, be these categories of discourse, categories of gender, etc. Thus in the case of strong classification, we have strong insulation between categories. In the case of strong classification, each category has its unique identity, unique voice, its own specialized rules of internal relations. In the case of weak classification, we have less specialized discourses, less specialized identities, less specialized voices." (Bernstein, 2000, p. 18).

After all, the use of Bernstein's concepts is quite common amongst mathematics educators (See Ensor, 1999; Dowling, 1998, Cooper & Dunne, 2000). Therefore I also had access to the way in which Bernstein's constructs were used in analyzing data. Based on the notion of classification, I categorized mathematics lessons which incorporated the everyday as weakly classified since they exhibited an "open relationship" between the mathematics and the everyday. Armed with these theoretical constructs, it was clear to me that what I was particularly interested in were learners' perspectives on mathematics lessons which could be categorized as weakly classified. In other words, weakly classified was a notion to describe lessons which incorporated the everyday.

2. THE LIMITATION OF CLASSIFICATION CONSTRUCT

I collected data from two schools, Umhlanga and Settlers. I paid attention to lessons in which mathematics teachers used activities which referenced what I regarded as the everyday. The following are two examples of tasks used by the two teachers whose lessons I observed. Task 1 is selected from a set of tasks used by the teacher in one school (Settlers) and task 2 is selected from a set of tasks used in another school (Umhlanga).

Task 1: *John's age is p years. Write down Sue's age in terms of p if Sue is 6 years younger than John.*

Task 2: *If the number of people suffering from AIDS in 2000 is 133.6 million and the world population is 6 000 million; calculate the percentage of people suffering from AIDS in the year 2000.*

Both tasks are characterized by an incorporation of the everyday in mathematics, they may thus be grouped together as weakly classified. However, grouping task 1 and task 2 in the same category conceals the different expressions used in presenting the tasks and the different ways in which learners may relate to each context. Firstly, in task 1, knowledge of mathematics symbols is assumed and in the task 2 such an assumption is not made. Secondly, even though John refers to a name of a real person, the claim that John is *p* years old bears no everyday sense. Categorizing these two tasks as weakly classified obscures these significant differences.

I then summoned Dowling's framework which provided a more precise and concrete language of description enabling a distinction between the two tasks cited above. In describing these tasks, Paul Dowling (1998) uses two categories: mode of expression and the nature of context drawn in. Tasks which have a highly classified mode of expression are those which communicate information in 'unambiguously mathematical' terms (Dowling, 1998, p. 135). Such tasks can either draw from the mathematics context or the everyday; in which case they will respectively be labelled 'esoteric' and 'descriptive'. Other tasks employ a weakly classified mode of expression and thus communicate information using non-mathematical expressions. Likewise, these tasks may also either draw from the mathematics or the everyday contexts; they will respectively be labelled "expressive" or 'public'. The four possible categories emerging from this discussion can be presented in the quadrant below.

	Strong classification of content	**Weak classification of content**
Strong classification of mode of expression	Esoteric Domain	Descriptive Domain
Weak classification of mode of expression	Expressive Domain	Public Domain

Figure 1: Categories produced from an interplay between mode of expression and content.

These categories enable a distinction between the two tasks cited above. Task 1 uses a strong classification of mode of expression, characterized by the use of symbols, and it also draws in the everyday context. Thus, it can be categorized as a descriptive domain task. Task 2 can be categorized as a public domain task because it employs a weakly classified presentation mode, characterized by the use of ordinary language and it also draws in the everyday.

By opening up a dialogue between the theoretical notion of 'classification' and the empirical data, it was possible to note that my initial theoretical construct of classification needed modification. Dowling's notions became important for my analysis. However, this model did not take into account the qualitative difference between the different types of the everyday; the point I attend to in the next section.

3. Different Types of the Everyday

My research interest was much more than what the everyday entailed in mathematics, it was mainly about the way in which learners related to and therefore viewed the role of the everyday. In other words, I was interested in the way a context resonated with the learners' experiences. Espousing the qualitative difference between contexts, Freudenthal (1970, p. 78) made the following observation; "When speaking about mathematics fraught with relations, I stressed the relations with a **lived-through reality** rather than with a **dead mock reality** that has been invented with the only purpose of serving as an example of application". (my emphasis)

I view these two Freudenthal-based categories as two opposite extremes. On the one extreme, 'dead mock reality' references the everyday in a way which is highly unlikely or impossible. An item, for example, which makes reference to an African-American president in the United States of America before 2004 is using a known concept (American president) inauthentically (there was never an African-American one before 2004). I use the term inauthentic for such contexts. On the other extreme, 'lived-through experience' makes reference to genuine or not far-fetched use of the everyday. For example, a task which makes reference to 'John who went fishing with his friends' is appealing to a context we know little about. However, the possibility of such an event cannot be confidently dismissed. To the extent that such a context is not obviously a make-belief, I will use the term authentic to describe it.

A context, authentic or inauthentic, may reference a scene or event which resonates with learners' experiences. These would include events that relate to the areas where learners stay and or which take place and are topical during the learners' lifetime. Such events are 'near' to the learners in terms of space (locality) and/or time (period of occurrence). Alternatively, a context may reference a scene or event which does not resonate with the learners' experiences either because it took place a long time ago or it took place in a place situated physically far from where the learners reside.

I have used the concept of 'near' similar to the way in which Royer (cited in Billet, 1998:8) uses it as a qualification for knowledge transfer. He, for example, regards the ability of a university lecturer to teach with ease in another university as a case of 'near transfer' since it permits deployment of skills to a similar context. In a similar way, Royer uses 'far' as another qualification for knowledge transfer. Carrying on with an example of a 'university lecturer', Royer regards a requirement of a university lecturer to teach at a vocational college or primary school as far knowledge transfer. This is because in this case, there will be a deployment of a skill to a novel situation. In sum, the concept of 'near' is related to familiarity or similarity and 'far' is related to novelty or unfamiliarity.

Using the concepts of authenticity/Inauthenticity and close/far to describe a context, the following four categories emerge

	AUTHENTIC	**INAUTHENTIC**
CLOSE	Authentic and near	Inauthentic and near
FAR	Authentic and Far	Inauthentic and far

Figure 2: Different categories of the everyday-non mathematical.

This model was provoked by the need to describe and differentiate between tasks such as (task 2 and task 3 below) which were included in different worksheets and different lessons in Umhlanga high school.

Task 2: *If the number of people suffering from AIDS in 2000 is 133.6 million and the world population is 6 000 million; calculate the percentage of people suffering from AIDS in the year 2000.*

Task 3: *How did the ancient Egyptians write the number 100?*

Using Dowling's model, the two tasks fit the public domain category since they draw in the everyday and they do not use an unambiguously mathematical mode of expression. However, the contexts drawn in have different appeals to learners. Task 3 references a context which is far (both in terms of time and place) and authentic (there is a particular way in which 100 was written in ancient Egypt). Task 2 draws in a near context of AIDS* even though it references inauthentic data. The use of Dowling's notions does not highlight the qualitative difference between these two tasks and therefore the different emotions they evoke amongst learners.

DISCUSSION

In this paper I have reflected on how a dialogical engagement between theoretical constructs and data shapes up these constructs. Whilst the concept of classification helped me locate the type of mathematics lessons I needed to focus on, I could not use it to distinguish between these lessons. Describing the mathematical activities as weakly classified failed to capture the different ways in which the everyday was incorporated in these activities. Dowling's constructs provided a more accurate language of description for the lessons; yet the different emotions evoked by the contexts in the tasks remained concealed. This led to a development of a new description based on authenticity/inauthenticity and near/far concepts.

The significance of opening up a dialogue between theory and practical settings has been echoed within the mathematics education field. In presenting this argument, Vithal and Valero (1998) and Adler and Lerman (2001) have voiced the need to grant the practical settings of the 'south' or 'developing countries' in order to better explain the dynamics in those settings. It is this dialogical engagement between theory and practice which is at the centre of Popper's thesis on conjectures and refutations. Theory divorced from practice becomes no more than a 'soothsaying practice' (Popper, 1972, p. 37). In this regard, Popper maintains that " A Marxist could not open a newspaper without finding on every page confirming evidence for his interpretation of history;" (1972, p. 35).

Instead of a lens, theory may be better described as a necessary starting point which becomes shaped up, modified and sometimes abandoned on the basis of data.

---

[*] Department of Education

[*] President Thabo Mbeki's questioning of the transmission of AIDS has provoked heated debates and reactions from AIDS activists at a national level. Umhlanga is located about 20 kilometres from a township in which an AIDS activist Gugu Dlamini was stoned to death for publicly declaring her HIV positive status.

REFERENCES

Adler, J. & Lerman, S (2003). Getting the description right and making it count: Ethical practice in mathematics education research. In A. J. Bishop, M. A. Clemens, C. Keitel, J. Kilpatrick & F. Leung (Eds.), *The second international handbook of mathematics education* (pp. 441–470).

Bernstein, B. B. (2000). *Class, codes and control.* London: Routledge and Kegan Paul.

Billet, S. (1998). Transfer and social practice. *Australian and New Zealand Journal of Vocational Education Research*, *6*(1), 1–5

Cooper, B. & Dunne, M. (2000). *Assessing children's mathematical knowledge: Social class, sex and problem-solving*. Buckingham: Open University Press.

Department of Education. (1998). Education Policy Document.

Dowling, P. C. (1998). *The sociology of mathematics education: Mathematical myths/pedagogic texts.* London: Falmer Press.

Freudenthal, H. (1975) *Mathematics as an educational task*. Dordrecht: Kluwer.

Martin, R. M. (1997). *Scientific thinking*. Ontario: Broadview Press.

Olivier, A. (1992). Handling pupils' misconceptions. In M. Moodley, R. Njisane & N. Presmeg (Eds.), *Mathematics education for in-service and pre-service teachers*. Pietermaritzburg: Shuter and Shooter.

Popper, K. R. (1972). *Conjectures and refutations*. London: Routledge.

Usher, P. (1996). Feminist approaches to research. In D.Scott & P. Usher (Eds.), *Understanding educational research*. London: Routledge.

Valero, P. & Vithal, R. (1998). Research methods of the "north" revisited from the "south". In A. Olivier & K. Newstead (Eds.), *Proceedings of the 22nd Conference of the International Group for the Psychology of Mathematics Education* (pp. 153–160).

USING RIGHTS-BASED FRAMEWORKS FOR RETHINKING TEACHER-DIRECTED PEDAGOGIES OF MATHEMATICS

Fiona Walls
James Cook University
<fiona.walls@jcu.edu.au>

This paper uses postmodern perspectives to demonstrate how, within the growing global discourse of human rights, accepted pedagogies of mathematics conflict with principles embodied in international conventions and declarations concerning children's rights, particularly their right to participation in all matters affecting their lives. It examines the ways in which discourses of mathematics education produce and sustain teacher-directed approaches to mathematical learning, and considers how such pedagogies compromise participation for young learners. It contemplates reframed educational discourse in which a *participant-determined pedagogy of mathematics* might more appropriately reflect the discourse of enhanced empowerment for children in the mathematics classroom.

Recent trends in mathematics education reflect a growing belief that mathematical achievement can be enhanced by reframing the sociomathematical norms that characterise traditional classroom practice. As learning theories such as social constructivism have become increasingly accepted in mathematics education, there has been a shifting emphasis from teacher as transmitter of mathematical knowledge, to teacher as facilitator of students' development of mathematical understandings. Accordingly, *context*, *relevance*, *meaningfulness*, *authenticity*, *richness* and *openness* of mathematical learning tasks have been increasingly considered as vital elements in inclusion and motivation of learners and in catering for diverse needs.

In spite of such changes in the discourse of mathematics education, fundamental relationships between teacher (adult) and learner (child) have persisted. Policy makers continue to assert that it is the teacher in a position of control who makes the difference in student achievement (Thrupp et al., 2003). There remains an unchallenged acceptance of traditional pedagogies in which the teacher, within the guidelines of a state-mandated curriculum, selects and manages students' learning tasks. This view is reinforced in a recent mathematics curriculum support document for teachers, (NZ Ministry of Education, 1997) which states that 'As the professional with expertise in both learning theory and curriculum, the teacher plays a pivotal role…by planning programmes where students' thinking and learning are of prime importance" (p. 21). A critique of teacher-directed pedagogies of mathematics may be timely given recent calls for rights-based democratic access through democratic mathematics education, for example Malloy, (2002) who suggests that "The idea of children having democratic access to powerful mathematics ideas is a human right" and "democratic education is collective in its goals and& Valero (2002) argue that "mathematics education becomes powerful in a cultural sense when it supports people's empowerment in relation to their life conditions." (p. 394). Democratic education advocates empowerment through participation. At present, the majority of the world's children have little agency in determining the path and nature of their mathematical learning within educational institutions (Apple & Beane, 1999; Gates & Vistro-Yu, 2002). The classroom itself may be regarded as a significant element of the *life conditions* of our children, and creating conditions of empowerment within the mathematics classroom must concern those who seek to "democratize" mathematics education.

Using statements gathered from teachers and students (Walls, 2003), and from teaching resource materials, this paper examines how discourses of mathematics education that produce and sustain teacher-directed task-oriented approaches to mathematics education run counter to the democratic principles of participation found in the discourse of children's rights. The paper raises issues about children's autonomy, entitlement to control their learning environment, and spontaneous determination of their own educational journeying, and considers the alternative discourses of rights-based participant-determined mathematical learning.

DOMINANT PEDAGOGIES OF MATHEMATICS: *TEACHER* AS *DIRECTOR*

Perhaps more noticeably than those of many other school subject areas, pedagogies of mathematics are characterized by a clearly delineated binary relationship between teacher and learner in which the teacher plays a dominant managerial role in the selection, assignation, and administration of mathematical tasks. Foucault (1977) refers to such modes of institutional organisation as techniques or apparatus of management. Within teacher-directed, task-driven pedagogies of mathematics, mathematical tasks are used for a variety of purposes including the introduction of new concepts, practice of previously learned skills, identification and grouping of children according to performance, or as a method of behaviour management. Mathematical tasks, variously referred to as *questions*, *activities*, *problems*, *exercises*, *lessons, examples*, *learning experiences, units*, *programmes of work, projects*, or *investigations*, appear in many forms including oral questions, quizzes, worksheets, textbook work, investigations, homework or test items.

Teacher-directed pedagogies of mathematics are characterized by their compulsory nature. As a compulsory and 'core' subject area in the education systems of most countries, mathematics education claims a significant proportion of children's schooling. Within teacher-directed mathematics programmes, teacher-selected tasks are themselves compulsory, and routinely exclude learners from the processes of task selection, design, and implementation.

During an ethnographic study in which ten children were tracked across three years of their middle primary schooling in New Zealand (Walls, 2003) the children were asked what they usually did at maths time. Typical responses from the children in the study spoke of the compulsory and teacher-determined nature of everyday tasks in their mathematical learning.

Jared: The teacher says, 'Go and get your maths books out.' And she writes stuff on the board for maths. (Late Year 3)

Georgina: We get into our [teacher-selected] groups and do the worksheet. (Mid Year 4)

Mitchell: You have to sit down and do some times tables or pluses or take away. (Late Year 5)

Over the three years of observation, mathematical learning experiences in the children's classrooms were found to consist almost exclusively of teacher-directed tasks including tests, small group instruction, and solo seat-bound activities based upon adult-devised worksheets, textbooks or questions on the board.

Teacher-directed pedagogies of mathematics appear to resist change. Brown (2001) describes how shifting expectations of what constitutes effective mathematics have produced an *opposition* between transmission (the old) and discovery (the new) conceptions of teaching mathematics, creating conflict for teachers between two seemingly distinct models. But although the nature and management of mathematical tasks may differ between these teaching approaches, the teacher-directed task-bound culture of mathematics classrooms within which teachers and learners are similarly produced and positioned, remains intact. The following video transcript of teacher/pupil interaction during a mathematics learning session in Jared's Year 5 classroom illustrates how the construction of the teacher as director was maintained within changing mathematics educational discourse.

Mr Waters: First of all this morning we're going to put up the title. *(Writes* 'Problem Solving' *on the board).* Underline it and miss a line. See if you've got your brains into gear. (*Writes on the board: (1) 2, 4, 6, 8,* □, □, □) A nice easy one to start off with. What you're going to do is complete the number pattern. *(Writes: (2) 3, 6, 9,* □, □, □). Fill in the numbers and continue it on. Maths is patterning, that's all it is. Complete the whole number pattern. *(Writes: (3) 5, 25, 45, 65,* □, □, □). They're going to get harder and harder. (*Looking at a child's book)* … Make sure you have the most important piece and that is the comma between, if you don't, your numbers will represent something else. You must set them out properly.

In this lesson, the learning experience was presented as *problem solving*, but through his use of the task-oriented expressions "you're going to", "you must", "make sure", the teacher positioned himself as the *taskmaster* whose role it was to allocate work and manage learners, emphasising the compulsory nature of the task, and the expectation that all children were to follow the same procedures. Teacher-directed pedagogies such as this were found in every classroom observed; teachers in the study displayed an unquestioning belief in and acceptance of their responsibility as selector and director of tasks, as evidenced by the following typical comments:

Mr Loch: At the moment I'm finding it's taking time for some kids to settle down, settle into a routine…kids just don't complete work and they're not used to actually getting through something. Finishing it off. That's something I'm very tough on. I like things to be completed. (Jessica's teacher, interview early Year 3)

Mrs Joiner: (*Writing about Rochelle*) She needs only a few reminders to complete set [mathematics] tasks. (Progress report for parents, early Year 3)

Mr Solomon: Georgina, I had to separate out from the others, for about four or five weeks I think it was. I gave her a desk over there by herself. *(Points to corner of classroom)* She was just far too distracted and didn't finish or get on with her [mathematics] work. (Interview, mid Year 3)

Ms Torrance: I think he [Dominic] would prefer working in a group… I would prefer him to work on his own. Independent [mathematics] tasks, he's not the best; he's very chatty. (Interview, mid Year 3)

In the classroom, teachers' direction often took the form of reinforcing protocols.

Ms Summers: *(To Peter)* You've finished! Doesn't it feel good when you've done it? (Classroom observation, late Year 3)

Ms Torrance: (*To the class*) We have some amazing speedsters who have got on their rollerblades and got their two sheets done already. (Dominic's teacher, classroom observation, late Year 4)

Ms Sierra: *(To a group*) You're supposed to do your own work … I don't want you talking, I want you to concentrate. (Liam's teacher, classroom observation, early Year 4)

Teacher/learner interactions in mathematics classrooms have been described by Doyle (1988) as a process in which "teachers affect tasks, and thus students' learning, by defining and structuring the work that students do, that is, by setting specifications for products and explaining processes that can be used to accomplish work" (p. 169). The pedagogical tradition of teachers' structuring of mathematical learning through a series of carefully selected and closely managed discrete *tasks,* may be regarded as an entrenched cultural feature of the mathematics classroom, regulated by a prevailing epistemological view of mathematics as a discipline comprised of specialised procedures based upon a body of universal principles which may be arranged in hierarchies of increasing complexity. In this view, mathematical truths can best be conveyed to the learner through a process of initiation in which the novice (child) is assigned increasingly difficult tasks by the expert (teacher) who has, through a similar process, acquired the same knowledge and skills. Task selection and management is thus regarded as the defining role of an effective mathematics teacher.

In recent times, greater focus has been placed on teachers' selection and management of mathematical tasks the promoting and guiding mathematical discussion, seen as a vital component of the learning process. Greater emphasis on meaningful contexts and of *thinking* and *working mathematically* is reflected in official curricula of many countries advocating pedagogical approaches based upon open-ended mathematical tasks, problem solving, and even problem posing.

Examples of recent attempts to provide students with mathematical tasks that are relevant and authentic can be found in the Rich Tasks for New Times approach of Queensland, and Realistic Mathematics Education of the Netherlands. But such innovations have continued to support the view that it is teacher direction that is central to the mathematical learning process. Carpenter et al (1997) for example, describe the teacher's role in *cognitively guided instruction* of children's mathematical learning as follows:

> Almost every minute, a teacher makes a decision about what to teach, how to teach, who to call on, how fast the lesson should move, how to respond to a child, and so on … because of the intimate knowledge of students that teachers have, no one else can make these immediate decisions about what to do in the classroom (p. 95).

Similarly, Ernest (2001), in describing a *critical mathematics,* says "Obviously teachers must decide what activities and projects would be best suited to their pupils, how often these kinds of activities can be done…" (p. 289) and provides teachers with a list of "suitable" topics. Although recent pedagogical shifts in mathematics education have strongly encouraged teachers to select or design tasks for interest or relevance, and increasingly expect or even compel children to participate by sharing their thinking as they undertake these tasks, it is seldom considered essential that children are consulted about the *context, content* or *efficacy* of such tasks. Irrespective of how *open* or *closed* the tasks may be, teacher-directed task-oriented pedagogies subtly or otherwise construct mathematical learning as a form of compulsory labour divided into discrete units of work which must be at least attempted and preferably completed by the learners, and by which learners' performances might be judged by the teacher.

Efforts to confer greater autonomy on young learners within educational institutions such as the learning-through-play philosophy of early childhood education, the child-centred learning movement of the 1970s, and inquiry-based learning of the 1980s and 1990s appear to have had little significant impact on teacher/learner relationships within mathematics education.

Recent international moves toward more expansive and *connected* mathematics have been offset by demands for greater specificity of learning outcomes. *Numeracy enhancement* projects in Australia, New Zealand, and the UK for example, support teacher-directed pedagogies through increasingly refined assessment tasks, enabling teachers to better "identify" children's mathematical learning stages, "diagnose" their weaknesses and strengths, and "prescribe" appropriate learning tasks. Such programmes operate in the belief that through intensive training including the use of effective tools of detection, teachers will be better equipped to make the most significant decisions about what mathematics their pupils will learn, when they will learn it, and how that learning will take place. Such approaches continue to suppress opportunities for learners to select learning contexts or to direct their own learning, and overlook significant learning factors such as children's friendships, understandings of the world, sensitivities, fascinations, passions, and aversions.

LEARNER-DETERMINED MATHEMATICS EDUCATION? CONSIDERING ALTERNATIVES

Alternative modes of children's learning are not difficult to find. Observations of the kinds of "informal" acquisition of knowledge and skills that occur outside of school settings, such as children learning to dive into the river with village wantok[i] (Efate, Vanuatu), or ride their skateboards in the street with a bunch of mates (Townsville, Australia), offer compelling models of learning that are neither teacher-directed nor task-dependent, rather they are *participant* or *learner-determined*. Children appear to flourish within such self-selected and self-directed experiential learning situations, in which learning takes the form of socially valued playing around. Within a self-selected social group, children as learners are supported to learn at their own pace, in their own time, and in a place of their choosing. They can start and stop whenever they like, they are enabled to discover and innovate, and they gain intrinsic satisfaction from their growing accomplishments. Learners challenge each other to take risks, monitor each others' progress, share strategies, and provide encouragement. "Mistakes" are accepted as a natural and even humorous part of learning. Above all, such learning is *embodied;* it engages the whole child – the cognitive, affective, motor-sensory and social "self".

Such observations might invite us to ponder how teacher-directed pedagogies of mathematics might play a significant role in widely recognised disaffection, marginalization and alienation in young learners' experiences of school mathematics, and to consider the merits of enhancing children's participation in their own mathematical learning. Support for a participant-determined pedagogy can be found in Pollard (1997) who describes how teachers might provide for a *negotiated curriculum,* arguing that "rather than reflect the judgments of the teacher alone, it builds on the interests and enthusiasms of the class" and noting that, "Children rarely fail to rise to the occasion if they are treated seriously. The motivational benefits of such an exercise are considerable" (p. 182). The children's thoughtful responses when asked during the research study how maths time could be better for them, confirm Pollard's assertions, while illustrating how teacher-directed pedagogies both defined and constrained mathematical learning for the children.

Researcher:	If you were the maths teacher what sorts of things would you have at maths time?
Jared:	Easy work…Playing games. (Late Year 3)
Jessica:	I'd like it if we did it together (Late Year 4)
Georgina:	Have more time, like we have half an hour on maths and we don't hardly have any time to do it. (Georgina, Mid Year 5)
Jessica:	Long enough for me to get stuck into it and start enjoying it. And then once I've started getting a bit bored, I think 'I want to finish this.' (Mid Year 5)
Dominic:	Just playing a bit more games. (Late Year 5)
Liam:	I wouldn't really do it [maths work] I'd just play the games. (Late Year 5)
Peter:	Um, probably more maths games and…more drawing things. (Mid Year 5)

Between the discourse of enhanced achievement through teacher-directed pedagogies of mathematics and the discourse of learner participation, efforts to increase learners' ownership can be discerned. The New Zealand Ministry Education (1997) for example encourages "allowing students to have some control over their own learning and assessment by involving them in planning learning and assessment activities" (p. 21). Hiebert et al., (1997) advocate learners' *adjustment* or *shaping* of mathematical the teacher-selected tasks while continuing to support the teacher's primary role in task selection. They advise teachers to "select tasks with goals in mind", and state that "although the selection of tasks does not require wildly creative or clever ideas, it does require careful thought about the mathematics landscape and about the way in which a series of tasks might lead students across a landscape" (p. 163). Community participation in negotiated curriculum content has also been suggested within the discourse of ethnomathematics as an effective approach for culturally distinct and traditionally marginalised groups (e.g., Lipka, 1994).

Writers such as Apple and Beane (1999), Cotton (2001), Skovsmose and Valero (2002) and Gates and Vistro-Yu (2003) examine intersecting discourses of democratic process and mathematics education, probing the dilemma that has challenged mathematics educators in recent times: valuing learners' right to freedom and independence on the one hand, and increased accountability for learners' progress by means of tighter control of the learning process, on the other. At the root of the dilemma lies educators' reluctance to entertain the notion that young learners have a legitimate role in determining *what* they learn and *how*. Davis (1996) captures this when stating that "a mathematical task should impose 'liberating constraints' which are intended to strike a balance between 'complete freedom' (which would seem to negate the need for schools in the first place) and no freedom at all" (p. 97).

DISCUSSION

Children's limited opportunities for participation within teacher-directed pedagogies of mathematics can be viewed as a *human rights* issue. The United Nations Charter of Universal Rights of 1947 identifies the rights of each human individual in terms of *needs*, including the need to belong, to feel safe, to be accepted and respected, and to be fully included in making decisions affecting their lives. These rights have been expressed specifically for children through the UN Convention on the Rights of the Child (CRC) of 1990, now ratified by 191 countries. The CRC upholds children's rights to participation. Article 12 confers "the child who is capable of forming his or her own views the right to express those views freely in all matters affecting the child", and Article 13 states "the child shall have the right to freedom of expression" (UNICEF, 2002, pp. 63–64). In their statement to the UN General Assembly's Special Session on Children in 2002, representatives from the Children's Forum issued a vision statement of a world in which children's rights are protected. It states, "We see the active participation of children: raised awareness and respect among people of all ages about every child's right to full and meaningful participation, in the spirit of the CRC, and children actively involved in decision-making at all levels and in planning, implementing, monitoring and evaluating all matters affecting the rights of the child" (UNICEF, 2002, p. 11).

Such statements imply that the rights of children to participate as self-determining citizens *in all areas that affect their lives* must include their education. Teacher-directed, task-driven mathematical learning cultures fail to recognize these principles. The exclusion of children in determining curriculum may be considered not only an abuse of children's rights to participate, but also as an instrument of cultural hegemony, as can be clearly seen throughout the island nations of the Pacific region where mathematics curricula closely adhere to colonisers' imported models.

The manner in which teachers select and 'set' tasks for learners, manage learners' engagement with the tasks, and use such tasks to determine what learners know and can do, says much about traditional relationships between adults and children. Feelings of disempowerment are captured in this child's telling statement:

> Dominic: Like, when I just get back from school I have to do, like, about four questions of (maths) homework and that really pisses me off. (Interview, late Year 4)

A changing relationship between the teacher and learner of mathematics is suggested by rights-based discourse. As Neyland (2004) argues, a postmodern ethical orientation to mathematics education "will shift the focus away from procedural compliance and onto the direct ethical relationship between teachers and their students" (p. 69). From a postmodern view, it is within discursive formations that such relationships are produced and maintained. Reframing the teacher/student relationship is therefore both contingent upon and made possible by changing educational discourse.

A compelling vision of child-inclusive schools is provided by the UNICEF (2003) report on the state of the world's children. It describes international efforts to establish *child-friendly* cultures of schooling, particularly in developing countries. One of the listed characteristics of a child-friendly school is: "it involves children in active *participatory* learning" (p. 89). It argues that a *human rights approach* is needed in efforts to improve conditions for children, in which "people are recognized as key actors in their own development, rather than passive recipients of commodities and services," and where "participation is both a means and a goal" (p. 93).

In focusing upon a discourse of *participant-determined* pedagogy, we might shift our gaze from child as *educational product* to child as *growing and valued member of a local community*, and child as *global citizen*. Within such discourse, a rights-based, participant-determined pedagogy of mathematics might embrace some of the following principles:

- Children have the right to negotiate, with help from parents and teachers and within national guidelines, a meaningful and relevant mathematics curriculum.
- Children have the right to engage in flexible mathematical learning situations collaboratively shaped with help from teachers, in an ongoing process.
- Children have the right to engage in mathematical learning situations whose broad goals (rather than specific outcomes) are mutually recognized by children and teachers as part of a multicultural mathematical landscape.
- Children have the right to engage in mathematical learning situations in their own time, at their own pace, and in a manner of their choosing.
- Children have the right to choose with whom to engage in mathematical learning situations, seeking support, information and assistance from a variety of sources, not just teacher or textbook.
- Children have the right to personal techniques of working mathematically.
- Children have the right to assess their own mathematical learning when and as they choose, supported by - rather than restricted to - collaboratively constructed assessment criteria.
- Children have the right to mathematical learning opportunities and assessment methods that operate to enhance the well-being of all children.

CONCLUSION

Moser et al. (2001) state that "the definition, interpretation and implementation of rights are dynamic processes that are inherently political in their nature" (p. 11). UNICEF's advocacy of child-friendly learning environments in which *children's rights as global citizens* are taken into account in line with the principles of the CRC, compels us to re-examine current pedagogies of mathematics from a rights-based and therefore essentially political, perspective. The almost universal practice of teacher-directed task management in mathematics education must be reconsidered within the discourse of children's rights to participation. Although some writers (e.g., Dowling, 2001; Vithal, 2003) caution that the aims of participative mathematics education - emancipation and empowerment of children – may be little more than *myth* since even the most well-intentioned intervention may serve to reinforce rather than redress existing inequalities, within international discourse that both increasingly recognizes the vulnerabilities of children and their need for greater protection, and values the contribution children can make to the development of local and global communities, the right of children to substantial involvement in determining their own learning has significant implications both as a growing ethical expectation and as a legal requirement of education. Osler and Starkey (2001) stress that "if schools are to ensure the greater participation of young people in decision making in line with the Convention on the Rights of the Child, schools must not only provide structures for participation, but also equip children with the skills to participate" (p. 100). As one of the most politicized of school learning areas, mathematics education must take a leading role in acknowledging the participatory principles of the CRC and considering its implications for classroom practice.

---

[i] "Wantok" literally means "one talk", referring to those related by a kinship and/or village spoken language

REFERENCES

Apple, M., & Beane, J. (1999). *Democratic schools: Lessons from the chalk face.* Buckingham, UK. Open University Press.

Brown, T. (2001). *Mathematics education and language*: *Interpreting hermeneutics and post-structuralism.* Dordrecht: Kluwer Academic Publishers.

Carpenter, T., Fennema, E., Loef Franke, M., Levi, L., & Empson, S. (1999). *Children's mathematics: Cognitively guided instruction.* Portsmouth, NH: Heinemann.

Cotton, T. (2001). Mathematics teaching in the real world. In P. Gates (Ed.), *Issues in mathematics teaching* (pp. 23–37). London: RoutledgeFalmer.

Dowling, P. (2001). Mathematics education in late modernity: Beyond myths and fragmentation. In B. Atweh (Ed.), *Sociocultural research on mathematics education* (pp. 19–36). Mahwah, NJ: Lawrence Erlbaum Associates.

Doyle, W. (1988). Work in mathematics classes: The context of students' thinking during instruction. *Educational Psychologist*, *23*(2), 167–180.

Davis, B. (1996). *Teaching mathematics: Toward a sound alternative*. New York: Garland.

Ernest, P. (2001). Critical mathematics education. In P. Gates (Ed.), *Issues in mathematics teaching* (pp. 277–293). London: RoutledgeFalmer.

Foucault, M. (1977). *Discipline and punish: The birth of the prison.* London: Allan Lane.

Gates, P., & Vistro-Yu, C. (2003). *Is* mathematics for all? In A. Bishop, M. Clements, C. Keitel, J. Kilpatrick & F. Leung (Eds.), *Second international handbook of mathematics education* (pp. 31–73). Dordrecht: Kluwer Academic Publishers.

Heibert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). Making sense: Teaching and learning mathematics with understanding. Portsmouth, NH: Heinemann.

Lipka, J. (1994). Culturally negotiated schooling: Toward a Yup'ik mathematics. *Journal of American Indian Education, 33*(3), 44–48.

Malloy, C. (2002). Democratic access to mathematics through democratic education: An introduction. In L. English (Ed.), *A handbook of international research in mathematics education* (pp. 17–26). Mahwah, NJ: Lawrence Erlbaum Associates.

Ministry of Education, (1997). *Developing mathematics programmes*. Wellington: Learning Media.

Moser, C., & Norton A., with Conway, T., Ferguson, C., & Vizard, P. (2001). To claim their rights: Livelihood security, human rights and sustainable development. Background Concept Paper, Workshop on Human Rights, Assets and Livelihood Security, and Sustainable Development. London: UK. 19–20 June, 2001.

Neyland, J. (2004). Toward a postmodern ethics of mathematics education. In M. Walshaw (Ed.), *Mathematics education within the postmodern* (pp. 55–73). Greenwich, CT: Information Age Publishing.

Osler, A., & Starkey, H. (2001). Legal perspectives on values, culture and education: Human rights, responsibilities and values in education. In J. Cairns, D. Lawton & R. Gardner, (Eds.), *Values, culture and education: World Yearbook of Education 2001* (pp. 85–103). London: Kogan Page Limited.

Pollard, A. (1997). *Reflective teaching in the primary school: A handbook for the classroom* (3rd. ed.). London: Cassell.

Skovsmose, O., & Valero, P. (2002). Democratic access to powerful mathematical ideas. In L. English (Ed.), *A handbook of international research in mathematics education* (pp. 383–408). Mahwah, NJ: Lawrence Erlbaum Associates.

Thrupp, M., Mansell, H., Hawksworth, L., & Harold, B. (2003). 'Schools can make a difference' – but do teachers, heads and governors really agree*? Oxford Review of Education*, *29*(4), 471–484.

UNICEF. (2003). *The state of the world's children 2004.* New York: The United Nations Children's Fund.

UNICEF. (2002). *A world fit for children.* Suva: UNICEF Pacific.

Vithal, R. (2003). *In search of a pedagogy of conflict and dialogue for mathematics education*. Dordrecht: Kluwer Academic Publishers.

Walls, F. (2003). *Sociomathematical worlds: The social world of children's mathematical learning in the middle primary years*. Unpublished PhD Dissertation, Victoria University of Wellington, New Zealand.

To be or not to be? Pre-service Teachers' Beliefs and Practices Towards Reform

Lyn Webb
Nelson Mandela Metropolitan University
<lyn.webb@nmmu.ac.za>

This study investigates the attitudes and beliefs of five pre-service Post-Graduate Certificate of Education (PGCE) teachers towards the outcomes-based teaching style required for the Further Education and Training (FET) band for Mathematics. The students completed the Standards Belief Instrument developed by Zollman and Mason and an instrument to investigate teachers' beliefs developed by Pehkonen and Törner. The student teachers were interviewed separately about their views of teaching mathematics and were given an opportunity to showcase their creative teaching styles in videotaped lessons. In this paper one of these student teacher's views are examined. During the course of a single lesson, this pre-service teacher displayed various, and at times opposing, teaching styles. It appears that Skott's (2004) claim that the motives of the teacher's activity do not necessarily depend on espoused beliefs, but emerge in the course of complex classroom interactions, is borne out in this study.

Introduction

South Africa is facing the introduction of a new outcomes-based curriculum in the Further Education and Training (FET) band in schools i.e., grades 10, 11 and 12 (Department of Education, 2002). According to Lappan (2000) the worldwide trend of curriculum change in schools creates a whole new role for teachers as they have to be effective in engaging learners in problems in context. But do teachers believe this approach will enhance learning, and do they endeavour to use a problem-solving approach in the classroom?

Studies concerning the beliefs of teachers and their practice in the classroom have been researched extensively over the years (Brodie, 2001; Ensor, 1998; Ernest, 1989; Hoyles, 1992; Lerman, 1986, 2002; Pehkonen & Torner, 2004; Thompson, 1992; Skott, 2001; Stoker, 2003). In this study the beliefs of five Postgraduate Certificate in Education (PGCE) students at the University of Port Elizabeth (UPE) were elicited using interviews, questionnaires, graphical and numerical self-evaluations and videotaping lessons in the classroom. The views of one particular student, Sina (not her real name), were interrogated in order to gauge whether her elicited beliefs were constant or contradictory, and whether her expressed beliefs were mirrored in her classroom practice.

Beliefs and Perspectives

Beliefs concerning the nature of mathematics can be seen on a continuum, from an 'absolutist' viewpoint, in which mathematical truth is unquestionable, certain and objective at one pole, to a 'fallibilist' viewpoint, in which mathematical knowledge can be seen as a social construction and is therefore fallible (as it can be revised and corrected), at the other pole (Ernest, 1989; Lerman, 1986).

Perceptions about learning mathematics can also be represented on a continuum —from mastery of skills to problem solving, and the opposite poles of a continuum of views concerning the teaching of mathematics can be represented by the notions of the teacher as instructor or the teacher as facilitator. (Lerman, 1986). According to Schoenfeld (1985) most students view mathematics as a body of knowledge to be memorized, despite the fact that teachers often emphasise the importance of understanding the subject. He also notes that learners experience neither understanding nor a perception of utility of the subject in practice. These findings have been supported by studies of South African teachers who, despite professing beliefs in a constructivist paradigm, used traditional approaches that led learners to see mathematics as a subject to be memorized (Stoker, 2003).

Pehkonen and Törner (2004) used qualitative and quantitative methods to achieve methodological triangulation to investigate teachers' beliefs about mathematics. They used the questionnaire, 'Conceptions of Teaching Mathematics', and a method of numerical and graphical self-estimation, both of which appear as appendices to this paper. The theoretical underpinning of their research is ascribed to the philosophy of Dionne (1984, in Pehkonen & Törner, 2004), who states that beliefs of the nature of mathematics can be divided into three perspectives, i.e., the 'Toolbox', 'System' and 'Process' perspectives

Table 1
Dionne's (1984) perspectives of the nature of Mathematics

Perspective	**Description**
Toolbox	Mathematics is seen as a mastery of skills. Doing mathematics involves calculating, using rules, following procedures and manipulating formulae.
System	Mathematics is a language of logic and rigour. Doing mathematics requires using a precise and concise language to express mathematical ideas.
Process	Mathematics is a constructive process where doing mathematics draws from real-life experiences and finds relationships between various constructs.

Ernest (1989) uses similar divisions, i.e., instrumentalist, platonist and problem-solving views of mathematics. The *instrumentalist* view sees mathematics as a set of unrelated but utilitarian rules and facts. Mathematics is an accumulation of facts, rules and skills that are to be used in the pursuance of some external end. The *platonist* view of mathematics is that of a static, but unified body of certain knowledge – implying that mathematics is discovered not created. The *problem-solving* view sees mathematics as a dynamic, continually expanding field of human creation, a cultural product, which is constantly being revised and constructed. These views impact on the implementation of a curriculum based on constructivist learning principles in the FET phase.

CORRELATION BETWEEN BELIEF AND PRACTICE

Research shows that there is not always a correlation between the beliefs teachers verbalize and their practice in the classroom (Ernest, 1989; Lerman, 1986, 2002). According to Ernest (1989) incompatibility is attributed to constraints and opportunities dictated by the social context of teaching. Lerman (1986) also feels that school-context factors are more likely to influence ways of teaching than attitudes and beliefs. Thompson (1992) cites some of the issues that complicate the relationship between beliefs and practices, such as the social context in the form of expectations by learners, parents and colleagues; authority and control; and evidence of mathematical understanding in learners.

Hoyles (1992) maintains that inconsistencies between beliefs and practices are accentuated when teachers are faced with an innovation, which in our case is the introduction of a contextual, problem solving approach to teaching mathematics (Department of Education, 2002). She takes a stance somewhat different to that of Ernest, Lerman and Thompson by advocating that a mismatch between beliefs and practices stems from 'situated beliefs', i.e., that situations, context and culture are co-producers of beliefs, and a mismatch between beliefs expressed outside the classroom and practices demonstrated inside the classroom should be expected. Ensor (1998) supports this notion, purporting that beliefs are not stable across contexts, and that differences in social situations result in multiple positioning of teachers. She suggests that the repertoires of knowledge and skills that one acquires could be called 'beliefs' which, in turn, are fore-grounded and back-grounded according to the context in which the person is operating at the time. Pehkonen and Törner (2004) maintain that beliefs are subject to continuous evaluation and change, a view echoed by Lerman (2002), who states that changes in beliefs affect practice and that change in practice, in turn, affects the beliefs of the practitioner.

An interesting slant to the research on beliefs is provided by Skott (2001), who maintains that there is the simultaneous existence of multiple, and possibly conflicting, communities of practice that emerge in the course of classroom interaction. He sees communities of practice developing where the contributions of both individuals and groups become accepted in the class and become part of the mathematical discourse. The role of the teacher is to sustain these individual and collective learning opportunities by adjusting his or her teaching style to each situation on the spot. He does not focus on the congruence or conflict between beliefs and practices, but attempts to disentangle the ways in which the multiple communities interact and frame the emergence of different strategies in teaching practice.

METHOD

In this study five pre-service PGCE teachers were given the Standards Belief Instrument (Zollman & Mason, 1992, in Furner, 2004) in order to gauge their attitudes towards reform in teaching (appendix A). They were also given a validated questionnaire developed and tested by Pehkonen (2004) on conceptions of teaching mathematics (appendix B). In addition they filled in a table developed by Pehkonen and Törner (2004) based on Dionne's perspectives concerning Toolbox, System and Process views of mathematics. They were asked to distribute a total of 30 points corresponding to their estimation of their 'real' teaching of mathematics and their 'ideal' teaching of mathematics using Dionne's Toolbox, System and Process definitions. The pre-service teachers were also asked to mark a point on an equilateral triangle with 'x' to indicate their 'real' teaching of mathematics and to indicate their 'ideal' teaching of mathematics with an 'o' (appendix C). The vertices of the triangle represented the Toolbox, Process and System perspectives on mathematics as propounded by Dionne (1984, in Pehkonen & Törner, 2004).

The students were interviewed about their views of teaching mathematics and were given an opportunity to showcase their teaching styles during a videotaped lesson in the classroom. For the purposes of this paper the views of one particular student, Sina (not her real name) are reported.

RESULTS

In the Standards Belief Questionnaire (Zollman &Mason, 1992, as cited in Furner, 2004) the scores (1= strongly disagree, 2= disagree, 3= agree, 4= strongly agree) were totalled and Sina scored 46, despite leaving out three questions relating to kindergarten and intermediate phase mathematics, which were not relevant to an FET teacher. This indicates that she has a tendency towards reform beliefs in teaching, encompassing beliefs that mathematics should be a meaningful, problem solving activity where active learning and good reasoning are encouraged.

During the interview, Sina re-iterated her commitment to teaching reform i.e., that she believed in a learner-centred, constructivist teaching approach. However, she made some contradictory statements, for example her statement;

> Developing the skill of critical analysis and problem solving through mathematics is very valuable in all facets of life… Free thinking is so important.

was contrasted with;

> I think that through repetition and practice certain things become automatic without parrot-style memorization.

This statement suggests that, although Sina subscribes to a problem solving philosophy (Process perspective), she sees repetition and mastery of skills as important in mathematics (Toolbox perspective).

Sina's response to the Pehkonen and Törner (2004) questionnaire indicated that she has an innovative, learner-centred approach to mathematics teaching. She was indecisive about the role of proofs and the role of visualization in teaching mathematics, as she scored 3 for questions 6 and 9, however, she was positive about using varied application exercises, problem solving, developing thinking skills and stressing understanding (she scored 1 or 2 for questions 1, 4, 7 and 12). The results when using both questionnaires (Standard Beliefs and Pehkonen and Törner's questionnaire) were consistent as Sina expressed a constructivist view of mathematics learning and teaching in both.

It is interesting to note that in the interview she was ambivalent about the role of understanding. When asked whether she felt it was more important to teach skills or emphasise learners' understanding when teaching, she answered;

> Neither. I know this is the easy way out, but it is true. I believe both are equally important and are used together most of the time.

In the self-evaluation table Sina misread the instructions and distributed 20 points instead of 30 between the Toolbox, System and Process perspectives of real and ideal teaching. She gauged her 'real' teaching of mathematics to be mainly Toolbox (9) > System (7) > Process (4). She viewed the 'ideal' teaching of mathematics to be Toolbox (8) = Process (8) > System (4). This correlated with the ideas expressed in the interview that both skills and understanding are of equal importance. This exercise also showed that she did not feel that her 'real' teaching style encouraged a constructive process.

In the visual representation of her 'real' teaching style, as opposed to 'ideal' teaching style, Sina positioned her 'real' teaching style (represented by x) midway between Toolbox and System, far away from a process approach, as is shown in Figure 1. Her view of an 'ideal' teaching style (represented by o) tended towards process, but remained midway between Process and Toolbox. This graphically mirrored her results in the self-evaluation table. An advantage of the graphic method is that one can draw a vector from the 'real' to the 'ideal' view to indicate how far from the 'ideal' the teacher views her 'real' teaching to fall. In this case Sina showed that she felt a balance between the three perspectives, leaning away from a system approach, was an ideal perspective.

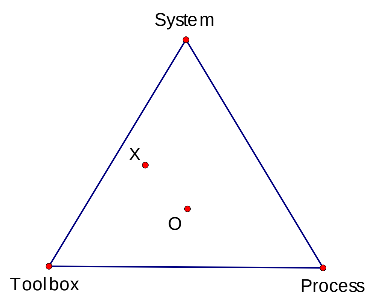


Figure 1: Sina's graphical representation of her 'real' (x) teaching style and her 'ideal' (o) teaching style.

On reviewing the videotape of her lesson in the classroom, two contrasting incidents happened where she enacted reform-based teaching methods and diametrically opposed traditional teacher-centred teaching methods.

In the first instance Sina was showing the whole class how to find the area of a circle if given the radius. She stated the formula and wrote it on the board. She emphasized to the whole class that;

> All these examples are exactly the same. Even if you do know, that's fine, we'll go over it all again. There's only one method to do all of these because they're all the same, just different numbers so if you get the method right you can do all of them – a, b, c, d.

Sina proceeded to go through a few examples finding the area of a circle before giving them more of the same exercises to complete. She also emphasized that the answers should all be expressed in m^2;

> …because if you don't you'll be wrong and you'll lose marks.

In this incident Sina demonstrates a Toolbox perspective of mathematics where mastery of skills and applying rules is emphasised.

In the second incident, Sina arranged the class in pairs and emphasized that this was a co-operative learning technique where discussion in pairs around solving the problem was essential. She gave them a problem and said;

> I am trying to get you to use all the things you have learned in the last two weeks in one sum.

She emphasised the process rather than the product by stating;

> I want you to hand it in so we can see where you are going wrong.

The problem involved a castle on a piece of circular land with a moat around it. Sina introduced a contextual element by discussing with the class the reason for a moat;

> In the olden days they had castles and they used to keep the baddies out with a moat. What happened when their friends came? They just put a drawbridge over it.

The problem involved finding the area of the rectangular castle, the area of the circular land and the area of the moat. Although the problem was a thinly disguised skills-based exercise, Sina had endeavoured to contextualise it and engage the learners in co-operative problem solving.

In the course of a single lesson Sina had used two opposite styles of teaching with the same learners. She had expected them to learn through a repetitious approach and a co-operative approach where the learners communicated in pairs in order to solve a contextual problem. In the interview afterwards she saw no disjuncture between the two perspectives.

CONCLUSION AND IMPLICATIONS

According to Brodie (2001), South African teachers can change considerably regarding mathematical knowledge and knowledge of pedagogy, but they have difficulty in changing their teaching practice towards methods of engaging learners in a learner-centred approach. Skott (2001) claims that the motives of the teacher's activity emerge in the course of classroom interactions and that previously espoused beliefs may become less significant, depending on the particular context at the time.

It is apparent that Sina's perspectives on mathematics were relatively constant whether she completed a questionnaire or self-evaluation task, as she consistently expressed reform-based views. However, her classroom practices reflected both a Toolbox approach and a Process approach—in Ernest (1989) and Lerman's (1986) terms she demonstrated an 'absolutist' viewpoint and a 'fallibilist' viewpoint—at different occasions during the same lesson. She emphasised mastery of skills at one stage and a problem solving approach at another stage of the lesson. She acted as both instructor and facilitator. She demonstrated both a traditional 'chalk-and-talk' style as well as a more innovative contextual, problem solving approach. Her classroom practices, therefore, were at times consistent and at times inconsistent with her verbalised beliefs—they changed according to the context.

As such the debate concerning the correlation between beliefs and practice continues, and I suggest that notions of context and situated beliefs have become increasingly more demanding in terms of recognition and focus when attempting to implement curriculum reform, such as demanded by the National Curriculum Statement for mathematics in South Africa (Department of Education, 2002).

REFERENCES

Brodie, K. (2001). Changing practices, changing knowledge: Towards mathematics pedagogical content knowledge in South Africa. *Pythagoras, 54*, 17–25.

Ensor, P. (1998). Teachers' beliefs and the 'problem' of the social. *Pythagoras, 46/47*, 3–7.

Ernest, P. (1989). The impact of beliefs on the teaching of mathematics. In P. Ernest (Ed.), *Mathematics teaching: The state of the art* (pp. 249–254). London: Falmer Press.

Furner, J. (2004). Implementing the National Council of Teachers of Mathematics Standards: A slow process. *Pythagoras, 59*, 46–56.

Hoyles, C. (1992). Illuminations and reflections: Teachers, methodologies and mathematics. In *Proceedings of the Sixteenth Psychology of Mathematics Education Conference,* 3, (pp. 263–286). Durham, USA.

Lappan, G. (2000). A vision of learning to teach for the 21st century. *School Science and Mathematics, 100*(6), 319–325.

Lerman, S. (1986). Alternative views of the nature of mathematics and their possible influence on the teaching of mathematics. PhD dissertation: University of London.

Lerman, S. (2002). Situating research on mathematics teachers' beliefs and on change. In G. Leder, E. Pehkonen & G. Törner (Eds), *Beliefs: A hidden variable in mathematics education?* (pp. 233–243).Netherlands: Kluwer Academic Publishers.

Department of Education. (2002). National Curriculum Statement: Grades 10–12 (Schools) Mathematics, South Africa: Department of Education.

Pehkonen E., & Törner, G. (2004). Methodological considerations on investigating teachers' beliefs of mathematics and its teaching, *Nordisk Matematikk Didadtikk, 9*(1), 21–49.

Schoenfeld, A. H. (1985). Students' beliefs about mathematics and their effects on mathematical performance: A questionnaire analysis. Paper presented at the annual meeting of the American Educational Research Association, Chicago.

Skott, J. (2004). The forced autonomy of mathematics teachers. *Educational Studies in Mathematics*, *55*(1-3), 227–257.

Skott, J. (2001). The emerging practices of a novice teacher: The roles of his school mathematics images. *Journal of Mathematics Teaching Education, 4*(1), 3–28.

Stoker. J. (2003). *An investigation of mathematics teachers' beliefs and practices following a professional development intervention based on constructivist principle*. Unpublished PhD dissertation: Curtin University of Technology.

Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127–146). New York: Macmillan Publishing Company.

APPENDIX A: STANDARDS' BELIEF INSTRUMENT

Directions: Shade in the answers that best describe your feeling about the following statements on the scantron grid provided. Use the following code:

1 = strongly disagree 2 = disagree 3 = agree 4 = strongly agree

1	Problem solving should be a SEPARATE, DISTINCT part of the mathematics curriculum.	1	2	3	4
2	Students should share their problem-solving thinking and approaches WITH OTHER STUDENTS	1	2	3	4
3	Mathematics can be thought of as a language that must be MEANINGFUL if students are to communicate and apply mathematics productively	1	2	3	4
4	A major goal of mathematics instruction is to help children develop the beliefs that THEY HAVE THE POWER to control their own success in mathematics.	1	2	3	4
5	Children should be encouraged to justify their solutions, thinking, and conjectures in a SINGLE way.	1	2	3	4
6	The study of mathematics should include opportunities of using mathematics in OTHER CURRICULUM AREAS.	1	2	3	4
7	The mathematics curriculum consists of several discrete strains such as computation, geometry, and measurement which can be best taught in ISOLATION.	1	2	3	4
8	Learning mathematics is a process in which students ABSORD INFORMATION, storing it easily retrievable fragments as a result of repeated practice and reinforcement.	1	2	3	4
9	Mathematics SHOULD be thought of as a COLLECTION of concepts, skills algorithms.	1	2	3	4
10	A demonstration of good reasoning should be regarded EVEN MORE THAN students' ability to find correct answers.	1	2	3	4
11	Appropriate calculators should be available to ALL STUDENTS at ALL TIMES.	1	2	3	4
12	Learning mathematics must be an ACTIVE PROCESS.	1	2	3	4

(Zollman & Mason, 1992, as cited in Furner, 2004, p. 56.)

APPENDIX B: A QUESTIONNAIRE FOR TEACHERS CONCEPTION OF TEACHING MATHEMATICS

Through the following questionnaire, we would like to get a profile of your ideas and conceptions concerning teaching mathematics. These are some statement on teaching mathematics. Circle the option which best describes your opinion.

1 = fully agree	2 = agree	3 = don't know	4 = disagree	5 = fully disagree

1	1	2	3	4	5	In teaching mathematics, one should use varied exercises and applications above all else.
2	1	2	3	4	5	Mathematics in school necessarily requires a concrete dimensions; abstract mathematics alone is not enough.
3	1	2	3	4	5	Logic is promoted in teaching mathematics, whereas creativity and originality are not stressed.
4	1	2	3	4	5	Problem orientation should be the core of teaching mathematics.
5	1	2	3	4	5	In teaching mathematics, finished products take priority, not the process by which they are achieved.
6	1	2	3	4	5	Doing mathematics means: working through the proofs carefully.
7	1	2	3	4	5	Teaching mathematics provides an excellent opportunity to promote the development of the pupils' thinking.
8	1	2	3	4	5	Mathematics teaching is especially meant for talented pupils.
9	1	2	3	4	5	One should always make sure to visualize aspects of teaching mathematics.
10	1	2	3	4	5	Indisputable formality takes priority in mathematics.
11	1	2	3	4	5	Learning calculation techniques is the core of teaching mathematics.
12	1	2	3	4	5	While doing mathematics, understanding the topic is the most important idea.
13	1	2	3	4	5	In teaching mathematics, one should often realize projects without subject limits.

(Pehkonen & Törner, 2004, p. 45.)

APPENDIX C

Starting point: A rough classification of mathematical views consists of the following three perspectives, which are part of every view of mathematics and the teaching of mathematics:

T Mathematics is a large toolbox: Doing mathematics means working with figures, applying rules and procedures and using formulas.
S Mathematics is a formal, rigorous system: Doing mathematics means providing evidence, arguing with clear and concise language and working to reach universal concepts.
P Mathematics is a constructive process: Doing mathematics means learning to think, deriving formulas, applying reality to Mathematics and working with concrete problems.

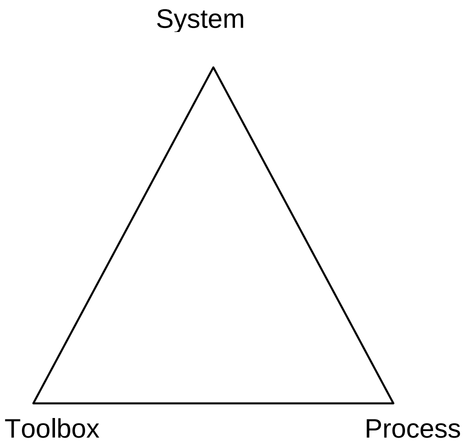
Question 1: Distribute a total of 30 points corresponding to your estimation of factors, T, S, and P in which you value your…

	T	S	P
… real teaching of mathematics			
… ideal teaching of mathematics			

For additional comments please use the reverse side of this page.

Question 2: Acknowledge your position on the three factors mentioned above by marking points within the equilateral triangle below.

x = real teaching of mathematics
o = ideal teaching of mathematics



For additional comments please use the reverse side of this page.
Thank you very much!
(Pehkonen & Törner, 2004, p. 46).

EDUCATION FOR SUSTAINABLE DEVELOPMENT: IMPLICATIONS FOR MATHEMATICS EDUCATION?

Keiko Yasukawa
University of Technology Sydney
<keiko.yasukawa@uts.edu.au>

In 2005, we have entered the UN Decade of Education for Sustainable Development. In this paper, I argue that mathematics educators and mathematics education researchers can make a contribution to educating for sustainable development. I make an important distinction between education *about* sustainable development, and education *for* sustainable development; the latter is the more important, but also the more difficult and challenging. The paper explores avenues forward by considering mathematics as a resource for imagining and creating new realities, and by proposing actor-network theory as an analytical tool for understanding the agency of mathematics in creating new realities.

This year marks the beginning of the United Nations Decade of Education for Sustainable Development 2005–2014. What, if any, implications does this have for mathematics education and mathematics education research? I will argue in this paper that while there are some "obvious" roles that mathematics education and mathematics education research have in education for sustainable development, there are some further roles that are less obvious and more complicated, because education *about* sustainable development is not equivalent to education *for* sustainable development. My premise is that mathematics educators and mathematics education researchers have contributions to make to, and we should be actively engaging with, education and educational research for sustainable development. Given that sustainability is about the relationship between the world now and into the future, I believe that we need to pay special attention to how we can engage young people in this project of sustainability.

Sustainable development is a concept that has become a familiar one (though still a contested one) since the 1987 publication of *Our Common Future*, better know to some as the Bruntland Report, arising out of the work of the World Commission on Environment and Development, chaired by the then Prime Minister of Norway Gro Harlem Bruntland. In this report, sustainable development was defined as:

> Development that meets the needs of the present, without compromising the ability of future generations to meet their own needs (WCED, 1987).

Thus sustainable development is about achieving social and environmental justice both now and into the future. And as expressed in the principles of Agenda 21 which emerged out of the 1992 United Nations Conference on the Environment and Development (also known as the Rio Earth Summit), sustainable development requires us to think globally and to act locally; that is, every action we take as individuals in our local contexts has global consequences. Agenda 21 became a key guiding principle for setting targets and developing strategies for sustainable development, and following the Earth Summit, individual nations were mandated to develop a Local Agenda 21 at various levels, for example, at national, state, and local council levels (Bennett, 2001).

The concern for a sustainable future dates further back in history than the Bruntland Report. Some would argue that the concept already existed through the notion of stewardship in many traditional cultures and philosophies (see for example, Bennett, 2001; Hay, 2002). Much of the literature on recent environmental movements attributes a key source of inspiration and stimulus from the publication of Rachel Carson's *Silent Spring* (1962) that exposed the devastating impact of unfettered uses of chemical pesticides in the Great Lakes region of the United States. Since then, there has been a significant growth in both popular movements and academic studies around the general themes of environmental justice and sustainable development. The academic studies have necessitated both an interdisciplinary approach to take account of the interactions between issues of quality of life of human beings and the natural environment in which we live, and the birth of new specialisations such as environmental engineering, environmental sociology, eco-design, eco-tourism and so forth, to generate new knowledge and practices in existing disciplines and professions.

> Sustainability is complex and complicated, with no single discipline definitively addressing either the problems or solutions: it incorporates technological, philosophical, economic, social, ecological, political and scientific dimensions. This may be illustrated through an examination of real-world issues or projects that are motivated by concerns over sustainability – for example, in Green architecture, eco-design, gender and development; integrated and sust*ainable transport; global citizenship; and lifelong learning* (Blewitt, 2004, p. 2).

Why should we, as mathematics educators, be concerned about education for sustainable development? And if we are concerned, what avenues are there for engagement and activism in this complex, complicated and interdisciplinary endeavour that needs to address the social and environmental justices issues of the current and future generations, both locally and globally? I consider three possibilities below.

ACKNOWLEDGING THE POWER OF MATHEMATICS

Mathematics is a powerful resource for describing the realities around us, including aspects of the social and physical environment in which we live. In particular, mathematics provides us with a tool for producing mathematical models of environmental processes such as: changes in the weather, population changes of endangered species, breakdown of different types of wastes; and of social trends such as changes in the distribution of wealth, levels of literacy and numeracy, access to services, and so forth. What various people have called critical mathematics, numeracy or mathemacy (Frankenstein, 1989; Johnston & Yasukawa, 2001; Skovsmose, 1994) can provide people with the skills and inclination to question how mathematical information is created, presented and used. Mathematics, and critical mathematics education in particular, therefore offer important knowledge and tools for gaining a critical perspective on social and environmental injustices represented by mathematical models.

As Davis and Hersh (1986) point out, mathematics can not only describe aspects of reality; it also has predictive and prescriptive functions. Thus mathematics is a tool for predicting or forecasting what could happen if certain controls or interventions were not prescribed to stop it from happening. In education about the future world, mathematics then has a very important part to play. Indeed one of the most powerful examples of the role of mathematics in describing, predicting and prescribing conditions in which we live, was the project in 1972 undertaken by a group of economists, scientists and businessmen calling themselves the Club of Rome; their goal was no less than to simulate the world system through a necessarily complex mathematical model. The simulation enabled them, with using what was advanced computing power at that time, to examine what the world would be like if the then current rate of growth continued. Their less than optimistic results and the caution against continuing with "business as usual" was published in *Limits to Growth* (Club of Rome, 1972). While some of their methods and predictions are now critiqued as simplistic, *Limits to Growth* did send a profound word of warning in the Western world about the pursuit of unlimited growth. In Australia, there has been a more recent exercise undertaken by the Commonwealth Scientific and Industrial Research Organisation (CSIRO) Division of Sustainable Ecosystems to construct several population, technology, resources, and environment scenarios to analyse Australia's options for the future (Foran & Poldy, 2002).

Another Australian example that illustrates the importance of critical mathematics as a resource for understanding the future impact of current practices relates to the uncovering of "lies, damned lies and economic models" (Hamilton, 2001) that were used by the Australian government to campaign against the endorsement of the 1997 Kyoto protocol on reducing greenhouse emissions. Through critical examination of the mathematical models that were used to argue the Government's case, Hamilton and his colleagues were able to uncover both the technical weaknesses of the model, and the interests of the people who constructed the model (interests groups in the coal industry), thus discrediting the claims made by the Government.

So mathematics is a powerful resource for students now and in the future for understanding the social and physical world they live in, and the predicaments associated with continuing to support the social and economic trends that we see in many countries today. Access to mathematical knowledge and skills is therefore an important part of education for sustainable development.

ENSURING ACCESS TO MATHEMATICS EDUCATION

As most of the participants in this conference would be aware, access to mathematical knowledge and skills remains a challenge for many groups of people around the world. The projects of many mathematics educators and mathematics education researchers in pursuing the democratic access of mathematical knowledge (see for example, Skovsmose & Valero, 2001; Penteado & Skovsmose, 2002) will therefore play a particularly important role in education for sustainable development.

It is not only because mathematics is a resource for students to learn about their social and physical worlds that access to and equity in mathematics education are important. It is also because mathematics remains a critical gatekeeper for access to higher levels of education generally, and for employment. Emerging research in mathematics and work (Bessot & Ridgway, 2000; Wedege, 2000; Zevenbergen, 2004) suggest that new technologies and new forms of work require different forms of numeracy that are more relevant to the competencies that are required in the workplace. They are supported by Castells' writing about work in the new *e-economy*:

> The e-economy cannot function without workers able to navigate, both technically and in terms of content, this deep sea of information, organising it, focusing it, and transforming it into specific knowledge, appropriate for the task and purpose of the work process.
>
> This kind of labor must be highly educated and able to take initiatives. Companies, large or small, depend on the quality and the autonomy of labor. Quality is not simply measured in years of education, but in type of education. Labor in the e-economy must be able to reprogram itself, in skills, knowledge and thinking according to changing tasks in an evolving business environment. Self-programmable labor requires a certain type of education, in which the stock of knowledge and information accumulated in the worker's mind can be expanded and modified throughout his or her working life. This has extraordinary consequences for the demands placed on the education system, both during the formative years, and during the constant re-training and re-learning processes that continue through adult life (Castells, 2001, p. 91).

Thus, there is ongoing research that is needed as work practices and expectations for entry in the workforce change. Furthermore, in many countries, including Australia, there have been changes to the labour market which have meant "the death of career, the decline of standard hours and the rise of casualisation" (Hamilton, 2003), and an increasing number of people are employed on short-term or casual contracts with limited career prospects (Watson, et al., 2003). In Australia, with changes to the industrial relations legislations over the last decade, the power of the trade unions have been severely curtailed, and an increasing number of workers are forced to negotiate their wages and conditions on an individual basis (Watson, et al., 2003). Thus being critically numerate is an even more important attribute for workers to access and remain in the contemporary workforce now, and if this trend continues, into the future.

MATHEMATICS EDUCATION *FOR* SUSTAINABLE DEVELOPMENT?

Both of the abovementioned reasons for arguing the role for mathematics education in this decade of education for sustainable development—mathematics as a powerful resource for modelling the world, and access to mathematics education as a critical prerequisite to an individual's chances for survival and success in society—are in some ways "obvious" ones, and many inroads have already been made in these areas by critical mathematics educators and researchers. I would argue, however, that both of these reasons are more to do with education *about* sustainable development, and not necessarily education *for* sustainable development. As a result of having access to mathematics education, and learning the theories and skills of mathematics, a person may be able to gain an insight into their social and physical world in ways that were previously not possible for them. They may even reflect on what they are now able to see with the aid of mathematics, and gain an understanding of the political, economic and cultural assumptions that have led to the state of the world as it is, and the future that it is heading to. But none of this may lead to the individual actively influencing the way the world is heading. That is, we cannot assume that simply because people are aware of the problems that face us and our future generations, they are going to take educated action to change the world into one that is more sustainable. One way of understanding this distinction between education *about* and *for* sustainable development is to consider the following comment about sustainability:

> Sustainability is both a practical and moral subject. It is interdisciplinary as much a matter of concern to the humanities (Said, 1993) as to the sciences. It is, at once, an inescapable dilemma of our time, a matter of study and reflection, and challenge to action. It raises questions about globalization and personal responsibility. It constitutes, in fact, all that a discipline calls for: a greater understanding and a basis for moral authority of knowledge (Cullingford, 2004, p. 250).

In other words, mathematics education for sustainable development will require what Mezirow calls *perspective transformation* whereby a person becomes critically aware of how their ways of understanding the world have been shaped by existing presuppositions, then reformulating those assumptions to generate "a more inclusive, discriminating, permeable, and integrative perspective; and [to make] decisions or otherwise [act] upon these new understandings" (Mezirow, 1990, p. 14). In some of the conceptions of critical mathematics, numeracy or mathemacy mentioned earlier, this link between reflection and action is implied. That is, critical mathematics/ numeracy/ mathemacy should empower people to take action to change the situation they see themselves in towards something that is closer to what they can aspire. This idea is discussed by Skovsmose (2002) when he argues that mathematics educators should perhaps pay less attention to theorising influences of students' backgrounds, and pay greater attention to their *foreground* in order to understand the *politics of learning obstacles*.

There are indeed some formidable learning obstacles, if by mathematics (or any) learning for sustainable development, we are expecting learners to gain understandings and skills to take action based on a moral stance. The German sociologist Beck has written about living in post-industrial society characterised as a *risk society* in which "the dark sides of progress increasingly come to dominate social debate. What no one saw and no one wanted – self-endangerment and the devastation of nature – is becoming the motive force of history" (1995, p. 2). In the risk society, Beck argues that there is a crisis of identity which is:

> … not being overcome by a productive turn from the passivity of 'one' to the activity of 'I'; rather people become vulnerable to the expansive grasp of flourishing sensation industries, religious movements, and political doctrines. Fun and joy, pain and tears, fantasy, memory, and attention to the moment, hearing, seeing, and feeling all lose their remaining traditional responsibilities for the self and are determined by facts driven by market-expanding fashions (Beck, 1995, p. 59).

In the Australian context, Hamilton talks about the *growth fetish* and that "social democracy is being superseded by a sort of market totalitarianism. When older people bemoan the corruption of modern politics, they nevertheless feel that it is a historical aberration impinging on the constancy of democratic rights and that in the end the people can still have their say. Disturbingly, younger people hear only the accusation that the system is incurably corrupt—and they believe it" (Hamilton, 2003, p. 21). The culture of consumerism must then be considered as a major force that can limit young people's engagement with acting *for* sustainable development.

A study on young people's consumption patterns (UNESCO & UNEP, 2001) provides some clues as to how the forces of consumerism can be understood. 5,322 survey responses from young people aged 18–25 in 24 countries led to the following conclusions:

> The young public in the survey believes that the environmental impact of consumption is linked to the use of products and the recycling process, rather than to shopping behaviour.
>
> They seem to prefer unorganised forms of everyday action to organised mobilisation as a strategy to improve the world.
>
> The young public shares many of the same values; however, the social aspects of sustainable consumption appear to be more important in Africa, Asia and Latin America as compared to other areas of the world (UNESCO & UNEP, 2001, p. 44).

While the report shows that young people are concerned about issues of sustainability, the report pays attention to the disjunction between this concern and young people's capacity to change some of their very behaviour that may be threatening the environment. A close link between consumer products and personal identity formation of young people emerges; this then has associated with it some challenges when a young person sees the need to change their consumption pattern for the greater project of sustainability. In another study about young people, based in Germany, Tully (2003) examined how technologies such as the cell (mobile) telephones, computers, the Internet and cars acquired meanings closely linked to their personal identify formation, and were very different to the utilitarian meanings that these technologies may hold for the older generations. Tully argues that young people's understanding of the relationship between technology and society is constructed through these technologies that are shaping who they are, rather than the big technologies of industrialisation such as nuclear power plants, large dams and factories that have both supported and generated critiques about the notion of technological determinism. Tully calls the latter category of technologies *Technology I* and the personal, "gadget" category of technologies *Technology II*. He argues that Technology I is based on rational, utilitarian purposes, whereas, Technology II is oriented towards emotional and experiential purposes.

How can mathematics (or any education) generate a critical response to patterns of behaviour that are so closely linked to young people's personal identity? Do mathematics educators have a role in posing such a challenge? Susan George (2004) claims that educators have precisely that responsibility when she argues in her book *Another world is possible if …* that that possibility exists "if educators educate", and "[t]hose who genuinely want to help the movement should study the rich and powerful, not the poor and powerless. … Although wealth and power are always in a better position to keep their secrets and hide their activities, …, any knowledge about them at all will be valuable to the movement. The poor and powerless already know what is wrong with their lives and those who want to help them should analyse the forces that keep them where they are" (George, 2004, p. 211).

LOCATING THE MATHEMATICS IN EDUCATION FOR SUSTAINABLE DEVELOPMENT

So how do we go about this project when there are suggestions that young people, particularly in affluent societies, are immersed and distracted by the lures of consumer goods and gadgets? How do we engage them in learning for sustainable development when the learning may lead to actions that threaten their lifestyles and identities? Where does mathematics fit in this already complicated project which may seem even more frightening to many students than "normal" mathematics which may be difficult, but at least leaves students' life outside the classroom alone. Here, we need to know that the reason that mathematics is such a powerful resource is not because it helps us to describe *what is,* but because it can also help us to imagine *what can be*, and in many cases help to shape and realise that imagination (Skovsmose, 1999). Skovsmose has called this creative dimension of mathematics as the *formatting power* of mathematics, and this power has been explored in the case of a particular "virtual" world that was imagined and realised through the resources afforded by classical number theory (Skovsmose & Yasukawa, 2004). I believe if mathematics can be examined with learners not only as a resource for critically understanding the world that is (and feeling doomed because it looks so bad), but also for imagining a world that can be, then we may be getting closer to educating *for* sustainable development.

Mathematics, however, is becoming increasingly invisible and packaged, as the study by Skovsmose and Yasukawa (2004) showed. It is packaged within technologies as algorithms which the user or consumer does not see nor understand. When young people see the relationship of technologies to society through consumer goods as Tully's study (2003) showed, and where these technologies become linked intimately with who they are and what they do, the mathematics that is embedded in these technologies become even more invisible. But all of these technologies needed mathematics to be imagined and realised, however invisible the mathematics has become to the end user. Mathematics is also a critical agent in the market system and how it operates to enrol people into a culture of consumerism. The way that the system can fine-tune its market for the goods that are produced requires calculations of production costs, affordability for the target market, the optimum life for the product, profit margins and so on. Again, the mathematics that is involved in bringing the gadgets to the young people may not be visible in its entirety, but without the mathematics, the market may fail to deliver what is attractive and affordable enough for the target market, while being dispensable and quick and cheap enough to produce for the producers.

Using mathematics as a resource for imagining a better, more sustainable world which is closely connected with, but which reaches beyond, the immediate lives of young people may require us and our learners to also examine the agency of mathematics in socio-technical processes. *Actor Network Theory* (ANT) (see for example, Latour, 1987; Law & Hassard, 1999) has gained acceptance (as well as critique) as a viable and inclusive resource for studying the interactions between human and non-human actors in socio-technical change processes. It is a way of understanding how various interests represented by human actors and indirectly by non-human actors such as technologies (including "gadgets", texts, management systems, and so forth) become enrolled into a dynamic network that gains momentum towards some end. In fact, Callon (1999) has successfully used ANT to analyse the market system as an actor network. In terms of understanding and using the resource of mathematics in bringing about sustainable socio-technical change, ANT may provide us with a perspective for understanding how mathematics is located in the process of imagination and realisation of what is imagined (Yasukawa, 2003). It will offer a way for young people to see how mathematics shape the systems and technologies that can shape their lives, and how mathematics interacts with other resources that are used to shape people's lives and the environment in which they live.

I started this paper with a premise that mathematics education and research have critical roles to play in educating for sustainable development. In conclusion, I believe that we must give considerations to the propositions that follow:

- Sustainability is about both social and environmental justice. Thus it requires an interdisciplinary inquiry involving learning and research across disciplines, as well as into informal knowledges that are shaping people's lives.
- Sustainability concerns the lives of both the current and future generations. Thus the project of sustainable development must be inclusive of intergenerational interests, and itself be sustainable.

- Sustainability will be experienced locally by individuals, but can only be achieved and sustained through globally shared visions and strategies.
- Consumerism is a powerful force that is limiting the power of young people to adopt lifestyles that are sympathetic to sustainability goals. Thus education for sustainable development must engage young people in imagining an alternative world that offers their aspirations to be realised without compromising those of others.
- Mathematics is a resource for imagining and realising an alternate reality. Education for sustainable development must build on this power.
- Mathematical theories are packaged and invisible in many technologies and systems. ANT may be a useful tool for excavating the mathematical agency in socio-technical systems, and understanding how mathematics is acting in such systems.
- Mathematics education for sustainable development will force teachers, researchers and students to step outside the security of traditional disciplinary boundaries, and put on different lenses to view mathematics, education, and research into mathematics education.

ACKNOWLEDGEMENTS

Thanks to my friend and colleague Betty Johnston for allowing me to verbally "brainstorm" the ideas for this paper to her when the ideas were still very confused. Having an audience like her at the earlier stage of writing a paper was very encouraging and helpful.

REFERENCES

Beck, U. (1995). *Ecological enlightenment: Essays on the politics of the risk society*. New Jersey: Humanities Press.

Bennett, D. (2001). Development of sustainability concepts in Australia. In J. Venning & J. Higgins (Eds.), *Towards sustainability: Emerging systems for informing sustainable development* (pp. 22–47). Sydney: UNSW Press.

Bessot, A., & Ridgway, J. (Eds.) (2000). *Education for mathematics in the workplace*. Dordrecht: Kuwer Academic Publishers.

Blewitt, J. (2004). Introduction. In J. Blewitt & C. Cullingford (Eds.), *The sustainability curriculum: The challenge for higher education* (pp. 1–9). London: Earthscan.

Callon, M. (1999). Actor-network theory – The market test. In J. Law & J. Hassard (Eds.), *Actor Network Theory and after*, (pp. 181–195). Oxford: Blackwell Publishers.

Carson, R. (1962). *Silent spring*. London: Penguin.

Castells, M. (2001). *The Internet galaxy: Reflections on the Internet, business and society*. Oxford: Oxford University Press.

Club of Rome. (1972). *Limits to growth*. New York: Universe Books.

Cullingford, C. (2004). Conclusion: The future – Is sustainability sustainable? In J. Blewitt & C. Cullingford (Eds.), *The sustainability curriculum: The challenge for higher education* (pp. 245–252). London: Earthscan.

Davis, P. J., & Hersh, R. (1986). *Descartes dream: The world according to mathematics*. London: Penguin Books.

Foran, B., & Poldy, F. (2002). *Future dilemmas: Options to 2050 for Australia's population, technology, resources and environment*. Retrieved 20 February, 2004, from http://www.cse.csiro.au/research/futures/futuredilemmas/index.htm

Frankenstein, M. (1989). *Relearning mathematics: A different third R – Radical maths*. London: Free Association Books.

George, S. (2004). *Another world is possible if …*. London: Verso.

Hamilton, C. (2003). *Growth fetish*. Sydney: Allen & Unwin.

Hamilton, C. (2001). *Running from the storm: The development of climate change policy in Australia*. Sydney: UNSW Press.

Hay, P. (2002). *Main currents in western environmental thought*. Sydney: UNSW Press.

Johnston, B., & Yasukawa, K. (2001). Numeracy: Negotiating the world through mathematics. In B. Atweh, H. Forgasz & B. Nebres (Eds.), *Sociocultural research on mathematics education* (pp. 279–294). New Jersey: Lawrence Erlbaum Associates.

Latour, B. (1987). *Science in action: How to follow scientists and engineers through society*. Milton Keynes: Open University Press.

Law, J., & Hassard, J. (Eds.) (1999). *Actor Network Theory and after*. Oxford: Blackwell Publishers.

Mezirow, J. (1990). How critical reflection triggers transformative learning. In J. Mezirow & Associates (Eds.), *Fostering critical reflection in adulthood*. (pp. 10–20). San Francisco: Jossey-Bass Publishers.

Penteado, M. G., & Skovsmose, O. (2002). *Risks include possibilities*. Centre for Research in Learning Mathematics, Danish University of Technology, Roskilde University, Aalborg University.

Skovsmose, O. (2002). *Students' foreground and the politics of learning obstacles*. Centre for Research in Learning Mathematics, Danish University of Technology, Roskilde University, Aalborg University.

Skovsmose, O. (1999). *Mathematical agency and social theorising*. Centre for Research in Learning Mathematics, Danish University of Technology, Roskilde University, Aalborg University.

Skovsmose, O. (1994). *Towards a philosophy of critical mathematics education*. Dordrecht: Kluwer Academic Publishers.

Skovsmose, O. ,& Valero, P. (2001). Democratic access to powerful mathematical ideas. In L. English (Ed.), *Handbook of international research in mathematics education* (pp. 383–407). New Jersey: Lawrence Erlbaum Associates.

Skovsmose, O. & Yasukawa, K. (2004). Formatting power of 'mathematics in a package': A challenge for social theorising? *Philosophy of Mathematics Education Journal*, *18*(October). Retrieved 20 January, 2005, from http://www.ex.ac.uk/~PErnest/pome18/contents.htm

Tully, C. (2003). Growing up in technological worlds: How modern technologies shape the everyday lives of young people. *Bulletin of Science, Technology & Society*, *23*(6), 444–456.

UNESCO (United Nations Educational, Scientific and Cultural Organisation) & UNEP (United Nations Environment Program). (2001). *Youth, sustainable consumption patterns and life styles*. Retrieved 20 January, 2005, from http://unesdoc.unesco.org/images/0012/001242/124238e.pdf

Watson, I., Buchanan, J., Campbelll, I., & Briggs, C. (2003). *Fragmented futures: New challenges in working life*. Sydney: The Federation Press.

WCED (World Commission on Environment and Development). (1987). *Our common future*. Oxford: Oxford University Press.

Wedege, T. (2000). Technology, competences and mathematics. In D. Coben, J. O'Donoghue & G. E. FitzSimons (Eds.), *Perspectives on adults learning mathematics: Research and practice* (pp.191–207). Dordrecht: Kluwer Academic Publishers.

Yasukawa, K. (2003). Towards a social studies of mathematics: Numeracy and Actor-network theory, in S. Kelly, B. Johnston & K. Yasukawa (Eds.), *The Adult Numeracy Handbook: Reframing adult numeracy in Australia* **(**pp. 99–107). Sydney: NSWALNARC and Language Australia.

Zevenbergen, R. (2004). Technologizing numeracy: Intergenerational differences in working mathematically in new times. *Educational Studies in Mathematics, 56*, 97–117.

HABITUS AND NEW NUMERACIES FOR CONTEMPORARY WORK

Robyn Zevenbergen
Charles Sturt University, Australia
<rzevenbergen@csu.edu.au>

School mathematics has been under criticism for some time in terms of the teaching approaches, curriculum and assessment methods. While a small proportion of students are successful in their study of school mathematics, many leave the institutions with adverse experiences, perceptions and learnings of mathematics. With considerable changes in the workplace and the world beyond schools, serious questions need to be posed as to what should constitute school knowledge and practice. Using Bourdieu's theoretical framing, this paper proposes that changed social conditions have resulted in the construction of new habitus among younger generations and that such habitus provide very different dispositions for viewing, working and interpreting social worlds – of schools and work. Drawing on data from a three-year project, it is argued that younger generations possess and approach numeracy practices in substantial different ways from older generations. The implications for school mathematics are considered.

Internationally, there is a concern about the literacy and numeracy levels of young people. Politicians, community members, employers and other sectors of the wider community bemoan what they see as declining standards. Reactions have varied but in many Western countries governments have instigated policies aimed at education and training sectors that focus on the development of basic skills. Such skills are frequently couched in conservative interpretations of basic knowledges and pedagogies and quite referred to as 'back to the basics' reforms. However, within these reforms, little consideration has been given to the changing social, economic and employment conditions and what skills, knowledges and dispositions are needed for contemporary times. This paper draws on data from a three-year project that investigated the numeracy needs and practices of contemporary work and concluded that young people approach numeracy work in quite different ways from their predecessors. To theorise this different orientation to work and numeracy, the work of Bourdieu is used. In particular, his notions of field, habitus, and capital have been most useful.

In structuring this paper, I have adopted a somewhat different structure to the traditional conference paper. First, I provide an overview of the project to give readers a sense of the outcomes arising from the various phases. The tensions in the data and field are discussed. An overview of Bourdieu's constructs are then provided and these are then used to theorise the project with the intention of reconceptualising 'basic skills' within a post-industrial framing.

PROJECT SUMMARY

The project was a three-year project aimed at identifying the numeracy practices in contemporary work with the explicit intention of improving numeracy learning for young people. In hindsight, it was premised on a deficit notion that young people were in some way/s hindered by their lack of numeracy understandings and that such deficits impinged on their capacity to participate in work and life effectively and efficiently. The research was broken into three distinct phases. Phase One employed a large survey in which 6 groups were targeted – employers, educators, and job placement officers were those people in senior positions who worked with young people as they entered the world of work. A second collective, young people of 22 years of age or younger were surveyed and included young people in school who worked in part-time positions, young people in work and young people seeking work. These two data sets - senior and junior participants – were then compared. Of the 60 variables, significant differences were noted in nine key items, of which 5 of these were related to numeracy, 3 to technology and 1 to literacy (Zevenbergen, 2004). No differences were noted between the two cohorts in generic skills. From the open-ended responses and follow up interviews, data indicated that the differences appeared to be related to how participants viewed basic skills. Senior participants appeared to perceive basic skills in terms of the three Rs where arithmetic and mental calculations were a key part of their understandings of core knowledge. In contrast, younger participants indicated that they saw that their tasks were more about problem solving, estimation, and thinking about tasks holistically and that mental calculations were best undertaken with technology. They tended to describe their dispositions to numeracy as being about deferring cognitive labour to technology so that estimating and problem solving were more critical to how they undertook their work.

The second phase of the research involved workshadowing young people in work as they went about their work. Using a stimulated recall methodology where the young people were photographed as they worked and then asked at the end of the workshadowing period (usually 3-5 days) what they were doing and how they were thinking as they went about their work. As reported elsewhere, this process confirmed the survey data whereby young employees deferred cognitive labour to technology and used problem solving techniques and estimation in their work in quite extensive ways. As this outcome could be theorized as being a consequence of newcomers or apprentices (Lave & Wenger, 1991) and that with experience they may undertake tasks differently, the data were then taken back to senior staff including those working with the workshadowing participants as well as extending the data set to a wider cross section of the community. The interviews with senior staff initially indicated that the young employees were taking 'shortcuts' in their work but employers confirmed that the strategies being used were effective and in many cases, good use of time and resources which had not been adopted formally by the company.

As the emerging data seemed to indicate that young people approached numeracy in different ways than expected by older generations, the final phase of the project involved a community

TENSIONS BETWEEN OUTCOMES AND THE FIELD

The project outcomes suggest that young people approach their numeracy practices in ways that are significantly different from their older peers. While such differences could be theorized as being due to novices and experts, the research sought to clarify the effectiveness of the strategies being used by young employees. Rather than focus on the minutiae, they prefer to work with a more holistic perspective—seeing the task as a whole—whereas older participants saw the task broken into many smaller tasks, of which one key one was often mental arithmetic. Millennials were comfortable deferring the cognitive labour to technology while older generations saw it as imperative that young employees were able to calculate and measure with high degrees of accuracy and that technology was not an integral component of their workplace. One employer was adamant that young staff did not need calculators on the shop floor. Rather, in his words, he claimed that "they already had a calculator, it was on their shoulders" (employer).

The differences in orientations to numeracy by the Millennials and non-Millennials may be seen as a difference in experience. However, when Millennials were interviewed who assumed senior positions in relevant fields to the Millennial worker, there was synergy between the Millennials descriptions rather than similarities between the positions held. For example, in the case of the builders, the employer was a very successful Millennial who employed many young people in his work. His views on how to undertake numeracy tasks were similar to his employees and he expressed disdain for how his older (in his words, "over 30") undertook their work. As such, what emerge from the data was very much a congruence among young people, i.e., Millennials and how they undertook numeracy practices and a congruence among those non-Millenials in how they saw numeracy practices. As such, it appeared that there was greater similarity based on ages than roles in positions. This was confirmed in both quantitative and qualitative data (Zevenbergen, 2004; Zevenbergen & Zevenbergen, 2003).

While the use of intergenerational differences has been a productive way to make sense of the data, it fails to advance the theoretical framing of the outcomes. In order to make deeper sense of the data, Bourdieu's theoretical framing offers considerable potential.

BOURDIEU'S THEORETICAL PROJECT

Bourdieu's work has been most useful in theorizing the intergenerational differences that have been observed in this project. In particular, his notions of field, habitus and cultural capital are most relevant. To a lesser extent, his notions of trajectory and symbolic violence are of value in rethinking the different orientations and rewards for engagement in contemporary workplaces. Each of these constructs are discussed below and linked to the project findings.

FIELD

In terms of the project, the field is that of "work" where the project sought to identify the numeracy practices across a range of contemporary workplaces. Mahar et al. (Mahar, Harker, & Wilkes, 1990) argue that "Fields are at all times defined by the set of objective relations of power between social positions which correspond to a system of objective relations between symbolic points… The structure of the field is defined at a given moment by the balance between these points and among the distributed capital." Within this framing, the field of work is defined by how those who control power relations (that is, employers and job placement officers) convey status on those who will gain and retain employment. For young people seeking work or retaining a position in work, this means that they are expected to show particular attributes that make them employable.

In his study of English employment training, Hodkinson (2001) similarly draws on Bourdieu to theorise the field and argues that the stakeholders are involved in a common game (using Bourdieu's game analogy), but where each of the stakeholders (employers, trainers, young people, parents, etc) are striving for different ends. He argues that "each stakeholder brings capital to the game … which gives them access to the power to influence the game" (p. 263). Those stakeholders who posses more capital—whether cultural, economic or symbolic—can exert more power to influence the game (or field). As Bourdieu (1984) argued, those players with the most resources are able to exert the most force over how the game is played. One only has to consider the power of the employer who can hire and fire as he/she desires (within limits) in comparison with the young person seeking work.

In the context of this study, the field of work is defined by the objective relations imposed by those in power, in this case employers and job placement officers and through the subjective relations of those within the field. For example, to gain access to positions, such as checkout operators in a supermarket or room attendants in a resort, young people needed to sit a suitability test. Whether implemented by employers or placement officers, the tests consisted of a range of questions, most of which were mathematically orientated. The test questions had little to do with the work to be undertaken—spatial pattern matching, ratio, operations with fractions and so one—so that access to work was constrained by particular objective relations. Similarly, for the potential employee, coming to sit these tests often created feelings of anxiety and disempowerment brought about by their school mathematics habitus which had positioned them as marginal students. Thus subjective relations also impacted on the field and who and how access was gained.

In terms of tensions within the study between the field and the outcomes, it became apparent that the objective structuring relations worked to marginalize and exclude some young people. Practices, such as the tests or those more centrally located within the worksite, often failed to recognize the strengths of the

Millennials. Those controlling the field expected particular practices to be undertaken, yet the participants in this study often worked in very different ways than those expected by the employer. For example, in one worksite, the employer wanted the young shopfitter to develop a pricing for a garden bed at the front of the workshop. The employer expected that the employee would develop a list of the goods required (plants, fill, retaining walls etc) and then phone various suppliers and then develop the most cost effective price. The employer anticipated that the task would take the remainder of the day. However, the employee was bored with the conversations at tea breaks so borrowed the secretary's computer during the break time, surfed the net for goods and prices, cut and pasted information and then collated it on a spreadsheet. At the end of the break, the quotation had been developed. This example was replicated in various manifestations across worksites where Millennials used technology in ways unanticipated by employers and in ways that were very cost-effective. While the skills and knowledges the Millennials brought with them to the workplace offered considerable potential to enhance sites, it was often ignored by employers or placement personnel. The state of relations between the various participants within the field determines what is seen as valued knowledge and practice. As Bourdieu (Bourdieu, in (Pierre, Bourdieu, & Wacquant, 1992, p. 99) argues, "…it is often the state of relations of force between the players that defines the structure of the field".

As the field currently exists, those in positions of power control and constrain the field so that particular dispositions, skills, attributes are valued while others ignored. Historically, those dispositions that were valued aligned with the modernist workplace and participants who displayed the desired dispositions were likely to be rewarded. However, with the changes in the wider social and global contexts, questions as to what constitutes valued knowledge within the field may be under challenge as the field moves towards post-industrial times. That is not to say that the field has changed to these new patterns of work, only that there is considerable movement in that direction. As this study has shown, currently, those in positions of power within the field are participants whose own dispositions to work and the values they hold have been shaped by their social conditions which, by and large, were those of the modernist workplace.

The social conditions to which those in power have been exposed have been exposed, create conditions for the construction of particular ways of seeing and viewing the world. This internalization of the culture to which one has been exposed, is what Bourdieu refers to as the habitus.

HABITUS

For Bourdieu (1979), "The habitus is a system of durable, transposable dispositions" which predispose the participant to act, think and behave in particular ways. While there has been some criticism of this construct as being deterministic, Bourdieu and others (Harker, 1984) argue that the habitus can change over time and across circumstances. The habitus is a product of history which is both of product of, and produces, individual and collective practices. Similarly, the habitus with which one enters a particular context can reshape practices within that context. Thus there is a mutual constitution of both habitus and context. For people entering the workplace, existing practices constrain what is possible and hence work to produce particular habitus that are bounded by that context. However, the same can be said for people as they enter an new context, there is considerable scope for them to change that context thus resulting in different circumstances that offer different potentials for alternative habitus constructions. For example, the young boatbuilder enters a workplace that is a product of particular histories. A particular context has changed over time and with those changes, different practices and habitus develop. The young worker entering the boatbuilding context has been shaped by different conditions some which will resonate with the workplace, while others may be quite different and hence offer potential to change existing practices.

In the context of this study, the habitus that the young workers brought to the workplace was shaped by the social conditions within which they have developed. For the participants in this study, we confined their age to 22 years or younger on the basis that this would mean that they would have been in the workplace for some time and have developed a repertoire of skills and knowledges that enabled them to be successful within that context. What had not been anticipated that this was also a defining age (within limits) of those young people who have grown up in a technology rich society. This generation has been called a number of names including Millennials, Generation Y, Gen Why, and Nexters. Unlike other generations, this generation is identified as substantially different from previous ones due to their immersion in a technological world. This has seen to create very different dispositions among this generation – technologically savvy, street smart, immediate gratification (due to technology), etc. While social commentators such as Mackay (1997) in Australia and Howe and Strauss (2000) in the USA have identified clear intergenerational differences, they have not framed them within Bourdieu's framework.

The intergenerational differences brought about through very different social conditions facilitate the construction of peculiar habitus for young people. In this case, the Millennial having grown up in particular social and economic conditions can be theorised as having considerable potential to construct a habitus that is akin to that of her/his peers. As Bourdieu is at pains to argue, this is not to argue that the habitus is a deterministic. It is to suggest that particular social conditions are likely to create particular habitus in young people. The habitus predisposes the participant to act and see the world in particular ways. For Bourdieu, his analyses were on classes but his model can be extended to generations. In the case of this study, what was observed was that the young participants were predisposed to use technology. They were seen to defer the cognitive labour to technology (whether cash registers, computers, or calculators), were more likely to estimate an expected outcome or process and then use the technology as a checking mechanism. Where problems arose, such as in the case of shop assistants and there was a discrepancy between the expected amount to be paid and the actual amount showing on the docket, the assistant did not go back and add the items. Rather, they would scan the docket for potential errors, identify the error and then rescan the item (if too many had been scanned) to remove the item. Thus their ways of working in terms of numeracy were influenced significantly by the technology. As one participant said,

> Shop Assistant: Why would I go and add all these items up. The register is faster, more accurate and more reliable. My job is really to see if they add up to something that looks right. My job is to keep the customer happy and the boss rich. It would take too long to do addition.

It is also important to consider that the technology behind modern cash registers is not just about adding up the amount to be tendered, but significantly about stock control and management. The participants in the study were acutely aware of this role.

The habitus of the young participants and their predisposition to use technology meant that they worked in very different ways from older generations. Not only were they less likely to perform mental calculations, but they were more likely to use estimation, and to problem solve more effectively than their employers and older peers. These dispositions can be seen in the examples provided to date. In terms of the habitus predisposing young people to use technology, older participants often commented on this disposition. In the comment below, offered by an employer in the retail industry, it becomes clear that not only do the young people he has employed display this disposition to use technology, but that young people have an affiliation with technology that may auger well in the future.

> Employer: Young people come into the workplace unafraid of technology. They don't have the respect for it that older people do. They see it as something they take for granted. As we get more technology here, I can see that we will need to employ more young people. They rush into learning with the new cash registers and don't really want to listen when we are training them. They are quite happy to play with them and make mistakes. Sometimes, this can create huge problems when they push wrong buttons and customers get angry but they don't seem too worried about the mistakes – only that the customers are yelling at them.

The dispositions highlighted by the employer (technology use, rushing in, failing to listen, willing to make mistakes i.e., risk taking) are characteristics that have been accredited to this generation in the intergenerational literature cited earlier.

An interesting comment that I will now follow up on is that where the employer notes that in the future "we will need to employ more young people". This comment suggests that the habitus of the Millennial may be something of value to the employers. In terms of Bourdieu's theoretical project, the concept of capital emerges as an important consideration.

CAPITAL

Bourdieu's theory cannot be considered as discrete entities but rather each concept interrelates with the others. Where status is gained in a field, it is via the accumulation of capital within that field. What is seen as capital in one field may not confer status in another. Consider the capital a speaker of BBC English has in the media and most everyday exchanges. This form of language becomes a form of capital that can be exchanged for goods—for example, money through salary as a news reporter. The more that one has of this form of language, the better the bartering power in salary negotiations. However, in another arena, such a counter-culture group such as young people who engage in hip-hop, this form of language will hold little value and hence be of little capital. In contrast, within this field, a different language will convey status and power, and hence be a form of capital.

Bourdieu (1983) proposed three main forms of capital: a) economic capital which is predominantly linked with, or convertible to, money and institutionalised into forms of property rights; b) cultural capital which may be converted to economic capital under particular conditions and institutionalised in the forms of educational qualifications; and c) social capital which may exist in the social connections that people have, and may be institutionalised in the form of nobility titles. Bourdieu (1991) uses the games analogy to describe capital. Using a card game as the metaphor, he argues that "The kinds of capital, like trumps in a game of cards, are powers which define the chances of profit in a given field" (p. 230). How well one succeeds in the game (or field) is determined by the "overall volume of the capital … and the composition of that capital" (p. 231).

Within the field of work, participants who have those dispositions (i.e. habitus) that are seen as valued by those in power, are more likely to be able to trade such dispositions for status. That is, in Bourdieuian terms, the habitus becomes a form of capital that can be exchanged for goods—whether salary or a job. Those who have more of the dispositions valued by the structuring practices within the field are more likely to be positioned more favourably than those who do not have such dispositions.

In terms of tensions with the project, what emerged was a difference in orientation to work. The habitus of the Millennials was different from that of the non-Millennials. While numeracy practices have been highlighted in this paper, other attributes were also noted—punctuality, attitude, dispositions to 'hard work', spending habits—all of which highlighted the differences between what was expected by employers, teachers and job placement officers and what Millennials brought to the workplace. As such, while the Millennials in this project had valuable dispositions that enabled them to work effectively within their workplaces, such dispositions were not recognised by those in positions of power in ways that could be exchanged for other goods.

AN EMERGING THEORETICAL MODEL

Using the games analogy of which Bourdieu was quite fond, it becomes possible to theorise the outcomes of this study through the distribution of capital. In terms of the study, what was observed is that the young participants, through their socialisation, had to come construct a post-industrial habitus. Through their experiences with technology and other aspects of their environments brought about through living in a post-industrial period, they had constructed very different habitus from that of other generations. In entering workplaces where there was still considerable numbers of non-Millennials in positions of power, the field was dominated and controlled by non-Millennials with their particular habitus which, by and large, were considerable less influenced by technology. This is not to say that their habitus was not technological-savvy, but it was likely to be a secondary habitus. That is, their primary habitus formed in their early years had to undergo considerable reconstruction if it were to be technologically-based. In contrast, the Millennials' primary habitus was constructed through their exposure to technology-rich milieux. However, as the study revealed in the third and final phase when findings were taken back to employers and other key stakeholders, there was a recognition that Millennial workers were different but that there was a strong propensity for non-Millennials to hold on to their value system. This system strongly valued mental computation, among other things, to the point where, as noted above, non-Millennials saw the use of calculators as a form of laziness or incompetence (as in earlier cited comment).

The objectified practices within the field, as established by those in positions of power, worked to identify and legitimate particular habitus. Those young people who have the habitus desired by the employers (and other key stakeholders) were more likely to be rewarded, in terms of employment and/or promotion. In this study, what was found was that many job selection processes focused on a very limited repertoire of skills. Many sites used tests to select staff, increasingly so with the numbers being handled by larger organizations. For example in one luxury hotel, the Human Resources department required all potential staff to sit a test. The test consisted mostly of items that were mathematical and/or logic-based. The personnel officer could not justify this test other than it was one we have used for some time. Given that many of the potential staff will not be undertaking work that required these knowledges, the purpose of the test becomes questionable. In the framework being developed for the project, it becomes possible to see that those in power were seeking particular attributes in their employees, most of which did not resonate with the tasks expected of them.

Of more concern to the project team is the future direction of workplace selection, retention and promotion. What has been observed is the failure of the field to recognise the habitus of Millennials that they bring to work. As noted, the Millennial who was able to take minimal time to construct the garden quote has yet to have such dispositions capitalised upon by the field. In this case, and many others, the Millennial worked in a way that was significantly different from that expected by the employer – deferring labour to technology – and in so doing, saved considerable time and money to the employer. As the study has found, Millennials have a habitus that predisposes them to defer cognitive labour to technology. Currently this disposition is not valued within the field, rather, mental calculation is seen as highly valued. As such, the habitus of the Millennial does not constitute a form of capital that is readily exchanged for status or power when those controlling power are non-Millennials. However, as workplaces change and more Millennials gain positions of power within the field, the field and its structuring practices are likely to change. Such shifts were evident in the few Millennials who are in positions of power, albeit marginal in relative terms to the field.

ACKNOWLEDGEMENTS

The Habitus *and New Numeracies for Contemporary Work* project was funded by the Australia Research Council through its Linkage Grants scheme. Industry partners include the Centrelink, Gold Coast City Council, Gold Coast TAFE, Queensland Studies Authority and SCISCO Work Pathways.

A special thanks is expressed to Kelly Zevenbergen who was the research assistant on this project. The researcher and industry partners thank her for the success of the project.

REFERENCES

Bourdieu, P. (1979). *Distinction: A social critique of the judgement of taste* (1984 ed.). Cambridge: Harvard University Press.

Bourdieu, P. (1983). The forms of capital. In J. G. Richardson (Ed.), *Handbook of theory and research for the sociology of education* (pp. 241–258). New York: Greenwood Press.

Bourdieu, P. (1991). *Language and symbolic power* (G. A. M. Raymond, Trans.). Cambridge: Polity Press.

Bourdieu, P., & Wacquant, L. J. D. (1992). *An invitation to reflexive sociology*. Cambridge: Polity.

Harker, R. K. (1984). On reproduction, habitus and education. *British Journal of Sociology of Education, 5*(2), 117–127.

Hodkinson, P. (2001). Use of habitus, capital and field in understanding young people's career decision making. In M. Grenfell & M. Kelly (Eds.), *Pierre Bourdieu: Language, culture and education* (pp. 259–269). Oxford: Peter Lang.

Howe, N., & Strauss, W. (2000). *Millennials rising: The next great generation*. New York: Vintage Books.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge: Cambridge University Press.

Mackay, H. (1997). *Generations: Baby boomers, their parents and their children*. Sydney: Macmillan.

Mahar, C., Harker, R., & Wilkes, C. (1990). The basic theoretical position. In R. Harker, C. Mahar & C. Wilkes (Eds.), *An introduction to the work of Pierre Bourdieu: The practice of theory* (pp. 1–25). Basingstoke: Macmillan Press.

Zevenbergen, R. (2004). Technologising numeracy: Intergenerational differences in working mathematically in New Times. *Educational Studies in Mathematics, 56*(1), 97–117.

Zevenbergen, R., & Zevenbergen, K. (2003). Numeracies for workplaces in New Times. In J. Searle, I. Yashin-Shaw & D. Roebuck (Eds.), *Proceedings of 11th Annual International Conference on Post-compulsory Education and Training* (Vol. 3, pp. 212–219). Brisbane: Centre for Learning Research, Griffith University.

CLASSROOM INTERACTIONS: SCAFFOLDING STUDENTS' LEARNING WITH ICTS

Robyn Zevenbergen
Charles Sturt University, Australia
<rzevenbergen@csu.edu.au>

Steve Lerman
London South Bank University, UK
<lermans@lsbu.ac.uk>

This paper draws on classroom data to explore how teachers scaffold the use of ICTs in mathematics classrooms. Drawing on recent work that explores the ways in which ICTs are potentially creating a new divide between social, cultural, linguistic and geographical groups; the role of pedagogy is examined. In this paper, we employ Bernstein's model to theorise the ways in which one teacher regulated students' use of ICTs within the Microworlds environment.

Since the Industrial Revolution scientific rationality has dominated Western thinking and provided a mechanism for access to power and status. Within global communities, radical social and economic changes brought about through the saturation of technology have resulted in very different patterns of work, status and power. Most significantly, access to technology has brought about changes to work, to national and global economies, so much so that the dominance of mathematics as a social filter is potentially superseded by technology. However, the digital divide is not exclusive of mathematics since much of the thinking and logic embedded within technology is influenced by mathematics.

PEDAGOGY AND THE DIGITAL DIVIDE

While there is a considerable literature emerging on a potential digital divide, there is a similar literature suggesting that ICTs may provide an equitable platform for all students. These contradictory positions are imbued with ideological positions regarding the potentiality of ICTs in learning. It is not our intention to analyse these literatures in this paper (see Zevenbergen, 2004 for a discussion on this), but rather to propose an alternative understanding of how teachers might use ICTs and hence their potential for learning and for equitable outcomes for students. Drawing on the literature on teachers' beliefs as they relate to equity, we propose that a teacher's beliefs about students and learning are powerful determinants in the learning outcomes. For example, the extensive work of Boaler in both the UK and US has shown that the ways in which teachers organise learning environments impacts significantly on the learning outcomes for students—whether these are cognitive, social or affective. How teachers come to organise these learning environments is influenced significantly by their own experiences and beliefs about how students come to learn mathematics. In her recent work in the US, Boaler (2004) reported on one school in the Bay District of San Fransciso. We have had the privilege of working with Boaler in this school and with the data (Boaler, Lerman, & Zevenbergen, 2004). What emerges for us so clearly is the commitment of the teachers to the students. They were firmly committed to providing a demanding mathematics curriculum for students who are considered the most disadvantaged in the schooling system—poor, African-American, Latino/a and/or Asian students. Using a program—Complex Instruction—when providing a quality learning environment, Boaler has shown that the students can learn complex mathematics in deep ways. The belief that the students could learn complex mathematics, and the provision of learning environments that supported such learning have met with considerable success. As Boaler showed, this school was one of the poorest performing schools in the state of California but has now raised achievement levels to well above state averages. Furthermore, the students' achievements do not map onto social background in the usual ways.

Similarly, within the Australian context, Indigenous students perform very poorly on state tests. This cohort of students is consistently at the bottom of all state testing schemes, often by significant differences with the non-Indigenous students. Rurality and remoteness further compound differences in performance as these usually correlate with social, cultural and linguistic factors. Chris Sarra (2005), an Indigenous principal, has shown that believing and reinforcing, with students and staff, that Indigenous students are intelligent and strong, scores on literacy and numeracy have improved dramatically as has attendance at school. Sarra argues that Indigenous students and communities need to believe that they are able to achieve and that schools are critical in providing a venue for this to happen. The achievements of Sarra are well documented in turning his school from an under-performing school to one achieving well above state average. His success with his community has been recognised through numerous awards (e.g., Queenslander of the Year 2004; Deadly Awards 2004).

In an extensive study of effective literacy practices in Victoria (Australia) schools, Hill and Rowe (1998) concluded by arguing that schools did not make a difference on student performance but rather individual teachers were the key determinant in student learning. This study has been controversial in that it challenges a very strong myth that is gaining considerable momentum in Australian schools in particular, that being, that private (independent, wealthy) schools offer better learning environments for students. This has resulted in a steady movement away from state schools whereby there are concerns about the ghetto-isation of public schools. Hill and Rowe (1998) have shown that individual teachers make a difference.

Rather than consider the digital divide as a block to access and success for some students, we propose that teachers can make a difference to student learning through the provision of learning environments. As in Boaler's and Sarra's work, believing that students can learn complex ideas through providing appropriate learning environments, suggests to us that it becomes important to document both practice and teacher beliefs about how best to provide for quality learning.

THEORETICAL FRAMING OF PRACTICE

In this paper we will be examining the practices of one creative and imaginative teacher. We will not be presenting an analysis of his students' learning, however.

Hence the focus of the analysis is on the process of teaching, for which we require a theoretical framework, a language with which to classify systematically the effects of what we describe. For this, we will draw on the work of the sociologist of education, Basil Bernstein. For Bernstein the dominant communicative principle in the classroom is the interactional which regulates 'the selection, organisation, sequencing, criteria and pacing of communication (oral, written, visual) together with the position, posture and dress of communicants' (1990, p 34). The communicative principle offers recognition and realisation rules which need to be acquired by communicants in order to achieve 'competence'.

Drawing on Bernstein's analysis we can attempt to characterise the interactions in Christian's classroom in terms of these parameters. What we know from other studies (e.g., Cooper & Dunne, 2000) is that the framing of the pedagogic interactions can range from strong to weak. In the latter case the pedagogy is invisible, i.e. the recognition and realisation rules are hidden from the students. Middle-class children, however, have generally acquired these rules from their home life and are therefore not disadvantaged by the weak framing, whereas working class children have not and therefore find themselves in a position where they cannot demonstrate their knowledge. This phenomenon is sometimes interpreted as calling for a return to traditional teaching, since here the framing is strong and the rules visible. We know, however, that for many other reasons such classrooms fail most students. Research shows that working within a progressive paradigm but mitigating the weak framing through strengthening some of the features of the pedagogy can make a substantial difference to the success of disadvantaged students (e.g., Morais, Fontinhas, & Neves, 1992).

THE RESEARCH PROJECT

Bearing these studies and exemplars in mind, over the past 2 years we have been involved in a number of research projects that are exploring teaching practice. One project has been documenting teaching practices where ICTs have been used to support numeracy learning. We have also been involved in working with schools on other research projects, including the teacher involved in the other project. As such, we have gained considerable insights into the ways in which he organises learning, the planning he undertakes, and the rationale for how he conceptualises pedagogy.

CLASSROOM ETHOS

The classroom from which the data are drawn is a multi-age classroom consisting of three year levels—Years 5, 6, and 7—so the students are aged from 10–13 years. The teaching space is a double teaching area with two teachers and approximately 58 students. The teachers co-plan their teaching within their classroom and across the sector (i.e., the two other multi-age classrooms at this level). The organization of the classroom is such that the teachers devote a full day's work to these units ("Rich Tasks" as described by Education Queensland 2003). One teacher works with students on aspects of the task that support the learnings needed for developing concepts while the other teacher works with the technological aspects of the unit. As such, at any particular time, approximately ⅓ to ½ of the class could be using ICTs. We have focused on this aspect of the classroom to document how the teachers scaffold student learning when using ICTs to support mathematical learnings.

In this classroom, Christian has established a practice which he describes as only giving them a part of the information that they need and then leaving them to 'discover' what they need. He is happy for the students to share their findings with other students rather than expecting all students to discover what is needed to solve the task. He describes his process as being akin to problem posing where he poses a task for the students who then are given some information which will help them to get started on the problem but will need to work through the task in order to discover what is needed to continue. This form of teaching is commonplace in this classroom.

Christian also offers other prompts to the students. These can be in the instructions or information that he has printed and placed on walls. This enables students to work independently when they are stuck at particular points. This also enables the teacher to work with the students who are working on the problem and not aspects of the task that could be achieved through other means (such as when students have not paid attention to instructions etc).

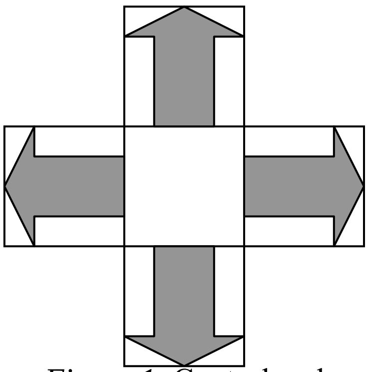


Figure 1: Control pad

In planning the overall Rich Task, the teachers have identified the particular skills and knowledges the students will need to be able to construct the final model. They have then broken these down into smaller tasks and embedded them into more meaningful contexts. One such task required the students to construct a dynamic model of the Solar System. To be able to construct the commands for the turtle to draw the solar system, the students need to be able to program the turtle which requires an understanding of the programming language. This includes directions and distances, as well as constructing shapes. In the extract taken for this paper, the teachers have planned the learning so that the students will be able to develop understandings of direction and distance. In the full day, the students constructed a replica of the game "Frogger" or "Hopper" where a frog hops across a road and river. The students need to construct a pad to control jumps up, down and across. To create the illusion of hopping, the commands are in units of ten with pauses. The students construct a control pad when four turtles have been used for each direction. This enables them to move the turtle around the screen with the arrow keys. This process developed their thinking and knowledge in relation to the programming knowledge while providing a context that engaged the students.

TEACHER BELIEFS AND PEDAGOGY

Over an extended period of time, one of us has been involved extensively with Christian and his school. Christian has taken a considerable leadership role within the school as a teacher-leader, that is, as a classroom teacher, he has been instrumental in instigating and leading many reforms. Most recently, he has been working with two key reforms—New Basics and Philosophy in Schools—which resonate with each other. New Basis is a reform in Queensland schools that radically reconceptualizes curriculum, pedagogy and assessment (Education Qld, 2005). Philosophy in Schools is a movement that exposes teachers and students to the use of philosophy to contest taken-for-granted views of the world through a particular strategy. As part of the New Basics reforms, schooling is broken into three-year blocks (Years 1–3; 4–6; 7–9; and 10–12) and within these blocks, students work on developing repertoires of practices (skills, knowledges, dispositions) that will support them in a rich learning experience (Rich Task). Within such tasks, there is considerable integration of knowledge across discipline areas (transdisciplinary knowledge).

In his approach to teaching, Christian has a disposition to "think outside the square" and decided that rather than make a model where there was papier maché models hanging from the ceiling, students would use computers to do it. Having no knowledge of Microworlds, he began researching how it worked and what he would need to know to support his students' learning.

The learning environment and how Christian had learned about computers became the catalyst for how he reorganised his classroom. Students work in pairs at the computers. There is a main computer that projects on to the white board. Christian works in a traditional mode at the front of the classroom, modelling the examples in a format that is typical of most classrooms. As examples are explained, students control the display computer so that a range of mediums are used simultaneously. Christian supports the students' control of the main computer for a number of reasons which he has progressively discussed over the projects. In his mind, he feels that he understands the basics of how the Microworlds environment works but (some of) the students are more advanced in their knowledge of the program. Usually these were the students who had been in Christian' class in previous years or who had access to technology in out-of-school contexts. These students had become very involved with the package and would spend any free time playing with the software so that they were highly competent with the commands and interface. At one point, he proposed that the students were more competent than him, so he felt that it was most appropriate for them to be in control of the technology. Christian also proposed that it was good role modelling to have students working with the technology in that it created a distribution of control and power within the classroom. Christian would often prompt those with the computer by indicating he was not sure what needed to happen knowing that the students would be able to work though the problem. He also felt that with the students working on the computer modelling the process in a digital format, he was able to provide other mediums for understanding the concepts that were embedded in the lesson.

In setting up the classroom, Christian was keen for the students to interact with each other and to share information. To establish an environment where the students could take risks, Christian saw it as critical that students felt safe in what they did, "that it was OK to make mistakes", "that it was OK to copy off the people next to you"; "that is was fine to ask for help from others"; and "that students needed to share their findings with others". He saw that many of these characteristics were ones that were embraced within the world beyond schools so felt that they should be encouraged in the environment that he was developing with the computers.

Planning

Teaching effectively often requires teachers to think through what is intended and potential barriers to learning (Mousley, Sullivan, & Zevenbergen, 2004). In planning for learning, Christian (and his teaching partner, Mary) undertook a backward mapping process that is commonly used in planning for Rich Tasks (Education Queensland, 2000) or mathematical investigations (Zevenbergen & Griffin, 2005) where the goal is considered and then teachers identify the necessary learning experiences that will create opportunities for students to learn the necessary skills, knowledges and/or processes that will enable them to complete the performance assessment item.

> There were many features of the program [Microworlds] that students would need to use to complete the project successfully. In addition, there were many new mathematical concepts that students would have to come to understand for successful completion. We didn't want to 'teach' these features and mathematical concepts in a formal expository way, rather we wanted to create a problem solving community where students could do as many steps in the problem solving process as they possibly could with teachers providing just the right amount of support at just the right time. With this in mind, we devised a series of small projects, which would enable students to master the necessary program features and mathematical concepts needed for the final major project. We arrived at these particular mini-projects after reflecting on the experiences we had had when we created our own dynamic model of the solar system (Zevenbergen & Judd, 2005).

USING TECHNOLOGY TO ENHANCE LEARNING

ICTs can be used in much the same way as other technologies, including calculators. The extensive literature on calculators has brought forward strong arguments for calculators to enhance deep understandings of many mathematical concepts (Groves, 1993; Ruthven & Chaplin, 1997; Stacey & Groves, 1996) across sectors of schooling. More recently, research on the use of graphing calculators has shown the potential of these technologies to enhance learning. The ways in which Christian plans the use of ICTs to support learning is such that the technology is integrated into the learning environment. His approach is shown below:

> Christian: We use technology in the classroom in the main trying to get it a seamless part of the curriculum. We don't look at the computers as an object of study, we look at them as an opportunity to further the classroom goals. Wherever possible, because we're working with Rich Tasks, the tasks that we elect to do have fairly prominent parts of them that need to be in some way constructed or developed through the use of computers.

From his approach, Christian sees technology as a tool to support learning rather than as an end of itself. This theme is consistently evident in his teaching approach.

THE TASK: CONSTRUCTING A CONTROL PAD USING MICROWORLDS

In this section we draw on an episode from a much larger unit. The unit requires the students to construct a dynamic scaled model of the Solar System using Microworlds. This was the first time the class had used this software package so some scaffolding was needed to support both the mathematical and technological learnings. In terms of the mathematics, the unit was rich in scale and ratio. Students had to construct scaled models of the planets, their moons, and the distance between the planets relative to each other and the sun. This knowledge then had to be transferred into LOGO language and then programmed so that the turtle would be able to draw the Solar System. The data used in this paper is a part of the Unit where the students are constructing the directional control for the Hopper game.

> T points to one of the quadrants and asks S to open up the arrow. Turtle commands appear on the screen
>
> T: OK, these are the instructions. Let's just see if you understand what this one stem of the instruction is saying here.
>
> [points to the first cluster of commands]
>
> T: It says, "Hey Turtle One, I'm talking to you". You can see that with the T1 and then the comma, it is actually saying something to the First turtle. It saying "set your heading to Zero , North [T points north, ie up with large ruler]; Move forward 10 paces {pointing to the next sequence in the command] and do that once.
>
> Teacher then explains that the turtle only needs to do this once or it will go for a lot longer. He then asks student to select the command and then copy.
>
> S copies the command, T waits and Ss chatter while the S does the command copy. T then points to the East arrow and explains that they are now going to look at

what they need to do to get the turtle moving to the East. Asks S to click on East arrow (to bring up command on screen)

T: If we paste in the instruction from before, what will we need to do to get it to go East?

[pauses for a considerable time]

Many students have their hands raised. Some students are working at computers. He stops and then suggests that they think about the question. He then rewords the question by asking:

T: My question is this, What part of the instructions do I need to change to make it go East?

Looks at the S at the computer who is unsure and seeks her input. No reponse, other students mumbling responses. T seeks input from other student who is not sure and offers an incorrect suggestion. T does not correct but stands with ruler pointing up (to represent North) and reminds students that when pointing up, this is zero degrees. He then rotates ruler through 90 degrees to show an Easterly direction

T: So if I move through this many degrees to be going East, what have I done?

Students offer a range of ideas, some debate as to what is being asked. Students talking as a group. Informal consensus emerges as a few students become more convinced that it requires a turn of 90 degrees. Christian stands and listens to students without commenting.

Boy: 90

T: OK, so change that to 90

Boy at computer is a bit slow as he changes the 0 to 90. Teacher asks if it is OK. Change to 90 is done and then teacher tells "Press OK". Teacher points to East arrow with the ruler.

T: OK, so now we have got our North and East Arrows working. So who wants to set up their own arrows now?

Ss indicate that they are ready to go

T: Alright then, away you go.

Ss move to desks and begin to work on their computers. The lesson continues with students working in pairs at the computers. As they create the command for the East button, there is some discussion on recalling the process. When they move to the South button, there is considerable discussion as students negotiate the changes to the commands they need for the southerly direction. As some students co-construct the new command – that being a change to 180 – there are gleeful comments coming from workstations as success is achieved. Students are happy to share their findings with others who ask. Other students continue to work in their pairs until they solve their own command instructions.

In this extract from the lesson, what can be seen is the Christian' embodiment of his beliefs about scaffolding learning.

As we mentioned above, we are examining the teaching of one gifted teacher who is successful in terms of equity not student learning. In order to be able to make justifiable and useful statements about such pedagogy we will now re-examine his teaching using the tools of Bernsteinian framework described above. Our intention is that this will serve as an illustration of the application of one systematic perspective. Clearly the study needs to be extended across a range of teachers and pedagogic forms. We will examine most but not all of the features of the framing; to say more would require a longer paper. Bernstein describes strengths by using + and – signs. Very weak framing, which is characteristic of reform classrooms, would be labelled F^{--}, whereas very strong framing, characteristic of traditional classrooms, is labelled F^{++}.

SEQUENCING

Christian carefully sequenced learning activities so that the overall goal of the teaching (a dynamic solar system constructed in Microworlds) was progressively developed through crafted activities (such as the Frogger example in this paper) which identified particular skills and knowledges that the students needed. These could be mathematical or technological, but in either case, there was a planned pathway for the students to progressively build their learnings from each session to the next. While Christian did not make these explicit to the students, the mini-tasks he developed, were seen as activities in and of themselves but more critically, developed the knowledge needed by the students for subsequent activities. The example provided in this paper is one of many sequenced activities that students undertook as they developed the repertoires needed for the final product. We would label this F^{+}.

CRITERIA

As an essential part of the teaching process, criteria were made explicit to the students. This is one of the key elements of the productive pedagogies framework (Education Queensland, 2000). Christian's class were given these criteria at the commencement of the overall task so that the students were acutely aware of expectations. Throughout the task, criteria were negotiated by the class – to clarify what was meant or to adjust what could be expected as the tasks progressed. This final point needs to be clarified in that the criteria were renegotiated as the demands of the task were realised. In some cases what were anticipated to be easily obtained outcomes were too challenging for the students whereas in other cases, students readily met and exceeded criteria. Throughout the process, students were an integral and central part to the explication and negotiation of criteria. Again we would label this F^{+}.

PACING OF COMMUNICATION (ORAL, WRITTEN, VISUAL)

From the extract and our extensive observations of this classroom, pacing was a key strategy employed by the teacher. Christian readily placed "cheat sheets", diagrams, charts and instructions around the classroom so as to enable students to work through the tasks in ways that were appropriate to them. He did not see such scaffolds as shortcuts as the ethos that had been developed in the classroom was one where students would use the written scaffolds when needed or when they felt overwhelmed. Christian recognised that over time, students engage at different levels—some days students would be highly motivated, other days less motivated.

Through the provision of the sheets, students chose the pathway they needed at particular times.

In terms of his teaching, Christian's oral pacing was quite different to the fast pace of many traditional classrooms. Christian, through his Philosophy training, strongly supported students having time to think before responding. The ethos of the classroom was one where students had learned to consider their responses. This strategy meant that students were given considerable time between the time a question was posed and the seeking of responses. We would label this F^-.

Position of Communicants

As noted earlier in this paper, the positioning of students in the classroom was one where they were in pairs at computers, on the floor or at the back of the room controlling the software. Christian would assume positions all over the classroom depending on the discussion and focus. The very different positions available to students facilitated a very different dynamic in the classroom from that of the more traditional, didactic environment. There was no clearly identified position of control—sometimes this was with Christian, sometimes with the students controlling the software, sometimes with the students on the floor or at desks. We would label this F^-.

Selection of Communication

Christian established the acceptable norms of communication, what was appropriate for students to say, when, and also the degree of control of the communication that he kept in his hands. Such a degree of control, though, is not that of the traditional classroom. We would label this F^+.

Summary

Although the general characteristic of Christian's classroom is that of a liberal-progressive pedagogy, some elements of the form of pedagogy are stronger than is typical. We would argue, in a similar way to that we used when describing the classrooms in Boaler's recent study (Boaler et al., 2004), this strengthening enables the rules for recognising what counts as the task and for realising an appropriate text to be available to a wider range of social backgrounds than is normally the case.

REFERENCES

Boaler, J. Lerman, S., & Zevenbergen, R. (2004). Symposium papers presented at the Annual Conference of the American Educational Research Association. San Diego, April.

Sarra, C. (2005) Keynote address. *Proceedings of the 4th Mathematics Education and Society Conference.* Brisbane: Centre for Learning Research, Griffith University.

Morais, A., Fontinhas, F., & Neves, I. P. (1992). Recognition and realisation rules in acquiring school science: the contribution of pedagogy and the social background of pupils. *British Journal of the Sociology of Education*, *13*(2), 247–70

Education Queensland. (2000). *New basics: Theory into practice*. Brisbane: GoPrint.

Groves, S. (1993). The effect of calculator use on third graders' solutions of real world division and multiplication problems. In Program Committee of the 17th PME Conference (Ed.), *PME XVII: Proceedings of the Seventeenth International Conference [for the] Psychology of Mathematics Education* (Vol. 2, pp. 9–16). Tsukuba, Ibaraki, Japan: University of Tsukuba.

Hill, P. W., & Rowe, K. J. (1998). Multilevel modelling in school effectiveness research. *School Effectiveness and School Improvement, 7*, 1–34.

Mousley, J., Sullivan, P., & Zevenbergen, R. (2004). Alternative learning trajectories. In P. I. R. Faraher & M. McLean (Eds.), *Proceedings of the 26th Annual Mathematics Education Research Group of Australasia Conference* (Vol. 2). Townsville: MERGA.

Ruthven, K., & Chaplin, D. (1997). The calculator as a cognitive tool: Upper-primary pupils tackling a realistic number problem. *International Journal of Computers for Mathematical Learning, 2*(93), 93–124.

Stacey, K., & Groves, S. (1996). Redefining early number concepts through calculator use. In J. Mulligan & M. Mitchelmore (Eds.), *Children's number learning* (pp. 205–225). Adelaide: Australian Association of Mathematics Teachers.

Zevenbergen, R., & Griffin, G. (2005). Financial numeracy: A rich investigation for Year 8 students. In R. Zevenbergen (Ed.), *Numeracy in the middle years*. Canberra: Australian Curriculum Studies Association.

Zevenbergen, R., & Judd, C. (2005). LOGO: Using technology to enhance numeracy learning. In R. Zevenbergen (Ed.), *Numeracy in the middle years*. Canberra: Australian Curriculum Studies Association.

LIST OF AUTHORS

LIST OF REVIEWERS