
**Proceedings of the Eighth International
Mathematics Education and Society
Conference**

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Swapna Mukhopadhyay and Brian Greer (Editors)

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Edited by Swapna Mukhopadhyay and Brian Greer

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MES8 Conference Logo

The MES8 logo design shows a basket by an unknown Klamath artist, courtesy of Portland Art Museum. The Klamath people are a group of Native American tribes of the Plateau culture area in Southern Oregon and Northern California (Klamathtribes.org).

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INTRODUCTION

Development and Aims of MES

The Mathematics Education and Society collective and series of conferences was initiated by Peter Gates and Tony Cotton who, with others, were concerned by the failure of existing organizations within the field to address the socio-political contexts of mathematics and mathematics education. A precursor was the series of three conferences organized under the title Political Dimensions of Mathematics Education.

The first Mathematics Education and Society conference took place in Nottingham, England, in 1998. At that conference, Ubiratan D'Ambrosio delivered the Paulo Freire Memorial Lecture as the opening keynote speech, and he will give the opening talk for MES8 on the first evening of MES8. Others who were at the first MES conference and will be at MES8 are Tony Cotton, Peter Gates, Robyn Jorgensen, Ole Skovsmose, and Keiko Yasukawa.

Subsequent MES conferences were held as follows:

- 2000: Montechoro, Portugal
- 2002: Helsingør, Denmark
- 2005: Queensland, Australia
- 2008: Albufeira, Portugal
- 2010: Berlin, Germany
- 2012: Cape Town, South Africa

Proceedings from previous conferences are available at www.MEScommunity.info

Political Context of MES

The program for the first Mathematics Education and Society conference included the following statement:

Recently education has become more overtly politicised throughout the world. In many cases this has been part of a conservative restoration systematically turning back reforms gained over decades, in other cases a form of new colonialism with developing democracies turning to Western academics for

answers to complex educational problems. Mathematics education can be seen as key in this politics of education, a politics that in many cases is leading to increasing social discrimination and injustice. Mathematical qualifications remain an accepted gatekeeper to employment, and thus managing success in mathematics becomes a way of managing the employment market. Mathematics education also contributes to the regeneration of an inequitable society through undemocratic and exclusive pedagogical practices that portray mathematics and mathematics education as absolute, authoritarian disciplines.

There is a need for wider discussion of the social and political dimensions of mathematics education, for disseminating theoretical frameworks, discussion of methodological issues, sharing of research, planning for action, and the development of a strong research network. The conference aims to bring together mathematics educators from around the world to provide such a forum as well as to offer a platform on which to build future collaborative activity.

The situation described has progressively worsened. The epicentre of such developments is the United States, but the seismic effects reach around the globe. Across the world, a great deal of political rhetoric concerning mathematics and science education proclaims the need to prepare students in mathematics and science to be “competitive in the global marketplace of the Information Age”. Moreover, in the United States in particular, but by no means exclusively, the rhetoric frames mathematics and science education as matters of national security. In the starkest possible contrast, Ubiratan D’Ambrosio has passionately drawn attention to the ethical responsibilities of mathematicians and mathematics educators in relation to the most important problem facing humankind, namely survival with dignity.

It is not sufficient to say, as it is common in our profession ... that we are fulfilling our commitment and responsibility to mankind “By doing good Mathematics” or “By being a good Mathematics teacher”. But doing good mathematics should be complemented with the question “What will be done with the Mathematics I am helping to develop?” and a good mathematics

teacher must always be asking “How will my students perform? Will they be conscious of their moral commitment in their professional life?” Our responsibilities include the uses society makes of our intellectual production and what is the influence we have in the behavior of future generations.

(<http://en.wikiversity.org/wiki/Ethics/Nonkilling/Mathematics>)

Proceedings of MES8

Apart from the invited plenary presentations, each contribution published in these Proceedings was peer-reviewed, with open review, by two experienced mathematics education researchers. In the case of differing recommendations whether or not to accept, a member of the International Board made the final decision. The production of the Proceedings was made possible through the cooperation of many participants in this and previous conferences who contributed their time to review papers. The challenges faced by some of participants from language backgrounds other than English to write their papers in English are acknowledged, as well as the time of colleagues who provided support for language improvement.

The reviewers were:

Julia Aguirre, Sikunder Ali, Annica Andersson, Bill Atweh, Margot Berger, Christer Bergsten, Karin Brodie, Dimitri Chassipis, Anna Chronaki, Marta Civil, Tony Cotton, Beatriz D’Ambrosio, Jeff Evans, Karen François, Peter Gates, Mellony Graven, Brian Greer, Guðný Gunnarsdóttir, Rochelle Gutierrez, Eric Gutstein, Beth Herbel-Eisenmann, Eva Jablonka, Robyn Jorgensen, Brian Lawler, Jackie Leonard, Steve Lerman, Danny Martin, Heather Mendick, Candia Morgan, Judit Moschkovich, Swapna Mukhopadhyay, Balbina Mutemba, Nirmala Naresh, Kathy Nolan, Daniel Orey, Alexandre Pais, Guðbjörg Pálsdóttir, Arthur Powell, Milton Rosa, Laurie Rubel, Wee Tiong Seah, Lauren Shookhoff, Ole Skovsmose, Joi Spencer, David Stinson, Hauke Strahler-Pohl, Jayasree Subramanian, Paula Valero, Anita Wager, David Wagner, Keiko Yasukawa.

Participants

At this conference there are participants from the following countries: Australia, Belgium, Brazil, Canada, Chile, Colombia, Denmark, France, Germany, Iceland, India, Israel, Korea, Mexico, New Zealand, Norway, Palestine, South Africa, Sweden, United Kingdom, United States of America.

Participants speak the following languages: American Sign Language, Arabic, Bengali, Catalan, Danish, Dutch, English, French, German, Greek, Hebrew, Hindi, Icelandic, Indonesian, Japanese, Korean, Mandarin, Maori, Norwegian, Persian, Portuguese, Spanish, Swedish, Tamil, Telugu, Turkish, Urdu.

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The logo of the conference is a basket made by an unknown person of the Klamath nation belonging to the part of the Americas where the conference is taking place. It symbolizes the pancultural nature of mathematical practices.

Swapna Mukhopadhyay and Brian Greer
June, 2015

OPENING ADDRESS

From Mathematics Education and Society to Mathematics Education and a Sustainable Civilization: A Threat, an Appeal, and a Proposal

Ubiratan D'Ambrosio

*Floods and fire and convulsions and ice-arrest
intervene between the great glamorous
civilizations of mankind. But nothing will ever
quench humanity and the human potentiality
to evolve something magnificent out of a renewed chaos.*

—D. H. Lawrence, *Fantasia of the Unconscious*, 1922

Throughout my career, I have been concerned with Social Justice. In 1993, I expressed my concerns with the survival of civilization on Earth with dignity for all (D'Ambrosio, 1993, p. 54):

Although the main concern of this meeting is Mathematics Education, I believe I will be allowed to subordinate my comments to a higher objective: the survival of civilization on Earth with dignity for all. This is not merely jargonizing. The world is threatened, not only by aggressions against nature and the environment. We are equally concerned with increasing violations of human dignity. We face more and more cases of life under fear, hatred and violation of the basic principles upon which civilization rests.

The concerns express our criticism of what is going on in the schools, of discriminatory practices, of bigotry. Regrettably, in some cases this behavior is tolerated and even stimulated by teachers, by administrators and by parents. Much has been said and researched about this situation. Mathematics is a central issue in the deplorable situation in schools, which affects particularly minorities, but also large sectors of the entire population. Now, the situation is even more damaging because of the rapid changes in the demographic pattern of major

urban centers. The papers of MES8, as of previous editions of MES, show that this is an urgent focus for our concerns and asks for our action.

But I increasingly realize that there is an even more urgent focus for educators, in particular for math educators. Our practices as educators aim at a future without bigotry, hatred, and fear and with dignity for all. **But there may be no future. Our existence, as a species, is threatened.** Our objectives must be even more than social justice and dignity for the human species, must be the survival of our species that is threatened by a societal breakdown, as it is strongly expressed by Martin Rees (2013):

The main threats to sustained human existence now come from people, not from nature. Ecological shocks that irreversibly degrade the biosphere could be triggered by the unsustainable demands of a growing world population. Fast-spreading pandemics would cause havoc in the megacities of the developing world. And political tensions will probably stem from scarcity of resources, aggravated by climate change. Equally worrying are the imponderable downsides of powerful new cyber-, bio-, and nanotechnologies. Indeed, we're entering an era when a few individuals could, via error or terror, trigger societal breakdown.

D. H. Lawrence inspires us with the hope that there will be a magnificent future for the new generations, with social justice and dignity for all. Mathematics is a powerful tool for reaching this future.

The Threat

I am impacted by research results from several scientific areas and by alerts from eminent analysts of the world scenario. Mathematician Mikhail L. Gromov, the 2009 Abel Prize laureate, says (Gromov, 2010):

Earth will run out of the basic resources, and we cannot predict what will happen after that. We will run out of water, air, soil, rare metals, not to mention oil. Everything will essentially come to an end within fifty years. What will happen after that? I am

scared. It may be okay if we find solutions, but if we don't then everything may come to an end very quickly. Mathematics may help to solve the problem, but if we are not successful, there will not be any mathematics left, I am afraid!

The eminent scientist Martin Rees, as quoted above, also alerts us for societal breakdown. Both are highly respected scholars and their alerts must be taken seriously.

In 2011, Christiane Rousseau, then Vice-President of the Executive Committee of IMU/International Mathematics Union, spoke about the growing interest in the science of sustainability and the attention IMU is devoting to this, as a response to an appeal of the ICSU/International Council of Scientific Unions. She says (Rousseau, 2011):

While it is not new that scientists are involved in the study of climate change and sustainability issues, a new feeling of emergency has developed. The warning signs are becoming more numerous that urgent action is needed if we want to save the planet from a disastrous future, since we may not be far from a point of no return: climate change with more extreme weather events, rising of the sea level with the melting of glaciers, shortage of food and water in the near future because of the increase of the world population and the climate change, loss of biodiversity, new epidemics or invasive species, etc.

Putting together all the alerts and reports and the serious concerns of responsible scientific bodies, we foresee a situation of chaos, out of which we, as educators, have to evolve to a sustainable civilization, with peace in all its dimension (individual, social, environmental and military) and societies with justice and dignity for all. But mathematics curricula ignores critical issues threatening the survival of civilization.

The Appeal

In the fifties, the world was threatened by a devastating nuclear conflict, as a consequence of the political context of the so called Cold War. The Doomsday Clock of the *Bulletin of Atomic Scientists*

was reaching midnight (as dramatized in the 1964 movie *Dr. Strangelove*, by Stanley Kubrick).

The extremely dangerous moment for the survival of civilization prompted the eminent scientists Bertrand Russell and Albert Einstein to write, in 1955, a *Manifesto*, which was endorsed by scholars from all over the World. This document became known as the *Pugwash Manifesto*' (<http://pugwash.org/1955/07/09/statement-manifesto/#more-1784>).

All the considerations of the *Manifesto* synthesize the dangerous foolishness of being attached and to fight for a creed or an ideology. I am strongly impressed by the appeal in the following paragraph:

We shall try to say no single word which should appeal to one group rather than to another. All, equally, are in peril, and, if the peril is understood, there is hope that they may collectively avert it. We have to learn **to think in a new way**.

It is impossible to deny that we need new thinking in Mathematics Education. Most of the proposals for new curricula aiming at social justice still insist in teaching the traditional subjects, mostly originated more than a century ago. The same as the movement of New Math, of post-WWII reforms, that was a rearrangement of classical subjects. The new powerful mathematics, developed during WWII — regrettably for destruction but potentially strong for peaceful purposes — were not incorporated in the New Math movements of the fifties. Contents were, as they still are, unchained to prepare students for more and more contents. This propaedeutic objective was justified by cognitively unsustainable theories. How was this related to the real daily life of students and communities?

The concern of mathematicians to update their priorities is well expressed by David Hilbert (2003, p. 5):

History teaches us the continuity of the development of science. We know that each era has its own problems, which in the era following or are resolved or put aside as without interest and replaced by new problems.

We, mathematics educators, should take this remark of David Hilbert as an appeal to think in a new way. Sameness does not improve the

poor state of Mathematics Education. I ask special attention to the dangerous illusion that better results in assessment and tests are relevant for real quality of education.

Important innovations in mathematics education are not producing significant results. National and international large scale assessments are discouraging, although we point to a few pockets of excellence. But how to reach the decision makers, that is, how to convey relevant mathematics to a large sector of society?

This is a challenge that must be faced in close cooperation of mathematicians and math educators. Again, citing Gromov in the same interview,

It is a very difficult question because we have to project mathematical ideas to people who work very far from mathematics—to people who make decisions in society.

I see as the main problem to get a structure of narrative and a language accessible to other experts and to the population as a whole, while maintaining rigorous standards, but not necessarily the rigor dominant in the epistemological cage of mathematics.

For some years, I have been using the concept of “epistemological cage” as a metaphor to describe knowledge systems. Traditional knowledge is like a birdcage. Birds living in the cage are fed by what is in the cage, they fly only in the space of the cage, they see and feel only what the wires of the cage allow. The birds in the cage communicate among themselves in a language proper to those that live in the cage, they breed and procreate, they repeat themselves. They can not see the color the cage is painted outside. A similar situation may happen with specialized scholars. The scholars in the cage develop their own jargon and adhere to rigorous methodological and ontological standards. To overcome academic sameness is a big challenge. It is common to see researchers subordinating their students to themes proposed by the advisors, restricting their space for creativity. The epistemological cage metaphor is similar to the *Salomon House* of Francis Bacon’s utopian work *New Atlantis*, 1627, and to the *Ivory Tower* in the History and Philosophy of Science.

Let us listen once more to Gromov:

I would not only focus on mathematics but on science and art and whatever can promote creative activity in young people.

When this develops, we may have some influence but not before that. Being inside our ivory tower, what can we say? We are inside this ivory tower, and we are very comfortable there. But we cannot really say much because we don't see the world as well enough either. We have to go out, but that is not so easy.

The Proposal

Creativity is the key point. We need new ideas, new approaches, to face the problems affecting the world. Our generation and our approaches are not producing the global changes to avoid total disaster. We need to **allow the new generations to think in a new way.**

For this we need a new conception of rigor, in which the integration of all sciences and other forms of knowledge, including mathematics, can be done spontaneously, without traumas of an episemological nature. This requires courage and audacity. Imre Lakatos observed that scientists must be revolutionary activists, characterized as those who believe that conceptual frames can be developed and replaced by better ones. Lakatos (1978, p. 20) himself does a sort of philosophical *mea culpa* by saying that “we’re the ones who create our prisons and we can also, critically, demolish them.”

The dominance of revolutionary visual effects techniques, for example using pictures and realistic computer-generated characters, is the domain of young people equipped with very affordable software, especially games, that allow them to create fantastic scenarios, fruits of fantasy and creativity.

The school must be a space not only for instruction, but primarily for socializing and for criticizing what is observed and felt in everyday life. This may stimulate creativity, leading to a new thinking.

These comments go beyond scientific knowledge and education. The same occurs in religion, in the arts, in language, in governance, in the economy and in other sectors of human activity. A simple example is the evolution of modalities of sport in the Olympic Games, an evolution of a social activity originated in Ancient Greece.

We are experiencing very rapid social and environmental degradation. A serious issue is the loss of credibility in academic knowledge, which is evidenced by the emergence of alternative proposals in many sectors which are under strict normative control, such as the health

sector, the religions, the political and economic sectors and the rejection of values-oriented behavior and practices and even of the exercise of citizenship. This affects profoundly the students which we expect to be attentive to our classrooms. They are, probably, more attentive to the credibility of academic knowledge and the previous generations capability of dealing with central problems.

The rejection of values-oriented behavior and practices and the challenge to citizenship may be relatively spontaneous, with demonstrations of protest and acts of vandalism and terrorism, but also planned, as the practice of organized crime, which have strict codes of behavior and well-defined goals. The scenarios of Ian Fleming's novels, featuring the equilibrated forces of the "Agent 007" confronting "Dr. No", "Goldfinger" and other leaders of the intellectual and materially equipped groups leading criminality, reveal an impressive organization. Common citizens, mainly youth, are observers and the victims of these confrontations and are ambiguous about sides to be taken. Both have convincing discourses.

Common citizens, mainly youth, reveal impatience and distrust, which can result in nihilism or in expectation of supernatural reward. This reminds me of the social phenomena of the *danse macabre* (dance of death), in the Middle Ages, which was an implicit recognition of "no hope" and of the impotence of established societal structures, such as religion and governance, to stop the spread of the plague. A similar phenomenon is seen in our societies, with the emergence and growth, since early 21st century, of unstructured groups, such as the *Anonymous*. These groups are well studied by cultural anthropologist Gabriella Coleman in her book *Hacker, Hoaxer, Whistleblower, Spy: The Many Faces of Anonymous* (Coleman, 2014). Historian of Science James Gleick gave the title *Today's Dead End Kids* to a provocative essay-review of this book (Gleick, 2014). We most probably have, among our students, many of the dead end kids. Generally, they are highly creative and very learned in all the aspects of the technologies of information and communication, and they are keen observers of the state of society and of the world. Some may be willing to play the ultimate game *Quintet*, the mathematical survival game central to Robert Altman's 1979 movie of the same name.

Mathematics Educators must be aware of the dramatic prospect facing humankind and should be careful when their students learn well how to play mathematical games implicit in curricula, but do not

reveal any **ethical thinking**. Mathematics is, probably, the discipline that is most distant from a general humanitarian ethics, other than the ethics of rigor characteristic of mathematics. We are preparing them for success, to do well in engineering (but many will develop lethal weapons), in economics (but many will improve the inhuman financial game which support capitalism and some may be CEOs of enterprises) and most of them will be co-opted as ordnance of government and big enterprises, acting as *blade runners* for the dominating class (as discussed in D'Ambrosio (2011) and see http://en.wikiversity.org/wiki/Nonkilling_Mathematics).

Current teaching lacks critical thinking and offers limited opportunity to be creative and to propose new modes of thought. From the cognitive viewpoint, current teaching ignores the fact that every individual possesses much knowledge, from different sources: from a not well explained genetic component to experiences accumulated from birth (even uterine stage) through each moment in life, in the form of individual or personal knowledge. All this is associated with traditions, including mythological explanations, and culture in general. All this constitute the broad personal knowledge of each individual, which is in permanent transformation. The dynamics of cultural encounters is an important factor for this transformation. The broad personal knowledge are the strategies for survival and transcendence.

Life is characterized by strategies for surviving (all the basic behaviors and doings, “how” to stay alive), common to all species, and for transcending (understanding and explaining facts and phenomena, going beyond survival and asking for “why”), unique in the species *homo*. The strategies for **survival** and **transcendence** are generated by each individual and, thanks to gregariousness and communication, are shared and socialized with others in a group, and constitute the **culture** of the group. (These issues about knowledge are discussed in the classic dilemma “nature vs nurture). All this is ignored in the traditional, mechanistic approach, to mathematics education.

In the seventies, I proposed the **Program Ethnomathematics**, which is a research program in the history and philosophy of mathematics, with pedagogical implications (see D'Ambrosio, 1985). The essence of the program is to deal with real situations and problems recurring to accumulated knowledge of the individual, as mentioned in previous paragraphs. To bring this to schools, there is a double challenge to contents and to methods. Contents and methods are

inseparable. Descartes made this very clear. The methods for pedagogical action in the Program Ethnomathematics rely on broad contents which have been accumulated by an individual as discussed above. Contents in traditional Mathematics Education are a frozen arrangement of theories and techniques developed and accumulated in academic environments, in Epistemological Cages and Ivory Towers. In traditional Mathematics Education, the organization within mathematical system may change, looking for shortcuts and for new organizations and for applications of techniques and theories. Teachers try to contextualize contents through problems formulated in terms of real life, but these are artificial situations created to serve the purpose of bringing mathematics to reality, indeed an artificial reality produced by the teacher to facilitate the learning process. What we may call “really real” situations and problems are there, waiting to be recognized and dealt with by the individual, in an individual *ad hoc* way. This was clearly discussed by Descartes in the *Discourse of the Method*. The *ad hoc* ways to deal with facts and phenomena, situations and problems, may develop into individual methods and shared, socialized and became methods. All this is very dynamic, in different levels of understanding.

I want to clarify that “really real” situations and problems are related to facts and phenomena as perceived by individuals, according to their sensorial and mental structures. I understand **reality** as everything material (**artifacts**) that exists, have existed or will exist, plus **mentifacts**, which are the imaginary, the illusions and delusions, the dreams and beliefs, even fiction, that belong to the mind of each individual. Mentifacts are inaccessible and inform only the individual who generated them, while artifacts are accessible and inform, through senses, every individual (D’Ambrosio, in press). An important role of education, in a very broad sense, is to help individuals to produce artifacts out of mentifacts. This is the step towards communication. Examples of this is to learn to speak, to draw, to count, to write and so on. Indeed, these are early evolutionary steps of every child, as well as evolutionary steps of the species *homo*.

How to bring all this to formal and informal schools? In the nineties I proposed **a new concept of curriculum**, a new trivium, structured to provide critical capabilities of communicating, critical capability of dealing and analyzing symbols and critical familiarity with technology, organized in three strands, which I call Literacy, Matheracy and Technoracy. This proposal was a discussion paper presented at the first

Mathematics Education and Society Conference in 1998, as the Paulo Freire Memorial Lecture (see also D'Ambrosio, 1999).

Literacy is the **critical** capability to communicate. It goes beyond reading, writing and counting. It includes also the competency of numeracy, which means operating and constructing and interpreting graphs, tables and other ways of informing other individuals. It includes also the understanding of the condensed language of codes. These competencies have, nowadays, much more to do with keys, visors and screens than with pencil and paper. There is no way for reverting this trend, the same as there was no successful censorship in preventing people to have access to books in the last 500 years. Getting information through the new media precedes the use of pencil and paper and numeracy, as well as reading, writing and counting, the traditional “three R’s”, must be dealt with calculators, computers and tablets.

Matheracy is the **critical** capability of drawing conclusions from data, inferring, proposing hypotheses and drawing conclusion. It is a first step towards an intellectual posture, which is almost completely absent in our school systems. Regrettably, even conceding that problem solving, modeling and projects can be seen in some mathematics classrooms, the main importance is given to numeracy, or the manipulation of numbers and operations. Matheracy is closer to the way Mathematics was present both in Classical Greece, in other cultural systems of Antiquity and nowadays in Indigenous cultures. The concern of matheracy was not restricted to counting and measuring, but with mythology, divination and philosophy. A critical view of History shows that matheracy incorporates a deeper reflection about man and the cosmos, about man and nature, about man and society and can not be restricted to a so called intellectual elite, as it has occurred in the past. Now, modern civilization, asks for this broad concept, to be present in classrooms of all age brackets, to citizenship in general.

Technoracy is the **critical** familiarity with technology. Of course, the operative aspects of it are, in most of the cases, inaccessible to the lay individual. But the basic ideas behind the technological devices, their possibilities and dangers, the morality supporting the use of technology, are essential questions to be raised among children in a very early age and in classrooms of all age brackets and citizens in general. History show us that ethics and values are intimately related to technological progress.

The three strands comprise a new trivium curriculum, which is closely related to the proposals of the Program Ethnomathematics (as is well discussed in a recent paper (Rosa & Orey, 2015)). Many will question “But where are the contents?” I believe that contents will emerge as a sub product of the process of providing these capabilities. There are doubts about the efficacy of linearly organized transmission of knowledge. We have to recognize that contents organized as syllabuses are obsolete. The dynamic evolution of the disciplines can not be contemplated in the traditional way. Contents will emerge from access to media, in a variety of ways (radio, TV, news papers and mainly internet), from classroom discussions about appealing issues and from projects. My ideas about this are inspired by the *Nine Chapters* of Chinese tradition, by Hans Freudenthal’s *Realistic Mathematics Education*, by *Mathematical Modelling* and by the *Method of Projects*.

As a Conclusion

This proposal gives the opportunity for students to identify the main threats to planetary civilization and to be creative in proposing new directions for a sustainable civilization.

Note

1. The name Pugwash comes from the village in Canada where the Manifesto was first endorsed by 17 scientists who had been granted Nobel Prizes. The scientists who first endorsed the Manifesto decided to meet each year to reflect on the state of the world and to propose measures to improve the overall scenario to secure peace in its four dimensions: military, environmental, social, and individual. The annual meetings are the Pugwash Conferences.

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PLENARY PAPERS

Over 68 Years with Mathematics: My Story of Healing from Modern Superstitions and Reclaiming my Sense of Being and Well-being

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Clarification: Since the 1970s I don't remember writing a single word whose meaning does not have connection to some experience (including my quoting stories told by others). This is in harmony with a conviction that started growing within me since I became aware of my illiterate mother's mathematics, and since I realized that her world formed a "universe" while mine claimed to be "universal". I can't think of a word that has universal meaning. Any word I use in this paper is contextual, and in most cases connected to experiences and actions. This new-found awareness led me to avoid teaching and thinking of mathematics out of context. In this paper, I start from the existential situations that I lived, and speak for myself, and invite others to tanaaqush (the closest meaning in English to tanaaqush in Arabic is: "mutually nurturing dialogue"). I am trying to make sense of my experience by saying honestly how I feel and perceive things. This opens oneself to being confronted. I write especially for young people in Arab countries, but also for youth in general: they have been victims of control, mainly through being constantly measured. In a very true sense, this paper is a statement of confessions.

Central Theme of the Paper

Just as we need to detoxify to remove harmful toxic substances from our bodies due to manufactured and processed foods, we need "detoxification" to remove harmful words, ideas, convictions, and perceptions from our minds due to manufactured and processed knowledge. Fasting or cleansing with organic food removes the harmful substances from our bodies toward a healthy one, in the same way that fasting and cleansing harmful toxic words and ideas from our minds leads to understanding

and reclaiming what is in harmony with well-being in relation to thinking. Detoxification of the mind from harmful substances is the central theme of this paper; it is basic to what I refer to as wisdom. Nurturing foods that go into our bodies and ideas that go into our minds will not be as effective if harmful substances and ideas remain within us. Cleansing the body and mind is crucial for healthy digestion and understanding, for our well-being¹. Digestion for the body is like understanding for the mind. 2-valued logic (every statement is either true or false, everything is either A or not-A, with no third alternative) that governs our thoughts, relationships, and actions is needed in machines and in controlling people, but harmful in helping pluralistic attitudes to flourish. Bush said, “If you’re not with us, you’re against us”; no third alternative. In education, you either succeed or fail; no third alternative. In contests you either win or lose; no third alternative. You are either developed or underdeveloped. On January 20, 1949, President Truman of the United States declared that people outside the US and Western Europe were underdeveloped and that the role of the US and Western Europe is to help us develop. Four-fifths of the world’s population, and thousands of cultures and civilizations, were dumped under one category: underdeveloped, with one solution that fits all: development!

The two active “viruses” that facilitate this harmful process in relation to mind are: believing that people are not equal² in intelligence *and* following what is dictated by someone in power without knowing why—and made to feel proud of it! The difference between today’s following what is dictated by someone in power and in earlier times is the fact that in earlier times it was labelled enslavement, whereas in today’s world, we are honored by certificates, jobs, and titles. For example, in studying and teaching mathematics, I was made to believe I was more intelligent than others, and I was rewarded by degrees, jobs, and titles, simply because I followed blindly what was dictated to me by some authority. I was made to feel proud through the praise and gains—which blinded me to see that all that I was doing was following instructions from authority. In other words, I was proud to be enslaved without knowing it! To me, the two viruses combined form the greatest danger we face today because they tear apart our internal immune systems (personal and communal) and defeat us from within. I don’t believe humanity can survive if these two viruses are not confronted. My greatest worry is related to those who are driven with good intentions but infected with these viruses—as I was.

Military Tanks and Think Tanks

We Palestinians have been living for decades under military occupation of the land, but we seem to be unaware of another occupation: that of the mind. Whereas occupation of the first kind was done via military tanks, the second took place via think tanks. I lived with “bombardments” on four fronts: military-political-economic machinery; knowledge-academic; religious dogma; and make-believe foods—all of them came from Europe and its settlements (mainly the US): the first via soldiers, politicians, and companies; the second via educational “missionaries”; the third via religious missionaries; and the fourth via food industries and corporations. Through them, I lost authenticity, self-rule, and my ability, right, and responsibility to make sense in relation to knowledge, religion, food, and community.

Put differently, my story with mathematics (and education in general) parallels my story with religion, politics, and food. They all embodied the same underlying logic: something claimed to be modern, better, and universal replaced something labelled obsolete, out of date, and local. Such replacements were referred to as progress, development, democracy, catching up, getting ahead, and civilization. I experienced occupation in all of them. I was made to believe that mathematics that came from London was universal; I was convinced that missionary Christianity was better than the local one; Israeli occupation of Palestine was labelled democratic; and I thought canned and packaged foods were best.

My healing from those occupations started with mathematics. It happened first in 1973 through my work with mathematics teachers and students in West Bank schools, then in 1976 when I realized that my illiterate mother knew and practiced mathematics in a way I was not able to do or comprehend. That was the intellectual “earthquake” in my life that shattered illusions and myths; an earthquake that shook the foundations of my academic understanding and liberated me at many levels. I felt emancipated like I never felt before or after.

The other big healing shake-up in my life was in relation to the source of a person’s worth (evaluation) when I read in 1997 Imam Ali’s statement in *Jabiz’s al-Bayaan wa al-Tabyeen*, a book written 1200 years ago (which I will discuss in more detail later). Healing in relation to religion took place through my attempts to reclaim the Palestinian Christ, which again was embedded in my mother’s

behavior and lifestyle. My healing from harmful food was instigated by my younger son Taamer and my wife Carmen, who each and together travelled their learning journeys towards well-being³. Healing from military occupation did not happen in the sense of ending it but of taking charge of our lives and managing our affairs—as best we could. It happened within the whole society twice: during the 1970s and in the first *intifada* (uprising) of 1987–91.

In my experience, the four types of occupation were, in essence, replacing something that had roots in life, community, and culture, and that embodied the spirit of regeneration, by something that was lifeless and more like plastic. In this paper, my focus is mainly on the role of words with manufactured meanings in occupying minds, and on the role of rooted words, with rich and diverse meanings, in the healing process.

Reviewing Education

My first experience with education (of which I am conscious) goes back to when I was 5 years old, when my parents decided to pull me out of the real garden around our house in Jerusalem and put me in kindergarten, believing that to be progress. I rebelled but lost. In my real garden, I learned a lot, including mathematics, without being taught and evaluated; it took place as an inherent biological ability in a real rich environment full of living things; full of joy and aliveness. Pulling me from what is real and putting me in what is artificial and claiming that to be progress was a main experience in my life up through the doctorate level.

Critical thinking is “thinking within context” which means rethinking (in light of experiences, observations, and reflections) what we acquire via traditions and via modern institutions. Reviewing education means to view again, not only what is visible but also, more importantly, what is invisible; to dig into foundations and the underlying logic. This is what I have been doing since 1971, which convinced me that modern education is not about learning but about winning and control. Official education (in particular, the mathematics that I studied and taught for many years) is crucial in this control. Occupation and control at the invisible level happen via words whose meanings do not stem from life but from licensed institutions and

experts, and via measurements along a vertical line, such as through grading, which is degrading. Words that embody inequality are crucial in domination; especially inequality in intelligence. This onslaught on human dignity is accepted as normal. The disease permeates all levels, including ranking universities. A professor is one who professes what has matured in one's life as a result of experiences, reflections, and making sense of it all, and is not ranked along a vertical measure. I always wondered: if Harvard is doing something that is obviously magnificent which people can see, why would it strive to be constantly Number 1?! What does that really mean?

I repeat: official education is not about learning but about control and winning. This has to stop. We cannot continue in a drugged stupor, believing that people's worth can be measured by numbers that are claimed to reflect something real. Corruption of food, drink, soil, air, sea, and relationships that we have today is not the result of ignorance but of design and planning. Until I was in my early twenties I used to eat four raw eggs every day; science⁴ was not yet advanced enough to corrupt eggs; now I don't dare eat one such egg!

Since I was 5 years old, I was bombarded by both bombs and words. The first destroyed the world around me; the second, the world inside me. This paper, however, distinguishes between two kinds of words: those whose purpose is to control, distract, and deform; and those that are rich in meaning and are in harmony with living wisely. Words of the first kind need superstitions, and claim to have universal meanings; those of the second kind need to be reclaimed, together with meanings that are contextual and action-related.

The British Left Palestine but Left Their Mathematics Behind

The mathematics textbook I studied at school (after the British left) was printed in London: Durell's "General Arithmetic for Schools". In the preface he wrote, "The character of this book has been determined by the belief that the primary object in the teaching of elementary arithmetic is to secure accuracy. In pure computation, the less a pupil has to think, the more likely is it that mistakes will be avoided." Problem on p. 373: "In a race of 100 yd., A beats B by 10 yd., and beats

C by 13 yd. By how much will B beat C in a race of 120 yd., assuming that A, B, C all run at constant speeds throughout?” The textbook had a chapter on “Shares and Stocks”. Problems I had to solve included: “(1) A man invests 450 pounds in Indian 2% stock at 69. Find, to the nearest penny, how much stock he buys and the income from it. (2) London Brick 8% (1 pound) shares stand at 36s., and Rio Tinto 5% shares stand at 4. Which investment gives the larger yield?” I got full marks on all tests and the teacher praised me in front of the class. I still don’t know the meaning of shares or stocks in an experiential sense; I have never even owned any. What I really learned were tricks to get the answer the teacher expected; I learned mechanical skills with no understanding – and that was labelled “knowledge” and I was labelled “intelligent”! My understanding was an illusion, my knowledge was ideological, and my intelligence was corrupted. I learned that to be successful I needed tricks to fool (unconsciously) not only my teacher but also myself. Praising me for being an intelligent student blinded me from seeing reality. I was praised by people who were deceived as I was while studying. How could I, at age 14, doubt a well-designed textbook printed in London, with a “great” empire behind it! Some may say the textbook I am referring to is old, and things today are much better. In fact, things have become worse. In UNESCO’s Newsletter Oct-Dec 2002, John Daniel (Assistant Director-General for Education) called for “McDonaldization” of education. He wrote, “The Massachusetts Institute of Technology has shown the way by making its own web materials available free. Let’s hope this heralds a worldwide movement to commoditize education for the common good.” As Durell (above) said, “the less a pupil has to think, the more likely is it that mistakes will be avoided.” E-learning is a modern version of treating learning as something ready-made that a person hands down to another.

The British left Palestine but left several things behind which were worse than military occupation, because non-military occupations (of minds, foods, living languages, source of one’s worth, biological abilities, sustainable patterns in living, and mediums of learning and community action that embody the spirit of regeneration) are invisible and make us perceive them as progress. One thing they left behind was mathematics. Luckily, some of our teachers brought their passions to teaching. When, for example, the subject of fractions came up, our mathematics teacher (Yousef Khouri) digressed to tell us in some

detail the inspiring story of why ancient Egyptians invented them. Due to flooding of the Nile, tax officials had to figure out the fraction (in terms of land and time) that was used by peasants; i.e., the invention was connected to justice. We also had as principal Khalil Abu Rayya who was fond of astronomy and created pictures in our imaginations about the universe, such as how mathematics was used to figure out the circumference of the earth in the library of Alexandria. From both, I learned unconsciously (and it appeared later in my work) that mathematics is not lifeless activity consisting of solving technical problems, proving uninteresting theorems, and spreading 2-valued logic, but a means to make sense and see interconnectedness in life. That was what attracted me to study and teach mathematics.

In addition to mathematics, the British left behind the idea of the nation-state (where borders of all kinds were installed in our lands and our minds); the flush toilet which flushes away our precious water and usable excrement; official education which flushes away what is valuable in our civilization and knowledges; banks and trucks which tear apart economic-social fabric; and rigid institutions that replaced social formations that were closer to sustainable patterns of living. We are still paying a high price for what they left behind. However, while we see the harm of military occupation, most people don't see the harm done by other kinds of occupation. I was 30 years old when I started seeing them, especially at the intellectual-perceptual-social level. Ignoring biological abilities (learning and healing), and killing immune systems (concerning our sense of self worth, for example) had devastating effects on us. (It is worth recalling Macaulay's strategy (1835) for Britain to rule India (which was applied in Palestine): despise local cultures; claim English culture is superior; show readiness to "help" people become civilized!).

Shattering Myths: "Discovering" my Illiterate Mother's Mathematics

At the peak of my career as a mathematics teacher/ educator (mid-1970s), I realized for the first time that my illiterate mother was an "illiterate mathematician" (as Richard Noss, Institute of Education, University of London described her after a talk I gave). That

“intellectual earthquake” shattered myths I had acquired in schools and universities. Her story appeared first in my doctoral dissertation, then in Fasheh (1990, 2002). Much of what I have written, spoken about, and done, since then, was inspired by her world. In the mid-1970s, I was head supervisor of mathematics education in West Bank schools, and teaching mathematics at Birzeit and Bethlehem universities, and was recognized as a distinguished mathematics teacher and educator. When I realized (in 1976) that my mother practiced mathematics in a way that was impossible for me to understand and do, her world saved me from many modern superstitions and taught me humility, plurality of knowledge, equality in intelligence, that best learning happens without teaching, that knowledge is action, and that official education tears apart the inner world of people and the social fabric in communities. I realized that her world was harmonious while mine was fragmented.

The purpose of my mathematics was to produce feelings of superiority and inferiority rather than equality, justice, usefulness, and living in harmony with nature. My realization of my mother’s mathematics healed me from the claim that mathematics requires higher intelligence than making a dress, writing a poem, farming a field, raising a child, sailing a boat, playing a drum, making a nutritious meal, or creating a happy home. Cutting a rectangular piece of cloth into pieces and forming a new whole that fits the body of the woman who brought the cloth is what she did for 50 years and was beyond my ability to comprehend. Whereas I studied and taught geometry that consisted of straight lines, triangles, circles, parabolas, etc., my mother made thousands of dresses that fit numerous women, each with her own geometry! Whereas I had to deal with problems and theorems about identical geometric figures, she made thousands of dresses no two of which were identical. She never needed 2-valued logic or the geometric set; all she needed was a measuring tape and a real woman. Whereas my mathematics is dogmatically precise, hers was flexibly precise; whereas my knowledge claims to be universal, hers formed a harmonious universe rooted in life and community. As an academic, I was very much like a merchant and a parrot: I brought readymade words and parroted them to students at a high price. The more such words were connected to the market, control, and power, the higher their price was.

My mind was so colonized that, when I realized my mother’s mathematics, the first thing I thought of was how to teach her what

I know and to find a sort of synthesis between my mathematics and hers. Soon, however, I realized that would have been like putting real flowers with plastic flowers in one vase. In so doing, the plastic flowers would outlive the real ones (just like my mathematics outlived my mother's). Real flowers wilt and die, plastic flowers go on looking the same over time and place; they go on standing straight looking powerful and eternal – just like my mathematics. I lived in the same place under four completely different political systems (British, Jordanian, Israeli, and Palestinian); the mathematics curriculum stayed aloof, it changed only in some insignificant symbols (Fasheh, 1997). If one gets a BA in mathematics in a Palestinian university, s/he is entitled to apply to any university in the world; it is the same worldwide—so shallow, meaningless, and fundamentalist. Just like plastic flowers, my mathematics had no taste, no smell, and did not move with the wind. A million plastic flowers put together for a hundred years can't produce a single plastic flower, whereas one seed of a real flower, after it withers and dies, can generate a million flowers. It is the spirit of regeneration that makes the difference between what is real and alive on the one hand, and what is make-believe on the other. In relation to medium, source of one's worth, and methodology, school mathematics is "frozen"; no matter what happens, it stays the same. We had modern mathematics dumped on us claiming it was different but it remained the same in relation to the aspects I mentioned above.

In short, my mathematics won over my illiterate mother's mathematics by dogma and not because it was superior; it won because of its connection to power wherein control, winning, and greed are governing values. My mathematics created within me feelings of superiority and a sense of pride that rested on falsehoods (such as the ones I mentioned earlier). In this sense, I find that even the word "indigenous" does not describe my mother's knowledge; it would be more accurate to say that her knowledge started from existential situations with attentiveness to details, including what women wanted. Some women brought the German magazine *Burda* and pointed to dresses in it that they wanted my mother to make for them.

One extremely important thing I learned from her was a meaning of identity greater than the identity of the logical principle of identity: the identity between her and what she understood—which is missing in "understanding" as used in universities, since experience is not essential. Without experience we cannot make interpretations.

Without experiencing the world of my mother, I would not have been able to make the interpretations I am making in this paper. Equating knowledge (in the sense of knowing professional terms, technical information, skills, and theories) with understanding (in the sense of identity between the person and what s/he understands) is a serious shortcoming in dominant education. We can certainly extricate and gain insights into nature/reality and into truths about them through manipulative experiments, but that is like squeezing out confessions by torturing someone—both are useful for those in power who want to dominate. What I am advocating here can be labeled a “return”, a return to self that begins with a turning inward. The “insights” I got in relation to my understanding of mathematics did not result from experiments (a darling in modern thinking) but from experiencing and observing my mother’s world. What is interesting is the fact that her understanding was a combination of experience, observation, and experimentation. In contrast, none of these were present in my studying of mathematics; it was mainly symbolic-technical knowledge. We need, however, to realize that we do not understand everything in life through experience. I need more than experience, for example, to understand the harm in foods I eat. But knowing the harm does not necessarily mean it is reflected in behavior. Understanding the harm should mean that knowledge becomes part of my behavior. What was significant in observing my mother’s world was the fact that I did not interfere, which is contrary to the usual purpose of observation: to know, judge, and change what one observes. Although, as I mentioned earlier, I observed and my first reaction was to interfere and try to connect the two kinds of mathematics, I soon realized that my obligation in her case was to preserve the situation the way it was and that my interference to change it would have been an act of violence. In this sense, anyone who gets knowledge (via universities, training programs, etc.) in order to change society is acting violently. Until the 1970s, I worked with the belief that my job was to transmit the knowledge I got to students and bring them to my level. However, the times I felt I was best as a teacher were when I did not have a subject matter that I was required to teach. This happened in various settings since 1971: mathematics and science clubs in schools; the “Mathematics in the Other Direction” course; Tamer Institute for Community Education; Arab Education Forum; working with women in Shufaat Refugee Camp, and with young men and women

in Dheisheh Refugee Camp, and currently with various other groups. When people ask, “What did you teach people in Dheisheh?” I say, “Nothing; that’s why they learned a lot, in freedom, personally and communally.” The most I gave them was not to feel less in the face of high-sounding titles and arrogant attitudes. Anyone who comes with the intention to help you change, I would recommend you ask him to leave or you yourself leave. Any help that is not reciprocal is demeaning.

Intuitive Mind, Rational Mind, and Equality of Intelligence

I learned many things from my mother: first and foremost, I learned from her the beauty of not being right and the freedom I felt as a result. I learned that her intuitive mind was more valuable than my rational mind in the sense that my mind can be replaced while hers cannot; it cannot be created by institutions and scientific ways. Her mind is almost sacred, mine is more like a commodity, a spare part that can be replaced by tens of people through a job advertisement in newspapers. I learned that a main task of my mind is to expose the illusions in much of what I had acquired in the institutional world as well as invite others who, like me, have been drugged by dominant ideologies, to rethink much of what they had acquired. “Equality of intelligence” belongs to the intuitive mind, which is usually buried under modern categories. Nurturing the intuitive mind is what I hope we will stress—as mathematics teachers and educators—in our Portland gathering.

Just think. With very few exceptions, every Arab child (regardless of skin color, gender, religion, or social and economic background) by the age of 2 or 3 speaks Arabic fluently, without textbooks, pedagogy, or evaluation! This is true with any other language; children learn the language at home and in the neighborhood at an early age. All they need is a real, rich, lively, interactive environment. *This fact shatters three myths: the myth of inequality, the myth that learning needs teaching, and the myth that children need to move from the simple to the complex and from one grade to another.* As Palestinians, we don’t have political-social-economic-legal equality; however, we can live and practice

equality in intelligence wherever we are. What is significant about this equality is that we don't demand it; we just practice it. The only obstacle is if we are drugged by inequality. Imagine what would happen if we start treating children in accordance with the faith that they are equal in intelligence and able to learn without teaching! How wonderful, liberating, and energizing that would be! It could be our gift to children around the world who suffer from the myth of inequality, used as the basis for racism⁵ and subduing people.

A fundamental challenge that we face in the world today is how to free ourselves from modern myths such as the ones mentioned above. The progress of Western civilization has been at the level of tools, not life. Hardly any aspect in life has improved in essence. Linking technological advancement with human progress deforms us⁶. It was very hard for teachers to accept my saying: "there is no child who is illogical". Schools plant seeds of inequality and powerlessness in children's minds. A basic task of experts is to make the past look backward and out of date. What I mean by "equality in intelligence" is that intelligence is a non-commensurable quality of humans (we can look at it as "mindprints", just like fingerprints, qualities of humans that cannot be compared along vertical scales). It is hard for many to accept what I am saying simply because we have been drugged to perceive modern myths as progress and scientific. Sitting on our behinds for 12 years looking at meaningless words and symbols (on boards, papers, screens), with no action and no context, and calling that learning, has caused much harm. Myths existed in other civilizations but the modern one is the first to measure intelligence, one's worthiness, and a country's development using numbers—and to claim that such measures reflect reality.

The Flush Toilet as an Embodiment of Modernity

Western civilization is the first to refer to itself as modern. Civilizations prior to the European one were given names (such as Indian), not adjectives. By calling it modern it was automatically given a universal status, making prior civilizations termed traditional, old and obsolete. Those who refuse such dichotomy were accused of

wanting to stay behind. The mind can easily be deceived by equating invention of tools with human progress. If we compare what we have gained and what we lost from sciences, the losses are much greater. I will use the flush toilet to illustrate. No doubt, it is a great scientific invention but devoid of wisdom. It flushes away precious and scarce water, excrement that should go back to the soil and pollutes the environment. We lose all that in order for the toilet to look clean! It is absurd, and in a place like Palestine, criminal. The flush toilet is not in harmony with the cycle of life and the spirit of regeneration. For those who ask about alternatives, I say there are many, but kept out of awareness because until now companies didn't make money from them. I first experienced a latrine toilet for ten days in Gustavo Esteva's home in Oaxaca, Mexico in 2001 (I didn't use water, there was no odor, and nutrients that were taken from the soil go back to it. It is called a "dry latrine"). Laziness and obsession with looks make us admire the flush toilet but remain blind to its disastrous consequences. This is similar to what modern education does: it flushes away useful rooted knowledge and sustainable lifestyles, and it flushes away wisdom as a guide to thought and action. The flush toilet is an excellent example in showing the difference between science and wisdom, and that science without wisdom is like a blind driver; he can go very fast but disaster awaits at the end. The role of mathematics here is similar to how girls in 7th grade (at Shufaat Refugee Camp schools where there were 4,000 students) used mathematics to figure out how much water is used daily. If each student used the toilet once, it meant about 40,000 liters of water were used daily by the 4,000 students!

Wisdom: The Core Idea in a Vision that Embodies Healing and Nurturing

A serious imbalance in life started when wisdom was locked in and the mind was put on the throne: the mind can "sail" very fast without a compass, and the compass of the mind is wisdom. Without wisdom as a companion, the mind can create tremendous dangers and crises such as the ones we live today. We harm nature (perhaps beyond ability to heal); we pollute human beings (body, mind, soul, and perception);

we tear apart social-spiritual fabric (requiring huge communal effort to be re-woven). An important aspect of wisdom is asking about consequences of what we intend to do and to refrain from doing it if it harms the 7th generation (as American Indians' wisdom expresses it). We need to do this in relation to teaching mathematics. Another crucial aspect of wisdom is being open to be confronted, the opposite of being arrogant which seems to be part of elite fields and institutions. In 1997, I established the Arab Education Forum at Harvard's Center for Middle Eastern Studies and directed it for 10 years. A main concern of mine was to reclaim wisdom in life, and learning in particular. I explored the idea in various ways and settings, including how mathematics and Arabic can help in this regard.

For more than 40 years, a main concern of mine has been the role of dominant words and beliefs in contributing to corruptions, problems, and crises that we live with today and, at the same time, searching for and reclaiming words that can heal us from modern myths.

The first time I was confronted and forced to rethink and heal from dominant harmful viruses was, as I said before, when I realized my mother's mathematics. Thus, when I hear phrases such as "marginalized people" and "empowerment of women", I say: look, it was my illiterate mother who empowered *me* (with a doctorate from Harvard) rather than the other way round! In addition, she didn't blindly follow instructions but embodied what Gandhi referred to as self-rule. In this sense, I was marginalized from life and needed the shelter of some institution. What is needed today is awakening, not development; protection, not progress; healing, not advancement; sharpening meanings, not parroting professional and academic categories. I want to stress again that a most important aspect of learning is confronting one's beliefs, convictions, habits, while doubting experts and professionals. Dialogue within a *muthanna* relationship (which I will soon explain) has been in my experience the best way to understand and confront the self. My mother (with whom I had the longest *muthanna*) taught me without words that it is not possible to understand myself and assimilate that understanding from my own perspective alone; I need a "you" in a *muthanna* relationship to do that. My relationships with my wife and younger son were *muthannas* that helped me confront myths in more than one field.

Healing from Feeling Proud to be Enslaved and from Enslaving Others

Believing people are equal in intelligence can be a remedy for many ills: social, political, and psychological at personal and communal levels. However, a serious question is raised here: why is it then so easy to deceive the mind? I was increasingly convinced that the explanation lies in three factors: absence of wisdom; use of words whose meanings do not stem from life and do not form images in imaginations (termed plastic words by Poerksen (1995)); and our salaries which depend on our willingness to be deceived. It is very possible that a teacher teaches for 20 years a subject he never uses in life, and doesn't have a personal reason for teaching it! It is a most worrying modern aspect. (I hope we will discuss this issue in MES8).

A person who follows a path outside what is permissible by academic authorities (which could be connected to political-economic powers) will be punished, which may take the form of belittling what he does, and may lead to expulsion, claiming (s)he is not scholarly enough (Cornel West, Ward Churchill, and Eileen del Rios are examples of what I am talking about here). There is no justification for not having room in educational centers to learn in freedom. For the past seven years I have been trying to convince at least one Palestinian university to allow 10 students who want to learn in accordance with the spirit of the House of Wisdom in Baghdad; I haven't succeeded yet, but will go on trying.

Rooted Words as Medium to Heal from Modern Myths and Superstitions

Where can we look for hope when a large portion of people in the world feels proud to be enslaved? Luckily, we don't need verbal empires to deal with academic empires. A thousand years ago, *An-Naffari* in Baghdad wrote, "The wider the vision, the fewer the words needed to express it." The vision I suggest is "equality in intelligence"⁷ using 2 words associated with mathematics—equality and intelligence—in a way that is harmonious with dignity and well-being. Believing in

inequality infects minds and relationships and justifies ills such as injustice. In contrast, believing in equality of intelligence can help in outwitting engines of domination. One can use stories as a means for healing, but since this requires a full paper to be devoted to it, I will only elaborate on three words. My hope in healing from modern myths lies in these three words that became flushed away in history, and rarely, if ever, are used anymore: *mujaawarah*, *yuhseen*, and *muth-anna*. I will elaborate on how the three words can contribute to what I am talking about here.

Mujaawarah

Mujaawarah refers to any group of people who freely decide to meet regularly and start from the existential situation where they happen to be, with no internal or external authority, in their quest to learn, understand, and act. A basic aspect in *mujaawarah* is both personal and communal freedom to learn and act—in harmony with wellbeing and wisdom. This was true about great centers of learning throughout history: the libraries of Alexandria, Gundeshapur, and Cordoba, and House of Wisdom in Baghdad. *Mujaawarah* cannot be used as a model as each *mujaawarah* grows in accordance with its inner dynamics, and interaction with its surroundings. I used it first during the 1970s (without using the word) starting with the voluntary work movement in the West Bank and “mathematics and science clubs”. Then I used it at Tamer Institute for Community Education (which I established during the first Palestinian *intifada*), and then with the Arab Education Forum (which I established within Harvard’s Center for Middle Eastern Studies). I use it today with various groups and settings.

I want to elaborate on “neighborhood committees” that sprang up spontaneously during the first *intifada* as manifestation of *mujaawarah*. I will focus on how Israel reacted. Whereas Israel did not mind our holding conferences condemning closure of educational institutions and demanding their re-opening, it was harsh on neighborhood committees. It took me a while to figure out why communal farming and learning are more dangerous than international conferences. The only explanation that made sense to me was the medium. Whereas licensed individuals and organizations formed the medium of conferences, the medium in neighborhood committees consisted

of people who thought and acted in freedom, with attentiveness to surroundings and to what they could do with what they have. I wrote an article entitled “Freedom of Thought and Expression or Freeing Thought and Expression?” The deeper freedom is one that frees our thinking and expression from dominant discourse.

Mujaawarahs that took the form of “mathematics and science clubs” in schools were the medium of learning mathematics and science. They started towards the end of 1973, where every student who wished to be in them, came with a question he wanted to explore. They met every Thursday and flourished until the Israeli military officer of education banned them in 1976. There was no curriculum and no evaluation; every person is both a student and a teacher. Personally and communally they learned about mathematics in freedom and in context.

A third example of *mujaawarah* worth mentioning is the courtyard around *Aqsa* Mosque in Jerusalem which was for 1400 years (until British occupation of Palestine in 1917) a commons, a place where people from all backgrounds, religions, and denominations met and conversed freely, and where children played together. It is worth mentioning here a story told by Wasif Jouhariyyah, a Palestinian who lived in Jerusalem under the Ottoman rule and then under British occupation, who wrote his memoirs under both periods. He mentions that one of the first regulations the British imposed in Jerusalem was in relation to entering the Mosque’s yard. They assigned days for Muslims, others for Christians, and still others for Jews—claiming they wanted to be sure that all parties got their rights! The spirit of *mujaawarah* (which characterized gatherings in the yard before British occupation) was replaced by *muhaawarah* (debate). That seemingly innocent rule contributed to sectarianism and divide and rule—very typical of the British.

Yuhsen

Imam Ali’s statement, “the worth of a person is what s/he *yuhsen*” with the various meanings of *yuhsen* in Arabic (what the person does well, useful, beautiful, respectful, and giving) is what I suggest as a basis for a person’s worth. I first read the statement in 1997, when I established the Arab Education Forum within Harvard’s Center for Middle Eastern Studies. I read it in *Aljabiz*’ book “*Albayan*

wa-Attabyeen” (written 1200 years ago); I don’t remember any statement that had as profound an impact on me. According to it, a person’s worth is not judged by professional committees and measurements that claim to be objective and universal, but by the five meanings embedded in the word *yuhseen*. The five meanings embody plurality, humility, contextual thinking, and reflection on the interconnectedness in life. According to it, every person naturally has worth, which is incomparable with that of others’. My mother’s worth was clear, in this sense: she made clothes that fit women to their satisfaction, and what she did was beautiful, useful, giving, and respectful. In contrast, I was deprived of such a rich and wise sense of worth. My teaching of mathematics was well done, but the other attributes were not essential; it was mainly a tool of power and control. My worth was shallow and connected to licensed institutions and professionals. Whereas a person’s worth in dominant practice is given by a number, one’s worth in *yuhseen* is given through relations and through interconnectedness of five dimensions.

Most probably, the way Imam Ali arrived at his wise principle lies in the fact that he abstracted it (a mathematical ability) from what he saw in various communities and cultures that he interacted with. He constantly travelled between Arabia and Iraq, meeting people from Iran, India, and other regions. He probably saw that it didn’t make any sense to think of the worth of people in only one way; every person’s “worth” is contextual, connected to action, and in relation to the people around. Naturally, his concept of the worth of people was pluralistic. A basic human right (which is not included in the Universal Declaration) is for people to be protected from being measured along vertical lines that claim to be universal. Imam Ali’s statement provides an alternative. Unlike a vertical measurement, such as in grades, *yuhseen* embodies diverse pluralistic meanings for a person’s worth. And instead of official committees who judge students without knowing them, the reference in *yuhseen* is a combination of one’s performance in action, and one’s relation to who and what is around; that is, instead of abstract numbers, the basis in *yuhseen* consists of actions and relationships. Instead of parroting ready answers, one’s worth in *yuhseen* is related to reciprocal relationships. Instead of competition, it is harmony that is basic. Instead of focusing on technical skills, information and so on, in *yuhseen* we start with what one does with passion, nurturing and enriching it.

Muthanna

Muthanna forms a good part of the grammatical structure and inner logic of Arabic (it has no synonym in any European language, except for ancient Greek). It reflects the richness that exists in every culture and, at the same time, shows the limitation of cultures. There is no culture that can encompass the totality of experience or have universal claims about life.

I first realized the logic and beauty of *muthanna* when I was invited to participate in a UNESCO conference (May, 2007) on intercultural dialogue. The concept paper UNESCO sent had this in it: “a very old philosophical debate [is] the relationship between the One and the Many”. When I read it I thought, “I don’t remember that I ever experienced being one or many; I always feel that I am made of *muthannas*”. *Muthanna* refers to a relation between two persons that becomes very important in the lives of both, and yet has a life of its own. It is neither legal nor intellectual nor economic nor social; it develops freely between the two. It is neither a couple nor dual—although the latter is usually used to refer to it. *Muthanna* does not perceive the other as non-I or as a person that is a copy of I, or a higher synthesis with I. Each person remains who s/he is, but a relation develops that becomes important to both. In this sense, it embodies a logic that is different from that of Aristotle and Hegel. It is also different from the logic of Descartes who said “I think, therefore I am.” In the logic of *muthanna*, “You are, therefore I am”—my existence depends on my relation with you. That’s why I believe that without *muthanna*, it is difficult to develop a healthy pluralistic attitude in living. This explains why a person like Samuel Huntington (1996, 2004) oscillates between conflict and integration; the limitation in his mind is due to the lack of *muthanna* in his language.

Instead of the “either... or...” logic, in *muthanna* the closest in mathematical symbolism would be $1 + 1 = 3$: person A, person B, and the relation (as a third creature).

In short, then, *mujawwarahs* are an alternative to controlling institutions; *yuhseen* is the alternative to degrading evaluation; and *muthanna* forms a relationship with a *you* rather than with “the other”. In addition, the dominant logic in mathematics (every statement is true or false) should give way to other perceptions. We already have in our region, some 700 years ago, a statement by *Jalaluddin Rumi*, “Beyond

right and wrong there is a field; I will meet you there". I hope that in Portland, we will meet— at least part of the time—in such a field. The three reclaimed archaic Arabic words and *Rumi's* statement are in harmony with "equality in intelligence", learning without teaching, the belief that a person's worth is incommensurable with that of others, personal and communal freedom and responsibility to learn, and the spirit of regeneration. They are crucial in liberating people from modern illusions. When a body gets sick, the healthy part rushes to start the process of healing. Without the healthy part, physicians and medicines can do little. Similarly, in order for societies to heal, we need to build on what is healthy in us. The three words and Rumi's statement form part of what is healthy in Arab culture. Revealing the beauty in Arab culture and inviting others to reveal the beauty in theirs can create a meaningful shift in our teaching of mathematics. However, there is a word of caution: what I am advocating here cannot be accomplished by force or at once. Dominant systems are entrenched deeply and for too long; we cannot fight them directly. Most of us today need to live two lives: one in the dominant world, the other in harmony with well-being and one's passion. I suggest to keep nurturing the second: use *mujaawarahs* as much as possible (rather than teams, committee, or classrooms); ask friends and children (starting with oneself): what do I/you do with passion and want to *yuhsen*? (rather than: what are you studying or doing?); and stress *muthannas* as the underlying logic in relations.

Mathematics, Society, and Politics: Concrete Experiences and Experiments

In this section, I would like to mention diverse examples of personal experiences concerning the relation between mathematics and society.

Mathematics as a medium of emancipation: The state of the world compels us to stop teaching mathematics as if we are living in fairyland. We are living in a world (designed by Francis Bacon among others) that is subdued by science and mathematics. These fields (in their present form) are corrupting both physical and human natures. We cannot talk about pluralism in living and keep 2-valued logic on a vertical line dominating our sense of a person's worth and intelligence,

and of the level of development of one's society. Corruption of minds paves the way for corruption in other aspects. The following are two examples using mathematics in emancipation (of self and students) during the 1970s. I already mentioned "mathematics and science clubs" as a way for such emancipation. The second example took place in 1979 when I introduced a course at Birzeit University for first-year science students entitled "Mathematics in the Other Direction" which I designed as a result of working with West Bank schools for five years and building it on what was ignored in mathematics (David Henderson, Cornell University, who was a visiting scholar at Birzeit University, taught the course). The essence of the course was noticing patterns, similarities, and the logic underlying different phenomena. I will mention two incidents to clarify. In one class, while we were talking about the logic underlying different phenomena, one student asked, "Isn't the logic of 'divide and rule' the one that underlies grading in schools?" I said, "Of course. I never thought of it this way but, yes, it is." The second incident: a student who was taking the course with me in 1980 was jailed by the Israelis for two months. When released, he came to ask how I was going to handle his absence in relation to the course. He started by telling me about his experience in jail. He said something fascinating: after interrogation, he was put with other political prisoners in a big room. Inspired by the course, he went around asking what the questions were during interrogation. He found that although the questions differed from one person to another, yet there was an underlying pattern to all interrogations. He put it in writing, and distributed it just in case students got arrested, to make them aware of the logic that underlies interrogations! After he said that, I told him I never thought the course would have meaning in that context; he did not have to do anything more for the course; his knowledge was manifested in action.

Another example from the 1970s was this: I would go around and ask children questions, such as: what is a point? The most fascinating answer I got was from a 7-year old girl: "a circle without a hole"! My first thought was she did not understand my question. Then I realized her amazing dynamic imagination: when a circle gets smaller and smaller, it eventually becomes a point when the hole disappears!

To help in emancipation, I do believe that we need to avoid teaching mathematics without context, so instead of textbooks, we will have "context-books"—at least during the first 6 grades, and let children

know that the world is interconnected. The Islamic calendar is connected to the moon; watching it over a period of time, and making sense of it, would help connect various aspects: mathematics, religion, geography, astronomy, and the calendar.

Useful knowledge: In 1967, I was teaching mathematics at Birzeit College in the West Bank. I lost my job as a result of the Israel-Arab war. Schools and universities were shut down. I realized the striking difference between my knowledge and my mother's. I needed an institution to hire me; outside of them, no one needed my mathematics. In contrast, my mother's knowledge was useful under all circumstances; yet, it was neither recognized nor appreciated in academia. In general, we teach in mathematics how to solve problems we don't have, not problems we live. For instance, we don't talk about how to use mathematics to unplug ourselves from the consumption pattern in living. During the first *intifada* I worked with groups of young people. I asked them to keep a detailed record of what their families bought that month, and how much of that was not needed. Most came back with the finding that at least 70% of what they bought they could have lived without.

Plastic Words (see above): The language used today contains more "plastic" words than ever before. If we choose for example the following four widely-used words: education, development, strategy, and planning, we notice that they behave like algebraic symbols x , y , z , and w , in the sense that we can put them in any arrangement and they appear to make sense! Expressions include: developing strategy for educational planning; planning strategy for educational development; strategic planning for educational development; strategic development in planning for education. We can form more such expressions using the four words. When reading such expressions, many get the feeling they don't understand them because they need professionals to explain them—just like students who think they are not good in mathematics because they are not smart enough. The truth may lie in that their minds refuse to accept symbols that don't create images in the imagination.

Being lucky: Although my life as Palestinian was full of destructions, forced movements, fragmentation, and being confined within borders, I still consider myself lucky in crucial respects: I lived most of my life without a nation-state and in the pre-development age; and the teacher I learned from most was an illiterate woman. I was

lucky because these aspects provided me with a worldview not attainable via institutions and professionals. I had to constantly rethink the meanings of words; I had to be responsible for doing what was needed to be done; and I learned how to live with what was available. Those aspects gave us energy, purpose, responsibility, meaning, aliveness, and inner strengths. I also feel fortunate because Arabic is the language I grew up with. I especially realized these blessings after 1993, when the World Bank was allowed to enter our society and create illusions in our life: a make-believe state, national curriculum, national banks, and national security forces. We lost living with hope and started living with expectations, and we lost living as *abaali* (where relationships among us were the most basic) and started living as citizens identified by national numbers connected to official institutions.

Meaning of values: In 10th grade, I had a classmate (Sami Haddadeen, who still lives in Amman, Jordan). We were close friends and we oscillated between being the first and second highest grades in our class. Some semesters he would be first, and others I would be. We did not think of that as important until one day a teacher said: "Let's see who will be first this semester". We felt uncomfortable but didn't know how to respond. We were too shy to tell the teacher that his comment was poisoning our friendship. Our friendship was most valuable to us and we didn't want anything to ruin it. Finally, we thought of something. We decided that I would put his name on my exam papers and he would put my name on his. We felt that method would be the best way to protect our friendship. One day, a teacher noticed it and told us we were cheating. We said we preferred to cheat in relation to our names rather than to our friendship. We were rebelling in an innocent way against competition. Measuring a person's worth by a number is a very effective tool in controlling behavior and relationships by shattering our inner worlds and tearing apart the social fabric among us.

The Arab Education Forum (which I established at Harvard's Center for Middle Eastern Studies). During the ten years at Harvard (1997-2007) I visited 29 countries and worked with various individuals, groups, and universities. Nothing made me worry about the future of humanity as much as with what is going on in academia. We academicians form a most timid and rigid group. In addition, we are fundamentalists believing that there is a single universal undifferentiated path for learning and progress. Life is looked at as applications of

our ideas and theories. Universities as places that have resources and environments where people who do useful things in life go to deepen and enrich their understanding of what they do are rare. They stress research rather than what a person searches for in life. *Ibn Arabi* 700 years ago said “you are what you search for”. Most of the research I did in my life was related to what I was searching for, some of which was for words, mediums, and sources of strength in us that are in harmony with wisdom. We have one word *babth* in Arabic for search and research; in life, the two are intermingled.

In my work at Harvard, it was Roger Owen, Director of the Center, who provided the space for me to do much of what I did in the 10 years I spent there. It is rare for an academic to see the importance of working within a vision rather than with goals. Owen is rare in the sense that he lives not only the title but also the meaning of the word “professor”: professing what he believes deep within him. Providing such space should be a main role of universities.

Final Remarks

There is no way to learn how to sail a boat other than by sailing one; it is the same with singing, swimming, cooking, driving, and climbing a tree. It is true in every aspect of life except in education where action and context are absent! Ending the tyranny of a single universal path for progress and learning is crucial to human survival; it should be looked at as one way, not the only or best. We need to reclaim part of the educational budget and use it in diverse mediums and settings.

I never tell anyone not to seek a traditional degree if one’s life depends on having one. What I am stressing here is to care about other paths which are crucial in living.

As I mentioned earlier, I say here what I feel deep inside and, at the same time, I am ready to be confronted. We can’t afford to go on living with illusions. I would label what I am writing here as “innocent”: I still look at life as interconnected, not fragmented and specialized; I still believe that people are equal in intelligence; I believe in *yuhseen*, not in smartness or excellence; and I believe in harmony, not perfection. Like a child, I cannot separate myself from my experience. My thinking, actions, understanding, and forms of expressions are intertwined and interconnected in a harmonious mix. I don’t believe

in universal meanings and solutions, but I do believe in universal interconnectedness.

Much of what I said in this paper (as I have already mentioned) is linked to the Arab culture. I would like to stress again that if a society gets sick, what is healthy in it is crucial in healing it. Words and statements that I mentioned are part of our sources of inner strength. Reclaiming them in our daily living reveals the beauty in Arab culture and invites others to reveal the beauty in theirs.

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Notes

1. In 2006, an Iranian friend sent me the following, said by an Imam more than 1000 years ago: “O God, pull me out of the darkness of illusion and bless me with the light of understanding”. What is interesting about it is that pulling a person out of illusions should accompany being blessed with the light of understanding. It is similar to getting rid of toxic substances in food to ensure healthy assimilation and digestion and to getting rid of harmful meanings and perceptions to ensure understanding. It is also interesting because it does not differentiate between educated and non-educated, literate and illiterate, but between one who lives with illusions and one who lives with understanding—it is possible for a person to have three PhDs and still live with illusions. How true the statement is today, where we live illusions in most fields: that the food we eat nurtures; mass media tell the truth; medicines heal; and courts do justice.

2. I decided to choose the word “equal” in “equal in intelligence” to bring out a meaning (of this widely used word) that is radically different from the mathematical meaning where it is usually used in relation to some measure. I am using it in the sense that

intelligences of people are equal in the fact that they cannot be measured! Mathematics cannot be used to measure intelligence without corrupting our perception and conception of it. What I mean by “equality in intelligence” is that intelligence is a non-commensurable quality of humans. We can look at it as a “mindprint” just like fingerprint, qualities of humans that cannot be compared along vertical scales; each is uniquely complete. Thus, no one has the right or authority to judge and measure the intelligence of another; such a person should be questioned. This is also true in relation to a person’s worth in the sense that it cannot be measured along a vertical line (which unfortunately is widely done under the name “evaluation”). For a different conception of a person’s worth see the discussion above about what a person *yubsen*—which cannot be compared; the worth of every person is unique.

3. My son, who at age 9 left school for 3 years, travelled for his learning journey on his own. Eventually, his focus revolved around providing raw organic vegan and vegetarian food through a small take-away restaurant in Taos, NM, and my wife accompanied him in his journey towards well-being.

4. I am against dominant science and dominant mathematics not in relation to their content but rather in relation to the medium they use and the basis on which they make their judgments. It is the practice that licensed professional and institutions have the authority to decide what is mathematics and what is not, what is science and what is not, and whether A is more intelligent than B. What I am against is giving such authority to a person or institution. No school or university would acknowledge that what my mother was doing is mathematics. My mother used a sewing machine made in the west, an invention of science. However, self-rule was her medium and the basis of her decisions and work; she only needed to enter into free mutually respectful agreements with women who came with pieces of cloth, seeking her to make dresses for them. There was no authority outside them that they needed to get their approval. The medium she used in her learning was *mujaawarab*; the medium she used in her dealing with women was mutual understanding... My medium was textbooks, licensed authorities, etc. In the science and mathematics clubs which I encouraged students to form, the medium was

mujaawarabs and staring with questions each of them had and which stemmed from experiences, experiments, and living. There was no dogma of any sort, no authority to tell them what to do or not to do. What is good about science? My problem in relation to this question is what we ignore; we ignore for example to ask: what do we lose from what we gain? We know that we gain a lot by using a mobile phone or a computer but we need to contemplate about what we lose and how to avoid, or at least lessen, the loss or harm. Is it possible to do what we want to do without such loss or harm? The medium here is crucial. What I am saying is that we cannot afford to go on teaching and doing science and mathematics without relating it to context, action, and consequences.

5. At the peak of the civil rights movement in the United States (1960s), Jensen (1969) of Berkeley University published a study that asserted that black peoples are intellectually inferior! It was published in the Harvard Educational Review—a good example of collaboration of top scholars and academic institutions, using “scientific” methods... all to show that black peoples are—genetically—intellectually inferior!

6. When a situation requires knowledge of tools, however, I definitely would seek someone well-versed in modern knowledge and science. For example, if I get a problem with my appendix or need a surgery in my eye, I would seek a physician who knows how to use modern tools. But if I seek well-being, I would look somewhere else, seek a wise person for example. Remember: we have a Ministry of Health in all countries but not a Ministry of Well-being; Schools of Health but not Schools of Well-being.

7. In my interactions with people, I keep stressing—both in my words and my work—that my dream revolves around that every child in Palestine should be free from the onslaught of the most dangerous inequality: labelled inequality in intelligence. I believe in equality in intelligence not as a result of proof but as a faith that guides my actions and interactions. Otherwise, I would accept and justify harm done to children in the name of being less intelligent. Believing in such equality necessarily puts the burden on us (as parents and educators) to search for why a certain child is having difficulty in living

in harmony with life; it also compels us to reclaim a most fundamental aspect in living: a pluralistic attitude.

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Weaving Social Justice in Elementary Mathematics

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Based on nearly three decades of teaching in inner city, bilingual elementary schools in a Midwest industrial city, the author suggests multiple ways to interject social justice issues and concepts into core mathematics curriculum that simultaneously draws on the children’s lives. Furthermore, the author uses anecdotal evidence to argue that this approach increases motivation and interest in the study of mathematics. Finally, the author asserts that even in an era of scripted, standard-based, data-saturated curricular mandates, social justice mathematics is both possible and necessary.

Corporate versus Social Justice Mathematics Teaching

For too long, too many educators in the United States have held the notion that teaching and politics must not mix. The dominant narrative is that teachers and teaching are to be “neutral” and “above” politics. Moreover teachers are frequently reminded in both explicit and subtle ways that they should acquiesce to political “realities” in broader social and political debates.

But that attitude is changing. I believe it’s changing in large part because of the international attacks on the public sector and public sector unions. It’s changing also because of a recognition by educators of the negative impact that edu-business is having on public education. And it’s changing because in no other time in history has the craft of teaching been under such sustained assault.

As a teacher activist in Milwaukee, Wisconsin, a medium sized industrial city that by many indexes is one of the most hyper-segregated (by race and poverty) in the nation (Schools and Communities United, 2014), I have witnessed both the intensification of those

attacks and the subsequent resistance. As Paulo Freire stated, “Conflict is the midwife of consciousness” (Horton & Freire, 1990, p. 176).

In the spring of 2011, teachers in the state of Wisconsin led an unprecedented workers’ uprising that challenged the attack by right-wing Republicans and their corporate backers on the issue of the right for public sector unions to collectively bargain. The legislation was qualitatively more punitive than any other so-called “right-to-work,” anti-union laws that exist in 23 states in the United States. Alongside the anti-union legislation, Wisconsin Governor Scott Walker made the largest budget cut to public education since the Great Depression. He finished off his educational agenda by essentially writing a blank check for programs that privatize education.

These attacks on public schools and teacher unions coincided with a push by “corporate reformers” to impose test-based accountability systems on students, teachers, and schools. Urged on by billionaire philanthropists and multinational textbook/testing companies, with the help of “non-profit” advocacy organizations like *Teach for America* and *Stand for Children* the federal and state government launched various initiatives that, in practice, are destroying the teaching profession through a toxic data-drenched cocktail of scripted curriculum and high-stakes standardized tests. Worst yet, a new generation of teachers are being inculcated with the notion that teaching should be “data-driven” instead of child driven; standardized, instead of culturally responsive; and scripted, instead of critical.

A key factor in this international infatuation with tests, technology and scripted curriculum is the growth of multinational corporations who have their eye on the K-12 education market in the U.S. and beyond. The U.S. market, now mainly spent in the public sector, is projected to be \$788.7 billion in 2015. “It’s really the last honeypot for Wall Street,” Donald Cohen, the executive director of In the Public Interest, told *The Nation* (Fang, 2014).

We must individually and collectively say “enough” to these attacks and join with parents and community partners to ensure that our teaching and schools foster deep understanding of our society and the disposition and skills necessary for students to be active citizens standing on the side of social justice.

This is what I have tried to do in my 30 years of teaching ten and eleven year olds. I’ve also tried to promote such a view of teaching in my work as an editor of the *Rethinking Schools* magazine and in my

role as President of the Milwaukee Teachers' Education Association, the largest local union in Wisconsin.

Shouldn't a Math Teacher Remain "Neutral"?

But shouldn't math teachers and curriculum just remain "neutral?"

Simply put, teaching math in a neutral manner is not possible. No math teaching—no teaching of any kind, for that matter—is actually "neutral," although teachers may think otherwise. As historian Howard Zinn (1970) once wrote: "In a world where justice is maldistributed, there is no such thing as a neutral or representative recapitulation of the facts."

For example: Let's say two teachers use word problems to teach double-digit multiplication and problem-solving skills. They each present a problem to their students. The first teacher presents this one:

A group of youth aged 14, 15, and 16 go to the store. Candy bars are on sale for 43¢ each. They buy a total of 14 candy bars. How much do they spend, not including tax?

The second teacher, meanwhile, offers a very different problem:

Factory workers aged 14, 15, and 16 in Honduras make McKids children's clothing for Wal-Mart. Each worker earns 43 cents an hour and works a 14-hour shift each day. How much does each worker make in one day, excluding fees deducted by employers?

While both problems are valid examples of applying multi-digit multiplication, each has more to say as well. The first example has a subtext of consumerism and unhealthy eating habits; the second has an explicit text of global awareness and empathy. Both are political, in that each highlights important social relations.

When teachers fail to include math problems that help students confront important global issues, or when they don't bring out the underlying implications of problems like the first example above about buying candy bars, these are political choices, whether the teachers recognize them as such or not (Gutstein & Peterson, 2005).

There are distinct differences between a biased curriculum and a

partisan one. Teaching is biased when it ignores multiple perspectives and does not allow interrogation of its own assumptions and propositions. Partisan teaching, on the other hand, invites diversity of opinion but does not lose sight of the aim of the curriculum: to alert students to global injustice, to seek explanations, and to encourage activism. This is the kind of teaching I hope teacher activists and teacher unions will promote (Bigelow & Peterson, 2002).

Why Social Justice Math?

A brief exchange in my fifth grade classroom where students are working in groups during “math time.”

“I like this kind of math!” José exclaimed as he and two classmates paused to answer my question “How’s it going?”

“Is this stuff really true or is it like the stuff in our math book that’s all made up?”

“And boring,” added Xavier. “This is interesting.”

“Good question,” I responded. “These data are accurate—based on research people and groups have done about pay and working conditions in sweatshops around the world. Some of the data is from the people who made the video we watched yesterday when we started the unit.”

The students in my fifth-grade, bilingual classroom were in the second day of a 4-day “sweatshops math” project from the book *Rethinking Globalization: Teaching for Justice in an Unjust World* (Bigelow & Peterson, 2002). My partner teacher – who was teaching the identical mini-unit in Spanish – and I had taken a break from the official mathematics curriculum, which had become tedious to both teacher and student.

Using teacher-provided information on sweatshops in 11 different countries, the students were calculating hourly, daily, weekly, and annual salaries while reflecting on the working conditions and age of the workers and the products produced. The students had to locate each country on the world map and make a large poster display including an analysis of data and a self-evaluation. Multiple-step problem solving for 10-year-old children is never easy, and there were some frustrations, but there was definitely no lack of motivation.

Integrating Social Justice, Mathematics, and Other Content Areas

Since the passage of the federal *No Child Left Behind Act* of 2001, the Milwaukee Public Schools, like those of many urban school districts, have become increasingly obsessed with data bits and standardized testing as the “silver bullet” to drive all curricular improvements. Even in historically progressive public schools like the one in which I have taught, La Escuela Fratney (Peterson, 2007), this obsession with data has meant increasing pressures on classroom teachers to “cover” the curriculum and to adhere closely to district wide curricular texts and pacing charts. These pressures have had the unfortunate effect of reducing both time and interest in curricular integration where mathematics, writing, social studies, science, and student interest might shape longer interdisciplinary units.

I’ve always held that educational reformers should advocate mathematics be taught in all subject areas, promoting “mathematics across the curriculum,” comparable to “writing across the curriculum.” Too often mathematics is segregated in schools and kept separate from the issues that confront students in their daily lives. The curriculum rarely encourages students to link mathematics and history, mathematics and politics, mathematics and literature – mathematics and people.

There are unfortunate consequences when mathematics is isolated. First, the not-so-subtle message is that mathematics is basically irrelevant, except for success in future mathematics classes, commercial transactions, or in select occupations. Second, students learn that mathematics is not connected to social reality in any substantive way. Thus students approach mathematics in the abstract and are rarely encouraged to seriously consider the social and ethical consequences of how mathematics is sometimes used in society. Third, if students are not taught how mathematics can be applied in their lives, they are robbed of an important tool to help them fully participate in society. An understanding of mathematics and how numbers and statistics can be interpreted is essential to effectively enter most debates on public issues such as welfare, unemployment, and the federal budget. For example, even though the minimum wage is higher than it has ever been, in relative terms it is the lowest in 40 years. But you need mathematics to understand this.

To be clear, I don't mean to imply that distinct mathematics skills aren't important—I am from the old school that holds that children need to learn their basic “math facts” in order to give them the mental fluidity to engage in much deeper and meaningful mathematics. Moreover, integrating mathematics with social studies does not necessarily make teaching more student-centered or content more concerned with issues of social justice. Those important components depend on the teacher's philosophical and pedagogical beliefs. My own beliefs require that I fully engage my students in understanding how to use mathematics to examine issues of social injustice and consider how we can use mathematics to make the world more just.

I have worked to integrate mathematics—from the simplest understanding of number concepts to more complex problem solving—with social studies, science, current events, reading and writing. In the interests of clarity (my classroom life is never so neatly ordered), I outline these approaches as: connecting mathematics to students' lives, linking mathematics and issues of equality, using mathematics to understand history, and integrating mathematics into action.

Connecting Mathematics to Students' Lives

For many teachers starting on this road of teaching mathematics for social justice, the first step is to build on what students bring into the classroom, and to connect curriculum to students' lives. Mathematics is a great way to do this. I usually start the year with kids exploring, in small groups, how mathematics is used in their homes and communities. They scour newspapers for numbers, cut them out, put them on poster paper and try to give sense to their meanings, which at times is difficult. They interview family members about how they use mathematics and write up their discoveries. As part of a beginning-of-the-year autobiography, they write an essay, “Numeric Me,” tying in all the numbers that connect to their lives, from height and weight, to the number of brothers and sisters they have, to addresses, phone numbers, and so forth. Over the years, I have pushed this assignment to go beyond the self and encourage the students early in the year to explore “math in the community” and “math in the world,” which at times is as simple as the population, but it has also gotten into more engaging issues such as the amount of money spent on the Iraq War

or the number of children without access to clean water. Some years I ask them to write a “history” of their experiences in mathematics classes, what they think about mathematics, and why.

This process starts a yearlong conversation on what we mean by mathematics and why it is important in our lives. As the class increasingly becomes sensitive to the use of numbers and mathematics in news articles, literature, and in everyday events, our discussions help them realize that mathematics is more than computation and definitions, but includes a range of concepts and topics—from geometry and measurements to ratios, percentages, and probability.

As part of the autobiography project we also construct a timeline. We start by putting the students’ birthdates and those of their parents and grandparents on a class timeline that circles the outer perimeter of my classroom (and which is used throughout the year to integrate dates that we come across in all subject areas). The students also make their own time lines—first of a typical day and then of their life. In these activities, students use reasoning skills to figure out relations between numbers, distance, time, fractions, and decimals.

I also use another beginning-of-the-year activity that not only builds mathematics skills but also fosters community and friendship. The whole class discusses what a survey or poll is and brainstorms questions that they would like to ask each other. After I model one survey, each student surveys their classmates on topics such as their national origin, their favorite fast food restaurant, music group, or football team, or what they think of our school’s peer mediation program. Each student tabulates his or her survey data, makes a bar graph displaying the results, and reflects in writing on what they have learned. Later in the year, they convert the data into fractions and percentages and make circle graphs. I encourage the students to draw conclusions from their data, and hypothesize about why the results are the way they are. They then present these conclusions orally and in writing.

This activity is particularly popular with my students, and they often ask to do more extensive surveys with broader groups of people. The activity lays the basis for more in-depth study of polling and statistics around issues such as sampling, randomness, bias, and error.

Linking Mathematics and Issues of Equality

To help my students understand that mathematics is a powerful and useful tool, I flood my classroom with examples of how mathematics is used in major controversies in their community and in society at large. I also integrate mathematics with social studies lessons to show how it can help us better understand the nature of social inequality. Kids are inherently interested in what is “fair,” and using mathematics to explore what is and isn’t fair is a great way to get them interested in all types of math concepts, from computation to fractions, percentages, ratios, averages, and graphing. Two such explorations that I often use is a world wealth simulation exercise and examinations of current events.

World Wealth Simulation (Bigelow & Peterson, 2002)

During the months of October and November, there is often lots of discussion of poverty and hunger in my classroom, related either to the UNICEF activities around Halloween or issues raised by the Thanksgiving holiday. These months provide a good time to use simulation exercises to help children understand the disparity of wealth in the United States and around the world. In one such lesson, I provide information on the distribution of population and wealth in the six continents, and then have children represent that information using different sets of colored chips. After working with students so they understand the data, we do a class simulation using a map of the world painted on our playground. Instead of chips to represent the population data we use the children themselves. I have them choose a slip of paper from a basket labelled “Chance of Birth” and the children go and stand in the corresponding area of the world—15 cram into Asia, 3 into Europe, 1 into the United States/Canada, 2 into Latin America, and 3 into Africa. One person in the group gets a nametag labelled “Negotiator” and another gets an empty paper bag labelled “wealth.” I then distribute the wealth, represented by cookies. I do so dramatically, with great flair, dropping eight cookies into the European wealth bag, and then turning south to Africa to give a half cookie to an equal

number of students (actually Africa has several hundred million more people than Europe). “Unfair! No fair!” students shout.

“Wait! Quiet!” I call out, “You’ll get your chance to discuss and debate this.” I tell the students that they are not to eat their wealth yet, but instead discuss as a group what strategy and arguments their negotiator might use. I then tell the negotiators, “Go visit other parts of the world and seek out what’s best for your people.” Depending on how restless the students become, the negotiating session lasts anywhere from 3 to 10 minutes. Sometimes the students representing North America are very generous and other times they sit and eat their cookies—without concern for what might happen later to them during recess. One year while Hector was holding the wealth bag for Asia, he proceeded to eat five of the six cookies for Asia. His “Asian” counterparts were furious, but after I prevented a popular uprising for cookies, we later reflected on how Hector’s actions—and the other students’ response—might be a closer representation of reality than my staged simulation. Wealth is not divided fairly anywhere in the world!

The activity is high energy. Emotions run raw. Tears sometimes flow. “That’s not fair!” is the most common refrain. Despite my rules, at times there is mass (illegal!) migration and even war. As an incentive to get my kids back into the classroom with the least amount of collateral damage, I let them know that there are more chocolate cookies in the classroom for anyone able to follow basic decorum on the way up.

Afterwards, we discuss the simulation and write about the activity. I know this activity has several limitations: (a) it doesn’t take into account huge inequality in wealth within individual countries, such as our own; (b) it seeks to describe a situation, not explain it; and (c) as a simulation, it can in no way re-enact the violence of poverty and hunger that kills tens of thousands of children daily. Nonetheless, these kinds of simulations can be powerful tools in motivating students to want to ask and answer the essential question, “Why?” Or in the words of some of my students: “Why does Asia have so many people and so little wealth?” “How did Europe and North America get to be so wealthy?” “Why are things so unfair?”

In order to compensate for the first limitation – the simulation’s blind spot on inequality within countries or continents, I usually follow up this activity with Peggy Kellogg’s (1998) simple “Ten

Chairs of Inequality” activity which has ten chairs each representing $\frac{1}{10}$ of the wealth of the United States and then 10 students each representing 10% of the US population. Through dialogue and my directing certain of the ten students to move into the appropriate chairs, students see that 10% of the population—represented by one student—occupies seven chairs, and his or her arm (which I say represents 1% of the US population) must stretch out over four chairs).

Each simulation not only connects mathematics to human beings and social reality but also it does so in a way that goes beyond paper and pencil exercises; it truly brings mathematics alive. It would take much less class time if I just tell my students about the world’s unequal distribution of wealth. But that wouldn’t have the same emotional impact as when they see classmates in the United States and Europe get so many more cookies even though they have so many fewer people.

Examining Current Events

Another way I help students analyze inequality is to use current events. News articles are filled with mathematics. Regularly, I photocopy a newspaper article and have students use yellow highlighters to highlight all numbers and number words. We then discuss their meaning, I pose problems using the numbers, and remind the students that only by being well skilled at mathematics will they be able to understand and influence the world.

For example, in small groups, students might examine data such as unemployment or job trends, convert the data into percentages, make comparisons, draw conclusions, and make graphs. This exercise is a great way to help students understand the power of percentages. They also use a computerized graph-making program; in doing so, they realize how the computer can be a valuable tool.

One group, for example, looked at news stories summarizing a university report on the 10,000 new jobs created in downtown Milwaukee due to commercial development. According to the report, African Americans held fewer than 8% of the new jobs, even though they lived in close proximity to downtown and accounted for 30% of the city’s population. In terms of the higher-paying managerial jobs, Latinas/os and African Americans combined held only 1%, while

white residents, who are overwhelmingly from the suburbs, took almost 80% of the new managerial jobs. Using these data, students made bar and pie graphs of the racial breakdown of people in different jobs and in the city population. They compared the graphs and drew conclusions.

The students then performed a role-play, with some students pretending to be representatives of community organizations trying to convince the mayor and major corporations to change their hiring practices. What began as a mathematics lesson quickly turned into a heated discussion of social policy. For instance, at one point a student argued that the new jobs should be split $\frac{1}{3}$ Black, $\frac{1}{3}$ Latina/o, and $\frac{1}{3}$ white, because those are the three principal ethnic groups in Milwaukee. Others, however, disagreed. Needless to say, this led to an extensive discussion of what is fair, of reasons why minorities had so few of the jobs created downtown, and what it would take for things to be different. And more importantly, in my mind, it once again demonstrated the power of understanding mathematics.

Using Mathematics to Understand History

It is important for students to be aware of whose voices they hear as they read history books or the newspaper, or watch a movie. Who gets to narrate history matters greatly, because it fundamentally shapes the readers' or viewers' perspective. We can analyze these issues with kids and help them become more critical readers of books and other media. In this process, mathematics plays an important role.

During our unit on the European invasion of the Americas, I have students in groups do a relatively easy, although depending on the book, somewhat tedious tabulation activity. Students analyze children's books on Columbus, tabulating whose views are represented. For instance, how many times do Columbus and his men present their perspective, versus the number of times the views of the Taíno Indians are presented? Using fractions and percentages the students make large graphs to demonstrate their findings and draw potential conclusions. Large visual displays—bar graphs made with sticky tape, for instance—are good points of reference to discuss and analyze. Math concepts of percentages, proportions, and comparisons can be used to help kids discuss the statistics they've uncovered and the graphs they've made.

A similar tabulation and use of percentages can be used to analyze popular TV shows for the number of “put-downs” versus “put-ups,” who is quoted or pictured in newspapers, stereotypes of females in popular cartoons, who is included in textbooks, and who is represented in the biography section of the school library. One year, when students were doing historical research project on a person who fought for social justice, two students noticed a huge discrepancy between what was in the new CD ROM encyclopedia about Harriet Tubman compared to Thomas Edison. Their discovery and class discussion led to a whole class project in examining bias in encyclopedias based on inches of type. Students did physical measurements, created bar graphs and compared data. A year later, after I was named the Wisconsin Elementary Teacher of the year, a right wing talk radio talk show host used that lesson, which I had written up in *Rethinking Schools*, as the grounds for asserting that I was committing “child abuse” in my classroom (Peterson, 2009a).

In the aftermath of the 2000 presidential election, in which the US Supreme Court selected George W Bush to be US president, I was talking about George Washington and slavery during social studies. I explained that while I respected him as a great general, I was critical that he owned 317 slaves. One student added that Thomas Jefferson also was a slave owner. And then, in part to be funny and in part expressing anger over vote fraud involving African Americans and the U.S. Supreme Court’s delivery of the presidency to George W. Bush, one of my students shouted, “Bush is a slave owner, too!”

“No, Bush doesn’t own slaves,” I calmly explained. “Slavery was ended in this country in 1865.”

Short exchanges such as this often pass quickly and we move onto another topic. But then one student asked, “Well, which presidents were slave owners?”

She had me stumped. “That’s a good question,” I said. “I don’t know.”

Thus began a combined social studies, math, and language arts project in which I learned along with my students, and which culminated in a fascinating exchange between my students and the publishers of their U.S. history textbook.

After I admitted that I had no clue exactly which presidents owned slaves, I threw the challenge back to the students. “How can we find out?” I asked.

“Look in a history book,” said one. “Check the Internet,” added another.

I realized that I had entered one of those “teachable moments,” when students show genuine interest in exploring a particular topic. Yet I had few materials about presidents and slaves, and no immediate idea of how to engage 25 students on the subject. I played for time.

First, I had a student write down the question—“Which presidents were slave owners?”—in our class notebook, “Questions We Have.” I then suggested that a few students form an “action research group,” which, in my classroom, means an ad hoc group of interested students researching a topic and then doing something with what they learn. I asked for volunteers willing to work during recess. Several boys raised their hands, surprising me because I would have guessed that some of them would have much preferred going outside to staying indoors researching.

After several weeks of research, class discussion, math and social studies lessons that involved extensive data crunching and graph construction one of my students summed up our mathematical findings in a letter to the publisher of the fifth grade US history textbook—which had no reference to any president owning a slave. She wrote:

I see that you do not mention that some of the presidents had slaves. But some of them did. Like George Washington had 317 slaves. So did Thomas Jefferson. He had 267 slaves. If you want to teach children the truth, then you should write the truth. The children should really know the truth about the Presidents. Or do you know the truth about the Presidents? Did you know that some of the Presidents owned slaves? Because my class has done research about the Presidents and slaves. We have found out that John Adams did not have no slaves. We have also found out that 10 out of 18 of the first Presidents did! In math we figured out that 69% of the years between 1789 and 1877 we had a President who had been a slave owner (Peterson, 2009b).

Connecting History to the Present

In this exercise, as the students and I study history, we pay particular attention to dates and data. I try to highlight numbers that relate to

social movements for equity and justice. For example, as we look at women's struggle for equality in the United States we try to imagine what it was like for the women's rights leader Susan B. Anthony to go to work as a teacher and get paid \$2.50 a week, exactly half the salary of the previous male teacher. Much can be done with such a statistic—from figuring out and graphing the difference on an annual or lifetime basis, to looking for wage differentials in other occupations and time periods, including the present. I have found children particularly interested in looking at wages paid to child workers—whether it be in coalmines or textile mills. We compare such wages to the price of commodities at the time, to wages of adult workers, and to wealth that was accumulated by owners of industry. Such historical connections can be easily linked to present-day concerns over U.S. child labor and minimum wage laws or to international concerns over multinational corporations exploiting child labor in Asia or Latin America to make consumer goods for worldwide markets.

One math/history connection that can range in sophistication, depending on the level of the students, is to look at who is represented in different occupations and areas of power in our society, and how that has changed over time. For example, students can figure out what percentage of the signers of the Constitution were slaveholders, common working people, women, wealthy individuals who held bonds, and so forth. A similar exercise would be to analyze U.S. Presidents, or the people our country has chosen to honor by putting their faces on currency and coins. Such historical number crunching can take a contemporary turn if the students analyze the gender and racial breakdown of the U.S. House and Senate, the editors of major newspapers, or the CEOs of the Fortune 500.

It's important for students to understand that such numbers are not permanent fixtures of our social structure, but have changed as result of social movements such as the Civil Rights and women's movements. To demonstrate this change, a teacher might have students tally the current percentage of African Americans or women in selected professional occupations and compare it to the 1960s, before the rise of affirmative action, through people's struggles.

The History of Mathematics—Whose History?

Another area is to teach the history of mathematics, pointing out the contributions to mathematical thought of various non-European cultures and civilizations. Greek mathematicians, for instance, were heavily influenced by their predecessors and counterparts in Africa and Asia. Arab mathematicians inspired European Renaissance scholars. The Mayans were one of the first peoples to develop the concept of zero and make sophisticated mathematical calculations. I have used a unit on the Mayan counting system with base 20 with my fifth-graders to demonstrate such sophistication and to help students expand their understanding of place value (Ortiz-Franco, 2005).

Integrating Mathematics into Action

Integrating mathematics into current events is a no-brainer; using mathematics to influence future events requires a bit more effort.

The “sweatshop math” project (previously mentioned) is used to focus on double digit multiplication, division, and problem solving skills. In this integrated mathematics, geography, reading, and writing project, I have students manipulate current data about pay and working conditions in various workplaces around the world and make a poster display their findings using a world map. They reflect on their findings and connect it to their own lives, particularly consumer items they buy. One year when I was preparing my students for the project, I put on the overhead some photos of child laborers—kids carrying bricks in India, child weavers in Pakistan, and a farm worker in Central America. We were discussing why families have to have their kids work when Osvaldo, a chunky Mexican-American boy raised his hand and said, “That’s what I used to do in Mexico. I worked with my grandma in the fields.”

The rest of the kids turned and looked at Osvaldo. “Oh really,” I said trying to quickly think how I would lead this discussion. “Tell us more.”

“Well, I worked in the fields with my grandma,” Osvaldo rarely contributed much to class discussions, not because he wasn’t smart or following along, but mainly because he was shy.

“How was it working in the fields?”

“It was OK.”

“Well I bet you wished you were going to school instead,” I half asked, half stated.

“No.”

“Yeah I wish I could work instead of going to school,” another student called out. The conversation wasn’t going in the direction that I had hoped.

I ignored the student’s comment and continued questioning Osvaldo. “Why didn’t you want to go to school?”

“Because the teachers were really mean in the school.”

“Mean?”

“Yeah, like if you misbehaved in school they put biting ants down your back and it really hurt!” Kids gasped, unbelieving. “It’s true!” Osvaldo said. “I didn’t want to go to school.”

“I wouldn’t want you to be treated like that either,” I said. And then in hopes of wrapping up this part of the discussion, I suggested that Osvaldo might want to write a poem about this experience working in the fields.

Later that day he came up to my desk and showed me a neatly written poem. It read:

A Five Year Old Boy

Based on the true life of Osvaldo

I am a 5-year-old boy working to support my family
I am a boy walking for a mile just to get clean water out of the
pozo
I am a boy carrying a sack full of caña more than a mile
I am a boy watching for snakes and scorpions while working
I am a boy carrying food for my farm animals
I am a boy working for hours in the sun
I am a boy working in pain in my back and arms and legs
I am a boy sleeping with pain and bruises on my fingers

“That’s great! Your words make me get a picture in my head and I feel like I am right there.” Osvaldo smiled. “Might you be interested in writing a second part to the poem about when your family came to the United States?” I asked.

Osvaldo shook his head no, and I understood. I never push kids to

tell more about how their families arrived in the United State than they want to. “Well, how about how you felt once you got here?”

The next morning Osvaldo showed me the second half of his poem:

I am a 5-year-old boy living in the United States
I am a boy eating strange food
I am a boy going to school
I am a boy seeing for my first time a show
I am a boy having my first game station
I am a boy seeing other people like African American, Puerto Ricans, and Americans
I am a boy living happily in my new home
I am a boy feeling sad for being far away from my grandma and grandpa
I am a boy getting chubbier each year for not working

At our author’s tea that Friday, Osvaldo shared his poem in its entirety with his classmates and received an extra loud round of applause.

Cost of War

That same year (2006)—like I have every year since the Iraq War started in 2003—I used the cost of our wars as a way to teach place value and the importance of understanding large numbers. Whether it’s having the students guess the population of the world or a large number I write down that has a lot of zeros, many students are quick to guess a whole range of (wrong) numbers (Peterson, 2005).

Despite the fact that my fifth graders have been taught place value throughout their elementary years, there is something about big numbers that lends itself to guessing. Perhaps it’s the omnipresent state lottery advertisements that tend to blur big numbers together. Or more likely it’s the fact that big numbers are just difficult to read, much less understand. Imagining a billion boggles my mind, whether I’m trying to fathom that number of galaxies swirling around the universe or the number of H₂O molecules in a drop of water.

With some work, though, kids can connect the large numbers both to their own lives and to bigger social issues like the cost of war. After a mini-math lesson on the cost of the Iraq War that included rather

tedious large group math calculations, my fifth graders calculated that the United States was spending about \$3,600 each second on the Iraq War.

“That’s six PS360s! [a video game system] every second!” Ben exclaimed. “That’s one heck of a lot of money.” Ben started going off on the “stupid war” and said some derogatory comments about Bush. I refocused the conversation saying that “name calling—even of a President you disagree with was not allowed in my classroom” but students were welcome to discuss and debate different points of view on the war and other important issues. A normally quiet child raised her hand and shared that her cousin was in the military and another boy shared that his sister was currently in Afghanistan. At one point, I asked students who had family or close friends in the military and a third of the 25 students raised their hands.

When one student mentioned that he thought it was silly to spend that much money on the war, I suggested that we take a look at a website that tallies the cost of the war. Any mention of the Internet sparks interest in my class so they all focused in on the classroom computer where we went to the website www.costofwar.com. Students were excited to see numbers streaming by fast on the screen. After showing a student how to freeze the tally, I choose one student to read off the number. As he did so digit by digit, I knew we had some work to do on understanding and reading large numbers. I wrote the time and then number on the whiteboard and as a class we practiced reading the number. In the process, I reviewed place values up to the hundred billions place.

The following day, I brought in a newspaper photo of a “die-in” from the previous night in which anti-war protesters laid on the steps of the federal building. I told the students I had participated, although some were skeptical because I was not part of the photo. That didn’t stop us though, and we checked the cost of war again and practiced reading the number.

On the third day, for our early morning activity, I had students calculate how much money had been spent in the last 3 days—and then by doing a very long division problem we determined that the United States spends about \$265,000,000 per day on the war. We checked out other parts of the website and found, for example, that the amount spent on the war could buy over 14 million 4-year college scholarships. “That’s a lot!” exclaimed several students, but in reality the number

was so large that it was near meaningless.

I decided to try to make the number more understandable by having the students figure out the cost in smaller units of times. Working in pairs and then as a large group and with a whole lot of discussion we calculated how many “tax dollars” are being spent in an hour, a minute, and then a second.

That’s when Ben shouted out the number of PS360s that could be bought each second—assuming, Ben clarified, “you buy the \$600, not the \$500 model.” Isaiah suggested that you could take those six and resell them on e-bay and make a lot more money. I could tell that my attempt at getting kids to reflect on the social cost of the war was losing out to their consumerist fascination with the latest video game technology. My frown must have communicated my disappointment because Ben quickly added that with all that extra money from the e-bay sales, we’d have all the more money to help people who needed it.

I made one last attempt to concretize the cost by having a student go to the part of the cost of war website on public education, that calculates the additional number of teachers (at that point, 2007, over 6 million) that could be hired for a year. I told the kids there are about 80,000 public schools in the United States, then as a class, we calculated that every school could have 75 more teachers—or each school getting two more teachers for the next 38 years. Given that we don’t have a music or gym teacher in our school, students related to that. “No offense, Mr. Bob,” one told me later. “You’re a good gym teacher, we’d just like to have a real one.”

The war had become a low-level conversation in my classroom. Occasionally, one of the students would come in and announce the new cost of the war from their Internet work they had done the night before.

The war also found its way into our school in different ways. In 2010, when district budget cuts threatened our school’s full-time librarian, I helped fourth and fifth graders form the “Rescue our Librarians Club,” which petitioned, testified in front of federal and local officials, and protested in favor of our librarian. I encouraged students to compare the cost of war with the cost of librarians. As I wrote later:

At the Rescue Our Librarians Club, we did some calculations. Assuming that wages and benefits for a librarian average \$75,000

a year, and that there are 95,000 public schools in the United States, the government would have to find \$7,125,000,000 to ensure a librarian for every school.

“Wow, that’s a lot!” one student exclaimed.

And it is. I directed the children to the website *The Cost of War* (costofwar.com), where we discovered that the United States spends nearly \$300 million daily on the wars in Iraq and Afghanistan. We figured out that funding all those librarians for a year was equivalent to 25 days of war spending.

The use of mathematics and the stark comparisons between the cost of war and students’ real needs further motivated students in their efforts (Peterson, 2010).

The year that Osvaldo revealed he had worked as a child laborer, I was scheduled to go to New York to give a keynote speech at a mathematics conference about how numbers count. I explained to my students about my trip and repeated what I had said all year: numbers are really important if we want to understand and change the world. I said if anyone wanted to write a poem that would show how numbers count, they could. Osvaldo addressed the poem to president Bush. He wrote:

Numbers Count

Numbers do count Mr. President
2,453 America soldiers are dying
17,648 soldiers are wounded
35,161 innocent Iraqi people are dying
281,864,948,707 dollars are wasted in the war.
Imagine how much we can do around the world
With that money!
We can feed the needy
Build houses
Give Scholarships
And much more
Do you care about the people?
I do.

Concluding Words

Of course the level of sophistication and complexity of the mathematics we use in our classrooms depends on the developmental level of our students. Teachers, however, too often underestimate what students are capable of doing. To the degree that I provide quality instruction, clear modelling, and purposeful activities, I am usually pleased with the enthusiasm with which my kids take on such math-based projects, and the success they have in doing them. I have observed in such social justice-based math projects and activities that the my students demonstrate considerably more “stick-with-it” attitudes, than in more mindless math assignments.

I have found that as a result of trying to implement “mathematics across the curriculum”—and in particular, integrating mathematics and social studies—my students’ interest and skill in mathematics have increased, in terms of both their understanding of basic concepts and of their ability to solve problems. Furthermore, they can better clarify social issues, understand the structures of society, and offer options for better social policies.

Even during an era of scripted curriculum and obsessive standardized testing, I believe that social justice math teaching is not only possible, but more necessary than ever. Teachers can weave social justice into even the most prescribed math lessons and find state or Common Core standards that connect to most uses of these types of mathematical questions. Math concepts and skills are universal. There is no reason why such concepts and skills can’t be taught in a social context, pushing students to examine issues of injustice.

Of course, this approach takes more time, preparation, and occasionally some delicate political manoeuvring. By working collectively – whether in school-based groups or through committees in our unions—teachers will likely succeed at this important work of interjecting social justice math into our scripted-curriculum dominated classrooms. By doing so we will become better teachers, feel that our work is more meaningful and, most importantly, give our students the tools they need to make this world—their world—a more just and sustainable planet.

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Curriculum and Critical Agency: Mediating Everyday Mathematics

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This paper traverses the path of critical education and reflection sought by activist academics who, in various capacities, strove to change curricula for science and mathematics education in India at the state and national levels. The National Literacy Campaign in the 90s had offered space to understand people's lived realities and mediate everyday math in the teaching learning processes, through a constructive critique of the hegemonic and alienating nature of the school subject. Subsequently, within the ambit of the National Curriculum Framework 2005 and the Right to Education Act 2009, the national primary math textbooks attempted to (re)mediate the experience of non-literate adults, and to address diverse children's knowledge through a (re)humanising pedagogy of empathy, despite the constraints of a large bureaucratic and increasingly neo-liberal state system.

Introduction

In the 70s, as part of a group of activist scientists, I was involved with curriculum development for the Hoshangabad Science Teaching Programme (HSTP) in rural government schools of Madhya Pradesh. It was the first inquiry-based science curriculum in the country, which forged compatibility between “academic credibility” and “relevance”, conducted through collaboration between voluntary activists, professional scientists, and school teachers. Taking cognisance of the majority of pupils who did not continue school after Class VIII, the curriculum, within the constraints of the State syllabus, gave priority to developing a critical scientific attitude for life, and even excluded concepts usually dictated by the discipline. The emphasis on low-cost and indigenously designed apparatus provided an incentive to teachers to exercise their own creative skills and symbolised an assertion of self-reliance. Pedagogically, it was also important for students to feel comfortable with the apparatus, to alleviate the sense of alienation and

mystification normally associated with science lessons, not just in rural but also in city schools. However, its critics used its philosophy of “low cost” to describe it as “low status” and label it as “backward science” (Rampal, 1992a). Political as well as parental pressure had often to be countered through discussions related to issues of science-technology-society in a developmental context. Incidentally, with the increasing neo-liberal pressures of a burgeoning IT industry, it now becomes even harder to advocate for low-tech humanistic pedagogies.

The programme had negotiated processes of participatory curriculum development and enactment with the government, and had sought several academics from across the country to be associated in various capacities, from among scientists, educationists, and later, social scientists, child developmentalists, linguists etc. However, the participation of mathematicians was then not similarly envisioned; even decades later, activist mathematicians are often found scouting for and nurturing the “gifted” or “mathematically talented” from among the ordinary millions of students, through joyful activities, math clubs or camps, but not interrogating the school subject. Reflecting from my personal experience, I find that science activism had informed debates and social action in the arena of education and science policy in the 70s and 80s, with several initiatives through voluntary peoples’ science groups, leading to a consolidation of the All India Peoples’ Science Network, immediately after the Union Carbide Gas Disaster in Bhopal. However, it was much later, in the early 90s, while working with unschooled youth and adults in the National Literacy Campaign, inspired by Freire’s critical pedagogy, that some of us began to relook at mathematics from a socio-cultural perspective.

Interestingly, as part of the HSTP even then in the 70s, along with the broader theme of measurement (of length, area, volume, and weight), a chapter on probability was consciously wrested from the traditional secondary school math curriculum, which was not open to restructuring, and developed as part of the middle school science programme. However, even with a focus on inquiry-based learning, the main thrust did not depart significantly from the normative understanding of probability, explored through activities, with only a few references to social contexts.

The chapter on chance and probability in the HSTP science text-cum-workbook began with the example of an erratic schedule

of a local passenger train, and drawing upon the daily dilemmas of commuters, gradually led learners to think about chance, prediction, and what is popularly perceived as “luck”. Through activities with coins, dice and *cowries*, and observations on how relative frequencies stabilise with large numbers, it introduced the idea of probabilistic thinking in estimating the percentage of seeds that germinate in a field, or sample testing for soil quality, tests for TB etc. The HSTP textbook encouraged students to work collaboratively in groups, raising questions, seeking tentative answers, collectively resolving doubts, experimenting, and deriving their own procedures using their observations. This participatory learning approach sharply departed from the individualistic image of the subject, through its hidden curriculum that enables some “talented” pupils to learn mathematics, while ensuring that the others understand that they cannot, with due deference towards those who can (Eggleston, 1977).

Interestingly, and perhaps predictably, the chapter on probability became the centre of a political controversy in 1985, even threatening the programme, when the science paper of the middle school State Board examination included two questions on probability, based on the context of “*satta*”, the local form of gambling. A politically motivated query was raised in the State Legislative Assembly by a member of the right wing conservative party in opposition. It strongly urged the closure of HSTP, alleging that the programme based on principles of “learning by doing” actually taught students to gamble! However, for the first time in the history of a state Legislative Assembly, an academic argument was presented through a substantial 25-page paper by an MLA (Member, Legislative Assembly), who defended the teaching of chance and probability, along the following lines:

- the historical origin of probability as a field of study lies in games of chance and gambling;
- HSTP does not promote gambling, on the contrary it leads students to understand the dangers of gambling by understanding the mathematics behind it;
- such politicisation of education can prove to be regressive for the scientific and technological development of a country.

The middle school teachers who framed the questions were served an official “show cause” notice but bravely and confidently defended their

action. As active resource teachers of the programme, they replied to the state Department of Education, explaining that having seen the terrible impact of gambling on the lives of daily wage earners in Hoshangabad district, they held that such questions enabled children to construct knowledge and make connections between probability and the slim odds of winning at gambling. In fact, the ability to challenge authority, as had been imbibed by the resource teachers and even students, through the democratic ethos of the programme, displeased large sections of the feudal bureaucracy and political establishment of the state.

Though in 2002, after thirty years, the state government did close HSTP owing to political considerations, much of the insights and experience gained continued to influence restructuring of education in other states of the country and, subsequently, at the national level as part of the National Curriculum Framework (NCF) 2005 (NCERT, 2005a). For the first time there was a large participation of progressive intellectuals and voluntary groups in developing the NCF 2005, its position papers, syllabi and textbooks (all freely available online), as a response to the agenda of cultural and religious imperialism of the previous government at the Centre. Since then this tension has continued to openly play up in the states, as and when there is a shift in government. Now with a change at the Centre there are again calls for a changed NCF and education policy. Academic bodies are seen promoting often contested chauvinistic claims glorifying ancient science and mathematics.

It is indeed a matter of concern why an enabling legacy, of developing curricula within the sociocultural context of learners and teachers, as was initiated in HSTP in the 70s, did not take us very far in mathematics education in the intervening four decades. The national textbooks for history and political science, as well as the primary textbooks, saw a major shift. However, in sharp contrast to the national primary math curriculum, which I shall discuss in more detail, the middle and high school curricula have continued to remain entrenched within a dehumanising disciplinary domain. This is despite the NCF 2005 and the historic Right to Education (RTE) Act 2009 for children 6-14 years, which mandates “learning through activities, discovery, and exploration in a child centred and child friendly manner” by “making the child free of fear, trauma and anxiety” (Section 29, GoI, 2009). The Act also posits continuous and comprehensive assessment and bans grade retention, any selection for

school admissions, or the conduct of competitive examinations at the elementary stage. However, the pattern of examinations remains tied to rote memorization of procedures and definitions, promoting hegemonic notions of mathematical “ability”. It intimidates children and also dissuades some creative teachers, as their efforts to encourage sense making get obliterated by the examination questions devoid of any meaning and contextual relevance.

The present curriculum for middle school math introduces normative probability in grade VII through conventional descriptions of coins and dice, with no attention to the epistemological and pedagogical complexity involved, nor to children’s subjective ideas and cultural values, through strong beliefs in luck, destiny, and fairness. We recently followed up on this theme in a doctoral study (Bhavna, 2013), to understand children’s probabilistic thinking and discursive practices through contextual tasks. We noted how they articulated their agency through school surveys (to predict and critically analyse patterns of child sex-ratio in the school families, interrogating patriarchal ideologies and gender preference practices), and using that knowledge we developed culturally responsive curricular units. In another ongoing doctoral study, we have seen how children of Class VIII in a government school, subjected to traditional school tests and declared “low achievers”, are able to perform sophisticated statistical analysis in culturally responsive assessment tasks, such as studying their Mid-Day Meal programme, or the daily wage labour data from local building contractors. In addition, the “high achievers”, too, discovered alternative ways of thinking and problem solving and posing from their peers; this significantly changed the group dynamics and scaffolded them to deeper and higher levels of engagement, from what they could earlier do in a routinely procedural fashion.

A preface to the Report “Learning without burden” (Government of India, 1993), a precursor to the NCF 2005, had noted that the problem of ‘drop-out’ results from those children being “pushed out” who, in a way, refuse to compromise with “non-comprehension”. This helped problematise the “deficit discourse”, with a shift towards the nature of comprehension and meaning making, relevance and participation, especially in the context of the curriculum. The Literacy Campaign in the 90s had also highlighted the “push out” phenomenon, stating that had elementary education been more inclusive and equitable in the decades after independence, the country would

not have been faced with abysmally low levels of literacy. The task of developing alternative numeracy curricula for the campaign had, for some of us, assumed a dual nature—exploring and mediating everyday knowledge while offering a constructive critique of schooled math.

Everyday Dilemmas: An Unschooled Mother, a Schooled Daughter

We got insights into the dilemmas of people's everyday lives and mathematical practices, as we interacted with Draupadi, while trying to help her daughter with primary school mathematics. Draupadi's background helps situate her learning as part of her life trajectory, while it “foregrounds” (Skovsmose, 2014) her daughter's experience at school. Draupadi, a middle-aged dalit (a neo-Buddhist belonging to a scheduled caste), had struggled initially to survive in a city through rag-picking and daily wage labour at construction sites. For some years, she had been living in a slum dwelling, working as a domestic help in several middle-class homes. She had missed out on school education during her turbulent childhood spent in a small village. At a very young age, she had had to take charge of her younger brother and sisters, after the death of her father and her mother's mental breakdown. Married at the age of thirteen, she had moved to the city with her husband and her siblings, in search of work. She learnt to read a little in her thirties, from one of her supportive domestic employers. The almost singular motivation of her life had been to send her children to school, to somehow release them from the vicious cycle of deprivation she found herself in. Much to her disappointment, schooling alienated her children, instilling a debilitating sense of failure. Her eldest daughter was “pushed out” in six years. Her son, who initially took interest and attended school regularly, gained recognition as an amateur percussionist in the local community, finally gave up, disillusioned after failing twice in the Class X examination; some years later, he took to alcohol like his late father, when summarily retrenched from a factory job despite his diligence. Sunita, her youngest daughter, was studying in Class V when we first met Draupadi, who desperately hoped that at least this bright-eyed child would complete school; but the odds were heavily against her.

Ten-year-old Sunita had been very upset at not being able to cope with school math and her teacher, who constantly reprimanded and humiliated her for it. She was totally confused by the algorithms taught at school, unable to use her own life experience the way her non-literate mother could effortlessly do. “See, if your mother wants to divide Rs.180 equally between her three children, how much would she give you?” we asked reassuringly. Sunita seemed confounded and stared blankly. Her unschooled mother laughingly provided the answer and wonderingly looked at her. Sunita, for her part, mindlessly manipulates numbers; when a problem is stated in words, she desperately tries to translate it into some mathematical operation between 180 and 3, and often cannot decide which operation to use, the “x” or the “+”, or some other. Even when she is told she has to divide 180 by 3, she can only visualise the school method of writing numbers in what is a meaningless and strange pattern to her (for ‘long division’)—bringing one down, writing something on the right side, some number above, subtracting, and so on.

We next ask Draupadi how much she would need if, instead, she had to give Rs. 180 to each. Again promptly, almost without batting an eyelid she smiles and says “Rs. 180 for three of them? ...that’s Rs. 540” without knowing any tables and without being able to write these figures down. How did she do it? First, she says, she pictures three 180s in her mind. From the last 180 she mentally gives 20 to each of the first two, “completing” 200 each time, and then added $200 + 200 + 140$. She proudly confesses that she challenged her husband, a skilled and literate mason, that she could perform complicated mental computations faster than he could do using his written methods. However, Draupadi does not try to teach her own ways to her daughter assuming that these and better ways are available to those who can write. Unfortunately, it is not so for young Sunita and millions like her. We realised this was not an isolated case of a “slow learner” and her “talented” mother. Most surveys with out of school children said they found school to be “uninteresting” and “difficult” and mathematics to be the “killer” subject. On the contrary, Draupadi, like most adults engaged in everyday mathematics, did not suffer from school-induced “math phobia” and within her everyday purposes was confident of applying a variety of mental algorithms (Nunes, Schliemann, & Carraher, 1993).

We noted that innumeracy was not necessarily linked to a lack of scriptural literacy, but more to the nature of the mathematics

curriculum, divorced from, and dismissive of, the learners' strategies and knowledge. Based on observations of everyday mathematical practices, we developed a Handbook on numeracy for the literacy volunteers, documenting the rich cultural repertoire of knowledge, and presenting an alternate curriculum for adult learners (Rampal, Ramanujam, & Saraswathi, 1998; 2000).

The National Literacy Campaign

During the 90s, Literacy Campaigns were conducted in over five hundred districts of the country, to include non-literate adults in the age group of 15-45 years. To provide a voluntary people's arm to work for the Campaigns as part of the National Literacy Mission of the government, the Peoples' Science Network had formed a special voluntary organisation called *Bharat Gyan Vigyan Samiti* (roughly translated as the Indian Knowledge and Science Committee). Many of us got involved in all the processes of the campaigns, at every possible level. The campaign was participatory, with local teams planning and implementing it under the aegis of the district administration. Teaching was carried out by millions of volunteers without any remuneration; the government bore a low cost (less than US\$ 2 per learner), while the community pitched in through volunteer teachers, space for literacy classes, food and shelter for the travelling theatre groups engaged in cultural mobilisation, and so on. For the first time in Indian education, the curriculum and primers were meant to be developed at the district level, in local languages, including some indigenous languages not scripted earlier. Despite the constraints, many districts did come up with their own primers and supplementary materials, especially in the post-literacy phase, based on local contexts such as the coal mines, migration, women's self-help groups etc.

We found that oral numeracy is more natural among unschooled adults than literacy, as they are more familiar with numbers, operations, measurements, etc. than with alphabets. However, in the traditional primers made by the State Resource Centres the literacy and numeracy content came together, the latter often appended to each lesson in an ad hoc manner. As far as reading and writing were concerned, there seemed to be some understanding that teaching would codify speech and verbal thought, through generative words,

and not start mechanically from alphabets. However, there was no such attempt to understand the lived “meaning of numbers” or to look for generative themes; the conventional primers laboured through numbers in an absurdly child-like and linear fashion, 1-10 in chapter 1, 11-20 in chapter 2, and so on. Pictorial illustrations were taken from children’s books, and learners were condescendingly asked to count eight ducks or five apples. Moreover, the national curriculum committee tended to “dilute” the expected competencies each time the poor mathematical performance of adults was brought up.

On the contrary, adults wanted to move on fast to more sophisticated and challenging tasks in mathematics, to help them deal with market transactions more confidently. Operations of addition, multiplication etc. made more sense as contextual problems, and adults were capable of doing “word problems” long before they learnt to read and write. However, conventional adult educators, influenced by the school myth of “word problems are more difficult” avoided these and other everyday calculations involving profit and loss, simple interest on loans, or even probability of winning or losing a lottery, considered important by the learners themselves. Similarly, adult educators were wary of fractions and did not try to understand how some adults comfortably negotiated through complex fractions as part of their work. In many southern states, intricate fractions exist in everyday vocabulary, and seem to have some link to the markedly intricate sound and beat patterns used in classical Carnatic music. In Tamil (the language of the state of Tamil Nadu) and Malayalam (of the state of Kerala), people speak of “half of one fourth” as “*araikaal*” or “three fourth of one eighths” as “*mukaal arakaa*”. This, interestingly, is not found in the languages or the music of the northern states. Similarly, “dichotomous divisibility” or repeated divisibility by two, significant in the counting systems (with base 12 or 16) or measures used by traditional societies, could be found in use with no place in the primers (Rampal, 2003a).

Learning while Doing: From the Everyday, to the Centre

Unlike traditional cognitive theories that isolate and distance the learning “mind” from its experience, theories of situated learning do

not separate thought, action, and feelings of the learners. While in the traditional case it is “learning” that is problematic, in the socio-cultural perspective “knowledge” becomes complex and problematic (Lave, 1996). We had become aware of some of these issues while working with adults coming to the Literacy Centres, and as part of developing alternate curricula for them we mediated their knowledge into the teaching learning processes. The campaign, where effective, had led to markedly increased demands for schooling of children with high enrolment in those districts. But the formal system remained rigid and unresponsive to their aspirations.

We had seen that adult learners engaged in a host of mathematical transactions, such as sorting, measurement, estimation, making patterns, etc. (Gerdes, 1985; Greer, Mukhopadhyay, Powell, & Nelso-Barber, 2009) as part of their activities related to life and labour. New ideas or skills needed to be reinterpreted through the learners’ own mediatory mechanisms, of assigning meaning to them, and, more importantly, testing them out in real-life settings. However, teaching practices used in traditional adult literacy classes never attempted to encourage such mediation and failed to engage learners in reflection of any kind.

Oral societies have invested tremendous effort and ingenuity in devising mnemonic techniques, such as *shlokas*, *mantras*, and *sutras*, rendered through elaborate rhythmic patterns, to memorise, preserve, and transmit their rich bodies of knowledge. A poetic ode to estimation and measurement, this old poem in Tamil reflects an amazing range of length measures from the atomic to an astronomical scale, through the use of rich life-world imagery. This excerpt from the English translation however does not give a sense of rhyme and the sounds of the original.

8 atoms	= 1 speck in the sun’s ray
8 specks in the sun’s ray	= 1 speck of cotton dust
8 cotton specks	= 1 hair point tip
8 hair tips	= 1 small sand particle
8 sand particles	= 1 small mustard seed
8 small mustard seeds	= 1 sesame seed
8 sesame seeds	= 1 paddy seed
8 paddy seeds	= 1 finger width
12 finger widths	= 1 span

2 spans	= 1 cubit
12 cubits	= 1 stick (kol)
500 “kols”	= 1 “koopidu dooram” (calling distance)
4 “koopidu dooram”	= 1 “kaadam” (about 1.2 kms.) and so on.....

The measure “*koopidu dooram*” or “calling distance” is known to have been used as a traditional measure in many early metrological systems, suggesting empirical knowledge that sound travels only a finite distance and different frequencies attenuate at different distances. Economic historians have noted that, unlike the metric system, which is wholly arbitrary and dependent on convention, folk measures should not be termed “conventional”; those are truly “representational” and therefore have a social “meaning”, determined through peoples’ activities (Kula, 1986). The dominance of qualitative “value” over purely quantitative considerations in the social thinking of pre-industrial societies is found to be striking. For instance, land measures were not directly “addable”, and accounted for the quality of soil, labour-time for tilling, or the amount of seed needed. Similarly, the “measure sold” was different in size from the “measure bought”; a heaped container is bought from the farmer while a flat one is sold to a customer at the same price, to take care of the costs of transaction; even now we are given 13 glass bangles at the rate of a dozen (to cover possible breakage). Such forms of measurement often co-exist with the standard systems but most educators refused to appreciate the ingenuity and significance of such measures, dismissing those as “crude”, inaccurate, or “primitive”.

As part of the teaching methods we devised, measurement and estimation began in the local measures before the metric system was introduced. For instance, the Numeracy Handbook (Rampal et al., 1998) for volunteers points out the different body measures used:

Extensive use of body measures is important, particularly for length. Exercises can be done with units like finger-width, finger-length, span, cubit, feet, fathom, etc. For depth, people often use units like ankle-deep, knee-deep, man-deep, etc. In addition, units like a *chatai*-length, a *dhoti*-length, length of fish etc. are also useful.

When it comes to volume measures, similar units are available such as:

- Taken as a pinch: salt, snuff, herbs, medicinal powder, etc.
- Scooped up by the fingers: mustard seeds, cumin seeds, *haldi* powder, etc.
- A handful: flour, pulses, sugar for sweets, peanuts, flowers, soil, etc.
- A palmful: “*prasad*” during ritual prayers received with a cupped palm, etc

The Metric *Mela*: A Festival of Estimation and Measurement

A plethora of creative activities to estimate and measure gave rise to the popular concept of a Metric/Math *Mela* (Fair) conducted by the volunteers and learners themselves, where the village turned out in all its fineries to participate. Incidentally, even the Test for the Literacy Campaign was organised as a large celebratory event where learners turned up in colourful clothes. The following excerpt from our Numeracy Handbook (1998, pp. 48-50) shows how the *mela* was conducted and encourages other volunteers to do so too:

Even as you approach the *mela*, you can hear the songs on the loudspeaker. You wish somebody would reduce the volume, but it is undeniable that the din caused does add to the festive atmosphere. Indeed it seems to go with the festoons, streamers, and the general riot of colour, with noisy children running about adding to the *mela* mood.

A couple of volunteers come forward to welcome you, assuring that you are about to have a “totally new” experience, and that this *mela* is entirely run by their students, the neo-literates of the village. You have heard this before, when they went door to door yesterday inviting everyone in the village to come, and even offered attractive prizes!



Figure 1: Festival goers. Source: *Numeracy Counts* (1998) Rampal, Ramanujam and Saraswathi)

Even as you join the queue, you ask why “metric” *mela*, but receive no clear response. The volunteers, young girls themselves, giggle a bit and are mysterious—you’ll find out soon. Even as you move along, you are intrigued by your red card, and you take a look at it. It has a big table with each row having a description and some blank entries. Passing over routine items like height, weight etc. you are intrigued by entries like “weight of a feather?”, “length of *lauki* (*gourd*)” etc. Wait a minute, what is that—“length of nose”?! They are not going to measure the length of your nose, are they!?

The Card

Name :	Male/Female	Card No..	Address :
My Height :	Cms	My Weight:	Kgs
Length of ‘ <i>lauki</i> ’ :		Length of Nose:	
Length of a chalk		Little finger:	
Weight of an egg:		Weight of the cabbage:	
Weight of a feather:			
Which has more water?:			
Volume of water in a bucket:		Volume of water in a bottle:	

The first stall you go to has a person with a measuring tape, who measures your height, enters it in your card and makes an entry

in her own register as well. In the next one, as expected, someone measures the length of your nose!

While you now have a pretty good idea of what the mela is about, and why it is a metric mela, the whole thing falls into place only in the evening, when there is a festive cultural programme followed by the much-awaited prize-giving ceremony. This is a virtual riot as there are prizes in a most interesting variety. There is one for the person with the longest nose, and one for the person with the shortest nose. The person who got the closest estimate for the weight of cabbage gets the cabbage itself as a prize, and similarly the one who got the length of the drumstick right gets the drumstick. Prizes for the tallest, shortest, heaviest, lightest ... Indeed there are prizes for almost everyone, and you get the prize for ... getting the weight of the chicken feather correctly!

The prizes are given away by the neo-literate who ran the stall, and this is in itself a novel experience for her and for the village. She also talks about how many got close to the answer, how many gave wild guesses (with some examples, causing much mirth). Interestingly, in all the stalls, though people gave their answer in whatever units they pleased, they have been converted to metric units before recording. Thus the term “metric mela!”.

The Meaning of Numbers

To begin with, learners were encouraged to consciously think of numbers in their lives, which could later lead to various group activities on issues of community resources through methods of participatory resource management. We had suggested to the volunteer teachers to begin with maintaining a number diary for their Literacy Centre. The teacher would ask them to make a simple number statement about themselves. For instance: “I weigh 53 kilograms”, “My uncle has eleven toes”, or “There are 7 people in my family”. The teacher would initiate a discussion with a series of questions:

- How many films do you remember having seen in the last five years?

- How many mangoes can you expect to buy for ten rupees?
- What is the number of the bus you take to go to from?
- What is the cost of one kilo of wheat?
- How many stars do you have on your *lehenga* (skirt)?
- How many buckets of water do you use every day?
- How many trees are there in your village?

The volunteer teacher was asked to give exercises based on the numbers that had arisen out of these discussions. Another exercise involved the teacher to say a number and ask the learners to respond with some objects of that quantity. For instance, the teacher would say “100” and a learner may respond with “100 jasmine flowers”; learners would also be encouraged to ask questions of each other, to look consciously for numbers and contribute to the growing collective database of familiar numbers.

Questions which required estimations of large numbers were deliberately posed:

- How many mangoes does a tree yield in a year?
- How many leaves does a typical mango tree have?
- How many *rotis* do you make in a year? How many do you eat yourself?
- How many stars are there in the sky?
- How many people live in our village?
- How many hairs do you have on your head?

Such discussions showed that learners understood that there is no limit to the growth of numbers. A challenging game took the form of bidding, “whatever number you tell me, I will tell you a greater number”. Gradually, when numbers got into the thousands, the teacher could go for much larger increments, and even when they did not know the exact relationships, say, between a thousand and a lakh (hundred thousand), it was enough to know the latter was much bigger. This honed their own estimation skills; some of them had seen estimation done during the leasing of fruit trees, where contractors would predict the likely yield of a tamarind or mango tree (Rampal, 2003b).

Exercises for large estimates were based on a series of Fermi-type questions. We began with posers such as, “What is the number of

cups of tea drunk this morning by the entire village?” and proceeded to discuss strategies to estimate. This further led to interesting group activities of planning, say, for a village feast, with estimates and quantities, and elaborate descriptions of the recipes. Needless to say, volunteer teachers were required to be trained to handle such sessions effectively. Some also conducted participatory resource mapping. They could estimate the number of children below the age of ten years, the number of cows and buffaloes, and so on, and later graduate to more systematic enquiries, which generated valuable databases. In several places detailed maps were drawn by the neo-literates themselves, and were used for watershed management programmes for the village. This helped the group develop a critical understanding of their world (Jordan, 2012), with the possibility of seeking more control over their lives. Indeed, a neo-literate woman who estimated the number of *rotis* she had cooked in her life claimed that the exercise changed her perspective about herself, her life and women’s work!

The National Primary Math Curriculum

Using the rights framework, tying quality to equity, which requires high expectations from, and opportunities for, *all* children to perform well (Boaler, 2008; Nasir and Cobb, 2007), we addressed “math for all” by restructuring the nature of primary mathematics. The NCF position paper (NCERT, 2006a) calls for a multiplicity of approaches, for liberating school math from the tyranny of the one “right” canonical answer, through learning environments that promote abilities for mathematization, invite participation, and offer *every* child a sense of success. A “participationist” (Sfard, 2008) vision of learning mathematics, unlike the acquisitionist approach, requires that learners begin by participating in collective mathematical discourses, of the home, community, or school, and progressively learn to communicate mathematically with themselves. The challenge of designing curricula for schools as diverse and ubiquitous as are in India is immensely daunting, to ensure representation of diverse discursive mathematical practices, through pedagogies of empathy that enable democratic participation (Rampal, 2010; 2013).

The syllabi and textbooks changed, though much of the challenge of changing classrooms remains. Given the resource starvation of

most of our elementary schools, textbooks form the only curriculum materials available for children and teachers, and these are being used by a large majority, leaving some private schools which prescribe books by private publishers. There are no concrete objects, games, or manipulatives in the classroom, no school library, and teachers' education does not steer them to mediate such resources. Low paid contractual appointments, frequent testing and a push towards privatisation further demoralises teachers especially those working in schools of the poor. Curricular reform involves sustained, unhurried and layered negotiation, within and outside the system, from policy documents to classroom practices, involving administrators, teachers, parents, teacher educators, and the media, to change mindsets about how children learn, how that may be assessed, and what basic provisions are conducive for that learning to happen. It has happened, at some places at some times, but is nevertheless an ongoing struggle.

As Chairperson of the NCERT (National Council of Education and Research and Training) Textbook Development Teams for the Primary Stage (Classes I-V), I had initially engaged with the writing of the syllabus (NCERT, 2005b) and textbooks for Environmental Studies (EVS), the subject that is meant to integrate Science with Social Studies. However, when the draft manuscript of the math textbook first came to me I was dismayed to see its didactic traditional form, contrary to the NCF. With barely three months to meet the publication deadline, I was requested to restructure the team and also work on it myself. As I look back, this was a crazy additional commitment to make, as another part-time voluntary task, along with my regular University teaching assignments. The urgency, however, allowed greater freedom within the bureaucratic institutional set-up, to select school teachers, teacher educators, academics, and also an array of sensitive artists, who broadly shared the social constructivist perspective, even if not necessarily exposed to critical math education. My experience with the numeracy curricular resources gave me the conviction to mediate children's everyday knowledge—for dealing with numbers, measurements, estimations, shapes, symmetries, and aesthetics—through contextual examples from art, craft, architecture and music.

Moreover, as we had done for the EVS textbooks, we sought real life protagonists to inspire and animate our thematic chapters. These included Kiran, who runs a junk shop in the city of Patna (The Junk-seller, Class IV), the masons - Jamaal, Kaalu and Piyaar - who built

the school in Murshidabad (Building with Bricks, Class IV), the cooperative of fisher-women running a fish-drying factory (Fish Tale, Class V), or Pedki Devi from Jharkhand who fought against her late husband's brothers, for publicly torturing her to "exorcise" her from being a "witch", to appropriate her land after his death (Time, Class III), and whom we had filmed during the post-literacy campaign.

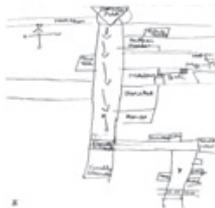
Its form needed to depart from that of the traditional textbook so that its voice resonated with more contextual "lived" resources (Gueudet, Pepin, & Trouche, 2012), through diverse genres of expressive narratives, folklore, auto/biographical stories, household recipes, travelogues, diaries, letters, records such as birth certificates, humour, fantasy, etc. even through tentative, tacit, and exploratory representations. We moved away from traditional inanimate illustrations which offer stunted, stereotyped, and monotonous images, to diverse representations including folk and tribal art, photographs, children's art, cartoons, and contemporary art informed by multicultural sensibilities. Each page was designed as a visual text, which could be processed by children in a non-linear manner. Though separate teachers' handbooks could not be developed, the textbook spoke to teachers through annotated comments. These notes shared reasons for dealing with concepts differently, observations about students' thinking, suggestions for out-of-class activities, encouraging them to look for lived resources in specific cultural contexts, persuading them to promote participation in place of the dominant modes of transmission.

Cross Disciplinary Themes

The National Curriculum Framework 2005 had recommended breaking down of the rigid boundaries between different subjects, to help develop a more holistic approach of learning from the child's environment and culture. This was attempted in the NCERT primary textbooks within the three subject areas of language, math, and environmental studies (EVS). For instance, the theme of "mapping", which traditionally comes under middle school geography without addressing children's understanding of several underlying concepts, was introduced across the primary subjects of language, EVS, and math. Some research had shown us that even privileged children (aged 8-10 years) exposed to maps at home and in school, drew maps of their own



Map I (age 13 yrs)



Map II (age 10 yrs)



Map III (age 8 yrs)

locality showing locations in a linear arrangement (Map II and III), though the actual road formed a closed loop. When asked, on site, why they made a road that kept going up the page, they confidently explained that while walking on that path they always went ahead and never turned back! The experience of walking on the road shaped the map they made, without their being able to imagine it from an aerial perspective. It was a 13 year old girl in the locality who drew a closed loop for the road (Map I) though the orientation and scale of her map was still impressionistic and very different from that of the cartographer.

Noting the real-life relevance of developing an understanding of mapping, which adults and even University students seem to fumble with, the primary math books included several chapters even though the syllabus (rather routinely framed) had not focused on it. Chapters were developed on the concepts of projections and perspective, aerial views of a site or object from different heights (as seen by a mouse riding on a hydrogen balloon), about directions, scaling, representation, etc. Iconic and pictorial maps were made for specific contexts, linked to narratives, such as children finding their way to the beautiful monument of Taj Mahal, which gradually progressed to abstract schematic representations. A chapter in Class V (NCERT, 2008) encourages the comparison of an iconic map with an aerial photograph, of India Gate on Raj Path, a well-known location in New Delhi which most children get a chance to hear about or watch on television during the Republic Day Parade. In addition, several creative formats were used, such as diaries or travelogues for a historical monument, visuals and pictorial maps, as well a treasure hunt game. Simultaneously, the EVS textbook developed the ideas of mapping by exploring a historical fort, or through a chapter “*Sunita Williams in Space*”, based on the true experiences of a NASA astronaut of Indian origin, who eloquently described her thoughts as she looked at the earth from space, and even poignantly

wondered where the “lines” or boundaries got drawn which, from there, could not be seen between India and its neighbouring countries.

Special Thematic Chapters: Authentic Contexts

We developed special thematic chapters to deal with varied issues of work, entrepreneurship, heritage, craft knowledge, history of monuments and pre-historic cave paintings, etc. using contexts that invoked and integrated concepts already learnt. For instance, *Building with Bricks* in Class IV (NCERT, 2007) begins with the true instance of a school being built by local masons, who are taken to visit the nearby mosque to observe the amazing variety of floor patterns built by their ancestors three hundred years back. They return inspired and make their own brick designs for the school courtyard. With modern bricks different from the older thinner ones, the masons generate different symmetries and patterns, which students are encouraged to analyse in photographs we had taken during our visit. The chapter goes on to measure a brick, to study its faces, see its projections and how (as the first example of a cuboid) it can be represented in two dimensions. It prompts students to observe photographs of other brick patterns from different parts of the country and analyse the designs in traditional architecture. There are examples from the work of Laurie Baker (without naming him), a Gandhian architect who devised low-cost environment-friendly buildings. It finally travels to a brick kiln, to understand the process through visuals; students are first introduced to the large number “one hundred thousand” (a lakh), as the number of brick kilns in the country.

The process of thinking of large numbers (as done in the numeracy curriculum), relatively and in familiar contexts, through orders of magnitude, is adopted throughout the books where, for example, they connect 100 with the scoring of a century by a famous cricketer, or are asked to recall where they have heard of a “lakh” (one hundred thousand). Similarly, the number one crore (ten million) is first introduced in the thematic chapter “The Fish Tale”, as “the number of people whose lives are related to fish—who catch fish, clean and sell them, make and repair nets, and boats, etc” (NCERT, 2008; p. 10). Indeed, “Asking students questions, such as, ‘How long does it take to count to 1,000?’ or ‘Have you lived more or less than 1,000 days?’ provides them an opportunity to think about 1,000 in a personal context, thus

helping them better understand the size of 1,000 in a variety of contexts” (Macintosh, Reys, & Reys, 1995, p. 216).

The Junk Seller is based on the true story of a young woman Kiran, who had, against all the odds of living in a poor, highly patriarchal, rural society, managed to set up her own enterprise in the city of Patna. She narrates her struggle, her early dislike of math in school and her acknowledgement of how it is now an integral part of her present vocation, which has indeed helped change her life and the situation of her family.

I studied in a Hindi medium school in my village. My father wanted girls to study like boys. I loved Hindi and Science, but I hated Maths! Today Maths is most useful for my work. I could never imagine this in school.

What about you? Do you also find Maths difficult? _____

What is the most difficult thing in your Maths book? _____

Which do you think is the easiest lesson? _____



When I was young my father died in an accident. So my mother worked as a servant in some houses. We had a difficult time. I had to leave school after Class VIII. I wanted to study more but my mother got me married. My husband’s family lived in a mud house. ... He had a tea stall.

I thought of starting my own business. I thought I should open a bangle shop or a tailor shop. But my uncle said that we could earn a lot by opening a junk shop.

People laughed and teased us about our work. They called it *ganda kaam* or “dirty business”. But I did not think so. I knew this idea would work (NCERT, 2007, pp. 60–61).

Through this visual narrative with on-site photographs, the unit deals with her loans, her junk sorting and selling, hiring of collectors, recycling of materials, etc. It challenges several prevailing notions of gender and mathematics, the stigma of “dirty work” attributed to certain castes and their supposed low position in society, and also the traditional focus on a “great man” as a role model. It inspires young women with a sense of “social agency” to develop their entrepreneurial abilities to transform lives. Later, Kiran proudly told me that many visitors come to her shop just to see the “textbook hero”.

Interestingly, this focus on cultural relevance and real life contexts caught the public imagination. Leading national newspapers and TV channels, which followed the development of the new textbooks through 2006–2008, enthusiastically reported on the primary math texts. Full page or lead stories, normally unusual to see on this theme, came with headings such as “NCERT’s Bold New Experiment Brings Maths Closer to Life”. This story began with:

Ever thought you could study geometry from brick patterns on the walls of a tomb in Murshidabad? Or arithmetic from a junk-seller in Patna? Well that’s what the new Class IV math textbook by NCERT is all about: maths and real-life (Mukherji, 2007).

I took this mediation with the media very seriously. As reporters called to get their stories, short of conducting a tutorial on the subject, I asked them to comparatively look at the old textbooks before our meeting, and then tell me the differences they saw. This generated a lot of interest in the primary math textbooks especially among young reporters, who kept recalling their own unpleasant experiences, and contributed in shaping public expectations about the reform.

The Notion of Time— Beyond Clocks and Calendars

Chapters on the concept of “time” in textbooks across countries routinely only deal with clocks and calendars, describing various units and devices of measuring time, and “informing” the child that a day

is made up of 24 hours, an hour of sixty minutes, while a week is 7 days, and so on. Why so? Indeed, if learning about “time” is viewed through a socio-cultural framework, a richer canvas can be drawn, to begin with the contexts in which children already know about time, also about subjective notions, while gradually scaffolding their understanding through progressively more challenging tasks and investigations.

The Class III (NCERT, 2006b) chapter on Time begins with asking children to correct some funnily jumbled up time markers used in a visual story, depicted in a folk art *pattachitra* style (which traditionally uses cyclic representations for time). Drawing upon childrens common sense and real-life observations and estimations of, for instance, how long it takes for a fruit to fall from a tree, for curd to set, for a litre of milk to boil, or a baby to come out of her mother’s stomach (a deliberate attempt at subversive humour), it leads them to think of several processes that elapse in different orders of time - in years, months, hours, minutes, or seconds. We ask:

Have you seen someone knitting a sweater? Or, someone weaving a cloth? Do try to find out from a potter how long it takes to make a pot. Also tell us if you take hours or minutes to have your bath! (Is it years since you last had one? Ha, Ha!).

Our voice tries not to be moralising, but empathetic, to help value the person, her craft and labour. Cultural narratives—about celebrating “a thousand full moons” as is done when a person completes eighty years, or about the “time line” of a woman’s life, when time flew, and when it stood still as she recalls the death of her husband, and how she fought against his scheming relatives who declared her a “witch” to snatch her land—have been incorporated through exercises that bring diverse lived experiences into the classroom. This also departs from the approach of curricular “infantilisation” or “Walt Disneyfication” (Giroux, 1996), which believes that children should be protected from the “harsh realities” and injustices of the real world—at least in textbooks, no matter what actually happens to them in the real world—and therefore resorts to the comforting contexts of comic cartoon characters (Rampal & Subramanian, 2012).

Concluding Thoughts

Conservative curricula allow at best a tokenistic approach where “celebratory” multicultural representations are limited to viewing diversity through the lens of the essentialised “other”, without critical engagement about issues of “difference”, discrimination and dominance. In fact, when some upper class teachers wondered if “witches” should be discussed with young children, it was pointed out that no questions were generally asked on the suitability of a Harry Potter book or even about viewing violent cartoons about witches on television; then why was the cursory mention of a real person falsely declared a “witch” so problematic? Instead, wasn’t it important and inspiring for children, rich and poor, to know that there were serious attempts to resist this exploitative practice against women?

Symbolic mediation between the home and school language is an important dimension of creating possibilities for meaning making, especially since mathematics (along with science) is increasingly being taught in English, though a regional language may be chosen for the social sciences. Even in the case of HSTP, where we worked in Hindi, the regional language of the state of Madhya Pradesh, I had sought a possible “orality” for the discourse of science (Rampal, 1992) realising that its formal “distanced” register did not ensure a bridging with the children’s languages. Working on the NCERT primary math textbooks we consciously forged an “articulation” (Skovsmose, 2012) of different discourses through multiple genres and did not concede to the premature use of technical terms where more accessible alternatives were available. We also gave voice to children’s imagery and intuitive, tentative ways of thinking, as in this excerpt from the chapter which takes them on a trip to the famous pre-historic cave paintings (NCERT, 2007, p. 28):

Anjan: What is Bhimbetka?

Ms. Raina: It’s a place with lots of cave paintings made by people ten thousand years ago.

Sumonto: Ten tho...uu...saa...nd years! I cannot even think of one thousand years back!

Gopi: Oh! One thousand years is a big thing. I cannot even think of one hundred years.

Gauri: I can think of 100 years because my father’s grandmother is 100 years old.

Manjeet: That means those caves are almost a hundred great grandmothers old!!
Everybody bursts into laughter – Ha! Ha! Ha!

Indeed we also found ourselves reworking the translations of the primary textbooks (from Hindi to English for EVS, and vice versa for math) which were scripted in alienating registers. However, articulations are an ongoing process; transitions from the home to school, primary to middle school, then on to high school, or from school to life beyond, at the University or the workplace, or the other way round, are not smooth.

Transacting a mathematics curriculum in a socio-cultural framework “to read the world” also requires what Freire (1970, p. 62) calls “problem posing pedagogies”, as distinct from problem solving ones, so that education “involves a constant unveiling of reality ... that strives for *critical intervention* in reality”. It requires distinguishing between using mathematics in real world settings, usually limited to shopping, travelling, or building, from those that ask students to critically investigate issues of injustice, through a sense of collective social agency (Gutstein, 2006). The “general notion that problems can be given ready-made to students is highly questionable. Instead, teaching through problem-solving implies acknowledging that problems arise for students as they attempt to achieve *their* goals in the classroom. The approach respects that students are the best judges of what they find problematic and encourages them to construct solutions that they find” (Cobb, Wood & Yackel, 1995, p. 222).

Effecting mediation between everyday and formal math is indeed challenging and involves working against valorisations of knowledges and voices (Abreu, Bishop, & Presmeg, 2001). In this case it certainly needed deliberation among the curricular team; also with the institutionally nominated traditional subject experts who resisted any change and, as an ongoing process, with teachers, parents, teacher educators, and administrators of the larger system. This constant mediation involves bridging or “making relationships visible” between multiple mathematical practices and sites of learning, such that it creates a “resource” for learners’ and teachers’ construction of meaning (Skovsmose, 2012, p. 352).

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Uncertainty, Pedagogical Imagination, Explorative Reasoning, Social Justice, and Critique

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It is autumn 2014. Well, actually it is not. It is springtime, because as I write this I am in Brazil, preparing my talk for the Eighth International Mathematics Education and Society conference (MES8). I was really happy to receive the invitation to make this talk. I was here in the very beginning, giving a Plenary in MES1 in Nottingham in 1998.

In my talk this time I will address the question: *What could critical mathematics education mean for different groups of students?* I am not going to argue for distinguishing between *critical mathematics education* and *mathematics education for social justice*. I believe we are dealing with two largely overlapping educational approaches. So the title of my talk could just as well be: *What could mathematics education for social justice mean for different groups of students?*

The text I am presenting here, however, is not the text for my talk. Instead in this paper, I address some philosophical issues that might only appear implicitly in my talk. Specifically, here I will reflect on the notions of uncertainty, pedagogical imagination, explorative reasoning, social justice, and critique.

Uncertainty

The notion of critique has deep philosophical roots. So has uncertainty, but that has rather different roots. I want, however, to point out profound connections between the two notions.

Let me acknowledge at the outset that my discussion of philosophy is confined to European philosophy. Considerations of the philosophical contributions of other cultures—indeed of the different ways in which “philosophy” might be diversely conceptualised—and

of the potential for deep cross-fertilisation, lie beyond the scope of this paper, though all of those issues are of profound relevance for what I am saying.

In Antiquity, Plato and Socrates were united in the fight against uncertainty. They did this by confronting scepticism as advocated by sophists such as Protagoras and Gorgias, both involved in heated dialogues with Socrates. Augustine and Thomas Aquinas also fought against uncertainty, this time in terms of doubt with respect to religious dogmas. It was feared such doubt would cause a dangerous opening for heresy.

René Descartes was deeply troubled by scepticism. He wanted to eliminate any possible space for scepticism by identifying a foundational epistemic bedrock that could not be shaken by any form of doubt. On this bedrock the whole edifice of knowledge should be raised. Immanuel Kant was also disturbed by scepticism, in particular as it had been formulated by David Hume. As a response, Kant wanted to provide a critique of the very structure of knowledge by identifying the universal categories which form that knowledge. Kant tried to demonstrate how such categories ensure that we can know mathematics, as well as universal natural laws, with certainty. Thus to Kant a critical activity served to identify the sources for epistemic certainty.

Karl Marx provided a new dimension to critical activities. Through his critique, he not only addressed political and economic ideas and assumptions, but also the very political and economic structures themselves. This way he formed a critique of the political economy as part of real political forces. According to Marx, such a critique should be based on scientific insight, and not just on well-intended opinions and initiatives. Anarchism, as for instance that formulated by Pierre-Joseph Proudhon, had provided a radical critique of a range of social issues. But Marx did not want to be associated with this form of critique, which he found superficial. According to Marx, critique needs to be based on a profound scientific analysis as illustrated by his own meticulous investigations of the logic of economic development. Thus different as they are, Descartes, Kant, and Marx all tried to connect critique with certainty.

I acknowledge the importance of the profound development of critical ideas. However, the notion of critique which I want to apply leaves aside any connection, implicit or explicit, with certainty. To me

one can criticise any assumed form of knowledge, assumptions, and ideology as well as any social, political, and economic institutions, but we cannot assume the existence of any solid platform for doing so. Whatever form of critique we conduct, it might include misunderstandings, limitations, and confusions. In this sense a critical activity becomes a profoundly uncertain activity. In fact I see *critique as an expression of uncertainty*.

However, I do not think we should let ourselves become paralysed by this observation. I do not subscribe to any version of the following argument: since we cannot be sure that the critique is appropriate, we better do nothing. Instead we actively have to face the *paradox of critique*: We are facing so many critical issues that we need to address, but at the same time we must acknowledge that our insight is too limited to do so in any adequate way. Any critique can be expected to be insufficient, but still, it remains a necessity.

When I talk about critical mathematics education I also acknowledge the paradox of critique. There are simply too many critical issues related to mathematics education that are in need of being addressed. But we do not have any adequate theoretical or practical basis for ensuring any such critique can be conducted adequately. This applies when we address: social exclusion through mathematics education; unjustified trust in mathematics-based information; strategies for providing empowerment through mathematical literacy; etc. Also critical mathematics education becomes an expression of uncertainty.

Pedagogical Imagination

Criticising something also means imagining that things could be different, because it is through a critique that one also expresses visions, hopes, and aspirations. Thus I see imaginations as a profound element of a critical activity, and imaginations carry with them a strong sense of uncertainty.

The first time I became aware of possible connections between imagination and critique was when I was reading *Sociological Imagination* by C Wright Mills. The idea of a sociological imagination is precisely to express alternatives to observed sociological facts. A sociological imagination provides conceptions of possible alternatives. If no alternatives to a particular sociological fact are identified,

the fact appears as a necessity. A sociological imagination, however, reveals that we are dealing with, not a necessity, but a contingency: it could be different. To reveal that certain facts are not necessities but contingencies is an important critical activity.

In connection with education I refer to pedagogical imagination. The idea is, however, the same: through a pedagogical imagination one tries to conceptualise alternatives to what is taking place – for instance in terms of ways of organising: interactions in the classroom, the content of the curriculum, the tasks set for homework, etc. A pedagogical imagination can help to reveal that certain educational facts are not necessities but contingencies. I refer to this form of revelation as a modulation of facts. A modulation indicates spaces for possible changes and I find that modulation forms a principal part of a critical activity.

The descriptive paradigm characterises much research in the social sciences as it also does in education. In this paradigm, neither sociological nor pedagogical imagination is assumed to play any role. According to this paradigm one has to research “what is the case” and not “what could be the case”. Positivism is an expression of the descriptive paradigm.

In *Naturalistic Inquiry* from 1985, Yvonna S. Lincoln and Egon G. Guba distance themselves from any form of positivism by pointing out what they refer to as the “myth of the positive given”. They do not assume the existence of an objective reality which can be uncovered through meticulous investigations. Instead they consider reality to be constructed and these ideas have already had a huge impact on much qualitative research in mathematics education.

Yet still, I feel that “naturalistic inquiry” as well as “positivist inquiry” belongs to the descriptive paradigm. While positivism focusses on what is assumed to be positively given, naturalism concentrates on what has been constructed. Neither of these two approaches includes imaginations as playing any important role in research. Neither positivism nor naturalism provide for modulations. Both represent an imagination-zero approach by concentrating on providing descriptions of “what is” and ignoring providing conceptions of “what could be”.

Modulations, however, constitute a defining element in Michel Foucault’s genealogy. Through detailed historical studies, he tries to show how different sociological facts emerge through historical

processes. He shows how facts become constructed through a dynamic of power. The processes are historic, they are contingent and so consequently what we observe could be different.

At times, Foucault has been compared to Kant with respect to epistemic profoundness, and this seems to be a valid comparison. Kant wanted to identify the principal categories that structure human knowledge. Foucault also wanted to identify a profound structuring of knowledge. But while Kant looked for this structuring in terms of eternal and pure categories, Foucault presented the structuring in terms of discourses that include presumptions, idiosyncrasies, historical particularities, as well complex networks of powers.

Foucault was deeply inspired by Friedrich Nietzsche's profound perspectivism. There does not exist any platform from which one can make any all-encompassing speech about the structures of the world, or about our knowledge about it. Ontology and epistemology operate within their own limited perspectives. According to Nietzsche, Kant was just playing with us when he assumed he was able to present universal categories for grasping the world. Big words from philosophers do not say anything about the world, they merely indicate in what direction the philosophers' own nose is pointing. This brought Nietzsche to recognise the profound dynamics of power that operate in any ontology and epistemology. With this inspiration, Foucault engaged in genealogical studies revealing this power dynamics within a range of domains.

Much research in education, and also in mathematics education, is inspired by Foucault and shares an interest in providing genealogies. What is taking place in mathematics education can be interpreted as formed through complex historical processes operating within a dynamic of social, political, economic, and discursive forces. This is certainly important to address. However, several Foucault-inspired approaches do not open space for pedagogical imaginations. In fact within much Foucault-inspired educational research, I have, several times, met the idea that providing suggestions for educational alternatives is an expression of romanticism. Thus genealogical studies might lead to the insight that certain facts are constructed within a given political and economic structure, and given this structure, we have to deal with necessary facts.

I agree that speculating about alternatives might be romantic. But still I do not want to assume any such version of an imagination-zero

approach. To me a genealogy may provide a first step in a modulation, but we have to engage in pedagogical imaginations as well. Thus I see genealogy and pedagogical imagination as two important features of a critical activity.

Explorative Reasoning

The relevance of pedagogical imaginations became clear to me during a period that I was involved in a project in South Africa supervising a group of PhD students in mathematics education. The project took place during the initial post-apartheid period (see, for instance, Vithal, 2010).

The South African educational system had been formed through decades of regimes of apartheid and colonisation. Naturally, it is important to address, also in post-apartheid research in mathematics education, what has taken place, but the PhD students I supervised certainly felt it imperative also to investigate what could be done *instead*. It appeared important to investigate, for instance, the dynamics of multicultural mathematics classrooms. But where should we go to observe such classrooms? What could be observed were only classrooms in a segregated educational system, since the educational patterns and structures cultivated during the apartheid period were not changed overnight. Certainly the actual *segregated* classrooms could be addressed through a naturalist approach; they could as well be subjected to a genealogy that would reveal how the brutal logic of apartheid has turned into the logic of schooling. But how were we to explore alternatives?

A pedagogical imagination appears an important part of making this step, but how could one in a more specific way research what is not the case? How were we to explore in more detail the features of a pedagogical imagination? This question brings me to the notion of *explorative reasoning* see Skovsmose, 2014b, and Skovsmose and Penteadó (in press), for additional discussions of researching possibilities).

I will illustrate what I have in mind here by a simple and constructed example (real-life examples from the South African project are, for instance, presented in Vithal, 2003). Let us then engage in a brief episode of pedagogical imagination, and let us just assume

that we are in the initial years of the post-apartheid period. We can imagine how issues about the geography of apartheid could become addressed in terms of maps showing the locations of different neighbourhoods. It could be a map of Durban showing the locations of white neighbourhoods and of black townships, and how Indian neighbourhoods have been inserted as buffer zones in between the two. We might imagine how this geography of apartheid becomes addressed in a multicultural mathematics classroom. We might also imagine how we could engage students in educational processes that provide multicultural understanding and respect.

We could imagine so many things, and our imaginations could certainly include illusions. This might only be an expression of the romantic. But we could try to qualify our pedagogical imaginations, by trying out at least something driven by this imagination. For instance, we could try to bring some children from different neighbourhoods together, at least for some lessons. We could try to engage them in examining some of the maps. However, what we in fact might be forced into doing would most likely turn out to become rather different from what we had imagined. The actual classroom activities we were conducting might need to be adjusted to the existing curriculum, to the priorities of the schools, to the parents' priorities, and to many other things.

But we did do something. Was it an experiment? Certainly not in the sense that we carefully planned what we were going to do. We did not get hold of parameters that might have an impact on what is taking place, and that needed to be considered when we interpret what we have observed. We were really not in control of anything.

Anyway, explorative reasoning takes as its departure what we did, but not in order to try to draw conclusions strictly *about* what took place. Through an explorative reasoning we try to qualify our pedagogical imagination. For instance, we might recognise that we must consider in greater detail the tensions that could emerge in a multicultural setting. We might become aware that some issues are experienced as too controversial, as they highlight the borders that might run through the multicultural classroom. We might recognise that some issues engage the students much better than others. We might qualify much better our ideas about reading maps. We might get new conceptions of dimensions of the geography of apartheid. We might identify other possibilities for exploring these dimensions

in mathematics education. In this sense explorative reasoning helps to qualify the content of our pedagogical imaginations.

Thus what we in fact did, we consider an open window through which we might consider better our pedagogical imaginations. Explorative reasoning draws from what has taken place, but is not directing itself towards conclusions about what has taken place. Through an explorative reasoning we try to investigate our conceptions of educational possibilities.

Social Justice

Like critique, the notion of justice is deeply rooted in philosophy. In the dialogue *The Republic*, Plato addresses the question: What is justice? Plato finds that it makes perfectly good sense to search for the proper definition of justice, as we are dealing with an entity belonging to the world of ideas. The sophists, however, negated the possibility of reaching any such definition. There does not exist anything such as justice. Thus since antiquity, the conception of justice has been part of philosophical controversies.

Thomas Aquinas interpreted justice from a religious perspective, as an expression of the will of God. Certainly Friedrich Nietzsche would oppose any attempts to define justice, and he might laugh loudly if presented with a notion such as “mathematics education for social justice”. Most likely he would associate conceptions of justice with a slave morality, as he did with conceptions of democracy. Within a liberal tradition the conception of justice has been addressed by John Locke and John Stuart Mill. They proposed a naturalistic perspective not drawing on any religious interpretations at all.

The very notion of “social justice”, however, only appeared in the 19th century, coined by Luigi Taparelli, a Jesuit who was deeply inspired by Aquinas and who emphasised the religious features of the concept. These ideas were explored further in *The Constitution Under Social Justice (Costituzione secondo la giustizia sociale)*, first published 1848 written by the catholic priest Antonio Rosmini-Serbati. Here we find the notion of social justice related to a range of ideas, reaching from social perspectives to strong religious assumptions.

A very important contribution to the discussion of social justice is represented by John Rawls in *A Theory of Justice*, first published in 1971.

Here Rawls provides a detailed review of the concept of justice, in particular as it has been elaborated in the liberal tradition and viewed through the lenses of analytic philosophy. However, Rawls brings his investigations beyond this tradition by acknowledging not only the descriptive part, but also the action part of social justice. It becomes a concept that calls not only for critical reflections but also for critical actions. I see *A Theory of Justice* as a most important contribution to the overall discussion of social justice.

Also within mathematics education one finds profound contributions to the overall conception of social justice. In *Reading and Writing the World with Mathematics: Toward a Pedagogy for Social Justice* from 2006, Eric Gutstein provides a careful interpretation of Paulo Freire's notions of "reading and writing the world". Gutstein (2006) illustrates what "reading and writing the world with mathematics" could mean. Thus he establishes a unique conceptual source for pursuing social justice through mathematics education, and illustrates the power of this source through a range of examples (see also Gutstein (2003, 2012)). In particular Gutstein brings us to see the importance of the activist feature of such an education. (For presentations of mathematics education for social justice, see, for instance, Gates, 2006; Sriraman (Ed.), 2008; and Wager and Stinson (Eds.), 2012. See also Skovsmose, 2011; and Skovsmose and Greer (Eds.), 2012.)

In the article "A Broad Concept of Social Justice" from 2012, Ubiratan D'Ambrosio provides a weighty reinterpretation of social justice. He leaves behind any of the elements of analytic philosophy that might be lingering in Rawl's *A Theory of Justice*. D'Ambrosio relates justice to the most basic conditions of human life. Thus he brings us to connect social justice to, for instance, freedom of choice, well-being, security, peace, and spiritual experiences. In this way he establishes a diversity of cultural and political features as integral parts of social justice. Simultaneously, D'Ambrosio relates social justice to the way mathematics does operate and may operate in society. In particular he emphasises that mathematics, as a most universal form of thinking, might help in providing ways of addressing some of the most universal problems of humanity, as for instance the technology-fabricated changes of nature.

To me conceptions of social justice have much to do with pedagogical imagination. While explorative reasoning might help to bring specificity to imaginations, interpretations of social justice might

provide broader scopes to these imaginations. Thus interpretations of social justice with reference to “reading and writing the world with mathematics” provide new directions for pedagogical imaginations. In a similar way, expressions like “freedom of choice”, “well-being”, and “peace” might open new landscapes of imaginations.

Several other notions, as wide open as social justice itself, can have similar functions. One can think of, for instance, autonomy, democracy, *conscientização*, responsiveness, and empowerment. They all belong to this extended family of notions that may provide fruitful soil for pedagogical imagination. (See Skovsmose and Penteadó (in press) for a discussion the importance of a notion like democracy for the formulation of pedagogical imaginations.)

Critique

As I have already emphasised I am not interested in making any distinction between mathematics education for social justice and critical mathematics education. In fact other general labels are in use as well such as “responsive mathematics education” (see, for instance, Greer, Mukhopadhyay, Powell, and Nelson-Barber (Eds.), 2009). Such trends in mathematics education belong to the same family, but let me now recapitulate some features of the notion of critique.

Uncertainty is to me an inevitable part of a critical activity. We should not try to eliminate uncertainty by assuming the existence of some unquestionable theoretical, philosophical, or political foundations upon which a critical activity can be conducted. I am deeply inspired by any concern for identifying and reacting to any form of suppression and injustice, but at the same time I want to leave behind any assumptions of the existence of an elevated basis for doing so. In particular I do not assume that the notion of social justice has a well-defined kernel.

Pedagogical imagination relates to hopes, visions, and aspirations. To me a critical activity is not only addressing what is the case, but also what could be instead. As a consequence, I find that pedagogical imagination plays an important role in any form of critical education. And to me explorative reasoning is important in order to provide specificity to such imaginations.

Social justice is one of the notions that can provide fuel to

pedagogical imaginations. However, notion of social justice is in permanent need of being re-interpreted. In fact one can think of it as not only an open, but also as a contested concept. It invites many different and also profoundly conflicting interpretations. To me, we need not abandon such contested notions, as they have important roles to play in providing new perspectives. In this way a notion like social justice provides fertile ground for pedagogical imaginations.

Critique of mathematics forms an integral part of a critical mathematical education. Thus one cannot assume that mathematics represents any intrinsic good qualities as a result of the very nature of mathematics. As any form of knowledge, mathematics constitutes an integral part of social, economic, and political power-structures. This applies to any form of mathematics, although maybe in rather different ways. This applies to industrial mathematics, applied mathematics, academic mathematics, insurance mathematics, ethnomathematics, as well as to any stipulated forms of critical mathematics.

Critique by means of mathematics also constitutes an integral part of critical mathematics education. Thus mathematics can establish powerful means for expressing a critique of a range of states of affairs. In particular, I can refer to many examples of reading and writing the world with mathematics. In many such cases the mathematical investigations contribute profoundly to critical investigations. In this way there can a lot of sense in talking about critical mathematics (see Frankenstein, 2012; and Powell 2012). I prefer, however, not to use this label, as it might invite the idea that such a version of mathematics is not itself deeply in need of critique.

Critique of critical mathematics education forms an integral part of critical mathematics education. Paul Ernest (2010) has pointed out and exemplified the importance of such a critique, and I agree with him (see as well Pais, 2012). Any critical activity needs to be addressed by a critique. There is no escape from this circularity, and certainly I am not going to refer to this as any vicious circle. It could instead be considered a very healthy circle, a virtuous circle. No critical approach can assume any form of self-justification. Critique could always be different.

Mistakes form an integral part of critical endeavours. This applies not only to critical mathematics education. We should always be aware that no critical approach takes everything into consideration. Critique is partial, temporary, and preliminary. Critique includes blind

spots and operates within particular perspectives. We have always to acknowledge the paradox of critique. Critique is crucial, but it could be mistaken.

Illusions form an integral part of critical activities, but I do not think of illusions as simply a negative feature of critique. I find that imaginations form an integral part of critique, and, as a consequence, one cannot prevent illusion from forming a part as well. Imaginations might turn out to be illusionary; they might also turn out to be feasible. There does not exist any well-defined way of clarifying this in advance. In this sense critique turns into a profound constructive activity, although still being uncertain.

And now that all this has been said, I am going to continue preparing my talk, and let me stick to the title: *What could critical mathematics education mean for different groups of students?*

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SYMPOSIA

Rituals: Connecting the Social and Disciplinary Aspects of Mathematics Classrooms

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In this symposium we invite participants to contribute their personal and cultural understandings of “ritual” and join us in discussing this construct and what it might reveal about mathematics teaching and learning. Symposium presenters have each found it useful to use ritual to describe or notice qualities of mathematics teaching and learning in empirical projects. We share common curiosities and motivations in our research agendas, but we also have points of departure from one another in our conception of ritual. We invite participants to join us in considering how our various and distinct perspectives might complement one another in our shared agenda of conducting mathematics education research that sheds light on the social, cultural, and political contexts that influence all mathematics teaching and learning.

Introduction and Rationale

This symposium takes as its starting point Erikson’s (1982) call to jointly consider the social and cognitive aspects of learning “if we are to develop an interactional theory of cognitive learning and teaching in social occasions” (p. 156). This statement is more than three decades old, and this call remains relevant, especially in mathematics

education. Studies that focus on the social dimensions of mathematics teaching and learning often neglect to consider the mathematical content being taught, while those looking at the mathematical content fail to consider the interactional aspects of teaching and learning. In bridging this gap, we have found Sfard's (2008) communicational theory, which conceptualizes mathematics learning as a particular form of social practice, an important step forward. In particular, we concentrate on her conceptualization of *ritual participation in mathematical learning*. The concept of ritual, which has been dealt with extensively in sociological and anthropological research, serves as a type of "conceptual boundary object" that enables us to connect the abundant research on social practices in the classroom with research on mathematical cognition and practice. All symposium presenters have found the notion of ritual to be helpful at capturing various "non-cognitive" aspects of mathematics teaching and learning. We have found that the notion of ritual helps us understand mathematics classrooms as spaces that are more than the sum of individual students' and teachers' knowledge and beliefs.

The general plan for the session is to begin by presenting our shared understandings and motivations and then highlighting the differences in our approaches. We have found that exploration of the distinctions has been generative to our own work, and we think that participants will similarly benefit in their own projects. To that end, we have invited a discussant whose work has touched upon the social aspects of classroom practice (e.g., Otten, Herbel-Eisenmann, & Cirillo, 2012), the disciplinary content in mathematics classrooms (e.g., Herbel-Eisenmann & Otten, 2011), and has used critical perspectives to consider mathematics classroom discourse (e.g., Herbel-Eisenmann & Wagner, 2010). We suspect that her insights will raise productive questions and serve to locate our theoretical and methodological dilemmas within wider challenges of mathematics education research.

Perspective One: Ritual as a Type of Disciplinary Practice

In this presentation, Einat will share her research on students' ritual participation in mathematical learning. Her framework draws on

Sfard's (2008) definition of ritual as performing routines for the sake of connecting with others (teachers, grownups, etc.) in contrast to explorative participation that is performed for the sake of describing the world mathematically. In exemplifying ritual participation, Einat will present two cases of 7th graders who studied under her instruction in an out-of-school course for 5 months. The first is Idit (Heyd-Metzuyanim, 2015), who, in addition to being instructed, was also followed for 2 years after the end of the course. Idit, who ended the course with high achievements, gradually became more ritualistic in her performance of mathematical routines, along with becoming extremely anxious about mathematics and failing her class at 9th grade. The case will be made that Idit's initial weakness in fractions at 7th grade, together with an educational environment at home and at school that encouraged procedural rule-following, provided fertile ground for the widening of her ritual participation.

The interactional aspect of ritual participation will be exemplified with the story of Dana (Heyd-Metzuyanim, 2013), who started the course with extremely low achievements and failed to progress despite intensive instruction. Close-up analysis revealed how Dana's ritual participation was, in fact, co-constructed by Dana and her teacher, Einat. Dana's case highlights the many factors (such as curriculum and parents' expectations) that, once a gap has been formed, hinder students from moving to explorative participation.

Broadening the Application of Perspective One: Ritual as a Characteristic of Both Teaching and Learning

In this presentation, Mellony and Einat will share their collaborative project in which they attempted to trace possible reasons for elementary school students' limited progress in numeracy in a low SES South African context. This was done through two lenses. One aimed to analyse two learners' (Mina and Rondaldo) ritual vs. explorative participation both in one-on-one interviews and in a small group "math club" lesson led by Mellony. The other lens examined the mathematical milieu in which these learners have been participating, through the analysis of a typical school-classroom lesson, similar to that in which

Mina and Ronaldo have been studying. We show that while Mina was acting in an extremely ritualized manner in the “math-club” lessons, Ronaldo was more explorative in his actions. However, the milieu, as seen in the school lesson, encouraged almost exclusively ritual participation. Thus while Mina was identified as a good and well-motivated student, Ronaldo was identified as an outcast or “trouble maker”. We conclude by drawing implications about the tenacious nature of rituals in the mathematics classroom and the challenges of changing them.

Perspective Two: Ritual as a Lens for Recognizing Culture in the Mathematics Classroom

In this presentation, Andrea will summarize her understanding of ritual, especially in contrast to the preceding perspective. McCloskey (2014) has defined ritual as that aspect of action that is *symbolic, traditionalized, formalized performance*. These four characteristics of ritual enable researchers to attend to the sociocultural nature of classroom practices. For Andrea, ritual is one way to “get at” the cultural nature of a practice because it is a mechanism through which cultural patterns may be perpetuated (Giddens, 1994), or, more hopefully, transformed (Turner, 1969).

By way of example, Andrea and colleagues (McCloskey, Lloyd, & Lynch, 2014) conducted an ethnographic study of a 5th-grade mathematics classroom in the United States. Using interviews, classroom observations, and artefact gathering and by applying ritual as an analytic framework, we uncovered tensions and contradictions at work in the classroom. For example, the phrase “Show your work” was used often by the teachers in an effort to encourage mathematical thinking, but we found instances where the phrase served to funnel mathematical thinking into particular forms. In this session, Andrea will share these and similar findings. She will also describe and seek feedback on theoretical and methodological challenges that arise, such as *How can we make claims about the cultural nature of a practice when the primary point of contact and data collection are the activities and attributes of individual teachers and students? How can humanities-oriented approaches help researchers make the postmodern turn?*

Plan for Symposium

More specifically, we plan to use the 90 minutes as listed below:

1. A brief introduction, including the goals, agenda and an overview of the three presentations (10 minutes)
2. Presentations, as detailed in abstracts above ($15 \times 3 = 45$ minutes total)
3. The discussant will offer feedback to individual presenters as well as to the ideas presented as a whole, and will invite participants' questions and ideas to constitute a larger discussion (10 minutes)
4. Symposium presenters will share recently collected data (video data and/or transcriptions) and invite participants to use one or both frameworks to analyze the data. Participants will be invited to reflect and offer critique on the applicability and usefulness of the frameworks. Possible questions include:
 - a. How useful is ritual for conducting research in mathematics education? What are the similarities and differences with other constructs in use in mathematics education research?
 - b. What are possible implications for practitioners from either or both perspectives of ritual?
 - c. What is foregrounded and what is obfuscated when ritual is used as way of thinking about mathematics teaching and learning? (25 minutes)

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Exploring Different Theoretical Frontiers

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Providing for a praxis of uncertainty, theoretical traditions that uncover how knowledge, power, and identity are interwoven and constituted in and through socio-cultural and -political discourses characterize the sociopolitical-turn moment in mathematics education research. Researchers who work in the sociopolitical-turn moment pull from a variety of theoretical perspectives most often located in the emancipate and/or deconstruct paradigms of inquiry. In this symposium, panelists discuss how different theoretical traditions available to researchers in the sociopolitical-turn moment provide new productive ways to think and rethink mathematics teaching and learning.

Aims of Symposium

[Text extracted and revised from Stinson and Walshaw (in press)]

This symposium aims to engage MES8 participants in a critical, interactive discussion about how theoretical traditions characterized as being in the sociopolitical-turn moment of mathematics education research provide new productive ways for researchers, teacher educators, classroom teachers, and policymakers to think and rethink mathematics teaching and learning. Panelists discuss, in turn, critical

theory, poststructural theory, feminist theory, and critical theories of race, and outline how these respective theoretical traditions provide different and uncertain possibilities for transforming mathematics teaching and learning into an empowering experience for all.

Relevance of Symposium

To make sense of the proliferation of theoretical traditions used in (Anglophone) mathematics education research since the 1970s, four distinct yet overlapping and simultaneously operating shifts or historical moments in mathematics education research have been identified: the process-product moment (1970s-), the interpretivist-constructivist moment (1980s-), the social-turn moment (mid 1980s-), and the sociopolitical-turn moment (2000s-) (Stinson & Bullock, 2012).

Providing for a praxis of uncertainty (Kincheloe & McLaren, 1994; Stinson & Bullock, 2012), theoretical traditions that undercover how knowledge, power, and identity are interwoven and constituted in and through socio-cultural and -political discourses characterize the sociopolitical-turn moment (Gutiérrez, 2013; also see de Freitas & Nolan, 2008; Valero & Zevenbergen, 2004). But here discourses are no longer mere words that might be heard or read but rather discursive practices that systematically form the possibilities (and impossibilities) of knowledge discourses, which, in the end, produce and reproduce régimes of truth (Foucault, 1969/1972, 1977/1980). Researchers who work in the sociopolitical-turn moment understand mathematics as a discursive formation (cf. Foucault, 1969/1972), made and remade within the sociopolitical contexts in which it is taught and learned; they adopt a degree of social consciousness and responsibility in their attempts to both understand and expose the wider social and political picture of mathematics and mathematics teaching and learning (Gates & Vistro-Yu, 2003; Jablonka, Wagner, & Walshaw, 2013). Acknowledging that no research is agenda-free, political or otherwise, these researchers do not rally around some common political agenda but rather understand that education is politics (cf. Skovsmose & Greer, 2012). And today, few disciplines are as politicized as (school) mathematics, as it is constituted in and through a dense web of sociopolitical power (Brown & Walshaw, 2012).

Researchers who work in the sociopolitical-turn moment pull

from a variety of theoretical perspectives most often located in the emancipate and/or deconstruct paradigms of inquiry (see Table 1 for a mapping of the moments of mathematics education research to broader paradigms of inquiry with a non-exhaustive listing of theoretical traditions). Although these paradigms of inquiry operate from different and often-argued incommensurable philosophical assumptions, both paradigms seek to open up the research text (de Freitas & Nolan, 2008), using theory as a vehicle for exposing different productive possibilities within mathematics education (Brown & Walshaw, 2012). Forging these different frontiers, researchers aim to open up “the fictions, fantasies, and plays of power inherent in mathematics education” (Walkerdine, 2004, p. viii) as they challenge the taken-for-granted assumptions and habitual ways of working and thinking that underlie much of “traditional” mathematics education research often located in the predict and/or understand paradigms.

As symposium panelists focus on theoretical traditions located in the emancipate and deconstruct paradigms, it is important to note two points. First, the term *different* rather than *new* is used to remind MES8 participants that the theoretical traditions discussed are neither new to the social sciences, generally, nor to education social science, specifically. They are, however, somewhat new to the research domain of mathematics education. Second, panelists are not suggesting that the theoretical traditions that are highlighted lead to a “better” or “best” way of conducting mathematics education research. These traditions do, however, disrupt the status quo by providing different (and uncertain) possibilities for producing different knowledge and producing knowledge differently (St. Pierre, 1997). It is also important to note that each paradigm of inquiry—predict, understand, emancipate, and deconstruct—comes with its own set of philosophical assumptions regarding truth, certainty, and logical consistency. Therefore, as panelists discuss different theoretical traditions located under the emancipate and deconstruct paradigms, our collective aim is not to “tell others what they must do” but rather to “shake up habitual ways of working and thinking” (Foucault, 1984/1996, 462–463).

Table 1
Moments of Mathematics Education Research & Paradigms of Inquiry

- Process–Product Moment (1970s–)→*Predict*
- Interpretivist–Constructivist Moment (1980s–)→*Understand*
- Social-turn Moment (mid 1980s–)→*Understand* (albeit, contextualized understanding) or *Emancipate* (or oscillate between the two)
- Sociopolitical-turn Moment (2000s–)→*Emancipate* or *Deconstruct* (or oscillate between the two)

PARADIGMS OF INQUIRY

Predict	Understand	Emancipate	BREAK	Deconstruct
*Positivist Experimental Quasi-experimental Mixed methods> Experimental Quasi-experimental Mixed methods>	*Interpretivist Social constructivist Radical constructivist Sociocultural> Phenomenological Ethnographic Symbolic Interaction	*Critical Freirian <Feminist> Critical Race Theory> LatCrit Theory> Critical Theories of Race> Critical mathematics Social justice mathematics Ethnomathematics Culturally specific/ responsive Mathematics	BREAK	*Poststructural/ Post-modern Post-critical Post-colonial Post-humanist Post-Freudian <Discourse Analysis

- * Indicates the term most commonly used
- < Indicates cross-paradigm movement
- The “Break” represents a hybrid, in-between space where the researcher might adopt a critical postmodern theoretical tradition (see Stinson & Bullock, 2012).
- *Paradigms of Inquiry* adapted from Lather and St. Pierre in Lather, 2006, p. 37.

PLAN OF SYMPOSIUM

Day 1	Day 2
<ul style="list-style-type: none"> • Coordinators’ Introductions • Panelists’ Presentations • Discussants’ Synthesis • Participants’ Q&A 	<ul style="list-style-type: none"> • Coordinators’ Introductions • Participants’ Working Group Breakout • Participants’ Working Group Report Out • What’s Next?

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STEM: Friend or Foe to Critical Mathematics Education?

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Should critical mathematics education (CME) embrace or reject the omnipresent STEM? We will outline some considerations and provide an interactive space for conversation on these and audience-motivated considerations. Audience participants are encouraged to come with responses to the session's title.

Overview

In the United States, and increasingly across the globe, science, technology, engineering, and mathematics (STEM) education is primarily linked to “vital preparation for today’s high-tech information economy” (Drew, 2011, p. 1) and has become a priority. However, the emphasis on human capital directly conflicts with goals of critical mathematics education (CME) (e.g. Gutstein 2008, Wolfmeyer, 2014). The confluence of actors promoting and implementing STEM also oppose more broadly the goals of critical education, especially via its conflation of reason, science, and technology, as this conflation contributes significantly to the ecological and social crises (e.g. Bowers, 1993; Martusewicz, Edmundson, & Lupinacci, 2011). This interactive symposium will first orient the audience to critical math education (e.g., Frankenstein, 1983; Gutstein, 2006; Skovsmose, 1994), then lay out the basics of STEM policies and discourses (e.g. Drew, policy statements). Next, we suggest that STEM conflicts with CME, and includes roundtable discussions for audience participation. We then argue the opposite: a robust optimism for STEM as a new site for CME. Finally, participants will share their own responses to the session title.

Section One: Conflicts between STEM and CME

To begin, we review a variety of work in CME (e.g. Frankenstein, 1983; Gutstein, 2006; Skovsmose, 1994). Next, each panellist will analyse an example of STEM conflicting with CME. Wolfmeyer discusses the emphasis that human capital plays in STEM education via his work-in-progress analysis of the STEM organization “Change the Equation,” via similar methodology used in Wolfmeyer (2014). He reveals a commitment to increased access to STEM careers, especially for women and people of color, and suggests this position is a soft-critical orientation to social justice in which economic relations of power are entirely ignored. To complement the social network analysis and results, Chesky presents her critical discourse analysis work on the media framing of STEM (Altheide & Michalowski, 1999; Fairclough, 1995; Lingard & Rawolle, 2004, Rogers, 2005). She reveals how U.S. news media portray STEM education as equitable practices that serve disadvantaged students, which misrepresents the objectives of STEM policy and conflates equity discourse to justify practices that are profoundly undemocratic (Gabbard 2000; Giroux, 2005; Martin, 2008). Next, Lupinacci presents an eco-critical (Martusewicz, 2001; Martusewicz et al., 2011) analysis of STEM, which broadens the CME perspective to address both social and ecological issues. We pose the inverse question: How can an ecological model, a heuristic for recognizing how we as humans culturally constitute and reconstitute understanding and how we might emphasize an ecological intelligence (Bowers, 2011)—or what Gregory Bateson refers to as an ecology of mind (Bateson, 1972; Bowers, 2011; Harries-Jones, 2002)—as the focal point for a subject inquiry be at the center of an integrated ecological inquiry? He focuses on the ways that mainstream STEM perpetuates the central cultural dualisms that subjugate.

Discussion Questions

1. Is STEM’s human capital emphasis, and especially its concern with increasing women and ethnic group success in STEM careers, in conflict with CME?
2. How does an eco-critical perspective engage previous critical math education scholarship; how does this broaden CME’s

- concern with STEM discursive practices?
3. What are other critiques of STEM policy, practice, discourse, and culture?

Section Two: Seeking the CME Space in STEM

Building off the session's devotion to *STEM-as-problem*, we next consider the possibilities of infiltration; despite our misgivings, how can we insert CME into STEM discourse, practices, and culture? In reviewing CME's application of mathematics content to social justice goals, Wolfmeyer merges mainstream STEM's interdisciplinary strengths (e.g. Straw, MacLeod, & Hart, 2012) with Gutstein's (2008) "reading and writing the world with mathematics." The interdisciplinary STEM space provides a fruitful setting for exploring social and ecological projects to be understood as CME. Chesky offers a distinct possibility for subverting STEM's economic objectives. Influenced by Badiou's (2003, 2005a, 2005b, 2008) theory of "event", Chesky argues that there is an aesthetic dimension of mathematics that is indeed part of the discourse of STEM. Following scholars (Sinclair, 2001; Tymoczko, 1993; Wang, 2001) who have begun to conceptualize an ontological awareness of mathematics, she explores techniques and curricula choices that insert the arts and ecology in STEM that meet both the dominant aims of STEM policy as well as the transformative visions of critical mathematics. Finally, Lupinacci proposes how the STEM space can challenge modernist Western industrial dualisms and their associated discursive influences to map an Either/Or onto how we—as educators and educational researchers—interpret the differences between STEM content and arts. The Deleuzian definition of difference offers a potential for reconceptualising STEM curriculum in support of social justice and sustainability, and these are linked to CME goals. Drawing from Deleuze (1994) and Bateson (1972), he examines the potential of pedagogical projects that recognize, respect, and represent difference as a valued productive and interpretive condition upon which everything exists. He introduces a theoretical approach to reconstituting STEM as inseparable from, rather than superior to, the arts (STEAM), with concrete examples.

Discussion Questions

1. Can the STEM space be successfully infiltrated by CME?
2. Do the examples of infiltrated-STEM as presented comport with mainstream STEM goals or truly subvert these?
3. What are other examples of infiltrating the STEM space?

Section Three: Discussion

The session's goal is to spark interest in, and dialogue with, the title question; participants come prepared with their own considerations about CME's place in STEM. We invite inquiries reflecting a broader conception of STEM. We hope the final discussion will critique STEM and present it as a space for infiltration. How are critical mathematics scholars engaging and working within STEM educational spaces and/or habits of mind? We encourage participants to draw from among the variety of frameworks appropriate to critical work, including critical race theory and critical disability studies and with goals that counter neoliberal projects and embrace community, democracy, Marxism, anarchism and/or anti-capitalism, for instance.

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Numeracy as Social Practice

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What do we mean by “numeracy as social practice”? There is a growing body of research that has suggested that, similarly to literacy, what counts as numeracy and what it means to engage in numeracy cannot be understood apart from the social, cultural, and political contexts in which the mathematical activity is embedded (e.g. Harris, 1991; Hoyles, Noss, Kent, & Bakker, 2010; Lave, 1988; Mukhopadhyay & Roth, 2012; Nunes, Schliemann, & Carraber, 1993; Reder & Brynner, 2009; Saxe, 1991; Street, Baker, & Tomlin, 2005). The symposium brings together researchers who adopt a social practices approach to their research in numeracy, and those who are interested in the implications of this approach to numeracy for research, pedagogy, and policy.

Aims of the Symposium

In this symposium, we explore how a social practices perspective of numeracy can contribute some critical insights to understanding and engaging in numeracy research, numeracy pedagogy, and educational policy. A social practices perspective focuses on what people do with numeracy through social interactions in particular contexts, rather than focussing on isolated mathematical skills. Numeracy practices can only be understood in relation to the social, cultural, historical, and political contexts in which they take place. The aims of the symposium are to 1) survey the range of theoretical resources

and methodologies that researchers of numeracy and mathematics are using to research numeracy/ mathematical practices; 2) discuss methodological challenges that arise in researching practices; and 3) examine the practical and political implications of findings from social practices studies of numeracy in research, pedagogy, and policy in mathematics education.

Rationale of the Symposium in Relation to MES

MES is a forum for exploring the social, cultural, political, and ethical dimensions of mathematics education. Viewing numeracy as a social practice is one way of making explicit how people's uses of mathematics (what we would call numeracy practices) are socio-culturally contingent. This perspective acknowledges and indeed focuses on ways in which certain numeracy practices are deemed more worthy than others and legitimised in official curricula and policies, while other practices are not. This perspective explicitly takes into account questions about power relations: whose knowledge counts, as well as ethical questions about inclusion and exclusion of people's knowledge based on potentially arbitrary measures of legitimacy.

There have been many MES papers that have interrogated mathematics/ numeracy as social practices or through compatible lenses as the social practices perspective (e.g. Baker, 1998; Coben, 1998, 2010; d'Ambrosio 1998; Evans & Tsatsaroni, 1998; Kaner, 2002; Knijnik 2008; Matos & dos Santos, 2008; Mukhopadhyay, 2013; Tomlin, Baker & Street, 2002). Across the contributions from these papers, we see studies drawing on theoretical resources including New Literacy Studies, Literacy as Social Practice (LSP), socio-cultural activity theory, Gramsci, ethnomathematics and discourse analysis. The papers report on studies of numeracy/ mathematical practices in diverse contexts (e.g. children's and adults' numeracy practices, home and school numeracy practices, numeracy in social movements). The symposium will enable researchers who bring these and other theoretical resources and research contexts to come together to discuss how the studies they have undertaken and the methodologies they employed provide particular insights into numeracy as social practice. It is a forum for a critical reflection on the possibilities afforded by social practices studies of numeracy.

Proposed Conduct of the Symposium

The symposium will begin with a brief survey of previous MES contributions to the concept of numeracy as a social practice, and will focus on identifying the theoretical resources that these contributions have drawn upon. It will then outline methodological implications of researching numeracy as a social practice, and the significance as well as the challenges of ethnographic approaches. Examples of ethnographic approaches used in different contexts will be presented as discussion starters. Thirdly, questions of how social practices studies of numeracy can inform pedagogy and policy will be posed, and participants will be invited to discuss both the practical as well as the political implications of their responses.

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**PROJECT
PRESENTATIONS**

Mathematics Teachers' Use of Data and Evidence in Practice: Intersection of Accountability and Agency

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Using data as an educational reform strategy has gained significant traction in recent years; however, there is little research that provides insight into what actually happens in practice for teachers. School and district accountability expectations commonly specify particular data and questions on which teachers should focus. In contrast, engaging in action research positions teachers as active change agents who pose their own questions and seek their own answers through data use. This project investigates the ways a group of mathematics teachers use data within the overlapping contexts of teacher accountability policies and their work as teacher researchers and offers insights into mathematics teachers' agency.

Introduction

In recent years, there has been a press for the use of data and evidence in schools, particularly in the United States, Australia, and England. There is, however, relatively little research that provides insight into the specific practices used by teachers or into the influences of school and policy contexts on teachers' daily uses of data (Little, 2012). Current accountability policies commonly require teachers to use particular data to answer questions posed by individuals outside the classroom (e.g., administrators, school boards). In contrast, when conducting action research, teachers actively pose their own questions and use data to seek their own answers to problems in their classroom practice. Therefore, examining teachers' work within the overlapping contexts of: (a) their school and its accountability policies, and (b) their work as teacher researchers, offers insights into potentially conflicting positions for teachers using data and evidence¹ in practice. This presentation is part of an on-going dissertation study that explores

the ways a group of mathematics teachers use and make sense of data and evidence and will address the questions: (a) *How do teachers discuss data and evidence?* and (b) *In what ways do issues of teacher agency surface as teachers discuss data and evidence over time?*

Relevant Literature

Teacher Data Use

Within the public discourse around data use there is a fundamental assumption that an increased focus on data is necessarily a positive reform strategy. Coburn and Turner (2011) define the process of data use as “what actually happens when individuals interact with assessments, test scores, and other forms of data in the course of their on going work” (p. 175). The ways in which educational stakeholders take up data-related activities involve complex relationships that can vary even within the same school (Datnow, Park, & Kennedy-Lewis, 2012). Additionally, individuals use and interpret data in ways that are strongly influenced by social interactions, worldviews, and power dynamics (Coburn, Toure, & Yamashita, 2009). Teachers regularly gather information from students to guide their instruction (e.g., assessments). An important distinction, however, lies in the framing of how teachers are positioned with respect to that data – Are teachers driven by data, or are they independent researchers posing their own questions and using data to answer them?

Teacher Action Research

Action research typically takes the form of a spiralling cycle of planning, action, data collection, and reflection. By framing teachers as individuals who pose their own questions and seek their own answers, the teacher action research paradigm positions teachers as active agents for change within their classrooms. Teachers are seen as co-constructors of knowledge, working together with colleagues or university researchers to investigate their own teaching, curriculum, policies, etc. Many have argued that the recognition of practitioners as valid generators of knowledge about their own practice has the

potential to challenge the status quo of the educational system in order to provide more equitable educational opportunities for students and empowered experiences for teachers (e.g., Jaworski, 1998).

Teacher Agency

Teachers experience agency when they feel they have the capacity to change the existing state of affairs and make choices based on personal goals, interest, and motivations (Ketelaar, Beijaard, Boshuizen, & Den Brok, 2012). I operationalize *teacher agency* as teachers' beliefs that they can act as independent agents. Teachers' agency can be impacted by both: (a) the different ways in which data-driven versus action research ideologies position teachers as more or less in control of educational change and (b) the social and cultural environments in which teachers use data.

Theoretical Framework

Coburn and Turner (2011) developed a theoretical framework to support the understanding of how teachers use data in practice, which includes four interrelated categories: (1) interventions to promote data use, (2) organizational and political context, (3) processes of data use, and (4) potential outcomes. Their framework is organized so that the processes of data use category, is situated within the surrounding category of organizational and political contexts. The first and fourth categories, interventions and outcomes, are represented as the inputs and outputs to the system, respectively. Coburn and Turner build primarily from Sensemaking Theory and interpretivist traditions, to which I add critical considerations of agency and power. The categories of this framework will be unpacked as a part of the presentation.

Method

Participants and Setting

This project investigates the practices of four mathematics teachers at

a linguistically and culturally diverse suburban middle school in the Midwest, US. The group is comprised of two seventh grade and two eighth grade teachers with teaching experience ranging from two to 30 years. All four teachers expressed a strong learning disposition and desire to improve their practice. For the past two academic years they have been engaged in departmental professional development focused on mathematics classroom discourse. During the current academic year the teachers will engage in cycles of action research based on the interests they cultivated during their previous two years of work. They will meet as an action research study group, along with the university facilitator and/or myself about once a month for two hours to share ideas and support one another in the process.

Data Sources

I will conduct individual interviews with the four teachers, their administrator, and curriculum and mathematics support personnel at least four times throughout the school year using a semi-structured interview protocol based on the Coburn and Turner (2011) framework. All interviews will be recorded and will serve as a data source for this investigation. Additionally video recordings of all of the action research study group sessions will serve as the second data source.

Data Analyses

I adopt a dialogic perspective of language use (i.e., Wortham, 2001) and I will use narrative discourse analysis techniques in order to determine how teachers discuss data and evidence. Through this analysis I will also gain insights into issues of teacher agency because narratives provide a means for teachers to construct their identities (e.g., Wortham, 2001) and the ways teachers construct their identities can offer insight into the ways they are experiencing teacher agency (e.g., Ketelaar et al., 2012).

In order to identify narratives I will use an open view of narrative structure that recognizes a variety of possible story openings, co-construction, and multiple story genres (Georgakopoulou, 2007). Narrative transcription will focus on the linguistic features of

narrative suggested by Gee (1991), dividing the text into lines and stanzas. I will examine each narrative using both the Coburn and Turner (2011) framework and the ways teachers position themselves through their stories (Wortham, 2001). I will then seek to understand the connections across the data sources and look for patterns in teachers' narratives across time, attending to the principle of emergence to make sense of how teachers' contributions build on one another (Wortham, 2001).

Relevance and Contributions to the Field

This project will contribute to the practice of and research on mathematics teaching and learning in multiple ways. First, it will provide much needed insight into the nuanced ways mathematics teachers use data in practice. Since teachers interact most directly and most consistently with students, they have a unique perspective to add to conversations among administrators and policy-makers regarding the data and evidence that are most informative for changing classroom practice. Second, this project has the potential to transform the field's understanding of the ways action research can impact teachers' capacity to change their practice using data and evidence. Overall, by understanding how teachers use and make sense of data and evidence, the proposed study can link the fields of teacher education research and educational policy to contribute to future supports for teachers to make effective use of the data to thereby improve mathematics instruction for students.

Notes

1. *Data* exist in a raw form without any inherent meaning, so whether or not data become meaningful information is dependent on the individual interacting with that data. In contrast, *evidence* is based on data, but used with a purpose. Evidence, therefore, is inherently tied to interpretation.

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Teachers Empowered to Advance Change in Mathematics (TEACH MATH)

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The overall goal of the TEACH MATH project is to transform preK-8 mathematics teacher preparation so that new generations of teachers will be equipped with powerful tools and strategies to increase mathematics learning and achievement in increasingly diverse public schools. To this end, the TEACH MATH project has focused on designing and researching the impact of a set of instructional modules for mathematics methods courses on prospective teachers' (PST) knowledge, dispositions, and practices.

Learning to be an effective mathematics teacher for diverse learners requires developing *knowledge, dispositions, and practices* that support capitalizing on children's cultural, linguistic, and community-based knowledge and experiences in mathematics instruction (e.g., Civil, 2007; Leonard, 2008). Teachers need to understand how children's *funds of knowledge* – the knowledge, skills, and experiences found in children's homes and communities – can support children's mathematical learning (Civil, 2002; González et al. 2001). Yet a gap often exists between the lived experiences of predominantly White and middle-class teachers and their ever more diverse students (Howard, 1999; Wiggins & Follo, 1999).

The Teachers Empowered to Advance Change in Mathematics (TEACH MATH) project has focused on designing a set of instructional modules for mathematics methods courses and researching their impact on PSTs' knowledge, dispositions, and practices. The research has occurred at six university sites that reflect a range of

geographic, demographic, and program contexts.

Instructional modules were created and implemented in elementary and middle school mathematics methods courses. These modules were informed by research and professional development materials on *Cognitively Guided Instruction* (Carpenter et al., 1999), community-based funds of knowledge for teaching mathematics (Civil, 2002, 2007), and Villegas and Lucas's (2002) six-strand framework for preparing culturally responsive teachers. More specifically, we designed the modules to include opportunities for PSTs to a) identify and build on children's mathematical thinking (CMT) and home/community funds of knowledge (CFoK), or together what we call children's multiple mathematical knowledge bases (MMKB), b) notice instructional and community practices that support preK-8 students' mathematical learning, and c) adapt curriculum materials and lesson plans to incorporate rich mathematical content, and children's MMKB (see also Bartell et al., 2010).

Brief Descriptions of Instructional Modules

In the *Community Mathematics Exploration (CME) module*, PSTs visited community locations and conducted community walks, at times guided by students and/or community members and then designed a problem solving-based mathematics lesson plan that built upon mathematical funds of knowledge in students' communities. The *Analysis of Classroom Practices (ACP) module* involved PSTs in using multiple lenses—teacher, learner, mathematical task, and power and participation—to critically evaluate mathematics lessons in various forms (e.g., written cases, video clips, curriculum materials, lesson plans, and enacted lessons). Finally, the *Mathematics Learning Case Study (MLC) module* was designed to support PSTs in expanding their thinking about children as mathematical learners. PSTs conducted problem solving interviews and interest inventories with individual children to learn about their experiences, interests, competencies, and perceptions about mathematics in the different contexts of their lives (school, after school, home, and community).

Key Results

Entry Points for PSTs

Across all sites, analysis revealed that many PSTs entered methods courses with the belief that connecting mathematical funds of knowledge was a valued teaching practice. Despite this initial agreement, we saw movement across the semester, with substantial percentages of PSTs voicing stronger agreement with this belief. Analyses of instructor reflections and PSTs' written work also suggested that while PSTs were very supportive of these ideas in the abstract, they tended to be vague, particularly initially, about what teachers might actually do to draw on these home and community resources. For example, when asked about how children's families and communities impact their of planning mathematics lessons, the majority of PSTs alluded to "making lessons students could relate to" without any specification of how one might learn about students' communities and their out-of-school experiences, or how one might use that knowledge to plan a lesson. We conjecture that, through their participation in TEACH MATH modules, PSTs acquired specific knowledge about mathematical resources available in children's homes and communities as well as specific ideas for using these resources in instruction (Foote et al., 2013).

Analysis of Lesson Planning Based on a Community Mathematics Exploration

Our analysis of CME projects found that PSTs entered the practice of making connections to children's MMKB in various ways and with varying specificity. For example, many lessons evidenced clear attention to children's mathematical thinking *or* children's funds of knowledge, but not both, suggesting that PSTs may have different entry points into this practice. We conjecture that some PSTs follow a trajectory in which they begin considering children's mathematical thinking joined with less demanding components of the practice, such as incorporating community contexts in problems, and then advance to identifying mathematical practices in the setting and considering

ways to connect to those practices in their instruction. Other PSTs connect to children and family practices in the community and yet include problem-solving tasks that require little mathematical reasoning, suggesting another possible path. An implication of this work is to consider, given a range of entry points, how mathematics teacher educators can help PSTs develop the practice of making connections to both CMT and CFoK (Aguirre, et al., 2013).

Analysis of Classroom Practices Through Multiple Mathematical Lenses

As part of the ACP module, PSTs engaged in a video lens activity four to five times during the semester, using a set of four “lenses” (teaching, learning, task, power and participation) to analyze representations of teaching and learning in videos. Results indicated that PSTs increased their depth and expanded their foci in noticing equitable instructional practices. In small group and whole class discussions, PSTs (a) shared their observations (often clarifying their own thoughts as they explained), (b) heard what their peers observed (often different from their own observations), and (c) posed and responded to questions (from both peers and MTE). We found that taken together, these experiences supported PSTs in noticing more details and becoming aware of the significance of these events (Roth McDuffie et al., 2014).

Analysis of PSTs’ Instructional Suggestions

The MLC module engaged PSTs in learning about a single student from their practicum classroom. Through various activities they learned about the students’ mathematical thinking and their interests, competencies, and resources. The PSTs then made instructional suggestions for furthering their student’s growth in mathematics. Results indicated that there were a variety of ways that PSTs made connections to one or more of students’ knowledge bases, including (a) assumptions about familiar or relevant contexts, (b) knowledge of familiar objects or activities, (c) mathematizing family practices, (d) identifying mathematics in activities that the child engages in,

and (e) specific details about children’s mathematical thinking. PSTs’ attention to these knowledge bases indicates that they were attending to particularities about students, and thus developing the powerful disposition to continue to do so (Turner, et al., 2015).

Concluding Comments

TEACH MATH attempts to support PSTs in exploring the social and cultural dimensions of children’s knowledge and experiences and to connect these to their mathematical thinking in instruction. In this way teachers are better prepared to understand students’ MMKB and design instruction that builds on them.

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After-school and into the Classroom: Beginning Secondary Mathematics Teachers' *Nos/otr@s* Relationships with Marginalized Students

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Service-learning experiences are opportunities for pre-service teachers mathematics teachers to make connections between coursework and practice and develop relationships with students. Under researched is how service-learning experiences relate to the first year of teaching mathematics. Using the Anzaldúan concept of nos/otr@s, this qualitative interview research investigates the process of developing teacher-student nos/otr@s relationships. Four beginning secondary mathematics teachers privilege these teacher-student relationships but encounter challenges developing and maintaining them. This study has implications for mathematics teacher educators, researchers, and mathematics teachers.

Introduction

With an increasingly diverse U.S. student population, mathematics teacher education programs consider a variety of approaches to prepare pre-service mathematics teachers to work effectively with marginalized students. Service-learning experiences are one such approach (Mule, 2010). Researchers show that these experiences help pre-service teachers make connections between coursework and practice, develop relationships with students, and interrogate their conceptions of marginalized students (Baldwin, Buchanan, & Rudisill, 2007). For pre-service mathematics teachers, service-learning experiences were also opportunities to deepen their mathematical knowledge and gain confidence using a variety of approaches to teach mathematics (Kirtman, 2008). What remains under-researched is how service-learning experiences relate to the first year of teaching. This

qualitative interview study will document the challenges and successes of four beginning secondary mathematics teachers as they develop and maintain relationships with marginalized students. I contend that the relationships that mathematics teachers develop with marginalized students inform how they teach mathematics.

Literature Review

The first year of teaching for beginning mathematics teachers is challenging. First, they are transitioning from the role of student to teacher (Lloyd, 2013). During this “semisurvival” phase (Lloyd, 2013, p. 104), they feel that they go from one challenge to the next (e.g., lesson planning, teaching, and classroom management). Second, experienced mathematics teachers seem to influence beginning mathematics teacher’s transition into the teaching profession and how they teach more so than their mathematics teacher education program (Ensor, 2001; Lloyd & Riley, 2013; Prescott & Cavanagh, 2008). Experienced mathematics teachers are perceived as efficient in their planning, effective in their teaching, and able to prepare students for standardized exams (Ensor, 2001; Prescott & Cavanagh, 2008). Wanting to be perceived as effective mathematics teachers, beginning mathematics teachers emulate transmission models of teaching that conflict with student-centered approaches and teaching for conceptual understanding promoted by their mathematics teacher education programs. Some beginning mathematics teachers feel that not until after their “probation” period ends that they can teach mathematics differently from their colleagues (Prescott & Cavanagh, 2008).

My research is guided by the following question: How do the visions/stances regarding relationships with marginalized students that pre-service secondary mathematics teachers developed during an after-school mathematics club relate to the teacher-student relationships they develop during their first year of teaching?

Theoretical Framework

My analysis is informed by the framework of *political conocimiento* (political knowledge for mathematics teaching) (Gutiérrez, 2012). She

asserts that mathematics teaching is a political act and that to teach marginalized students effectively it is important to seek solidarity and advocate for them. Seeking solidarity is grounded by the Anzaldúan concept of *nos/otr@s*. *Nos/otr@s* relationships (with the slash) are in constant negotiation. Those engaged in these relationships seek solidarity with one another, are respectful of each other's lived experiences, and accept multiple perspectives (Gutiérrez, 2012). Neither person is expected to adopt the other's values or impose one's values on others. Some may refuse to enter into this kind of relationship by maintaining their own perspective and refusing to accept those of others (*nosotr@s* relationships - without the slash) (Gutiérrez, 2012). *Nos/otr@s* relationships may benefit mathematics teachers because acknowledging, respecting, and integrating the lived experiences of marginalized students may facilitate engaging them to do mathematics.

Methodology

Four beginning mathematics teachers (three white females and one Latino male) were enrolled in our equity-based mathematics teacher education program. All volunteered for three semesters, five times per semester, at *I Do Mathematics* (IDM) an after-school mathematics club for Black and Latin@ middle school students. Currently, they are teaching secondary mathematics.

This qualitative interview study (Kvale & Brinkman, 2009) consists of four semi-structured interviews during the participants' first year of teaching mathematics. The lead author will conduct two interviews during the fall semester and two in the spring semester. Interviews will be transcribed and coded. Two concept-driven codes that will be used are challenges beginning teachers encounter and the kinds of relationships developed with colleagues; two data-driven codes are how they develop relationships with students and the use of (or challenges of) IDM-type activities.

Results / Discussion

I am currently conducting this research. I will briefly refer to previous findings and the literature to offer some anticipated findings.

[IDM] gave scholars a chance to practice building relationships and practice asking questions and it was... the applied part of all of our discussions that we would have in seminar (Rose)

Rose's statement reflects those of the other four participants. She recognized the opportunity to practice "building relationships" with middle school students at IDM. This seems to indicate participants had the opportunity to develop *nos/otr@s* relationships at IDM. Rose's statement parallels findings that service-learning experiences were opportunities to develop relationships with marginalized students and make connections to their courses (Baldwin et al., 2007).

I was learning from them; they were learning from me; we were helping each other out with these... activities (Annie)

Annie, similarly to other participants, described her experience of co-constructing knowledge with the middle school student at IDM. Although she facilitated activities, by stating "I was learning from them," she also expressed that she was the middle school student's peer by stating, "we were helping each other out". This is contrast to most pre-service mathematics teachers' experience where they maintained the role of the mathematical expert (Kirtman, 2008). Annie's experiences seems to indicate that she and others participants had opportunities to co-construct knowledge as a result of *nos/otr@s* relationships with students at IDM.

My vision is to, even under these rules [Common Core State Standards / Curriculum] that the school has set over me, still bring... a game, math questions, math challenges... [as] a way for students to think and construct ideas (Daniel)

As a beginning mathematics teacher, this participant will attempt to resist the "rules" imposed on him. The goal of using mathematical "games, questions, and challenges" is for students to "think and construct" mathematical ideas. His experiences at IDM seem to anticipate his approach for teaching mathematics through games and challenges. These mathematical challenges may help develop *nos/otr@s* relationships as a way to engage marginalized students in the practice of constructing mathematical knowledge. This is in slight contrast to

the literature where pre-service mathematic teachers predict changes to homework, practice, and teaching (Kirtman, 2008) but never refer to using mathematical games, questions, or challenges.

As a result of these findings, I expect to document how these participants seek opportunities to: develop *nos/otr@s* relationships with marginalized students; learn from students; and integrate games and challenges during their first year of teaching mathematics. I also expect to find, as the literature suggests, that they will encounter challenges teaching (Lloyd, 2013; Prescott & Cavanagh, 2008). In particular, I want to understand how the relationships with more experienced mathematics teachers will influence the teaching practices of the beginning mathematics teachers and potentially impact how they develop teacher-student relationships. As challenging as the beginning mathematics teacher experience will be, I expect to find examples of *nos/otr@s* relationships as a way to engage marginalized students in their mathematics classroom. Participants may not be deterred even if more experienced mathematics teachers do not privilege *nos/otr@s* relationships.

Conclusion / Implications

This study raises questions regarding how pre-service secondary mathematics teachers transition into the role of beginning mathematics teachers. As researchers document the challenges beginning mathematics teachers encounter and the limited influence of teacher education seem to have, what are ways that teacher educators can support beginning mathematics teachers to teach mathematics that is aligned with their stance for developing *nos/otr@s* relationships with marginalized students? Another question raised is how can experienced mathematics teachers support beginning mathematics teachers in their pursuit of developing *nos/otr@s* relationships? These are complex and difficult questions for mathematics teacher researchers, educators, and experienced mathematics teachers to consider.

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Indigenization of Mathematics Curriculum: An Evolving Experience

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Background

This is my first semester in a Doctor of Philosophy in Education program at the University of Regina. Recently, through reading, I have been reminded that although “I cannot change others; I can work at changing myself” (Mason, 2002, p. iv). Hence, I am using doctoral course work to explore concepts that I have ignored, resisted or avoided in the past. This semester such exploration has occasioned opportunities to focus my learning on Indigenous ways of knowing.

Context

Indigenization of Saskatchewan Curriculum for Mathematics is an idea in its infancy and my recent involvement is accidental. In late August, at the beginning of the 2014-2015 school year, in-service for local public school teachers focussed on treaty education. “The Treaty Essential Learnings” include topics and concepts (the treaties, the treaty relationship, the historical context of treaties, worldviews, symbolism in treaty-making, and contemporary treaty issues) that “students would be expected to learn through the provincial education system, K-12” (Office of the Treaty Commissioner, 2008, p. 3). In-service sessions led a few mathematics teachers, from various secondary schools to seek guidance in incorporating Indigenous understandings into their lessons. As a result, one night while working on a university course research assignment in understanding Indigenization of mathematics curriculum, Jordan (my assignment partner) shared that secondary school working groups for various subjects had begun to form and that he was attending the first mathematics working group meeting the following day. Due to my interest in relating Indigenous ways of knowing and mathematics, I took the

facilitator's contact information and consequently I asked her permission to join the working group. Suggested goals for the mathematics working group included: having group members share how they are or plan to Indigenize mathematics courses; exploring possibilities for Indigenization; discussing the work that has already been developed in each of the mathematics content areas; and, attaching work to curriculum outcomes (or core concepts) so that teachers might leave the meetings with some ideas for classroom use (Bellegarde, 2014).

Initial Project Ideas

At the first mathematics working group meeting, members mentioned that connections between tipis and mathematics have been noticed by a few educators; however, those educators conveyed that they had not had the time to write about them. This mentioning ignited my interest in trying to link tipis and mathematics twofold: to contribute to the perceived needs of the group, and to submit a micro-project proposal about the experience for the other required university course in which I was enrolled.

First Approach

In my past teaching practices, when planning for mathematics lessons, I started by looking at the related curriculum outcomes (foundational objectives at the time) to see what was supposed to be taught. This time, in trying to think about lesson planning more holistically, I decided I would figure out mathematical notions from the tipi, and then try to connect those notions to curriculum outcomes. I hoped this approach would assist in the creation of a more illustrative resource to use when teaching in future and possibly to share with other working group teachers. Because I have never collaboratively planned mathematics lessons, I tried to see what mathematics I could link to the tipi before expanding/enhancing my ideas through collaborations with other educators or elders. Essentially I know nothing about tipis, or tipi raising so I considered how I might look at local Aboriginal Perspectives tipi raising videos (<http://aboriginalperspectives.ca/videos.shtml>) rather than just books or online resources about

tipis. I thought this approach might create some tacit understandings of tipis that could not be learned from printed texts. I knew I could not do this kind of work alone and yet I felt compelled to start in such a manner, perhaps because I have been taught to learn independently and to try to see what I can figure out before asking others to expand/enhance my understandings. However, I must also admit that I do not know how to go about learning the proper etiquette for collaborating with elders, which is something I hope to learn through participation as a member of the mathematics-working group.

Second Approach

In reading work by Barton (2008) I started to understand two things about linking Indigenous ways of knowing and mathematics: 1) that “it is often difficult to write in one language the equivalent of what is being said in another” (p. 7); and 2) that even though mathematical language in Maori was developed collaboratively with Maori educators and elders “...mathematics education seemed to be a vehicle that led to the subtle corruption of the ethos of the Maori language” (p. 4). These words resonated with me, and altered the course of my thinking. I questioned if it was appropriate to compile all of the mathematical understandings that I could figure out from the tipi before the next mathematics working group meeting so that I could be more knowledgeable when talking with the group. I began to consider whether the mathematical notions of the tipi would somehow be better represented more holistically as a new set of mathematics curriculum outcomes or perhaps even be part of a new ‘Indigenous Mathematics’ course within the Saskatchewan Curriculum. However, I had also considered that perhaps the mathematical notions within the tipi should be left undocumented and thereby uncorrupted. I was lost but I was very aware that something about the First and Second Approaches felt wrong. Through my past experiences in studying Complexity, I learned that dividing systems into constituent parts destroyed relational meanings between parts. Thus, in writing the second draft of the micro-project, which also turned out to be the first draft of this Mathematics Education and Society (MES) conference proposal, I expressed my disappointment in thinking that I would be partitioning the tipi to create disconnected mathematical understandings. I shared

this disappointment with a few of the university professors through the submission for feedback of the draft MES proposal.

Third Approach

Then, almost simultaneously, two experiences led to broader understanding. In preparing for Jordan's and my assignment presentation I came across the word 'participation' and it resonated with me. That same day, an associate professor graciously gave me feedback with regard to my MES conference proposal. She asked, "Have you considered beginning with actually learning how to set up a tipi with the participants?" (Pete, 2014). I realized my approach to relating tipis and mathematics needed adjusting and that I needed to start by participating in the actual activity of tipi raising with elders, if possible. However, this idea was fleeting. The cyclical nature of these experiences allowed me to more deeply and respectfully consider that "as a non-Aboriginal Canadian, it is not my place to reformulate the *Framework* so that it embraces an Aboriginal perspective" (Aikenhead, 2010, p. 389).

Project Purpose and Methodology

The purpose of my course-based micro-project is no longer about linking tipis and mathematics. I might ask at the next mathematics working group meeting if anyone thinks it a good idea to collaboratively explore links between tipis and mathematics and whether I might be able to participate in such collaborations if exploration begins, but this is no longer my research focus. Instead I want to compare my experiences, in trying to relate Indigenous ways of knowing and mathematics, with experiences from consenting working group members or other teachers who have also tried to link Indigenous ways of knowing and mathematics. Ermine (1995) writes that "individuals and society can be transformed by identifying and reaffirming learning processes based on subjective experiences and introspection" (p. 102). Thus (as not so subtly suggested through the sharing of an article by another professor who had read the draft MES proposal) it is necessary for my growth and development as a researcher to

continue this critical autoethnographic self-study and first-person narrative representation of my experience in relating Indigenous ways of knowing and mathematics curriculum (Pennington & Brock, 2012).

Methods and Analysis

Through conversations with consenting educators from either the mathematics working group and/or my elective university course I hope to gather stories about the following questions: What experiences brought you to wanting to relate mathematics or mathematics education and Indigenous ways of knowing? What has it been like in trying to learn to Indigenize mathematics curriculum or lessons? What does the phrase ‘to Indigenize mathematics or mathematics curriculum’ mean to you? What have you tried? How did you conceive what you tried? Have there been roadblocks or detours along the way? In comparing research participant data with my answers to these questions, I hope to access new insights and understandings of approaches in collaborating and in relating Indigenous ways of knowing and mathematics.

Pennigton and Brock (2012) suggest the analysis of the data rests with the researcher to revisit and to critically reinterpret experiences within social contexts. I must regularly use reflexive practices to analyse my ‘whiteness’ or ‘racism’ - in other words, my position of privilege and my awareness and empathy for another’s situation. I must also share these reflections and stories within a supportive community where discussions can emerge so I can continue to grow and develop not only my ways of being a researcher but also my ‘ways of being.’ Though there will be times when I need guidance and redirection, “no one [can] dictate the path that must be followed” (Ermine, 1995, p. 108).

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Mapuche Ethnomathematics: Mathematical Learning's Promotion from Cultural Knowledge¹

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The objective of this research project is to create activities that favour the learning process of the Mapuche People's ethnomathematics, answering the need to revalue their knowledge and spread it within Chile's traditional Educational System. The implemented methodology is qualitative, with a strong emphasis on content analysis, ethnography, and participant observation. The expected results are centred in reporting Mapuche ethnomathematics, constitute a culturally pertinent design of teaching activity, and inform about a concrete teaching activity that favours the learning of ethnomathematical elements that are relevant to the Mapuche people.

The centre of interest of this project is in Mapuche³ students and their mathematical education in a formal context in Chile. The problem is that, for years, children have been instructed omitting their diversity and richness of knowledge that constitutes them in a particular community. In addition, our native people have not only not had the right to see their knowledge integrated in a national curriculum, but also new knowledge was imposed as unique and true, isolating them to an inferiority status in the national educational standards.

In the actual educational system, Mapuche boys are forced to attend a school that not only does not offer them an education for the conservation of their roots, but also evaluates and classifies them as slow students when learning prescribed knowledge and in “proper” schoolwork, since both occidental and Mapuche logic are not explicit (Quintriqueo & McGinity, 2009). Nevertheless, as stated by D'Ambrosio (1985), referring to other native cultures, the “experts” tend to omit this reality, pointing out that these low results are solely due to the poverty context in which the students are immersed. Therefore, they discard the possibility of giving them a mathematical

education starting from the development of their practices, knowledge, languages, and codes that clearly state the ways in which they do mathematics.

In this way, knowledge in the mathematical area has been one of the most affected, given that the existence of cultural mathematical knowledge is not officially recognised⁴, ignoring the fundamental step of evaluating its relevance to mathematic education. Accordingly, this project attempts to emphasise the relationship between mathematics and the Mapuche People's culture.

At a national level, the Ministry of Education, conscious of the need to contextualize the national program to the reality of the native people, created in 2003 a curricular document oriented to the adequacy of contents to an environment that favour the Mapuche people, Aymara and Licarantay. Therefore, each subject possesses an appropriate contextualization, which in mathematics is oriented to a rural reality with elements of the indigenous people that only an educator who knows the culture will comprehend (Huencho, 2012).

The theoretical element supporting the project is based in Ethnomathematics as a field of research that has contributed to comprehend that mathematical knowledge has developed in diverse cultural groups (indigenous people, associations, etc.), to explain and represent the reality that surrounds them (D'Ambrosio, 2012). First, it is necessary to know the mathematical elements fundamental to the Mapuche people from acknowledging their primary activities, then move towards a proposal associated to the promotion of the learning process through the people's mathematical knowledge.

In this way, we have analysed two research sources, which are centred in ethnomathematics, that are focused on the rescue of mathematical knowledge of a native sociocultural group and the use of this knowledge in the classroom.

The first group object is comprehension of how studies of academic relevance develop an emic and etic methodology to be introduced in the cultural knowledge of indigenous people; these studies centre their methodology in ethnographic work or historiographies to rebuild ethnomathematic knowledge. The second group shows how ethnomathematical knowledge is valid to be presented in the classroom, mainly because of its similarity to the contents that the academic curriculum proposes, informing the best results that indigenous and non-indigenous achieve. Little is said about who decides

the implemented didactic model and if it is culturally relevant, in the same way that evaluations used to measure the learning processes of the students leave an empty space concerning the characteristics that these instruments possess. A high percentage of the research is placing ethnomathematics at the service of the Mathematic Education.

So, the main objective of this research is to create activities that favour the learning of Mapuche ethnomathematics in a culturally relevant way. To achieve this goal, it is necessary to fulfil the following specific goals: (1) Identify and organize the ethnomathematics knowledge of the Mapuche people, (2) Identify the processes to transfer knowledge developed by the Mapuche people, (3) Contrast the ethnomathematical and transfer knowledge with *kimches* (wise people) of the Mapuche community, (4) Design activities that favor the learning of Mapuche's ethnomathematics, (5) Validate the design of activities in diverse Mapuche communities.

The methodological proposal to develop this project is based on a qualitative focus group divided into two phases. The first phase develops three of the first specific objectives, starting with an analysis of the contents of the documents with historical relevance to the Mapuche people, anthropological documents, the work of linguists, historians, and narrators in their own history, that enlighten the normal life of the people where we could find the foundations of their ethnomathematics and the ways in which the transference of knowledge is achieved from the wise people to the community in general. The identified information is contrasted with the Mapuche community and their *kimches* (wise people) with the idea to achieve a higher degree of comprehension of the interpretation made to the historical documents, then, through ethnography, semistructured interviews, and participant observation, we expect to gather relevant information, which will give the research sense in the actual world.

The objective is to contrast, but also gather new elements that could be a contribution to the construction of activities that favor the learning around relevant activities to the community, because of their implication to the past, the present, and future of the Mapuche people.

The comprehension of the knowledge transition process of the ethnomathematics and the activities with meaning to the Mapuche people, will allow us to move on to the second phase of the research in which the fourth and fifth objectives will be addressed. Therefore, the

period to design the activities for learning starts. So different tables will be set up to dialog with the community, teachers who know the culture, and the researcher, with the object of formulating a design of activities that favor the learning of Mapuche ethnomathematics. We will take notes of the formulated decisions, which will serve as a foundation for the design and proposal of the learning activity.

Finally, the discussion table will evaluate the pertinence of the crafted activity through a discussed design, and different validation sessions will be carried out with Mapuche students from different communities. The objective is to validate their effectiveness as long as they meet the objectives that the activity proposes and culturally pertinent learning instances regarding the teaching method and the content.

The proposed methodological design covers three type of data. The first comes from written documents; the second comes from the transcribed interviews and discussion tables; the last one comes from execution of the teaching activities for which we will register each of the sessions. These sources of data will be analysed with the software Atlas.ti since it uses Grounded Theory to classify, structure, and analyse large amounts of text audio and video making easier the interpretation of such, given their hierarchical system of codification, variable definition, and weight analysis to the selected units of analysis.

This research seeks reporting elements that help rescue a culture that has been “historically denied and prejudiced”, not only in the educative system, but also in the economic production and the social relationship between Mapuche and Non-Mapuche people. Therefore, this investigation wants to be a contribution in three dimensions: report the Mapuche ethnomathematics and constitute it a basic element that favors multiple new researches in the education area; build an activity design which favours a culturally pertinent teaching dynamic; and finally, inform concrete teaching activity which favors the learning process of ethnomathematical elements that are relevant to the Mapuche people.

Notes

1. This research Project is part of the work that I must develop to opt for a PhD in Educational Sciences.
2. Mapuche. Teacher of mathematics. Masters Degree in Education. And student of the PhD Program in Education at Pontificia Universidad Católica de Chile.
3. The Mapuche people represent 9.06% of the total population and the 81.8% of the total indigenous population (INE, 2012) (National Institute of Statistics).
4. Adalberto Salas (1980) is the only one who has developed a numeric system, used by the Mapuche people, from a linguistic perspective.

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Using Cogenerative Dialogue to Incorporate Students' Perspectives about Their Experiences in a Mathematics Classroom in an Urban School

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The consistent underachievement of African American students in mathematics warrants an investigation into their perspectives of the types of teaching and learning that promote effective learning environments for them. There is a growing body of research that examines students' perspectives of their experiences in mathematics classrooms (Stinson, 2008; Terry & McGee, 2012). This paper adopts a critical perspective of an on-going empirical study to showcase the use of students' perspective in a middle school mathematics classroom. This study will help gain a better understanding of how to better support students through their experiences in their mathematics classroom with cogenerative dialogue. As such, this study has important implications for students' success in mathematics in urban schools.

Introduction

There is a missed opportunity for students to offer potential suggestions for what they believe helps them navigate learning in their school and classrooms. The perspectives of underrepresented and culturally diverse students in urban schools are not consistently acknowledged and valued. When given the opportunity, students may have much to say back to their teachers, administrators and schools about their experiences.

This on-going dissertation study will explore the nature of how a teacher and her students participated in cogenerative dialogue (hereafter, cogen) to enhance students' opportunities to learn mathematics over time. Cogen can be defined as the "use [of] current understandings to describe what has happened, identify problems,

articulate problems in terms of contradictions, and frame options that provide us with new and increased choices for enhancing teaching and learning” (Roth & Tobin, 2002, p. 252). The use of cogen may provide insight into the use of students’ perspectives in mathematics classrooms and could potentially enhance students’ mathematical experience and learning. Examining how, and in what ways, teachers and students participate in cogen may reveal mathematics’ teacher strengths, provide insight into how they can better promote their students’ mathematics learning, and reveal ways their teacher might be inhibiting student success.

Significance of this Work

Haberman (1991) asserted that teachers should seek to actively involve students in activities that engage them in the classroom. An understanding of schooling from students’ perspectives may help schools to identify and target critical areas, which may, in turn, direct educators to focus on ways to promote better student experiences (Mitra & Gross, 2009).

Relevant Literature

Student Perspectives

Empowering African American students to do their best work requires placing their voices at the center of the discussion of what works for them in the classroom (Howard, 2001). Scholars who have successfully examined African American students have addressed the issues of race and how it played a role in students’ mathematical background (Stinson, 2008), examined students’ mathematics identity (Martin, 2000), and revealed how high achieving African American males rely on support networks inside and outside of the classroom (Terry & McGee, 2012).

Cogenerative Dialogue

Cogen works best when all participants, especially students, feel that by participation they are provided with opportunities to actively improve or alter their learning conditions in the classroom (Martin, 2006). These conversations are aimed at identifying and reviewing what seems to work and what does not, especially the practices that may disadvantage certain students and truncate their learning (Roth & Tobin, 2002). Although there is not official protocol highlighting the ways to engage with cogen, typical structures include the teacher soliciting and collecting student feedback about their experiences in their classroom, and then discussing the feedback explicitly with the students. Studies have highlighted the benefits of the use of cogen for improving the quality of teaching and learning (Lehner, 2007), and providing empowering experiences for students (Martin, 2006).

Research Question

This on-going project seeks to answer the following questions: 1) What are African American students' experiences when their mathematics teacher engages them in cogen? 2) What feedback do students provide their mathematics teacher with during cogen?

Theoretical Framework

Critical theory is a type of social theory oriented toward critiquing and changing society as a whole. It is difficult to define because, as Kincheloe and McLaren (2005) noted, there are several critical theories that continue to evolve. Critical theory debunks the idea that there is one way of seeing and knowing. As Swaminathan (2007, p. 22) explained, "Consequently, to address savage inequalities and meet social justice objectives, it is crucial to create spaces in schools for students' voices to be heard and taken into account in structuring educational experiences".

Critical Pedagogy

Within the larger framework of critical theory exists critical pedagogy that focuses on listening to students and allowing their voices to be heard and embraced in the classroom. Critical pedagogy affords an opportunity to connect to students' experiences. Utilizing student perspectives can provide a democratic learning environment in the classroom. This paper will pay attention to counter-hegemonic practices in the classroom, which refers to centering the voices and experiences of those who have historically existed within the margins of mainstream institutions – namely African American students in mathematics classrooms.

Methods

Setting and Participants

This study was conducted in a 7th grade Pre-Algebra classroom at Westside Middle and High School in an urban community in the Midwest. The school enrolls approximately 1700 students per year. In 2013, 95% of the students were African American or Latina/o and about 88% of the students received free or reduced-priced lunch. To gain in-depth perspectives of minority student experiences, the data used in this study highlight focus-group sessions with 12 African American and Latina/o students who were enrolled in the same pre-algebra classroom. Their teacher, a White middle-class woman, wanted to improve communication with her students through the use of student feedback.

Data Sources and Analyses

The data sources for this study included student feedback forms, video-recorded classroom observations and audio-recorded focus groups sessions (Krueger & Casey, 2000). The student feedback forms were co-constructed by the teacher and the author. The focus group questions and student feedback forms included questions about students' mathematical understanding and any pedagogical changes

that occurred in the classroom. I observed the classroom an average of three times a week and transcribed video data each time that the teacher engaged in cogen with the students. I held two 90-minute focus group sessions before and after the teacher held cogen sessions. During these 90-minute sessions, I asked students to speak and reflect on their experiences with cogen. On average, six different students were present at each session.

Data will be analyzed by identifying emerging patterns and responses from the cogen sessions and focus group data, leading to the establishment of codes and the identification and creation of thematic categories that highlight the key findings (Miles & Huberman, 1994). I will analyze the conversations during the cogen sessions and focus group data, paying attention to what the students are reiterating. I will also pay attention to student feedback forms that could provide more information about the students' experiences before, during, and after engaging in cogen sessions with their teacher.

Preliminary Findings

This on-going analysis suggests that the students encouraged the teacher to use more hands-on activities, and less teacher-centered instruction. Because their teacher solicited feedback, the students also noted that she cared about them. More information about the conversations during cogen will be shared in the presentation.

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Power in Preschools: How to Support Teachers in the Unpacking Process

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New norms and values introduced into preschools through curricula changes require teachers to adapt their teaching. However, in making adjustments in order to increase children's agency, teachers can lose sight of how they wield their own power. In this project, a professional development facilitator worked with five Swedish teachers. Although the preschool teachers considered their primary role to be carers who resisted interfering in children's explorations, the initial analysis suggests that they were wielding considerable power in their interpretations of what was occurring. Consequently, it has become important for the professional development facilitator to unpack with teachers the power that they wield in order to develop their teaching. This project description discusses initial ideas for doing this.

Teachers' Understandings of Children's Agency

Preschool as an institution is framed by and organized by the institution's norms and values and therefore has a profound effect on children's possibilities for childhood (James & Prout, 2001). As norms and values change within a society, and thus within an institution, then childhoods will also change. In Swedish preschools (Emilsson, 2007) as elsewhere (Broström et al., 2014), the government policy, as represented in curricula (Skolverket, 2011), has advocated an increase in children's agency in their learning. Previous research had shown that teachers generally had a tight control over what occurs (Emilsson, 2007). This has seen to be problematic and instead it is currently being suggested that "by supporting play without dominating or disrupting it, teachers can aid children's learning and development" (Sandberg & Arlemalm-Hagser, 2011). In this way, Swedish preschools are

considered to contribute to young children becoming citizens who can understand how to take actions in society.

From a mathematics education research perspective (for example Edo, Planas, & Badillo, 2009), the teachers' role in activities is crucial for learning. However, in navigating between old and new norms and values, teachers may struggle with what their roles should be, particularly if they are unaware of the power issues, or how changes in their teaching might affect power relations. For example, Ebrahim (2011) identified four strategies that children in four South African early childhood centres used to enact their agency "resistance, avoidance, ignoring and collaboration" (p. 124). Of these strategies, only collaboration might be considered by teachers as a positive contribution to children's learning of mathematics. Yet if increasing children's agency is to be promoted in preschools, then teachers may need to rethink their own perceptions of how children can enact their agency.

Following the emphasis in the Swedish preschool curriculum on activities for children (Skolverket, 2011), our approach is to develop the teachers' knowledge and skills in regard to using mathematics activities that support children's engagement. However, as discussed in the next section, as the project developed it became clear to the professional developer that in trying to promote children's agency teachers were not recognising their own wielding of power. This led the professional developer to also consider her own use of power when working with the teachers. At the core of this power issue is how pre-school teachers interpret children's actions and act on those interpretations and how the professional developer brings these to the fore.

The Project and How Shifting Norms and Values Change Power Relations

One of the authors, Dorota, is involved in a research project with five pre-school teachers from two preschools in a small town in southern Sweden. The children that the teachers work with are between one and three and half years old, The teachers decided to video-record different situations so that they could observe and reflect on possibilities for discovering and making visible children's own ways of learning. The teachers watch the videos with the professional developer,

sometimes in small groups and sometimes with everyone together and discuss what children are doing.

As the teachers wanted to observe the children and what they did, often activities are set up for the children but in ways in which the teacher has limited involvement. Children had several opportunities to engage in the same activities. One of the outcomes for the teachers of video-recording the children several times doing the same activities was that they could see that children often focused on different aspects of the activity.

For example, in an activity to do with light and shadows, which can be considered as connected to understandings of spatial orientation, the children needed some time to become aware of and utilise the on/off button on the torches before they were able to focus on projecting their shadows on the wall. The teachers were surprised that the time and repetitions were not a hindrance to children's learning as they did not become bored in the way that the teachers had expected. The teachers became aware that by controlling how many times they allowed children to do the same activity, they affected the likelihood that children would learn.

At other times, teachers did not seem aware that they exercising power through their observations. For example, during one episode, a young boy put the torch at the top of a cardboard roll focussing the light into a narrow spot. The other children called for light. He consequently went and turned the room light on. The teacher saw this action as a destruction of the shadow-making activity. From the professional developer's perspective, it was by making this interpretation the teacher had wielded their power to determine which children were legitimately using their agency, by making shadows, and those who were not, as in the boy who turned the room light on. This raised the issue of how to make teachers aware of their use of power in situating some of these young children as learners and others as not which could then limit the legitimising of children's agency (James, 2007).

The Issue of Power

The focus of the project for the professional developer, at least, has moved from the teachers' pedagogical skills, to investigating how

power relations can change. One of the reasons for considering different power relations is that the teachers' awareness can affect children's own actions and the outcomes from these. There are issues to do with how children are considered active participants in activities by positioning their exploring and learning in positive or negative terms. The possibilities for mathematical learning are framed by materials, tools, and equipment chosen by a teacher. In these ways, children are given access to discover the processes of learning. In order to begin an exploration of power with teachers, it is important to acknowledge what the professional developer considers to be power relations because this definition affects how she will work with teachers. This is because it is her use of power that will affect the teachers' learning possibilities (Meaney, 2004).

The concept of power has taken different routes in mathematics education (Valero, 2004). One example has to do with the way that mathematics itself is considered to be powerful. An attempt to overcome the hegemony of Western mathematics has been the introduction of ethnomathematics, which takes into account the use of mathematics in different cultural contexts (D'Ambrosio, 2010). In one way, this can be considered an alternative stance to traditional school mathematics. In the Swedish preschool curriculum, the mathematical understandings that preschools should offer to children through activities are based on Bishop's (1988) six categories of mathematical activities (see Utbildningsdepartementet, 2010). This provides one possibility for discussing with teachers how different conceptions of mathematics may be considered by society as more or less powerful forms of knowledge.

Another way of considering power is related to the relationships between people (see Meaney, 2004). Foucault (Gordon, 1980, p. 198) stated that "[i]n reality power means relations, a more-or-less organized, hierarchical, co-ordinated cluster of relations". Therefore, power can be seen as a "...relational capacity of social actors to position themselves in different situations and through the use of various resources of power." (Valero, 2004, p. 11).

At the moment, the professional developer in collaboration with the other authors of this paper are working on how to support teachers to become aware of their perceptions of mathematics, of children, and what are legitimate learning behaviors that affect possibilities for taking children's agency in their learning seriously.

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Analyzing the Cultural Responsiveness of Two Mathematics Units

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This paper describes a project presentation on the application of an analytical tool to assess the cultural responsiveness of two mathematical units developed in a bilingual mathematics club. Results provide initial considerations to assess culturally responsive mathematical tasks. Future steps and implications are discussed.

In this session we will discuss an on-going project that investigates Culturally Responsive Mathematics Learning Contexts (CRMLCs). Specifically, we explore the curricular features to design and establish CRMLCs in out-of-school settings with bilingual Latina/o students in the United States.

Setting

The out-of-school program, *Los Rayos* mathematics club, brought together bilingual Latina/o undergraduate students (mostly pre-service teachers, henceforth referred to as PSTs), Latina/o elementary students, and Spanish-speaking mothers to work collaboratively on open-ended mathematics tasks that required students to experiment, develop multiple strategies, and communicate their reasoning. This program, created through the Center for the Mathematics Education of Latinas/os (CEMELA), was designed to investigate the linguistic and cultural resources bilingual Latina/o students use and that support their mathematics learning. We focus our investigation in a time period during which noticeable shifts were documented in student engagement, language use, and ways in which PSTs used students' culture as an instructional resource.

During the first half of this time period, the curriculum included a collection of adapted mathematical tasks that emphasized problem solving and included non-routine problems that focused on topics such as fractions, logic, geometry, patterns, etc. The tasks did not build on one another. They were grouped by topic and were placed in binders, so everyone referred to them as the “binder activities.” Student groups, each with a PST as facilitator, chose the tasks to work on in each session. On the second half of this time period, however, the context and content of the mathematical activities changed. This time, all students were invited to work on the same project, whose mathematical goal was to support children’s proportional reasoning. This project, which participants came to refer to as the “recipes project,” consisted of sequenced activities designed to ultimately lead each group to create a recipe and prepare a dish for *Los Rayos*’ end-of-year party. Project activities involved measuring and creating orange juice combinations, students reasoned and described why some combinations tasted more orangey; making mole with a group of mothers who posed mathematical problems in the process; creating a perfect recipe that each group chose; developing a budget and buying recipe ingredients at a local grocery store; scaling up the perfect recipe to serve all participants in the party; and finally making the dish for the party.

The two distinct activities mentioned above could be considered as discrete mathematical units since the topics and goals of each aimed at the understanding of different topics. Though both units were enacted at the same educational environment or physical space, the quality of student and PSTs’ participation greatly varied between the two units. As mentioned earlier, several shifts were documented in each period. A major documented shift was the participants’ language use. Specifically, during the binder activities, participants used predominantly English while in the recipes project, they predominantly used Spanish (Vomvoridi-Ivanovic, 2009). Another documented shift was the ways in which PSTs used cultural references as an instructional resource. Specifically, when working on the binder activities, PSTs alluded to cultural references only in non-mathematical contexts, while in the recipes project culture was used as a resource in mathematical contexts (Vomvoridi-Ivanovic, 2012). Finally, another shift included students’ mathematical engagement.

Student engagement in recipes project was analyzed on how student proportional reasoning was supported. Results yielded that

student reasoning and engagement were enriched through multiple relations. These relationships were initially constructed through social and cultural dimensions, but they intensified over time into a mathematical dimension. The mathematical engagement was essentially relational. All participants' roles were transformed simultaneously through the design, preparation, and enlargement of the original recipes; proportional reasoning was simply part of the activity (Domínguez, LópezLeiva, & Khisty, 2014).

Overall the point that has captured our attention the most is the increased use of Spanish in mathematical activity during the recipes project. This shift in language use is an indicator of a learning context that apparently was more culturally responsive than the one in the binder activities. Here we investigate the ways in which the recipes project promoted a more culturally-responsive, mathematics-learning context.

Framework

Cultural responsive mathematics teaching includes teachers' understanding of the socio-political context of the teaching and learning in their classroom (Aguirre & Zavala, 2013). The inclusion of culturally responsive approaches deems family activities as mathematical resources available to students and teachers to support mathematics learning (Civil, 2007), and community practices can be leveraged to help contextualize and extend student mathematical understanding (Díez-Palomar, Simic-Muller, & Varley, 2007). In addition, this culturally responsive approach also capitalizes on authentic problem-solving contexts (Turner & Strawhun, 2007).

Methods and Findings

In order to investigate learning contexts culturally responsiveness, we chose to use the Culturally Responsive Mathematics Teaching (CRMT) Lesson Analysis Tool created by the TEACH math project (Aguirre, Drake, Bartell, Foote, McDuffie, & Turner (2012); (<http://www.mathconnect.hs.iastate.edu/Instruments.html>). This analytical tool is designed for professional development purposes so as

to support lesson/unit design and implementation with equity and power dimensions in mind. For this study, we used this tool to analyze a small subset of our data. Specifically, we analyzed four videos that captured *Los Rayos* group's interactions during two sessions in both the binder activities and the recipes project. Our findings confirm greater cultural responsiveness for the recipes project.

Although we found the CRMT Lesson Analysis tool to be useful in distinguishing between the two units (binders and family recipes), there are some areas in which the descriptors included in this tool seem limiting to capture the differences in richness of the experience that students had during each unit. For example, in both analysed sessions of the recipes unit, there is a kind of engagement, or student enthusiasm, that the descriptors included in the tool seem to fall short in capturing. It is not only about the fair and distributed participation, but also about the quality of participation which the CRMT tool seems unable to fully capture, despite earning full points according to the description provided in the rubric. Perhaps, an explanation for this limitation is that *participation* is a concept that implies a situated task or activity, and evaluating participation independently from its context seems counterproductive.

Another issue we found regards the tool's category five: academic language support for ELLs. As is the case with most classrooms (the intended context of the CRMT tool), there is emphasis on English as *the* mathematical language. The fact of including "different representations" does not acknowledge explicitly the use of other languages, which is evident in the case of the *Los Rayos* students, especially during the recipes unit. Multimodality could be used during instruction in any language to build understanding, but the explicit acknowledgement of the use of another language reflects not only other resources to understand mathematics, but also the inclusion of other practices and identities in mathematics.

Regarding categories 2 and 3 (depth of mathematical knowledge and mathematical discourse, respectively), our analysis highlights that the quality of mathematical discourse and the kind of reasoning that students experience during the two units is not only different in quantity, but in quality. For example, in recipes unit, we see a mathematical reasoning built on through social interactions and linked ideas over time. These provide a robustness of mathematical understanding that is not evident in the binders unit. In the binders unit, the short

engagement with the task seems to provide a level of reasoning that is transitory and irrelevant to the collective of the working team. In the recipe unit, the reasoning and mathematical actions of one person matter to the collective because it is connected to the collective. This takes the quality of mathematical discourse to a different level, as the interactions and exchanges seem to be more functionally and socially purposeful than in the binders unit. Thus, the complexity of the mathematical reasoning and discourse is also social, not only mathematical. Here is when the description included in the CRMT analytical tool, “students’ reasoning, explanations, and arguments demonstrate fullness and complexity of understanding,” seems arbitrary in, and vague around, what the “fullness and complexity” of knowledge and understanding mean.

During the session, we will present examples of our analysis using the proposed tool. Our goal is to engage the audience in thinking and conversing about the subtleties of designing and implementing culturally responsive mathematical tasks for unique contexts, and ways to capture the depth of particular practices and curricular designs.

Notes

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Interplay of Artistic Identities and Mathematical Dispositions at an Art Crating Company

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We share preliminary findings from our pilot investigation of mathematical engagement and dispositions of workers at an art crating company. We aim to contribute to conceptualizations of artistic identities and productive mathematical dispositions through case studies of participants who primarily see themselves as artists. While they do not consider themselves to be “math people,” their work is steeped in complex mathematical reasoning. Findings from the study contribute to the design of mathematics learning settings that support productive mathematical dispositions.

Background and Framing

Students often experience mathematics as an activity distinct from their lives. Ladson-Billings (1997) argued that in the U.S. mathematics is commonly seen as a discipline in which some are capable of engaging while others are not. This makes it culturally acceptable for individuals to be “bad at” mathematics. Additionally, there might be misalignments between students’ identities and the characteristics of successful mathematics learners. Boaler and Greeno (2000) made it clear that how mathematics is taught has deep effects on how students view what it means to do mathematics, and what constitutes the process of learning mathematics. They analyzed interviews comparing students’ experiences in AP Calculus classes that employed two very different forms of instruction: didactic and discussion-based. In the didactic classes students saw mathematics as “closed” in the sense that there was one right answer with no room for interpretation. These students listed qualities like “perseverance” and “obedience” as characteristics of successful students, and explained their affinity for mathematics based on how well their views of themselves aligned

with these qualities. Boaler and Greeno worried that many students are alienated from mathematics because they are not willing to author identities compatible with didactic classroom settings.

Boaler and Greeno (2000) drew from Holland and her colleagues' (1998) work on "positional identity," describing it as "the way in which people comprehend and enact their positions in the worlds in which they live" (p. 173). Positional identities are "relative to socially identified others, one's sense of place, and entitlement" (Holland et al., 1998, p. 125). Another important aspect of identity is that which relates to participation in cultural activities, or what Holland and her colleagues call "figured worlds." Figured worlds are socially constructed systems of activity where actors, artifacts, and events are attributed particular meaning and interpreted in predictable ways by participants. Boaler and Greeno's (2000) findings revealed the potential mismatches between students' positional identities as learners and the figured worlds of their mathematics classrooms; at the same time their results raised questions about the interplay between students' experiences in the figured worlds of their mathematics classes and their developing identities as mathematics students.

Through ethnographic case studies of two fifth graders' mathematics learning, Jackson (2009) provided some insight into how mathematics and mathematics learning were constructed through instruction, and how "social identities" of students were simultaneously (and reciprocally) constructed. By social identity Jackson was referring to how students viewed and positioned themselves and how others viewed and positioned them in the local classroom setting. The mathematical social identities constructed by and for the students were influenced by how mathematics was constructed at the school and in the class, and vice versa.

This view of identity, consistent with Holland and her colleagues' (1998), is the one we use in this study. Identity, in this sense, plays a crucial role in learning by influencing how students understand themselves as learners and as capable mathematicians. It is clear, then, that mathematics learning settings must support the development of identities that are better aligned with learning and doing mathematics in both the short and long term (Cobb & Hodge, 2002).

A critical aspect of students' identities are their dispositions, or "ideas about, values of, and ways of participating" (Gresalfi & Cobb, 2006), for doing and learning mathematics. Productive mathematical

dispositions have been associated with seeing mathematics as a sensible domain that one can reason about, and seeing mathematics learning as a process one is capable of (e.g., Kilpatrick, Swafford, & Findell, 2001). This involves opportunities for students to engage and identify as competent mathematicians, and to see themselves as participants in and authors of mathematical activity (Boaler & Greeno, 2000; Gresalfi & Cobb, 2006). These opportunities can be supported through classroom mathematical practices (Gresalfi 2009).

To contribute to the design of settings that support students' construction of productive mathematical dispositions while remaining respectful of students' existing identities, we are working towards developing more nuanced understandings of what productive mathematical dispositions might look like, and how they might intersect or interact with identities affiliated with other disciplines. In particular, we investigate artists and artistic identities of workers at an art crating company, which builds custom crates for art objects. Many of the employees at the company are artists, working at the company in order to make a living in a setting related to their chosen (but not-yet profitable) art professions. In this project, we attend to the artistic identities and mathematical dispositions of the workers, with the goal of developing rich descriptions of the artistic identities of workers at the company, and how these artists view their own mathematics activity, engagement, and learning.

Study Design, Methods of Analysis, and Significance

We share case studies of six workers at the company. Cases include employees from three departments: design, shop, and packing. Designers manage projects, assessing the goals and constraints of the job, measuring art objects, and developing specifications for crates. They produce a representation of the crates called a "cut sheet," which is passed on to the crate shop. Crate shop employees cut and assemble the major parts of the crate exterior. This, along with the cut sheet, is transferred to the packing department, where packers create the protective and supportive interior of the crate. Work activity in all three departments is saturated with different forms of spatial reasoning,

including complex measuring practices, spatial organization of materials, and transformation of spatial representations across different scales and contexts. Workers recognize their activity as deeply mathematical and dominated by problem solving events, and a fair amount of expertise (and often collaborative discussion) is involved in interpreting a cut sheet and executing its instructions.

While our focal participants see themselves as capable of doing and learning the mathematics relevant to their jobs, many of them were not successful in school mathematics and do not identify as mathematically inclined. Art and art creation figure largely in their world of crate design, construction, and packing, as does the embedded mathematical activity in which they engage. The setting is permeated by artistic meanings and significance for these workers; at the same time, mathematical competence remains critical to their success, and their dispositions toward doing and learning mathematics have consequences for their work. As such, the study centers on three main research questions: RQ₁) What are characteristics of the workers' artistic identities? RQ₂) How is doing mathematics and the process of mathematical learning socially constructed in this workplace? RQ₃) What are characteristics of workers' mathematical dispositions in the context of their work? What is the interplay between workers' artistic identities and their mathematical dispositions?

We make comparisons along two dimensions. First, each department has different goals for work, and engages in substantially different spatial practices. Investigating workers from each department allows us to discover nuances in mathematical engagement. Second, we choose one relative newcomer to the job and one relative old-timer for each department. This provides comparative leverage for making conjectures about "expert" spatial practices and how learning is organized, speaking to what it means to do mathematics as well as processes of learning.

We use primarily ethnographic methods, including participant observation and interviews. Data include field notes, video recordings, and artifacts from the field. As the study progresses and mathematical practices are defined and characterized, fieldwork shifts to selective observations of work more directly related to these practices. Analysis of fieldnotes provides insight into how mathematics and learning is socially constructed, from the workers' perspective (RQ₂).

Initial interviews focus on workers' histories of participation in

art, mathematics, and the company. Later interviews will include “job simulation” tasks (Scribner, 1986), allowing us to test conjectures about their mathematical practices and characteristics of their mathematical dispositions as they solve problems (RQ₃). Video-elicited interviews will serve to provide member-checks as well as additional data regarding all research questions. Video also serves as data for microethnographic analysis of work and mathematical practices. The frequent collaborative consulting interactions that take place make mathematical practices and learning (RQ₂) visible for analysis. These interactions also allow for analysis of how workers position themselves and each other in relation to mathematical activity (RQ₁, RQ₂, RQ₃).

Significance

Research has suggested what certain features of productive mathematics dispositions might consist of, like conceptual agency and authority (Gresalfi & Cobb, 2006). However, this study can contribute understandings of how individuals who strongly identify with other disciplines might come to also have productive mathematical dispositions. In other words, the findings could be thought of as contributing possible trajectories of learning identities—specifically, how different prior dispositions can be built upon to support, eventually, positive mathematical identities.

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Virtually There (Again): Internship E-advisors and Professional Learning Communities in Mathematics Teacher Education

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In the research being discussed in this presentation, a faculty advisor (also the researcher) created a digitally-enhanced internship experience, featuring a teacher-intern-faculty advisor (TIFA) learning community. The research goals were two-fold: (1) to understand more about 'best practices' (i.e. meaningful and sustainable practices based in blended learning environments) for teacher education field experiences and becoming a mathematics teacher, and (2) to disrupt traditional notions of teacher education programs as places to 'train' and 'prepare' teachers, with field experience being viewed as the 'supervised' enactment of these preparation techniques.

Introduction

At the 2010 MES meeting, I presented a paper entitled *Virtually there: Introducing the internship e-advisor in mathematics teacher education* (Nolan, 2010). In that presentation, I reported on research into the design and use of desktop video conferencing in the mentoring of secondary mathematics interns (that is, pre-service teachers engaged in their internship, or extended field experience). Overall, the key objective of that project was to develop a working model for creating and sustaining an ongoing, synchronous dialogue between faculty advisors and assigned interns during a four-month internship field experience in schools.

Four years later, that project has evolved: its goals have become more critical, its methodology turned more introspective with the use of self-study data, and its analysis strategies now informed by key concepts of Bourdieu's social field theory. The goal of this presentation is

to provide an overview of how that same research project has evolved and taken on a life of its own in its present form as an internship professional learning community. By creating a multi-dimensional model for the internship (field) experience, the research integrates more reflexive, critical approaches to learning to teach, and teaching to learn, mathematics.

Purpose and Context for Research

Initially (back in 2009), the internship project was developed out of a desire to understand how the use of various technologies could (1) establish a continuum between pre-internship university courses and the internship field experience (studying theory-practice transitions in becoming a mathematics teacher) and, (2) reduce the burden of travel and labour costs associated with supporting faculty advisor travel between the university and schools. That initial internship model focused primarily on highlighting the promises of virtual mentoring in the development of mathematics teachers. The project was built on the premise that using video conferencing technologies would allow (even encourage) sustained contact between the faculty advisor and her interns, thus enhancing the role of the faculty advisor in pre-service teachers' process of becoming (a mathematics teacher). Four years of research and refinement of the model has produced results that could be seen to challenge this premise. At the same time, however, the model and its original goals have evolved into a more collaborative and reflective community of practice.

In the current internship model, the faculty advisor (also the researcher) facilitates a digitally-enhanced internship experience, featuring a professional learning community. The model includes the integration of an enhanced lesson and video study approach to professional development and a digital 'e-advisor' component to intern supervision (Fernandez, 2003; Gorman, Mark, & Nikula, 2010; Nolan, 2011), including the use of multiple technologies such as desktop video conferencing, video flip-cameras, and online collaboration and discussion forums. I refer to this enhanced internship model as the Teacher-Intern-Faculty Advisor (TIFA) Learning Community Professional Development model. The TIFA learning community consists of three (3) interns, their cooperating teachers, and me as

faculty advisor (also researcher and teacher educator). The model requires all participants (interns, cooperating teachers, and faculty advisor) to engage in lesson study experiences, to videotape mathematics lessons, to participate in an online learning community and to meet 4 times (one day each) during the 4-month internship semester for professional development activities (for example, lesson study and video analysis) and, as part of the research component, for interviews and reflective focus groups.

As mentioned, the goals of the research have evolved over the years. Firstly, in the current research project I now seek to understand more about ‘best practices’ (i.e. meaningful and sustainable practices based in blended learning environments) for teacher education field experiences and becoming a mathematics teacher. Secondly, I conduct the research with a goal of disrupting traditional notions of teacher education programs as places to ‘train’ and ‘prepare’ teachers, with field experience being viewed as the ‘supervised’ enactment of these preparation techniques. Data is presently being analysed through the lens of Bourdieu’s social field theory with a goal of critiquing the network of relations and discursive practices that support (and (re)produce) traditional practices in teacher education programs and associated field experiences. This paper and presentation focuses primarily on the first goal, describing the learning community internship model in its present form and how it has evolved over the years from that reported on at MES 2010.

Research Methodologies and Theoretical Framework

The full study informing this paper challenges and disrupts traditional discourses of teacher education programs and associated field experience, tracing the intersections of identity, agency and reflexivity in mathematics teacher education using Bourdieu’s sociological theory (see, for example, Bourdieu, 1977). It does so primarily using a self-study methodological framework. By studying my own professional practice, I am in a position to interrogate the discourses shaping my roles and practices as a teacher educator and faculty advisor. In this brief paper, it is not possible to delve into self study data or to draw

on specific concepts of Bourdieu's social field theory to analyze and interpret data (generally, Bourdieu's concepts require substantial background discussion before using them). Instead, I direct the reader to other publications where I outline Bourdieu's social field theory concepts specific to the larger research study (see, for example, Nolan, 2012).

Research Methods and Data

The research project involved multiple and diverse forms of data, focusing on researcher self-study reflections and the reflective contributions of 'others' involved in the project. Data collection for this TIFA internship project included individual and group interviews with interns and cooperating teachers, conducted both in person and through video conferences. The interviews generally took place immediately following each professional development day where we engaged in lesson and video study as a community. For the purposes of both data collection and to enact my role as a faculty advisor, I used multiple technologies to maintain a relationship with the TIFA community between our face-to-face meetings.

The data gathered through individual and group interviews asked interns and cooperating teachers to discuss how (or if) the TIFA learning community had an influence on them, as being and/or becoming teachers. Due to limited space and scope of this paper, I present only two example quotations drawn from the lengthier group interview transcript to represent the significance and influence of this internship learning community model.

I'm finding it's really—like the whole internship process—this is my first time I've had an intern... But I know this is making me a better teacher. And I'm thinking about things I'm saying. And just talking, about teaching and everything. It's been great. It's been awesome. And watching the videotape... It's just nice to watch a video and see what other people are doing in their class. I think this is phenomenal. (Cooperating Teacher 1, Oct. 2013)

Yeah, I agree. I think as becoming a teacher, it's beneficial watching your own videos because, you know, you post conference, you

talk about it and then you go back and see what you just talked about. But it's also extremely beneficial watching other people's videos because there's the classroom management techniques or there's the questioning techniques or just seeing from a different perspective another classroom setting or atmosphere. I think that's extremely beneficial especially as a new teacher to see what classes you could have... (Intern 1, Dec. 2013)

Closing Thoughts

As the model has evolved over the years, I have faced many challenges in designing and implementing this TIFA professional learning community internship model. Despite these challenges, it is becoming apparent that I am realizing the new goals of the project. The model is disrupting traditional notions of teacher education programs as places to 'train' and 'prepare' teachers by establishing an ongoing collaboration between the university (faculty advisor and interns) and the schools (cooperating teachers and interns). The data indicates that the TIFA triads—interns, cooperating teachers and faculty advisor—have embraced the view that the model represents valuable mentoring and professional development for 'being' and 'becoming' teachers. I understand more about 'best practices' (i.e. meaningful and sustainable practices based in blended learning environments) for teacher education field experience and becoming a mathematics teacher as I see the experience changing both the model and my role within it. As I continue to evolve and adapt this model for future TIFA learning communities, I maintain that a critical goal of teacher education "is not to simplify the experience of those learning to teach, but to complicate their experience to the point where they are forced to think, forced to encounter the Other..." (Marble, 2012, p. 29).

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Critical Professional Identity of Pre-service Teachers: Introducing Theories of Equity in a College Algebra Class

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Pre-service primary school teachers in a college algebra class modelled high school graduation rate data while learning several theoretical perspectives on educational equity. Qualitative analysis of the students' writing reveals a strong interest in allocating responsibility for educational disparities towards teachers and school administrators. This may signal emerging identities of teachers as agents of change. "Positioning" statements that summarize relationships, goals and life trajectories of students, families, and educators varied from compassionate and self-reflective to stereotypical. Sociocultural theories of teacher development suggest that these students are at an early stage of learning about and navigating across significant professional positions in education.

Introduction

Pre-service teachers whose training addresses theories of educational disparities have a foundation for continuing, powerful self-reflection. We report on an interdisciplinary research-in-practice project in which pre-service teachers learn about educational equity theories at an early stage—their second undergraduate semester—in a college algebra class. We discuss ways in which an interdisciplinary algebra module that places analysis of achievement gap data within the context of several educational theories allows pre-service teachers to express emergent professional identities.

Background

Pre-service primary school teachers in a college algebra class

completed a module on equity mathematics based on high school graduation rates by ethnicity and by income level in the state of Minnesota, USA. The module included a lecture on three theories that address educational disparities: anthropologist Oscar Lewis' theory of the culture of poverty (1966), the funds of knowledge approach (Moll, Amanti, Neff & Gonzalez, 1992), and Delpit's position on institutional racism (2012). Students modelled trends in graduation using algebra or spreadsheets and they learned a professional equity calculation known as the risk ratio, in which the risk of not graduating for the member of a particular group is compared to that of all other groups. Students were asked to react to their mathematical work from the standpoint of the educational theories. In the final section of the assignment, students were asked to pose a question or scenario regarding equity in education and to answer it using both mathematics and educational theories. Suggested examples included:

- Is the graduation gap worse when you look at race data or socioeconomic status data?
- Describe an educational program that might improve graduation rates. Create a reasonable estimate of how much improvement you might observe.
- Would you recommend a different measure of student success? Discuss in detail how you would collect information, how you would analyze it, and why your method would be better.

Conceptual Framework

This study draws on a sociocultural perspective on teacher identity research, focusing on the interactions among the individual, culture, and society (Grootenboer, Smith, & Lowrie, 2006). It attempts to understand the shift from an individual, cognitive framework to a transformation which happens through participation in a community of practice involving other educators (Gainsburg, 2012). A sociocultural perspective promotes “identity formation as being “steered” by society with the individual attempting to “navigate predetermined passages” (Grootenboer, Smith, & Lowrie, 2006, p.613). In this initial report, we present students' emerging identities as they interpret graduation rates from the perspective of educational theorists. Students' reactions to

the various theorists offered a window to understand their emerging identities as prospective teachers at an early stage in their training.

Emerging Themes

We used a hybrid coding method on 22 completed essays that included students' mathematical and interpretive work. We used grounded theory to identify statements regarding responsibilities for educational equity (Charmaz, 2006). We coded students' interdisciplinary analysis using an established rubric for interdisciplinary writing (Boix-Mansilla, Duraisingh, Wolfe, & Haynes, 2009). To receive a high rating in interdisciplinary analysis, a student's work had to demonstrate active use of mathematics, explicit and thoughtful reference to an educational theory, and it had to state a result or position that depended deeply on both of these.

Responsibility for Equity Outcomes

We identified students' written statements that allocated responsibility for educational disparities. Under our current coding protocol in this work-in-progress, out of 22 essays, two essays indicated that students bear some responsibility for educational disparities (code RESP-S) and fourteen essays identified teachers and school administrators (code RESP-ED). Nine of the essays indicated that social structures account for educational disparities (code RESP-SOC).

Teaching the same to everyone is clearly not working.
[RESP-ED].

I do not think it is completely the school or teacher's fault. I think that it is partially the student lack of support from home or self-motivation [RESP-S].

Given the prominence of personal responsibility as a cultural value in middle-class America, the willingness to consider social structure as an explanation for social outcomes was somewhat surprising.

Positioning Self, Educators, Students, and Families

We used “positioning” as a theme for portrayals of self, educator, students, and families in terms of belief, behaviour, and action. Ten of the essays offered positionings of the student’s self as a future educator or of educators in general (code POS-ED). Fifteen of the essays offered positionings of students and their families (code POS-FAM; a few could arguably be re-classified as responsibility statements).

As a future teacher, like all future teachers, I hope to change the trends. [POS-ED].

I think stereotypes play a large part in it—people don’t expect non-Whites to graduate as much as they expect Whites to graduate. [POS-ED or RESP-SOC].

A Mexican student may have a higher risk factor living with 1st generation parents. [POS-FAM].

A major issue for low-income and students of color is that they do not have a positive role model in their life. [POS-FAM].

Students sometimes aligned themselves with Delpit’s social criticism or the Funds of Knowledge celebration of student knowledge, or acknowledged their own privileged status, yet elsewhere expressed a troubling stereotype of students and their families.

Interdisciplinary Synthesis of Mathematics and Educational Theory

We rated only two of the essays as displaying a high level of interdisciplinary synthesis. Both of these students described educational intervention programs, one that the student had observed through volunteer teaching and one that the student had read about elsewhere. Both students estimated potential improvements due to

the interventions using either graduation rates or risk ratio. Two additional essays were rated as “emerging,” and 18 were rated as minimal. Many students relied more strongly on either mathematics or on educational theory in their interdisciplinary commentary, so that insufficient synthesis resulted in a lower analytical rating. The interdisciplinary component of the assignment was accessible but challenging; this pedagogy merits more attention from pedagogical researchers.

Discussion

Pre-service teachers modelled educational disparities, and then allocated responsibility to teachers and administrators more often than to students and families. This seems to represent a desire to be an agent for change through the practice of education. In contrast, when pre-service teachers wrote about students and families, they tended to use both positioning statements and responsibility statements. Positioning statements were a way to practice an opinion about families and students, to generalise about their relationships and trajectories and in general, or to construct an ideological framework for understanding them.

In many cases, positioning and responsibility statements were compassionate. In other cases, they expressed stereotypes that one hopes would become more complex as the pre-service teacher moves through their academic program and initial professional practice. Giving voice to an emerging professional identity, even one that admits stereotypes, is an opportunity to become aware of the complexity of the social field of schooling.

Conclusion

This interdisciplinary algebra assignment makes pre-service teachers aware of the severity of educational disparities in their state and encourages them to begin establishing a theory-based perspective on them. These early-stage identities were complex, professionally-engaged and contradictory. Interdisciplinary algebra can reduce slightly what Britzman calls the “fragmentations” inherent in teacher

education curriculum: 'the compartmentalization of knowledge; the separation of content from pedagogy; the separation of knowledge from interests; and the separation of theory and practice' (1991, p.33). By studying equity issues in an algebra class, students have support for using mathematics critically in a professionally relevant way.

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Toward a Decolonizing Pedagogical Perspective for Mathematics Teacher Education

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During the past few decades, there has been considerable attention to the newly emergent field of mathematics teacher education and research regarding mathematics teacher education. Despite this, there is still great need for critical lenses on these developments, and particularly, to question the extent to which conventional perspectives may exclude opportunities for prospective teachers to unpack and examine the ways in which their “ideological postures” toward teaching inform, often unconsciously, their emergent practice. Our purpose for this proposal is to present and solicit feedback regarding our Decolonizing Pedagogy Framework and its applicability for reconceptualising mathematics teacher education and counter imperialist aims of education practice.

Introduction

During the past few decades, there has been considerable attention to the newly emergent field of mathematics teacher education and research regarding mathematics teacher education (Adler, Ball, Krainer, Lin, & Novotna, 2005; Hill, Rowan, & Ball, 2005; Silverman & Thompson, 2008). Within this discursive space, much of the attention has centered on issues of teachers’ beliefs, their opportunities to develop content knowledge for teaching, and even more foundationally, what knowledge(s) constitute the breadth and depth of what teachers need to know and do toward enacting quality classroom instruction. Despite the increased attentiveness generally, there is still considerable need for critical lenses on these developments, and particularly, to question the extent to which conventional perspectives may exclude other ways in which, for instance, cultural knowledge is valued within mathematics teaching and learning contexts (Nasir,

Hand, & Taylor, 2008). Moreover, there is a central need to counter the prevailing tendency to exclude opportunities for prospective teachers to unpack and examine the ways in which their “ideological postures” toward teaching inform “often unconsciously, their perceptions and actions when working with linguistic-minority and other politically, socially, and economically subordinated students” (Bartolomé, 2004, p. 97).

Although we acknowledge the limitations of our perspective as situated within the U.S. context and amid a field currently “dominated by small-scale studies in English-speaking countries” (Adler, et al., 2005, p. 360), we hope that our nascent reframing of mathematics teacher education here will reinvigorate the central question of how knowledge(s) for teaching mathematics will be regarded, whose knowledge(s) will be included, and how we can do this work in ways that subvert the imperialistic tendencies of education. Furthermore, although there has been considerable attention to mathematics teacher education programs—particularly in the context of reform-oriented movements—very little attention has been focused on the ways in which that work engages or questions ways in which the “massification” of mathematics as a school subject may influence the ways in which teacher education is conceptualized and practiced (Adler, et al., p. 360; also see Gutierrez, 2012, for a notable exception).

Purpose

Our purpose for this proposal is to present our emergent framework and solicit feedback from the MES community toward refining the ideas for use in mathematics teacher education research and practice (See Figure 1). Informed largely by a social-foundations lens—and one that does not often intersect with conventional mathematics education scholarship—we hope that this presentation affords an opportunity for an open, mutually reciprocal exchange of ideas.

About the Framework

Through an analysis of U.S. schools as locations of both oppression and opportunity, the framework presented here inverts normalized

beliefs around access and equity. We see this as a move situated squarely within a sociopolitical-turning moment in mathematics education. While traditional models are founded on the premise that schools provide equitable opportunities and that all oppression is interpersonal, this framework aims to trouble that notion—one that has become the basis for social-turn reforms in mathematics education research and practice. Steeped in a social-foundations analysis of school systems, this framework articulates the structural inequities that undergird U.S. schools and names opportunities as interpersonal spaces of resistance, interruption, and humanity. Based on evidence that schools are fundamentally inequitable, this umbrella theorizing about teacher learning for equity in education builds capacity for hope in practitioners reclaiming their classroom and school spaces. Our aim, again, is to gain some insights from the MES community toward more carefully integrating this framing with the particular concerns and developments of mathematics teacher education.

The framework consists of a fluid developmental sequence of five interconnected and nonlinear spaces of learning—from framing a decolonizing analysis of public education, through developing a professional stance of critical teaching and engaged praxis, to attending to mathematics-specific identity construction, and ultimately landing in a metacognitive naming of one's own framework for teaching mathematics. In examining the scope of the phases, it is clear that at the heart of the design is the intention to reestablish critical lenses through the reinfusion of social foundations into teacher preparation—and math teaching, particularly. The decolonizing analysis pervades the entirety of the framework and establishes a lens for teachers, students, and facilitators to analyze experiences in contemporary schooling from sociohistorical, -political, and critical stances. Once the foundation has been laid, the task turns to engaging prospective teachers in the development of a critical professional stance to enact these principles in careful ways. With slight concessions to the national (U.S.) climate, the third space is one in which teachers can be prepared to embrace a deeply intellectual process in their students and in the design of their content instruction. In the next phase, prospective teacher address the need to resist ascribing to students motivations that are historically rendered and pathologized, but instead look deeply into sources of meaning and matrices of understanding that manifest themselves uniquely in learning styles and

complex constructions of identity. Ultimately, candidates must have such an in-depth working knowledge of the framework beyond their own experience of it as students that they can use it as the skeletal system for their ongoing development as practicing teachers.

The framework was co-generatively developed through a grounded theory methodology and realized through engaged and collective practice in community schools. It is intended to be returned to communities through teacher practice, as it works to sustain radical teachers through long-term engagement in public education with the development of individualized stances of resistance and resilience. Future research directions include PAR studies with framework graduates in urban classrooms navigating implications of the framework in teaching and learning in light of classroom and school settings.

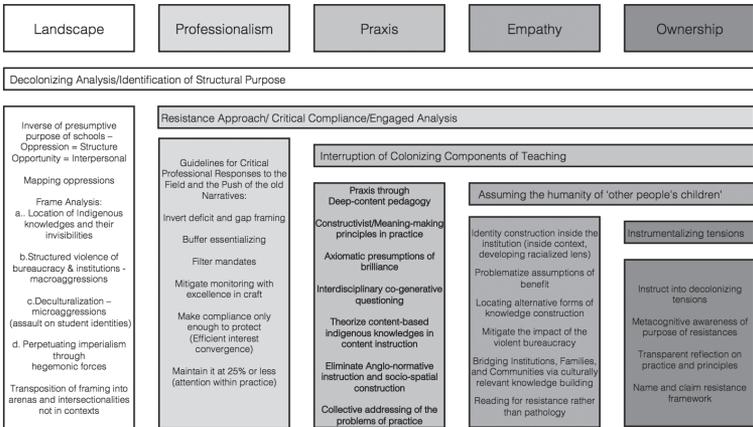


Figure 1. Decolonizing Pedagogy Framework

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Finding the Way: Cultural Revival through Mathematics Education

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The colonisation of many Indigenous communities, particularly in the nineteenth century, has led to a loss of cultural knowledge as the colonisers' knowledge became the valued knowledge in educational institutions. In this project we are exploring some of the challenges attached to revitalising cultural knowledge through mathematics lessons. Although school curricula for Indigenous students often highlights the need for traditional cultural knowledge to be given equivalent status, very few projects have done so successfully. We use examples of spatial orientation to illustrate some of the issues that surround our efforts to revitalise cultural knowledge through mathematics education.

The Rights of Indigenous People

Although colonisation of Indigenous communities continues to be prevalent throughout the world, in 2007, the United Nations adopted the Declaration on the Rights of Indigenous Peoples. Article 31 indicates that Indigenous people should be able to protect their culture. New Zealand was one of only four countries which opposed the adoption of this declaration (May, 2011). New Zealand has had a long history of suppressing the linguistic and cultural rights of its indigenous people.

In Aotearoa/New Zealand the Indigenous Māori language and culture were excluded from schooling by a range of national policies until the 1970s whereby the Māori language was recognised as an endangered language (Fishman, 1991).

In response to the parlous state of the language, in the 1980s, various initiatives were launched that focused on revitalising Maori knowledge and language, including Maori-medium schools

(McMurchy-Pilkington, Trinick & Meaney, 2013). A key component of the development of Māori-medium schooling has been the development of Māori-medium mathematics. Although mathematics education has successfully contributed to the revival of *te reo Māori*, the Indigenous language (Meaney, Trinick & Fairhall, 2012), the same cannot be said for resurrecting Māori knowledge. Although there is interest in how this can be done, this is still very much a work in progress. In this project description, we begin the process by identifying some macro and micro issues which can affect the reintroduction of traditional cultural knowledge into the classroom.

Ethnomathematics

In the past 20 years or so, the issue of using cultural knowledge within mathematics classrooms has been frequently considered under the auspices of ethnomathematics. D'Ambrosio (1992) described a research program in ethnomathematics as “the study of the generation, organisation, transmission, dissemination and the use of jargons, codes, styles of reasoning, practices, results and methods” (p. 1183). To date little research has investigated how these ideas can be incorporated into mathematics classrooms (Meaney, 2002). The exception would be the work done with the Yup'ik in Alaska by Lipka, Yanez, Andrew-Ihrke, and Adam (2009). Lipka et al., (2009) summarized many of the assumptions on which their ethnomathematical pedagogy was based:

The assumptions include that students will gain increased access to the math curriculum because they can identify with the curriculum and pedagogy on multiple levels, from familiar contexts to familiar knowledge, and that they will have multiple ways of engaging with the material. ... Further it is assumed that the inclusion of local knowledge, language and culture may well have a positive effect on students' identity that will be different from the typically reported process of schooling that marginalizes so many AI/AN (American Indian/Alaskan Native) students. (p. 266)

Researchers in other parts of the world acknowledge similar assumptions in regard to the benefits of ethnomathematics (Meaney 2002).

Nevertheless, ethnomathematics has been criticised. For example, the valuing of a practice only if it can be labelled as Western mathematics has been questioned (Jablonka & Gellert, 2010). Barton (2004) stated that although ethnomathematics provides opportunities to reconsider how aspects of Western mathematics are perceived, labelling cultural activities as “mathematics” was problematic. Similarly, Pais (2011) suggested that although learners may engage in a range of activities, these activities must be recognized as mathematics to become mathematics.

Macro Issues with Revival of Cultural Knowledge

For Māori-medium schools, there are a number of factors which hinder the inclusion of traditional cultural knowledge into mathematics lessons. One of these is that government-funded schools are required to implement state mandated curricula based on Western mathematics (Meaney et al., 2012).

Another challenge is that the original practice may no longer be in general use in the community. For example, in discussions with teachers in the Māori-immersion school in which Uenuku, one of the authors, a principal, it was found that many did not understand or know about traditional spatial frameworks. They did not know why the cardinal directions in *te reo Māori* were orientated to East/West rather than to North/South (Meaney et al., 2012). They were constrained by their reliance on Western mathematical concepts, which underpin resources, the curricula etc.

Without local experts who can provide necessary input about the traditional knowledge, teachers have had to turn to historical documents to ascertain what the knowledge might have been. Although documentation is more readily available over the internet (see for example, <http://teaurere.org.nz/Navigation.htm>), much work is left to individual teachers to build the knowledge base from which mathematics lessons can be planned. This takes time and effort that may be beyond an individual teacher’s possibility.

In the next section we discuss some of the issues facing Uenuku’s school in attempting to incorporate traditional knowledge about spatial orientation.

Spatial Orientation

Different components of “spatial ability” have been identified, each emphasising different aspects. Bishop (1980) suggested that the major abilities of spatial thinking that are commonly addressed are spatial *orientation* and spatial visualisation. Spatial orientation is the ability to understand and operate on relationships between objects in space. Spatial *visualisation* enables a person to carry out mental manipulations of two- and three-dimensional objects (Clements & Sarama, 2009).

At te Kura Kaupapa Māori o te Koutu, we have been investigating students’ understanding of spatial orientation and possibilities for introducing activities based on cultural knowledge. This is the beginning of an ongoing project and the results should be considered initial results only.

In 2013, we interviewed children from grades 2-8 (6 year olds to 12 years old). The results indicated that very few children could relate the cardinal points to specific directions (by pointing to where they thought each point was). Only the older children had a sense where places were in relationship to where they stood and none had any knowledge of the wind names as determiners of direction- the traditional way of discussing direction. Consequently, a major challenge is to have students become aware of traditional Māori spatial frameworks and ways to orientate.

In 2014, students in Year 8 were asked to create a map of their local area from memory, adding significant cultural sites, place names, places of significance and a scale. They then compared their maps with that of Tuki and Ngāhuruhuru, two young men who were kidnapped from Northland, New Zealand, in 1793 and taken to Norfolk Island. While there, Tuki drew a map of New Zealand for Governor King (Binney, 2004). Understandably, the area that Tuki knew best and thus represented in the map was disproportionate in size and area to the rest of New Zealand, and the most detailed. Their teacher, in reflecting on the lesson, stated:

Most of the students’ maps of Rotorua reflected Tuki’s perceptions of scale etc. For example, the area where the students were familiar with and lived was out of scale with the rest of their map. Lake Rotorua varied between 5km and 35km. They surprised

me in the depth of cultural detail they were able to add to their maps. They then appreciated the idiosyncratic nature of Tuki's map.

Although only a beginning, using historical documents that indicate the sorts of traditional knowledge known to Māori has potential to support students learning about the relationship between traditional Māori knowledge and traditional Western knowledge. In this situation, one set of knowledge is not preferred over another, rather the different kinds of problems connected to each, and the overlap between them, becomes an area of discussion and reflection for students which should support their learning.

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