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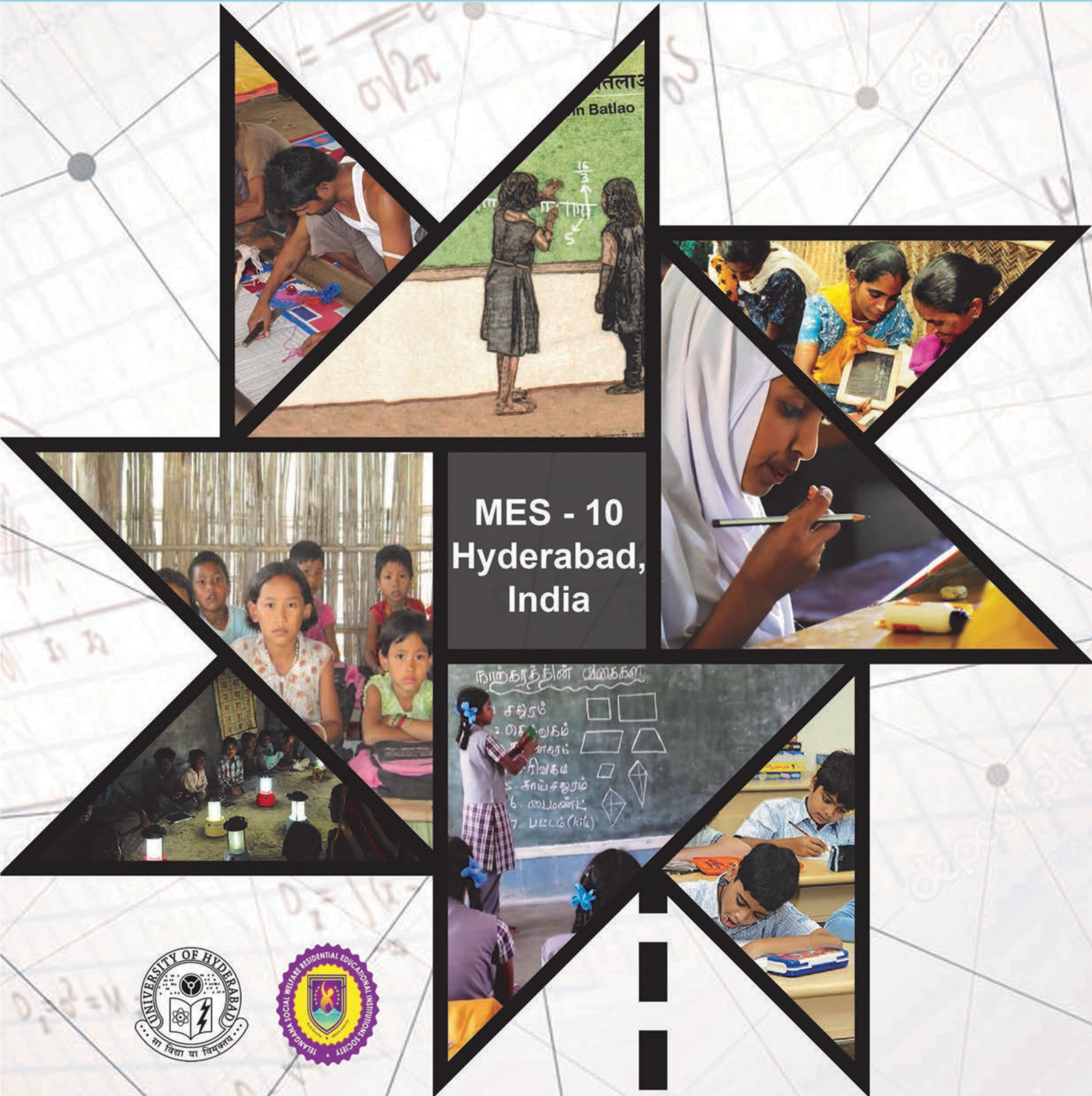

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PROCEEDINGS OF THE TENTH INTERNATIONAL MATHEMATICS EDUCATION AND SOCIETY CONFERENCE

Edited by

Jayasree Subramanian

Hyderabad, India

January 28th to February 2nd, 2019

MES10

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MES10 Conference logo was designed to show the diversity of sites and contexts in which mathematics figures and mathematics education takes place in India: artists and artisans in whose work mathematics is embedded, women learning mathematics in adult education programmes, privileged children in urban setting with access to technology at their desks, socioeconomically marginalised children attending night school, sitting on the floor with lanterns to provide light, students in rural classrooms with bamboo walls. The logo bringing out the linguistic, religious, sociocultural, economic and regional differences of learners was designed by Mohd Junaid Siddique and Murchana Roychoudury.

MES 10 International Committee

Steering Committee

Local Organising Committee

Acknowledgement

The conference organisers would like to acknowledge the support of University of Hyderabad and Telangana Social Welfare Residential Educational Institutions Society, Telangana.

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INTRODUCTION TO MES10

Jayasree Subramanian

The tenth International Mathematics Education and Society Conference is part of a series of conferences organized by the MES community. The MES conferences seek to bring together academics and activists across the world, engaging with the socioeconomic, cultural, linguistics, racial, political and ethical dimensions of mathematics education and provide a platform for them share their thoughts and experiences and learn from each other and also to build and strong research network on mathematics education and society. The first MES conference (MEAS as it was called originally) was organised in 1998. MES conferences arose out of a dissatisfaction with the then existing conferences as they failed to address these issues in their entirety. The first MES conference was organised by Peter Gates and Antony Cotton from 6th to 11th September 1998, in Nottingham and it also departed from the standard conference format by allowing more opportunity for participants to discuss the plenary lectures and the papers presented in the conference. The MES conferences the followed are:

MES2 held in Montechoro, Portugal from March 26th to 31st, 2000

MES3 held in Helsingor, Denmark, from April 2nd to 7th, 2002

MES4 held in Gold Coast, Australia, from July 2nd to 6th, 2004,

MES5 held in Albufeira, Portugal from February 16th to 21st, 2008

MES6 held in Berlin, Germany from March 20th to 25th, 2010

MES7 held in Cape Town, South Africa from April 2nd to 7th, 2013

MES8 held in Portland, Oregon, USA from June 21st to 26th, 2015 and

MES9 held in Volos, Greece from April 7th to 12th, 2017.

More details about the conferences and also the proceedings can be accessed at https://www.mescommunity.info/ . Several participants of MES have continued as

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members of the MES community since its very beginning and have contributed to it through a continued interaction between conferences. The sense of belonging to the community is further strengthened by the open review policy of MES, which allows the reviewer and the authors to interact and have a productive engagement. MES also acknowledges the linguistic diversity of the participants and provides language support in a way that strives to retain the authentic voice of the author. Over the years MES conferences have provided scope for several participants to discuss their projects and get valuable suggestions, collaborate across countries to enquire in to themes and how they configure in different contexts and evolve alternative approaches in mathematics education research.

HOSTING MES10 IN HYDERABAD, INDIA

The idea of hosting MES in India stems from the deep conviction that if any country needs an MES conference, India needs it more, given the multitude of differences and hierarchies that shape education in India. It is the first time that an international conference in mathematics education is being hosted in India. This is also the first time that a MES conference is being hosted in an Asian country. We hoped that holding the conference in India would make it possible for a larger participation and eventually a representation of Asian countries in MES. We have had a limited success in realising this as we have participants from at least five Asian countries. A feature common to most of the Asian countries and that which poses a major challenge to education and mathematics education in particular, is the multitude of hierarchies and diversities including linguistic, religious, cultural, regional and economic diversity that operate in these countries.

India is a vast country with wide diversity in language, caste, tribe, class, ethnicity, region and religion; it is largely patriarchal with pockets of matrilineal practices that are fast disappearing. There are several hundreds of castes which are hierarchically organized in a complicated way and continue to play a significant role in the everyday life of a person determining one’s opportunities for education and employment. The adivasis (the indigenous people of India/ the tribals) who are outside the Hindu caste system face continued threat in the post independent India as they are forced out of the forests and rendered poor and at the mercy of world far removed from theirs.

There are about 780 living languages in India. Post Independent India reorganized the states on the basis of languages spoken by people and so 22 languages are recognized as official languages in the 29 states of India. Also, with increasing migration for labour, most of the cities in India are becoming multilingual in a layered fashion. At one level the educated middle class move from several part of the country into metropolitan cities and sidestep the language issue by accessing education in English medium for their children. At another level the migrant labourers from low income background with limited or no education, accessing education in government funded schools face severe challenges because of language.

Regional differences again are very wide with metropolitan cities at one end and the remotest villages and tribal pockets with no proper road, no electricity, badly run schools with teachers who themselves never had the opportunity to get good education at the other end. With less than 20% of youth having access to higher education and less than 10% of the jobs in the formal sector, mathematics education in India could be the ideal testing ground for alternative curricula in mathematics. But research in education receives practically no attention from the state, thus ensuring that the benefit of education goes to those who are already privileged. The efforts of the focus group in primary mathematics curriculum development in the year 2005 is perhaps a solitary exception to that.

India's record in excellence in modern mathematics is far superior to its record in equity in mathematics education. A cursory look at the premier research institutions, central and state universities and other institutions of higher education would be sufficient to establish this fact: premier research institutions have their share of world class mathematicians and they are over represented by men from dominant caste category. At the school level, we have anecdotal evidence to believe that, a large percentage of those who fail in the school final examination, fail in mathematics and a

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significant percentage of them belong to religious minorities, marginalised castes, tribes and class from rural India as well as urban slums. We do not have any statistics on the caste wise performance of students in the school final examination. We do not know if, among the privileged and the not so privileged sections, gender plays a role in determining students' performance or opportunities in mathematics. We do not know to what extent and in what ways language issues mediate success in mathematics education. We do not know if students from urban slum fare as well as the students from rural middle-class homes. We know very little about the difficulties faced by students in learning algebra and geometry and about the approaches to teaching that works better.

If India, which is still struggling to get all its children to school and with huge diversities making its terrain of education far too uneven, really cares to realize its slogan of 'mathematics for all' it would be obvious that it must invest in mathematics education research. For we cannot cure an illness that we do not understand. Lack of interest on the part of the state to invest in mathematics education research should therefore be seen as a political act determined to keep the marginalized where they are. In such a scenario, hosting the conference in India not only gives us a scope to engage with issues in mathematics education that are central to us but also opens up the possibility for taking up collaborative research and also becoming part of the MES community.

A BRIEF DESCRIPTION OF MES10 PROGRAMME

MES10 will have an inaugural address by a renowned academic and the editor of the prestigious *Economic and Political Weekly*, Professor Gopal Guru. This will be followed by a talk focusing on the issues and challenges in mathematics education by Professor Farida Abdulla Khan who has years of engagement with the domain. The four plenary speakers are Prof. George Joseph Gheverghese from UK, Prof. Rochelle Gituerrez from USA, Prof, Tony Trinick from New Zealand and Tania Cabral and Roberto Baldino from Brazil. The plenary lectures will be followed by responses from

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two participants of which one is from India. Following this we will break into groups and discuss the plenary paper and the responses for one hour and get back to the plenary speakers with question. The plenary events will take up the first half of the day.

In the seconf half of the day, we will have presentations by participants. MES10 has 7 Symposia, 11 projects, 65 Research Papers, and 10 Posters. These will be presented in the parallel sessions in the afternoons.

ACKNOWLEDGEMENT

The Proceeding of MES10 could not have been brought out without the support of international committee members who were very generous with their time in advising and guiding us in carrying out the activities and also the members of the MES community who agreed to review several papers and provided detailed and constructive comments. In addition, some of the reviewers also volunteered to provide language support to those who requested for support. The continued interaction over a period of nearly two years, the unfailing support at every level and the team work has made the experience of organizing the conference very memorable. I would like to thank the members of the IC, the reviewers and those who provided language support.

In organizing the conference, I faced a very difficult situation, the details of which I will not go into here. It suffices to say that without the support of Beth Herbal-Eisenmann in her capacity as the Convenor of MES, Peter Gates, David Wagner, Brian Greer and Swapna Mukhopadyay it would have been impossible for me to procced with organizing the conference. I would also like to thank David Kollosche and Anna Chronaki for his support in setting up the web page.

I would like to thank the members of the Steering Committee for their support and advice. Their help in arriving at a list of plenary speakers and the two who will give the inaugural address was very crucial.

I would like to thank my co-chair Prof. Prajit Basu who readily agreed to host the conference in University of Hyderabad though the Centre for Knowledge Culture and

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Innovation Studies got the Vice-Chancellor's permission. I would also thank him for booking the auditorium and guesthouses well in advance. I would like to thank my advisor Prof. Rajat Tandon and the members of the Local Organizing Committee who agreed to meet many times, helped in every possible way to make the conference a success.

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The list of reviewers for MES10:

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I would like to my alma mater the University of Hyderabad for hosting the conference and providing all the support. I would like to thank the Telangana Social

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MES10 Participants

Australia, Austria, Belgium, Bhutan, Brazil, Canada, Chile, Columbia, Finland, Germany, Greece, India, Nepal, New Zealand, Norway, South Africa, Sweden, Turkey, United Kingdom, the United States of America

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INAUGURAL ADDRESS

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RETHINKING MATHEMATICS EDUCATION IN THE CHANGING CONTEXT OF THE EDUCATION SECTOR IN INDIA

Farida Abdulla Khan

The Indian state that came into being as a sovereign democratic republic in 1947, set itself on a course of modernization and development with equality and social justice as guiding principles. Education was an important part of the developmental project and was meant to be one of the major tools of social transformation in a society polarized by hierarchies of class, caste, region etc. Although the seriousness with which the project of social justice was implemented within the educational sector in the early decades after independence can be questioned, it was nevertheless grounded in a socialist programme and the primary responsibility of provision was seen to rest with the state. A major shift in economic policy towards liberalization in the 1980s and 90s however has changed the educational landscape. The steady withdrawal of the state from major social sectors has serious implications for the education of the poor and the marginalized. This presentation draws attention to these shifts in educational policy and practice in India, and its consequences within the field of mathematics education given its status within the educational system and the tremendous power it has acquired to determine children's life chances in several ways.

Education through mass schooling, as an essentially modernist and democratic project was meant to equalise access and learning, and although this project has not been entirely unsuccessful, the everyday realities of the systems and structures of schooling have been far from ideal. The generally unproblematic acceptance of education as simply and always a means of positive social change masks the ambiguities and the processes that in fact often contribute to the inequalities and unjust outcomes that it is meant to overcome. There exists also a fairly long critical tradition of theory and analysis that has investigated the ways in which education mirrors and reproduces social and economic hierarchy. This tradition allows more complex readings of education and inequality (Ball, 2004).[i] It is within these critiques and the social concerns of class, gender, ethnicity and a host of other parameters that voices within mathematics education research are beginning to critically examine and investigate the social, cultural and political contexts of mathematics education.

THE INDIAN SCENARIO AND EDUCATION

Although the Indian subcontinent with its variety of regions, languages, religions and ethnicities has a long history, the modern democratic nation state was established only a little over 70 years ago after a long struggle for independence from more than a century of colonial rule. In 1947, Jawaharlal Nehru, the first prime minister of the newly independent nation declared: "The future beckons to

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us. Whither go and what shall be our endeavour? To bring freedom and opportunity to the common man, to the peasants and workers of India; to fight and end poverty and ignorance and disease; to build up a prosperous, democratic and progressive nation and to create social, economic and political institutions which will ensure justice and fullness of life to every man and woman" (quoted by Jeffery, 2005). Social justice and equality are also central tenets of the Indian constitution along with guarantees against all discriminatory practices in the name of caste, class, gender, religion etc. Yet 70 years after the adoption of the constitution, India remains a deeply divided, hierarchical society where, "Even as material prosperity and technical achievement reach dizzy heights, under the global capitalist politico-economic order, gender and class-caste inequalities have grown more polarized and patterns of domination and subjugation more entrenched. For a significant proportion of the population restructuring economics and cultural upheavals have spelt dispossession and displacement, alienation and poverty, indignity and insecurity in an overall context of social conflict and political and ethnic violence" (Velaskar, 2010, p.59).

Education, as an instrument of social change, figured prominently in the rhetoric of development and modernisation and up until the 1980s both educational policy and the general thrust in education was focussed on equality, social justice and concern for a just order. Although the rhetoric never quite translated into adequate action, there was concern for a commitment to remove "social, gender and rural-urban disparities". At the time of independence in 1947, barely 14% of the population was literate and only one out of every three children in the age group of 6 to 14 was in school. Providing schools and access therefore became the dominating activity and the focus while glaring inequalities and an increasingly differentiated system of education in India failed to come under scrutiny or to be analysed seriously or systematically. (Nambissan, 2015). The primary push through these early decades was for access – to basic literacy and to schooling although largely through a perspective of deficit. The effort of the state was on enrolment, transition and school completion rates. It has been observed that the political class which held the power to shape educational policy implementation was also strategically placed to distribute patronage and exploit the system for personal gain. (Kaviraj, 1997, quoted by Velaskar)

The Nehruvian ideals of a socialist politics and a planned economy with a commitment to the idea of social justice was the early model that formed the basis of policy till well into the 1980s. During this period the state was the major provider of elementary education and a vast network of elementary schools was put into place across the country. The entry of global capital in the 1980s and the subsequent fiscal crisis of early 1990s changed the course of the Indian economy and policies associated with it. This resulted in the liberalisation of the national economy, followed by structural adjustment in a number of spheres with serious implications for the social sector and public provision of health and education. A

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henceforth openly capitalist agenda with an increasingly neo-liberal stance began to replace the earlier model of state socialism. The entry of international donor agencies in the education sector allowed space for international educational concerns to be reflected both in policy and allotments. The combination of an open embrace of a neo-liberal agenda and a free market, along with a steady shift towards a conservative majoritarian politics and the rise of the BJP through the 1990s and into the 2000s has led to a further shrinking of socialist concerns and progressive spaces.

Although global concerns for 'education for all' especially in the developing countries are a priority within this shift, the concepts of quality and inclusion acquire a different meaning within a market-oriented model of education. In the context of global interest in education and investment in developing countries, debates about needs for reform and change have become more internationalised, with an increased participation of non-state actors especially in the elementary education sector.

The pressure from donor agencies and the structural adjustment of the 1990s and a renewed urgency to get children into schools, pawned a whole series of short term and longer time-bound "missions" and projects within education that sought more efficient remedies for better performance and greater enrolment. Although these measures were successful in increasing access - the results in numbers are quite impressive - the more profound changes that have been established have not benefited those who most need educational support. An already polarized system of elite private schools for the rich and government school system for the poor started to become even more stratified and the public system that served the more marginalised itself became more hierarchical with the establishment of alternative schools supported by communities, with poor infrastructure and less qualified teachers, accompanied by laying off of regular teachers and large-scale appointment of para teachers in several states.

And all this in the context of an overwhelmingly large population of the poor and marginalized. According to figures of the World Bank, as late as 2010, a reported 32.7% people in India were living below the international poverty line of $ 1.25 per day and 68.7% on less than $ 2 per day with someof the worst indicators for child health and nutrition. Also "over represented amongst the poor are communities that have historically been disadvantaged due to their caste, location as 'ex-untouchable' (Scheduled Caste) marginalised ethnicities (Scheduled Tribes) and minority status (Muslims)" amongst others. (Nambissan, 2015, p.3)

Since the early 2000s there is a much greater tendency to allowing private players into this sector with a growing advocacy for public-private partnerships as a viable model. As late as 1994, 90% of elementary school students were enrolled in either government schools (67%) or in government aided schools (221%) and only 9.8% of children were enrolled in private unaided schools which indicates

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that the government was the primary provider of education for the majority of children in the country. By 2013 however the proportion of private unaided schools had increased to 17.4% (Quoted by Nambissan, 2015). The implications of state withdrawal from this sector for the poor and the marginalised within a socially and economically deeply hierarchical society are bound to be disastrous. In fact research and reviews of these changes and their impact on the ground furnishes little cause for optimism.

Another significant and disturbing phenomenon since the early 2000s has been the proliferation of what were peddled as "cost efficient" budget schools for the poor in India and other developing countries. Powerful advocacy for educational markets for the poor has been traced to the UK and USA to think tanks, foundations and organisations that propagate a free market philosophy and are linked to pro-choice organisations in India. Ball and Nambissan (2011) have traced the links between these networks, the businesses and business philanthropists and intellectual entrepreneurs that have set up advocacy groups and were early actors in the campaign for school vouchers and parental choice for the poor. This network now includes corporate players, investment companies and venture capitalists looking to invest money in new businesses in India and elsewhere. This is the for-profit sector looking for new markets and investing in budget schools. The details of this are murky and research is available to look into the detail[ii]. A corollary of the emphasis on basic skills and minimum learning is also manifested in the search for marks and in the shadow system of private tuitions at considerable cost for those who can least afford it. (Majumdar, 2015). "Education that is projected as good for poor/low-income groups" according to Nambissan "is minimalistic in terms of the role of the teacher, and curriculum transacted, with the 'quality' of these processes demonstrated in terms of narrow learner outcomes. Of particular concern is the is the aggressive advocacy for creating the deskilled teacher of the poor through training models such as para skilling where young people are drilled and directed to perform the role of teachers who will transact a narrow set of standardised, homogenised and mechanical skills under tight management controls" (2015, p. 24)

Reviewing the discourse and reform around the issue of quality in an article in 2010, Padma Velaskar notes that "Whatever the equity rhetoric, the underlying unifying principle of reforms is the market" and warns that "the new global emphasis on the quality of education and its emulation by developing nations must be located in this changed context" (2010, p. 75) On the basis of review of research she mentions five significant inequalities that have resulted from the reform efforts: 1. Persistence of iniquitous access, 2.worsening levels of basic provision, 3. Polarisation of systemic hierarchy under privatization 4. Persistence of internal processes of domination and 5. Accentuation of inequalities of attainment. The review leads her to conclude that the "roots of educational inequality lie complexly located in the historical processes of caste, class,

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ethnicity and gender stratification and state policy that institutionalises particular forms of educational systems".

THE ROLE OF MATHEMATICS

Within this quintessentially modernist institution of formal schooling, the sciences and mathematics occupy an important place within the curriculum. The social sciences and the humanities have been more easily accessible to critiques, steeped as they are within the human and the social element. The attempt to locate the physical sciences and most of all the discipline of mathematics within the social and especially the political, is a much more difficult task. The power of the received status of these disciplines is such that there is an intuitive resistance to associating their knowledge to a social component, not only by the practitioners of these disciplines but in the popular imagination. As Valero states "For many people it is astonishing that mathematics can be thought of in relation to something "social" such as power relationships, political affairs and actions and values and forms of living such as democracy" (p.1)

This importance of mathematics as an integral part of the school curriculum has never been questioned, and debates have usually only focussed on the content of the curriculum and how it is to be structured. One universal fact about school mathematics is that a majority of students have difficulties with it, and that teaching mathematics poses a bigger challenge than does any other school subject. In India, and it seems to be true of most parts of the world, students have stronger reactions and feelings about mathematics than any other school subject and failure in mathematics is a major reason for children dropping out of schools. It is these aspects of mathematics that inspire researchers but also pose a challenge to policy makers and curriculum developers. The question of whether different mathematics should be taught to different students has also been debated with varying degrees of unease. The distinction between the study of mathematics as a subject in its own right or as a key skill and as of functional value in the study of other subjects has also been discussed.

In a volume on "Rethinking the Mathematics Curriculum", Anderson (1999) argues that "appreciation of (mathematics) should not be reserved for the tiny minority who are destined to become professional mathematicians but should be part of the education of all 'generally educated citizens'" (p.6) and goes on to state that "Driven by all kinds of demands, from innovative industries to arms developments and high-tech projects such as the space race, Given this exalted status that the discipline occupies, 'the socio-political turn in Mathematics education' and the seriousness with which it is being embraced (although still in limited circles) is a welcome one. mathematicians are pushing forward the boundaries of their subject to the extent that no individual can be knowledgeable about more than a part" (p.19). Dorfler's (1999) chapter in the same volume declares that the utility of mathematics lies not just in its 'application' but the

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‘thinking tools’ it provides, and Brown characterises mathematics as: a) furthering the aims of society and b) furthering the aims of the individual.

The concern with social issues in mathematics education has not been entirely absent but has figured largely with reference to learning, teaching and and predominantly in discussions around differences in achievement. It is only more recently that issues of exclusion, inequity and social justice are being discussed within mathematics education itself with a call for a more nuanced understanding and a more complex analyses within larger social and political contexts. This comes from a variety of perspectives including the Marxist and what has been termed ‘the socio-political turn in Mathematics education’.

According to Guiterrez (2013), “the socio-political turn signals the shift in theoretical perspectives that see knowledge power and identity as interwoven and arising from (and constituted within) social discourses”. She further states that “Those who have taken the socio-political turn seek not just to better *understand* mathematics education in all of its social forms but to *transform* mathematics education in ways that privilege more socially just practices.” (p. 40) She mentions the influences as coming from a) Critical theory that influenced the development of critical pedagogy, b) Latcrit theory that focusses on racism within the American context with a focus on deconstructing race and the relations of power and oppression implicated within it and finally, c) post-structuralism as a later theoretical trend that has influenced math education and offers theoretical tools for analyses. Bullock has advocated an intersectional approach cutting across various *isms* with a focus on the “pursuit of justice”. To this end she advises “intentional collaboration among critical mathematics educators through” what she calls “justice communities” and “moving towards justice by directly confronting the multiplicative effects of injustice and oppression” (Bullock)

IN CONCLUSION

Notwithstanding these recent developments and the absolutely valid claim of bringing their awareness both into teaching and research, the question of how they can be integrated and consciously suffuse both research and practice of math education remains an unresolved issue. With concerns for my location in the context of schools, education and the shifting political agendas of the Indian state, and the earlier discussion on the state of the school education sector in India, I want to end my presentation with a few suggestions and raise a few questions that can be used for discussion.

In an interesting review of the socio-political turn, while ruing the turn away from a Marxist analysis, Pais (2014) questions the prominence given within the socio-political turn to concepts of *identity* and *power* with the idea of moving beyond the Marxist idea of social justice based on inequalities generated by capitalism, as well as the move from examining school structures and institutions to examining discourses and social interactions. On the basis of a review of research

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on issues of equity on mathematics education, Pais argues that failure in school mathematics is not a contingent occurrence but a necessary feature of the system of schooling and posits exclusion as a problem of the system itself, the democratic and inclusionary rhetoric of the system notwithstanding. 'The problem of equity requires a fundamental societal change, which we experience as impossible' he states, and adds that the 'problem of equity goes beyond mathematics education'. He further claims that 'by realising that exclusion is something inherent to school, we realise that ending exclusion implies finishing schooling as we know it' (p.1080).

I agree with Pais (and of course the many stalwarts who have critically engaged with education and opened up its seemingly benign agenda to critiques on economic, political and social fronts), in stating that a capitalist, market oriented and neo-liberal agenda of education is necessarily weighted in favour of the privileged within these systems. Having encountered Bourdieu, surely one cannot gloss over the reproductive function of schooling and the extreme and unrelenting efforts needed by the poor, and the socially and culturally marginalised to achieve success within this system. I am referring here also to a society, social structures and a political context where defying a caste marker, displaying a religious symbol or crossing a gender prescription is a matter of life and death, to say nothing of all the humiliations that grinding poverty in conjunction with a host of other parameters of oppression and marginalisation impose on large proportions of our population across the country.

However, I would also argue that while not losing sight of the oppressive structures under which schooling functions, the use of discourse as a category of analysis along with the concepts of power and identity allow us to peel off the layers of obfuscation that mask the reality of oppressive structures. It is precisely in the act of unmasking the nuanced ways in which structures oppress and exclude children from schooling that any attempts at creating a truly inclusive system of education can become possible. So while we struggle to create other systems of schooling that function in truly emancipatory ways, we can begin by recognizing the power that math education wields in creating exclusion.

Elsewhere I have discussed the gatekeeping power of school mathematics and the ways in which it determines life chances where the poor and the disenfranchised are bound to lose. (Khan, 2015) It is precisely for these reasons that mathematics educators need to be aware of the politics of power and the intersections that render student populations most vulnerable. While resisting this power on larger structural and social platforms, I argue that keeping the political and social implications in mind, mathematics educators, teachers and researchers also need to understand and unravel the mystique and power of mathematics in ways that every student in the classroom irrespective of the discriminations and marginalizations they face, has the chance of at least being an equal player in the competition that the present school system creates. The first section of this paper

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is meant to illustrate just how difficult that struggle is for the most vulnerable and how the neo liberal agenda and thinking in education is making it more difficult with each move.

In the context of a neoliberal and globally powerful assault on education, I wish then to raise the following questions:

1. If schooling and accreditation is to determine life chances, who has the power to challenge this authority?
2. If the most dispossessed are also at the lowest rungs of the accreditation what kind of future are they looking at and how can mathematics education intervene?
3. If mathematics is a vital and non-negotiable part of this accreditation and validation system, what kind of mathematics can the most marginalised hope to master and how do they hope to compete?
4. If competition is the norm – who can afford to step out of it with least harm – surely not the already dispossessed?
5. How do we then address questions of what mathematics to teach and how to enable students to understand, critique and challenge it?

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[i] The new sociology of education in the UK and similar trends in the USA initiated more complex understandings of inequality in education and much current critical research in education acknowledges this tradition. Some prominent figures (amongst many) are Bernstein (1971) and Young and Whitty (1977) in the UK; Apple (1979), Anyon (1980) and Giroux (1983) in the USA. See volume edited by S.J. Ball (2004) for a review.

[ii] See Nambissan & Ball, 2011, and Nambissan 2015 for a detailed review and analysis of the advocacy and establishment of budget private schools for the poor in developing countries.

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III PLENARY PAPERS

THE SOCIAL TURN AND ITS BIG ENEMY: A LEAP FORWARD

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Abstract: The socio-political movement called the Social Turn in mathematics education has alerted people to the use of mathematics as a factor of social exclusion. The political efficacy of the movement as a driver of change relies on the degree of indignation that it is able to provoke. We suggest that, instead of a having long discussion to define a minimal positive agenda on which we all could agree, perhaps we can set a maximal negative agenda by eliciting a discourse that we all would spontaneous and promptly reject. In this article, we create a character that would utter such discourse, and call it the Big Enemy, paraphrasing George Orwell. It is an entity incapable of showing any indignation when confronted with the charges of the Social Turn; on the contrary, it maintains that school is as it should be. We present a hypothetical dialogue between BE and Mrs. Smith, a teacher and researcher devoted to the directives of the Social Turn. Our argument is based on economic principles and Lacanian psychoanalysis.

INTRODUCTION (By Roberto Ribeiro Baldino)

Ana Chronaki (2017) has very nicely refreshed our memories about the foundation of MES. During these twenty years, our identity has been cast into aphorisms like "a space for people who are concerned with the failures of existing institutions and structures to address the socio-political aspects of mathematics and mathematics education" (Chronaki, 2017, p. 9). This identity has been broad enough to harbor different conceptions of "socio-political" among the multitude of conceptions to which the one thousand pages of MES-9 Proceedings give witness. Therefore, I feel the need to make precise how I understood what our identity would be when Peter Gates proposed MEAS (MES-1) during the PME in Finland.

At that time, globalizing Capital was already triggering a ruthless struggle against all forms of opposition. This struggle has become more severe today. In mathematics education, it materializes itself through the strict control of publication vehicles so that no novelty will emerge that challenges the general trend of neoliberal globalization. In the name of welcoming socio-political aspects, a cohort of reviewers submits articles to castrating requirements. What is the focus of the paper? Is the argument well-grounded on a thorough revision of literature? What is the article's contribution to improving practices? How helpful is the conclusion? What is the research methodology? What is its theoretical framework? What has been the impact of the authors mentioned in the references?

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Having to ground a new idea on existing literature resembles the quadrature of the circle. How can an author like Hegel ever cause an 'impact' if he is rejected for not having caused an impact thus far? Besides, being 'helpful' subjects any criticism to a Hollywood-style happy end. Due to the obtuse, or perhaps exceedingly smart, scrutiny of reviewers, the criticism of existing practices is filtered out and new ideas are foreclosed. Research on research, research on failure and on economics has been suggested - Pais (2016, 2014) and Pais & Valero (2014) - but has produced little eco.

Because of vigilant scrutiny, great part of recent literature in mathematics education consists of reports of local improvements, bulletins of victory, juggling with new words over old issues, substitution of recently established fashions by even newer ones. Literature on mathematics education hovers uncritically over the reality of schools and classrooms. Nevertheless, this literature is cast into an academically perfect format; some papers rely on an intimidating amount of references to transform quantity into quality. Despite the huge investment in mathematics education (including journals, research grants, help for congresses, paid-for access to articles, exorbitantly expensive yearbooks) mathematics in the school continues to be haunted by failure and anxiety. Classrooms remain almost untouched.

My enthusiasm to participate in MEAS has been grounded on the hope of constructing a community that addresses social issues from the point of view of challenging the traditional current research in mathematics education. In MEAS (MES-1) Peter Gates launched a prophetic question: what measures should we undertake to keep MES from being absorbed by the trends of global capitalism and becoming just one more space for the reproduction of sameness. At that time, my naïve answer was: if this happens, we can always found a new one. I do not think that we have already reached this point. I imagined that MES would have a progressive characteristic, so that each meeting would articulate with the previous one rather than with the world's political agenda of the moment. I imagined that authors would present their main articles here and send reproductions elsewhere.

Therefore, in the present intervention we seek to take MES-9 as our basic reference. We were glad to find papers in MES-9 that pointed towards the initial concerns present at MEAS. D'souza (2017), Subramanian (2017) and Sadafule & Berntsen (2017) offer us dramatic reports of schools and classroom reality: they show us where and how exclusion through school and mathematics happens. These reports make evident that mathematics education is not an 'interdisciplinary area of knowledge' subject to external control; on the contrary, the exclusionary object that we are required to deal with in our classrooms is itself an *interdisciplinary object.* Accordingly, we authorize ourselves to gather knowledge from other areas to account for our activity as researchers-teachers. We do not have to submit to specialists what we learn and apply from sociology, economy, psychoanalysis or political science. This is the identity that I hoped MES would have. The essentiality of exclusion made evident in India is present in the U.S., Europe or South America. Failure and exclusion will be the main focus of our address today.

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THE BIG ENEMY

The foundation of MES is concomitant with what we consider as an important political movement in mathematics education, referred to as the *Social Turn* (Lerman 2000, Gutiérrez, 2010). This movement constructively criticizes mathematics education for insufficiently addressing questions such as equity, gender, ethnicity, achievement gap, identity and power. "Without such critique, mathematics education as a field is in danger of stagnation, unable to address the realities of global citizens" (Gutiérrez, 2000:24). This movement has generated a huge amount of literature in the last decades. Its political efficacy is grounded on the *degree of indignation* that it is able to raise when people are confronted with the role of mathematics as a social segregator. However, imagine an interlocutor who consciously rejects any indignation, recognizes the criticism of the Social Turn but simply replies, 'of course, why not?'

In *Ghost*, the 1990 film by Jerry Zucker, the efficiency of the main character (Patrick Swayze) as a ghost depended on how well he could scare people. However, the villain (Tony Goldwyn) simply assumed that he was dealing with a ghost and was not scared at all. Following this line, we propose a character who refuses to show any indignation in the face of Social Turn charges and argues that *this is how things should be.* How would we deal with such an enemy?

Should we try to find a minimum common ground to anchor our struggle? Perhaps, but, as Hegel teaches us, any positivity inevitably negates itself in the process of enunciation that he called dialectics. Pais (2016) shows how the use of 'dialectics' in recent mathematics education literature has deviated from the meaning it had in Hegel and Marx. Intentionally or not, such deviation weakens the Marxist discourse. On the contrary, relying on the original Hegelian meaning, Pais & Costa (2017) show how the official UNESCO discourse of *global citizenship education* aimed at uniting everyone around positive values actually, ends up by reinforcing the neoliberal ideology at the level of school practices.

Therefore we suggest that perhaps we might unite, not around something on which we all promptly agree, but around something we *promptly reject*. Can we discern an entity that we can immediately recognize as our common enemy? Greer et al (2017) produced an excellent short text that counts as a balance sheet and a manifesto of the Social Turn; they end with a gloomy phrase: "We have no choice but to continue the struggle" (p. 162). On the contrary, we contend that choosing to fight off a common enemy, may signify a leap forward in our struggle for the values of the Social Turn.

We will endeavor to show that the discourse of such an entity already exists, sparsely enunciated but never fully claimed by anybody. Bourdieu & Passeron (1970) teach us that as long as this entity remains half-hidden, its power increases. Accordingly, if we bring it into fully enunciation and force it to assume its identity, it will certainly become weaker and make a visible target for our multi-sided attack. We therefore postulate an entity that would remain aloof of all the indignation the Social Turn expects from its interlocutors. Paraphrasing George Orwell's "Big Brother", we will call this entity the

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Big Enemy (BE). It will be what Žižek calls a cynical "manipulative authority", for which the symbolic order is a fiction and "merely uses it as a means of manipulation" (Žižek, 2002:251).

We will introduce BE, first through its reaction to the Social Turn as expressed by Geer et al (2017) and later, through a dialogue with Mrs. Smith, a hypothetical character condensing the values of this movement. We hope that our expositive strategy will indicate how to reinforce what we think are the open flanks of the Social Turn.

BE's REACTION TO THE SOCIAL TURN

Geer et al: How do we forge a critical community of educators and activists who can invert (and subvert) the hegemonic pyramid to realize 'mathematics for the majority'? (159).

BE: Of course, I subscribe to that. Mathematics for the majority means that it is not for all. A certain amount of failure should remain as necessary to enhance the merit of those who manage to learn it. Economists also know that a certain amount of unemployment is beneficial.

Greer et al: Pushing poor countries into PISA, along with hysterical chest-beating by the media on poor performance, demoralizes those that fight for basic provisions for public education despite abysmally low resources (161).

BE: Public education is a waste of resources. Due to different abilities grounded on genetic heritage, poor performance by some is unavoidable. Average poor performance in mathematics is a sure indicator of the backwardness of a country and the media has the duty to alert its population.

Greer et al: This serves the interests of the privatization lobby and the billion dollar market share of 'low-fee private schools', actively promoted in developing countries, with low-paid unqualified teachers and programmed tablets (iPads) to deliver outcomes (161).

BE: The privatization lobby is right, education should be completely private; only private capital is fast enough to recognize the needs and adapt education to market demands. Besides, education has a cost, which is paid with taxes collected from everyone. Teaching platforms based on technology provide the necessary uniform quality of the product that schools deliver to society. They allow reducing the cost of teacher's salaries for the befit of all. Teachers should limit themselves to verifying whether the students have followed the prescribed timetables. With excessive qualification, some will be willing to introduce changes and become unhappy if they are prevented from doing so. Moreover, the standardization obtained by programmed tablets allows qualified students to be recruited from developing countries as well.

Greer et al: With more than half the children not able to complete secondary school, the State Board examinations (Class X and XII) serve to support the hegemony of mathematics as 'gate keeper', for future professional opportunities (161).

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BE: Of course, mathematics provides a precise measurement of acquired knowledge and alleviates teachers' consciousness of the dilemmas they face when they fail a student. It is natural that some children will not complete secondary school; this is due to a law of nature that Darwin called survival of the fittest. School must allocate people to different professions; lesser jobs must also be filled.

Greer et al: (2017): "recognizing that traditional knowledge and indigenous cultural heritage have value and validity in their own right and capacity to both define and promote development" [i] (160).

BE: Of course, all available social resources, including from marginalized ethnic groups, should be included in a developmental project. Economic development needs efforts from everybody, no matter how small their contribution. "Despite all the talk of pollution, global warming and climate change (…) when the moment comes to choose between economic growth and ecological stability, politicians, CEOs and voters almost always prefer growth" (Harari, 2015:23).

At this point the reader can already guess the identity of BE. But let us proceed a little further. To the following statements, BE would simply reply: that is exactly how it should be. Since the speaker cannot control the aphorisms of what she says, BE may read the aphorisms of the Social Turn in the following way.

BE: *Mathematics for the majority?* Of course, the majority should learn mathematics up to the point of recognizing that the numerical data presented by government agencies are above any possibility of fallacy. There is no need to go beyond that; the needed experts will find their way to the top anyhow. *To become citizens*? Of course, people should become good citizens, obey the laws and endure vicissitudes in times of crisis and save surplus in normal times. *Struggle for social justice*? Of course, social justice should protect heritage and risky investments from crisis that threaten bankruptcy and plunging the whole of society into chaos. *Global citizenship education*? Of course, "critical democratic approaches to global citizenship education (…) occur within a wider society that is 'reproducing powerful corporate cosmopolitan ideals entrenched in a set of neoliberal and knowledge-economy norms' (Marshall, 2011, p. 424 in Pais & Costa, 2017).

BE CHALLENGES MRS. SMITH: CAPITAL NEEDS YOU

Mrs. Smith is deeply impressed by BE's insolence. This entity is pure evil, she thinks. For more than three decades, Mrs. Smith has been an exemplary teacher and researcher. Some accepted ways of running schools have always disgusted her. She enthusiastically embraced the humanist concerns of the Social Turn and the *global citizenship education* directives as a blessed pathway in a world of injustices. She is very dedicated to her students and works hard for many hours, far beyond what is expected from her. Being a Kardecist, when someone reminds her that public service

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will never reward her or even recognize her extra effort, she timidly replies, *I do it out of kindness* (Duval, 2018). Mrs. Smith is indeed a beautiful soul. Now, she is appalled at BE's discourse. The evil in the world has become too strong, she thinks. All my research devoted to promoting justice and equity with respect to gender and ethnicity has just been nullified. There is nothing that I can do… perhaps I should retire…, she concluded.

On the next day, upon entering her office, she becomes aware of a strange presence. *Who is there?* she asks. The answer comes clearly into her mind: *It is I, your BE*. She feels a cold shiver run through her spine but controls herself. *What do you want?* Again the answer is clear: *I need you; please do not retire*. Mrs. Smith was perplexed. Pure evil needs me*?* She redresses herself and decides to accept the dialogue, to confront the intruder as she used to do when an evil spirit showed up in the Kardecist meetings. *Well, tell me who you are.*

BE: My name is Capital, proceeds the entity. My essence is a circular movement in which I change forms from commodity to money, back to commodity, and so on. At each turn, I buy human work in the form of expenditure of muscles and nerves and crystallize it into a commodity. Crystalized human work is what you call *value.*

Mrs. Smith: Do you mean that you are made out of captured human work?

BE: Precisely. This is not a dreadful will of mine. It is just that if I stop my money-commodity metamorphosis, I stop circulating, I stop growing, I fade and vanish. I only exist while in movement. "The modern economy needs constant and indefinite growth in order to survive. If growth ever stops, the economy won't settle down to some cosy equilibrium; it will fall to pieces" (Harari, 2015: 59).

Mrs. Smith: All right, that's enough. Just as you came, you may leave, said Mrs. Smith, hoping to end the conversation and expel the intruder.

BE: Going away is not that simple, replied the entity. I exist as a natural law in the world, much before the advent of men. When a dog buries a bone, it is saving a piece of me [ii]. I cannot avoid being present at every single act of reproduction of life. My desire is always satisfied. In 1989 people realized that mine is the only possible mode of such production. What they are on the verge of finding out is that I am not at all interested in (re)producing their lives. Human beings are an irrational burden that I still have to carry "Computers powerful enough to understand and overcome the mechanisms of aging and death will probably also be powerful enough to replace humans in any and all tasks" (Harari, 2015: 76). For me, only my growth matters.

Again, a shiver runs through Mrs. Smith's spine. This entity was not the specter of someone who had once being alive and subjected to human desire. On the contrary, its desire seems fully transparent to it; growth is all that matters. She had never dealt with an entity that transcended death.

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Mrs. Smith: Well, I am a school teacher and a researcher in mathematics education. What, precisely do you want from me?

BE: I want you to go on doing what you do. For many centuries I managed to hide that *labor-power* is one of my commodity forms, until a certain Karl Marx discovered that I buy this commodity, pay its price in the market and use it to produce more value than it costs me. He called it *surplus-value*. More recently a certain Lacan invented an 'object of desire' that he called small-*a* and related it to me. "The object *a* was the place Marx revealed, uncovered, as surplus-value" Lacan, 2007:201).

Mrs. Smith: I see. This is why people say that you exploit others for your own benefit...

BE: Exactly. I collect this *surplus-value* for my own growth. It took more 150 years for people to find out that I also have a special form, a labor-power whose place of production is the school apparatus at all levels; they are calling it the *qualified-labor-power* (Baldino & Cabral, 2013).

Mrs. Smith: Do you mean that school produces one of your forms? That it reproduces you? Do you mean that school is a place of economic production?

BE: Yes, of course. Many people seem dissatisfied with my mode of production, especially some who operate the school apparatus. Each of them picks one of my features and criticizes and tries to change it. In the end, the sum of their efforts is null and my will stands. Their hasty effort reminds me of the poor motorcyclist who turned his coat backwards because the zipper was broken. He fell, the ambulance came and found him dead. “How come?” asked the first responders, “from so simple a fall…” People around explained: “apparently he was ok, but had twisted his neck, so we tried to straighten it out; we heard a ‘crack’ and he died.”

Mrs. Smith could not help smiling, not at the joke, which she already knew, but because she had never heard of spirits telling jokes.

Mrs. Smith: Commoditizing education is one of the criticisms addressed to you.

BE: That’s because people do not realize what school is for. After graduation, the qualified-labor-power produced here will be exchanged for salaries in the market, for a lifetime. "Today we assume that you learn a profession in your teens and twenties, and then spend the rest of your life in that line of work" (Harari, 2015:30). Qualified-labor-power has use and exchange values and, like any other commodity, must have a seal of quality. It must conform to a uniform pattern. Through legislation and curricula the state ensures that equal kinds of qualified-labor-power have the same use and exchange values: the graduation programs must contain the same number of hours of crystallized human work and the product must pass quality tests according to the same syllabi.

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Mrs. Smith: That is a one-sided view of the school. You cannot reduce our work to the production of qualified-labor-power.

BE: I am not reducing, I am just adding. I admit that looking into the economic role of schooling may displease you. Nevertheless, your students own an initially simple labor power that they set aside in consignation to participate in an economic enterprise. During class time they are forbidden by law to sell this labor-power, but they take possession of it again at graduation when they will become free to sell it again, thenceforth for a higher salary.

Mrs. Smith: That is almost a truism. Everybody knows that school certificates lead to better economic conditions.

BE: Indeed; then there's no harm in saying it. Besides, it might help you to understand why you cannot simply dismiss me. You and your students work to increase the value of your qualified-labor-power. "You are the product of the university, and you prove that you are the surplus-value (…) which is that you leave here, yourselves, equivalent to more or fewer credit points" (Lacan, 2007:201). Teachers own a piece of me and you take good care not to devaluate it. The value of your qualified-labor-power is more real than your subjective feelings about yourself. School provides your students with the necessary experience to recognize me inside themselves. "You come here to gain credit points for yourselves" (ibid). You, dear Mrs. Smith, have been very successful in this game, haven't you?

Mrs. Smith avoided responding to the insolence. Are you saying that my students are capitalists?

BE: Nearly, yes; they are learning to become capitalists. In the school practices they play a double role, both as *workers and as capitalists.* They act as workers when they study and seek to increase the use-value of their qualified-labor-power; they experience the capitalist position when they realize that they are increasing the exchange-value of a commodity that belongs to them and is circulating through the school. As you know, they seek to produce maximum value with least effort, don't they?

Mrs. Smith: Do you mean that I am teaching my students to become capitalists?

BE: Precisely.

Noticing a certain nervousness in Mrs. Smith's voice, BE adds…

BE: Relax, there is nothing you can do to avoid it. My school is structured to stage this experience. Accordingly, the teachers' role is also doubled. You play a sacerdotal role when you sit together with the students to help them, but you also play a judicial role, when you decide about their promotion.

Mrs. Smith : I see, so you ask me not to retire in order to keep the school going.

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BE: Not quite. If you retire, other people will replace you. However, they will see their activity as being rather formal, whereas you believe in what you do. You truly believe that you can make justice when you make a decision about an assessment.

Mrs. Smith becomes nervous, as if she had been accused of perpetuating injustice. She protested.

Mrs. Smith: I am always very careful, she said, when I evaluate my students. I try to measure what they have learned in a precise and trustful way. I am eager to apply results from recent research on brain science, as my colleague Elizabeth de Freitas (2017) has indicated. Bioscience may provide me with a precise evaluation of knowledge and increase the reliability of my judgment (Amalric & Dehaene, 2017; Butterworth, 2017).

BE: That is precisely what I mean: you are special for me because you believe that your summative judicial function is a consequence of your teaching and evaluation. Your behavior is exemplary to other people so they will also believe it. This belief is of uttermost importance for me. Among you, there was a certain Lacan who explained that I must remain hidden in order to function as an object of desire. My hidden role stimulates the students to love me. You help me by dissimulating school failure as being necessary, and making students believe that it is entirely due to their own fault.

Ms. Smith protested again. What do you mean? I have always fought for mathematics for all. Is failure at school necessary?

BE: Certainly. Everybody works in the production of qualified-labor-power; their effort builds up the institution's sign-value. However, only students who get certificates collect the work done by all students. Appropriation of the work done by one’s colleague is the most important experience that I need the school to provide to the students. Again, a certain Marx called it seizure of *surplus-value.* Your earnest desire for justice is very important to make the appropriation of surplus value go unnoticed, as if it were a natural consequence of the *credit system* (Baldino & Cabral, 1998, Vinner, 1997).

Mrs. Smith was upset. Pure evil not only needed her but had been using her from the start. BE continued.

BE: As a researcher you also perform a split role. You must try to promote real change in the school and you must keep and increase the value of your qualified-labor-power, by publishing regularly. The Social Turn movement has opened possibilities of publishing research far beyond their effect in actual classrooms. Students have the opportunity to live the worker's and the capitalist's experience; teachers are required to experience the sacerdotal and the judicial functions; researchers have the opportunity to experience the ideology of improvement and the drive to transform school. The dominant ideology only recognizes the first one of each of these splits, namely, the

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student's function as workers, the sacerdotal function of teachers and the good-will intentions of researchers. The other three are economic functions that must remain covert: namely the student's capitalist function, the teachers' summative function and the researchers' self-promoting function.

Mrs. Smith: Everybody knows these functions; they are not covert; it is simply that it is not necessary to mention them.

BE: Not quite. Not only it is not necessary to mention them, but it is necessary *not to mention them*. Teachers must select according to the ideology of ableism (D'souza, 2017) and establish benchmarks for success and failure. Students participate in the competition game; winners learn to love me, losers learn to respect me. Both experience the *jouissance* of owning me. They must keep me circulating and growing in the school practices. They must not suspect of my presence in school. Insofar as I participate unnoticed, people may align their unconscious desire around their qualified-labor-power, which is one of my forms. They make it the semblance of what a certain Lacan calls the *object-a*, the cause of their desire. The most important result of school practices is that *graduates are the ones who have best learned to love me.* You, yourself, are a good example.

Mrs. Smith had always thought that teachers are selected precisely because they are the ones who most love school practices. Now she realizes that this love also addresses Capital. She fights off an inadmissible shadow of hatred in her beautiful soul. BE noticed it and continued.

BE: Hatred is not negation of love; it is only an inversion. Graduates in the movement called Social Turn seem to hate some of my aspects. However, since my control over the world is global, I am able to transform their actions into profitable enterprises. The value of their qualified-labor-power grows and I circulate in the air tickets they buy and the hotels they pay to participate in congresses, in the grants and fellowships that they get as researchers and graduate advisors. Publishing houses sell books, journals and special editions, they charge for online access to articles that criticize me. This makes me circulate. The Social Turn is a kind of disease that I keep under control, to maintain my façade of democratic freedom.

Mrs. Smith was appalled. She remembered a fragment of a paper that only now made sense to her. School practices are *algorithms* (Harari, 2015) "that operate below the user interface that is invisible to practitioners. Hence it turns out that the mask of description conceals an underlying prescription and is further veiling the human influence" (Lensing, 2017:679).

Reading Mrs. Smith's mind, BE warns:

BE: Do not trust this "human influence". Human desires are already the effect of my algorithms. Mathematics relies on arithmetic, arithmetic relies on numbers, numbers are a logical development of quantity, and quantity is the

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> result of the dialectics of the one and the many (Hegel, 1966). *I am the One*. Do not expect to resort to what you call 'human influence'; it is already under the control of my algorithms. Straehler-Pohl (2017) explains how mathematics contributes to making gadgets and immediately effaces its vestiges. He calls it "de|mathematization". This is just a particular case of what he could call de|capitalization. You do not notice my vestige in your desire.

Mrs. Smith sinks in dismay and BE exits.

Actually, BE is not a novelty of ours. Marx referred to Capital as an automatic subject (*automatisches Subjekt*) whose supporters (*Träger*) are the capitalists. We have just taken a step further and noticed that globalization has turned Capital into an *entity capable of speech and action*. Lacan says that, for common sense, God is a universal eye that is brought to bear on all our actions. In this sense, Capital is a much more powerful god: it determines the meaning of whatever we say and do (Baldino & Cabral, 2017).

> If we ever have the power to engineer death and pain out of our system, that same power will probably be sufficient to engineer our system in almost any manner we like, and manipulate our organs, emotions and intelligence in myriad ways (Harari, 2015:50).

Replacing "we" with "I" we get a typical BE discourse, the discourse of a *crazy god*. Can we resist it? Perhaps, but according to Butler, Laclau and Žižek (2000) we have to take into account that its power over us completes itself when we hope to be endowed with a hard inner core that would resist such subject repression "While the subject is pushed to suppose an ontological self and search for it, she is also simultaneously pushed to manufacture it" (Lensing, 2017: 680).

COMMENTS FROM OUR THEORETICAL FRAMEWORK

From the perspective developed thus far, the Social Turn appears as a compromise with, rather than as critique of current mathematics education research. From this perspective, the Social Turn may be subjected to an ideology criticism such as Pais & Costa (2017) have applied to UNESCO's global citizenship discourse: "the implementation of this discourse in schools and higher education institutions seems to be thwarted by neoliberal practices, marked by a market rationality" (ibid, 2).

On this issue, we should ask, with Žižek: does the critique that presupposes ideology as a false consciousness "still apply to today's world? Is it still operating today?" (Žižek, 1999: 29). The answer is no, insofar as the cynical reason takes the stage. "The cynical subject is quite aware of the distance between the ideological mask and the social reality, but he nonetheless insists upon the mask" (ibid). Such a "cynical subject" is precisely what we attempted to elicit with the fictitious entity called BE. Against such a cynical position, the classical ideology critique of unmasking the particular interests behind the ideology is powerless.

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However, our presentation of BE did not lead to this dead end. Why? On the contrary, our presentation indicates that cynical reason can still undergo criticism. How can it be done? Žižek provides us the answer: "The cynical reason (…) leaves untouched the fundamental level of ideological fantasy, the level on which ideology structures the social reality itself" (ibid, 30). The social reality that is left untouched both by the Social Turn and BE, as well as by 'global citizenship' and 'neo-liberalism' is already structured by the ideological mask of "real abstraction" (Sohn Rethel, 1978).

In our case, real abstraction, the built-in ideology structuring reality, is the fantasy that disguises the promotional procedure moment as assessment. Once this fantasy is revealed as covering the moment of seizure of surplus-value, as happened in the final part of the dialogue with Mrs. Smith, BE loses its ground and becomes constrained to defending *exploitation as such*. "The most unbearable thing for the cynical position is to see transgressing the law in an open way, announced, that is, raising the transgression to an ethical principle" [iii] (Žižek, 90: 75).

The destructive effect of obliging BE to fully state its intention may be exemplified by the 1949 Gregory Ratoff film, Cagliostro, inspired on Alexandre Dumas' novel Joseph Balsamo. In one of the last scenes, the villain Cagliostro (Orson Welles) who masters almost superhuman hypnotic powers, falls in turn under a hypnotic trance commanded by his former master, Dr. Mesmer (Charles Goldner). Cagliostro is forced into fully stating his will: "Power! Power!" Our endeavor is to make BE confess its intention: "Growth! Growth!", regardless of the survival of humankind and the planet. Once this enemy is well elicited and we evaluate its strength, we may become united around a common agenda and the social turn will have its leap forward.

A FINAL WORD

As we finish writing this article we hear of the last school shooting, this time in Santa Fe, Texas. The news comes only on the tenth page of our daily newspaper, with a small call-up in the front page! It is perhaps worth remembering what we said in MES2 and repeated in Baldino & Cabral (2013:82).

> Hereby we can find an answer to Althusser's question: "*Why is the educational apparatus in fact the dominant Ideological State Apparatus (ISA) in capitalist social formations, and how does it function?*" (Althusser, 1970 p. 93). We retrieve our answer of many years ago: "It is because at school the student learns, above all, to participate in and accept the conditions of production and seizure of surplus-value, the work done by one's fellow men" (Baldino, 1998, p. 77).

It is time to start conjecturing that the routine school massacres actually target attacks on the capitalist mode of production.

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NOTES

[i] Quoted from UNESCO.

[ii] Perhaps, this is extending too much the law according to which man's anatomy explains the ape's anatomy, just like saying that a straight line segment is a rectangle with zero height. Would Marx subscribe to it?

[iii] This phrase does not appear in the English translation.

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DIFFERENT WAYS OF KNOWING: STYLES OF ARGUMENT IN GREEK AND INDIAN MATHEMATICAL TRADITIONS

George Joseph

In this talk, I propose to examine critically the ways in which different mathematical traditions of the past have been characterised by historians of mathematics. A litmus test of a valid mathematical practice today is 'proof' and a number of criticisms have been levied against certain traditions because of the perceived absence or lack of rigour in their proof procedures (seen today as the litmus test of whether we are "doing" real mathematics or doing it well). By taking specific examples, I propose to examine the validity of these criticisms and indicate the relevance of alternative proof traditions today. Further, such an examination is often compounded by certain deeply seated historiographic bias in Western scholarship, a complex product of colonialism and hellenophilia. While there is no intention to rehearse the argument relating to the nature and origin of this bias, it is important that this bias be recognised and countered in any attempt to "recover and reclaim the world history of science".

INTRODUCTION

"East is East and West is West: Never the Twain shall Meet" is a well-known quotation of Rudyard Kipling. Does this apply to mathematics or at least mathematics before the European Renaissance? In general terms, the mathematics of Renaissance Europe was influenced primarily by Greek mathematics through the media of Arab mathematics, while Indian mathematics, despite influences from outside, established a unique tradition. In Indian mathematics there was no conflict between, on the one hand, visual demonstration and numerical calculation and, on the other, proof by deduction. Whereas in Greek mathematics only proof by axiomatic deduction was held to have any universal validity.

Numerical validation or empirical demonstration notions of proof became common in Indian mathematics from around 500 CE. There was no conflict between *pramana* (validation) on the one hand and empirical demonstration or numerical calculation on the other. Deduction was an integral part of *pramana*, but it was not accepted that the exclusion of the empirical somehow conferred a superior, infallible status on deduction. And numerical calculation was not regarded as an inferior sort of activity. The function of rationale was to provide an understanding of the practical techniques of calculation, not to establish some supposedly universal and incontestable truths.

In contrast, the current mathematical proof tradition that has its origins in Greek mathematics is perceived as a thought experiment using two valued-logic to: i) verify a result, ii) communicate and persuade, iii) discover a result, and iv)

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systematize results into a deductive system. Such a proof paradigm is needed because the dissonance of pure mathematics from everyday reality is such that the analysis needs to be precise and rigorous. Further, the need for mathematics to be a stable body of knowledge (as opposed to theories in the physical sciences) requires this proof paradigm. However, these considerations do not normally affect every-day mathematics. Here we can utilise informal methods of proof and not be so concerned about the conventional proof paradigm.

CHARACTERISING THE MATHEMATICS OF ANCIENT GREECE AND INDIA

The Greek Tradition

The beginnings of Greek mathematics are often traced to the founding of the Pythagorean school with its peculiar mixture of mysticism and scientific curiosity. In experiments with strings of different lengths, the Pythagoreans arrived at a definition of musical notes in terms of the ratios of these strings.[1] It was a simple extension of the concept of ratio to define it next in terms of the relation between two line segments (and by extension a parallel relation to point/line/area/solid) and thereby establish an equivalence between geometric and arithmetic conceptions of ratio.

Two consequences followed: (i) the principle was established that it was from measurability that 'countability' stemmed and (ii) a number has a figurative quality over and above its magnitude. Arising from (ii) came the practice of classifying numbers into square, rectangular, triangular, pentagonal and other figurative numbers: a tradition peculiar to Greeks and not found in other contemporary traditions. From (i) emerged the fundamental criterion for the constitution of the realm of *arithmos* (i.e. numbers): commensurability. Or in other words, for a number to exist it had to have a geometric dimension, expressed as the ratio of two commensurable numbers.

But soon in the realm of *arithmos*, the Pythagoreans came across lengths which were incommensurable in determining the mean proportion of two sides of a rectangle needed to convert it into a square of area equal to that of the rectangle. It was also noticed that the diagonal of a unit square, which is square root of 2, is incommensurable with the side. And so is the ratio of circumference of a circle to its diameter which is now denoted as π.

The discovery of incommensurability caused a great scandal among the Pythagoreans. As the story goes, a pupil of Pythagoras, Hippasus of Metapontum, perished at sea as a result of revealing this discovery. Memory of this scandal is retained in some of the terminology of modern mathematics. Numbers which are expressible as a ratio of two integers are called *rational* numbers, whereas numbers such as the diagonal of a unit square or π, not expressible as a ratio, are known as *irrational* (i.e., 'un-ratio-able'). It is interesting in this context that the etymology of the word "rationalism" comes from the Latin word ratio, which is a translation of

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the Greek word “logos” meaning mathematical ratio, symbolising reason itself.

Thus Pythagorean difficulties with incommensurability arose from the attempt to establish a close correspondence between geometric and arithmetic quantities, the result being a heavy emphasis on a geometric interpretation of the irrationality of numbers. Because of this geometric bias, the early Greeks just did not recognise irrational numbers and consequently operations with numbers were reduced to a narrow geometric realm, robbing them of considerable potency in arithmetic.

An important trait of Pythagorean philosophy was its atomistic outlook: Number was perceived as constituted out of an indivisible unit, a ‘monad’ or primordial atom. The Greeks imagined that the world and its bodies were built "out of numbers". Even Euclid, whose acceptance of the Eleatic tradition (examined later in this paper) helped to release the subject from the straitjacket imposed by incommensurability, still considered number as created from monads with a pre-existent reality.

Central to the Greek mathematical tradition was a method of demonstrating the validity of a mathematical result, the "method of indirect proof". The method (also known as method of *reductio ad absurdum*) is also central to modern mathematics. Behind it lies two assumptions:

(i) A statement (A) has a singular negation (not-A) and, by corollary the principle of double negation holds, so that not not-A is equal to A.

(ii) If it can be demonstrated that "not-A" is logically inconsistent with A, then A must hold (or must be valid).

The origin of this method of proof is in the teaching of the Eleatic philosopher, Parmenides, a younger contemporary of Pythagoras and his pupil Zeno. A central plank of his philosophy was that Truth cannot be grasped by means of sense perception but only by reason (*logos*). And the method devised to prove the existence of this realm was the method of *reductio ad absurdum*.

Among the earliest and the best known application of this method of reasoning was in Zeno's paradoxes.[2] This method posited the existence of a singular negation of a statement so that the statement and its negation had mutually exclusive non-interpenetrating hypothetical existence. Thus the truth of any statement can be demonstrated if its negation can be disproved and it can be disproved by showing logical inconsistencies which follows if the negation is assumed to be true.

This method opens up a realm of existence which is an ideal realm in that truth is not arrived at through its empirical accessibility but through a particular "ideation" procedure. Here we see the radical anti-empiricism of Eleatic philosophy: a rejection of empiricism since no empirical test can be devised to check whether or not a hypothesis is correct since all physical experience must in principle be rejected as false or unreliable. Though Eleatic philosophers had very little to do with mathematics directly, still the method they evolved from purely philosophical

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arguments played a decisive role in the further evolution of Greek mathematics and remains central to modern mathematics.

Two centuries later in Euclid's elements we find the complete internalisation of Eleatic tradition within mathematics. There was one fundamental tension between the Eleatics and Pythagoreans which posed considerable problems for attempting synthesis. This was the preoccupation of Pythagoreans with a conception of space, which is ‘sensuous’ in origin. For Pythagoreans number not only had relation with space but also with "material". Eleatic philosophers on the other hand denied the existence of space. This apparent irreconcilable nature of the Pythagorean and Eleatic position was reconciled by the Platonic synthesis which climaxed in Euclid's *Elements*.

Plato's philosophy (427-347 BCE) was influenced by Pythagorean mathematics as well as by Eleatic philosophy. Plato inherited from the Eleatic tradition the distrust for ‘sensuous’ experience which he relegated to the realm of appearance and impermanence. For him numbers were separable from the object of the senses so that they existed apart from perceptible things as a separate realm of being. Since counting of objects of the senses conformed to a "non-sensible" idea of number, differences between counting and the science of numbers became a vital distinction in Platonism.

The immediate consequence of this distinction, at least within the Platonic tradition, was the exclusion of all computational problems from the realm of the pure science of *arithmos*. The science of *arithmos* in its ideal form was already quite well developed by the time of Plato; and so Plato had good reason to regard arithmetic as the primary science of his time. It was with the conception of space in geometry that Plato had problems. Awareness of space is associated with location of things which move, change, are created and destroyed in it. And it was precisely this dynamism that Plato wanted to avoid. The attempt to develop an extra-sensory conception of space marked the beginning of theoretical geometry. Plato believed that comprehension of space was not by sense perception but by a kind of ‘reasoning". On the one hand, it was eternal and indestructible, and on the other it was intimately bound up with the phenomena of the perceptible world.

For Plato, the aim of geometry should be to gain knowledge of eternal reality. Arithmetic was based on "one", its ratios, etc. These were ideal and completely abstract forms and well suited for the Platonic ethos. There was, however, no such ideal starting point for geometry. Hence the idealisation of geometry, which was demanded by Platonic connections needed different logical structure than of Pythagorean arithmetic. It was here that Eleatic method entered into mathematics. The formulation of the axiomatic foundation of geometry was a result of Platonic tradition which culminated in Euclid.[3] In Euclid's *Elements* the material of geometry (lines, intersection points, angle, geometrical figures, etc.) were ideal entities whose visible counterparts were merely crude representations. The definitions were

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intended to eliminate as many "sensible" features as possible from geometry; the axioms and postulates were also aimed at making the foundations of the subject purely abstract. Euclid defined a point as "that which has no part" and a line as " a length without a width". Euclid avoided any mention of motion unlike Proclus, a later commentator, who defined a line as the "uniform" and "undeviating flowing of a point". What was sought was the realm of ideas in itself whose simplicity, consistency and beauty were the sole reason for their "truthfulness". The realm of ideas was the true realm of being.

Properties of this realm were sought using the Eleatic method which involved demonstrating the existence of the properties of ideal forms by showing the inconsistency of their negation. From Plutarch, we know that for Euclid the ultimate objective of his geometry was to investigate the Platonic solids. The Platonic solids were elemental ideal objects which played a central role in Plato's cosmology. He based the distinctiveness of four fundamental elements (earth, fire, air and water) on the four regular ideal solids: earth-cube (because cube is a stable solid), fire-pyramid (because fire rises up as symbolised by a pyramid), water-icosahedrons and air-octahedron.[4] Physical bodies were identified with ideal geometrical forms in accordance with the Platonic thesis of identity of idea and being. Platonic matter was held to be a kind of body lacking all qualities except ideal forms which physical bodies imitate. The conception of causality within the Platonic paradigm was one of linkage of underlying ideal forms independent of time. This concept of causality later played an important role in the evolution of Kepler's laws of celestial mechanics.

Another significant feature of Euclidean geometry was its avoidance of the difficulties faced by the Pythagoreans with their discovery of incommensurability. It may be remembered that Pythagorean atomism and the equivalence implied between geometry and arithmetic inevitably lead to the problem of incommensurability which stalled any further development. Notions about the existence of mathematical objects or the criteria for legitimation of mathematical objects which Pythagoreans upheld were much more stringent than those of Euclid. The criteria for "existence" which Euclid adopted from the Eleatic tradition transcended this impasse leaving behind the problem of incommensurability.

To summarise, the distinctive feature of Greek mathematics which explain both its strengths and limitations are:

(i) Objects of mathematics were either elemental, "complete in themselves" like points, lines or numbers such as Pythagorean monad and Plato's One and the Indefinite Dyad or "derived" such as ratios, etc.

(ii) The criterion for "existence" of mathematical truth was different for the Pythagoreans and Euclid. For the Pythagoreans, it was the principle of constructability (genesis) and for Euclid it was the criterion of indirect proof. As Euclid reinterpreted Pythagorean mathematics in his own terms and the

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bulk of Greek mathematics is known through his writings, we regard the methodology of indirect proof as a dominant characteristic of Greek mathematics.

Together with these characteristics went the anti-empiricism and the geometric inclination of the Greek contribution to mathematics.

The Indian Tradition

The conception of numbers was very different in India. Unlike the Pythagoreans and Euclid, numbers were not regarded as made up of primordial atoms but as entities whose value depended on their efficacy for mathematical operations. It is this outlook that facilitated the creation of a place value system containing fractions, negative numbers and zero. As early as the seventh century CE, the Indian mathematician\astronomer, Brahmagupta, was considering zero and negative numbers at par with other numbers by formulating explicit rules for arithmetical operations with such numbers. By emphasizing the primacy of operations in determining the existence of numbers, Indian (and for that matter Chinese) mathematics steered clear of any problem caused by incommensurability.

There were other differences between the Greek and the Indian traditions. While both traditions contained different methods of demonstrating the validity of a certain mathematical result, the "method of indirect proof" which was central to Greek mathematics was alien to Indian tradition.[5] But a method, known as ***tarka***, was however used in the Indian tradition to show the <u>non-existence</u> of certain entities.[6]

In not accepting the method of indirect proof as a valid means for establishing the existence of an entity (which existence is not even in principle established via direct means of proof), the Indian mathematicians took what today would be known as the constructivist approach to the issue of mathematical existence. But the Indian logician did more than merely disallow certain existence proofs. The general Indian philosophical position is in fact one of completely eliminating from logical discourse all reference to such 'unlocatable' entities whose existence is not even in principle accessible to direct means of verification. This appears to be the position adopted by Indian mathematicians. And "it is for this reason that many an 'existence theorem' (where all that has been proved is that the non-existence of a hypothetical entity is incompatible with the accepted set of postulates) of Greek mathematics would not be considered as significant or even meaningful in Indian mathematics" (Srinivas, 1987, p. 12).

There were other interesting differences between the Greek and Indian logic in so far as they were applicable to mathematics. The Indian logic was structured as an important adjunct of epistemological analysis and was therefore both formal and material, deductive as well as inductive. Indian logic did not owe much to the development of mathematical reasoning, as was the case in Greece, but probably arose from medical and linguistic debates. The main theoretical concern of Indian

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linguistics was constructability of accomplished entities of language. Thus, about two thousand five hundred years ago, Panini, in a book entitled *Astadhyaya*, offered what must be the first attempt of a structural analysis of a language.

An indirect consequence of Panini's efforts to increase the linguistic facility of Sanskrit became apparent in the character of scientific and mathematical literature of the period. This may be brought out by comparing the grammar of Sanskrit with the geometry of Euclid - a particularly apposite comparison since, whereas mathematics grow out of philosophy in ancient Greece it was partly an outcome of linguistic development in India.

The geometry of Euclid's *Elements* starts with a few definitions, axioms and postulates and then proceeds to build up an imposing structure of closely interlinked theorems, each of which is logically coherent and complete. In a similar fashion, Panini began his study of Sanskrit by taking about seventeen hundred basic building blocks - vowels, consonants, nouns, pronouns and verbs, and so on - and went on to group them into various classes. With these roots and some appropriate suffixes and prefixes, he constructed compound words by a process not dissimilar to the way in which one specifies a function in modern mathematics. Consequently, the linguistic facility of the language came to be reflected in the character of mathematical literature and reasoning in India. Indeed, it may even be argued that the algebraic character of ancient Indian mathematics is but a by-product of the well-established linguistic tradition of representing numbers by words.

It was this outlook that led to zero and negative numbers being considered as the same as other numbers when it came to operations with them. The word *karani* was used to describe the surd (or root) of any number whether it was rational or irrational. While a rational-irrational classification did not develop, there existed a classification on the basis of exact and inexact numbers. In a *Sulbasutra* text by Apastamba (c. 400 BCE) on constructing a circle equal to the area of a square, appears the concept of *stulagnana* or approximate value for the ratio of circumference to diameter (π). Constant revisions to the implicit value of π became a continuing tradition in Indian mathematics from the *Sulbasutra* value equal to 3.088 around 500 BC, through Aryabhata (c. CE 500) who gave its value as 62832/20000 = 3.1416, eventually leCEing to the highly accurate value correct to eleven place of decimal attributed to Madhava (c. CE 1350) using a finite approximation to the infinite series for π.

The idea of the realm of numbers as homogeneous with respect to arithmetical operations is an essential prerequisite for the development of algebra, since to operate with provisionally indeterminate of unknown quantity, assumes that such an operation is no different from working with determinate and known quantity. The central imagery behind Indian algebra (called *Bijaganita*) is that if one sows a seed (*bija*) in the field of numbers (*ksetra*), one picks the fruit (*phala*) irrespective of what the field contains. It is interesting that from the time of the Bakhshali Manuscript (variously dated from 200 – 800 CE) the terms *bija* and *phala* were

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used as technical mathematical terms to describe the unknown and solution respectively.

Thus the algebraic mode of argument has been a distinctive feature of Indian mathematics in spite of variations in their approach and conception. In essence, the tradition is fundamentally distinct from the abstract geometrical mode of argument characteristic of the Greeks. though, given that the Greek word "geometria" means "measurement of land", Greek geometry began as empirical, but got idealised early on. In India, geometry never got "idealised" in the way it did in Greece. Geometry was largely "concrete" and empirical in character. The concrete character is best exemplified in a popular method of "proof" which they shared with Chinese mathematics. The essence of their method involves what we would call today "dissection and reassembly" is based on two commonsense assumptions:

(i) Both the area of a plane figure and the volume of a solid remain the same under rigid translation to another place, and

(ii) If a plane figure or solid is cut into several sections, the sum of the areas or volumes of the sections is equal to the area of or volume of the original figure or solid.

The reasoning behind this approach was very different from that behind Euclidean geometry, but the method was often just as effective, as shown in the discussion of different 'proofs' of the right-angled triangle theorem (or Pythagorean Theorem) in a later section.

Indian geometry has, however an algebraic character which is best seen in the genesis of trigonometry there. Because of their geometric emphasis, the Greeks used chords in their astronomical calculations, whereas the Indians had developed the notion of sines and versines (i.e., 1□− cosine of an angle) as early as CE 500. Aryabhata was perhaps the first Indian astronomer to give a special name to these functions and draw up a table of sines for each degree. Approximation formulae were developed for these functions, culminating in the construction of sine tables in Kerala during the 15th century where the values in almost all cases are correct to the eight or ninth decimal place, an accuracy not to be achieved in Europe for another two hundred years. The Indian work in this area influenced the development of Arab trigonometry with al-Battani (b. 858 CE) using the sines regularly with a clear recognition of their superiority over Greek chords. The Arabs introduced certain new functions including tangent, cotangent, secant and cosecant functions.

The Nature of Proof

Consider the word 'proof' in the sense that Lakatos (1976) uses it to mean a 'thought experiment' which suggests a decomposition of the original conjecture into sub- conjectures or lemmas, thus embedding it in a possibly quite distinct body of knowledge.'

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In a broad sense any ‘proof’ has psychological, social and logical features (Resnick, 1992, pp. 15-17). The psychological task is to convince its readers of its conclusions. The notation and the way in which the argument is formulated, organized, and presented determines whether the proof succeeds at this task. Yet success in convincing an audience does not necessary mean that the proof is free of error. Proofs make certain claims about mathematical objects.[7] Understanding such claims requires training and the more ‘advanced’ the mathematics the longer the training required. Nowhere is this training more important than in the comprehension of the logical framework in which the proof is embedded. Therefore it is important to distinguish between the psychological and logical powers of a proof. A logically impeccable proof could appear obscure and unconvincing because the audience have not acquired through training a satisfactory understanding of the mathematical objects of which the claims are being made in the first place.

It is the third feature of a mathematical proof that is often ignored. Proofs are social and cultural artefacts. They evolve in a particular social and cultural context. And it is important since we might tend to forget that part of finding out how a proof works includes finding out how its intended audience (the author included) comes of follow it. This is further complicated by the fact that proofs are context-bound – not only in relation to a proof’s language and notation but also its reasoning and data (or the uses to which a mathematical result is put to).

As an illustration, consider the nature of the Indian proof (or *upapatti*). For a period going back to about 2000 years, a great deal of attention in Indian mathematics was laid on providing an *upapatti* (roughly translated as ‘convincing demonstration’) for every mathematical result. In fact some of these *upapattis* were noted by European scholars of Indian mathematics up to the first half of the nineteenth century. For example, in one of the early English translations (1817) of parts of *Brahmasphuta Siddhanta* of Brahmagupta (b. CE 598) and of *Lilavati* and *Bijaganita* of Bhaskara II (b. CE 1114), Colebrook gives in the form of footnotes a number of *upapattis* from commentators and calls them demonstrations. Similarly, Whish (1835) who brought to the attention of a wider public Kerala work on infinite series for circular trigonometric functions showed *upapattis* from a commentary on Bhaskara II’s *Lilavati* entitled *Yuktibhasa* (c. CE 1550) which related to the Pythagorean theorem. One of these will be discussed in a later section.

One of the main reasons for our lack of comprehension, not merely of the notion of proof, but also of the entire methodology of Indian mathematics, is the scant attention so far paid to these commentaries which seemed to have played at least as great a role in the exposition of the subject as the original text itself. It is no wonder that mathematicians of the calibre of Bhaskara II and Nilakantha (a sixteenth century Kerala mathematician/astronomer) wrote not only major original treatises but also erudite commentaries on either their own works or on important works of

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an earlier period. It is in such commentaries that one finds detailed *upapattis* for results and processes discussed in the original texts and more general discussion of the methodological and philosophical issues concerning Indian astronomy and mathematics.

As an illustration, consider the commentaries of Ganesha Daivajna (b. CE 1507). According to Ganesha, *ganita* (used both as a generic word to describe the subject of mathematics as well as used in a specialized sense to describe calculation) is mainly of two types: *vykataganita* and *avykataganita* . *Vykataganita* is that branch of *ganita* which employs clearly laid out procedures or algorithms well-known for general use. This is in contrast to *avyaktaganita* which is distinguished form the first type by including procedure that use indeterminate or unknown quantities in the process of solution. The unknown quantities were referred to by terms such as *yavat tavat* (i.e., 'as much as') and different colours (*varna)* denoted by abbreviations such as *ka* (for *kalaka* or black), *ni (*for *nilaka* or blue) etc, just as in modern algebra unknowns are denoted by symbols *x, y, z,….* , etc.

In a chapter on the solution of quadratic equation from his *Bijaganita*, Bhaskara II poses the following problem:

> Say what is the hypotenuse of a plane figure in which the side and upright are equal to 15 and twenty? Also show the *upapatti* of the received mode of computation.

Later he adds:

> The demonstration that follows is two fold in each case: one geometric (*ksetra*) and the other algebraic (*avyakta*)…The algebraic demonstration must be shown to those who do not understand the geometric one (and vice versa)

We will now proceed to the next section to consider the 'different ways of knowing' the Pythagorean theorem in the two mathematical traditions: the Greek and the Indian mathematical traditions

PROOFS IN GREEK AND INDIAN MATHEMATICS: THE PYTHAGOREAN THEOREM

The Greek Proof

In Euclid's *Elements*, the Pythagorean Theorem makes its appearance as Proposition 47 in Book I. For his proof he uses the result that parallelograms are double the triangles with the same base and between the same parallels (Proposition 41). In Figure I below, the smaller of the two rectangles that constitute the largest square is double the area of the triangle sharing a common base with it and has the same area as the smallest square that is double the area of the triangle that is congruent to the first triangle. An analogous result holds for the other rectangle forming part of the largest square and which in turn can be equated to the second larger square through their corresponding congruent triangles. Hence, the proof is

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complete in that the two rectangular sections of the largest squares add up the square on the hypotenuse which is equivalent to the sum of the squares on the other two sides of the right-angled triangle ABC. A simpler proof is possible by applying the properties of similar triangles. However, this was not available to Euclid since he only discussed these properties later (in Books V and VI).

Euclid's proof is a good example of a proof based on axiomatic deductive inference.[8] The axioms arise from the propositions that he has already proved previously regarding (i) congruence of triangles ["If two triangles have two sides equal to two sides respectively, and have the angles contained by the equal sides also equal, the two triangles are congruent(Proposition 4, Book 1)] and (ii) a parallelogram on the same base and in the same parallels as a triangle is double the triangle (Proposition Based on these propositions he makes certain deductions with respect to the relationship between the three sides of a right-angled triangle.

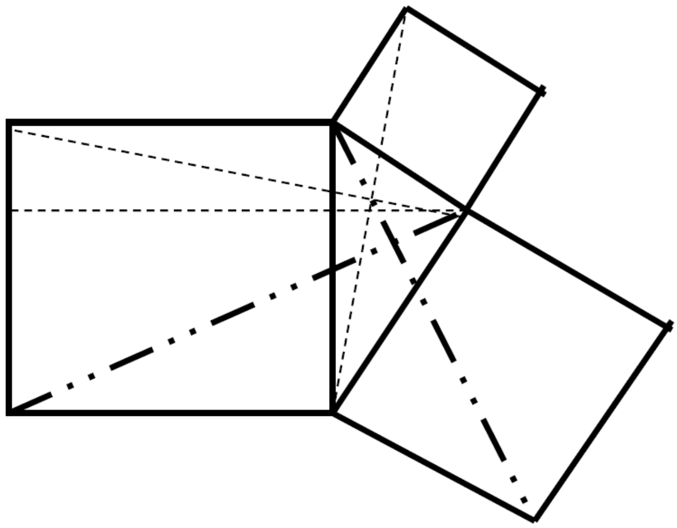

Figure 1

THE INDIAN PROOF: DIFFERENT PROCEDURES AND ONE RESULT

As a preliminary illustration of the common principle of 'Dissection and Reassembly', consider the geometric proof of an algebraic identity. It is found in a commentary on Bhaskara II's works by Krisna Daivajna (1606) who proposed to establish the validity of the identity $a^2 - b^2 = (a - b)(a + b)$ using Figure II below. No written commentary is present in the 'proof'. For sake of convenience and clarity, we use the our present system of labelling.

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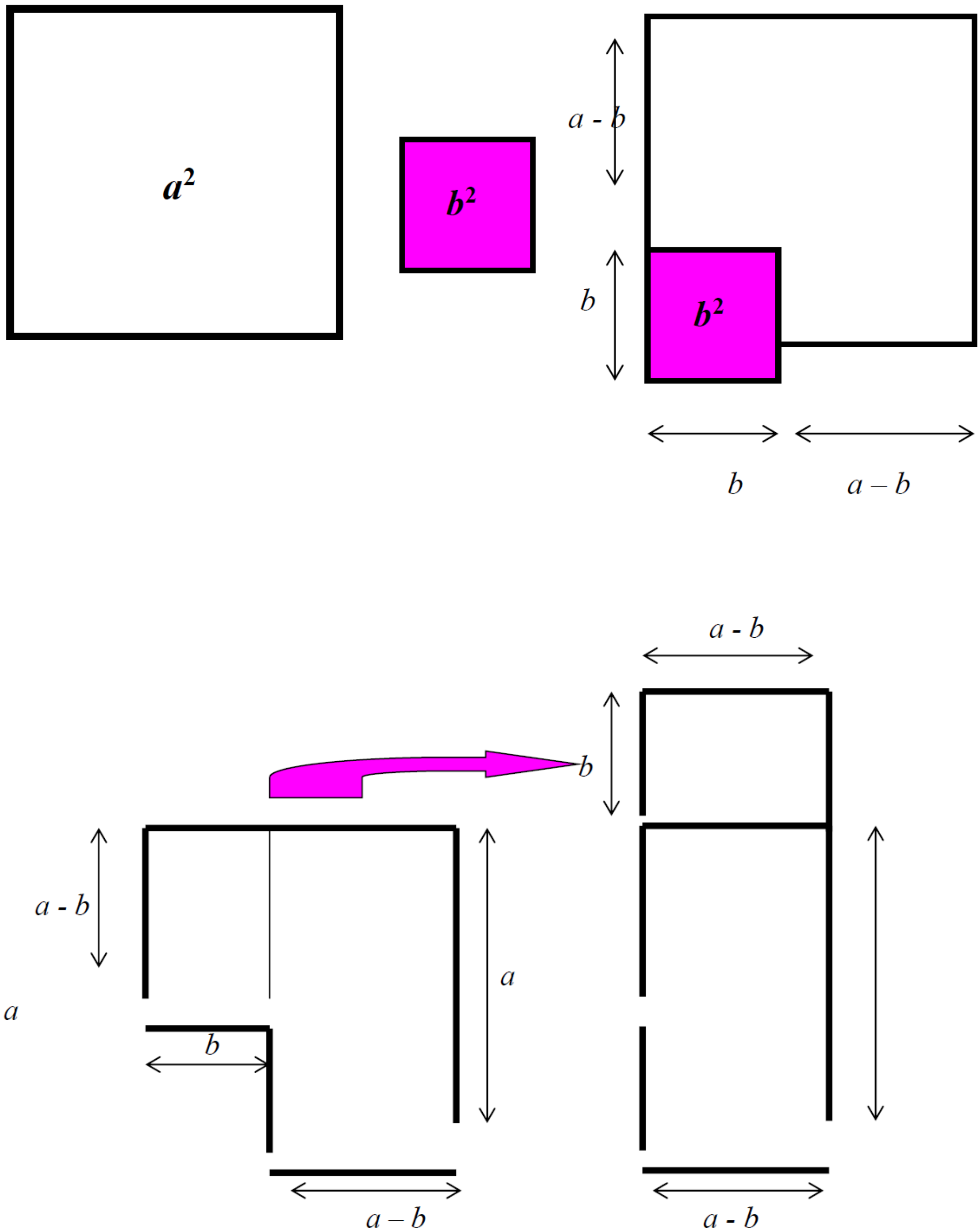

Figure 2

Hence $\boldsymbol{a^2 - b^2 = (a - b)(a + b)}$

In his commentary ***Buddhivilasini*** on Bhaskara II's *Lilavati*, the 16th century CE mathematician/astronomer Ganesa Daivajna has the following demonstration of the Pythagorean Theorem. In Figure III below three triangles congruent to a given right angled triangle ABC are placed in such a way to form a square with side BC with a

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square ‘hole’ at its centre – the square ‘hole’ has side equal to (AC – AB). Bhaskara’s text contains only the diagram and the word ‘Behold’ suggesting that the diagram itself indicates the proof without the need for any explanation. However, Ganesa offers a succinct explanation. The area of each of the four congruent right angled triangles given above is ½ AB.AC. Also, the area of the bigger square = Area of the smaller square + the area of the four triangles. So it would follow that

$$BC^2 = (AC - AB)^2 + 2AB.AC \text{ or } \mathbf{BC^2 = AC^2 + AB^2.}$$

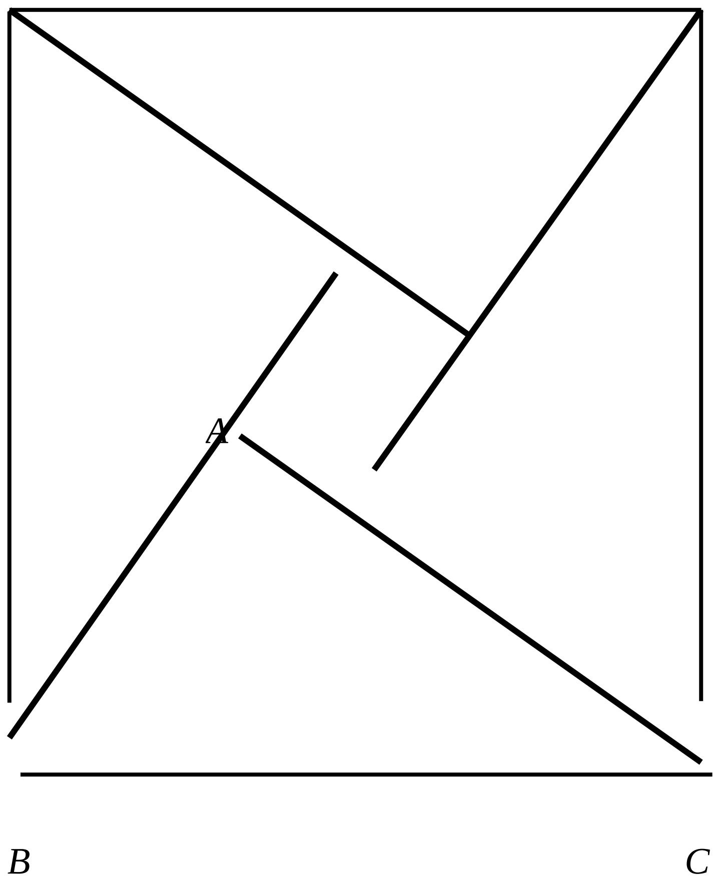

Figure 3

As one can see this proof relies on the reader to be persuaded about the square configuration of the perimeter and in the centre. Such a persuasion is possible at an *intuitional* level (i.e., Bhaskara II ‘s visual demonstration) or at a rhetoric level as in this case or where, in the case of the Chinese text, *Zhoubi Suanjing*, dating back to at least several hundred years BC, a process of ‘piling up the rectangles’ is used.[9] The Chinese proof consists of Figure IV which is a slight modification of Bhaskara’s diagram together with a short commentary. The four outer-half rectangles (or 2 rectangles) which constitute a total area of 24 square units in the simple example taken of a (3,4,5) sided triangle is subtracted from the larger square of area 49 to leave an area of 25 square units which is the square of the hypotenuse

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in this case. In other words,

$$\mathbf{(3 + 4)^2 - 2(3 \times 4) = 3^2 + 4^2 = 5^2}$$

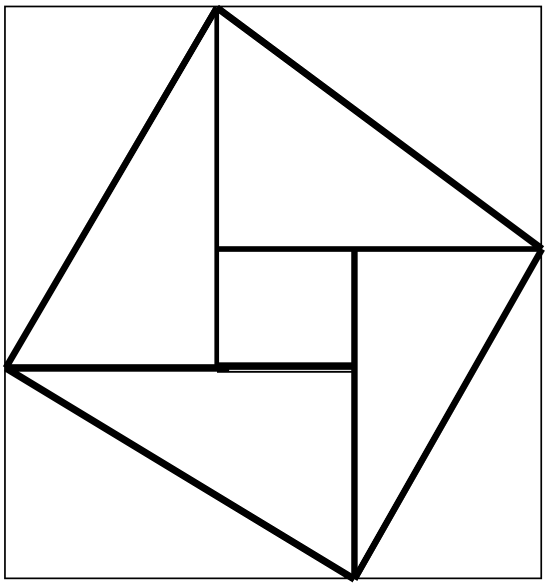
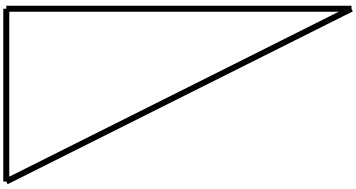


Figure 4

The extension of this ‘proof’ to a general case of other right-angled triangles was achieved in different ways by Chao Chun Ching and Liu Hui, two commentators of *Zhoubi Suanjing* who lived in the third `century CE. The work of Liu Hui was particularly interesting since he did not refer to the diagram in the original text. The method he used was also a variant of the ‘dissection and reassembly’ principle.

A 16th century text of Kerala (Indian) mathematics, the *Yuktibhasa* of JystCEheva, has an ***‘action visual proof’*** of the Pythagorean result that is dependent only on the use of the eye.

(i) Construct a right-angled triangle



(ii) Next, construct the squares on the smaller sides

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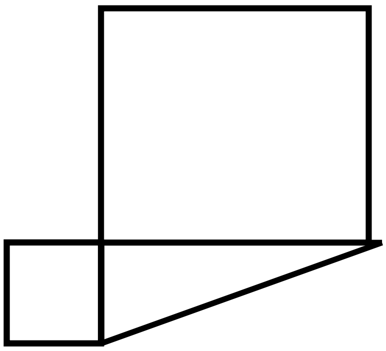
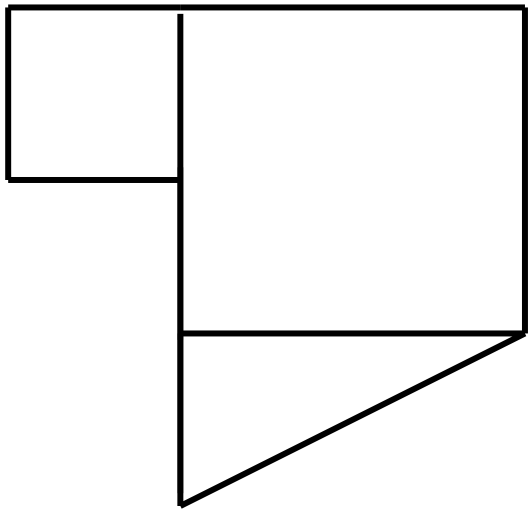
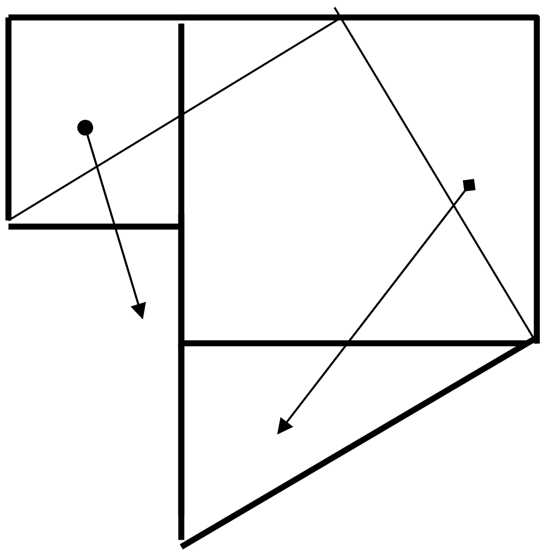


(iii) Move the smaller square and align it with larger one as shown below:



(iv) Construct two triangles congruent to original triangle giving the configuration:



(v) Cut, move and paste these two constructed triangles to form square on hypotenuse:

This proves in a convincing way that the sum of the squares on the smaller two sides of a right angled triangle is equal to the square on the hypotenuse.

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Finally, an *upapatti* from Ganesa's commentary on Bhaskara's *Bijaganita* that comes closer to the spirit of the Euclidean proof than the ones discussed above. In the Figure V below, ABC is a right-angled triangle. CE is a perpendicular dropped from A to BC. Since triangles ABC and CEB are similar,

$$\frac{AB}{BC} = \frac{BD}{AB} \Rightarrow BD = \frac{AB^2}{BC}$$

Also from similar triangles ABC and CEC,

$$DC = \frac{AC^2}{BC}$$

So $$BC = BD + DC = \frac{AB^2 + AC^2}{BC}$$

Or $$BC^2 = AB^2 + AC^2$$

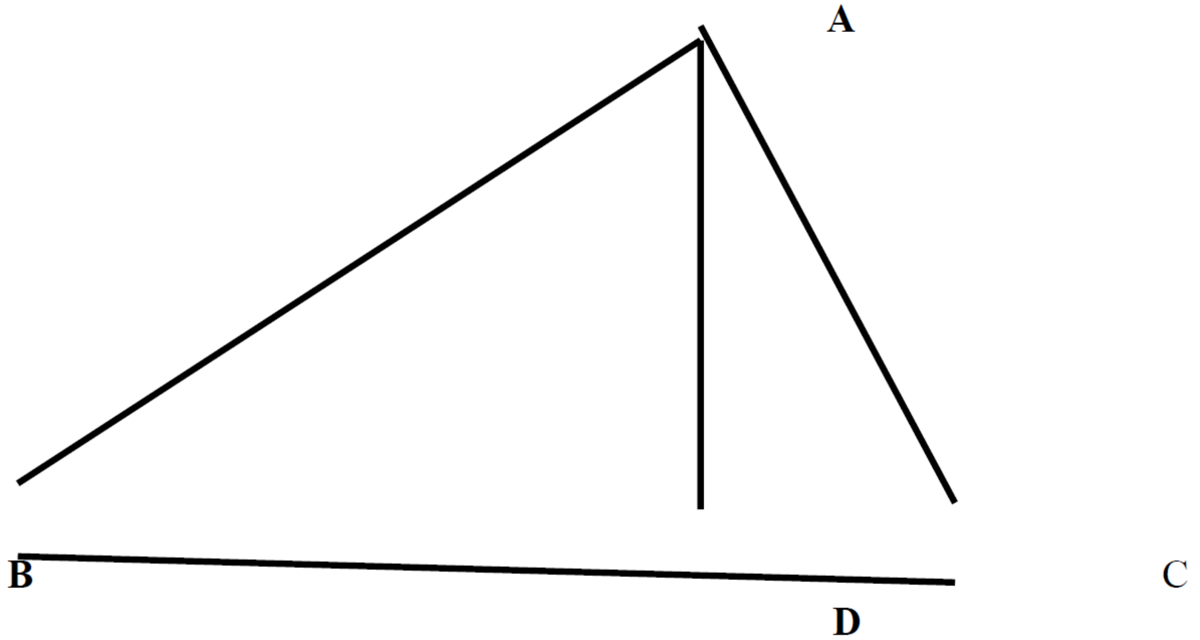

Figure 5

In these examples we see that Indian mathematicians used the whole range of informal proof techniques from proof by visualisation to generic proof to deductive proof. That this approach did not disadvantage them is testified by the fact that numerical methods of the calculus were discovered in Kerala some 200 years before Newton and Leibniz.

What is also clear from the examples discussed is that the notion of *upapatti* (proof) is significantly different from the notion of proof as understood in the Greek or even the modern traditions in mathematics. Sarasvati Amma (1979) sums up the difference in the following way:

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There is an important difference between the Indian proofs and their Greek counterparts. The Indian aim was not to build up an edifice of geometry on a few self-evident axioms, but to convince the intelligent student of the validity of the theorem so that the visual demonstration was quite an accepted form of proof…

Another characteristic of Indian mathematics makes it differ profoundly from Greek mathematics [is that] knowledge for its own sake did not appeal to the Indian mind. Every discipline (*sastra*) must have a purpose. (Sarasvati Amma, 1979, p.3)

CONCLUSION

The non-recognition of the foundational conceptions and methodologies of non-European mathematical traditions has restricted our understanding of the nature and potentialities of mathematics. Can we seriously believe that we could come to grips with the foundational tensions in modern mathematics without recognizing the deeper cultural determined ideological differences that went into the creation of this mathematics: stress on becoming (dynamic) versus stress on being (static), constructability versus indirect proof, empiricism versus idealism …the polarities are many and hardly ever discussed.

ENDNOTES

[1] The traditional though highly improbably story has it that Pythagoras made his inference from observing the variations in sound of anvils of different sizes.

[2] Zeno of Elea (c. 490 BCE), a pupil of Parmenides, proposed a number of paradoxes of which four dealt with motion. Of the two opposing views on space and time then, the first assumed both as infinitely divisible in which case motion is continuous and smooth and the other that they were made up of indivisible small intervals so that motion is discontinuous, taking the form of a succession of tiny jumps. The paradoxes were directed against both views. And the apparent difficulty of resolving these paradoxes had a profound effect on the subsequent development of Greek and European mathematics.

[3] The line of argument is more complicated than this sentence implies. For Plato, a mind that had no sense perceptions could still reason, i.e., produce geometric truth. Aristotle, on the other hand, believed that a mind could only be deductive after sense-perception. This is the point where Descartes defines his reasoning for the foundation of mathematics and physics in his "Meditations”. The famous "I think therefore I am" comes from Descartes completely emptying his mind of all thoughts\perceptions. This is where the Eurocentric foundations for mathematics after the Renaissance came from, i.e., a rejection of Aristotle's ideas (which had been suitably modified and incorporated into the Christian ideologies) and a return to a more "pure" and geometric world view. It is also relevant that at this time European society needed science more than ever as the bourgeoise began to emerge. This eventually led to the undermining of the power of the church and enhancing the power of the state.

[4] It is possible that there might have existed an earlier scheme:

Earth-cube ==> water-icosahedron ==> air-octahedron ==> fire-sphere

This scheme is more consistent with a progressive approximation from a cube to a sphere while in the later scheme the shape of the fire breaks the whole pattern. There is a possibility that the

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later scheme is a philological corruption of a more rational earlier scheme since the fire is the most volatile and mobile element it ought to be spherical rather than pyramidal

[5] If mathematical entities are regarded as "being" or "non-being", then the "existence" criteria would require the logic of indirect proof (using the principle of excluded middle). But if entities are regarded as "accomplished" or "unaccomplished" then the "existence" criteria would be constructability. Here we see an important difference between foundational basis of Greek mathematics with its formal logic in which the central principle is the method of indirect proof and that of Indian mathematics with its algorithmic logic and the central principle is that of constructability. For a further discussion of the role of indirect proof in Indian mathematics, see Joseph (1994)

[6] Thus in showing that a negative number cannot have a square root a commentator, Krisna Daivajna (c. 1600 CE), used the method of indirect proof. For a further discussion of Daivajna's mode of argument, see Joseph (1994)

[7] The nature of mathematical objects determines how we make contact with them. If mathematical objects are based on the Euclidean ideas of atomistic and an object-oriented view of space (points, lines, planes and solids) this will be in complete contrast to a Navajo idea of spaced as neither subdivided nor objectified and where everything is in motion (Bishop, 1990, p. 51) The crucial point is that ideas of proof are culturally created and be understood within that culture resisting any temptation to make crude comparisons across cultures and ways of deciding between ideas.

[8] To account for the beginning of the axiomatic-deductive method in Greek mathematics a common hypothesis takes the following lines: the shocking and stressful discovery of incommensurability led to a "logical scandal" which then gives rise to a real "crisis of foundations", serving as the springboard for a thorough rebuilding of the whole mathematical method (Van Der Waerden 1983; Neugebauer 1957; Boyer 1968, Wilder 1968). There were other discrepancies (such as the rules for the computation of areas) found in old Babylonian mathematics that could have contributed to the same end product (Van der Waerden (1983). Other explanations for the genesis of axiomatic deductive methods have been sought in what are perceived as peculiarly Greek characteristics. The Greeks were philosophers and lovers of the beauty (Kline 1954) or philosophers and thinkers (Heath,1921). Also, ‘externalist’ explanations focus on social factors, as the slave-based ground of Greek society and economics, or the role of democracy (Farrington (1946, 1947), Boyer (1968), to explain Greek love both for the abstract (typical sign of idleness!) and for the dialectics (typical democratic tool!). In the last few years there have been a number of contributions about the philosophical background of the 'method question', underlining the Aristotelian, Platonic and Eleatic legacy among its roots. Aristotle's role cannot be denied for his 'syllogism" and for his idea of "principles"-based knowledge, Plato's Academia and its idealism were the real cradle for the reform of the mathematics foundations, and to Parmenides the first dialectics and the formal non-contradiction principle can be ascribed. This ‘internalist’ explanation in recent years has been suggested by Knorr (1975) and Szabo (1960), outlining an interpretation of the new method's birth thoroughly inner to a 'philosophical' debate.

[9] For a discussion of Chinese proof s in *Zhoubi Suanjing* and other sources see Joseph (2011, 248- 252)

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MATHEMATX: TOWARDS A WAY OF BEING

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In practicing and making their knowledges of the world, people make themselves, the societies and the spaces they inhabit. In moving and acting they perform the knowledge spaces, they create trails, they 'know as they go' through the cognitive and physical landscape. (Turnbull, 2009, p. 3)

Coming to know is a mirroring or a re-enactment process where we understand Nishnaabeg epistemology to be concerned with embodied knowledge animated, collectively, and lived out in a way in which our reality, nationhood and existence is continually reborn through both time and space. This requires a union of both emotional knowledge and intellectual knowledge in a profoundly personal and intimate spiritual context. Coming to know is an intimate process, the unfolding of the relationship with the spiritual world. (Simpson, 2014, p. 15)

Mathematx is an activity that cannot be extracted from the living being(s) in the process of solving problems and/or experiencing joy--the mathematxn…Living mathematx means both that we live a version of mathematx as well as we are a living version of mathematx…Living mathematx means moving through the world with other living beings, acknowledging, appreciating, and reciprocating the patterns produced. (Gutiérrez, 2017a, p. 17-18)

In Living Mathematx (Gutiérrez, 2017a), I introduced three Indigenous concepts that have guided my work over the years—In Lak'ech, Nepantla, and reciprocity—and suggested they serve as guiding principles of a new practice of mathematics. I emphasized the benefits to mathematics as a growing discipline and to addressing our planetary crisis. In doing so, I sidestepped the challenges inherent in creating an ecology of knowledges, especially as it relates to the experiences of Indigenous peoples. In this paper, I seek to analyze some of the specific ways that Western mathematics in US schools operates as a form of dispossession and how mathematx addresses the need for Indigenous people to continue to remake themselves while interacting with non-Indigenous people.

As a Chicanx scholar with Rarámuri roots, I offer my perspective to help guarantee that the next 7 generations continue to recognize and know themselves as Indigenous. While some of the knowledge of Anyáwari[1] might help to better protect the planet, I do not suggest that Indigenous people owe anything to white settlers in terms of a set of tools to be coopted or a process for healing the relationship between humans and land or water. Our intelligence and bodies are not resources or labor. Any strengthening of the relationship between Indigenous people and land, water, animal,

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and plant nations is important for our own flourishing. In that sense, I present my thoughts here to extend the argument I began in Living Mathematx. I offer an analysis that applies primarily to North America, United States in particular. However, I imagine that readers in other parts of the world may find the analysis useful, as it pertains to the experiences of others in settler-colonial societies. The perspective I offer is my own. It comes out of my interpretation of life experiences, including stories that have been shared with me, as well as academic literature I have read.

In this paper, I begin by describing particular ways in which mathematics (education) often operates as dispossession and, therefore, violence, for people who are Indigenous. Instead of centering whitestream/Western/Eurocentric mathematics, I argue for a form that respects and supports Indigenous worldviews among others. This form of mathematics seeks, acknowledges, and creates patterns and relationships that solve problems and offer joy—something I refer to as mathematx. In doing so, I illustrate some connections between mathematx and enactivism in mathematics education. Finally, I highlight how mathematx might be fostered in out-of-school or community spaces as a form of "presencing" that reattaches people to lands and waters. Throughout the paper, I use whitestream, Western, and Eurocentric interchangeably to highlight the fact that a focus on Europe or the West hides connections to power and imperialism in the US. Moreover, while the term Western can convey what has been sanctioned within the discipline of mathematics, it is not solely Western in that many concepts and "advances" were stolen from other peoples and lands throughout history.

There are many ways in which Indigenous people have experienced the process of dispossession (e.g. of language, culture, land). For example, the Indian Removal Act of 1830 allowed the US government to steal lands from Indigenous nations in order to gain access to valuable "natural resources" such as gold (Bowes, 2017; Dunbar-Ortiz, 2015). Although many resisted, several Indigenous nations (and mixed race and Black slaves who lived among them) were transplanted from the Southeastern part of the US to west of the Mississippi River. The most famous of these forced relocations was known as the Trail of Tears where members of the Aniyunwiya (Cherokee), Muscogee, Seminole, Chikashaw (Chickasaw), Chahta (Choctaw), and Ponca nations were required to march on foot to new lands and where upwards of 8,000 of them died of exposure, disease, and starvation. This kind of forced removal of Indigenous bodies from lands occurred again for those who are Chicanx as part of the Mexican Repatriation in the 1930s. That is, during the Great Depression, out of fear that we were taking scarce jobs and using up government assistance, 1.8 million "Mexicans" and "Mexican Americans" were shamefully and illegally forced by the US government to return to México, even though an estimated 60 percent had never lived there (Balderrama & Rodríguez, 2006; Wagner, 2017). While this history of stolen lands is relatively well known, some of the most severe forms of dispossession have occurred through the education system.

Throughout North America, during the late 19th and mid 20th century, as part of a partnership between Christian churches and the US and Canadian governments,

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hundreds of thousands of Indigenous children were taken from their families and forced to attend Indian Residential Schools where the main objective was to assimilate them into white society, a process otherwise known as "Kill the Indian and save the man" (Adams, 1995; Pratt, 1892). In essence, in order to become fully human in the eyes of broader society, Indigenous children were required to lose their knowings (e.g., languages) and ways of honoring life. In Canada, this practice continued through 1996. In the United States, by 2007, most but not all of the boarding schools had closed. Similar patterns of Christian churches opening schools to "civilize" Indigenous children occurred throughout the world, including in Australia, New Zealand, Central/South America and the Caribbean, Scandinavia, the Russian Federation, Asia, Africa, and the Middle East. In these other countries and continents, sometimes the elites of an Indigenous group or those with the highest potential for assimilation were targeted to ensure they returned to their communities and perpetuated colonial powers (Smith, 2009). This history is horrendous and remains a part of all of our lives today (Bombay et al., 2014).

1. MATHEMATICS AS DISPOSSESSION

Eurocentric mathematics does not capture complete systems and knowings—ways we recognize and remake ourselves as Indigenous.

School Mathematics

Although the education system as a whole has inflicted incredible damage on Indigenous peoples, I argue that mathematics both as a school subject and as a practice in society has played a powerful role in dispossession, by valuing and sanctioning only certain kinds of ontologies and epistemologies that side with colonialism. To be clear, I am not suggesting there is a clean separation between knowings that are Indigenous (often viewed as traditional[2] and local) versus Eurocentric (often viewed as scientific and universal), as the two have coevolved and will continue to do so. In fact, many of the things known and credited to Europeans or white settlers came from contact with Indigenous peoples who shared their understandings of the world. Moreover, as with any cultural group, there is often more variance within a given group than between groups, as Indigenous people hold different roles in society, know different things, and have different views based on their experiences and connections with specific lands and waters. It is this diversity and dynamic response to our world that has allowed Indigenous peoples to replicate our survivance (Vizenor, 2008) generation after generation. In addition, presenting Western and Indigenous knowledges as "opposites" has been the strategy of colonizers who suggest Indigenous knowledges are in contrast to Western ones that have served as the foundation for progress in the modern world (Harding, 2008). So while there is no such thing as a single Indigenous "way of knowing" or "worldview," there are underlying epistemologies (ways of knowing) and ontologies (views of reality) that differ from modern Western/Northern/Eurocentric[3] views and that contribute to Indigenous systems (Battiste, 2002, 2008; Cajete, 2000).

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It is these underlying differences that I investigate, in part, to highlight relations of power.

Throughout this paper, I use "knowings" in place of "knowledge" or "intelligence" to highlight the fact that anything known is always an ongoing act—a way of being—that connects humans with our animal, plant, land, water, and other relatives in the physical and spiritual worlds. Knowings are not things; they are a process of deep engagement. Knowings are not developed and permanent; they need to be renewed regularly. In doing so, they renew persons. Anishinaabe scholar, Leanne Betasamosake Simpson (2017), reminds us that Indigenous knowings draw upon and link past, present, and future at the same time. For me, Indigenous knowings are always plural, as they can never be known by only one; they are always embedded in a network and are shared as complete "systems" employed to empower Indigenous people. In fact, Mi'kmaq scholar Marie Battiste (2002) reminds us that "Indigenous Knowledges…fill the ethical and knowledge gaps of Eurocentric knowledges" (p. 5). As such, my use of the word "knowings" implies that I am talking about systems that are both robust and flexible because they are dynamic, and that such knowings come by way of various pedagogies, including observing, listening, making, singing, dreaming, participating in ceremonies, and proper asking. Later in this paper, when I speak of learning, I will expand upon the concept of self-determination to explain why knowings are plural but the meaning that one makes from interactions could be conceived as individual.

In developing my argument about dispossession through mathematics, I found myself relying heavily upon Indigenous scholars writing about values, views of reality, and knowings in a more holistic manner than empirical studies of teaching and learning mathematics. In academic studies of teaching and learning in Indigenous communities, regardless of whether they occurred in schools, few researchers focused specifically on mathematics. Of those that did, the descriptions were thin or relied largely on applying taxonomies and looking for Western equivalents. Many of these studies were written in the 1980s or 1990s and perpetuated essentialized or deficit-based views of Indigenous students. I do not include those results here.

Many of the researchers studying Indigenous people and mathematics were not, themselves, Indigenous and that raised questions about how a non-Indigenous lens might have affected the findings (e.g., How was something identified as "Indigenous" and worthy of reporting out? What was ignored because it did not fit with Western rationality or how data is collected/valued in the academy? What was not shared with these non-Indigenous researchers in order to protect sacred knowings and ceremonies?). Some of those non-Indigenous researchers were people who had been taken into the homes/lives of Indigenous communities or spent many years, decades for some, in a specific community[4]. But, few studies focusing exclusively on mathematics in Indigenous communities were written by people who identified themselves as Indigenous[5]. In some ways this makes sense, as Indigenous knowings do not separate mathematics from a way of life (Cajete, 2000). As more graduate programs produce Indigenous scholars, we are likely to see a growing body of research

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that critiques the methods, evidence, and findings currently employed in the mathematics education literature, and offers fresh perspectives on what we refer to today as (modern Western/Northern/Eurocentric) mathematics.

Recognizing that Indigenous knowings are complete systems, I found myself questioning whether the very act of trying to understand mathematics in relation to Indigenous knowings was, itself, a form of dispossession. I acknowledge those challenges and contradictions here. There is as much variation between Indigenous nations as there is between Indigenous and non-Indigenous people. Not enough studies in mathematics education exist to help flesh out the nuances of a given nation. Moreover, many Indigenous knowings are transmitted from one generation to the next orally through story. As such, these knowings are purposefully not accessible to people outside of a particular nation. The process of reviewing academic literature relied upon extracting only those parts that related to what we think of as mathematics, thereby carrying with it the vestiges of colonization. And, yet, because of the powerful role of modern/Western/Northern/Eurocentric mathematics in schooling and society and the need to name oppression outright, I decided it was still a useful exercise. I attempted to focus on notions of mathematx and holistic notions of mathematics as opposed to simply numbers, counting, and geometry. Also, where relevant, I attended to literature outside of mathematics to highlight what was erased.

School mathematics serves not just as a gatekeeper for further STEM careers, it often serves to wash out the informal and natural ways people do mathematics outside of school. This is true for Indigenous and non-Indigenous children alike. For many students, schooling can instill a sense of failure and dependency, stripping people of knowings they already possess (Valenzuela, 1999; Gerdes, 1988). This separation of school and home/community is especially harmful to tradespeople, artisans, and mothers (Conner, 2005; Harding, 2008; Walkerdine, 1990). For Indigenous students in particular, we must ask: "What is lost in mathematics as a whole, and in global society, if we continue to dispossess people of valuable knowings that support them to be whole—to continually remake themselves and fulfill their responsibilities—and that have the potential to enrich the lives of others, including our other-than-human relatives?

I am not alone in suggesting that participation in school mathematics is often incongruent with Indigenous ways of knowing. See for example, Bishop, (1990); Wagner & Lunney-Borden (2015); Meaney et al., (2013); among others. This paper is not an exhaustive review of the literature on mathematics and Indigenous people. Instead, I focus on a few concepts that are dominant in Western mathematics, and therefore US school mathematics, to highlight dispossession.

Objects, Counting, and Capital

When it comes to counting and measuring in school mathematics, students are largely required to view the world as composed of discrete objects with enduring qualities. This conception of "objects" is often paired with the collection of those objects for possession and accumulation of wealth. In fact, some have argued that the invention

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of number comes from Western notions of individual ownership of objects and capital (Matthews et al., 2005). Yet, Indigenous knowings consider life as always in relation, not separate from humans or others. For example, Rarámuri knowings recognize that trees are not ours; that is, they cannot be owned, but we must protect them for the next 7 generations. So, the concept of owning animate objects and accumulating them for (individual) wealth can be viewed as violating relationships and the complexities they embody. And yet, many mathematics textbooks present word problems in consumer contexts that presume students understand and value capitalism (Frankenstein, 1990; 1995). Throughout history, number systems were important for astronomy, calendrics, divination (religion), not necessarily wealth (Closs, 2001). However, US students are rarely required to apply knowings they have of these systems to the practice of mathematics.

In many Indigenous communities, the redistribution of wealth is an important process. Various ceremonies involve giving away of wealth and taking "only enough" or "no more than half" and leaving some for others. Indigenous knowings highlight our ongoing connections with the previous 7 generations and the next 7 generations, compelling people to consider an act as not just affecting one's own desires or one's community today, but also in past and future. Kórima is an important word in Rarámuri tradition, whereby people who have more are expected to share with those who have less. This concept is tied to the idea that we are all related and that we have a responsibility to each other. With respect to mathematics, Wagner & Lunney Borden (2015) studied one Mi'kmaw community and found that measuring involved levels of precision that were tied to context and were appropriate for their products to be useful and beautiful, relying upon common sense rather than particular dimensions or forms of counting. Their research suggested that one knows how much is "enough" with ones eyes or body and through repeated practice. The importance of such knowings is, in part, tied to values of not wasting and to social responsibility. In this sense, the ethics of the practice—for what reason one is measuring—is as important as the practice itself.

In contrast, one of the eight mathematical practices in the US Common Core State Standards is Attending to Precision (Governors Association, 2009). They state:

> Mathematically proficient students try to communicate precisely to others. They use clear definitions…in their reasoning…They state the meaning of the symbols they choose…are careful about specifying units of measure, and labeling axes to clarify the correspondence with the quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision.

However, for many Indigenous nations, being "precise" in this way is not necessarily a sought-after goal. Being required to attend to precision could come at the cost of ignoring complexity. In most of the mathematics education literature, examples of appropriate degrees of precision tend to focus upon such things as final place value when multiplying decimals and whether it makes sense to talk about balloons in

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fractions or integers. While there is room for interpretation around the language of "degree of precision," there is no mention of ethics or purposes for attending to precision. Moreover, requiring students to "state the meaning of symbols," "specify units of measure," or "calculate accurately" as part of their reasoning process may cause students to abandon other more holistic ways of measuring or counting that make sense and that support them to continually consider whether the amounts they are using are "enough," whether they fit a systemic approach to understanding, and if the purposes for conducting such measurements are proper (e.g. towards redistributing wealth). That is, as systems, Indigenous knowings cannot be separated from ethics. Whereas there is evidence that many graduates of advanced mathematics and science programs need to be taught how to apply their understandings to real world contexts and to consider the ethical implications of their work (Hersh, 2014; Resnik, 2005), the idea of context and ethics are already embedded in Indigenous knowings.

The notion of essentialized objects possessing enduring qualities can be traced, in part, to language. I mentioned earlier the history of Indigenous people being stripped of their languages through boarding schools. Being dispossessed of language means losing the complexity and values/principles embedded in language. Let me explain. Language structures the way we think of everything. For example, in the English language, almost all non-humans are referred to as "it," regardless of whether living or the result of industrial processes like metalwork or plastics. English speakers use "it" to describe chairs, spiders, bushes, wind, and computers. Exceptions are made for non-humans we love, such as pets, cars, or boats, to which we might refer with gendered pronouns. But, for the most part, English language structures how people think of themselves as separate from, often superior to, non-humans in a way that normalizes hierarchies.

Language also affects how we conceive of number or quantity. For example, in studying the Yoruba of West Africa, Watson (1990) found that the Yoruba language highlights the quality of things rather than individual objects with lasting qualities across all time and space. For example, water is considered as "having the properties of water" or "watermatter." Such knowings rely upon different logics that do not assume the world is distinct from humans or that mathematics is a form of representing the world as a "complete and given entity" (Verran, 2000). In English, we begin by breaking everything up into units in the world as a means to count. Australian Aboriginal peoples have been found to use a formalized recursive representation of kinship as the major integrative standardized form similar to how Western societies use the formalized recursion of tallying—number—as an integrative standardized form of knowledge (Verran, 2000). This recursive representation connotes the nature of relations and allows a naming and way of ordering the world. As such, it becomes a way of encoding social order and includes ethics. Because Indigenous knowings are maintained through language, we must protect both the language and the knowings.

Caswell and colleagues (2018) found that in Anishinaabemowin[6], geometrical properties are reflective of actions that were taken to produce a representation. For example, the word for square literally means "it squares itself" which is an action

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performed rather than an enduring property of the object. Ardoch Algonquin First Nation member and Anishinaabemowin teacher Marjolaine LaPointe recounts,

I was thinking of this word, kakadeyaa, which means a square, but it's not a noun. It's a verb which means it's squaring itself. And, kakadeyaa has this k sound which is a cutting, a grouping, a separating, and when we break the language down into those sound-based parts, we start thinking in three dimensions as opposed to binaries of "this is this and this is that," and we see relationships between many things. And the language really lends itself to spatial reasoning and math inquiry. That word also implies that what you're looking at has grouped or separated itself in a way that is different from its natural state. Which is kind of what a square is because if you look out onto the land, what we see are not really squares—we see things that are all kinds of different shapes but that very harsh squaring off is an odd thing to see. And, so that's why we have a very specific word for it. (p. 86)

Other scholars studying mathematics learning have found similar patterns of action in the language of Rotinonhson:ni (Doolittle, 2006) and Mi'kmaw communities (Lunney Borden, 2011), as many Indigenous languages are highly verb-based. The heavy reliance on verbs, as opposed to nouns, is consistent with the understanding that everything is in motion. In fact, scholars who have studied Runasimi[7] and Diné bizaad[8] highlight the complexity required of speakers who need to string together sounds to create a verb. Such work requires encoding "not only tense, aspect, number-/person-of-subject, but also such grammatical notions as transitivity, causation, modification and internal arguments" (Courtney & Saville-Troike, 2002, p. 624). In creating verbs, the speaker must consider <u>all</u> of the possible variations that could be uttered and why particular ones would be more appropriate. Because these word parts can be strung together in various orders, thereby providing a multitude of forms, one could argue that Runasimi and Diné bizaad speakers are doing combinatorics throughout their day as they create verbs and communicate with others. The reliance on an understanding of combinatorics allowed Diné bizaad speakers to successfully serve as "code talkers" for the US government during World War II. Other Indigenous languages that are highly verb-based share some of these dynamic properties.

Indigenous ways of knowing support learners to see the world as comprised of actions and relationships, rather than enduring qualities that are representative of discrete objects. Doing so allows the learner to be poised to see patterns as dynamic processes (reflective of past/present/future and related to our responsibilities) rather than static representations. In terms of learning, students in the study by Caswell and colleagues may be more likely to see squares as having the potential of shifting (back) into rectangles or stretching into other forms of polygons that are less "regular," thereby providing the foundation to view a relationship between all quadrilaterals or polygons. This dynamic view is important for addressing one of the problems that plagues the mathematical learning for many US students—their sense that mathematics is a set of disconnected facts or topics. In fact, many mathematics teachers have incorporated computer software to help students imagine a more dynamic quality to spatial

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representations or functional relationships. Another approach might be to help non-Indigenous students understand, from the outset, the concept of objects as action-oriented and in ethical relationship with others.

The Spiritual World and Religion

Indigenous knowings are held and continue to grow through various mechanisms, including ceremonies. Although Indigenous peoples have been forced to practice traditional ceremonies in hiding or under persecution, they have maintained them precisely because of their importance in establishing balance in the world. That is, ceremonies are the basis of remembering to remember beauty and wounds, to maintaining and renewing patterns that are foundational to our flourishing, to honor our original instructions. Yet, in the United States, it was not until the American Indian Religious Freedom Act of 1978 that Indigenous people were allowed to practice freely. Lakota scholar Vine Deloria (1992) reminds us that even when other practices have been eliminated, traditional ceremonies have persisted. He states,

> …the ceremonies have very little to do with individual or tribal prosperity. Their underlying theme is one of gratitude expressed on behalf of all forms of life. They act to complete and renew the entire and complete cycle of life, ultimately including the whole cosmos preset in its specific realizations, so that in the last analysis one might describe ceremonials as the cosmos becoming thankfully aware of itself. (Deloria, 1992, p. 280).

Ceremonies allow humans to have a connection with the spiritual world so that new revelations can be imparted to local people about their place and circumstances. The goal is to be at one with the cosmos. Yet, nowhere in school mathematics has space been carved out for spirituality to be acknowledged or practiced.

This separation of mathematics from anything spiritual serves as a form of dispossession of that ongoing connection, a denial of emotional life and the ability to fulfill one's responsibilities. I use the term spiritual not to connote New Age practices but to highlight life forms that are part of Indigenous systems of knowing and that are beyond whitestream rationality. I also note that many things count as ceremonies. For example, growing up, whenever I or any of my three siblings got hurt, my mother would take the injured part of our body and rub softly in circles, saying, "Sana, sana, colita de rana; si no sanas hoy, sanarás mañana[9]." She was practicing a tradition handed down over generations. My interpretation is that our Anyáwari had given my great grandmother a story that honored frogs as our animal relatives and recognized their superior ability to regenerate body parts that we, as humans, could not. My mother, perhaps in her own way, also added that we have the power to heal ourselves. I would imagine myself as frog and attempt to regenerate the broken skin. Every time we hurt ourselves, the cultural practice reminds us of our place in this world. I have practiced the same ritual with my own children.

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Mathematicians who study the sociology and history of mathematics have argued that throughout the world, mathematics developed with strong spiritual and religious connections, even though today it is taught very much from a Western perspective (Bishop, 1990). One such example comes from India. For example, the fields medalist Manjul Bhargava, draws upon ancient Sanskrit[10] poetry based on Virahanka numbers where the rule for determining them was written around 600A.D. (Singh, 1985). Virahanka numbers were recorded in a linguistics textbook and helped poets express their thoughts with rhythms based on combinations of long and short syllables (Bhargava, 2016). Similar rhythms are found in music. As a result of Western imperialism, these numbers are now commonly referred to as "Fibonacci numbers." Having been raised by a father who is a Sanskrit scholar and deeply grounded in such knowings, when Bhargava today gets stuck on a mathematics problem and seeks insights, he turns not to algebraic symbols and writing but to tabla music (Klarreich, 2014).

Indigenous ways of knowing take on many forms other than the scientific method or conjectures and proof used in Eurocentric mathematics. In fact, people learn through stories, observation (of humans and non-human persons), and dreams (Hermes, 2000). For example, Srinivasa Ramanujan made major contributions to the fields of mathematical analysis, infinite series, number theory and continued fractions. With almost no formal schooling, he provided answers and insights to problems considered unsolvable (Ono & Aczel, 2016). In contrast to the forms of proofs that were used in England and that were sanctioned by the mathematics community, Ramanujan argued that his knowledge came to him by way of messages in his dreams from the goddess Namagiri. Yet, because this connection to spirits is not viewed as objective, rational or logical, it is also not seen as intelligent (Urrieta, 2003). I would add that it is not seen as mathematical. As such, Ramanujan died before the mathematics community could verify his work through proof and fully appreciate his contributions.

The importance of a spiritual connection in knowings is why Indigenous scholars seek land as pedagogy, not land-based pedagogy (Simpson, 2014). Treating land and water as our relatives—as belonging to nations with important lessons to offer—is important for maintaining ourselves. In this sense, land as pedagogy becomes a life practice. This practice is different than studying concepts "on" or "in" lands/waters as if they are simply situated there, as opposed to life forces of lands/waters. Such a practice recognizes that land has memory. Whereas Western science asks us to learn about plants and water systems, Indigenous science asks us to learn from plants and water nations (Bang et al., 2014). Mathematics is similar in that the patterns that other-than-human nations teach us are not simply about their physical representations or the ways they take up space or use tools. Such nations also have lessons to share about gratitude, reciprocity, and patience, based upon time scales that span minutes and centuries at the same time. In this sense, they are modeling how to solve problems and bring joy. These are patterns we can appreciate, reference ourselves to, and adopt.

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Binaries, Bodies, and Identities

Modern Western/Northern/Eurocentric mathematics is based on agreed upon postulates and axioms. The "law of non-contradiction" states that every proposition cannot be both true and false at the same time[11]. This law leads to the "law of the excluded middle" suggesting that every proposition has to be either true or false; there is no middle ground. For example, a number is either rational or irrational, even or odd, but not both. This kind of singular logic supports the idea that individuals are either man or woman, either rational or irrational, which easily leads to either deserving of respect or not, thereby reflecting the idea that hierarchies are normal. Yet, Indigenous knowings are non-hierarchical; they have acknowledged and often honored 2-spirit individuals who simultaneously walk in two worlds, many of whom maintain a special connection with the spiritual realm (Simpson, 2017). This acknowledgement of people who are 2-spirit is similar to the use of Latinx, where "x" is a variable that reflects individuals as having identities that reflect gender fluidity, rather than following patriarchal lines. With its emphasis on 2-value Boolean logic, Eurocentric school mathematics dispossesses Indigenous people of the complexity of their knowings and dispossesses in particular 2-spirit and queer bodies, which I refer to as 2SQ[12], of their full identities. Our animal and plant relatives teach us the fluidity of gender and sexual orientation. For example, some Black, Grey, Grizzly, and Polar bears, as well as Beluga whales, possess both boy and girl bodies; clownfish are able to transform between male and female based on the social environment. In fact, gender in animal nations does not dictate body, behavior, or life history. "When a gender binary does exist, it tends to be slight and sometimes reverses gender stereotypes…there are often more than two genders and multiple types of males and females" (Roughgarden, 2004, p. 6). Therefore, we must consider the relationship between Indigenous human knowings and the knowings of our plant and animal relatives.

A binary logic also supports the myth that "some people are good at mathematics, while others are not," thereby justifying fuller access to life's resources for some people over others. Elsewhere, I've discussed this matter, suggesting that is it somewhat arbitrary that expertise in mathematics (rationality and abstraction in particular) serves as a proxy for intelligence in society and thereby confers status and economic benefits when other sensibilities (e.g., thinking holistically, being artistic, relating well to others) are equally viable constructs of intelligence (Gutiérrez, 2017b JUME).

Fuzzy sets and fuzzy logic are exceptions to the notion of the excluded middle. Here, statements need not be true or false but may have a degree of truth between zero and one, thereby representing a kind of probability. In fact, some researchers have suggested a connection between Indigenous ecology management and fuzzy logic, something I will explore later in this paper. Unfortunately, fuzzy sets and fuzzy logic only come into play in higher levels of mathematics and engineering-based applications to which only very specialized students are exposed. It is important to

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explore how notions of fluidity, which are embraced in Indigenous systems of knowing, might play out further in mathematics.

The concept of binaries also plays out in school mathematics through the body, reflecting the underlying sensibilities and tools important for doing mathematics. In most US mathematics classrooms, as well as research on mathematics teaching and learning, there is a great emphasis placed on cognition. The learner is thought to develop the mind in mathematics. For the Rarámuri nation, the body (the chest in particular) is where the souls are housed, whereas the mind is housed in the head. The two are connected and always in relation, making it virtually impossible to interact with only mind and not soul. Yet, with few exceptions, school mathematics does not expect students to draw upon their bodies or the senses (e.g., intuition or rimúma[13]). In fact, many of the students with whom I have spoken have suggested that mathematics classrooms require that you leave your body at the door. Ways of knowing that come largely through logic involve the brain, but so many other parts of the body are also important for understanding the concept of pattern in mathematics.

With respect to the body and identities, educational researchers have examined how gender, race, and culture are implicated in the social construction of mathematics because only certain identities are seen as normal. Levya (2017) has articulated well the concept of masculinization in mathematics and the importance of intersectional analyses. He suggests that heterosexism and heteronormativity marginalize students who do not conform to gender norms. Focusing on girls in particular, Hottinger (2016) notes, "We simply cannot reconcile the cultural construction of femininity with the construction of mathematical subjectivity" and in doing so, she suggests we implicitly ask girls to choose their femininity or their mathematical prowess. I seek to extend that argument by suggesting it is not just that mathematics feels inaccessible to people who are womxn[14] and 2SQ, it robs them of their specific and important knowings passed to them through generations.

Hottinger (2016) has suggested that a Western version of mathematics denies access for people of color to view themselves as mathematical. She argues this happens through textbooks that present mathematics through internalist histories (suggesting mathematics was discovered, rather than created by humans) or by exernalist histories (that give agency to humans for creating mathematics, but normalize the ideal mathematician with Western culture, abstraction, and masculinity). I largely agree with Hottinger's argument and have taken a stronger stand in many talks I have given on rehumanizing mathematics, suggesting mathematics in school often serves as an act of violence that is dehumanizing to most students, and in particular to students who are Latinx, Black, and Indigenous, by privileging such things as logic over intuition, speed over process, and algebra/arithmetic over other forms of mathematics (Gutiérrez, 2018a, b, c). For me, school mathematics carries out this violence by preventing students from being whole (Gutiérrez, 2013).

However, I read Hottinger's underlying goal as addressing exclusion—getting women and students of color to see themselves as mathematical subjects, as doers of

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mathematics, as capable of mathematics. If she sees dispossession, it is over what she refers to as "mathematical subjectivity." My overarching goal is not simply to allow Indigenous children to see themselves as mathematical during school. Rather, I aim to support such children to see themselves as Indigenous inside and outside of school, to be whole and attached to land and water. It is a subtle but important difference, as the first perspective, in some ways, suggests an approach that has occurred within the ethnomathematics movement in schools, and is present in some of the responses to the Truth & Reconciliation Report in Canada (TRCC, 2015). That is, a Eurocentric frame has tended to be used as the measuring stick by which we evaluate other cultures' practices as "mathematics." In this way, we look for examples and ways that other cultures are mathematical. From Hottinger's perspective, we help girls reconcile their success with their understandings of their femininity and we acknowledge/include them in mathematics classrooms and histories. The goal seems to be on developing an existence proof about other people doing mathematics rather than understanding how other forms of performing mathematics can make mathematics, overall, more vibrant and how other epistemologies and ontologies can further support Indigenous people in becoming themselves. In essence, many efforts in mathematics education push mathematics into the culture of Indigenous people, to find mathematics in Indigenous people's practices. Instead, we should be considering how Indigenous practices and first principles might be incorporated into our understanding of mathematics. To be clear, I am not saying that Indigenous learners should not have access to Western mathematics, but they should understand how the current form has been used as a tool for imperialism. As long as we continue to use the same metrics and principles, we will get the same mathematics, the same mathematicians and, unfortunately, the same relationships with other nations and the planet.

Variables and Forms of Analysis

Eurocentric ways of knowing tend to focus on a small number of variables and understanding them quantitatively, especially in order to create models that help humans optimize outcomes that can control the environment. Blackfoot scholar Leeroy Little Bear (2011) reminds us that in focusing on a small number of variables, we often miss the big picture. Indigenous knowings can attend to a small number of variables (i.e. taking the measurement of only one body part in order to make clothing because notions of ratio and similarity can be applied). However, they rely heavily on a different kind of logic (Verran, 2000; 2001) and emphasize a holistic view of large amounts of variables and understanding them qualitatively (Berkes & Berkes, 2000; 2009). In alignment with this realization, some researchers interested in better supporting Indigenous students have suggested the need to move from a focus on sequential teaching of number and algorithm to holistic teaching of pattern and structure (Matthews et al., 2005).

Although counting is important in any culture, for many Indigenous nations, there is a greater emphasis placed on number as having meaning and significance, understanding signs and relationships between things (Berkes & Berkes, 2009). Just as computers are

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able to compile and read a lot more information and more quickly than our human brains can, they cannot always make sense of it in ways that are consistent with human ways of knowing. That is why big data can only get us so far; such data can point to correlations, but not causality or meaning.

As aforementioned, Indigenous ways of knowing have been seen to parallel fuzzy logic, also known as many-valued logic, which concerns itself with partial truths. Fuzzy logic is used in electrical and computer systems that need to be responsive to changing conditions. Such logic follows a "rule of thumb" because the system does not have sharp boundaries or because information is sometimes incomplete. Berkes & Berkes (2009) lay out an important comparison between Indigenous ways of knowing and fuzzy logic that I find useful. They note that Indigenous knowings bring in large amounts of qualitative information to address the complexity of ecologies, which are neither bounded nor fixed. In essence, many Indigenous communities apply a way of interacting with the environment that is consistent with fuzzy logic. That is, they prescribe ways of moving through the world with classifications that transition gradually rather than abruptly. Indigenous knowings are based on human judgment, for example the degree to which something fits a category rather than on rules for defining it within or outside of the category, as a member or non-member. This emphasis on human judgment over following rigid rules underlies many of the teachings in Indigenous communities that raise children to do what is "right" versus what is "correct." There is always a holistic picture that needs to be considered. Moreover, these knowings are local, meaning context can change what helps define the rule of thumb as well as what actions should occur as a result of that rule. This element of judgment is qualitative rather than quantitative, but there is a kind of quantification in the collective memory of people and various samplings throughout the day, year, and across generations.

What counts as evidence, repeatability, and quantification is different within Indigenous nations. To be clear, I am not saying that Indigenous people are not interested in quantifying, but it involves a different process and embodies a different meaning. Quantifying is not about breaking things up into discrete units and measuring. It is not about counting the number of sick animals in a region, but more about the degree to which something is normal or abnormal, given the many years in observing and being in contact with animal nations (Berkes & Berkes, 2009). This process involves extensive sampling that also attends to new information and relies differentially upon Elder knowings as well as collective knowings in the community. As such, conducting one-time measures with the greatest accuracy may not be a sought-after goal.

In school, learning to assign variables and apply linear thinking could be considered childish for Indigenous students raised with community knowings because such thinking reduces the complexity of the situation. That is, to follow school mathematics, many Indigenous children may feel they are robbed of more sophisticated or complete ways of knowing or ways of communicating those knowings to others. When students

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are asked to "show one's work" and the work is not derived from linear or binary thinking that can be recorded in sanctioned ways or when the knowing is so obvious or is simply "felt" and does not need to be written down, students can feel they have been made to look dumb. The complexity of Indigenous ways of analyzing data calls into question the overreliance on quantifying, "precise" measuring, and the use of variables to represent concepts that is common in US mathematics classrooms.

When schools attempt to understand what and how students know things based primarily on their ability to notate it correctly with paper-pencil tools or symbols, they may miss much more sophisticated ways in which students come to know. Schools may convince themselves that they have ascertained the thinking process of the student when they ask them to show their work or when they conduct clinical interviews. However, such practices may simply encourage students to "leave behind" other ways of accessing information or teach them to "play the game" of school mathematics well enough to be seen as legitimate. As such, understanding the limitations of written variables and quantitative models for Indigenous students is not only important for their own flourishing, but also opens the door to reflect on how schools may miss the complexity of knowing that non-Indigenous students also experience.

To be clear, I am not saying qualitative measuring is better than quantitative or that the two are completely distinct. However, quantification and precision have been the cornerstones of a Western mathematics that erases other ways of thinking. There is a need for both qualitative-centric and quantitative-centric approaches for further developing complex knowings.

Universality and Abstraction

Although there is some overlap, the <u>universality</u> of things emphasized in Eurocentric traditions (Gleiser, 2015), does not align well with the <u>unity</u> of things, viewing everything as related, emphasized in Indigenous communities. By "universal," I mean applicable to all purposes, conditions, and situations. By "unified," I mean coherent or connected in a way to be seen as a unit. Universalism, with a focus on enduring truths, erases the connections and relations that individuals and nations have to each other, as parts are viewed as largely interchangeable. Earlier, I mentioned the importance of enduring ceremonies and maintaining lands and waters. However, even this aspect does not reflect universality. All meanings of sacred lands have not been identified in a way that remains static over time. All spirits are alive and sacred locations, as well as their meanings, could change (Deloria, 1992).

Abstraction is one tool that is utilized to show how things are generalizable and, therefore, universal. Within Western mathematics, a certain special case implicitly asserts something about a general theory. For example, understanding something about finite groups, cyclic groups, or symmetric groups underscores the concept of group theory and the properties of groups. Each of these special cases illuminates something unique. However, those special cases do not negate the presence or the "truth" of the general case. Universality is maintained.

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Mathematics is determined by the meanings we assign to terms. For example, with the transitivity of identity proposition, we say that if a = b and b = c, then necessarily a = c. However, for many purposes in life, “a” may serve to substitute “b” in some occasions but might not work to substitute for “c.” For example, a mother may be equivalent to a father when considering the need for a signature for a child under 18, but not when considering the ability to breastfeed. One might question if a is truly equal to b, why do we have both? Would both exist if they weren’t truly different in some way? However, the transitivity of identity cannot be disproved by empirical evidence (e.g., place-based relationships with others) because it is taken to be true as a proposition, based upon its definition. It is the truth we have agreed upon apriori.

In fact, Eurocentric mathematics reasoning (logic) does not help us prove anything new. Rather, it is a process for making explicit what simply was implicit in the premises that we set down in our original postulates. That is, Eurocentric mathematics does not help us develop new knowings based on empirical data; it only allows us to validate and name such knowledge. As such, it is really most applicable to arithmetic and disciplines that derive from arithmetic, which tend to be the focus of US school mathematics. It does not capture well topology, geometry, and fuzzy logic.

The valuing of abstraction over context in mathematics is evident in the naming of the world around us. Even when shorelines are not quite perfect fractals, when Virahanka numbers do not fit all pinecone species, when phyllotaxis and the golden ratio do not fit all leaf placements on plants or a human’s sense of proper proportions for rectangles, (e.g., Cooke, 2006; Hoge, 1997), academic mathematicians continue to convince others that these patterns are important and enduring algebraic or geometric imperatives that reflect universality. Settlers have used abstraction to develop models or formulas that will apply in most if not all circumstances, often for the purposes of controlling one’s environment. Yet, even academic mathematicians have suggested universality is simply a myth that mathematicians protect (Hersh, 2014; Gleiser, 2015).

I do not mean to imply that abstraction is not important at times or that it is not a value of Indigenous peoples. In fact, most Indigenous stories rely upon an aesthetic that values metaphor. That is, a story tells you of a particular event or idea, but also relates on many levels to things not particular to the context, for example highlighting first principles such as reciprocity, relationships, self-determination, or the importance of making. In this sense, the story is both concrete and abstract, offering different meanings depending upon the one hearing it. The learner must draw their own conclusions from stories. Many creation stories follow this pattern and support the metaphoric mind that departs from rationality and offers creativity and intuition as important processes (Cajete, 2000) that relate to self-determination (Simpson, 2017; Alfred; 1999). However, abstraction as a form of metaphor is not normally taken up in a US mathematics classroom.

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Evaluation, Learning and Self Determination

When it comes to schooling, we cannot separate the practice of evaluation from definitions of success or learning. That is, when we say we are evaluating students' learning, we are offering an implicit definition of what it means to be educated. Researchers have noted the distinction between education (European culture) and educación (Mexicanx/Chicanx culture). The former involves formal academic training and book smarts, while the latter focuses on moral development, forms of respect, and knowing one's place in the world (Elenes, et al., 2001, Reese, et al., 1995; Valdés, 1996). Similarly, Tewa scholar Gregory Cajete has suggested that being "educated" means knowing one's gifts and how to give them in the world (Cajete, 2000). In this sense, education involves a shift from the individual and what they know to how a person is received in community, especially because all knowings are based on relationships. These distinct definitions of education imply different forms of intelligence and, therefore, different logics. The emphasis on forms of respect, knowing one's place in the world, and knowing how to give one's gifts suggests that education cannot be separated from doing. Education is an active form of knowing, as it requires being received in the right way, being understood by others, and being able to live in a good way. That is, education in Indigenous communities cannot be separated from ethics, politics, government, or what might otherwise be considered simply social interactions.

For many Indigenous nations, self-determination is key. For me, this is exemplified in the manner in which the Rarámuri nation has chosen to remain isolated, living in the Copper Canyon, and specifically within the caves of mountains in the winter months. When colonizers arrived, our Anyáwari escaped to the mountains where the terrain was difficult for colonizers to follow. Many Rarámuri remain there, independent of the state of Chiahuahua and the colonizers' ways, even though outsiders argue they have given up access to government services like healthcare or city infrastructure. Yet, creativity is required when resources are scarce, which taps into the metaphoric mind. The spirit world helps guide us on our lifelong journey of learning, so the need for schooling from the government is less important because we are surrounded by many teachers who are our relations and because the value of self governance is critical. Cajete (2000) reminds us that we must have "face" (identity, our story is our word), "heart" (passion for constant learning and the work that it will entail), and "foundation" (our gifts and talents that help us do the work of learning). While I mentioned earlier that all knowings are plural because they are always reflecting our relations with others, self-determination recognizes that the process of knowing is also subjective, and, therefore, assumes meanings are not universal but must make sense for an individual in her connection to lands and waters.

Consent is another important characteristic of self-determination for Indigenous nations. Consent means all beings are involved and informed (Simpson, 2017). Some scholars have referred to this autonomy as "mental sovereignty," being able to maintain ways of knowing that are deeply personal (Zuni-Cruz, 2008) and help you remain

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whole. Part of learning and fulfilling one's responsibilities involves making sense of the stories and teachings presented to you throughout your life. These lessons cannot be imposed on a person externally as is often seen in the coercive relationship between teacher and students in many mathematics classrooms across the US. Instead, an individual, supported by community, must use all of the pedagogic tools available (e.g., observation, listening, making, singing, proper asking, dreaming) to learn and share one's gifts. Having to make meaning on their own helps Indigenous children learn to be self-sufficient and able to solve problems of their own or in their communities.

If children experience dominance and non-consent as normal parts of school, then that normalization can be used against them by those in power. Simpson articulates this well,

> Within the context of settler colonialism, Indigenous peoples are not seen as worthy recipients of consent, informed or otherwise, and part of being colonized is having to engage in all kinds of processes on a daily basis that, given a choice, we likely wouldn't consent to. (Simpson, 2014, p. 161)

For me, mathematics plays a particular role in non-consent by introducing rules that, for many students seem arbitrary. That is, these rules generally appear in textbooks from out of nowhere or are pronounced to students by the teacher. Learning to follow arbitrary rules that are created by others is the first step in forming a compliant citizenry (Gutiérrez, 2018b).

When we understand how mathematics is implicated in the process of dispossessing Indigenous people of an elaborate system of knowings that are systemic, complete, and support flourishing, we can recognize the importance of taking ethical action. Such action involves not only restructuring schooling so that it better supports such systemic knowings to benefit Indigenous people, but also making non-Indigenous people more aware of potential knowings they could appreciate and possibly learn. Doing so sets the stage for non-Indigenous people to become conscious of knowings they also may have been forced to leave behind in the process of becoming legitimate participants in a high status subject like mathematics or in a technological society. As such, understanding mathematics as dispossession is important for both Indigenous and non-Indigenous people. When it happens within the constraints of schools, I refer to this process of acknowledging dispossession and attempting to bring back that which has been erased by schooling as "rehumanizing mathematics" (Gutiérrez, 2018a).

Summary

I have argued that school mathematics serves as a form of dispossession for Indigenous people. I name this dispossession as a kind of slow violence (Nixon, 2011) that continues to accumulate and take its toll on specific bodies, in particular. Rhee and Subedi (2014) articulate this well, "When your (ways of) being and knowing are constantly delegitimized, disrespected, marginalized, inferiorized, attacked, erased, and/or destroyed, how do you continue to be?" (p. 340) This goes to the heart of continuing to remake oneself as Indigenous. And, yet, this form of violence is crafted

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and sold as "rights" to Indigenous communities. It comes in the form of students' rights to a STEM career and its accompanying higher income, the right to equity and the closing of the achievement gap, and the right for students to have access to a "rigorous" curriculum that will prepare them for a technological society. When this messaging is successful, people relinquish their mental sovereignty; humans become labor in a technological society; and mathematics operates in protection of capital.

My experience as a mathematics teacher and a teacher educator has shown me that, when a child is not doing well in school mathematics, even mothers can end up siding with the system, confessing they, also, were never good at mathematics, as if it were a genetic trait passed onto the child rather than a reflection of a school system that does not know how to value the complexity of one's knowings. We should question what is lost when Indigenous children are required to view the world through a 2-value logic, a privileging of a particular form of abstraction through written symbols, universality, static objects as separate from humans and with enduring truths, and a form of mathematics that operates outside of spirituality, ethics, or individual self-determination. In trying to encourage Indigenous students to engage in Eurocentric school mathematics because their lives or their communities will be fuller with more mathematics, they are implicitly being fed a message about the superior position of the Western world. That message has lingering effects when people believe society is advancing in a linear fashion and that only more mathematics, science, and technology will help speed up that progress.

Indigenous knowings support students to see the world with a logic that moves beyond binary framings to value fluidity and probability, concepts that might strengthen our understanding of fuzzy logics or new mathematical concepts because everything is in motion. The goal is not to develop lists of Indigenous vs. Western, but to understand, from specific examples, how schooling can feel alienating and limiting rather than affirming and expanding. To be clear, I am not calling for an "anything goes" kind of mathematics. A haphazard version of mathematics would not have allowed my ancestors to survive and navigate the stars. The forms of mathematics that have proven to be useful for both problem solving and joy have persisted and helped us navigate this place.

Mathematics in Society

Indigenous people are affected by conceptions of mathematics not only in schools but in interactions in broader society. Based in a kind of binary logic, mathematics in broader society further supports dispossession as a way to name the world, categorize, and relate to other nations around us. Time and space are presented through a Eurocentric logic as linear. Yet, Indigenous peoples have always recognized that we participate in past present and future together, as all things are in motion. Mathematicians who have studied Indigenous cultures throughout the world have corroborated a variety of non-linear perspectives on time and space (e.g., Ascher, 2002).

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I do not mean to suggest that <u>all</u> Indigenous peoples (ancient or modern) live in harmony with their natural surroundings. However, the kinds of cultural practices that have helped us survive have tended to be ones that support the idea of humans as members of an ecological system that shows respect for lands, waters, and other relatives; these cultural practices are grounded in a deep and evolving understanding of complex relationships and processes. Although based on a different set of logics, both rigor and replicability factor strongly into Indigenous knowings.

I mentioned earlier the limitations of binary thinking for solving complex mathematics problems in the classroom. Those same limitations arise in navigating life. Depicting the world as binary (e.g., people with "disabilities" v. ones without) normalizes divisions and hierarchies and becomes the tool for arguing that some humans are more deserving of their status, as has been shown with respect to animacy, queerness, and dis/ability. In fact, distinguishing humans from nature is what often serves as the rationalization for humans to exploit animal and plant nations for their own needs. Non-Indigenous humans have continued to justify their superiority over animals and nature through such arbitrary markers as "opposable thumbs" that have allowed them to be "tool users." Yet, Western scientific communities are slowly catching up to what many Indigenous peoples have known for centuries about our relatives' use of tools. For example, kites and falcons, also known as firehawks, have been documented as carrying burning sticks from one location to another in order to spread fire (Nicholas, 2018). Doing so allows the birds to take advantage of fleeing rodents and reptiles that are the source of their food supply. Starlings fly in murmurations as a way to protect against predators and to be more efficient in feeding (Butler, 2009). Anyone who has watched these murmurations as they undulate and flow like bodies of water through the sky know they are performing a sophisticated social algorithm that surpasses the skills of humans. And, plants have been found to use decision-making systems similar to human brains (Topham et al., 2017). The idea that there is some distinct dividing line between humans and our plant, water, land, and animal relatives seems unjustifiable by any science or mathematics.

Historically, Eurocentric ideas of measurement, length, and area have been used for deciding trade and land allotment (Bishop, 1990). Those ideas were forced upon Indigenous populations. For example, Cartesian geometries and conceptions of "land as object" have factored into the creation of maps that have served to separate Indigenous peoples from lands/waters through laws and policies. Such government mapping projects, based largely in 2-value logic, have historically served to delineate lands as either sacred or not, thereby allowing only certain lands to be protected or occupied by Indigenous people, and opening the door for destruction or occupation of other lands by settlers (Bryan, 2009). Yet, as my elders have taught me, maps are carried in our bodies, not on paper.

During the 2004-2005 academic year, my husband, three children, and I lived in Zacatecas, México. I had gone as a Fulbright scholar to study how mathematics was taught in secundarias (secondary schools) and my children attended school there for

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the year. In preparation for our trip, and because we had driven our car to México, we purchased several road maps, thinking they would help us get around more easily. Zacatecas is a World Heritage city, located in a narrow valley at the foot of el Cerro de la Bufa. The city is home to Wixáratari (Huichol) nation who farm the hills, are experts at elaborate beadwork recognized throughout the world, and who lead large pilgrimages across the state to perform ceremonies. Zacatecas' roads were constructed to follow the paths of rivers and mountainous terrain. My experience getting directions in Zacatecas highlighted how useless those maps were because buildings were not numbered or sequential and because lands did not expand linearly or by way of Cartesian coordinates. Successfully navigating the streets of Zacatecas required regularly stopping to request guidance from individuals on the street. Their directions never included street names or numbers, perhaps because it was assumed one could figure out those details upon approaching, but more likely because their relationships with these places were not of a symbolic/numeric form. They knew of these places by having attended an event there, having met with others there, having walked those particular lands. Their knowledge was of the form of "conocimiento" that was personal and attached, not distant and objective. Looking back, I would argue that even in 2004, many Mexicanx and Wixáratari people rejected maps as a way of relating to land, a politics of refusal.

Eurocentric mathematics also serves as a means to exclude, a tool of imperialism. That is, Western mathematics in North America has been the basis of arbitrary metrics and laws that fit the settlers' needs. For example, the US government has developed a minimum blood quantum for Indigenous people to remain on a tribal registry and qualify for rights or governmental assistance. Such laws prevent children of parents from different nations from maintaining their blood quantum for a given nation and, thereby allow the government to continue to decrease the overall Indigenous population. This was the case for friends of mine who one partner was Anishinaabe and the other Dakota. They were living on a reservation and contributing to language revitalization efforts there, accepted as members of the Anishinaabe community. However, their child was in jeopardy of not being recognized as "Native" by the US government because of the metrics used. The same Eurocentric logic that maintains a minimum blood quantum also has factored into the one drop rule for people who are Black, a metric that throughout history has positioned them as subordinate, from accumulating wealth, and/or as being justified as property that can be owned by others. When it suits the settler, Eurocentric mathematics is employed as the value-free logic that justifies binaries and hierarchies.

Survivance

The Survey of Earned Doctorates conducted by the National Science Foundation (NSF, 2018) indicates that .002 percent of all doctorates in 2016 were earned by individuals who identified as "American Indian" or "Alaska Native." In the combined fields of mathematics and computer science, the percentage is .0017; in all of the combined math-related fields such as engineering, earth and physical sciences, and life sciences,

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the percentage is .0026. Those numbers have remained more or less the same since 1996. Before continuing to read, I ask you to pause and take in those numbers. What story do those percentages tell us? If there is a problem, what is it? And what are possible paths to a solution?

There is a plethora of studies discussing the "Indian mathematics problem," but few provide a critical analysis. For exceptions, see Hankes (1998) and Leap and colleagues (1982). Whitestream perspectives in the US suggest that the low performance and lack of participation indicate one of two problems broadly conceived as "the achievement gap" (Gutiérrez, 2008): 1) such individuals lack the intellect or resources to gain entrance into rigorous mathematics programs and/or 2) there is a leaky pipeline (i.e., such individuals are not adequately supported to complete programs they have started). And, while few scholars today see the problem as reflective of intellect or capacity, Indigenous peoples continue to be positioned as in need of paternalistic care.

Viewing the problem as an "achievement gap" that needs to be closed comes primarily from valuing participation in a society built upon individual achievement, accumulation of wealth, and domination of the natural world. From a different point of view, the low percentages of Indigenous people earning a doctorate in mathematics and related fields may simply highlight how those fields currently offer little in the way of maintaining or remaking oneself. Simpson (2017) refers to this flight from colonial structures as Biiskabiyang, the process of returning to ourselves for resurgence. Having been a member of Society for the Advancement of Chicanos and Native Americans in Science (SACNAS) for over three decades, I have heard many stories from Elders who persisted in their fields. Many of them were the only Indigenous person in the sciences at their university and many carved out new disciplines as a result of weaving together Indigenous knowings with forms of science sanctioned by the academy. They did so because they refused to operate solely within the settler's view of science. See, for example, Chicanx scholar Eloy Rodriguez, who in the 1980s co-founded the fields of zoopharmacognosy (the study of how animals use plants to self-medicate and prevent diseases) and chemo-ornithology (the chemical ecology of insect-bird-plant interactions) and Tewa scholar Gregory Cajete who, in the 1970s, developed new understandings of science and science education at the Institute of American Indian Arts.

But, for as many of those Indigenous scientists who were successful by the standards of the academy, many cautioned me about the challenges of trying to maintain oneself as whole while participating in the sciences and the vigilance I would need to employ. I am especially grateful to have heard many stories from Aniyunwiya scholar Thurman Hornbuckle about our ancestors, their resistance, and the importance of looking to our animal and plant relatives for proper medicines. Over the years, he shared how it was often a struggle to participate in a university that values laboratories funded by outside grants, and therefore particular forms of evidence, while he had a more complex understanding of our animal, plant, and land relatives that came from a different

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wisdom. He reminded me of the importance of refusal and the strength we have to resist and walk in two worlds.

Simpson offers the story of Waawaashkeshiwag[15] who, having been overhunted and exploited, retreated from humans in order to renew themselves as a nation. One could argue that animals, plants, water, and other nations also find ways to withdraw and then renew themselves by reclaiming parts of the urban landscape that colonizers have attempted to sterilize (Bang et al., 2014). Following this metaphor, we witness similar retreat by humans historically. Adams (1995) studied the boarding school experience for Native students and found,

> Indian students were anything but passive recipients of the curriculum of civilization. When choosing the path of resistance, they bolted the institution, torched buildings, and engaged in a multitude of schemes to undermine the school program. Even the response of accommodation was frequently little more than a conscious and strategic adaptation to the hard rock of historical circumstance, a pragmatic recognition that one's Indianness would increasingly have to be defended and negotiated in the face of relentless hegemonic forces" (p. 336).

With respect to mathematics, we see retreat in the form of choosing not to go along in the mathematics classroom, asking, "When are we ever going to use this?"; interrupting the teacher; refusing to do work that seems meaningless; expressing and affirming our bodies by taking up space; or failing to take exams that do not value our histories, our futures, or those of the next 7 generations. In many US classrooms, there is an underlying emphasis on competition, achieving as an individual, and publicly displaying intelligence. When Indigenous students place greater value on maintaining relationships with peers (Pewewardy, 2001) over getting the right answer in order to please the teacher or to "look smart," schools can easily misunderstand the depth of knowings that Indigenous children carry with them.

Because Indigenous knowings are already complete systems, choosing not to participate in school mathematics may be less deleterious than maintaining community knowings and not benefitting from the complementary knowledge that schools can sometimes offer. This is especially true, given that schooling as a colonizing institution perpetuates power dynamics by sanctioning a limited set of concepts regarding what counts as truth or how one can know. In fact, educational researchers have studied students who "dropped out" of high school and who were cast as "failures" by the school system and society (Fine, 1990). Yet, upon further examination, these "drop outs" maintained better mental health than those who stayed in school, thereby calling into question which students were really successful. The ongoing push to get more Indigenous people into STEM fields could be viewed as simply neocolonialism. Learning Native languages, not mathematics, is arguably a more pressing concern for most Indigenous communities. This is true because a loss of language is a loss of spirit.

Most commentaries about the relationship between Indigenous people and mathematics are centered on underrepresentation and the need to close the gap

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(Gutiérrez, 2008). However, given the forms of dispossession I have outlined, we might, instead, view such "underrepresentation" as a politics of refusal, a productive strategy that has led to the protection of Indigenous knowings and pedagogies as well as survivance of nations (Vizenor, 2008). Rather than seeking recognition by settlers—earning degrees in mathematics in order to show intelligence—many of us turn inward, to our communities, to renew ourselves by strengthening our holistic and systemic ways of viewing patterns in our relations. Such patterns provide guidance for how to re-attach to land, water, our languages, and each other. In this sense, our refusal becomes generative. By interrogating some of the ways that school mathematics serves as a form of dispossession, we can see how Eurocentric mathematics is not universal at all, as it fails to apply to the experiences and knowings of Indigenous peoples.

We learned from Waawaashkeshiwag's retreat how not to take for granted our relationship with animal nations. Similarly, the retreat of Indigenous people from mathematics offers important lessons for non-Indigenous people about the limitations of current practices in the discipline and in society.

2. LIVING MATHEMATX

If we want to create a different future, we need to live a different present.

What might it look like to have a form of mathematics that does not side with colonialism? In Living Mathematx (LM), I argued <u>against</u> a Eurocentric form of mathematics that seeks to explain everything and <u>for</u> an ecology of knowledges as a new goal for mathematics (Gutiérrez, 2017a). I explained,

> Ecology of knowledges does not follow a single abstract universal hierarchy among knowledges. Rather, it sees knowledge practices as context dependent. In that sense, it recognizes that different knowledges can address our understanding and ability to relate to one another depending upon our different purposes (e.g., the ways we aim to connect, the problems we seek to solve, the ways we invite joy into our lives) (Little Bear 2009). For example, by seeking to be predictive, generalizable, reductionist, and quantifiable in nature, Western perspectives tend to privilege knowledge as a form of (re)presentation and explanation of reality (Aikenhead and Michell 2011). Yet, given the global crises we face, we might be better served by knowledge as action—a form of intervention (Santos 2007; Andreotti 2011).

The idea of knowings coming into interaction purposefully reflects the aforementioned findings that, to a certain extent, all knowledge is local. Putting knowings into interaction allows us the opportunity to see the contributions of different perspectives. In some ways, my call for an ecology of knowledges is consistent with theory building in science. In Star's (1988) view, "Scientific theory building is deeply heterogeneous: Different viewpoints are constantly being adduced and reconciled…each actor, site, or node of a scientific community has a viewpoint, a partial truth consisting of local beliefs, local practices, local constants, and resources, none of which are fully verifiable across all sites. The aggregation of all viewpoints is the source of the robustness of science." (p. 46)

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While an ecology of knowledges might apply to a range of disciplines often referred to as "social[16]" versus "natural" sciences, I was articulating a vision for mathematics in particular, something I refer to as mathematx. I stated,

> I am arguing that within mathematics, we might acknowledge and value an epistemology of knowledges. That is, mathematically, we might come to see that different ways of knowing, different knowers, and different forms of knowledge are all legitimate, partial, and interdependent.

In developing mathematx, I drew upon three Indigenous concepts that were important in my upbringing: 1) In Lak'ech—recognizing we are all related; seeing others in us and us in others; breaking binaries, 2) Nepantla—Nahua metaphysics that values multiplicity, uncertainty, and movement (e.g., gender fluidity; abstraction in relation to context), and 3) reciprocity— recognizing and fulfilling our obligations to others, where consent is always embedded. Mathematx seeks, acknowledges, and creates patterns that solve problems and offer joy.

My goal was to move beyond a practice of mathematics where humans remained at the center and where a single ontological or epistemological perspective dominated. Among other reasons, I developed the term mathematx (pronounced mathemaTESH) to distinguish it from mathematics as a stand-alone discipline; to highlight the politics of hierarchies of knowledge; to apply a variable that supported movement, gender fluidity and, like the work of Malcolm X, signaled an erasure of persons in the X; and to work against the epistemological arrogance of non-Indigenous humans who too often believe they have rightful dominion over society and the natural world.

Nepantla is a key part of mathematx as it attends to Nahuatl metaphysics—recognizing that we are both <u>in</u> the world and we <u>are</u> the world, not separate from it. As such, whenever we practice mathematics, or preferably live mathematx, we are also (re)making mathematics/mathematx. When we are conscious of our place in this world and our goals, we are better able to not just (re)present the world through mathematics, we purposefully co-evolve the world.

Previously, I referred to relations between these knowledges as "inter-knowledges." In this paper, in order to highlight ways of knowing as action and to honor different nations' ongoing collective wisdom, I use the term "international knowings." With this phrase, I seek to capture the relationships between both knowers of different nations and between the knowings they employ. That is, we could speak of international knowings that include, but are not limited to, the following:

- knowings between different nations of Indigenous peoples
- knowings between Indigenous human nations and non-Indigenous human nations
- knowings between human nations and nations of plants, waters, animals, and/or lands
- knowings between different plant nations

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- knowings between different animal nations
- knowings between plant nations and animal nations
- knowings between water nations and land nations

In LM, I highlighted the concepts of Nepantla and nos/otrx [17] (Anzaldúa, 2000; Anzaldúa & Keating, 2002; Gutiérrez, 2012) to discuss multiple views of reality and the interdependence between people (Gutiérrez, 2015). That is, the "us" and "them" are not separate or consumed one by the other, but intertwined in solidarity. I also highlighted the idea of Two-Eyed Seeing (Hatcher et. al, 2009), whereby one eye sees with Western scientific knowledge while the other sees with Indigenous knowledge and, together, both eyes help see better. This is similar to the argument I was making where I suggested that as Nepantlerx who constantly lives with one foot in one world and the other foot in another, we embody interdisciplinarity and a new way of framing reality for mathematics and science (Gutiérrez, 2007). Building on that work, the kind of international knowing I am calling for today occurs not just through two eyes, but through many, and is consistent with what Bartlett et al., (2012) have referred to as 4-Eyed Seeing or 10-Eyed Seeing.

I also argued that mathematx is a way of being because it is a form of action, a way of intervening in colonialism. This form of action aligns with Indigenous worldviews of being at one with the cosmos. Whereas mathematics is a human practice that reflects the agendas, priorities, and framings that participants bring to it (Gutiérrez, 2010/2013); mathematx is an international practice.

> Because mathematx acknowledges that all persons will seek, acknowledge, and create patterns differently in order to solve problems and experience joy in life, multiple knowledges are valued and sought. These multiple knowledges are important, given that all knowledge is partial and each offers us a different angle and understanding on the world. The goal is <u>not</u> to work towards a summative understanding, as if by simply adding the different knowledges we will have a complete or perfect view. Rather, our work is to locate ourselves in others and others in us, as we attempt to understand our world through patterns. Doing mathematics in this way offers us the opportunity to unlearn our epistemological arrogance. The concept of reciprocity draws upon complementarity in recognizing that different knowledges contribute something others do not. Mathematx nurtures a view of mathematics that always considers strengths and limitations for particular purposes." (p. 21)

In fact, the value of locally developed knowings is important for offering a variety of perspectives. "Though knowledge systems may differ in their epistemologies, methodologies, logics, cognitive structures, or socioeconomic contexts, a characteristic that they all share is their localness." (Watson-Verran & Turnbull, 1995, p. 346) Acknowledging such localness works against the "god trick," a view outside of the system where science happens in some positionless space where everything can be viewed (Haraway, 1991).

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I also argued that local perspectives are important not just for the practice of mathematx but for those who practice it because we are influenced by the actions we perform, especially as they affirm the relations we have with others.

> Not only must we: a) be conscious of the ways mathematics can dominate and b) constantly question what counts as mathematics and who decides, we must also c) think about how we, as living beings, practice mathematics as we interact with others and ourselves. As we begin to reimagine mathematics, we have the opportunity to reimagine the mathematician—who is considered a mathematician as well as how are mathematicians influenced by the mathematics they do? (p. 4)

That is, living mathematx seeks to weave together different ways of knowing and different knowers, a form of attachment to others that strengthens the network.

> When we move from a global universal mathematics to a form of mathematx, whereby we acknowledge epistemological pluralism and are guided by first principles of In Lak'ech, reciprocity, and Nepantla, we are likely to see changes in not only mathematical activity (and products) but also in mathematxns. (p. 21)

If we recognize that we carry with us the knowings of the previous 7 generations and the future of 7 generations in the way we "present" the world, we can understand how mathematx is both a way of being and a way of intervening in the world. In using "present," I mean a form of being present, what Simpson (2017) refers to as "presencing" to create a future. Such presencing is based on repeated recognition of our relatives so that they may, in turn, recognize us. In doing so, we deepen our relationships through reciprocity; we fulfill our obligations; and we remake ourselves. This is what I meant when I said earlier that knowings need to be remade, and in doing so, they remake us.

Mathematics in "nature" has often been the backdrop to arguments for the importance of abstraction because these patterns seem to pre-exist outside of humans and therefore are enduring truths. On the other hand, living mathematx views plant, animal, and other nations as performing patterns and, therefore, as both abstract (covering global spaces) and not abstract (attending to local spaces), reflective of a Nahua metaphysics. The idea of attending to local spaces highlights the fact that no two pinecones perform Virahanka numbers in the exact same way. It is this two-fold contribution of living mathematx—performativity (intervention) and the inclusion of other-than-human persons—that is valuable for me.

Within the field of mathematics education, there is little that captures the performativity and inclusion of more-than-human persons. However, enactivism comes close. It underscores the idea that everything is interconnected (Varela, 1991). That is, there is no way to talk about something the learner knows without considering how that learner has interacted with or influenced their environment (e.g., others). Enactivism sees learners and their environments (including other learners) as co-

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evolving (Towers & Martin, 2015; Martin & Towers, 2015). Therefore, all knowing is action.

In speaking about the process of recognizing and performing patterns as a form of mathematics, I further suggested,

> From a philosophical perspective, perhaps it is neither that we have come to appreciate the "natural" patterns present in plants, animals, and rocks, as Platonists would have us believe (i.e., that they have taught us patterns that were programmed within them or that they developed), nor that we simply project our own aesthetics onto our living cousins (i.e., that we see the mathematics we want to see in our environment) as Realists would have us believe. More likely, our relations and the tensions between us provide the multiple lenses on reality and instability. We are constantly in motion like a Nepantlerx.

What I meant in that passage is if we are trying to locate some kind of content of mathematics or knowing, it lies neither exclusively outside of us (in plants, animals, land) or exclusively within us (in our minds and our chosen aesthetics); rather mathematics happens in the interaction (the relations and tensions between us). In many ways, this view is consistent with the work of enactivism and the concept of structural coupling (Maturana & Varela, 1992). For enactivists, knowledge is not in the head of a person or in the contents of a book, but rather in the interaction.

More than just point out the importance of the interaction, Maturana and Varela's early work (1980) discussed the ethics of such action.

> …when a human being makes the choice of a particular way of living, apparent in the realization of a particular set of social relations, he makes a basic ethical choice in which he validates a world for himself and for others that he has explicitly or implicitly accepted as partners in living. Accordingly, the fundamental ethical problem that a man faces as an observer-member of a society is the ethical justification of the particular relations of surrender of autonomy and individuality that he demands from himself and from other members of the society that he generates and validates with his conduct. (p. xxvi)

Although many mathematics education scholars have used Varela's research on embodied mind to develop the concept of "embodied mathematics," much of this work is linguistic in root, not biological[18]. Such studies tend to use neuroscience to confirm cognition and body changes. Some scholars have argued that mathematical concepts arise from bodily experiences (Lakoff & Nunez, 2000). For enactivists, the body is not simply controlled by the brain; it is its own animate system (diPaolo, 2010)—cited in Reid & Mgombelo (2015). This is consistent with the aforementioned framing of the body within Rarámuri nation. The core of the body is controlled in the chest where the souls are located. These souls have sensibilities that also contribute to the system of the person.

In terms of capturing mathematx, what is lacking in enactivism in mathematics education is the spiritual aspect—that we are doing this for a particular purpose, with

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particular guiding principles to maintain connections and be in cooperation with our relatives in order to uphold our responsibilities. Moreover, enactivism does not explicitly address the fact that all past and future are already part of the present because anyáwari and spirits are with us at all times.

Mathematx affirms not just that we co-evolve the world, but we are actively learning from our relatives who are producing patterns around us, and sharing those patterns with us, when we are open to listening. Potawatomi scholar Robin Kimmerer reminds us,

> Our stories from the oldest days tell about the time when all beings shared a common language—thrushes, trees, mosses, and humans. But that language has been long forgotten. So we learn each other's stories by looking, by watching each other's way of living. (Kimmerer, 2003).

When we listen and observe deeply, we see that the patterns that are produced reflect a kind of interdependence between nations. Some might say this is aesthetics and ecology intertwined. That is, some patterns are beautiful to observe and might also provide advantages for maintaining nations. Goldenrod and asters growing together are an example (Kimmerer, 2014). The bright yellow juxtaposed with royal purple is pleasing to the eye partly because yellow and purple are complementary on the color wheel, but the color combination also attracts more pollinators when they are together as opposed to separate. As such, goldenrod and asters offer the lessons of beauty and cooperation to those who take note of the pattern.

Mathematx is important because it reflects the idea of seeing through the eyes of our relatives. That is, by seeing "you in me" and "me in you" (In Lak'ech), we strengthen our connections with others and we remake ourselves. So, where might we focus our gaze in choosing which patterns to observe in our relatives? Ojibwe, Lakota, Choctaw, Little Shell Band of Chippewa-Cree, Miami, and Navajo scholars Megan Bang, Lawrence Curley, Adam Kessel, Ananda Marin, Eli Suzukovich, and George Strack suggest we could consider how water sees Shikaakwa (Chicago) and how asema (tobacco) sees it (Bang et al., 2014). In urban areas, what are the patterns that have been erased by humans and that our plant relatives live to fill in? How do wetlands "fill in" and "(re)become" or "restore/y" themselves (Bang et. al, 2014)? How does moss do this?

Moss is a plant that follows her own non-binary logic. Moss can transition between dead and alive by the mere addition of water. As such, moss is algorithm, a process that solves problems. What can we learn from the patterns that moss live? How do other plant nations see cities? Trees live and die every year, yet their lives reflect decades, even centuries, of networked presence. Their thinking and breathing is deep and slow, a commitment to cooperation, crafting patterns that restore/y them into the landscape and make them resilient. How do we learn to listen to what our relatives are telling us? Land reminds us that we are spatial thinkers and that we have navigated

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space with our bodies. As such, reference to land, not time, is important for restore/ying ourselves into the world.

While others have talked about science/ecology or language revitalization, I'm suggesting that mathematx is seeing how our plant, animal, and land relatives relate to each other and work together to restore/y themselves and ourselves back into place. To honor them, we need to attempt to live those patterns ourselves, not just in educating, but in organizing for ourselves and reaching out to those non-Indigenous people to whom we are also related. When it comes to actively reclaiming our rightful connection to lands and waters, how do we spread ourselves to fill in spaces? We might apply the lessons we learn from our relatives to projects such as building networks for activism in our communities (brown, 2017). For example, in thinking about how the "whole" is moving in a direction together, while "individuals" are also responding to local stimuli, what lessons do murmurations of starlings hold for us? That is, how does remaining tightly connected to the next six closest relatives allow us to be interwoven and attached to a larger body? How does being connected to that larger body allow us to be flexible and yet robust, fending off predators or those in opposition to our movement? Any local intervention operates in a particular way that is always in relation to what is happening at the global level. Just as our plant relatives learn to relate with non-Native plants or other nations, how do we respond when we come into contact with non-Indigenous human relatives? This is not our primary concern, but it does matter along the way.

3. STRENGTHENING KNOWINGS THROUGH LEARNING SPACES

Real change will not come from within colonial structures like our current schooling system. Decolonization will not happen in a classroom. We can only do our best with the constraints that schools provide while we keep our eye on a much larger vision. Even if schools attempted to incorporate Indigenous knowings, they are unlikely to capture the diversity of different nations. History has shown us that attempts to bring culture into schools have often led to essentialism and a focus on "heroes and holidays." Like learning a new language, mathematx must be lived in relation to others, not sterilized or generalized through textbooks and lesson plans.

What happens to those of us who have trusted the schooling system to offer us or our children useful and meaningful education only to end up feeling cheated? How do we make sense of a system that says it cares about mathematics for every child, but only acknowledges the child who fits within the system? How can we be simultaneously suspicious of hope as a sign of complacency or compliance with the present (Duggan & Muñoz, 2009) while also willing to reimagine a better future for us and our relatives with this planet? How might living within this tension reflect Nepantla? Can we imagine a future where mathematicians no longer side with colonialism (e.g., accumulation of capital, dispossession, military forces and imperialism)? How do we connect Indigenous ways of knowing with Western and other ways of knowing without subsuming one under the other? In Lak'ech helps guide us to think about how others are a version of us and we are a version of them, to look for shared solidarities, not

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commensurability or equivalence. These are important questions with which we must grapple. Let us consider how this work has been approached in various schooling spaces.

As a result of the Indian Residential Schools Settlement Agreement, in 2008, Canada created the Truth and Reconciliation Commission (TRC), tasked with summarizing the history and impacts of the Indian residential school. Their 2015 report offered 94 “calls to action” and put into place a national dialogue on reconciliation and reparations for First Nations people. Seeking to take up the call to action, a group of mathematics educators (both Indigenous and non-Indigenous) with experience in teaching, research, and administration related to Indigenous education developed the Revisioning Reclaiming Reconciling School Mathematics (RRRSM) project. The RRRSM project builds upon an Indigenous framework that values respect, collaboration, and reconciliation and offers a mission and vision for how to move forward in the Saskatchewan province of Canada. Informed by the culture-based renewal of school science program that occurred in the period 2008-2014, the goals are to move beyond an acultural presentation of school mathematics and to develop a “culture based Western school mathematics” that includes the following cultural components:

1) the culture of traditional mathematics: its history, ideologies, values, and presuppositions;
2) everyday practices in the lives of citizens and in occupations and professions that explicitly or implicitly involve either traditional mathematics content or analogues of it (i.e., math-in-action);
3) powerful explicit or implicit influences of traditional mathematics content on society, for which there are political, economic, social and ethical consequences (i.e., another type of math-in-action);
4) traditional and contemporary Indigenous mathematizing found in local Indigenous communities, which can be interpreted as analogous to certain Western mathematics content. Therefore, these examples of Indigenous mathematizing can be learned by students and can be translated into traditional mathematics content, in non-appropriating and non-tokenistic ways, in order to teach in mathematics classes, for the benefit of all students.

One powerful aspect of this approach is it seeks to address the politics of knowledge, history of mathematics, and raises issues of ethics. Supporting Indigenous and non-Indigenous learners to understand the histories and politics of mathematics with a critical lens is key for identifying how school mathematics often colludes with colonialism and for understanding how different knowings might come into conversation.

The focus on a culture-based Western mathematics along side of Indigenous principles is consistent with the idea of reconciliation put forth in the TRC report. Yet, Indigenous scholars have highlighted the important distinction between reconciliation that assumes mutual reciprocity (change from within) and one that assumes Indigenous peoples will seek recognition by white settlers. So, in the eyes of the TRC, where is the place of

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Indigenous pattern seeking/making and joy when it does not have an “analogous” Western version? My review of some of the forms of mathematics that are present in Indigenous systems that are not present in Western mathematics highlights that not only is it impossible to simply overlay Indigenous knowings onto Eurocentric ones, doing so would involve further politics and violence to Indigenous peoples. This violence happens because in looking to understand Indigenous practices, many scholars following the Western tradition strip away the “unnecessary” parts (e.g., rituals, language, connection to spiritual world) so they can be more easily utilized or generalized into other contexts. In this sense, “…only useful Indigenous knowledge systems become worthy of protection” (Agrawal, 2002, p. 291). Yet, once they have been extracted and generalized (sanitized), they are no longer Indigenous or helpful for Indigenous peoples.

What might it look like for schools to try to do this work when so many teachers do not have adequate knowledge of Indigenous languages, the history of First Nations, history of mathematics, an understanding of the politics of mathematics, and are not Indigenous themselves? RRRSM has relied heavily upon approaches used in the culture-based renewal of the school science program. For example, they seek to collaborate with, rather than consult, Elders. Even so, it is difficult to see schools as the primary sites for protection and proliferation of Indigenous knowings. Schools are rarely the kinds of places where students are supported to meaningfully learn from peers, to understand their views, in part, to understand themselves and what it means to be in relation with others. Even if we could ensure that a diversity and depth of Indigenous knowings would be taken up in schools, the fact that schools generally carve the day up into distinct subjects (e.g., math, English, art) and age groups and are primarily taught by middle class white females who grew up in the suburbs means a major problem for implementation. Resources might also need to take place in local communities. In other words, if some projects are focusing their efforts on what schools could do, what kinds of community-led “third spaces” might also be supported (economically) by the government? I am thankful for the ʻōlelo noʻeau[19] that Linda Furuto, who directs the Ethnomathematics Program at the University of Hawai’i at Manoa shared with me: "ʻAʻohe pau ka ʻike i ka hālau hoʻokahi” (All knowledge is not taught in the same school). It is important for Indigenous communities to be able to create our own protected learning spaces, consistent with self-determination and mental sovereignty, that operate outside of school systems.

Other educational institutions have explored ways of making their learning experiences consistent with Indigenous perspectives and developing knowledge useful for their communities by using existing educational institutions as spaces for learning. For example, in the late 1990s, in an effort to assert self-determination and nationhood, Indigenous leaders cultivated a body of Native charter schools where students learn through land centered literacies that serve as the basis of aloha ʻaina (Goodyear-Ka’opua, 2013). Similar schools were developed by partnering institutions of higher education with tribal education, such as Arizona State University’s approach with a

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doctoral cohort in their School of Social Transformation (Huaman & Brayboy, 2017). The Akwesasne Aboriginal Science and Mathematics Pilot Project was another important effort and focused on balancing Western and Rotinonhson:ni (Mohawk) ways of knowing. Importantly, they placed Rotinonhson:ni ways of knowing at the center and allowed Western concepts to expand and reinforce learning (LaFrance, 1994). The Northwest Indian College has positioned Lummi students and faculty as experts who can rethink mathematics assessments, course content, and pedagogy (Bunton, Cook, & Tamburini, 2018). The program takes seriously the idea that students need to be their whole selves by emphasizing the mathematical knowledge and skills that students already possess as a result of their lived cultures and histories. Also building upon the TRC "call to action," and as part of the Math for Young Children Project, Anishinaabemowin teachers, First Nation education counselors, and Elders created and carried out a professional development program that supported early years mathematics teachers to rethink their teaching (Caswell, et al., 2018). Grounded in the Anishinaabemowin word Gaa-maamiwi-asigaginendamowin[20], their work seeks to support language revitalization while working on children's spatial reasoning and geometric thinking. A key aspect of the project included Anishinaabemowin teachers and classroom (mathematics) teachers co-teaching. Similar professional development efforts are underway in other places. For example, based upon the Polynesian Voyaging Society's Hokule'a, the University of Hawai'i at Manoa recently launched the first in the world degree-granting program in ethnomathematics (Furuto, 2018). Taking a kind of "grow your own" approach, the program seeks to professionally develop a new cadre of teachers who could usher the work forward.

One lesson we learn from the range of programs that seek to center Indigenous knowings is they were many years in the making and had Indigenous leaders who fought to design and manage the programs. Another lesson is teachers are viewed as cultural actors who possess deep political clarity about their work, many recruited from the communities of students they seek to serve. Sometimes referred to as kinship teaching because "all parts are related and taught in context" (Mohatt, 1994, p. 177), this work is complex. The challenge/problem is that these schools are still funded and operate under the existing settler state which determines what, how, and when students learn. As a result, not all schools intending to support Indigenous students are successful at avoiding the culture/academic divide that implies one either fully assimilates and gains academic knowledge or remains isolated within "traditional" Indigenous frameworks (Mohatt, 1994), painting the identity of "Indigenous intellectual" as a contradiction (Hermes, 2000) or freezing Indigenous knowledge into a static, traditional/ancient (read inferior) form. However, when we view culture as living and changing, as relational, then the notion of these dichotomies no longer makes sense. Learning from how different programs have navigated the politics of developing and maintaining their sites seems an important research area upon which the mathematics education community should focus. Given that womxn and 2SQ people have borne the brunt of dispossession when it comes to mathematics, how might they

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be more centrally located in community-based learning spaces? I have seen little written from this angle.

Developing intuition in both schooling and community-based learning spaces could be a fierce means of resistance to the constant portrayals of such knowings as inferior precisely because they are not rational, logical, or male. When children are taught to listen to their intuition, it is a form of rejecting the colonizer's way of naming the world and what is important. It is a way of protecting not just the mind, but the body, from a message that says "You are wrong." When Indigenous children learn to listen to their intuition, they are guided to identify their gifts and learn how to share those with others. In doing so, we move away from recognition as the goal and we get closer to the kind of mutual reciprocity of which Coulthard spoke. When non-Indigenous children learn to value intuition and other ways of knowing, they can begin to reverse the epistemological arrogance that schooling or society may have implanted. Acknowledging the importance and connectedness of different ways of knowing would mean that non-Indigenous students would be expected to learn more than logical, rational thinking in order to be recognized in mathematics. The process of encouraging intuition may serve as a means for countering dispossession by cultivating connections.

In helping Indigenous youth understand changing lands and waters in the Pacific Northwest, Chicano/P'urépecha and Ojibwe scholars Barajas-Lopez and Bang (2018) speak of this re-attachment through the art of making pottery. Run through a Native American STEAM camp that operates outside of school, they highlight the mathematical practices and concepts employed in clay work. Such work firmly recognize clay making as a "longstanding cultural practice with deep nature-culture relations that are enacted within the context of family and communal responsibilities" (p. 16). Their work helped Indigenous youth embrace the mathematics of the natural world, including "symmetry, shapes, angles, tessellations, lines, and curves in plant relatives" (p. 17) and to take note of the growth patterns they observed. Perhaps more importantly, it helped remind youth that mathematical knowledge is not separate from cultural activity. In this sense, they were helping Indigenous youth live mathematx by reattaching to land.

The value of a goal like living mathematx is our approach can be holistic. Seeking, acknowledging, and creating patterns for problem solving and joy do not get us into the bind of trying to map what currently counts as Western mathematics onto Indigenous knowings. Mathematx respects that such knowings are already complete systems and cannot easily be captured by previous definitions of mathematics. My goal in proposing mathematx is not just oppositional (locating sites for resistance), but seeks to create something different of our own (a generative politics of refusal). As such, this work will occur most robustly outside of schools. As aforementioned, decolonization does not happen in a classroom. It happens when people are re-rooted to lands and waters—when relationships between nations of animals, plants, humans, waters, and lands are strengthened and connected to the spiritual world. It does not

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occur when we define a mathematics curriculum that is supposed to fit all students. But, mathematx can offer schools a longer term vision.

As Indigenous people, we have a rich history of creating our own education spaces when the education system has failed us and our children. History shows that with respect to Indigenous education, those who possess power have exercised it not to the betterment of those with less power, but for protecting their own rights. Just as 'Traditional Ecological Knowledge" has often been acknowledged as only those parts useful for settlers or whitestream society, we have witnessed within the mathematics education community the development of lists of stripped down or culture free "effective mathematics practices" that are meant to be universal and, therefore, appropriate for all learners. As such, we are reminded that we must keep our eye on both 1) practices that can occur within Indigenous communities and that are more reflective of living mathematx, as well as 2) practices that occur within systems of schooling that might be more reflective of rehumanizing mathematics (Gutiérrez, 2018a, b). Doing so requires supporting Indigenous people to organize and develop additional spaces for our own learning while continuing to demand more humane practices for our children in schools. Whereas mathematx seeks to create a connection and appreciation for knowers who belong to human nations in connection with nations of animals, plants, water, land, and the spiritual world, rehumanizing mathematics seeks to bring these knowings into dialogue within the constraints of schooling. Both have as their goal intersubjectivity and connection, a kind of reattachment that seeks to counter dispossession. Living mathematx seeks to support attachment of humans to our more-than-human relatives, while rehumanizing mathematics seeks to support humans (Indigenous and non-) to reattach to each other.

I have argued that mathematics in school and society operates as a form of dispossession. To be clear, I am naming a process that is active and ongoing, but also incomplete. Being able to recognize dispossession does not imply that Indigenous people have been completely stripped of vital knowings. In fact, various scholars make this point clear, suggesting that in the work of resistance and resurgence, Indigenous people will continue to stand strong and remake ourselves, "as we've always done" (Simpson, 2017). Battiste (2002) reminds us that although Canadian schools and society have attempted to take our knowings, they are always latent, waiting for the proper environment to bloom again. She cites the words of President Hampton of Saskatchewan Indian Federated College,

> The Europeans took our land, our lives, and our children like the winter snow takes the grass. The loss is painful but the seed lives in spite of the snow. In the fall of the year, the grass dies and drops its seed to lie hidden under the snow. Perhaps the snow thinks the seed has vanished, but it lives on hidden, or blowing in the wind, or clinging to the plant's leg of progress. How does the acorn unfold into an oak? Deep inside itself it knows--and we are not different. We know deep inside ourselves the pattern of life. (p. 28-29).

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In Chicanx culture, there is a well-known teaching with which I was raised and that parallels the message of President Hampton. Quisieron enterrarnos sin saber que éramos semilla. (They tried to bury us; they didn't know we were seeds.) Like moss who can transition between death and life with the addition of water, we are poised to reattach to land, water, and other nations in ways that not only allow Indigenous peoples to flourish and to live mathematx, but our remaking opens the door for more international knowings that can benefit the mathematical education of non-Indigenous peoples.

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[1] Anyáwari is the Rarámuri word for ancestors.
[2] Although Traditional Ecological Knowledge (TEK) is commonly used in science studies, I avoid the term for two reasons: 1) "traditional" perpetuates the stereotype that Indigenous knowledge is static and based in the past, and 2) TEK has tended to emphasize the empirical knowledge of lands and waters in a way that reduces complexity to forms that can be commodified for settler purposes.
[3] I use the term modern/Western/Northern/Eurocentric to highlight that there are differences in Western views depending upon if we are talking about the global north or south. I include the term Eurocentric to establish that while the views are predominant in Europe, there has been a blending of views (a co-evolution) with Indigenous views. Yet, Europeans have gained the credit for establishing such intelligences in mathematics. The term "modern" highlights the fact that some ancient European societies align with Indigenous worldviews and practices that continue today. Although "modern" is often used in contrast to "traditional" to mean technologically sophisticated and contributing to progress, I use it to point to a particular point in mathematical history.
[4] See, for example, the work of Beverly Caswell, Jerry Lipka, or Lisa Lunney Borden.
[5] Noted exceptions in the US and Canada include Edward Doolittle (Mohawk professor of mathematics), Zachariah Bunton (Lummi instructor of college mathematics), Filiberto Barajas-López (Chicano/P'urépecha professor of mathematics education), Florence Glanfield (Cree professor of mathematics education), Jason Jones (Anishinaabemowin coordinator), Tracey Jones Kabatay (Anishinaabe educational counsellor), Marjorie LaPointe (Anishinaabemowin teacher), Belin Tsinnajinnie (Navajo professor of mathematics), and Judith Hankes (Ojibwe professor of mathematics education).
[6] Anishinaabemowin is the word for Anishinaabe (also known as Ojibwe) language.
[7] Runasimi is the language of Quechua people.
[8] Diné bizaad is the word for Diné (Navajo) language.
[9] Translation: "Heal, heal, frog's tail. If you don't heal today, you will tomorrow."
[10] Sanskrit language is used to communicate religious and philosophical texts in Hinduism, Jainism, Buddhism, and Sikhism.
[11] Noted exceptions are the Liar's paradox and related Quine's paradox. Intuitionist logic also rejects the law of excluded middle because the only mathematical objects with which we can talk are those that can be constructed.
[12] I borrow this naming from Leanne Betasamosake Simpson (2017).
[13] Rimúma is the Rarámuri word for dream.
[14] I use the term womxn in place of "women" to decenter the idea that womxn are merely the opposites of, or extensions of, "men." I use an x to allow for fluidity in what counts as womxnhood.
[15] Waawaashkeshiwag is the Anishinaabe word for deer.
[16] I place the terms *social* and *natural* in quotations to highlight the construction of such categories. In fact, there is nothing completely "natural" about the natural sciences that can be clearly distinguished from anything we might call social. See, for example, the work of Bruno Latour and Steve Woolgar (1979) who show how scientific facts are constructed.

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[17] Previously, I used the term nos/otras coined by Anzaldúa and nos/otr@s from my work in 2012. However, I choose the term nos/otrx here to reflect the status of language that currently honors 2SQ identities. See also Gerardo et al., (2014) for an analysis of nos/otrx in relationships occurring in an after-school bilingual mathematics club.
[18] Noted exceptions include the body of work by Jo Towers and Lyndon Martin.
[19] Hawaiian proverb.
[20] Gaa-maamiwi-asigaginendamowin loosely translates "gathering to learn and do mathematics together, collectively performing useful action."

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MATHEMATICS EDUCATION: ITS ROLE IN THE REVITALISATION OF INDIGENOUS LANGUAGES AND CULTURES

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Scholars writing from social justice perspectives have previously identified a range of issues associated with the imposition of Western mathematics on Indigenous groups. Collectively, some of the major concerns are: the claim that mathematics is culture free, when in fact mathematics has been one of the most powerful weapons in the imposition of Western culture (Bishop, 1990, 1991); that the political dimension of mathematics is often ignored in regard to marginalised cultures which raises ethical and moral issues (D'Ambrosio, 1985); and that education systems often assume that students from minority groups should be taught subjects such as mathematics only through a majority or dominant language (Barwell, 2003). This is because the language used in schools, as in wider society, is closely bound up with issues of "access, power and dominance" (Barwell, 2003, p. 2).

Paradoxically, schooling in Aotearoa/NZ and mathematics education have played a pivotal role in first supporting te reo Māori loss and then in the modern era, its revitalisation. Māori mathematics educators have utilised mathematics as a vehicle to support the revitalisation of the language. This has been aided by the high status of mathematics in the school curriculum in Aotearoa/NZ generally. I report on our various first-hand experiences, initially at the grassroots level of teaching and then the national level, developing lexicon, curricula and teacher capacity to teach mathematics in the medium of Māori. Drawing on the work of Nancy Fraser (1995, 2003, 2005), I critique these major mathematics education developments from a social justice perspective.

INTRODUCTION

In this paper, I report on how Indigenous mathematics educators have utilised national policy imperatives, such as the high status of mathematics in the general education community, to support the revitalisation of the Māori language and culture. Scholars writing from social justice perspectives have previously identified a range of issues associated with the imposition of Western mathematics on Indigenous groups such as Māori. Some of these issues are:

- The claim that mathematics is culture free (Bishop, 1990), which includes the dominance, until relatively recently, of a Eurocentric narrative for the history of academic mathematics (Joseph, 1992, 1997).

- The poor performance of Indigenous students in many colonised countries being attributed to a range of deficits within the cultural group, rather than because of cultural dissonance (Powell & Frankenstein, 1997).
- The political dimension of mathematics being ignored (D'Ambrosio, 1985, 1999), including how a curriculum is structured and what it contains (Doerrer & Pulley, 2010). A curriculum is determined as much by what it omits as what it contains—thus, it is political and often discriminatory—particularly against minorities. For example, an education system that assumes students from minority groups should be taught subjects such as mathematics only through a majority or dominant language is discriminatory (Barwell, 2003). Thus, decisions about the language of instruction is closely bound up with issues of "access, power and dominance" (Barwell, 2003, p. 2).

However, there is limited research that shows how Indigenous communities have overcome some of these issues. In this paper, I outline some of the work that has occurred around the development and implementation of a Māori-medium mathematics curriculum that has contributed to the achieving of Māori community aspirations for saving their language from extinction and for improving Māori students' achievement in mathematics. To do this, I describe first-hand experiences, initially at the grassroots level of teaching and then at the national level, developing lexicon, curricula and teacher capacity to teach mathematics in the medium of Māori. This is not to say that everything has been achieved, and consequently, some remaining issues and tensions are also discussed.

Before describing the development of Māori-medium mathematics and its role in language and cultural revitalisation, I contextualise the Māori language and education system and the circumstances that led to the Indigenous language of New Zealand, the Māori language, becoming endangered and faced with extinction (Spolsky, 2003).

MĀORI LANGUAGE DECLINE

Te reo Māori, an Eastern Polynesian language, is the only Indigenous language of Aotearoa/NZ. While there are different dialects, tribal dialects are mutually intelligible to all fluent speakers of *te reo Māori*. Unlike other more diverse Indigenous language contexts internationally, such as those in India, Australia or Brazil where there are many languages and some with very small communities of speakers, our challenges are not as great.

The conventional history of the language is that it was brought to Aotearoa/NZ by Polynesian seafarers migrating, most likely, from the areas of Tahiti and southern Rarotonga (Harlow, 2005). *Te reo Māori* evolved over several hundred years in Aotearoa/NZ in isolation from other languages, with many adaptations made to the language in response to a temperate climate and an environment different from that of its tropical homelands (Harlow, 2005). If the language dies out in Aotearoa/NZ, there is not a pool of speakers from another location or country to replenish the current pool of speakers.

When the first missionaries and settlers arrived in the 1800s, Māori had a robust system for educating their children to ensure the survival of their communities in Aotearoa/NZ (Riini & Riini, 1993). After 1840, with more and more European settlers arriving and the creation of a British colony, European forms of government and schooling were established. Resources were explicitly produced in 1846 to support arithmetic taught in the medium of Māori; these included multiplication and money tables for use in missionary schools (Barton, Fairhall, & Trinick, 1998; Williams, 1975). The arithmetic terms used in the text were primarily "loanwords" from English and transliterated simply by giving them a Māori phonology (Trinick, 1999). Simon (1998) argued that the hegemonic function of the missionary schools in the early 1800s was to provide a formalised context to assimilate Māori communities into European beliefs, attitudes and practices, with the intent to "civilise" the Māori population. Ironically, in the early missionary schools, the assimilatory function, primarily cultural and religious, was conducted using *te reo Māori* as the language of instruction (Smith, 1990). As Europeans gained political control, the English language gained prestige and quickly became the language of wider communication and the dominant language of Aotearoa/NZ. By the late 1880s, a number of policies had been introduced that implicitly supported a shift in schooling for Māori students from *te reo Māori* to English (Penetito, 2010; Simon, 1998). This included formal policy that banned the use of *te reo Māori* in all areas of the school such as the playgrounds as well as the classroom. The imposition of this hegemonic model, that is, speakers of one language (English) imposing it on another language (Māori) speaking community, can be traced to ideologies and beliefs that influenced Eurocentric education at the time. As in other situations of colonisation, these educational policies reflected strong assimilationist attitudes that linked proficiency in English to supposed best outcomes for Māori (May & Hill, 2005). European attitudes towards Indigenous languages were based on three central premises: the certainty that bilingualism is onerous; contempt for subordinated, non-standard languages; and a belief in linguistic "survival of the fittest", a social Darwinian view of language (Trinick, 2015).

From the 1860s onwards, schooling in all subjects including arithmetic was taught exclusively in English, generally decontextualised and based on European cultural and economic practices such as trade. The mathematics curriculum was narrow in scope—primarily arithmetic to prepare Māori students for manual labour roles. Students who aspired to secondary school education were required to pass tests, taken in the medium of English—including arithmetic. *Te reo Māori* was often regarded as the prime obstacle to the progress of Māori children in schooling (Benton, 1981). The inevitable result was the marginalisation of *te reo Māori* within education and, by extension, its atrophying over time as a standardised educational language.

Nevertheless, up until the 1950s, most Māori people lived in linguistically isolated rural communities where *te reo Māori* remained the principal language. From the 1950s onwards, the Māori population rapidly became urbanised in response to economic and social struggles in rural areas. This saw Māori move from being a 90% rural population to 80% urban in less than 20 years (Benton, 1981). In urban areas, there was a greater likelihood of Māori mixing with English-language speakers in most (if not all) workplaces (Chrisp, 2005).

Over time, the external forces of industrialisation, urbanisation and overt and covert language policies created internal psychological forces that discouraged Māori families from speaking *te reo Māori* to their children or grandchildren (May, 2012). Many Māori, including my grandmother and others of her generation from working-class backgrounds, believed opportunities for employment and commerce would be open only to those fully proficient in the dominant language—English. It was not that they did not support the learning of *te reo Māori*, but the belief that English-medium education was important for their children and grandchildren for employment was pervasive. As a consequence of these overt (English-only schooling) and covert (English-only workplaces) from the 1940s to the 1960s, the number of Māori children speaking Māori plummeted from an estimated 96% to 26% (Benton, 1981). Alarmingly, this shift occurred in almost one generation showing how quickly language shift can occur in contexts such as Aotearoa/NZ.

It was against this background of rapid and significant language loss that the Māori community resurrected Māori-medium education in Aotearoa/NZ (May & Hill, 2005), albeit in a very different form from the pre-colonisation days.

MĀORI LANGUAGE REVITALISATION: EVOLUTION OF MĀORI-MEDIUM SCHOOLING

At the point of the reintroduction of *te reo Māori* as the language of teaching and learning in the early 1980s, there was no education policy and/or plan for *te reo Māori* use in Aotearoa/NZ (Peddie, 2003). Schooling in New Zealand has been controlled centrally by the government through the Ministry of Education since the early colonial days. As such, schools have been required to follow the various national mathematics curricula. As a consequence, these early Māori-medium schools were required to follow the English-medium mathematics syllabi for schools (see Department of Education, 1985, 1987). On the other hand, the *reo Māori* lexicon and resource development to enable schools to teach in the Indigenous language was highly localised, responsibility having fallen to principals, staff and family communities of individual schools (Benton, 1984).

At the time, *te reo Māori* revitalisation schooling efforts were gaining momentum in the 1980s, a neo-liberal transformation began in Aotearoa/NZ with a raft of reforms, centred particularly on how state institutions including education were to be structured and managed (Olssen & Mathews, 1997). These reforms were underpinned by a shift in

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ideology to a more market-oriented perspective (Olssen & Mathews, 1997). This resulted in major education policy reforms, including considerable curriculum reform in areas such as mathematics (McMurchy-Pilkington & Trinick, 2002). Initially in the curriculum reform process, no consideration was given to the needs of schools teaching in the medium of Māori (McMurchy-Pilkington, Trinick & Meaney, 2013). This caused considerable consternation in the Māori-medium schooling community (McMurchy-Pilkington & Trinick, 2002). Subsequently, the Minister of Education agreed in the early 1990s to develop a specific Māori-medium mathematics curriculum—*Pāngarau i roto i te Marautanga o Aotearoa* (see Te Tāhuhu o te Mātauranga (2009) and originally published in 1996) (McMurchy-Pilkington et al., 2013). While on one level the Minister's acquiescence was surprising, the development of a numerate society has been the goal of the Aotearoa/NZ education system for some time (McMurchy-Pilkington et al., 2013). Thus, this numeracy strategy extended into Māori-medium education.

In the following sections, I explore how mathematics (as one of the predominant subjects used in schooling in this country) came to have a pivotal role in the modern era, of supporting the reclamation of *te reo Maori*, a reversal of over 150 years of educational policy that promoted European culture and language. However, to show how this occurred and what tensions still exist as a result of the compromises made in the last 40 years, I have used Nancy Fraser's (2003, 2005) ideas on social justice.

THEORETICAL FRAMEWORK

This analysis of the development of Māori-medium mathematics education for schooling draws on Nancy Fraser's (2003, 2005) theorisation of the bases of social injustice. Her earlier iterations of the model (Fraser, 1995), were built around a binary recognition/distribution paradigm of cultural and economic injustices. The division of social justice into distinct domains has been contentious, particularly by those who would argue that they are indivisible (e.g., Young, 2008). However, Fraser (2003, 2005) contends that disentangling the different domains of justice is crucial if we are to understand the match (or mismatch) between inequalities and strategies to address them.

Cultural-symbolic injustices, Fraser argues, are associated with "representation, interpretation, and communication, and manifest in cultural domination, nonrecognition, and disrespect" (Fraser 1995, p. 71). The cultural/symbolic injustices can be conceptualised as "ideologies and norms that classify some groups of people as worth less respect than others" (Nelund, 2011, p. 63). These injustices require a politics of recognition. This can involve strategies of affirmation, whereby minority groups such as Māori attempt to affirm their identity through Māori-medium schooling.

Socioeconomic injustices, on the other hand, involve exploitation, economic marginalisation and are associated with unequal distribution of material resources between groups in a society, which Fraser (2005) often refers to as "maldistribution".

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These injustices require a politics of redistribution that attempt to reduce the obstacles caused by socioeconomic inequalities such as reallocating resources to decrease the imbalance or deficit.

Fraser (2005) added a third dimension, the political dimension of representation, as a reaction to the forces of globalization when it became clear that who was making decisions was becoming increasingly important. She argues that representation-related injustices, or "political voicelessness", are becoming increasingly important to consider in struggles for justice and democracy in a globalising world (Fraser, 2005).

Issues of representation occur for Indigenous groups at many levels of society including education. Initially, the Māori-medium schooling movement saw their evolution as a means of developing cultural and political autonomy/emancipation from the New Zealand State education system (Smith, 1990). However, this aspiration has become somewhat muted as government funding (redistribution) is frequently followed by state control (representation).

Fraser (2005) argues that all three dimensions are mutually entwined and reciprocally influence and reinforce each other, but none are reducible to the other. Efforts to work towards social justice must involve all three dimensions (Fraser, 2005). Fraser (2003, 2005) viewed social justice from the perspective of participatory parity—how are we to participate as equals.

Fraser's (2003, 2005) arguments are based on examples of injustices of gender, race, and class in society at large and within nation states (Cazden, 2012). This paper looks at how might redistribution, recognition and representation apply when considering the development of Māori-medium mathematics education, in particular the critical corner stones of development of the lexicon (mathematics language), the curriculum, and the teachers.

THE ELABORATION OF THE MĀORI LANGUAGE TO TEACH (WESTERN) MATHEMATICS

As noted by Cooper (1989), the (re)introduction of an Indigenous or minority language into the modern educational domain and its subsequent elaboration and standardisation to enable it to operate in new domains can take place amid conflicting interests. This was certainly the case in Aotearoa/NZ where *te reo Māori* had been explicitly excluded from the schooling domain for over 150 years. Drawing on Nancy Fraser's (2003, 2005) three dimensions, this section critiques the process of development and the at-times conflicting goals that influenced the codification and lexication of Māori-medium mathematics terminology. It also specifically examines the roles, policies and beliefs of the agents, including the two State agencies, involved in the process.

When Māori-medium schooling was initiated in the early 1980s, there were few mathematics terms available in *te reo Māori* for schooling. The few mathematics terms were predominately loanwords adopted from English. Examples of loanwords include *numa* for number, *whika* for figure, and *meiha* for measure. In the early Māori-medium schools, vocabulary development was largely informal, involving elders, teachers and community working together to establish a corpus of terms for daily classroom use rather than any formal, top-down approach (Trinick, 2015).

However, by the early 1990s, the proliferation of localised word lists from different regions and the publication of different wordlists in *te reo Māori* for the same mathematics terms, raised educational and Māori language change issues at the national level (Trinick & May, 2013). Consequently, the Māori Language Commission, created by the *Māori language Act 1987* to support national language revitalisation, and the Ministry of Education, the agency responsible for supporting and resourcing schooling became involved (Trinick, 2015). The Māori Language Commission was concerned with unwanted changes to *te reo Māori* language, whilst the Ministry of Education was concerned at the need to develop a corpus of terms to enable the development of a national Māori-medium mathematics curriculum up to upper secondary school. For the previous 10 years, the Ministry of Education was ambivalent to lexicon development work in schools (Barton et al., 1998). At that point in time, there was a general agreement by the various agents and agencies to use specific word creation principles and to standardise a list for use in government publications, and as a reference point for teachers (Trinick, 2015).

From 2000 onward, word creation of mathematics terms has defaulted to the Ministry of Education when the Language Commission withdrew from this sort of work. Each Māori-medium mathematics initiative, launched primarily as an outcome of the government's numeracy strategy, has resulted in further expansion of the corpus of terms.

Nevertheless, there was an early recognition that the standardisation of a corpus of terms by itself was "not sufficient" (Trinick, 2015). As Halliday (1978) argued, there is much more to the development of a mathematics register than just lexical expansion. What was also required were new and/or different ways of expressing mathematical ideas and concepts in *te reo Māori*. In English, these forms of expressions have developed incrementally over time as an outcome of social interactions, including functional use in classrooms. Without such a timeframe, groups funded by the Ministry of Education have made a conscious and deliberate effort to accelerate the process, including the development of specific resources to exemplify and model syntactic structures to express mathematics ideas in *te reo Māori* (Trinick, 2015).

When considering the early lexicon and mathematics register development, let us consider Fraser's dimensions (2003, 2005) into how social justice was implemented. Initially, Māori teachers and the community used their own agency (Fraser's representation - albeit limited) to meet their needs to learn mathematics. There was no support (recognition) from the Ministry of Education—the State agency charged with supporting schools. The dynamics changed when the Ministry of Education acquiesced to demands for a Māori-medium curriculum—thus, State agencies assumed some financial responsibility for further lexicon development. Therefore, a modicum of representation was provided as a result of Māori advocacy and agitation which ensured equality of distribution of mathematics curriculum. The catalyst for this was the eventual recognition of the importance of lexical development in *te reo Maori*. This was because agencies such as the Ministry of Education directed by State policy imperatives were concerned with developing State mandated curricula, which could not be done without a standardised corpus of terms.

The linguistic strategy that the Māori Language Commission and Māori mathematics educators adopted to create terms gave considerable recognition to Māori culture. The strategy of adopting phonologically-adapted terms from English was rejected and existing terms formed in this way were purged. From that time onwards, mathematics terms have been created by changing the meaning and/or function of existing Māori language terms (Trinick, 2015). For example, *koeko* traditionally had only an adjectival meaning "tapering to a point". In the development of the mathematics lexicon, it was transformed into a noun to mean "cone." In other examples, terms which had fallen out of use were resurrected. For example, *ine* the traditional term for measure was resurrected and *meiha* (transliteration of 'measure') was purged.

So in terms of developing the lexicon for mathematics, all three dimensions of Fraser's (2003, 2005) model were present, albeit at different times in the development process. We see redistribution in the form of the resources provided by the Ministry of Education to support the lexication of mathematics terms, where previously no support existed. While this view of redistribution does not fundamentally change income (re)distribution for Māori, it does shift the pendulum to a more equitable distribution of resources to support the specific needs of Māori-medium schooling. Instead of borrowing terms, it was decided to resurrect *te reo Māori* terms that had fallen out of use, or expand the meaning of *te reo Māori* terms in use in everyday language by giving them an explicit mathematical meaning. While a Western mathematics meaning was added, the original meaning was retained for everyday usage—giving some cultural recognition to the mathematics lexicon. This strategy was determined solely by Māori based on a political imperative to privilege *te reo Māori* rather than English.

As noted, all three conditions of representation, recognition and redistribution are necessary for participatory parity. It was not until the Ministry of Education provided the financial resources and devolved the authority to develop the lexicon to the Māori community, that all three dimensions of Fraser's (2003, 2005) model were present and a resemblance of social justice in regard to the elaboration of the language to teach mathematics was achieved.

MĀORI-MEDIUM MATHEMATICS CURRICULUM DEVELOPMENT

During this early period of re-establishing te *reo Māori* as the language of teaching and learning for schooling in the 1980s, Māori-medium mathematics was based on mainstream (English-medium) mathematics programmes. These New Zealand mathematics programmes at the time were directed loosely by a number of syllabi and guidelines based variously on the new maths reform movement in mathematics education out of the United States in the 1960s (see Herrera & Owens, 2001 for discussion on these reforms) and on the "back to basics" movement in the late 1970s (Shearer, 2002). This scenario was to be maintained for the next 10 years, maintaining unequal distribution of resources and denying participatory parity in terms of meeting the needs of Māori-medium education. As noted, mathematics education in new schools changed significantly in the 1990s with the development of prescriptive curricula as a consequence of the new paradigm influencing educational policy.

Thus, as an outcome of general New Zealand curriculum developments in the early 1990s, a new mathematics curriculum eventually emerged in the medium of Māori. This was the first time in the long history of schooling and curriculum development in Aotearoa/NZ that Māori educationalists were given some authority, however limited, to develop the first ever State curricula for Māori-medium education. Māori were provided with the financial resources (distribution) but with caveats attached (limited representation) essentially related to neo-liberal economic theory. While written in *te reo Maori* and thus supporting lexical enhancement, it had to be based on the general *New Zealand Curriculum* framework which was very similar to those in other Western countries. Thus, the distribution element was focussed on distributing Western mathematics knowledge as the valued commodity, and that this view of social justice was required to be distributed equally. To achieve this equal distribution, the Ministry of Education had specific conceptions about how the curriculum development process would be undertaken and what the finished curricula would look like (McMurchy-Pilkington et al., 2013).

On the other hand, Māori-medium mathematics curriculum developers saw an opportunity to co-opt the development of a Māori-medium mathematics curriculum to serve their communities aspirations, including the development of a Māori-medium mathematics register (McMurchy-Pilkington et al., 2013). The Māori-medium curriculum designers were focussed on the representation element of social justice. Their issue about what should be recognised highlighted the issue of who should be making decisions that would affect the Māori community.

However, while the enhancement of the mathematics register was beneficial to meeting the community's aspirations, there was minimal recognition by the State in regard to revitalising Māori knowledge in the curriculum. What was required of the developers was essentially a translation of the English-language version, thus significantly reducing cultural recognition. One of the consequences of this cultural denial is that even 20 years later, the sector is still struggling to incorporate Māori mathematical practices into school mathematics, which is why language rights by themselves do not guarantee emancipatory forms of education.

Trinick, as cited in McMurchy-Pilkington & Trinick (2002), stated that the writing of the curriculum "legitimised the teaching of mathematics in Māori . . . led to teacher, advisor and resource teacher of Māori professional development . . . that suited their specific needs [and] many Māori were involved in mathematics education debate" (p. 36). The determination by Māori to revitalise their language saw them take advantage of the spaces that had opened up in the development process, making the process a more enabling one, even within the heavy contractual constraints placed on them by the State (McMurchy-Pilkington et al., 2013). Their manipulation of the system to ensure that their aspirations for language revitalisation can be seen as showing how representation affects social justice. However, the Ministry of Education's reluctance to relinquish control of what should be distributed had implications for the inclusion of Māori cultural knowledge.

In 2006, the government decided to revise the 1996 version of the Māori-medium mathematics curriculum. While the basic structure of the 1996 mathematics curriculum was to be maintained, the earlier restrictive requirements, for example, that it had to be a translation of the English version, were removed (McMurchy-Pilkington et al., 2013). There had been a number of political and educational changes over the previous 13 years that facilitated this change. While the basic tenet of neo-liberal ideology lived on and underpinned the revision of the curricula from 2006–2008, the capacity to develop Māori-medium curriculum had expanded significantly over the intervening decade and the "Ministry of Education appeared more accommodating of difference" (McMurchy-Pilkington et al., 2013, p. 357). However, Māori political determination was still very limited.

From the government's point of view, the impetus and state support for developing Māori-medium curricula can be linked to the need to be seen as distributing the highly-valued subject of mathematics equally to all students through having the same school curriculum. Thus, their view of social justice was focussed on distribution, without acknowledging that Māori should be represented in the decision making about what kind of mathematics curricula would best suit their community aspirations. Too often Māori interests are marginalised and reinterpreted to conform to preconceived notions centred on what is good for all. As noted by Fraser (2005), the transference of material resources to the maligned groups does not in itself remedy issues of representation. Often these remedies leave intact the conditions that create power inequalities, such as that that exists between Māori-medium schooling and the State.

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TEACHER DEVELOPMENT ISSUES

An important issue confronting Indigenous schooling movements around the world attempting to revitalise their endangered language is frequently that of teacher supply (Trinick, 2015). For Māori-medium education, the issue of teacher supply followed similar patterns of ambivalence by the Ministry of Education to that of the lexicon and curriculum. Initially, the evolving Māori-medium schooling movement in the 1980s were left to their own devices to staff their schools. Consequently, a number of schools relied upon native speakers (generally elders) from their own local communities. Unless they had formal teacher qualifications, they received minimal (if any) payment, a situation not unlike other Indigenous schooling contexts (see Owens, Edmonds-Wathen, & Bino, 2015 for examples). In this early development phase, there was no redistribution of financial resource to address the Māori-medium teacher supply issue. Teacher training institutions remained fixated on the needs of the English-medium sector. This was to eventually change, if somewhat diffident to the challenges of the Māori-medium schooling sector.

By the early 1990s, various initial teacher education programmes, under pressure from the schooling sector, responded by developing culturally responsive type programmes. While Māori culture was acknowledged and even given some emphasis, the programmes were not aimed at developing *te reo Māori* proficiency to the level required to teach in Māori-medium schools and most of the programmes were in the medium of English (Stewart, Trinick, & Dale, 2017). One of the arguments used by the teacher education providers was that there was a limited number of Māori-medium specialists with the requisite university qualifications to teach at the tertiary level (Murphy, McKinley, & Bright, 2008).

However, by 2008, 12 programmes defined themselves as Māori-medium (see Hōhepa, Hāwera, Tamatea, & Heaton, 2014), although the number of applicants still did not meet teacher supply demands and the financial sustainability of these programmes is constantly being questioned. The teacher education sector is funded by the Tertiary Education Commission, which has no specific funding formulae for Māori-medium education. Any shortfall in funds must be covered by the institute concerned. By presuming that social justice is achieved by funding all teacher education programmes the same, the Tertiary Education Commission does not recognise the importance of Māori-medium teacher education for supporting Māori community aspirations. Therefore, one of the ongoing issues facing Māori-medium teacher education is economic which relates to the redistribution dimension (Fraser, 2005). The government does not fund them any differently from their English-medium programmes. The political dimension (representation) of Māori-medium teacher education is still very limited.

As with the schooling sector, the teacher education sector policy is dominated by the needs of the English-medium schooling sector. However, on the positive side, there is now a pathway into Māori-medium teaching where 20 years ago none existed, thus giving some recognition to the needs of Māori-medium schooling. The growth in Māori-medium

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teacher education has provided a career pathway for Māori academics and researchers. This is important to ensure the ongoing robustness of Māori-medium schooling movement that strives to maintain the vitality of the language and culture, but also to ensure its relevance in a modern global society.

MĀORI-MEDIUM MATHEMATICS EDUCATION; ATTEMPTS TO MAKE EDUCATION MORE JUST

The language of schooling even in marginalised language groups is not often considered from social justice perspectives, but generally from linguistic rights perspectives. What this paper shows is that schooling in Aotearoa/NZ is paradoxically situated as not only one of the main causes of language loss but also inequality, but also the solution to these very same inequalities. Māori-medium education has strived to address these injustices.

When considering Fraser's (2003, 2005) redistribution dimension, Māori-medium mathematics education has been significantly more resourced than other Māori-medium curriculum areas because of the role of mathematics as a government educational priority area and also, relatedly, because of its high-stakes positioning in the education system. As an outcome of these various education policy initiatives, there has been a considerable corpus expansion, first of mathematics terms and then of the mathematics register, through the development of specialised resources, discussions in communities of practice and functional use in the classroom (Trinick, 2015). A standardised corpus of terms enabling the teaching of mathematics to senior levels of secondary school has emerged over the past 30 years or so when previously none existed. This corpus of terms now appears to be in general use in mathematics education.

When considering Fraser's dimensions (2003, 2005), there has been some recognition to support language revitalisation efforts by validating *te reo Māori* as the medium of instruction, including the teaching of mathematics to at least Year 13. This has helped raise the status of Māori-medium schooling. However, from its inception in the 1980s, most in the Māori-medium sector have argued that the most desirable approach to school and community participation in Indigenous education is autonomy and self-determination (Smith, 1990). However, this has not totally been achieved, because Māori-medium schooling is State funded and thus they are required to account to the State in various ways, including implementing State curricula, being staffed by State credentialed teachers and so on. Therefore, from Frasers (2005) perspective, representation is still very limited because Māori-medium schools are still very subordinate to the State.

However, the wider revitalisation of *te reo Māori* since the 1970s has largely taken place in education and been regarded internationally as a success story (May & Hill, 2005). We argue that mathematics education has played an important role in Māori-medium schooling in raising the status of the language and thus supporting language revitalisation.

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RESPONSE TO PLENARY PAPERS

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AN EXPENSIVE EXTERNAL PEACE AND THE ROLE OF MATHEMATICS EDUCATION IN RECLAIMING HUMANITY AND DIGNITY: A RESPONSE TO TRINICK

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Hegemony is characterised by Fontana (1993) as being produced by societal structures that exhibit the power to 'translate the interests and values of a specific social group [the ruling class] into general, "common" values and interests' (p. 141).Social structures, such as education systems, have been active contributors to hegemony by continually maintaining and reinforcing dominant ideologies through standardised state curricula and policies.

Ever since the establishment of White/Western dominance across broad swathes of the planet, education – including supposedly value- and culture-free mathematics education– has been (mis)used to legitimise and operationalise the values, language, relationships and interests of the settlers, including their political presence and political power. The assertion that Western mathematics is both universal and ethically neutral has been used as a powerful instrument of Western dominance, largely through the claim to be mathematically educating various indigenous peoples(Fasheh, 1990; May & Hill, 2005).

The propagation of Western existence through schools over an extended enough period has contributed to the erosion and even the loss of indigenous languages and cultures, as was the case in the historical event presented by Tony Trinick. In his paper, within two major historical parts, Trinick illustrated the ways in which education in general and mathematics education in particular have played a crucial role in extending and reinforcing Western dominance and in undermining te reo Māori, a language and associated culture that only began to be revitalised relatively recently.

I shall start my response by stating what I found to be particularly fascinating in his paper – that is, the underlying element of hope running through a narrative of regression, change and renewal. No social reality is free of struggle and injustice. Viewed from a broader perspective, his paper sheds light on how, notwithstanding a history of palpable injustice, it is possible for the colonised to once again be and grow. I respond to his paper by explaining my view of the unfolding of the history of this people over the course of two centuries, a history that has witnessed the Māori reclaim the power that had been taken from them and the humanity and dignity that had been denied them.

A FORCED AND EXPENSIVE PEACE

In the initial part of his paper, Trinick explains that, in the 1800s when the first missionaries and settlers arrived, the Māori had a robust system of education that was

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conducted in the te reo Māori language and that helped to ensure the survival of their culture and communities. I believe that this education system promoted harmony within the various Māori communities and, as such, was a system of peace. Yet the establishment of a British colony disturbed this peace through the institutionalisation of European forms of education that were centrally controlled by the government and that eventually imposed formal policies banning the use of te reo Māori in all school subjects and requiring that all schools and students follow the national mathematics curricula. From the late 1800s onward, all school subjects including arithmetic were taught exclusively in English and in accordance with British culture and interests. The focus of the mathematics curriculum was primarily non-arithmetic in order to prepare Māori students for the labour force. English, replacing Te reo Māori, became the language of success, prestige and economic security. A new and contrived external peace was achieved. Children went to school, adhered to the policies and learnt mathematics – but with no consideration provided for the Māori medium and culture, nor for their humanity. An expensive peace!

I ask what Munir Fasheh(1998) asked 'Which is More Fundamental: Outward Peace or Being True to Our Humanity?'.As Munir asserts,

> One of the worst types of war is one which takes the form of marginalising and silencing people. At the same time, however, one way of achieving external peace is by silencing people. But silencing people unjustly means either killing the internal human flame in them or creating an internal 'war' within each individual who feels the injustice. In this sense, peace and dignity don't always go hand in hand (p. 1).

Trinick's paper illuminated the fact that, for the Māori, being true to their humanity was more fundamental. Imposing political and institutional control to the extent of suppressing and even eviscerating the communal language and culture is tantamount to crushing humanity. Loss of language is not limited to the exchange of one series of words to another and still expressing the same realities, perceptions and ways of being. As mentioned by countless scholars in the field of mathematics education, language is more than a means of communication. Language is the accumulations of the histories, wisdom and perception of many generations passed along to the future. Loss of language of a community is loss of its ways of being.

Fasheh believes that proclaiming one's humanity and dignity includes 'expressing one's voice and being ready to be punished for it, which means leading to violence' (p. 1). When the Māori initially expressed their voices, ideological conflict (including violence) followed. They attempted to affirm their identity and dignity through the establishment of Māori-medium schooling. Trinick explains that, during its early stage, Māori-medium schooling was a means of reclaiming the Māori's culture and their emancipation from New Zealand's state education system. Nevertheless, in Trinick's view, the key question became 'how are we to participate as equals?' when all of the funding was controlled by the New Zealand government and when representation-related injustices of 'political voicelessness' were still being

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perpetuated. As Trinick explains, after over 150 years of excluding Aotearoa/NZ, the(re)introduction of te reo Māori into the state-funded educational system took place in the midst of conflicting interests and goals, including the power of state agencies like the Ministry of Education and the survival of Māori communities. Yet, after almost a century of conflict within the system of education in general and within the system of mathematics education in particular, the state's need for national curricula for Māori-medium schools led to the Ministry of Education's need for a list of mathematical terms. Phonologically-adapted terms from English were rejected and new mathematical terms were created by changing the meaning and/or the function of existing terms in the Māori language (Trinick, 2015), an approach that provided not only linguistic continuity but also cultural recognition.

Mathematics education first played a key role in establishing and perpetuating European hegemony over the Māori communities and then played a key role in reclaiming these communities' hegemonically-made-invisible culture. The Māoris' loss of external peace through the near eradication out their language and culture has, almost 200 years later, been largely reversed. The Māori people's assertion of their dignity and humanity is an example of the victory of hope – the hope that, by bearing true allegiance to universal human values, a long-suppressed people do regain their voice and proclaim their unique identity.

A NARRATIVE OF HOPE – IN UNDERSTANDING THE SUFFERING OF OTHERS

In a conversation between Edward Said and Daniel Barenboim (see the link below), Said praised what he called a 'rare thing about Daniel [Barenboim]', specifically that 'he is a person who can understand and experience the suffering of others'. Similarly, Richard Rorty in his book Philosophy and Social Hope explains that no one can hope a utopian society. Rather, all we can do is supplement our pre-existing societies, not by increasing our philosophical sophistication but instead 'simply by having our attention called to the harm we have been doing without noticing that we are doing it' (p. 237). Hope, for Rorty, is more than a state of mind and a goal in action. Rather, he understands hope as a narrative, 'a story that serves as a promise or reason for expecting a better future'. I believe, Trinick's historical account is a narrative of hope and progress, by highlighting the harm in colonising that remained un-noticed for decades.

Decolonisation and regaining communities' dignity are struggles not only for Māori but for many others. In a hope for a more just and equitable future, I like to re-submit Rorty's invitation of a hope: "to sew together a very large, elaborate, polychrome quilt" between members of different communities with a thousand little stitches—"to invoke a thousand little commonalities between their members". To minimize differences, one difference at a time. Tony's story is an example of little stitches made by mathematic education :)

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GROUNDING STYLES AND ARGUMENTS IN PRACTICES: A RESPONSE TO GEORGE JOSEPH

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In this brief note, I have tried to underline the importance of situating styles of reasoning and argumentation in mathematical cultures in the context of institutional learning and work. Using this opportunity, I have also highlighted how retrieving and reconstruction of mathematical practices is possible given that we open ourselves to the possibilities of diverse computational cultures available to us in the form of records of practices of different practitioners in history.

The main task of the social historian of mathematics is to historically ground styles of mathematical reasoning that are usually considered context free. The resources available for the historian to undertake this task from within the mathematical texts are sparse. Rarely do such texts reveal the social context that enabled the systematization of computational knowledge. It is no surprise then, that historians of mathematics keep looking for the 'mathematician' in past cultures when such a socially distinct identity had no place, or they look for 'mathematical equations' in texts which did not follow that logic. In both instances, the questions asked by historians of mathematics have merely served as vectors of translation. When traditions are invoked, as national or civilisational, texts become their resources, often obscuring the practitioners and their practices which went in the making of the texts. Curiously, the thought-worlds of the authors of these texts are mobilized to seek elements or even foundations of these traditions, where the quest for generalization inevitably clouds the mundane world of the practitioners who were contemporaries if not fellow travellers of the canonical masters of invoked traditions.

What if we ground these texts as records of practices of real people, who observed, recorded and systematized how their contemporaries as members of occupations, trades and institutions performed knowledge tasks as part of their contexts of learning and working? The context of work of the practitioner in the social hierarchy of production and distribution provides us with the space to explore the communication and systematization of mathematical skills. Thus we might begin by acknowledging that the orientation of specialized knowledge was conditioned by the social hierarchies under which such knowledge circulated. If the working worlds of the practitioners come to the fore, then is it possible to get a better idea about the sensibilities that Prof. Verghese invokes such as in the so called Indian tradition, the primacy of the operations determined the ways with numbers, where the functioning rationale was to provide an understanding of the practical techniques of calculation and not establish universal truths, and that numerical calculation was not considered an 'inferior activity'. Could such characterisations be ascribed to particular professions in particular place and time in history before they could be ascribed

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national or civilizational traits, even if they serve heuristic purpose to clarify difference in styles and arguments between cultures of computational practice.

In an ongoing effort to retrieve and reconstruct the histories of practitioner's mathematics in India, opening up the field of computational practice to places of work and learning has brought out interesting sources for study. Through this programme, we aim to contextualize the locally embedded logics of mathematical practice and the contextual motivations for the development and presentation of mathematical texts. This approach has some important lessons for us to learn from vernacular sources that cover the complex continuum of practices in the spheres of education, accounting, commerce, bureaucracy, artisanal practice and theoretical pursuit of knowledge. This continuum forms a comprehensive social context, and prevents us from identifying one kind of sources (Sanskrit) as the only 'national tradition'.

These sources that we have identified and involved in archiving and publishing will provide us with the means to analyse how mathematical practices and texts functioned as social and cognitive agents, linking (or setting apart) schools, workshops, civil administration, markets and centres of higher learning. These materials exist in almost all Indian languages and include elementary number primers, arithmetic tables which were products of teaching and learning practices in the elementary schools from almost all the regions of India. For long considered artefacts of a computational past, historians of mathematics have only recently acknowledged the historical significance of tables as important vehicles for making mathematical learning and representation possible.[i] Manuscripts of accounts and records of merchant practices constitute a very significant portion of different archival collections, both private and public. State related revenue accounts, land revenue details, temple accounts, private income expenditure accounts and several kinds of registers of the public offices are the commonly known types. The *mahajani* accounting manuals in parts of North India and Gujarat, the *nakarattar* merchant community's specialised accounting practices as found in their registers are numerous and testify to the diverse computational practices. Another realm that testifies to forms of transmission among the practitioners is the domain of work involving artisanal communities and specialised craftsmen like carpenters, sculptors, goldsmiths, etc. Although the practice of apprenticeship in various crafts and artisanal work varies with the nature of the profession and the community, the nature of mathematical engagement across such crafts involves learning on the job. The world of craft work is steeped in measuring and proportions, creating new material forms through the acts of constant measuring and materially inscribing proportions. In the approach of the traditional 'Indian sculptor', measure and proportion function as integral parts of a mnemonically coded system for transmitting artistic forms over time. This system was also a "constructive" one relying on the use of visual devices consisting of drawn lines and points. The initial delineation of such devices is the first step of a learned procedure or technique of execution which translated the two

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dimensional drawing to a three dimensional sculpture. The complex process of this translation was guided by a mnemonic process where in the artist memorizes the volumes of his forms and their dynamic relationships to one another in conjunction with a working technique that facilitates their realization. Through this the practitioner of sacred geometry, linked measure and form – proportion and morphology.[ii]Along with these materials, there are also available in large numbers, mathematical treatises in the regional languages of India.

The *Subhankari* texts in Bengal have mathematical content recorded in *arjya* verses that deal with computations in activities related to agriculture and commerce. These texts primarily involved the learning of the *arjyas*, with verses of two kinds – first, those which once memorised and understood, facilitated computation when working with indigenous units of measure and currency, and second, verses that posed simple mathematical problems. They often also contained multiplication tables. The teaching of the *Subhankari* continued well into the nineteenth century in Bengal, but has been largely been ignored by the Indian historians of mathematics since 'they did not care to deal with subjects as elementary counting skills and popular negotiations with weights and measures or how children were taught elementary numerical skills.'[iii]
Najaf Haider has unearthed a very important set of texts from the seventeenth century known as *Dastur-u- Amal*, written in Persian, the language of administration of the Mughal Empire. These texts were written by accountants and record keepers and aimed at those who wished to be trained in professional record keeping and accounting. The technical parts of these texts usually contain six themes: 'numbers, notions of time and calendars, accountancy, record keeping, the duties of government officials.[iv] The first section on arithmetic and computation includes 'multiplication tables, calculation of crop-yields, salaries, wages, rates of interest, surface areas, surface areas suitable for land, cloth, stone, wood and so on, and tables to calculate agricultural land, units of weights and measures such as those for jewellers, goldsmiths, and grocers, currency exchange rates, and tables for calculating fractions of money.' The subsequent sections deal with calendrical time-measures, and principles of accountancy, denoting a transition from the Indian style of accountancy into an adaptation of the Iranian style, which used a special technique of writing known as *siyaq*. These manuals also contain actual or illustrative records of the Mughal state while a few actually do have a prescriptive list of virtues of good conduct that would suit an ideal accountant-professional in seventeenth century Mughal period.

In the Odiya speaking region, we have found a huge corpus of manuscripts where mathematical procedures are recorded as musical compositions in verses among other kinds of treatises, and yet again deal with the activities of measuring and counting in an agrarian and mercantile world. The Tamil and Malayalam speaking regions have a similar corpus of texts called the *Kaṇakkatikāram* texts. One of the striking features of orientation of Tamil texts like the *Kaṇakkatikāram*, *Kaṇita Nūl* and the *Āstāṉa Kōlākalam* is the scheme they use to classify the computational field. Computation

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rules are not organized in terms of the types of rules to be employed to deal with the requirements of the computational context, but in terms of the objects involved in computation – Land, Grain, Rice, Gold, Stone, Wages and so on. Computation rules are object centered and not classified by mathematical operation - not as rules for addition, subtraction, division, multiplication, squaring and cubing. This distinct mode of classification points to a definite orientation to practice of particular occupational groups, but of course, only to be generalized as a scheme that someone has to learn in that social context of the early modern Tamil region. It is evident that these three texts stand in the cusp of two spheres of transmission, that of apprenticeship and learning, wherein the work of particular practitioners was organized and oriented towards a pedagogic situation that might have involved both apprenticeship or learning on the job as well as in the elementary school.

ORIENTATION IN PRACTICE

The *Kaṇakkatikāram, Kaṇita Nūl* [v] and the *Āstāṉa Kōlākalam* [vi] follow a scheme of classification of the computational arena that indicate the normative techniques employed to solve problems arising out of situations where in objects need to be measured, counted and assessed. But is this merely a function of textual organization or is it a schema that makes ready association with the work of the practitioner? It is important to discern the practitioner to make sense of the orientation of the computational methods and problems presented in these texts. Measuring land, weighing gold, computing wages, measuring and distributing produce, estimating and computing time constitute the orientation of all these three texts. Delineation of measures and their conversions and all kinds of problems involving them constitute the primary focus in these texts. Problem situations determined classification of the computational field. I will briefly point out typical problem situations using the 1783 manuscript of *Kaṇakkatikāram* to be able to identify the different actors involved in such situations – the people present in contexts who were involved in activities that were mathematical.

The titles of the sections themselves reveal how close the problem situations were rooted in the economic and social life of that society. They contain in them activities of different practitioners engaged in the production, distribution and regulation of resources in the community. In the *Kaṇakkatikāram's* [vii] section dealing with Land, there are thirty verses that deal with practical mensuration involving land, where the measuring rod is constantly present. The person who did the concrete activity of measurement is of course present (if one wants to acknowledge his function), but the problems are addressed not to him, but to someone who would assign a quantified or computed measure to that material activity.

In the next section on Gold the first nineteen verses deal with persons who deal with gold – assessing the fineness of gold and selling it, but in the next fourteen verses we see problem situations where gold assumes the form of money and brings in cultivators as sharecroppers and tax payers in the village land; wage workers ploughing land and other manual labour; persons who had to share profit in

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proportion to income and in a couple of problems even salaried workers who receive money from the treasury come up. Following this there are twelve verses in the section dealing with measuring produce, primarily rice. In problems that deal with conversion of paddy in to rice, the cultivator is constantly present trying to exchange rice with money using various units of volume measures. But the interesting personality in this section is the person who actually pounds rice in a rule of three type of problem, about calculating how many breaths in relation to the number of times pounded, would yield how much rice. Then, there are verses which deal with problems of the stone mason carving out pillars and grinding stones; Verses then bring up the job of the water regulator, or the person in charge of irrigating land from the tank using different number of sluices and the time it takes to do so.

The next set of verses present problems involving land and grain, primarily involving proportions but they bring up multiple personalities – the agricultural labourer once more (except that here, his wages are in fractional quantities and land measured in terms of fractional measures), creditors involved in land related transactions; the labourer who carries grains; the water lifter and his wages; person who deals with commodities like oil, ghee and milk; people of two settlements betting on a fund, people paying tributes; the money lender; the conch maker; the customs duty collector; the courier or the messenger; the singer, the dancer, person who assesses productivity of land and the live stock trader and even the gambler. Almost all of the problems in these thirty-odd verses involve the problem of proportions using the rule of three, five and seven.

It is interesting that a mathematical text could bring in almost all actors involved in the economic life of the eighteenth-century Tamil region through spheres of transactions that entailed measuring and counting. The carpenter making standard sized planks from timber and the weaver dyeing clothes of particular strengths do not appear in this particular manuscript, but they do appear in the *Kaṇita Nūl*. Another interesting aspect in these texts is that the computations dealt with real world transactions in relation to land and its produce, which constitute the dominant concern; the world of the merchant or the trader is relatively less represented. The mathematics of the world of the merchant largely came under the rubric of ‘general problems’ which is not ‘commercial arithmetic’, a common enough assumption for any kind of problems that does not deal with the astronomical, in the Indian history of mathematics. If all the above economic actors were engaged in activities that involved computational moments as part of their working lives, can they be entitled to being called numerate practitioners whose styles of reasoning and argumentation as recorded in these texts testify to prevalent cultures of computation that were diverse across the country.

These sources assume significance for a social history of mathematics in a caste society. Eighteenth-century Tamil society denied access to institutional education to the labouring caste groups with only the landed and the artisanal castes gaining access to what were called the *tiṇṇai* schools. But the *veṭṭiyāṉ*, who belonged to the

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untouchable and physically segregated caste was the person who physically measured land in the contemporary revenue administration hierarchy. Did he not know how to count what he was measuring, holding the measuring rod, walking up and down tracts of land? Or was it only the person who he was reporting to, the village accountant, the immediate superior authority who would be numerate? Did the worker who measured grains not know how to deal with the measuring vessels and the counts that they yielded? Did not the water regulator, the *nīrkkāraṉ* (sometimes called the *kampukaṭṭi*) who regulated tank based irrigation know how to keep time and assess extent of land in numbers? Would they have not learnt from within the family or on the job, just as the *kaṇakkuppiḷḷai* did, to cultivate their ways with numbers? What would constitute 'numeracy' in such a context, especially when modes of learning were in an oral environment and writing was only one instant in the learning process in that period? Or still further, how might we understand the notion of apprenticeship, which concerns itself with transmission of knowledge in relation to only certain caste bound trades and professions, or particular kinds of work?

How did these subaltern life worlds shape contemporary knowledge production, be it that of the local practitioner or of the dignified tradition, say the computational astronomical variety? Could context free knowledge emerge in a socially fractured society? To raise such a question is to propose that the history of knowledge in caste societies will have to reorient their central concerns towards the relationship between the mind and the hand in knowledge production in fundamental ways. The history of the practitioner's knowledge forms is but a beginning in this direction.

If we situate these diverse cultures of computations in the histories of work, occupation, institutional learning in history, the textures of their orientation in practice offers us a better understanding of the context in which forms of mathematical practices were cultivated and nurtured. Acknowledging multiple ways of doing and thinking within the frame of invoked national traditions is as important as identifying and arguing for difference and multiplicity across national traditions. These histories together have a lot to offer for the practitioners of mathematics education.

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NAVIGATING MATHEMATXON SELFSAME LAND THROUGH THE SETTLER-NATIVE-SLAVETRIAD IN THE U.S. CONTEXT: A RESPONSE TO ROCHELLE GUTIÉRREZ

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In this response, I interrogated the theoritizations and meanings of land, relationships to land, and non/humanness within living mathematx and discussed how mathematx may inadvertently reinscribe anti-blackness. Through a full elaboration of settler colonial logics, namely, the triad of settler-native-slave, I located tensions in mathematx when navigating Black geographies and more-than-human geographies.

MATHEMATX, THE LAND, AND THENON-HUMAN

As an Indigenous project, mathematx is fundamentally a philosophical movement in which "the land must once again *become* the pedagogy" (Simpson, 2014, p. 14). The *relationship to land* comprises the epistemologies, ontologies, and cosmologies upon which mathematx is constructed as a sense-making activity and experience related to quantity, logics, spatiality, and pattern. While mathematx–as theorized by Gutiérrez— is borne out of three Indigenous concepts (i.e., Lak'ech, Nepantla, and reciporcity), the relationship to land is its driving force and logic. At its core, mathematx seeks to (re)establish meaningful connections to the land, water, humans, and non-humans alike. In fact, it is through *interaction* with this ecology of knowledges that mathematx emerges. Said simply, mathematx is lived through relationship and that relationship is premised on the land. Essential to mathematx is the de-centering of humans and the inclusion of more-than-human persons—not as objects or entities to learn *about* but as our relatives to learn *from*. Mathematx provides a compelling possibility of "presencing" or creating new futurities and relations among land, water, plant, Indigenous, and non-Indigenous nations. In response to this compelling image, I wish to interrogate whether the current theoritizations and meaning(s) of land, relationships to the land, and non/humanness include Black children and youth in the U.S. context, as a particular and uniquely important non-Indigenous community. To make a finer point, I wish to highlight how mathematx may inadvertently reinscribe anti-blackness by not qualifying meanings of land and opening itself to "the problematics of centering more-than-human worlds…while leaving unexamined differential human vulnerabilities and responsibility in anthropogenic places" (Nxumalo, 2018, p. 14).

COMPLICATING 'LAND AS PEDAGOGY': INTRODUCING THE SETTLER-NATIVE-SLAVE TRIAD

Given the importance of the relationship to land, mathematx is more than a philosophical movement. It is a political movement that seeks to deconstruct settler

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colonialism. Settler colonialism has served to restructure peoples' relationship to land and this restructuring can perhaps best be understood through the *triad of settler-native-slave* in the U.S. colonial society. While often treated as 'a rather simple idea,' I find it useful to (re)elaborate on the project of settler colonialism in order to effectively highlight the potential problematics of land and land relations within mathematx.

Dotson (2018) describes settler colonialism in this way:

> It is a kind of colonialism that occurs when people show up at an already populated space and come to stay. It involves complicated and, at times, twisting doctrines of discovery that operate in epic feats of forgetting and merciless logics of elimination that authorize many registers of genocide that can include anything from unimaginable scales of violence to the violence of forced assimilation. (p. 4)

To effect settler colonialism, Indigenous people must be destroyed and disappeared; the elimination of Indigenous people must be methodical, providing the unfamiliar settler with the necessary expertise and labour to survive (Wolfe, 2006). Dotson (2018) summarizes Wolfe in this way: "In order for settlers to come and stay, Indigenous peoples needed to be removed, but not too quickly and not all at once" (p. 4).Within the logic of elimination, Indigenous people must be dispossessed as prior claimants. However, in this arrangement of elimination, the demands of labour must still be satisfied because labour(that is, abundant labour) is critical to the settler colonizer. In the USA, labour involved "the subjugation and forced labour of chattel slaves, whose bodies and lives become the property, and who are kept landless" (Tuck & Yang, 2012). Tuck and Yang explain the imperilled position of the slave within the settler colonial triad:

> Slavery in settler colonial contexts is distinct from other forms of indenture whereby *excess labor* is extracted from persons. First, chattels are commodities of labor and therefore it is the slave's *person* that isexcess. Second, unlike workers who may aspire to own land, the slave's very presence on the land is already an excess that must be dislocated. Thus, the slave is a desirable commodity but the person underneath is imprisonabe, punishable, and murderable. The violence of keeping/killing the chattel slaves makes them deathlike monsters in the settler imagination. (p. 6)

The logic of elimination makes clear the imbrication of dispossession and enslavement within the settler colonial project. Such an assertion does not seek to equate the "native" and the "slave," or somehow make Black folk 'more' innocent in the dispossession of native lands. As Dotson (2018) states, "[T]here is no innocence in settler colonial USA" (p. 3). This elaboration of settler colonialism is a pivot toward complicating settler-native dynamic by acknowledging the use of slaves and indentured servants in building this particular society. This elaboration is a move to invoke a triad of settler-native-slave and assert there are very different relationships to the 'selfsame' (or exact same) land.

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‘PEDAGOGY OF LAND’ FOR THE UNGEOGRAPHIC?

Focusing on the “slave” within the settler-native-slave triad in U.S. society, some argue there has been an alienation from the land, due to a catastrophic severing to a ‘motherland,’ a precarious Middle Passage to an ‘unknown’ land, and exploitative relations to the ‘discovered’ land through forced labour. There has been a persistent questioning of African American peoples’ relationship to the land, particularly among Afro pessimists, such as Fred Moten (2013),who essentially described Black people as landless or ‘ungeographic’. For example, Moten asserts the following:

> [W]hat distinguishes the sovereign, the settler, and even the savage from the slave is precisely that they share “a capacity for time and space coherence. At every scale—the soul, the body, the group, the land, and the universe—they can both practice cartography, and although at every scale their maps are radically incompatible, their respective ‘mapness’ is never in question. This capacity for cartographic coherence is the thing itself, that which secures subjectivity for both the Settler and the ‘Savage’ and articulates them to one another in a network of connections, transfers and displacements” (Wilderson 2010: 181).

Summarizing Wilderson (2010), Moten argues that both the settler and savage are endowed with a geography and cartographic practices; it is the slave without cartographic practices, i.e., ungeographic. This triadic entanglement to land is clarified by Tuck, Guess, and Sultan who write:

> The Savage, though a “genocided object” (p. 51) is afforded an ontology (afforded by a genealogy derived from land, presumably or at least prior occupation of it) whereas the Slave, positioned within a grammar of suffering (p. 11), is denied an ontology so long as she is denied freedom. (p. 6)

The assertion of Black populations as ungeographic has been roundly challenged by expanding imaginations and meanings of geographic capacity and land relations (e.g., McKittrick, 2006; Nxumalo, 2018; Nxumalo & Cedillo, 2017; Tuck, Guess, & Sultan, 2014; Tuck et al., 2014). Black feminist, Katherine McKittrick (2006)asserts a new concept of geography and connections to place that work both in parallel and orthogonal to a traditional “cartographic, positivist, and imperialist” geography(p. xiv).McKittrick asserts complex understandings of geographies that challenge normative relations to place, land, and space through a decolonial poetics, and refuse “stories of damaged place relations, surveillance, and absenting” (Nxumalo and Cedillo, 2018).

In the case of mathematx in which land becomes pedagogy, there must be legible Black relations to the land for Black child mathematx learning. Mathematx requires such land stories for the Black child to participate in this philosophical and political movement. Fortunately, the Black/Land project (2017) is engaged in this very work of “collecting, (re)remembering, and presencing Black land stories that interrupt view of Black life in North America as ungeographic while also refusing to dwell only in tales of damage-centered Black relations to land” (Nxumalo & Cedillo, 2017, p.

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106).Still, these stories are marked by their scarcity and require specific investment to establish ties to either traditional mathematics or practices in mathematx.

TROUBLING POSTHUMAN GEOGRAPHIES

Posthuman or more-than-human geographies are central to mathematx, insofar as mathematx seeks to decenter humans in mathematical practice.In doing so, mathematx challenges any construct of separateness from between human cultures andmore-than-human natures. Again, the triad of settler-native-slave creates tensions within the theoretical framework of mathematx, given the a priori construction of the slave as a nonhuman within settler colonial project.(Recall Tuck & Yang's assertion that the "person" of the slave (not the labor) within settler colonial system is an excess and, thus, the slave's humanity is a contested and contingent identity.)Given the tensions within this triadic arrangement, Nxumalo (2018) through Sylvia Wynter's work describes the necessity of interrupting human exceptionalism, while troubling a universalized view of the human. Other Black feminist scholars, such as Zakiiya Jackson, build on Wynter's mandate and "question whether posthuman theories might perhaps inadvertently [reinstate] transcendentalist Eurocentrism in leaving largely unexamined, ongoing, and uneven racialized ordering within the normative 'human' that these [posthumanist] perspectives seek to decenter" (Nxumalo & Cedillo, 2017, p. 108).While Nxumalo & Cedillo laud the possibilities of Black feminist geographies, it is an open and uneasy question as to how the uneven geographies and unlivablity experienced by marginalized children can be sufficiently problematized within mathematx, and simultaneously dislodge humanist child-centered views. I, perhaps unadvisedly, desire for the legibility of Black children within the category of human and lack the radical imaginaries required of the futurities invited by mathematx.

CONCLUDING THOUGHTS

Here, I want to be clear on my stance. Black people, Black children, and Black youth are *not* ungeographic. As McKittrick (2006)writes, "Black matters are spatial matters" (p. xii) and spatial matters are indeed land matters. This does not presuppose that land matters are simple matters. The dominant epistemological resource for making sense of Black people's relationship to land is the institution of slavery in the USA and, thus, a damage-centered narrative precedes the desire-centered narrative of mathematx. Therefore, mathematx implicates other projects—a far-reaching Black land project, as well as (although this is less clear to me) the unfinished project of Black humanism. Such projects must be achieved coterminously within mathematx if this philosophical and political movement seeks to avoid being yet another mathematics education project complicit in the perpetuation of anti-blackness. Mathematx must ambitiously hold uneven land relations between human and more-than-human on the selfsame land.

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CONFRONTING THE BIG ENEMY: REACTION TO CABRAL AND BALDINO

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As exemplified by MES, some pockets of self-identified critical mathematics educators have sought to place mathematics education within its historical, cultural, social, and political – in short, human – contexts. In reaction to the parable of Mrs. Smith and the Big Enemy, I suggest paying more attention to the specific ways in which capital and mathematics education are symbiotic, and to positive cases of teachers and researchers becoming politicized. The looming possibilities of various catastrophic occurrences imply the need for urgent action.

HUMANIZING INQUIRY IN MATHEMATICS EDUCATION

As Mathematics Education emerged as a field of inquiry, it originally largely drew on developmental and cognitive psychology (as reflected in the formation of the International Group for Psychology of Mathematics Education in 1976). Subsequent development has been marked by "diversification of influential disciplines and related methodologies – broadly speaking, the balancing of technical disciplines by human disciplines such as sociology, sociolinguistics, anthropology, psychoanalysis, and of formal statistical methods by interpretative methods of research and analysis." (Greer & Skovsmose, 2012).

As indicated by Cabral and Baldino, the humanization of mathematics education, with the social and sociopolitical turns, has penetrated parts of the field. Nevertheless, the penetration of educational power structures remains limited. For example, President Bush set up a National Mathematics Advisory Panel in 2008 consisting of an "unholy alliance" of narrowly empirical psychologists, psychometricians, mathematicians, political activists, and others, that released a series of reports based on a positivist characterization of research while ignoring almost all work within mathematics education as we would recognize it (Greer, 2012). Moreover, the footprints of the military-industrial-academic alliance are easily discernible.

Mainstream mathematics education largely remains in a complacent state of disorder (Straehler-Pohl, Bohlmann, and Pais, 2017).

TELLING OF STORIES

In the present time, with about 35% of Americans loving Big Brother, and being prepared to believe that 2 + 2 = 5 (or sometimes 3) rational argument seems inadequate, so we may turn to psychoanalysis and literary forms such as science fiction, humour, satire. George Orwell's 1984 is a particularly powerful example, introducing Big Brother, Newspeak, Doublethink, and the memory hole, manifestations of all of which it is easy to find in the contemporary post-truth world.

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The Big Enemy reminds me of Nick Naylor, central figure of the movie "Thank You For Smoking", who is a lobbyist for "the Academy of Tobacco Studies" that reports research showing that cigarettes are not harmful to health. As a satire, it is arguably more powerful than a direct appeal to rationality. For example, Indian cigarette packets carry not only verbal health warnings but also gruesome pictures of cancerous lungs, yet about a million die each year from tobacco-related diseases. (Parenthetically, one of the first "successes" of Edward Bernays, nephew of Freud and the father of the PR and advertising industry was in greatly increasing sales of cigarettes to women by sloganizing them as "beacons of freedom").

SPECIFICALLY, *MATHEMATICS* EDUCATION

Cabral and Baldino's parable about Mrs. Smith and BE is about education in general. What might be said about *mathematics* education in particular? For example, mathematics is surely the subject that produces most alienated learning (Lave & McDermott, 2002). How is mathematics education implicated in producing docile humans without critical disposition or agency in relation to the ways in which mathematics is used to control our lives? Might we not say that a major mechanism for teaching children to accept the abstraction of capital is by teaching them to accept the abstraction of number? How can mathematicians be persuaded that mathematics education is about more than passing on some subset of the body of fossilized mathematical knowledge?

WHAT TEACHERS KNOW

Mrs. Smith is portrayed as an innocent, lacking insight and agency. Is that fair? Perhaps, overall, but there are extenuating circumstances. During teacher training (in many countries) a teacher may have been told about "the social turn" but, as pointed out by Donald Macedo, a close collaborator of Paulo Freire, is extremely unlikely to have experienced a course on ideology. Further, any lingering effects of exposure to progressive positions are likely to be washed out after a short period in the day-to-day reality of school, where a teacher is kept so busy that there is little time for reflection. Andthere are financial practicalities (in "Thank you for smoking", Nick Naylor refers to "the need to pay the mortgage" as "the yuppie Nuremberg defense").

Nevertheless, in terms of examples in the context of the USA, there are teachers who have figured out, through reading critical writings on education (e.g. Baldwin, 1963), and through critical observation of what is the case, that – for example – education has become corporatized, in particular through the assessment(or "accessment" (McDermott & Hall, 2007))industry, and that the legislative program called "No Child Left Behind" has left millions of children behind. Teachers in the state of Washington refused to administer tests and have organized national and international resistance (Hagopian, 2014), in which their students have joined them. Activist teachers in Chicago have taken control of their union and have organized strikes and other actions, in solidarity with students, parents, and communities. Rico Gutstein has been active in that struggle, as well as teaching high school students how to use

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mathematics to interrogate the sociopolitical issues important to their lives (Gutstein, 2006).One more example is Rethinking Schools, a grass-roots organization of activist teachers (see, e.g., Peterson, 2015).

AND WHAT WILL MRS. SMITH DO THEN?

Having read about the end of innocence for Mrs. Smith, one wonders what she will do then. To quote Pais (2012):

> Some will say that such an awareness of the problem takes us to a deadlock. Indeed, by realizing that exclusion is something inherent to the school system we realize that to end exclusion means to end schooling as we know it. In the current matrix of world social organization this does not seem possible. Thus … what should be done?

Pais presents strong arguments against acting without a deep analysis of the problem (for which see Straehler-Pohl, Bohlmann, & Pais, 2017).It is folly to proceed with the "same old, same old" until we know that what we are attempting is on solid foundations. However, as Amartya Sen (2009) points out in "The idea of justice", antislavery activists did not need to wait for a complete theory of justice to take action. And Freire expressed the need for balance when he said (1972, p. 41) that "reflection without action is sheer verbalism, 'armchair revolution', whereas action without reflection is 'pure activism', that is action for action's sake."

Further, how much time to we have for contemplation? Time is running out to avert climate catastrophe. The Doomsday Clock is at 11.58 (this is the symbol whereby the Bulletin of Atomic Scientists represents a judgment of how close we are to nuclear holocaust). Global capitalism could crash any time soon, perhaps as the unforeseen result of the interaction of millions of micro-transactions controlled by computers, or just because it is the ultimate Ponzi scheme ("if something cannot go on forever, it will stop" as economists put it).

So, it is reasonable to ask, what do I suggest? As a personal position, I will continue, as a form of political activism among others, my attempts to critique mathematics education and to persuade others to do the same.

CARRYING ON

In Greer, Gutierrez, Gutstein, Mukhopadhyay, and Rampal (2017) we stated that "We have no choice but to continue the struggle". Speaking for myself, this position stems from a sense of indignation – at many things. Is it folly to continue when the situation seems hopeless? It would be very easy to stop trying, stop learning, enjoy the company of friends, travel, wine, literature, art, movies…

In a remarkable testimony, the mathematician/activist Chandler Davis (2015), apologized on behalf of his generation for not doing enough to combat the nuclear Arms Race, in particular the acceptance of the Mutually Assured Destruction (MAD) doctrine (portrayed in the movie "Dr. Strangelove", and reminiscent of the slogan WAR IS PEACE on the Ministry of Truth in 1984). In particular, referring to Von Neumann's endorsement of the doctrine, he asks "How could I, as a junior scientist,

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be disloyal to this mathematically argued theory from one of the world's greatest mathematicians?".

Speaking to the younger generation, he posed a challenge:

> … you must rouse yourself to better efforts than we managed just to have any future, I implore you: have the wisdom and the strength not merely to survive but to survive proudly and happily. To choose the best future.

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CRAFTING NEW DISPOSITIONS IN MATHEMATICS: CHALLENGING DISPOSSESSION A RESPONSE TO ROCHELLE GUTIÉRREZ

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The official form of mathematics causes dispossesion, and Gutiérrez argues for alternative 'knowings', as ongoing processes of deep engagement, through pedagogies of caring and sharing. This response revisits theories of 'situated activity' that allow a reformulation of learning, through relations between individuals, historical traditions, materials and their transformations within environment. Contrary to her suggestion on the process of theory building in science, this paper calls for a crafting of dispositions that challenge dispossession in the teaching of both science and mathematics, by unveiling unacknowledged histories that continue to obscure artisanal or indigenous knowings, pushing majorities to the margins, as the neo-liberal discourse forges new heirarchies between 'skills' and 'knowledge'.

In her paper 'Mathematx: Towards a Way of Being', Gutiérrez looks at how Western/Eurocentric mathematics acts as a form of dispossesion, while arguing for an alternative that respects and supports Indigenous worldviews among others. She calls for ways of 'knowings that seek, acknowledge, and create patterns and relationships that solve problems and offer joy'. As a Chicanx scholar with Rarámuri roots, she raises concerns about efforts that look for mathematics in indigenous practices, rather than incorporating indigenous principles and practices into the discipline of mathematics. The aim is not to create further binaries, separating the indigenous from non-indigenous, but to reframe school mathematics for all, a crafting of new dispositions to challenge the agenda of dispossession.

Western/Eurocentric mathematics convey 'official knowledge' sanctioned by the discipline, but tend to hide its relations to power and imperialism, and obscure its history where people's knowledge has been 'stolen' or appropriated over time. Knowledge is not a product for permanence, but a plurality of ongoing processes of deep engagement, Gutiérrez speaks of 'knowings' as a shared way of being that connects 'humans with animal, plant, land, water, and all other relatives'. Knowings need to be renewed, to renew selves, to survive and sustain. Knowings are robust and flexible systems because they are dynamic, and are sought through pedagogies of caring and sharing, which include observing, wondering, working, asking, listening, crafting, singing, bonding, reflecting, dreaming, participating in ceremonies…..

Situated Learning and *Doing* Mathematics

Gutiérrez indicates enactivism as closest to viewing knowings as interactions, where learners and their environments co-evolve. I suggest we reflect on theories of situated

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learning, which span an entire spectrum of life, from informal to formal settings. These give an interdisciplinary base to understand how people *do* mathematics, in diverse and disparate social and cultural contexts. Unlike traditional cognitive theories that isolate and distance the ‘mind’ from its experience, theories of situated learning do not separate thought, action and feelings. Traditionally knowledge is seen as something that is imported across a boundary, from the external to the internal. Theories of cognitive ‘acquisition’ - of knowledge, concepts, skills, etc. - tend to separate the actor from the ‘acquired knowledge’ as well as the social context. Socio-cultural studies of learning move away from the narrow focus on ‘transmission and internalisation’ of existing knowledge, towards understanding what constitutes new knowledge, as a collective invention, adopted by people when faced with contradictions that hamper an activity (Lave, 1996). More importantly, situated theories suggest that ‘learning disability’, a label which inceasingly encompasses large numbers of children in the global ‘business of learning’ (Rampal, 2018), could relate to those who face not increasingly more difficult but more *arbitrary* tasks at school, with little meaning and purpose for a probable later use in life. However, the same children might perform complicated mathematical tasks, as part of the context of their activity. The use of ‘activity’ as the unit of analysis allows a reformulation of the relation between the individual and the socio-cultural environment (Rogoff, 1995). An ‘activity’ is shaped by the active contributions from individuals, their social partners, historical traditions and also materials and their transformations. The interdependence between the individual, the task, and the environment needs to be understood through conscious ‘foregrounding’ of specific activities, without losing the holistic background. Thus we may foreground either a single person learning or the whole community, but must not assume them to be separate from the activity.

Dewey said “If the living, experiencing being is an intimate participant in the activities of the world ... then knowledge is a mode of participation (Dewey, 1916, p. 393)”. “By doing his share in the associated activity, the individual appropriates the purpose which actuates it, becomes familiar with its methods and subject matters, acquires needed skill, and is saturated with its emotional spirit” (p.26). Vygotsky stressed on the inherent *transformation of the learner and what is learned* during the process. Using the notion of ‘participatory appropriation’, cognitive development is thus seen as a dynamic, active and mutual process. There is an important difference between perspectives of ‘participatory appropriation’ and ‘internalization’ in their assumptions about time. In internalization, time is rigidly segmented into past, present and future, and treated as separate. Then critical problems arise about understanding the process of ‘transmission of knowledge’ across time - about how individuals store memories of the past, about how those are retrieved and used in the present, with plans made for and executed in the future. However ‘participatory appropriation’ (Rogoff, 1995, p.150) looks at learning as an ongoing activity. People participate in activities and learn to handle subsequent activities based on their involvement in previous events. Time does not need to be segmented into past, present and future, and learning is not viewed as internalization of stored units, from

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external exposure to knowledge or skills. As a person acts on the basis of previous experience, the past is present in the participation, and contributes to the event by having prepared the ground for it. The present event is therefore different from what it would have been if the previous events had not occurred. *Transformations* that occur during participation are developmental changes in particular directions, but are locally defined, and do not need universally specified end points or 'milestones'.

Gutiérrez suggests that dispossession is closely tied to schooling's 'milestones' or 'learning ourcomes' and its purposes of measuring success. Cultures that attribute collective responsibility to 'education' and intelligence, include 'forms of respect, knowing one's place in the world, and knowing how to give one's gifts', and associate knowing with action. Indeed, cross cultural notions of 'social intelligence' highly valued in collectivistic communities, include wisdom, leadership and social responsibility as attributes of an 'intelligent' person, contrary to the narrow idea of 'technical intelligence' promoted by schooling. Thus, the rural Zambian concept of *Nzelu* corresponds to wisdom and responsibility not for people who use their intelligence for selfish purposes, but only those who use it in socially productive ways (Serpell and Hatano, 1997). Our own work on culturally responsive assessments in mathematics has shown that students who may perform poorly in standard test questions on data handling, show a better understanding when engaged in assessment tasks which give them a larger sense of purpose, such as recording and analysing the Mid Day Meal data for their school.

Hidden Knowledges in Science and Mathematics

Gutiérrez states that her call for an ecology of knowledges is consistent with theory building in sciences, and quotes (Star, 1988, 48) :"Scientific theory building is deeply heterogeneous: Different viewpoints are constantly being adduced and reconciled… The aggregation of all viewpoints is the source of the robustness of science ". The reference is not listed, but this view on theory building in science is problematic.

The development of modern science is based on several knowledge traditions, from cultures surveyed by the voyages of discovery or were colonies of Europe. However, in the development of modern science, these traditions were consciously delegitimised or even 'cognitively lost'. Schools today can act as sites of production of new knowledge and of relegitimisation and appraisal of these lost traditions. There is a need to reclaim a 'cognitive debt', perhaps similar to the social movement for an audit of the immeasurable 'ecological debt' of colonising countries, through centuries of exploitation of mineral and other natural resources of the Third World, which has resulted in its present economic debt (Rampal, 2009).

Curiosity about nature was not only kindled but systematically formalised and nurtured in every civilisational area, with significant cross-transmission and cross-fertilisation of knowledge. Goonatilake (1998, p67) calls for new attempts to 'mine civilisational knowledge', through a rich array of techniques, metaphors and intellectual solutions. "The Scientific Revolution began after the voyages of

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discovery, and aspects of both projects interacted with each other, namely, the search for science and the search of the Europeans' "other". The imperialist perspective that accompanied both events soon began to assert the superiority and exclusivity of everything considered European. Soon, these attitudes crystallized,... into a view that other cultures were inherently incapable of intellectual work that goes today under the rubric of science". However, studies over the last few decades indicate a continuum of knowings; for instance, systems of plant classification developed by folk classifiers or modern scientists are found to be quite similar. Indeed Linnaeus, the founder of the modern classificatory system "had access to a wider store of plant samples, created through European expansion into the rest of the world" (Ibid, 70).

The teaching of science fails to acknowledge its 'cognitive debt' to many civilsational knowledge traditions, and also hides the historical contribution of 'artisanal epsitemologies' (Smith, 2004) that have shaped the discipline. Recent alternative histories of science have documented the contributions of several anonymous actors, who understood nature through their hands and senses, not just through texts and mind. These include early women agriculturists, and numerous communities of artisans, such as foundry men, miners, ship builders, farmers, carpenters, engravers, etc. Indeed Paracelsus is called the first 'people's scientist' who practised medicine in a poor Swiss village around A.D. 1500, worked among miners to treat their occupational diseases, gathered knowledge from common people such as barbers, bath attendants, mid wives, magicians, etc, and even taught at the University in a Swiss German dialect, not Latin as tradition required. While such socio-cultural and political histories now constitute a rich field of Science Studies, a similar arena is waiting to be developed in mathematics. Valero reminded the MES9 Conference that: "An alternative cultural history of mathematics would first take mathematics as being no different than any other kind of human or cultural production. It would rather ask questions about the conditions of possibility of the production of mathematical concepts and techniques within a network of materialities, political struggles, economic arrangements, moral and philosophical debates, forms of government of the time and space configurations in which they emerged. While such types of histories have been frequent for the natural sciences, they are rarer for mathematics. Such an invitation pushes the researcher outside of the comfortable boundaries of the internalistic understanding of mathematics and places such endeavor in the messiness of the making of human life and culture" (Valero, 2017, p137).

Interesting insights into people's history of mathematics are given by economic historians who show for instance that metric measures are wholly arbitrary and dependent on convention, unlike folk measures, which in fact should not be called 'conventional'; those are truly representational, and therefore have a social meaning, determined through peoples' activities (Kula, 1986). There is a striking dominance of qualitative 'value' over purely quantitative considerations in the social thinking of pre-industrial societies. Land measures were not directly 'addable' and were meant to

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account for the different quality of soil, labour-time for tilling, or the amount of seed needed. Similarly, the ‘measure sold’ was different in size from the ‘measure bought’; a heaped container is bought from the farmer while a flat one is sold to a customer at the same price, to take care of the costs of transaction; even now we get 13 glass bangles at the rate of a dozen (to cover possible breakage). Such artisanal forms of measurement continue to co-exist in India along with the standard systems, but most mathematics educators refuse to appreciate their ingenuity and significance, dismissing them as crude, inaccurate or ‘primitive’. In my own Plenary at MES8 I discussed our efforts at incorporating people’s knowledges in the national primary mathematics textbooks in India (Rampal, 2015). We remind ourselves yet again, that the collective challenge to reframe our knowings and craft new dispositions for mathematics education, still lie ahead…..

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LANGUAGE IDEOLOGIES FOR UNDERSTANDING LANGUAGE FOR SCHOOL MATHEMATICS: A RESPONSE TO TRINICK

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Trinick provides a valuable social justice perspective on the relationship between the development and implementation of a Māori*-medium mathematics curriculum and the revitalisation of* te reo Māori *in* Aotearoa/NZ*. Aiming to create a conversation across contexts with varying language infrastructures, I offer the notion of language ideologies to strengthen our understanding of what may be conflicting interests in debates about language for learning mathematics. For the notion of language ideologies I draw on the work of South African scholar Carolyn McKinney and her co-authors. Considering beliefs about language as well as mathematics, mathematics education and schooling is crucial, since language is key to meaning-making in mathematics and hence to equity in mathematics education.*

INTRODUCTION

Trinick (2019) uses Nancy Fraser's concepts to provide a valuable social justice perspective on the relationship between the development and implementation of a Māori-medium mathematics curriculum and the revitalisation of *te reo Māori* in Aotearoa/NZ. He points to the revitalisation of *te reo Māori* using agency within existing schooling and mathematics education structures. Through Māori-medium mathematics, access to the dominant Western mathematics is provided, thus paradoxically reproducing this dominance. Trinick (2019) points to the "conflicting interests" of agents such as the community and state education authorities, concluding that there remain "issues and tensions".

As a South African mathematics education scholar, a number of questions about mathematics education in Aotearoa/NZ emerged in my reading of Trinick (2019). Indeed, each reader brings to this paper their positioning in a context with particular *language infrastructures* (Setati, Adler, Reed & Bapoo, 2002). Aiming to create a conversation across these contexts, I offer scholarship on *language ideologies* – as given meaning in South Africa – to aid our understanding of contestation between agents. This notion supplements the mathematics, mathematics education and schooling ideologies identified by Trinick (2019), for example, the view of mathematics as culture free, the high status of mathematics education based on Western mathematics, and neoliberal economic ideas in schooling.

"FIGHTING TO BE ME": LANGUAGE CONTESTATION IN SOUTH AFRICA

In 2016 female school students staged protests against discriminatory practices at two state high schools in South Africa. The English-medium schools, located in wealthy urban suburbs, were classified as white schools under apartheid but were officially

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opened to all races from 1996.[1] During these protests, which featured widely in the press and on social media, black students reported not being permitted to use African languages in and out of the classroom, being told to straighten their hair rather than wear natural hair styles, and being told they are "too loud" (Nemakonde, 2016; "Sans Souci Girls' High School pupils protest", 2016). A student was quoted as saying she was "fighting to be me" ("Sans Souci Girls' High School pupils protest", 2016). At one school a student referred to her school "dompas" in which her behaviour was recorded as merits or demerits ("Sans Souci Girls' High School pupils protest", 2016). The term "dompas" is a colloquial term for a type of internal passport, originating in colonial times, that black South Africans had to carry during apartheid and that was used to police their movement within the country (Savage, 1986).

At both schools the protesting students were observed by private security or police (Nemakonde, 2016; "Sans Souci Girls' High School pupils protest", 2016). These protests were certainly not the first time discriminatory practices at former white schools in South Africa were raised (Christie & McKinney, 2017). Taken together, these events pose challenging questions regarding South African schooling: After two decades of democracy, how did we come to a point where students are protesting linguistic and racial discrimination, while literally being 'policed'?

Indeed, the 1996 South African constitution and post-apartheid policies aimed to address the colonial and apartheid mechanisms that had entrenched such discrimination. For example, to promote multilingualism, official language status was extended from English and Afrikaans to include nine African languages: Sesotho, Sepedi, Setswana, Tshivenda, Xitsonga, isiNdebele, isiXhosa, siSwati, and isiZulu (Setati, 2005). McKinney (2017) notes that initial language in education policy gave schools responsibility for deciding how to promote multilingualism, for example, by selecting the 'Language of Learning and Teaching' (LoLT) and by providing opportunities for students to use other languages at school.[2] A new school curriculum in 2010 recognised 'mother tongue' as the LoLT from Grades 1 to 3 (with English as 'additional language'), and prescribed a switch to English as LoLT in Grade 4. Success in this school system requires students to be "at minimum bilingual in the language resources of their homes and the kind of monolingual English resources required of schooling" (McKinney & Tyler, 2018, p. 5). This, in a country in which less than 10% of the population reports English as 'home language' (Statistics South Africa, 2011) and in which multilingual classrooms are the norm. English remains the dominant language at schools, with the majority of students learning mathematics in a language they are still learning (Setati, Reed, Adler & Bapoo, 2002).

LANGUAGE IDEOLOGIES IN SOUTH AFRICAN SCHOOLING

South African scholar Carolyn McKinney suggests that, to answer the posed question "How did we get here?", that is, to understand what language is valued and how we position one another according to our language use, we need to consider *language ideologies*.[3] McKinney (2017, p. 19) notes that these extend beyond what just an individual believes about language to "the sets of beliefs, values and cultural frames

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that continually circulate in society, informing the ways in which language is conceptualized as well as how it is used" (citing Makoe & McKinney, 2014, p. 659).

A dominant language ideology in South Africa is the idea of language as unitary, stable, and clearly boundaried (McKinney, 2017). This monoglossic view of language informs the naming of distinct languages such as 'English' and 'isiXhosa', the association of an individual with a 'mother tongue' and 'additional language(s)', and debates about the standardization of the lexicon and register of a language. Makoni (1998) argues that the named African languages valued in the constitution are actually colonial constructions used for social classification during the colonial and apartheid eras. Indeed, as the student protests suggest, classification by the related constructs of language and race persist. A related language ideology dominant in this context is that of monolingualism as the norm (McKinney, 2007).

In South Africa, where the apartheid-era African language medium of instruction policy for black primary schools was regarded as preventing access to English and where English continues to dominate the linguistic market (Bourdieu, 1977), these two dominant ideologies point to what is 'normal' language use. McKinney (2017) refers to this as *anglonormativity*; the 'normal' language user is someone who is proficient in 'standard' English, while the 'deviant', 'deficient' or 'problematic' language user is someone who lacks proficiency in 'standard' English, speaks with a particular accent or switches languages. Thus agents such as parents advocate for their children to be taught mathematics in English (Setati, 2008), irrespective of whether it enables meaning-making in mathematics, or whether the student can "be me" in the classroom.

The language ideologies discussed here can be identified in views on school mathematics. For example, as noted by Tyler (2016), clear distinctions are made between the mathematics lexicon and register and 'everyday' language. Also, it is expected that student learning of the former will proceed unidirectionally from spoken to written language, from 'everyday' to 'mathematical' language and, in multilingual classrooms, from the 'home language' to 'standard' English.

These dominant language ideologies can be contrasted with the less common views of language as heteroglossic and of plurilingualism as the norm. McKinney (2017), following Bakhtin (1981), defines heteroglossia as "the complex, simultaneous use of a diverse range of registers, voices, named languages or codes" (p. 22), which form part of a multimodal repertoire for meaning-making in a particular context. As noted by Tyler (2016), heteroglossia has been demonstrated in practice in South African mathematics classrooms in the form of *translanguaging*. Her study of translanguaging points to multidirectional – not unidirectional – shifts between registers, languages, modes, and discourses in the learning of mathematics.

TALKING BACK TO THE AOTEAROA/NZ CONTEXT

I now talk back to the context of Aotearoa/NZ by posing questions to mathematics education scholars in that context. I supplement the mathematics, mathematics

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education and schooling ideologies identified by Trinick (2019), with language ideologies as discussed. The latter are crucial for they have "profound consequences" (McKinney, 2017, p. 18) for how we use language to teach mathematics and, since "language is the most significant tool for a child to access the curriculum" (p. 43), for equity in mathematics education.

- What language ideologies inform the "interests" of the agents described by Trinick? And with what consequences for shifts in policy and practice?
- Trinick identifies members of indigenous communities and state education bodies as key agents in decisions regarding language use. To what extent might other agents such as students, teachers and parents inform decisions?
- Trinick focuses on the development of lexicon and register in the development of the Māori mathematics curriculum. Were there developments with respect to modes, discourses, voices and identities in this process?
- Trinick chooses to focus on the development of the indigenous *language*. To what extent might language use in his context be related to constructs of race, class, and/or geography (rural vs. urban), as it is in South African schooling?
- How might the ideology of mathematics as neutral and culture free reinforce the practice of using curriculum reforms from elsewhere in the production of the Aotearoa/NZ mathematics curriculum in both English and *te reo Māori*? What opportunities might be created for agents to question what counts as the dominant mathematics and to interrogate relations of power (McKinney, 2017)?
- Trinick focuses on the revitalisation of *te reo Māori*. Given that language lies at the heart of equity in mathematics education, what can we learn about the contribution of the curriculum to "improving Māori students' achievement in mathematics" as mentioned briefly by Trinick in his introduction?

It is hoped that, with answers to questions such as these, we might move towards a point where indigenous communities might "carry [their] sense of confidence wherever [they] are without the indignity of having to justify and fight for it" (Ndebele, 2016).

NOTES

1. Given my focus on past and present language policy and practice in South Africa, I use the apartheid-era racial classifications of 'black' and 'white', which are still used to report educational performance in South Africa.

2. I use single scare quotes for terms such as this to signal their ideological nature.

3. McKinney and her co-authors draw on the work of scholars such as Bakhtin, Blackledge, Blommaert, Creese, Foucault, and Woolard to study language ideologies at work in South Africa.

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CONSTRAINTS ON MATHEMATICS EDUCATION REFORM IN INDIA: A RESPONSE TO CABRAL AND BALDINO

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In my response, I briefly describe mathematics curriculum reform in India, given the importance of curriculum reform for social justice agendas. The larger constraints on reform imposed by the board examinations lead to mathematics being seen as a subject to be managed rather than some of value that must be learned. This serves the purpose of having a public that is compliant, uncritical and uninformed.

Cabral and Baldino (this volume) criticise mathematics education research (MER) for perpetrating an illusory optimism by ignoring structural constraints that severely limit any real improvement on the ground. Their pointed criticisms will find a mark: mathematics education researchers issue bulletins of victories even though they are limited and local, substitute recent fashions by even newer ones, intimidate with an array of references, etc., while making almost no difference to the core problem of failure and anxiety over mathematics in our schools. The criticism is aimed at making researchers self-reflective, in which it admittedly achieves some success. It makes some of us, who have been advocating for growing a community of mathematics education researchers in India, pause and wonder if we are uncorking a genie that will only worsen the problem. But we must press on nevertheless and "have no choice but to continue the struggle" (Greer et al., 2017, quoted in Cabral and Baldino, this volume).

A community of mathematics education researchers does exist in India, but is miniscule for a country of 1.3 billion. It is difficult to ascribe to this small community a dominant paradigm or a governing ideology. Cabral and Baldino's criticism could be seen as applying more widely to the community engaged in mathematics education reform, rather than just MER. This is how I read the interesting exchange between *Big Enemy* (BE) and Mrs. Smith in their article (this volume). Mrs. Smith has truly embraced the concern for social justice, works hard towards achieving it for her students and is motivated by kindness. Yet she ends up implementing BE's agenda, not only when she is trying honestly to evaluate her students and assign grades differentially, but also when she is engaging with and helping them. The evaluative function assigns value differentially to students as part of a certification process, while her "inclusive" approach ensures that all (or most) students contribute their labour to the process. Mrs. Smith becomes willy nilly an instrument in the process by which some students appropriate the work done by all, and refashion the surplus value into a form of capital that Cabral and Baldino call "qualified labour power". Thus the locus of play of capital in school education is the dialectic of engagement and resistance, of participation coerced by the school authorities and opting out by the marginalised students, all of which serves to solidify entrenched social stratification as sociologists of education have argued. In seeking to ensure "mathematics for all", well meaning

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teachers and authorities squeeze out the option of seceding, which improves the efficiency of schooling in solidifying stratification. At least, this is my interpretation of the argument made by Cabral and Baldino. At the larger level, the dialectic plays out through the certification processes instituted by the board exams, which I will come to later.

Before that, a quick overview of the efforts at reforming the school mathematics curriculum in India and their underlying motivations is in order. Mathematics education reforms have a longer and larger presence than MER in the Indian context. The key policy documents produced in post-Independence India, over a period of 70 years, all gave primacy to mathematics in the school curriculum. However, as Khan (2015) points out, there were important shifts in how the goal of mathematics education was envisioned, especially in the most recent curriculum reform document – the National Curriculum Framework 2005 (NCF 2005). In the early policy documents, mathematics was seen as a foundational discipline for science and technology, as well as for business and finance, and hence important to the overall goal of national development. The NCF 2005 introduced a new vision of mathematics "not solely as abstract disciplinary knowledge that can serve the cause of a modern industrialized and technologically driven economy but as tools of rational and critical thinking and enquiry, to serve individual curiosity and a quest for knowledge" (Khan, 2015: 1528). The new vision of mathematics education was combined with a conception of constructivist pedagogy, which was new in the Indian context, but resonated with earlier calls for "joyful learning" and "activity based learning".

There were multiple developments leading up to the NCF 2005. Two of these are important to mention, and they arise from very different sources. First, the Indian economy, relatively insulated for several decades following a policy of economic self-reliance, and a cold war dictated foreign policy of non-alignment, began to open up its economy to globalization in the early 1990s. In the wake of this development, foreign donor agencies propelled a major intervention in primary education to "improve quality". Several states revised their textbooks to make them child-friendly and activity based. These efforts resulted largely in more attractively packaged, "fun-filled" textbooks and workbooks, which were not necessarily concerned with the mathematics that the students engaged in. From a different direction, voluntary organisations working at the grassroots from the mid-1970s aided by university academics, developed new curricula for primary school mathematics. Added to this was the impact of the adult literacy campaign of the 1990s. These efforts culminated in the new textbooks developed under the NCF 2005 (Rampal, 2015; Mukherjee & Varma, 2015). Thus, while the donor agencies may have created a space for reform, the leadership for it was largely provided by those who had participated in grassroots level curriculum development and literacy campaigns. The momentum generated by these two divergent lines of work carried over into curriculum reform by several states across India in the last two decades.

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A close look at the primary mathematics textbooks produced under the NCF 2005, reveals the influence of the MER strands of ethnomathematics and critical mathematics education. (For details, see Anita Rampal's plenary paper in MES-8: Rampal, 2015). However, as widely recognized, curriculum frameworks and textbooks alone are insufficient to create change. Rampal acknowledges that several challenges remain, including the lack of print and other learning resources and underpaid contract teachers who face the pressure of frequent testing and administrative changes.

Beyond the primary level, the mathematics curriculum and textbooks do not show much influence of ethnomathematics or of critical pedagogy. While teaching and curriculum design experiments targeted at the middle and secondary students have been done elsewhere (e.g. Gutstein & Peterson, 2005), these are yet to acquire a significant presence in India. Further, there is divergence in the views on what direction the reform of the middle and secondary mathematics curricula must take. Khan (2015), emphasising a social constructivist perspective, argues that there is "a need to socialise children into a formal system of knowledge in the mathematical classroom, and into a knowledge system that is validated by a social community of mathematics experts for them to become active participants in the creation of knowledge". This aligns with the views expressed by Gutstein and Peterson: "our commitment is for students to learn the necessary mathematics to deal with and get past the unjust gates in front of them and the mathematics necessary to tear down the gates entirely. The stronger students' grasp of mathematics, the better equipped they are to comprehend and change the world" (2005: xii).

On the other hand, some researchers have examined what the teaching of formal mathematics entails for the marginalized, rural students, especially those from lower castes and other disadvantaged communities of first generation learners (Subramanian, Umar & Varma, 2015). Drawing on their descriptions of the material conditions of learning in rural and remote schools, and of the students' actual mathematical learning, the authors question the value of teaching such students operations on polynomials or geometry. Factors such as under-resourced conditions, teachers' preparedness, etc., are unlikely to improve significantly even over a decade, the time span in which a cohort enters and leaves school. Subramanian, Umar and Varma therefore ask if it is ethical to foist a uniform curriculum based on "powerful mathematics" on all Indian students. Resolving these tensions will be possible only as more experience is gained in innovative curricular experiments with diverse groups of Indian students. Such experiments and ensuing curriculum reforms are insufficient to ensure that failure, anxiety and alienation disappear from school classrooms, but they are a necessary step which can enable change in this direction. A curriculum backed by learning resources that is available and communicates a different vision of education is an essential weapon in the struggle for change.

Beyond the curriculum, the major structural constraints in the Indian education system are the school certification and college entry examinations, the pressure points through which larger social and political pressures impact the education system. The

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examinations in turn create a backward pressure that shapes both the content and format of school testing and teaching, all the way down to the primary school. Key assessment reforms introduced through policies and legislation, such as no detention till Class 8, continuous and comprehensive evaluation done by the teacher in class, and no streaming till Class 10 have all been pushed back strongly and are likely to be abandoned in the next round of policy changes.

Let us take the case of streaming, which is operationalised through two levels of the mathematics for the Class 10 board examination, the first big educational challenge in an Indian adolescent's life. Several years ago, the education minister of the state of Maharashtra expressed frustration at the fact that high school students were spending nearly all their time studying for mathematics and neglecting other subjects. In 2008, at his initiative, the state introduced an easier "general mathematics" paper as an option for the Class 10 board exam. This was despite the strong opposition to having two levels of mathematics up to Class 10 in the NCF 2005, which had argued that, given the existing structural inequities in the education system, the lower level would become the default option for students from a less privileged background. Indeed, some years after Maharashtra introduced the two level mathematics exam, we met the principal of a school catering to students from a religious minority. He was not even aware that there were two levels of mathematics, because all the students in his school were automatically enrolled into the lower level track. This came at a price, because such students would not be able to enter engineering, architecture and other technical streams of post-secondary education.

After experimenting with the two level exams in mathematics for several years, two years ago, the Maharashtra Government scrapped the system. The arguments given were that there was no significant improvement in the pass percentage, and that fewer students were opting for the easier maths option every year. A former chairman of the exam board suggested that instead of a separate paper, enough easy questions must be incorporated in the common paper, so that weaker students could pass the exam (Pednekar, 2018). As it turned out, in 2018, the year when the easy option was scrapped, the pass percentage went up by 0.7 percentage points to 86.4%. In the meanwhile, the Central Board of Secondary Education, overseen by the Central govt., has just announced two levels of Class 10 mathematics exam starting next year!

The high rates of passing in the board exams do not necessarily indicate successful learning outcomes. As the former chairman's remarks indicate, a major concern is to keep the level of the exam easy so that large pass percentages can be ensured. Thus mathematics, for the overwhelming majority is a subject to be managed rather than learnt. Easier exam questions, generous internal marking, rote learning techniques such as hundreds of hours of tedious practice, are some of the ways in which mathematics is managed. What is the impact of the culture of "managing" mathematics in order to pass the exam, on the educational outcomes for students from marginalised groups? Such a culture can be an effective instrument to ensure that the public remains compliant, uncritical and uninformed.

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In a culture that is not appreciative of the value of precise and quantified information in the public domain, the powers that be can resort to convenient manipulation of seemingly objective facts. For example, a monitoring think tank agency that conducts monthly sample surveys, reported an overall employment growth of only 1.8 million in 2017. A member of the Prime Minister's Economic Advisory Council, used the *same data* and estimated the number as 15 million. It turned out that the latter had bracketed out the steep *fall* in employment among certain age groups ("When Surjit Singh Balla", 2018). It requires a widespread mathematical literacy to create adequate pressure for the production, authentication and publicly sharing of critical data that affect the welfare of the masses. Such literacy is necessary for indignation, even outrage at the state of affairs, to emerge.

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SYMPOSIA

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RACE, MATHEMATICS EDUCATION, AND SOCIETY: TOWARD GLOBAL PERSPECTIVES

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Abstract: Racial structures and racism, in all their forms, are ubiquitous and persistent on a global scale. Mathematics education, both as a social institution and as a field of social practices, is situated within the racial histories, prevailing racial orders, and the permutations of social meanings for race in various societies. This symposium aims to create a space within MES-10 for cross-national, critical discussions about constructs and questions regarding race and racialization, building on an initial discussion (anti)Blackness within mathematics education and expanding the conversation to other manifestations of race and racialization in different global contexts. Facilitators will foster whole- and small-group discussion of these issues.

AIMS OF THE SYMPOSIUM

This symposium aims to create a space within MES-10 for cross-national, critical discussions about constructs and questions regarding race and mathematics education. Although the panellists are based in the United States where Blackness and anti- Blackness have manifested ideologically and materially in different ways to shape all aspects of U.S. society, the aim here is to build a broader global understanding of these and related concepts in relation to mathematics education. The facilitators recognize that Blackness may not be an easily or universally indexable concept but we assert that there are modes of racialization in operation in most geopolitical contexts such as the caste system in India, xenophobic discourses in Europe, and colonial legacies in the Pacific. We also recognize that our present understanding of Blackness and anti-Blacknessare limited by our status as U.S. scholars. Therefore, we seek to understand Blackness, anti-Blackness, and white supremacy in other contexts. We will begin by defining Blackness and anti-Blackness as concepts in the U.S. context only as an impetus for introducing and complicating structural conditions in other national contexts. Facilitators will aim to introduce and foster whole- and small-group discussion, but the central aim is for participants and facilitators alike to explore discourses of race and racialization in mathematics education across the globe.

RELEVANCE OF THE SYMPOSIUM

The ubiquity and persistence of racial structures and racism, in all their forms on a global scale, stems from the fact that the meanings for race and racial categories are created, politically contested, and recreated in any given geopolitical context as a way to maintain boundaries of difference related to domination and oppression. We contend that mathematics education as a social institution and a field of social

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practices is situated within the racial histories, prevailing racial orders, and web of social meanings for race in different societies.

In the United States, for example, Martin (2011, 2013) discussed how the 'new math' reform of the 1950s and 1960s which sought to enlist the 'best and the brightest' (i.e. white males) was also a sociopolitical project that promoted nationalism and xenophobia while also unfolding in a civil rights context characterized by legal exclusion of and discrimination against African Americans. More contemporarily, Martin (2013) characterized mathematics education as a social project that has always been put in service to the prevailing racial projects of U.S. society. Scholars, including Gutstein (2009), have linked mathematics education reforms to nationalist and neoliberal politics that perpetuate racist capitalism. Others (e.g., Battey & Leyva,2016; Martin,2008) have explored whiteness as an ideology that mediates research and knowledge production, policy, and practice in mathematics education.

We are encouraged to see that the topic of race and mathematics is receiving increased attention in particular locations, namely the UnitedStates (e.g.,Larnell, 2016; Martin,2009; Martin, Anderson, Shah, 2018; Spencer & Hand,2015; Parks & Schmeichel,2012; Price & Moore,2016; Shah & Leonardo,2017), SouthAfrica(e.g.,Khuzwayo,2005; Mosimege,2000; 2012), Colombia (e.g., Valencia, 2016; Valoyes-Chávez,2015,2017), and Brazil (e.g.,DaSilva,2008,2016; Gomesda Silva & Powell,2016). We are also encouraged by, and seek to catalyse, collaborations among scholars in different parts of the world to compare and contrast the racial contexts of mathematics education and to develop a broader understanding of how race and racial structures operate within mathematics education (e.g., Valoyes-Chávez & Martin,2016). However, our critical review of the extant research literature suggests that attention to race and mathematics education in other racialized contexts has been limited. In many European countries, for example, related phenomena of mass migration and displacement and tensions around cultural and religious differences have continued to surface expressions of White supremacy, nationalism, xenophobia, anti-Black racism, and other forms of racial oppression. Yet the specific manifestations of these unfoldings and their implications for mathematics education remain largely unexplored.In this symposium, we seek to surface these and other issues, and to foster discussions of similarities and differences across the globe.

Two keys to broadening the discussion of race in mathematics education are (1) to discuss how the meanings for race have been negotiated within and across global contexts, and (2) to develop understandings the racial projects that give rise to these meanings and racial dynamics. Having participants consider these two points makesit possible and reasonable to ask questions such as: what are the implications of the emergence of a far-right, conservative racial project for mathematics education in Denmark and for immigrant families and their children who find themselves under attack by the right-wing Danish People's Party? How do experiences with everyday racism by Malays and Indians in Singapore play out in the context of mathematics education? What are the implications for mathematics education of the maintenance

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of white supremacy in Brazil, a country that prides itself on maintaining a racial democracy? How do caste and race intersect to impact mathematics education for Dalits and African-descended Siddis in India? And What are the racialized conditions and histories of mathematics education for Aboriginals in Australia (post White Australia policy)? Maoris in New Zealand? Native Fijians in Fiji? Across all of these contexts, how do practices and policies that maintain white privilege or promote racial purity manifest in different ways to benefit those at the top of the racial hierarchy?

With these kinds of open questions in mind, this workshop has the potential to generate discussion in the following areas, which we hope participants will consider after the workshop is concluded:

- Specifications of how White supremacy, nationalism, colonialism, xenophobia, or anti- Black racism, for example, shape mathematics education research, policy, and practice in specific geopolitical contexts.
- Context-specific examples of how mathematics teaching and learning in schools and classrooms are shaped by or resist racism and racialization processes.
- Intersectional analyses of race with ethnicity, gender, class, religion, caste, ability, in the context of mathematics education.
- Explication of different theoretical approaches to studying race and racialization processes in mathematics education across global contexts.
- Explication of research methods and approaches to studying race and racialization processes in mathematics education.

FORMAT OF THE SYMPOSIUM

We intend the sessions to take on a working group format in which participants actively negotiate the meanings and articulations of race and racialization in mathematics education contexts around the world by offering examples of how these are manifested. The symposium would include two sessions (i.e., over two days) in which participants will discuss and interact in small and whole groups. Discussions across the two days will be prompted by the use of prompts such as: (1) Describe two or three forces of oppression or stratification, including racial stratification and oppression, in your region or country that are based on beliefs about the inherent superiority of one group in relation to others, (2) Provide some examples of how mathematics education plays out differently for groups who are thought to be in superior and inferior positions in your region or country. In what ways are these experiences consequential?, and (3) Describe context-specific consequences of the relationship between discourses about mathematics achievement as they intersect with racialized discourses and other identity-contingent discourses (e.g., regarding gender or sexuality or physical ability).

Each day's session will include transitions to small-group discussions, first within national and global regions and later across national and regional contexts.

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Discussions will begin with prompts such as: (1) Describe two or three forces of oppression or stratification, including racial stratification and oppression, in your region or country that are based on beliefs about the inherent superiority of one group in relation to others,(2)Provide some examples of how mathematics education plays out differently for groups who are thought to be in superior and inferior positions in your region or country. In what ways are these realities consequential?,(3)Describe the language and discourses used to signal difference in your country that reflect beliefs about the inherent superiority or inferiority of some groups, and (4) Moving forward, what might research on race and racialization look like in different global contexts?

Specific take-aways for participants will include: (1) conceptual language for discussing race and mathematics education, (2) greater awareness of extant research on race and mathematics education in particular global contexts, and (3)images of how processes of racialization may intersect with other systems of oppressions, including sexism, heterosexism, and ableism.

Although the two-session sequence will allow participants to progressively develop ideas and understandings about race and racialization process within and across global- regional contexts, the conversions are sequenced to allow new attendees to join either day of symposium if not both.

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UNFOLDING GLOBAL/LOCAL POLICIES, PRACTICES AND/OR HYBRIDS IN MATHEMATICS EDUCATION WORLDWIDE:
UTOPIAS, PLEASURES, PRESSURES AND CONFLICTS

Symposium by: Anna Chronaki *with Gill Adams, Melissa Andrade, Gustavo Bruno, Fufy Demissie, Renato Marcone, Aldo Para, Hilary Povey, Dalene Swanson, Paola Valero, Ayse Yolcu*

AIM AND RATIONALE

The utopian dream of a global world is not new. It can be traced back in the 15th century navigators and 19th century colonial times and post world war peace pleas but also even earlier in ancient myths and religion narratives on the morals of living on earth. In our times, processes of globalisation are hastened by increased techno-culture, free-capitalist economy, migration, environmental calamity and war. During the last decades a global world imagery has taken momentum in how the lives of children, youth, families and educators could be reconfigured through the launching of 'global citizenship education' as strategic areas for curricular organisation by institutional bodies such as UNESCO or OECD. Explicit goals for an increased globalised, internationalised, cosmopolitan and urban worldview across nation-states, provinces and indigenous communities can be substantiated (for some) in the context of mathematics education practices. Based on the (false) epistemological assumption that mathematics remains a neutral and universal language, mathematics education becomes easily figured as the space for crafting a citizenship subjectivity for globality. In this context, one needs to problematize what is at stake when mathematics education curricula become (or not) framed within discourses of global citizenship education? How the 'future' of mathematics education can be reconfigured when the rhetoric of global citizenship education meets diverse localities? And moreover, what are the effects of such political imageries in diverse localities for children, teachers and materials, as well as for markets, economy and policies?

The purpose of the symposium is twofold. It is, *first*, to discuss how such utopian dreams of global, transnational and cosmopolitan citizenship become entangled or disentangled with/in discourses of mathematics education in diverse localities and diverse cultures, nations, languages and bodies including a variety of practices and communities. Presenters and participants will bring forward projects that unfold certain historiographies that map the effects of discourses of global citizenship in mathematics education by attempting to reinstate its colonial, de-colonial and post-colonial politics. This might imply an encounter of ideological critique of such curricular endeavours, of problematizing processes of globalisation in specific localities, or discussing its potential affirmative politics for reclaiming what mathematics education might regenerate out of this conflictual discursive materiality. And, *second*, the symposium will try to create and perform an offhand presentation that can engage the conference participants based on visual materials and narratives brought by presenters and participants in either digital or physical forms. These can be fragments of varied media,

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artefacts, or, even art-based performances that will denote or interrogate the hybrid presence of 'global' citizenship education discourses in 'local' (and vice versa) mathematics education practices such as curriculum activities, exam-, guidelines but also contemporary artefacts, as well as cultural representations in well known movies, literature, poetry, games etc. The aim of such an endeavour will be to problematise the meanings and effects of discourses on 'global' or 'local' mathematics education policies, practices or hybrids and explore their effects not only for the formal -the written norms- but also for the hidden school curriculum -the unwritten norms- or even the informal leisure practices beyond the walls of schooling.

ORGANISATION

The symposium will involve 9 presentations in two sessions of approximately 90 minutes each. Each session will start with a short discussion of the issues orchestrated by the chair and followed with short presentations of 10 minutes each. This will allow time for discussion, group-work and workshop-based involvement with participants.

SESSION A: GLOBAL/LOCAL POLICIES and PRACTICES

This session will start with a short introduction to the symposium, its aims and rational. The focus here will be to discuss the effects of utopian visions of globalization and current conceptions of global learning for democratic citizenship on mathematics education policies and practices that need to be interrogated and critiqued.

1.Global curriculum policies for (un)democratizing school mathematics, by Paola Valero, University of Stockholm, Sweden: Mathematics education is increasingly controlled by policy embedded within visions of globalization. The tendency towards the regulation of national curricula to direct the work of teachers towards improving students' achievement is often presented as a matter of equity, access and democracy. Based on the discussion of changes in policy in Sweden, Denmark and Norway, I will illustrate how the appeal to a new democratization with mathematics is rather creating the opposite effect of more differentiation, classification and potential exclusion among students. The mechanisms for making of mathematics a key tool for human capital growth and economic competitiveness clearly show the current predicaments of (un)democratization.

2.The 'Disadvantage' Child as a Thread for Economy by Melissa Andrade Molina, freelance, Chile: The low-performer as characterized by institutional bodies—i.e. OECD—raises issues of exclusion as it correlates students' performance in PISA with some individual "risk factors". Underachievement in mathematics is explained not solely in terms of abilities, skills, mathematical proficiency or students' grades, but also ethnicity, language, economic class or social and economic standing. The homogenizing status of the "all" in policy reports enables to assume that differences amongst students can be minimized or erased if all students have the same opportunities, making them responsible for their own failure or success in school and in their future. Such a narrative naturalizes discourses that exacerbate inequality. Thus,

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the underachiever, fabricated as a low performer who cannot meet the ‘factors’, is thus subjected as a risk for society and a thread for economy.

3. You deserve to be part of this World! By Renato Marcone, Federal University of Sao Paulo, Brazil: The inclusion of people with disability was never an issue of core concern for the international Mathematics Education community and was discussed mainly in the confines of Special Needs Education. However, today it is frequently addressed as a response to a Brazilian policy for people with disability in the public university. It has been noted how, despite exceptions, such inclusive policies still resemble globalization and colonization practices. Specifically, the parameter of normality is always provided by the ‘normal’ group by, simultaneously, defining the ‘abnormal’ to be made included. The prerogative of defining disability belongs to a vision of ‘normality’ held by institutions like WHO (Marcone, 2015). One of the core questions is: What are the consequences of inclusion of people with a disability without a dialogical construction of a definition about disability? It is almost like, without a Global and official definition of disability, ‘those people’ cannot become citizens of the World

4. *Critical global citizenship mathematics education and political responsibility in ‘glocal’ context, By Dalene Swanson, University of Stirling, Scotland*. I collaborated in the very first co-produced truly transnational, transdisciplinary online course on global citizenship, which was hosted at the University of British Columbia in 2004, when the term ‘global citizenship’ was in its infancy and yet to be defined. The initiative marked a serious attempt to engage global citizenship critically and stave off neoliberal determinations of it. I subsequently brought critical global citizenship to bear on mathematics education, in recognition of mathematics education’s ‘political responsibility’ to a viable, alternative, renewed and sustainable future. Recently, I was awarded funding to recruit a PhD student. The ESRC-SGSSS PhD studentship, on ‘Global Citizenship Mathematics Education’, explores controversial global issues within and through school mathematics education (in Scotland and globally), embracing a glocalising pedagogy and praxis for mathematics education. I will discuss the affordances, limitations, sensitivities and imperatives of doing critical global citizenship mathematics education at this political moment in our global history.

SESSION B: GLOBAL/LOCAL PRACTICES and HYBRIDS

This session will continue discussing the effects of globalization visions in local policies and practices of mathematics education but will also try to unravel how these become hybridized producing potential affirmative curricular action.

1. Curricular action: a possible role in mathematics education for Philosophy for Children. By Hilary Povey, Gill Adams and Fufy Demissie, Sheffield Hallam University, UK: This contribution springs out of an attempt to engage critically with mathematics education practice in schools through the vehicle of a European project, PiCaM (Project in Citizenship and Mathematics), co-funded through the ERASMUS + Programme of the European Union (Project number 2017-1-UK01-KA201-036675).

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Working with 10-12-year old and their teachers, the project involves participants in Germany, Greece, Portugal and England. It is the ambition of the project to contribute to combating discrimination, segregation and racism, validating the cultural history and supporting the education of all, including disadvantaged groups and migrant children. The particular focus here is on the role of P4C and the extent to which it may support *either* a neo-liberal interpretation of citizenship and global competence *and/or* a critical citizenship linked to affirmative politics. What is the potential of P4C to encourage political compliance to conventional ways of understanding knowledge? To valorise a rationalist, scientistic approach to the world? Or to foster ideological critique?

2. Is it possible to open the cage from the inside? By Aldo Para, freelance, Colombia: Indigenous education is the scenario par excellence for tensions among local and global forms of knowledge. Tensions are even stronger when mathematical knowledge is focused. Although the need for a balance between the access to global knowledge and the respect for local indigenous traditions has been agreed, it is common to suspect that any localization of the global in indigenous contexts is merely a neocolonial move. I argue that this conflict arises because the desired balance is framed under the logic of an epistemological hegemony. This logic is evident in images in which the global "is unfolded" in the local, minorities are "included" into majoritarian societies and the gap to be closed is the one that "separate them from the normal". I will comment on the experience of 'educación propia', an indigenous model of education developed in Colombia to call into question such hegemony and to develop the idea of multiple and rooted modernities (Harding, 2008) in which the locus of enunciation is displaced.

3.'Mathematics Recovery' or a SMBH in Mathematics Education by Gustavo Bruno, Autonomous University of Madrid, Spain: Curricular contents, expected achievements and assessment criteria are thoroughly detailed, for each mathematics course in secondary education, in official documents of the Community of Madrid, Spain (especially the "Official Bulletin", BOCM by its initials in Spanish). But the persistent failure of students isn't addressed in detail. Along the "official" mathematics courses, the BOCM offers optional courses called Mathematics Recovery (MR), to deal with the ever present, endemic failure. The configuration of MR courses is left to the criteria of teachers and institutions. In actual school practice, MR courses become vortices of nonsense in the relations between students, teachers, contents and assessment. In this presentation, I argue that these nonsensical vortices of MR are in fact fundamental to make sense of the political whole of ME. The political equilibrium of ME requires black holes such as MR, as every major galaxy in the known universe is sustained by a super-massive black hole (SMBH) close to its very center.

4.*Traveling of the Problem Solving Child: (Inter)nationalizing School Mathematics in Turkey. By Ayse Yolcu, Hacettepe University, Turkey:* Problem solving is usually considered as central to mathematics education, taken as a competency to deal with real-life situations mathematically for reflective citizenship (OECD, 2013). Nevertheless, problem solving is not merely about mathematics but also about making particular subjectivities, such as problem solving child, who knows how to act and plan

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in rational ways to bring progress, development and civilization in a world of uncertainty (Popkewitz, 2008). Turkey is not an exclusive in those desires to make world certain, predictable and governable. Following a global step, mathematics curriculum underwent a significant reform (MNE, 2005). In these internationalizing efforts, practices focus on processes where children actively participate in constructing their own knowledge while solving problems of daily life. I discuss the internationalization of school mathematics within Turkey not as a copy other programs but, how it is re-connected with cultural and political context that is encountered with while traveling (Said, 1983). The confrontation is not reductive or assimilative, but enables further production of discourses. In this presentation, the principles that historically configure the problem solving child of Turkey is explored.

5.Provincialising a Critical Global Citizenship Mathematics Education in the Context of Greece: By Anna Chronaki, University of Thessaly and University of Malmö. Current pleas for a critical global citizenship mathematics curriculum tend to argue for an ethical encounter of difference as race, gender, class, disability, migration so that to safeguard equity. Grounded in the rhetoric of 'development' as equity through quality, mathematics education becomes a colonial project. A project, that, by and large, must serve to 'develop' children as future citizens, who in turn, will serve to sustain a 'developed' society (Chronaki, 2011) or will work towards its 'development'. This logic entails the logic of 'fear' (Massumi, 2005) of not-developing enough according to inter/national measures. The present paper will try to consider the effects of such figurations for Greece's current precarious temporality as a south European indebted province of the so-called 'first world', but firmly, rooted at the borders of its ancient past and contemporary Balkan or Mediterranean hybrid identities.

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JOINING THE PIECES OF THE TIVAEVAE TO ENACT STRENGTH-BASED MATHEMATICS LEARNING IN AOTEAROA, NEW ZEALAND.

Roberta Hunter   Jodie Hunter   Trevor Bills   Bronwyn Gibbs

Massey University

In this symposium we draw on a strength-based philosophy to illustrate pedagogical practices which counter the racial disparities and colour-blindness Māori and Pāsifika and other marginalised students experience in mathematics classrooms in Aotearoa, New Zealand. We present through the voices of the teacher educators, teachers and students how they view teaching and learning in classrooms in which Māori and Pāsifika and other marginalised students are positioned more equitably. Our goal is to develop interactive discussion with other participants around issues of equity and social justice in strength-based mathematics classrooms.

RATIONALE

Within Aotearoa, New Zealand, a large percentage of Māori and Pāsifika students attend what Frankenberg, Siegel-Hawley, and Wang (2010) describe as hyper-segregated[1] schools. Within these schools, the student population is segregated by both culture, ethnicity and socio-economic class. Here within Aotearoa, New Zealand, as in other parts of the world, school segregation is confluent with a number of structural and systemic inequalities. These include less qualified and experienced teachers, a high level of teacher transience, less money spent per student and poorer funded buildings and facilities, as well as teacher-held deficit views of the students and their whānau[2] (Hunter & Hunter, 2018; Rubie-Davies, 2016). Nevertheless, when student achievement as conventionally measured is lower than those in more advantaged schools, blame is placed on students themselves and their whānau, attributed to many erroneous reasons (including a lack of parental or whānau interest and investment in education, a lack of ability, and a poor attitude and drive in the students). Rubie-Davies explains that these deficit views affect the opportunities the students have to learn because of the low expectations held by teachers that results in remediation and procedural rote teaching pitched at a low level of challenge. A deficit view also allows blame to be placed on students and whānau rather than on teachers and the inequitable education system they are positioned within.

In Aotearoa, New Zealand the cultures of Māori, Pāsifika peoples and other ethnically diverse students, rather than being viewed as strengths which can be drawn on to support, nurture and empower these students as mathematical learners, have for too long been viewed in deficit ways. Likewise, mathematics research and education has not addressed the ways in which racial disparities continue to be perpetrated within the structural and relational inequities of the state schooling system. Furthermore, the de-

[1] Hyper-segregated schools are those in which at least 90% of it's student population from racial/ethnic minority groups or 90% white.
[2] Family and the wider local community

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silencing of race continues in what Martin (2009) describes as colour-blindness and whiteness. Within our extensive and in-depth practice-based research we note what Lewis (2004) terms racial ascription is common in many schools in Aotearoa, New Zealand. Through racial ascription the dominant cultural group (Pākeha[3]) in schools are distinguished from the ethnically diverse students using common markers of otherness. These include such factors as culture, language, skin colour and socio-economic status; all of which are used to denote perceived lowered status of this group of learners by teachers and other students. Our goal in this symposium is to present a research-based argument which shifts away from a deficit and negative view to one that is focused on the teaching of mathematics within a strength-based cultural lens, the results of which are analysed through use of Pāsifika methodology.

AIM

In this symposium we draw on a strength-based, and research-informed project which is embedded within hyper-segregated schools which many Māori and Pāsifika students attend. The transformative approach (Developing Mathematical Inquiry Communities) we describe is embedded within culturally responsive and sustaining teaching and learning of mathematics. Within this project all participants are supported to reconceptualise persistent and long-held notions about who can learn mathematics successfully. This approach explicitly builds on and extends the cultural and social understandings and values of Pāsifika students in order to provide them with more equitable learning opportunities. We aim to illustrate the complexities and challenges, and shifts in practices and perceptions over time, as teaching and learning of mathematics is repositioned as an equitable social justice endeavour.

To enhance our analytical lens, we introduce and draw on Pāsifika methodology through the use of Hodges's (2000) metaphoric Cook Island *tivaevae* (quilt) model. This model directly focuses on the beliefs and values held by Pāsifika peoples (Hunter et al., 2016). Within this model five values—aokotai (collaboration), tu akangateitei (respect), uriuri kite (reciprocity), tu inangaro (relationships), and akari kite (shared vision)—are emphasised as key to underpinning culturally responsive and sustaining pedagogy for Māori and Pāsifika learners.

PLAN FOR THE SYMPOSIUM

Our intent across the four papers is to illustrate through the voices of teacher educators, students, and teachers what happens when mathematics is engaged with, and used, as a tool for social justice and equity for Pāsifika students in Aotearoa New Zealand. The questions we ask are:

What does the teaching and learning of mathematics within a strength-based frame look like?

[3] The term Pākeha is a Te reo Māori language term for New Zealanders of European descent. In more recent times the term has also used to refer inclusively either to a fair-skinned person or any other non-Māori New Zealander. .

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What key factors are involved and how do these differ from more traditional mathematics classrooms?

How do teachers and students view teaching and learning in strength-based mathematics classrooms?

In exploring these questions our central goal is to examine and explain what happens for Māori and Pāsifika students in mathematics classrooms when researchers, teacher educators, teachers, and students engage in what Freire (1970/2000) described as problem-posing education. Within this frame Freire described a need for a humanist liberating education that has students "develop their power to perceive critically *the way they exist* in the world *with which* and *in which* they find themselves" (p. 83).

At the same time, we want to engage interactively with the participants in discussion and so we will invite participants to join us in discussing how strength-based mathematics can confront such difficult and confronting issues as racism and how it can support the de-silencing of race in mathematics classrooms. The structure of the sessions will feature four presenters each giving a short presentation from the perspective of the different groups of participants (teacher educators, teachers, and students) engaged in Developing Mathematical Inquiry Communities to explore what happens when Māori and Pāsifika and other marginalised students in Aotearoa New Zealand are provided with opportunities to learn within a strength-based mathematics programme.

The 90-minute symposium will be structured as follows:

1. Welcome and brief introduction of the goals of the symposium.
2. Symposium organisers will each present a short paper.
    - Bronwyn Gibbs.
      Bringing transformative change through engaging in a strength-based teaching and learning mathematics programme.
    - Trevor Bills.
      Identifying the key factors of a strength-based approach which support Māori and Pāsifika students equitable access to mathematics learning
    - Dr Roberta Hunter.
      Teachers and teacher educators working together in professional learning to bring generative change in culturally responsive and sustaining practices
    - Dr Jodie Hunter.
      Listening to the voices of students as a window to understand how a strength-based approach serves as a mechanism to counter what Martin (2009) describes as colour-blind racial ideologies and practices.
3. Short question and answer session about the presentations.
4. An interactive session around the notion of strength-based pedagogies.

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5. Conclusion and plans for further collaboration.

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ONTOLOGICAL POSSIBILITIES OF MATHEMATICS IN MATHEMATICS EDUCATION: AN ACTIVITY-BASED EXPLORATION AND DISCUSSION

Jasmine Y. Ma, Maisie Gholson, Rochelle Gutiérrez, Molly L. Kelton, Maxine McKinney de Royston

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This symposium engages participants in activities and discussion designed to investigate three cases that lead toward reimagining a more humane, equitable, and sustainable mathematics for mathematics education.

AIMS OF THE SYMPOSIUM

In the US, school mathematics is, at best, a human construction that reflects the knowledge and value systems of a relatively small set of cultures. At worst, it is an extension of white institutional space (Martin, 2013) that is effectively a form of structural and symbolic violence that has little to do with learning and everything to do with socializing and racializing (if not traumatizing) youth into limited futures. In this symposium we propose to host a discussion of how the field of mathematics education might continue to (re)imagine a more equitable, more humane mathematics, grounded in three considerations proposed by symposium presenters. We hope to engage the diverse, international participants of MES in unearthing assumptions that we, as a field, typically make in studying mathematics, mathematics teaching and learning, and mathematics education, encouraging us to consider more humane conceptualizations.

RATIONALE

Mathematics education reform in the US has tended to target two rough areas of change for what mathematics to include in schooling and how to support learning. The first involves curricular content, or what should be prioritized in US K12 math (and in what order). Such an approach has largely left what counts as "mathematics" unquestioned, and in the hands of academic mathematicians. More well-known examples of this approach include "the new math" (Phillips, 2014), and the introduction of the Common Core Standards (National Governors Association, 2010), but many, more minor introductions of new content or sequencing also fit into this category (e.g., learning trajectories). The second area of change keeps curricular content intact, but focuses on what it means to understand or know mathematics, and how to support this kind of learning. This category spans a wide variety of research including a focus on reasoning and sense making (NCTM, 2009); the introduction of new technologies as standard supports for learning (Kaput, 2001); and efforts like Funds of Knowledge (Civil, 2002; González et al., 2001) and critical pedagogy (Gutstein, 2006), that argue for providing access to learners by valuing and leveraging their existing cultural practices and knowledges, and simultaneously

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empowering them with mathematical tools for futures related to citizenry, insurgency, or critical consciousness. This latter category, while more explicitly scrutinizing the cultural and political systems that (re)produce educational injustices, tends to leave mathematics content untouched. For example, the tension between academic mathematics recognizable by researchers and valued by teachers and the authentic home and community practices of students and their families was central to the Funds of Knowledge work in mathematics (Civil, 2002).

We argue that a different focus of research, the one to be explored in this symposium, has the most promise in making systemic change. Scholarship in this category, which we will refer to as *what counts as mathematics*, raises serious questions about Western academic mathematics as the primary (and usually only) mathematics that should be taught in school, eschewing what Gutiérrez (2002) called a "dominant mathematics"—one that systematically and predictably excludes particular populations of learners. Some versions of an alternative mathematics that decenters Western academic mathematics include those investigated in ethnomathematics, which invests in a sociocultural and historical analysis of the mathematical practices of indigenous cultural groups (e.g., Ascher, 2002; Zalslavsky, 1973), although such analysis is not without critique (e.g., Pais, 2013). Similarly, scholarship in cognitive anthropology and the learning sciences have investigated mathematics that occurs in professional workplaces (e.g., Masingila, 1994) and everyday settings (e.g., Lave, 1988). While these studies illuminate a broader notion of mathematical knowledge and practice than that typically valued in mathematics education in the US, they tend to be leveraged to improve aspects of typical school mathematics (such as discourse practices, or resources learners use) and stop short of reconceptualizing what a more expansive and humane mathematics might look like; instead, the goals and logic of academic mathematics remain uninterrogated and solidly in place within US schools (Gutiérrez & Dixon-Román, 2011).

We believe though, that work involving what counts as mathematics holds significant promise for an equitable, sustainable, and humane mathematics education. The epistemological assumptions of these studies, rooted in ethnography, resist the imposition of a priori notions of valued mathematics. Instead, they share a commitment to honoring the lived experiences of nondominant communities and what Lave (1988) called "just plain folk," and to looking for and understanding the mathematics in their lives. The three cases to be discussed in this symposium take related, but varied orientations to the reconceptualization of mathematics.

Gholson's perspective disrupts adult-centered notions of mathematics as a finite body of knowledge, a collection of topics, or a set of practices. She offers the possibility of mathematics from the perspective of the child, particularly Black girl children from an urban center of the United States, from the perspective of what James, Jenks, and Prout (1998) describe as the "the tribal child." In the words of James and colleagues, "Children are not understood as 'cultural dopes' [in this view]; theorists do not begin from the premise that they [children] have only a misguided, mythological,

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superficial, or irrational understanding of the rules of social life. Their worlds are real locations, as are those of adults, and the demand is that they be understood in those terms" (p. 29). We concede to this demand through an activity in which participants will engage in children's mathematics *as children* and imagine what kind of experience or object mathematics might be within the world of Black girlhood.

Ma and Kelton's case takes up learning as a cross-setting phenomenon (Jackson, 2011), seeking to reconceptualize mathematics as practices that travel, transform, and reposition themselves and learners across contexts. This multi-sited perspective looks at learners' experiences of particular mathematics in order to characterize how these practices might be conceptualized when taken as life-wide rather than site-specific. As an example, we investigate proportional reasoning as an aesthetic practice across video consumption and production activities.

Gutiérrez brings a third perspective, borrowing existing concepts from Indigenous cultures that might help us rethink a multiplicity of mathematics. She argues that we begin with a set of guiding principles—In Lak'ech, reciprocity, and Nepantla—as a way of "seeking, acknowledging, and creating patterns for the purpose of solving problems (e.g., survival) and experiencing joy" (p. 15), what she refers to as mathematx (Gutiérrez, 2017). In doing so, we have the opportunity to learn from other-than-human persons and to rethink our relationships with them and with each other.

In this symposium, we pose the question, what are possible routes, as mathematics education researchers, for reimagining both what can and should count as school mathematics, and the kinds of engagements we should design for our students to support mathematical learning? Where, when, and how should we look to help us develop new conceptualizations of what might count as valuable mathematics for compulsory education and/or sustainable futures? How can these reconceptualizations offer a new construction of mathematics that disrupts dehumanizing and limiting disciplinary logics and perspectives on learning?

SYMPOSIUM STRUCTURE

The symposium will begin with a brief (10 min) introduction of framing questions and the three cases for reimagining mathematics. Then participants will explore each perspective through a hands-on activity designed for an experience of relevant phenomena, followed by the discussion of video data (30 min each). We will conclude with a discussion that brings the three cases together, considering goals and tensions involved in reimagining mathematics for mathematics education.

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“CRISIS" - THE NEW NORMAL: FAKE (POST-FACTUAL) MATHEMATICS EDUCATION

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INTRODUCTION AND AIM

Crisis emanates not only from economic and political aspects of one’s social location, but it comes at the intersection of constant inaccessibility, denial, prevention, prejudice, sustainability risks that emerge from one’s disadvantaged conditions that have presumptively become “normal” conditions of living. The crisis conditions become so normalised and seasoned that those living in it often do not see them as crisis. Such conditions can emerge from living in constant poverty and hunger, constant denial of rights and access by the State (or a foreign military occupation), traversing difficult terrains of sustained conflict situations, fear of bullets and physical, psychological and financial trauma. Many of our lives are entangled around crisis all the while and everywhere when living within crisis has become a social norm.

Set in the above backdrop, this symposium will be a sequel to the symposium on “Crisis” in MES-9 by the same group of presenters, which critically discussed problematising the notion of crisis within mathematics education. It is important to note that, although this symposium is a sequel of the MES-9 symposium on Crisis, it is not a continuation. It is rather inspired by our work during and after the MES-9 Crisis symposium. This means, anyone is welcome to participate, including those who did not take part in the previous edition. This symposium in MES-10 will engage participants to deconstruct theoretical traditions and their absence to unpack crisis situations that have become normalised and seasoned in “developing” and “underdeveloped” world contexts. This time we will explore the capricious combination of the presence and absence of mathematical knowledge in our national contexts. This combination is another clear feature of our crisis.

The idea for the present symposium emerged from a few conversations in our WhatsApp chat group – which we have been following since the MES-9 symposium preparation days. We were talking about fake news during 2018 presidential elections in Colombia and Brazil (and other events), the huge influence over the results and wondering why this was becoming a norm and what the agency of Mathematics Education in this situation is.

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What can we do? What is the common concern with respect to Mathematics Education? We were scared about the poor reading comprehension and with the absence of facts checking. The absence of verifiable facts. There are gossips and memes flooding the Internet. How do we then weave these new phenomena with Mathematics Education? How does mathematics education enable us to make sense of the new "normal"? Is this really a "new crisis" or is the world seeing this as "crisis" just because the West has started seeing it as crisis! Symposium session will begin with each participant presenting examples of how mathematics could be absolute or useless, important to the comprehension of facts or just ignored, giving place for the so-called fake news by unpacking "crisis". This set of presentations will be followed by inviting the participants to analyse the examples and our hypothesis.

We saw important relations that we would like to discuss during this symposium. For example, those memes and gossips usually fight for the creation of narratives around official statistics, voting intention, survey results, number of unemployed, and so on, to present a fictitious picture. A liberal government will say that money for public education is an expense. Instead, a socialist government will call the same thing as an investment. The battle is for the narrative, the discourse, or the naturalization of a certain discourse. However, in general, we do not see any use (or abuse) of mathematics. It is just a bunch of memes, parodies and no (or no-true) arguments. There are no numbers. There are no reasons. There are no mathematical models. It is the absence of any measurable facts. We are talking about the social networks and the black press through WhatsApp chains and youtubers. We are not talking about the mainstream established media (CNN, Rede Globo, BBC, etc.). Is the mathematical argument "losing" its power?

We believe that one possible answer is "sort of". It is the appealing of the emotional side: "make America great again"; "America first"; "Peace without impunity"; "do not turn into Venezuela", and the list goes on. It is the propagation of fear and hate that suspend any critical judgment. Do not reflect, do not think, just react against the imminent thread. Defend your life and run. It is the opposite of the cold and timeless reasoning of mathematics. We find cases like the Trump's denial of global warming just because the winter was longer this year or the image of Venezuela as hell while Colombia is doing worst in almost many indicators. Authority and fear triumph over reason. Numbers and facts cannot be trusted, only images, feelings can.

It is paradoxical that critical scholars have fought against the uncritical faith in mathematics and now the problem is that there is a uncritical suspicion in mathematics. We wonder if the Mathematics Education community played some part in this battle. Part of the Mathematics Education community always criticizes the ideology of certainty, but now that it is gone from some

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places, some of us are afraid. Now the dominant discourse is to bypass and ignore scientific facts. So, we have succeeded in spreading doubts on mathematics! Too many doubts! Should we celebrate? Or, it is like the saying: "be careful with your desires, they can come true"? Which sort of mixing of mathematising and de-mathematising acts is operating? Ideology of certainty is on and off randomly? In older times that ideology was working continuously, now it has breaks.

Nevertheless, we do not think we should equate "doubts on maths" with "prohibition of maths". It is not because of doubting, or critiquing mathematics that we have a dismissal of mathematics. It is about being aware and making aware, but the critique in social agenda has stopped. It has moved from blind faith to rejection without any hesitation. Perhaps the lack of critical awareness about mathematics might have also contributed to its rejection. How can we work towards ensuring that critical awareness not only counters blind faith but also counters blind dismissal?

We would also like to discuss to what extent the critiques coming from socio-cultural and socio-political approaches to the omnipotence of mathematical knowledge have been capitalized by hegemonic forces as well. Not in the sense of instrumentalize or appropriate in bureaucratic ways the statements of Critical Mathematics Education, but in the sense to foster within public sphere an automatic rejection or mistrust on arguments based on mathematics. Such rejection is a strong revival of the *intrinsic dissonance* of mathematics, unfolded by Skovsmose & Valero (1999). We will collect a gallery of episodes in which the use of mathematics was banned, dismissed or replaced in our countries.

Our contention is that we are not being successful at promoting a critical distance towards mathematics, and that many of our arguments against universality and neutrality have been trivialized and turned back against its original intention. Fear has been effective as a partisan discourse. We need to understand and operate within this mechanism of de-mathematising public discussion. We need to begin to spin in a different direction to rescue the critical perspective on mathematics.

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LANGUAGE DIVERSITY AS A RESOURCE FOR KNOWLEDGE ABOUT MATHEMATICS EDUCATION AND SOCIETY

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A diverse MES community, by ethnicity, geography, language and so on, can potentially strengthen our knowledge about mathematics education and society. Yet it is possible that certain socially situated research practices might constrain the what, how, who and where of our epistemological positions, research questions, methodologies, writing, and knowledge. This symposium focuses on diversity in the language resources of researcher and research participant or between researchers. It explores how language practices, for example the dominance of English, might constrain knowledge production, how individuals navigate these constraints, and how this diversity may be a resource for building knowledge and an ethical community.

A DIVERSE MES COMMUNITY: OPENINGS AND CLOSINGS

The MES community focuses on the historical, social, political and ethical dimensions of mathematics education, and aims to provide a space for discussion and collaboration amongst researchers and practitioners across contexts. These contexts are characterized by social, cultural, gender, geographical, ethnic, political, linguistic, and other differences. Scholars have acknowledged the potential of this diversity for strengthening research (e.g. Ernest, 2009). However, there are concerns about the extent to which the dominance of certain socially situated research practices might constrain or "compress" (Atweh & Clarkson, 2001, p. 80) the what, how, who and where of epistemological positions, research questions, methodologies, writing, and knowledge in global education research communities (e.g. Apple, 2001; Atweh & Clarkson, 2001; Ernest, 2009; Meaney, 2013). Examples of such practices are research funding models, the dominance of English as the language of communication, the location of conferences and publishers in the global North, and the valued journal and conference genres.

In our experience, this concern moves in and out of focus in MES scholarship and the practice of the MES conference itself. Indeed, responding to these concerns is both a challenge and "risk" (Meaney, 2013, p. 78). On the one hand these research practices are political, structural and ideological (Ernest, 2009; Meaney, 2013). For example, *anglonormativity* or the dominance of standard English constructs multilingualism (and not monolingualism) as a problem, a construction that in some contexts is racialised and classed (McKinney, 2017). In addition, a challenge to the hegemony of some practices has to be accompanied by *design* (Janks, 2010) to rewrite the script. Otherwise, we risk simply reproducing the dominance we seek to challenge. On the other hand, as argued by Ernest (2009), there is a need to "make space for the

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personal" (p. 74) in this structural analysis, recognising the roles, motivations and agency of individuals in navigating these practices.

In this symposium we seek to pursue with others the risky task of exploring the knowledge building practices of research on mathematics education and society in contexts of linguistic diversity. This diversity may lie in the linguistic resources of researcher and research participant or of researchers themselves (e.g. Andersson & le Roux, 2017; Meaney, 2013; Setati, 2003). As we argue next, this diversity cannot be considered in isolation from the other forms such as geography, ethnicity, and so on.

THOUGHT-PIECES: LANGUAGE, PLACE, BEING, KNOWLEDGE AND POWER

Language and "home"

In a research interview the student Sipho (a pseudonym), a working class, black African student at an elite, English-medium university in South Africa, described his experience of academic writing: "With writing you are not expected to come from your home" (Thesen, 2015, p. 421, citing Nomdo, 2006). With this reference to "home", Sipho establishes a link between his language use, place and sense of belonging therein, and being. For as Thesen (2015) argues, in postcolonial contexts, this association with "home" necessarily racializes the identity of the writer.

Using Sipho's comment to explore academic writing, Thesen (2015) asks: Who might be the "you" referred to by Sipho? Is it possible/necessary to feel one belongs in the academy? Since language is not neutral and what people say or write involves weighing up one's commitment to what one brings, ones imagined future, and who one hopes to be in the future, what are the implications for knowledge-making? What are the implications for this process if engaging in a particular language practice involves having "sold out or lost out on something valuable" (p. 424)?

Language and "reality"

Analysing the "African novel", Ashcroft (2014, p. 75) uses extracts from Ben Okri's (1998) novel *Infinite Riches* to elucidate the "writing" of African history by colonial powers. In this novel, Okri (1998) describes the writing of the Governor General, a symbol of colonial power:

> He rewrote the space in which I slept. […] He changed the names of places which were older than the places themselves. He redesigned the phonality of African names, softened consonants, flattened the vowels. By altering the sound of the names he altered their meaning and affected the destiny of the named […] The renamed things lost their old reality. (1998, pp. 125-126)

Extracts such as this, Ashcroft (2014) argues, show how in the process of "knowing the other" (p. 75), the colonial power changed the place, experience, and future of the colonised. Indeed, Okri (1998) emphasises the power of language in this rewriting: "with a stroke of his splendid calligraphic style he invested us with life" (p. 127), life that began with colonisation.

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LANGUAGE, PLACE, BEING, KNOWLEDGE, AND POWER IN MATHEMATICS EDUCATION RESEARCH

In this symposium we explore with conference participants relations between the language practices of the community and the how, what, who and where of research on mathematics education and society. We ask questions such as:

1. Who gets to bring their “home” to research on mathematics education and society, and to “(re)write” this space? How do individuals navigate the social conditions that structure their research practice?
2. In the process of “knowing the other”, what meanings are made visible? What meanings might we be “losing out on”? And with what consequences for what we “know” about mathematics education and society?
3. How might the diverse language repertoires of the MES community be a resource that both strengthens our knowledge of mathematics education and society, and allows us to live the ethical practice of mathematics education to which the community aspires?

SYMPOSIUM STRUCTURE

The introduction to the symposium (5 mins), will be followed by brief input in which presenters will draw on their own research histories – as it pertains to linguistic diversity in the resources of researcher and research participant or between researchers themselves – to respond to the thought-pieces and the related questions posed (30 minutes). Presenters will respond in a language of their choice, with written pieces in English provided. Symposium participants will then be asked to bring their own histories to these thought-pieces and questions in small group discussion and plenary report backs (45 minutes). A discussant will round off the session by reflecting on the breadth of the contributions (10 minutes).

Luz Valoyes-Chávez: Last year I joined a group of scholars to discuss math problem solving from different perspectives. Although the seminar was held in Chile, a Spanish-speaking country, the official language was English. As I started my presentation, I freaked out. I could not module coherently my ideas. I felt dumb and dull and could not face my colleagues the following day. I decided I would not present in English anymore. This episode has made think deeply about notions of diversity, inclusion and integration in the field of mathematics education and particularly inside the research community. At this symposium, I would like to discuss about these reflections.

Jayasree Subramanian: The language of teaching and learning at the higher education level as well as reading and writing at the research level in India happens in English. However, school education, preservice and in-service teacher training happen most often in the regional languages. This means any action research needs to be visualized and formulated in English, carried out in a regional language and reported back in English. I carried out an action research to test out an alternate approach to teaching and learning fractions at the primary level in a region where I

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could barely speak the local language Hindi. The classroom transactions were therefore carried out by my team members who speak Hindi. But their comfort in English is very limited which means when the research was reported I had to speak for my team members rather than have them speak for themselves. It posed major ethical challenge for me because the research could not have been carried out without their significant role in realizing my vision into action. Was I right in formulating and reporting on their behalf?

Lisa Darragh: During my time as an English-speaking researcher in Spanish-speaking Chile I found myself enmeshed in a new language-power dynamic. Locally I was marginalised due to being part of the language minority, and this generated many challenges during the research process. Yet within the wider, global research community, I held a position of power being a native speaker of the dominant academic language. Whilst I could give assistance to my colleagues with their international conference and journal submissions, this position was conflicted, perhaps maintaining the oppressive power structures that come with language dominance. In having a "home" away from home am I contributing to research colonisation?

Annica Andersson: As a researcher, I meet students and teachers whose loads, languages, cultures, experiences etc. I don't share. I have been struggling with ethical questions about what to share in my research writings from these peoples' stories. In addition, language concerns make it even more complicated. I have reflected about how I as a researcher translate a students' second language Swedish, with the particular loads the students' choices of words brings, into my second language English, in an academic "correct" way. Then, what do I answer journal reviewers who asks me to "clean my transcripts so the language becomes clear and explicit in English?" or the reviewer who commented "This [research text] is clearly not written by an English native speaker". Boldly put, I have recently been thinking that we as researchers maybe shouldn't share others' stories at all as there are risk and troubles for others' and me that follows these decisions. At this symposium, I would enjoy to talk about these reflections and hear your views.

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AVAILABLE POSITIONS, IDENTITIES AND DISCOURSES IN MATHEMATICS CLASSROOMS

David Wagner	Annica Andersson	Beth Herbel-Eisenmann
University of New Brunswick	University of South-Eastern Norway	Michigan State University

This symposium will engage participants in discussion and reflection relating to the positionings and identities available for people to take up in mathematics classrooms and the related available discourses. We will also consider our constraints as researchers to recognize available identities, positionings, and discourses. Participants will reflect on their experiences with these concepts in and out of research contexts.

SYMPOSIUM FOCUS

Together and separately, the three of us have analysed the positioning and identities of students and teachers in mathematics classrooms. We have become increasingly interested in the way research using these constructs uses the word available. There are many instances of researchers referring to “available identities”, “available discourses” and “available positions” in the literature but we have not found these constructs sufficiently conceptualised. And so we invite our colleagues to join our conversation about these constructs in this symposium.

A number of theories connect to the questions we bring to the symposium. In particular, positioning theory says that people draw on known storylines to interpret their interactions and to position the people in the interaction (e.g., Davies & Harré,1999). Contexts make some storylines available and others not. Within a storyline, certain positions are available while others are not. Similarly, in research on identity it is often important to identify discourses at play and the identities taken up by people. Again, what discourses are available and what identities are available within these discourses is often an important part of the analysis. The questions of focus for the symposium:

1. When students navigate mathematics classroom interactions, what positions and identities are available to them to take up? This question hinges on the available storylines and discourses.

2. How do our positions and identities as researchers constrain our analysis of mathematics classroom interaction? How do they impact what storylines, discourses, positions, and identities are available for us to notice?

3. How can we as researchers expand our vision to recognize more or deeper storylines, discourses, positions, and identities? In other words, how can we see things that have not been seeable for us?

4. How can mathematics classrooms be constructed to make more and/or “better” positions and identities available for students?

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These questions are not new to us. Beth and Dave have raised questions like this before—for example, in an earlier unpublished draft of their elaboration on positioning theory used in mathematics education (Herbel-Eisenmann, Wagner, Johnson, Suh & Figueras, 2015), they criticized the progenitors of positioning theory who “did not state how they knew what the available positions might be within any given storyline, except that one knew these based on the grand narratives or stories we live by.” The questions also surfaced in Annica and Dave’s work on a forthcoming article. They decided to avoid the words ‘available identities’ and to leave the questions for future work. Because they are big questions. This led the three of us to bring this question to MES for discussion. We already know that available identities and positions are contingent on context. We expect that this situationspecific aspect of available identities and positioning will draw our conversation to some specificities of mathematics classroom contexts. We have also asked ourselves what might govern availability in a mathematics classroom context. Would there perhaps be a super-storyline or super-discourse that governs what is available?

We note that our claim about the lack of theorization and abundant use of “available xxx” is based on extensive literature searching, which we will not fully report on here for lack of space. We provide a couple of examples, however, that are perhaps the closest we have found to addressing these questions about theorizing these ideas in mathematics education. Evans, Morgan and Tsatsaroni (2006) interrogated group work in a mathematics classroom. Their analysis “identifies positions available to subjects in the specific setting [a mathematics classroom], using Bernstein's sociological approach to pedagogic discourse” (p. 209). The paper indeed identifies positions that are available and notes how they are related to different discourses, but is not clear on what makes a discourse or a position available. Nasir (2014), who has studied mathematics classrooms, described how students “simply take up available identities for which there are significant identity resources in both the local community and the broader society” (p. 143). She went on to point “out that identities always consist of the raw material that people find in the social contexts around them and occur in social interaction with others” (p. 143), but in this book she identified identities that are available in particular contexts without going into detail about how certain identities may be available or not in a given context.

SYMPOSIUM ACTIVITIES

The symposium will begin with an overview of the focus questions and how they are important to key theories used in our field. We will reflect on and discuss an experience common to all participants, drawing on something from the opening of MES or an agora. We will invite participants to describe the positions and identities that were available and taken up by them, and relate these to available discourses and storylines available. Are there discourses, identities and positions available at MES now that have previously been unavailable? Are different articles accepted/refused now than a decade ago? Whose texts are/were included and/or excluded?

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Then, to help us think about how new positions and identities may become available for a person, we will form small groups to reflect on personal experiences. We will each think about a position, identity, storyline or discourse that we can recognize now, but which we remember not being able to recognize some time ago or an interaction when others assumed particular available identities/positions for us. Or perhaps we see an identity/position with a different perspective. We will reflect on what made it possible for us to see that which we could not see before. Possible areas for reflection might include gender, race, ability/disability, age, social class, caste, religion, immigration/emigration, language, literature/myth, etc.

For example, while at a national mathematics education conference in the U.S., Beth had a meeting in the evening near the bar with a senior male colleague to talk about some issues related to committee work for the organization. After the discussion, they both walked back to a large table of people they knew who were also attending the conference. Another senior male professor who Beth had never been formally introduced to (but knew from past conferences/presentations) looked at the male senior colleague and said, "Who's your girlfriend, [name]?" and laughed. Beth replied by introducing herself, stating her professional rank and affiliation, and telling the person that they would be attending a small meeting together soon. Another example: when Dave lived in Swaziland he got a little experience of what it is like to be vulnerable as a minority, but he is aware that aspects of his privilege made him less vulnerable than minorities in other contexts.

Finally, to apply this important reflection to our research practices, we will ask participants in their small groups to consider some of their research data or a memory of a mathematics classroom experience. For this it would be beneficial for participants to bring with them some transcripts, photos, or other data from their research if possible. We chose not to use our own data because we want participants to be familiar with the contextual details of the situations used for reflection.

In this second group work set, we will each ask ourselves the focus questions above. What positions and identities are available for the mathematics classroom participants to take up? etc. The tricky part will be to overcome the challenge of seeing the unseeable. In order to push ourselves to notice the constraints in our analysis (what positions, identities, discourses and storylines are available for us to see), we can think about what we can see that we think other researchers or participants in the classroom might not be able to see and why. And we can think about what we can see now, but which we think we would not have seen in a similar situation some years ago. What makes it possible for us to see such things now?

For example, in a research interview about ten years ago Ara, a 15-year-old Turkish man, pointed out to Annica that his interviewers, as Swedish people, would not be able to understand immigrants (he used other words) like him (see Andersson & le Roux, 2017). When conducting interviews in similar situations last year, Annica noticed that she did not heard statements in that vein. Reasons for one person saying this and others not might be that there are now new identities available for

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newcomers or other identities no longer available, or discourses that allow people to talk about language and mathematics learning in ways that have not previously been available. Another example: Beth and Dave found that a new form of discourse analysis (lexical bundle work) made it possible for them to see interaction as strange though it once seemed natural (Herbel-Eisenmann & Wagner, 2010).

We aim for the small group interaction to be rich and meaningful in itself. Some of what we learn in the small group interaction will be shared in the large group, but we will not rehash everything. After these two sets of small group work and reporting back to the large group, we will bring two further questions forward for all.

1. What does this mean for research? We think of ethics, methodology, theory, …

2. What might we all do to sustain this conversation and thus bring greater depth to our own research and to our field, toward goals of humanizing this work?

We reiterate here that participants are encouraged to bring a piece of data/text from recent work/reflection. This may be an artifact (e.g., transcript, photo, …) or a vivid reflection on an experience in a mathematics classroom.

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PROJECT PRESENTATIONS

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MIGRATION AND DIALOGUE IN MATHEMATICS CLASSROOMS

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The presence of migrant children in mathematics classrooms can create challenges for students and teachers, who may experience tensions, transformation and adaptation. Our recently launched research project aims to generate accounts of the experiences of student migration in Canadian mathematics classrooms, as well as the experiences of their teachers. We propose multi-layered dialogues between students, teachers and parents to highlight and investigate the ways in which the members of a classroom community make sense of the interrelationship between their cultures and the ways in which they act to enhance the teaching and learning of mathematics.

CONTEXT

Jansen (2016) characterises migration as a change of residence, across international borders, as well as within countries. By bringing cultures into contact, migration influences values in both directions, modifying all individuals concerned, and creating new social structures (Gasper & Truong, 2010). In the context of mathematics classrooms, constituting, maintaining and modifying such new social structures can result in complexity and confusion for both learners and teachers. One aim of our research is to obtain accounts of some of the experiences of migrant students in Canadian mathematics classrooms. A second aim is to create dialogue between teachers and students to investigate how the sharing of cultures-historical repertoires of all members of a classroom community might help teachers and students in the process of mutual integration, and enhance the learning and teaching of mathematics for the students. Gutiérrez and Rogoff (2003) describe cultural-historical repertoires as ways of engaging in activities that are based on observing and participating in cultural practices. Our research is located in upper elementary schools and addresses the following questions: 1. What is the nature of migrant students' cultural-historical repertoires of mathematics? 2. How do the teachers of migrant students experience migrant students' experiences of the cultural-historical repertoires of mathematics in their mathematics classrooms? 3. How do teachers of migrant students adapt their practice in response to migrant students' experiences of the cultural-historical repertoires of mathematics in their mathematics classrooms?

MIGRATION, EDUCATION AND MATHEMATICS

Much research within our field has studied the multifaceted effects of migration on the learning and teaching of mathematics. Research that investigates teachers' experiences of migration in their mathematics classrooms frequently identifies various tensions. Many teachers do not feel prepared to work with migrant students, due to challenges relating to cultural, social and linguistic differences (Civil, 2007). To reduce the apparent cultural 'gap', studies suggest bringing students' knowledge and experiences from of out of school into the mathematics classroom (Zevenbergen, 2008). Integration of and learning from students' experiences can be challenging,

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since teachers' conceptualisations of the issues of migration are directly influenced by their own historical and cultural backgrounds (Nieto, 2015).

Research has also identified challenges faced by students as a result of migrating into mathematics classrooms. The predominant underlying assumption is that migrant students are in need of support. A deficit-oriented view suggests that the mathematical performance of migrant students is lower than that of their non-immigrant counterparts, due to socio-economic inequalities, linguistic barriers and cultural differences (Bishop, 2002). Some studies conducted in the field of migration research suggest that migrant students' mathematical performances depend ultimately on the cultural influences they are exposed to and the level of cultural connected-ness they encounter from the host and from home societies (Ogbu, 1992). Hence, the underlying factor that affects children's schooling is not necessarily linguistic and cultural differences; it is the nature of the relationship between cultures (Ogbu, 1992). The lack of negotiation between school culture and students' home cultures is thus an obstacle to migrant students' learning of mathematics (Borden & Wagner, 2010). There is little research that investigates migrant students' experiences in mathematics classrooms. We see these experiences as resulting from the relation between cultural-historical repertoires. Our study proposes to examine these relations by providing a space for dialogue between students and teachers.

CULTURAL-HISTORICAL REPERTOIRES IN DIALOGUE

Our theoretical perspective conceptualises interaction between cultural practices in mathematics classrooms. We draw on a relational approach derived, in particular, from the work of Bakhtin (1981). In mathematics classrooms, teachers and students participate in mathematical discourses, including ways of talking, gesturing, acting and interpreting. The collection of practices available to a participant we refer to as their cultural-historical repertoire, because these practices are derived from participation in a shared cultural milieu, including a shared history of collective participation. The formation of these repertoires is based on observation and participation (Gutiérrez & Rogoff, 2003) and, over time, these cultural-historical repertoires change as a result of dialogic encounters with the repertoires of others. We see the relation between different cultural-historical repertoires as *dialogic*, since they are mutually constituting and defining. Our notion of dialogue derives from Bakthin's, in which meanings of words (and ways of doing mathematics) are situated and related to their surrounding (i.e., before and after) utterances or actions (Bakhtin, 1981). Expanding this view of "meaning-in-relation" to specific cultural practices, we see that a cultural-historical repertoire (e.g., a method of adding fractions) similarly arises from its relations with other practices (e.g., students' previous experiences of fractions). From this perspective, the notion of migration in mathematics classrooms as a dialogical cultural-historical exchange between all students–those from migrant backgrounds and their classmates–as well as their teachers. Mathematics classroom interactions are a site of dialogue in which meaning (of mathematics and of culture) arises relationally through the encounters of all the members with each other. These

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encounters are shaped by forces which privilege some standard ways of doing mathematics on the one hand (centripetal forces) and forces that constantly produce multiple different practices and meanings (centrifugal forces) (see Barwell, 2018).

METHODOLOGY: LAYERS OF DIALOGUE

To address our research questions, we plan to create a multi-layered, multi-vocal, dialogic setting, so that the experiences of parents, students and teachers can be shared. We assume that these voices will be the basis for dialogue among the different cultural-historical repertoires of the members of mathematics classrooms in 10 elementary schools. Our research design draws on the multivocal structure outlined by Tobin et al. (2009) and subsequently adopted in mathematics education by Reid et al. (2015). This approach involves participants (in this case, students) collecting and representing information about their own experiences (and that of their parents), which are then shared with others (in this case, their teachers). The resulting interaction provides researchers with insights into participants' understandings of each other's cultural-historical repertoires. In different parts of the study, therefore, we will engage students and teachers as co-researchers. As co-researchers, students and teachers will be asked to take notes about the mathematical practices of their classrooms. To compare the cultural-historical repertoires of students and teachers across different settings, we propose 5 layers of dialogic interaction, involving 5 students and 5 teachers in each of 10 schools: 1. *Voices of parents* – students will interview their parents about their mathematical practices; 2. *Voices of each student* – each student participant will take notes once per week for 4 weeks; members of the research team will then interview each student; 3. *Student synthesis* – in a one-time collaboration, the five students in each school work to create an artefact (text, video, graphic or a combination) representing a synthesis of the experiences they have recorded in their notes; 4. *Dialogue between teachers and students*- In each school, five teachers will engage with the artefacts made by the students; 5. *Voices of teachers* – the five teachers will take notes of their experiences in the 4 weeks following their encounter with the students' artefact in order to record changes in their own cultural-historical repertoires. This approach is designed to facilitate dialogic encounters without the researchers needing to directly interpret students' or teachers' cultures.

FINAL REMARKS

We believe dialogue between mathematical cultural-historical repertoires is an evolving, open-ended, mutually respectful process between teachers, children, parents, and communities. Our study seeks to highlight the potential richness afforded by migration, by listening to students' voices and promoting a non-authoritarian dialogue among students and with their teachers. An original aspect of this study is to stimulate a multivocal, intercultural dialogue for students and teachers to see not only the cultural-historical repertoires of others, but also to reflect on their own practices. By the time of the MES meeting in February 2019, we hope to have some initial data

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from at least one school and to be able to report on the initial implementation of our research design in one school.

ACKNOWLEDGEMENTS

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CRITICAL EARLY CHILDHOOD MATHEMATICS FOR CHILDREN OF COLOR

Melissa Adams, DeAndrea Jones, Theodore Chao

Abstract: Early Childhood mathematics education often focuses only on counting, cardinality, and pattern recognition. Through a university/school partnership, we detail how a veteran prekindergarten teacher of color, working in a school serving a community of color in the United States, utilized democratic and inquiry-based early childhood pedagogical frameworks to mathematically empower her 4 and 5-year-old students. In this project session, we focus on the impact that a critical approach to mathematics education had on this veteran teacher's school and practice, with reflective discussion led by the teacher herself.

INTRODUCTION

Within the United States, the research on critical mathematics teaching practices that humanize and empower children is only just emerging, particularly in the area of early childhood education. Parks (2015) and Wager (2013) have exposed the current obsession with early childhood numeracy, which is heavily pushed onto children of color, as just another mechanism to limit children's creativity and inhibit the development of rich mathematical strategies. Instead, these scholars offer play-based mathematical learning as an antidote to the standards and accountability-focused obsession that has infiltrated U.S. schools. Ward (2017) and Authors (2016; 2017) take this construct of play-based mathematics one step further, connecting it to critical pedagogy. We argue that even the youngest thinkers can utilize mathematics as a critical tool of inquiry, that exposes and confronts oppression.

But how do Prekindergarten teachers actually enact and use critical pedagogy within their practice in conjunction with the development of rich mathematical strategies and creativity? In this session, we outline a three-month professional development partnership with an entire elementary school serving a low-income community of color in the United States of America. We highlight how this Cognitively Guided Instruction (CGI) (Carpenter et al.,1999) based professional development empowered a veteran prekindergarten teacher to try out new practices in her approach to mathematics teaching. And, we reflect on how these practices impacted not only the children's mathematical learning, but also the beliefs and practices of the teacher herself. Further we will present critical units 1 designed to put these beliefs and practices into action during the 2018-2019 school year.

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FRAMEWORK

This session details a professional development university/school partnership in Ohio, a large, midwestern state in the United States, that focused on supporting early childhood and elementary teachers to embrace and enact critical mathematics pedagogy. In general, when it comes to the mathematics learning occurring in early childhood classrooms, there is a tendency to focus on the development of single concepts or skills (eg. counting, identifying coins) rather than building the foundations for deep mathematical thinking and conceptual understanding. This tendency can limit students' development of reasoning and mathematical communication skills (such as argumentation and justification of ideas) that are keys to mathematical achievement (Papic, 2013). The state of Ohio's learning and development standards for Prekindergarten separate mathematics into a general knowledge category and, in fact, consist of a list of single concepts and skills (Ohio Department of Education, 2011)that students are meant to develop on a trajectory towards reaching Kindergarten content standards, but do not include underlying mathematical processes or communication skills . This inherently limits both the quality of instruction and the content students have access to and requires teachers to think beyond mandated standards if they are to promote mathematical thinking that can truly impact students' mathematical trajectories.

This professional development partnership intended to help teachers utilize and innovative research-based mathematics teaching practices. These practices served to mathematically empower students at an elementary school serving a predominantly low-income, Black community. In addition to CGI, we also engaged in teacher education experiences that focused on rehumanizing mathematics (Gutierrez, 2018), connecting mathematics identities to racial identities (Aguirre, Mayfield-Ingram, & Martin, 2014), and play-based early childhood mathematical thinking (Parks, 2015).

METHOD

Background

In this professional development partnership, two teacher educators worked with a group of 22 early childhood and elementary mathematics teachers over three months. The first teacher educator is a veteran Latina bilingual elementary teacher who primarily served Latinx students in whom she hoped to instill pride in themselves, their families, their language and their culture. To that end, the mathematics class centered student thinking and was deliberately structured to be culturally affirming--centering issues of social justice with CGI strategies employed deliberately in order to foment mathematical community (Author, 2018).

The second teacher educator is a veteran elementary mathematics teacher educator who identifies as Asian American and whose research focuses on ways to support and empower teachers of color.

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In this session, we focus on working with one veteran prekindergarten teacher who has worked relentlessly to empower her children of color to recognize racism, confront oppression, and continually love and care for their world and their community. This teacher accomplishes this through allowing children to make decisions about what kinds of mathematics they want to learn and how it connects to their community knowledge (Authors, 2017) and using mathematics for her children to unpack critical details within U.S. Black History (Authors, 2016).

Project Enactment

This partnership involved teachers collecting video from their classrooms for detailed feedback and discussion, the observation of model lessons enacted by the primary teacher educator, and opportunities to co-plan and co-teach with the primary teacher educator. The detailed work between the primary teacher educator and the prekindergarten teacher focused on four goals: (1) Moving from rules and memorization to centering critical thinking and deep understanding; (2) Belief in the tremendous capacity of young children and enacting new strategies that maximize that capacity; (3) Moving from seeing some children as being more capable of problem solving and critical thinking to serving the whole class; and (4) Envisioning mathematics beyond dictated standards and within units that emphasize social justice themes and student interests.

With an understanding of the politics of our curriculum, the veteran educator was searching for a method which will teach all of her students and produce high levels of growth. Since, we believe that students have a high capacity to learn at this age and are excited and eager to learn, the veteran educator in this project consistently added to the prescribed curriculum out of an ethical sense that student's learning opportunities should not be limited due to politics.

This project will follow attempts to add student thinking-centered mathematics and problem-solving within critical and child-interest centered units (e.g., AfroLatinx identity; music, rhythm and movement) throughout the course of a year, documenting both the shift in teacher pedagogy and practice and the outcomes in terms of student learning and engagement.

DISCUSSION

We approach this work as U.S.based early childhood mathematics teachers and educators who would like to learn more about how to enact critical stances from scholars outside the U.S. Our experience has shown us that early childhood mathematics education in the U.S. is heavily skills driven, which does not acknowledge the brilliance and critical capacity of young children, particularly children of color.

We hope that this session will only allow us to share the work and thinking we have engaged in through forming university/school-based partnerships with the expressed purpose of empowering children of color. We also hope this session will allow us to

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engage in deep thinking and discussion around ways to align our work with global constructs, particularly philosophies in early childhood education that come from non-western and non-Eurocentric images of the child.

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USING SHOW AND TELL SOFTWARE TO EXPLORE MĀORI WAYS OF COMMUNICATING MATHEMATICALLY IN MĀORI-MEDIUM PĀNGARAU/MATHEMATICS CLASSROOMS

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Māori-medium pāngarau/mathematics classrooms, within the wider Māori-medium education context, are a space for revitalizing Māori language and culture. The language of instruction is an endangered minority language and is also a second language for many Māori-medium students and teachers. Added to this complexity, is the newness of the pāngarau terms and register. In order to address these pedagogical and linguistic challenges, there is growing interest and use of technology to support Māori-medium teaching and learning programmes, including pāngarau/mathematics. This project uses Show and Tell software to identify ways in which Māori-medium pāngarau students could utilise the specialized language of the pāngarau register to communicate mathematically through a uniquely Māori cultural lens.

MĀORI-MEDIUM CONTEXT

Many students and teachers in Māori-medium pāngarau/mathematics classrooms are second language (L2) learners of te reo Māori (Christensen, 2004; May, Hill & Tiakiwai, 2004; Trinick, Meaney & Fairhall, 2014). Further adding to the linguistic complexity of the Māori-medium pāngarau learning environment is the newness of the pāngarau/mathematics terms and register. Significant lexical development work has been conducted over the last 30 years (McMurchy- Pilkington et al., 2013, Trinick, 2015). Often pāngarau teachers may not have had access to the pāngarau register in their own schooling or initial teacher education (Christensen, 2004; Meaney et al., 2012). Therefore, pāngarau students who move schools or even classrooms may have to contend with unfamiliar terminology.

A considerable body of research argues that the explicit teaching of mathematical language can simultaneously support the acquisition of mathematical knowledge (Dowker, Bala, & Lloyd, 2008; Hunter, 2005; Pimm, 1987; Pitvorec, Willey, & Khisty, 2011; Schleppegrell, 2007). It is argued that in order to understand mathematics, students must understand the language of mathematics (Usiskin, 2012). International second language acquisition research suggests that judicious use of the first language (L1), which is English in the case of many Māori-medium students, can encourage the acquisition of the L2, in this case te reo Māori (Cummins, 2000). However, many Level 1 Māori-medium immersion programmes (81%–100% language immersion), adhere to the language separation policy outlined in *Te Aho Matua* (Department of Internal Affairs, 1999) which advocates the separation of languages by location, time and subject. This reflects the Māori-centric ethos of L1 Māori-medium immersion programmes, where English may be taught as a separate language subject but is not used as a language of instruction.

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This project explores a pedagogical approach that supports this Māori-centric approach to language revitalization and utilizes a system of capturing pāngarau language and content/concepts as they are being acquired. The study aims to provide opportunities for pāngarau teachers and students to investigate and reflect on their own development of the language and conceptual ideas necessary to communicate mathematically through the specialized terms of the pāngarau register and language resources unique to the Māori language.

USING SHOW AND TELL SOFTWARE IN MĀORI-MEDIUM PĀNGARAU CLASSROOMS

Show and Tell or screencasting software applications (apps) provide opportunities for students and teachers to capture their mathematical communications using multiple representations and share these with others (Williamson-Leadley & Ingram, 2013; Thomas, 2017). These types of apps have a whiteboard feature that students can either draw on or type into, the ability to capture or upload images and the ability to record audio or video explanations in real-time as they are being created or communicated. They can also choose which representations they wish to use and connect these representations to create explanations of contexts, strategies and solution methods for solving problems.

By allowing students to choose from and capture multiple mathematical representations in this way, Show and Tell apps provide opportunities for teachers to gain insight into students' mathematical reasoning. This provides insight into students' thought processes and problem-solving strategies (Peltenburg, Van den Heuvel-Panhuizen, & Doig, 2009) and also the students' mathematical reality and how this reality has been constructed (von. Glasersfeld and Steffe, 1991) which includes the language and cultural lens they are using. This is significant for Māori-medium pāngarau teachers and students who are L2 learners of the Māori language and may be struggling to communicate mathematical concepts using a singular language mode (Allen, 2015).

PROJECT OUTLINE AND DATA COLLECTION

The student work sample presented in Figure 1 below provides an example of the pedagogical approach described. The data was collected by a pāngarau/mathematics facilitator in a Māori-medium primary school. The facilitator worked with a group of seven Year 4–5 students for three hours per week over a period of 15 weeks. During the course of the study, students constructed their own pakitau (story problems) and used a range of mathematical representations to explain their solution methods. The students used My Mediasite Personal Capture software[1], to combine video recorded explanations of their story problems and static images of their solution methods. The

[1] For further information on My Mediasite Personal Capture software see: http://www.sonicfoundry.com/mediasite/capture/mymediasite/

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student work sample below shows an image that the student took of their workbook. The transcript is taken from the video recording the student made of themselves explaining their pakitau (story problem). In this case, the work sample shows slight differences between the written/oral explanations and the drawn/written diagram and equation.

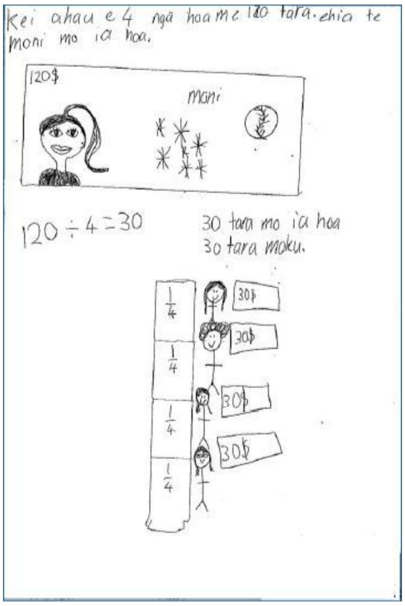

Figure 1: Student work sample (Allen, 2015 p.77)

Transcript:

Student: Kei ahau e whā ngā hoa me tahi rau rua tekau tāra. E hia te moni mō ia hoa? (tah) … Toru tekau tāra mō ia hoa. Toru tekau tāra mōku.

English translation: I have four friends and one hundred and twenty dollars. How much money would each friend receive? Thirty dollars for each friend. Thirty dollars for me.

DISCUSSION

The diagram and equation clearly show that $120 is divided equally amongst four people. However, the story problem in the written and oral explanation seems to imply that the money was shared amongst five people, the four friends and the student herself. While the diagram and equation could have been considered self-contained (Cummins, 2000; Gibbons 2002) and mathematically accurate, the ambiguity in the context of the story problem provided could indicate the need for further language modelling. In the example above the prefix: *toko-* (used with numbers 2-9) has not been used to indicate a group of people. The plural possessive determiner: *ōku* has also been used without the pronoun: *mātou* which could be used to indicate that the student is including herself within the group of four people. These are examples of resources within the Māori language that the student could use to add clarity to the story problem context. It is also possible that the student could change the amount of money being divided in the initial context to $150 and adjust the diagram accordingly. While these alterations to the story problem and diagram could improve the accuracy of the explanation, they do not fully

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utilise the language resources described above. This project aims to use Show and Tell software to develop a pedagogical model that identifies and addresses next steps in improving the accuracy of mathematical communication in Māori-medium pāngarau classrooms through utilizing resources unique to the Māori language.

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MATHEMATICS EDUCATION OUT IN THE RURAL SCAPE: *EXPERIMENTING WITH RADICAL DEMOCRACY FOR COMMONS*

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Abstract: The purpose of this paper is to discuss a pedagogic experimentation in the rural scape at a Greek village that tends to challenge the neoliberal and capitalist politics of economic austerity in the area. A group of young educated people in their 30s instead of migrating to the 'West' for a secure job have opted to return at their place of origin for a way of living around the values of a common ownership economy. They experiment with a vision of radical democratic pedagogy as vital for life. It unfolds in their relations, first with children as part of an after-school workshop and second with the adults in varied interactions as part of sharing knowledge for manual work in the land and with hand-made constructions. As such mathematical ideas, skills and competences are being entangled in diverse opportunities of life reorganisation around what they perceive as their commons.

INTRODUCTION

Due to an increased societal and environment crisis, in recent years, there has been a need for radical political engagement and action around collective reorganisation that involves experimentation with alternative economies and pedagogies. In many parts of the world, coalitions of parents, students, educators and activists have sought to confront and challenge the intensification of privatisation and austerity measures in both economy and public education through organised occupations of educational spaces, educational strikes, testing boycotts, and mass demonstrations, but also through vital projects of coming together and reclaiming human rights and dignity for accessing what is considered as 'common' goods. A central challenge for educational theorists and activists has been how to reconcile the need to defend and strengthen public educational institutions while simultaneously trying to find a new language and set of principles from which to reimagine these institutions in ways that do not reproduce their historical and/or present limitations. Questions posed with increasing frequency are: How might we reconstruct common educational institutions against their imaginative and political enclosure? How educational institutions and local economies may work together around commons? And, how could cultural capital based on academic knowledge, such as the deployment of varied forms of mathematical skills or competences and mathematising processes, might contribute in this endeavour?

In Greece, specifically at the precarious times of crises, a move is being performed, mainly by young people, from private and public institutions to common ownership collectives where 'common' αγαθά (αγαθόν=good) such as land, water, food, education, arts, crafts and technologies are commonly used, shared, preserved, protected or even produced (Kostakis, 2013, Kioupkiolis, 2014, Pechtelides, 2016). Such projects denote the urge for creating new spaces for economy (i.e. οικονομία: ο

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νόμος του οίκου: home rule) redirected from a high dependence on the state and the markets –held responsible for the 'tragedy of commons'- towards a more self-organised community based on democratic participation, equity, respect and autonomy (Ostrom, 1990, De Angelis, 2017). We want to argue, by means of this study, that for this to happen, community self-organisation around economy as 'commons' and pedagogy in the form of a radical democratic pedagogy need to go hand-in-hand and work closely. Specifically, two levels of creating pedagogical spaces can be noted, *first* amongst people in the community around issues of reclaiming their 'commons', and *second* with children and youth around varied micro-contexts of activity for commons.

This is the case of a collective of five young educated people in their 30s who returned to the rural scape of a Greek village, after having studied and worked in urban states, with the vision of contributing to a life that challenges economic and pedagogic austerity through radical democratic participation. The village is located in a northern mountainy area and is now inhabited mostly by Albanian immigrants or elders with most younger population having immigrated in 'west' countries. Despite precarious conditions, the collective opted to return at their place of origin aiming to create a space for a democratic commitment with land, knowledge and people. They have spent a few years working with the land and launching an after-school workshop where children from 3 to 17 year olds engage in manual activity, crafts and constructions, as well as, outdoor walks and coming together festivities. A core aspect of their co-engagement with children and adults in the rural scape is their focus in a serious experimentation with the potential of radical democracy in the village at a temporality of crisis.

In this context, we enacted with an ethnographic study in order to explore how such a commitment may affect pedagogic relations and, in turn, how pedagogy, and mathematics pedagogy in particular, may contribute towards envisioning this radical democratic project. Although mathematics education, along with other specialised subject areas was not the specific focus of the after-school workshop, we have denoted the tacit, but yet agentive, presence of mathematical knowledge (see Chronaki, 2010) in the realm of working with children towards creating new realities through making new objects and generating new activity. However, our experience of being there and attempting to interpret what has happened with children's knowledge in relation to contemporary social theorising in mathematics education troubles any essentialist onto-epistemologies of knowledge/power in democratic relations and raises a number of questions that we wish to discuss in this project presentation.

THE STUDY: A PEDAGOGIC EXPERIMENT FOR COMMONS

The study took place in an after-school workshop organised by this collective in a 700 inhabitant's village located at the rural scape of one of the high mountains in Greece (see table 1). People are mainly elders or immigrants from Albania who work in small-scale, yet globalised, agriculture domain whilst most educated youth have departed abroad or closer cities for securing a job in domains of high specialisation. The building where the collective lives and works is located a bit outside of the village and they

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make use of an old school provided by the municipality. Renovations involved the construction of yards, playgrounds and garden space and emphasis was placed on reorganising space and time to accommodate communal life and openness for children to visit and stay with them. Work was geared towards land cultivation around sustainable environmental oriented principles for their everyday living. Specifically, their philosophy involved cutting down consumption needs, becoming less demanding, recycling, reusing and producing the necessary quantity and quality of goods they needed for food, as well as for making their own constructions.

Table 1. Varied views of the location and the after/school housing

The after-school workshop, based on practical, hands on or manual work and outdoor activity, provided a micro-scale pedagogical laboratory for children to engage with similar values of living. Children were involved in making practices where they are responsible of constructing their own games using wheels, planks, ropes or any available material. Children created a balance game by placing wheels at some distance and then a plank to make a beam of balance. At other times, they created a tend to use in outdoor activity or crafts that involved knitting and jewellery making. Gardening, domestic animal care and constructions in relation to these are of high priority.

Table 2. Children's after-school engagement with activity around commons

Woodwork provided also a context for children to develop explicitly skills that cut across competences in varied areas including mathematics as Ermis explains:

> 'If you get any wood first you definitely have to go through these four five stages which are punching, cutting, grinding and joining woods together. Counting and marking on the woods is a very important part in which mathematics can be found. So, there is all the design and at the same time there is planning on paper what we are going to do'.

From a mathematics education perspective, all these micro-practices can be seen as providing occasions to discern the embedded and embodied mathematical activity in which one denotes the growth of skills, competences, reasoning, critique, language and concepts as shown by Alan Bishop (1998), Ole Skovsmose (1994), Rico Gutstein (2012) and Rik Pinxten (2016). However, one may wonder how the discursive context

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of such this experimentation with economy as commons in which the mathematics education of children is being placed might matter, in what ways and for whom? As well as, what emergent subjectivities for children and adults are being co/configured?

		

Table 3. Woodwork and the making of constructions

PLACING MATHEMATICS EDUCATION WITH/IN A RADICAL DEMOCRATIC EXPERIMENTATION OF A PEDAGOGY FOR COMMONS

Whilst the notion of democracy is an almost impossibility, its semantic tensions can be traced in early Greek and Roman times in the idea of 'demos' as an expression of people in the city holding power or 'civitas' where life is constituted through a system of rights (see Etienne Balibar's reading of democracy in Greek and Latin etymology). The assumption of demos' engagement directly in the political life contests a view of democracy as a constitutional or juridicial-legal process applied unproblematically without considering a wider nexus of conflictual positions due to an increased globalised, neoliberal and capitalist world. Radical democrats in the 1950s and 60s like Cornelious Castoriadis (1991) attempt to reconnect democratic theory to the notion of the political as a socio-material order defined by antagonistic discursive positions forming the constitutive ground of politics. Laclau and Moufee (2001) have advanced this into an ongoing antagonistic process for achieving anti-essentialist and plural positions. To this, Hard and Negri (2000) suggest the Deleuzian notions of rhizome and multitude as alternative ways of envisioning the maturation of social movements as nomadic collective subjectivities. Such vision may bring forward a new political subject that is yet to be configured through radical democratic efforts. We could see, in the realm of our study, that the everyday efforts of the young people collective upon their return to the village are constantly geared towards a strategic re-organisation for a political subjectivity focused on valuing the protection of commons.

How, then, we might realise the placing of mathematics education in this realm? How could a radical democratic experimentation of a pedagogy for commons may require (or not) mathematical knowledge? In what ways cultural capital through academic knowledge as mathematical skills and competences may participate in this endeavour? At this place, our study may suggest two considerations. *First,* a radical democratic pedagogy embedded in a local context of economy for commons needs first of all to espouse relations of radical ethics of equal participation that respects people and land. Respect for people and land go together. They are based on awareness of how the tragedy of commons has come about and its effects on global and local contexts. This

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double gesture of respect needs to denounce a top/down relation with expertise and experts, but still resorts on a process of articulation that involves mathematics in the forms of de/re/constructing ideas and objects, problem posing and solving, reasoning as ir/rational argumentation im/pure logic for the need to protect economy as commons. For example, we have been aware of discussions where members of the collective have tried to convince people in the village of sustainable ways of cultivating the land or, of how global capitalism has enforced ways of working that harm local land and these become re/contextualised in ways of working with children in the after-school workshop. *Second,* mathematical knowledge such as skills and competences for making constructions and even for articulating a viable reasoning process evolve not as formal education but through emergent activity where 'more knowledgeable others' share knowledge and expertise upon demand in a non-hierarchical manner. This was evident at a number of occasions when groups of children across ages were involved into making complex constructions (see woodwork in table 3) but also when they had to articulate arguments for convincing each-other. One may ask how such knowledge is being circulated, shared amongst the bodies of children and adults and how does it contribute towards reconfiguring collective subjectivities.

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Articulating Critical Numeracy: A Numeracy of Resistance

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Abstract: The purpose of this paper is to: (1) to elucidate an emerging analytical framework of critical numeracy that works to problematize the teaching and learning of numbers; and (2) to use this framework to propose and enact numeracy so that people understand numbers and numerating as a social and political activity. Numeracy must be analyzed from a sociopolitical stance to revitalize a contextualized understanding and principled purpose towards a transformative project with number.

The purpose of this paper is twofold: (1) to elucidate an emerging analytical framework for a critical numeracy that works to problematize the teaching and learning of number; and (2) to use this framework to propose and enact numeracy so that people understand number and numerating as a social and political activity. The logics of numeracy must be analyzed from a sociopolitical (Valero, 2004) stance. It must be situated, contextualized, historicized, politicized and most importantly, must be reconnected to a creative life force that is rooted in all human activity. I draw upon works that address Eurocentrism (Anderson, 1990; Amin, 1989) and a presumed objectiveness of science that rests on coloniality (Quijano, 2007; Powell & Frankenstein, 1997) and underdevelopment (Rodney, 2011). Numeracy can be understood as a location for political organizing, much like projects grounded in critical pedagogy. These projects transgress disciplinary boundaries and are exemplified in publications such as *Rethinking Mathematics* (Gutstein & Peterson, 2013). Such examples could be deepened in elementary schools and teacher education settings, provoking researchers and educators to engage in a praxis that opposes a toxic and manufactured notion of the world by means of mathematics education (Fasheh, 2015).

My intention is to go behind/beyond the disciplinary silos from which the concept of number is put forth. The social and political process of numerating (i.e., acts of creating and understanding/giving meaning to numbers) is already embedded in the mathematics education policy and curriculum, in forms such as high stakes standardized tests (Au, 2008), and also embedded in the political-economic processes shaping social life, monopoly finance capitalism (Foster, 2016).

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MATH IS SOCIAL

Mathematics is an integral part of the theoretical enactments of science, a diversely developing praxis of knowing and being in the world (Ferreirós, 2016). This implies that science is not universal but diverse in relation to socio-political developments and histories globally. The development of mathematics education (Powell & Frankenstein, 1997), along with a particular form of numeracy, has unfolded within broader socio-political histories of colonialism, capitalism and imperialism, from which a Western modern science has developed and expanded (Mutegi, 2011).

SITUATING NUMERACY

Numeracy is a socially based activity embedded within historically derived social relations that integrates math and ways of being. Currently, it is conceptualized in relationship to mathematics education and in a variety of contexts that renders its meaning in ambiguous ways (Coben, 2003). Numeracy is not synonymous with mathematics education even though there are shared principles and historical roots. Whereas mathematics is widely considered abstract, formal, and symbolic, numeracy is posited as contextual, intuitive, and integrated. The growing call for numeracy in the form of mental disposition toward facility with numbers and mathematics is of particular importance in this time of profit-driven data analytics and algorithms. How does a preparation of mental dispositions connect ways-of-being to political identities and/or subjectivities? The mandate for numeracy in this historical moment has widespread implications across disciplines. By attending to logics of coloniality as processes of social differentiation, and underdevelopment as a means of value extraction, we can delve deeply into numeracy and numeration systems. Numeracy, a tool of political control, is ambiguously conceptualized in relation to mathematics education and within a variety of contexts (Coben, 2003). Yet while its uses and constructions are contested, numeracy is posited to be necessary.

What numeracy is, how it is shaped, and what it does, are indicative of dialectical praxis in social life. Within capitalist logic, everything is numerated, commodified, financialized, and conjoined to algorithms in an effort to accumulate capital. Whereas numeracy has been forwarded as a measure of social development —in terms of numerate logic, skill, mindset, and disposition—the enactments of numerating focuses on processes where conceptions and material conditions in the world collide.

ACTS OF NUMERATING AND SYSTEMS OF NUMERATION

Numeracy and quantitative literacy in the aforementioned framework maintain an ethos connected to dispositions and ways of being. However, we can investigate the material activities and realities that accompany acts of numerating and systems of

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numeration. Numerating is a social activity and indicative of political relationships. Number, readily employed in the activities of numeracy, is commonly referred to and connected to static entities, or things, which are concrete, objective, and embodying of universalizing truth. Number is conceived of and related to an object, as an unchanging thing. The symbols and meanings designating number remain uninterrogated and taken for granted. However, by shifting our focus from a designation of number to activities, or socio-political agentive forces, of numerating as an activity process, we can attend to contradictions in society.

Numbers mediate ontological levels so that things which are counted are somehow constructed or verifiable to be real. Yet, these acts of ennumeration are political in essence. Just as dominating and colonial powers asserted the use of number as a form of power, numbers in a system of generalized commodity production underscore invasion, subjugation, and control. Ultimately, number plays a significant role in a process of alienation. At the same time number serves as a form separating communal connective sense among people. Number is a social practice, situated in an on-going process of knowledge production, and is inherently political, signifying an ontology of becoming.

NUMERATING AND HISTORICAL BECOMING

In order to extend a contextualized analysis of number, numerating and numeracy, we must shift a focus from teaching and learning to knowing and transformation. In order to build a critically transformative stance to numeracy, I draw and extend upon situated knowledge, as oriented by Haraway (Carpenter, 2012). She positions that all knowledge is partial, located, grounded, subjective, and has a common materiality in its configuration. I also draw on Carpenter's (2012) assertion of experience and learning, to extend a theory of numerating that must go beyond representation and reimagine a possibility of transforming experience. We must develop and connect our understanding of experience (which is simultaneously individual, social, and global) in order to change the conditions that we live in. In this way, number can be indicative of relationality in that it infuses difference, contradiction and possibility.

One purpose in developing a critical numeracy will be to analyze the dialectical relationship between a consciousness and praxis of number in society. Conceptualizing and articulating a praxis for social transformation connects ways of knowing, what is real, and ways of being. This project seeks to link number and acts of numerating to numeracy in a global capitalist system in order to develop a critical consciousness. Freire’s (2000) conception of *conscientização* interlinks the act of knowing with material realities and lived experiences. The act of knowing number must in turn be linked to historicized material realities and lived experiences as a

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historical becoming (Stetsenko, 2011). This is inextricably linked to the shape, structure, and processes of production in the material world. A critical stance toward numeracy posits a possibility to understand and analyze number and numeration in conversation with those processes of capital accumulation and underdevelopment discussed earlier. This mode of investigation necessitates work in and beyond the concept of number, numeracy and mathematics.

HOW DOES CONSCIOUSNESS RELATE TO PRAXIS AND BECOMING?

Consciousness is engendered through critical praxis and transformative becoming. Contradictions form the basis of movement/shifts, meaning material, physical and social realities are in constant motion. These shifts are not inevitable and do not achieve a stable balance, yet are indicative of a continual becoming (Stetsenko, 2010). It is through an active engagement in these contradictions that praxis and consciousness converge into becoming and possibility. Consciousness of number and numerating is in this manner linked to political activity and signified through a connection between knowing and becoming. Following this, the dominant assertion of numeracy must be de-linked from the logics inherent in the configuration and positionality of math education, the same logics of underdevelopment and coloniality.

I assert that Numeracy, a socially based activity embedded within historically derived social relations, integrates math, numbers/abstractions, and ways of being. A critical transformative numeracy links an analysis of contradictions between labor and capital to political struggle and also to a critical consciousness functioning to (re)form what is real, what we know, and what is human.

What we can actively work to do through school, classroom, and communities is to mobilize people, to see schools and classrooms as an organizing space, and to practice curriculum as intricately connected to struggle and resistance. Numeracy can be situated through a critical transdisciplinary approach, from place based to popular education, expanding “liberation” from the confines of a classroom and sanctioned math curriculum. Thus when we teach and learn colonial history we can expand our engagement in order to know, connect and transform our understanding of number in a relational manner. The relational view illuminates both capitalist logic and coloniality as on-going enactments of dispossession through a variety of means. In order to do this, we must be able to envision the potentialities of a grounded, agentic, and transformative numeracy as educators, activists, and learners.

The conception of liberation, of human, and of the political mentioned above should be rooted in life (Fasheh, 2015) and social relations. A numeracy that is critical can also unite these convictions with Marx’s phrase “for each according to their ability, to each according to their needs”, revitalizing a contextualized understanding and

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principled purpose to a transformative project with number. Only from this place can we begin a program to explicitly affirm that agency is what makes numeracy count.

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CULTURAL INSIDER AND OUTSIDER PERSPECTIVES ON RESEARCH IN MATHEMATICS EDUCATION

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Abstract: Within mathematics education research, issues of equity for students from diverse backgrounds have received increasing attention. This discussion paper presents perspectives from two researchers as to what it means to be a cultural insider and outsider undertaking research in mathematics education. Experiences from their work which examines how young indigenous students engage in early algebraic thinking is used to examine this phenomenon. In particular, we present some of the affordances and constraints of our own cultural backgrounds (Cook Islander – Insider and Australian- outsider) when undertaking research.

INTRODUCTION

In recent years, many Western countries have had a changing student population that is increasingly diverse. Within mathematics education research, issues of equity for students from diverse backgrounds have received increasing attention (e.g., Hunter & Hunter, 2017; Louie, 2017). Louie (2017) argues that mathematics classrooms are built on a 'culture of exclusion' whereby narrow definitions of what it means to do mathematics shape student access to mathematics. In both New Zealand and Australia, specific groups of students are marginalised within schooling and mathematics classrooms. These include Pāsifika students in New Zealand and indigenous Aboriginal students in Australia. Our work in the field of early algebraic reasoning focuses on both Pāsifika students in New Zealand and indigenous Aboriginal students in Australia. Specifically, in this paper we focus on the affordances and constraints of our own cultural backgrounds when working with diverse students with the first author being a second generation New Zealander of Cook Island heritage and the second author being first generation Australian of non-Indigenous heritage.

LITERATURE REVIEW

Relevant to work focused on equity for diverse students in mathematics classrooms is the notion of insider and outsider status of researchers. Differing groups of people have sets of values, beliefs, behaviours, and knowledge that they draw upon. Researchers who are viewed as cultural insiders have commonalities with research participants sharing common culture, social background, and/or language (Bishop, 2008). Some researchers (e.g., Bishop, 2008; Liamputtong, 2010) argue that insiders have an advantage in understanding shared cultural and social values, beliefs, and knowledge. In relation to mathematics classrooms, this may include a shared understanding of values underlying behaviour or knowledge of patterns from cultural artefacts. Cultural outsiders come from different parts of the community and may hold different values, beliefs, and perspectives than the research participants. This

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may cause difficulties in accessing and understanding the values, beliefs, and knowledge of the community. However, a counter argument is cultural outsiders have the ability to view what would be seen as a common phenomenon by a cultural insider and analyse this with more depth (Liamputtong, 2010).

Other researchers (e.g., Dwyer & Buckle, 2009; Kerstetter, 2012) critique the notion of a strict "insider/outsider" dichotomy instead arguing that we should take the perspective of "the space in-between". In this view, all researchers are positioned in a space between complete insiders or complete outsiders and occupy different spaces depending on the context of the research project. This space in-between is multi-dimensional including identity, cultural background, and the relationship to the research participants (Kerstetter, 2012). In this paper, we begin to draw on these notions in considering our positions as researchers in mathematics education and work in the field of early algebraic reasoning with diverse students.

CULTURAL INSIDER PERSPECTIVE

Many studies in early algebra, in particular pattern generalisation, have been conducted with students from dominant cultures with tasks drawing on geometric patterns set in a mathematical context (e.g., dots, squares, tiles). Of the studies where students are from culturally diverse groups, the tasks presented generally draw on personal contexts of the students (e.g., seating guests for dinner - Carraher, Martinez, & Schliemann, 2008), or shared contexts with the teacher and student (e.g., animals, desk plans – Miller, 2016). In the mathematics education research space, these are readily recognised as legitimate mathematical growing patterns without requiring justification of whether they are a mathematical task or a cultural artefact. In contrast, there appears to be little research which draws on culturally connected patterns (e.g., weaving baskets; cultural dances; quilting) as mathematical growing patterns.

My initial impetus for thinking about the intersection of my cultural background and research work in early algebra was interview data collected from Pāsifika students in an ongoing professional learning and development project 'Developing Mathematical Inquiry Communities' (Hunter & Hunter, 2017). Specifically, interviews with Pāsifika students aged 11-13 years who frequently positioned their cultural identity as non-mathematical, for example "it makes me feel different because Tokelauans don't do maths" (Hunter & Hunter, 2017, p. 4). Although not often evident in the mathematical tasks used in New Zealand classrooms (Averill, 2012), Pāsifika people come from a long history of mathematics including navigation, construction, and craftwork and design as a few examples.

As a researcher of Cook Island heritage, I view the patterns inherent within our cultural artefacts as existing mathematical patterns. Both my own heritage and work within schools and communities with children and families from Pacific Island nations results in familiarity with the funds of knowledge (e.g., tivaevae design, cultural performance, hair-cutting ceremonies) held by Pāsifika communities. The affordances that I can bring into my work include the knowledge of cultural patterns

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from different Pacific Island nations, how these are developed and created, what the items are used for, and their cultural significance. In a New Zealand and Australasian context this is celebrated by Pāsifika people, the recognition of our culture as mathematical and opportunities for our children to recognise that mathematics is intricately linked to their cultural background. I also have the affordance of being able to interpret the ways in which teachers and students draw on values during interactions in the classroom from a Pāsifika view-point and make sense of particular behaviour (e.g., respect shown through silence and a lack of eye contact).

However, a key constraint as an insider researcher is an initial lack of shared perspectives with researchers from dominant cultures or other countries. For example, with my insider knowledge, I know patterns in cultural items (e.g., tivaevae, tapa cloth) are an inherent part of our life in New Zealand as Pāsifika people and assume others share this knowledge. Tapa cloth is used as mats, wall hangings, clothing, and in a range of settings, home, church, school, and for celebrations. However, this is not shared knowledge outside of New Zealand and therefore I am questioned and asked to justify whether this is "really" a familiar pattern for Pāsifika students. Similarly, these artefacts may not be recognised as already mathematical by researchers from a dominant culture, for example, recent review feedback argued that using tasks based on a cultural artefact (e.g., tapa cloth, see figure 1), meant that it was "recontextualised as something different – an illustration of patterns". Finally, a lack of shared understanding of values and how these may play out in a classroom setting can result in a difference of interpretation when behaviour of students is interpreted from a Western view-point rather than a Pāsifika view-point.

Figure 1. Tapa cloth

CULTURAL OUTSIDER PERSPECTIVE

The majority of the research that I have undertaken in early algebraic thinking has been with students from cultural backgrounds where I am considered an outsider (e.g., Miller, 2016). In particular, I have worked with Aboriginal students in Australia and Pāsifika students in New Zealand to build an understanding of how young students engage with and express their early algebraic thinking in patterning contexts. As a non-Indigenous education researcher, I understand that, although my own culture deeply influences the perceptions of the world around me, those views are not a defining assessment. Thus, undertaking research as an outsider requires the building

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of relationships and continuous dialogue with knowledgeable others to understand the shared space in which I am working which proves to be an affordance of better understanding the complexities of teaching and learning mathematics.

Despite the drive to draw mathematics from Indigenous culture, at times it can be challenging for an outsider to design tasks that are authentic and culturally empowering for students. Generalising growing patterns is an abstract concept to both Indigenous and non-Indigenous students and contexts used to teach this are often drawn from traditional mathematical representations (e.g., dots, squares, tiles). Prior to and during my PhD, I had ongoing discussions with Indigenous educators to determine mathematical growing patterns that could be seen from their culture that I could use in the classroom with young primary school students. They identified that their culture is rich in patterns including dance, art and kinship models. However, it was determined that a majority of these patterns, (e.g., kinship models and art) would be an inappropriate context for a non-Indigenous teacher/researcher to make connections or allusions to. So, it was decided that the best context to draw on was a shared context for both the students and myself; the school and natural environment.

Drawing from a shared context proved to be an engaging and powerful way for young indigenous students to learn and engage with generalising mathematical growing patterns, however, it has been challenged by researchers beyond the Australian/New Zealand context. For example, I have been challenged in international forums for not implementing a culturally responsive pedagogical approach as I have not used or drawn directly from Aboriginal culture – in particular Aboriginal artwork. While I acknowledge there is a richness in artwork that provides a platform for exploring mathematical patterns, I find drawing on these artefacts particularly challenging. As an outsider, I feel it is inappropriate, at least in the context of the research I had undertaken, to reduce an indigenous art piece which tells a rich personal story to a traditional mathematical concept.

Despite being a cultural outsider, this also has affordances when working together with cultural insiders or more knowledgeable others. An example of this is the current work I am undertaking with Jodie Hunter examining how Pāsifika students develop early algebraic thinking. At times, there are cultural nuances that require further or deeper understanding for myself, this presents an opportunity for consideration and analysis of the learning interactions with the students from both of us as researchers. For example, what is sometimes seen as a typical student interaction for an insider may be not typical for the outsider. As researchers working jointly together, we can examine the cultural values that influence the students' mathematical discourse while students are explaining their mathematical thinking and justifying their generalisations to their peers. In my research in Australia, this has been particularly apparent in the non-verbal cues students use to communicate with each other when expressing their pattern generalisations (Miller, 2016).

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CONCLUSION

This paper outlines our initial thoughts in relation to the affordances and constraints of being cultural insiders or outsiders while undertaking research in the field of early algebra with diverse groups of students. In both positions, there are constraints in relation to shared knowledge and perspectives. However, these constraints can also be an affordance by offering us the opportunity to examine and analyse more closely our perspectives and interpretations. We view this paper as the beginning of an examination of how a shared space can look for early algebra research when culture and cultural perspectives are bought to the forefront, both to inform researchers and empower young students at risk of marginalisation.

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MATHEMATICS, EDUCATION, AND ANARCHISM

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Abstract: Anti-oppressive frameworks for mathematics education fail to disrupt hierarchies of mathematics and the role of the educator. In this paper I begin to explore what a mathematics education freed from authoritarian relations might entail.

If *mutual aid* and *self-assertion of the individual*[1] are defining qualities of the progression of human relations (Kropotkin, 1892/2016), there is reason to consider how education and specifically mathematics education might be organized in such an anarchistic future (Restivo, 1998). Although Suissa (2010) and Wolfmeyer (2012) argue that an anarchist education within a statist society is a necessary force toward becoming stateless, I proceed here with emphasis on the qualities of an education in that ideal stateless (or anarchist) society.

Current anti-oppressive and justice orientations in mathematics education fail to disrupt the hierarchies of White Supremacy embedded in what is taken as Mathematics (Lawler, 2016). Further, the teacher is oft-positioned as an enlightened leader (Wolfmeyer, 2012), knowing what's best for the students and community and declaring all must follow their lead. Marxist and similar perspectives assume or explicitly assert the need for hierarchy in the educational process. Such orientations rely on an "enlightened elite who hold what they consider to be objective truth for how society currently functions and how it will be transformed" (p. 45).

In this brief paper, I embark on a project to conceptualize the place of mathematics in an anarchistic education. I aim to consider specific elements of Mathematics and Education that would be fundamentally different in an anarchistic education. First, I establish what I mean by anarchism.

ANARCHISM

In this paper I follow Kropotkin's (1892/2016) conception of anarchism as one emphasizing the social sciences over the political sciences; I place emphasis on interpersonal relationships over community-organizing relationships (the state). I consider anarchism as a mode of human organization and social self-determination, rooted in the experiencing of daily life (Suissa, 2010).

All forms of anarchism express profound skepticism toward skewed, coercive, and exploitative power relations, rejecting various forms of oppression, such those of class, race, gender, sexual orientation, religion, etc. Rather, anarchism aims to maximize both individual autonomy and collectivist freedom, leading to the reduction of fixed hierarchies that systematically privilege some people over others.

There are a variety of anarchist orientations, characterized by placing more weight on one rather than the other side of a polarization of freedom versus equality, often distinguished by individualist and social anarchists. Individualists place emphasis on

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the rational individual, as morally and intellectually sovereign (Suissa, 2010). The social anarchist views individual freedom as conceptually connected with social equality, emphasizing community, cooperation, and mutual aid. “I am not myself free or human until or unless I recognize the freedom and humanity of all my fellow men” (Bakunin, in Suissa, 2010, p. 44).

In short, *anarchism is the absence of hierarchy*, social, economic, or political. It describes a relationship among people that minimizes if not eliminates coercive structures or interactions in all areas of human life, taking seriously the possibility of an equal and free society, organized on the basis of *cooperation*, *mutual aid*, and *freedom from hierarchy*.

MATHEMATICS

As did Restivo (1998) at MEAS 20 years ago, it is incumbent to consider a notion of mathematics compatible with an anarchistic way of living. What would be conceived as a mathematics beholden to the principles of cooperation, mutual aid, and freedom from hierarchy?

A fundamental shift required in an anarchistic orientation to mathematics is to unseat the dominant ontological view of mathematics as something that exists external to the knower (Lawler, 2016). Such an orientation fails anarchist thought in two ways: (1) it removes authorship from the individual or community of knowers, and (2) it allows for hierarchy of knowing—some know more of “the” mathematics than others. Such an epistemological pluralism (Turkle & Papert, 1992) allows for many ways of knowing and engaging in mathematical activity. When everyone is perceived to do mathematics, authority emerges from below, rather than above.

Secondly, an anarchistic view of mathematics would reject it as a linear, hierarchical subject. A third shift, related to the first two, is to focus on the humanity and communalism of both individual and collective participation in mathematical activity. Collective generation of (mathematical) knowledge is the foundation by which we make and re-make our selves and our collectives (Restivo, 1998).

EDUCATION

When people are viewed as authors of mathematics and knowledge generation as collective activity, we recognize that education is a human activity, a practice in its own right. Too often education is defined in service of something else (at present, a capitalist economy), as a subordinate practice. An thus, education functions “to perpetuate social and economic injustice” (Ward, 1973).

Anarchist values reject the notion of a universal, compulsory, state-controlled education (Suissa, 2010). Any organized effort for education should be voluntary, functional, temporary, and small (Ward, 1973); furthermore should be designed without externally-defined aims (Suissa, 2010). Activity we might name to be educational would emerge spontaneously, based upon community need or interest. This is not to reject the possibility of school buildings or classrooms, but to emphasize

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the voluntary nature of participation, by those who may be called student or teacher. Further the activity would be temporary, until the learners determined the educational activity was no longer necessary. Inquiries or topics that bind collective mathematical activity (education) arise from student or teacher interest. Mathematical inquiry, or education, could begin with a community-based problem to be solved, could be a gathering of people who enjoy engaging in mathematical activity, or may be a group who've determined they need to understand some particular topic more thoroughly.

Such a form of organizing for mathematics education defines a relationship between teachers and students characterized well by Paulo Freire (2005):

> Through dialogue, the teacher-of-the-students and the students-of-the-teacher cease to exist and a new term emerges: teacher-student with students-teachers. The teacher is no longer merely the one-who-teaches, but one who is himself taught in dialogue with the students, who in turn while being taught also teach. They become jointly responsible for a process in which all grow.

An anarchist's mathematics education flattens the relationship between student and teacher. Topics and timelines for study, as well as knowledge are co-generated. Order emerges spontaneously for educational events spontaneously among persistently re-organizing groups, in which topics for inquiry are identified by the leaners, premises well aligned with Ward's (1973) social ideals of anarchism.

A final characteristic of the anarchist "classroom" is the pursuit to understand and respond to oppression within the learners' community, whereby individuals, acting together, directly pursue not only the prevention of injustices, but also the creation of alternative social relations free of hierarchy and domination.

CONCLUSION

An anarchistic mathematics education would be organized on the basis of cooperation, mutual aid, and freedom from hierarchy. Hopeful disruptions of the present hierarchal structure of a Eurocentric mathematics has been offered by many, recently Berry (2018) and Gutiérrez (2017).

Berry (2018) argues that mathematics education is rooted in a colonizing frame, based on separation, hierarchy, status, and competition. A decolonizing pedagogy would be a communal approach in which (1) teachers and learners interact in mutual relationships, (2) resources are shared within the community, and (3) learners are participating members of the community. The values espoused by Berry align well with an anarchistic mathematics education, but important elements are underdeveloped—specifically attention to what counts as mathematics and the role of education. Gutiérrez' (2017) has explicitly pushed back against an authoritarian conception of mathematics. She explicitly names *mathematx* as "ideas and methods of thinking that are specially suited for developing insights and strategies to challenge hierarchical systems' (p. 35).

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Suissa (2010) helps us place reshaped views of mathematics into the context of education. She argued that a social-anarchist orientation to education which "systematically promoted and emphasized cooperation, solidarity and mutual aid" would undermine the values underlying the capitalist state, and thus "would both encourage the flourishing of these innate human propensities and inspire people to form social alliances and movements aimed at furthering the social revolution" (p. 32). These projects affirm we can imagine an anti-oppressive mathematics and mathematics education differently.

NOTES

1. In short, these two premises can be restated as *cooperation* and *freedom from hierarchy*, operating in a complex relationship: when a mutual aid collective loses its basic character it impedes progress, and the individual must assert themself against the bonds imposed by this institution. The drive of man to have "two opposed instincts; egoism and sociability. He is both more ferocious in his egoism than the most ferocious beasts and more sociable than the bees and ants" (Bakunin in Suissa, 2010).

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SELF-ASSESSMENT IN UNIVERSITY MATHEMATICS: SHAKING THE POWER STRUCTURES THROUGH UNIVERSAL DESIGN

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Perhaps nowhere in the field of university mathematics learning are power structures so clearly visible as in the field of assessment. Here, these structures are shaken through summative self-assessment. Digital Self-Assessment research project aims to enhance ownership of mathematics learning through innovative assessment, but also to promote inclusion with Universal Design for Assessment. The vast data gathered in the project provides insight into the empowering effect of self-assessment.

INTRODUCTION

Today's undergraduate students are increasingly more diverse in their background. At the same time, large courses with hundreds of students are taught, this applying to mathematics as well. In mathematics, however, the teaching practices have remained largely untouched: Burton & Haines (1997) have suggested that the mathematics community is resistant to change of the learning environments. They even argue that the community should be responsible for the lack of innovative assessment at university level, which, according to recent studies have shown to be highly based on closed book exams in undergraduate level mathematics (Iannone, & Simpson, 2015). As a result, assessment in higher education (HE) in general is highly based in power. Although not determining students' future choices as strongly as the grades gained during the lower educational levels (those affecting to the very option of entering university), these grades still have an influence on students' lives.

Assessment is known to greatly affect learning for decades (e.g. Biggs, 2011). Through assessment the teachers can wield power that affects the cognitive and emotional processes of their students. Students' understanding of their own learning processes, as well as their self-efficacy are subject to the external power by the grade giving authority. Talbot conceptualizes power by using Foucault's metaphor: "*Power is deployed by those who are in a position to define and categorise, to include and exclude*" (Talbot et al., 2003, p. 2). Recent study found that students in HE feel strongly negative emotions towards assessment methods that were out of their control (Wass et al., 2018). Power lies in the epistemological structures, as defined by Tan (2004): this kind of broader institutional power exists beyond interpersonal episodes. Epistemological power is seen when students, even though given more autonomy on assessment, seek to learn according to their perception of what the teacher desires. Finally, the project draws on Foucauldian analysis based on his idea of *disciplinary power*; power found on *discourses* that underlie our daily life (Foucault, 1977).

Promoting equity is an important goal itself. However, we should acknowledge the fact that greater equality of power between students and staff also enhances learning (e.g.

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Nicol, & Macfarlane-Dick, 2005). We argue that shaking the power structures of assessment is especially important in the field of mathematics, a science known to discriminate minority groups such as women (Sumpter, 2016), ethnic groups (Gutierrez, 2008) and students with disabilities (Woodward, & Montague, 2000).

UNIVERSAL DESIGN FOR SELF-ASSESSMENT

Self-assessment is a popular practice for enhancing student empowerment in HE (Tan, 2004). Broadly, student self-assessment (SSA) can be defined as a process involving reflection and monitoring student's own work (Brown, et al., 2015). While self-assessment as a formal assessment method has been studied for decades, the focus has mainly been on determining whether SSA is *accurate*; or, in other words, whether self-assessment is in line with tutor or peer assessment (for a review, see Brown, et al., 2015). Indeed, there is a general agreement that *validity* is the core element of assessment. Many researchers (e.g. Andrade, & Du, 2007) suggest that SSA should be only used in a formative way; namely, that SSA should not count towards students' grades because of cheating and lessening the focus on learning. Perhaps this is why SSA practices very rarely offer students real power over their grades. Other researchers as well argue that when students are given power over their grades, the accuracy of their self-assessment lowers (e.g. Tejeiro, et al., 2010). But these studies have failed to empower students in a way Taras (2001) sees it. In Tejeiro et al's (ibid.) study, the real power was held by the teacher, as the students had a chance to evaluate only 5 % of their own grade. In line with Taras (ibid.), we argue that in order to truly foster the ownership of learning, SSA has to be used in summative ways.

While previous research reports attempts to empower students through innovative assessment (e.g. Taras, 2001), rarely have these arrangements been based on inclusivity. What Taras asks is: How to empower students? We broaden this question: How to empower *all of the students*, knowing they come from diverse backgrounds? In order to answer this, we apply the theory of Universal Design. Universal Design for Assessment (UDA; Thurlow, Johnstone, & Ketterlin-Geller, 2015) aims to make assessment accessible and inclusive for all learners. UDA can be defined as an integrated system with a broad spectrum of possible supports so as to provide the best environment in which to capture student knowledge and skills.

Previous research on UDA does not take power structures into account. While UDA has been applied to make test items more accessible, we argue that only through combining this theory with innovative assessment leads to true empowerment. While SSA has been implemented in HE in order to enhance student empowerment, only UDA makes sure that this shaking of power structures truly involves everyone.

THE DIGITAL SELF-ASSESSMENT (DISA) PROJECT

Started in the University of Helsinki in 2017, the Digital Self-Assessment (DISA) research project seeks to reform assessment environments in order to promote self-reflection and ownership of learning. We replaced the traditional course exam with summative SSA - after practicing SSA formatively, students evaluated their own

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course grades. Like Taras (2001), we emphasized various forms of feedback. We created an assessment environment in an undergraduate linear algebra course reflecting the principles of UDA. Peer and tutor feedback was provided in a digital environment. The lectures were based on discussion, while the students were encouraged to participate in an open learning space where feedback and support was always available through student tutors. The general research aim in the project is: How to support students' autonomy when there are over 400 diverse participants in the course?

What we have learned through vast interview and survey data is that shaking the power structures with summative SSA greatly affected learning of our students. The quantitative data showed that summative SSA and deep learning of mathematics were connected (Nieminen, et al., 2018). Further ongoing analysis showed that most of the students felt empowered through ownership of learning - "*finally learning for myself, not for an exam*" is a common theme in our data.

However, this kind of an empowering assessment was also described as "*weird*" and "*new*". The students largely reported that assessing mathematics skills is not easy since they have never done that before. Also, many students described their low mathematical self-efficacy creating barriers for reflective SSA. These issues raise concerns about *epistemological and disciplinary power*. The students in our study were aware of teacher's authority in the assessment process. This was most clearly seen during an interview with a student who told that when she was asked to assess her grade and justify it, she simply wrote in "*the teacher can decide which grade I earn*". On the contrary to this, one student with self-reported learning disabilities described her empowerment through SSA: she was able to separate her *skills* and her *self* and thus she was able to reflect on her learning honestly. Overall, DISA has been able to empower students by challenging traditional views on assessment. Further analysis is needed to study the process of empowerment acts in underrepresented student groups.

Shaking the power structures with summative SSA and UDA offers an insight into other underlying power structures not only related to assessment. Who are truly empowered through assessment and who are not - and why is that? To sum our point, we quote Taras (2001, p. 612): "*If we want students to take responsibility, then we have to allow them to do so*." However, support is needed in order to enhance this kind of empowerment in the diversity of learners.

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INITIAL REPORT FROM THE PROJECT IN CITIZENSHIP AND MATHEMATICS (PICAM): MORAL AND POLITICAL DILEMMAS

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Sheffield Hallam University[a] and the University of Thessaly[b]

This is an initial report from the PiCaM project reflecting some of the partners' concerns. We aim to raise within the MES community troubling questions about the nature of such projects - engulfed in EU discourses of citizenship and school learning in response to globality - whilst nevertheless seeking to find a way of acting in the world and trying to find, however limited and partial, an answer to the question: "what is to be done?" We are aware that equally troubling questions could be raised about the notions of mathematics itself and of mathematics education but these are not the focus of this report.

INTRODUCTION AND PURPOSE

PiCaM (http://www.citizenship-and-mathematics.eu/) is co-funded through the ERASMUS + Programme of the European Union (Project number 2017-1-UK01-KA201-036675) and therefore was initially framed in response to the discourse of an EU call for action on global learning. However, it involves six partners[i] and at least some of us are committed to a mathematics education that challenges neo-liberal assumptions about citizenship and problematises the concept of global learning. In presenting this report at MES alongside some of the curriculum artefacts the project has begun to produce, we seek critical feedback on designing a curriculum unit under such a call from a mathematics education community whose shared values make sense to us.

Under what circumstances, if any, is such an application for funding, and the inevitable compromises involved, justified?

We take the designing of curriculum artefacts for teachers to be in itself problematic inasmuch as they may be conceived of as unchangeable or as being the subject of an other's specific intentionality.

How, if at all, is it possible for them rather to be conceived as openings to create spaces for interaction or spaces for dialogicality or even as creative experimentations that allow teachers and children from the margins to tell their own stories?

If so, what ethical dilemmas remain?

THE PICAM PROPOSAL

We begin by quoting the introduction to the proposal which outlines its intentions:

> PiCaM is designed to embed critical global learning in the teaching of mathematics in school for young people aged 10-12 years. Using the core curriculum as a vehicle,

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> materials will be devised and tested that embed global learning content and participatory approaches in the teaching of mathematics. Attention will be paid to both mathematics and global learning curriculum content with the learning mediated through appropriate and inclusive pedagogies, supporting an innovative, participatory, integrated approach to developing social, civic and intercultural competences and critical thinking…The project will connect up teachers and children across the partnership allowing for networking and the exchange of ideas and understanding.

Is it possible to frame global learning and social, civic and intercultural competences in ways that support an emancipatory project for both teachers and children?

In what ways can school-based pedagogies ever become aware of their inclusive and exclusive potentials?

Although we recognise that citizenship is a slippery, dangerous and contested concept, we invoked it from the beginning and referred to notions of citizenship that extend beyond the nation state but nevertheless bought into the European project:

> …in recent patterns of cross-border migration into Europe, children have escaped poverty and war and now face the long struggle of adapting to life in a new country. Children in host countries also need adaptive skills to face the challenges and seize the opportunities that increased globalisation entails. Critical global learning supports the understanding of ourselves as citizens, both world-wide and, in particular, as citizens of Europe. There is a need to build social cohesion on the basis of shared values of inclusivity and equity for all.

Has the phrase "citizens-of-the-world" been colonised beyond redemption?

What characteristics cluster around the notion we are trying to invoke of critical citizenship in contemporary times and places of globality?

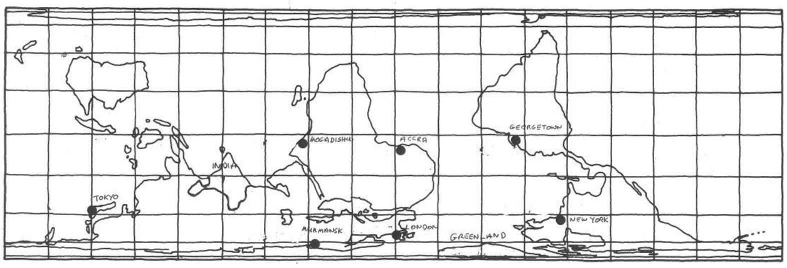
PICAM MATHEMATICS CURRICULUM MATERIALS

In this section, we give very brief descriptions of the draft curriculum materials produced thus far, each connected to an image that seems to say something about the activity.

What role can curriculum materials have in a pedagogy genuinely focused on the needs of children and teachers?

If we move from the situated and local to more general curricula artefacts and pedagogic interventions, how do we move beyond reproducing the same cultural, linguistic, gendered, raced and ethnic stereotypes?

Mapping our world with mathematics



The intention is to help learners understand that the taken-for-granted way we see the world is shaped by

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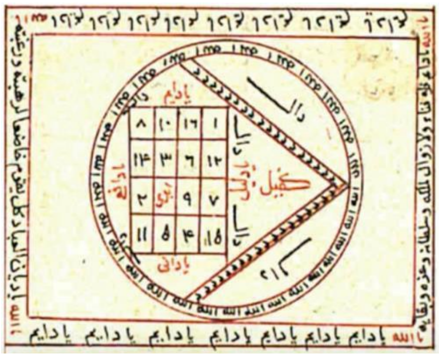
historical forces - relationships of domination and dominium and the experience of colonialism. Concerned with the mathematics of map making it questions north/south, Eurocentrism and the construction of the nation state.

Fair and square: magic, Vedic and Latin squares



Through exploring magic, Latin and Vedic squares, learners are encouraged to develop a pluralistic and historically informed cultural perspective on mathematics, to think of pattern and balance as ways of making sense of the world and to understand themselves as enquirers who pose problems and who work with others to develop their thinking.

Global crisis and local solidarity: debt versus money as commons

Contemporary hegemonic ways of understanding money and debt are interrogated and critiqued. The role of mathematics in promoting neo-liberal values is considered as are the ways in which unproblematised uses of mathematics have contributed to the global crisis. The notion of commons as shared resources in which each stakeholder has an equal interest is explored and the TEM local currency in Volos, Greece is studied in order to attempt to understand how these reframe goods, services and relations.

Designing the world around me: mathematics and cultural inspiration in design

Drawing on a range of cultural heritages with a particular emphasis on the role of Islamic art, the role of mathematics in design is considered. How symbols can operate to express values is explored and encouragement of and respect for the learning community fostered. Ideas of difference and connectedness are investigated. Throughout there are opportunities for exploring how symbolic representations are underpinned by ideas and beliefs

Playing and making: Creating spaces for becoming together

This activity creates spaces, inside and outside the school context, for children to collaborate and share experiences amongst themselves and with others through playing mathematical games or making mathematical crafts. The game or the craft becomes a way to access the complexity of living together in the urban landscape; and the *mathematics*

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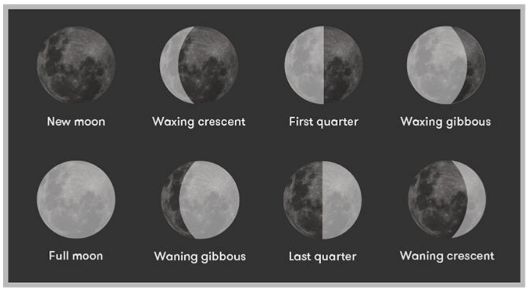
of the game or the craft becomes a way of signifying connections between words, bodies and algorithms.

Mathematical bodies

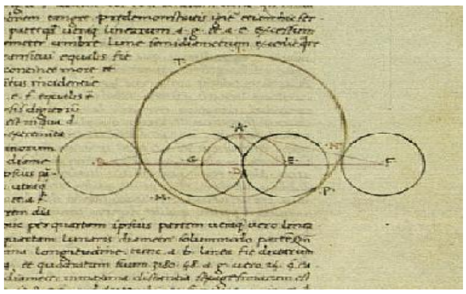
This activity is rather different from the others in that the material does not have any content linked specifically with global education. Its function is solely related to pedagogical considerations. Here mathematics is explored as embodied and playful and experienced together. The activity is designed to build a learning group where everybody matters and everyone has an equal role to play. It offers an embodied experience of some simple mathematics

Further resources are at the initial planning stage: an exploration of time, currently focused on understanding the solar (Gregorian) and lunar (Hijri) calendars but having the potential to interrogate the contemporary understandings of time and their source in industrialisation; material related to land use and growth which raise ecological considerations; and gender oppression in ancient and contemporary societies explored through two narratives, one of Hypatia the Alexandrian mathematician and the other of the black women working for NASA in the early 1960s.



Do any of these topics have the potential to fulfil the (somewhat utopian) aims of the authors?

Can we conceive of them as having a life of their own in the hands of teachers and children producing unpredictable openings for encountering the multiple other?



How do images operate to reinforce or destabilise understandings?

Or, how do they trouble (or not) dominant views of mathematics, critical mathematics education and citizenship or globality?

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MEANING IN MATHEMATICS EDUCATION AND UNIVERSITY STUDENTS WITH DISABILITIES: CONCEPTUAL ENCOUNTERS

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The goal of the research is to investigate the construction of meaning in Mathematics Education, which contemplates the encounters of four concepts: intentionality, foreground, personal meaning and image-action, focusing on Students Target Audiences Special Education. The data production is composed of semi-structured interviews with this group of students and will be analysed according to the theoretical purposes presented, with elements of content analysis and data organized by categories and themes. The methodological procedures will be based on qualitative approaches. The idea is to identify obstacles that hamper students' mathematical learning.

BACKGROUND

With this paper, we present a literature review of ideas from theorists who discuss meaning in Mathematics Education, with our interpretations of what we consider the four concepts encounters: intentionality and foreground (according to Ole Skovsmose), personal meaning (according to Maike Vollstedt) and the action-image, by admitting a particular connection between them. We understand as image-action a complement to the study of meaning, and this concept will be developed by us during of the research. We also consider that the image that the person makes of himself has to do with the self-judgment proposed by Vollstedt (2011). This image may be promising or not for the future of the student. Perhaps the experiences in mathematics experienced by the students are destroyed; perhaps the personal meaning for mathematical studies is ruined. Within these four concepts, the interest is to understand the meaning that is built (or not) by special education target public students (EPAEE)[1].

Assuming students have the opportunity to construct their own knowledge, with the development of pedagogical practices that consider the unique form of their interaction with the world and the educational specificities of each, the following question emerges: permeated by physical, intellectual, emotional and communication differences, what are the Meanings in Mathematics Education that are constructed (or not) by EPAEE? In this research these meanings are analyzed according to the conceptual encounters: intentionality, foreground, personal meaning and image-action. What is the meaning, for example, for the student, when doing some mathematical

[1] Students with disabilities: In Portuguese EPAEE is the abbreviation of Special Education Target Public Students.

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exercises? In order to answer these and other questions, we present the theoretical perspective that will support the research.

THEORETICAL FRAMEWORK

According to the "Inclusive Program Guidance Document"[2] (Brasil, 2013, p.10), a program that discusses the accessibility in Higher Education of students with disabilities, this group of students in Higher Education has increased by 70% in 2012 compared to 1998. For the authors of this document, the conceptions of inclusive education are motivating the transformations of Brazilian education, including in higher education.

Therefore, we understand the need for studies that may help overcome school difficulties, pointing to a greater interest in these discussions, presenting that the inclusion of the EPAEE in higher education can be performed with better conditions for learning, since the number of students who dream of the possibility of a promising professional future is increasing. Thus, we emphasize the need for research that discuss the obstacles that impede access to and continuity of the studies[3] of this group of students in Brazilian Universities, and that contribute to their learning. This need clarifies the research we propose, which aims to investigate the construction of Meaning in Mathematics Education, which contemplates the meeting of four theoretical concepts: intentionality, foreground, personal meaning and action-image, focusing on the EPAEE whom attend higher education.

It is necessary to understand the concept of Meaning. In the book "Meaning in Mathematics Education", organized by Kilpatrick, Hoyles and Skovsmose (2005), several authors discuss the concept of Meaning in Mathematics Education. For them, the theory of Meaning has developed in many directions, philosophical or not, psychological or not, complex or not. Meaning in Mathematics Education can refer to concepts and tasks. Skovsmose (2008, p.22) considers that "references also include motives for actions; in other words, include the context for locating the purpose of an action", which is performed by the student in mathematics classes. The references may be related to real life situations which, using pedagogical strategies in an environment conducive to learning, students produce different forms of interpretations of meanings.

In addition, the meaning in Mathematics Education may be related to the foreground of students, as Skovsmose (2014) explains who understands the construction of the foreground expression to designate a complex combination of social, economic, cultural, and political contexts related to people. For Skovsmose (2014), foreground can signify a space for directing intentionality. In fact, we

[2] In Portuguese the name is "Documento Orientador do Programa Incluir"
[3] In Brazil there are some public policies to help students finishing their studies, for example scholarships and pedagogical helps.

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understand the importance of paying attention to the intentions of the students, because this fact can keep the student from accomplishing the proposed school tasks, due, for example, to other priorities or concerns. We can then connect intentionality with the opportunities, longings, expectations, anxieties, hopes, obstacles, joys or sorrows related to the future of the individual that make up the foreground notions.

Taking as true the statement that intentionality *is directed to*, then the action of a student, for example, in performing or not the mathematical tasks proposed by the teacher, have to do with the intentionality in this execution: perhaps the student decides positively to recognize in the learning of this content, an important element in the composition of their professional future; or maybe he likes math and wants to improve his school performance; maybe you have sympathy for the teacher and want to please him. Intentionality thus maintains a dialogue with the concepts of meaning and foreground. And, as Skovsmose (2014, p.83) justifies, "I suggest relating the discussion of the meaning of actions to intentions and foregrounds and the meaning of learning intentions-in-learning and foregrounds-for-learning". This construction can be personal, as the author Maike Vollstedt (2011) proposes in her studies of personal meaning.

The Personal meaning, according to Vollstedt (2011), can be understood as expressiveness, object and action when the focus is the student's mathematical learning. The author also clarifies that one of the challenges for Mathematics Education is to present convincing answers to the questions that address the meaning for the student, given the presented disciplinary content. The personal meaning of the student depends on the context in which it is linked and can form part of the mathematical formulations and the educational and psychological development of the person. It has to do with the intentionality and relevance to the student faced with an object (mathematical) or an action (learning). Therefore, a study that seems to us important in the mathematical approaches in the classroom, specifically those of the higher education with EPAEE.

Then, considering Skovsmose's (2015) statement that intentionality can be intuited in approaches, for example as an imagination, perhaps the student decides by not solving the exercise (action), because of the image he makes of himself, possibly obstructed. In a preliminary view, I consider the concept image-action as a complement related to the meaning in Mathematics Education. The image that the person makes of herself or himself has to do with self-determination proposed by Vollstedt (2011), with possibilities of relating or not with learning. In the following section we present the study's intended methods and analysis.

METHODS AND ANALYSIS

In order to respond to the research problem, the option is based on qualitative approaches, since we intend to understand the subjective nature of the object to be analyzed. In other words, to discuss and analyze, interpret and understand the opinion and behavior of the university students with disabilities, we intend to conduct

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interviews which will be recorded in audio and video and later transcribed in narrative format. Next, discussions will be presented by the researcher, choosing questions with pertinent elements to those addressed in the study and considering the look of the researcher. For example, to get to know the interviewee, his or her childhood and background, we will present some images and ask them to choose the one that represents their emotions; however, if the intention is to analyze the participants´ feelings about Mathematics we will ask her or him to choose an animal that represents these[4]. We show it in images (1):

Figure 1: elaborated by author

The idea is that the interviewee feels invited to express their feelings, thoughts, the meanings that are constructed (or not) in relation to mathematical studies, wishes and dreams of the future and the place occupied by Mathematics in this context. We also understand that this meaning can be revealed through speech, but not only. It is one way to get this information from the participants. Therefore, to facilitate communication with the participants of the research, we will use different forms of interaction, since, among the students, we will be able to find students who need initiatives that favor this accessibility. These different forms of interaction will be the use of images, photographs, sounds, clippings of newspapers and magazines, words or phrases expressed in written, spoken or signaled form (Brazilian Sign Language for the case of deaf students).

The first action for the analysis of the interviews will be to identify the four theoretical concepts present in the speeches, gestures, expressions and presentations of the students. Elements of content analysis will be used because they are recommended for studies that involve motivations, values, attitudes, beliefs, and the unveiling of ideologies that often do not present themselves in a concise and clear way at first sight. Secondly, the sorted data will be organized by categories established after the interviewing with the participants and then restructured into a typology, constructed according to the orientation to personal meaning, intentionality, foreground and image-action, all related to mathematics.

[4] David Kollosche in his research uses this idea relating mathematical feelings to an animal. For more information see Kollosche (2017).

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It is hoped, with the research, to identify obstacles that impede the learning of the students with disabilities, proposing alternatives that can help them in the construction of meanings in Mathematical Education, bringing elements and understanding the beliefs, dreams, attitudes, values and motivations in relation to mathematical knowledge of university students with disabilities, responding to the questions proposed with the study.

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STUDENTS IN SOCIAL OPPRESSION: A STUDY OF MEANING IN MATHEMATICAL LEARNING THROUGH FOREGROUNDS AND EMPOWERMENT

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Abstract: This project is based on Critical Mathematics Education. It reveals concerns about mathematical learning in social, cultural, and political issues. Our focus is to discuss student's meanings about this learning, especially ones in social oppression, and to relate them to their foregrounds and empowerment possibilities. We aim to answer the following research question: What are the meanings in mathematical learning for students in social oppression, and possibilities for reworking foregrounds? Finally, we emphasize that for methodology we will be inspired by ethnographic research and participant observation.

INTRODUCTION AND RATIONALE

This work rests in the field of mathematics education, having as main theoretical line Critical Mathematical Education (CME), represented especially by Skovsmose (1994). In an earlier Master's level study, we used this perspective, as well as on Ethnomathematics and Pedagogy of Paulo Freire, dedicating ourselves to study the presence of some key concepts in critical investigations (Soares, 2008). In this new project, we intend to highlight some concepts that are linked to the concept of meaning.

According to Kaiser (2008), Skovsmose (2016), and Kilpatrick et al. (2005), the act of learning mathematics has a subjective meaning, and this meaning is associated with many factors and interpretations. Skovsmose adds that the concept of meaning in math classes for students is not restricted to references to other mathematical notions, or familiarity with the concepts, or participation in inquiring process, or even to the uses and applications of mathematics in real life. In fact, For Skovsmose, the meaning of mathematical knowledge for students is a political issue, especially related to the perspectives of action that such meaning mobilizes. Thus, Skovsmose reports that he tries

> [...] to provide a politicisation of the discussion of meaning by relating meaning to conditions and prospects for actions. I am going to pay attention to the socio-political formation of the horizons towards which actions might be directed. I see meaning as a way of acting, and students' experiences of meaning related to their conditions for completing such actions. (Skovsmose, 2016, p.38)

In this way, the concept of meaning for a person is linked to his/her horizons of action, to the conditions and perspectives for that person's action. In this sense, meaning is

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related to the concepts of “foreground” (Skovsmose, 1994) and “background” (D'Ambrosio, 1990) .

Foreground (Skovsmose, 1994, 2012, 2014, 2016) is a concept that refers to a person's future perspectives, full of possibilities and obstructions, hopes and fears. On the other hand, a person’s scope of aspirations is related to his/her background (D'Ambrosio, 1990), that is, to life experiences, cultural environment, realized dreams and the frustrated ones. In the case of students in social oppression, I mean, with some social condition that somehow suppresses their freedom or dignity (such as low income, minority groups in relation to race, gender, disability, etc.), the background may have an even greater weight. But even though the foreground (or foregrounds) of a person (or a group) is related to past experiences, and these experiences leave traces and bring directions to the future, his/her foreground is an opaque, uncertain concept, susceptible to change. We hypothesize that the aspirations present in the foreground of the students interfere with the meaning attributed to mathematical learning in the school, because they influence in their perspectives of life.

Finally, we emphasize that the concept of empowerment will also be important to us. Freire (1986) highlighted the role of critical reflection and awareness in the process of social empowerment, especially in Latin America. Thus, we could reflect on this process of empowerment for people, groups or communities in a social oppression.

Summarizing, in this project we propose a look at CME, with regard to the concepts already mentioned: meaning, foreground and empowerment. We are interested in a Latin American context on the above-mentioned concepts.

AIMS

Based on the reflections that were presented previously, we established the research question, revealed in the following text:

What are the meanings in mathematical learning of students in social oppression, their foregrounds, and possibilities for reworking foregrounds and empowerment?

And these are the specific aims:

• Understand the meanings attributed to mathematical learning by students in the regions surveyed;

• Investigate their foregrounds and relate them to the meanings that have been investigated;

• Identify possibilities of re-elaboration of foregrounds to the empowerment of the research students.

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METHODOLOGY AND PROCEDURES

We will carry out a study of social groups from 2 Latin American countries. One of them will be Brazil, justified by our experience and desire to develop studies with groups with local social oppression. The other Latin American country will probably be Colombia, and both groups will be near a peripheral community and near a large urban centre.

A qualitative field research will be carried out, with inspiration in ethnographic research, as well as participant observation, because in this type of methodology the researcher is in the middle of the groups or communities surveyed, not only observing how interacting with him (Angrosino, 2009).

For the production of data, in the first moment we will follow some maths classes of schools of the researched places, as well as to access didactic materials and students' notebooks. The objective is to be comfortable with the mathematics classes developed in the region, their methods and objectives.

In a second moment, we will use our main instrument of data production: the interview. Students groups will be formed from each of the researched sites, so that from a semi-structured interview the information about foreground and meaning emerge. Still, the researcher can count on other triggers of the discussions, such as images, texts, videos and student materials. The interviews will be recorded and filmed, because just as the speeches, the expressions and intonations are also important. In a third moment, some of these students who participated in the groups will be interviewed individually, because we understand that new data emerge when the answers are not given in a group.

FINAL REFLECTIONS

We understand that the development of the present work can establish important relations between mathematical learning and its social and political character, since it aims to identify the relationship between the students in social oppression’s foreground and their meanings of learning mathematics. Likewise, the social, political and cultural aspects that was addressed here can contribute greatly to studies on social empowerment in general, and in particular, to Mathematical Education studies in Latin America.

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MATHEMATICS IN ACTION AND PROBLEM BASED LEARNING: POSSIBILITIES IN HIGHER EDUCATION

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Abstract: This paper presents an ongoing research project related to mathematics education in higher education. The main goal is to understand how aspects of mathematics in action can be explored in the teaching and learning of calculus from the perspective of Problem Based Learning. This proposal is supported by a qualitative approach. The production of data will be constituted from the development of a problem that will be carried out by a group of students in a Civil Engineering course. We intend to contribute with discussions about the challenges related to the field of Mathematics Education at the higher level.

INTRODUCTION

In recent decades, concerns about the challenges and perspectives of higher education have been the subject of discussions both on national and international levels. The aspects involving the teaching and learning processes, as well as the understanding and use of the knowledge constructed in the different courses, are linked to the future actions of the individuals in our society, be they personal or professional.

Particularly, a growing concern associated with teaching and learning of calculus has been evident in the Brazilian academic scenario. Skills associated with this field of knowledge are increasingly required for a competent performance of individuals in society (Rezende, 2003).

According to directives that guide the educational bases of higher education in Brazil (Brazilian Ministry of Education, 2001), mathematics in university curricula aims to: promote skills and competences derived from the relations between this area and other fields of knowledge; encourage the use and analysis of contemporary issues, contemplating aspects of reality; provide an education that contributes to the understanding of the impact of certain decisions, supported by mathematical knowledge, in different contexts, global and social, among others. However, often these aims seem not to be given attention in conventional classrooms, since the focus is more on the application of conceptual techniques, supported by the reproduction of exercises.

Mathematics is not only practiced in academic or professional fields; it is also part of varied everyday situations and, therefore, it is necessary to explore them with approaches that lead to reflection. Educational environments can mobilize reflections on the knowledge produced and the effects they have on the context in which they are applied.

Thus, this study seeks to explore learning located within problem situations. In order to do so, the research project in question is supported by work with on a specific

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problem, based on the perspective of Problem Based Learning (PBL). Thus, the research question that guides this study is**: How can the aspects of mathematics in action be explored in calculus teaching, from the perspective of Problem Based Learning?**

PBL is an educational approach that operates by presenting one or more problem(s) to small groups of students. The purpose being to encourage student participation, aiming to combine theoretical knowledge with its possible practical applications, such as those related to the professional field. The perspective adopted is based on three fundamental principles: the learning, which is organized around the problems; the content, which aims to encompass interdisciplinary learning; the social approach, characterized by working in groups, stimulating aspects such as dialogue, communication, reflection, collaboration, etc.

Thus, PBL is not seen here as a problem-solving method, but rather as a type of action that can mobilize different knowledge and skills, including problem solving. Discussions about mathematics that is at work in our real-life contexts can be explored in higher education, stimulating reflections on the use and applications of constructed knowledge.

In this research, it is understood that PBL can be a teaching strategy that stimulates and contributes with discussions about the use and application of mathematical concepts in real world contexts. Such a connection, supported by the use of problems, can help the teaching and learning of mathematics in the university context.

THEORETICAL FRAMEWORK

The understanding that mathematics is associated with a variety of situations and practices, be they social, economic, or political, for example, is linked to the concept called mathematics in action. This term is related to the concerns of Critical Mathematics Education, inspired by Ole Skovsmose and, this author, is one of the main theoretical references of the project.

In this sense, some critical questions related to the field of mathematics can be asked: What are the different roles of mathematical education in our society? What kind of opportunities can mathematics education offer students? How can an individual's background influence future expectations? How can students be enabled to reflect on the impact of a certain mathematics-based decision? How can mathematics be linked to a university education that can lead to specialization, without worrying about sociocultural issues?

In the midst of such questions, this research is intended to address concerns about how mathematical conceptions can be projected in real world contexts. To investigate these concerns and understand how the aspects of mathematics in action can be explored in higher education, the organization of this proposal will be through the PBL.

Mathematics in action refers to "those practices that include mathematics as a constituent part of themselves, eg, technological innovation, production and

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automation, management and decision making, financial transactions, risk estimates, analysis cost-effectiveness, etc." (Skovsmose, 2008, p.51). According to the author, all these practices contain in themselves actions based on mathematics and therefore there is a need to reflect on the role of mathematics therein. Mathematical knowledge should not only serve to help students learn certain forms of knowledge and techniques but should also invite them to understand how these forms of knowledge and how these techniques are put into action in specific contexts. "Actions can be dangerous, courageous, benevolent, meritorious, etc. and, likewise, actions based on mathematics can also be so "(Skovsmose, 2014,p.89).

Different aspects regarding performances based on mathematics may exist, and thus Skovsmose states five aspects of mathematics in action. These are: technological imagination, hypothetical reasoning, legitimation or justification, realization and dissolution of responsibility.

Technological imagination refers to the possibilities of exploring the development and the alternatives of a project supported in an imaginary scenario; hypothetical reasoning addresses and assesses the consequences of an imaginary scenario, raising hypotheses and investigating the likely outcomes of something not yet realized; legitimacy or justification can support certain actions, from mathematical arguments; the realization, in which one can observe the use of mathematical models, portraysthe presence of mathematics in the day to day; the dissolution of responsibility brings to the fore discussions on ethical issues and relations between knowledge and power.

In addition to using the work of Skovsmose, theoretical foundations associated with the PBL studies and the university mathematics proposed by Ole Ravn Christensen and Paola Valero, both from the University of Aalborg, Denmark, will be used in the research design. The approaches given by these authors turn to sociocultural approaches, which intend to go beyond concerns related only to a theory-practice duality in classrooms.

Thus, the perspective of PBL to be used in this research is understood as a possibility to invite students to reflect on how certain mathematical concepts, studied in the universities, can be related to varied contexts.

METHODS AND ANALYSIS

The proposed research will be based on a qualitative approach and on the development of a problem proposed in the classroom, from the perspective of PBL. It is characterized as a case study because it will be carried out in a specific context, where the features of the specificity are essentially part of the analysis.

The production of data will be done in a federal education institution of the city of São Paulo, with students in a Civil Engineering course. The curriculum structure of this course is inspired by the principles of the PBL, according to information that appears in the official documents of the institution. We established a partnership with an

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effective teacher in this institution, and we intend to promote eight face-to-face weekly meetings with students, in contrast to regular classes.

To do so, we will invite incoming students in the Engineering course to participate in this investigation. They will form a group, which may have five to seven members. The intention is that they can discuss and propose solutions to a given problem based on different knowledge, in particular, knowledge that involves the understanding of limit.

From the presentation of the problem, the students will have the possibility to analyze it and discuss it in a group. In the proposed approach, they can build hypotheses, carry out a study and construct action plans, share knowledge, and work collaboratively. Different tasks will be delegated to the members of the group, who assume rotating roles, such as rapporteur and secretary. Together, decisions will be made and proposals for solutions to the problem will be outlined. The evaluative processes, which include, for example, the feasibility analysis of the solution and the performance of each student and the group, can be done at the end of the problem development. With this group work, based on the perspective of the PBL, discussions will be conducted that contemplate the projections of mathematics in a real context, that is, of mathematics in action.

In this research, the data will be produced by means of audio recordings, field diary notes and three semi-structured interviews: two of them will be done with the students, one before and another after the problem is developed, and the other will be directed to the teacher.

In the analysis of the data the theoretical bases/concepts related to the five aspects of mathematics in action, proposed by Ole Skovsmose and previously mentioned, will be used.

Proposing a scenario approaching the five aspects of mathematics in action can promote understanding of how mathematical techniques and tools are put into action in a real context. We intend to contribute with studies aimed at mathematical education in higher education, as well as to foster other possibilities for research on such subjects.

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RESEARCH PAPERS

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THE DESIRED MATH TEACHER IN UNIVERSITIES' STUDY AND ASSESSMENT GUIDES

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Becoming mathematics teachers' study guides are institutional texts which here form the base for a discourse analysis. We aimed to traced the "images of the desired mathematics teachers", a concept and theoretical framing which distinguishes becoming teachers as both a subject and as subjected to discourses of power which constitute categories such as for example a 'teacher' and a 'good teacher'. The found images of desires for becoming "communicative teachers", "analytical/reflective teachers" and "principled teachers" invites us to picture and discuss a certain kind of future teachers. The most striking desire we did not find aligns with an "imagined image of the social just and politically aware teacher".

INTRODUCTION

Course outlines, including assessment criteria, labelled study guides are texts formulated to inform becoming mathematics teachers about what is expected of them in their different on-campus university courses during their teacher education. These are interesting documents as they shape the making of, or the images of the desires institutions formulate for becoming teachers. These institutional texts here form the base for a discourse analysis, conducted as a smaller part of a large state-funded research study of Swedish mathematics teacher education, a teacher education which is politically hot and questioned at this historical point in time. For example, the OECD recommendations for Sweden is to raise the quality of teacher education (Lindblad et al., 2015), a topic which is discussed politically in government and frequently in media.

We analysed these different institutional study guides that are handed out to all students at the start of each mathematics teacher education course. With this quest, we intended to further understand the discursive institutional images of the desired becoming mathematics teachers. We also asked what images that were missing in these documents. The research question we posed is "What are the images of the desired mathematics teacher, reflected in institutional university courses study guides and assessment criteria?" These images not only reflect what is desired outcomes at the local university. Even if Swedish universities have a freedom to create their own courses, they are required to formulate the local goals in relation to national goals (SFS 1993:100) as there are also regular expectations of the texts (Prop. 2009/10:165.)

THE DESIRED TEACHER IN THEORY

The concept and theoretical framing of this research is *the images of the desired teacher* which distinguishes the teacher as both a subject and as subjected to discourses of power which constitute categories such as for example a 'teacher' and a 'good teacher'.

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> The research and the international agencies are promoting enunciations and statements - discourses - around an ideal image of the mathematics teacher. The discursive formations respond to particular requirements and demands of spatiotemporal conditions. This ideal image of the mathematics teacher is transformed into the 'must be' of the teacher, a desired subject, producing regimes of truth, power relations, which normalize forms of thinking about concrete teachers. (Montecino &Valero,2015, p. 797)

We define *discourse* as "practices that systematically form the objects of which they speak' (Foucault 1972, p. 49); thus, in this instance, the becoming mathematics teachers, are determined by the way that theys are spoken, or in this case written, into being in the local institutional documents.

THE DESIRED TEACHER IN PREVIOUS MATHEMATICS EDUCATION RESAERCH

Mathematical knowledge and teaching

Using a conceptual framework developed from the work by Rusznyak and Bertram (2015), showing if the valued application is of *reasoned judgement* or *preferred technique* in students' practicum protocols, Christensen and Österling (forthcoming) showed how images of a desired teachers are painted in Rwanda, South Africa, Sweden, Canada, Singapore and Great Britain. Four images of a desired teacher are reflected in these particular practicum protocols. Within the valued applications category the image of *the knowledgeable teacher* "who will teach well because of her knowledge base, but the image remains silent about what happens when one does not know how to apply this knowledge in practice". Second, within the reasoned judgement category dominates an image as "that of *the constantly improving teacher*, always seeking to adjust practice to new content knowledge, new pedagogical theories or new policies, this teacher is in a constant cycle of reflecting and adjusting.". Third, "*the successful teacher"* ensure that the *preferred outcomes* are reached, through attending to planning (transformation) and in particular to instructional practices. How the outcomes are achieved remains unclear, since neither the knowledge base nor the implementation practice are made explicit in the practicum protocols. Lastly, the forth found image reflects a *"knowledge-transforming teacher"* who "is assumed to have content knowledge as well as know-how on making this accessible to students through careful and considerate adaptation of content to the level of learners.". These images were not ranked in any way; however, each conveys a discourse for what characterises a becoming good teacher and what is less important in the practicum documents.

Taking a short historical view[1] on desired teachers, we have acknowledged examples as the early *scholar-teacher*, who not only knew how to manage the classroom, but could "think for herself, apply disciplined knowledge, and act as an agent of cultural renewal" (Connell, 2009, p. 216). From the 1970s, a desired *collegial professional* is described, who creates a teaching environment with collaborations, risk-taking and

[1] For a more in-depth historical overview, see Christensen and Österling (forthcoming)

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continuous improvement (Gainsburg, 2012). Lately, critical voices have been raised warning for a shift to a desired *measured competent teacher* in a culture of assessing teacher effectiveness (Hargreaves, 2000). An upcoming trend in Sweden, previously noted in the UK and the US (Arbaugh et al., 2015) is an image of a *de-professionalisation* of teachers (Beach & Bagley, 2013), which was noted as significant for this study.

Resonating with the *constantly improving teacher* above Montecino and Valero, (2017, p. 150) showed, and critiqued, through a Foucault-inspired discourse analysis, the making of mathematics teachers in OECD and UNESCO documents. They conclude that the making of desired teacher becomes a *sales agent for mathematics* through a *never-ending teacher education:*

> The idea of permanent training is operating as part of a dispositive by setting diverse forms of control, discourses, and forces. Consequently, the mathematics teacher is condemned to be incomplete and to have constant deficits to overcome, since society and the market will always be setting new requirements, demands, and urgencies that the teacher must face.

Gates and Jorgensen (2009, p. 164) underline that a political perspective on teacher education would ask "how teacher education plays a part in the furtherance of a practice which evidently works against the interests of many learners. Significantly, such socially unjust practices are not imposed upon teachers; they are enacted by them, and believed by them to be essential and natural". In this citation, we recognise an image of a desired *social just and politically aware teacher,* an image to be developed further.

Social justice and equity

Teachers are not only expected to know their mathematics, how to teach mathematics and orchestrate a classroom. They are also expected to be aware of and strive for an inclusive teaching. The responsibility for a change of the underperformance of marginalized students in mathematics education is an important part of teachers' assignments. I.e., in the US context, Gutiérrez et al. (2008) concludes that:

> […] the Association of Mathematics Teacher Educator (AMTE), the National Council of Supervisors of Mathematics (NCSM), and the National Council of Teachers of Mathematics (NCTM), three national organizations that support teacher educators, mathematics teachers, and teacher leaders, have made equity a priority for their organizations.

In fact, JMTE published two special issues on social justice and mathematics teacher education in 2009, where Gates and Zevenbergen (2009, p. 161) argued for "foregrounding social justice in mathematics teacher education". In addition, in 2011 an article authored by eight prominent teacher education researchers argued a need for "foregrounding equity in mathematics teacher education" (Strutchens et al., 2011). Another way of addressing social justice issues may be expectations to use

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mathematics per se for developing awareness of, critiquing, and working to change societal injustices (Andersson & Wagner, 2017; Gutstein 2006; Skovsmose, 2013). Expectations could be that becoming teachers should be confident with a culturally responsive teaching (i.e. Parker, Bartell & Novak, 2015) or critical mathematics education (i.e. Andersson, 2011; Gutstein, 2006; Skovsmose, 2011). However, Nolan (2009) questions the "marriage" between social justice and mathematics education and questions if "counselling" is required for the relationship:

> In mathematics teacher education, I am interested in working toward such an ideological shift—one that opens spaces of empowering possibilities and potentialities for prospective mathematics teachers and their students. […] Therefore, I continue to ''direct students' eyes'' to modes of critical questioning and deconstruction—an approach that, thus far, has resulted in more frustration and self-questioning than empowerment. I continue to grapple with decoding the research theory of mathematics teacher education and social justice […] such that I might create the conditions for a practice that is not centred on deploying mathematics as a neutral tool to analyse socially unjust facts and figures, but instead centred on critiquing the story-lines that weave throughout teacher education.

METHODOLOGY

We emphasise that the image of the desired teacher in the analysed texts are not concrete individuals. As Montecino and Valero (2017, p.137) described the case:

> The "mathematics teacher" that we discuss here is not a concrete individual of flesh and bone. It is a discursive construction, where power is actualized in articulating ways of thinking about desired forms of being, and where the meaning of and expectations for the mathematics teacher is configured and negotiated. […] these ways of reasoning frame possibilities of being and becoming.

We underline that the results should be taken as indications, as we here account for a study with data from study guides from all mathematics education courses, but from one university. Hence, the images of the desired mathematics teachers we found needs to be verified or contradicted with data from other mathematics teacher programs, within and outside of Sweden. This data is collected but not yet analysed.

SETTING THE CONTEXT

Swedish teacher education is allocated at 24 universities, spread over the country. They are all part of a rather decentralised system as all higher education in Sweden is. The frameworks are laid out by the Parliament and the Government together, but they are not regulated in detail. A consequence of this relatively decentralised system is that the educational programmes, even those leading to the same degree, may differ greatly from institution to institution. The Swedish Higher Education Act contains provisions and descriptions of degree fulfilments that the academic institutions offering initial teacher education need to take into consideration. The National Agency for Higher Education (NAHE) is responsible for continuous teacher educations inspections.

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The Government bill for the *"Best in Class"* of Teacher education was initiated in April 2010. It presented a new teacher education program focusing four professional degrees: preschool degree, compulsory school degree, subject teacher degree and vocational teacher degree.

Preschool class and Years 1-3 compulsory school degree: The 240 ECTS credits program give teachers broad knowledge for teaching most subjects, enhancing knowledge of reading and writing and advanced mathematics for younger children.

Years 4-6 compulsory school degree: The 240 ECTS credits program will inspire future teachers to study both the extent and the depth of the subjects. Studies in Swedish, Mathematics and English are compulsory, while other subjects are optional.

Years 7-9 and upper secondary subject teacher degree: This programme is longer and correspond to 270 ECTS credits and gives teachers skills to teach two or three subjects.

The students in the programs receive study guides, including course outlines and assessment criteria, at the start of each course. We analysed all the on-campus university courses' study guides for mathematics and mathematics education courses in the above programs in one university[2].

ANALYSIS

We here account very briefly for how we conducted the discursive analysis (Fairclough, 2013; Halliday, 1978). The texts were first analysed through marking all verbs indicating actions. Secondly, the verbs were analysed in the specific context they were written. Hence, we categorised both the number of different verbs and then categorised the verbs in their specific context. We here give two examples from the course guide for the very first course for becoming grade 4-6 mathematics teachers to exemplify the categorisation and translation process[3]:

The first example is the phrase "You are responsible for your education" [Du ansvarar för din utbildning]. The Swedish verb "ansvarar" is translated into "beeing responsible" and then categorised into verb groups indicating "relational" [relationella] and "disciplining" [diciplinerande]. Together with other verbs, analysed in their context, this group of verbs and phrases developed into an image of a desired principled teacher.

Second, a number of verbs as "describe" [beskriva], "explain" [förklara], "account for" [redogöra], "reason" [resonera] and "motivate" [motivera] were categorised as communicative verbs and in context connected to either:

a) Being able to communicate mathematics per se as in "communicate mathematics with support of different representations" [kommunicera matematik med hjälp av olika representations- och uttrycksformer]

[2] For an analysis of the practicum guides, see Christensen and Österling (forthcoming).

[3] The translation of the Swedish verbs and document phrases into English are made by the authors.

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b) Accounting for someone else's mathematical knowledge as in "describe pupils knowledge in number sense and arithmetic in a formative way" [beskriva elevers kunskaper i taluppfattning och aritmetik utifrån ett formativt syfte]
c) Use personal communication skills in teaching situations as in "analyse and use different types of communication and interactions in teaching including digital tools" [analysera och använda olika typer av kommunikation och interaktion i undervisningen inklusive digitala verktyg]

Different phrases with verbs categorised as communicative developed into the image of a desired communicative teacher.

Lastly, we went back to the literature and asked what other researchers previously had found, described or imagined, that was missing in our findings. We then re-analysed the data, looking for what might not be there, or what we may have missed in our analysis. These last (not) findings are accounted for in the discussion section.

RESULTS

Found images

The one outstanding, most common image, in all the mathematics teacher study guides and course outlines including assessment criteria, is of a desired *communicative teacher*. A mathematics teacher shall be able to describe, explain, account for, reason and motivate mathematics, mathematics learning, personal learning and development, mathematics teaching, planning and assessment in all imaginable communicational ways as orally, written and with technical devices. It is noted that it is not specifically communicated what language(s) teachers shall be able to communicate, however implicitly it is the Swedish language as the administrative authority language. This relates to findings by Skog and Andersson (2015) who found that a language discourse in mathematics teacher education disempowered a group of immigrant students.

The second most common image was of the desired *analytical/reflective teacher* who analyses, reflects, identifies and problematizes mathematics, mathematical teaching and pupils' learning of mathematics. Examples of the goals students should meet are to be able to "analyse steering documents and teaching materials" [analysera styrdokument och läromedel] and "reflect on perceptions of knowledge, knowledge progression and qualities of knowledge" [reflektera över kunskapssyn, kunskapsprogression och kunskapskvaliteter]. An interesting finding is that the word "critically" in connection with a verb is only used in becoming upper secondary teachers' course guides. For example; "plan and critically reflect on how teaching sequences may develop pupils' mathematical abilities" [planera och kritiskt reflektera över hur undervisningssekvenser kan utveckla elevers matematiska förmågor].

Then we found an image of the desired *principled teacher*; which we imagine stays in tension with a possible rebellious, risk taking or questioning teacher student. Statements, usually in an imperative form, as "you are expected to work with it 40

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h/week" [du förväntas arbeta med den 40 tim/vecka]; "if you haven't participated in the collaborative work prior to an oral presentation or examination you have to do it together with other students the next time the course is on" [Om du inte deltagit i förarbetet inför någon muntlig redovisning eller examination, får du göra den tillsammans med andra studenter nästa gång kursen går]; "You are responsible for your education" [Du ansvarar för din utbildning] and it is "your responsibility to know the existing rules" [ditt ansvar är att känna till de regler som finns]. These statements were common in some study guides, but invisible in others. We ask ourselves to what extent and in which direction students are affected by these kind of statements, and also how they play out together with goals such as being "critically reflective". Nevertheless, this discourse is problematic of several reasons and we wonder about the desire for principled teachers at this historical point in time, specifically in relation the findings accounted for above by Nolan (2009) and Montecino and Valero (2017).

In one of the guides, we found traces of an image of a desired *exercising teacher* who "trains herself in explaining a mathematical content" [träna sig i att förklara ett matematikinnehåll] and "trains herself in planning lesson activities" [träna sig i att planera lektionsaktiviteter]. When translating Swedish into English, we are aware of the different discourses and the subjectivities they invite to in the English language when talking about becoming mathematics teachers' "teacher education" in contrast to the phrase "teacher training". Here we notice traces of "teacher training" also in the Swedish language in one of the course guides. In our context, this might be part of the discourse of a de-professionalisation of teachers (Beach & Bagley, 2013).

When looking specifically at verbs for mathematics teaching; we notice that an image of a *planning teacher* is desirable. Neither classroom orchestrating, classroom management or other "teaching verbs" are noticed, only the verb planning which is commonly used in all courses.

Lastly, small traces were found of an image of a particular kind of desired *social justice aware and reflecting teacher*. This image is specifically outlined in the becoming upper secondary mathematics students' course guides but mostly absent in the lower grades. In statements exemplified with "identify and account for relevant rules in the Administration Act, the Schools Act, the Discrimination Act, the European Law and how they are connected" [identifiera och redogöra för relevanta regler i regeringsformen kap. 1-2, förvaltningslagen, skollagen, diskrimineringslagen, europarätten och dessas inbördes sammanhang]; "account for and reflect on work-ethical aspects concerning teachers' approaches/attitudes to pupils and parents" [redogöra för och reflektera över yrkesetiska aspekter rörande lärarens förhållningssätt till elever och föräldrar] and "critically review the school systems significance for the reproduction of social inequalities and for the elimination of those" [kritiskt granska skolsystemets betydelse för återskapande av sociala ojämlikheter liksom för undanröjandet av sådana], an image of a teacher being aware of social injustices and

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being able to reflect on these injustices in steering documents is required. How to actually act, teach, assess or interact in classrooms, schools or with parents in social just ways, or to teach in inclusive ways, is not addressed in the texts.

DISCUSSION

Montecino and Valero (2017, p. 135) argued after a discourse analysis of OECD and UN documents that:

> […] nowadays cultural thesis about who the mathematics teacher should be are framed in a double bind of the teacher as a policy product and as a sales agent. Narratives about the mathematics teacher are made possible within a dispositive of control, which makes mathematics education and mathematics teachers the cornerstone for realizing current market-oriented, competitive, and globalized societies.

With this argument in mind, and in relation to the images of the desired teachers (the *communicative teacher, analytical/reflective teacher* and the *principled teacher*) that we found in these institutional study and assessment guides, we wonder about images that does not exist in the guides but still previously have been addressed in research?

The most striking desire that we *not* found aligns with the image of *the social just and politically aware teacher*. At this historical point in time, where challenges of marginalisation, migration, class and gender is discussed and broadcasted more than ever, we would have expected images of teachers having knowledge about history, the challenges, the possibilities and/or what an inclusive and culturally responsible teaching both in general and specifically in mathematics might mean to students, their learning and their well-being in mathematics classrooms. This desire was almost absent in the becoming primary mathematics teachers guides, but mentioned in upper secondary as these becoming teachers are expected to be able to reflect on injustices in steering documents, but only implicitly in teaching.

In addition, in dialogue with published research we expected to trace images of a desired caring teacher (c.f. Noddings, 2013). However, no statements were found with words connected to empathy, compassion, caring for people or caring relationships. However, we found several examples that could be placed in a category of caring about the mathematics; exemplified by the goal for becoming grade 4-6 teachers "reflect on including teaching methods and didactical solutions that promote pupils' mathematical development" [Reflektera över inkluderande undervisningsmetoder och didaktiska lösningar som främjar elevers matematikutveckling]. In the course guides for becoming teachers in the lower grades (1-3 and 4-6), we categorised an image of a desired *relations-expert teacher*, a teacher who collaborates, invites fellow students to conversations, partake in group work and adheres to ethical rules formulated by the study groups. However, verbs indicating care for people and/or relationships were not found in the guides for becoming mathematics subject teachers.

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At last we address absent possible images of the risk-taking teachers, the inquiring teachers or the open-minded teachers. The found images of desires for becoming *communicative teachers, analytical/reflective teachers* and *principled teachers* invites us to picture a certain kind of future teachers. We ask ourselves if we will be able to find teachers who challenge discourses, open new not-yet-imagined doors for students, inquires and are curious about mathematics, mathematics learning and their students in the future, who maybe develops skills for inquiry and research? Will we find teachers who understand the importance of balancing different aspects of our lives—intellectual, physical, and emotional—to achieve well-being for ourselves and others, and who recognize our interdependence with other people and with the world in which we live?

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DIGITIZED NATIONAL TESTS IN MATHEMATICS: A WAY OF INCREASING AND SECURING EQUITY?

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On one hand, the Swedish governing discourse on equity in the context of digitizing education portrays modernization, progress and democracy as a foundation in the equity work. On the other hand, in the context of digitized tests, equity is rather framed within a neoliberal logic while related to all individuals' possibilities of choosing a 'good life', and to compete on equal terms. Not all disadvantaged groups are the target, though. It is mainly boys who are supposed be given better grades, and, in addition, students with disabilities who are supposed to (as far as possible) be able to have the opportunity to show their knowledge during the test. Language or socioeconomically diverse settings are not mentioned with regard to digitized national tests.

INTRODUCTION

Today, common 'vehicles' for development and learning in school are digitization and technology (Hylén, 2013). Data use in education is a regime of governing who derives from the idea of educational transparency (Prøitz, Mausethagen & Skedsmo, 2017). Regulatory practices trough increased surveillance is rooted in a neoliberal logic and might increase inequalities (Apple, 2000). At the same time as digitalization are frequently used in measuring and securing educational quality, the effects from digitization are understudied (Goodman, Seymor & Andersson, 2015). Another vehicle for progress and well-fare in Sweden, besides digitizing, is the extensive and national assessment of students' knowledge. These tests are both the object of governing strategies and part of the means to govern the education system (Ozga, 2009). National assessment is mandatory in primary school in Sweden, in the years 3, 6 and 9, and have until now been performed with paper and pencil. The two vehicles, national assessment and digitizing, are now put together by the government as the Swedish National Agency for Education has been given the assignment to digitize all Swedish national tests by the year 2022. This course of action is, among other things, assumed to enhance equity and fairness (prop. 2017/18:14; U2017/03739/GV).

Despite the use of equity as a frequent educational argument, while addressing change and development, the meaning of the notion of equity in relation to digital testing is often shadowed and multitudinous and might actually mean very different things (Espinoza, 2007). However, in the long run, the interpreted and expressed meaning of equity will direct the actions of and approaches towards individuals, since values are connected to concepts (Llewelyn & Mendick, 2011). To take account of what policymakers mean by and how the concept of equity is communicated is of great interest since this governs what equity is and could be. The purpose of this paper is to contribute with knowledge regarding aspects of equity in the governing of digitizing

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national mathematics tests in Sweden. The investigation is guided by the following research questions. RQ1: What discourses of equity are constructed in policy texts on digitization of National Tests (NT) in mathematics in Sweden? RQ2: What do these discourses of equity contribute to?

EQUITY IN EDUCATION

To legitimate valuable knowledge is and has been a social issue (Berger & Luckmann, 1967). Features like what knowledge is being taught; what is going to be assessed; and what the organizational arrangements are, are embedded in political power structures (Hutmacher, 2001). Unterhalter (2001) draws out how equity can be understood in education, while pointing to the ideal of respect for all. She also addresses that the term equity is not clearly defined in research literature, leading to normative assumptions about equity in education. In line with the definition of equity in the Oxford English Dictionary (2018) we define the term to be 'the quality of being equal and fair'. Thus, equity has to be thought of as a process of making equal and fair, in other words, "equality turned into an action" (Unterhalter, 2001, p. 416). Furthermore, Unterhalter suggests that equity can be understood as a process from 'above', the 'middle' or 'below'. In this paper, equity is pursued from 'above', in the Swedish policy texts we are examining, as a way to govern equity and fairness actions according to rules. This is especially interesting to scrutinize since the Swedish education system is presumed to allocate resources fair and equal, to ensure the same opportunities for all, regardless of birth, social class and ethnicity. (National Agency for Education, 2018)

EQUITY AND DIGITAL ASSESSMENT IN MATHEMATICS

Results from digitized national test are considered to increase transparency and allow discharge through the assessment in the sense of surveillance inspections (Thompson & Cook, 2015). This approach holds the belief that the development of surveillance technologies overrides the capacity of human capital to ensure validity, equity and quality (Piketty, 2014). Researchers have previously pointed out how testimonials derive from the preference that teachers and knowledge need to be monitored in order to be legitimate (Mickwitz, 2015). This can in the digitized test-form also be applied to the test constructors since data on the validity and suitability of the construction will be gathered in the digital system. O'Keeffe (2017) argues that through this, digital samples not only collect and produce data, they also make sense of what capability or skill might be.

There are some comparisons made regarding how computer-based versus paper and pencil-based tests affect different students' opportunities to achieve and participate. Spiezia (2009) notes that students from families with larger economic, cultural, social and technical resources are getting better results at the same level of IT use compared to students with lower socio-economic backgrounds. In addition to this, a Norwegian study has shown that in addition to socio-economic background, language background and motivation is crucial (Hatlevik, Ottestad & Throndsen, 2015). There are other studies (see, for example, Shapley, Sheehan, Maloney & Caranikas-Walker, 2009)

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which show the opposite, i.e. that socio-economically disadvantaged students achieve equally beneficial mathematics results as students from more advantaged conditions when using digital tools. In addition, Shapley et. als' (2009) point out that both student groups were far more technically skilled than students who went to schools that were not as computer-intensive.

A parallel can be drawn to a study showing that students' prerequisites play a part in the ability of the digital examination to function formatively. Students with higher goal achievement had better opportunities to demonstrate their mathematical skills in a digital examination. An important element was the feedback the pupils received and that it adapted to the students' ability (Faber, Luyten & Visscher, 2017). In addition to this type of adaptation, it is crucial that the examination/test conforms to the students' ability in manners that allows them to access the content. This, together with the opportunity to take the test at one's own pace, seem to be beneficial for digital mathematics test (Landau, Russell, Gourgey, Erin, & Cowan, 2003). Research has also shown that the results of computer-based tests are similar to the ones reached with paper and pencils with the same students (Siozos, Palaigeorgiou, Triantafyllakos & Despotakis, 2009). Another comparison between the two forms has been made in Singapore with 11-12-year olds. A conclusion was that students' viso-spatial thinking and ability had a greater impact on the digital test (Logan, 2015).

THEORETICAL FRAMING

Practices in institutionalized fields, like making (political) decisions and writing policy texts, are in this paper understood in terms of discursive practices (Foucault, 1988). Policy texts on education intend to steer curriculum and classroom practices, and curriculum can be considered to be the system's view of the ideal situation in which it functions (Cummings, 2013). Governmental decisions and documents are in this paper understood as *inscription devices* for the inherent meaning and values held by the concept equity in the context of digitized tests (Popkewitz, 2004). Following from this, these texts are inscribing meaning (see Popkewitz, 2004), norms and values into other discursive practices as for example the practice of assessing knowledge. Thereby, they govern prerequisites in relation to students' participation. Educational reforms are in this way grounded in and built on visions of what is desired in society. Statements in policy texts embody ideas about "how to see, think and act" (Popkewitz, 2012, p. 177) towards people, and thereby they function as governing technologies. That is, they construct certain kinds of people, who that person is and should be. Consequently, they work their ways into the lives of people, for example students, and reinforce a way of thinking of oneself (Foucault, 1983). The notion of discourse is in this paper used to explore how equity is thought about and acted on in the texts on Digitization of the National Tests in Mathematics (from now on abbreviated DiNTM).

EXAMINING POLICY TEXTS

Initially we selected eight governmental texts including the preparatory work, the decision and assignment of implementation given to the Swedish National Agency for

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Education (see table 1). In addition, the overall national strategy to digitalize the Swedish school is included, as it precedes and have a strong bearing on the digitalizing of national tests, and also the proposition following from an evaluation of the national assessment and in which equity was highlighted. The policy texts are viewed as discursive practices (Foucault, 1972), since texts build on existing discourses and are written within the discursive practice to write policy texts. All sections of the texts referring to equity in the context of digitized tests were selected, as well as sections that addressed equity without mentioning the term digitization specifically.

Table 1: Overview of the governmental documents analyzed in this paper

No	Kind of source and title	Short description
1	Memo: *Nationella prov: rättvisa, likvärdiga, digitala.*	A PM, where the government announce the proposition on an investigation of the national assessment system in regard to fairness, equality and digitalization.
2	Proposition: *Nationella prov: rättvisa, likvärdiga, digitala.* (Prop. 2017/18:14).	62 pages of proposition, the same as was announced in the PM above
3	Memo: *National strategi for digitaliseringen av undervisningen*	A PM, where the government announce a national digitalization strategy of education, stating that children and students have to reach a high level of digital competence, which is connected to equity.
4	Information: *För ett hållbart digitaliserat Sverige – en digitaliseringsstrategi*	Information about a strategy about how Sweden will be the best in the world to take advantage of the potential of digitization trough the goals: digital - competence, -security, -innovation, -management and – infrastructure.
5	Strategy*: För ett hållbart digitaliserat Sverige – en digitaliseringsstrategi.* (N2017/03643/D)	14 pages strategy about equal access with regards to students needs and prerequisites, and effective use of technologies. Writings about digital equity are included.
6	Memo: *De nationella proven digitaliseras*	PM regarding the governmental decision to digitalize the national tests and the approval of the previous mentioned proposition (prop 2017/18:14)
7	Memo: *Uppdrag att digitalisera de nationella proven mm.*	A PM to the National Agency of Education, which has to ensure the accessibility of the national tests and that they can be used by all students.
8	Assignment: *Uppdrag att digitalisera de nationella proven mm.* U2017/03739/GV	4 pages assignment stating that digital test will lead to a higher level of equity because the grades become fairer. The assessment system has to be modernized.

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In a first analytical step, we repeatedly and carefully read the selected texts, independently by the each other, with the aim of summarizing and organizing. The texts were then compared and scrutinized, looking for patterns, contradictions and similarities. Key words that often appeared were identified. The second step included thematization of the key words. The theoretical constructs *discourse* (Foucault, 1970/1993; 1972) was used to elaborate on how equity in the context of DiNTM is discursively constructed through the policy texts. Specifically, we construed discourses that order and shape how equity is to be realized in DiNTM. Below, we address RQ 1: that is, the equity discourses we construed based on the analysis of the policy texts, while drawing on quotes from the policy texts to support our results.

CONSTRUED EQUITY DISCOURSES

It is no doubt the Swedish government and parliament address digitizing and democracy as intrinsically dependent on each other. For example, the first line in the National Strategy for Digitizing Education is: “Digital competence is basically a democracy issue” (text (t.) 4, p. 3). According to the press message the aim of the digitizing education is that:

> Sweden will be the best in the world to take advantage of the potential of digitizing. Education policy has an important role to play in achieving this ambition. The government has therefore developed a national digitization strategy for the school system. (text from memorandum on the web, no pages indicated)

In the digitizing strategy (t. 5), the government describes a foundation for continued work to use the potentials of digitization to raise both students’ achievement and *increase equity* in the school system. However, when analyzing the policy documents in relation to digitizing tests, equity is discursively constructed in various ways. Four discourses were construed from how policy texts constructed the concept of equity:

1. Equity as threatened and deficit

Equity is constructed as *threatened* by flaws in the assessment system and as constituting the very legal rights in the assessment of knowledge. The minister in charge fabricates flaws as the reason for digitizing tests, and the modernized digital system as a solution leading to a higher level of equity as the grades become fairer:

> … now we will get better order and remedy when grading and in student’s knowledge assessment. There are now good conditions for the assessment of test results, and in the end assessment can be done in a more equal way for all students. Digitizing the national tests is an important and long-awaited modernization. (t. 6, PM on the web, no pages indicated)

The flaws are partly due to teachers as producers of inequity since teachers’ judgments are described to be too mild when assessing their own students:

> teachers who assess their own students’ test answers tend to make generous assessments and put relatively high test scores (t. 2, p. 13).

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This is described as supposedly more common if the teacher works alone and also something to contravene with the opportunity in digital tests to make students anonymous:

> … in order for the pupils to perceive less that the assessment is unfair, the proposition suggests that student solutions of the national tests should be unidentified in the assessment in cases where the tests have been carried out on computer (t. 2, p. 48)

Equity is also described as *needed* between boys and girls, through gender equality in regard to the tests' impact on grading, and for students with disabilities. Equity for the former group is at the same time described as a possible threat by the system, which is connected to how the test might be designed.

> In order to ensure that students with disabilities will take part of the positive effects, the technical solutions must be designed to suit all students and that the student's individual needs will be carefully investigated. (t. 2, p. 31)

2. Equity as access

The design of the system might threaten equity as described in the discourse above. Continuing this argument, for students with disabilities, equity is further emphasized as access. Adaptations and also that the technical and digital setting suits everyone, which is described as crucial. Equity in the digital production and transformation of the test is relying on international, special educational and digital expertise.

> Additionally, there should be experts in the group that take particular account of the needs that students with disabilities may have. (t. 2, p. 40)

Equity as access is also stated in text 8 as students with disabilities are addressed, after mentioning "all students':

> ... strive to ensure the accessibility and usefulness of the test for all students, including students with disabilities, in order not to limit the student's ability to demonstrate his or her knowledge in the test situation. (t. 8, p. 3)

3. Equity as justice in grading

Equity is discursively constructed from the documents as justice in grading, something that needs to be monitored and fixed through the governing of teachers' assessment of the tests. The tests will have the sole purpose of governing assessment towards equality and not be used for evaluating teaching. The tests are thereby being enhanced for the purpose of a governing technology of grading and assessment:

> As the national tests will have the purpose of being supportive for grading, the criterions for evaluation will be the same in the correction of national tests and grading. (t. 2, p. 28)

Equity is overall expected to be achieved if the teachers' knowledge about students and the relations between them, is taken out of the equation when answers are evaluated.

The equity is then described as increased if the tests should be self-correcting instead:

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> Student solutions of national tests should be assessed by someone other than the teaching teacher. Student solutions on digitized national tests should be unidentified. (t. 2, p. 14)

The need for taking these steps is referred to as flaws in stability over time and differences between tests in regard to how they are assessed and how heavy they are considered for the grades in the subject. A levelling of these aspects would especially contribute to equity in grading in favor of boys since teachers are more likely to give a girl a higher grade, regardless of the test:

> Unidentified student solutions are deemed to increase the possibility to equity between boys and girls…girls are in a higher degree getting a higher final grade then the grade on the test. (t. 2, p. 22)

Another statement is that the equity will be better secured in the future, since the tests' sole purpose will be to determine the grade, and not, like previously, to also contribute to an evaluation of the teaching

4. Equity as equal competition

A final goal of the above depicted access and justness, in order to achieve equity, is to improve equity in regard to competition. Equity is thereby discursively constructed as being about fair competition. This would ensure that the students are competing on equal terms regarding higher education.

> This provides for increased legal certainty for the students when they can compete on the basis of more equivalent conditions when applying for higher education. (t. 2, p. 30)

DISCUSSING DISCOURSES OF EQUITY

We are here addressing RQ 2. Four discourses were demarcated in the texts. What do these discourses of equity contribute to?

These discourses contribute to a narrative in which equity initially is threatened and viewed as missing, and something that firstly needs to be met through creating access to the tests for all students. Further on in the narrative, is a pull away of teachers and relations from the evaluation. This will supposedly lead to just grades and the essential objective: equal opportunities to compete in future life and higher education. In relation to digitized tests, equity is both a mean and a goal.

However, the analysis of the policy texts denotes that f. ex. the discourse 'equity as threatened and deficit' relies on a normative assumption (Unterhalter, 2001) about teachers not being fair enough in their assessment of their own students. They are made-up as too generous in the texts. The teachers have to depersonalize themselves to make the assessment fair. Regarding an imagined increased transparency and discharge through the assessment, we see a risk that the processes of digitizing the national test will become a full-scale surveillance checkup (Thompson & Cook, 2015), also of the not trusted teachers. We can also see a risk of Shapley et. al.'s (2009) point, that student groups who are more used to technology than others will gain more on DiNTM than students who not are used to digitized tests or digital tools. The discourse

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‘equity as access’ will not be realized at all if not, as described in the digitizing strategy of education, all students have the opportunity to develop digital competencies. Thus, and counter productively, an expected positive impact of digitization may be negative to certain groups of students. Our argument, leaning on Siozos et al.’s (2009) findings, is that the policy texts’ reason for equity is not sustainable.

In the new national system for assessment, equity seems to be week and in need of fixing. Equity “is” corrupted by relations and personal or identity factors. Especially ‘the lonely teacher’ is a “problem”. Equity is further emphasized as something that the teacher produces and not something the student “has”: a circumstance that leads to possible serious future consequences and to a view of students as passive receivers without agency. Thus, equity is constructed as something that needs to be governed by the state - and trough disciplining the teacher, and possibly by teachers governing each other. The access is further described as dependent on needs and prerequisites, which implies that equity is not at all sameness. At the same time equity is frequently described as sameness – same opportunities to display knowledge, same grading, same opportunities to be evaluated, same judgement in assessing tasks etc. All together the concept of equity is ambiguous, and it is actually not defined the schools a very interpretable and wide assignment to meet.

In the Swedish context of digitizing the national tests in mathematics, equity is rather framed within a neoliberal logic while related to all individuals’ possibilities of choosing the ‘good life’, and to compete on equal terms (Llewellyn & Mendick, 2011). A further exploration of how the tests are constructed and thereafter realized in the various settings would be an interesting follow-up study. As well as an in-depth study of what happens in the classroom and the assessment situation when the tests goes digital, and then paying attention to different prerequisites in the schools’ culture and the individuals’ experiences and competencies. Especially since effects from the digitized classroom is very under-researched (Goodman, Seymor & Andersson, 2015).

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NON-TYPICAL LEARNING SITES: A *PLATFORM* WHERE FOREGROUND INTERPLAYS WITH BACKGROUND

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After watching the groups of boys studying at the stairs of a suburban railway station in Mumbai for a while, we decided to talk to those students to understand their choice for such an unusual place of study. Adopting an eclectic exploratory approach, this paper argues that such impromptu places of collective learning, motivation is often guided by the prospects of inter and intra peer group learning and tends to redefine young students' foregrounds. This paper hypothesises that this railway platform provides opportunities to those whose performance in formal education remained poor and to those who could not afford the market of private tuitions thereby missing out on future scope. Albeit the gender disparity, such impromptu places of learning also tend to resist the enculturation process of the dominant cultural capital as well.

THE PLATFORM 3

Those stairs at that railway station leading to the platform number 3 always amused us. For there was always present a few groups of boys – but curiously, never a girl, – studying while sitting on a newspaper or on plastic sheets neatly placed on the shaded steps or on the mezzanine floor or even on the platform itself. Located on one side of a crowded suburban railway station in Mumbai, platform 3 remains vacant almost the whole day except between 8 am and 9 am when two local trains arrive and depart from this platform. This is a terminal platform and for the rest of the day it serves to different groups of students who come here and study any time of the day, even during late evenings. Platform 3 is a little away from the main platforms 1 and 2. There is a foot over-bridge that connects the main station and the three platforms with the backside entrance/exit. The stairs leading to platform 3 are connected to this foot over-bridge. The stairs are well shaded and illuminated and, in the evenings, it is well lit by the tube-lights. There is a police barrack across the tracks, officiated by the Railway Protection Force (RPF) personnel on the other side of platform 3 with some vacant land in between. The number and size of the groups studying on platform 3 varied depending on the time of the day and the time of the year. It is especially crowded close to some important examinations when students come to platform 3 for academic preparations. These are large-scale high stake public examinations in which a huge number of aspirants appear making these examinations highly competitive. These examinations are highly sought after for some of these are the gateways to technical courses like those of engineering, chartered accountancy or business management. There were groups of boys who prepared for such school-end or university entrance tests. We wondered and asked ourselves "why do they study at this place?" "Even at late hours?" "What are the reasons that draw these students here?" As education researchers it is interesting to explore and understand why those boys were studying at such a non-typical place and in what ways do their will (or

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interest) guide their decision. Further, we explore whether such efforts by a section of students and citizenry have any implication for the policy planners, city planners and educators. The questions of access, equity and social justice arise and this paper attempts to discuss these issues in the backdrop of above setting.

Adopting an eclectic exploratory approach, we argue that at such impromptu places of collective learning, students' motivation which is often guided by the prospects of inter and intra peer group learning and tends to redefine children's foregrounds (Skovsmose, 2012) who handle their financial disadvantaged conditions to come out of their background, empowering themselves and achieving autonomy (Freire, 1996). The tussle between the wishful foreground and the modest, pulling behind background emerges which forces students congregate at one non-typical place to collectively attempt for a different future. Statistics suggests that background defines possibilities and frames foregrounds, but it reveals only tendencies since background is fixed while foregrounds are widely open (Skovsmose, 2012). We discuss the emergent picture by analysing the influence of platform 3 on students' backgrounds as well as on their foregrounds and the role that mathematics plays in this process.

FOREGROUND: ACCESS TO QUALITY MATHEMATICS EDUCATION

Barriers for learning are often individualised and those obstacles are considered as part of a social and cultural background attached to the person (see Skovsmose, 2005). Much in the same way in the caste-based socially stratified Indian context, children's social location often guides and limits their foregrounds. The fault, then, is on the individual and not on the system. Her or his background would define the kind of future they would get without much choice. A complex combination of children's "experiences and interpretations of possibilities, tendencies, propensities, obstructions, barriers, and hindrances" form what Skovsmose refers to as foreground (2014, p.5). He argues that the wishes of the individual can affect her or his future along with the social and cultural background. Hence, a “ruined” foreground might enhance the idea of failure while a strong and hopeful foreground might influence future framing in a constructive way. The concept of foreground comes to tell that a background with social and/or cultural hindrances is not an impediment, rather it says that the existence of opportunities depends on how an individual perceives them, often not in an objective manner. Effectively, children's foreground emerges from the "socially determined opportunities" available to them.

In India, there has been “historical and sociological reasons behind inequities and differences in education and learning” (Bose & Kantha, 2014, p. 1073). Pedagogic efforts often do not take into consideration the socio-economic or socio-cultural backgrounds of the learners coming from different social locations. Influence of socio-economic conditions on education, particularly mathematics education, remains a major determinant of educational attainment. That is, economic conditions remain a major factor that determines living conditions and access to education. In some parts of India, social hierarchy (caste-based social strata, class) in terms of social standing and socio-cultural factors play an important role in determining access to education

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(Bose & Kantha, 2014; Thorat & Neuman, 2012). Access to technical higher education is seen as a gateway to future job opportunities and other social benefits, for example, betterment of social location with vertical mobility in the social status. However, such access is determined by a stringent filtering process in the form of public examinations. Often, mathematics and a strong foundation in it comes as a requirement for such tests. So much coveted are these examinations that many entrepreneurs privately run institutes to coach aspirants against hefty amounts of money which triggers the access and social justice questions. Even for the school-end board examinations, pupils go for extra training and coaching by paying hefty tuition fees. Such arrangements filter learners from low-income families out of the race. This is an example of how students' socio-economic conditions socially determine opportunities for them which in turn shapes their foregrounds. But in some cases, as it unfolds in our story below, learners' foregrounds too reshape their social opportunities and hence their access to higher education. We argue that this social phenomenon is a reply to the access question.

CULTURAL CAPITAL AND THE QUESTION OF ACCESS

Educational attainment particularly those of the dominant group can be perceived as one form of the cultural capital. Bourdieu equates dominant group's cultural capital with economic capital since dominant institutions (financial, educational and others) are structured in a way to favour those who already possess such cultural capital which is deemed as legitimate by the dominant hegemony. (Haralambos and Heald, 2013). Bourdieu argues that the "success of all school education depends fundamentally on the education previously accomplished in the early years of life" (Haralambos and Heald, 2013, p. 258-259). Cultural capital in Indian society is not evenly distributed through the class structure which largely accounts for the class differences in educational attainment. Students from upper class or dominant background begin with an advantage having been socialised into the dominant culture right from the beginning. Learners coming from such dominant background have access to the dominant cultural capital and embedded skills and knowledge which help them build on their later years of education. Compared to this the underprivileged position of the learners from marginalised background is "legitimated by their educational failure" (p. 259). Historically, the dominant groups have acquired cultural capital passed on through generations which is legitimated through social structures such as educational institutions constituted in a way to function as a gatekeeper for those who do not possess this capital. As a result, the advantage of the educational institutional knowledge eludes those who do not possess this cultural capital. Ironically, knowledge of mathematics is seen as an important component of such cultural capital and mathematical knowledge therefore acts as a "gatekeeper" (Skovsmose, 2005) to future avenues and opportunities.

Cultural capital can also be related to the "class-based socialization of culturally relevant skills, abilities, tastes, preferences, or norms that act as a form of currency in the social realm" (Winkle-Wagner, 2010, p.5). Winkle-Wagner claims that "cultural

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capital can be grasped as those culturally based resources that can act as a form of capital" such as, knowledge about educational institutions (schools), educational credentials, and so on. Bourdieu's construct of cultural capital demonstrates that one's culture can act as a "power resource" in social settings where one can exchange cultural knowledge which is inherently structured to disallow a few others (Bordieu & Passeron, 1964). Using this construct, we argue that learners coming from the working class or marginalised background are more likely to fail the examination due to lack of possession of dominant cultural capital (sound education capital). This phenomenon illuminates the concern that platform 3 presents where opportunities generated through cooperation between inter and intra peer groups, learners from disadvantaged and marginalised backgrounds aim to cross the barriers of examinations and dream for achieving what was perceived as out of reach.

Embodied cultural capital possessed by the children in a community is often in the form of cultural habits which are transmitted from family elders to children. For example, developing a familiarity with schools and institutions is part of enculturating cultural habits that develops a sense of self-study among the children. In a similar way, presence of cultural capital or absence of it among parents guide them where to send their children for education, if at all. Absence of such capital at home but availability in the community around help young learners find their own ways to accumulate such capital themselves. Platform 3 as a non-typical learning space reflects this trend among the learners. Nonetheless, this trend raises a question: is platform 3 helping to maintain the status quo of the capitalist ways of knowledge production by pursuing the cultural capital or fighting against it by sharing the cultural capital among those who do not otherwise have the means for acquiring them? We look at reflecting on this question – and further – from the story we gathered at platform 3.

THE SETTLEMENT AND ITS PEOPLE

We visited the low-income settlement located in the neighbourhood of platform 3 from where the groups of boys came. This settlement like most others in Mumbai is densely populated with small tenements clustered together on both sides of narrow and dark lanes. The tenements are low in height (around 8 feet), mostly single-roomed and single-storeyed. The rooms sizes vary between 5 to 8 feet in width and 8 to 12 feet in length. Almost every tenement has electric connection and access to television. It was around dusk when we visited the settlement and observed that women in the households were busy in the routine chores, namely, cleaning of floors, washing utensils, preparing to cook and so on, showing an explicit sexual division of labour. This gave us a clue about the fact that there were no girls studying at platform 3. Some young children (aged around 7-8 years) were running down the lanes. The settlement is located near the suburban railway tracks and there is a road highway further north while a concrete boundary wall on the south that separated it from the naval land. There is an open area on the west with a concrete floor and a concrete dais structure where political meetings are held, or community festivals are

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celebrated. The open area is where community elders were seen talking leisurely with acquaintances and relatives and young boys – but no girls again – were seen mostly running around. There were a few vegetable vendors and a few cookie sellers who had spread their ware on the ground for selling. It was evening time and the place was bustling with activities with occasional loud music coming from nearby stationary stores.

While returning, we met two older men from the settlement sitting in an open area and talking to each other. We approached them and discussed the objective of our visit. The two gentlemen enthusiastically explained that it was disturbance and lack of space at home that drive young boys to go to platform 3 for studying. They added that the platform had a shaded staircase with bright tube lights and since boys of different age groups visit it, they help each other academically. This viewpoint was reflective of the community's approval to alternative learning sites such as platform 3 and to the learning practice involved therein. We argue in the analysis how this practice is linked with the transmission of cultural capital possessed by the learners.

We visited platform 3 for piloting an initial interaction with the groups of boys studying there. These interactions helped us build an understanding about the group, their social location and socio-economic conditions of their families, their foregrounds and other driving forces that brought them to platform 3. After initial analysis of the pilot observation and based on the pilot interview, we visited the place again a few months later and had a focus group discussion with one such group. We used a few guiding questions and encouraged the boys to respond freely. We visited one group at a time, introduced ourselves besides making the purpose of our visit clear and discussed the study. The focus group discussion was audio recorded after

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taking verbal consent from the respondents. All the five boys were in the age group of 16-18 years and they all spoke in Hindi – their first language. They all belonged to immigrant families from north India and lived in the nearby low-income settlements. None of them was a native settler and their first (home) language was different from the local language Marathi.

Most of these learners regularly came to platform 3 were pursuing either a higher secondary course (Grades 11 and 12) in science or commerce, or a bachelor's degree in commerce. Some learners had science and engineering background. Most of these learners discussed that they often encountered difficulty in mathematics and accounting. Inter and intra peer group exchanges and discussions helped them elucidate their problems and to learn about how to find solutions. We noted that it was with an aspiration of doing better in mathematics that some students came to platform 3. A few students also expressed their aspiration to become a successful engineer while a few others aspired to become chartered accountants, and set-up business firms and become more successful than Ambani (A renowned multi-billionaire businessman in India who also lives in Mumbai). To fulfil these aspiration of students, they believed mathematics play an important role.

We adopted the thematic notes that we produced from the interview logs for analysing our story. These themes focused on foreground and access. The thematic notes were individually reviewed by the three of us separately, compared and the differences were reconciled.

HOW DID THE STORY UNFOLD?

As we strolled through the platform 3, we met teenage boys studying and discussing in groups. In one group, there were five boys preparing for the on-going Grade 12 examinations. We approached them first and they spoke about their current studies as gateway to the future welfare. We observed that there were different heterogeneous groups who helped their juniors in shaping of their learning/aspirations. Aspirations and foregrounds (Skovsmose, 2012) of most boys had a direct relation with their father's profession and none of them mentioned their mothers, sisters or any other woman as examples to follow – rather only males in their families (see Table 1).

Respondent	Father's occupation	Mother's occupation
Bishu	Contractor	
Suman	Truck driver	
Param	Deceased, Brother: Engineer	
Kamal	Rickshaw driver	
Deepak	Tailoring (stitching) work	

Table 1: Profile of the respondents

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The following sections discuss the socio-critical ramifications emerging from such alternative approaches towards achieving one's/group's foregrounds and outline the possibilities that non-typical learning sites like platform 3 offer.

Building cultural capital: Peer group learning

Intra and inter group sharing and discussing happened within and between group members. Students often help each other in their own groups, for example, one of the day we visited them, Bishu and Suman had come together to platform 3 at 5 o' clock in the morning, when the sky was still dark, to clear doubts about their English exam scheduled later in the day. Inter-group learning happened when younger boys sought help from other groups of engineering or chartered accountancy students or from those who were preparing for such examinations and come to platform 3 for their studies. Elder boys readily helped younger ones by addressing their queries. Corroborating this, Suman emphasised that they come to platform 3 to study because they know they would get help from the seniors. This allows the community of students in platform 3 to build a cultural capital through such peer learning much in the similar way as it happens in other communities elsewhere. Access to such knowledge capital draws more students in the group. We refer to this arrangement as a physical learning network. We argue here using Bourdieu's notion of cultural capital (Bordieu & Passeron, 1964) that the physical learning network that platform 3 provides makes it possible for the learner to aspire to acquire the elusive “educational capital”. Their foreground encourages the learners to make that giant leap and cross the barrier to gather that “legitimate” capital which is often otherwise seen as beyond reach.

Better conducive environment and a “so what?” questioning

We started this paper with a scenario, the platform 3 and the boys studying there, and some questions about it, e.g.: “why they study at this place?” “Even at odd hours?” “What are the reasons that draw these students here?” “Do such efforts by a section of students and citizenry have any implication for the policy planners, city planners and educators?” “Why there are no girls studying there?”. During the study, we realized how complex the answers could be and how big and embracing these questions and this work are. It was possible to understand a little about the reasons they are studying there, but we did not get any data about the reasons why we could not find girls studying there or implications for policy and city planners and educators, which can become an aim for our future work.

The complexity of the research showed up strongly during a virtual meeting with the authors, in India and in Brazil. One of us raised a question about the economic conditions having emerged as a major factor, determining living conditions and access to education. The question was “If the boys had proper structure in their homes, would they still come to the platform 3 to study anyway?" Based on students' utterances like, “yahan acchi padhai ho jati hai” [good study happens here] and “jaldi

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yaad ho jata hai” [memorising becomes easier], we tended to believe they would, that the platform 3 was playing a strong role on their learning process. Still, the same student, during the same conversation, says “that their home environment was not conducive for studies”. He explained that he cannot “concentrate” on studies “while at home”. The small tenements in the settlement are often shared by many family members and relatives leaving less space or environment for them to focus on studies. There was loud music or neighbours talking to each other which discouraged these boys from studying at home. If some tenements also undertook income generating activities, then that left no scope for concentrating on studies till late in the evening.

One can argue that the students are happy to be at platform 3 for studying and meeting friends. Students mentioned meeting friends and opportunities for playing games like cricket and other recreational activities as reasons behind congregating at platform 3. Coming to the platform offers them an open space and an opportunity to take time off from the regular chores. On the other hand, one could also say it is romanticizing the situation, as the real issues are, for example, the lack of appropriate housing for all in a big city like Mumbai, the lack of appropriate open spaces for them to engage in sports activity, and the lack of economic conditions to avoid teenage and child labour were other motivating agents that brought them to such non-typical learning sites.

OPPORTUNITIES FOR CHANGE

Platform 3 provides opportunities for those whose performance in formal education remained poor or those who could not afford the capitalist market of private tuitions thereby missing out on future scope (see, Bourdieu, 1973). Platform 3 allows construction of a non-institutional arrangement where discursive practices take place to not just help each other but also to comprehend, understand and move ahead. Platform 3 in a similar way provides an opportunity to students from low socio-economic background to build and transmit a kind of cultural capital through such peer learning. Therefore, platform 3 is a platform for extending social justice by creating a scope for distribution of opportunities and privileges within the society.

They are the boys who have seen the world of contradiction – on one hand, huge economic disparities of Mumbai while on the other hand their own location as working-class immigrants living in a low-income settlement surrounded by posh localities around, where privileges are not distributed equally or not at all. But these boys are inspired by the other world of Mumbai which is much more fancier and want to live in a better way. Education is one of the important tools for turning their aspiration into a reality. In India, right to education has been enacted as a fundamental right but only for children between 6 and 14 years of age. However, even that doesn't ensure access to quality education and the cascading effect moves up through higher grades. There are limitations of access to resources (school, library) and the cultural capital. Through platform 3 these boys explore their access out of arrangements in broader social arena despite their adverse social location and

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lack of access to their aspirations. They choose a public place/space for building the foundation of their future keeping in mind their present based on equity. They share, learn from each other and cooperate each other.

RAISING ANXIETY: RESISTANCE AND DEFICIENCIALISM

Platform 3 also offers resistance to the elusive cultural capital which learners try to redefine in their own ways. They gather and learn from inter and intra peer groups by involving no financial implications which otherwise is a necessary component for acquiring the dominant cultural capital. An impact study on the learners of platform 3 in future could corroborate such claim in this direction. It would be interesting to understand how learners' foreground shaped their future career placements and whether their background impeded such career movements or not.

Nevertheless, an urgent issue has emerged from our study and it has become a future aim of research for us: what about the gender stereotype observed in this study? In other words, why we do not find girls studying at platform 3? And why the boys we interviewed have never mentioned women from their families as a model to pursue? According to Marcone (forthcoming) one could see this matter as a silence produced by an exclusionist environment and our research is a step towards creating awareness and bringing anxiety over this issue. Those are aspects of a concept that Marcone calls as Deficiencialism, in which false disabilities are produced, for example, women are seen as not being able to perform male tasks, or a blind person not being able to learn mathematics, and so on. In this sense, platform 3 mirrors a very complex social structure. Marcone & Skovsmose (2014) would say that it fights exclusion but also produces exclusion at the same time.

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CRITICAL ANALYSIS OF WHITENESS IN A MATHEMATICS EDUCATION PROSEMINAR SYLLABUS

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In this paper, I contribute to examination of racial hegemony and the reproduction of that hegemony within a large Doctoral program in mathematics education in the United States. Drawing on the literature and methods of Critical Race Theory and Critical Whiteness Studies, I examine the syllabus of a required proseminar course in mathematics education for evidence of the ways that Whiteness is present in and reproduced by the syllabus. Findings relate to (1) the presumption of hegemonic Mathematics, (2) the normalization of Cognitive Psychology over other research paradigms, (3) ways in which the absence or presence of certain words fails to critically wrestle with White Supremacy, and (4) the dominance of White voices in the required readings. Possible implications for ongoing program design are shared.

My purpose in this writing is to examine racial hegemony (i.e. White Supremacy) and reproduction of that hegemony within the syllabus of a proseminar course required as a part of a mathematics education doctoral program in the United States. I do this in the pursuit of three larger goals: (1) to enhance my own understanding of such hegemony as it might exist in mathematics education doctoral programs more broadly (Richardson, 2000), thus informing my potential future work in the design and revision of such programs; (2) to contribute to the reflective awareness of others who are responsible for the design and revision of such programs; and (3) to invite conversation around reflective analysis of such hegemony as it exists in the enculturating machinery of the field of mathematics education. In order to perform this analysis, I make use of the tools and methods of Critical Race Theory (Delgado & Stefancic, 2001) and Critical Whiteness Studies (Leonardo, 2009). I use the term Whiteness to refer to both the material and symbolic assemblages that systematically and systemically privilege members of the White social group over others (Lewis, 2004).

I have chosen to focus on careful analysis of a single syllabus from a single mathematics education doctoral program in order to allow for careful attention to fine detail. Selection of a syllabus from a program in the United States was intentional, given the role of the United States in broader global hegemony as well as my own positionality as a graduate student in the United States who may one day be responsible for the design and revision of such programs. Although selection of a single syllabus from a single program entails limitations to generalizability, there are several reasons to consider it probable that observations regarding racial hegemony in this one syllabus will be emblematic of the presence of racial hegemony in mathematics Ph.D. programs more broadly: (1) White supremacy is a global phenomenon (Leonardo, 2009); (2) the selected syllabus is from a course that is required of all mathematics education graduate students enrolled in one of the largest and most highly recommended mathematics

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education programs in the United States (Reys & Dossey, 2008), so the program might be conceived of as directly responsible for the enculturation and development of a nontrivial subset of future contributors to the field (consider Picower, 2009) and as a program from which others might conceivably draw inspiration; and (3) in looking over the program requirements and the syllabi for coursework that fulfils the same role (i.e. introduction to the broad range of domains of research in mathematics education) at other large programs in our field in the United States, I have found that many require a substantively similar course for graduation.

POSITIONALITY, STUDY CONTEXT, AND INTERACTION

Given the entangled nature of positionality and the spaces we occupy and examine, it is necessary to treat these features of my analysis in concert. With that in mind, I will first describe my own positionality, then expand on the layered nature of the syllabus under investigation as well as some possible interactions between my positionality and this layered space.

Positionality is not simply a state of being; it is a dynamic, interactive quality that actively affects (and perhaps effects) the way you interact with the world and the way the world interacts with you (e.g. Kendi, 2016; Peshkin, 1982). Thus, although it is necessary in stating ones positionality to describe your nominal state of being, it is even more important to try to anticipate, notice, dissect, and describe how this state of being interacts with and constructs your lived world. With that in mind, while I will name my positionality, I will try to avoid the confessional approach historically adopted by many researchers, instead focusing on its dynamic co-construction with the space I am critically analysing.

Given the intricate role intersectionality plays in identity and positionality (Crenshaw, 1991; Gholson, 2016), I will open by attempting to state my positionality in as much of its layered complexity as I am able. I am White, male, cis-gender, heterosexual husband and father from rural United States. I was raised by upper-middle class parents in primarily working-class White locations. I am an urban teacher and a teacher-educator, and I have received substantial mathematical enculturation and higher education enculturation. I am also a mathematics education graduate student who intends to pursue a career as a mathematics education researcher. It is worth noting that many of the categories by which I have identified myself are blurred or variable in nature and are not static within time or space. For example, although I identify as and am positioned as White, I am aware that positioning in this social group varies by location (e.g. Daniel, 1949) and time; members of my family several generations older than myself would not have been received as White given their German, Scottish, and Irish origins (Painter, 2010; Roedigger, 1991/2007, 2006).

Given that I am performing a critical analysis of a syllabus from a required proseminar course in mathematics education offered by a mathematics education doctoral program, I will now attempt to identify some of the possible interactions between my positionality and that layered space. I will begin by considering the larger

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spaces of university and graduate program in which the syllabus is embedded and inextricably connected. For convenience, I will hereafter refer to the university as Big University (BU) and the graduate program as the Mathematics Education Program (MEP). As a White male, I would occupy a position of privilege both at BU and within the MEP, in both the broader hegemonic sense (e.g. countrywide racial and gender narrative) and the more local sense that White men comprise most of the student body and positions of power within the program. BU has more than 30,000 [1] enrolled students, of which 80% are undergraduates. For both the undergraduate student population and the total student population, roughly 80% of the students are White, 15% are international, 10% are Black, 5% are Hispanic, 5% are Asian, 3% are Multiracial, with the remaining student population including Native Americans, Native Alaskans, Pacific Islanders, and those who did not report a race. The gender breakdown of BU is close to 50-50, with slightly more women than men (Big University, 2016). It is worth noting that this data seems to have been sourced from surveys which oversimplify racial and gender constructs. The MEP itself has more than 15 graduate students, including roughly 16% Women of Color, 16% Men of Color, 28% White Women, and 44% White Men (Mathematic Education Program, 2017b). The graduate program also has more than 15 faculty, including no Women of Color, 20% Men of Color, 55% White Women, and 35% White Men (Mathematics Education Program, 2017a). Although these numbers show more White Women than White Men, positions of power such as program director are occupied by White Men. Thus, my positionality as a White Male would place me in a position of privilege in this space. Consequently, I must take extra care and apply a critical lens to try to perceive the ways some members of this program might be marginalized in ways I would not be.

The proseminar course itself is part of a two-semester course sequence intended for first or second years students in the MEP, as well as students from other programs who have interests in mathematics education. There is no requisite order to the two courses. As described in the syllabus, the purposes of this two-semester course sequence are to: (1) introduce central themes, concepts, and paradigms of research in mathematics education; (2) develop the scholarly literacy of students; (3) support the development of the individual research interests of students; (4) create a community of learners and scholars amongst the students; and (5) facilitate student participation in the broader community of mathematics education researchers. The sequence is designed to cover eight broad domains of mathematics education research: (1) learning, (2) teaching, (3) assessment, (4) policy, (5) curriculum, (6) discourse, (7) equity, and (8) teacher education. The syllabus I am analyzing is for the course that focuses on the former four domains.

I am presently a second-year doctoral student at a similarly large program in mathematics education in the United States. I have taken both of the mathematics education proseminar courses required in my own program, whose syllabi are substantively similar to the one under investigation. Consequently, I am likely to be viewing this syllabus from a perspective between that of a faculty member and a

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student, but closer to that of a student; though I am being enculturated into the mindset of a faculty member as a part of my membership my university's mathematics education program, I remain a student in the early years of doctoral work.

CRITICAL ANALYSIS OF A PROSEMINAR IN MATHEMATICS EDUCATION SYLLABUS

This critical analysis will be framed through two tenets of Critical Race Theory: (1) challenges to claims of color blindness and neutrality, and (2) experiential knowledge (e.g. Delgado & Stefancic, 2001; Sleeter, 2016). The tenet of color blindness (Bonilla-Silva, 2006, 2015) prompts us to interrogate how structures, broadly defined, that seem neutral actually reinforce Whiteness and White interests. The tenet of experiential knowledge calls upon us to ask whose voices are being heard. Both of these tenets help draw attention to the ways White Supremacy and racism continue to operate in an ostensibly "post-racial" society (Bonilla-Silva, 2015), often through localized discourses and events that act as pieces of larger social narratives which construct and reify White Supremacy (Martin, 2009).

Challenges to claims of color blindness and neutrality

The tenet of color blindness prompts us to interrogate how structures, individuals, objects, or practices that seem neutral actually reinforce Whiteness and White interests. In my critical analysis of a syllabus for a proseminar in mathematics education course, I have identified several aspects of the syllabus which deserve unpacking in light of this tenet: (1) Mathematics and Mathematical epistemology as value-neutral, (2) the domains of research as racially neutral, (3) cognitive psychology as racially neutral, and (4) the use or absence of certain words in the syllabus.

Capital-M Mathematics, the culturally privileged mathematics practiced by Mathematicians in the United States and taught in schools across the country and beyond, is commonly perceived to be objective and value-neutral (Ernest, 1991; Martin, 2009). However, substantial evidence exists to the contrary (e.g. Almeida & Joseph, 2004; Burton, 1999; D'Ambrosio, 1985; Ernest, 1991; Fendler, 2014; Joseph, 1987; Lakatos, 1976; Martin, 2009; Restivo, 2017; Sinclair, 2009; Wells, 1990). With this in mind, it is notable that none of the domains of research demand challenge to this assumption, and few of the required readings explicitly challenge it, with no required readings foregrounding a challenge to it. The eight domains of research covered across the two proseminars are: (1) learning, (2) teaching, (3) assessment, (4) policy, (5) curriculum, (6) discourse, (7) equity, and (8) teacher education, none of which demand a challenge to hegemonic perception of Mathematics as value-neutral. Even the domain of equity, which draws attention to social constructs such as race and gender, only demands that these differences be recognized in people and not that they be recognized in Mathematics itself. Consider, for example, the National Council of Teachers of Mathematics' position on access and equity in mathematics education, which states, "Addressing equity and access includes both ensuring that all students attain mathematics proficiency and increasing the numbers of students from all racial, ethnic,

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linguistic, gender, and socioeconomic groups who attain the highest levels of mathematics achievement.” (2018) This position statement addresses how certain nominal categorizations of people correlate with Mathematics Achievement, but does not interrogate Mathematics itself as racialized (or gendered, etc.). For contrast, consider a hypothetical alternate version of this syllabus which treats the broad array of epistemologies of mathematics as a domain of research and requires anthropological, philosophical, and historical analyses of mathematics as social practice rather than objective truth, thereby demanding that Mathematics be recognized as *a mathematics* rather than *The Mathematics*.

Cognitive psychology is the dominant paradigm of mathematics education research expressed in the syllabus. In the syllabus, 7 weeks of required readings are devoted entirely to work performed in the cognitive psychology paradigm (e.g. Schoenfeld, Smith, & Arcavi, 1993; this paper, whose primary title is simply “Learning,” provides a characterization of a student’s mathematical “knowledge structures” over the course of her interactions with a learning environment), 1 week to the situative paradigm (e.g. Putnam & Borko, 2000), 3 weeks to sociocultural paradigms (e.g. Nasir, 2002), and 2 weeks to a mixture of paradigms. One of the two weeks with mixed readings provides a roughly even mix of cognitive psychology and sociocultural perspective, while the other is cognitive psychology dominant by a ratio of 3:1 with the only sociocultural piece presented as being antagonistic. This focus on cognitive psychology is notable because cognitive psychology is, at its core, a paradigm that understates or overlooks race (Martin, 2009). Cognitive psychology perceives cognition as something exists entirely within the confines of the minds of individuals, relegating the dynamic social construction of race to mere context rather than an element of social cognition. As a result, cognitive psychology does not demand that one consider race or Whiteness, except as names by which to disaggregate data or variables to be controlled for. Sociocultural and situative perspectives, in contrast, perceive race as an element of cognition, worthy of investigation in of itself rather than worthy of consideration as a contextual variable.

The use or absence of certain words in the syllabus demonstrates that the syllabus fails to critically wrestle with concepts of race and whiteness. Race(ism) only appears twice in the entire syllabus, once in the course calendar and once in the reference list, both times as a portion of a single article title: “Learning discourses of race and mathematics in classroom interaction: A poststructural perspective” (Shah & Leonardo, 2016). Ethnic(ity) and gender similarly each appear only twice in the syllabus, once in the course calendar and once in the reference list, both as a portion of the title of a single article: “SES, ethnic, and gender differences in young children’s informal addition and subtraction: A clinical interview investigation” (Ginsburg & Pappas, 2004). Words like white(ness), sexuality, and belief do not appear at all. In contrast, the word “research” appears 46 times, including 18 times outside of the reference list. Similarly, the words “scholar(ly)” and “academic” appear 33 and 14 times respectively, in each case 12 times outside of the reference list. For example, the

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very first sentence of the syllabus reads, "The proseminar in mathematics education course sequence is designed to introduce you to the *scholarly* discipline of mathematics education and help you begin the process of becoming a *researcher* and *scholar* of mathematics education" (italicized for emphasis). Given the preponderance of White faculty in the graduate program in which this syllabus is nested and in postsecondary institutions more broadly (NCES, 2017), each of these three words are not colorblind; they connote Whiteness. As a final point of interest, I note a line from the course participation expectations in the syllabus: "During our class discussions, **it is very important that you make your voice heard**." (emphasis present in original syllabus) This behavior is more normalized as socially acceptable for those of certain social groups (e.g. white, cisgender male) than others (Langer-Osuna, 2011), but this tension is not openly acknowledged in the syllabus, thus masking Whiteness and racialization through color blind language.

As can be seen in even these few examples, this syllabus serves to enculturate new mathematics education Ph.D. students into a White (male) view of mathematics through local contribution to broader color-blind and post-racial social narratives (Martin, 2009). This syllabus does not challenge the presumed color-blindness of Mathematics or the de-emphasis of race that cognitive psychology entails. By avoiding the naming of race, and the naming or recognizing various hegemonies more generally, this syllabus normalizes and thus perpetuates them (Bonilla-Silva, 2006, 2015).

Experiential knowledge

The tenet of experiential knowledge calls upon us to ask whose voices are being heard. In the case of this syllabus, those voices are overwhelmingly White. Constructs of racial identity and positioning are socially constructed (Gould, 1981/1996) and can vary from place to place. Since extant literature from the authors in the syllabus does not provide information about how all of them self-identify, and given the fact that the syllabus I am analysing is drawn from a program in the United States, I chose to proceed in a manner that would reflect how these authors might be positioned by those who have spent the majority of their lives living in the United States. I gathered pictures of the faces of all authors, save one (George Stannic) whose picture I could not find. In cases where an author was required more than one week, I included their face the corresponding number of times. Though these pictures reveal nothing about how the authors identify or how they are positioned by others in their professional or everyday lives, they can be used as a rough tool to infer their racial positioning in the context of present day United States, thus giving some sense of whose voices we are hearing. In reviewing these pictures with a colleague, we counted 30 faces that we positioned as White Men, 25 that we positioned as White Women, 5 that we positioned as Men of Color, 6 that we positioned as Women of Color, and one who we positioned as White and know, from conversations with faculty, to identify as non-cisgender. Even when taking into account the limitations and weaknesses of positioning authors by pictures of their face, it is notable that more than 80% of the faces we observed we positioned as White. It is further notable that seven weeks of required readings contained only

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White faces, and only the week nominally dedicated to sociocultural and critical perspectives contained more voices of Color than White voices. It seems that, according to this syllabus, if you are a scholar of Color in the United States then the only academic space in mathematics education where your presence is normalized is in sociocultural and critical work; all other spaces are for White scholars.

It is also worth noting that, even for those articles that are authored by People of Color, their voices are filtered through methodological and rhetorical norms of the field of educational research. The methodocentrism of educational research virtually demands that all work adopt quantitative or qualitative methodologies (Weaver & Snaza, 2017), leaving out a broad array of approaches to making sense of the complexity of race and Whiteness. The rhetoricocentrism [2] of journals, the careful policing of language within our field which involves the avoidance of aesthetically grounded language and metaphor (APA, 2009; Coe, 1994), further limits voices of Color and opposes efforts to convey the violence of racism (and other hegemonies). The mere naming of Whiteness, or the use of methodologies or language that convey the cruelty of Racism and other hegemonies, can be enough to see a paper immediately rejected from potential publication.

CONCLUSION

A United States mathematics education proseminar syllabus that directly confronts Whiteness in the discipline would be likely to meet with heavy resistance (DiAngelo, 2011). However, I believe that directly confronting Whiteness within the syllabus would be an act of interest convergence that would strengthen the discipline as a whole (Kendi, 2016). The incorporation of more varied authorial voices and the normalization of research paradigms and epistemological stances that treat race as more than mere context would simultaneously invite more varied student voices into the broad array of spaces in our discipline and give those students the types of conceptual tools necessary to push our discipline towards contributing substantively to changes that empower students and teachers in our program and beyond. By demanding challenges to Whiteness within the very structure of the syllabus and the program it supports, “whiteness can be constructively confronted” (Sleeter, 2016, p. 166).

NOTES

1. Specific numeric information regarding the population of the University and graduate program has been modified to help anonymize the University and graduate program, but care has been taken to not alter values in ways that might alter interpretation, e.g. the relative size and ratio of population subsets are preserved.

2. What we learn when we learn a genre, such as the genre of academic writing, “…is not just a pattern of forms or even a method of achieving our own ends. We learn, more importantly, what ends we may have…” (Miller, 1984, p. 165). Although I adopt the word “rhetoricocentrism” to describe this phenomena in order to draw intentional parallels to conversations around methodocentrism and ethnocentrism, genre theorists rather evocatively name it *the tyranny of genre* (Coe, 1994).

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RHETORICAL EXPERIMENT AND EMPATHY: MATHEMATICS AND MATHEMATICAL PRACTICE AMONGST THE NACIREMA

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In this paper, I experiment with repurposing anthropological satire (Miner, 1956) as a rhetorical method of conveying the idea of "gaze," in an emotionally honest way, in the context of ethnomathematics (Pais, 2011; Pais, 2013). In particular, I present an ethnomathematical analysis of the mathematics and mathematical practice of the Nacirema, a North American group living in the territory between the Canadian Cree, the Yaqui and Tarahumare of Mexico, and the Carib and Arawak of the Antilles. Findings relate to the ritualized nature of Nacirema mathematics, and to the foundational belief underlying Nacirema mathematics that the dynamic, embodied, and contextualized mind is intellectually frail. Goals underlying this analysis center on evocation of empathy and actively pushing back against restrictive rhetorical norms.

INTRODUCTION

Consider for a moment someone who has little to no personal experience of marginalization (i.e. White, cis-male, heterosexual, etc.). How might we, in the context of sanctioned mathematics education research, convey to such a person what is meant by *gaze*, in the sense *Male gaze, White gaze*, or of anthropological *gazing upon the other* (e.g. Foucault, 1973; Pais, 2013)?

This question has been the driving force underlying this paper, though I myself did not become fully conscious of this until recently; I initially framed this paper as expressing a critique of ethnomathematics in the non-normalized form of anthropological satire (Miner, 1956), but in the months since then I have become aware that the real foundation of the paper was not rhetorical experiment in critique, but instead rhetorical experiment in empathy.

Empathy, our capacity to try to experience life through the paradigms and frames of another, is tightly tied to equity research and activism. It is often (always?) empathy that draws us to pursue the cause of equity within and outside of research (Foote & Bartell, 2011), and I would argue that part of our goal as researchers is to experience empathy and then to convey that empathy to readers of our published work (to be clear, I mean *empathy* and not *sympathy*; the latter is othering). We want to convey as genuinely as possible a human experience, the lived reality of a culture or case or phenomenon constructed and experienced by humans; this is, in a sense, part of what thick description aims to do (Geertz, 1973). However, this is profoundly nontrivial for at least two reasons: (1) Normative language in many spheres of academia actively recommends against empathic language (APA, 2009; Rosenberg, 1960), and (2) efforts to convey empathy are necessarily filtered through both the author and the reader (Geertz, 1988).

The latter half of this paper is devoted to a work of ethnomathematical satire, an experiment in the use of satire as a rhetorical tool for evoking empathy in the field of mathematics education. This work of satire attempts to *gaze* upon hegemonic mathematics

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and mathematics education, creating a sense of alienation in a hypothetical reader who might have few conscious experiences of marginalization and perceives as normal the matter-of-factness of mainstream Western (Joseph, 1987) mathematics teaching. The reader is meant to feel uncomfortable, not for the sake of feeling uncomfortable in of itself, but instead for the sake of giving the reader a foothold by which to more sincerely empathize with marginalized communities subject to such *gazing*.

POSITIONALITY AND MOTIVATION

As I mentioned above, efforts to convey empathy are necessarily filtered through both an author and a reader, so I would be remiss not to unpack in more detail my positionality as an author and how that dynamically co-constructs a relationship with the specific hypothetical reader I envision. Although I do have some personal experiences with marginalization (e.g. I have a sensory processing disorder and I have predominantly existed as a non-Christian in spaces where that was considered wholly unacceptable), I nonetheless remain a White, heterosexual, cis-Male born and raised in the United States, and consequently my first experiences encountering the notion of *gaze* in research were experienced with some emotional distance. I understood functionally what *gaze* was, but it did not immediately resonate with me emotionally; I had no empathic foothold (at least none that I was conscious of at the time) by which to understand *gaze* in an emotionally honest way, and the writers whose work I was reading were restricted in their ability to convey the emotional reality of *gaze* by the rhetorical norms of their field (APA, 2009; Coe, 1994; Rosenberg, 1960). Here, the words of Rosenberg (1960) seem relevant:

> What takes people in about the scientists is the grand display they make of the machinery by which their petty findings are dredged up—explanations in technical language of how the data was collected and how much of it, how different factors were weighted, what safeguards were built against distorted interpretation, etc. Actually, instead of a portrait of society, one gets a view of the sociologist's workshop with some charts on the wall. (Rosenberg, 1960)

It was this emotional disconnect with the meaning of *gaze* that led me to write the latter half of this paper. I was, ultimately, trying to make my own sense of what it might feel like to be *gazed upon* (Richardson, 2000), but I was also trying to do so in a way: (1) that would be accessible to others of similarly privileged positionality, and (2) that would actively operate against the rhetoricocentrism I perceived in our field (e.g. Leonardo, 2016).

Rosenberg (1960), in the quote above, is critiquing the ways in which the rhetorical approach of researchers, the very way they write to convey their findings, can act as a hindrance in spite of the underlying value of that research. Just as methodologies can be productively viewed as tools in a toolbox, rhetorical approach might be productively viewed in a similar way, with different rhetorical styles better suited to highlighting different observations. Just as methodocentrism can problematically restrict the sense-making machinery of researchers (Weaver & Snaza, 2017), rhetoricocentrism (Coe, 1994) can problematically restrict modes by which the

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findings and wonderings of researchers might be expressed. With all of this in mind, I hope you will join me for this rhetorical experiment.

MATHEMATICS AND MATHEMATICAL PRACTICE: CONCEPT, PEDAGOGY, AND CURRICULUM AMONGST THE NACIREMA [1]

Across the cultures of this great world there exist mathematics of astounding variety (D'Ambrosio, 1980; D'Ambrosio, 1985). Performed, conveyed, and taught in radically different ways, one might speculate, as Miner (1956), that if all the logically possible combinations of mathematical "behavior have not been found somewhere in the world… they must be present in some yet undescribed tribe." (p. 503) Indeed, it seems that the similarities across the array of mathematics might be fewer in number than the differences (Bishop, 1988). With this in mind, the strange mathematical epistemologies and practices of the Nacirema present a fascinating example of the extremes to which human mathematical behavior can go.

In his seminal ethnographic study of Nacirema body ritual, Miner (1956) introduces their people as follows:

> They are a North American group living in the territory between the Canadian Cree, the Yaqui and Tarahumare of Mexico, and the Carib and Arawak of the Antilles. Little is known of their origin, although tradition states that they came from the east. According to Nacirema mythology, their nation was originated by a culture hero, Notgnihsaw, who is otherwise known for two great feats of strength–the throwing of a piece of wampum across the river Pa-To-Mac and the chopping down of a cherry tree in which the Spirit of Truth resided. (p. 503) [2]

The unifying claim underlying Miner's (1956) work is that Nacirema body ritual is built on the belief that the human body is ugly and weak, and that their only hope to avert these characteristics is through the powerful and religious influence of ritual and ceremony. As I will expand on here, Nacirema mathematics is similarly ritualized, and common Nacirema mathematical practice seems to be built on the corollary that the dynamic, embodied, and contextualized mind is intellectually frail and foolish.

For children of the Nacirema tribe, the majority of their formally acknowledged mathematical enculturation occurs outside of the spaces in which mathematics is applied [3]. On roughly half the days of a given year, children travel en masse to specially designated shrines to knowledge where they participate in the ritual of Loohcs. [4] Children are funnelled into rooms illuminated with fluorescent glow, walls physically and metaphorically closing them off from the outside world. According to established norms of these shrines, the children make their way into rooms in groups of 20-25 (OECD, 2017) and arrange themselves in meticulous rows, positioned so that their eyes all face the front of the room, much as they might be accustomed to doing in one of the many religious shrines that adorn their landscape. Standing at the front of these rooms, a specially designated medicine woman or man leads these children

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through a series of ritualized tasks. Days commonly begin with a chant celebrating a ceremonial piece of cloth in which resides the spirit of the Nacirema people.

The ritual of Loohcs is comprised of several ‘stanzas’, each of which focuses on a particular array of valued cultural knowledge. The stanza on which we will focus is devoted to mathematical practice, though the nature of the mathematics and its practice may look unfamiliar and strange to outsiders.

Typically, in the mathematical stanza of Loohcs, the medicine woman [5] will relate in expository fashion some desired piece of cultural information that children are expected to commit to memory. The medicine woman will then lead the children through some decontextualized computations, where one specific sequence of steps to arrive at the computation is conveyed; this sequence of steps is typically described in terms of how symbols are manipulated (e.g. “carry the one”) rather than how the underlying quantities or concepts are reasoned through (e.g. “when we add 8 ones and 9 ones, we can regroup to get 1 ten with 7 ones”). The stanza then concludes by having children work independently to reproduce these manipulations of symbols on problems that are identical, save for the swapping out of a few numbers for a few other numbers. This sequence of exposition, guided example, and individual recreation of example is further reflected and reinforced in the curricular materials of the Nacirema (e.g. UCSMP, 2007), sacred texts passed through the hands of many children on their voyage to adulthood. This ritualistic and expository approach to learning has been described by researchers using the metaphor of transmission (Resnick & Ford, 1981), and this focus on step-by-step symbolic manipulation has been nominalized with words like “procedural” or “instrumental” (Hiebert & LeFevre, 1986; Skemp, 1976), or relegated the status “proto-mathematics” (Thomas, 1996). More open-ended mathematical exploration is not normalized behavior within the stanza of Loohcs, and medicine women who try to incorporate it can be subject to substantial social backlash and estrangement by children and their parents as well as by other members of their society (e.g. Packard Humanities Institute, 1999).

As the medicine woman, and later the students, work through their mathematical manipulations, the commonly convey their symbolic acrobatics in writing. The medicine women write numbers on the wall at the front of the room, and the children write them upon their parchment. Numeric values, both written and spoken, are typically expressed in the Nacirema base-1010 system [6] rather than in the common base-10 or base-10000 of the technological age. Nacirema associate base-1010 with their fingers, a group of appendages treated with some mysticism and seen as inextricable from the practice of counting (Betts, 2015; NRC, 2001). Curiously, Nacirema measurement systems seem to be built off of base-11, base-100, base-1100 (sometimes also associated with fingers since each hand has 1100 phalanges), and base-10000 structures, though even in these cases written and spoken numerals are translated to base-1010.

As days and weeks wear on, the medicine women strive in their meetings with students to progress from performing arithmetic operations on small numbers to

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performing arithmetic on large numbers (CCSSI, 2018; NRC, 2001). This valuing of large quantity over small elicits a feeling of distance, with small values that connote familiarity being relegated to a position of relatively little interest. This sense of distance is further reflected in the very language of the Nacirema, where differences amongst small numbers have little to no impact on grammatical construction; this is further complemented by other aspects of Nacirema language, such as their relatively primitive and coarse-grained vocabulary for denoting familial relationships [7].

As the medicine women perform and share mathematical exercises with children, the notion of equality takes on a curious meaning. Nacirema curricular texts frequently place the equal sign at the end of decontextualized arithmetic expressions which are meant to be evaluated by the child, and on Nacirema electronic abaci the equal sign results in the evaluation of entered expressions (Li, Ding, Capraro, & Capraro, 2008) [8]. Thus, rather than conveying a sense of "sameness," equality becomes a verb, a command to perform a computation.

Through the mathematical stanza of Loohcs, notion of mathematical relation also begin to take on a curious form. Conceptual relationship amongst shapes, numbers, and more are repeatedly expressed with the aid of tree diagrams in rigid hierarchical form. It seems that Nacirema view the world statically and focus on parts and wholes rather than flux and process, and they view space and location as static qualities, the latter sometimes represented as a tuple associating an object with a specified point in space [9]. This hierarchical structure of conceptual relations is itself static, and seems to focus more on the contents of parts of the hierarchy as opposed to the relationships between these contents [10].

This hierarchical structure is present in Nacirema society and in the very structure of the ritual of Loohcs. In society, Nacirema people are stratified by social constructs such as class, race, gender, sexuality, and religion. In the ritual of Loohcs, medicine women regularly attach specific numbers and letters to the children they teach, systematically sorting them according to culturally valued metrics. As touched on previously, these metrics are commonly decontextualized and procedural in nature. In fact, contextualized metrics are frowned upon in Nacirema society, as the Nacirema people believe that contextualization and humanization introduces bias that is otherwise absent, given the perceived foolishness of the contextualized mind.

I conclude here by reiterating my grounding claims: Nacirema mathematics is ritualized, and common Nacirema mathematical practice is built on the belief that the dynamic, embodied, and contextualized mind is intellectually frail and foolish. The ritualization is visible in the ritual of Loohcs, the culturally formalized structure of the mathematical stanza of Loohcs and its associated curricular resources, the mysticism surrounding fingers, and the formalized stratification of children through the ritual of Loohcs. The rejection of the dynamic, embodied, and contextualized mind as weak and foolish is visible in the literal and metaphorical separation of children from context during the ritual of Loohcs; the use of procedural protomathematical concept, pedagogy, and curricula in the mathematical stanza of Loohcs; the valued abstraction

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and decontextualization of fingers in counting; the static view of space and hierarchy in Nacirema mathematics; and the decontextualized metrics used in the stratification of Nacirema children. Further study of Nacirema mathematical practice and belief could be invaluable to the field, as recording and analyzing their strange ways could help give us insight into our own.

NOTES

1. “Nacirema” is American spelled backwards. Much in this narrative might be construed as “gazing upon,” which is intentional.

2. The geographies mentioned by Miner here are specifying that the place under study lies between Canada and Mexico, and the myths he mentions are pieces of common folk knowledge throughout the United States.

3. Compare the American schooling system to the sorts of learning in practice observed in other cultures (e.g. Carraher, Carraher, & Schliemann, 1985; Saxe, 1988). In American culture, everyday mathematical learning and problem solving is relegated lower status than its decontextualized analogue (Sierpinska, 1995; Thomas, 1996).

4. Loohcs, which is school spelled backwards, is being used to play upon the satire of schooling.

5. Although I am using female pronouns as the “catch-all” pronouns in this paper, in Nacirema culture, male pronouns are commonly used when a speaker is referring to all genders, e.g. saying “Hey guys” when addressing a mixed gender audience

6. To draw attention to the subjective choice of how to express numbers, I chose to present the numbers in this paragraph in a potentially unfamiliar form. Specifically, they are expressed in base-2 (binary), a system where a ten is comprised of two objects, a hundred is comprised of two tens (i.e. four objects), and so on. Various cultures have made use of a startling array of numerical systems, including base-5, base-20, base-27, base-60, and systems that are fundamentally unary (base-1), as well as the aforementioned base-3, base-4, and base-12 systems (e.g. Lancy, 1978; Zaslavsky, 1973).

7. Beyond distinguishing singular and plural, very few grammatical changes accompany the use of small numbers in American culture. Regarding intrafamilial relationships, “uncle” can refer to a mother’s brother or a father’s brother, and “cousin” can refer to an astounding array of relationships. Compare this to the language of the Aborigines, for example, where there is much more linguistic focus on small numbers and finer grained language for familial relationships (Bishop, 1988; Harris, 1980).

8. The tendency to portray the equal sign as a symbol meaning “compute” is less common in the curricular materials of some other countries such as China (Li et al., 2008).

9. Compare this to Navajo mathematical conceptualization, which views the world in terms of processes, events, and fluxes rather than parts and wholes, and views space as dynamic rather than static (Bishop, 1988; Pinxten, van Dooren, & Harvey, 1983).

10. It has been noted that in many societies in Papua New Guinea, classification is not hierarchical; instead, it can be described “pairing” rather than “taxonomizing” (Bishop, 1988).

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TEACHER AGENCY IN PROFESSIONAL LEARNING COMMUNITIES

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This paper investigates how teachers' professional agency influenced their decisions to participate in or withdraw from professional learning communities. Data from interviews with eighteen teachers and principals are examined in relation to five key features of professional learning communities: focus, long-term inquiry, collaboration, leadership support and trust. Agency is conceptualized in terms of stepping up to participate in communities, even in challenging circumstances and in pushing back against the community given the circumstances. Both forms of agency are visible in our data but they may have different consequences.

INTRODUCTION

During the past decade, professional learning communities (PLCs) have drawn the attention of teacher educators. PLCs are groups of teachers who come together to engage in regular, systematic and sustained cycles of inquiry-based learning (Katz, Earl, & Ben Jaafar, 2009; Stoll, Bolam, McMahon, Wallace, & Thomas, 2006). PLCs provide spaces where teachers can reflect together systematically, facilitate collective and sustainable shifts in their practice, and make and justify decisions based on their professional expertise, the realities in their local contexts and the knowledge base.

Teacher development policy in South Africa views the role of PLCs as providing "the setting and necessary support for groups of classroom teachers, school managers and subject advisors to participate collectively in determining their own developmental trajectories, and to set up activities that will drive their development" (Department of Basic Education & Department of Higher Education and Training, 2011, p.23). Teachers are positioned as professionals, with statements such as: teachers should "take control of their own development"; "individual teachers will be able to highlight areas of weakness, and use expertise within the PLCs to help address their difficulties" and "PLCs will assist teachers to integrate their own professional knowledge with the latest research-based knowledge about content and practice" (p.14), suggesting that teachers will come together in PLCs as collective agents, driving their own learning.

While much of the research into PLCs positions teachers as agents, both individually and collectively, the role of agency in relation to teachers' work in PLCs has not been explicitly examined. Drawing from a project that developed and researched PLCs among high school mathematics teachers in South Africa, this paper explores how teachers' professional agency constrained and afforded their participation in PLCs.

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SUSTAINING PLCs

A number of key characteristics for sustaining PLCs have been identified: focus, long-term inquiry, collaboration, leadership support and trust. PLCs need a clear and shared focus, which should challenge members to go beyond what they know, should be broad enough to leverage change and yet not too broad that the collaboration becomes diffuse (Katz et al., 2009). Who chooses the focus and how it is interpreted has implications for the kinds of communities that develop, for teacher agency and for sustainability.

Successful communities engage in long-term, systematic and rigorous inquiry, often based on classroom or school data (Boudett, City, & Murnane, 2008). Ongoing inquiry into classroom-based questions can create possibilities for meaningful learning among teachers, and can work against the often fragmented professional development programmes that teachers are exposed to (Borko, 2004). Resources such as time and space are crucial for long-term engagement in communities.

Collaboration is important not only because individuals learn best through collaborating but because a key aim of PLCs is to produce collectively generated shifts in practice (Stoll et al., 2006). Such shifts are more sustainable for schools, and teachers working together to achieve a common goal are more likely to be successful than individual teachers working alone.

Much research suggests that the quality of leadership support for PLCs is important in achieving robust communities. Leadership support stems from the principal and includes other senior management in the school, such as deputy principals and heads of department. This support includes providing space and time for the communities to function and other resources where necessary. Most importantly, good leadership creates a vision for ongoing professional learning within the school (Katz et al., 2009; Stephan, Akyuz, McManus, & Smith, 2012; Stoll et al., 2006). Leadership is also important within the community and the research suggests that good facilitation of community conversations is key to the sustainability and success of communities (Earl & Timperley, 2009; Katz & Dack, 2013).

Trust is an important part of collaboration and productive learning relationships. Learning together requires teachers to challenge each other's ideas and practices and trust helps to avoid defensiveness and conflict, while supporting challenge and disagreement. A key role for community facilitators is to support the community to develop trust. Research suggests that where there are strong hierarchical relationships within schools and where teacher morale is low, the trust required to sustain PLCs may be difficult to attain (Schechter, 2012; Wong, 2010). Relationships that involve teacher agency in relation to the systems in which they work, and their PLCs, are important if we are to understand how to support teachers to work with and build their communities.

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TEACHER AGENCY

There is a long history of work on agency with a general understanding that while agency is strongly afforded and constrained by structural systems, it can support people to challenge, innovate and resist the status quo. As Buchanan (2015) argues:

> Agency is the socioculturally mediated capacity to act. Individuals are neither free agents nor completely socially determined products. There is choice, but the options available to choose from are shaped by larger force relations (p.704).

Research on teacher professional agency argues that agency is an emergent phenomenon of actor-situation relations, is something that people do, rather than have (Biesta, Priestly, & Robinson, 2015), is enacted in relation to particular work contexts, dominant discourses and power relationships (Buchanan, 2015; Etelapelto, Vahasantanene, Hokka, & Paloniemi, 2013) and “can manifest itself in various ways, not merely as entering into and suggesting new work practices, but also as maintaining existing practices, or struggling against suggested changes” (Etelapelto et al., 2013, p.61). Buchanan identifies two kinds of teacher agency: “stepping up” and “pushing back”, where stepping up involves teachers going beyond the perceived expectations of their roles, taking up additional roles, such as coaching or leadership roles, or trying out new ideas; and pushing back involves rejecting or re-configuring practices and policies with which they do not agree.

In relation to PLCs, teachers’ stepping up might include: developing useful foci of inquiry, finding time and space to engage in inquiry with colleagues, seeking support of school leadership as they try to change practice, taking on leadership roles in communities, and finding ways to develop trust in and with their communities. Pushing back might include: not supporting particular foci of inquiry, creating fragmented inquiry in the PLC, supporting mistrust in relation to the PLC, and withdrawing from the PLC. The analysis in this paper explores how teachers who withdrew from the project in the earlier stages and those who stayed with the project enacted their agency in different ways.

PROJECT AND RESEARCH DESIGN

The focus of inquiry in the project was the reasoning underlying learners’ mathematical errors, which was chosen because all mathematics teachers deal with learner errors, however, very few delve into the thinking behind such errors. Learner errors are often built on partially valid mathematical reasoning and making that reasoning explicit for teachers and learners can help teachers to value learners’ current mathematical thinking and develop stronger mathematical thinking (Smith, DiSessa, & Roschelle, 1993).

A sequence of activities, where teachers analysed learner errors, formed the basis of the PLC work: test analysis; learner interviews; curriculum mapping; choosing leverage concepts; readings and discussion; planning lessons together; teaching the planned lessons; and videotaping and reflecting on the lessons together (see author

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ref for more detail). Although the activities were set up before the project started, areas of choice and flexibility for PLCs were built in, making the project somewhat adaptive (Koellner & Jacobs, 2015). A key area of flexibility was for PLCs to choose the areas of mathematics content to work on, based on their analyses of learner errors in their schools. Communities also adapted activities and sometimes left some out.

During the four years of the project (2011-2014), twelve schools from two districts participated, some joining in 2011 and others in 2012. The districts were selected purposively: they were close to each other and there were schools in each district that were interested in participating. Schools within the districts were selected if three or more mathematics teachers were interested in participating in the communities and if they served mainly learners of low or low to mid socio-economic status. In 2011 and 2012, six schools left the project, and in some schools that stayed in the project, some teachers left, while others joined. Over the four years, 50 teachers participated, 22 for three years or more. An important empirical question became: why did some teachers and schools choose to leave the project and others choose to stay. A key theoretical question is how teachers' choices to stay or leave were produced by and reflected their professional agency.

The study employed a qualitative methodology in order to understand the teachers' perspectives on their decisions to participate in or withdraw from the project. Interviews were conducted in three schools that had withdrawn from the project and three schools still participating in the project, with some teachers in the latter schools having withdrawn. The principal or deputy principal in each school was interviewed, as well as six participating and seven withdrawn teachers.

The interviews were conducted by a graduate intern, who had not met the teachers or principals previously. The interviews were semi-structured in order to allow for some similarity across while also allowing for the interviewer to explore the different participants' perspectives more deeply, and included questions about: the teachers' and principals' expectations of the project; whether their expectations had been met; their experiences of the benefits and challenges of the project; and the support that the teachers had received from the school management and the project facilitators. The interviews lasted between 30 and 45 minutes, were audio-recorded and transcribed.

Initial themes for analysis were developed in relation to the interview questions and an initial summary was developed for each set of data: principals, participating teachers and withdrawn teachers. As the themes were developed, they came to reflect the five characteristics of PLCs discussed earlier in the paper. The author wrote a first draft of the data analysis and then re-read all the interviews to check that all important data had been included, disconfirming data had been reconciled, all claims could be backed up by the original data and all quotes were correctly captured.

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School	Joined project	Left project	Participants interviewed
B	January 2011		DP, 2PTs, 2WTs
C	January 2011	August 2011	P, 1WT
G	January 2012		P, 2PTs, 1WT
I	January 2012		P, 2PTs
J	January 2012	May 2012	P, 1WT
K	January 2012	March 2012	DP, 2WT

Table 1: Sample (P=Principal, DP=Deputy Principal, PT=participating teacher, WT=withdrawn teacher).

FINDINGS

The findings distinguish between the participating and withdrawn schools in interesting ways and are presented in relation to the five key features of successful PLCs, with implications drawn out for teacher professional agency.

Focus

Participating teachers mainly found the focus useful and relevant while withdrawn teachers expressed concerns. Typical comments from participating teachers show that the teachers appreciated taking time to understand how their learners were thinking, why they made errors and that learners could come to understand and correct their own errors:

> *The reason I become so interested is the critical analysis of the learners' mistakes ... we don't have only just to mark the learners' work, just rushing through, we should look at the reasons behind their work* (participating teacher, G_2)

> *Being able to get the reasons behind learners' answers, I can now at least try to ask them…to keep on probing the learners until they realize their mistakes. And in some instances correct the errors, their own errors* (participating teacher, B_1)

Two withdrawn teachers articulated their misgivings with the focus and benefits from the project very clearly, saying that they could not see the relevance of working with learner errors.

> *Where, what, how do I benefit from this ... it was nice arguing, identifying some of those errors made by learners, trying to think why they made these mistakes, how, why, you know, those different views from learners, people justifying those wrong answers, It was fun, it was fun, but you know, in as much as it was fun ... I couldn't link what we were doing with what we are doing in the classrooms* (withdrawn teacher, J)

> *With error checking and all that stuff ... and then most of us teaching grade twelve, how can we address that* (withdrawn teacher, K_2)

Teacher J enjoyed the sessions she attended but did not see enough benefits for her to want to continue with the project. Teacher K_2 argued that the focus on errors took teachers back to the basics, which should have been covered in earlier grades and

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which he did not have time for. Noting the level of under-preparedness of his Grade 11 and 12 learners left him demoralized as he was unsure how to address the errors.

Three of the principals (B, I, J) knew that the communities were investigating learner errors and argued that this focus should help the teachers to teach more effectively and improve learner achievement. One principal (C) knew that the teachers were analyzing tasks and test items but was not sure about the reasons for the activities. Two principals (G, K) said that they did not know what happened in the communities but assumed that the facilitators were helping the teachers with their content knowledge and to teach better.

So while a number of the participating teachers were inspired by the focus on learner errors and could be said to be stepping up – taking the externally suggested focus and making it their own, most of the withdrawn teachers pushed back on a focus they found to be irrelevant (although perhaps enjoyable) and demoralizing. The participating teachers' agency supported them to think about new ways of seeing learners while the withdrawn teachers chose not to develop this possibility further.

Long-term inquiry

All of the teachers cited time as a key challenge to participation: finding time to meet as a group and finding time to do the work of the project outside of meetings, for example the project readings or personal reflections on videotaped lessons. The following are typical comments:

> *Time is always a problem (laughs), especially for us as teachers because we always see ourselves as working overtime* (participating teacher, I_2)

> *The most pressure we had was of the time...the timing… Because during the week afternoon, we are tired. By the time you reach here you don't want somebody to come and say, let's think* (withdrawn teacher, C)

Related to time are teachers' workloads, which were cited by six of the withdrawn teachers (B_3, B_4, C, G_3, J, K_2) and four of the participating teachers (B_1, G_1, I_1, I_2). The withdrawn teachers in Schools J and K noted that they have many and big classes, requiring a lot of marking and leaving little time for other activities. The principals confirmed that the teachers do have heavy workloads, particularly in School J.

Five withdrawn teachers (B_3, B_4, J, K_1, K_2) and two participating teachers (B_2, I_2) distinguished clearly between the project work and their everyday school work, calling the project work extra work and arguing that their contractual obligations to the school must take priority over any additional professional development work.

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Schools have their own programmes in place, so you find that sometimes [facilitator] will come, you know, and by that time there's a staff meeting, or there's an intervention, you know. It kind of compromised the whole programme (withdrawn teacher, B_3)

I would give my core business better preference, and then the remainder of the time would be utilised for things relating to the project (participating teacher, B_2)

Although time and workload were issues for all of the teachers, a key distinguishing feature between the two groups was that the participating teachers made time for the project:

Because what I've also learnt is that here on earth there's no time, but one has to make time for anything after all (participating teacher, B_2)

But as for me I consider it as pressure that I can't do anything about, other than finding a better way to deal with … I must just find myself time (participating teacher, I_2)

The participating teachers were able to find time to meet, sometimes with the support of the school leadership and sometimes without it, and found ways to manage the additional pressure of the project work. All of the principals in the participating schools (B, G, I) noted that the teachers made great efforts to attend the community meetings. This level of agency was not easy to sustain but they could see connections between the PLC work and their regular schoolwork and they found that the activities supported their learning for the classroom. Some of the withdrawn teachers enjoyed the early activities but could not see the immediate benefits for their schoolwork. This made them less likely to resolve the time challenges, their agency was to prioritise their scarce resources for their school work.

Collaboration

Collaboration was described as a goal and benefit by all of the participating teachers and four of the withdrawn teachers (B_3, B_4, C, J), with some typical comments being:

It's the sharing idea part of it, the debates, you know, the questions, and also learning (participating teacher, B_2)

The discussions are so enriching …I learnt a lot (participating teacher, I_1)

The withdrawn teacher from School C argued that only two of the four teachers were strongly committed to the project, which reduced the benefits of collaboration:

We were four teachers teaching maths. Then one teacher refused to be involved. So we were three. In the three of us, the other one was so committed [to other aspects of his work]. *So most of the time it was me and* [name]. *And it was of no use, just the two of us*

So while all the teachers acknowledged the benefits of collaboration in the PLCs and wanted that kind of collaboration, the PLCs only worked for those schools and teachers who could step up to maintain the collaboration. This suggests that in the collective work of teacher learning, the agency needs to be shared, teachers work together for the benefit of all and if not all want to do so, it may lead to push back from others.

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Leadership support

Support from school leadership was mixed, clearly distinguishing between the participating and withdrawn teachers. At school I the principal was interested in the project, talked to the teachers about it and gave the teachers time off from other duties to do the project work. The principal also attended some meetings to "*show my interest*" and could talk about some of the substantive activities in the project. The deputy principal at School B had some knowledge of the project, having participated in the previous year. He sometimes excused the teachers from administrative duties to give them time for the project.

Of the three principals at the withdrawn schools, two allocated the project as the responsibility of the mathematics Heads of Departments (J, K) and only one principal (C) maintained any contact with the teachers about their participation, in that he spoke with the teachers about the project and tried to encourage their participation. The teacher from School J said that while the school leadership wanted the project in the school because it reflected well on the school, the teachers experienced problems, which is why they withdrew. The teacher from School C noted that the principal supported the project and encouraged them to attend but the head of department did not support the project and did not participate herself, which demoralized the others.

This data supports the literature that argues that support from school leadership is important in sustaining PLCs (Katz et al., 2009; Stoll et al., 2006). While one principal actively supported the communities, four principals were content to leave the support and management of the communities to the project facilitator. For the most part in South Africa, principals' roles are conceived of in terms of management rather than academic leadership, leaving many principals themselves with few resources to support academic learning among their colleagues. The conflict at Schools J and K between that school leadership and the teachers comes out of the hierarchical nature of South African schools and the difficulties teachers may experience in expressing their own agency. Teachers are unlikely to express critical views, explain their needs nor ask for explicit support from principals; rather they might accept requests on the surface but then refuse in other ways.

Trust

The project facilitators worked hard to develop trust in the communities and the data shows that they were successful. Four of the participating teachers (B_1, G_1, I_1, I_2) commented on the openness and trust among the teachers in the communities. A typical comment was:

> *We can talk to the other community members freely without, how can I say, stage fright. We are confident because we are talking with colleagues, knowing that no-one is judging you* (participating teacher, B_1)

One of the participating teachers (I_2) raised a challenge to trust – fear of being judged and evaluated on her teaching, particularly because she was videotaped:

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I don't know about videorising [sic]. *I think we're coming from an era where people were critical about you, they were looking at all the bad things that you were doing ... Now, we understand you're videorising it so that we can see ourselves developing. But somehow, at the back of our minds, it's like, is it true? Are they not hiding something from us?*

This teacher's concern came from her experiences during apartheid, where school inspection was judgmental and inspectors held a lot of power over teachers' careers. Although she knows that the system has shifted in favour of development, her prior experiences make it difficult for deeper trust of the system. The teachers were given the choice of whether to be videotaped and this teacher chose to be videotaped (as did almost all the teachers), suggesting that for her, the balance between trust and concern was, at this time, weighted in favour of trust.

For other teachers and schools, lack of trust was a key element in their withdrawal. The teachers in School K mistrusted the research focus of the project.

Because at the end of the day, we would see it as a research which benefits someone and not us really (withdrawn teacher, K_2)

They were thinking it is a personal research ... it sort of gave an impression that there will be your small jealousies that also come into the picture to say, but, he will use me to conduct a research and he ends up being an achiever of this particular area after having used me (Deputy Principal, K)[1]

For the teachers who continued to participate, trust and agency came together to support participation. Even thought there were some doubts, and most likely always will be, they fact that they were able to trust much of the process supported their agency in participating. This is a major "step up", requiring courage, time commitments and a strong sense that the project would be beneficial. For those who withdrew, major mistrust of the project's motives supported a push back.

CONCLUSIONS

The analysis has shown that teachers' decisions to stay with or leave the project were produced by and reflected their professional agency. That a significant number of teachers and schools chose to stay with the project suggests that there is a strong sense of professional agency among teachers wanting to improve their practice and understand their learners better. These teachers found the time for the project and their participation both reflected and produced trust in the project, albeit with some nuance.

At the same time, there were a significant number of teachers whose professional agency supported them to push back against the project and to withdraw. Mistrust of the project agenda co-produced discomfort with each of the other key characteristics of PLCs. If the project's focus on errors was merely a means to support the research agenda, it could not be trusted as a valid focus for professional learning and would

[1] The facilitators were doing research in the project for higher degrees. They were open about this with the schools and it seems that in this case, the teachers perceived the major benefit being for the research and not their development.

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limit the teachers' ability to continue with the curriculum because of the gaps in learners' knowledge. There was some tension between the school leadership and the teachers in the withdrawn schools – adding to mistrust of the motives for the project.

So both sets of teachers expressed their professional agency in relation to the project, some choosing to stay and others choosing to leave, influenced by both systemic, personal and community considerations. A question remains as to whether both forms of agency promote professional learning. This question cannot be answered empirically here, because while we have data that shows that many of the participating teachers improved how they engaged with learners' mathematical errors (author ref), we have no data on what the withdrawn teachers did. So, another key question about teacher agency must relate to the consequences of teacher choices, and the extent to which their agency promotes the growth and learning of their learners.

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PROBLEMATIZING "MATHEMATICS EDUCATION FOR SOCIAL JUSTICE"

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Abstract: In this presentation we explore conceptions of social justice and of mathematics education for social justice (MEfSJ). We recover examples from research literature that both attempt a definition and/or link MEfSJ to a set of concerns related to gender equity, access, democracy, ethnicity, and social background. Other examples show a link between MEfSJ and notions of Critical Mathematics Education/Pedagogy. And other examples show more or less substantial challenges to, and tensions in, our attempt to conceptualize MEfSJ. Positioning ourselves in a socio-political understanding of ME, we also explore the origins of the notion of social justice in the 19th century. Finally, we attempt to articulate that historical understanding of social justice with a socio-political approach on ME, testing that articulation with some present-day concerns and research examples.

Introduction: "MEfSJ", a heterogeneous set of concerns

Atweh (2004, 2007) points out that although, the expression "social justice" (SJ) appears related to issues of gender, ethnicity/multiculturalism, socio- economic factors, and inequity in access and failure, in Mathematics Education (ME) research spaces there isn't a precise definition of SJ itself. This lack of theoretical engagement has not stopped certain changes in policies and practices, however. Atweh indeed attempts a multidimensional concept of SJ, but he admits that "due to the limited theorization of the concept from within mathematics education, we will rely on works from outside the discipline itself" (Atweh, 2007, p.4).

Mathematics Education for Social Justice (MEfSJ), as theoretical and practical concern, has been somewhat addressed in the works of Frankenstein (1983, 2001), with the notions of critical mathematics pedagogy and the idea of Critical Mathematics Literacy Curriculum. Similarly, Gutstein (2012) uses the suggestive label of "Mathematics as a weapon in the struggle". He describes his work as teaching mathematics for social justice or critical mathematics, but also affirms that:

> I have adopted this more radical phrasing because, in 2007, then-President Bush travelled to South America and claimed that the U.S. was promoting "social justice" there. When I heard that, I remembered that others can appropriate our language, and so I starting using words that resist easy cooptation. Briefly, I mean by this that, "students need to be prepared through their mathematics education to investigate and critique injustice, and to challenge, in words and actions, oppressive structures and acts" Gutstein (2012, pp 24).

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We understand that in these perspectives, mathematics knowledge is not only technical or contemplative but also political, in the sense that it can help to reveal, understand and fight social injustices (inequalities). Mathematics is a knowledge that can be used by the dominant classes to oppress but that could (or should?) be re-claimed by the oppressed sectors in their struggle for emancipation and self-determination.

Skovsmose (1994) articulates the notion of the formatting power of mathematics. The contemporary world is, in substantial ways, normalized through the language of mathematics and its algorithms, especially regarding the technologies of information and communication. Mathematics can generate the critical structures of society as much as read them. Participation as a critical citizen in a democracy requires mathematical knowledge capable of addressing the critical structures of society. So, Skovsmose considers that the whole movement of MEfSJ is an example of Critical Mathematics Education (CME), or that MEfSJ relates to a critical stance on mathematics education (Alrø et al., 2010).

Skovsmose & Valero (2005) consider that reflecting on the relation between ME and SJ (whose meaning they approximate to democracy, or equity/equality, or inclusion) is a challenging issue. The challenges come from the complexity of the informational society at any scale (local and global), the difficulty of defining mathematics and mathematics education, and the ambiguities of the idea of "democracy" (and SJ, we add!). But there are also challenges from factual and conceptual-linguistic disagreements between ME and SJ. There is the possibility of intrinsic resonance between ME and democracy/SJ if "proper teaching and learning" empowers students. There is the possibility of dissonance between ME and SJ, because evidence shows that in many cases ME has functioned as a mechanism of systematic exclusion of students on the grounds of gender, ethnicity, social class, etc.

Christiansen (2007) analyzes theoretical and practical tensions in the acting-research of a ME for Democracy (a perspective that we also, intuitively, would link to MEfSJ). About the idea of mathematical competencies for democratic participation, she observes that the value of mathematical statements and their relation to the issues of "critical citizenship", do not function in the same way in Denmark as in other parts of the world. In post-Apartheid South Africa, the most significant challenge was furthering equality in access and living conditions.

More disgressions: Mathematics Education as political

The picture becomes even more complex. At least Pais (2012), Rasmussen (2010), Straehler-Pohl (2015), and more recently Gellert (2017), from different approaches, address the idea that mathematics education, as part of the school system, is a device of social selection and stratification, a credit system, a technology for the production and allocation of human resources. Andrade-Molina and Valero (2017) and Valero (2017) have addressed also the molding of subjectivities through mathematics education. Gutiérrez (2013) contends that:

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> If, as a field, we are not willing to recognize the political nature of mathematics education or the fact that teaching and learning are negotiated practices that implicate our identities, we might as well give up on all of this “talk” about equity. (Gutiérrez, 2013, p. 27)

In Madrid, for example, we perceive these traits of mathematics education through the serious joke of the “sciences person vs letters person”. To be a “sciences person” means being apparently good in physics, chemistry, biology, but mostly, good with numbers, good at math... (in school, that is!). But to be a “letters persons” could be a wide range of things: from liking Latin and Greek, to being good in economy, to just being awful at math (in school) and choosing any life path as far away from math as possible. Also, in the Spanish unconscious collective, the identification as a “letters nation|culture|country ‘lagging behind’ the more advanced science nations|culture|countries” could be a cultural trait. Our research interest also considers how this idiosyncratic subjectification process in Spanish culture could be linked to ME, and why it is never understood as an issue of SJ in ME research.

A notion that can allow us to reflect on this political nature of ME is what Valero (2010) proposes as a “network of social practices of mathematics education”. From that perspective, the classroom and the school practice are not the only object of research on ME, the context of the classroom is not merely interference or an influence in an otherwise clean practice. School, classroom and “con-text” (as Valero expresses in the mentioned article) are mutually constitutive. Relation is real, constitutive of the phenomena of study, and not purely a modal or extrinsic denomination. Consequently, there are a number of new, potentially legitimate research concerns in ME, which may include: research discourse and practice, its contradictions and complicities with policy-making spaces, public perceptions and mass media depictions, “common-sense” statements (such as “mathematics for all”), (international) assessment practices… and the interweaving and connections through which these nodes define each other, by their relative positions and tensions, etc.

So far, we adhere to the perspective of ME (in our western societies, in Spain in particular), as a governmental procedure, a social engineering, a subjectifying technology (in the line of a Foucauldian approach and the extensive work of Valero in that frame, e.g., Valero, 2017). Our research interest is, thus, about the domestication of mind and bodies, the establishing of certain truths, and the governmentality through the far-reaching, inherently political, technology of ME. Some of those cannot always be addressed in preexisting categories of gender, race, ethnicity, socio-economic background, religion. Those are legitimate concerns, but perhaps we are looking for a non-existing Leviathan, a mind-body frame, the desired citizen (in the line of, e.g., Andrade-Molina, 2017).

So, the search for a somewhat satisfying formulation of SJ to address these research concerns, and to articulate it with a political understanding of ME, carried us to the origins of the discussion about SJ in the 19th century.

The historical setting of the origins of *social justice*.

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In the book titled Saggio Teoretico di Dritto Naturale Appoggiato sul Fatto (Theoretical Treatise on Natural Law Based on Fact) (1840), and in different essays, articles and monographs, the Jesuit priest Luigi Taparelli d'Azeglio (1793-1862) studied political concepts such as social justice, representation, sovereignty, nationality, and, most crucially, solidarity and subsidiarity. He was the founder of the journal La Civiltà Cattolica in 1850. As Behr (2004) argues, the historical setting in which he started these theoretical developments was the pressure of dramatic social change and political violence, the unstoppable force of the industrial revolution, the emergence of the liberal nation-states across Europe. For the Italian peninsula, the evolving milieu was the attempt of political unification, known as the Risorgimento.

In this situation, and with the effects of widespread social inequity already generating great social unrest all throughout Europe (which would culminate in the dramatic revolutions of 1848), the competing political and economic models were, amongst others, Comte's Positivism, Stuart/Mill's Liberalism/Utilitarianism, and later, Marx and Engel's dialectical materialism. According to Behr (2004), Taparelli's intention was to develop a theoretical understanding of the social changes that could go beyond the pious platitudes. Traditional religious discourse was open to attacks from the privileged class, because criticizing greed was seen as promoting revolution, and advocating the patience of the proletariat left it open to the typical accusations of religion being the opiate of the masses.

At the Collegio Romano in the 1820's, Gioacchino Pecchi, future Pope Leo XIII and founder of what would be called Catholic Social Teaching in the encyclical Rerum Novarum, was one of Taparelli's students. So, as Behr (2004) argues, Taparelli has fair reason to be considered one of the fathers of Catholic Social Teaching, as many fundamental notions are of his elaboration. At the same time, his brother Massimo Taparelli d'Azeglio was a liberal politician and one of the most notable public figures of the Italian Risorgimento. The events of the time and their mutual correspondence show they were fierce enemies in the political terrain and loving brothers.

Considerations of a classical natural-law approach

Taparelli (1866) understood that human beings are rational animals, provided of intelligence and free will. Throughout many philosophical arguments, that implies the development of a classical moral theory (virtues, perfection, fulfillment), a theory of human rights and duties, and of course a theory of the nature of society. As Behr (2004) explains, he developed this approach to prevent the extremes of the individualist and collectivist perspectives, that is, society as the mere aggregate of individual atoms or as a whole that supersedes and absorbs the parts.

To our understanding, the key is the Aristotelian notion of *zoon politikon* (*political animal*, a fundamental concept that appears both in the Nicomachean Ethics and the Politics), although not directly addressed by Taparelli. This notion means that humans are not merely social animals like others in the natural world but, because of intelligence and free will, humans naturally tend to the polis, i.e., the politically

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organized society/community. The notion of political here should be understood against the background of Aristotelian philosophy, encompassing more than what we understand today as ballots, elections, ideologies and mechanisms of power.

That doesn't take away the "power" dimension in the idea of politics (because the polis is organized with an authority), but for Aristotle, the notion had a lot more to do with the public dimension of the moral development of the human person, in the polis or the city-state, and with the striving for the common good. Even more, the main raison d'être of the state or any other kind of civic or political association, (or more precisely, of the authority of any political entity), is the provision of the common good. If it fails in that end (for incompetence, corruption, ignorance, or whatever), the authority is null.

> But in this way, the authority of the consorzio will not be any more a true authority, nor will the consorzio be free in that case; once deprived of its own being, it will be more than a mass of individuals enclosed in certain limits of space. (…) Indeed, what is authority? It is the right to order the society to (the common) good; then, the less exposed to disorder the reason of the superior, the more true and pure his authority will be (Taparelli, 1866, para. 707; our translation).

The Socio-Political theory and social justice

For Taparelli, as a matter of historical fact and natural law, every human society beyond a certain size (bigger than a family or partnership, for example) is constituted by other societies. These intermediate societies in which human life flourishes he called consorzi (consortia). Moreover, he calls a society large enough to provide for the common good of its members (which today we would identify as a city, a full state, or even the Greek city-state) a protarchy. Other forms of intermediate societies are the deutarchies, a kind of larger, more autonomous consortia.

How can the deutarchies form a protarchy? They can do it either by composition (by natural, free or forceful association) or by division of a protarchy. This formation may come to be through different historical or political processes. The deutarchies can either pre-exist the protarchy or come to be after it. Every kind of society, at every level, is given its unity of being by its proper ends (its own "common good"), its own authority and its own operation (Taparelli, 1866, para. 689).

The critical principles for understanding the rightful articulation of those levels of society, i.e., social justice (which are rooted in a natural-law anthropological view) are: *socialità* (sociality, which later became identified with solidarity) and *dritto ipotattico* (hypotactic right or law, derived from a grammatical expression, hypotaxis, which refers to the rules of subordination between clauses in a complex sentence), which was later assimilated under the label of *subsidiarity*.

By sociality Taparelli meant the rights and, especially, the duties to others. More concretely, the principle of sociality can be synthetized, according to Behr (2003) as the duty to seek the good of others. And by subsidiarity, Taparelli meant:

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> Every consortium must conserve its own unity in such a way as to not lose the unity of the larger whole; and every higher society must provide for the unity of the larger whole without destroying the unity of the consortia. (…) given the facts of the association, it would be as against nature for the consortia to reject the unity of the social whole as it would be for the whole to abolish the consortium.…(Taparelli, 1866, para. 694-695).

Taparelli called this kind of complex society a "hipotactic association". Of course, he conceded that in this arrangement, the consortia and the protarchy lose certain independence and specific freedom, but for both individual persons and societies of the whole, that means the possibility of enjoying a greater liberty and achieving a good greater and more fulfilling than what they could achieve on their own. In this way, they could enjoy the participation in the common good, that is, the republic.

Social justice is, thus, according to Behr (2004), a norm and habitus, (a social virtue, an arrangement of the political whole, a habitus of collective action), directed towards the common good, according to the principles of sociality and subsidiarity.

Taparelli was clear in admitting that these definitions only made sense in the concrete, historical and actual configurations for each society. But he argued that sociality and subsidiarity were evident from a natural-law argument and as a matter of fact, as an historical reality.

One example of a completely opposite view of the state is the classical Leviathan of Hobbes. As Behr (2004, p. 15) cites, Hobbes considered that the intermediary associations between the state and individuals were a "divisive" influence in the unity of the body politic, as "worms in the entrails of man". That view is a logical consequence of an individualistic-egoistical anthropology, in which by norm the state must enforce society, and opposite to the zoon politikon of Aristotle. Behr (2004), citing Taparelli in an 1859 article of La Civiltà Cattolica, says:

> Once these associations, that form the organic parts of the larger society are destroyed, society finds itself essentially altered, reduced to a powder of scattered atoms, lacking all cohesion, all special function, and chained to total dependence on the supreme force [the state], a heap of purely passive, inorganic molecules. (Behr, 2004, p.8)

After all this analysis, our main question is: can we still speak about Mathematics Education for Social Justice, without the concern of just using a "hollow incantation" (Hayek, 1976)? We will attempt to articulate the use of the expression SJ in a precise, meaningful way, from a socio-political stance in ME; and exemplify it with actual and ongoing research projects and perspectives.

ME and *Social Justice*: homogenizing society, piercing through the *consortia*.

Following the Foucauldian approaches to subjectification and governmentality, the aseptic, neutral, instrumental trait of mathematics prevalent in ME discourse and practice promotes the adoption of an individualistic perspective in the desired citizen and worker (Valero, 2017, p.11); the inception (Andrade-Molina, 2017, p. 183) of a neo-liberal world-view. Mathematics becomes at the same time feared, perhaps

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rejected, but accepted by its symbolic power (Sáenz & García, 2015, p. 26). The individualistic pursuit of success or acceptance of failure is epitomized in the way ME school practice, and the mathematization of the entire credit system in education, operates and divides students.

This double character of ME as technology of subjectification and government points towards a process of control and homogenization of society, at the level of individualities, accepted truths, and access to resources. Of course, a high-tech homogenized society of individualistic, atomized personalities is at odds with the notion of hypotactic association. It's almost the complete opposite of what Taparelli had in mind when he crafted the concept of social justice (SJ) with all its subtleties.

The era of the strong, centralized nation-states, during which the notion of SJ was first formulated, is arguably gone. But the capitalistic system (with its homogenizing tendencies and its essentially individualistic anthropological premise) persists as strong as ever, albeit in mutated forms. In that sense, Pais (2011, p.29), citing Žižek, calls capital "the concrete universal". For us, it means that it can be still relevant to consider the Taparellian notion of SJ in the present state-of-things of universalized capitalism (as *the* paradigm of production of good and services), and the role of ME as a (small but key) tool for the reproduction of that "concrete universal".

ME, as a political technology, is thus at odds with the idea of a pluralistic society of societies, arranged in SJ. It's hard to imagine that molecular individuals, passively (or not) allocated to different roles would have a tendency to "act critically and actively as citizens" - not as isolated, abstract individuals, but by the forming of consortia. Let's see two concrete examples for testing this complicated analysis.

The first: in a recent (2017) Service-Learning experience, addressing the question of the "Sciences person vs Letters person" divide, we asked some students if having "shaky" grades in mathematics equated to being a "letters person". One of the students said: "how could you be a 'sciences' person, if you are bad at math"? "Bad" at math implies a non-sciences person, in an absurdly artificial division of sciences vs letters that does not correlate to the concrete, actual composition of society. An artificial category that pre-conditions how the people understand themselves and the society in which they live, regulating top-to-bottom the ways in which they could (maybe, if ever) associate in consortia to participate in political life.

The second: on a more historical note, Apaza (2017) has researched on the ME issues of the Ayllus of the High-Andean regions of Perú. The Ayllus are traditional communities of familiar base and pre-Incan ancestry. Apaza proposed a reinterpretation of the Yupana, a traditional kind of abacus of the High-Andean cultures, for promoting a ME experience. Under disguise of practice-oriented PhD work, Apaza (2017, p. 88-104) distinguished between the Western rationalistic, and the High-Andean relational, rationalities. Implicit there was, of course, the issue of (post)colonialism, not only by the "force of bayonets" but by the imposition of foreign norms of reason and regimes of truth.

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The perennial low-achievement in ME of the High-Andean Ayllus is both a symptom and an excuse: a symptom of the homogenizing pressure to adapt the foreign norm of reason and to leave behind the historical culture, political rights, and with those, lands and material goods. The Ayllus are, in any crazy Taparellian dream we could have, deutarchies, pre-existing the colonial process and the international homogenizing trend of capitalistic globalization. ME is a small, but significant, mechanism of that whole assimilation-disarticulation process, piercing directly the unity of that deutarchy, a real societal order. The excuse is the failure/achievement, the improvement of mathematics achievement through the replacement, with modern ME, of the mathematic and scientific culture proper to the pre-colonial Andean civilizations. The failure correlates to the label of deficient and primitive pre-assigned to the High-Andean cultures, which ME kindly corroborates.

ME and *SJ*: Teachers and Researchers.

Another anxiety for which we intend an articulation, between a socio-political stance on ME and SJ, is the relation between ME research and teaching practice. Valero (2010), with the key notion of ME as a network of social practices, uses the metaphor of the context as "that surrounding accompanying and constituting the text, [which] does not fall inside the research gaze" (p.8, our emphasis), and explains the prevalence of abstract models in ME research practices such as the didactic triangle. Pais (2017) also considers that ME research lives in a world of its own making, in a state of narcissism, but at the same time, with the irony of "research's beautiful souls" (p. 59), he points to the un-assumed complicity of ME research with policy making spaces of dubious interests.

The socio-political issue that draws our attention is a kind of paradox: ME research seems to live in the clouds in regards to actual school practice, its recipes and recommendations don't work to overcome systemic failure, and the school practice is very static along the years. Yet ME research practices, many times close to the policy making spaces, are actually very influential in school practice through devices such as external assessment, curricular developments, and discourses about the wonders of the use-value of mathematical knowledge, its neutrality and objectivity.

A school is, no doubt, a consortium. However, historically, many schools were created precisely by the nation-states of Taparelli's years, as consortia for the fabrication of certain social order. So, the question is: is it truly SJ the way in which researchers in ME at the same time remain detached, assess, and regulate teaching practice (or teachers), thereby influencing policy making? We know the relation between ME research and ME teaching is not the same in every country or community. So, what could a proper, rightful, just, relation between the two kinds of practices be? Is it possible to imagine that ME, composed of persons and other material goods, is a kind of hypotactic association, with a unity of order towards a common good? That would imply a deep reflection on the role of researcher and teacher and the relations between them.

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Of course, that is nothing new to say. But the situation in which researchers are mostly (if at all) responsible towards policy making spaces, economical agents, their superiors and fellow researchers and institutions, but never towards the persons and consortia which are affected by their practice, is an issue of flagrant social injustice. The hierarchy of researchers as superiors in the food chain to both teachers and students is assumed and implicit, but not justified.

Final Thoughts

We conclude with some policy recommendations: there is need for more funding, resources and research, improved curricula and teacher training, teachers and students must…Nah.

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ACTIONS WITHIN THE ZONE OF PROXIMAL DEVELOPMENT: REPRESENTATIONS OF MATHEMATICS CONCEPT BY KOREAN TEACHERS

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The present study is premised on the assumption that Vygotsky's Zone of Proximal Development (ZPD) and representation of concepts provide a powerful theoretical lenses for analysing teachers' goals and actions that aims to extend the depth of students' mathematical understandings from a socio-cultural perspective. We analysed key episodes of teaching from two Korean mathematics lessons. Three significant findings emerge from a preliminary data analyses. Firstly, both teachers provided clear instructions to access students' prior knowledge. Secondly, teachers use effective open-ended tasks to elicit student engagement. Thirdly, participating teachers sustained student engagement with prior mathematics concepts by challenging them constantly. The above actions provide insights into the nature of ZPD in Korean mathematics classrooms.

CONTEXT AND RESEARCH PROBLEM

There is an emerging consensus that teaching mathematics, among others, must aim at supporting the development of understanding of mathematics concepts and procedures, and the use of this set of knowledge to search for solution and further elaboration of concepts. The focus on teaching for understanding is reflected in the goals of learning standards that are outlined in a number of major mathematics curriculum documents such as the Australian national curriculum (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2018) and Principle and Standards for School Mathematics (National Council for Teaching Mathematics [NCTM], 2000).

This study was motivated by our desire to better understand and learn from mathematics teaching practices of an emerging eastern Asian country, namely the Republic of Korea. This country will be referred to as Korea in this report. It is widely acknowledged that Korean students have been consistently performing well in international assessments of mathematics such as Trends in International Mathematics and Science Study (TIMSS). Korean students have been ranked among the top four countries in mathematics for the past 20 Years on the basis of scores in TIMSS. The reasons underpinning this continuing trend in their performance are much discussed and debated but little understood. The present study is set against the above context of attempting to explain the high performance of Korean students and the impact of teaching that supports mathematics learning.

We commenced the study on the assumption that Korean teachers, particularly their teaching approaches play a critical role in students' performance in and attitude towards mathematics. The goal of our larger study is to generate rich descriptions of

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tasks and strategies that Korean mathematics teachers employ to better engage and challenge their students. In this report we provide preliminary data based on the observation of two lessons of mathematics in lower and upper primary grades.

THEORETICAL FRAMEWORK

ZPD and Representation

The research questions and analyses of data are framed within the constructs of Zone of Proximal Development and Representation in order to understand the social setting, goals and action of the participants. According to Vygotsky (1978) ZPD is the distance between a child’s independent problem solving capability and the higher level of performance that can be achieved with expert guidance. But what is the nature of this higher level of performance and how could an expert engage the learner in order improve access to knowledge and skills that underpins that performance? These questions are addressed via the framework of Representations. In this study, the expert guidance is provided by our Korean teachers.

The aim of the study reported here was to capture lesson goals and key events in a lesson that Korean teachers used in order to engage and extend their students to move into the ZPD. In particular, our interest was to generate fine- grained analyses of knowledge schemas that were generated by the students with assistance by teachers’ actions. The past three decades have witnessed considerable research about the role of representations as semiotic systems in teaching and learning mathematics. While there are differences in the way this notion is conceptualised (English, 1997, 2015), there is consensus that it provides an effective theoretical tool to better understand teachers’ practices and students’ growth of mathematical understanding. A commonly used interpretation is that representation is a model or artefact that can be used to denote another entity. For example, symbols and visuals are representation of mathematics concepts (Goldin, 2002).

More recent conceptualisations emphasise the process and products dimensions of representation (Kaput, 1998). Thus, representation is cognitive process that involves learners constructing and interpreting mathematical concepts. As a consequence of that process, learners develop a representation or model that is a product. For example, part-whole relations in a fraction can be represented in symbolic and visual forms (products). However, in order to create these forms, students need to construct links (process) between the *concepts* of parts and wholes. A second aspect of the process is the translation of one representation to another and the articulation of the relationship among the representations as argued by Janvier (1987). From a semiotic perspective, representations constitute ‘sign vehicles’ (Presmeg, 2016) which stand for objects of mathematics. In the present study, we use the term representation to refer to these sign vehicles that buttress mathematics concepts and provide a window into teachers’ and students’ thinking processes.

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RESEARCH QUESTIONS

The following two questions guided the study:

What is nature of tasks that Korean teacher utilise to engage students in the mathematics classroom?

What do the above tasks reveal about representations that Korean teachers regard as effective in supporting deep mathematics understandings?

METHODOLOGY

Design

We used a case study design to examine tasks and activities in Korean mathematics classrooms. This design has been argued to support research that aims to conduct in-depth study of events that unfold during real -life activity (Yin, 2009). It is appropriate to study situations where the researcher has little or no control over. As our aim was both descriptive and explorative of representations by teachers during the course of normal teaching sessions, we decided to adopt a case-study approach to seek data to answer our research questions.

Participants

Two teachers and their students from Korean primary schools participated in the study. Both the schools were located in the greater Seoul metropolitan area. School A and B were located in high and low SES areas respectively. Teacher Eui (pseudonym) was from School A, and she was teaching Year 2 students (8- year-olds). The second teacher, Hae (pseudonym), was from School B. Hae had Year 6 students (12-year-olds). Both were volunteers and we had official approval from the school principals for the study.

Procedure and data

The primary source of data was classroom observations of mathematics lessons. Prior to classroom observation, the two teachers and their principals were briefed about the purpose of the study, its value is helping students learn mathematics and our desire to understand the unique strengths of Korean mathematics practices.

The lessons were conducted in Korean language. The lead researcher (Chinnappan) video-taped the lessons each lasting about 40 minutes. The videos were analysed for key events that were adjudged to be rich in representations. This phase of data analyses was led by Korean researcher and mathematics educator, Prof Shin of the Seoul National University of Education. The key lesson events were then discussed with two experienced Korean primary teachers for their comments. We provide artefacts from the lessons and commentary that was based on translated comments from the two experienced teachers and our participants, Eui and Hae.

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DATA AND ANALYSIS

Year 2 Class (Teacher Eui)

Grouping is fundamental to understanding the concept of quantity or ‘how many’ in a number. Equally important in this understanding is how the quantity is expressed in the representation of a number. The lesson focus was for students to understand the grouping idea which is the basis of the Hindu-Arabic numeration system. In the Hindu-Arabic numeration system, place value is a central concept and the place of the number in the notation of a number indicates its value. Equally, this value is determined by groupings of tens. For example, in the number 23, number 2 is sitting in the tens place and it stands for 2 groups of ten or twenty. Thus, groupings in tens and the place of a numeral in the number are two key concepts that students use not only in making sense of numbers but also number operations.

Eue was attempting to increase students’ access to the concept of grouping that underpins the both the quantity and expression of that quantity in the Hindu- Arabic via three external representations by drawing on the Egyptian Numeration system. The Egyptian Numeration utilised hieroglyphs for numerals but based on the base 10 system. Although they developed separate symbols for one unit, one ten, one hundred, one thousand, one ten thousand, one hundred thousand, and one million, these symbols were built on groups of ten. Figure 1 shows Eui introducing her students to these symbolic representations. In Figure 2, we see Eui demonstrating the relationship between Egyptian and Hindu-Arabic representations of 12 and 1, the addition of these to make 13, a further representation in the Egyptian format.

Artefact 1 (Teacher Eui)

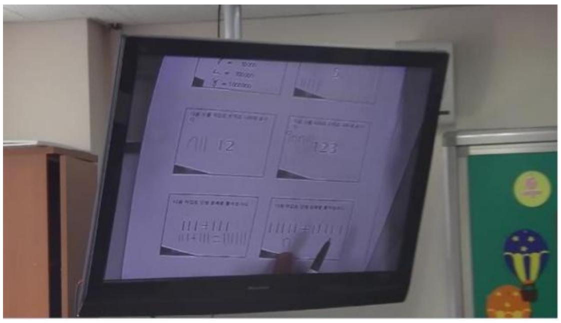

Fig 1

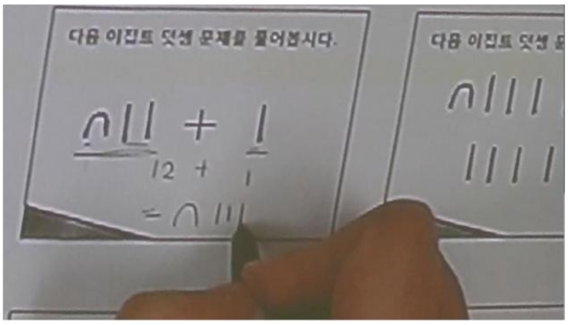

Fig 2

Artefact 2 (Student Work Samples from Eui’s class)

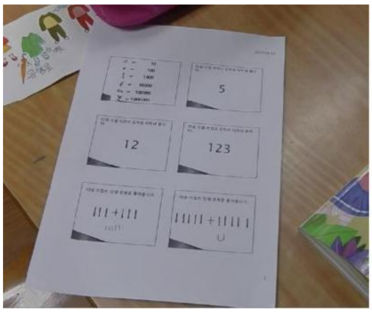

Fig 3

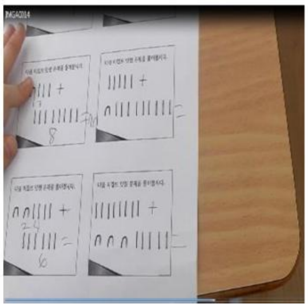

Fig 4

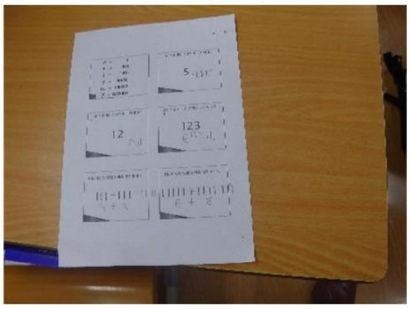

Fig 5

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Figures 3, 4 and 5 show work sample of students indicating Hindu-Arabic symbolic representations of 12 and 123 and students converting them to Egyptian symbol representation. Grouping in tens is a common running concepts in all three work samples.

Year 8 Class (Teacher Hae)

This lesson was based on and extended students' prior knowledge of formulae for circumference of a circle and area of rectangle. They also had learnt properties of circles, parallelograms and rectangles. The goal of this lesson was get students to develop the area formula for circle by drawing on the above prior knowledge. It was an open-ended investigative group problem-solving activity. Students were given about 10 minutes to share ideas about ways to determine area of a circle. This phase generated a number of strategies from the students.

Following the above search for potential solutions, students were provided with a worksheet with sectors of a circle (Figure 6). They were then asked to use the sectors and develop a plan that could assist them find area of a circle from which these sector were drawn. As the number of sectors increased from 8, 16, 32 and so on, the resulting figure resembled a parallelogram. Hae posed the question (Figure 7) about the nature of the figure if students continued the process with increasing number of sectors. The expectation was that the resulting figure would be a rectangle. The next major part of the solution was to link area of the rectangle with that of the circle whose sectors were drawn from. Figures 8 and 9 show the work samples from students. The work samples show that students finding the area of the rectangle, and reasoning that that the length and width of the rectangle is πr (½ of circumference of circle) and radius (r) respectively. Thus, the area of the rectangle was equal to πr multiplied by π (πr^2). It is important to point out that the students do not use π in the area formula for circle in Year 8 Korean classroom. Instead they denote π by its value.

There were three steps that involves representation by the actions of Hae. Firstly, students has to represent sectors of circle into parallelogram. Secondly, parallelogram had to re-represented as a rectangle. Finally, area of the rectangle had to be represented as area of circle. Throughout the lesson, Hae was active in pushing students to return to their prior knowledge and use that knowledge in the search for the solution through a cyclic process of representation and re- representation all of which were cognitively demanding but powerful activities supportive of deep understanding.

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Artefact 3 (Teacher Hae)

Fig 6

Fig 7

Artefact 4 (Student Work Samples from Hae's class)

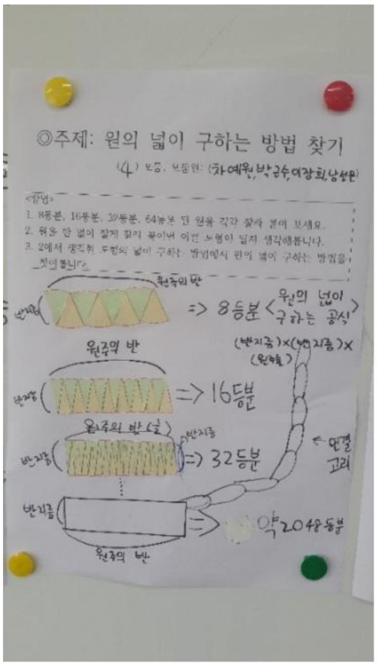

Fig 8

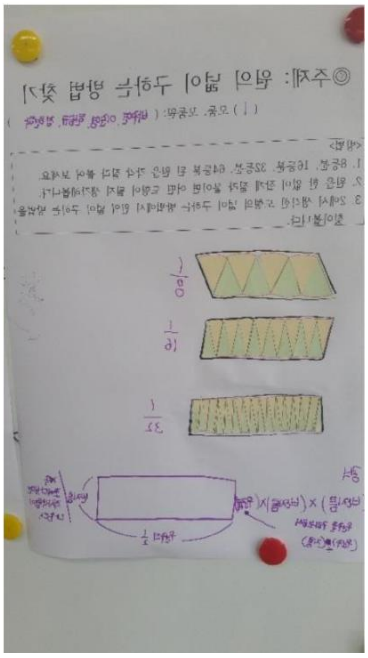

Fig 9

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DISCUSSION AND CONCLUSION

The aim of the study was to generate a nuanced description and analyses of how Korean teachers, in general, challenge and extend their students' understanding of mathematics. The frameworks ZPD and Representation provided a powerful lenses through which to gain insight into goals and actions of the teachers and identifying knowledge that was played out as students engaged with the problems. Both teachers acted as knowledgeable adults in helping students access the required prior knowledge but it was left to the students to exploit that knowledge in ways that helped develop problem solutions. Thus, while guidance was instrumental, the teacher tended to be critical observers in monitoring the actions of their students.

Literature suggests that representation can be a process and product. We have evidence, at least on the basis of learning events from two classes of mathematics, from different schools, to argue that our participating Korean teachers used sophisticated and powerful representations of numeration and measurement concepts to guide their students' engagement. Further research is warranted across different schools, year levels and mathematics concepts before one is able to generalize how widespread is the use of representation in Korean classroom practices and the quality of such representations.

Both participating teachers were deliberate in providing clear instructions to assist students activate relevant prior knowledge in order to make progress with tasks at hand. Secondly, teachers used effective open-ended tasks to elicit student engagement and monitor shifts in their understandings (Presmeg, 2002), albeit it was more visible with the year 8 class. Eui and Hae sustained student engagement with prior mathematics concepts by challenging them constantly. Finally, there is indirect evidence that teachers accessed high levels of content and pedagogical content knowledge in order to drive the representation-based learning activities. In their analysis of teachers' knowledge for teaching mathematics, Ball, Thames and Phelps (2008) and, more recently, Stylianou (2010) argued that representations constitute an important fact of their pedagogical content knowledge. We suggest that, at least in the case of our two teachers, Korean mathematics practices place high premium on knowledge exploration and extension, and flexible utilization of knowledge in the problem space.

While there is support here about the utilization of representations, how student exploit those representations is an issue if we are to fully appreciate their pedagogical value. As suggested by Kaput (1989), availability of representations is not sufficient for children to take advantage of that representation. Exploiting a given representation of concepts involve learners having to examine the underlying mathematics concepts and relations, and rules governing the use of that representation. In the case of Eui and Hae, we sensed actions that scaffolded students to access and exploit a given representation.

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As flagged earlier in setting the context for the study, our larger goal is to better understand Korean teachers' actions and associated knowledge that appear to support the high achievement levels of their students in TIMSS program. The tasks that were analysed in this preliminary study may not me directly associated with or similar to those faced by students in TIMSS. This is a limitation and in future studies we plan to address this issue.

ACKNOWLEDGEMENTS

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DE/MATHEMATISING THE POLITICAL IN MATHEMATICS EDUCATION: *A DE/POSTCOLONIAL CRITIQUE*

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Abstract: Various interpretations have been given to the double gesture of de/mathematising in relation to a variety of social thematic contexts demarcating 'the political' in mathematics education. By way of theoretical intervention, we offer the beginnings of a de/postcolonial critique in response to such programmes of work, while recognising the important contributions they have made to advancing complex political approaches to mathematics education as a pedagogic praxis. Social thematic approaches of de/mathematising have promoted ways of envisioning mathematical activity and agency. Our critique is both celebration of these diverse initiatives, as well as, a way of moving these conversations forward in newer, alternative, politico-epistemological directions. In a dialogue amongst us, based on our previous works, we reconsider 'the political' in mathematics education through a de/postcolonial critique.

INTRODUCTION

Much has been said on the nature of mathematising and demathematising and their necessary underpinnings beyond a conception of 'horizontal and vertical mathematisation' (Gellert & Jablonka, 2007). In this respect, parallel working interpretations of mathematising can be defined as: *first*, a process that renders a set of activities increasingly mathematical, as well as, *second*, the increased political influence of the voice of mathematics and mathematics education (and STEM) in the social domain. Dialectically, demathematising would have the dual reverse effect of deminishing the mathematical effects or qualities in a set of activities, and/or its political diminishment. These multiple, dialectical understandings are constantly at play in the way in which they concomitantly and often contradictorily inform each other. An example of this may be mathematics and mathematics education's increasing technological and economic utilitarianism under global neoliberal governance, which has the dual effect of entrenching its perceived importance while in servitude to specific technoscientific and economic agendas (Skovsmose, 2006, Chronaki, 2009, 2010, 2011, Swanson, 2017a).

In this paper, we attempt to move beyond current understandings of what it means to mathematise in relation to varied social thematic contexts at the service of mathematics education. These social thematic contexts span from realistic mathematics education, to real world activity, critical mathematics education (CME), Ethnomathematics, arts or craft-based mathematics, playful contexts, media-based materials, pseudo-contexts based on word problems or varied themes of mathematics in action. By way of theoretical intervention, we offer an entrée to a de/postcolonial critique in response to such programmes of work, recognising the important contributions they have made to

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advance complex didactic and socio-political approaches to mathematics education as praxis in relation to society, and the way in which they have promoted alternative ways of envisioning mathematical activity. They have encouraged us to think beyond the confines of school, classroom and curriculum as containers of knowledge and knowledge circulation, ways that envision mathematical pedagogy as relating to people and their histories in present complex societies. Nevertheless, we provide some distinction from these programmes of work by attempting to move these conversations forward in newer, alternative politico-epistemological directions, ones which more centrally consider critical de/postcolonial perspectives.

While socio-political projects are still relatively recent in mathematics education (Ernest, Sriraman & Ernest, 2016), there is a need to explore how a de/postcolonial critique might offer opportunities to centre ethical, democratic and (geo)political considerations in de/mathematising activity, while bringing forward concerns about social and economic development, culture, race, class, ability, gender, and global or local (in)justices to bear on arguments in relation to mathematics and mathematics education discourses. We suggest that a de/postcolonial critique can usefully provide theoretical concepts that enable us to speak of ontological and epistemic considerations *politically* in mathematics education in-between global and local contexts.

MATHEMATISING, DEMATHEMATISING AND THE POLITICAL

From the time of Galileo who argued that the book of nature is written in the language of mathematics, to Freudenthal who coined the word 'mathematising', a dominant conception of mathematics as the 'Queen of the Sciences' has pervaded discourses in the public domain, and these inheritances largely remain in educational contexts where mathematics is taught. Pervasively, in the school setting, pseudo contexts of 'real-life' have often served as exemplars of mathematising as if providing an easy straight-forward entry to 'the real'. In the many educational and social contexts, mathematics often has been divined as revealing Truth. Its reification within Enlightenment discourses has perpetuated such dominance (Swanson, 2005), thus giving rise to critical conversations about the potential benefits and dangers of de/mathematising within society in the context of a world structured according to socio-economic, epistemic, embodied and political hierarchies and widespread inequalities of every form (Ernest, Sriraman & Ernest, 2016).

The need to open up diverse meanings and spaces for conversations about the nature of de/mathematising has become ever more urgent in the face of the perpetuation of a widespread singular logic structured around a hyper-pragmatic, economically-informed 'reality' and pervasive neoliberal 'common sense' (Hall and O'Shea, 2013). In this light, we argue that there is a need to see de/mathematising as a broad processual, interactive and evocative space where discourse, power, and 'the body' come to influence ecologies of knowing and being by way of coming to know the world through mathematics (Swanson, 2013a; Chronaki, 2009, 2010, 2011). The process of mathematising is therefore unavoidably political, and cannot escape such influences and positionings through a call to objectivity and the lure of certainty (Swanson, 2005).

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In another sense, the process of doing mathematics often works like religion, turning de/mathematization a matter of morals (Restivo, 2009).

De/mathematising's necessarily political nature is a condition we purposively embrace rather than attempt to render as neutral, which we argue acts as a political positioning in itself. Foregrounding the acknowledgement that mathematising activities are informed by relations of power and cultural-historiographical investments, it is in the understanding of this purposive political act that we bring critical de/postcolonial perspectives to bear on such conversations. We are not following an expected paradigm of academic engagement by offering 'solutions', but rather attempting to grapple with complexity in problematising the myriad of issues at hand and in opening up alternative conversations about what it means to mathematise and what are its many effects in contemporary society in this political moment. The effects of de/mathematising social activities can be traced to some degree through the effects of power in which mathematics education discourses and practices cohere, constructing particular 'regimes of truth' (Foucault, 1980), through the evocative power of context (Bernstein, 2000) and its politics. Yet, the ethical implications of power dynamics are often left unattended in the literature, with some attention being given to Levinasian perspectives for example (Maheux, Swanson & Khan, 2012), referencing structural exclusion (Swanson, Yu & Mouroutsou, 2017b; Jörgensen, Gates, & Roper, 2014), or exploring the affirmative potential of minor gestures (Chronaki, submitted).

The move to understand de/mathematisation is one which begins to frame mathematics education in its many socio-political contexts. Bringing 'the political' together with 'mathematics' or 'mathematics education' may seem a surprising, radical or even 'irresponsible' gesture, within the framework of modernist hegemony. Yet, there are multiple ways in which one might respond to the question of what it might mean to (de)politicize mathematics education, if taken within a wider conception of mathematics and mathematics education's role in contemporary global relations. One view would be a deconstructive move as an intervention to a dominant narrative of mathematics' claim to being able to objectively describe nature and the workings of the universe, which is itself a position that has had political effects in the way it has granted mathematics authority and power (Swanson, 2005). This authority has lent credence and power to mathematics education, extending its 'formatting power' (Skovsmose and Yasukawa, 2004) in advancing globalizing modernism through its practices. In the same vein, mathematics and hence mathematics education's claim to neutrality is a politicizing of mathematics, while claiming the opposite. But more importantly, the prevalence with which mathematics educational tasks have displayed an avoidance of contexts of an explicitly political nature, such as addressing issues around disenfranchisement of certain groups within the nation state, oppression of ethnic minorities or women, or controversies over climate change, amongst others, speaks to its always-already politicized nature. The political nature of mathematics is also internal to its structures and practices, and heated debates over 'grouping by

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ability' and 'mixed ability grouping' (Swanson, Yu, & Mouroutsou, 2017b) are only one example of how fraught issues around exclusion and inequality in mathematics classrooms can be, let alone drawing political parallels to the ways in which these practices help undermine democracy in society. The poor attention given to issues of racism in mathematics education is a political issue desperately demanding attention, and the broader injustices of mathematics education in its implications to sustaining gender inequality as well as economic and social oppressions of the Global South require interrogation from feminist de/postcolonial positions (Chronaki, 2009, 2011). Not only is mathematics education already political by the nature of the power struggles in which it is embroiled, but in the sense of the injustices and absences discussed here, it can be argued that it owes a 'political responsibility' to the search for more viable, sustainable and more just alternative futures (Swanson, 2017).

Mathematics education's disavowal of becoming implicated in the singular logic of global modernist hegemony, needs undoing through a dual approach: First, by making its always-already political nature explicit; and second by politicising it in a way that holds it responsible for the current global condition. The way in which we engage the political counts however, and approaches that afford a deeper critical sociology of mathematics education in respect of global/local relations and becomes increasingly urgent given the crisis of global social and ecological instability. This opens up a space for de/postcolonial critique that may attend to the localizing issues of oppression with a view to its global effects, as well as, the global effects that impact on local conditions and communities, and with a focus on their intricate interconnectedness. Here, arguments related to mathematics education's political implications in global structural injustices can advance the argument for political accountability through a language of ethics and democracy, whether pertaining to the social, economic or ecological and ethical discourses.

ETHICS OF MATHEMATISING AND DEMOCRACY

Considering ethics in terms of rights and democracy, many areas of theoretical interest to mathematising as social processes often see the advocacy of mathematics as an automatic good (Swanson, 2013a), albeit that the manner and nature of mathematising and pedagogy count. Within these terms, the effects on people's lives and ecology are understated. Here, much advocacy of mathematising from these perspectives leaves fundamental assumptions unquestioned and unquestionable. A critical relationship with democracy for mathematics education (Skovsmose & Valero, 2001) involves an active (re)direction of its intents and purposes. What is seldom asked, however, is the question of whether choosing *not* to participate in experiences of mathematics education or its (re)direction were itself also a critical relationship with mathematics education. Seldom is the view held that the refusal and disobedience to the evocative power of mathematics is also a democratic action. Swanson & Appelbaum (2012) argue that mathematics education for democracy and development must take seriously specific acts of refusal that directly confront the construction of inequality common in most development contexts. They argue that globalisation and development discourses,

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via citizenship and nationalism, construct oppressive relationships with learners and mathematics education. Such relationships are coercive and based on assumptions of the inherent goodness of learning mathematics and of mathematising as a virtue or the right to mathematics education is one and the same as the expectation to do so, for the person and/or society's own good. In a similar vein, Chronaki & Kollosche (2018) discuss the case of a female student in a secondary school who opts towards refusing mathematics as an act of opposing a process of being schooled in an austere context. However, seldom is the action of refusal to participate in mathematising activities understood in the light of a refusal to participate in mathematics education's colonising and/or globalising neo-liberal gaze.

Bringing Jacques Rancière's (2009) notion of 'radical equality' to mathematics education theory helps to advance the ethical and emancipatory position of intentional disregard for ideological narratives such as the ones produced by dominant development mathematics education discourses (Swanson, 2013b). Consequently, by reconsidering the assumptions behind mathematics education in its global development context, one can reframe refusal, disengagement, disobedience or resistance not as deficit or failure but as a critical position of radical equality in relation to arguments on mathematising, access and choice (Swanson and Appelbaum, 2012, Chronaki, submitted). This brings us back to the issue of a politics of mathematics education, where mathematics education is recognised as operating within a geography of global relations. Mathematics education is, of course, a contested terrain with competing values, ideals, and intentions attributed to it, but its general role in advancing economic development within a frame of globalising modernism speaks to a particular ensemble of discourses that have political effect in the global social domain, construing particular relations of power that operate on micro, meso, and macro levels. We argue that a feminist de/postcolonial framing could provide insights and interventions in coming to understand how such power dynamics operate on a global scale, with particular implications for 'the local' that may act as forms of symbolic violence against communities (Bourdieu, 1990).

DE/POSTCOLONIALITY AND MATHEMATICS EDUCATION

The origins of postcolonial studies in the field of science and technology, as Harding explains (1998), can be traced back to the 1940s when a West Indian historian looked at how the immense profits from Caribbean plantations had played such a crucial role in making European industrialisation possible. This early investigation revealed how the British had intentionally destroyed the Indian textile industry in order to create a market for imported British textiles. Postcolonial studies have helped to reveal that imperial control has driven the politics of scientific knowledge historically, likewise postcolonial scholars have undermined the assumption of a single universally-valid scientific and technological tradition by offering evidence of alternate, localised ways of knowing scientifically. Furthermore, they have documented how the modern European 'utopia' of a perfectly coherent account of nature's regularity and order is beginning to take on the character of 'tragedy of the commons' (Lloyd, 1833; Hardin,

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1968, for a review see Chronaki, 2009). In other ways, Swanson (2013a) has noted how the promotion of the study of mathematics and science has been touted as necessary in the modernist call to 'fix Africa', thus not only contributing to ongoing deficit views of the continent, but advancing the colonialist project in that context rather than undoing it. Under this mantra, mathematical and scientific discourses not only "enclose an African commons but (they) reproduce a colonized Other under the auspices of benevolence" (p, 337). In referencing Neil Turok's message that "the next Einstein will be African", a particular postcolonial patronage is produced towards the African continent through the authority vested in Western science and mathematics:

'In his TED broadcast, Turok (2008) promotes the mathematical sciences for "talented young Africans" as a panacea to all Africa's 'ills' and claims that "by unlocking and nurturing the continent's creative potential, we can create a change in Africa's future".[i] Here 'Africa's future' depends on access to Western-endorsed mathematics.[ii] Mathematics has the power to 'know' what is best for Africa. If more 'talented young Africans' (and here 'talent' assumes exclusively 'mathematical talent', reifying this form over others) succeeded at mathematics, then Africa might be "fixed". For Africa to be awarded a construct of 'success', and only in Western-European terms, it must produce an Einstein. In other words, talented Africans must mimic Western scientific heroes. They need to emulate Western-European scientific discourses that have sole currency in the global modernization project. There is no other way to be 'successful' other than in these terms and judged from the dominant Western-European gaze'. (Swanson, 2013a, p. 338)

Throughout the years, the categories of 'woman' or 'black' have become the subject of an extensive literature mainly through the accounts of travellers, missionaries and colonial officials. Chronaki (2009) discusses how Andrea Cornwall (2005), in her review of postcolonial feminist studies in Africa during the last three decades, explains that efforts to 'read' women range from studies that tend to define women as invisible, weak, and powerless to studies that challenge stereotypical assumptions about women's ability to participate in economics, mathematics and politics. Such representations are often firmly— but tacitly—located in a Western feminist perspective and evoke contradictory images, while their relevance and utility have been increasingly questioned by activists and academics. Postcolonial feminisms differ from the liberal, radical or socialist feminisms as they focus mainly on conceiving gendered and power relations within global political, economic and social programmes. They interrogate the assumption that the liberal pursuit of progress, development and colonialism are distinct and dominant projects. Thus, the question is how the distinctive concerns of postcolonial feminisms call for distinctive approaches to questions of science and technology, and call for a revisiting of children and adults' relation to mathematising via a feminist de/postcolonial lens (a discussion considering Spivak, Harding and Haraway can be located in Chronaki, 2009, 2011).

While anticolonialism has been touted as having some relevance to mathematics

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education discourses, there has been little attention given to postcolonial and decolonial thinking in the ways in which it can offer an epistemic critique in relation to the nature of mathematical knowledge and the process of mathematising, informed by colonial relations and politics of knowing. Institutional neo-liberal demands for fulfilling modernisation agendas have set the terms for global economies by increasing, in analogy, the monitoring and regulating of individuals, groups and targeted communities. Such measures serve to perpetuate the global neo-colonial project.

The current conception of societal development, framed as 'economic progress' within the neo-colonial project, has excluded a range of other possible meanings and ways of engagement. This has been the experience of mathematics education in its increasing standardisation across the globe in assessment regimes, curricula, and pedagogy. This 'standardisation' has been invested in power, suppressing the cultural and localised ways of knowing in majority world contexts or global South via 'development' agendas, while installing the values, codes and epistemic relations of the minority world or global North 'as universal'. Development as a concept presumes a *need* for development on the part of the targeted communities and the individual through adequate pedagogy (Chronaki, 2011). In this sense, any development programme situates the subjects and the communities that are ostensibly aided as 'lacking' and in need of assistance. At the same time, political discourses within developing countries often frame the needs of their (often black and/or female) citizens in terms of deficit and economic lack, blaming their citizens for their own and the country's economic 'failures' for which national school mathematics results become the weapon (Chronaki, 2011, Swanson, 2013b; Swanson & Appelbaum, 2012).

Considering the global social imaginary of the current neoliberal world, it may be timely to bring some de/postcolonial theoretical concepts to bear on mathematics education in global development contexts in providing a geo-political focus that more centrally considers the role of the nation-state, the geo-political imaginaries of empire and the broader neocolonial/neoliberal global(ising) condition in respect of mathematics education in global context. For example, in a study with Roma young girls aged 10 to 12, it was noted how the context of selling and buying goods as a thematic approach for arithmetic became an explicit colonial act (Chronaki, 2005, 2011a). A preliminary analysis of episodes of Roma girls' interaction with the teacher revealed how gendered and racial discourses became instrumental in valorising the importance of those girls' engagement with the arithmetic task. Specifically, the teacher comments on how learning certain arithmetic operations well will protect them from thieves, and on how the process of learning such operations requires a certain disciplining of the body and mind and avoiding a focus on beauty, eating or chatting. Whilst the girls laughed at these comments, from a feminist postcolonial viewpoint, one may critique how the teacher so easily employ certain racialised (i.e. Roma people are thieves) or gendered (i.e. Roma girls think of marriage and not work) stereotypes

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in order to convince them of the necessity of doing mathematics. Through this, such arguments turn, easily, into colonializing assaults on Roma girls' identities.

Some post/decolonial ideas valuable to move forward a critique in mathematics education are inscribed around such foci as (for example): centre-periphery discourses, loss and exile, disavowal and dispossession, epistemic violence, epistemic hegemony, epistemic suppression, epistemic racism, abyssal thinking, representation and voice in geo-political context, othering and exoticism, global social and ecological injustices, discourses on dominance and the subaltern, benevolence and salvationist discourses, global/local a/symmetrical relations, cultural imperialism; and the problem of 'dividing the world' into distinct categories or dichotomies (East/West; South/North; developing/developed; margins/centre; majority world/minority world). These can offer opportunities to provide frames of reference with which to converse with mathematics education from a wider geo-political and global justice-oriented perspectives that pave the ground for both a process of decoloniality and postcolonial critique (Chronaki, 2008; 2011, Swanson, 2013a,b; 2017).

CONCLUSION

As already mentioned, mathematising and demathematising have been given some attention in relation to social processes of mathematics education via such work programmes as ethnomathematics, critical mathematics education, or, realistic mathematics amongst many others. They have done much to underscore an interpretation of mathematics as being invested in cultural, historical, economic and social norms and values. Critical mathematics education, in particular, has pushed the conversation forward in considering mathematics in its broader political enterprise, but the theoretical concepts borne from de/postcolonial thought situate conversations on mathematics education in terms of contestations between global political imaginaries, whilst bringing into play the epistemic and ontological implications of such political considerations in the local contexts. A de/postcolonial critique begins to reverse the symbolic violence of Northern -emanating discourses within the mathematics education field, by introducing the thought of theorists, such as Spivak, Harding, Haraway, Mignolo, and Quijani, that hail from the global South. It opens up the opportunity to consider global ethics and democracy in relation to mathematising activities and discourses. As such, it also brings in the sphere of the geo-political while attending to the local or individual level when considering culture, gender, socio-economics and class, amongst other difference discourses, in historical and political contexts and their investments in global social relations of power.

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[i] See Dowling's 1998 critique of Paulus Gerdes' assumptions in the use of such ethno-mathematical language applied to another mathematical and African cultural context.

[ii] See Bishop, 1990, on mathematics as a form of Western imperialism.

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RACIALIZED SYMBOLIC VIOLENCE – AN EXAMPLE

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Abstract: Racialized symbolic violence is an (evolving) theoretical framework grounded in Bourdieu's social reproduction theory. This framework, when utilized in analysis of qualitative data, identifies tacit societal narratives that shape people's lives but typically go unexamined. We acknowledge that race is at the center of many of these narratives, and our aim is to bring it to the forefront of analysis. In this paper, we present analysis of the views and experiences of student tracking applied to case study data from a New York City Teaching Fellow in secondary mathematics.

INTRODUCTION

This paper is a follow up to a recent research paper (Cooley, Brantlinger, & Hannaford-Simpson, under review) in which the authors put forth the concept of *racialized symbolic violence*, a nascent theoretical tool for understanding tacit acts of domination against people of color in mathematics education. The theoretical tool is further elucidated in this paper to show its adaptability to research in mathematics education; it is forwarded with an example from our data on the issue of tracking in mathematics and the question posed to teachers: who has the ability to learn mathematics? Qualitative case study research methods are employed to examine how alternative teacher certification program (ATCP) mathematics teachers understand and react to tracking, and to describe their perceptions of student mathematics ability in their responses to open-ended interview questions about tracking. The research question posed is: *In what ways do ATCP mathematics teachers make sense of student tracking in NYC public schools and how are these racialized?*

CONTEXT FOR THE STUDY

NYC has led national trends in the increasing segregation of U.S. public schools. In 2012, the *NY Times* reported that half of NYC's 1600-plus public schools were over 90% Black and Latino, and among the most segregated schools in the country (Fessenden, 2012). Most NYC public schools are high-poverty and serve a majority of low-income students of color and ethnic minorities.

The NYC Teaching Fellows (NYCTF) program describes itself as one of the "most selective" ATCPs whose goal is "to bring great teachers to the students who need them most" in high-needs subjects such as math (NYCTF, 2017). Since 2000, upwards of 25,000 select ATCP teachers have entered NYC public schools, including *more than 70%* of all new secondary math teachers (NYCTF, 2014; TFA, 2017).

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Background Context: Racialization of Tracking in Mathematics

Among the entrenched, tacit narratives in school mathematics is the notion that math ability is innate and placement in high tracks is strictly meritocratic. This is important not least of all because of the pervasiveness of tracking; 75% of all U.S. students are tracked in math, a fairly constant statistic for over two decades (Loveless, 2016). While educators debate whether tracking is beneficial or harmful for some or all students, research strongly suggests that it is racialized. For example, as of 2009, Black students composed 16.7% of the U.S. student population but just 9.8% of those in honors or gifted programs. Similarly, Latino students were 22.3% of all U.S. students but only 15.4% of those enrolled in gifted services (USDOE, 2010). Additionally, Black and Latino students are much less likely than white or Asian students to be enrolled in Algebra I in the 8th grade, a commonly accepted benchmark for completing AP Calculus (USDOE, 2014). Further, Black students are over-represented in low tracks (Mickelson, 2001). Research points to racial bias as a reason for these persistent disparities. For example, Grissom and Redding (2016) explain that even among students with high standardized test scores,

> Black students are less likely to be assigned to gifted services in both math and reading, a pattern that persists when controlling for other background factors, such as health and socioeconomic status, and characteristics of classrooms and schools (p. 1).

Research has shown that subjective decision-making influences who is deemed worthy of honors and high tracks, as do race, behavior, social class, language, and gender (e.g., Anderson & Tate, 2008; Berry, 2005; Mosqueda, 2010; Oakes, 1990). Berry (2005) argues that teachers' judgments about student ability for high-track math classes, conscious or unconscious, seem to be based on behavior, i.e. good math students are passive. Referencing Boykin (1986), Berry persuasively argues that some African American children have strong psychological and behavioral verve, an "intellectual and behavioral vibrancy evidenced by engagement in high-energy activities" (p.15) which is in conflict with school math power structures.

The process for placing students in high track math most often begins with teacher referral (Donovan & Cross, 2002). This can disadvantage students of color if a teacher holds lower expectations for them (Elhoweris, Mutua, Alsheikh, & Holloway, 2005) Grissom and Redding (2016) explain that,

> Teachers of color are more likely than White teachers to exercise discretion on behalf of students from their same racial or ethnic background - and similarly for White teachers and White students - such that students' probabilities of being assigned are higher with own- race teachers. Researchers have, in fact, provided suggestive evidence of such a relationship, finding that Black and Hispanic students are better represented in gifted-and- talented programs in schools that employ larger numbers of Black and Hispanic teachers (e.g., Nicholson-Crotty, Grissom, & Nicholson-Crotty, 2011; Rocha & Hawes, 2009) (p. 2).

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Given the prevalence and high stakes of tracking and evidence of racial bias, it would seem ATCP math teachers, predominantly from high-tracked backgrounds, would be exposed to the debates in order to make informed opinions and to reflect on their own possible biases. Without this edification, the hegemony of math tracking and its exclusion of many students of color continues to proliferate without critical consideration of its impact on society and on students of color in particular.

RACIALIZED SYMBOLIC VIOLENCE: A FRAMEWORK FOR ANALYSIS

Racialized symbolic violence is an evolving theoretical tool that draws from Bourdieu's theory of cultural reproduction (Bourdieu, 1986) and a growing field of research on the racialization of mathematics education (Martin, Anderson, & Shah, 2017). We recognize race as a social, political, and historically contingent construct; however, "race" is real in that racial categories serve to maintain power, or lack of power, among groups of people (Nasir et al., 2009). Racism refers to a social schema "in which economic, political, social, and ideological levels are partially structured by the placement of actors in racial categories or races" (Bonilla-Silva, 2001, p. 37).

Symbolic Violence: Bourdieu in Mathematics Education Research

Bourdieu's theory of cultural reproduction centers on the struggles related to power relations between players in a field, such as the field of mathematics education. Players agree that the "game" is worth playing, and the rules they collectively ascribe to are the basis of truth and that which is valued (Bourdieu,1990). We focus on three aspects of Bourdieu's conceptual toolkit: doxa, misrecognition, and symbolic violence. *Doxa* is a set of implicit narratives, or the rules of the game, whose central tenet is the (mis)recognition of arbitrary power relations (e.g. class, race, gender) that exist within a field as natural and the way things are supposed to be, thus perpetuating their power (Bourdieu, 1986). For example, "standardized test scores represent intelligence" could be considered a doxic belief in the field of school math. *Misrecognition* is the manner in which actors in the field unconsciously view doxa; they accept its tacit narratives as legitimate and reinforce their arbitrary power structures as valid. This misrecognition, or normalization of the dominant culture as superior, is what Bourdieu called *symbolic violence* (Bourdieu, 1998). As an illustration, William, Bartholomew, and Reay (2004), report of a development that went from the students talking about "getting" a certain score on a standardized math exam to "being" a certain score, demonstrating how these messages are internalized and result in symbolic violence:

> Students were increasingly valued not for their personal qualities, but rather for what they could contribute to the targets set…by the school district….the results of these assessments came to be bound up with not just what kinds of careers might be open to them, but who they were now, who they could be, and even their moral worth (p. 58).

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This leap from acceptance of dominate hierarchical structures as natural to the construction of self-worth demonstrates the power of symbolic violence as a tool to understand how discourse can affect learning, teaching, and identity formation.

Symbolic violence is *racialized* when the constructed meanings for race become highly salient in the doxa (Martin, 2006). *Racialized symbolic violence* in math education includes the misrecognition of doxic beliefs that favor white middle- class culture and values, reproduce racialized societal narratives about ability and competence, and conceal their power structures in the normalization of whiteness.

Racialization of Mathematics Education

While the "mathematics for all" rhetoric is widely embraced, race, racism, context, identity, and power are rarely considered in math education research or policy documents when they are in fact central to the learning and teaching for all children, and for Black, Latino and other minority students specifically (Larnell, 2016; Martin, 2006, 2007, 2013; Pais & Valero, 2012; Valero, 2004).

However, an emerging group of mathematics education researchers and scholars focused on the intersections of math education with race, class, identity, and power are calling for liberatory mathematics education (Martin, 2018), to turn away from deficit views and focus on the success and brilliance of Black children (e.g., Berry, 2008; Martin, 2009; Martin, 2018), and a revolution in mathematics education (Gutiérrez, 2017, 2018). While their voices are still relatively small in number, the mounting call for a complete rethinking of the structures of mathematics education so as to break with antiblackness and white supremacy is growing. We believe the data provided here add to the evidence that the current mathematics education structures are not serving students of color and ethnic minorities in urban public schools.

METHODS

Study Design

We utilized case study research methods for the design and analysis. The data is drawn from a 2-year study of 9 NYCTF math fellows trained at one of 4 colleges. From 2006-08, we observed and videotaped each of the fellows in their classes 15-20 times, conducted post-observation reflections and interviews, collected course materials, and conducted two in-depth, semi-structured interviews in 2006 and 2007. We focus on one case study here and her views and experiences with tracking.

Participant

We chose Karen as the focus of this analysis as she is representative of a good- sized cross section of NYCTF math fellows entering in the years 2006-07 (Brantlinger & Cooley, under review). From a Midwest, upper-middle-class white family, Karen was 24 years old and attended a select state university on the East coast for undergraduate studies and a prestigious university outside of the U.S. for her MA.

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She majored in International Communications and Religious Studies respectively. This paper is based on observations of her first year teaching. Karen was at a middle school serving 97% Black and Latino students and 75% students from low-income families[1]. There we observed her teaching Math A, which included algebra and a wide range of topics, and the course required for graduation. Though Math A was designed as a three-semester course, Karen's class was to complete it in two and then take the standardized exam. Due to the truncated time, she was under the impression at the beginning of the school year that this was an honors class. There was some form of tracking in the school, although the criteria were vague. The class we observed reflected the demographics of the school and held about 30 students.

FINDINGS

Doxic Beliefs of Tracking

Karen believed a teacher's role was to impart knowledge via direct teaching and that good students, like her, were passive receptacles. She presented herself as an honors student, earned by merit, misrecognizing her educational opportunities as ability. Her students, who did not have the same white, upper-middle-class educational circumstances, were misrecognized by Karen as incapable and apathetic.

[1] Information taken from 2005/06 NYC DoE School Report Cards.

She revealed bias against low-tracked students, framing it as being empathetic:

> [H]ow depressing was it if somebody put you in the lowest reading group as a 2nd grader and you figured that out? Well, then you're gonna throw up your hands and be like,"Well, I'm the dumb one" (2006 interview).

Karen referred to a common doxic belief about low-track students, i.e. they lack intelligence. In these remarks, and other similar ones, she reinforced her belief that tracking is strictly ability-based and not subjective. In this way, however unwittingly, she protected her prestigious status as a high-tracked student. However, when asked about tracking in her classes, Karen responded by discussing behavior and not merit:

> R: How do you feel about tracking?
>
> K: Well, it caused a giant disparity in my classes. It was horrible actually. One class was just very behaved…. [A]nd the well-behaved kids got stuck in one class and then the other two classes were nightmares. So, it just was bad. If we had mixed up a bunch of them maybe all the classes would have been a little bit more tolerable.
>
> R: So, you would want to do that?

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> K: Yeah. I would have loved if some of my good kids were mixed in with some of those, like, Rikers Island kids (2007 interview).

Reflecting her (white, suburban) norms, Karen framed tracking in terms of student behavior and how it affected her ability to manage classes, similar to research that shows tracking is based on characteristics other than ability. Good students behaved in ways familiar to her, i.e. quietly receiving math information imparted by a teacher. She elevated her criticism with racialized language; her low-track Black and Latino students became "Rikers Island kids," referencing the infamous NYC prison, revealing a blatant racial bias, calling to mind the school-to-prison pipeline. This callous remark served to distance Karen from her students, reducing them to the status of the Other, and thrusting them even further, to the realm of outcast, prisoner.

Once a Test Score, Always a Test Score

Karen began with little knowledge of the school, students, or math and tracking in general. However, she quickly adapted to calling students by the numbers they received on the standardized exams: "I've been looking at my school's achievements and I think it said 80% of the school is a two or a one. So, we're knowing that they're not up to standards in terms of achievement" (2006 interview). She did not seem to have a full understanding of what the rating system was based on. However, she planned to use her students' previous test scores to group them within her classes:

> R: Do you think you need to internally track them in order to have different activities?
>
> K: Yeah, probably. I know at my school they all have files somewhere - this is what my professor has told me…. So, [there are] files on these kids [that] will tell me their previous math grades….and in this 1, 2, 3, 4 system what numbers they've gotten. They're tested twice a year, I believe [my professor] said….I'll be able to see what their numbers had been in the last few years…. I can sort of use those as a gauge right off the bat as who is gonna be my high achievers from past experience and kind of work on that (2006 interview).

We see from above that test scores were used to decide not only what students had achieved, but also what they were *capable* of achieving in the future. In this way, she reinforced the doxic belief about the value and power of testing that works against students, particularly students of color. Referring to them as numbers was acceptable and, as with the casual comparison to prisoners, it distanced her from the students.

> R: What do the scores mean?
> K: Well, it said it was at 1, 2, 3 and 4 and 4 is way above grade level, 3 is meeting and slightly above grade level, 2 is not meeting but approaching and 1 is far below. And the majority are 1's and some are 2's, a few 3's sprinkled in, no 4's in the whole school from what I saw in 7th and 8th, which is shocking and scary (2006 interview).

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Symbolic violence is manifested when students, in particular students who were low-income Black and Latino, who do not have the educational advantages or access to capitals necessary to score at or above their grade level in math, internalize the doxic beliefs that they have failed already and cannot catch up. Karen overtly repeated these narratives, messages that permeate society and that we have all been exposed to. Those unprepared to score well were perceived as a burden:

> K: …our school is one year away from being under review. So, this year they have to pass the test…. So, every math teacher was here over break teaching to try to get them ready for the test. But all it was, was the insane children that can barely sit still during class period wreaking havoc for a week over break (Feb. 2007).

Note that Karen referred to the students needing extra help as "the insane children that can barely sit still," again referencing behavior and not math ability.

Karen's Narrative: The Honors Class That Wasn't

As an illustration of how assumptions about students' tracked position may affect teacher beliefs, we offer the story of Karen's perceptions of her students. These students were expected to complete the Math A curriculum in one year as opposed to the standard three semesters. She assumed they were honor students. She initially perceived better behavior in this class, which she related to the rank:

> K: I just wish that all my other classes were as functional as this one. I feel like this class is my savior as I can try new things and do more challenging activities and not worry too much if something goes wrong because the class will not disintegrate into chaos. I use this class as my trial class most of the time (Nov. 2006).

> K: This is the only true honors class in the building. So, they need this class to get threes or fours to bring the school's average up (Jan. 2007).

Karen assumed her honors class would be passive as she was as a student and would have the requisite capitals to pass an exam. She placed similar assumptions on low-track students, disparaging their behavior and math ability: "But then my other classes are totally insane and they need almost one-on-one tutoring to get anything" (Feb. 2007). As the year progressed, her assumptions clashed with reality. Students did not behave or respond to the math as she had anticipated. Indeed, she found them "loud" and "out of control." When asked what this meant, she replied:

> Just like not doing what I ask. Like I guess there wasn't a lot of fighting….but I consider the out of control as in not doing anything in my classroom when I've asked repeatedly to get something done. I was surprised at the level of apathy that 13 year-olds would have and just didn't care about it (August 2007 interview).

In the excerpt above, we see that Karen expected complete silence. When students did not respond to her procedural direct teaching style, she blamed them as "apathetic," a common doxic belief about Black and Latino students in the

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American milieu. That is, when students did not conform to her white suburban norms or did not demonstrate math capital, Karen's dissonance quickly gave way to racial stereotypes. As it became clear that likely few of them would pass the exam, we saw Karen attempt to stop any of the students from taking the exam, the results of which would have been a reflection on her teaching:

> R: They are going to [take the exam] in June?
>
> K: No, but I'm told that they are, but I'm going to tell them later that they can't. They're never going to be ready; not if I stopped for two months and reviewed (Feb. 2007).

By late spring Karen had abandoned all hope of getting the students successfully through the curriculum and prepared for the exam:

> R: Are you giving the students the Math A exam in June?
>
> K: We're supposed to. I'm just going to pretend that I forgot about it, nobody else is talking to me about it. There's absolutely no way they're ready for it (April 2007).

We argue that in a white suburban community, a teacher would not consider opting students out of a standardized exam without parental permission or administrative knowledge. Relying on the perceived lack of parental power, she assumed no one would protest. In earlier research papers, Karen's lack of math knowledge and racial competency were documented (Cooley, unpublished; Cooley et al., under review). However, her shortcomings were not considered as having any impact on student learning; it was the fault of students for being "apathetic" and unlikely to attend college or study math. Karen rationalized that the lack of student success was a result of not actually being honors students, i.e. low-level students who could not learn:

> R: So, you are giving the Math A exam after all?
>
> K: …[N]ow [the principal] ordered the test and he said they're taking it…. They don't all have to take it, somebody has to take it and I was like, this isn't even the honors class in school. It's the honors class in the academy. The honors class in school is somebody else's and they never force her to teach Math A. So, the honors kids…aren't going to be taking this test. So, I told [the Math Chair] like what happens if they all fail. Because I'm a first-year teacher….and you're trying to squish 3 semesters into 2, in a class that's not really advanced. They're just 8th graders. And they said, oh well…Now I don't really care because I'm leaving. So, you're not gonna hold me responsible for this train wreck (May 2007).

Karen's dissonance with the "train wreck" of her teaching experience was soothed by blaming students' innate inability to learn math. Her assessment of students as "just 8th graders," is indicative of the superlative status given to those placed in high tracks. In the end, Karen described spending 5 weeks preparing for the exam by tutoring 4 or

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5 students whom she thought could pass, while the rest completed portfolios to use as evidence that they had learned some math and could, perhaps, be allowed to go to the next grade level. All of her students took the exam; few of them passed.

The following summer, the researcher followed up with Karen about her views of tracking. She often spoke of how much she had enjoyed being high-tracked herself in school. However, Karen indicated that there were no students at her school worthy of the resources of being high-tracked, as they were all “twos”:

> R: And so why doesn’t [tracking] translate over to your own teaching situation?
>
> K: Probably because thinking about [the school], there’s no group of children that I saw that were so far above the rest that they should have been separated….80 or 90% of…. Students were level 2 so they were all basically just a little variant of below average (2007).

Karen’s comments illustrate the barriers test scores place for students who have not scored at or above average and, indeed, how the resources and attention they need are denied to them as a result. Further, being a high-tracked student had opened opportunities for Karen and she described enjoying the experience. However, her opinion was that none of her students were worthy of such elite educational settings.

Summary of Findings and Discussion of Racialized Symbolic Violence

Karen was unfamiliar with standardized testing as it was not part of her Midwestern, white, suburban educational experience. However, she adapted quickly to a scoring system that preserved her privilege and validated the manner in which she was credentialed. She held strong preconceived notions about low- and high-tracked students and intelligence. Yet, she responded to questions about tracking by discussing student behavior, providing support to research that suggests teachers recommend students for tracks based on behavioral (i.e. white, middle-class) norms rather than ability (Berry, 2005). When Karen initially believed her students to be typical high-tracked students, i.e. like her, she understood their abilities and needs through what Bourdieu and Passeron (1977, p. 202) call “aristocratism of talent” wherein it is tacitly understood that those who are high-tracked deserve the quality of their educational setting and it should be preserved, while low-tracked students deserve a lack of opportunity. For example, Karen’s argument for detracking countered a common argument espoused by teachers; those who support tracking often do so in order to confine behavioral issues from “spreading” to those in higher sets (Archer et al., 2018). Karen flipped this narrative and instead wanted students who behaved more like her to “rub off” on her “Riker’s Island kids.” This can be understood by the fact that Karen categorized the entire student body as undeserving of high tracks and, thus, behavior management was the most important issue.

Archer et al. (2018) found that tracking was so naturalized that even those relegated to low tracks did not question its inequity. This is supported by

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Karen's comments about student apathy, marking a sense of defeat and acceptance of failure, demonstrating the potential for racialized symbolic violence caused by students' hierarchical placement in the classrooms of ATCP math teachers.

Karen began to realize that her students were not having success and her narrative changed accordingly. Rather than taking responsibility or trying to understand them as students who could learn math, she relied on typical racial stereotypes to construct them as behaviorally deviant and intellectually incapable, a "train wreck." Her initial reaction was to employ passive aggressive techniques of avoidance. Her belief that she could deny students an opportunity to take the exam without parental permission demonstrated how little she valued her Black and Latino students' families. As Welner and Burris (2006) assert, "when parents of low-track students are politically invisible, they are too easily ignored" (p. 97).

Poor test results could have threatened Karen's reputation as a "good" teacher or even her job. As James (2015) discusses, "misrecognition is "functional" rather than simply aberrant or some sort of unintended by-product" (p. 100). Karen decided that society, and the school administration, did not expect many students to pass. Therefore, she selected 4 or 5 students to be worthy of tutoring while the others were given a perfunctory option to perhaps be allowed to move to the next grade level.

Karen demonstrated a lack of empathy when she realized she would be leaving and would not have to deal with consequences of the "train wreck." This also demonstrated a potential detriment to the students' future mathematical progress. Archer et al. (2018) note that following a Bourdieusian perspective, "from the point of view of societal elites, the 'wastage' of working-class and Black talent that is generated by such [tracking] practices is a small price to pay for social reproduction" (p. 137). Archer et al. (2018) further found that the implications of tracking are not limited to attainment but also to self-confidence and identity. If, as a result of Karen's jockeying with taking or not taking the exam and projecting a sense of failure about the students, they had less self-confidence and viewed themselves as not having math ability, their "have not" positions would become internalized, establishing math as "not for the likes of us," and manifesting racialized symbolic violence.

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NOTES FROM THE FIELD: CREATIVITY KIDNAPPED

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Abstract: The essay discusses the process through which the subject of creativity has been limited in scope by the imposition of external professional standards. The primary focus of creativity research in mathematics is on the relationship of creativity with giftedness, which habitually is determined by college preparatory exams such as the scholastic achievement tests 'SAT' and other fluency-intelligence exams. This focus eliminates attention to the creativity of the 'rank and file' learners to the degree that almost nothing is known about their creativity. That's the content of Creativity Kidnapped. The second part of the essay offers the avenue for Liberation of Creativity, which is anchored in the concept of bisociation by Koestler, (1964), that is in the creativity of the Aha!Moment - the experience of creative insight.

INTRODUCTION

This theoretical essay investigates two questions:

1. What is the process through which creativity has been kidnapped by the network of professional standards and judgements, and
2. How to liberate creativity through the change in the theoretical and practical approach?

This essay is the outgrowth of reflections by the Teaching-Research Team of the Bronx about its attempts to introduce STEM creativity into the NSF INCLUDES network of chosen (circa 100) STEM projects aimed at the increase of participation by the "underrepresented" student population in US.

The TR Team of the Bronx has been investigating creativity for 8 years and the NSF INCLUDES program attracted our attention because not a single project took creativity as the principal medium through which to increase student participation in STEM careers, and only 8% of the projects' abstracts listed creativity as a by-product of their projects. This absence of attention to creativity is especially striking at the background of increasing number of voices from industry which list it as one of the primary expectation for new hires. Creativity and Innovation have become the buzz words of 21st century, but Mathematics Education is trailing far behind especially in the context of "underserved".

The approach we take here in addressing these questions is aptly characterized by mathematics researchers as critical mathematics pedagogy, which is an approach to mathematics education that includes a practical and philosophical commitment to liberation (Tutak et al,2011). Approaches that involve critical mathematics pedagogy give special attention to the social, political, cultural and economic contexts of oppression, as they can be understood through mathematics (Frankenstain, 1983).

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They also analyse the role that mathematics plays in producing and maintaining potentially oppressive social, political, cultural or economic structures (Skovmose, 1994). Critical mathematics pedagogy demands that critique is connected to action promoting more just and equitable social, political or economic reform.

In the Section 2 we review research on creativity and describe the process by which educational profession had 'kidnapped' the knowledge about creativity for the benefit of knowledge of giftedness, which is traditionally defined by the high scores in high stake standardized tests such as SAT or IQ scores. The kidnapping of creativity has left researchers and practitioners without an approach which can address the creativity of "underrepresented and underserved". Section 3 focuses on the process of liberating creativity with the help of bisociation theory of Arthur Koestler, as the theory of 'creativity of and for all'.

CREATIVITY KIDNAPPED: RESEARCH ON CREATIVITY

Liljedahl (2013) traces much of the research on creativity in mathematics to the work of Henry Poincare noting that his thoughts, *"....stands to this day as the most insightful and reflective instances of illumination as well as one of the most thorough treatments of the topic of mathematical discussion, creativity and invention".* Koestler (1964) quotes long passages of Poincare to portray a metaphysical quality of 'intuition' that is intimately involved in the creative process and the need for all students to experience their own intuition within this creative process. Liljedahl (2013) notes that Hadamard combines the inspiration of Poincare with Gestalt theory to advance a stage theory of creativity involving; preparation, incubation, illumination and finally verification which is employed to this day in understanding the creative process. Koestler (1964) postulates a specific mechanism 'bisociation' to refer to the creative process that underlies the illumination 'Aha' of moment, which bridges the gap between often unconscious-intuition and the conscious problem solving process. For Koestler as well as Liljedahl (2013) this process has an affective component. Research on creativity however, soon departed from its roots on 'intuition' leading to illumination and instead became focused only on the novel or original products of creativity and methods to assess potential for such thought. Thus, creativity research in mathematics education often referred to as little 'c' creativity as opposed to 'C' creativity of research mathematicians has reduced to the relationship between creativity and gifted students. Recent publications on creativity (Leikin and Sriraman, 2016, Leikin et. al. (2017)) emphasize the creativity of the gifted students defined through fluency-achievement on standardized tests such as the Stanford-Binet IQ test, (Leikin et. al. (2017), Karp (2017)). According to Wagner and Zimmerman (1986) two other tests commonly used to identify giftedness by high achievement are: The Scholastic Aptitude Test (SAT) and the Hamburger Test fur Mathematische Begabung, 'HTMB', a set of seven problems designed especially for talent search. Research on gifted students often focuses on their original or novel products as measured by tests based upon ideas of two American Psychologists: (1) Joy P. Guilford's distinction between convergent and divergent thinking and (2) Ellis P.

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Torrance's work focusing on characteristics such as fluency, flexibility and novelty. We note that Koestler (1964) agrees with the novel criteria while he disapproves of fluency as being something one can learn without understanding and is sceptical about flexibility if it is learned without understanding.

This means that giftedness is traditionally found among students who are very good in school measures of achievement. This approach filters out those students who might be creatively gifted, but who are not fluent in mathematics language and procedures. What does it mean to be mathematically creative, but not fluent in mathematical language? It often means finding oneself in a remedial classroom filled with many other talented students who don't know their own creative gift, thus are destroying it under emotional or material poverty stress. And that's why Prabhu (2016) notes on the basis of her teaching-research experience that *"the creativity in teaching remedial mathematics is teaching gifted students how to access their own creativity".*

A noticeable assault on creativity started in 2001 when Anderson, L & Krathwohl, D. with their team published the revised Bloom taxonomy based on the new research conducted between 1995-2000 (Anderson et al 2001). The process resulted in significant changes in the original Bloom taxonomy. The comparison of the two is shown in Fig. 1 below. There are three essential changes in the revised taxonomy: (1) use of verbs instead of nouns, (2) changing Synthesis to Creativity and (3) the change of order between from Synthesis→ Evaluation in the original Bloom's taxonomy to the Evaluation→ Creativity in the revised one.

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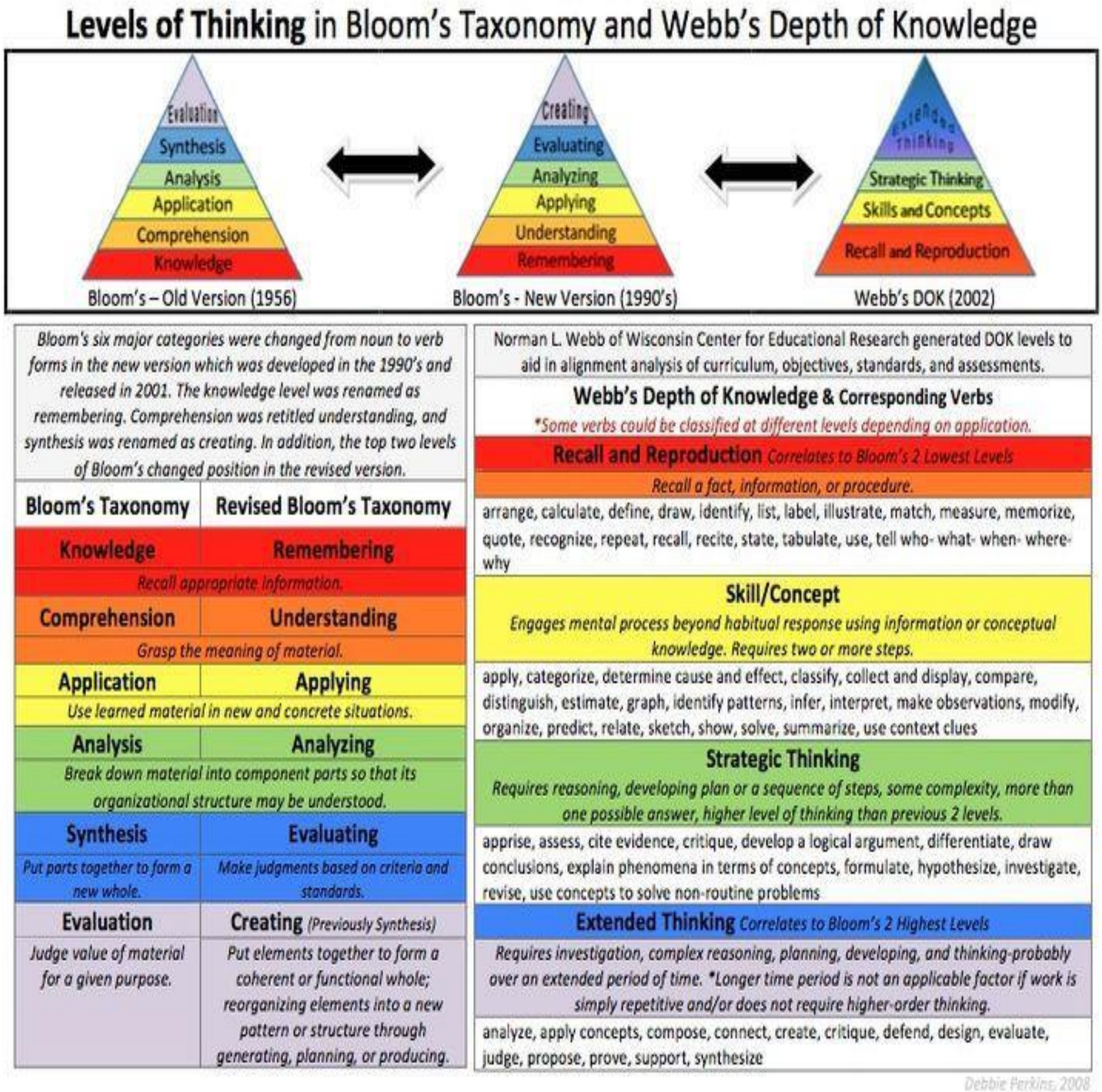

Figure 1 Bloom's revised Taxonomy

The main issue as it relates to kidnapping creativity is not in the point (1) nor (2) - creativity often involves, of course, synthesis. The kidnapping process starts in the point (3) – the change of order between Evaluation and Synthesis/Creativity in the revised taxonomy. Note that Evaluation in the original taxonomy, which comes after Synthesis is characterized by the statement "*Judge value of* [obtained] *material for a given purpose*" while the evaluation in the revised one introduces a different element, significantly, before reaching creativity of the pyramid: "*make judgements based on criteria and standards*". In other words, whereas Synthesis/creativity was the matter solely between the creator and the purpose of the creation in Bloom's taxonomy, the revised taxonomy imposed the limitation upon creativity by requiring it to be in agreement with professional standards and criteria outside of the creative process before being considered as creative thought. The adherence to the standards and rules was placed in advance of promoting learner's thinking, thus creativity-insight is not

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considered part of the learning process until one is able to judge the quality of one's thought products against external standards. This further removes the focus of educators on the process of creativity and insight towards the quality of the product. For this reason, this framework requires individual fluency as a pre-requisite to creativity and supports the myth that only gifted (very fluent) students can be creative by divorcing inspiring products of thought from the often messy process often filled with failure and wrong insights that occurs within both research and classroom learning. Learning and creativity within a social situation is supported by students presenting, explaining, clarifying and elaborating on their own and other's ideas, such student reasoning and critical analysis is at the heart of a creative learning process (Herschkowitz et. al. (2017), Baker (2016) Prabhu (2016)). Indeed, incorrect insights often give rise to 'teaching moments' within the classroom Dias (2016). Another situation in which the creative process occurs well before fully conscious judgements are visible is during the transition from spontaneous intuitive knowledge to more algebraic structural thought, this teaching-learning process often requires conscious thought on semi-conscious actions Baker et. al. (2016).

Keyung Hee Kim (2012; 2008) has made recently a significant and relevant observation of the decrease in Creative Thinking Scores on the Torrance Test of Creative Thinking. The significant decrease of Fluency and Originality scores was observed between 1990 and 2008. The largest decrease was for the kindergarteners through third graders; the second largest decrease was for four through six graders (p.292). These very students have now become the majority of the population entering community colleges across the nation. Kim (2008) reviews the studies and theories that have shown that once underachievers are placed in an environment that fosters their creative needs with motivation, mentors, understanding, freedom, and responsibility, they can become highly productive. She notes that "*many gifted students are underachievers and up to 30% of high school dropouts may be highly gifted*". We suggest the imposition of professional standards upon the definition of creativity has eliminated a sizable sample of creative 'rank and file' students from professional research on creativity. Sriraman et al (2011) point out that understanding of "*The role of creativity within mathematics education with students who do not consider themselves gifted is essentially non-existent (p.120).*" We not only don't know the nature of creativity of 'rank and file' students, but we also don't have tools to investigated it. The tools developed in the context of mathematical giftedness cannot fit the tools needed here (Czarnocha et al, 2016).

CREATIVITY LIBERATED

The act of creation is... The act of liberation...

The defeat of habit by originality.

Arthur Koestler, *The Act of Creation*

The search for "creativity of and for all" is vital to increase participation of females and underserved students in STEM fields. Our own team, the Teaching-Research

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Team of the Bronx has focused attention on the commonly known Aha! or Eureka experience as the manifestation of creativity in our classes. Vrunda Prabhu (2016) coordinated the bisociation theory of Aha!Moment of Koestler (1964) with classroom events in remedial mathematics, showing, together with Liljedahl (2013), its high positive motivational and cognitive value. The definition of bisociation as *the spontaneous leap of insight which connects two or more unconnected frame of reference*, (Koestler, 1964, p.45) makes us experience reality along two planes at once, and offers hints how to facilitate Aha!Moment. Dias (2016) studies how a creative learning environment can reach a multiple repeating student allowing him to pass a college mathematics class, Sheffield (2017) chronicles the struggle of one student's transition from 'school induced retardation' to giftedness. Baker et. al.. (2016) reviews the work of constructivist researcher to solve the 'learning paradox' that asks how one builds new abstract structure out of nothing and integrates this with Koestler's bisociation theory to study the process of shared learning experience to create meaning of new math concepts within a classroom setting. Hersckowitz et. al. (2017) studied peer-peer teacher led dialogue to create shared meaning within a classroom.

(Czarnocha, 2018) analysed the issue of the extend of internalization during 'Aha' moments within socially shared as well as individual settings by measuring the Depth of Knowledge (DoK) reached during the moment of insight The DoK assessment is based upon the coordination of bisociativity with stage theories of development by Piaget, in particular the Piaget Garcia 'PG' Triad of conceptual development (Piaget and Garcia, 1987). The question here is how to free creativity from extraneous standards and criteria and thus, a focus on the nature of the creative insight itself is essential. Since the intrinsic quality of bisociative-insight is connecting unconnected frames of reference or matrices of thought, that is building a schema of thinking, the natural approach is to look upon it from the viewpoint of the development of the schema. That's why the PG Triad as the theory of schema development is so useful as a DoK assessment tool. It measures content of insight as the change in understanding during the 'leap' of insight. The analysis of an Aha! Moments reached by students in remedial mathematics classrooms (Czarnocha, 2018) shows the inadequacy of standard taxonomies of Bloom, Anderson and Krathwohl revised taxonomy of Bloom or DoK instrument such as done by Webb (2002) due to their static character. Absence of a dynamical instrument to trace the development of concepts from one level to another makes it difficult to analyse increased levels of understanding due to insights. The process of understanding present during an Aha!Moment may involve several dynamic cycles of movement from elementary understanding of mathematical concepts through their analysis to the creative synthesis, all possibly in an instant. Coordination of bisociativity within the work of Piaget-Garcia triad offers an instrument of analysis responding to the need to assess and analyse student's creative learning.

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A central observation which allows for the investigations of creativity of every student was stated by Shriki (2010) who indicates two of the main reasons for the absence of research into creativity of the rank and file: *"(1) The significance of creativity in school mathematics is often minimized because it is not formally assessed on standardized tests, which are designed to measure mathematical learning. (2) The problem with relating to* students' work as 'creative' is rooted in the definition of creativity as a useful, novel, or unique product...Although according to the traditional view of creativity; students' work would not be considered as creative, the researchers agree that students' discovery may still be considered creative if we examine the issue of creativity from a personal point of view, namely, whether the students' discoveries were new for them." This important broadening of the domain of creativity to include its subjective component is supported by several statements by mathematicians and scientists based on experience:

(1) According to French mathematician Hadamard: "*Between the work of the student who tries to solve a problem in geometry or algebra and a work of invention, one can say that there is only the difference of degree, the difference of a level, both works being of similar nature*" Hadamard, (1945, p.104).

(2) Applebaum and Saul, (2009) observe "*a remedial algebra student can exhibit creativity as often and as clearly as an advanced calculus student" after which they assert* (Assertion III) "*Creativity in mathematics can be found at any level of the subject matter, and at any level of mastery*."

(3) Koestler asserts "*minor subjective bisociative processes…are the vehicles of untutored learning* (p.658)." The closest classroom approximation to such conditions of untutored learning, allowing students to 'experience creativity for themselves' is the Discovery method of teaching in many of its variations, i.e., inquiry and/or guided discovery, problem solving, project based learning, or faculty/student research. The TR Team of the Bronx has studied Koestler bisociative creativity within the framework of Polya's problem solving and inquiry based learning focusing on the synthesis of planes of reference. For example through the coordination of different actions, the reversal or a process of the application of an action to more structural or abstract object outside the students comfort zone, and the comparison of different strategies-actions to solve a problem. Questions that arise are (1) where does creativity come into play within critical thinking during shared learning in a school environment? and (2)when does bisociative experiences within a social situation lead to internalization of knowledge?

CONCLUSIONS

Steffe (2017) muses over the fate of the 'constructivist revolution' to reform mathematics education in the USA noting that the common core state standards-mathematics 'CCSS-M' has put an end to the 'math wars' that developed after constructivist pedagogy was implemented in California. However, as noted by Sheffield (2017) the CCSS-M does not list creativity as a standard and arguably the

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rigid standards leave little place for it. We argue that creativity within the classroom dialogue is paramount to reviving interest in mathematics and that the affective component of creativity is a vital in this effort. There is much to be done, issues that arise and continue to obscure the focus on the creative process of learning within the STEM classrooms and STEM experiences include: (1)the continued focus on criteria for giftedness within academia and mathematics education as a pre-requisite for creativity, (2) absence of research knowledge of the creativity of 'rank and file' especially for female and underserved students, (3) the lack of interest in government agencies with creativity that sponsor STEM projects to assist underserved and students (4) lack of inclusion of creativity within standards for education and (5) educators views that thought processes are creative only after they have been completed and verified against standards. The common theme for these decisions is the systematic and systemic process of eliminating access to creativity from the 'rank and file' learners by imposing, consciously or unconsciously, criteria extraneous to creativity, which limit creativity research within the teaching practice. In response, the TR Team of the Bronx (Prabhu and Czarnocha, 2014) proposed the bisociation theory of Aha!Moment of Koestler (1964) as the tool for the democratization of creativity. As a final comment we would like to offer the following observation. Both the decrease in creativity observed by Kim (2012) and work leading to the revised Bloom taxonomy started in the first half of the nineties decade of the last century. That is also the decade of the final world-wide victory of uniform globalization, which eliminated the previous highly disconnected bipolar world. Since the conditions for the creativity of Aha!Moment is the gap between two or more separate frames of reference one can conjecture that the process of kidnapping creativity was initiated exactly during the process of global uniformity.

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INTERROGATING DEFICIT VIEWS: STUDENTS AND TEACHERS OF MATHEMATICS IN REFORM CONTEXTS

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Within global neoliberal politics, teachers are constructed as being in need of reform, and within mathematics education, we perpetuate this notion through designing professional development and research projects promoting teacher change. With this framing, it is easy to blame teachers for the ills of mathematics education. Here I report on the ways in which teachers similarly blame their students, view them as problematic, and discursively use them as a reason not to engage with problem solving mathematics. Yet problematic students are produced in the wider, socio-political context and teachers are themselves constrained by this production. I ask in what other ways do we generate problematic and deficit views of various groups in the educational project.

INTRODUCTION

Mathematics education plays a key role in neoliberal politics (Montecino, 2018; Valero, 2017), as can be seen through the Organisation for Economic Cooperation and Development (OECD) interest in mathematics curricula and achievement world-wide. The publication of results from Program for International Student Achievement (PISA) pit nations against each other and provide them an impetus to improve their ranking, or at least maintain their position. Other OECD studies such as the Teacher Education Development Study in Mathematics (TEDS-M, e.g. Kaiser & Blömeke, 2013) focus attention on teachers via the quality of teacher preparation programmes. Driving such initiatives is the idea of mathematics as crucial in the knowledge-based economy.

This 'need' for mathematics and mathematically literate citizens runs alongside a discourse of *reform* in mathematics education (e.g. Gellert, Espinoza, & Barbé, 2013). Typically, this reform calls for teaching which is more student-centred, based on problem solving, and often involving collaborative group work (see for e.g. Koellner, Jacobs, & Borko, 2011). Underlying these reform drives is the assumption, often stated explicitly, that mathematics education requires fixing, and this generates the need for curricular reform, redevelopment of initial teacher education, and programmes of professional development (PD). Note that most of these solutions to the problem of mathematics tends to position the *teachers* as being in need of reform (Graven, 2012), and as responsible for producing the desired results (Robertson, 2012).

Reforming teachers is a widespread project. In the U.S., 99% of teachers attend PD each year (Sztajn, Borko, & Smith, 2017), and this is likely similar in many other nations, particularly when performance evaluations are connected to PD. Typically it is *us*, mathematics educators, who provide this PD and thus we too contribute to the notion of teachers as being in need of reform. We may further this discourse when we investigate the success and sustainability of our PD programmes. In fact, we may produce the notion of teachers as problematic through our very framing of the research

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questions, for example by looking at whether teachers have increased their knowledge, changed beliefs or formed more appropriate identities (Skott, van Zoest, & Gellert, 2013).

The study I report on here was similarly concerned with success and sustainability of a programme of PD in reform-style mathematics teaching, and I acknowledge my own entanglement in the discursive web. My project followed 15 primary school teachers from Chile in the years following their participation in a yearlong PD program promoting the teaching of mathematics using non-routine, collaborative problem solving. In an effort to avoid a deficit framing, I began with a view of teachers as professionals (Day, 2002). In other words, I assumed that the teachers in my study would engage with ideas of best practice and would wish to take on board new pedagogy to enhance the learning experiences of their students (see for critique: Montecino, 2018). This viewpoint enabled me to explore the contextual (including local, personal, social and political) aspects affecting the teachers' uptake of the PD. Correspondingly, one of my research questions centred on the constraints and affordances on teachers' implementation of collaborative non-routine problem solving in their mathematics classrooms. The other research question aimed to understand teacher identity change post PD.

It was whilst undertaking my study that I was struck by the abundance of *deficit discourse*. Those delivering the PD programme sometimes referred to the teachers negatively, framing them as problems in a reflection of the global politics described above. Yet the teachers in turn described the students as problematic in much the same manner. From my perspective, it appeared that by taking on such views they could all absolve their responsibility - for any failure of PD or failure of teaching. For the purpose of this paper, I wish to interrogate this tendency to take a deficit view on other groups in the educational project.

In order to explore this question, I will begin by taking a post-structural approach to re-examine some data obtained from my research project. Specifically, I look at the ways in which teachers working in mid to low socioeconomic populations discursively constructed their mathematics students as being problems. I will juxtapose these findings with further theoretical consideration of the ways teachers may be similarly constructed as deficit in their resistance to reform and imagine the wider repercussions for the educational project.

PRODUCING THE SUBJECT

The theoretical framework I use in this paper is post-structural (see also Mendick, 2017), in particular I consider the discursive production of the subject. Foucault (1972) provides us a language for understanding the way in which discourses construct reality and that subjects are made in discourse. Butler (1997) describes the subject as *performatively* constituted and recognises the subject as agentic in this process. That is, "subjection consists precisely in this fundamental dependency on a discourse we never chose but that, paradoxically, initiates and sustains our agency" (Butler, 1997,

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p.2). This means that while the subject is made in discourse, they may also exercise agency in the making up of themselves, albeit within constraints.

Within education, Youdell (2006, 2011) examines educational exclusions in neoliberal schooling practices. Here we are looking at the making of "student" and the "mathematics teacher" in a variety of discourses. These include discourses of mathematics teaching reform, neoliberal educational discourses of accountability, discourses of poverty and of special educational needs. Such discourses work to constitute what it means to be a mathematics teacher, or a particular type of learner.

DEFICIT VIEWS

Within our discipline, there is already some evidence of the deficit views we may hold of students. It appears that some groups of students in particular bear the disproportionate brunt of deficit framing. These are the lower ability students (in contexts where there is grouping based on 'ability'), students from low socio-economic situations, students labelled with special educational needs, and generally students already marginalised in some other way.

Straehler-Pohl and colleagues (2014) discuss how academic expectations may differ depending on socio-economic conditions leading to inequity of schooling experiences. Specifically, they examined the mathematics education of low achieving students in an inner-city school in Barcelona where the majority tend to come from socio-economically disadvantaged backgrounds. Their investigation during the first few weeks of school found the differing practices and differing linguistic registers within different ability streams served to reproduce inequalities. Relatedly, teacher expectations, which impact achievement, are lower and more negative for indigenous and minority group students than they are for white students (Turner & Rubie-Davies, 2015). Turner and Rubie-Davies' (2015) study also found streaming to impact on expectations, but additionally found mathematics teachers to hold deficit beliefs about some students and a tendency to deny ethnicity by 'not noticing' students' racial and cultural backgrounds.

Whilst research on race, gender, and socioeconomic status is relatively common in mathematics education, there is much less published regarding students labelled with learning disabilities (Borgioli, 2008). This group, called students with "special rights" by Gervasoni and Lindenskov (2011) and including all students who underperform in mathematics, have not historically had access to high-quality mathematics education. Within this context, Healy and Powell (2012) argue against equating difference with deficiency and promote an understanding of ´disadvantage' by looking at the way in which the dominant social groups define what is 'normal'.

Others within mathematics education have acknowledged the deficit framing of teachers. Montecino (2018) examines the production of the teacher within neoliberal discourse, arguing that the "mathematics teacher is always configured to be outdated" (p. 153) and must invest in permanent PD. In the context of the UK, Hardy (2009) uses a critical discourse oriented approach to examine the construction of primary

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mathematics teachers in the UK as problematic. She argues the policy context produces 'truths' about mathematics teachers, such as their need to improve, lack of knowledge and even references to internal states such as confidence to 'explain' the problem. "Regardless of further training or 'top up' undertaken by teachers, they always emerge with the wrong sort of knowledge" (Hardy, 2009, p. 191).

Melanie Graven (2012) similarly discusses the construction of teachers as being in need of "fixing" within the discursive world of educational change. She notes "[s]tories of teachers as deficient and in need of radical change shut down the space for teacher learning" through alienation from the learning process and a negation of their current identities (p. 129). Similarly, within a theoretical discussion of PD, Gellert and colleagues (2013) posit that the major mode of "knowledge dynamics" in PD is accumulation. "You know more/less, enough/not enough" and argue that a deficit perspective regarding teacher knowledge is inherent in the literature (p. 536).

Something this literature has in common, whether looking at deficit views of students or acknowledging deficit views of teachers, is the portrayal of these views as incorrect or at least misguided. Particular students are themselves blamed for poor achievement in mathematics and teachers are blamed for poor uptake of reform or student achievement; and some literature interrogates these deficit views. In this paper, I aim similarly to interrogate these deficit views, but I hope to do so without transferring the deficit frame to the holder of the misguided view. I conclude by raising the question of the prevalence of deficit views in education and consider our own culpability in the practice of passing the blame.

BACKGROUND CONTEXT

In 2012, the Chilean government published a new primary curriculum with a number of changes; in mathematics, much greater emphasis was placed on problem solving (MINEDUC, 2012). Primary teachers in Chile are not typically used to providing for non-routine problem solving in their mathematics programme (Felmer, Perdomo-Díaz, Giaconi, & Espinoza, 2015) and typically teach in a very teacher directed, 'traditional' manner (Radovic & Preiss, 2010). The reform demanded by the curricular changes thus required teachers to make considerable adaptations to their regular teaching. With this in mind, many universities in Chile have provided PD for schools at all levels.

This study stems from one such example; a year-long PD in non-routine, collaborative problem solving teaching for primary school teachers of mathematics, and follows reform style PD operating in many other nations (e.g. Koellner et al., 2011). The programme began in 2015 and the participants were teachers who taught in either public or private voucher primary schools, in mid to low socioeconomic areas of Chile. The majority were based in Santiago but others were in rural schools or located in smaller cities along Chile. Teachers of grades 5 to 8 were typically mathematics specialists and those who taught grades 1 to 4 were generalists.

It is important to understand the contextual realities for the teachers who participated in the PD. Class sizes in the voucher schools were typically of 45 students whilst public

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schools, initially also large, were facing ever increasing drops in enrolment. Public schools suffered from insufficient resources whilst teachers in voucher schools did not have job security. All teachers faced the pressures of nation-wide exams, which contribute to their teacher evaluations. Much of these conditions relate to the neoliberal educational context – a marketised form of schooling trialled first in Chile before being adopted incrementally elsewhere globally (Contreras, Sepúlveda, & Bustos, 2010; Valenzuela, Bellei, & de los Ríos, 2014).

DATA COLLECTION AND ANALYSIS METHODS

All 140 teachers who completed the PD were invited to my follow-up study and 15 volunteered. Throughout 2016, these teachers engaged in a series of email interviews in which I sent reflective questions via email and teachers responded the following week via either email, the whatsapp cellphone application, or telephone. I developed each subsequent set of email questions based on the responses to earlier series - some were standard for all teachers and some were specific to the individual. Some examples of questions include: "What challenges do you think you may face in the teaching of problem solving?"; "Tell me about the students in your class this year?"; "Tell me about a problem solving activity you have done this year? And how did the students respond?"; "Are there any students for whom the problem solving activity is particularly beneficial?". Mid-way through the year I visited each teacher's classroom during a mathematics lesson to gain a better understanding of their context and to generate further questions for subsequent email interviews.

In 2017, the 15 teachers were invited to continue with the second phase of the project which involved an additional classroom observation and an in-person interview. Ten of the 15 participated in this second phase. Here my research focus was more on identity enactment as mathematics teachers, although I also asked some questions to further elucidate the themes which had emerged from phase one. I translated all responses into English and asked bilingual colleagues to check the translations.

To analyse the data for this paper I extracted all the comments made by teachers in relation to students, whilst maintaining a connection to the context in which it was said. I wanted to explore my perception that teachers called upon and contributed to a deficit discourse when speaking of their students. I unpacked these comments using a poststructuralist discursive approach. This approach entailed examining the production of the subject as described in the theoretical framework above. I did this analysis with both the Spanish and English versions of the text together, to ensure that meaning was not lost – either in the translation to English nor in my imperfect understanding of the Spanish.

CONSTRUCTING THE STUDENT AS PROBLEMATIC AND LACKING

When I asked primary school teachers what problems or limitations they may face in their teaching of mathematics I was somewhat surprised at the number of comments related to the students. Consider the following responses:

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> The difficulty I will face is the low academic self-esteem that some students have, they believe that they can't succeed if they are not encouraged during problem solving. Another issue is that I work with a high percentage of students with special needs. (Rosario[1], grade 3-4 teacher, voucher school).

> There are children [in my class] more afraid to take risks and they try to solve problems in a conventional way (Lucia, grade 2 teacher, voucher school).

> The [children in] the new course I was assigned are not used to this type of work (Elyacuano, grade 4 teacher, public school).

> That many children are not used to reasoning, or to working alone, without the supervision of an adult. They have a hard time separating from the teacher. Teamwork, which is always a problem, because as they are small children, they are very individualistic and self-centered, because of their age (Maria Sofia, grade 1 teacher, voucher school).

> The big problem is that the children are used to the structured work of their book (Javiera, grade 4 teacher, voucher school).

These quotes all position the students as being problematic, variously due to low self-esteem, unwillingness to take risks, or being unused to independent or unstructured work. The students, thus labelled, then provide reasons for not teaching collaborative, non-routine problem solving in mathematics. In other words, the students are *used discursively* to construct constraints on the teaching of problem solving.

It is easy to read such statements and dismiss them all as simply being excuses, examples of the ways in which teachers resist change. It is easy to follow this reading with an interpretation of the teachers as being themselves problematic in the reform mission for mathematics education. What I wish to focus on in this paper is the way in which the wider socioeconomic and political context contribute to the production of deficit students (and later to the similar production of the teacher). This requires a setting aside of blame and an unpacking of the ways in which teachers describe their students with an attention to the wider social and political context of this production.

After the emergence of students as being a key 'constraint' in their development of mathematics, I followed up on this result in a later email with the open question: "Tell me about the mathematics students you have in your classes this year?" The responses gave some clarity to the ways in which students' socioeconomic background, diagnosis practices of special needs, and the political construct of 'vulnerability' all worked together to construct the problematic and deficit student.

Teachers often brought up the students' socioeconomic background as an example of deficiency. Sometimes teachers directly connected background to learning in their description of the students. For example, Maribel connects being 'low' in sociocultural *and* educational level and connects characteristics in the home to genetic characteristics:

> At an academic level they are quite, they are quite low in their sociocultural and educational level too. They are, they do not learn much, you try, they, they learn, because every child

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[1] All names are pseudonyms

> learns, but sometimes they do not learn enough because of different characteristics, the home, genetic problems of learning (Maribel, grade 5-8 teacher, public school).

Such statements resonate with the literature showing how students from lower socioeconomic backgrounds are more often attributed with learning difficulties and experience lowered academic expectations from their teachers (Turner & Rubie-Davies, 2015). It appears common to make assumptions about the home life of students from these demographics, as highlighted by Maika's comment:

> There are many who do not have pedagogical support at home because both parents work or several families are separated, presenting emotional problems and low motivation to overcome their difficulties. Therefore, they do not have a routine that helps them to fulfil their school duties properly. (Maika, grade 6-8 teacher, public school)

Such comments demonstrate *discourses of poverty* at play. These discourses align lack of parental support, broken homes, emotional problems and poor motivation with poverty. Youdell (2011) describes how the discursive framing of deprivation 'disarticulates' causes of deprivation from wider political, social and economic structures and correspondingly the causes of lower levels of attainment from practices of education (see p. 13). Maika's comment illustrates this discursive framing well.

In Chile, as elsewhere, an institutionalised labelling practice rewrites the discourse of poverty and converts it to numbers. Consider the statement made by Cindy below.

> The school in which I work is a school that has a high percentage of priority children; more than fifty percent of the school are priority children. It is a school with a high level of vulnerability. Eh, they have many threats in their surroundings, due to delinquency, drug trafficking, child abandonment, absence of care or concern for [the children]. That makes them fit into the vulnerable category. (Lucia, grade 2 teacher, voucher school)

The demographics of the school are captured as a percentage through a count of the number of "priority" children. The additional label of "vulnerability" also speaks volumes. In Chile children are labelled as "vulnerable" based on a metric including their socio-economic background as well as other measures (e.g health, number of parents in the home) to calculate their risk of school failure or dropout. This labelling practice converts the child's context or background, to something very much inscribed on the body – this discourse of *vulnerability* conjures up the idea of easily injured, as a *soft* body that is easily harmed. Further, schools are then given a vulnerability index, based on the percentage of children who are labelled vulnerable (see: Infante, Matus, Paulsen, Salazar, & Vizcarra, 2013, for a discussion of the production of vulnerability in Chilean students' visual narratives). This political practice provides teachers with a language to label the students (and the entire school) as inherently problematic.

Another institutionalised labelling practice in Chile, and worldwide, is the labelling of children with special needs. In 2009 the "Differentiated Grant for Children with Special Needs Law" (Decreto #170) was passed, defining the student with special needs and promoting their inclusion in schools through increased funding. The definition of special needs stipulated in the law actually includes poor performance in mathematics (Darragh & Valoyes-Chavez, 2017), demonstrating how this subject is

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particularly implicated in the politics of labelling. It is in schools' interest to have students labelled with special needs as it generates further funding for the school. A consequence of these practices is that additional staff have entered the school environment in Chile specifically for the purpose of 'diagnosing' and 'treating' children labelled with special needs. Children are withdrawn from the classroom for both these purposes. The terms used here come from a *medical discourse*, positioning the students as 'sick'.

The classroom teacher may then take up this labelling practice. Consider Marcela's comment:

> There are 12 children with attention deficit untreated ... There are many students with specific needs, but it is very difficult to treat them all. (Marcela, grade 4 teacher, public school)

The 12 children, as Claudia explained elsewhere in the interview, had not all been professionally 'diagnosed', rather the teacher herself recognised them as having special educational needs. I suggest that this demonstrates how the insertion of other professionals in schooling and the medicalised discourse facilitate the problematizing of the students as being a problem (in this case as a medical problem). (See also Darragh & Valoyes-Chavez, 2017)

To summarise, we can see how policy and social contexts work to produce a deficit or problematic view of students through discourses of poverty and politics of labelling practices. We must therefore reconsider our blame of the teacher in this production.

CONSTRUCTING PROBLEMATIC TEACHERS

As suggested earlier, it is very easy to read the teachers who speak of their students in these ways as being themselves problematic. It is certainly challenging to write about, and criticise, the teachers' construction of their students in such ways without contributing to a deficit view of teachers. Yet if we can understand the seemingly problematic nature of students as being a product of the social, political and historical context – then we must view the seemingly problematic teachers in the same way.

The very study I present here has its' roots in a notion of the teacher as being a problem. Chile's poor performance in international tests have directly led to curriculum reform (with greater emphasis on problem solving) and teacher reform by way of PD. When I interrogate teachers' framing of students as problematic, I must also interrogate my own framing of the teacher as problematic for this very act.

QUESTIONS FOR DISCUSSION

By way of conclusion, I wish to open up a few questions for discussion. What other groups in the educational project are similarly produced as deficit? What might be the consequences of this production? To what extent are we, as mathematics educators, implicated in the deficit framing of various groups of people within education? Finally, and especially: How might the act of research itself perpetuate deficit frames?

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CULTURE AND LANGUAGE IN THE PRACTICE OF MATHEMATICS TEACHING IN INDIA

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The concern about inability of children to learn school mathematics has grown since education for all become the norm. Failure cuts across diversity, yet it afflicts the under-privileged children not at the center of the hegemonic main stream more. Since most individuals and communities efficiently use some form of mathematics, the fear of school mathematics is worrying. There are suggestions that community mathematics is different from prevailing mathematics but school mathematics does not know or ignores this knowledge and hence excludes them from learning. School program derived from hegemonic formal mathematics has no link to context, language, culture and everyday life of people. We examine the policy formulations and challenges that lie in the including cultures, languages and the sets of knowledges, in school mathematics.

BACKGROUND

Children find mathematics difficult to cope, a high failure rate makes a large number of children scared of dealing with it. They are relieved to be free of it. It carries high prestige and for those who do well, it is a mark of greater intellectual capability. We know that the children from disadvantaged backgrounds on the average struggle with it more. The challenge thus is to ensure that more children, indeed all learn mathematics. Linked issues are should we teach all children the same mathematics and how much should it be? Is the mathematics we teach, its nature, its way of working, its ideas etc. elite and not aligned to the common people? We consider this in two parts: The first on ideas about what is mathematics and the second on the variety of experiences and circumstances of children. Keeping these in mind how do we use the diversity in background meaningfully? The focus being on pragmatic possibilities within current mathematics programs and the difficulties and unresolved questions within.

This paper is based on experience of the author on development of frameworks , syllabi, textbooks and in preparing teachers for elementary and secondary school mathematics in Indian state education departments and the NCERT. The NCF 2005 and its position paper NCF PP 2006, as the academic basis of all syllabi, text books, classroom transactions and teacher preparation programs in India are the base documents for the discussion.

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Curricular Directives in India

After introduction of the ten plus two system mathematics is compulsory up to class X in Indian schools. While, the strand of usefulness for life has been particularly emphasised since National curriculum framework of 2000 (NCF 2000), it is also indicated in the X year frameworks of 1975 and 1988. Yet, these emphasise mathematical ideas principally to enable higher studies including in science. The 1975 and 1988 documents do not mention the relationship of mathematics to language or any cultural dimensions. Even science is linked only as a modernisation project to challenge obscurantism and develop scientific temper. NCF 2000 emphasises retention and consolidation of cultural and traditional aspects. With a core aim being to develop sense of responsibility and loyalty all these see education as a means to develop assimilable citizens. The documents do not deliberate the linkages with mathematics believing it to be location free. The only concern being to distribute it across classes. NCF 2000 overtly emphasises culture, Indianness with shades of social constructivism and suggests cultural specific pedagogies. It argues for non-uniform and non-mechanistic way of teaching-learning. It argues that even if the information processed by learners is broadly similar. the content can vary. The examples presented are story-telling, dramatics, puppetry, folk play, community living etc. focusing on joyful learning and involvement of school with local people, festivals, functions and life styles. (NCF 2000, 41)

NCF 2005 is more nuanced in cultural and linguistic dimensions. Mathematical ideas and their direct applications is separated from the way of perceiving the world and overall cognitive abilities. It suggests mathematics through mathematisation would develop abilities for perceiving the world in many different ways both materially and inferentially in the same way as ability to read and literacy provides a wider view and science provides scientific temper. For example the ability to visualise the appearance of an object when transformed under an operation, estimate what can fit in to an area, fragments a piece can be divided into, estimate the numbers required for different situations. It would through the ability of working with and understanding proofs help in identifying what can be logically deduced from certain statements. The ability to understand data would help analyse cause- effect relationship and isolate unrelated factors and help develop models to understand phenomena and make judgments. The better understanding of probabilities and data distribution would alter ideas of chance that influence our attitudes. One can argue that logical ability from this and the experience of generalisation, abstraction and proof would make for less gullible and less rash persons. Since the NCF 05 or the position paper does not really clarify what it means by mathematisation the implications of its development can be considered as wide as these.

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WHAT IS MATHEMATICS?

To consider the role of language and culture in teaching of mathematics we need to know what mathematics we are talking about, its nature and how it relates to life, language and culture of children. Mathematics is considered a different kind of discipline as less driven by context and being more abstract with a more hierarchical concept organisation implying later mathematical ideas are built on earlier learnt ideas and objects of mathematics. The context and experience of mathematics and the way quantity and space are thought about implies individuals and communities have different strategies. A person engaged in a particular vocation or a task acquires the associated

'concepts and procedures' and competently handles related situations cannot generalise. Contextual mathematics, its possibilities and limitations need to be examined in wider contexts. (Khan, 2004 & Ramanujam, 2016)

The experientially acquired ability to deal with quantity, space, forms of representation, intuitive modelling encompasses elements and shades of mathematics that remain limited to the context. It has also been argued that mathematical ideas of communities are perhaps differently structured implying that many different mathematics exist. (Kilpatrick et. al., 2001 in Presmeg, N (2005, 48) Using community mathematics to teach diverse children becomes complicated made worse as structure of community mathematics is often not evident. They cannot be generalised and used for new situations. Here a comparison with language would be relevant.

Language and Mathematics

Children acquire language through interaction with their social group, community and environment. Her cognitive abilities are similarly acquired and consolidated. Some of these emerge as capabilities in mathematical engagements whose scope is determined by the extent of the tasks and exposures she gets. For languages perhaps a basic grammatical edifice develops and the child is able to extend it to wider situations when required giving her extraordinary linguistic creativity and ability to absorb new ideas in her language. She manages the rule governed arbitrariness and both constitutes and is constituted by her language and culture. Her facility and capability develop as she interacts with new situations and circumstances. Tomasello finds intentionality of the child to learn and the joint intentionality with the teacher to make learning possible essential. Even if we do not accept the entire thesis, the importance for learning of the child and the teacher understanding the purposes and sharing them cannot be over-emphasised. (Tomasello, 2001)

The language taught in school is a different language or a formal version of known language. School expects to develop ability to read printed words, absorb oral or written texts with new and complex ideas. By extending reading school expects to break the shackles imposed by the limited exposure domain, free

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children from boundaries and allow knowledge exploration and imagination about how things can be different and possibilities that can be conceived. The debate on language, how it should be taught and what should be the medium for other subjects enriches conversation but in any case developing a greater felicity and capability in any language extends boundaries and possibilities of thought and imagination. Is mathematics similar and extends the mind and should children formally learn mathematics in school? Mathematics appears more difficult and not naturally learnable, necessitating sharply defined purposes for its teaching and learning by all. And the linked question as to what mathematics, the same for all or different for different children? Till what stage should common mathematics be taught, with what content and manner of transaction. The NCF PP 2006 on mathematics wants children to use mathematics naturally and fluently.

THE CURRENT STATEMENT OF MATHEMATICS TEACHING IN INDIA

NCF PP 2006 says, "In developing a child's inner resources, the role that mathematics plays is mostly about thinking. Clarity of thought and pursuing assumptions to logical conclusions is central to the mathematical enterprise." Aim developing ability abstract with mathematisation in all students, not merely utilitarian and experiential mathematics. Due diverse backgrounds, universal mathematics must be situated in the "lived centrality" of lives of children. Child must feel included and matter more than the subject. Transaction must engage each student to strengthens her resources and curriculum should assume success for all children. Curricula that assume failure would fail. Mathematical objects are related to and manifest in our lives but are un- linked to particular contexts or physical materials, thus program must prepare students to ensure they understand notion of mathematical statement and acceptable proofs. Mathematical ideas maybe generalised and abstracted from life but are represented in abstract objects and ideas. At secondary level proof, argumentation, modelling, data handling and the inter-relationships among abstract ideas e.g. geometrical visualisation of algebraic understanding, are essential. Children must understand precisely and sharply abstract mathematical entities and link that with life experiences. While they hunt shapes around they must recognise that circle and other shapes are boundaries that separate inner and outer parts. The upper primary child must go beyond number sense and engage fluently with number patterns, algebraic notation, number relationships, seek patterns and connections in the wide world around. She must use algebraic notation, a succinct and compact language efficiently. (NCF position paper on mathematics 2006). Mathematics program must develop logical thinking, abstraction, imagination, visualisation etc. through a robust system that helps all students and reflects awareness of the existing knowledge of children and penetration in to our lives. Quantification in life requires numeration, estimation, comparison, scaling, knowledge of the operations and similarly spatial competence needs conceptual

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ability, confidence in spatial relations like transformations, visualising, mapping and projections. The implication, deal with mathematical ideas from all domains in an informal way but go beyond that. Keep concrete materials as important but move to generalising rules and exploring inter-relationships. Concrete experiences may help in learnability but they do not represent mathematics. Mathematical ideas begin from precise presentations of underlying concepts and their consistent use to build a hierarchically structured understanding enabling creation and use of ideas to form new ideas. Learners must find own solutions to unfamiliar problems, create new problems, explore mathematical entities, add to what is given and not be restricted to a fixed set of definitions, explanations, formulas or solutions. (NCF PP 2006) Mathematical ideas must go beyond materials and facts. As Presmeg N says a developed number sense means ability to think of new numbers and their relationships. Just using algorithms is not a reflection of number system knowledge. Knowledge requires understanding why each algorithms work and be able to create new ones. The out-of-school mathematics must come in to classrooms so that ideas become meaningful for students. However, it must not trivialise the inherent mathematical concepts. (Presmeg N (2005, 46)). This is consistent with the argument that numbers always linked to material situations, would limit development of mathematical concepts. (Kilpatrick et. al. (2001) in Presmeg (2005, 48)) Linking shapes, lines and points to 3 D objects, 2 D pictures, sketches and projections obscures the concept. Circle considered as a bangle or a coin rather than a locus of equidistant points, is a misconception.

The question is that is this present mathematics really needed? How does it help a child and her community? Why is extended ability to use numbers in her contexts not sufficient?

Can mathematical ideas of the community become central? Can mathematics un-burdened by symbols and their grammar focused on use, be sufficient? Studies and anecdotes indicate children have difficulty in learning objects and concepts not sufficiently present in their life. For example they have discomfort with negative numbers, letter numbers, algebra, sin x & cos x, notion of general proofs, 'complex' mathematical sentences using symbols, mathematical terminologies, problems that require a multiple step process. All these are 'abstract' and uncommon in life. How much of this abstraction must be universally accessible and can we restrict curricular expectation to that? This however, may conflict with the principle of equitable opportunities and liable to the criticism of poorly judged human capabilities. The complicated relationship between the ability to comprehend and assimilate and the manner it is taught makes the choice of deciding what to include and what to leave out as obtuse, difficult. The NCF is concerned that current school mathematics is inaccessible to many children, largely from deprived backgrounds making them afraid of mathematics. It emphasises inclusion of experiences but reiterates the goal of decontextualised abstraction. It sees the shift in learning as a way to involve the non-participating majority but not as a dilution

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of standards as such a shift may demotivate the 'bright' minority. (Position Paper on Mathematics 2006)

To answer, why and what in school mathematics, on one side is trivialisation that everything is mathematics and on the other the squeezing out of community knowledge for formalism. Including community mathematics in school requires a researcher and cannot be done by a teacher. (Millory (1992, pp. 11–13)) as cited in Presmeg N (2005, 8)) This difficulty being that the ways of community in dealing with and organising the world may not be codified or codifiable; essential for it being transact-able (if at all) over diverse classrooms. It is not clear that even the possibility of many mathematices is accepted, how will it be included in the classroom.

In terms of how to teach mathematics, the position paper and notes to teacher in NCERT text books suggest recognition of the experiences and formulations of children and opportunity for them to articulate and work on these. It expects children to build their own logical formulations and use their own language to express them. What more should be included has to be spelt out more concretely? Is it different 'mathematics' or different ways of learning? Is logical thinking, abstraction, imagination, visualisation for abstraction, building logic, organising and analysing ideas, visualising new relationships etc. needed? Like language a window to viewing knowledge and life differently? Mathematics is not like language. Connections and patterns across different societies or phenomena in languages are naturally observable and less abstract than mathematics as they relate to experience. Is then the question about mathematics curriculum or for the entire edifice of abstract and de-contextualised knowledge. This has to be seen in the context of the political implications of hegemony not merely pedagogy to recognise and resolve separately.

What to do about this?

Humans learn best through familiar contexts engaging with the world using their knowledge. Teacher support is essential to extend their control over concepts. For all children to learn formally diversity in children and teachers suggests variety of ways as essential. The varied experiences, support and aspirations that children come with, necessitate flexible programs. Teachers' capability, motivation, mathematical knowledge, self-confidence and desire to engage with children also differ. They can not create equivalent learning possibilities. The nature of schools and the facilities makes for differing ambiences. The task is to make possible over a large domain, inclusion of language, life experience, culture for each child to build her arguments. Can a mathematics class room make children recognise that what they do relates to reality? Can we embed maths and problems in meaningful and enjoyable situations ? The position paper stresses reduction in fear and relationship of the taught mathematics to life experience of children. While recognising the abstract nature of mathematical ideas, it emphasises initial development through engagement with

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concrete contexts. It affirms that in any learning, including of mathematics, experience of the learner is central. Classrooms need include children's creativity and multiple strategies. They must value children's articulation and logical formulations and demand that children create tasks, questions and problems.

The NCF PP 2006 mentions language and culture of the children as important yet fails to substantively address the issue leaving the implications of the statements unspecified. It argues for sensitivity to languages of all children in textbook and hence a multiplicity of textbooks, yet there is no explication on languages or level of decentralisation of text books. Is just translation required or particular materials for each children group to extend their knowledge? Are text books and syllabi enough or something else is needed? The paper stresses developing mathematical ability by problem solving using children's knowledge, suggesting children need to decode presented problem, identify knowledge required, break up the problem, evolve her strategy, solve it and have opportunity to present her ideas and solution. Mathematics must link to real experience with opportunities to create and formulate problems and use in class-room methods learnt in the community for appreciation of relevance of mathematics to life.

We however, know that, in life mathematical ideas are not as they appear in school mathematics and later. (Walkerdine, 1988 as cited in Presmeg, 2005) Everyday mathematics is different from school mathematics. It is often not useable in another context and certainly not in all situations even those with similar elements. A newspaper vendor can mentally keep track of the papers sold and the collected money but finds it difficult to do similar mathematical processes when in another shop, say kirana shop. So while the forms of generalisation, the way of reaching to and communicating answers are different, each child learn and uses practical mathematics. They acquire the abilities and develop own strategies too. So mathematics teaching has to begin with as related to experiences of children and useful in daily life transactions, but it must move away from experiences as the ideas themselves are inherently abstract and ideal constructs This again suggests absence of easy answers to how this relationship can happen in class-rooms.

The NCF PP 2006 seems to struggle with this and fails to find concrete and convincing connections to be built upon. It is not able to point out the mathematics emerging from cultural grounding appreciation and how it can help acquisition of underlying concepts and is unable to show other connections of everyday or contextual mathematics and what concepts and procedures can the teacher extract from them. To operationalise this research and subsequently teacher development is needed. Norma Presmeg, arguing for cultural and language inclusion in the mathematics classroom underlines this need. (Presmeg, 2005, 47-50)

Repeated studies and analysis suggest that the children from certain backgrounds have an advantage in mathematics learning. While, the poor and minority background children do possess some informal mathematical abilities and seem to match the abilities of other children from dominant and powerful

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backgrounds on these, they lag behind in others. This 'immaturity of mathematical development' (sic) probably accounts for problems encountered in simple mathematical tasks and word problems. Papers in this field emphasise need for research to find contexts that are not just physical. Meaningfulness is not merely realism or concreteness. The ability to develop appropriately flexible mathematical understanding requires multiple re- contextualisations which need to be constructed and created in imaginations as well. This is not easy particularly in the context of the circumstances of the schools, classrooms and teachers as extraction of mathematics from lives of the children and community is not easy unless already known and extraction shown or indicated earlier. (Kilpatrick et. al., 2001 173-175)

Teachers are not researchers or mathematicians to see and appropriately utilise possibilities. They cannot ask or answer questions about difference in idea of space and number in the culture they are interacting with and its linkage to school mathematics? They cannot see possible connections easily or identify whether difference is conceptual or only in name and description.

A teacher in classrooms cannot explore different basic mathematical axioms and construct alternative ideas based on them. The pragmatic possibility is ensuring maximum use of children languages with use of their words and concepts for engaging with newer concepts and grapple with the subject to gradually increase scope for learning more concepts. Enable them to keep pace with those with more learning enabling backgrounds. Interactive classrooms will ensure collective mathematics learning rather than being individual projects. Interaction will restrict extent of domination, control and exclusion enabling a challenge to the dominant formulations. In a sense they learn what those dominating know and learn to interact with them and at least partially challenge them. For this the mathematics accepted as known knowledge has to be transacted leaving only manner of this transaction so that children do not feel oppressed.

In conclusion while we have to be sensitive to the language and culture question in transactions and also simultaneously to questions of domination and hegemony, school mathematics has to be within the frame of current abstract and not merely context driven mathematics knowledge. It has to assume certain epistemic roots and analysis of what mathematics is. Remaining within that is not what makes school mathematics alien and incomprehensible. While language and mathematics both are acquired in context and society, are the basis for comprehension and expression of ideas and linked to the construction of thought in the mind yet they are very different. The ease of creating new ideas and the fluent usage to describe and understand new phenomena is radically more in language. Each language is potentially as capable as the other and can perform all functions. We therefore in principal may argue that all children should be taught their language and in it but that argument cannot be made for mathematics at the moment. Without significant research in some of these areas, mathematics in schools would remain acculturating rather than transforming through

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acculturation. Such a shift would also require major systemic and structural modifications.

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THE SOCIAL MODEL OF DISABILITY AND CRITICAL MATHEMATICS EDUCATION

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Abstract: This paper reports a five year long and ongoing research study with blind children in a learning centre located in Mumbai. On recognizing that disability as well as mathematics are political in nature made me realize that the question of teaching mathematics to blind children is not merely a question of didactics but a social justice concern. In this paper, I locate Disability studies and Mathematics Education within the Sociopolitical arena and explore how critical theories within each field of inquiry may inform the other in the process of addressing the question of teaching mathematics to blind children. To achieve this end, I develop a theoretical framework by bringing together critical insights from the Social Model of Disability and Critical Mathematics Education.

INTRODUCTION: THEORY AND LITERATURE

The field of research concerning the teaching of mathematics to blind children is largely assumed to fall under the category of special education or inclusion. While special education was originally considered synonymous with having separate schools for students with disabilities, by the 1980's the discourse of mainstreaming would find prominence in special education research (Bratlinger, 1997). Mainstreaming, which can be understood as *integrating* students with disabilities into the *mainstream* system, has been considered problematic for being underpinned by the assumption that a *lesser* system (of special education) has to adhere to the norms of the dominant mainstream (Idol, 1997). Research in special education is currently dominated by a discourse on inclusion which typically refers to, as Idol (1997) describes, a form of education in which a student with special education needs attends the general school program for the whole of the school day rather than being partially trained in a special education program.

Even though the debate on inclusion has offered alternatives to segregationist and even mainstreaming approaches towards special education, it has broadly been based an understanding of disability as individual pathology. Subsequently we find research studies around inclusion aimed at prescribing how teachers and students may be trained and prepared to deal with students with disabilities in their classroom (For example, (Idol, 1997)).

The field of Disability Studies and in particular, the *Social Model of Disability,* challenges such assumptions by arguing that disability is a social construct. The social model of disability begins with the rejection of the idea that disability is an outcome of individual pathology, and locates the problems of disability within society. Members of the Union of the Physically Impaired Against Segregation

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(UPIAS, 1976), who developed the perspective, declared in their *Fundamental Principles of Disability* document that

> it is society which disables physically impaired people. Disability is something imposed on top of our impairments, by the way we are unnecessarily isolated and excluded from full participation in society. Disabled people are therefore an oppressed group in society (p. 4).

Through the social model, disability (different from impairment) can be seen as a window into, the internal contradictions or maladjustment of an apparently functioning society. While the social model brought Marxist thinking to Disability studies around the 1970s, the theorization of disability from a Marxist standpoint had found expression even in 1919 with Helen Keller (1920) who described how "the blind man, [is] a *symptom* of social maladjustment (p. 38)." While disabled people were perceived as external intrusions that disrupt an otherwise well adjusted society and could be controlled through exclusion, Helen Keller argued the reverse.

Just as the social model highlights the sociopolitical nature of disability, which is otherwise perceived as natural and neutral, critical theories on mathematics education reveal the sociopolitical character of mathematics (also predominantly understood as apolitical). Critical Mathematics Education (CME), as a field of research, shows that mathematics plays a central role in sociopolitical processes (Skovsmose and Borba, 2004) by, for example, providing means and justifications for certain forms of inclusion and exclusion (Skovsmose, 2005).

To acknowledge the political role played by mathematics as a means of exclusion entails rethinking what we understand by inclusion/exclusion. I agree with Marcone and Skovsmose (2014) who argue for inclusion and exclusion to be understood as a discursive duality, rather than part of a binary concept where inclusion is uncritically taken for granted as good and exclusion as bad. The discourse on inclusion must address, for example, the question raised by Figueiras et al. (2016), namely, inclusion into what order of things?

A sociopolitical understanding of disability highlights the necessity for a critical approach to teaching mathematics to blind children (read: Critical Mathematics Education or CME) and thereby raises many challenges. Skovsmose (2016) could hardly find any study that explicitly referred to mathematics education for social justice in a context regarding blind students. Skovsmose subsequently asks, "What could reading and writing the world with mathematics mean for blind students?" Skovsmose recognizes that doing so would be a challenge for blind students, for among other reasons, due to "difficulties that arise from the relationship between Braille and mathematical symbols."

However, the problems with such a formulation of CME is that firstly, the mathematics with which we may want blind children "to read and write the world" may further reinforce their "otherness". Secondly, the relationship between "Braille and mathematical symbols," as Skovsmose speaks of, presents the view that there

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exists a “normal” way of writing mathematical symbols, and the “other” way, namely, Braille as done by blind children. Also, the dependence on “the available technology” constructs a blind child as a “potentially full human” rather than an already full human being to begin with.

From a CME standpoint, Skovsmose (2014) argues that for mathematics education to work in support of democracy, “the microsociety of the mathematics classrooms must also show aspects of democracy (p. 4).” But what could a democratic classroom mean when it includes blind and mentally challenged children studyingalongside the so called gifted children along with an “expert” teacher in a society where mathematics plays a sociopolitical role?

To critique the supposed neutrality and universality of mathematics Pais (2013a) narrates his exploration into ethnomathematics, that, combined with critical mathematics education holds the potential to question the role of mathematics in the school curriculum and, mathematics itself as a culturally bounded field of knowledge. Ethnomathematics, he recollects, provided the “epistemological critique of the enduring belief in the universality and neutrality of mathematics knowledge (p. 2).”

Although disabled children do not constitute an ethnic group, they too are affected by the epistemological hegemony of mainstream mathematics for which a critique can come from ethnomathematics. As Pais highlights, the importance of ethnomathematics is not so much related with the study and of “other” mathematics but with “it’s critique of academic mathematics itself, through a social, historical, political and economic analysis of how mathematics has become what it is today (Pais, 2013a).”

However, as Pais then points out, in the classroom, ethnomathematics gets stripped off of its emancipatory core and is reduced to a learning device devoid of any critical reflection on the sociopolitical aspects of academic mathematics. And when local knowledge is brought to school it gets decontextualized from the conditions that justify the emergence and use of this knowledge (Pais, 2013a, 2013b; Knijnik, 2012). Pais dismisses the role of CME in providing a “solution for problems that by their very nature are economic and political (p. 5).”

Pais’ targets his critiques at didactic practices based on how CME (including ethnomathematics) has been applied. However, CME necessitates “reinventions” of critical pedagogies in given contexts (Frankenstein, 1983). In the context of teaching mathematics to blind children, while locating disability as well as mathematics within the (socio) political realm, the question that needs to be asked is, *How may Critical Mathematics Education be “reinvented” in view of taking a socio*political *approach towards teaching mathematics to blind children?* And simultaneously, how may CME inform Disability Studies, considering the significant role played by mathematics education in processes of exclusion and disablement?

These concerns were central to my research work and was conducted at a nearby study centre for blind children in Mumbai.

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BACKGROUND TO THE STUDY

The study reported in this paper was carried at the *Vivek Education Foundation, School for the Blind*, a study centre that caters to partially/blind students most of who attend “normal” schools (with blackboards and teachers with no knowledge of Braille). A few students do not go to any school but take help of the teachers in the centre to prepare for their open schooling exams. Around 45 students are registered with the centre, almost all of who are from lower to lower middle class backgrounds. Prior to being a part of the study centre, these children would be confined to their homes and barely receive any education. The National Association for the Blind (NAB) would send teachers to individual children’s homes where they would be taught for two hours, twice a week which was clearly insufficient given that in addition to the curriculum they also needed to be taught Braille, Abacus, the tactile geometry kit, etc. One of the teachers, Ms Kanchan who is affiliated to NAB and teaches at the centre, recalled how even the children’s siblings excluded them from playing on the pretext that they might get hurt. It involved a relentless effort on the part of Ms. Kanchan, to ensure that the children were no longer isolated in their homes and be part of this learning centre. The owners of the centre would also struggle to get the children get admitted into the neighbourhood school amidst severe resistance from the school authorities.

The motivation to visit the school was not to pursue formal research but out of curiosity of having a study centre in our vicinity. But, during a conversation with the centre manager and teachers, they had expressed their need for someone who could teach music and english speaking. Returning with my guitar and they being satisfied with my guitar playing and singing, we were offered a two-hours time slot on Saturdays from 11am to 1pm to engage the children with music, and other recreational activities. As time progressed, we would also volunteer during exam seasons to tutor the children. Each child was be assigned a volunteer teacher to tutor them. I would also audio record my readings for them.

While tutoring the children, it was common for students to digress from the topic and open up about their personal experiences. My initial recorded observations had taken place fortuitously when one of the students, Rani (a 9 standard student from the centre) spoke of her experiences in two different schools. The audio recorder was running with the purpose of recording her textbook and my teaching for her use later. Her narrative provided a powerfully critical perspective on disability and society that I would not have been able to capture had I taken a standard interview. Rani contrasted her experiences in two different school settings, one in which she wasdiscriminated against and the other in which she would claim to not “feel at all, different,” although both the schools were “normal” schools.

RANI’S NARRATIVE

Rani began sharing her experience in her school:

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> **Rani**: Society has not, till even now, accepted blind people. …their thoughts, mindset is not there, to help. …I don't play with them, (They think that) "this will happen to her, that will happen to her." That's why I'm made to sit separately. …I have received (sports) medals; meaning everyone, blind as well as normal children would receive medals. …I showed that in school. Even still they would not know that in her also there is talent. …till now, their thinking hasn't changed.

Rani expressed howdespite having received medals she would not be considered talented enough to be included. Had Rani ended her narrative here, it could be argued that Rani's experience of discrimination could have been an inevitable outcome of being a blind girl. However, Rani continued by stating that she was not discriminated (made to feel different) in her previous school. And neither was her friend, Rudra who also studies at the centre.

> **Rani**: …my school before this, …was very good. I did not at all feel different. …And even Rudra (another student of the centre). They cooperate. …And right in the front they keep me. Like in sports, etc. ...In this school so much discrimination doesn't happen.

Rani claimed to not feel different at all and would attribute the reasons to her friends and her teacher for cooperating with her. It was clear to Rani that her ill-treatment was not an outcome of her blindness but rather how society deals with blindness. Referring back to her current school, she continued narrating how she was discriminated in her new school being excluded:

> **Rani**: But this private school, they [discriminate], very much …keeping me separate. …it might be with me also, I didn't manage to be involved.

Rani highlighted that the school was a "Private school." Unlike her previous school, here fees had to be paid. Rani's experiences with discrimination in this school had in fact, begun right from the day of admission when the school initially denied her entry on the pretext of her blindness.

By comparing her experiences in two different setups, it was evident for Rani that her enablement and disablement had more to do with her social environment than her blindness. It was clear for Rani that it was not her disability that led to her exclusion but rather it was exclusion and discrimination against her that disabled her in the private school.

This incident occurred before deciding to carry out a research study in the centre. In fact, the thought of pursuing a research project in this field as proposed by my supervisor made me uncomfortable since I could not see my role as anything other than an able-bodied man from a privileged position seeking ways of "helping" marginalized blind children adapt to his visuonormative and disabling world and its mathematics. However, chancing upon the social model (Oliver, 1990) provided me an alternate view of the problem, although the pedagogical and other implications was yet to be explored given the dearth of literature in this area.

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So at the end of the children's exams, through a discussion with the centre teachers, I proposed the idea that we have a mathematics summer camp. The centre teachers and students were happy with my proposal.

FIRST MATHS CAMP

The camp conducted at the study centre included a series of mathematics lessons. We were three researchers) and 15 students aged between 9 and 20 years. The children were either partially or completely blind. All spoke Marathi, and Hindi was their second language. The sessions were audio recorded and observations were noted. The discussion were carried out in Hindi (since we found Marathi difficult).

We sat in a circle on the floor and asked the children their difficulties in mathematics. Through the course of the discussions, we decided that our session would be around the topic of divisibility.

We began discussing multiplication tables and followed it exploring rules for division by 2. Even after figuring out the rule (of seeing the units place), on asking if a number is even, the students still preferred to actually divide the number by 2. Not wanting the children to do so, I presented a bigger number: 33333333330, while emphasizing the reason for doing so. Although the children understood my question they found it more exciting to actually dividing the number.

Rani explained my question in Marathi to her friends. But the children either kept discussing how big the number is or explaining the question to their peers. Monica was not comfortable speaking out and would speak to Shikha who would present Monica's views to the class. Often, I tried to move on with the topic, but the children ignored me and saw to it that their friends understood whatever was being discussed.

The session also included a discussion on addition and subtraction. And towards the end of the day's session, Faizan explained to me why some students, like his friend Sheikh, did not understand the problems I was be posing:

> **Faizan**: what is odd and what is even, he (Sheikh) doesn't know. We can see that if this goes by 2 and if we takes 4-5 examples then we can say that for bigger numbers it will be divisible by 2. Then only will it come in the definition. But like logically, this will go here, he hasn't learnt. He gave his third standard exam after which he went directly to give his tenth, and there too he did not take mathematics. From the basics you will need to teach.

Faizan also shared the backgrounds of other children who had also not gone through the regular schooling route, to get me to consider those factors. But I thought that such details may be irrelevant to the understanding of basic number operations which we carry out while, say, shopping:

> **Rossi:** That is true but …you all do use numbers. Like when you go to the shop, then you get some idea that …
>
> **Faizan**: It's not a matter of idea. He has the idea. Because now as per what he remembers …he will divide and say, yes, it can be divided …But

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> (in the context of going to the shop) he has to also count money, …products (saamaan) are also there. Price is also mentioned. So there he can do it. But here you are saying, only this is given. And he wants to count the other. If all three are given then he will be able to do it. In practice, all factors are present, how much should be subtracted, how much (change) should be returned, how much is there with him, all those factors are there.

Faizan knew that Sheikh had the idea of operating with numbers in real life. He articulated all the factors that operate in the context of making a purchase that involves "subtraction" - a price is mentioned on the product, the buyer gives some money to the shopkeeper; the shopkeeper hands over a part of the balance, and observing how much value the purchaser has with them while keeping in mind the price of the product keeps returning change until the exchange is balanced. Faizan would point out how by reducing the context into a simple mathematical operation, I would be making some children feel lost. The inclusive and democratic character of our mathematics session had less to do with the available assistive tools and my teaching approach and more to do with a spirit of cooperation among the children by which every child would be taken along as the discussions proceeded while also having some control over the pace of the discussion.

"WHERE DID -1, -2 COME FROM?"

For our second mathematics camp, held months later, I planned to quickly revise natural numbers, odd and even numbers and to move on to more difficult topics. To provide a context for talking about odd and even natural numbers, I used the example of objects like ice-creams and balloons, since they cannot be divided into fractions.

We started with the properties of odd and even natural numbers. However, on asking about zero, the students agreed that it is both odd as well as even - they argued that zero leaves no remainder when divided by 2 thus making it even. But, with zero, we have nothing to divide (by 2) and thus it is odd. However, Faizan expressed discomfort with including zero with odd numbers by sharing his observations that: odd+/-odd is even, even+/-even is even, odd+/-even is odd. Stating that this is how *he* made his definition ("*aisa meine definition banaya*") he proceeded to state that since 3−3 is zero, 0 is an even number. I was happy that the children were now convinced. Trying to move on to negative numbers, I asked about -4:

> **Rossi:** -4. Is it an even number or an odd number?
>
> **Faizan**: Sir, before that if we think that this -1, -2, why did it come? Because only after that would we know whether it is an even number or an odd number. ...there has to be a reason. …like we did for even and odd numbers. …we found out that we can divide things…in two parts…we can evenly divide, *so we gave them the name, even numbers.* Similarly, if they cannot be divided, we gave them the name, "odd numbers." So something might have happened so that this -1, -2 came.
>
> **Rossi**: We can think with examples of where -1, -2 could be useful.

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Faizan: Sir, like houses…like there are storeys on the top…there's a ground floor…below that also they build something, like basement…Below that also they make two, three floors…like we had gone to the theatre… Parking lot was in the basement…So in the lift…after zero, we saw -1, -2…

The discussion that continued emphasized the need to conceptualize odd and even numbers. For Faizan, it was necessary to know the historica lroots of a concept, to answer questions related to it. I however had simply wanted to complete my lesson plan by linking what a student said, to the concept I wished to teach. So I continued:

Rossi: …if we look at the different floors of the building, it's necessary for us to have -1, -2. …if we go to the second floor, and if we go down by even numbers, then through even numbers, we reach zero, which is the ground floor, right? And below that also, if we go two floors lower, then you reach -2 which is also even …

Faizan: So this means that -4 is also an even number.

The children were convinced that -4 is best categorized as an even number - These numbers fit into a continuous pattern of alternating even and odd numbers, whether read backwards or forwards. However, Faizan interrupted:

Faizan: Sir, its very old! So how could those people think that in the future such houses would be built?…They must have made (negative numbers) with something else …we are coming up with these right now because in front of us we have these made…so they could not have made this -1, -2 after that…Because if we try to fit these into our example, it is fine…but what I'm thinking is that this (-1, -2, ...) is many years old. So that time also houses (like this) must have been there. So why would they have thought that in the future, such houses would be there where -1, -2 …we need to look at another context…

I posed the question to the other students. But Faizan kept thinking aloud:

Faizan: …Maybe at that time,…such advanced technology wasn not there…(after a long pause) Ah! It might have happened that at the Harappan civilization houses had wells where …where water would automatically change…It was advanced but perhaps negative numbers were even before that…So how could that have come into being…

While after a few discussions, we ended the day's session with the consensus that 0, -2, -4, …are even numbers, this agreement was be challenged the following session when Faizan pointed out that unlike even numbers, 0 has a distinct property - if you keep dividing any even number (2, 4, 6, …) by 2, you will sooner or later arrive at an odd number. This does not happen with zero. The children now insisted that zero was a special kind of number that cannot be simply categorized with other even numbers.

DISCUSSION

Faizan argued that mathematical concepts are not to be taken as the starting point of a mathematics discussion or as though existing *a priori* and to be fitted into an

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example, but are developed by people for a reason. He inquired into the possible history of numbers, developed definitions based on his observations of the properties of numbers and also challenged, for example, the evenness of zero. The students transformed the nature of our mathematics teaching sessions in which no one approached mathematical ideas from a reified standpoint. The children took ownership of their mathematics and would redefine the very norms of mathematics learning by also bringing their own mathematization to, the learning process.

Often, by ignoring me to ensure that their friends were not left behind, the students effectively resisted the kind power that I was unintentionally exerting that would have otherwise served to further reinforce the understanding that exclusion is inevitable among a diverse group of children.

The distinctive character of the student's responses to my teaching can be better understood by contrasting it with, for example, typical resource-rich mathematics classrooms of unaided private schools in Mumbai. Khanna (2017) presents a sociological study of one such classroom in which he observes "…the eagerness of the boys at the front to contest and answer questions in a flash." Khanna attributes this to "a strong spirit of competition and rivalry" that might have emanated "from competition among elite schools in the region, as parents from relatively privileged backgrounds (p. 25)." In comparison our sessions were dominated by a spirit of cooperation and empathy through which the students ensured that none of their friends were excluded from participating in our discussions. And this has implications beyond education aimed at only physically/mentally challenged student.

CONCLUDING THOUGHTS

The study suggested that to move towards genuine inclusion it is necessary to not only recognize how social arrangements and practices produce disablement, but to identify and resist different manifestations of ableism, for example, modes of thought and values that resist and pathologize "difference" as though a symptom to be treated or tolerated from the standpoint of a normativity. And this difference includes the different ways of mathematizing which may lead to mathematical ideas that may appear to deviate from the 'canonical' body of mathematical knowledge (for example different definitions of even numbers). The different nature of our classroom interactions challenged the authority of the teacher (the researcher) who came with assumption that in a diverse classroom it is inevitable that some children will be left behind. In this way, "difference" proved to be political. The students' expressions of differences serve to actively resist power that was exerted through mathematical interaction at different levels - from expecting that some students would naturally be left behind, to expecting all students to arrive at the same body of knowledge.

For children to "read" (critically interpret) and "write" (politically engage with) the world through mathematics, it is crucial that children first build a sense of ownership of the mathematics they use. And in this regard, we as mathematics educators can draw from the emancipatory ideals of ethnomathematics while reinventing CME in our critical pedagogies.

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Rani’s demonstrated that exclusion is the cause rather than a consequence of disability. Also, disabled children are understood as mere recipients of inclusion. However, the field observations demonstrated that the children themselves worry about and critically engage in the project of inclusion. And this occurred at multiple levels. For example, being the only blind girl in a “normal” private school, Rani took up the responsibility to ensure her own inclusion. Thereafter, in our mathematics sessions, she saw to it that all her friends participated in our discussions by translating and contextualizing the ideas, etc.

The children had created an inclusive subculture lead by values of empathy and cooperation within a society dominated by a culture of rivalry, competition and merit. The struggles of the teachers of the centre played a huge role in creating this alternative space in addition to getting the children admitted in regular “normal” schools. The study centre being less influenced by the sociopolitical forces inherent in typical normal school ensured that the children were not only a part of a learning community but would freely question the norms of society. This inclusiveness of the centre was expressed in the ways in which the diversity among the children was managed by the students themselves.

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UNDERSTANDING LEARNING OF MATHEMATICS TEACHERS THROUGH POSITIONING THEORY:INTERPLAY OF COLLABORATION AND TRUST

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Abstract: The educational reform efforts highlight collaboration among the teachers as a way to improve quality offering of education in the schools. The collaborative aspects are well integrated into the guidelines of reform curriculums indicated through phrases namely "exploring ideas together", "inquiry learning", "working in teams", "learning from each other" etc. The present study explores learning of primary mathematics teachers within context of teacher training sessions where they work in collaborative settings in order to understand and implement principles and pedagogy proposed by the New Curriculum in Portugal (ME, 2007). Through this study, I am searching for analytical tools that can provide evidence about teachers learning within these teacher-training sessions where teachers collaboratively engage in processes of learning about their changing professional needs of mathematics teaching and learning.

In this paper, I will present the case of a teacher (Fan). I will utilize positioning theory developed by Rom Harré and Luk Van Langenhove for illustrating how this teacher takes particular positions as a way to engage in the processes of doing professional mathematical learning as promoted through the training sessions of the program. Here I will argue that positioning theory as a conceptual tool can help us explore link between trust and collaboration since trust is a important link with collaboration between teachers and support opportunities for learning from each others (Tschannen- Moran, 2001). The trust is defined through five different facades defined previsously by Tschannen-Moran (2001) as a the way teachers attempt to use pronouns when they are narrating about there own practice.

BACKGROUND OF THE STUDY

This is a critical time for teacher education around the world. For example, OECD (2005) report *"Teachers matter"* emphasized the relation of students' success and quality of teaching. Despite of the theoretical discourse, the results are still not encouraging. The concern for quality on teaching has brought critical implications for teacher education and for research on the connection between teacher education and students' success. However, researching the connection between teacher education and students' success faces into many challenges. One of these challenges is how to develop and sustain teachers' quality and ensure that all teachers continue to engage in the processes of learning that allow them to meet their professional needs in an effective manner. Researching teacher

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development/learning requires paying attention to many factors, among others, including researching teaching situations/acts, beliefs and practices that generating particular kinds of learning for the teachers. Here one can bring a close eye on practices and processes where teachers respond to particular kinds of learning situations that teacher educators create for teachers to be ready to tackle professional demands in order to teach effectively. For example, The *OECD Handbook for Innovative Learning Environments (2017)* has clearly highlighted learning of the teachers as collaboration. This is also suggested within mathematics research area as well (Jaworski, 1998; Sowder, 2007). Through this case, I will provide an account of how primary mathematics teachers position themselves through their participation in the training sessions as part of professional development specially, I explore whether positioning theory as a possibility to seek trust between teachers as a way to promote collaboration between them. To respond to this question, I examine the dynamics between the teachers, through discursive standpoint. The collaboration between teachers should be seen as a vital component of teachers' continuous learning (Mitchell *et al.*, 2018). In these way, as positioning theory is perceived as a continuously negotiation of meanings. Since the positioning of the teachers is highly dynamic, it becomes difficult to determine the different position. However, through the examination and exploration of rights and duties, it is possible to analyze how teachers speak and behave in certain circumstances, particularly through analyzing the way pronouns are used.

METHODOLOGICAL AND ANALYTICAL CONSIDERATIONS

A strong motivation for the present study emerges from my own experience as a teacher educator where I worked with Mathematics Primary teachers from 2008-2012 under the work of Education Program (PFCM). This Teacher Education Program recognized some of the ideas suggested by educational research regarding professional development as mathematics teachers and curriculum development in mathematics. Within the frame of PFCM special emphasis was given to the actual professional needs of the participant teachers through their active involvement in this education program and strongly supported these teachers to work in groups where they could exchange their ideas in order to generate the possibility of deep reflections. In particular, the PFCM suggested the following principles and activities to provide quality professional development opportunities to the participant teachers: training workshops for the practicing teachers, join experimentation of the designed mathematical tasks in the classroom accompanied by the trainer and reflection on classes with the trainer and later in the respective working group and promoting teamwork in a way that groups should be consisting of teachers from the same school or the same group. The program consisted of monitoring sessions (with planning, observation and reflection of lessons in teacher classroom) and 15 group-training sessions, where reflection continued and also discussion of mathematics and its didactic based on tasks proposed by the teacher educator, while taking the collaborative work as a assumption. I adopt a qualitative methodology, approaching

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dynamic content analysis as a way to write up my study inspired by the work of Quivy and Campenhoudt (1998). In this type of analyses, the concern is mainly with the vocabulary and the structure of the sentence. It is through positioning moves that people assume certain positions to themselves and others. Moreover, I wanted to trace how teachers take dynamic stances or positions to engage in the processes of learning to appropriate offerings as exhibited through demands that the New Curriculum imposes on them. Here one can recognize the idea that helping the teacher to teach effectively is not only a question of developing habits of thinking as teachers but also more about putting into action (in their practice) whatever they already know (Darling-Hammond et al., 2005). Finally, to complement my observations I made interviews of four teachers involved in my study in order to clarify their own professional trajectory (motivations to choose the profession, what kind of professional development courses they took).

PROFESSIONAL DEVELOPMENT OF THE TEACHERS

Professional development can be viewed as an opportunity for teachers to come together and engage in the processes of working with each other to find approaches or solutions to challenges that they are facing within their professional and personal concerns connected to their professional practices of teaching and learning of mathematics. This can also lead to the needs to create networks for working together and learning from each other. Nowadays, networks can provide many ways of developing professionally (Eraut, 2007; Sowder, 2007). Moreover, it has also been studied that professional development of the teachers is related with learning successes of students (Adler, 2000; Guskey & Huberman, 1995; Sowder, 2007), but still much is unknown about the nature and quality of teacher learning that takes place during the processes of professional development (Sowder, 2007). Research related to professional development also points to the learning that takes place in work processes, particularly in the relationships established with peers (Eraut, 2007; Sowder 2007). These relationships may be established in person or online. Here the professional learning is not only about individual motivation, but it depends on the characteristics of the knowledge bases in the profession. In teaching, teachers usually have a wide range of concerns. So, teachers are reflective by the nature of their job, as compared with other professions (Schön, 1983).

LEARNING OF THE TEACHERS

Accordingly to the roots of learning, one can pursue basically two different perspectives: in Latin, with the meaning of holding, or from the Indo-European root, with the meaning of a track or furrow. "At various times in the evolution of our language it might have been understood as following a track, continuing, coming to know, or perhaps even getting into a rut"(Catania, 1998, p. 1). About the learning of the teachers, it become an imperative in these days, when society, economy and politics and so, education and curriculum demands are

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changing so quickly. Since Schulman's categorization about knowledge (1986) many researchers have since then identified more sub-categories of the knowledge. Blending collaboration and learning is becoming so far a globalized trend for every profession. Seeking learning of the teachers through collaboration is so far a need, ultimately to have more success with the students, as it shows highly correlated through PISA (OECD, 2018).

POSITIONING THEORY AS A WAY TO ELABORATING TRUST BETWEEN TEACHERS

I have used the approach of positioning theory as proposed by Rom Harré and Luk Van Langenhove (1999) in order to illustrate how these teachers take particular positions in terms of relating with each other (as co-participants of training sessions) as a way they learn in the training sessions . Within this context, I explore the following two research questions:

i) How can we analyze the teachers discourse in the context of Education Program of mathematics teachers through positioning theory?

ii) How positioning theory, in turn, can illustrate the intergroup dynamic as teachers influencing each other?

I consider positioning theory in the present study as a useful tool to analyze teachers discourse. Here, the underlying idea is that different actors have different positions of influence, which draws attention to the "dynamic stability between actors' positions, the social force of what they say and do, and the storylines that are instantiated in the saying and doings of each episode"(Harré & Van Langenhove, 1999, p.10). Positioning theory is also the "study of local moral orders" because it's "mutual and contestable rights and obligations of speaking and acting"(Harré & van Langenhove, 1999, p.1). While dealing with their own rights and obligations, how can teachers participate in a learning community, while stepping outside their own comfort zone? Since identity formation according to Wenger (1998) is a "lifelong process whose phases and rhythms change as the world changes"(p.263). Implicitly the social environment is a key point, since identity is negotiated all the time. In addition, because actors belong to multiple communities, their actions can become quite different depending on the nature and composition of the group (Wenger, 1998). This point to the fact that fluctuation in positions that different actors take exists. To address this aspect, positioning theory (Harré & Van Langenhove, 1999) will serve the purpose to capture the fluctuating aspects of the positions, but it is still "a starting point for reflecting upon the many different aspects of social life"(Harré & Van Langenhove, 1999, p.10). Since, the discourse of the teachers is a relational work, examining the dynamics of the interaction between teachers is a way to explore how teachers are using pronouns for example. Addressing trust between teachers is an important idea, since institutions with high level of trust; the participants are more comfortable to invest their energies (Tschannen-Moran,

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2001). Positioning theory aims to explain how positions and storylines together enable possible actions and meanings between teachers, in today's world it is vital for any organization to have social capital to handle and tackle the unexpected issues.

FINDINGS

Here I will share a case about the learning of a primary mathematics teacher (Fan) in order to conceive her learning in terms of change in reflections and possible effects act of sharing of her learning on the learning of her other colleague within the same training session. She had studied mathematics education as an undergraduate. At the time of training, she already had 15 years teaching experience. She was in charge for the teachers group (grades 5 and 6), while representing them in pedagogical teams with members representing the different sections of the school.

Here I want to present Fan's positioning related to interpreting changing reflections on the work of her students through the tasks for supporting the learning of students in mathematics. She often presented the tasks that she had developed in her classroom with confidence, probably related with the fact that she was in charge for the teachers with grades 5 and 6 (in other words, it seems it's part of her duty, to perform as good as possible). In the presented case she tried to predict mistakes that her students could make in handling the demands of the mathematical tasks that she invited them to work on. She tried to modify her direction of teaching depending on the emerging learning needs and the reactions of the students to the presented learning tasks. Her reflections on the tasks that she developed in relation to anticipating mathematics learning of the students, she shared difficulties that she experienced in terms of her students' reactions while putting these mathematical tasks into practice in her classroom. Here she associated the difficulties that students face with the tasks as an indicator of her displeasure with her own teaching and with herself:

> Fan: When I presented the task, my goal was that they should be able to calculate the area of the rectangle, while using fractions. When we were planning the task, we were very careful choosing the fractions. For example, they could see that 3 half's are equal to 1,5 m...but my surprise was really that they couldn't recall the formula to calculate the area of the rectangle. Despite my careful plan, I couldn't predict this. Two students, they added the values [instead of multiplying two measurements]. Honestly I felt disappointed with myself! (Session number 5, 07.01.2011)

She made this commentary in education session number 5. She expressed her disappointment with herself, even when she was thinking, that she was careful with her plan, while trying to predict children's difficulties. Here one can notice that her positioning was shifting from an experienced teacher (with confidence and certainty on her judgment) to a teacher that can dynamically anticipate children's action in an ongoing manner and stay watchful to the emerging varied learning demands of the students in a real classroom situations. Here the changed position of the teacher allow her to become open for new learning for herself and for her students

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and to be ready to accept her own failures as part of her approach to engage continually with her own learning as a professional teacher. This indicates that positions of a teacher can change, but at the same time they can be temporary. But, since she points out her disappointment while accepting her own failure, this admittance of her disappointment within the group reflections allowed other fellow participants of the workshop to revisit their own positioning in relation to their practices. Before the described episode, teachers used often the pronoun “I”, when describing their own teaching experiences. After the described episode, teachers where reporting their own “failures” while using more often the pronoun “we”. Probably because of Fan status, since she was representing some of them, there were also teachers from low primary schools (grade 1-4), this represented a turning-point for the group. Since participants are more comfortable and able to invest their energies in contributing to organizational goals when they feel trust (Tschannen-Moran, 2001), this seems to point the fact that when Fan pointed her own failure, she trusted the group. Latter, in the same session, Fan interacted with another colleague, while trying to reinforce the "us" pronoun:

> Iba : Sometimes I look like a kindergarten teacher, because I create such a story around the [mathematical] context. I create a story around or create a kind of dolls. I'm sorry but in not specialized in mathematics...
>
> Fan : Yes, it's very important for all of us to know. Sometimes I also feel ridiculous. (Session number 5, 07.01.2011)

Interestingly Fan support's Iba's effort to use stories, and she makes a moral assumptions, assuming that "I also feel ridiculous", positioning herself in a reflexive way. But at the same time, Fan is assuming the importance of Iba's opinion for the entire group.

In another episode with a different teacher (Lim) as he was reflecting on a work of students in the training session, interestingly he assumes a different tone, as he usually was referring to his work with "I". In other words, change of positioning of one teacher (Fan) can influence the positioning of other teachers (as per the case of Lim) or consequently they can also change their positioning, in a reciprocal way. In the followed episode he is describing an episode with his 1st grade pupils, about sitting in a room, according to the number they have received previously:

> Lim: (...) since we worked a lot in the number line, I was expecting that at least someone would ask me where was the chair number zero. But we couldn't predict that they would be stuck with adding 5+5+5=15, instead of 10+5=15 for example. (Session number 6, 21.01.2011)

Interestingly, when he is describing the task, something that he feels as a "failure" act, through students’ answers, he moved his positions from using the pronoun "I" to "we". This is just one example that occurred after Fans' episode. More cases came

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after this one. One can argue that due to the rights' and duties, maybe experiencing the openness of Fan, hierarchically responsible for most of the group, the other members start the too share more openness, there less successful episodes.

One can perceive the transformation from using "I" or "Me" to "We" or "Us", particularly evident in describing not so well succeed episodes. Harré and Langenhove (1999) called this positioning to "I" or "me" as a self positioning, and "we" or "us" as the "other". After this episode, there were more "failure episodes", but carefully used as a collective episode, while using the pronoun “we”. Even if there is still a doubt about the legitimacy about the “we” as a trust, still recognizing “failure” episodes, shows benevolence, openness, but also reliability and honesty. According to Tschannen-Moran (2014), these are facets of trust between teachers and students. In the discussion part, it’s further developed, applying the facets to the teachers exclusively. In this way, when one person positioned herself, others through interaction, also positioned themselves. In this way, positioning is dynamic and reciprocal. In other words, positions taken by teachers, despite of highly dynamic, can also be deterministic. That is, taking positions can be temporary, but we cannot ignore the fact, that positions can be sensed by others. Moreover, positioning theory is a highly dynamic process, one can witness turning points when the positioning of most of the group changes. In other words, positioning theory can provide insights about how members of a particular group can influence each other. Here one can observe the fact that teachers turned slowly to position themselves as being part of the group. Despite knowing that professional development is also being able to learn in a teacher-training program, the "feeling belonging to the group" takes time. As recognized by Harré (2012) the center of cognition shifts from individual to the local group. The positioning theory through analyzing the use of pronouns can provide us with a useful tool to understand teachers’ positions. In the present case, the movement from "I" to "we" was evidence of taking the position as “other”. In this way, one can argue that positioning oneself in a group includes the positioning of others as well.

DISCUSSION

In relations to other theoretical perspectives such as narrative identity (Clandinin & Connely, 1991), positioning theory offers a broader perspective to the analyses of positioning that actors take within group settings. Concepts of the positioning theory such as shared storylines and local moral orders can be applied specifically to the learning that occurs within interaction and dynamics within the groups. Groups’ dynamics can be interpreted as a mode of learning that can occur among and in between teachers. Here different positions in a small group can be seen as an act of negotiation and how members of a group negotiate their relations in discursive practices. While negotiating, members of a particular group can position themselves according to the interpretation that individual make about the moral aspects of social behavior. For example, through the change of pronouns from “I” to "we" and "us" in the form of talking about something difficult (as elaborated through this study),

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the positioning theory research into small groups can bring our attention to different multiple aspects of group interaction and group processes and the dynamic learning that can occur in the group settings. Since teachers are afraid about others critics (Hargreaves, 1995), the jump from "I" to "we" could at the same time represent a changing direction. The "we" pronoun seems to offer an interesting perspective to the problem-solving processes in a collaborative mode, so in this way, collaboration seems linked with the trust, as per suggestion some studies (Mitchell et al. 2018; Tschannen-Moran, 2001). One hypothesis is that other teachers understood Fans' benevolence, as a way to trust. Particularly, since the benevolence was coming from the teacher in charge of mathematics on 5th and 6th grade. In this way, benevolence is reciprocated by others. Through positioning theory, one can see professional development of the teachers as a slow, dynamic and reciprocal work in progress. Accepting less good things should be part of the learning process according to today's postmodern perspective world (Hargreaves, 1995). On the other hand, learning in professional settings of the teachers is at the same time, a "tacit dimension" part of the professional knowledge, as people are often unaware that they do learning through their work (Eraut, 2007). Learning here is in parallel with the trust between the teachers to narrate less achieved tasks with their students', which presupposes a collaborative mode. But still, how can we argue about teachers learning within positioning theory is still an open question, but from the Latin root where learning had the main idea of "holding" and through the analyses of the pronouns, one can assume that learning collaboratively is related with the idea of the group as a "we", sort of developing slowly a group identity. But in other hand, following the Indo-European origin, it could mean following the "group track". In the described episodes, one can claim that being more benevolent, open and honesty seems to provide to the other teacher more reliability. One of the critiques is precisely how much positioning and so far learning is intentional since the constraints of belonging in a teacher's trainer group exist. It means, in the end there will be a certificate proving teachers` capacity to participate in this kind of a program. Beyond the obvious question, that in the same conditions, another group of teachers will have different experiences. One can also argue that the presented case provides a way to seek some possible "turning-points". Experience here should be understood as positioning, with the constraints, the kind of positions that exist here will not necessarily be found at other places and times. This is the simple reason why Harré and Langenhove (1999) define position in terms of rights, duties and obligations, in other words, "moral" properties are also momentarily specified. Clearly, teachers are constantly engaged in self-positioning, but also, while positioning the others. Highly dynamic and reciprocal, it also generates more questions for empirical research. For example, once it begins to open the acceptance of the "failures" as a group responsibility, is it for sure, that it is a turning-point? Since teachers feel the reliability, it requires creating trusting communities, where teachers should act with consistency and fairness, but still it seems difficult to suggest that acts can be pursued as turning-points. Also accepting "failures" and sharing them seems to be a trust movement on the teachers. First of all, it shows

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benevolence from the teacher side, it presupposes that teachers are responding sensitively to their needs. At the same time, the accuracy of information that they are sharing is also one of the facets of trust pointed by Tschannen-Moran (2014), in her study about fostering trust with students. During pointing their own failures, teachers seem more open to help or to collaborate with each other. It seems a positive feedback, more trust develops more collaboration; and more collaboration seems to provide more trust. The learning disposition seems be related with collaboration while discussing the issues faced in their classrooms and in this way teachers develop professional learning according to their openness to share. Adopting an open-hearted stance in relation to students on a nonjudgmental attitude and the same time listening well is a way to seek trust according to Tschannen-Moran (2014). The positioning theory allows us to capture intergroup relations, in a collaboratively ever-changing storylines, from different and particular positions. The word "position" has been used in many ways, in social and psychology writings. Particularly in recent years it has become a way to analyze mediated interactions between people, from their own individual standpoint. But positioning as a discursive practice requires seeking all individual points from a certain collaborative community. Since the concept of learning is also becoming more and more dependent on social influences, seeking the learning of the teachers, with the lent's of possible turning-points, linking with the use of pronouns seems a plausible way to further develop about the learning of the teachers. How we can address the trust within the positioning theory in teachers education still needs to be more discussed in further research, specifically relating the using of the pronouns as possible turning-points as a collaboration between teachers. Since in these episodes, the positioning of the teachers changed, at the same time, this changing can be temporary. The main reason to reinforce the collaboration between teachers is that mainly distrust is still likely to have a deleterious effect on communication (Tschannen-Moran, 2001). But since collaborative is rather than a choice, research in teacher education should put all the efforts to find out ways to foster the communication process between teachers in order to support the teachers learning.

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VALUES AND BEAUTY IN MATH EDUCATION

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In this article I argue for a philosophical training of teachers in mathematics to make them critical to their own subject. The same as other teaching objects (e.g. chemistry, biology, geography and literature) mathematics is embedded in a culture, in a society, in a history, and in a school curriculum that is value-laden and power sensitive. Although most curricula spend few room for philosophical questions in favour of the purely mathematical subject, I'll argue that a mathematics teacher with a philosophical training is better prepared to work with pupils who raise general or philosophical questions about the subject being taught. Moreover, I'll show that a philosophical approach of teaching mathematics enhances mathematical literacy as defined by the OECD. Mathematics is part of pupils' capacity of critical citizenship.

INTRODUCTION

Let me introduce some ideas on a philosophy of mathematics with an anecdote that happened to me a couple of weeks ago. A nephew of mine sent me a message pointing to the fact that an article I wrote for the newspaper was hanging on the wall at the mathematics classroom (Image 1). The article (a column) was indeed written a couple of years before and published by a Flemish newspaper (De Standaard, April 25th, 2013). I first will recall what I wrote in the article which was written for the broad public and second I will try to understand or explain why the article was now hanging in the mathematics classroom already for a couple of years (2013-2018) (Image 2). The title of the article is 'The power of the number' (François, 2013). In the short abstract I refer to mathematics as the powerful language of pure reason of human being. Although mathematics is related to pure reason, at the same time one can observe that people all over the world give added meaning and value to numbers since the origins of mathematics. I was thinking of Pythagoras' philosophy of mathematics (I will explain below) but also nowadays people add values to numbers, e.g. 13 is the number of bad luck, to name just one of them and best known in western countries.

In the following section I first will give a brief overview of the idea that mathematics is the purest language in which we have to describe or to define 'nature'. Secondly, I discuss values of mathematics with special attention to the value of beauty in mathematics. Finally, I'll discuss the added value of integrating a broad perspective on mathematics in the curriculum of mathematics education to enhance teachers' competencies and abilities to teach the pupils according the description of mathematical literacy as given by the OECC (OECD, 2010)

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Image 1: Messenger message from pupil in mathematics classroom (nephew of the author) saying “who’s hanging at my classroom?! 😆”. Credits: Karen François

Image 2: Article from the Flemish newspaper (de Standaard, 25 April 2013) hanging on the wall at the mathematics classroom in a Flemish school, grade three in secondary education (pupils age 15-17 years old). Credits: Jeroen François

LANGUAGE OF NATURE

The expression that “the book of nature is written in the language of mathematics” is dedicated to Galileo who worked hard to capture the variety of the physical world into a one-dimensional language that expresses the essence of nature: the law(s) of nature expressed in the language of mathematics. The idea to grasp the essence of nature (instead of describing its variety) is as old as philosophy of science. Although we had different perspectives on the way nature has to be studied and represented (e.g. the

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concept of the universal flux expressed by Heraclitus' quote 'It is not possible to step twice into the same river' (Marcovich, 1967)); Pythagoras' ideas about the essence of nature became mainstream in epistemology. Pythagoras identified number as the foundation of all things expressed as 'All is number'; 'The whole thing is a number'; 'One is God' or 'One is substance and not a predicate of something else' (Aristotle, 1984; Barker, 1994; Wertheim, 1995). This philosophical idea became the core of Modern Science at the beginning of the 17th century and was further generalized from then. The idea was elaborated on by Descartes who identified mathematics as both, the language in which knowledge has to be expressed but also as the method through which we can achieve certain knowledge about nature. Descartes (1966) handed down the regulations (*regulae*) for how to represent and to get to know things (*objecta*) properly. He gave birth to a new method he applied to his study on Optics, Geometry and Meteorology, for which he wrote an introduction in 1637 'Discourse on Method' (Descartes, 1953). Based on his investigations in geometry, Descartes determined intuition and deduction as the two core operations through which reason achieves its goal; grasping truths in an immediate way and be straightforwardly deducible from such claims (the same way as conclusions of mathematical proofs are deducible from the premises). Descartes first inquired on this method in his 'Regulae Ad Directionem Ingenii' or 'Regulations Concerning the Intellectual Activities' (Descartes, 1966).

In the following paragraph I'll briefly investigate the principals of this method as a method that is value laden (a more extensive analysis can be found in François & De Sutter, 2004). Therefore, I have to analyse the first three rules of Descartes' Regulae.

> **Rule 1** The purpose of any intellectual inquiry should be to reach solid and true judgments about everything that occurs.
>
> **Rule 2** We should attend only to those objects of which our minds appear to be capable of having certain and indubitable cognition.
>
> **Rule 3** Concerning the things proposed, one ought not to look at what others might have thought or at what any one might have conjectured, but only at what we can either clearly and evidently intuit or deduce with certainty; for in no other way can knowledge be acquired. (Descartes, 1966)

In his first rule, Descartes announces his program, laying out the purposes of his *Regulae*. In the second rule, Descartes describes the epistemological constraints put on obtaining certain knowledge and in rule 3, the role of mathematics within the sciences is highlighted. This selection of what kind of things to know seems to be one of neutrality and objectivity. Rudolf Boehm (2002) distinguishes two kinds of truths. He makes the difference between *logical* truth and *topical* truth, the former referring to true knowledge obtained by applying a logical deductive method, the later, the topical truth refers to the question of which objects *should* or are *interesting to* be known. In Descartes' *Regulae*, this question is reduced to the method itself; are qualified only those objects of which our minds are capable of attaining indubitable cognition (rule 2). Objectivity is one of the aspects that mathematics can provide. Although objectivity

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or generality seems to be a neutral position, as such it *is* a position and thus a certain perspective which is described as being a value (Bishop, 1988; Chemla, 2016; Ernest, 2016). In the following section I'll elaborate on the notion of values in mathematics.

VALUES OF MATHEMATICS

Recent studies on mathematics and values are related to the analysis of values in pure and applied mathematics (Bishop, 1988; Chemla, Chorlay & Rabouin, 2016; Ernest, 1991, 2016; Shulman, 2018) and more specific to the research field of mathematics and statistics education emphasizing the interrelation with values pupils or teachers add to mathematics (Queiroz, Monteiro, Carvalho & François, 2017; Dawkins & Weber, 2017; Leavy & Hourigan, 2018). Bishop (1988) was one of the first philosophers of mathematics who emphasised the importance of analysing and unveiling hidden values in mathematics. He presents six different sets of ideals and values associated with mathematics (differentiating between six categories in three dimensions). The (i) dimension of ideology includes (ia) the value of rationalism and (ib) objectism; the (ii) dimension of sentiment includes (iia) the value of control and (iib) progress; and the (iii) dimension of sociology relates to (iiia) the value of openness and (iiib) mystery. Bishop analyses and relates all six values to mathematics. *Rationalism* is the expression of aiming at the highest level of certainty (as expressed by Descartes and explained in the former section). *Objectivism* is based on inanimate objects, phenomena and humans. Mathematics is used to *control* the environment, e.g. the use of algorithms to control big data generated by human practices. Mathematics relates to the promises of development, change, growth and thus *progress* (e.g. the European policy statement 'knowledge for growth' and Science, Technology, Engineering, Mathematics (STEM) initiatives). The openness and mystery of mathematics seem paradoxical. Mathematics is *open* because mathematical truths, propositions and ideas can be examined by all people (or let's say mathematicians in the field). The same time, mathematics feels as a *mystery*, not only by people in the street or pupils at secondary education, given that they know so little about the subject; but also mathematicians can feel mystified about mathematics because the unknown, the question of where mathematics comes from, the conjectures or unproved propositions that are believed to be truth. In his later work, Bishop, Clarke, Corrigan & Gunstone (2006) studied values in mathematics and science education based on an empirical design. He adapted the initial framework used for the interpretation of values in mathematics education by recasting the value of objectism as empiricism to accommodate scientists. Research findings suggest that empiricism and rationalism are the most important values for both mathematics and science. More recent work on common values in mathematics and science was done by Chemla et al. (2016). They studied the different meanings and the role of generality in the history of mathematics and in the sciences. Interesting to see how generality was prized in performing mathematical problem solving as in performing mathematical proof. This work gives a historical and intercultural perspective on how different 'collectives' have valued *generality*. These studies give strong argument of the relation between values and mathematics (and the sciences) and this from the early beginnings

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of the practices. In his early work on philosophy of mathematics, Ernest (1991) inquires about the inherent values in mathematics. Abstract is valued above concrete, formal above informal, objective above subjective, justification above discovery, rationality above intuition, reason above emotion, general above particular, theory above practice, the work of the brain above the work of the hand, and so on. In his recent work Ernest (2016) distinguishes between epistemological, ontological, aesthetical, and ethical values. Both can be overt or covert where open values are explicitly acknowledged by mathematicians; covert values are tacit, hidden or generally unacknowledged by the mathematical community. Open values are truth, purity, universalism, objectivism, rationalism and utility; in contrast, covert values are objectivism and ethics. Objectivism is perceived as value free. Ethics includes separatism, openness, fairness and democracy. Ethics in Mathematics (EiM) is a research field of growing interest and much has to be done within this domain. An interesting example of the work that is done the last months can be find at the website of the international conference on 'Ethics in Mathematics', held at Cambridge University, April 2018. The community is a collection of mathematicians, and experts from outside mathematics with relevant experience and knowledge (EiM, 2018). The study of values in mathematics has direct relevance for the teaching and learning of mathematics. Ernest (2018) describes the failure and the success cycle in the learning of mathematics where three elements develop together. Negative attitudes to mathematics such as poor confidence and mathematics self-concept or possible mathematics anxiety has a negative impact on the learning. Pupils experience reduced persistence and learning opportunities, and they will probably avoid mathematics. Disengagement in turn generates mathematical failure due to repeated lack of success and failure at mathematical tasks and tests. These three elements are connected to each other, once the circle is started, it becomes a self-reinforcing cycle. The same mechanism is at place in the success cycle where positive attitudes improve the engagement and learning which in turn increases mathematical success.

Within the field of mathematics and statistics education, increasing attention is also paid to the role of affective factors in the teaching and learning process. Attention is given to different aspects of affection e.g. feelings, emotions, beliefs, attitudes, conceptions and values. Queiroz et al. (2017) give empirical evidence that affective factors are at place when interpreting statistical data. They argue that hidden values can better be uncovered to give full meaning to the influence of interpreting data. Dawkins & Weber (2017) analyse in a theoretical paper the concept of mathematical proof in terms of their values such as establishing a priori truth, decontextualized reasoning, increasing mathematical understanding and maintaining consistent standards for acceptable reasoning across domains. They argue that these values may influence students' apprenticeship into proving practice. Students who do not accept these values will likely find proving practice confusing and problematic.

Also, teachers' beliefs can affect the learning process. Leavy & Hourigan (2018) investigated pre-service teachers' beliefs about the nature of mathematics and about their value of mathematics. The study reveals that participants reported generally

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positive beliefs about the value and the enjoyment in doing mathematics. They further reported that it is possible for mathematics education programmes to change and to stimulate improvement in beliefs and attitudes toward mathematics.

What is mostly overseen and more an issue of popular writings on numerology is the connection of values, virtues or even ethical considerations to numbers and geometrical figures. Guedj (2002) gives many examples of numbers that are beautiful (see the Pythagoras example below), complex (Zeno's paradox) or the representation of order (numbers that represent the harmony of heaven). Although many mathematicians may believe that mathematics is pure and value-free and indeed some would argue that this is one of the defining characteristics of mathematics (Shulman, 2018), I'll give historical and empirical evidence that numbers, as the building blocks of mathematics (from an intuitionist foundational perspective), are connected to values that have real impact on societal matters.

It may seem quite strange to add value to numbers although this attitude is as old as the practice of dealing with numbers (e.g. as was the case with Pythagoras and Nostradamus amongst others). Until today, even in Western societies, numbers are still associated with values. A nice example is the logo of Brussels Airlines which consists of *fourteen* dots (representing the letter 'b' referring to Brussels and Belgium and visualizing the network of the runway during takeoff and landing). The logo as original designed (November 2006) consisted of *thirteen* dots which was criticized because of the connotation of thirteen with 'failure' and 'unhappiness'. Brussels airline decided to redesign the logo by adding one more dot (BBC, 2007) (see image 3).

brussels
airlines

Image 3: Painters have been adding an extra dot to the Brussels airline's logo (BBC, 2007) and Image 4: Brussels airline's logo (Brussels Airlines, 2018)

There are indeed cultural differences. In China for example, *four* is the number of bad luck and the number *fourteen* is even worse. Chinese pronunciation of four (si) sounds the same as the character for 'dead' (Cajori, 1993); the pronunciation of fourteen (shi-si) sounds the same as 'want to be dead'. But this was no argument for Brussels airline to again change the logo existing of fourteen dots.

Until today, there are still nice examples on how values connected to numbers play a role in our daily life. Even high rational companies take into account possible impact of the use of the number 13. Level 13 of row 13 is time to time missing (see Image 5).

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Image 5: number 12 and 14 indicating the row on the plane Fokker 100 Carpatair company (credits Karen François)

THE SPECIAL VALUE OF BEAUTY

One of the values that is well known and largely discussed and illustrated is the value of beauty (Ernest, 2016). Aesthetics is central to mathematics and the claim that some aspects of mathematics are beautiful is often heard in relation to patterns based on symmetry (Weyl, 1952). The analysis of mathematical artefacts based on symmetry has an old tradition and goes back to the work of Pythagoras (6-5 century B.C.). In his work, symmetry was a central property when adding value to numbers (Weyl, 1952). Numbers were represented by peddles placed in sand making specific configurations that represented geometric figures. Numbers were also "divided into classes: odd, even, even-times-even, odd-times-odd, prime and composite, perfect, friendly, triangular, square, pentagonal, etc." (Struik, 1967: 41-42) These configuration of numbers is called figurative numbers. A nice example where symmetry has a decisive role in adding value to numbers is the division of odd and even numbers and their respective value. Odd numbers were labeled as *male* or determined, even numbers as *female* or undetermined (Aristoteles, 1984: 1560). Looking at the figures below one can understand how philosophers (in times of full development of patriarchy) came up with these ideas (François, 2008).

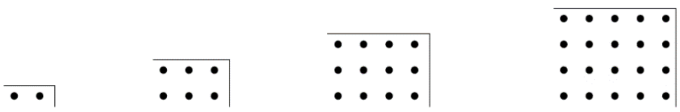

Figure 1. Even figurative numbers: female or undetermined (2 symmetry axes)

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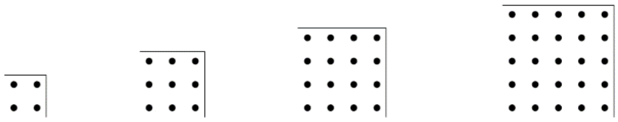

Figure 2. Odd figurative numbers: male or determined (4 symmetry axes)

A rectangle (having two symmetry axes) is less perfect than a square (which has four symmetry axes) and thus less valued because of the lower amount of symmetry axes. Indeed, the most perfect geometrical figure is the circle. Plato (*Timaeus*, 34 b) argues the perfectibility of the circle because of its infinite amount of symmetry axes. The same argument was used by Birkhoff (1933) when he coined the formula of the beauty of (geometrical/mathematical) objects where M stands for 'esthetic measure'; O for 'order' (or the amount of symmetry axes) and C stands for 'Complexity' (or the number of corners). $M = \frac{O}{C}$ Applied to rectangles, squares and circles we have the following:

Rectangle	**Square**	**Circle**
$M = \frac{O}{C} = \frac{2}{4} = \frac{1}{2}$	$M = \frac{O}{C} = \frac{4}{4} = 1$	$M = \frac{O}{C} = \frac{\infty}{0} = \infty$

Figure 3. Formula of the beauty of rectangles, squares and circles

Symmetry was a central mathematical concept at the dawn of time and the origin mathematical practice. It was discussed and used in mathematics and mathematical practices to interpret numbers and to give (esthetic) value to mathematical objects.

Image 6. The bee pendant 1800-1700 B.C. Iraklion Museum. Credits Karen François

Weyl (1952) explores the idea of symmetry starting from the vague notion of symmetry as the harmony of proportions to develop the geometric concept of geometry in all its variation to finally rise to the mathematical idea of "the invariance of a configuration of elements under a group of automorphic transformations" (Weyl, 1952: i) Bilateral symmetry (or the symmetry of left and right) appears as the first case of a geometric concept that refers to the operation of reflection and rotation. It is also mentioned that this kind of symmetry reflects the structure of the human body (Weyl, 1952: 8). Given the fact that mathematical practice originates from human practitioners with a human bilateral body can give rise to the idea of the universality of the concept. Bilateral symmetry has been observed since the ancient practices of arts and mathematics. The bee pendant (a gold ornament from Malia, Crete, consisting of two bees depositing a drop honey in their honeycomb) dating from 1800-1700 B.C.

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is a nice example of bilateral symmetry in the arts from the ancient time (see image 6). The importance of symmetry as a geometrical concept is known from the work of the ancient Greeks. Modern mathematics provided tools to define the concept of symmetry. It is defined by using the Cartesian coordinate system. Symmetry is prevailing if we have “invariance with respect to transition from one Cartesian coordinate system to another; this symmetry comes from the rotational symmetry of space and is expressed by the group of geometric rotations about 0” (Weyl, 1952: 134). The orthogonal coordinate system is named after Descartes who introduced the method of rectilinear coordinates, providing a link between geometry and algebra. As these coordinates are the foundation of analytic geometry, Descartes can be considered as the founding father of analytic geometry, combining geometry and algebra. (Kline, 1990). All pupils are confronted with the Cartesian coordinate system from the early beginning of their school career. From primary education until doctoral training pupils, students, and PhD candidates have to work with the coordinate system to represent and to interpret data. In the final section I’ll discuss the added value of dealing with values in mathematics classrooms.

DISCUSSION: ADDED VALUE OF A PHILOSOPHY OF MATHEMATICS

Let me introduce this section with a second anecdote. At the beginning of my career I was a teacher at secondary schools. Although my topic was philosophy and ethics (compulsory part in the curriculum for all pupils at secondary schools in Flanders – two hours a week) I sometimes referred to other topics of the curriculum. Sciences, mathematics, geography and chemistry where my favourites (and clearly related to my educational background since my first degree was in sciences). One day I discussed about geometry as it was taught to the students. I told the students about the fact that the geometry in the curriculum was just 'a' geometry, namely Euclidean geometry, and that there were other geometries. I thought of Lobachevsky and Riemann (from the 19th century). I suggested to discuss this issue with their mathematics teacher to know the true facts of different geometries. Unfortunately, the week after, the students came to me telling that the mathematics teacher did not understand what it was about. I realised that mathematics is widely perceived and taught as a pure technical and value free science without a history nor a cultural aspect. Consequently, mathematics curricula are focussing on the technical aspect of doing mathematics (François, 2007). It had never been my intention to put the mathematics teacher in an uncomfortable situation. However, I was convinced that the teacher training in mathematics had failed. If a mathematics curriculum wants to meet the international standards of mathematical literacy, it has to go beyond the pure technical aspects of mathematics. In this paper I argued about the embeddedness of mathematics into history and culture by discussing the value of mathematics (i) as the language of nature arguing its importance within epistemology and the value of objectivism; (ii) based on theoretical and empirical investigations regarding values in mathematics(education); and (iii) investigating the special value of beauty based on symmetry. If we want to teach the next generation of mathematicians and of critical citizen with mathematical literacy competences,

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mathematics education curricula need to go beyond a pure technical training. ‘Constructive, engaged and reflective citizens’ need a well-informed education to be prepared ‘to make the well-founded judgments and decisions’ (OECD, 2010). I recall basic statements from OECD (2010) which changed a bit over time. Although added value to critical and constructive citizenship was emphasised from 1999. “Mathematical literacy is an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen” (OECD, 1999). The concept of mathematical literacy was reformulated in 2010 as follows: “Mathematical literacy is an individual’s capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts, and tools to describe, explain, and predict phenomena. It assists individuals to recognize the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens” (OECD 2010). The way how the concept of literacy is used and applied in international comparative research (e.g. PISA) is discussed and criticised because its uncritical application to the most divers countries and backgrounds. A technically oriented curriculum leads us indeed to the competence of mathematical literacy in a narrow interpretation, excluding the emphasis on critical citizenship. Mathematics as a human and cultural practice with a broad application in society deserves a broad context in which it is taught. Based on this broad interpretation and framing of mathematics critical competences can be developed. If teachers want to answer the broader (philosophical, historical, cultural) questions of pupils, they will have to receive a better teacher training that pays attention to the philosophical, historical and cultural embedding of mathematics. Only this way will we be able to meet the expectations of the broad meaning of mathematical literacy, a basic right for every student and by extension by every citizen.

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SCIENCE TECHNOLOGY ENGINEERING MATHEMATICS (STEM) LAND: FACTORS AND INTERVENTIONS INFLUENCING CHILDREN'S ATTITUDE TOWARDS MATHEMATICS

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Abstract: *The dislike and fear of Mathematics in children is well documented in literature (Daniel, 1969). Further, literature suggests that children's interest in Mathematics decreases from elementary to high school (Köller, Baumert and Schnabel, 2001). Many practising teachers also tend to believe this. However, a survey with the children in a rural STEM land indicated that their interest in mathematics had been retained or increased from when they were in 5th grade. In this paper, we look at*

1) Is there a co-relation of interest in Mathematics with how well children do in their curricular examinations?

2) We also describe the learning environment and interventions at STEM land and the response of children to these. The questions we focus on:
Will freedom to plan their work help?
Does opportunity to choose working individually or with peers change the learning environment?
Does access to games and puzzles give a broader perspective of Mathematics and will it lead playful learning?
How interested are children in using materials in Mathematics that make abstract ideas concrete?
How interested are children creating projects demonstrate mastery of concepts?

CONTEXT AND INTRODUCTION

This is an Action Research project by a team of engineers who wanted to study our interventions as we implemented them to find out whether these methods and materials would increase children's interest, or exam results, in two rural STEM centres run in two outreach schools of Auroville – Udavi School and Isai Ambalam School. Both schools aspire towards the holistic development of the child and the managements are progressive. The children attending come from villages surrounding Auroville.

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Udavi School follows the Tamil Nadu state board syllabus and we work with 47 children from 7th to 9th intensively for 6 hours/week for all their Mathematics (Math) classes. A few children also come for an activity class in STEM. Other subjects are handled in their regular classrooms. Isai Ambalam School follows the central board syllabus (CBSE) where we work with 48 children from 3rd to 7th grades intensively for 6 hours/week during the Environmental Sciences (EVS) and Math classes. In demographics, the occupation of parents in both schools is unskilled labor (35%), skilled labor (55%) and salaried workers (10%). The predominant community accessing Udavi School is MBC (Most Backward Caste) and accessing Isai Ambalam School is SC (Scheduled Caste). This paper focuses on STEM land at Udavi School and on Mathematics.

At STEM land our goal is to develop the values of responsibility, equality and the courage-to-create in children. At STEM land in Udavi school the children learn to take responsibility for learning and address not only their curriculum, but also create projects that demonstrate their mastery on topics learned (Ranganathan, 2015) and also learn electronics, programming, etc. They do this through self-learning, peer-learning in multi-grade environments and through interactions with facilitators. Children have assessments once in a week on their chosen goals to help them with seeing their progress in addition to their regular examinations from the school. STEM land is also open to anyone to come and learn electronics, programming, Mathematics, puzzles and strategy games. The activities of STEM land that help create a collaborative learning environment are documented in detail elsewhere (Ranganathan, 2018). This paper focuses on specific interventions as listed in the abstract at Udavi.

Motivation of the paper: Change of interest in Mathematics

We asked the children "Rate your interest in Mathematics from the time you were in 5th grade to now". They rated this on a Likert Emberling scale of 1-10 with 1 indicating a strong decrease, 5 a retention and 10 a strong increase in interest.

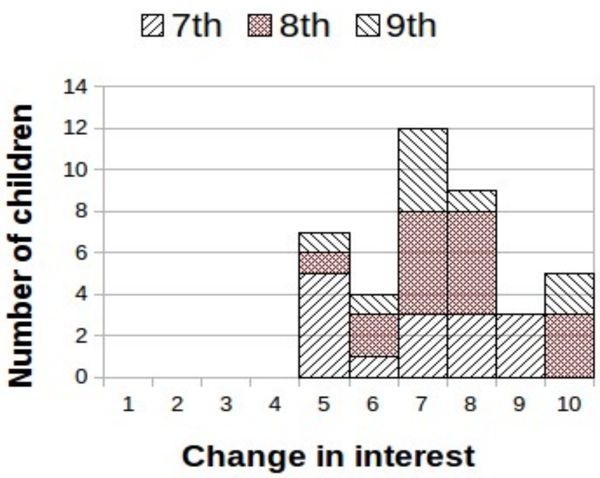

Figure 1: Change in interest in Mathematics from 5th grade for children in 7th, 8th and 9th grade.

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The result of the survey was interesting for us, because it indicated an increase in interest when literature suggests otherwise(Köller, Baumert and Schnabel, 2001). In this paper we look at what could be special about the environment at STEM land that causes interest to increase.

PHILOSOPHIES UNDERLYING STEM LAND

The philosophy underlying the approach for STEM land is based on the principles of progressive and constructivist thinkers like Jerome Bruner in the United States, Sri Aurobindo and Mukunda in India and many others briefly described here.

Constructivist Education Theory (Bruner, 1960) indicates that knowledge is not delivered into the learner (whether child or adult) but recreated by the learner on his or her own. Children actively construct their knowledge by connecting new knowledge to what they already know.

In India, Sri Aurobindo (Aurobindo, 1910) says that nothing can be taught, but the teacher can guide, support and encourage a child in the process of learning, enabling them to evolve towards perfection. More recently, Mukunda (Mukunda, 2009) describes the three aspects of learning that are relevant to schools – conceptual knowledge, procedural knowledge and higher order reasoning. Conceptual knowledge (and change), she states, greatly benefit from constructivist approaches.

Taking a specific aspect of STEM of Maths, the National Curricular Framework (NCF 2005) (Pal, et al., 2005) states that the 'useful' capabilities relating to numeracy, number operations, measurements, decimals and percentages are only a narrow goal of Maths education. The higher purpose of Mathematics, it says, is Mathematization: the understanding and application of Mathematics in different situations with a focus on abstraction, patient problem solving and logical thinking. Meeting this goal requires a fundamental change in the approach used in schools. It requires classrooms to move away from simplistic 'sums' to more complex problem solving and contexts. It requires a shift in conversations in the classroom from the 'right answer' to considering and discovering approaches to problem solving. In a similar fashion NCF treats the development of scientific inquiry as more important than the knowledge of scientific facts.

LITERATURE ON ATTITUDES TOWARDS MATHEMATICS

The role of attitudes in learning Mathematics (Daniel, 1969) has been explored earlier. It has been found that there are many students have a fear of Mathematics, and dislike mathematical activities. Some students completely hate Mathematics, some fear making mistakes. Daniel also suggest that school should be a place where tasks are made more attractive and require educational programs to be more flexible and individualized.

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Conceptualization of academic interest (Köller, Baumert and Schnabel, 2001). This paper says that although interest is usually considered important antecedent to successful academic learning empirical data suggests that this assumption is weak. While doing research in different schools and colleges in different countries the researcher found that interest in Mathematics continues to decrease from Grade 7 onward.

The psychological research and theory of (Silver, 2004) suggests that by having students learn through the experience of solving problems, they can learn both content and thinking strategies. When a problem doesn't have a single correct answer students work in collaborative groups to solve a problem which increases their peer learning. PBL (Problem Based Learning) increases self-directed learning which helps them to solve real life problems. Second, students must be able to set learning goals, identifying what they need to learn more about for the task they are engaged in. Third, they must be able to plan their learning and select appropriate learning strategies. The final goal of PBL is to help students become intrinsically motivated by their own interests, challenges, or sense of satisfaction.

CORRELATION WITH CURRICULAR EXAMINATIONS:

Here are the results and analysis of some of the surveys conducted with 45 children in STEM land. 7th Graders had experienced STEM Land for six months, 8th graders for one and half years and 9th graders for two and half years. Ten STEM land facilitators conducted the survey. The survey of 14 questions was on their experience of various interventions of STEM land. The survey was conducted in a one-on-one interview with children (in English and Tamil as requested by the child). The researchers, who are also the teachers conducted this survey.

Linking with the original research of how children view mathematics (Daniel, 1969) we asked:

How is Mathematics different from other subjects?

In each class from 7th to 9th grade, there were about 15 to 16 students. In every class around 65% of the children said that Mathematics is difficult compared to other subjects. Among the children who found it difficult some found numbers and calculations confusing, others found it difficult to remember formulas. One child even said that Mathematics is the most difficult subject among all the subjects. 25% of the children said that they are able to notice that they use math in other subjects and said that it helps in life. Very few children said that there is no difference between Mathematics and other subjects. Only 8% of them said that they are interested in Mathematics and it is easy to understand. Some children even distinguished Mathematics as a subject from what they do at STEM land.

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Correlation between interest and curricular examinations

Using the data on change of interest in Mathematics for each child from 7th to 9th grade and we calculated the Pearson Correlation Coefficient (PCC) with how well they did in their curricular examinations. We found that for 7th standard the PCC was -0.4329 which means the answer they gave for the interest level and the marks they have scored in exam was inversely related. For 8th standard the PCC was 0.222142 which is positive but very weak. For 9th standard the PCC was 0.171023 which was also weak. Similar to what we see in literature (Köller, 2001) we do not find a strong correlation between how children do well in examinations and their interest in Mathematics in the limited sample (47 children) in STEM land.

INTERVENTIONS AT STEM LAND AND OBSERVATIONS:

We focus on a few specific interventions and the response of children to these here.

Intervention: The freedom given to plan their work

At STEM land, we believe children are responsible for their learning and for their growth. A software was created at STEM land to support children create a plan and track their progress. Children create a plan of what they are going to learn each week. They are assessed each week on the goals they work towards and children can track their progress visually as a pie chart.

Response

We asked "*How satisfied do you generally feel at the end of a week? On a scale of 1-10 with 10 being fully satisfied and 1 being not satisfied*". This question was asked to understand how students were responsible for their learning. Figure 2 shows that almost all the children are satisfied on working towards their goals.

We also asked what they did when goals were not met. Most said that they work harder and seek support from peers and facilitators. Some children said that they work at home to complete what they set out to do. Only a few mentioned that they felt sad and were not able to proceed to the next topic.

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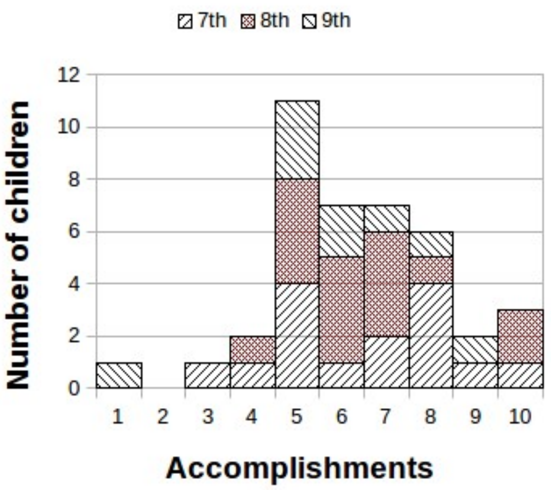

Figure 2: How accomplished children generally feel at the end of the week

Intervention: Access to games and puzzles that give a broader perspective of Mathematics and are joyful

At STEM land we have an active environment of challenging ourselves with many puzzles and strategy games. The puzzles include physical disentanglement puzzles, rubiks cubes, etc. The games include strategy games such as Abalone, Othello, etc. These are as often checked out and used by children (even if it is for a short time) as laptops. There is a fairly large group of children who can solve the rubiks cube.

Response

We asked children "*How stressed or joyful do you feel at STEM land (1=stressed, 5=neutral and 10=joyful)?*". We found that most of the children felt joyful being at STEM land.

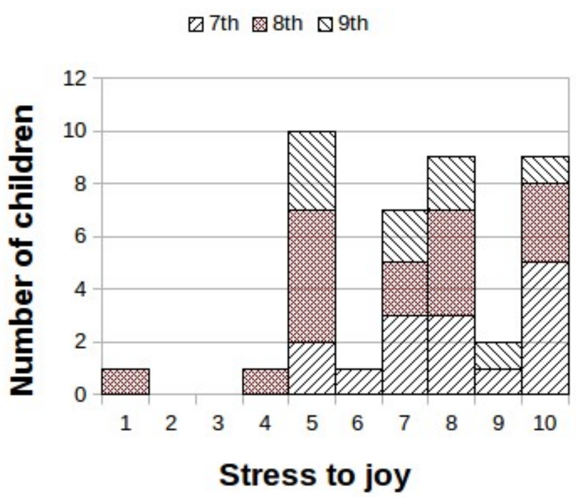

Figure 3: How Joyful children feel working at STEM land (1=stressed, 5=neutral and 10=joyful).

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We also asked "*Where and how do you use math in your day-to-day life?*" Children furnished numerous examples about how they use mathematical concepts in real-life: some mentioned that they use it while shopping and to calculate monthly expenses for their house.

One child said that he uses Mathematics to change his perspective, when he had only

10 minutes to complete a task he changes this into 600 seconds and this helps him calm down and complete his work. He said this had increased his confidence level.

Intervention: Choice of working individually or with peers

At STEM land children have a choice of working individually or in groups. This allows for individual learning as well as peer-to-peer learning. In addition, in a week there are four to five multi-grade classes. Where younger children and elder children work together and collaborate in learning.

Response

The response to a question of "How often do you collaborate on a scale of 1 to 10 (1-rarely, 5-often, 10- always)" is shown below. This indicates that most of the children often collaborate with others, working in groups and sharing what they have learned.

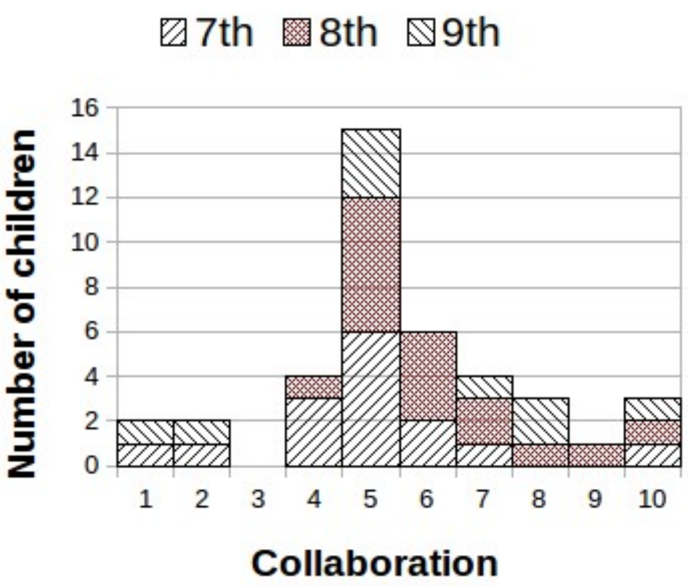

Figure 4: How often do I work with other children (1 – rarely, 5 – often, 10 – always).

When children were asked to list what they learned from peers they said they learned to solve problems that they don't know; Math concepts like square root, factorization; strategy games like Othello, Abalon; to solve the Rubiks cube; programming in Scratch, Geogebra and Alice; mind-storms (robotics), and electronics like makey makey, soldering. Some children also mentioned that they learned how to effectively use STEM land including using laptops, checking-in and out materials,

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filling plans for the week, taking up responsibilities for specific materials at STEM land and even not being afraid and asking for help when you don't know something.

Intervention: Access and use of materials in Mathematics to make abstract ideas concrete

In STEM land children have access to a wide variety of materials that make mathematical concepts concrete including Montessori, Jodo Gyan, etc.

Response

We asked children "*What is your interest in learning mathematical concepts using materials in STEM Land on a scale of 1 to 10. 1 being not interested, 5 being interested and 10 being very interested"*. The average score was close to 8.5 and none of the children in all three grades went below a score of 5 on using materials.

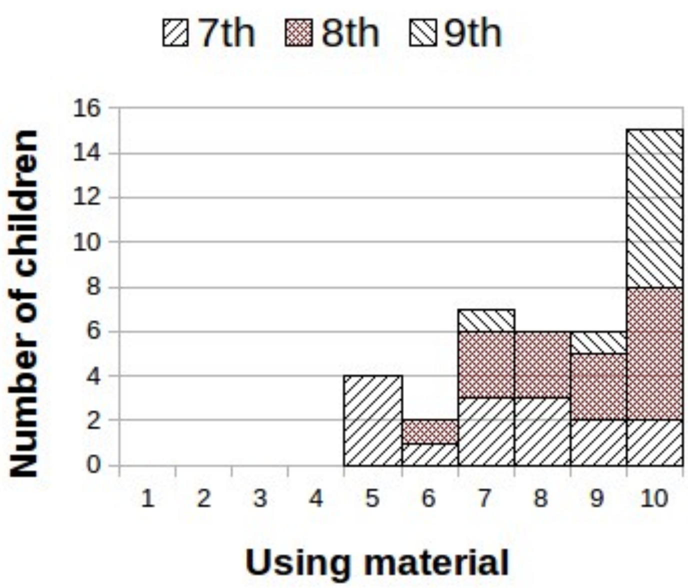

Figure 5: Interest is using materials to learn mathematical concepts.

Intervention: Creating projects that demonstrate their mastery of concepts

In STEM land children are encouraged to create projects while learning mathematical concepts. This is done by the facilitators themselves creating projects, allowing children to work across grades. Children take their own idea to create a project that they learned in their chapter. Projects can be both physical e.g. creating a sets game or in software e.g. creating a visualization of a concept or an interactive project or game.

Response

We asked children "*How interested are you in creating projects on a scale of 1 to 10 (1 not interested and10 very interested)*". Most children said they like learning through projects. Children mentioned that when they make projects they get clarity

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on the concept. Some mentioned that they are able to solve problems easily after making a project. They are able to learn new things like programming while doing a project.

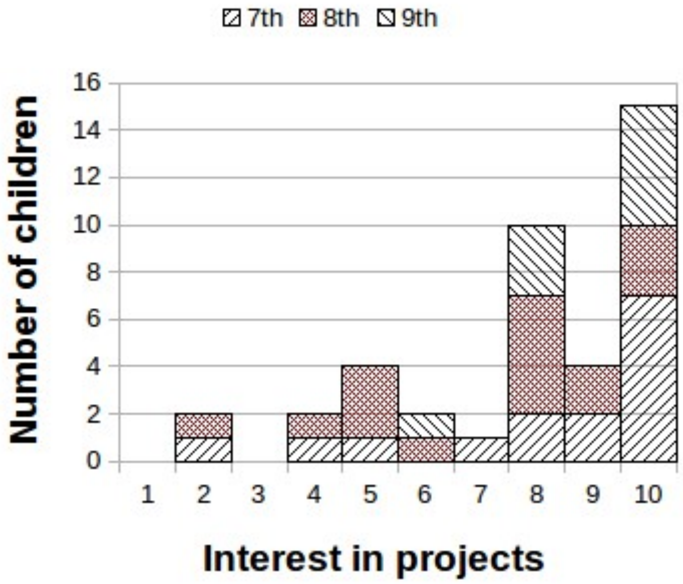

Figure 6: Interest in making projects (1-not interested, 5 – interested, 10- very interested).

Some children mentioned that while doing a project they face many problems and they are able to break down big problem into smaller problem. While working on a project children are able to think about how to proceed step by step. Having completed the project, if they forget the concept, they are able to revisit their project they themselves made to remember that concept.

CONCLUSIONS

Children have retained or increased their interest in Mathematics when they come to STEM land.

Their interest is inversely or only weakly related to their ability to do well in curricular examinations. This implies that even those who do poorly find something of interest at STEM land.

We examine various interventions and their responses

the freedom to plan their work – gives them a sense creating a plan and accomplishment.

choice of working individually or with peers – allows them to work as they most effectively can.

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access to games and puzzles that give a broader perspective of Mathematics and are joyful – makes it fun being in STEM land.

access and use of materials in Mathematics to make abstract ideas concrete –

helps make mathematics more accessible.

creating projects that demonstrate their mastery of concepts – helps them express their creativity.

We continue to explore various interventions that alter children's attitude towards Mathematics.

ACKNOWLEDGEMENTS

We thank Aura Semiconductor Private Limited, Asha for Education, Udavi School, Isai Ambalam School, SDZ and SAIIER for their continuous support to STEM land that made this research possible. We acknowledge Vaidegi Gunasekar for initiating this research, Bala Anand for supporting us for this research, Heidi Watts and Anita Komanduri, Muthukumaran for their guidance in this paper.

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SHAPING MATHEMATICS CLASSROOM THROUGH FUNDS OF KNOWLEDGE

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The main intent of this paper is to explore the concept of Funds of Knowledge (FOK) in relation to contextual teaching in mathematics. This paper discusses the basic idea of FOK from the lens of community of practice and socio historical perspective. It unveils the need of FOK in mathematics along with key concepts. It also explains how FOK is quintessence for the well-being and existence of any society. Having this approach, teachers and educators can support and assimilate students' mathematics learning at home as well as school activities.

INTRODUCTION

In the teaching-learning process of Mathematics, 'contextual' or 'situational' problems have earned attention and consideration from all stakeholders of mathematics education community. Identification and utilisation of mathematical concepts dealt in out- of- school situations (into mathematics classroom) at all stages of schooling has become a significant aim of mathematics education at national level (NCF, 2005). Similarly, underlying principal of mathematics education emphasise on proficiency to relate studied mathematical concepts in diversified situations (NCTM, 2000). Context forms an important pedagogical tool to substantiate the learning and understanding of mathematics. Research has also proven that learners are able to transfer mathematical knowledge to problem solving tasks in everyday life settings (e.g., Carraher et al 1985; Graumann,2011; Muller & Burkhardt, 2007). This means, students' own experiences are an important source to understand mathematical concepts and context can offer context related explanations only (Van den Heuvel-Panhuizen,2005). Sometimes, contextual tasks also prove to be equally problematic as de-contextualized problems for students (Verschaffel et.al, 2000) if these contextual tasks are from unfamiliar situations dipped into real world based pseudo puzzles that demands only near, not far transfer of learning.

In a classroom, mathematical questions that are fenced with contextual connotation may not have common element of students' experiential world. In such situations, students do not get success to unscramble the situation as expected and carry on explaining any mathematical problem in a normal manner. In fact, students' understanding, interpretation and implementation of 'context' in simulated tasks are different from what teachers/ educators think so (Carraher and Schliemann, 2002). This is a big challenge to provide contextual tasks to school students in a meaningful manner.

The aim of school mathematics cannot be restricted only in the sphere of mathematical knowledge, contextual or real life settings and drill and practice of

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mathematical exercises. One of the major aims of school mathematics is to facilitate far transfer of learning, where students are competent enough to unravel problem solving tasks in familiar and unfamiliar situations both. They are able to recall or reproduce procedural skills (with or without standard algorithms), factual information with conceptual understanding, even those that are not practiced/studied recently.

CONTEMPORARY TRENDS

Linking mathematics to the real world cannot be as simple as it seems to be. To cultivate critical thinking and transfer of learning in mathematics, students' participation in social, cultural and institutional practices need to be considered as unit of analysis.

COMMUNITY OF PRACTICE

Forman (2003) suggests that sociocultural approaches to learning can take care of such movements since Vygotskian notions of human thinking is based on society in its origin. He explained how instructional practices are connected to learning outcomes. Mathematics teaching learning process is social as well as communicative in nature and entails the establishment of a classroom community of practice (Lave & Wenger, 1991). Community means set of people with similar type of activities and common ways of thinking, views, values and belief system.

In a way, all classrooms are communities of practice, yet classroom communities vary according to the learning practices approved and justified by members of that community that is accepted by teachers and students (Boaler, 1999). In case of mathematics, mathematicians, mathematics educators, pre service, in service and para service mathematics teachers along with learners of all grades of school system make one community. In this community, mathematics is involved by teaching mathematics, learning mathematics, and doing research either into mathematics or into teaching or learning mathematics.

In conventional mathematics classroom, community of practice is explained in terms of tabula rasa, where teachers write on black board and students copy as it is without having any mistake. Students have to listen to what teacher explains to them with the help of text book. Teachers give lectures and students abide by them as followers without having any query or argument. Rote memorization, completion of text book exercises, drill and home practice of same problems discussed in classrooms are only means to comprehend mathematical concepts.

On the other hand, reform based classrooms believe in open approaches, where discussion, critical analysis and peer collaboration are appreciated to recommend and shield mathematical ideas and conjectures. There is no single authority and students are expected to argue and defend their thought processes with relevant explanations and examples. Such classrooms are considered as communities of mathematical inquiry (Richards, 1991).

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SOCIO-CULTURAL APPROACH

The standpoint of a sociocultural approach is totally contrasting from other cognitive theories of learning, which emphasize on individualism. In fact, a sociocultural theory never supports the individual construction of knowledge, rather it encourages to examine higher mental processes (like: voluntary attention, intentional memory, planning, logical thought, and problem solving) of human being in a cultural, institutional, and historical settings.

The advocates of sociocultural theory realise that learning means to procure the culture of learning of particular society (where learning takes place). Students should have belongingness towards cultural values that prevail in that (social) context of learning. Knowledge is co-constructed through interaction within the social sphere and collaboration with experienced peer of same social milieu. Social contact and interaction act as important factors in the development of cognition. Initially, interaction happens within the society members (normally parents, closed relatives and friends, as they are ambassadors of the culture and also the channel that help them to adjust in that culture with the help of language), on the social (inter-psychological) plane and after words, within the individual as an intra-psychological plane where assimilation and internalization appear along with personal interest and values added to knowledge.

> Social interaction plays a fundamental role in the development of cognition. Vygotsky believed everything is learned on two levels. First, through interaction with others, and then integrated into the individual's mental structure. Every function in the child's cultural development appears twice: first, on the social level, and later, on the individual level; first, between people (inter-psychological) and then inside the child (intra-psychological). This applies equally to voluntary attention, to logical memory, and to the formation of concepts. All the higher functions originate as actual relationships between individuals. (Vygotsky, 1978, p.57).

According to this, learning contexts out of the school situation like homes and communities where students live and participate in numerous activities with adults and peer, all scaffold to understand higher cognitive tasks and cannot be ignored at any cost.

FUNDS OF KNOWLEDGE PERSPECTIVE

This simply forces one to subscribe to outside learning context of the classroom, like students' involvement in home activities and association with community events. Also, one should know how such lived practices have an effect on classroom practices. The integration of 'community of practice' and 'socio cultural' approach clarifies that the students' community should be active enough and should in numerous social events that pile up Funds of Knowledge (FOK). Students' FOK comprises of vast and different domain of activities in their contexts (Velez-Ibanez & Greenberg, 1992). In line with socio-cultural theory, anthropological point of view and according to educational perspective, Moll et al. (1992) developed Funds

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of Knowledge. “Funds of Knowledge are historically accumulated and culturally developed bodies of knowledge and skills meaningfully put to use as household and other community resource” (Moll et al., 1992).

Funds of knowledge embrace abilities, skills, practices, ideas, as well as different forms of knowledge needed for the implementation and welfare of a family. This concept requires that the families stimulate the learning process and it may not be apparent in case of regular teaching. Mostly, classroom proceedings are aloof in nature but household activities cannot occur in isolation. Connection with more households and societies via distinct social networks helps to understand how parents encourage and assist their children’s learning.

The works of Luis Moll and his colleagues in Arizona had paid attention to the immigrant children and developed an approach to make use of their diversities into “pedagogical assets” (Moll & Gonzalez, 1997, p. 89). They found that families had ample information about agriculture, animal husbandry, water distribution, ranch economics, mining, medicines etc. and insisted to use these valuable resources of information to promote pedagogical development instead of their deficits (in which these children were examined and marked).

> In a Vygotskian sense, this suggests that children’s informal daily interactions provide a bank of experiences from which to draw upon, and from which students could develop more formal, scientific and conceptual knowledge in school-based learning activities. (Hedges et al., 2011).

In FOK, teachers gather first-hand information by visiting houses, observing different activities held in household and interviewing families. For one family or particular group of families, funds of knowledge are identified and gathered to make changes accordingly in pedagogical strategies. The aim is to integrate students’ out-of-school funds of knowledge to enrich learning opportunities by tapping into their scholastic funds of knowledge to boost up educational progress.

The value of using FOK is to set up “zones of possibilities” (Moll & Greenberg, 1992, p. 327) to carry home knowledge (in other words, students’ own understanding of various tasks, i.e., everyday knowledge) into the school system. Once the teacher becomes aware of social and cultural assets of household and institutions, their belief systems, perceptions and attitudes towards working-class students transforms from cultural and social deficit to cultural and social assets. They can make use this capital for the preparation of novel classroom programmes.

EDUCATIONAL INVESTMENT IN MATHEMATICS EDUCATION

Since ancient times, mathematics has retained an eminent social value. Mathematics has always been considered as an indispensable and important component of school system (Powell, 2002). Taking into the consideration of high value of mathematics, researchers have explored realistic ways to examine to uncover the mathematics embedded in the culturally situated everyday experiences. Several researchers have attempted to examine realistic methods through which this transfer takes place.

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González et al., (2001) explored how mathematical understanding develops while working/doing work at home and also how it can be used in classroom instruction. They discovered mathematical concepts in cooking, sewing, construction, and time management. However, family members were proficient in their tasks but unable to see mathematics in those tasks such as in sewing.

A few examples are discussed to understand the picture of FOK in the Indian classroom. Baker's (2007) work in India showed how FOK is useful in Adult Numeracy programme. He noticed that women were using local flask known as "*barajja*" (can have a kilogram of water) and a paili (can contain 9 to 11 kg) to measure different produce such as ghee, wheat and sesame seeds. They had to sell their produce to shopkeepers. Although their containers were accurate to measure capacity but women were using them for measuring weight. The densities of ghee and sesame are different, so their weights would be different. These women were skilful in bargain to sell as they knew the difference between standard measure and non-standard measure, weights of their produces and distinction between capacity and measures. They never took less money from shopkeeper for their produces. These type of understandings and proficiencies (to deal) nurtured in well-established social and cultural background. These practices were used as FOK (Baker, 2005) in adult mathematics education programme and obtained positive results in learning outcomes.

In India, the mathematical knowledge of child vendors (newspaper seller and betel leaves seller) and school going children has been studied by Farida Khan (2004). She found that due to the everyday practices, child vendors were able to recall and use addition and subtraction to solve word problems easily in comparison to school students.

In the book 'Numeracy Counts', Rampal et al., (1998) pen down cultural heritage of folk mathematics of different societies in India. They found that unschooled adults increased their knowledge and skills, since they follow pragmatic approach of 'learning while doing' mathematics. In similar line, K. Subramaniam (2010) emphasized that Indian culture and tradition contains mathematics in its deep roots. It should be properly explored and bring into the student community. He cited examples of 'kuttaka'- a way of asking puzzles and giving solutions by Mushari community. Although this community is low in the social hierarchy, still, it has preserved ancient Indian algebraic technique in mathematics whose description can be found in 'Aryabhatiyam' of the 5th Century CE.

CONCLUSION

The common argument has emerged from these studies that situated practices immersed in FOK, carried out by members of (particular) community were unlike the traditional classroom culture. The social practices provided meaningful context to proceed ahead. Transporting everyday mathematics or real life mathematics into

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the school curriculum highly demands bringing in abundant social and cultural mathematics from wider sections of society.

Initially, teachers were ethnographic researchers and visited students' homes and applied results from observations and series of interviews to give better instruction to students (Moll et al., 1992). In present scenario, parents, members of diversified societies and communities can participate as equitable companions in the integration of indigenous way and western way of knowing in the teaching-learning of mathematics together with mathematics educators.

This article has looked over how tapping cultural and social capital (i.e., FOK) provide a great potential to prepare (marginalized) students for their academic education.

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HOW DO STUDENTS MAKE USE OF THEIR MATHEMATICAL KNOWING WHEN THEY READ THE WORLD?

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Abstract: This article discusses some central connections between Skovsmose's concept of mathemacy and Gutstein's concepts of reading and writing the world with mathematics. The indicated connections are used to introduce a discursive analytical approach which can be used to analyze critical mathematical discourse. The approach is used to explore the relationship between mathematics and critical sense-making in relation to two excerpts from a conversation about mathematics and climate change taking place in a teacher education course. ***.*** *The analysis hints at the possible usefulness of further developing both critical mathematics education theory and methodology by widening the focus from an individual-centred conception of literacy to a more process and collective oriented focus.*

INTRODUCTION

Reading and writing the world with mathematics (Gutstein, 2003, 2006, 2016) and Mathemacy (Skovsmose, 1992, 1994) represent two different conceptions of critical mathematical literacy respectively within the Mathematics for Social Justice (MSJ) tradition and the Critical Mathematics Education (CME) traditions.

A shared aim of the two authors is that mathematics education becomes able to facilitate the development of critical mathematical literacy. A literacy that supports the critical citizens to identify, evaluate, judge and act on social issues using mathematics to transform ideological and social conditions (Gutstein, 2006), (Skovsmose, 1994)

A recurrent theme of exploration in both MSJ and CME remains how developing critical mathematical literacy might be possible to facilitate within the current organization of the subject of mathematics in schools. Both Gutstein (2006) and Skovsmose (1992) ascribe value to the mathematical skills and knowledge currently attended to in mathematics education. However, they also find it to be an insufficient basis for a truly critical mathematical literacy that can attend to the root causes of social issues.

In this paper, I approach issues of mathematical literacy from both a theoretical and empirical entry point. First, I present a theoretical background to critical mathematical literacy which calls out central connections between Skovsmose and Gutstein's work. Inspired by Barbosa (2006), who applied Skovsmose's terminology as a form of discursive analysis, I then build on these connections to suggest a united discursive analytical approach. The approach is then explored in relation to two excerpts where student teachers and a lecturer talk about the social significance of mathematical prognosis for climate change. The analysis of the excerpts particularly attends to the

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relationship between the students mathematical and critical sense making. Finally, I discuss questions that arise from the analysis and suggest how this might lead to further modifications of the approach and how it indicates future venues for research on critical mathematical literacy.

CONNECTING READING AND WRITING THE WORLD WITH MATHEMATICS AND MATHEMACY

Skovsmose concept of Mathemacy

Mathemacy is a form of literacy, a critical mathematical competence, suggested by Skovsmose as a parallel (1994, p.24-27) to Freire's ideas about critical textual literacy (Freire's work is a theme in the following section). Particularly mathemacy constitutes a central and significant part of the democratic competence (1990, p.124) which is needed in the modern technological society which is highly structured by mathematical technologies. (Skovsmose, 1992) Mathemacy concerns «*critical readings*» of «socio-political contexts» (Alrø, Skovsmose, 2002, p.136). Skovsmose (1990) describes some of the aims of critically reading, or critically reflecting about, mathematics in relation to social phenomena. These are *«to understand the main principles in the "mechanisms" of the development of society although they may be "hidden" and difficult to identify. Especially, we must be able to understand the functions of applications of mathematics. For instance, we must understand how decisions (economical, political, . . .) are influenced by mathematical model building processes. »* (p.11)

Skovsmose suggests that mathemacy can analytically be divided into three competences (1994, p.115); *Mathematical knowing* relates to mathematical knowledge skills and procedures; *Technological knowing* is associated with a competence in mathematical modelling (p.35) and in general the application of mathematics; *Reflective knowing* is a meta competence that relates to evaluations of applications of mathematics in society (1994, p.35) like mathematical model use and judgements of actions (2011, p.71) based on mathematics. Such evaluations and judgements, for instance, relate to reflections about whether models are in fact able to account for the complexity of an issue in a meaningful way and whether model predictions are trustworthy. Reflective knowing is the critical component of mathemacy and includes the knowledge needed *"for discussing the nature of models as well as the criteria used in their constructions, applications and evaluations."* (1990, p.124) The evaluation of mathematical models is central to a democratic competence as models are central in decision making throughout society (1992); The structuring of social reality (1992) as well as the discursive framing and management of social issues (2011, 2012). Reflective knowing is also related to dispositions and abilities to reflect. Accordingly, Skovsmose talks about reflective knowing as a process and uses it synonymously with words such as 'reflections', 'reflecting' (1994, p.101-102) and critique which can all be understood as ways of "giving thought to action" (2011, p.71-72). Skovsmose

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consciously uses the word ‘knowing’ in relation to the three competencies to emphasize that these are open concepts which cannot be explicated fully e.g. as authorized bodies of knowledge. (1994, p.101-102).

Gutstein’s ‘Reading and writing the world with mathematics’

Gutstein (2003, 2006, 2016), like Skovsmose, builds on Freire in his framework of mathematics for social justice. Centrally he builds on Freire’s idea that the development of textual literacy, *reading the word*, should be developed dialectically together with sociopolitical consciousness- *reading the world*. (Gutstein, 2003, p.44; 2006, p.24) At a personal level writing, the world can be thought of as the process of developing ones reading of the world. At a social level ‘writing’ refers to acting in the world.

In Gutstein (2003; 2006)’s adaptation of Freire’s terminology, the focus point is how reading and writing the *mathematical* word (making sense of respectively doing mathematics)- like textual literacy- is related to reading and writing the world. As practical examples of ‘reading the world with mathematics’, Gutstein (2006) accounts how his students would do various mini-projects. In these projects, students were required to analyze data (reading and writing the word) and provide mathematical arguments for instance to make sense of whether police traffic visitations where racially biased. Gutstein notes that reading and writing the world (with mathematics) is a developmental process (2006, p.27) that over time builds students mathematical literacy (p.45). During this process students, beyond developing mathematical capabilities, also build faith in their possibility of facilitating social change -writing the world with mathematics- which can be considered a form of social agency.

Reading and writing the word and the world with mathematics is related to three knowledge categories. *Community knowledge* relates to and is created by the people affected by social injustice and includes "(…) social relations, community life in all its complexity (…) perspectives and interpretations of the world" (p.200). It also includes so-called informal mathematical knowledge which is produced and used by adults and children outside school in their lives. *Critical knowledge* includes both general critical knowledge e.g. knowledge about the historical origins of social inequalities as well as critical mathematical knowledge- how to read the world with mathematics. *Classical knowledge* includes the skills associated with being a skilled mathematical problem solver; the mathematics needed to pass high stakes tests and the knowledge needed to pursue mathematically related careers. Reading and writing the world thus relies on both mathematical and non-mathematical knowledge bases and the process of *reading the world* in social practice can importantly be understood as that of people making use of various literacies (Gutstein, 2016)

Connections between the terminology of the frameworks and how it relates to discourse.

Gutstein and Skovsmose’s frameworks have a shared ideological and theoretical basis in Freire (e.g. 1970/2000). At a conceptual level Gutstein’s and Skovsmose’s ‘knowledge’ and ‘knowing’ categories both relate to a (mathematics) educational and

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social context. However, their organization differs and the knowing and knowledge categories can accordingly not be put in a 1-1 correspondence. Gutstein's three types of knowledge all include mathematics but also extend from a mathematical knowledge domain into other forms of knowing. Skovsmose's mathematical and technical knowledge is more narrowly focused on mathematical content learning traditionally focused on in mathematics education than Gutstein's categories- respectively on learning mathematics as an aim in itself and learning mathematics as a technological tool. Accordingly, we might understand mathematical and technological knowing as a subset of classical knowledge.

Skovsmose (2014) has also explicitly related mathemacy to Gutstein's idea of reading and writing the world with mathematics:

> *Similar to literacy, mathemacy refers not only to a capacity in dealing with mathematical notions and ideas but also to a capacity in interpreting sociopolitical phenomena and acting in a mathematized society. Thus, mathemacy combines a capacity in reading and writing mathematics with a capacity in reading and writing the world (see Gutstein 2006).* (p.153)

I believe that the first sentence in the quote is an analytical division within Mathemacy: A) *"dealing with mathematical notions and ideas"*- is a declaration of mathematical and technological knowing; And B) *"a capacity in interpreting sociopolitical phenomena and acting in a mathematized society"* – is a declaration of reflective knowing. In the second sentence, I suggest that the first half of the sentence: a) "*mathemacy combines a capacity in reading and writing mathematics* (…)- Skovsmose is equating mathematical and technological knowing with reading and writing the mathematical word. In the second half of the sentence, he further connects this b) *"with a capacity in reading and writing the world"*. All in all, I interpret Skovsmose as saying that mathematical and technological knowing can be considered as synonymous terms to reading and writing the mathematical word (A & a); And reflective knowing can be considered synonymous to reading and writing the world with mathematics (B & b).

As we have seen Skovsmose's and Gutstein's ideas have many similarities. By combining ideas and concepts from both of their works also get a rich language for studying the practical relationship between mathematical and critical discourse. The metaphors of reading and writing are useful for discursive analytical purposes as they make a valuable distinction between the processes of 'making sense of' mathematics and the social world (reading) respectively creating new meaning and acting facilitated by one's understanding (writing). In relation to the process of reading the mathematical word, I suggest that we could integrate the terminology of 'reading and writing' and that of 'knowings' and then talk about 'mathematical and technological readings of the (mathematical) word/world'. While Gutstein uses 'reading and writing the world' to refer to critical readings, I suggest that it might from an analytical perspective be useful to distinguishes between 'reading the world' and 'critically reading the world' in a discursive analysis.

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DATA AND METHOD

The two excerpts I will analyze using the combined terminology are taken from a video transcript of a conversation concerning a temperature prognosis of the Intergovernmental Panel on Climate Change (figure 1) discussed in a teacher education seminar at the masters' level in Norway. Eight student teachers and a lecturer were participants in the seminar. A previous variant of the activity is described in (Hauge et al. 2015).

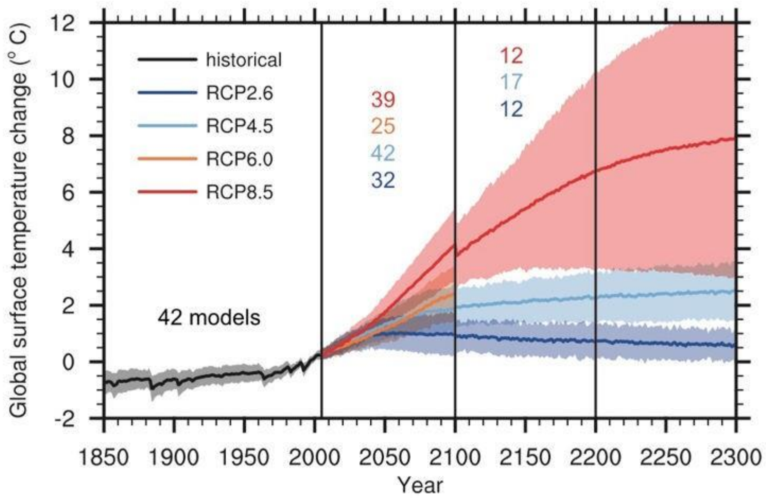

Figure 1 (IPCC, 2014, p.89)

The prognosis which the students and the lecturer discussed presents the modelled development of the global average mean temperature as a function of different greenhouse gas emission rates (RCP - representative concentration pathways). Each 'scenario' (the colored shading) indicates the 90% confidence interval of the model predictions for each RCP value. The colored numbers denote the number of models for each segment of the predictions and solid colored lines indicate the mean of the predictions. Changes in global average temperature are indicated relative to the period 1986-2005. *(IPCC, 2014, p. 89)*

To make the analysis transparent to the reader the excerpts are presented first in verbatim form and then in a paraphrased form which maintains references to social contexts and mathematics and distills the interpreted argument of the utterance; Finally, an analysis is carried out based on the paraphrasing. In Barbosa (2006)'s analysis of discourse building on knowings, the unit of analysis are sequences of talk, discussions, which he labels with one of the three knowings. In my analysis, I further attend to individual utterings. Barbosa defines talk related to building mathematical models as technological while talk is 'just' mathematical when it relates to *"ideas from pure mathematics"* (p.297). In my categorization of technological readings, I extend the scope from Barbosa's focus on the model as a technology to also include non-critical model related talk about the context. For instance, referencing a value from a model prognosis to talk about a future temperature I suggest is a technological reading which

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depends on the act and skill of reading of the graph (a mathematical reading). The following analysis explicates both the technological and the underlying mathematical reading. The analysis of *how* or *if* the world is read critically is based on a holistic interpretation of whether the reading is primarily concerned with mathematical/technological issues or whether it also concerns evaluation or judgement related to a social context.

ANALYSIS

Following an extended conversational focus on visual features of the plot (Figure 1) and the possible modelling processes related to its construction the lecturer asks the students about their perception about the prognosis social significance- Jan is the first to respond.

150 JAN:	Well, this one indicates that in 300 years, the temperature may have risen by as much as 7 degrees. If we continue along the red one. And I think I've seen documentaries in which they believe that if the temperature rises by more than 5 degrees, we then go... Or, the earth goes on, but those living on it don't. So, hopefully this is not the scenario we are aiming for. So, it's an estimate of how the future will be in terms of whether we will survive and whether we'll manage and be capable of survival and we'll have a population of 11 billion people on earth, then it's better if the planet is not 6 degrees hotter. Then it's better to go for the blue alternative. That's the goal we should aim for.
151 LECTURER:	But is that what we are doing?
152 JAN:	Probably not!

Jan indicates that the plot concerns the future viability of life on earth: *"this one indicates that in 300 years (...) the earth goes on, but those living on it don't."*. He states that an increase of more than five degrees will end life: *"if the temperature rises by more than 5 degrees (...) the earth goes on, but those living on it don't."*. He states that the red scenario suggests an increase of more than five degrees while the blue scenarios stay below. He then concludes that the blue emission rates should be worked towards: *"(...) the temperature may have risen by as much as 7 degrees. If we continue along the red one. (...) Then it's better to go for the blue alternative".*

Jan is making a mathematical reading of the word- the red graph- when he reads off the temperature increase as being "as much as 7 degrees". Jan seems to use his reading of 7 degrees- which might correspond to the red mean- as a descriptor for the graph as to say something about the physical context of climate change; how much the temperature will likely rise. Jan is thus using the graph as part of a technological reading of the context. The plot in isolation does not like a text have a beginning and an end or even a correct direction of reading. It is therefore too simple to say that Jan is reading the mathematical word. Rather it would be appropriate to say that he is also writing the word since he has to engage in a mathematical process to extract significant information from the graph and communicate the essence of this information to the

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other listeners. Jan is particularly calling out the red graph and the 7-degree increase because of its societal significance- such an increase is related to human extinction. Calling out this consequence builds on a critical knowledge (Gutstein, 2006) about climate-science- that five degrees is (he believes) a so-called tipping point for human life. Jan's evaluates the red scenarios description of the future as fatal based on his technological reading and writing of the word. In this sense, Jan is critically reading the world with mathematics as he is making sense of and evaluating the social reality which the mathematics describes. When Jan says, "That's the goal we should aim for", he is also motivating action based on the graph for himself and the others which can be related to writing the world.

Dialogue as a collective reading of the world

Immediately following Jan and the lecturer's exchange [150-152], Arne is given the word by the lecturer:

154 ARNE:	But if you look at the graph from before, or until now at least. Then, based on my mathematical skills, it is most likely that you end up with one of the two graphs at the bottom. There is little that... If you set aside the research that has been done, if you only look at the graphs, then there is little that suggests that it will suddenly go through the roof, like the red one does.

Arne suggests that we might ignore climate research and use mathematical skills to identify the likely future: "(…)If you set aside the research that has been done, if you only look at the graphs(…)" He indicates that if we compare the historical trend in data with the scenarios predictions the blue scenarios are most likely while the red scenario is unlikely as it grows to fast "(…)if you look at the graph from before until now (…) there is little that suggests that it will suddenly go through the roof, like the red one does."

Arne's utterance indicates that he has visually made a mathematical reading of the trend in the historical data. His utterance also indicates that he has made a technological reading by comparing the scenarios in terms of their fit with the historic trend. It is difficult to say if Arne is reading the world beyond making a comment about how the temperature will likely change. This "physical" reading of the world shows dependence on mathematical knowledge about a graphs trend and technological knowledge about how to use extrapolated trends to evaluate model predictions.

The immediacy of Arne's utterance to Jan's and the initiation of Arne's utterance with the word "but" -which might indicate disagreement- however suggests that Arne is not just making an argument about the plot but is also referring to Jan's utterance. If this is the case, then Arne's statement can be understood as a critique of Jan's reading of the red scenario- stating that it is mathematically unjustified. This could either be a technological critique of which model scenario is trustworthy and/or a comment that humans are not heading for extinction and that a change in behavior is unnecessary. The first of these critiques is a (technological) reading of the word/world and the second is a critical reading of the world. A further analysis of the dialogical context

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might give further hints, however, it is also possible that the analytical ambiguity persists. Further, it is possible that the ambiguity is not analytical but that the interlocutors are disagreeing about what they *are* or *would like to* be debating.

DISCUSSION

Skovsmose's theory of mathemacy and Gutstein's theory of reading and writing the world with mathematics are largely compatible. I have argued that critical reflections, reflective knowing and critique in Skovsmose's terminology corresponded with the process of reading and writing the world in Gutstein's work. Furthermore, Skovsmose's mathematical and technological knowing was found to be comparable to a subset of what Gutstein's termed as classical knowledge. Based on the close ties between the two frameworks a discursive analytical approach was suggested. The approach was explored in an analysis of discourse that specified what the mathematical word was; How it was read and written based on mathematical and technological competence; And how this facilitated reading and writing the world both in a critical and non-critical sense.

The analysis of Jan's uttering showed that the analytical approach could be applied to a practice level to study how students make use of different forms of mathematical knowing in unison with other domains of knowledge to read the world. It was shown that Jan's critical reading of the world was dependent on both technological knowing and mathematical knowing. However, Jan's technological reading of the word was also shown ultimately to be dependent on his critical knowledge relating to climate science. Without this critical knowledge, Jan would not have been able to critically read and write the world- calling out the red scenario as fatal as to motivate a different line of action. Jan's critical knowledge related both to the *purpose* of the mathematical and technical readings but also the need for these particular readings. In relation to a mathematics educational context, I find that this supports Gutstein's theoretical placement of critical knowledge as a foundation for reading the world. It also exemplifies and supports the thesis that writing the mathematical word and the world can be concurrent and interdependent processes as both Freire and Gutstein suggested. This is a further argument in support of mathematics education moving beyond a narrow focus to mathematical content learning isolated from socio-political contexts. If students are to become confident and fluent in the complex process of reading and writing the world with mathematics as critical citizens, they have to build experiences with reading the world and further reflect on these experiences.

However, as the attempted analysis of Arne's statement revealed applying the analysis is not always a straightforward process, as there is no guarantee that talk can unambiguously be labeled as a particular type of reading or writing. Arne's utterance in itself was found to lack clear evidence which allowed it to be demarked as just a technological reading of the word or as a critical reading of the world. Informed by indications that Arne's statement was connected to Jan's uttering the analysis was

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expanded to explain Arne's uttering based on the larger dialogue. The analysis shows that although Arne's statement is not explicitly critical like Jan's it might meaningfully be considered as a critical reading when we take the conversational context into account. I suggest that the analysis of the exchange indicates that we have to consider "criticality" as being more than an absolute property of individual utterings but instead as a quality which relates to the context of dialogue and how it is experienced.

This should have consequences both for the analysis of reading processes but maybe also the way that theory emphasizes the *individual* as a critical agent in theories of critical literacy. Methodologically I suggest that an analysis of reading and writing should recognize when utterances are made as part of a conversation. That is to recognize how utterances relate to each other and how they refer to collectively familiar events outside the educational context. Readings of the world should be seen as part of a (critical) dialogue with other interlocutors' and events. This might be facilitated by introducing a third category which somehow relates to the process of reading the word of the other interlocutors. This might provide a better basis for describing the interaction between deliberators in developing and expressing their readings of the word and the world. Such a dialogical focus to critical literacy and particularly reading and writing with mathematics might better be able to capture how collective deliberations develop in a dialectic manner. Introducing an element of dialectics or dialogical context also seems fitting for an analysis that aims to reveal *if*, *when* and *how* processes of reading are actually *experienced* as having significance; Whether they appear 'critical' to the interlocutors'. Gutstein's work importantly concerns moving from reading and writing only the word to also reading and writing the world with mathematics. Mathematics education has a responsibility to address social issues (Abtahi, Gøtze, Steffensen, Hauge & Barwell, 2017) particularly when these, like climate change, are heavily influenced by how mathematics is read and written (Barwell, 2013). A critical educator recognizing the role of mathematics in society and the responsibility of mathematics education might very well be faced with questions related to the current analysis; *Do our conversations address critical aspects of issues? How do I support and facilitate critical conversations?* And *How might mathematics facilitate (or hinder) critical reflections?* Studies of reading/writing processes could enable further development of a didactical theory that relates both to the teaching and learning practice of CME and MSJ and facilitates a deeper understanding of how people in general are and could use mathematics to read and write their social world.

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STEM AND THE RACE BETWEEN EDUCATION AND CATASTROPHE

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Abstract: Given ongoing threats to human civilization, I contend that solutions from within Science, Technology, Engineering, and Mathematics are inadequate to solve what are essentially human problems. Among the human aspects that should pervade mathematics education if it is to contribute to averting catastrophe, I reflect on Social Studies, Technoscepticism, Environmentalism, and Multidiversity, an alternative version of STEM.

INTRODUCTION

Just less than a century ago, H. G. Wells (1920) declared that "Civilization is in a race between education and catastrophe". In this paper I discuss the responsibility in this regard of *STEM* education, that being an acronym I have constructed, standing for Social Studies, Technoscepticism, Environmentalism, and Multidiversity. It is intended as a contrast to the standard STEM of Science, Technology, Engineering, and Mathematics (and I use *STEM* and STEM throughout to disambiguate). I recently discovered that Andrew Hacker (2016), in contesting the dominant importance of STEM, similarly proposed the alternative acronym PATH (Philosophy, Art, Theology, History – or Poetry, Anthropology, Theater, Humanities). In his book "The Math Myth and other STEM delusions", Hacker interrogates the unreasonable political influence afforded to STEM, in particular Mathematics, most specifically the notion that everyone should study significant amounts of abstract mathematics. Most crucially he argues that failure to meet artificial assessment barriers halts advancement in careers that benefit society but do not depend on competence in formal mathematics.

It is very easy to find statements along the lines that STEM “is essential for our future prosperity and to our nation in order to develop competitiveness and play a vital role in a global economy” (just Google "importance of STEM education in India" which is how I found this particular statement, but it could have come from almost any country). In arguing that STEM education has an important, but not predominant, responsibility in the race to avoid catastrophe, I start by acknowledging the achievements of STEM, including the development of the computer on which I am typing this and that gives me access to information for this paper, and the means of travel that allow people from all over the world to gather in Hyderabad for this conference. The improvement in health care due to scientific advances should also be acknowledged (with some important reservations). On the other hand STEM has been centrally implicated in war and other forms of violence. A central point of this paper is that the critical problems facing humankind are primarily human, not technical, in nature.

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I present comments on each of the components of *STEM*, with implications for mathematics education, then discuss the overarching question of the radical transformation of what is construed as "truth".

SOCIAL STUDIES

"Social Studies" is the term used in the United States for "the integrated study of multiple fields of social science and the humanities, including history, geography, and political science". In 1920, when "An Outline of History" was published, the "war to end all wars" had just ended. The seeds had already been sewn for the major world conflicts dominating the twentieth century and into the present: the Russian Revolution had recently happened, the British and the French were carving up the remains of the Ottoman Empire, the Treaty of Versailles had arguably prepared the conditions that gave rise to the Second World War. In 1920, Gandhi launched the Non-cooperation Movement in India.

More broadly, we could characterize Social Studies as dealing with understanding of Others, across time, cultures, ideologies, and other aspects of human multidiversity. A century after the statement by Wells, we face many catastrophes, of which climate change and nuclear war are but the most obvious. My central point is that averting these catastrophes will not be achieved through technical solutions alone, but will require human solutions, and that this realization should be reflected in education.

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Mathematics education has a natural place within Social Studies, as defined above. Most obviously, mathematics as a discipline has its own history, which can be told in relation to its roles in society, and not just in terms of great mathematicians and theorems. For example, the development of probability as a branch of mathematics is inseparable from sociopolitical issues and conceptions, as clearly analysed by Hacking (1990). And scholars such as George (1992) and Raju (2007) have deconstructed the Eurocentric narrative of the history of academic mathematics. (The history of mathematics education, and its entanglement with politics etc. is also relevant and important but I will not try to deal with that here.)

An obvious theme of intersectionality lies in applications of statistical analysis to social and political phenomena. By way of a historical example, Florence Nightingale is recognized for her use of statistical data (including graphical representations that she invented) to persuade the British government to change the way in which it treated wounded soldiers during the Crimean War. What is less well known in that in the latter part of her life she acted as an advisor to the British government on many issues of social reform, digesting masses of statistics and exposing "all the needless deaths she could see from the evidence of statistical reports" (Bostridge, 2004). including many from famine in India.

In relation to curriculum, accordingly, a strong case can be made that statistics and data analysis should be an alternative to the traditional algebra/calculus trajectory.

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Moreover, in the light of the complexity of systems characteristic of our physical and social worlds, there should be more weight given to learning about systems dynamics, including at the level of secondary school (Fisher, 2017).

Beyond analysis of statistical data, we may point to mathematical modelling. Davis (2015) discusses the part played by Game Theory, as developed by Von Neumann in particular, in justifying the Mutually Assured Doctrine (MAD) position that guided policy in the Cold War, and stated that:

> How could I, as a junior scientist, be disloyal to this mathematically argued theory from one of the world's greatest mathematicians ... a theory accepted by the high command of both superpowers? Here's what I said instead: If your rival attacks you with nuclear weapons, you should put all your forces into preventing retaliation.

(And for a discussion of the Indian/Pakistani nuclear race see Ghosh (1999)).

TECHNOSCEPTICISM

Since I coined (surely not alone) the term, technoscepticism has gained a great deal in topicality. Here I consider three aspects, the first being a backlash against "antisocial media". For example, Chamath Palihapitiya, a former senior executive of Facebook, referred to "tools that are ripping apart the social fabric of how society works… eroding the core foundations of how people behave by and between each other". (www.theverge.com/2017/12/11/16761016/former-facebook-exec-ripping-apart-society). A leading technology enthusiast turned critic, Sherry Turkle, has concluded that "the younger generation is becoming too consumed in their digital lives resulting in lack of communication with family, friends, and teachers" with far-reaching social consequences, including a decline in empathy, the capacity for understanding and valorizing others. (Turkle, 2015).

My second point is what I call the supreme irony of the "Information Age", namely that we have access to information but insufficient means to determine which parts of that information are, in many complex senses (and see below), true. Recently, Italy added a new item to schools' curriculum: recognizing fake news (www.npr.org/2017/10/31/561041307/italy-takes-aim-at-fake-news-with-new-curriculum-for-high-school-students).

Most generally and importantly, my third point is the contention that technical solutions are not enough to solve human problems. Skovsmose (e.g. 2005, Part 2) discusses "Mathematics in action" and, in particular, draws attention to what he calls "formatting" i.e. the many ways, often not benign, aspects of our lives are controlled by models, typically computerized, over which we have no control and to which we generally have no access. In similar vein, a recent book by Cathy O'Neil (2017) called "Weapons of Math Destruction" describes in detail how computerized models with massive consequences for the lives of many people have built-in and uninterrogated assumptions that simpify building the models, and thereby result in social injustice.

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A central implication for mathematics education that can be drawn is that children should be taught about the nature of mathematical modelling, such as that:

• the motivations of the people constructing the model should be considered

• how are the implications of the model communicated to those it affects?

• a model is crucially dependent on the assumptions on which it is based

• the application of mathematical models based on analysis of abstract structures beyond their domain of applicability is fraught with danger (consider the example of the MAD doctrine being underpinned by Game Theory, as discussed above).

ENVIRONMENTALISM

There are many aspects of environmental degradation, including deforestation, management of water and food, and the degree of pollution. However, the most pressing, in the sense that it poses an existential threat to civilization, is climate change. At the time of writing, a report has just been issued ranking countries in terms of danger from climate change; the first four are: India, Pakistan, the Philippines, and Bangladesh (www.weforum.org/ agenda/2018/03/india-most-vulnerable-country-to-climate-change).

The issue of climate change is perhaps the clearest example of the central point I am trying to make. On the one hand, there is no lack of first-class scientific work collecting data, documenting what is happening, constructing predictive models, and recommending courses of action. On the other hand, there is a situation in the United States where stated positions on climate change are a matter of party affiliation, rather than weighing of the evidence. That is a human problem, not a technical problem. As another example, there are well-developed models capable of guiding sustainable fishing, yet overfishing occurs in most parts of the world, including India (see www.theguardian.com/environment/2017/jan/31/bay-bengal-depleted-fish-stocks-pollution-climate-change-migration).

A vast topic, on which I only touch here, is the reservoir of traditional knowledge gained over centuries by people living in all kinds of environments. Just by way of one example, there are sustainable forms of water storage, including in India (see: www.ted.com/talks/anupam_mishra_the_ancient_ingenuity_of_water_harvesting/discussion).

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The obvious contribution of mathematics education is in teaching how to interpret multivariate data and complex systems (Coles, Barwell, Cotton, Winter, & Brown, 2013). In this endeavor, new forms of dynamic computer displays are very powerful aids. For example, it is feasible for secondary school students to achieve a grasp of System Dynamics through the use of software (Fisher, 2017). Such representations

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also have great potential for education of the public in general, and politicians in particular (www.dur.ac.uk/smart.centre/).

MULTIDIVERSITY

Humanity is diverse in multiple ways, including language, ways of life, epistemologies, knowledge systems, ways of conceptualizing and interacting with the environment. This multidiversity is being eroded by globalization (Westernization), as warned against in a study of the spatial epistemology of the Navajo people:

> Through a systematic superimposition of the world view and thought system of the West on traditional non-Western systems of thought and action all over the world, a tremendous uniformization is taking hold…The risks we take on a worldwide scale, and the impoverishment we witness is – evolutionarily speaking – quite frightening. (Pinxten, Van Dooren, & Harvey, 1983, p. 174-5)

The globalization of education has been painstakingly analyzed by Joel Spring. He points to phenomena that he labels economization, corporatization, and "audit state" by which he means "the use of performance standards to assess government programs, including the use of standardized assessments to evaluate educational performance" (Spring, 2015, p. xiv). (After a 10-year boycott, India will participate in PISA again in 2021).

... and mathematics education

From the Ethnomathematical perspective, multidiversity is embodied in the human practices that can be characterized as mathematical (Mukhopadhyay & Roth, 2012). That includes the practices of mathematics as formal discipline, within which there is also diversity (Hersh (2006), and see "Aspects of Mathematical Pluralism" by Chakraborty and Sirker at: journalofmathematicsandculture.files.wordpress.com/2016/07/mathematical_pluralism-chakraborty-and-sirker-final-10-1.pdf).

The multidiversity of such practices is generally not reflected in school mathematics, where powerful forces are driving global uniformization. As an extreme example, consider "Cambridge Mathematics" (www.cambridgemaths.org) which aims to provide a "world-class" mathematics education for all students in the world aged 5-19. It is hard to see how such an arrogant, indeed imperialistic, aim could possibly address multidiversity.

By contrast, the position I take is that mathematics education, whilst reflecting certain universalistic aspects of mathematics must be embedded in historical, cultural, social, and political – in short, human – contexts and speak to the lived experiences of students (e.g. Rampal, 2015; Subramanian, 2015).

TRUTH

“Let us learn the truth and spread it as far and wide as our circumstances allow. For the truth is the greatest weapon we have”. That is the continuation of the quotation from Wells with which I began. Contrast his hope with the following:

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> After much discussion, debate, and research, the Oxford Dictionaries Word of the Year 2016 is "post-truth" – an adjective defined as "relating to or denoting circumstances in which objective facts are less influential in shaping public opinion than appeals to emotion and personal belief" (Oxford English Dictionary, Nov 8, 2016)

Historically, we may trace this phenomenon to the book by a nephew of Freud called "Propaganda" (Bernays, 1928). Earlier, Bernays had worked on World War I propaganda in the US Committee on Public Information (as did, for example, Rudyard Kipling in England). Shortly afterwards, aware of the negative connotations of the word, Bernays coined the term "public relations" and was instrumental in developing that science. The stance that truth is irrelevant, that what matters is what people can be persuaded of, reflects the ethics of the PR/advertising industry that feeds capitalism by persuading people that they want things and should buy them, irrespective of whether they need them.

The issues raised have already been discussed in terms of the importance of technoscepticism. In the context of historical revisionism in India, with particular reference to mathematics, see Raju on Vedic mathematics (www.thehindu .com/opinion/op-ed/nothing-vedic-in-vedic-maths/article6373689.ece)

"Truth" is a value held dear by mathematicians, but in a very restricted sense, intimately linked with faith in absolute proof and certainty. I will venture the statement that "truth" is one of many cases where the use of the same word obscures fundamental differences, as in mathematicians' claims that learning mathematics helps people to think and to solve problems. By contrast, I assert that mathematicians and even more so mathematics educators have a responsibility to address truth in a complex human and not merely formal way. My analysis of how epidemiological estimates of excess deaths in Iraq following the invasion of 2003 (Greer, 2008) were interpreted by politicians, pundits, and statisticians provides ample evidence of disagreements about methodological issues among statisticians (arguably appropriate and reasonable but indicative of complexity) and naive post-truth reactions even among those who might have been expected to know better (including the mathematician John Allen Paulos).

To return to the example of the mathematical underpinning of the MAD doctrine, what is true, mathematically, is that the theorems within game theory follow from the axioms. But what about the axioms and what about the range of applicability?

And what is regarded as truth varies historically, including in mathematics.

FINAL COMMENTS

STEM has produced both wonders and horrors, and mathematics is centrally implicated in both (D'Ambrosio, 2015). Mathematicians and mathematics educators have ethical responsibities to do what they can to work for, in Ubi's words, "survival of civilization on Earth with dignity for all". STEM assuredly has vital roles, in the development of new ways of generating energy, for example, but technological

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solutions alone will not suffice. Johansen (2014) pointed out that the reduction of carbon footprints by individuals is trivial in comparison with the carbon footprint of war (which could be considered the ultimate case of trying to solve a human problem by technical means).

Accordingly, what is suggested is a symbiotic intertwining of STEM education with *STEM* education as envisaged in this paper.

Davis (2015) posed a challenge to young people:

> ... instead of passing on to the next generations a well functioning world, my cohort is leaving one that seems hell-bent for disaster. Maybe the disaster of nuclear war, maybe the disaster of environmental collapse: which will win the honour of delivering our death stroke? ... Yet in this great jeopardy, when you must rouse yourselves to better efforts than we managed just to have any future, I implore you: have the wisdom and the strength not merely to survive but to survive proudly and happily. To choose the best future.

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TENSIONS BETWEEN REPRESENTATIONS AND ASSUMPTIONS IN MATHEMATICS TEACHING

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Mathematics teaching and mathematics teachers are part of cultural, societal, and educational structures. These structures, and different actors within the structures, construct mathematics teaching differently and influence the scope of action that teachers hold. To explore the mechanisms behind this influence, Fairclough's concepts of representations and assumptions were used to analyze common themes in interviews with six Swedish mathematics teachers. Results showed that there is diversity in ways of representing, and that three groups of actors are visible in the representations: teachers, official actors, and students and parents. Results also revealed tensions between representations and assumptions that have consequences for teachers' considerations and decisions about their mathematics teaching.

INTRODUCTION

Mathematics teaching is a situated, cultural activity that differs between cultures, countries, schools, and classrooms. No matter where the teaching takes place, the mathematics teacher is the one who encounters the students and is assigned the responsibility of teaching. However, teachers do not operate in isolation from the outside world: they "are part of the larger cultural politics of schooling and education" (Montecino & Valero, 2015, p. 794). The extent to which the teacher is mandated to organize the teaching varies between countries. Swedish teachers, for example, have a high degree of freedom to organize their teaching.

Whether a teacher works in a country where she has a high degree or low degree of freedom, she makes decisions about teaching that influence her students' experiences in the mathematics classroom and in their lives. The decisions are made in a cultural context and are influenced by, among other things, the teachers view of mathematics, students, and mathematics teaching. However, results from an interview study in Sweden have revealed that decisions made by other actors, and opinions and ideas that others possess, influence the decisions teachers make. Sometimes, tensions exist between views/ideas/values held by different actors, and these tensions can lead to decisions about teaching that contradict the teacher's own views or those held by research or the curriculum (Grundén, 2017).

In summary, the story, or rather, the stories of mathematics teaching have been written by many authors. In this paper, yet another chapter is added by sharing results from an interview study that reveals ideas, representations, about three aspects of mathematics teaching: special needs, text books, and organization. In addition, tensions between representations are visible. The aim of the paper is to present different representations and tensions that arise among them and discuss possible consequences for mathematics teaching.

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BACKGROUND

Acknowledging that "mathematics teachers are part of the larger cultural politics of schooling and education" (Montecino & Valero, 2015, p. 794) implies that focusing on the mathematics classroom is not enough. We also need to know more about the ways the cultural politics of schooling and education is put into practice.

Montecino and Valero (2015) and Montecino (2017) argued that society desires a special kind of mathematics teacher and attempts to homogenize and govern teachers through, for example, teacher education and teacher development programs. In such programs, teachers are fabricated to follow imposed norms and standards, regardless of each individual teacher's context and experiences (Montecino & Valero, 2015). Not only are teachers considered to be products that should "convergence toward the ideas, ideals, rationalities, and subjectives that are being promoted in society" (Montecino & Valero, 2015, p. 804), but they are also agents that should promote ideas and shape students through mathematics (Montecino & Valero, 2015; Montecino, 2017). In this view, mathematics teachers are seen "as a means for progress and for the success of society" (Montecino & Valero, 2015, p. 804). Based on these views, regardless of how "free" teachers may be with regard to planning and organizing their teaching, they are highly influenced by various societal discourses.

Another example of how different actors influence and shape the story of mathematics teaching was presented in a Canadian news media study by Barwell and Abtahi (2015). Based on the assumption that news reporting "shapes the way in which mathematics education is understood by the general public" (p. 298), which, thus, influences public policy and the mathematics education context, they argued that the binary dominant frame of traditional teaching versus discovery learning found in national newspapers need to be discussed.

Yet another example of differences in views about mathematics teaching concerns how the mathematical content is divided and presented in mathematics teaching. Mulligan, Mitchelmore, English, and Crevensten (2013) emphasized that focusing on patterns and structures in early years is beneficial for students when later learning mathematics. However, results from Grundén (2018) showed that teachers often organize their teaching according to the headlines in the textbook they use, which results in mathematical content being divided into areas such as numbers, geometry, algebra, etc. Patterns and structures are, in this way, not foregrounded but hidden within other areas; therefore, the benefits Mulligan et al. (2013) demonstrated that one achieves by focusing on patterns and structures might disappear.

Mathematics teaching in Sweden

In Sweden, the curriculum does not govern how and what to teach in detail. The Swedish curriculum merely provides overall aims and the content that should be covered in 3-year cycles. Hence, Swedish teachers have, at least in theory, a high

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degree of freedom; there is more space for their ideas, as well as the ideas of other decision-makers related to mathematics teaching.

Different authors write different stories about mathematics teaching and learning (Österling, Grundén, & Andersson, 2015). In a study about what students find important when learning mathematics, results showed that values of different practices in mathematics education are sometimes at conflict. Österling et al. (2015) used Bishop's mathematical values to show that the Swedish curriculum advocates rationalism and openness, while students value the opposite, namely control and objectism. The values of the students align with what is common practice in Swedish classrooms and what is emphasized in Swedish culture.

However, ideas are neither constant nor isolated. They influence each other and are influenced by each other. The ideas that dominate and have the greatest influence on teaching vary. In a Swedish context, where teachers have a high degree of freedom, one could imagine that a teacher's ideas about mathematics teaching primarily influence the teaching that takes place in the classroom. However, examples from a Swedish interview study have indicated that ideas from other actors influence teachers to such a great extent that they sometimes make decisions about their teaching that run counter to their own ideas (Grundén, 2017).

METHODOLOGY

People represent parts of the world in their process of meaning-making. The same area of the world might be represented by different perspectives (Fairclough, 2003). According to Fairclough (2001, p. 2), "practices are socially constructed by representations." Hence, the practice of mathematics teaching is socially constructed by representations of parts of the world that relate to mathematics teaching. Different actors, including teachers, students, and school leaders, might represent mathematics teaching and areas related to mathematics teaching differently. By embracing the idea of representations as building blocks in social practices, knowing more about ways of representing becomes part of exploring the social practice of mathematics teaching. However, not only what is said but also what is unsaid is of importance in the construction of a social practice. In the practice of mathematics teaching, as in other practices, there are assumptions, i.e., "meanings which are shared and taken as given" (Fairclough, 2003, p. 55), that are also of importance for the construction of the practice of mathematics teaching. Since the practice of mathematics teaching is constructed of different representations and assumptions, it seems reasonable to think that tensions between different representations and assumptions may arise. In this paper, tensions are thought to occur when different representations and/or assumptions meet. Therefore, the interviews with teachers are used to explore representation, assumptions, and potential tensions between them. In the analysis, assumptions are thought of as representations not connected to an actor.

In a previous study (Grundén, 2017), six Swedish mathematics teachers were interviewed about planning in mathematics teaching. Two of them teach school years

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1-3, and four of them teach school years 7-9. The interviews were conducted based on notes about issues related to their planning in mathematics teaching that each teacher had made two weeks prior to the interview. The interviews were recorded and transcribed, and the material was used to make a content analysis to find themes in the stories of the six teachers. For the purpose of this paper, three of the themes that arose from the material were chosen: special needs, text books, and organization. The themes were chosen based on the observation that teachers referred to different actors within them. Each theme was analyzed based on three questions: What representations are visible? Whose representations are visible? What tensions between representations are visible?

RESULTS

In the following section, different actors' representations of each of the three themes: special needs, text books, and organization, and aspects of the themes are reported. After the review of the representations, a section will follow about prominent tensions that were visible in the teachers' stories within the three themes.

Representations

The analysis of the statements sorted under the special needs theme revealed representations of special needs themselves, students, teaching, and ability grouping. When it comes to special needs, teachers represented it as two-folded. On the one hand, special needs refer to students in need of challenges, described as students who, in addition to the normal teaching, need additional tasks and challenges. On the other hand, special needs can also refer to students in need of support. The students in need of support are represented differently by different teachers, but in summary, it can be said that the students are described as either weak, unwilling, lazy, and/or slow, or in need of a different kind of teaching. The representation of students as weak, unwilling, lazy and/or slow also imply adapting one's teaching, but the adapted teaching is represented as "more of the same," either by giving them the same number of tasks but more time to do them or giving students more tasks and more time.

When teachers represented students as in need of a different kind of teaching, they described, for example, how mathematics problems are used; this allows all the students to be able start working with them, use multiple forms to show and describe the mathematical content, or for adapted instructions addressed to certain students. Another way of adapting one's teaching is to use different forms of ability grouping, which was represented by the teachers either as fixed groups where students were divided based on their mathematical ability, or as temporary grouping that was made based on ability and circumstances, such as tasks, access to teachers, and the student's decisions. There were also statements in the interviews where teachers expressed taken-for-granted assumptions about ability grouping. For example, one teacher said, "Nowadays, you are not allowed to use ability grouping with children," and thereby, referred to a representation with no specific actor.

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In the interviews, text books were represented in various ways. Among the teachers, text books were represented as a resource used as a frame for planning, a bank of tasks, and a tool. Text books were also represented as obstacles, such as a cause of differentiation among students, like when a teacher said, “Students spread out when they work in their text books, some make a lot of tasks, so you lose `control´ over the students.” Text books were also represented as something that should not be used so frequently in mathematics teaching. Assumptions about text books were also visible when a teacher expressed that completing the text books was an unspoken expectation that she felt.

In relation to students, text books were represented by the teachers as reference books for students and either as the main resource or as one of multiple resources that students work with during lessons. One teacher referred to how students represent text books by saying that younger students equalize [school] mathematics with text books. Hence, for them, having mathematics lessons is working with their text books.

Within the category of organization, several representations were visible. The organization itself is represented both as dividing the mathematical content into areas and distributing them over the school year(s), and as organizing and structuring sequences of lessons and individual lessons. Although the curriculum was emphasized as the base for organizing the content, there were examples of teachers working in municipalities where there is a fixed plan for all students in a specific age group. However, it seems common that the content is divided into areas that complement the headlines in text books. However, this way of representing the organization is challenged by teachers who problematize learning mathematics in fixed areas and those who emphasize working thematically with other subjects. The organization of content is also represented as moving from concrete to abstract and as setting a minimum knowledge for each area that all students should learn. One way of representing the organization is to talk about it as “first this, then that,” i.e., students have to finish one area, and the tasks (often in the text book) that are part of that area have to be done before going on to the next area.

When it comes to organizing and structuring sequences of lessons and single lessons, there are two main issues that are raised: balance between working with text books and varied teaching, and assessments. For some teachers, text books and students working individually with tasks in the text books form the base of mathematics teaching, and organizing the teaching is equivalent to deciding what part of the text book to use. However, various other teachers represented organization as deciding how to vary their teaching and use different resources, group arrangements, and activities during the lesson(s). This variation was expressed by one teacher as, “Switching between different ways to work with the mathematical content. Being outside working practically, inside having an introduction, problem solving, students working with text books, etc.” Two teachers referred to two ways of representing organization as *the old way* and *the new way*.

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According to two of the teachers, students' representation of organization consists of a wish for structured and teacher-led lessons with instructions, followed by individual work, and, ultimately, an examination. However, another teacher emphasized that students used to represent the organization in this "old way" but nowadays are "almost demanding" to do something else.

There are different ways of thinking about time regarding organization. A teacher for younger students emphasized how she organizes her mathematics teaching, so that she also can take advantage of moments and see learning possibilities in situations outside the mathematics classroom. In addition, a teacher for older students emphasized that she thinks outside the lessons when she organizes her teaching. She wants students to work to catch up on what they, for various reasons, did not do during the lessons, or to obtain extra help from a special teacher. Her students, on the other hand, have expressed to her that she should only organize the teaching during the lessons.

An essential part of organizing mathematics teaching is, according to the teachers, considering and making decisions about assessments. Assessments are represented as examinations that finish an area, collecting and assessing students' task solutions and answers and grasping what students have learned during the lessons. According to one teacher, assessments are also represented as checking what students have done, in terms of how many tasks the student has completed.

Actors

Based on the analytical question, "Whose representations are visible?" actors were identified and categorized into three groups: teacher(s), official actors, and students and parents. The dominant group in the interviews were the teachers, and they formulate representations as their own, as, for example "I always think […] that there is a minimum number of tasks in text books that all students have to work with," and "I do not think that it is bad to be bounded to text books."

The second group of actors, official actors such as policy documents, researchers, school leaders, and Skolverket [National Agency of Education], and their representations were visible in statements like, "The municipality has decided that all 3rd graders must complete the AG1 Diamant [a specific test from the National Agency of Education] within three minutes," and "Then we have the curriculum […] The competences. We must train competences".

When it comes to students as actors, their representations were reproduced in teachers' stories about what individual students had said, or what students regarded as a group. The ways of representing that originated from the parents were not clearly stated but were rather expressed through feelings of what the parents thought. Examples of statements within the category of students and parents are, "I almost think the kids demand [to do other things than work individually with text books]," and "…what I think is that good students and parents would say that it is wrong [to have a special needs perspective on all teaching]."

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Tensions

As shown above, there are several examples of different representations at play. However, the results indicated that different ways to represent a part of the world do not automatically cause tensions to occur. Tensions occur when different representations meet in a situation where all of them are taken into consideration and are part of a decision-making process. In relation to the material analyzed for the purpose of this paper, many tensions were found, three of which are presented here.

Several interviews revealed a tension between the assumption that ability grouping is bad or not allowed and the teachers' ways of representing ability grouping as beneficial for student learning. One teacher expressed this tension by saying, "I think we would like to make different groups, but nowadays, you are not allowed to group children based on ability." In another case, one teacher said, "I cannot even agree that these children [with special needs] would feel bad about having their own group." In these statements, the teacher's own representation clashed with the assumption. The second teacher also incorporated a presumed student representation: "It is also not fun [as a student] to sit in a group and not understand anything." In this quotation, the teacher expressed that ability grouping by students is represented as an opportunity to be included.

Another noticeable example of tensions regarded text books. There seems to be an assumption that text books are bad, and that the use of them should be reduced. However, as seen above, the teachers represented them in a more positive way. One example is from a teacher who said the following:

> Actually, I don't think that being bounded to the text book is bad, although I know that it does not sound good to say so. Everyone's goal is to not be bounded to text books, but then I feel I would not cope not being bounded to text books, because for me, it is a great support to [help me] keep in mind the things that I know the students need to reach goals in mathematics. There I take the liberty to…

In the quotation, assumptions are visible in "…I know that it does not sound good to say so," and "Everyone's goal is not to be bounded to text books." The teacher's representations are visible: "I don´t think that being bounded to the text book is bad," and "Because for me it is a great support." There is also a sign of how the teacher handles the tension: "There I take the liberty to…" Another teacher also expressed the same tension in other words. A different teacher also involved the students' presumed way of representing text books when she said that younger students equalize having mathematics in school with working individually in their text books; thereby, the students represent text books as school mathematics.

Furthermore, in the theme of organization, there is a prominent example of tensions where a teacher talks about how she wants to vary her teaching and include discussions, practical work, and reasoning. Her students failed a test and, in a discussion with the students about how to move forward, the students told her that they wanted extremely structured teaching, with her giving instructions and telling

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them what to do; their job is to do what she tells them to do and ask questions when they do not understand. The students also wanted a test every week. Although the teacher decided to teach the way the students want, she mentioned telling the students that she is embarrassed about the teaching, because that is not the way one should teach; it is what she calls "old-fashioned teaching." The students' responses demonstrated that they thought the teaching was great. In this example, it seems that the teacher's representation of organization of mathematics teaching is in line with assumptions, but there is a tension between the teacher's representation and assumptions and the students' representation.

DISCUSSION

Findings from this study showed that there are multiple ways to represent the three themes of special needs, text books, organization. The different representations were addressed to three groups of actors: teachers, official actors, and students and parents. In addition to the representations, assumptions that complemented and contradicted the actors' representations were found. Findings also revealed that when contradictory representations and assumptions are confronted in situations, such as a teacher's decision-making process, tensions arise that might lead to decisions that contradict the representations that teachers formulate as their own.

The results indicated an assumption about ability grouping as being "forbidden" and a tension between this assumption and the teachers' representation of ability grouping. Turning to policy documents in Sweden, the assumption is not supported. The school law states that all students should be given opportunities to develop in accordance with their abilities, but nothing has been said about how the teaching should be organized to offer students these opportunities. Based on a review of teaching, the Swedish Schools Inspectorate has emphasized the frequent tendency to use ability grouping as problematic (Skolinspektionen, 2010). Furthermore, the disadvantages of ability grouping have been emphasized by a teachers' union (Lindgren, 2010). Hence, the official representation in Sweden appears to be that ability grouping should not occur. Although the teachers referred to assumptions and official representations, they also resisted them and opened up possibilities for ability grouping in their teaching by expressing different ways to group students based on their abilities. The official representation is embraced by having all students in the same classroom, although the teachers still, in a way, adapt teaching to their own representations and group students based on their abilities (e.g., normal group, low-achieving students, high-achieving or gifted students, etc.). Some students cope/survive/succeed in mathematics education, no matter what the teaching looks like. However, the students who do not are the ones who are most in favor of adapting teaching to research about inclusion and special needs. Otherwise, they end up losing when already established assumptions and representations about mathematics teaching govern how the teaching is organized.

Based on the interviews, one can generalize and say that the emerged assumptions positioned text books as negative, and whose existence can be questioned. On the

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contrary, the teachers' ways of representing text books in mathematics are positive for various reasons, which result in tension when the teacher needs to decide to what extent the text book shall be used. In Sweden, text books are published by commercial companies, and teachers (or groups of teachers) decide what text books to buy. This adds another actor of importance for mathematics teaching. To be able to sell, the companies' ways of representing text books in mathematics should not cause too many tensions with the teachers' representations. Consequently, it seems reasonable to think that teachers want something familiar, which might make it harder to propose new ideas and develop mathematics teaching.

When a teacher who advocates varied mathematics teaching complementary to a large teacher development initiative discussed her students asking for traditional mathematics teaching, the students expressed a way to represent organization. In this case, tensions between the teacher's representation and the students' representation led to the teacher choosing to organize her teaching in line with the students' requests. An alternative interpretation might be that the teacher had learned how to "talk the talk," so that her representations were actually the official ones. Through the requests from the students, the teacher was empowered to resist the official representations and practice the teaching that she wanted.

Some representations were categorized as the teacher's representation. However, one can ask if it is possible to talk about a teacher's own representations. Representations expressed as "I think…" and "My opinion is…" might actually be fruitfully imposed on the teachers and part of what Montecino and Valero (2015) and Montecino (2017) called the homogenization and fabrication of teachers. Nevertheless, there were examples when teachers resisted official representations, which indicates that teachers have the power to resist. However, regardless of whether the teacher is seen as a fabricated product or agentic actor, discussions regarding different actors and their assumptions and representations are important for a greater understanding of mathematics teaching.

IMPLICATIONS FOR FURTHER RESEARCH

In interviews, teachers could talk about assumptions beyond what they personally think, official representations, and the representations of students and parents alike. These assumptions are unspoken, but real in the sense that they influence the decisions that teachers make. It seems reasonable to think that assumptions also influence politicians, researchers, and other actors responsible for the official ideas, in addition to students and parents who have opinions about mathematics teaching. A greater awareness about these assumptions could involve their incorporation in discussions; perhaps also knowing more about these assumptions would lead to better conditions to resist them. Resisting such ideas may be a factor to change mathematics teaching to become a more inclusive, tolerant, and equal part of students' lives.

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PROBLEMATIZING "THINKING" IN MATH EDUCATION

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In this paper, we trace the problematic ubiquity of "thinking" discourse to the invisibility of whiteness in math education, with a particular focus on teaching and teacher preparation. To make this theoretical connection, first we discuss and problematize the over reliance on "thinking" and related phrases, such as "What do you think," in math education. Next, we argue that the term "thinking" as it gets used in classroom discourse, indexes the dominant hegemonic epistemology of Cartesian dualism. In turn, the Cartesian grip on mathematics education both re-inscribes and conceals a racial hierarchy of mathematical ability that privileges whiteness (Martin, 2009). We call for future research to further unpack the linkages among Cartesian epistemology, mathematics, and race, and we consider potential implications for teaching and teacher preparation.

INTRODUCTION: "THINKING" DISCOURSE IN MATH EDUCATION

"Who thinks that they could explain their thinking for question one?" – Deborah Ball to her fifth-Grade math class

"There is no right or wrong answer. We're only interested in your thinking. … You've done a fantastic job of explaining your thinking!" – Mathematics education researcher to a middle school student

Thinking and related phrases, such as *What do you think?*, are ubiquitous in K-12 math classrooms (Herbel-Eisenmann, Wagner, & Cortes, 2010). They are assumed to be pedagogically neutral or positive. This paper is part of ongoing research that challenges and problematizes this assumption.

The narrow focus on "thinking" and related notions like abstraction and manipulating symbols that is pervasive in US classrooms can lead teachers to overlook or fail to recognize other authentic forms of knowing in mathematics. In a previous study, Gutiérrez (2015) conducted a year-long participatory ethnography of a secondary math class. The analysis in that study focused on Mr. Matthew Lam, a teacher who repeatedly presented his class with the metaphorical equation "math=thinking= liberation," and the underlying logic of this equation appeared to guide his instructional decisions. One of Mr. Lam's primary pedagogical objectives was to support student thinking—*critical* thinking in particular. At various points throughout the project, he stated that he associates certain types of mathematical thinking (e.g., generalizing) with the ability to "see," "critique," and "understand systems," which is something that he sought to foster in his students as part of his social justice pedagogy (Gutiérrez, 2015). In his instruction, Mr. Lam occasionally conflated mathematical reasoning with using symbols, emphasizing "abstractions" that involve "letters" as central to mathematical thinking. At times, this negatively influenced his students' agency because it overlooked other authentic forms of knowledge and

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knowing that did not rely on "abstraction" or manipulating symbols as evidence of mathematical "thinking." For instance, some students constructed generalizations (that were mathematically correct) using an array of semiotic resources such as verbal speech, rhythm, gesture, repetition, and non-conventional representations that were never sanctioned by the teacher (Gutiérrez, 2015). If students' emergent ways of knowing and different forms of knowledge go unrecognized, this can lead to narrow or even distorted notions of what counts as mathematics.

Mr. Lam did not come to this view of mathematics as abstract thinking on his own. This narrow view of mathematics is widespread and often taken for granted in K-12 classrooms, with Mr. Lam's case illustrating the potential consequences for students. In a linguistic analysis of 148 classroom transcripts involving eight different math teachers in a variety of school contexts, Herbel-Eisenmann, Wagner, & Cortes (2010) reported that the second most frequent group of words found in their data was *What do you think*. This particular phrase, which they refer to as a "lexical bundle," had 198 instances spread across 8 classrooms. (The most frequent bundle was a directive, *I want you to*, which had 333 instances spread across those same 8 classrooms). These findings indicate that notions of math-as-*thinking* are quite common in classroom discourse. Hence, given the apparent ubiquity of "thinking," the objectives of this paper are twofold: (1) to argue that this hidden aspect of mathematics education is not just a semantic issue but a substantial point that could have a tremendous impact on teaching and teacher preparation; and (2) to suggest an approach that can support pre-service teachers to uncover and disrupt hierarchical views of what counts as mathematics.

THE CARTESIAN GRIP ON MATHEMATICS EDUCATION

At the heart of this paper is an earnest question. Can mathematics pedagogy survive without *thinking*? We urge the community of educators and researchers to reconsider the impact of thinking discourse in mathematics teaching and learning.

The use of the term "thinking" is not necessary for facilitating authentic learning experiences and fostering conceptual understanding. On the contrary, there is some empirical evidence that suggests the constant use of this term in classroom discourse reinforces a Cartesian mind-body dualism[1] that *interferes* with learning mathematics (cf. Cook & Brown, 1999), as in the case of Mr. Lam and his students (Gutiérrez, 2015). Functioning just under the radar, the Cartesian view of mathematics privileges knowledge "possessed" or "acquired" by the individual over group participation and social practice (cf. Sfard, 1998), which severely underestimates the importance of social interaction in conceptual learning (Radford, 2003; Sfard, 2007; Vygotsky, 1978). Cartesian dualism also promotes stereotypical notions of mathematics based primarily on analytic, decontextualized reasoning which, in turn, promote false notions of mathematics education as an *a*cultural and *a*historical enterprise. Critical educational scholars have repeatedly argued against the idea of mathematics education as politically neutral because it ignores the discursive

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as well as structural dimensions in the organization of learning (Nasir, Hand, & Taylor, 2008; Valero & Zevenbergen, 2004; Warburton, 2015).

To clarify, this paper *does not* argue that thinking is absent during mathematical activity or problem solving. We do argue that there are powerful and pervasive social-historical narratives that *equate* mathematics to thinking, such that “math=thinking.” Moreover, these narratives posit that *pure* thinking (i.e., mental activity that is disconnected from perceptual or bodily activity) carried out by the *individual* (i.e., conceptualized as either completely divorced from or minimally influenced by social context) is what drives mathematical discovery and therefore learning. The Cartesian view of mathematics reproduces a stereotype of mathematics as *tantamount* to pure thinking. Math learning is believed to be mostly an “amodal” phenomenon, a way of reflecting on the world that begins and ends ‘in the head’. The *process* of mathematical problem solving is to ‘think about stuff’, and the final products that *result* from that ‘pure’ mental activity are other clearer/logical/cogent ways of thinking. Mr. Lam’s pedagogical views draw from these narratives about mathematics, as he explained during an interview when asked to speak about his view of math-as-thinking:

> Mr. Lam: This is about thinking… change the name of the course from math to THINKING. […] Thinking happens as you do things that are concrete, but the thinking itself is abstract and that’s what mathematics is all about, the abstract, you know, building your capacity to abstract.

In his words, Mr. Lam implied that his main charge, as a mathematics teacher, is to support students in building their “capacity to abstract” because *that’s* what mathematics is all about.

Indeed, an exclusive focus on math-as-*thinking* lays out a constrained space of what counts as mathematics that ignores the fundamental roles that the human body, interaction, discourse, and signs play in mathematics learning (Abrahamson & Trninic, 2015; Alibali & Nathan, 2012; Nemirovsky, 2003; Radford, 2003; Wertsch, 1998). Furthermore, this problematic conception of math is fundamentally linked to another socially constructed narrative: math-as-*intelligence*. Through the prism of Cartesian epistemology, mathematical ability is often seen as an index of general intelligence, rather than simply the development of skill or competence within certain genres of semiotic activity. In their analysis of race and mathematical practices, Shah and Leonardo (2017) further unpack the notion of math-as-intelligence and point to one of its limitations:

> The practice of doing school mathematics in the U.S. context is associated with a particular set of meanings. Mathematics is typically viewed as the most difficult of all subjects; only the elite few are thought to possess the innate capacity to understand mathematics (Schoenfeld, 2002). Perhaps as a result, mathematical ability and intellectual capacity are believed to go hand in hand (Ernest, 1991). And because mathematical

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ability is viewed as innate, it tends not to be seen as something that can develop through concerted effort over time. (p. 60)

In sum, the word "thinking" as it gets used in classroom discourse, both indexes and re-inscribes a narrow conception of what counts as mathematics rooted in Cartesian epistemology that privileges certain ways of knowing. In turn, classroom discourse that subscribes to Cartesian precepts overlaps with racial-mathematical discourse (Shah, 2017; Shah & Leonardo, 2017) that serves to create a "racial hierarchy of mathematical ability" (Martin, qtd. in Shah, 2017, p. 11). Danny Martin (2009) proposes that this hierarchy is one in which "students who are identified as Asian and White are placed at the top, and students identified as African American, Native American, and Latino are assigned to the bottom" (p. 315). In this way, whiteness and white privilege are particularly well-hidden and maintained by mathematics education, and these intersecting discourses pose real challenges not only in research (Martin, 2009) but in teacher education as well (Warburton, 2015). As a response, we find pushing on constraining views of mathematics as thinking as a concrete way to analyze whiteness and to disrupt it.

MATHEMATICS EDUCATION RESEARCH AS COMPLICIT WITH THE MATH-AS-THINKING REGIME

Through the use of everyday terms such as "thinking," Cartesian dualism is baked into the everyday language of mathematics education. This results in the reproduction of hierarchical category systems and phraseologies related to both mathematical ability and intelligence (e.g., "fast" versus "slow kids"; "advanced" versus "below basic" courses; and "failure" versus "success" with problem-solving) that have pernicious effects on learning and student identity (Horn, 2007; Larnell, Boston, & Bragelman, 2014; Ruthven, 1987). Furthermore, the implications of these hierarchies extend beyond the classroom. We maintain that constructs stemming from psychology—such as "mindset," "grit," and "smartness"—are actually derivative of Cartesian dualism. Whereas certain research constructs are intended to be useful tools for investigating and improving mathematics education, they in fact "tighten" the grip and delimit opportunities for learning, because these, too, are *relational* constructs (Shah, 2017) based on "differences" that have been demarcated, historically, along racial lines. Therefore, these constructs, although believed to describe non-apparent, psychological properties of individual students so as to identify leverage points to bolster "smartness," for example (Leonardo & Broderick, 2011), instead serve to reproduce and strengthen the racial–mathematical hierarchy due to the tight, three-way link between mathematical ability, intelligence, and race.

MATHEMATICS TEACHER PREPARATION AND FUTURE RESEARCH

To loosen the Cartesian grip on mathematics education, we suggest broadening the space of what counts as mathematical activity beyond a narrow focus on thinking. We offer several strategies for teaching and teacher preparation intended to support a more expansive view of mathematics. One specific suggestion is to avoid certain

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types of statements during classroom discourse, such as *What do you think?* and *Can you explain your thinking?* In a recent report (Gutiérrez, 2018), we describe a series of prompting strategies stemming from the research literature on embodied cognition that emphasize action and perception (Abrahamson & Trninic, 2015; Alibali & Nathan, 2012; Nemirovsky, 2003) as a way of disrupting Cartesian epistemology and stereotyped notions of mathematics that ignore crucial bodily activity. Problematizing the over reliance of "thinking," while at the same time drawing on other semiotic and embodied measures, can shift the discourse away from math-as-*thinking* to math-as-*action*, which creates robust opportunities to learn. For teachers or instructors of mathematics, avoiding "thinking" may lower the threshold to gesture (cf. Kelly, Byrne, & Holler, 2011), possibly improving communication with learners (see also Alibali et al., 2013).

In the remainder of this paper, we discuss how teacher preparation might support pre-service teachers (PSTs) to re-conceptualize math-as-action and, along those lines, we suggest possible avenues for future research. PSTs' relationships with mathematics have been shaped by pervasive and constraining views of mathematics as thinking previously outlined in this paper. For secondary teacher candidates, these views of mathematics may not have been personally problematic in that these teachers succeeded enough to pursue mathematics under a math-as-thinking paradigm. For secondary teachers, broadening what mathematics is and what it means to be good at it may feel threatening. Adopting a broader, inclusive view of mathematics involves letting go of one's sense of oneself as 'uniquely' competent at mathematics. We argue that this type of internalized cognitive meritocracy is linked to discourses of whiteness that preserve the racial hierarchy of mathematical ability.

For elementary teacher candidates who may not have had positive experiences with mathematics in their own K-12 education, narrow conceptions of mathematics and mathematical competence can have lasting consequences for how they engage (or choose not to engage) with mathematics (Wood, 1988). As one elementary PST, Alejandra, commented in a math pedagogy course reflection paper, "All throughout primary school, there seemed to only be a very specific set of skills that were viewed as an asset in math. Since I did not seem to do math in the way that I was 'supposed to', I felt that I did not possess any skills of value." Though Alejandra does not specifically reference math as thinking, the "very specific set of skills" that were valued in her classrooms, and her awareness that she did not do math the way she was "supposed to" speak to the constrained space of mathematics that was available to her and the impact this had on her sense of competence. For both secondary and elementary PSTs, these deeply cultural notions of what counts as mathematics, rooted in Cartesian assumptions about the nature of mathematics, coupled with social structures and practices that privilege whiteness, can impact how future teachers relate to mathematics and to their students.

But there is hope. If we aim to prepare future teachers to disrupt dominant hierarchies of power and privilege by broadening the space of doing mathematics beyond math

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as thinking, we argue that we need to support teacher candidates to *experience* mathematics as an open space in which every human being has something meaningful to contribute. During the Math Pedagogy course Alejandra took during her elementary preparation program (taught by the second author), she had ample opportunities to engage in rich mathematics and to see herself as a competent doer of mathematics. In her reflection paper at the end of the course, Alejandra described her own transformation from seeing herself as someone who "did not have much to contribute" to seeing that the way she approached mathematics could be a resource for her peers, writing, "I suddenly realized I had contributed by: representing quantities in different ways, making connections, using tools, asking questions, and explaining strategies. It was such a revelation to see that so much of what I had done and the way I had thought about the math, was in fact a strength that had brought both my group and myself to a deeper place of thinking and understanding." In Alejandra's reflection, she describes mathematics as an expansive space that includes, but is not limited to *thinking*, and she sees herself as someone who has strengths to bring to this space.

We find it instructive to compare Alejandra's view of mathematics with Mr. Lam's view, which we discussed above. Whereas Mr. Lam's perspective seems to privilege math-as-thinking in ways that constrain how he engages with his students, Alejandra's new understanding of mathematics as both thinking and action supported more expansive ways of seeing her students. She writes, "It made me realize that I had not paid close enough attention to all of the unique ways that my students were *thinking about* and *doing* math [emphasis added]…so, I redefined for myself, what being smart in math really meant. As I began to observe all my students more closely, I came to find a plethora of strengths and varied ways of thinking that I had overlooked before (or had not known to look for)." In our estimation, it appears that what counts as mathematics for Alejandra is quite different than what counts as mathematics for Mr. Lam. For Mr. Lam, the dominant view of math-as-thinking represents a constraining discourse that lays out mathematics as a cognitive terrain that can only be traversed by the most "capable" students and "abstract thinkers." But for Alejandra, her reflections suggest that she sees mathematics as a more open space that includes the rich and diverse ways her students engage with and make sense of mathematics. For Alejandra, expanding beyond the math-as-thinking paradigm opened up new ways for her to relate to mathematics and new ways to see her students as competent doers of mathematics.

One of Alejandra's classmates, Lena, also wrote about the role of doing mathematics in shifting her ideas of what counts as mathematics and explicitly connected her more expansive view of mathematics to issues of equity:

> "Another specific aspect of the math groups that was very new to me was the notion that everyone has something important to contribute to math instruction. While I believe that every human being has unique and amazing aspects to them in general, for some reason I had never extended this personal belief about humanity to the ways that people engage in

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> mathematics…Most of us came to this program with a high degree of interest in and commitment to social justice and equity, and yet for most of us, thinking of all of our students as being brilliant in math has been a big transition. What is it that makes mathematics this rarefied subject, as if our notions of equality don't apply in math class?" (Lena, Final Reflection Paper, 2018)

Lena critically reflects on the role of mathematics in perpetuating hierarchies and inequity, and she implicitly pushes on the tight coupling of math, thinking, and intelligence when she refers to math as a "rarefied subject" where "notions of equality don't apply." These new ways of understanding mathematics and their students became possible for Lena and Alejandra because they had repeated opportunities to participate in mathematical tasks that invited in multiple ways of knowing and of engaging with mathematics.

In this paper, we found it insightful to juxtapose cases of elementary preservice teachers' views of mathematics with those of a secondary teacher, Mr. Lam. We observed a number of differences across our cases, and this analysis led us to theorize different aspects of the math-as-thinking regime and to consider what it might take to disrupt this regime. In comparing these two cases, our intention is not to oversimplify the complex factors and contexts that impact how teachers conceptualize mathematics. Instead, we use these examples to illustrate how different understandings of mathematics affect how teachers make sense of students' mathematical actions and to suggest how we might foster more expansive views of mathematics. Our experiences with Alejandra, Lena, and the other PSTs we have worked with during teacher preparation suggest that providing PSTs with mathematical experiences that push past dominant views of math-as-thinking has rich potential for supporting equity-focused teacher learning. There is still much to be learned about the ways that the Cartesian grip on math education influences elementary and secondary teachers views of what mathematics is and about what we might do to disrupt this paradigm in teacher preparation. Future research can systematically examine how math-as-thinking and math-as-action afford and constrain opportunities for mathematical learning in K-12 classrooms and in teacher preparation. Learning more about the specific ways math-as-thinking functions in a variety of settings can inform new approaches in teacher preparation that seek to broaden the landscape of what counts as mathematics to invite in diverse ways of knowing and of engaging with mathematics.

NOTES

1. Cartesian epistemology is a branch of philosophy attributed to René Descartes and his study of knowledge. Cartesian dualism refers to his concept of the mind and the body as two completely different types of substances (immaterial vs. material) that interact.

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USING GRAPHIC ELICITATION METHODS TO TALK ABOUT SOCIAL IDENTITIES AND MATHEMATICS

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In this paper, I examine the potential for a particular representation of multidimensional identities and alternative interview methods to elicit students' multidimensional social identities and mathematics identities. The analysis looks at what happened when I interviewed eight secondary mathematics students about their multidimensional mathematics and social identities using graphic elicitation methods. Further, I raise ethical and political considerations that arise from the inherent tensions associated with conducting research where social identities, such as race and gender, are concerned, particularly when an interviewer from a dominant group generates and interprets data from less powerful participants.

Increasingly, research in mathematics education attends to how social identities – race, ethnicity, gender, class, etc. – influence students' mathematics identities, or how students see themselves and are perceived by others as capable learners and doers of mathematics (e.g., Gholson, 2016; Langer-Osuna, 2011; Nasir, Snyder, Shah, & Ross, 2013). Accompanying this growing body of work on race and mathematics identities, gender and mathematics identities, etc. are calls for addressing students' intersecting social identities alongside mathematics identities (e.g., race, gender, and mathematics identities; Leyva, 2017). Understanding intersections of multiple social identities with mathematics identities will depend both on researchers' capacity to conceptualize models for multidimensional identities and the effectiveness of attending to these multidimensions throughout various phases of the research.

In this paper, I examine the potential for a particular representation of multidimensional identities and alternative interview methods to elicit students' multidimensional social and mathematics identities. Because we cannot be sure that our methods are "operating as we would wish from the point of view of the interviewee rather than our own unless we analyse what happens in interviews" (Edwards, 1990, p. 476), this analysis looks at what happened when I interviewed eight secondary mathematics students about their multidimensional mathematics and social identities. Given the socially constructed nature of research, the identities of the researcher and the participants, in addition to the methods adopted, shaped the data and findings generated by the study (Archer, 2002). Thus, any effort to understand intersections of multidimensional social identities and mathematics identities also raises important considerations of power in the negotiations among researcher and participants. This paper, therefore, also addresses ethical and political considerations that arise from the inherent tensions associated with conducting research where social identities, such as race and culture, are concerned (Milner, 2007).

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RESEARCHING MULTIDIMENSIONAL IDENTITIES

I aimed to understand students' multidimensional identities, including their mathematics identities and social identities. Further, I sought to consider how students leveraged their multidimensional identities in mathematics classroom interactions, namely, how particular identities became salient as students interacted with mathematics, with their peers while doing mathematics, and with their mathematics teacher. Doing so, required attention to the power dynamics at play across students' varied identities. In other words, particular social identities might privilege students while others might marginalize them during mathematics classroom interactions. For example, a Black boy in mathematics might experience privilege (i.e., elevated academic status) because of his gender while simultaneously experiencing marginalization (i.e., lowered academic status) due to his race.

Eliciting the interplay of students' multidimensional social identities and their mathematics identities through interviews presented significant challenges for two reasons. First, individuals belonging to more than one oppressed group can resolve their identities in different ways (Jones & McEwen, 2000). Thus, students could vary in their ability to recognize and describe their multidimensional social identities, particularly in relation to their mathematics identities. For example, some students might identify with only one aspect of self (either because others had assigned them that social identity or because they particularly identified with that oppression), or they might identify with multiple social identities but only in a segmented way (i.e., only in particular contexts, which may not include the mathematics classroom) (Jones & McEwen, 2000).

A second challenge to bringing multidimensional identities into the conversation during interviews arose because of similarities and differences in the social identities of the interviewer and interviewees, which can exacerbate problems of "exploitation and alienation within the research process, particularly with regard to the way data are elicited from less powerful participants and are interpreted by dominant group researchers" (Archer, 2002, p. 110). As a white, middle class adult, I held a more powerful social position than the young adults, who were mostly people of Color and from working class families. For example, my white identity could, but would not necessarily, silence discussions about racism because interviewees of Color might be suspicious about my perspectives or might try to avoid unpleasant situations or potentially awkward topics (Archer, 2002).

In the remainder of this section, I elaborate on both of these challenges and discuss the methods intended to make space for discussions of multidimensional identities during the interviews.

Using graphic elicitation methods to model multidimensional identities

I developed a graphic elicitation method to guide the interview because simple visual tasks can help interviewees attend to aspects of their identities (social and mathematical) that they might neglect when offering only verbal descriptions of self

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(Bagnoli, 2009; Freeman, 2017). In other words, a “graphic elicitation tool may encourage a holistic narration of self, and also help overcome silences, including those aspects of one’s life that might for some reason be sensitive and difficult to be related in words” (Bagnoli, 2009, p. 566). Further, centering graphic elicitation of identities in the interviewing process can challenge dominant interview narratives by allowing those who are often oppressed and silenced an alternative way to amplify their voices (Freeman, 2017).

The graphic elicitation method I developed drew on Jones and McEwen’s (2000) study in which they conceptualized multiple intersecting identities by depicting a *core identity* – those most valued personal attributes that are somewhat hidden from others – encircled by *outside identities* – those personal attributes that are potentially less meaningful but can easily be named by others. In this model (Figure 1), outside identities included race, culture, class, religion, gender, and sexual orientation, and the proximity to the core identity indicated personal value of those attributes. The model suggests that the locations of outside identities in relation to the core identity are fluid because those identities are situated within contexts, such as family background, sociocultural conditions, current experiences, and career decisions and life planning, which influence the centrality of outside identities (Jones & McEwen, 2000).

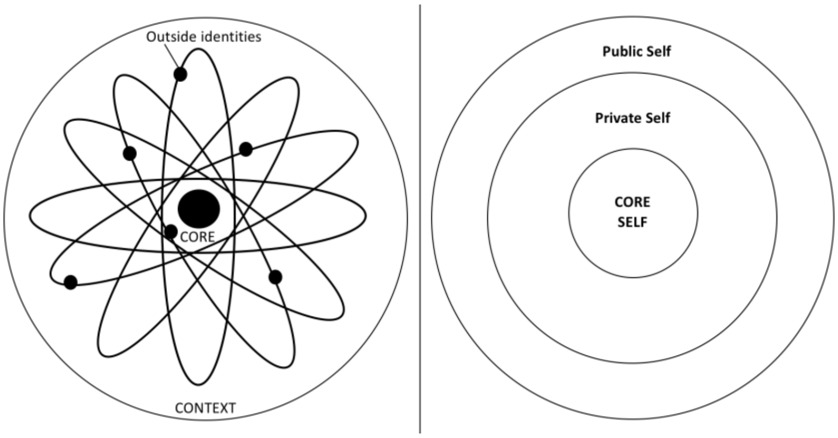

Figure 1: Models for multidimensional identities. The model on the left shows how Jones and McEwen (2000) represented multidimensional identities. The model on the right shows the model used for graphic elicitations methods during interviews.

Modifying Jones and McEwen’s (2000) layered and circular model, I sought to allow interviewees to bring in self-identified aspects of self when creating their graphic model (versus only asking them to specifically reflect on race, class, etc.). As in the original model, however, I wanted interviewees to show their different identities in relation to a core identity and outside identities. Thus, I adopted a common graphic elicitation method depicted using concentric circles, namely relational maps. Relational maps are typically used to allow interviewees to show their relationships with others in their lives (Bagnoli, 2009), but I drew on this model to allow interviewees to show the relationships between their core and outside identities. This modified model, henceforth identities map, for representing multidimensional

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identities allowed participants to select any aspect of self and represent their intersectional identities as related to core self, private self, and public self (Figure 1). More specifically, the following prompts helped interviewees make sense of these different levels of identities. *Core self*: What is most important about who you are as a mathematics learner / person? *Private self:* What are some things about who you are as a mathematics learner / person that others might not know? *Public self:* How do others see you as a mathematics learner / person?

To account for the identities maps' inability to show identities as fluid and to account for the influence of contexts on identities, I designed the interview prompts to focus students' attention on particular contexts as they engaged in graphic elicitation. First, I directed students to create their identities maps by asking: *Who are you in Mrs. Stone's class? Who are you as a mathematics student?* (Prompt 1). Students completed their identities maps prior to the interview, representing core self, private self, and public self in relation to being a mathematics learner in a particular context, and I designed specific questions for each individual interview to probe for more information about students' mathematics identities. During the interview, I asked students to engage in graphic elicitation again: *Who are you outside of mathematics or academic classes?* (Prompt 2). I directed students to either create a new identities map or to modify their mathematics identities map (e.g., add to it, cross out parts of it) in relation to being a person, and I probed for additional information about these identities and their contexts. Finally, I asked students once again to modify their identities map by selecting from a list of words describing social identities (i.e., outside identities, Jones & McEwen, 2000) and adding only those words that they thought described them to their identities maps (Prompt 3). This list included the following social identities with examples: race (e.g., white, Black, Latinx, Asian/Asian American, other); gender (male, female, other); class (e.g., poor, working class, middle class, rich); sexual orientation (heterosexual, homosexual, other); religion (e.g., Christian, Muslim); and ability (e.g., able bodied, disabled, learning disabled, gifted). Then, I asked them to reflect on whether or not those social identities mattered in the context of mathematics.

RESEARCH CONTEXT AND PARTICIPANTS

This paper discusses data from a larger doctoral study that sought to understand students' mathematics experiences and identities when engaged with an innovative approach to equitable mathematics teaching (for details about this instructional approach, see Harper, 2017). The study took place at an open-enrollment, STEM-themed magnet school located in a low-income area of a small city in the Midwestern United States. The school's mission emphasizes technology-driven (1:1 student to laptop computer ratio) project-based learning, in which students collaborated on projects to explore and solve authentic, real-world tasks or problems, using ideas, knowledge, and skills across a range of disciplines. This newly founded magnet school enrolled approximately 400 7th-10th graders from the local community at the time of the study. Schoolwide enrollment was 43% Black, 24% White, 18% Latinx, 9% Asian/Asian American, and 6% bi- or multi-racial, and the median household income

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for the school district was 22% lower than the county's median household income. In the larger study, I drew on ethnographic methods aimed at understanding classrooms from students' perspectives (Anderson-Levitt, 2006) across the 2015-2016 academic year in one 9th grade geometry class with a racial/ethnic composition that reflected the schools' demographics. The teacher, Mrs. Stone, is a white woman, who was in her fourth year of teaching mathematics during the study. Mrs. Stone is committed to ensuring equity and social justice in her mathematics teaching, and she and I have collaborated towards that goal since 2013.

This paper focuses on end-of-the-year interviews, which were specifically designed to give students an opportunity to discuss their mathematics identities and social identities in order to validate emerging interpretations of students' multidimensional identities from the larger analysis (Bagnoli, 2009). I conducted these interviews with eight students. I purposely selected student participants for the interviews so that, collectively, they represented varied intersections of race/ethnicity and genders. The participants were: Carley [1] (Black [2] Girl); Dante (Black Boy); George (White Boy), Monique (Black Girl); Nemo (Black Girl); Rosy (Korean American Girl); Simba (Black Girl); and Tino (Latino Boy).

TALKING ABOUT SOCIAL IDENTITIES

When using the identity maps to talk about self in mathematics class (Prompt 1) and outside of academic classes (Prompt 2), students described aspects of their personality that were relevant across both contexts (e.g., "like telling my ideas" [Rosy]; "shy" [Simba]). They also talked about aspect of self that only applied in mathematics class (e.g., "work hard and stay on task" [Carley]) or only applied outside of academic classes (e.g., "a cool person" [Dante]). None of the students, however, spontaneously brought up dimensions of their social identities related to race, gender, class, etc. until after I explicitly prompted them (Prompt 3). Once prompted, students talked about different dimensions of their social identities with varying level of detail.

In my first two interviews (Monique and Dante), I prompted students to consider social identities without discussing my own social identities. During all subsequent interviews, I shared my own social identities with students in order to clarify my prompt and to ease the discomfort of talking about taboo topics (i.e., race). First, I talked about how most people would see me as a White, middle class female, and thus, I would put those social identities in "public self." Then, I shared that most people do not know that I grew up poor, and I would add that to "private self." Finally, I explained that being a woman is an important part of what shapes who I am, meaning I would also write that in "core self." In the last three interviews (Nemo, Rosy, and George), I added an additional detail about myself to try to further ease discomfort with taking about race. I shared with students that I grew up in a biracial family, with a Black stepfather and biracial siblings. I talked about how most people do not know that about me but it significantly shaped who I am (i.e., "private self").

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Below, I discuss how students talked about each of the prompted social identities and give more insight into how the conversation was negotiated between interviewer and interviewee. Further, for each social identity, I share details about how the students responded when asked to reflect on whether or not the discussed social identities mattered in the context of mathematics.

Talking about race

All eight students talked with me about racial identities. Together, the conversations and additions to the identity maps shed light on the complexity of students' racial identities and misunderstandings about those identities. George was the only student who discussed and mapped his racial identity in a way that aligned with the racial identity pre-determined by me and the teacher. George said, "They know I'm a white male," and he wrote, "white" in the "public self" circle. For others, conversations and mappings revealed aspects of racial identities that were previously unknown.

In particular, I learned about students' biracial identities: Nemo (Black/Asian), Dante (Black/white), Carley (Black/white), and Rosy (Asian American/white). Both Nemo and Dante named biracial identities, and the placement of these identities on the map provided additional details. For example, Nemo said, "I'm Black and Asian," and wrote "Black" in the "public self" circle and "Asian" in the "private self" circle, thus indicating that her biracial identity was unknown to others. Both Carley and Rosy elaborated verbally on their biracial identities, confirming others' lacked awareness of this aspect of their identities. Carley shared, "Most people probably just think that I'm all Black, African American, but my mom is white and my dad is Black," and Rosy said, "Everyone thinks I'm white but I actually have a mom who is born in Asia."

Although not biracial, Tino and Simba shared details about how their racial identities were misunderstood by others. Tino shared, "This is what they don't know about me… I'm Latino, and people see me as White. I'm from Argentina." In contrast, Simba discussed how people assumed she was biracial: "Some people think I am [Asian] because of my eyes and so I just tell others that I am. If they ask me what am I mixed with, I say Asian." Monique's racial identity was also misunderstood by others but in a different way from her peers. She was the only students who chose not to write a racial identity on her identity map, and she did not bring up race until I asked her if she identified as a particular race, to which she responded "No."

Conversations about race were the most robust of all conversations about social identities. Thus, it was unsurprising that students expressed rather strong feelings about whether or not racial identity mattered in the context of mathematics. Dante, Tino, Simba, George, Nemo, and Monique all believed that race did not matter in mathematics. These students expressed a sentiment very similar to Dante's: "No…No…It's just a race and how your family is built. You can't really change it. Be proud of who you are." Rosy and Carley, however, shared that race mattered in mathematics. Carley expressed awareness of racial stereotypes in mathematics:

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> Some people might think that because you're a certain gender or race that you may not know as much. Most people think that Asians or Japanese people know a lot. Sometime that may not be the case.

Rosy described how race was the biggest factor impacting social interactions in the mathematics classroom but not the content of mathematics itself:

> I think race would be the most ... Biggest one because this is just a problem in all classes, all cities, everywhere in the world that I wish could just be resolved…It would just kind of affect the social part of any class, not so much how it would affect math though.

Talking about class

Seven students (all except Carley) talked about their class identities. Most students described a public self that was middle class, but similarly to talking about race, talking about and mapping class shed light on the complexity of class identities. Namely, students shared how they thought others might perceive them as being wealthier than they actually are. For example, Simba shared, "Some people think I got bank just because I wear different sneakers every day...I'm not. I wish. I would call myself middle class." Nemo elaborated, "I guess sometimes we were both middle class and working class. I'll put this on the inside [private self]." Rosy was the only student who provided information about her family life that related to her class identity:

> Others might think I'm a middle class or rich, so, I'm actually working class and middle class… My family, we kind of struggle a lot, it brings a lot of problems to my parents. My mom and her boyfriend was arguing and my dad's all stressed out because money and payments.

Only Monique and Rosy brought up class specifically when I asked whether or not social identities mattered for mathematics. They both agreed that it did not matter. Rosy said, "Poor, working, middle, rich, I don't think that would matter." Monique focused on the social interactions based on class: "To me even if they're rich, I'm still going to be nice to them. I'm not going to treat them any different."

Talking about gender

Six students (Carley, Dante, George, Nemo, Rosy, and Tino) talked about and mapped their gender identities. Simba only wrote "female" on her identity map, and Monique chose not to add a gender identity to her identity map. All students (except Monique) chose to place their gender identities in the "public self" circle, and discussion of gender was brief. For example, Nemo said, "I'm a female." In this way, students expressed a more straightforward perception of their gender identities compared to their class and racial identities.

Similarly to how they talked about race and mathematics, only Carley (see excerpt in section on race) and Rosy expressed a belief that gender mattered in the context of mathematics. Rosy shared that gender might impact social interactions in a mathematics classroom, but she did not think gender was a big of an issue:

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> I think gender may be a little ... Well, that's another one of those big things out in the world…I don't really know anything about it ... I think that's something that could possibly come up in classes between a boy and a girl arguing ... Like socially between people in classes.

In a few of the interviews, I shared how my own gender identity mattered in the context of mathematics:

> When I'm in math classes at college, often times I'm the only female in the class. That becomes really important, and I think it influences how I interact with people because I am the only female in the class.

None of the students, however, expressed a similar view of the role of gender in mathematics or took up talking about gender-based experiences in mathematics.

Talking about ability

Five students talked about ability as part of their social identities (Monique, Nemo, Rosy, Carley, and Simba). Ability was the only social identity that students connected to mathematics class without my prompting. For example, Simba shared, “Is there such thing as math dyslexia? Yeah. I knew I had it. I knew it. I just don't get math. It makes my brain all discombobulated.” Nemo told me about her reading disability: “Reading-wise, I read really slow. I just don't tell nobody. I just would rather not read.” This was relevant to mathematics classroom because the project-based context often required students to read and write alongside doing mathematics.

Talking about other social identities

Only George, Rosy, and Tino talked about religion, and both Rosy and Tino elaborated on how religious identity might matter in academic contexts. Rosy thought religious identity probably would not matter in mathematics, but she said, “I think that's something that would matter in biology when they're talking about how humans came to be because we were just talking about that.” Tino talked about how religious identity might matter in any class, including mathematics “because if the teacher tries to say something about another religion and kind of advertises or wants people to be that religion, I won't like that. They can be fired for that.”

Monique was the only student who talked about sexual orientation. Although she did not share her own sexual orientation, she talked about how she did not consider that or other social identity as important for who she is as a person:

> For sexuality or disability, I really don't care about that either. Some of my friends, they're bi[sexual]and then the special needs kids here, I'm nice to them and talk to them every time they talk to me. I think this is important because I think people need to treat people nice.

CONCLUSION

The use of graphic elicitation in interviews about mathematics identity opened spaces for students to talk about their various social identities. At a minimum, these identity

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maps were helpful in revealing students' self-identifications and aspects of self they saw as most important to discuss (Freeman, 2017). Beyond naming students' social identities, these maps also provided insights into those social identities that students have grappled with most – race and class – and those social identities they saw as most influential in the context of mathematics – race and ability.

Use of graphic elicitation alone, however, was insufficient for opening spaces for students to talk about their social identities. It is noteworthy that Monique was reluctant to name her social identities and that I shared the least with her about my own social identities. As other researchers (e.g., Archer, 2002; Bagnoli, 2009) have noted, disclosing more about myself often encouraged students to share more about their own identities, but sharing my own personal experiences did not always prompt students to elaborate on their social identities, with gender in particular. Further, the strong hierarchy imposed by the concentric circles may have constrained some students' efforts to communicate their identities. Thus, it is crucial to keep in mind that these methods (and any interview methods) generate data that may not have been present without researcher input, which may more strongly influence some responses than others (Bagnoli, 2009).

Ethical interviewing and reporting of interview data entails making every effort to present the researcher's analysis and the participants' perspective, especially when those perspectives differ (as was the case with gender and mathematics in this study) (Anderson & le Roux, 2017; Milner, 2007). Much has been written about matching interviewer and interviewees in order to mitigate the researcher's ability to understand the participants' perspectives. For example, some argue "only women are experientially equipped to conduct research with women participants" (Archer, 2002, p. 109). Social identities, however, are multidimensional, and differences among interviewer and interviewees are inevitable. For example, differences in class can undermine gender and racial solidarity (Johnson-Bailey, 1999). Further, researchers with similar backgrounds can conflate their own experiences with the participant's experiences (Archer, 2002). Thus, every researcher has a responsibility to strive for more effective ways of negotiating spaces for the voices and perspectives of research participants.

NOTES

1. All names are pseudonyms. Most students chose their own pseudonyms.

2. Race/ethnic identities are more complex than suggested in the summary table. Several students are bi- or multi-racial, or their racial/ethnic identities were misrepresented by others. These complexities are discussed later in the paper, but the table lists the racial/ethnic identities based on students' primary self-identification.

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APPROACHING FAKE NEWS IN MATHEMATICS EDUCATION

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Abstract: Fake news is considered a threat to democracy. How can mathematics education take a responsibility? What challenges should we prepare for? In this paper I present literature related to fake news on how to define it and how to understand it as a problem. I discuss this in relation to both the status of quantified information and the decreasing trust in science, and further articulate some challenges, dilemmas and paradoxes. The intention of the paper is to start a discussion on how mathematics education can approach the topic of fake new. I present some suggestions on what could be included in a mathematics classroom and suggestions for further research.

INTRODUCTION

> The Fake News is working overtime. Just reported that, despite the tremendous success we are having with the economy & all things else, 91% of the Network News about me is negative (Fake). Why do we work so hard in working with the media when it is corrupt? Take away credentials? (Trump 2018).

President Donald Trump has indeed contributed to put fake news on the agenda. Fake news has been argued to be a threat to democratic values, and internet and social media, Twitter and Facebook in particular, have been blamed. Often, fake news is considered as specific kinds of fabricated news, appealing to people's sentiments (see for instance Gelfert 2018). President Trump, however, uses the term rhetorically to dismiss his opponents. A tweet response to the above tweet from Trump provides a related explanation: "*President Donald Trump finally admits that "fake news" just means news he doesn't like*" (Linddara 2018).

A related term to fake news is *post-truth*, which the Oxford Dictionary announced as the word of the year in 2016. The term was defined as "*relating to or denoting circumstances in which objective facts are less influential in shaping public opinion than appeals to emotion and personal belief*". Common for both terms is the idea of appealing to people's sentiments and beliefs, where facts become less significant. However, the term "truth" in post-truth has been criticized, reasoning that politics is not about truth (Berghel 2017). This brings us to a key point: the challenge of labelling something as real or fake, true or false, because it may be impossible to decide and because of the many shades between.

Numbers play an important role in politics, policy development and decision-making, so that competences and skills related to mathematics are relevant for facing news and fake news with a critical attitude. A significant part of the literature on critical mathematics education is dedicated to discussing the importance of critical reflections to critical citizenship and democracy. Leaning on this and on literature on theory of

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science, this paper aims at starting a discussion on how mathematics education can take a responsibility in facing problems of fake news. A key aspect, I argue, is distinguishing between fabricated, twisted, incorrect and uncertain numbers, but also recognising associated challenges, dilemmas and paradoxes.

I start out with referring to how fake news has been discussed in the literature and how it is understood. I then turn to literature discussing the status of quantified knowledge and information. Drawing on these perspectives, I articulate and discuss some challenges related to i) separating fake numbers from twisted, incorrect and uncertain numbers, ii) balancing values and "facts" and iii) the challenge of agreeing on what sources are trustworthy. Finally, I suggest what mathematics education for fake news may look like.

FAKE NEWS

Fake news has been defined in various ways. Gelfert (2018) pointed out that fake news is often related to either dissemination of fabricated news or to dismissing of news (as for instance in Trump's tweet above). Fake news does not only cover news that are entirely fabricated, but may be real news mixed with misinformation, or the context or connection may be false. Some denote news presented in political satire as fake news (Gelfert 2018).

Berghel (2017) divides fake news into three categories: (i) Fake news where the source of the story is open and is thus easier to evaluate, (ii) fake news that are based on sources who are anonymous, and where this is justified by claiming it protects freedom of speech. The content from such sources is challenging to evaluate and dismiss. (iii) Fake news that look authentic, but are fabricated news. Gelfert (2018) argues that fake news should rather denote news fabricated with the intention to manipulate the reader for certain purposes, and should thus be separated from gossip, rumour and hoaxes because their intention may merely be to misinform. He suggested the following definition: "Fake news is the deliberate presentation of (typically) false or misleading claims as news, where the claims are misleading by design." (Gelfert 2018, p. 108).

A study on how fake news spread on Twitter, tweets connected to the US presidential election in 2016, found that half of the fake news originated from ordinary users, although tweets often included links to non-credible news websites (Jang et al. 2018). In contrast to real news stories, fake news were often modified before retweeted. The selected fake news were all about hurting either Hillary Clinton or Donald Trump, and satsified thereby Gelfert's (2018) criteria of being misleading by design.

Fake news is associated with a range of challenges. One concern is that there is an expectation that news from newspapers and news programs are news that can be trusted, although the reliability and motives of news should always be scrutinised (Walton 1997). Fake news take advantage of this by mimicking the design and style of real news. Fake news can therefore cause confusion about current events and issues

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(Nelson & Teneja Viotty 2018). A concern is that news are fabricated to align with people's sentiments and beliefs, which then may strengthen these beliefs and thereby reduce people's willingness to negotiate beliefs (Gelfert 2018). However, there is disagreement on whether fake news actually is a significant problem. Nelson and Teneja Viotty (2018) concluded that only a low percentage of people actually visited sites with fake news, and this group consisted of heavy internet users who read news from a range of sources. Yet, the authors suggested that future research should address the consequences of fake news for democracy.

Nelson and Teneja Viotty (2018) suggested that the problem with fake news is not the fake news itself, but the readers' capability and opportunity to judge the credibility of news. Herein lies a challenge. De Keersmaker and Roets (2017) examined how people adjusted their judgment when they received correct information and learned that the initial information was false. Their results indicated that the adjustment of attitudes depended on people's cognitive abilities (reasoning, remembering, understanding and problem solving). This implies that the effect of fake news cannot or is not easy to be undone.

In line with the findings of de Keersmaker and Roets (2017), Allcott and Gentzkow (2017) suggested that there is a need to help increase people's cognitive ability to distinguish between fact and fiction. People can also be supported in finding and evaluating the origin of fake news since social media, blog accounts and websites are mostly responsible for creating fake news (Ma et al., 2016). Tandoc Jr et al. (2017) carried out a survey in Singapore to understand how people check when they come across news they were unsure of whether was correct - whether it was the source or they were unsure of the message itself. Respondents informed that they incidentally could stumble upon alternative sources on internet they could compare news with, or they noticed friends on social media sharing the news. Intentionally, they searched on Google, visited targeted web sites or asked friends, families or experts. They also reported that they drew on their own insights and experiences, and evaluated the source and language of information.

Above insights and perspectives support the idea that education and research on fake news are relevant for addressing capacities related to critical mathematics eduucations and democratic citizenship. However, the question of how to increase students' capability in evaluating the trustworthiness of numbers in news is not straight forward of several reasons. I discuss some related challenges, dilemmas and paradoxes, where I draw on literature on the status of quantified knowledge, in particular from theory of science.

THE STATUS OF QUANTIFIED INFORMATION IN SOCIETY

Besides fake news on the presidential elections in 2016, climate change and immigration are topics where accusations of fake news or false information are common. Numbers often accompany media stories on both topics. Numbers have the

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appearance of representing objective facts and plays a key role in politics and in science. Lord Kelvin, whom the temperature metric is named after, has supposedly claimed the following (AZquotes, not dated):

> When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge of it is of a meagre and unsatisfactory kind.

This illustrates the importance quantifications had, and still have, for defining quality in science. Mathematics based argumentation is crucial also in politics, policy and decision making. Porter (1996) described how quantification became a central means in seeking objective justice after the French revolution. It became important to collect population data, as for instance birth mortality, life expectancy and income, but also to standardise metrics so that all farmers could receive the same price for their produce. However, the standardisation showed to introduce some problems. One was that some farmers included a portion of hay together with the grain since the price was still the same. This caused the development of quality categories, which again initiated new problems. Likewise, Porter described how cost-benefit analyses were developed to ensure a just river dam policy for local farmers in the US. These examples illustrated that while the intention of basing decisions on numbers was to obtain an objective and just decision making and a just society, the quantities were concealing injustice. Similarly, Merry (2011) argued that a range of indicators in policymaking increased injustice due to limitations of such quantifications.

Declining trust in science

Recent surveys in the EU and in Norway suggest a concern that trust in science may be decreasing. For instance, almost half of the respondents in the Norwegian survey agreed that research results are often paid by the industry or by the authorities and thereby not trustworthy (RCN 2017a). The president of the Research Council of Norway requested researchers to improve their communication on the complexity of their research and to be explicit about their role when they present their research in the media (RCN 2017b). Three surveys in Europe concluded that most people have trust in science, but knowledge lacks on the reason for distrust (Euroscientist 2017).

Saltelli and Funtowicz (2017) claimed that science is under pressure between two crises, which the researchers themselves are responsible for: people's lack of trust in science and the lack of reproducible results in science for policy. They argued that the lack of trust is due to researchers and experts giving confident advice, for instance within finance, nutrition and medicine, but where the positive effects have been exaggerated, evidence have been in dispute and/or examples of corruption have been demonstrated. The other reason is examples of evidence-based policy that have been a failure because it has relied on research being correct, while the system it represented was too complex for the methods to grasp. They further argued that conflicts on facts in the media, as for instance on climate change, influence people's trust in science. The

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solution to avoid these problems, they claimed, is to find new science practices, in particular to engage citizens in participatory processes (Saltelli & Funtowicz 2017)

CHALLENGES, DILLEMMAS AND PARADOXES

The challenge of separating uncertain, wrong, twisted and fake numbers

The distrust in the media, in science and the Porter's examples suggests that the problem with numbers is not just whether they are fake/fabricated or twisted (misleading on purpose). The complexity of the climate system, including human impacts, will necessarily make predictions about climate uncertain. Global warming is sometimes called fake news, and indeed, there are people who accuse climate scientists for fabricating numbers as part of a conspiracy. In hindsight, the Intergovernmental Panel on Climate Change (IPCC 2013) has admitted that specific assumptions turned out not be correct. You could perhaps say that the resulting numbers were wrong, but because of the complexity, IPCC numbers are bound to be somewhat uncertain, predictions in particular. IPCC (2013) has indeed developed a system for communicating uncertainty and reliability.

It would be fruitful to distinguish uncertain numbers from wrong numbers, which again is something different from twisted and fabricated numbers. Skovsmose (1994) introduced the term *formatting power of mathematics* to denote how mathematics can change our perception of reality, for instance what is considered a social just distribution of benefits. The formatting power is not necessarily good or bad, intentional or non-intentional, which the problems Porter (1996) pointed to indicate. Skovsmose (1994) emphasised that critical reflections on the formatting power is crucial for critical citizenship and democracy. These ideas have been drawn on in mathematics education on climate change (Barwell 2013), and in particular in relation to understanding different facets of uncertainty (Hauge & Barwell 2017).

Fake news and the science crisis announced by Saltelli and Funtowicz (2017) may not go well together. Fake news, personal opinions and sentiments may become more influential if researchers and experts are less trusted. The suggested solution to the distrust - citizen participation in various stages of the research processes and policy making - may give leeway to more fabricated and dishonest information and may seem like a paradox. On the other hand, such inclusive and democratic processes may give more trust to the end product. One challenge is to allow critical inquiry in a way that appreciates uncertain knowledge as still relevant and something different from wrong knowledge or fake news.

The dilemma of accepting values while dismissing fake news

Numbers are fabricated in fake news with the intention to strengthen or alter people's beliefs. The numbers are thus attached to specific values. Also uncertain numbers may be value-loaded when they are applied in politics, policy development or decision-

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making. For instance, since the climate system is complex, the model of the climate system needs to be simplified in some ways. These choices will influence an estimate, an assessment or prediction in some or other way. Due to unknown features, it may be impossible to say how or how much. The influence of choices may strengthen argumentation in favour of a specific value stance or political opinion, and in disfavour of others. An implication is that science and its accompanied numbers is not value-free in complex and controversial issues (Funtowicz & Ravetz 1993). How to deal with associated uncertainties is a value question and in many cases a political decision.

Indeed, politics is about values: What is a good society? What should be prioritised in politics: climate change, refugees or other societal needs? What and who decides the border of acceptable values? In practice, it may also be challenging to justify a drawn line between acceptable value-loaded numbers and non-acceptable fake numbers.

The paradox of fake news and trust

I will illustrate the paradox of trust with an example about immigration. There is a video on YouTube arguing how Muslims will outnumber Christians in Europe with the present immigration policy (friendofmuslim 2009). The video presents a range of numbers on Muslim immigration, how these numbers increased over the years, mixed with expected fertility rates. The numbers, presented as facts with documentation, and a map of Europe constitute the main features of the video, but are accompanied by dark colours and music appealing to people's fear. Three years later, BBC produced a similar video with counter arguments including numbers and a map (BBC 2012). The BBC video commented on each of the numbers in the first video, correcting or questioning numbers, sources and assumptions. Was the first video fake news? It meets the criteria of appealing to people's sentiments, and part of the information BBC argues is wrong, so the question is whether BBC can be trusted and whether Gelfert's (2018) criterion is satisfied - that the numbers are twisted on purpose with an intention to mislead people. Although this seems to be the case, it is precarious to make claims about other people's intentions. Numbers can simply be wrong. It should also be noted that the comments on YouTube following the BBC video, include numerous critical statements and arguments about the BBC numbers and why BBC should not be trusted.

This suggests that people tend to choose what sources they consider trustworthy. Subsequently, some people will not be convinced by counter facts or by certain sources labelled as trustworthy. So, when for instance researchers claim they know what news are fake, as for instance Jang et al. (2018) who checked the reality in their selected news through fact-checking websites, it may not convince everyone that these websites govern the truth. In the end, it is about who/what we trust, because we cannot deconstruct all information and evaluate every detail. For instance, climate science is too advanced for me to fully evaluate its quality. At best, I can evaluate how climate science is presented in light of information from other sources on quality, uncertainty, assumptions and principles on how, for instance, the IPCC expert panel works in constructing an assessment. A central aspect of this is what sources and pieces of information I trust.

SUGGESTIONS FOR EDUCATION DEALING WITH FAKE NEWS

The aims of teaching about fake news and how to deal with fake news is to promote democratic values and capacities. Now follow some ideas and suggestions on what teaching and learning might be. These are also suggestions for further research, as there is limited research on students working with fake news in mathematics classrooms.

- *Learning about fake news because it has become a common term*. This includes definitions of fake news and facilitating an awareness of how social media distribute fake news and how fake news is different from fabricated news without manipulative elements, unintentional wrong news and news with limited perspectives. Students can find examples from the media, reflect on presented numbers, discuss whether it is possible to judge whether the story is fake news or not and discuss problems with fake news.
- *Learning that applied mathematics is seldom about correct or incorrect answers*. Students can work with mathematical models to experience that choices in their modelling affect model results. Empirical examples from critical mathematics education include modelling a fair distribution of child support to families (Skovsmose 1994) and fair distribution of seeds to poor farmers (Barbosa 2006). Discussing statistics, numbers and graphs in the media has also been promoted as valuable for experiencing uncertain numbers (Watson 2004; Hauge & Barwell 2017; Hauge et al. 2015). The aim is to foster the ability to understand the difference between uncertain, wrong and fake numbers.
- *Learning mathematical concepts and mathematics based argumentation.*

  > In the East, it could be the COLDEST New Year's Eve on record. Perhaps we could use a little bit of that good old Global Warming that our Country, but not other countries, was going to pay TRILLIONS OF DOLLARS to protect against. Bundle up! (Trump, 2017).

  Natural variation is an important concept for understanding climate change, and differentiating between uncertainty concepts would be useful (Hauge 2016a). Other suggested concepts are risk (Barwell 2013; Hauge 2016b) and linear regression together with its limitations, since it is a very common tool in predictions. Students can experience building mathematics based argumentation themselves, as for instance developing and discussing mathematics based arguments related to racial justice and mortgage (Gutstein 2013).
- *Learning and discussing the value of democracy.* This includes the value of free elections, institutions that ensure that laws apply equally to all citizens and the protection of human rights, where freedom of speech is a human right, but also a benefit to society through critical and active participation in society.

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- *Allowing discussions on controversial issues.* Hess and McAvoy (2015) found that discussing controversial political issues in the classroom had several benefits: the students became more confident, knowledgeable and interested in politics, and were more likely to discuss with people who disagreed. Of course, this should be done with respect and proper argumentation. I also suggest that this paper's challenges, dilemmas and paradoxes are discussed. This requires:
- *Teaching and learning with real-life topics*. Real-life examples can easily be found on internet, for instance the immigration video described above. The examples should be on topics students care about, and where they can demonstrate agency through drawing on knowledge and experience outside school. Topics may require a cooperation across school subjects, since mathematics, social science, language and others might be relevant.
- *Source criticism* is of course central. The topic is included in school curricula in Norway, but could probably be expanded to match the media development.
- *Learning about argumentation structures.* There are several argumentation frameworks students can work with to raise awareness and help evaluate the strength and reliability of mathematics based argumentation. The Toulmin (2003) model is one, which is a logical structure characterising the function of different statements in argumentation. My colleague Kjersti Breivega applied the Toulmin model during a course for teachers on how to work with the refugee crisis, immigration and conspiracies in Norwegian language and mathematics classes. Her examples were statements from the friendofmuslim (2009) video described above. Wood (2000) provided another framework, consisting of seven types of arguments. The three most relevant are factual, political and ethical arguments. The two latter are value based and lead to conclusions on how to act, and can be supported by factual arguments. He claims that value stances are not often explicitly articulated in political and ethical arguments because they are more efficient when they are taken for granted. Twitters are very short with little space for supporting information. Making values explicit in media discussions could therefore be an exercise in classrooms in search for the root of disagreements. Students can work with fake news examples to learn to recognise manipulative features and understand underlying intentions. Another key aspect is to look for how uncertainty in numbers is presented, if at all: through statistical measures or discussing limitations and drawbacks.

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COLLECTIVE CONSTRUCTION OF HETEROGLOSSIA IN A KOREAN PRIMARY SCHOOL GEOMETRY LESSON

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In a Korean primary school geometry lesson, the students and the teacher collaborate to manage the tension between formal and informal ways of speaking. We trace this collaboration using Barwell's development of Bakhtinian standardising (centripetal) and diversifying (centrifugal) discourses. The students and the teacher used four types of linguistic and expressive resources to influence the heteroglossic quality of the lesson: the presence or absence of honorific grammatical markers, exuberant metaphors that personified geometrical shapes, formal curricular terminology vs. informal speech, and gesture and other forms of bodily motion. The class used these features to co-construct the shifting boundary between centripetal and centrifugal speech.

INTRODUCTION

Recent Korean mathematics curriculum recommends that classroom lessons incorporate students' lived experiences. This reflects a shift from a transmission conception of learning to one that is collective, subjective, grounded in activity and emotion, and multi-modal (Radford, Schubring, & Seeger, 2011). However, curriculum and professional development offer teachers little guidance on how to draw out their students' experiences and incorporate them into math lessons. Teaching and learning are "part of a same process, connected by interrelated processes of signifying and meaning-making" (Radford et al., 2011, p. 149). Teachers often experience this tension as a "dilemma of mediation" in which they must balance formal mathematical expression with students' diverse and informal ways of expressing understanding (Adler, 2001; Barwell & Pimm, 2016). Given the recent Korean curricular reforms, the dilemma of mediation is likely to figure prominently in classroom discourse.

To better understand how new teaching recommendations influence classroom discourse, we use Barwell's approach to Bakhtin's theory of language, that classroom speech shifts between centripetal, standardising discourses and centrifugal, diversifying ones (Barwell, 2014; Bakhtin, 1981). Because these ways of speaking are posed as opposites, it raises the research question of how classroom discourse shifts from one to the other, who manages the shift, and what elements of language are associated with the shift. A classroom discourse analysis of a full-class discussion in a third grade geometry lesson in South Korea yielded two observations about these research questions. First, we found that the teacher is not fully in control of shifts between centripetal and centrifugal ways of speaking in full-class discussions. The teacher and the students collaborate to manage this tension. Second, we identified four linguistic and expressive modes involved in this process: honorific grammatical markers vs. plain language, metaphors that personify geometrical shapes, formal vs.

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informal speech, and gesture and other forms of bodily motion. This paper describes the ways in which a teacher and her students collaborate, using a variety of linguistic devices, to shift between centripetal and centrifugal ways of speaking during productive mathematical conversations.

THEORETICAL FRAMEWORK

Barwell (2014) describes a tension in mathematics classrooms between unitary and diversifying languages using Bakhtin's theory (1981). Unitary language "gives expression to forces working toward concrete verbal and ideological unification and centralisation" (Bakhtin, 1981, p. 271 in Barwell, 2014, p. 913). These institutionally-authorised language forms are always interlaid with less dominant voices: regional language forms, alternative political opinions, informal ways of phrasing ideas, and so on. The unifying, centripetal forces include any socio-ideological context which forces language use into certain forms; in contrast, the centrifugal forces refer to peripheral languages, the languages diverse groups of people use within their community, or informal language, a socially-constructed formal language. Heteroglossia refers to the multidimensional and multi-layered characteristics of language (Bakhtin, 1981; Barwell, 2014).

This language theory suggests that unitary language is socially constructed and that various socio-ideological dimensions are involved in this process, so that languages are ever-changing in use (Bakhtin, 1981). In this sense, it is important to identify the formal features that allow speech to be regarded as either centripetal or centrifugal. Drawing on Barwell's (2014) analysis of language use and tensions arising in mathematics classrooms, this study considers the language forms and vocabulary promoted by the curriculum as a benchmark of formal language and as one of the centripetal forces. Following examples in Setati (2001), we consider formal language to be words and phrases emphasised in local curriculum documents, like "shape" ("도형", "모양") and "right angle," because the curriculum guidebook directs teachers to use these words and encourage students to learn and get used to them.

Because this study was conducted in a Korean language classroom, it is important to note that the Korean language uses several types of optional grammatical markers on nouns, verbs, and adjectives that express formality and relative social status of speakers. In this paper, we use the term "honorific" to refer collectively to several types of deferential markers presented in the Korean language transcript. The choice of honorific or plain form in Korean language is explained by a variety of contextual and interactional dimensions, and not by relative social status alone (Park, 2012). Based on the school context and policy which will be explained in the next section, we regard the honorific form as a centripetal language force.

Informal language refers to phrasing that could occur in many settings, not just mathematical ones. This includes spontaneous metaphors. The use of metaphors can be pedagogically useful to both students and teachers during a mathematics class, regarding the expression of a mathematical image and reason (Abrahamson, Gutiérrez,

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& Baddorf, 2012). Metaphors and gesture are similar in this sense. Alibali and Nathan (2012) found that students often expressed new conceptual knowledge in gestures before expressing themselves verbally. Representational gestures can indicate both perceptions and habitual action (Alibali & Nathan, 2012) and seem to be the most prominent in the following selections.

METHODS

Because this study is concerned with the teacher's and students' collaborative construction of meaning, a classroom discourse analysis provides a useful perspective to examine language in use in the classroom. Classroom discourse research approaches help to study diverse language resources that students and teachers bring to classroom interactions (Gee, 2011; Rymes, 2015). The data collection occurred from May 2017 to July 2017, and the first author observed the math class two or three times a week, for 11 weeks total and 26 lessons. The first author also kept a field notebook during the observation, focusing on the atmosphere of the classroom conversations, student involvement, and drawings and writing on the board during class activities.

The project was conducted in a classroom of 19 third graders and their homeroom teacher in a primary school in one of the biggest cities in South Korea. There were five students who acquired Korean as a second language and whose first language was Chinese, but they spoke fluent Korean. The teacher had been teaching for five years as a tenured public primary school teacher. She was interested in making her classroom a humanising space where the students respected each other and could be comfortable sharing their thoughts and different identities, as well as expressing themselves. At the time of the classroom observation, it was the third month of the school year, since a school year starts in March in Korea. Although the Korean language culture does not expect students to use honorific language with their classmates, this school encourages students and teachers to use honorific language with each other. Teachers and parents in the school community agreed that using honorific language in the school would generate a respectful environment and culture.

In South Korea, the curriculum is implemented at a national level. Although there is no standardised test in this school district, formal and informal assessments are implemented based on the curriculum. In the third grade geometry chapter, students learn the basics of two-dimensional shapes and their rotation based on their geometry lessons from previous years about the basic concepts of two-dimensional shapes such as round, triangular, and quadrangular shapes. This chapter also plays a role in laying the groundwork for learning congruence and symmetry of shapes in the fifth and sixth grades. The objectives of the chapter analysed in this research are for students to understand characteristics of diverse triangles and quadrangles and, in the unit presented in this paper, to predict and check the result of a shape's rotation.

DATA SOURCE AND ANALYSIS

Classroom discourse data sources include (1) the recordings from 26 lessons, as well as a transcription of audio component; (2) field notes from the lesson observations, as

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well as an informal teacher interview before and after each lesson about the lesson plan and teaching; and (3) the textbook, teacher guidebook, and worksheets used in the lesson. The audio components of all of the lessons were transcribed in Korean during the data collection period, and the transcription was checked against the video component for accuracy. The students' and the teacher's gestures or any body movement involved during their interactions are described in brackets. The discussion of this paper relies mostly on the data source of classroom discourse recordings, but we compared other data sources, such as field notes and teacher interviews, to check interpretations.

One of the most notable features of communication in this classroom was the rich metaphors that the students used to describe shapes and their movement. We also noticed that honorific language forms were not strictly maintained during the conversation, despite the recommended school policy and the culture of Korean language use. We looked for additional linguistic modalities that might be involved with centripetal and centrifugal ways of speaking, such as grammatical animacy.

The excerpts below are drawn from a single day's lesson on two–dimensional shapes. The specific day of the lesson was chosen after a preliminary analysis of the four language modalities for the whole of the transcript. As the teacher consistently drew out students' own interpretations of the concepts throughout the lesson, the students more actively disclosed their thoughts. Overall, this day's lesson showed especially rich and active student involvement. The selected excerpts in this paper clearly represent the different modes of language use that governed shifts between centripetal and centrifugal ways of speaking, as befitting the limited space of this paper. We reread the whole transcript to check any counter examples of the selections and analysis.

The selections are transcribed in Korean in order to show grammatical features, like honorific or plain language. In the transcription, bold font indicates words that carry an honorific grammatical marker based on Korean language use; italic font indicates a metaphor or personification or, in one case, a grammatical marker of personification; and underlining indicates the use of formal vocabulary as specified in curriculum documents. These four language varieties were identified and decided upon during the data analysis process, not predetermined during data collection.

RESULTS

During this lesson, the teacher and the students used all of these expressive modalities to collaboratively construct the changing boundary between centripetal and centrifugal language forces. In the selections that follow, we trace the ways in which both students and the teacher used these language forms to ensure that the lesson engaged both mathematics and the students' lively envisioning of the geometrical figure.

In the initial exchanges in the discussion, the teacher asked students to describe a rotation of a shape  into the shape , with the initial shape drawn on the board. In line 1, the teacher questioned Ryeong-a, using a mixed form of honorific language. She used plain form to call the student's name but switched to honorific form to ask a

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question. The combination of honorifics with the academic term “shape” introduced the discussion in a somewhat formal manner. However, as we see in lines 2, 3, 5, and 6, the teacher and the students were not restricted to using honorific language despite the school’s encouragement to do so.

In line 3, the teacher introduced another grammatical form that might have had some influence on the future conversation. The teacher referred to the shape as 얘, a noun form with an animacy grammatical marker, translated as “this person.” Animacy markers typically refer to living things in Korean, and often, mathematical geometrical shapes would be marked differently, as non-animate. While we cannot explain precisely why the conversation followed its subsequent trajectory, after line 3, the students introduced the most remarkable aspect of the discussion—the abundant metaphors of personification that began with the sense that the rotated shape was sleeping. We used italics to mark animacy in line 3 as a type of personification, but in the remaining cases, we used italics to note direct metaphors of personification.

1 Teacher: Let’s see...Ryeong-a, what <u>shape</u> **is made**?
교사: 자…령아야 어떤 모양이 **됩니까**?

2 Ryeong-a: Well…These two are moving to here…that part at the bottom…*[Pointing at the two arms of the shape* F *on the board with her thumb and index finger, and then rotating right, clockwise, to indicate .]*
령아: 음…이쪽 두개는 이쪽(오른쪽)으로 가고…밑에 있는건…

3 Teacher: Ah, *this person* moves here [*Pointing at the picture* F *on the board with finger]*, and *this person* comes here (), right?
교사: 아... *얘가* 이렇게 가고, *얘*가..이렇게 온다고?

4 Hyo-suk: …**has to** *sleep*.
효석: *자야* **돼요.**

5 Teacher: Ah, is *sleeping F* <u>shape</u> drawn?
교사: 아, *잠자는 에프* <u>모양이</u> 나온다고?

6 Hyo-in: This way. *[With her arms raised up.]*
효인: 이렇게.

7 Teacher: In this way? *[Smiling while mirroring Hyo-in’s gesture, two arms raised up.]*
교사: 이렇게 한다고?

The use of honorifics and academic language at the beginning of the discussion represents a centripetal moment, oriented more towards standardised forms of discourse recommended at the school. In subsequent lines, though, all participants introduced, confirmed, and elaborated centrifugal, diversifying discourse moves. Hyo-suk extended the teacher’s use of animacy by personifying the shape as a human being, who can sleep, and the teacher accepted and revoiced this metaphor in line 5. Ryeong-a and Hyo-suk both offered verbal descriptions and gestures that the teacher revoiced in a question form and mirrored in lines 3 and 7. This is a centrifugal language feature

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because students expressed new knowledge in gestures before they expressed it using precise speech (Alibali & Nathan, 2012).

Still, the shift in this selection from centripetal to centrifugal voicing was not complete considering the curriculum vocabulary which the students were expected to learn and practice. In line 5, even as the teacher confirms the “sleeping” metaphor with interrogative form, instead of correcting the metaphor, she added the formal academic term “shape” to the description. This description aligns with the teacher’s dilemma of when and how to lead learners from their informal talk to formal spoken mathematics (Adler, 2001; Setati, 2001). These examples demonstrate the delicacy in which the teacher and students worked together to create a mathematical sense-making process that relied on both diversifying and unitary language.

In the following selection, a little later in the lesson, the teacher pressed students to become more precise in their mathematical descriptions in a discussion of congruency and rotation based on the language use presented and suggested in the curriculum. Students had been using single-word responses, such as “turning” and “left.” In line 26, the teacher accepted a student’s response of “left” and used it to induce a more detailed and academic answer.

26 Teacher: Right, and how many right angles on the left?
교사: 그렇지, 왼쪽으로 몇 직각?

27 Students: Three right angles!
학생들: 세직각!

28 Teacher: Correct! Three right angles on the left…?
교사: 그렇지! 왼쪽으로 이거 세직각…?

In this excerpt, the teacher particularly shifted towards centripetal discourse forms as the students worked towards more precise descriptions. This selection represents a tendency that occurs in other moments in the discussion. The teacher tended to use plain, non-honorific form when the audiences of the utterances were individual students and when she talked about the shape. Similarly, when students answered and talked about the shape, they used plain form with a non-honorific noun form ending. Just after this excerpt, Hyo-in responded with “a right angle on the right,” which is another correct but more advanced answer since it requires the skill to predict a position of the shape with the opposite directional rotation. This shows that the teacher and students could both introduce centripetal discourse without fully invoking another standardising speech form, honorific markers. In the next excerpt, figurative language is featured prominently, especially personification.

32 Teacher: The below? What is that?
교사: 밑으로? 뭐가 밑으로?

33 Hyo-in: It’s under *narcosis* and *the head* goes below. *[Indicating with her head moving downwards.]*
효인: *수면상태*인데 *머리*가 이렇게 밑으로 가.

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34 Teacher: Ah, *the head* goes down this way. *[Referencing the initial shape drawn on the board to describe the result of the rotation,]*

교사: 아, *머리*가 밑으로 가고.

35 Ji-woo: *Headstand* upside down [compared] to the previous one. ()

지우: *물구나무*, 아까 전꺼랑 반대로.

In line 33, Hyo-in added information using the personification of the shape with a representational gesture (Alibali & Nathan, 2012). By personifying the shape, she was able to refer to each angle and part of the shape as a body part, and this seemed to help her explain the position in more detail. Furthermore, the teacher accepted Hyo-in's personification by revoicing (line 34), and Ji-woo named the position "headstand," also referring to the previous position of the shape (line 35). Whereas selections 1 and 2 can be seen as collaborative shifts from centripetal to centrifugal and back to centripetal voicing, Ji-woo's participation in line 35 places these two tendencies in dynamic tension with each other. Ji-woo's "headstand" metaphor personifies a rotational movement that is a conjectural explanation of the mathematical activity.

Line 40 demonstrates the teacher's tendency to use honorific markers when addressing the class as a whole or referring to the collective work of the class. In line 40, she started her evaluative utterance with honorific form but switched to plain form when she invited students into the problem-solving process.

40 Teacher: You**'re correct.** The previous one **is correct**. You did a good job. This? What can we do with this? *[Pointing to the shape on the board.]*

교사: **맞네요,** 아까 전에 생각 한 것 **맞네요**. 잘 찾네. 이거를? 이거를 어떻게 하라고?

41 Hyo-in: This way, flip over. *[Flipping over her right hand]*

효인: 이렇게, 뒤집으라고.

42 Teacher: This way? How…? *[Mirroring Hyo-in's flipping over gesture with a transparent film on the board where the shape was drawn]*

교사: 이렇게? 어떻게…?

The teacher's honorific form choice is consistent with Park (2012)'s finding that the use of a more formal speech style indicates the teacher's awareness and recognition of the audience and her institutional identity as a teacher. In line 40, the teacher plays an institutional role by evaluating the students' work but switches to informal language to introduce a new task, signaling the teacher's less formal and friendlier status (Park, 2012). When Hyo-in responded to the question with a gesture (line 41), she flipped her hand while mentioning "flipping," and the teacher revoiced and mirrored this action, but she requested additional information about "how" to flip the shape. In this subtle moment, the teacher accepts the student's centrifugal form of participation but introduces some centripetal force into the conversation.

52 Hyo-in: It (he/she) **is *sleeping*** like chopsticks.

효인: 젓가락 처럼 하고 **수면을 하고 있어요.**

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53 Yeon-ji: **Teacher,** the *head* and *arms* **seem** *aching*...here...because *this person sleeps* this way. *[Stretching her arms and positioning her head downward at the same time to simulate shape]*

연지: **선생님,** *머리*하고 *팔*이 *아플 것* **같은데요...**여기...이렇게 하고 *자니까.*

54 Teacher: Ah, here. And are these *eyes*, *nose*, and *mouth*? *[Touching points on the shape.]*

교사: 아...여기... 여기가 *눈코입*이야?

55 Students: **Yes.**

학생들: **네.**

In lines 52 to 55, metaphors of personification occur abundantly. Although the teacher had used plain form in her comments just before line 52, the students introduced honorific markers, perhaps as a bid to gain the conversational floor and to introduce new elaborations of the metaphor of a sleeping, head-standing shape. This is consistent with Hyo-suk's initial honorific offering of the "sleeping" metaphor in line 3, as well. Yeon-ji used honorifics and gesture to add an emotional dimension of "aching" to the personified shape (line 53). The teacher collaborated with the students by suggesting additional attributes to the personified shape. The teacher's gestures in line 54 contribute some precision, and perhaps slight centripetal movement, by identifying key reference points on the shape. Still, this phase of the discussion seems mostly to represent centrifugal movements towards elaboration of the metaphor of personification through the reversal of the teacher and the students' roles because the teacher offers a conjecture that the students approve (lines 54-55).

The final excerpt shows the conversation as it approached the end of the lesson.

60 Jeonghoon: **Teacher,** *that person* **seems not** to *breathe* well.

정훈: **선생님,** 왠지 *숨을 못 쉴 것* **같아요.**

61 Teacher: *This person* has *eyes*, a *nose*, and a *mouth*, *standing right* at the first moment, then this way, doing a *headstand* and then *lying* beside. Is it okay? *[Displaying and rotating the shape on the board with a transparent film.]*

교사: 자 *얘*가 *눈코입*이 있는...처음에는 똑바로 *서있다가*, 이렇게 했다가, *물구나무 섰다가*...그 다음엔 옆으로 *누웠어*. 이건 괜찮아?

In line 60, Jeong-hoon echoed Yeon-ji's technique in line 53 by using honorifics to continue the emotional and empathetic elaboration of the metaphor of the personified shape, describing it as breathing poorly. Discursively, the teacher's story in line 61 is rather centrifugal because she summarised several of the previous metaphors—her own and other students'—into a story about the shape's rotation. The teacher's action, using a transparent film to check the orientation of different versions of the shape, maintains some centripetal character because the act of verification represents a standardising disciplinary value. Here, as in selection 3, the teacher and students co-construct a moment in which centripetal and centrifugal sense-making are held in tension with each other, using multiple expressive features of language and motion.

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DISCUSSION

This conversation tended to emphasise diversifying, heteroglossic discourses, but it included many reversals of the expected roles. The teacher supported, revoiced, and mimicked students' gestures, informal speech, and metaphors of personification. Students sometimes introduced the honorific markers that school policy recommended, not only to show respect and deference, but also to introduce elaborations of a metaphor that might not have seemed to be mathematical on the surface. In fact, interviews with the teacher revealed that she was experiencing the dilemma of mediation during this lesson on congruency of shapes. During the data collection, she often worried whether her lessons were mathematical enough, since classroom conversations often consisted of abundant metaphors rather than mathematical language.

In this study, both the teacher and the students frequently introduced or otherwise supported centrifugal ways of speaking. Although we would expect the teacher to introduce centripetal speech into the conversation, at times the students were responsible for this more standardising speech. To describe this collaboration, we attend to several linguistic and discursive features. Throughout the lesson, (1) the students and the teacher shifted between honorifics and plain language depending on their intentions, the subject of a sentence, and the audience of a comment. (2) They also used formal, academic terms drawn from curriculum, informal expressions including metaphors, and gestures to express mathematical concepts. Finally, (3) another type of informal speech that occurs in our selections is spontaneous metaphors that treat a geometrical shape as if it were a sleeping, moving, feeling person.

This study illustrates the ways in which the language use of the students and the teacher in a math class crosses the boundary between centripetal and centrifugal forces. The previous studies of math classroom discourse mainly concern language use in multilingual classrooms (Adler, 2001; Barwell, 2014; Setati, 2001). This study demonstrates that research on heteroglossia in a classroom where the teacher only uses one language during instruction, but some of the students are multilingual, is also relevant. Additionally, it contributes to the growing body of discourse research on mathematical classroom learning in a non-European language.

CONCLUSION

Theoretically, through this multimodal analysis which attends to honorific markers, informal speech, elaborate metaphors, and gesture, we can understand the ways in which the teacher and students colluded to fulfil in complex and shifting ways the tension between mathematical and school policy expectations, and students' joy in learning mathematics. This study shows that, despite the limited space for students' authority in the classroom, the students still bring their own interpretations and experiences while taking up various language modes in this process. Considering that the teacher still has the most authority in directing the conversation, this calls for additional resources and support for teachers to learn to mediate discourse while acknowledging students' ways of expressing their knowledge and thoughts.

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HOW MATHEMATICAL HABITUS SHAPES SPATIAL REASONING RESPONSES

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This paper reports on one aspect of a three-year spatial reasoning project. Three schools from diverse social and geographical locations engaged in a set of spatially-challenging tasks. The responses were distinctly different across sites. We frame these observed differences as influenced by the habitus of the learners as a function of their socio-geographical location. The responses provide educators with a way of framing individual differences within a positive assessment framework, rather than a deficit model that could be derived from expected school mathematics responses. We argue that such reckoning fails to acknowledge different ways of seeing and being.

BACKGROUND TO THE PROJECT

For almost 50 years now it has been recognized that achievement in mathematics is influenced by factors such as gender and socio-economic status. Student equity groups in Australia including Indigenous students, rural and remote students, and students from low socioeconomic backgrounds, consistently perform lower than their more advantaged peers on international testing such as PISA (Thomson, De Bortoli, & Underwood, 2016). As we have mentioned elsewhere (Lowrie & Jorgensen, 2018) there have been many targeted intervention programs aimed at decreasing this achievement gap, most of which focus on the number strand of mathematics. Despite these intervention programs, the achievement gap still persists, suggesting a broader approach to intervention programs is needed.

Recognizing the need for a new approach to intervention programs, the current project focuses on spatial reasoning as a targeted intervention for students' mathematics performance. This approach is based on research that suggests there is a strong association between spatial reasoning and the fields of mathematics (Logan & Lowrie, 2017) and STEM (e.g. Uttal, Miller, & Newcombe, 2013; Wai, Lubinski, & Benbow, 2009). Spatial reasoning, or more broadly spatial thinking, involves an awareness of space, the way we represent and interpret spatial information and then make decisions (National Research Council, 2006). Some examples of this in the mathematics curriculum include thinking about distance, coordinates and dimension; encoding and decoding diagrams such as maps and graphics; orientating ourselves in new environments; and reading maps. Targeted spatial reasoning programs in primary schools have shown improvement in both spatial reasoning and mathematics (Lowrie, Logan, & Ramful, 2017).

This research project is being undertaken in four phases, focusing on student groups who are identified as being disadvantaged in mathematics (students from rural and remote locations, Indigenous students, and low SES students) as well as with a comparison student group in an urban high SES school. A key point of difference in this project is the recognition that students have a diverse range of skills in spatial

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reasoning that are influenced by their experiences in and out of school. Recognising and drawing on these diverse skills are crucial to developing students' school mathematics skills. As such, the early phases of this project focused on gaining an understanding of students' in and out of school spatial reasoning skills. A tailored spatial reasoning teaching program that draws on the unique skills of each student group will be developed for teachers to implement in their classrooms. The influence on students' spatial reasoning skills, and mathematics achievement will be measured before and after the training program is completed. It is the early phases that we report on in this paper.

SHAPING A SPATIAL HABITUS

In framing the emerging results, we draw on the work of Pierre Bourdieu to better understand the responses. For Bourdieu (1990), the habitus is a set of transposable dispositions that can be the embodiment of culture. The habitus shapes the ways of viewing and acting in the social world but also the ways in which the social world can be shaped and transformed. It is a dynamic process and is not, as some have argued (Giroux, 1983) as being fixed and unchangeable. The habitus can be reshaped by experiences. For our project, the concept of habitus is invaluable as we try to make sense of our data.

The habitus with which learners enter the mathematics classroom has been shaped by their familiar experiences. Some students come to mathematics classrooms with experiences that align with the practices of school mathematics and are better able to engage with the spoken and unspoken rules of the culture of the mathematics classroom. As a result, these learners are more likely to be seen as successful learners of mathematics, largely due to their familiarity of the practices of mathematics than some innate ability per se. Consider the middle-class English-speaker whose language practices align with the language and discursive practices of school mathematics. For these learners, coming into the mathematics classroom is an easy transition. In contrast, for Indigenous learners whose counting system may be one, two, three, big mob or for those Indigenous learners whose home language may be Pitjantjatjara, for example, there is no comparative terms and much of quantity is signified by intonation. For example, in talking about relative distance, the speaker may say "It is a long way" or if the distance is much further it is signified by "It is a looooong way" where there is greater intonation on the term *long*. While providing directions, the speaker would also engage with gestures by tilting their head to signify direction while using their head to signify the length of the distance as well. For these students, there is a greater disparity between home and school so there is greater need for a reconstituting the home habitus to the school mathematics habitus if the learner is to be seen as a successful learner of school mathematics. One way that this reconstitution occurs is through the development of a two-ways learning so that learners can walk in both worlds – their home world/culture alongside the school/western world/culture.

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In coming to understand our data, we have used the concept of habitus to consider the responses being made by the learners. In the spatial tasks, we recognise that the primary (spatial) habitus of the learners is shaped by their out-of-school experiences. In rural and remote contexts, the geography is remarkably different from urban/city contexts. Navigating around these vastly different geographies requires different skill sets and ways of noticing. However, some of these skill sets are not part of the recognised school curriculum and there has been a propensity to dismiss other ways of knowing when they do not align with the hegemonic knowledge represented in and through school practices.

Our intent in this project is to explore ways of knowing spatially and what differences, if any, do people have in terms of their ways of doing, knowing and noticing spatial phenomenon.

SPATIAL REASONING TASK

The larger project has drawn on three types of spatial reasoning. The aim of these tasks was to gain an understanding of the spatial knowledges of different student equity groups. There are many spatial tests that are designed to understand students' spatial skills, however, spatial skills are also engaged with, and used, in everyday environments (Hegarty, Montello, Richardson, Ishikawa, & Lovelace, 2006). With this in mind, the aim of the activities was to gain an understanding of students' day-to-day use of spatial skills.

The tasks focused on understanding the strategies that students used to complete the spatial activities, the spatial information they drew on, and the different ways they represented spatial information. The tasks were designed to be open ended to enable students to demonstrate their differing approaches without restriction. The tasks were also designed to relate to a 'real life' scenario or to be contextually relevant, rather than a school-based testing situation, as a way of exploring if context influences students' spatial skills.

The three tasks were based on three constructs of spatial reasoning, spatial orientation, spatial visualization, and mental rotation. In this paper we focus on the findings from the spatial orientation activity. Spatial orientation includes two main elements. The first element is perspective taking. This involves using your position in space as a point of reference for considering the location of other objects, such as whether the object may be beside you, or above or below you (Ramful, Lowrie, & Logan, 2017) and how those objects might look from different angles and perspectives (Hegarty & Waller, 2005). The second element is spatial navigation and wayfinding. This involves perceiving spatial information from the environment around us, generating and keeping spatial representations in memory, and using and manipulating these representations to navigate environments (Wolbers & Hegarty, 2010). This can happen in real life or with external representations such as maps. These are all key skills used when thinking about mapping tasks.

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METHOD

Data were drawn from three distinct sites, namely: a remote Indigenous school; a rural school; and an inner-city multicultural school. These three schools are quite different in terms of their social and geographical dimensions. It is not our intent to ‘compare and contrast’ the schools. By contrast, we are concerned with emerging results that provide us with an understanding of how spatial geography influences spatial reasoning. Specifically, we explore the application of Bourdieu’s notion of habitus to frame the responses and to create a new understanding of ‘spatial habitus’ in which to embed our emerging findings.

Data were collection from participants in Years 3 and 5 (8-9 year olds and 10-11 year olds respectively). The numbers of students from any school varies depending on the size of the classroom, ranging from 5-25 students for any of the year cohorts. Where numbers have been small at a school, we have included a second school of the same demographics to increase the number of respondents so that a more valid understanding of learners’ ways of spatially thinking, representing and being can be developed.

Learners were asked to complete the tasks then explain how they get from home-school. The depth of data varied considerably depending on the learners, their confidence to speak with the researchers and their fluency in English language. We provided some students with prompts such as ‘can you tell me about x feature’ to encourage them to describe their route. Student work samples were photographed and stored digitally. Video recordings were taken of the students as they worked through the tasks. The latter has been a valuable tool in documenting the processes students undertake as they work through some of the tasks. For the data presented in this paper, we draw on the maps created by the students and their explanation of how they go to and from school.

The spatial orientation task involved students visually representing their path from home to school, and school to home, then describing verbally their path from home to school. Depending on the learners’ familiarity with representation, some scaffolding was provided to enable the learners’ entry into the task. Simple prompts, such as draw your house first, where is the school, what do you see as you go on your way to school, were used for some learners who initially struggled to draw a map. Preliminary analysis of the representations identified some unique features of the maps drawn by students in each school.

INITIAL OBSERVATIONS

Remote Indigenous School

A key focus of the remote Indigenous students’ maps was community. Students drew meaningful points in their community with their path from home to school/school to home being drawn around this. Students focused on their relatives’ homes when drawing the map, and did not always draw roads, instead just a line around the community to indicate their path. These maps were also mostly arranged around the

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page, not in a linear way. They tended to take a global view of their environment, not just focus on the route.

What was of even greater interest to us was that when these maps were compared with Google maps of the community, there was a great synergy in the representations. This suggests to us that there was an acute awareness of the geographical layout of the community, including scale and representation. While not entirely accurate, it is notable that the markers (family houses, school, store, health office, power station, etc.) were drawn on the map. This suggests that there are important markers in the learners' lives and life in community that they chose to include in their representations.

An interesting feature of a small number of these maps was the way they represented the roads in the community. While the community did not have any bitumen roads with marked lines, a few students drew the 'highway' with dotted lines. For us, this seemed to show that they were aware of (some) Western mapping conventions and had incorporated these into their representations.

Rural School

There were two noticeable types of maps the rural students drew, with the variation appearing to be due to how close the students lived to the school. Those that lived close to the school focused on drawing a large amount of their community, not just their path to and from school with familiar landmarks also identified such as the teacher's house, their friends' houses, and the main street shops. Those that lived further from town drew a map where their path was often represented by a line with features along the way identified such as their mailbox, neighbour's house, friend's house, railway, and in one case, even a sign with the distance to the nearest main centre. Some students also opted to put their map on two pages as they felt it did not fit on one page. Students in the rural community, especially those that live in town, were aware of features of their community beyond the path they take to and from school. There were similarities between the Remote Indigenous students' maps and the rural students' maps in that both groups showed more awareness of their surroundings. Natural landmarks were more prominent in the rural students' maps.

Urban Multi-Cultural School

Students in the urban school focused on the path to and from school, then included features they see directly on their way. They tended to be from an egocentric point of view, focusing only on their route. So for example, they identified houses on either side of the road, or the shops, or traffic lights. Most identified what was in their immediate path, however some did identify the bigger picture outside their direct path. Most of the houses were not identifiable as familiar houses to the students. The verbal descriptions of many of the year five students involved a lot of directional language (left, right, straight) with the teacher identifying that they had focused on this recently in class.

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The urban learners focused on the markers along the route with minimal interaction beyond the route. As such, their maps were more myopic than the learners from more expansive geographical locations. We suggest that this might reflect their experiences where they have not really deviated from the route and may have little knowledge of the spaces beyond the route.

SO WHAT DOES THIS MEAN? DEVELOPING AN UNDERSTANDING OF A *SPATIAL HABITUS*

From the data that we have been collected to date, we are attempting to theorise the outcomes as a reflection of the spatial habitus of the learners. Drawing on Bourdieu's theorization of habitus, the geographical experiences of the learners has the potential to create spatial understandings that may be reflected through the responses offered by the participants in this study. Their movements in and through a particular geographical space create opportunities for understandings of space. For learners who live in expansive geographical locations – such as farming regions or remote deserts – the physical location is very different from urban dwellers.

In creating this new understanding of a spatial habitus, we suggest that the lifeworlds of the learners and their physical location in a particular geographical (and cultural) space enables the embodiment of a spatial habitus. This spatial habitus, just as Bourdieu (1990) theorized, is shaped by their experiences and provides a way of seeing and acting in the social, and physical, world. This notion of habitus thus provides a way of interpreting the data that has been collected.

The Influence of Geography

For learners in remote or rural there are vast expanses of land where there are few changes in the geography, subtle differences in the landscapes become poignant in navigating through the landscape. With the absence of manmade constructions, and a clearer view of the landscape, there is a greater potential to develop a holistic understanding since the landscape can be seen. As we observed in the data from the learners living in these expansive physical worlds, their spatial habitus provided them with a holistic understanding of their ways to, and from, school. These learners tended to show a very profound sense of their physical world as represented in the detail of their drawings. In contrast, the spatial habitus of the urban learners was far more myopic and consisted on some land marks but their understandings of their route to and from school was constrained by relative markers such as *left* and *right*.

The Influences of Culture

We are also cognizant that culture may play a considerable role in the construction of the spatial habitus. In considering the lifeworlds of the learners, aspects of their social worlds are likely to influence how they see their physical spaces. The work in the remote Indigenous community has been particularly powerful for us to think about this. In their maps, the students were very particular about whose homes were represented and, in their interviews, talked about the people who lived in these homes. Typically there were family members. The homes were drawn with great care

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and precision. It would seem to us that the family homes were a considerable influence in shaping the experiences of these learners. This suggests that they are influential in developing a spatial habitus due to the significance of family in the lifeworlds of these Indigenous students.

As we have found (Jorgensen & Lowrie, 2018), these maps also have a very accurate scale with the real world. When these maps were compared with a google map of the community, there was a high degree of accuracy in the scale representation. As other research (Watson & Chambers, 1989) has shown, Indigenous Australians develop an acute sense of their country (landscapes), so it is not too surprising to confirm their research. The remote Indigenous students had a very accurate sense of their community and could represent this with a high degree of spatial acuity. This sense of space was not evident in the urban learners' representations or talk. Their sense of space seemed to be far more relational and arbitrary. What is new is to consider this profound sense of space alongside culture.

TOWARDS AN UNDERSTANDING OF A SPATIAL HABITUS

In this paper, we have been playing with the notion of a spatial habitus as a tool to understand the responses we have observed in our data collection. The use of a concept such as spatial habitus will allow us to think about the differences we have observed across our cohorts within a productive framework and one which is not focusing on deficits where such deficits are seen as different from the standard school ways of spatial being. Although Bourdieu did not explicitly write about a spatial habitus, our data suggest that the lifeworlds of our participants appear to shape their ways of seeing and acting in the socio/spatial worlds. As we suggest, the data indicate these worlds may have become internalised to create quite nuanced spatial habitus that are contingent with the spatio-cultural worlds of our learners.

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INCLUSIVE MATHEMATICS EDUCATION IN THE GERMAN-SPEAKING COMMUNITY: THE POLITICS OF THE EMERGENCE OF A RESEARCH FIELD

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Abstract: In the German-speaking countries, the number of publications on inclusive mathematics education has increased severely since the ratification of the UN Convention on the Rights of Persons with Disabilities in 2009. This study is based on a literature survey and reports that inclusive mathematics education is focussing one-sidedly on open learning environments, while special needs of students are seldom taken into consideration. It also addresses the uncritical dogmatisation of inclusion and tolerance of stigmatisation through mathematics education. Eventually, the emergence of inclusive mathematics education as a research field is discussed from a systemic perspective.

Kurzzusammenfassung: Die Anzahl der Veröffentlichungen zu inklusiven Mathematikunterricht in den deutschsprachigen Ländern hat seit der Ratifikation der UN-Behindertenrechtskonvention im Jahre 2009 deutlich zugenommen. Diese Studie basiert auf einem Literaturbericht und legt dar, dass die Mathematikdidaktik einseitig offene Lernumgebungen fokussiert, während besondere Bedürfnisse der Schüler selten in den Blick geraten. Sie diskutiert außerdem die unkritische Dogmatisierung von Inklusion und die Toleranz von Stigmatisierung durch den Mathematikunterricht. Schließlich wird die Entstehung des Inklusionsdiskurses innerhalb der Mathematikdidaktik als Forschungsfeld aus einer systemischen Perspektive diskutiert.

INTRODUCTION

Traditionally, Germany, Austria and Switzerland do not only have highly segregating school systems, usually allocating students to three different tracks of normal schools after the fourth year of schooling, they also operate professionalised networks of special-needs schools for the exclusive education of students with special needs. These schools are usually organised around one or several special needs foci. Germany distinguishes between the foci learning impairment, mental development, emotional and social development, language development, physical and motoric development, hearing impairment, visual impairment, and illnesses. With the ratification of the United Nations Convention on the Rights of Persons with Disabilities in 2009, Germany, and in similar processes also Austria and Switzerland, committed themselves to the provision of an inclusive school system. Thereby, research and educational policy tend to use a wide understanding of inclusion:

> [I]t is imperative to consider the different dimensions of diversity. That includes disability in the sense of the Convention on the Rights of Persons with Disabilities as well as special starting conditions such as language, social background, cultural and religious orientation, gender and special giftedness and talents. (KMK, 2015, p. 1, my translation)

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In the course of the development which started in 2009, mathematics education has seen the establishment of inclusive mathematics education as a field of scholarly inquiry in the German-speaking community. This development will be the object of this study.

This study can first of all be understood as an attempt in 'researching research' (Pais & Valero, 2012). Just as mathematics education originated from an integration of mathematical, pedagogical and psychological theories and was limited to such theoretical perspectives until connections for further disciplines were established (Kilpatrick, 1992), every emerging field runs the risk that the theories originally underlying it 'comprise particular choices in terms of analytic filters that we apply, governed by underlying ideological motivations and trends of which we are not always aware' (Brown, 2008, p. 249). Consequently, research should always be considered a political act which can and should be questioned critically, not only in the micro-cosmos of specific studies or theories, but as a whole. In this vein, this study can be understood as an opportunity to pause and look back on the work that has been done, on its conditions, assumptions, emphases and results, in order to gain orientation for the future. At the same time, the development of inclusive mathematics education in the German-speaking community can be studied as an example of an emerging field within mathematics education. Thereby, I am fully aware that most readers will not be familiar with German traditions in mathematics education. That is why the following analysis will not engage in intra-German discussions but present very general observations which might – to some extend – also apply to other countries and other developments of fields of academic study.

The core of this study is a literature survey with a multi-dimensional categorisation to represent the publications in the field of inclusive mathematics education in German-speaking countries in an assessable form. On that basis, the strengths and weaknesses of the contemporary field will be discussed, especially its over-emphasis of open learning environments and its uncritical stance towards inclusion and stigmatisation. In the end, the discussion returns to the politics of the emergence of the field.

LITERATURE SURVEY

Providing an objective overview of the literature in the field proved to be demanding as it required intensive decision-making as to which publications to consider and which not to consider. As I wanted to make sure that all contributions were published under the influence of the UN convention, I started my literature survey in the year 2011, and had it end in 2017, the last completed year before the preparation of this study. In order to delimit the field, I restricted the study to German publications only, even though a few Germanophone authors also or predominantly published in English. I searched library and research databases as well as Google Scholar for monographs (including doctoral disserations), book chapters and journal articles including the keywords 'inclusion' and 'inclusive' in relation to 'mathematics' in the titles (using the German expressions), and went through all results for references to further publications on inclusive mathematics education. All publications found were analysed for the school

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type in focus (primary or secondary), for the style of publication (research, overviews, best practice, and case reports), and for the focus of the contribution. Every sixth contribution focusses on ‘mathematical giftedness’. However, as this focus has a more than 30-year-old tradition in Germany which initially had little to do with inclusive education, and as it is nearly impossible to determine where inclusive mathematics education begins in this branch, I decided to exclude this focus from the following analysis. Another nearly 7% of all the publications found focus on diagnostics. As these contributions do not present specific diagnostic tools for students with special needs but discuss the application of general tools in inclusive settings, it was likewise hard to demarcate that line of research from general diagnostics, leading me to exclude this focus as well. Finally, I excluded another 5% of all findings as they focus on dyscalculia, a field also much older than the idea of inclusive mathematics education and difficult to distinguish from it. The remaining foci are listed in Table 1 below.

Focus	Re	Ov	BP	CR	abs.	rel.
Open learning environments	10	7	38	-	55	49%
Teacher education for inclusion	5	2	7	1	15	13%
Migration	8	2	-	-	10	9%
Language diversity (w/o migration)	5	5	1		11	10%
Emotional and social development	-	-	-	1	1	1%
Visual impairment	2	1	-	1	4	4%
Hearing impairment	-	-	1	-	1	1%
Learning impairment	1	-	1	1	3	3%
Mental development	2	-	-	-	2	2%
Physical and motoric development	-	-	-	1	1	1%
Other or several foci	-	5	2	1	8	7%
In total	33	22	50	6	111	100%

Table 1: Numbers of occurrences of German publications on inclusive mathematics education in 2011–2017, classified by inclusive focus and by style of publication. (Re – research, Ov – overviews, BP – best practice, CR – case reports)

ONE-SIDED RESEARCH FOCUS

It is striking that 49% of the publications focus on open learning environments. These are collections of problems that touch a common mathematical content and allow for multiple solutions paths and problem-solving. Open learning environments

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> include different levels of difficulties in a natural way, so that it is possible to work on different levels. Learners have options to choose, for example, the ways of solution, the materials, the tasks and the representations. (Scherer & Hähn, 2017, p. 25, my translation)

Indeed, there is an evidence-based consensus that heterogeneous learning groups profit from the possibility of in-class differentiation (e.g., Scherer, 1995). However, an interview study with teachers on the possibility of inclusion of mathematics education revealed that many teachers consider it impossible to teach mathematics inclusively or feel badly prepared for that task (Korff, 2015). Teacher students have been shown to express similar concerns, although their confidence in managing inclusive mathematics education can be raised significantly by appropriate coursework (Korff, 2016). Thereby, teachers and researchers alike are well aware that the inclusion of students with special needs might have them end up with special tasks which are fundamentally different from the task of the regular group, leading to a 'microexclusion' in spite of an achieved 'macroinclusion' (Faustino, Queiroz Moura, Gomes da Silva, Muzinatti, & Skovsmose, 2017). Therefore, the provision of a variety of open learning environments for the use in inclusive mathematics education is an important product of the research community.

Nonetheless, the documented focus on open learning environment also constitutes a fundamental problem. It runs danger of becoming the dominant form in which inclusive mathematics education is thought. Why else would an author refer to open learning environments and then claim that 'inclusion in mathematics education can be made possible with easy means' (Grohmann, 2014, p. 51, my translation)? Inclusion through open learning environments uncritically assumes that inclusion is more or less achieved by allowing various speeds of learning. For example, Andrea Peter-Koop's (2016) proposal that open learning environments should not only allow work on a 'basis level' but should include two degrees of 'support levels' and two degrees of 'expansion levels', each with appropriate tasks and material, documents a tendency to think inclusion one-dimensionally along the ease and speed of learning processes. Admittedly, 70% of the students with diagnosed special needs have been labelled the foci learning impairment, mental development, or emotional and social development (numbers from 2014, Klemm, 2015), and in those cases, it is usually assumed that the intellectual development in mathematics equals that of regular students, albeit delayed (cf. Moser Opitz, 2016). Yet, such a perspective on inclusion does not ask for the specific conditions of learners with special needs, who indeed might experience and approach mathematics very differently. Focussed research on the special needs of students with special needs foci in mathematics education add up to only 11% of the publications on inclusive mathematics education and provide hardly any answers for students with specific special needs. As a consequence, the contents of inclusive mathematics education and the order in which those contents are discussed stay those

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which were chosen for the idealised regular students, eventually merely adapted for the in-coming diversity. To my knowledge, only Klaus Rödler's (2016) approach constitutes an exception in proposing to begin elementary instruction with the introduction of multiplication as a concept that is new and challenging for nearly every student.

For the analysis of the emergence of the field of inclusive mathematics education, it is essential to ask for the reasons of the one-sided dominance of open learning environments in the field. In the Germanophone mathematics education community, open learning environments have a more than 40-year-old research tradition (Häsel-Weide, 2015). The concept is well known among teachers, although not widely established in school. Traditionally, this branch of research advocated the use of open learning environments for all students and had no special focus on inclusion. Still today, some authors legitimise the use of open learning environments 'for all students' in inclusive settings by positive effects of learning outcomes rather than by the wish for inclusive education (e.g., Scherer & Hähn, 2017, p. 25). The mechanisms behind that branch of research can be understood as part of what Sverker Lundin (2012) calls the 'standard critique of mathematics education' (p. 74) in the sense that school practice is constantly criticised, provoking more research, in our case on open learning environments, but eventually remains more or less the same. Formulated polemically, inclusive mathematics education provided an opportunity for scholars to cast their old ideas on open learning environments into new publications, to attract external funds, and to impose their ideas on school with renewed authority. Tellingly, this focus of inclusive mathematics education has the lowest rate of research output with less than a fifth of all publications presenting new insights. The vast majority of contributions on learning environments are best practice reports, proposing that academia already knows enough and only needs to communicate its insights. Even though open learning environments have a lot to offer, they require further research, and still they cannot be the only answer to inclusion.

INCLUSION AS A DOGMA

In the German publications, political directives in favour of inclusion and romantic ideals of all children learning together happily are regularly held as warrant enough to justify inclusive mathematics education and discard any critical considerations or alternative forms of dealing with heterogeneity. Under the 111 publications that lay the basis for this analysis, not one sets out to critically discuss the idea of inclusive mathematics education. Thereby, already the idea of inclusion through open learning environment provokes a wide range of critical questions: *In how far is it generally possible to design a logically-structured course in mathematics in the form of open learning environments? In how far is it even possible to address similar contents at highly differentiated levels without essentially altering the contents?*

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In contrast to the dogmatism with which inclusion is met in mathematics education research, German inclusive pedagogy has witnessed intensives debates concerning the chances and dangers of inclusion. For example, Bernd Ahrbeck (2014) argues that the discussion on inclusion in school is too emotional and normative and not sufficiently based on empirical evidence. Regarding the teaching of students with various special needs by unspecialised teachers in inclusive settings instead of the teaching of students with one special need focus by specialised teachers, Ahrbeck warns: "Special needs education is in danger of lowering its standards, for which a high price will have to be paid, first and foremost by the affected children themselves" (p. 9, my translation). And concerning open learning environments, Jürgen Budde (2015) explains that meeting every learner's individual needs will eventually stand in conflict to mutual work on the same topic. In mathematics education, the central question would not only be in how far inclusion is possible and desirable, but, above all, on the basis of which normative orientation such a question could find an answer in the first place. Eventually, the question of inclusion in mathematics is a (not yet) well-informed political decision, which mathematics educators should not leave to politicians and follow all too willingly but co-organise more actively and critically.

STIGMATISIATION THROUGH INCLUSIVE MATHEMATICS EDUCATION

School, and mathematics education in particular, has the social function of assessing students for later selection and allocation (Kollosche, 2018). If students with special needs do not get marked, they will have to be labelled as 'different' in order to be comparable to other applicants to the job market. Thereby, the definition of the labels and the rights connected to each of them are necessarily arbitrary. For example, the special needs focus in Germany is granted if a learner's IQ scores below 70; with only 1 more point on the IQ score, the same child might be labelled as 'normal'. As different diagnostic methods are applied in the 16 federal states of Germany, the proportion of students with diagnosed special needs varies from as low as 5.4% in one state up to 10.8% in another (Klemm, 2015). To some extent, it is coincidental whether a struggling student is diagnosed a learning impairment or labelled as intellectually impaired. Equally fluent differences are established when deciding where other special needs foci such as hearing impairment begin. However, the consequences for affected students can be severe. They are henceforth branded as handicapped, a stigma that has been shown to have negative effects on self-efficacy, achievement and self-confidence. Admittedly, they may also receive individualised support to ease their learning of mathematics. But if these benefits outweigh the effects of stigmatisation is hard to say and subject to intensive discussions on inclusive education research (e.g., Arishi, Boyle, & Lauchlan, 2017). In any case, voices from inclusive pedagogy are already demanding a decategorisation and positioning themselves against an industry that relies on a market of students who require special care (Frances, 2013).

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Statistical data from Germany reveals that the proportion of students in exclusive education at special needs schools has remained almost constant from 2010 to 2014, while the proportion of students in inclusive education has increased from 1.2% of the student population in 2010 to 2.1% in 2014 (Klemm, 2015). While some scholars celebrate this increase of inclusively educated students from 19% of all students with special needs in 2010 to 31% in 2014, others are astonished by the sudden increase of the proportion of students with special needs. Often, the policy of inclusive education does not mean that less students are education in exclusion at special needs schools, but that more students are diagnosed with special needs. In fact, it may be argued that inclusive education in Germany has yet failed to considerably improve the situation of learners who are still educated in an exclusive system; instead, it has produced additional thousands of learners who will be stigmatised as problems for the education system, receive specialised assessment and equate to more funding or personal support for their teachers.

While mathematics education obviously does play a role in that stigmatisation business, and possibly a central one, given the close neighbourhood to general intelligence which is often attributed to mathematics, German publications on inclusive mathematics education do not address the problem at all. With their one-sided focus on open learning environments and the concomitant ignorance of the challenges of specific special needs in inclusive mathematics education, researchers in mathematics education have played their part in keeping the traditionally excluded students out of inclusive school. At the same time, they have profited from an increased demand for inclusion in regular schools which was produced by the intensified labelling of students. Without a critical position concerning the increase of the number of students who are diagnosed to have special needs, mathematics education is willingly taken part in a development that might not lie in the interest of students and teachers. For example, the literature survey showed that diagnostic tools, which were designed for regular students, are uncritically applied to students with special needs. If such tools are able to inform teachers about the special condition of a learner or rather document the student's deviation from the norm and legitimise stigmatisation, is a delicate question. To take another example, it is established that managing open learning environments requires socio-linguistic and meta-cognitive abilities that are unequally distributed among learners (Kirschner, Sweller, & Clark, 2006; Theule Lubienski, 2000). So, while open learning environments might be a pragmatic solution allowing a large variety of students to learn on a shared topic, it might simultaneously establish new and less visible forms of exclusion along other axes of differences.

In socio-political studies, mathematics education has generally been shown to disadvantage students along ethnicity, gender, migration background, and social class (Jurdak, Vithal, Freitas, Gates, & Kollosche, 2016), thus establishing new lines of

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exclusion. What is more, processes of stigmatisation as ‘unfit for understanding mathematics’, which create forms of exclusion which are predominantly based in the mathematics classroom, have been identified in empirical studies (Kollosche, 2017; Lange, 2009). All in all, it is fair to say that mathematics education leads to discrimination and exclusion, and it is at least surprising that such processes are not discussed in publications on inclusive mathematics education. An exception to the general phenomenon of stigmatisation through mathematics education is the research on dyscalculia which has recently expressed awareness for the fact that problems with basic arithmetic is not a medical issue but a case of failed teaching (Gaidoschik, 2010; Meyerhöfer, 2011).

CONCLUSION

In terms of inclusive mathematics education in the German-speaking countries, it has been shown that the discourse relies heavily on open learning environments, whose potentials are not yet fully understood, but without any doubt limited. This reliance threatens to mask more substantial questions, especially how mathematics and mathematics education can interact with students with specific special needs. Also, the German field will have to face critical questions concerning the potentials and limitations of inclusion in general as well as concerning the role that mathematics education plays in stigmatisation processes.

Returning to the political analysis of the emergence of the field of inclusive mathematics education in the German-speaking countries, it can be argued that the increased political focus on inclusion, which is also expressed by a considerable funding, has not yet led the academic basis to provide insights that allow for a wide inclusion. Instead, it has supported a specific group of colleagues, particularly those from primary school education who had already worked with open learning environments and could quickly – presumably more quickly than colleagues embarking on less prepared tracks – present first ideas and intervention programs. While the contributions on inclusive learning environments are an important piece in the puzzle, this development might have led to a situation in which many scholars are resting on their success rather than promoting critical questions and developing new lines of research. This situation might change drastically if policy makers, alarmed by the stagnation of inclusion rates, decide to take a more critical stance themselves and fund research on inclusive mathematics education more purposefully.

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CORE AND PERIPHERAL PRACTICES AND BELIEFS OF TEACHERS IN THE CONTEXT OF CURRICULUM REFORM

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Abstract:11 Indian in-service mathematics teachers' beliefs and preferred practices about mathematics, its teaching and students have been analysed by triangulating teachers' responses to a questionnaire with those expressed in semi-structured interviews. It was found that reform based practices of using activities to support mathematical explanations and reasoning were added peripherally in teachers' repertoire while the core practices of memorising rules, procedures and their shortcuts remain unchanged in their priority. The role of supporting beliefs strengthened by culturally accepted notions of mathematics teaching is discussed in establishing these practices as core practices, while the reform discourse focused on 'telling' teachers to teach constructively without developing adequate knowledge or revisiting beliefs necessary for adopting reform based pedagogy.

INTRODUCTION

In the Indian context, the efforts related to bringing educational reform in teaching of mathematics has focused on developing frameworks like National Curriculum Framework (National Council For Educational Research and Training [NCERT], 2005), National Curriculum Framework for Teacher Education (National Council for Teacher Education [NCTE], 2009), revision of textbooks, imparting training to in- service teachers through workshops adopting the cascade model and issuing circulars to the teachers to comply with the vision. The reform efforts have underlined the importance of moving beyond the textbook centred teaching with the focus on learning mechanical procedures to developing students' power of mathematisation and reasoning and connecting the knowledge to life outside the school (NCERT, 2006). However, inadequate teacher preparation and lack of idea of how a reform based pedagogy looks like has led to teachers not being able to implement the reform based ideas in their classroom. This paper argues that pedagogy is influenced by beliefs held by teachers which are in turn influenced by the sociocultural and political context in which they are situated. The reform discourse introduces conflicts in teachers' discourse and practice but these conflicts cannot be resolved unless the teachers develop a shared vision of reform and use their agency to make informed choices in the classroom rather than implementing reform pedagogy because it is prescribed by the reform documents.

THEORETICAL FRAMEWORK AND RESEARCH REVIEW

Several research studies have indicated that focus on change in teaching strategies without taking teacher thinking or beliefs into consideration results in

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teachers making superficial changes which do not lead to any significant change in student learning opportunities as beliefs guide the choice of practices (Cohen & Ball, 1990). Pajares (1992) proposed that they can be classified as core and peripheral beliefs, explaining teachers' choice of practice. In this paper, I extend this framework to posit the relation between core and peripheral practices of a group of teachers with their core and peripheral beliefs. Teachers' beliefs and reported practice can be considered along the continuum, having transmission view of teaching mathematics at one end and the student-centred view at the other. The core beliefs of the individual teacher or for the majority of teachers in a group may indicate whether the beliefs and practice are nearer to the transmission end or the student-centred end of the continuum.

Relationship between beliefs and practice have been found to be a complex one where they influence each other rather than having a linear relationship (Guskey, 2000). Much of the literature discussing beliefs considers beliefs to be cognitive in nature. However, Wilson and Cooney (2002) identified the need to consider a group of teachers as unit of analysis for studying change in beliefs indicating the need to look at beliefs through the sociocultural lens. Several studies indicate that beliefs might be situate in nature as they vary from one context to other (Hoyles, 1992). However, few studies have analysed the influence of sociocultural factors like systemic expectations in times of educational reform on teachers' beliefs and practice. This points to a gap in the research literature where most studies having traced an individual teachers' change in beliefs, knowledge and practice without taking into consideration the sociological aspects like institutional culture as well as cultural beliefs prevalent in the society.

The studies done in Indian and Asian context on teachers' beliefs and practice highlight the differences in the way teachers think about exemplary practices for teaching mathematics. e.g. Bryan, Wang, Perry, Wong and Cai (2007) found that eastern teachers considered memorisation to be an intermediate step to understanding and classroom practices are "shaped by cultural, environmental and societal assumptions". In Indian context study by Dewan (2009) found that the most commonly prevalent beliefs are that mathematics is a body of knowledge mainly about the four basic operations and that not all children are capable of learning mathematics. Thus, while beliefs may be influenced by culture, they may or may not be sensitive towards issues like equity and addressing diversity among students.

The theoretical framework in this paper situates the individual teachers' beliefs, knowledge and practice within the institutional culture on one hand and the culture within the professional development context on the other. The two contexts may or may not have consistent messages about what mathematics is and how it is to be taught. The institutional culture is characterised by the interaction with peers and administrators that gives validation to the pedagogical

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efforts by the individual teacher while the same role may be played by teacher educators in the professional development context. Both these contexts are situated in the larger educational culture and may exhibit a variance in the type of beliefs, practice and knowledge. The larger educational culture also may or may not be in coherence with the ideas of folk pedagogy and cultural norms for teaching mathematics, especially in the context of educational reform (See Figure 1).

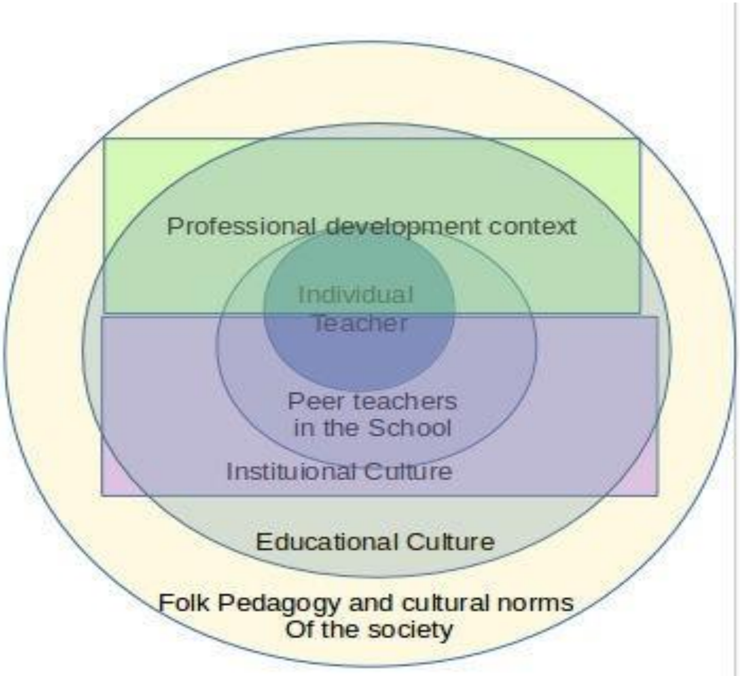

Figure 1: Interaction between individual teachers' beliefs, practice and knowledge with socio-cultural context

THE STUDY

The findings reported here are part of a larger study from 2009-2011 on collaborating with teachers to develop student-centred classroom practices aimed at teaching mathematics for understanding. The study had different phases: professional development workshops and collaborative follow-up of classroom teaching by the researcher. Participants in the study were 11 mathematics teachers (5 primary teachers referred to as P1 to P5 and 6 middle grades teachers referred as M1 to M6) from a nation-wide government school system which catered to mostly to government officials and students from low socio-economic background. Purposive selection of teachers was made through nominations by principals. The teachers were asked to attend the professional development workshop through the formal order given by the education department. All the teachers who participated had more than 15 years experience of teaching and were between the age range of 39 to 50 years. There were only two male model school teachers (M4, M6) and there was no male primary teacher. All the

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teachers had the bachelor degree in mathematics or science, along with the professional degree of B.Ed while a few had a masters degree in mathematics.

Data collection and analysis

Data about teachers' beliefs and practices were collected through Likert type written questionnaires (balanced scale having both positive and negative statements) and detailed individual semi-structured interviews during the first professional development workshop. The questionnaire had six parts focusing on teachers' beliefs about mathematics, its teaching and learning, frequency of practices adopted, beliefs about self, beliefs about students, and teachers' personal data. Content validity of both questionnaire and interview was done by experts (researchers, teacher educators) and changes were incorporated as per suggestions. The interview had prompts based on the items given in the questionnaire as well as further probing questions. Teachers took one hour to complete the questionnaire. Each interview took approximately one hour and was audio recorded. The circulars issued by the headquarters were analysed along with other curricular documents. Observations of teachers in schools and conversations with the headmasters, principals and other teachers in schools also informed the analysis.

Descriptive analysis of the data from teachers' responses to the questionnaire was compared with the teachers' responses in the interview. Transcripts of all the interviews were prepared and were coded for emergent themes based on qualitative analysis approach suggested by Miles and Huberman (1994). These emergent codes were then discussed with another researcher along with transcripts of interviews and were revised to arrive at a final list of codes which was used to analyse all interviews. The analysis was written up as memos while juxtaposing and comparing this analysis with analysis of curricular documents and observations and conversations in the schools.

RESULTS

In the first section, the core practices and beliefs indicated from triangulation of questionnaire and interview data is discussed. In the second section of results I discuss the factors influencing the implementation of curricular reform as gleamed from the interviews of teachers, analysis of curricular documents and a few observations in schools.

Teachers' core and peripheral practices and beliefs

The data from the questionnaire and the interviews of the teachers participating in the workshop indicate that teachers' views were more aligned to a transmission view of teaching and procedural focus in mathematics. While the questionnaire responses by themselves indicated a more positive alignment towards student-centred teaching and focus on reasoning, teachers' description and examples during the interview focus on showing procedures, repeated practice, getting

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correct answers and indicated a lack of knowledge of connections between concepts and procedures. Analysing the consistency of responses in the questionnaire and interview, along with teachers' articulation of priority and supporting beliefs indicated that these practices are the core practices for teaching maths for this group of teachers as most of the teachers reported using these practices. Use of activities, connection with daily life, engaging in reasoning, giving an opportunity to students to voice their thinking remain peripheral practices, for which teachers had lesser priority and were used for the purpose of making mathematics interesting and not necessarily to develop an understanding of mathematics. In the following paragraphs I give a few examples and description of the core and peripheral practices and evidences from analysis of questionnaire and interviews.

The most prominent core practices among teachers was to focus on memorising procedures by showing procedures, doing repeated practice and avoiding mistakes. Evidence for this was indicated from consistent response in questionnaire and interview and examples and description of practices in the interview. For example, teachers' response in the questionnaire and interview were consistent for items like "In the beginning of the class, I show students how to solve a particular problem and then give similar problems to practice from the textbook". Additionally, 10 of the teachers gave similar problems to practice after an activity indicating that purpose of the activity was to introduce the method of solving the problem.

> My typical mathematics period involves giving explanation by examples, doing activities from the textbook and then solving questions by students. (P2)

> You have to stop at a simple problem. You tell them to practice only this…. So if I am giving 50 problems, children with no help at home will be able to do only 20. Only the easier ones... I want to stop at that. Let them practice these 20 again…. I have to bring them to passing level. (M2)

The excerpts from the interview indicate how teachers adopted the practice of repeating the classwork as homework for students, activity as a pretext for showing procedures to be followed up by doing similar problems and giving easier problems to practice for weak students.

Teachers reported how they avoided students' mistakes by explaining the steps of the procedure carefully. When asked about how they responded to wrong answers, most teachers' responses indicated that they rarely made efforts to understand why the student had made the mistake and 10 of the teachers reported that they immediately corrected the mistakes. The strategies that teachers adopted for addressing student errors was to indicate where the correction had to be done, repeating the steps of the procedure and asking students to do repeated practice. Teachers also asked 'good' students to 'explain' the steps to the students who have made mistakes. Most teachers described in the interview, how they try to elicit the

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correct solution from students by focusing on textual cues and showing or recalling solutions to similar problems. P2 described an activity which she designed to develop the understanding of the words for denoting addition like “total”, “altogether” for grade 1 children by mixing beads from smaller bowl to a bigger bowl and asking students to use these words. She determined the success of lesson by students being able to identify the correct operation while solving problems. The middle school teachers too focused on typical questions that they predict would come in the examinations (See excerpt below).

> I make students repeat the questions if they are important like linear equations, age problem, upstream-downstream questions because I want to make them familiar with the sentence language. I am doing on the blackboard and they are copying so whether they understood or not, I can know only when they do it again. (M3)

Teachers also focused on the speed for solving problems and introduced shortcuts to help students. Shortcuts are the strategies through which number of steps can be reduced in a procedure, for e.g., cancellation while dividing whole number or multiplying fractions. There were five teachers out of eleven who believed that a good student of mathematics should be able to do mathematics problems quickly and accurately. Except for M1 and P3, all teachers felt that shortcuts should be taught. Some teachers, including primary teachers, admitted to teaching shortcuts and gave the justification on the basis of the utility of shortcuts in competitive exams. M5 though admitting to teaching shortcuts, felt that they do not help in understanding as children “skip the steps in shortcuts... and make mistakes”. He believed that shortcuts should be taught only in higher classes, since they help students perform in multiple choice questions. He felt that knowing a shortcut is beneficial when students have to respond to one-mark questions requiring lengthy steps.

Analysis of teachers’ interview indicated that they had incorporated key slogans into their discourse like “activity based method”, “play-way method” which they felt were advocated by the new textbooks based on curriculum reform documents. The teachers reported that they used activities for different purposes, like an introduction to a concept, creating interest in students, and for developing the understanding of concepts. However, by using “activity” teachers meant the use of a visual or concrete material or a context in the class to illustrate an example or a task, which typically involved demonstration of the procedure by the teacher followed by the students’ repeating what the teacher has demonstrated. In contrast, activity in the textbook was given to engage students in thinking about some aspect of the concept and coming up with their own ideas and solutions. In the interviews, only one teacher elaborated on how activities can help in developing understanding or eliciting students’ ideas (P1, see excerpt below) although all described how they use activities for introduction of the chapter. Some teachers, when they used the phrase “introducing the concept” through the use of activity meant showing the procedure using drawings or

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manipulative. Most teachers used the activity given in the textbook and a few primary teachers designed activities for students. Since most teachers attributed less priority to student engagement and use activity as a proxy for showing procedures it has been termed as a peripheral practice.

> This is more practical [new textbook]. Examples are given from life.... Child will never forget with this.... One method will not do. 2-3 methods will definitely come…. Some problems we will give to students and will ask how it will be done. (P1)

Another peripheral practice was focus on explanations and reasoning while doing mathematics. Most teachers considered explanations as synonymous to telling the steps of the procedure. Middle school teachers agreed that reasoning is important in questionnaire but they took examples from only geometric proofs to discuss reasoning in interviews. When asked how they will explain Pythagoras theorem, most described the verification activity through cutting and pasting shapes as given in the textbook. Most primary teachers felt that their students will not be able to appreciate conceptual explanations or are capable of reasoning. When asked how they explain the division algorithm, they described the procedure and all (except P1) were not able to elaborate on the conceptual basis of the division.

> We talk of divisibility rules.. you have an explanation for it. But is the explanation necessary for the child. (This) is the question we have to think of answering. The child may not appreciate the explanation. We try to go to his level, reason it out with him but to a fifth class child 100 tens [means] 10 hundred he is not going to appreciate. (M6)

The practice of establishing connections with daily life was highlighted as an important practice in the curriculum reform agenda. Although, teachers indicated high frequency of use of these practices in the questionnaire, the analysis of interviews indicated that teachers were not able to give meaningful examples of how mathematics can be connected to daily life. All teachers agreed that mathematics should be connected to daily life but they also had doubts that it might get confusing for students or time consuming. The examples or contexts shared by teachers in the interview indicated that very few teachers used contexts to elicit students' knowledge of daily life or to discuss key mathematical concepts and meanings. Only a few were able to provide examples that illustrated meaningful connections between mathematics and daily life indicating that it is a peripheral practice which teachers find difficult to align with their focus on teaching procedures.

The tools also probed teachers' views towards equity in classroom participation and learning opportunities. Giving an opportunity to students to share ideas and giving attention and support to students from a socioeconomically weak background, support establishing of equity. On the other hand, practices like labelling students, giving lower level questions to weak students to practice so that they pass the exams constrained the establishment of equity. Teachers' response

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to questionnaire items indicated that they have a positive attitude towards adopting practices to support students' autonomy. However, interview responses indicated that they did not prioritise these practices. Only a few teachers were able to give examples of students' ideas that came up in the classroom. Teachers M3, M4 and P2, said in the interview that students could come up with the procedures of their own, but after probing deeper, it was found that they felt that students can come to know different procedures from elsewhere (magazines, parents). They believed that only a few intelligent students can discover procedures and thus most students have to be taught.

> Children bring their original ideas, their world is very big. He learns from a lot of things other than the teacher in the school, e.g. internet, father in a specific profession. Very few children able to give justification and explanation on their own. (P1)

These excerpts indicate that teachers' beliefs about students' capability hinder the way in which teachers conceptualise opportunities for students to share their ideas or solutions.

Factors influencing implementation of curricular reform

The curriculum documents discuss how educational experiences of the teachers in schools as well as university lacked the aspect of engaging in problem solving, examining concepts from different point of view, making connections and providing reasoning and justification (NCERT, 2006, NCERT 2005). These ideas were introduced to teachers through in-service workshops. Examination of a manual for in-service workshop designed by headquarters of the education system of the participant teachers revealed the focus on re-examining content, pedagogy, communication skills among others along with the excerpt from the curriculum document specifying "connecting knowledge to life outside school" and "ensuring that learning is shifted away from rote methods" as one of the guiding principles for teaching. The participating teachers had already undergone prior training which had introduced them to the ideas of reform based pedagogy for teaching mathematics and circulars had been issued for all the schools, asking teachers to comply with reform based pedagogy (see excerpt below).

> Teachers must understand the pedagogical orientation to the course materials and organise the children's classroom experiences in a manner that permits them to construct knowledge on their own. There is a great thrust to distinguish knowledge from information and to perceive teaching as a professional activity, not as coaching for memorisation or as transmission of facts. Schools have to make every attempt to empower the teachers accordingly so that the curriculum transation truly reflects the shift in the educational paradigm as envisaged in NCF-2005. (Circular no. 1.2007 dated 1/1/2007)

Teachers' attempt to make changes in their pedagogy were influenced by the how administrators interacted with the teachers. While talking about their best

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lesson, many teachers recounted the lesson they had prepared and delivered for the inspectors or headmistress for inspection. The evaluation rubric used by the inspectors noted if the class was interactive or not and to what extent the teaching aids were used but did not have any aspect to notice about the opportunities students got to share their own ideas and solutions and to what extent the explanation, justification and reasoning for solutions was given in the classroom. The administrators themselves lacked the deep knowledge of the way classrooms with pedagogy in coherence with the new curriculum framework will look like and in light of being accountable to student pass percentage in examinations approved the way practices were added peripherally in teachers' repertoire.

In the interviews, while teachers acknowledged the change in the textbooks from focusing mainly on calculations to inclusion of activities and colourful pictures, the change in their teaching as reported was limited to increase in student participation through activities and solving problems on board after showing procedures. It did not reflect explicit attempts to connect knowledge to life outside schools or to elicit students' ideas and solutions. When talking about how they see themselves as a mathematics teacher, most teachers talked about the high pass percentage of students that they have achieved in the previous years indicating that their sense of identity as a teacher was highly dependent on students' marks in examination. This also related to the emphasis that they had placed on teaching questions that are likely to come in examination. One teacher (M1) discussed how examinations worked against the objective defined by the curriculum reform.

> If you want to implement then exams-tests should not be there. After every one month, unit test is there. It will be common question paper set by others. Then the teacher has to get 100% result. It's a rat race. (M1)

Teachers also recounted how they faced problems in teaching through the new textbooks, as parents and tuition teachers familiar with the more traditional way of teaching mathematics, teach the students methods and solutions ahead of the class. So when teachers engaged the class in doing activities and tasks from the textbooks, students with parental help or tuitions share the learnt solutions and also limit the opportunities for other students who do not get any help to engage in developing understanding of mathematics.

CONCLUSION

The findings indicate that the change in curriculum did not bring about a significant change in teachers' practice since teachers had integrated new practices as peripheral practices like doing activities and increasing student participation without moving beyond the focus on procedures. The purpose of these practices was still learning of the procedures and teachers avoided discussion of conceptual aspects while implementing them. Similarly focus on reasoning,

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connection to daily lives and equity in student participation were also practices that were peripheral since the teachers do not prioritise them over practices for teaching procedures. Teachers preference for these core practices indicated that the core beliefs held by teachers about mathematics, teaching and students were more consistent with the transmission view of teaching and procedural view of mathematics.

Why these practices exist as core can be explained by them being supported by more than one core belief. For example, showing procedures as a core practice may be support- ed by core beliefs of mathematics as procedures as well as the belief that students are not capable of coming up with solutions on their own and need to be told the procedure. Core beliefs, thus, together form a coherent stable structure, as these beliefs are in alignment with each other. Besides internal cognitive factors, the sociocultural and political factors also played an important role in imparting stability to core beliefs and continued use of core practices. The practice of showing procedures and repeated practice has been established as a cultural norm within and beyond the classrooms, shrinking the space for teachers to try out new practices. Even though textbooks had changed due to curriculum reform, little had changed in assessment system in terms of focus on reasoning and valuing students' solutions, supporting further the adoption of core practices. Lack of support structures within the school system also aggravated the issue although curriculum reform was discussed in the professional development context with the teachers. These aspects make core practices stable and difficult to change in the light of educational reform. The larger educational culture itself can be considered under flux where teacher educators, administrators and teachers together are trying to make sense of how the classrooms based on reform ideas would look like. However, teachers have the least power in determining ways to reform the pedagogy as they are considered as "implementers" of curriculum reform and there is lack of avenues to discuss and develop a shared vision of curricular reform.

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DISCUSSING MATHEMATICS TEACHER EDUCATION FOR LANGUAGE DIVERSITY

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As part of a large research project about supporting preservice teachers to learn about teaching argumentation for critical mathematics education in multilingual classrooms, we outline a framework for considering the knowledge, skills and practices that we, as teacher educators, consider mathematics teachers need. Our objective for describing such a framework is to provide a discussion document for teacher educators, primarily at our institution, but also for others who aim to improve their mathematics teacher education practices and want to determine theoretically how to navigate the complexity of changing our practices.

TEACHER EDUCATION AND MULTILINGUAL MATHEMATICS CLASS

Recently, we were funded to investigate how the compulsory mathematics teacher education courses for Grades 1-7 at our institution could be improved so that preservice teachers learn appropriate knowledge, skills and practices to teach argumentation for critical mathematics education (LATACME) in multilingual classrooms. We consider that aspects of the project are likely to be relevant for other programmes, both in Norway and elsewhere. In Norway, preservice teachers have complained that they are not receiving adequate input about how to teach subjects, such as mathematics, in multilingual classrooms (Thomassen, 2016). Internationally, there is an awareness that mathematics teacher education programmes should include understandings about how to work with language diversity at the school level (see Aguirre et al., 2013; Essien, Chitera, & Planas, 2016; McLeman, Fernandes, & McNulty, 2012; Thompson, Kersaint, Vorster, Webb, & Van der Walt, 2016). However, research has shown that it is difficult to provide programmes that situates language diversity as a resource and challenges preservice teachers' deficit views about language diverse students (de Araujo, I, Smith, & Sakow, 2015; McLeman et al., 2012; Taylor & Sobel, 2001).

In this discussion paper, we set out our assumptions about the knowledge, skills and practices that preservice teachers need for working in multilingual mathematics classrooms. It is important that we clarify our assumptions in order to interrogate our research-based decisions about how to adjust both teaching and research. In the next section, we describe a framework that sets out our assumptions. We, then, describe the background for each of the different components.

THE LATACME FRAMEWORK

The LATACME framework (see Table 1) highlights two responsibilities that we consider teachers in mathematics classrooms have, focussing on their relationship to argumentation for critical mathematics education for multilingual students. To fulfil

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these responsibilities, we consider preservice teachers need to take on three specific roles: teacher; learner; and advocate.

Responsibilities / Roles	Facilitating the exploring and learning of mathematics	Facilitating the exploring and learning about the world through mathematics
Teacher	Knowing how mathematical topics and mathematical argumentation can be developed where languages are considered a resource.	Knowing how to develop connections between critical mathematics education and argumentation.
Learner	Learning from multilingual students about their understanding of mathematical topics and argumentation.	Learning about critical mathematics education issues of interest and importance to multilingual students and their communities.
Advocate	Knowing how to provide input about mathematics education, including argumentation, to (multilingual) parents, school communities and government.	Knowing how to advocate that students need to use of mathematical arguments in order to explore the world.

Table 1: The LATACME framework: setting out the knowledge, skills and practices that preservice teachers need for working in multilingual mathematics classrooms

In the next sections, we first discuss the responsibilities and then the roles. Although, separated in this discussion, we considered that preservice teachers need to understand how they operate together. Working with the complexity of mathematics education is necessary in order to achieve “morally informed and committed action” (Hardy & Rönnermann, 2011, p. 464) that is valuable for both individuals and for society as a whole. We acknowledge that our descriptions use normative language, but hope that this does not deter these ideas being challenged.

RESPONSIBILITIES

Teachers have professional responsibilities to the students in their classrooms and to the investment that society makes in education. As generalist teachers in Grades 1-7, our preservice teachers need to combine specific mathematics-education responsibilities with professional responsibilities connected to other subjects and their holistic work as teachers. We consider that there are two mathematics-education-specific responsibilities connected to being teachers in Grades 1-7. The first is to facilitate students´ possibilities, including multilingual students´ possibilities, for exploring and learning about mathematics. This is linked to school mathematics acting as a gatekeeper for jobs and further study. The second is to do with supporting

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students to explore and learn about the world with mathematics, which we consider to be closely related to critical mathematics education. As Skovsmose (1994) stated, “the teaching and learning process should be oriented towards the goal of providing students with opportunities to develop their critical competence in the form of qualifications necessary for participation in further democratisation processes in society” (p. 61).

Mathematics continues to operate as a gatekeeper in many Western societies, in that mathematical qualifications are required for entry to further study or job opportunities (Ernest, 2002). Yet, immigrant students, including in Norway, have lower scores on PISA tests, particularly if they use another language at home than the language of instruction (Chiu & Xihua, 2008). Therefore, teachers have a particular responsibility for supporting immigrant students to have the best possibilities for gaining the requisite mathematical knowledge, including that of mathematical argumentation, to fulfil these gatekeeper requirements.

Mathematical argumentation is important in fulfilling this gatekeeper function because it is considered a core component of school mathematics, due to its strong connection to proof (Enge & Valenta, 2015) and because it is through argumentation that students show they have mastered the conventions of school mathematics and belong to the community of successful learners (Cobb & Hodge, 2002). In reviewing earlier research, Kleve (2015) suggested that not all Norwegian school students would have equal access to essential mathematical genres, what she called secondary discourses, because the students came with a range of different everyday conversation styles, or primary discourses, which were more or less in alignment with secondary discourses. Similarly, Kempert, Saalbach, and Hardy (2011) suggested that lack of fluency in the language of instruction could have an impact on bilingual students possibilities for understanding the “definitions, explanations, and argumentations” (p. 548) needed for solving mathematics word problems, which have a particular structure. Thus, there is a need for teachers to understand how some groups of students have their opportunities to learn mathematics reduced because of an unfilled need to systematically develop mathematical argumentation skills (Erath, Prediger, Quasthoff, & Heller, 2018) and that multilingual students bring with them existing language resources that can be utilised in their learning (Planas, 2018).

The second responsibility in the framework is about using mathematics to explore and learn about the world. In describing what he called “social empowerment”, Ernest (2002) stated that learning mathematics should result in students being “able to understand and begin to answer important questions relating to a broad range of social uses and abuses of mathematics” (p. 6). Thus, multilingual students, like other students, need to learn to critique existing societal issues with mathematics and promote their ideas through argumentation. Yet, moving mathematical arguments into societal conversations is not straight forward (Aguilar & Blomhøj, 2016) and for some students the unfamiliarity of the societal contexts may affect their willingness to engage in these types of arguments (see for example, Lubienski, 2007). Students

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coming from an immigrant background may not have the same interest in or familiarity with the societal contexts chosen by a teacher.

As teacher educators, we need to discuss and clarify our understandings about the responsibilities of mathematics teachers for providing relevant mathematics education. At the same time, we accept that it is not easy for educators to resolve the inherent tension between the two responsibilities, described by Jablonka and Gellert (2010) as “a pedagogy of access and a pedagogy of dissent” (p. 43), nor for learners to gain both because of the high level of reflection required (Powell & Brantlinger, 2008). Still, it remains important for teacher educators to focus on this complexity through reflective discussion about: what constitutes school mathematics; what kind of outcomes it is supposed to achieve; what is said about it; how it is conducted; in which ways does it expect people to act towards each other (Franke, Kazemi, & Battey, 2007).

ROLES

In order for the responsibilities to be achieved, we consider that preservice teachers need to take on three roles: teacher, learner, and advocate. Each role contributes different sets of knowledge, skills and practices to mathematics classroom teaching.

Teacher

The role of teacher is the one that most preservice teachers would expect to have in mathematics classes (see for example, Meaney & Lange, 2012). The knowledge, skills and practices, needed for teachers to fulfil their responsibility to increase students´ possibilities to explore and learn about mathematics, has been labelled pedagogical content knowledge (PCK) (Hill, Ball, & Schilling, 2008). PCK consists of both knowledge of the subject, in this case mathematics, and knowledge of how best to teach that subject, such as awareness of misconceptions students might have and how to overcome them (Shulman, 1986). The content of teacher education courses is often discussed in terms of PCK, reflecting the idea´s origin as supporting teacher educators to determine what they should focus on with preservice teachers (Shulman, 1986).

Although preservice teachers´ mathematics content knowledge has been much investigated, less research has been done on the PCK that teachers need for improving school students´ mathematical argumentation in multilingual classrooms, particularly in regard to using students´ multiple languages. Enge and Valenta (2015) found that Norwegian preservice teachers struggled with providing appropriate mathematical argumentation because they did not define the mathematical objects or seemed able to choose and use appropriate representations, such as algebra, in their argument. A lack of clarity about their own mathematical argumentation could affect the possibilities they offer their students for exploring and learning mathematics. This could be the case for multilingual students, whose needs and language resources may be different to their monolingual peers, which Planas and Civil (2013) described as

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the tension between "the simultaneous need to reinforce and improve the language of instruction and that of sharing knowledge in the home dominant language" (p. 374).

Based on research done in Māori settings, Meaney, Trinick, and Fairhall (2011) used PCK to identify the knowledge that mathematics teachers need about language diversity. We adapted their points about the inclusion of language issues in mathematical PCK to highlight how PCK could relate to mathematical argumentation:

- Knowledge of mathematical language argumentation
- Knowledge about students' mathematical argumentation including how to use students´ multilingual language resources
- Knowledge about teaching mathematical argumentation to (multilingual) students which utilised their current languages as resources.

The knowledge, skills and practices connected to teaching argumentation also include knowing about how resources such as digital technologies can affect the possibilities for mathematical argumentation. For example, Wegerif (2004) found that ICT can facilitate and direct students´ mathematical arguments towards subject matter learning and Wegerif and De Laat (2011) argued that ICT can be seen as a facilitator opening and shaping spaces for argumentation that otherwise would not be there.

Nevertheless, we do not consider PCK, as it is commonly defined, as adequate for addressing the second responsibility of supporting students to explore and learn about the world using mathematics. In fact, PCK can over-emphasise the importance of learning mathematics only for the gate-keeper function. In summarising research on critical mathematics education, Meaney and Lange (2013) identified several examples where teachers and students resisted the inclusion of real-life experiences into discussions because they considered the only valid arguments were mathematical ones. Therefore, teachers need appropriate skills, knowledge and practices that provide multilingual students with opportunities to explore their everyday experiences with mathematics. For example, as a result of a teaching intervention English and Watters (2005) found that young children could blend their everyday knowledge with their mathematical knowledge in developing arguments and justifications about modelling problems. This suggests that teachers can change their conceptions about including students everyday knowledge into mathematical argumentation. Thus, the specific needs of working within a multilingual classroom, where students have a range of backgrounds and experiences, require teacher educators to support preservice teachers to gain a broader range of teaching skills, knowledge and practices.

Learner

As well the role of teachers, preservice teachers have the role of learners, so that they can understand multilingual students´ mathematical thinking and the skills and knowledge they bring to both mathematical argumentation and argumentation that

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uses mathematics. Walshaw and Anthony (2008) emphasised that “a context that supports the growth of students' mathematical identities and competencies builds on students' responses, shapes the reasoning and thinking to an appropriate level, and moves ideas and solutions toward a satisfactory conclusion” (p. 539). As learners, preservice teachers need skills to identify what their students know as a basis for then developing their possibilities to explore and learn mathematics and the world with mathematics.

For multilingual students´ learning opportunities to appropriately utilise their cultural background, preservice teachers need to know how argumentation can be culturally shaped. Luykx, Cuevas, Lambert, and Lee (2005) noted the importance of understanding the impact of cultural communication patterns, “students from non-mainstream cultures often consider it rude or combative to address points made in the previous person's contribution, defend one's arguments with logic and evidence, or look for anomalies in another person's statement” (p. 127). Thus, finding out about multilingual students' language resources, including cultural understandings, is an important component in preservice teachers´ roles as learners. To confront stereotypes, preservice teachers need to learn from students and their families about their experiences and expectations so that their teaching will be better informed.

In order to fulfil the second responsibility, preservice teachers also need to gain skills, knowledge and practices to learn about students´ interests, which could be used as appropriate contexts for critical mathematics education. Political contexts have been identified as motivational for students´ learning mathematics, because “mathematics can be used to legitimise and justify political decisions that directly and significantly affect the social dynamics of some communities and the lives of their inhabitants” (Aguilar & Blomhøj, 2016, p. 257). However, in a review of research on ethnomathematics and critical mathematics (Meaney & Lange, 2013), the contexts used in mathematics lessons were almost always chosen by the teacher, based on assumptions about what students were interested in. As teacher educators who value the importance of the second responsibility, we need to assist preservice teachers to learn from their students, and their families, about their interests so that mathematics can add value to students´ learning (Trinick, Meaney, & Fairhall, 2017).

Advocate

Although rarely noted – PCK makes no mention of it at all – preservice teachers may need to be advocates for their multilingual students ´ rights to engage in mathematical argumentation and argumentation using mathematics in many different circumstances. As Anthony and Walshaw (2009) stated, “major innovation and genuine reform require aligning the efforts of all those involved in students’ mathematical development: teachers, principals, teacher educators, researchers, parents, specialist support services, school boards, policy makers, and the students themselves” (p. 27).

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For example, Planas and Civil (2013) discussed how “students whose dominant language is not the language of instruction may withdraw from participating in whole-class discussions and defer to the students whose dominant language is that of instruction” (p. 375). Therefore, a teacher may need to advocate for multilingual students´ right to use the full range of their language resources with other students.

Given that accepting the second responsibility about the need for preservice teachers to provide students with opportunities to explore and learn about the world with mathematics is not generally perceived as part of mathematics education, there may be a need to advocate for teaching methods which emphasis the role of argumentation in multilingual mathematics classrooms. Research by Graue and Smith (1996) showed how established parental understandings about what constitutes mathematics education both affected their children´s views and were difficult to change. Graue and Smith (1996) strongly suggested that a more dialogic approach to implementing teaching reforms was needed. Yet, preservice teachers may not consider it their responsibility to instigate a dialogic approach by, for example, contacting parents outside of set parent-teacher evenings, in which the focus is mostly on students´ achievements (Meaney, 2013). Thus, their teacher education programmes need to provide them with alternative ways to advocate for new teaching practices with parents.

Preservice teachers may also need to advocate for their students in the wider society. In Scandinavia, Lange (2008) and Svensson, Meaney, and Norén (2014) documented how media discussions affected the views of teachers and the general public about multilingual students´ possibilities to learn mathematics. Therefore, teachers may need to learn how to raise alternative discussions in the public sphere about how and what multilingual students should learn in mathematics lessons.

Yet developing preservice teachers´ skills, knowledge and practices to be appropriate advocates for their multilingual students at all levels of education is not common in mathematics teacher education programmes. Therefore, there is a need for us, as teacher educators, to reflect on how they can make this part of what these programmes.

CONCLUSION

In this discussion paper, we have presented a framework (see Table 1) that outlines the knowledge, skills and practices connected to the responsibilities and roles that preservice teachers in our compulsory mathematics teacher education programme need so that multilingual students have the greatest chances for learning mathematics. As teacher educators, we need to discuss whether these are the knowledge, skills and practices needed by preservice teachers. Certainly, we must extend our expectations beyond preservice teachers relaying curriculum requirements if we are serious about multilingual students exploring and learning about mathematics and exploring and learning about the world through mathematics. We see the framework as providing opportunities to discuss whether other responsibilities or roles or other ways of

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thinking about our work as mathematics teacher educators would be more appropriate.

Discussions about the role and responsibilities as being more or less appropriate will only be the first step in instigating a teacher education programme that takes seriously the need to support multilingual students as having with them a range of resources connected to their languages, experiences and aspirations. Although previous research outlines the difficulties in changing our teacher education courses, we anticipate that this will be an exciting component of our upcoming research project.

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AN ECOLOGICAL PERSPECTIVE ON THE REPRODUCTION OF DEFICIT DISCOURSES IN MATHEMATICS EDUCATION

Nicole L. Louie and Aditya P. Adiredja

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Abstract: The scholarly traditions that inform mathematics education research include those that examine learning and sense-making from a cognitive perspective as well as those that investigate issues of culture and power as they shape and are shaped by learning. At times, these two traditions have been cast as conflicting or competing. In this paper, we bring them together to propose an ecological model of cultural reproduction in mathematics education in the US that reveals (1) the interplay of cultural and cognitive processes in the reproduction of deficit discourses about students of color, and (2) the dominance of these discourses in classrooms, schools, and academic research.

INTRODUCTION

The scholarly traditions that inform mathematics education research include those that examine learning and sense-making from a cognitive perspective as well as those that investigate issues of culture and power as they shape and are shaped by learning. At times, these two traditions have been cast as conflicting or competing (Martin, Gholson, & Leonard, 2010). In this paper, we bring them together to propose an ecological model of the reproduction of deficit discourses within mathematics education.

By *deficit discourses,* we mean storylines that position some groups of people as inferior to others, which have been repeated enough to take on an air of common sense. We focus here on deficit discourses about the intellectual and especially mathematical ability of students of color [1] in the United States (US) (Martin, 2009; Valencia, 2010). Examples include "Asians are good at math" (which is fundamentally relational, framing mathematical success as natural for Asians and Asian Americans and unnatural for other people of color; see Shah, 2017) and the narrative "Latinas aren't good at math" (documented by Leyva, 2016). Our perspective is that any interpretation or assessment of ability or success is subjective and partial. Further, dynamics of power and privilege are salient in such interpretations, and their use to position students from marginalized groups as problematic or inferior constitutes *epistemological violence* against those students and groups (Teo, 2008, p. 57). There are myriad consequences spanning everyday classroom interactions (e.g., who is called on during class discussions), educational policies (e.g., policies that determine who gets access to rich and rigorous curricula versus drill and recitation), and the distribution of resources and opportunities throughout society (e.g., who is seen as a good candidate for college admissions and employment).

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In this paper, we theorize processes through which deficit narratives continue to be reproduced despite a longstanding national discourse about "equity" and "mathematics for all" (e.g., National Council of Teachers of Mathematics, 1989, 2014), by both teachers and researchers who seem to intend to support mathematics learning for all students. Our work is situated in the US, where race is a salient category around which social hierarchies are organized. Each of us has employed critical perspectives to examine processes of learning in mathematics education in our prior work, but with different objects of inquiry and through different lenses. In what follows, we describe the intellectual spaces in which we have worked and some of the contributions and limitations of our prior thinking and writing. We then synthesize our perspectives in the form of a model of cultural reproduction in mathematics education in the US that reveals (1) the interplay of cultural and cognitive processes in the reproduction of deficit discourses about students of color, and (2) the dominance of these discourses in classrooms, schools, and academic research.

THE POLITICS OF RESEARCH ON MATHEMATICAL SENSE-MAKING

Aditya Adiredja

In my work I have focused on uncovering the intersections between traditional research about mathematical sense making and equity research. Informed by sociopolitical perspectives in mathematics education (Gutiérrez, 2013; Valero, 2004), including Critical Race Theory (CRT; Delgado & Stefancic, 2001), I have been investigating the interrelatedness of knowledge, power, identity, and social discourse in research about sense-making. This work has highlighted the power of deficit narratives as part of social discourse in determining the kinds of knowledge and people that are valued in mathematics. CRT scholars have focused on challenging deficit *master narratives* about Black and Latinx students and on critiquing the privileging of White students in meritocracy and other seemingly "objective" standards to assess students (DeCuir & Dixon, 2004; Ladson-Billings & Tate, 1995; Solórzano & Yosso, 2002). As Nelson (2001) defines them, deficit master narratives are socially circulated and reified stories that suppress morally relevant details about a person or a group with the impact of disrespecting and/or misrepresenting such person or group. These narratives often serve as a script to make sense of experiences of members of the group and justify interacting with them in particular ways (Nasir, Snyder, Shah, & Ross, 2013; Nelson, 2001; Stanley, 2007). The narratives we included in the introduction are examples of deficit master narratives about students of color.

In examining these issues, I have developed a model for the self-sustaining system of deficit narratives about students of color in mathematics in research (Adiredja, in press). In this model (Figure 1), I identify the influence of (a) deficit master narratives about the intellectual ability of students of color in society at large, established through deficit-oriented research and broader systemic racism, and (b) deficit narratives about all students' mathematical sense making, rooted in a research tradition that focuses on students' misconceptions, difficulties, and failures to conform to normative ways of thinking. Because of their dominance, these narratives orient researchers to (c) a deficit

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perspective when they observe or analyze mathematical work, especially work by students of color. As Teo (2008) has highlighted, this perspective then produces (d) a deficit interpretation of the specific mathematical work, which in turn produces (e) a deficit story, reified in published research, about the mathematical (in)ability of students of color. This story feeds back into dominant narratives, reinforcing them as true.

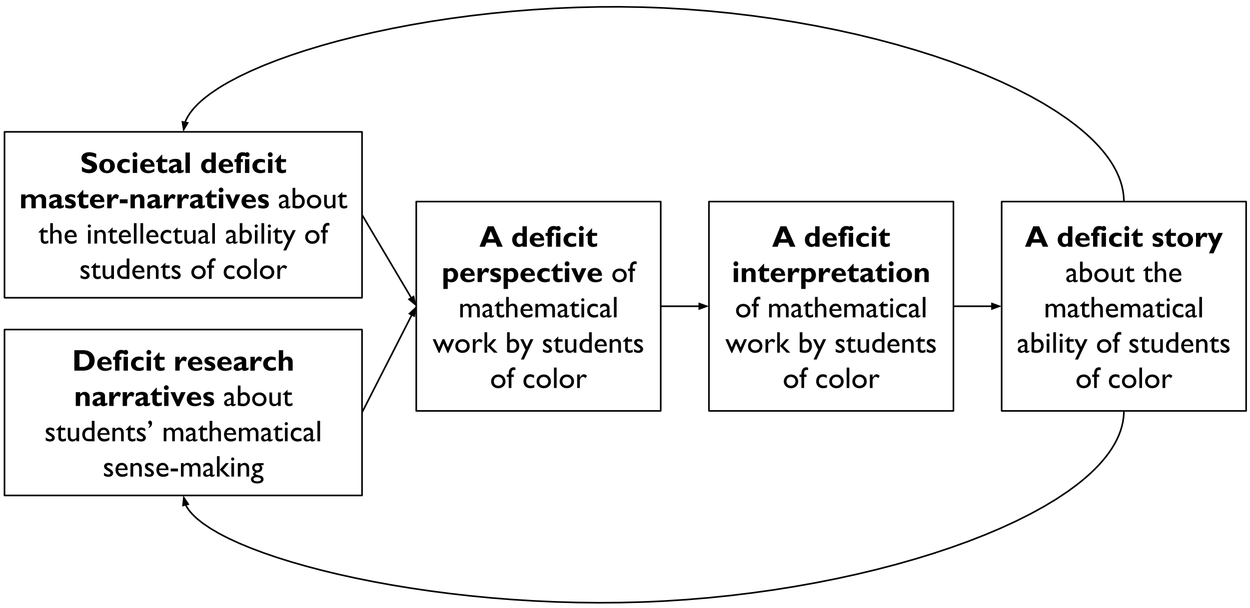

Figure 1: The self-sustaining system of deficit narratives about students of color in mathematics. (Figure adapted from Adiredja, in press)

The diagram highlights the active role that individual researchers can play in the process of reproduction. The first two boxes of deficit narratives on the left are existing artifacts in society and research. The next three boxes to their right represent a researcher's role in possibly sustaining the system. I posit that the process of reproduction begins with the use of deficit perspectives to interpret the mathematical work of students of color. I define a deficit perspective as a perspective that focuses on inherent problems in students' mathematical knowledge and attributes those problems to deficits and faults of the students. The deficit perspective presumes that without (or even with) instructional intervention, students' understanding of mathematical ideas are flawed. In Adiredja (in press), I identify from existing research some of the supporting tenets of this perspective, which include the over-privileging of formal knowledge and the demand for the use of coherent and formal mathematical language. I illustrate the detrimental impact of using such seemingly objective standards in analyzing the mathematical work of a Latina undergraduate student by contrasting it with an anti-deficit approach.

The model was partially informed by my practice as a mathematics professor. However, the details of an analogous model for teaching require effort to articulate. Through the current collaboration, we are able to unpack some of these details. Our

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collaborative work also highlights the importance of local communities of practice in reproducing deficit narratives, which I had not fully considered.

CULTURE AND IDEOLOGY IN TEACHER SENSE-MAKING

Nicole Louie

My work has focused on teachers of mathematics, in particular, teachers who are working toward an explicit equity agenda. I began my first major research project wanting to learn more about how teachers' local communities of practice (e.g., high school math departments and course teams) supported their members to make sense of and enact equitable mathematics instruction. Disrupting hierarchies of mathematical ability and their racialization was central to the teachers' stated goals, and they were active participants in a professional development program designed with that purpose. Yet I found that even the teachers who were most successful with respect to that goal were caught in the tensions and contradictions created by what I have come to call *the culture of exclusion in mathematics education* (Louie, 2017).

In Louie (2018), I investigated why the culture of exclusion was so difficult for teachers to escape, drawing on the case of a teacher I called Amanda. Amanda was highly skilled in noticing and naming diverse mathematical strengths in her students, virtually none of whom fit stereotypes about who is "good at math" (their skin color, the accents with which they spoke, their test scores). But in spite of her skill, Amanda explained that colleagues, administrators, and standardized assessment systems made her wonder whether she was "overcompensating" for students' deficiencies, focusing on ways of being smart that were "fake." That is, the dominant culture manifested in varied and abundant ways to resist Amanda's attempts to challenge it, including sowing doubt in her own mind. Many of these ways were cloaked as objective or neutral, particularly when it came to discourses that circumscribed what counted as "real" math and "real" mathematical ability, which were instrumental in sustaining hierarchies (consistent with Nasir, Cabana, Shreve, Woodbury, & Louie, 2014; Parks, 2010). In other studies of the relationships between the dominant culture of exclusion, the local cultures of particular schools, and individual teachers' classroom practice (Louie, 2016, 2017), I have also found that teachers frequently reproduced exclusionary discourses about what it means to be "good at math" and who can attain this status, seemingly without conscious awareness. Despite the teachers' good intentions, the nature of the culture of exclusion is that it makes some forms of practice sensible and automatic while making others illogical, difficult, or invisible.

A difficulty I have faced in my research to date is that it has provided limited opportunities to highlight teachers' agency and capacity to alter the culture of exclusion. I have analyzed the social networks that some teachers organized to support them in transformative work, but I have paid less attention to the specific processes whereby teachers participate in actual transformation. This is one affordance of the present collaboration.

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AN ECOLOGICAL MODEL OF CULTURAL REPRODUCTION IN MATHEMATICS EDUCATION

In reading one another's work, we found similarities in how we understood the reproduction of deficit narratives by mathematics education researchers and mathematics teachers, though we had used different terminology (deficit master narratives, perspectives, interpretations, and stories in Adiredja's work, and dominant culture, exclusionary frames, and ideologies in Louie's work). This was striking to us because it pointed to the pervasiveness of deficit discourses in the culture of mathematics education. Further discussion allowed us to notice differences in emphasis that were generative for our thinking. We decided to synthesize our perspectives in a unified model of the reproduction of deficit discourses in mathematics education, which we present here (see Figure 2). Below, we highlight four aspects of the model.

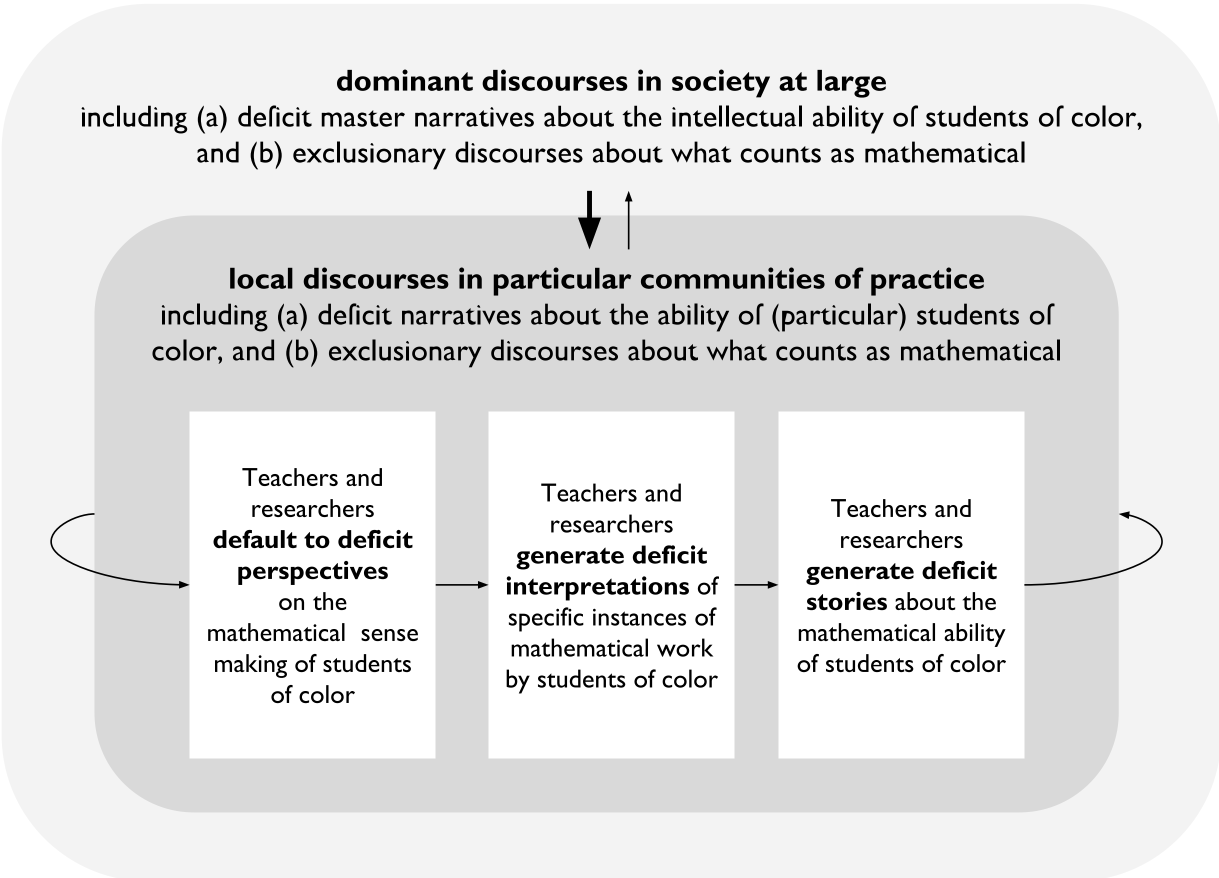

Figure 2: How deficit discourses about students of color in mathematics education are reproduced: an ecological model.

An ecological perspective

One feature of this model is that it takes an ecological perspective (Bronfenbrenner, 1977). That is, it frames human learning and sensemaking as embedded in nested systems, including the immediate settings of everyday interaction as well as the broader cultural norms and ideologies that pattern these settings. Thus, the outermost layer of Figure 2 focuses on dominant discourses impacting students of color, specifically

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(a) racialized deficit master narratives about the intellectual ability of students of color and (b) seemingly deracialized exclusionary discourses about what counts as mathematical activity and what counts as mathematical ability, which interact and contribute to the deficit positioning of students of color.

These dominant discourses come alive for people through their participation in social practices (Nasir et al., 2013). This is reflected in the middle layer of Figure 2, which focuses on local discourses as they are negotiated in specific communities of practice (cf. Wenger, 1998). Mathematics teachers, for example, are embedded in school faculties and subject-matter departments; researchers who study how students make sense of mathematical concepts are embedded in communities of scholars engaged in a shared conversation via conferences, research publications, and so on. In addition to having shared deficit narratives about students' mathematical understanding (e.g., students struggle with particular concepts), a research community also shares particular standards and values, which might influence the reproduction of deficit narratives. Adiredja (in press) identified different tenets of deficit perspectives, which include some of the elements of exclusionary framing of mathematical activity that Louie (2017) identified.

Dominant discourses also take on localized and specific meanings in teachers' and researchers' communities, as community members participate in enacting, sustaining, and potentially altering them. For example, educators might participate in policies that effectively exclude Black from honors programs, even if race is not explicit in those policies, or researchers might produce and uncritically consume articles that report on the mathematical "underachievement" of Latinx youth, sustaining a discourse of students of color intellectually and mathematically inferior to white peers. Local narratives about specific children of color might also develop, positioning them as students who "aren't motivated" or "don't do abstract thinking."

The interplay of agency and structure in reproducing deficit discourses

Dominant discourses are powerful. They shape the ways that we understand and act in the world, often without our conscious awareness, normalizing some responses while making others strange. Yet they do not completely determine how we think or act, and the relationship between dominant discourses and local ones is reciprocal if not equal (indicated by the different sizes of the arrows in Figure 2). There is always the possibility of alteration, so that communities and even individuals may challenge master narratives through resistance and counter storytelling (Delgado & Stefancic, 2001; Giroux, Lankshear, McLaren, & Peters, 1996; Solórzano & Yosso, 2002). For example, teachers at Railside High created a culture within their mathematics department in which mathematical ability was redefined to include not just accuracy and fluency with standard algorithms but also reasoning, justification, questioning, creativity, invention, and more, supporting students from racially and socioeconomically diverse backgrounds to succeed (Nasir et al., 2014; see also Gutiérrez, 1999).

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Most of the time, however, people do not challenge dominant discourses but instead reproduce them. Unpacking how this process occurs is important for identifying leverage points at which deficit narratives can be interrupted. It also helps us understand how good intentions might not stop people from deploying deficit narratives and thereby contributing to marginalization and oppression. The three boxes in the center of our model represent this process. We extend the model from Adiredja (in press) to illustrate how both researchers and teachers sustain deficit discourses.

Like researchers, teachers develop stories about particular students of color based on specific professional interactions. In the classroom, they make necessarily quick assessments of students' mathematical work, comparing students to one another and making judgments about who is smart, who "gets it," and who needs help. Researchers engage in similar classification, albeit at a more leisurely pace, interpreting differences between the work of this student and that student, both students and what the researchers consider correct or ideal. In making sense of students' mathematical work, neither teachers nor researchers are immune to either discourses about students of color in general or local narratives about particular individuals.

People frequently employ deficit discourses not because they intend to but because they are so commonplace as to appear neutral. They might ask seemingly objective questions such as, did the student's work evidence any familiar misconceptions? Was there any error in arithmetic or symbolic representation? Did they misuse certain words? Were there inconsistencies in their thinking? These questions focus on the deficits in students' mathematical thinking. Not only do their answers tend to lead to deficit interpretations of students' mathematical work, but they also miss students' intellectual work and contributions (Adiredja, in press).

This is particularly detrimental when the targets of interpretation are students of color whose thinking is not traditionally valued, or those who have yet to learn to display their thinking in ways that are commonly recognized as valid (e.g., using formal mathematical language, in English; Civil, 2011). This can dangerously turn into generalizations about a student's mathematical (in)ability (e.g., labeling someone "high" or "low," an "A student" or a "B student"). We emphasize that this process tends to be sub-conscious or implicit. Without intending to adopt a deficit perspective, people often default to one, and deficit interpretations and stories result.

The importance of multiple local contexts

Above, we alluded to the importance of local narratives in our model. Although they are not represented clearly in Figure 2, we also wish to highlight the variety of local contexts in which any given teacher or researcher participates, which may give rise to various local narratives. In Louie's (2018) work, for example, Amanda was part of a high school mathematics department in which the prevailing narrative was that "real math" was about following algorithms, manipulating symbols, and getting the right answer quickly. She was also part of a supportive community of educators who, like her, drew on Complex Instruction (CI; Cohen & Lotan, 1997; Nasir et al., 2014) to

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redefine mathematical competence and disrupt racialized ability hierarchies. Amanda's case highlights that a teacher's work is situated in different local communities, some of which may naturalize deficit narratives and some of which may challenge them.

Researchers and research projects are similarly situated within multiple communities of practice. One relevant community is what we might call the mainstream American mathematics education research community, which has sometimes positioned work on race and power as decidedly outside its purview (Martin, Gholson, & Leonard, 2010). Yet there are also communities of education researchers that center the study of racial power dynamics as they affect various levels of the education system, many of whom we have cited in this paper. Different research communities have their own theoretical traditions and values, which may normalize or question deficit discourses.

DISCUSSION AND CONCLUSION

Our goal with this paper has been to synthesize our perspectives on cultural reproduction in mathematics education in the US. The proposed model extends the previous work that each of us has done, theorizing the process of reproduction of deficit discourses in both teaching and research. It emphasizes the importance of dominant and local discourses in informing individual teachers and researchers in reproducing and sustaining a system that positions Black, Latinx, Indigenous, and some Asians as mathematically inferior, while simultaneously framing such positioning as neutral, objective, and well-grounded in evidence. Our collaboration highlights the political nature of work that happens *throughout* mathematics education. Rather than isolating deficit narratives as the problem of one sector, we show that both teaching and research are shaped by power and ideology in mundane, taken-for-granted ways.

While it has not been the focus of the current paper, a natural extension of this work is theorizing processes by which deficit master and local narratives can be challenged, from an ecological perspective. Each of us has separately begun unpacking such processes in cognitive research from an anti-deficit perspective and in teaching that frames mathematical activity and mathematical ability inclusively, respectively. We intend to continue our synthesis to incorporate these anti-deficit processes.

As we move forward, we are interested in learning from scholars whose work is grounded in other countries. Is it the case that systems of deficit narratives similar to those we have described exist in other contexts, despite differences in how social hierarchies are organized (e.g., different intersections of race, ethnicity, class, caste, gender, and sexuality)? What can we learn about the strengths and limitations of our model from cases from across the globe—and for that matter, from more intersectional analyses of cases from the US? We will continue to investigate the degree to which deficit narratives are reified in research and practice, and their amenability to counter-stories. We hope that our work can contribute to ongoing conversations in the international mathematics education community about inequity and social justice.

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NOTES

1. We focus on Black, Latinx, Indigenous, and those Asian populations in the US that tend to be discursively positioned at the bottom of the racial hierarchy of mathematical ability (Martin, 2009). The "model minority" narrative notwithstanding, members of many Asian and Asian American groups experiences similar discrimination and prejudice as other people of color in the US.

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NATURE OF MATHEMATICS AND PEDAGOGICAL PRACTICES

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Abstract: I experienced that learner's views about the nature of mathematics has been influential in their mathematics learning. Students' interest to learn mathematics and teachers' interest to teach mathematics also depend on their beliefs and thoughts about the nature of mathematics. Similarly, parents' encouragement and motivation for their children to study mathematics also depends on the parents' view regarding the nature of Mathematics. Previous research study has shown that nature of mathematics affects in/directly the teaching and learning process. Thus, this paper has discussed the constructed notions of nature of mathematics based on my experience of teaching and learning mathematics; also, the literature related to nature of mathematics. Furthermore, I have discussed teaching practices which have been shaped by the different types of nature of mathematics.

Key words: Nature of mathematics, Pedagogical practices, 21st century

SETTING THE SCENE

Mathematics has always been taught from the very beginning of the school level with the basic skills of addition, subtraction, multiplication, and division among others. In my experience of learning mathematics, it was interesting to learn in lower grades rather than in upper grades. Now a days, also same types of experience might be experienced by the students because of its nature as well as learner's beliefs and thoughts regarding mathematics. People have very different ways of understanding as well as illustrating the particular pieces of mathematics (Thurston, 2006). However, Students' and teacher's perception of the nature of mathematics have great influence in teaching and learning process. In the case of students, their thoughts and perceptions of the nature of mathematics play a vital role in their learning in which learning is more dependent on the interest of learner and those interest arises according to the basis of their thinking as well as experiences. Likewise, it applies in mathematics teacher and mathematics curriculum designers as well; On the other hand, the people from the society also play a vital role in shaping the beliefs, thinking, perception and nature of mathematics which in/directly affects children's learning. Dossey (1992) also mentioned that perception of the nature and role of mathematics held by the society has a major influence on the development of school mathematics curriculum, instruction, and research.

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In the context of Nepal, most of the mathematics teachers have been habitual to respond that, it is difficult, abstract, memory- based, exact, important in the future, there are no activities, it is important for the exam, etc. to the different epistemological questions related to mathematics raised by the students. In my experience of teaching mathematics at school and learning mathematics from school to University, only a few of the teachers and stakeholders have been talking about contextual mathematics, activities, connection to the real-life situation. So, the perception and beliefs about mathematics perceived by the teachers and parents are transferring onto the students which either enables or constraints to achieve the students learning outcomes in mathematics. Thus, engaging with mathematics depends on the teachers, students, school management and other stake holder's perception of the nature of mathematics. Therefore, the main purpose of writing this paper is to unpack my understanding about the nature of mathematics based upon the literature and my experience of learning and teaching as well as its impact in teaching and learning in the hope that it may enable me as well as other mathematics educators to transform my and their beliefs towards nature of mathematics from absolutist to falibilist as well as to improve teaching practices. So, I have at first discussed mathematics and its nature, its characteristics, etc. Second, I have discussed the historical background of the nature of mathematics that shows how mathematics has a place to discuss, its nature, as well as what are the basis to discuss the nature of mathematics. Third, I have discussed the nature of mathematics from the learner's point of view that I got from literature. Finally, I have engaged on how nature of mathematics effects, positively or negatively, in the teaching and learning process of the same.

NATURE OF MATHEMATICS

In this 21st century, there have been discussions about the nature of mathematics. It has been talked about by different mathematicians, researchers, teachers, students, and other practitioners who are working in the field of mathematics education. Thoughts or perception about nature of mathematics is related to their beliefs, perception towards mathematics are related to how they understand the mathematics, as well as how people use the mathematics in real life context. Since I started my career as a mathematics teacher, my experience of learning mathematics is changing. How I learned mathematics was becoming important rather than why and what I learned. My students, however, still memorized the formula, a mathematical problem without connecting to the day to day practices and real-life examples. Then, I realized that my experience of learning is totally guided by the absolutist nature (Ernest, 1991) of mathematical knowledge. Ernest (1991) mentioned that the absolutist nature of mathematics portrays knowledge as the certain and unchangeable truth, mathematical knowledge is made up of absolute truth and represent the unique realm of certain knowledge. Actually, my experience of learning mathematics in school level, as well as Intermediate and Bachelors, was guided by the absolutist nature of mathematics and lies on Platonist views of mathematics or school of logicism. Accordingly, in school level our teachers were focused on solving routine and readymade problems rather than understanding the problem, context and creating the problem situation. The same was true of higher education as well: meaningless mathematical theorems were to be memorized in a deductive way. Similarly, absolutist nature of mathematical knowledge is based on axioms and definition (Ernest, 1991) which is fixed, unchangeable and objective.

After completing my Bachelor's degree, I enrolled in Kathmandu University School of Education (Kusoed) for Masters in Mathematics Education. At that time, I got the opportunity to learn mathematics in a more practical way. I also became aware of the pedagogical practices. The different literatures on mathematics education helped me to practice new approaches in teaching, like, activity based, collaborative, etc. In my teaching career, it has helped me to link the mathematics to the day to day practices. Similarly, I came to know about the Fallibilist nature of mathematics. In which Ernest (1991) mentioned that mathematical truth is fallible and corrigible and open to revision. It also challenges the absolutist nature of mathematics. Fallibilist nature of mathematics focused on revision, revisiting, and claimed that knowledge is subjective. Fallibilists views mathematics as a human invention rather than a discovery. The major source of fallibilist nature of mathematics is Wittgenstein's philosophy where he described mathematics as a *language game* (Lerman, 1990), means that mathematics is created through communication. Mathematics is associated with our day to day communication and interaction.

While talking about the nature of mathematics different researchers have presented their views about the mathematics in different ways. In this context, Pant and Luitel (2016) mentioned that mathematics is a problem-solving game in which there are many routine mathematical problems focusing on algorithmic process aiming for the right answers in our mathematics textbook.

Similarly, Luitel (2012) has discussed the nature of mathematics as impure knowledge. While doing this he subscribed the soil metaphor which connects the relationship between people, culture, and land. Thus, here the word impure used to represent the local knowledge, informal mathematics, cultural artifacts that help to connect mathematics and human practice. Further, Luitel (2012) mentioned that, mathematics is an im/pure knowledge system which helps to empower the inclusive nature of mathematics. Whereas absolutist nature of mathematics is pure, and fallible nature of mathematics is impure. Likewise, D'Ambrosio (2001) mentioned that ethnomathematics is used to express the relation between culture and mathematics. His view was highly focused on cultural nature of mathematics. This is also popular in Nepal. More than one hundred different ethnic groups and many of them have rich mathematical practices in their day to day practice (Pant & Luitel, 2016). These kinds of practices help to promote cultural nature of mathematics or ethnomathematics. D'Ambrosio (1990) defines *ethno* as related to distinct cultural groups identified by cultural traditions, codes, symbols, myths, and specific ways of reasoning and inferring (as cited on Pant & Luitel, 2016). Thus, cultural nature of mathematics has

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been talked about as the mathematics that helps to connect human beings and mathematics.

Moreover, the government of New Zealand, Ministry of education (2006), mentioned in their school curriculum that the nature of mathematics is the exploration and uses of patterns and relationships in quantities, space, and time, way of thinking and solving problems that equips students with effective means for investigating, interpreting, explaining, and making sense of the world in which they like (as cited in Loveridge, Tylor, Sharma, and Hawera, 2005). These perspectives on nature of mathematics enables students to develop the ability to think creatively, critically, strategically, and logically.

The research conducted by the Loveridge et. al (2005) in title students' perspectives on the nature of mathematics. They articulated the different views of nature of mathematics given by the students that I mentioned inside in the rectangular box. This is really interesting to know that different students think differently about the nature of mathematics. This is constructed on the basis of their experience of learning mathematics as well as interaction with teachers, parents, friends and other stakeholders. These types of research enable teachers to think about their existing pedagogy in teaching mathematics as well as motivating teachers to teach mathematics in the more effective way.

> Math is not just about numbers; math is something that you can make really fun, especially with geometry and symmetry, because you can draw shapes and draw characters that you like. (Year 5–6). Patterns because plus is minus and plus is times and times is division and division are fractions and fractions are decimals and decimals are percentages and it goes on and on (Year 2– 4). Math is like something you use every day (year 7 – 8). (Loveridge et al. ,2005)

Above discussion on nature of Mathematics shows that there are multiple ways to explain the nature of mathematics. We need to be aware of all the perspectives of the nature of mathematics and must try to articulate which is more relevant and appropriate in this 21st century. In my point of view, cultural nature of mathematics enables students and teacher to make mathematics contextual. Cultural nature of mathematics also supports the nature of mathematics, mathematics as a body of im/pure knowledge.

HISTORICAL BACKGROUND OF NATURE OF MATHEMATICS

Talking about the nature of mathematics is not a new agenda. It had been discussed even before the fourth century. Plato and his student Aristotle are the first who provided the space to discuss nature of mathematics. From Plato's point of view, objects of mathematics had an existence of their own, behind the mind, in the external world (Dossey, 1992). As a mathematics student from the school level, now I am realizing that my schooling was shaped by Plato's point of view. In my schooling, I thought that mathematics was beyond of our thinking, abstract and discovered. In this context, a student of Plato, Aristotle, had different views. His views of mathematics

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were not based on a theory of an external, independent, unobservable body of knowledge but were based on experienced reality where knowledge is obtained from the experimentation, observation, and abstraction (Dossey, 1992). From this, I want to say that the observation of any object differs from person to person. This is based on their experience or related to how they perceive. Thus, there is no objective truth related to mathematics or there is no any objective answer to what mathematics is. What is mathematical knowledge? How is the mathematical knowledge constructed? Etc.

In the mid-19th and 20th century, new problems with the appearances of paradoxes in the real number system and the set theory were encountered, which has created other ways of thinking regarding mathematics. Dossey (1992) mentioned that to deal with such problems new views of mathematics arose, namely the school of logicism, school of intuition and school of formalism. School of logicism was founded by German mathematician Gottlob Frege in 1884. School of logicism is more related to Plato's views of mathematics. In which the (Rotman, 2006) mentioned that Platonist views mathematics is neither a formal and meaningless game nor some kind of languages mental construction, but a science, a public discipline concerned to discover and validate objective or logical truth. So, it depends on logic, proposition, mathematical symbols, objective subject matter, etc. Further, the proponents of logicism set out to show that mathematical propositions could be expressed as completely general propositions whose truth followed from their system rather than from their interpretation in a specific contextual setting (Dossey, 1992). Logicism claims that all the mathematical truth can be proved from the axioms and the rules of inference of logic alone (Ernest, 1991). Similarly, the Dutch mathematician I. E. J Brouwer started to think about the intuitionist school which accepts only the mathematics that could be developed from the natural numbers forwarded through the mental activities of constructive proofs. Moreover, intuitionist picture of mathematical assertation and proofs depends on the coherence and acceptability of what it means by an effective procedure (Rotman, 2006). Thus, intuitionist focus or claim that a certain mental construction has been carried out on mathematics instead of already established law and logic.

> School of thought created in the early 1900s to deal with the paradoxes discovered in the late 19th century advanced the discussion of the nature of mathematics, yet none of them provided a widely adopted foundation for the nature of mathematics all three of them tended to view the contents of mathematics as products (Dossey, 1992. P. 41)

School of formalism was introduced by German mathematician David Hilbert. He said that mathematics is arising from intuition, based on objects that could at least be considered as having concrete representation in the mind, mathematics was made up of the formal axiomatic structures developed to rid classical mathematics and its shortcomings. Formalist claim that pure mathematics can be expressed as uninterpreted formal systems, in which truths of mathematics are represented by formal theorem (Ernest, 1991)

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PEDAGOGICAL PRACTICES IN MATHEMATICS CLASSROOM

I agree with the statement of Thompson (1992) who advocates that a person's understanding of the nature of mathematics predicates that person's view of how teaching should take place in the classroom. It means that pedagogical practices of mathematics teacher are guided by their beliefs towards the nature of mathematics. In this context, Presmeg (2002) has argued that beliefs about the nature of mathematics either enable or constrain the bridging process between everyday practices and school mathematics (as cited in Loveridge et al., 2005). Thus, the connection between day to day mathematics, cultural artifacts, etc. are dependent on the teachers', as well as students' beliefs regarding the nature of mathematics. However, the authors of the mathematics textbook mentioned overloaded content which challenges to introduce student friendly teaching pedagogy in mathematics teaching. In my experience of teaching in the traditional school, my belief was shaped by the absolutist view of Mathematics in which I was guided by the concept "*one size fits all" pedagogy* (Stinson, Bidwell, & Powell, 2012) thereby, presenting myself as an authoritarian teacher delivering lectures related to facts and theorems, solving the mathematically routine problem step-by-step and asking students to solve the mathematical problem to prepare for the test. When I joined a progressive school, as well as Kathmandu University School of education (Kusoed) I got the opportunity to work with progressive educators, professors, and lecturers who always motivated us to critique the existing pedagogy and to change teaching pedagogy from 'one size fits all' to 'humanizing pedagogy' that values students and teacher, problem posing pedagogy, etc.

Accordingly, I have attended training, workshops, seminars and conferences, related to mathematics which were very helpful for me to modify my thinking about the beliefs and nature of mathematics that enable me to enhance activity-based instruction, collaborative approach, critical pedagogy, as well as experience- based learning which is highly fallibility in nature of mathematics. Moreover, sometimes students' and parents' beliefs towards the nature of mathematics affects in shaping the pedagogical practices because the teachers are deprived to fulfill the interest of parents and society as well. Thus, teachers may not get the freedom to play inside the classroom with their own way and strategy.

CONCLUSION

In this review, I looked into the nature of mathematics. It is difficult to define what mathematics is because mathematics as a subject is like a personality of human beings that can be looked from different angles. However, it is true that people's perception and beliefs towards the nature of mathematics is determined by their experience of learning as well teaching mathematics. Similarly, how people explain the nature of mathematics explains how they understand it. Accordingly, teaching approaches and assessment system of the teacher is governed by their beliefs about the nature of mathematics. The policy makers, curriculum developers, and

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others who are working in the field of mathematics have played a role to shape the belief (negative or positive) towards nature of mathematics. Moreover, the nature of mathematics like, fallible, impure and cultural enables inclusive education and provides equity and justice for marginalized groups of students as well as enables the teacher to incorporate critical pedagogy, activity-based approach, collaborative approach, etc. in teaching and learning process. Students are taken to be the center of the learning and teachers are like environment creator and facilitator, and the curriculum is guided by the images of the curriculum like *curriculum as experience, agenda for social reconstruction, currire* which has been discussed by Schubert (1996). Whereas, nature of mathematics like absolutist, pure, orients teacher to teach objective knowledge, facts, postulates, formula through rote and memorization thereby the marginalized group of students is blocked from learning. Teachers are being the center of teaching and learning activities. A standardized test is the ways of assessing student's achievement and the curriculum is guided by the images of the curriculum like *Curriculum as content or subject matter, curriculum as planned activities, a curriculum as a cultural reproduction* which has been discussed by Schubert (1996).

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SYMBOL AS METAPHOR AND METONYMY: A SOCIOLOGICAL PERSPECTIVE ON MATHEMATICAL SYMBOLISM

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Abstract: This paper uses the framework drawn from the writings of Jacques Lacan to analyse the nature of mathematical symbolism. In particular the debate between Newton and Leibniz over the notion and the notation of the differential is examined using the Lacanian categories of metaphor and metonymy. Focussing on a specific aspect of mathematical practice, the way in which symbols are handled by its practitioners, it attempts to underline the character of mathematics as a distinctive form of sociality. The analysis investigates a specific question. What do controversies over and reactions to mathematical symbols tell us about mathematics as a social practice? The argument shows that symbols are not just aides which people use in mathematics rather the very raw materials on which the practice is performed.

INTRODUCTION

Through numerous case studies science and technology studies have contributed immensely to demystify the experimental and laboratory sciences by problematising the idealised images of scientists and laboratories and carrying out *in situ* observations of material practices of actors, both humans and non-humans, through which the discourse of science takes shape (Latour, 1987; Latour and Woolgar, 1979). Having said that it is remarkable there is a huge dearth of similar studies of non-experimental disciplines such as mathematics and formal logic and that have been are few and far between[1]. Given the battery of concepts and techniques developed in the past decades to study scientific activity from a sociological perspective one would have expected similar studies of mathematics which also needs to be de-mythicised and understood along broadly identical lines. The other problem is that even when mathematics is looked at sociologically the social dimension is just externally appended to what is primarily taken as a non-social terrain of logic and formal reasoning. This is quite evident in Claude Rosental's (2008) otherwise remarkable study where the author demonstrates that controversies in formal logic are often negotiated and settled by concerns which properly speaking have nothing to do with logic itself. This is surprising given a long and rich tradition in social anthropology of studying logic and cognition which demonstrates that logic is not only externally but internally and pre-eminently social[2]. This paper is offered as a contribution to address this gap in the science and technology studies paradigm and also departs from that tradition in seeking to underscore the nature of mathematics itself as a distinctive kind of sociality. It borrows from science studies a close attention to the role of

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controversies and debates in the transmission of scientific knowledge and also takes a further step to underscore the concern with symbols in mathematics as a sui generis or autonomous concern that can be sociologically investigated. Thus the paper attempts to understand mathematics not just as one scientific activity amongst others but as a social whole in itself whose coordinates can be ethnographically or historically mapped. With that end in sight let us first briefly review the approach adopted in this paper.

The line followed in this paper attempts to apply the sociology of symbolism latent in Lacan's classic essay *The Instance of the Letter in the Unconscious, or Reason since Freud* (Lacan, 2006, p. 412-444) to the historical example presented here. It is a deliberate attempt to call it sociology of symbolism because in what follows the Lacanian categories of metaphor and metonymy are given an anthropological twist guided by Bhrigupati Singh's treatment of Georges Dumézil's categories of *Mitra* and *Varuna* in a recent paper (Dumézil, 1988; Singh, 2014, 159-187). Singh (2014) in search of appropriate concepts that could respond to the pressures of his ethnography asks us to see Mitra and Varuna, two modes of sovereign power, not as rigid structural categories but as 'ambivalent potentialities' inherent in a bipolar idea of sovereignty (Singh, 2014, p.166). He explains how the complex give and take between power and life can be ethnographically mapped through two potential tendencies of Mitra (contract) and Varuna (coercion) that are inherent in the encounter between the state and its subjects. Also, it is suggested that privileging one mode of power over the other will not be true to the insights offered by the ethnographic attentiveness that an anthropologist brings to the field. In a similar vein in this paper Lacanian categories of metaphor and metonymy are read not as separate formal mechanisms but as intensities and affects which are released when people doing mathematics create new symbols during their mathematical endeavours. In particular an attempt has been made to understand with the help of metonymy and metaphor as intensities or potentialities to understand the debate between Newton and Leibniz over the differential notion and notation. The paper seeks to bring out how following their reactions not only adds to our understanding of mathematical symbolism but also helps us in understanding mathematics itself as a form of life. Not getting too ahead of the argument let us first review what Lacan says about metonymy and metaphor before moving on to the historical example.

There are at least two important lessons to be learned about the tropes of metaphor and metonymy from Jacques Lacan's (2006) classic essay. One is that metonymy prepares the ground for metaphor to emerge. As in the line *the fog comes on little cat feet* the metaphor saying that something (fog) is something else (cat feet) carries such sublime force because the ground for it to emerge is already prepared through the metonymic relationship between feet (part) and cat (whole). Thus, metonymy enables the substitution of feet for fog by first transferring the idea of stealth from the little cat to its feet. The second lesson is that the bar of

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signification between the signifier and the signified is maintained in the case of metonymy suggesting that different signifiers can be used to convey the same meaning whereas in the case of metaphor the signifier crosses the bar of signification and 'stuffs' the signified thus creating new or extra signification[3]. Interpreting it differently it can be said that metonymy leads to the displacement of signifiers in the desire for meaning whereas metaphor attests to the autonomy of the signifier as it overshadows the signified thus indicating a shift in registers.

Using these two key lessons this paper follows the discussions around the symbol for the ordinary derivative, *dx*, between Newton and Leibniz. It is shown that whereas Newton was too caught up with the metonymic specificity of his fluxion notation and was thus unable see the significance of the *differential* in its own right, Leibniz could sense the conceptual crossing over by proposing a more metaphorical symbol. In the conclusion a sociological perspective on mathematical symbolism is outlined and it is suggested that various controversies over symbols that litter the history of mathematics can be understood in a new light using the framework described in this paper. Also, as part of the conclusion itself the possibility of exploring mathematics as a *form of life* is hinted at and some preliminary remarks are offered with regard to the same (Wittgenstein, 1953, p. 174).

NEWTON, LEIBNIZ AND THE DIFFERENTIAL NOTATION

Leibniz's penchant for accurate and carefully thought out symbols has been duly noted by various historians of mathematics[4]. Indeed, the extent to which he pondered over good symbolism is remarkable to say the least and this distinguished him significantly from his contemporaries. But more importantly, for our purposes, Leibniz revelled greatly in metaphysical speculation and his painstakingly worked out symbols played a pivotal part in these efforts of his. In fact, his metaphysical system is underpinned by complex mathematical themes and to grasp its full bearing it is necessary to understand how it is animated by them (Duffy, 2013).

More recently Deleuze and others have tried to draw the implications of this interlacing of the metaphysical with the mathematical in Leibniz's system. These discussions invite us to examine how do notations bear, invoke or are inseparable from the 'ideas' and what ideas themselves are in the first place (Deleuze, 1990; 2014; Smith, 2012). However not going into those details, let us see how Leibniz defines the concept of the differential to come straight to the issues I am trying to raise:

> Here *dx* means the element, i.e. the (instantaneous) increment or decrement, of the (continually) increasing quantity x. It is also called difference, namely the difference between two proximate x's which differ by an element (or by an assignable), the one originating from the other, as the other increases or decreases (momentaneously). (Leibniz quoted in Duffy, 2013, p.10)

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Note how the idea or concept of the differential seems identical with the notation *dx* in Leibniz's above formulation such that the two are taken to be the same although it goes without saying that it is only after considerable deliberation and sustained correspondence with fellow mathematicians that Leibniz settled with this notation. Now, what is it that the *dx* symbol stands for? As is clear from the above quote Leibniz designates infinitely small difference (instantaneously/momentaneously) with the symbol *dx* and in the process gives it a distinct quantitative or discrete sense. The difference for Leibniz matters for what it is, i.e. as pure difference. In this way the idea of infinitesimal difference is given a meaning which is primary and precedes any determination by already given and determined entities. This expresses a profound metaphysical stance which can be stated in simpler terms. Consider this apt reconstruction of Leibniz's position by a recent commentator where his crucial ideas regarding difference and continuity are brought to the fore:

> What is at stake in infinite analysis is not so much the fact that there is an actually existing set of infinite elements in the world. The problem lies elsewhere. For if there are two elements- for example, Adam the sinner and Eve the temptress-there is still a difference between these two elements. What then does it mean to say that there is a continuity between the seduction of Eve and Adam's sin (and not simply an identity)? It means that the relation between the two elements is an infinitely small relation, or rather, that *the difference between the two is a difference that tends to disappear*. This is the definition of the continuum: continuity is defined as the act of a difference in so far as the difference tends to disappear – *continuity is a disappearing or vanishing difference*. (Emphasis in the original)(Smith, 2012, p. 52)

In Leibniz's notation the differential relation is designated as $\frac{dy}{dx}$, which expresses the ratio of the infinitesimals *dy* (infinitesimal change in the value of the dependent variable y) and *dx* (infinitesimal change in the value of the independent variable x). There is an engaging theme to be followed about the primacy of relations (hinted at in the above quote) in Leibniz's ontology but I'll not follow that lead here for that will take us far away from our central concern. Rather I seek to draw the reader's attention to the manifestly *algebraic* treatment of the differential and the differential relation in Leibniz's discourse. This means that there is a tendency present in Leibniz to treat differentials as autonomous algebraic objects and this aspect of his thinking, i.e. bypassing geometrical intuition and demonstrating the possibility of doing calculus with infinitesimal numbers, troubled Newton the most.

These autonomous symbolic entities were simply *fictional* for Newton for he seriously doubted their intuitive and experiential availability for thought. He rather developed his calculus by giving primacy to the idea of *motion* and instead of thinking of quantities as independent algebraic entities he thought of them as resulting from change and variation with time.

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These continuously varying quantities like area, volume, distance etc., he called as *fluents* and their rate of change with time he called *fluxions*. What mattered more for Newton was the sense of physical intuition and the Leibnizian hypothesis of infinitesimally small numbers went clearly against this sense. To be sure ideas of 'instant', 'instantaneous' played as important a part in Newton's system as they did in Leibniz's, but instantaneous change was approached by Newton in terms of the physical intuition of limits, as a *tending* towards zero of the fluxion, and never was this infinitely small interval given an infinitely small number or quantity to represent it[5]. In fact, the question of infinitely small never arose at all! For Newton the difference was small but always *real* (and not infinitesimally small) and the smaller it got progressively, it mattered less and less for practical purposes. It is important to note that though these versions of calculus, of Newton and Leibniz, yielded identical results in practical application they differed significantly in their theoretical and philosophical aspects. More specifically for our purpose where Leibniz hypothesised infinitesimals such as dx, dy etc. Newton had no need of such quantities and it entailed a significant difference in their respective notations as well. As against the $\frac{df}{dt}$ of Leibniz, Newton had a 'dotage' notation for the fluxion i.e. $\dot{f}$. Florian Cajori points out:

> Newton's earliest use of dots, "pricked letters", to indicate velocities or fluxions is found on a leaf dated May 20,1665…The earliest printed account of Newton's fluxional notation appeared from his pen in the Latin edition of Wallis' *Algebra,2* where one finds not only the fluxionary symbols $\dot{x}$, $\ddot{x}$,…, but also the fluxions of a fraction and radical, thus $\frac{yy}{(b\dot{-}x)}$ $\sqrt{((aa\dot{-}x)^{\cdot}})$. (Cajori, 1993, p.197).

I am not interested in tracking the historical trajectory of how the *d*-ism of Leibniz triumphed over Newton's 'dotage' notation for I believe the bottom line of the point I am trying to make is clear by now. Controversy between Newton and Leibniz over 'better' notations, part of the larger controversy regarding who invented calculus first, reveals that there is more to notations than meets the eye or perhaps we need to take more seriously what meets the eye! Whether they are Newton's pricked letters or Leibniz's symbols it is impossible to miss that they indubitably carry recognisable metaphysical or metaphorical traces. Moreover it is their very *materiality* as signs, as peculiar inscriptions on paper, and the distinct physical sense that they connote (pricked, infinitely small,..) which constitutes them as objects for thought and gives them something more than a simple representational character. They problematise the very idea of representation in their wake. It is not important for us that this debate is considered settled for all intents and purposes in favour of Leibniz's supposedly more *logical* notations for the fact that this controversy happened, itself is very important for understanding the nature of mathematical symbolism.

Now, what does this debate tell us about the questions that we posed in the introduction? I would suggest, Leibniz's notation triumphed over Newton's

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because it enabled metaphoric ‘crossing over’ to the next level, a theme which we have discussed earlier. As pointed out before, Leibniz’s differential notation and the fact it can be expressed as a ratio anticipated the arithmetisation of calculus, which would take another century or so to be fully underway. The key metaphor involved here was taking infinitesimals as atoms or discrete quantities. This made possible and simplified a whole range of operations leading to a quick advancement of calculus as an independent branch of mathematics.

On the other hand Newton’s dotage notation was not so pliable. Rather than opening a realm of possibilities it was too caught up in its visual peculiarity. It’s metonymic contextuality was on the higher side overshadowing its metaphoric conceptuality. Reader’s must note that it was not as if Leibniz and Newton were entirely clear about this. There was no direct and easily demonstrable relationship between their metaphysical and theological views and the respective notations they came up with. It is rather that notations themselves became, to use Greimas’s terms, *actants* in their own right. Specifically, the potential for expansion was hidden in Leibniz’s notation itself and was certainly not foreseen by him in a clear form[6].

Yes, there was an elective affinity between Leibniz’s intellectualist conception of god, his theoretical bent of mind, attested by his engagement with drawing genealogies for nobility, his forays into natural philosophy, theology and metaphysics etc and the notation that he eventually used. Something similar could be said about Newton. His fluxion notation was less metaphysical and more practical, which again has elective affinity with, his volitional conception of god, his forays into alchemy and other such pursuits, his greater acumen in what became modern physics, and his supervision of coinage of the English realm![7] But that was it. With the benefit of hindsight and wealth of historical analyses at our disposal we can perhaps square it all, but both Leibniz and Newton were greatly ambiguous about their respective methods, notations and philosophies. We have read too much into their intentions and perspectives but if we look a little closely it emerges how hesitant and at times clueless they were of their fundamental methods and techniques. The point that I wish to underscore is simply that the debate and the context in which it took place is a fertile ground to analyse how metaphoric and metonymic intensities travel in mathematics and how symbolism comes to play a deeply constitutive role in its discourse. To reiterate, the agency of symbols precludes the possibility of any mathematical subject controlling them and sociologists of mathematics are best advised to see how they acquire lives of their own. It is remarkable how with this subtle but important change in perspective the entire debate is cast into a new light and new facts and interpretations present themselves. Moreover it suggests a possible entry point into the mesh of mathematical practices from an anthropological perspective. Same framework could be used for the iota symbol, *i,* for complex

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numbers but rather than exploring that equally enthralling historical episode let us move towards a tentative conclusion.

CONCLUSION

Veena Das (1998) in her subtle critique of Saul Kripke points out that he overrates the importance of agreement on rules rooted in the appeal to community in his interpretation of Wittgenstein's solution to the sceptical paradox. Rather taking a cue from Stanley Cavell she points out that, agreement *in* a form of life 'is a much more complicated affair in which there is an entanglement of rules, customs, habits, examples, and practices and that we cannot attach salvational importance to any one of these in questions pertaining to the inheritance of culture' (Das, 1998, p.176). These insights have been put to use in this paper by setting forth that agreement on or belief in mathematical assertions or symbols cannot be taken for granted. Moreover it is shown that agreement in mathematics, taken both as a language and as a form of life (Wittgenstein, 1953), can be seen to be growing around the complex process of the mutual absorption of what may be variously called mental, cognitive or intuitive and the social.

The description offered in this paper also demonstrates that the concern with mathematical symbols is an important concern *in itself*. This is so because mathematical symbols are not simply representative of abstract ideas they are also *constitutive* of them. This is particularly true of mathematical ideas which are complex and are difficult to grasp intuitively. The argument made in the paper shows that the work of constituting such concepts is played by mathematical symbols and other forms of writing which are unique to mathematics. It can also be added that ironically mathematical symbols simplify complex operations by *enabling* less thinking because the function of thinking is delegated to them. Clearly then there is more to symbols than just representation.

To reiterate the point that has emerged through the paper, aligning the two axes of metonymy and metaphor respectively with the twin senses of form and life into which the expression 'form of life' in Wittgenstein (1953) can be broken one gets a simple and tentative structure through which various controversies over symbols in mathematics can be analysed and mathematics itself can be seen as a form of life[8]. It has been shown in this paper how in one context metonymic displacement fails to lead towards the emergence of the metaphoric potential of the symbol (Newton) and in the other this process takes place remarkably well such that a vague and intuitive shift to a different register is given a concrete form through the agency of the symbol (Leibniz). I would like to end this paper by emphasising three key points which can be taken up for further clarification and discussion. Firstly, the framework drawn from Lacan can help us to revisit controversies in mathematics over symbols and to recast them in a new light. The figure of metonymy helps us to recognise the displacement of symbols and serves to direct our attention to the context and the figure of metaphor helps us to see

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how new ideas finally emerge out of their inchoate and rudimentary forms. Secondly, these two tropes also enable us to focus upon the symbols themselves because as we have seen symbols acquire an agency and life of their own, wriggling free of the mathematicians who invented them. Thirdly, the paper also proposes that this framework can be used to study mathematics as a form of life, both historically and ethnographically. The purport of this to understand how the cognitive and the social are mutually absorbed into each other within mathematics taken as a vibrant and lively social practice.

Lastly, I would like to end this paper by making some comments relevant for mathematics education. I hope the paper has demonstrated that as much mathematics is a cognitive activity it is also social. The intertwining of the cognitive and the social emerges most clearly in the complex and multiform process of mathematisation. The historical episode that I have recounted shows how mathematical knowledge proceeds through encounters between signs and individuals. In terms of mathematics education this shows that we must think about learning in mathematics beyond the framework of representation. As Deleuze tells us, 'learning takes place not in the relation between representation and an action (reproduction of the same) but in the relation between a sign and a response (encounter with the other)' (Deleuze, 2014, p. 27). More concretely learners should be allowed to see and experience for themselves what symbolisation does in mathematics. This is important because it will reorient learning mathematics towards learning how to mathematise and to think mathematically. Grasping the meaning and purpose of symbolisation is extremely significant because as I have repeatedly asserted in the paper symbols are not only representative but also constitutive of ideas. Also it reduces the sense of *arbitrariness* that students often feel while learning mathematics. Symbols like that of the differential or epsilon-delta notations might come across as mechanical contortions but if the symbolism is carefully grasped students will see how they are internally motivated and necessary. It goes without saying that this would considerably enhance the pleasure and purpose of learning the subject.

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Notes

[1] One could think of early studies by Bloor (1973) and Livingston (1986), although strictly speaking these studies do not follow the science studies framework. Recent studies more in tune with the science studies paradigm are MacKenzie (2001) and Netz (1999). For an early philosophical classic see, Lakatos (1976).

[2] See in particular Descola (1994) and Douglas (1986). Both try to address the classic Durkheimian problematic of understanding what factors are responsible in 'forming the intellect itself' (Durkheim, 1965, pp. 21).

[3] In Lacan's text metonymy is symbolised as follows: $f(S \dots S')S \cong S(-)s$. Here displacement is indicated by '…'. This displacement is from one signifier (S) to another (S') such that the bar between the signifier (S) and signified (s) is maintained. And metaphor is symbolised as follows: $f\left(\frac{S'}{S}\right)S \cong S(+)s$. It can be taken as symbolising that signifier S'replaces the signifier S such that the bar between the signifier (S) and the signified (s) is crossed leading to the creation of new meaning (+). See Lacan (2006, pp. 428-429).

[4] See for example Florian Cajori's (1993) monumental work on mathematical notations and Carl Boyer's (1959) influential work on the history of calculus.

[5] As Roy Wagner points out, "The limit here is that of a continuous and finite process. The evanescent quantities (short spans of time and space toward the end of motion) indeed decrease without end—that is, indefinitely, with no final state. Indeed, their disappearance at zero is not, considered a finite state, as zero is viewed as absence, rather than an existential state (similarly, contemporary mainstream mathematics considers the sequence of increasing integers as having no final state). But the ratios, whose limit is of interest here, span a finite and continuous range. The limit is simply the end of this range. The operative metaphor here carries the notion of "limit" in the sense of end or edge from the domain of geometric magnitudes to that of ratios, such as velocity" (Wagner 2012: 132).

[6] This shows that a mathematical symbol has an agency in its own right.
[7] For these historical and biographical details see, Boyer (1959).
[8] As Veena Das and Clara Han in a recent interpretation point out, 'Cavell's analysis of the two separate dimensions of the expression *form* and *life* and Das's elaboration of the idea of *naturalness* …help us see the two aspects of the expression form and life as nestled in each other: socio-cultural differences, or the form that human existence takes, as well as the way in which the social and the natural mutually absorb each other' (emphasis in the original) (Das and Han, 2016, pp. 3). A slight inflection that has been given to these ideas in this paper is that the theme of the mutual absorption of the natural and the social has been altered to the mutual absorption of the intuitive, cognitive or mental and the social. No doubt these ideas and the interpretation they have been put to need further fleshing out but as this is not the major objective of this paper or rather the objective of this paper has been to prepare a ground for these ideas to emerge they have only been suggested and briefly explored in the conclusion. Even the structure that is outlined needs to be refined further but at least it suggests a creative possibility to be used in further opening the mathematical practice for anthropological research.

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MATHEMATICS OF EXCLUDED WOMEN
A PERSONAL NARRATIVE

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Exclusion definitely affects a women's mathematics. Some classification of the factors that shape the unique features associated can definitely be specified. On the basis of these considerations I propose a new partial theoretical model for accessing the connection between a woman's attitude towards and practice of research in mathematics, and the discrimination faced by her. My approach to the subject has been very non-standard and unique with much focus on representing reasoning and knowledge. In this narrative research (or reflective paper), I try to explain the shaping of my mathematics at the research level over many years in the clouds of many exclusions in the perspective of the model mentioned.

INTRODUCTION: BASIC FRAMEWORK

Exclusion and its consequences have their hierarchies that vary with one's perception of sociological context and related assumptions. Women as a class are uniformly excluded relative to a level of exclusion. Trans women and lesbians face much higher levels of exclusion at all stages of their life. These exclusions affect most aspects of life, learning and creative activities of all kinds and in particular research level mathematics.

In feminist and sociological studies, it is well known that women react to systemic discrimination and suppression in different ways (by way of becoming door mats or slaves, door mats that play with the contradictions of capitalism-patriarchy nexus, through cautious self-centered negotiations, applied feminist approaches, overt rebellion, activism in the tradition of the personal being political, uncompromising rebellion, and others). Often their work in fields that involve intellectual labor are colored by an urge to highlight their suffering or an urge to break free. A classification of such subtle modes of expression can be computed with words and will be referred to as *proactive feminist attitude*. I think that these relate to attitudes and ways adopted by women in mathematical research as well. Admittedly, the available studies do not go into the fine details of relatively successful researchers - because the level of interdisciplinary interaction necessary would be hard to attain. Most studies on STEM research and gender tend to focus on passive conditioning attitudes alone (as, for example, in (Leslie, 2015)). Some narrative accounts (Subramanian, 2017) are available – but these have not been optimized for illustrating the nature of sexism and discrimination. In contrast, attitude of students towards mathematics can be accessed by the three dimensional model (involving emotional dimension, vision of mathematics, and perceived competence) (Zan, 2013).

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One can be bold and work within frameworks and problems suggested by others, or one can be bolder in seeking out newer problems or one can construct frameworks that question the existing status qua or one can break free from the institutional chains of the research ecosystem - the possibilities are finite. Attitude of researchers in mathematics towards research has largely been governed by the whims and fancies of funding agencies and the limited interests of the people-in-charge. People who try to work outside the traditional order can be labeled rebellious. Such work may or may not involve additional philosophical insight on part of the researcher. Most trans women are women who have endured systemic suppression from society to the point that they have lost hope of fitting in any way - if these women love research in mathematics then their mathematics is likely to be rebellious.

The above two paragraphs can be regarded as a new proposal for a partial theoretical model for accessing the connection between a woman's attitude towards and practice of research in mathematics, and the discrimination faced by her. I will refer to this as the *Discrimination responsive Attitude modulated Practice of Research*- (APRD) model. The present paper is a contribution to the rebellious category in the model – Does it relate to the practice of research? Further aspects of the model will be developed separately.

Narrative research is common in feminist studies, grounded theory and is also well known in mathematics education research as in (Connelly et.al, 1990). In fact, the three dimensional model (Zan, 2013) was arrived at through focused narratives. This paper is in the form of a personal narrative, in which I also seek to find connections for the APRD model mentioned earlier - am the only trans woman in mathematics research in the country.

I am a leading researcher in the foundations of rough sets (a formal approach to vagueness), algebra, logic and allied fields (but my mathematical career has been a very nonstandard one and am sort of a polymath). I transitioned relatively late and am also a lesbian. In this brief narrative (with many explanations), I will also try and highlight important aspects of my development and of being a trans woman and a lesbian in mathematics at different levels.

Writing this narrative was difficult for a number of reasons. Mathematics is everywhere and any context that involves enough mathematical insight should qualify. That in turn includes things like knowledge representation, feminism and related reasoning. I am not too sure about the boundaries that I have set in relation to this aspect. Not many readers may be aware of the intricacies of gender transition and related issues. So I have explained these in the following section before delving into research in mathematics and related politics.

GENERALITIES ON SEX, GENDER AND DYSPHORIA

This subsection is intended to clarify the terminology and concepts of sex, gender and sexuality used in this paper. From a modern scientific perspective, the sex (or biological sex) of a person is best seen as a tuple of parameters corresponding to

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hormonal, brain, clinical, chromosomal, physical and epigenetical sex, and more. Gender is plural term that refers to *gender identity, gender expression and gender as a social construct.* Serano (2013) clarifies much on these. It is important to distinguish between the terms in any discourse.

Gender identity of a person is the person's innate sense of gender and is intrinsic to the person in question. There are studies that show that it is strongly influenced by prenatal development. It has been shown to be hardwired modulo different assumptions and connections with genetics are also known. There are studies ((Spizzirret et. Al, 2018), for example) that also relate gender identity to brain structure, hormones in brain, and other modern biological markers of sex.

A person's understanding of their own gender identity and coherent gender expression has direct connections with their well-being. But the problem of determining a person's gender identity is not an easy one due to the machinations of patriarchal power structures. People tend to develop their own dialectics to different extents and these share many patterns. Mostly when people are assigned wrong gender at birth and have sufficient dysphoria do they want to express the incoherence with their gender identity. The patriarchy and discriminatory power structures try to interfere even with this process by imposing various strictures on identity, expression and punishment mechanisms that amount to proactive gas lighting and criminal oppression. In my opinion, the idea of '*sufficient*' in '*sufficient dysphoria*' can be characterized in terms of possible sets of attributes (Mani, 2014C). Typical transition procedures for a binary-identified trans woman includes HRT (hormone replacement therapy), laser treatment and GCS (gender confirmation surgeries). Recovery from GCS procedures is painful and warrant medical leave for few months.

An alarmingly large percentage of trans women actually attempt suicide at some stage of their life because of social exclusion, isolation, discrimination and loneliness– this is indicated in the US survey for example (James, et. al, 2016). I have dealt with the nature and existence of connections between loneliness, exclusion, sexuality and suicidal tendencies in a recent paper (Mani, 2016). It is also known that loneliness can have a number of negative consequences on humans that include: reduced lifespan, heath problems, lowered level of trust levels in others, feelings of social incompetence, victim mentality and self-consciousness. A closely related concept is that of *depersonalization* (Costa et. al, 2016; Jones, 2017). It can be regarded as a common symptom of gender dysphoria. A person experiencing depersonalization would feel that certain feelings and experiences of oneself or the world as unreal. Depersonalization does not alter perception of reality but induces a level of detachment from the world and numbing of emotions.

MY TRANSITION IN BRIEF

A dense account of my transition can be found in my blog post (Mani, 2014). Basically I had gender dysphoria since my childhood. Related symptoms and depression intensified in middle school. I did well in my school final examinations

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though. Subsequently I became severely deviant from prescribed "course work" - diversifying into higher mathematics, physics, psychology and politics that did not quite belong to the higher secondary (HS) syllabus. I could not cope with dysphoria and my performances in the exams were erratic - obtained a rank in medical entrance examinations and bad result in the HS examination. I worked with untreated depression for many years – till I discovered the importance of medicines myself. My B.Sc. results were also affected by depression. Dysphoria was however not the only reason for my severe nonstandard style of academics – the system was defective.

I had good access to books, libraries and journals even during those days. Possibly, because of my problems, I became pretty knowledgeable in psychology and psychiatry. The limited information about trans-sexuality in “high-quality” journals that I could find then was not good. The literature on lesbian trans women were not widely known. But I managed to work out my gender identity and orientation correctly, and even consulted a psychiatrist for clearances for HRT (Hormone Replacement Therapy) and GCS (Gender Confirmation Surgery). But could not proceed because of opposition at home, and the psychiatrist was also not modern enough. My plans were stalled then. My first suicide attempt happened a few months after this. My parents have always been of a conservative, highly superstitious, religious, casteist, bigoted, narcissist and brainwashed type serving the interests of the patriarchy and capitalism. They have always been a hindrance.

I got into research in mathematics independently after B.Sc. and a PG diploma in statistics from ISI. I was elected as a member of the Calcutta Mathematical Society shortly after that. Also I did not accept an offer to join Prof C.S. Sheshadri’s team in Chennai during the time. More aspects of my research career are in a separate section in this narrative. Over time I published a part of my rather large amount of my research in many top grade international journals. I also did a master’s degree in mathematics after publishing over 150+ pages. As of now the figure is over 1050.

Much later, when I had to decide between suicide and living in late 2012, I decided to transition fully. My medical and legal transition happened smoothly in subsequent years. The NALSA judgment (NALSA, 2014) simplified things tremendously. Social transition was also easy because I had been introverted and unsocial all through my pre-transition years. My relationship with my ex-girlfriend and girlfriend was also problem free because we are all lesbians, feminists and rather mature. The long distance aspect has been an issue with my girlfriend.

The physical part of my transition was very easy for me as I was already good looking, femme, healthy, athletic, response to hormones was excellent, had no complications, a partial class and an Asian advantage. Suicidal tendencies vanished completely on commencement of HRT itself but am yet to fully come out of the grips of dysphoria related adaptations of pre-transition period (Mani, 2014A).

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I did a late doctoral degree in mathematics during my transition period too. I faced plenty of bureaucratic delays at various stages of it. The review period for my~300 page thesis was also extraordinarily long.

As far as my sexual orientation is concerned, during my school days, I was essentially a lesbian. I have always been sexually attracted to a large spectrum of femme women. Pre-HRT, I was also attracted to so-called "boys" -- femme types bordering on butch. So I used to think that I had bisexual tendencies. However I have always held that "any sufficient aggregation of masculine features in a person" is a sign of decadence. So much about boys and men. I have always been mature enough not to translate this into general misandry. My preferences shifted towards larger classes of women during and after HRT. Overall since my high school days, I have always identified as a lesbian and being a lesbian is not just about my orientation but is part of my identity. It did affect socialization, but I am not mixing that up with dysphoria related reasons.

I never had any real role models, though I have always admired a lot of women (mostly older) and often abstracted and adapted their positive traits in my own way.

EARLY SCHOOL

I cannot remember much of my days in primary school, but I was an excellent student with a good collection of merit cards in science and mathematics. There were five types of exercise books - for mathematics there were ones with small and big check patterns. The ones with pink big check patterns were more beautiful and easier to write into. When I was around three years old, I remember studying a number of patterns found in my surroundings. Things like brick walls without plaster, open dimly lit spaces, complex objects, and possibly related emotions. I have no recollections of people or any sound that I may have heard.

I remember doing more of algebra and arithmetic in secondary school. Collaboration was not really encouraged in schools and it was mainly up to individuals to understand from the teachers instructions and/or the thing in books and possibly reflect on them. I have always been introverted and so I liked to work alone. I do not remember lessons or maybe their physical context. But I do have some recollections of solving problems. But I am not saying that it was like “I solve; therefore I am :-)”. All science subjects including mathematics were my favorites and I did like experimenting beyond what was in the curricula and even at home. Further I had access to a rather large collection of books on science (especially physics) and fiction (all in the English language).

RESEARCH CAREER: AN APPROXIMATE ACCOUNT

Initially my research interests in mathematics included analysis, fixed point theory, summability, semi-group theory and partial algebra mainly. I was pursuing all this without any formal association with an institute and through self study. I had access to plenty of journals and books as always. I ended up with a substantially large and unusual background in research-level mathematics (and philosophy too). During this time, I had a few opportunities like joining a research group in algebraic geometry

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(mainly). But I did not join them. I was not too interested in publications at this stage and was always working alone. I suffered from depression, gender dysphoria and suicidal tendencies during this phase - but was not too clear about life goals. Also I was into part-time teaching in mathematics and science during the latter part of this phase.

Gradually my research interests shifted to algebra, rough sets, vagueness and logic. Most of my work is on foundational issues and I invent/use very imaginative and therefore risky methods and approaches. I have published many excellent papers before, during and after transition during this phase. One line of research in which I have been very successful is in formalizing causation in the contexts of formal approaches to vagueness in a granular way. I will not digress into other subareas and technicalities here.

That is despite managing two careers simultaneously. I have also been involved in part-time projects in soft/statistical computing especially after 2004 and have been an active free software activist and contributor since much earlier. I should also mention that during most of my pre-transition days, the basic plan was to achieve as much as is possible before opting for suicide - because I saw no open doors. It was basically a decision and given the substantial level of dysphoria including disconnect from the world, it was a reasoned decision rather than an emotional one. I changed this decision primarily because of an activist mindset and additional knowledge about the dynamics involved, though it is true that 2012 was also an academically relatively successful year for myself. Previously I used to think that thirty plus was too late an age for proper transition under the circumstances.

My productivity rate during my pre-transition period has been less than that after the start of my transition in late 2012. Transition also involves adjusting one's lifestyle in puberty-like state - the previous statement stands with or without this statement. From my experience, I can say that HRT helps one concentrate and think better, and this holds even during the beginning stages of HRT.

Employment Question

Pre-transition, I never saw anything in full-time jobs in teaching or other sectors. Working for the corporate sector is death, the idea of working full time in the corporate sector is based on decadent capitalist values, teaching jobs are ill conceived, jobs outside academics are less sensible and most importantly "*you do not need the property papers of a building to jump off it in the near future*". And there has always been the need to evolve alternative leftist strategies in the decadent Indian economy. Somehow the dynamic part-time opportunities in the teaching, short-term project, freelance academic and statistical/soft computing consultancy sectors fell in place to an extent. The main advantage was that I could devote more time to research. Yes, I have always looked down on the slave mentality of the middle and upper middle classes on the job question.

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Post and during transition, I have also been involved with course development work for foreign universities and massive online open courses. But I do face mountains of discrimination, and some transphobia.

COLLABORATION AT RESEARCH LEVEL

A researcher can collaborate with others in the research context by way of reading, reflecting and re-purposing parts of sets of published recent research done by other researchers in the same subarea and in the context of the same problem with due acknowledgment. This statement can be modified by possibly replacing

- "reading, reflecting, and re-purposing" with "rejecting", "adapting", "improving", "initiating" and many other terms,
- "published recent research" with "published research over a long period of time", "new research" and many other terms,
- "same subarea" with "related subarea", "apparently unrelated area" and others,
- "same problem" with "related problem", "unrelated problem", "orthogonal problem" and others,
- "with due acknowledgment" with "with partial acknowledgment", "jointly with some others" and others, and
- "others in the research context" by "specific researchers in the research context".

In fact researchers can collaborate with others in a number of ways and these ways form questionable hierarchies based on questionable rationales of evaluation of collaboration.

In this scheme of things, I have never really collaborated at the level of producing joint papers (except for a joint paper in an allied field to date). All of my research papers have been authored by myself alone. This has to do with my style and approach to research that I have explained earlier. The trans aspect in the sociopolitical conditions has had a clear influence on this. The latter is very important - there are a large number of trans women in academia with regular and regular-high-impact research careers (see for example (Conway, 2005). It is also true that I have been part of academic groups like the Calcutta Logic Circle and others for many years.

I do not always do research with the sole intention of publishing them and a substantial part remain unpublished. In the early phase, I was not particularly good in communicating and interested in publishing and was too independent to collaborate. In the pre-transition phase after this my collaboration level has been much better as indicated in publications, participation in conferences, peer review, societies, workshops and other academic events.

Some people (mostly men) were definitely bigoted, misogynist and trans-phobic towards me during and post transition. Some others took time to get adjusted to the facts. While some others were excellent. See my article on trans inclusion (Mani A, 2014C) for more light on these remarks - being a functional feminist is an essential

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requirement for survival. I do face discrimination in India in particular, because not everybody is well educated or rational. Even then I have had much impact on the mathematics, logic, philosophy, computational intelligence and computer science communities, So much of inclusive sex and gender sensitive education has never been seen in the country. Otherwise I have improved on my level of collaboration.

TRANSITION AND RESEARCH STYLE

Most of the problems/theories that I work on are of a foundational nature, but I do keep track of possible practical applications. Understanding vagueness involves plenty of philosophy, related logics and insight in practical problems. This works well with my approach in which I place much stress on exactness, formalizability and beauty of proofs and justification. In my pre-transition state, the problems of dysphoria, depression, a certain amount of depersonalization (that was part of my gender dysphoria) did slow down my work. I preferred to work in a less connected way relative to other people's work and simultaneously my work was colored by an urgency during my pre-transition days. Basically I was simply concerned with the research in question and nothing much. But it is also true that I substantially improved my research communication over time.

As mentioned before depersonalization in pre-transition state is part of dysphoria and related to loneliness and suicidal tendencies. This has also been examined to some extent in the available literature. Risk in the context of research can be understood in multiple ways. One useful definition can be in terms of potential reader’s capabilities to comprehend the research being communicated. Sometimes I become so innovative that reviewers get floored. At other times, I have taken risk in accordance with my pre-transition goal to maximize quality of output (relative to myself) because of 'limited time'.

During and after transition, I had to adjust research strategies, improve communication and to an extent work harder on connections. This can be seen in my research work in and after 2013. I have also succeeded in extending my mathematico-logical approach to subjects like feminism.

Some studies indicate that HRT can be expected to affect brain functions. Many of its good effects are known, but not much is known about the impact on research- level mathematics. From my experience, I can say that it improves clarity of thinking, total volume of both numeric, symbolic and linguistic computations, emotional content of communications – this matters in mathematics too, imagination (some of my recent papers involve more imaginative attacks but am also getting mature with time) and memory power too (this is sometimes reflected in the scale of interdisciplinary tasks that I have done).

CONCLUDING REMARKS

I hope that this narrative paper helps in understanding some of the inter-sectional aspects of gender diversity and its impact on women in mathematics. Because of the

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discriminatory social order and a history of severe anti-trans discrimination and genderism, it is trivial that trans women need reservation and additional support in the education sector – this narrative confirms as much.

The proposed APRD model obviously requires additional work. From the narrative, it is possible to abstract subjective values corresponding to motivation, genderism faced, other discrimination faced, quality of work, social attitude, research style, and other parameters, research-attitude and pro-active feminist attitude. Of these, the parameters *research-attitude* and *pro-active feminist attitude* have the potential to predict parts of the others. This is significant.

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HAPPY AND HEALTHY FAMILIES! FINANCIAL MATHEMATICS AND THE MAKING OF THE HOMUS OECONOMICUS

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Which notions of subject do financial mathematics activities in Brazilian high school textbooks offer to students? Using Foucauldian tools such as discourse analysis and the idea of the constitution of the subject, we have analyzed the textbooks approved by the Brazilian National Textbook Program in 2015. The discourse that connects ideas of "good" economic practices and financial "health" suggests that, if the instructions contained in the activities, a financial healthy life will be achieved. This discourse presents the family as company that produces human capital, constituting each member as a self-entrepreneur, responsible for the success or failure of the company-family. Economic success is also linked to happiness. Financial mathematical activities subjectify students to be inserted in a society governed by neoliberal policies.

INTRODUCTION

In this paper, we advance further some of the results in Coradetti (2017), in which she analyzed discourses in the mathematics textbooks, focusing on the theme of financial mathematics. One of the main results evidenced in the previous study was that the approach to the importance of financial mathematics in people's lives led to norms on how to produce a happy family, a happiness that is directly related to the adequate management of money, through a similar family constitution to the constitution of a well-managed and efficient company, formed by economically useful and enterprising individuals (Coradetti & Silva, 2017).

The financial crisis of 2008 as a world event that affected many countries —and people— triggered numerous reactions, among those the absolute necessity to provide financial education. In Brazil, initiatives for financial education were located, among others, in the mathematics curriculum. Indeed, the criteria for the production of official textbooks made public by the Brazilian National Textbook Program (BNTP) stated that:

> One of the distinctive features of the recent changes is the increasing use of mathematics in everyday social practices - purchases and sales, loans, credit, bank accounts, insurance and many others - as well as in scientific or technological activities. Especially in the daily life of the citizen, the repercussions of the new technological resources of the computer and of the calculator, both widely diffused in all the social means are evident (Brazil, 2013, p. 57).

The "everyday social practices" mentioned in the above document include instructions on the best way to consume. The document is emphatic and normative about the

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importance of the textbooks to include these social practices, emphasizing that "a mathematical education appropriate to the final phase of Basic Education can not neglect the above mentioned aspects" (Brazil, 2013, p. 58). As a result, financial mathematics became an area of the mathematics curriculum that brings together the aspirations of financial literacy and of mathematics education.

At first sight, it would be difficult to question the desirability of such initiative. At the same time, the question emerges, about the directions in which the financial mathematics capable child gets conducted. In this paper we problematize how statements on financial mathematics, present in Brazilian high school textbooks, constitute particular notions of who are and what characterizes the financially sound child and family. We analyse how financial mathematics appearing in the problems and illustrations articulate the transmission of scientific knowledge as the tool to rationally plan the future, with a republican morality (Tröhler, Popkewitz, & Labaree, 2011) that makes individuals and families the basic unit of economic stability. The idea of sound economic behaviour —controlled and regulated with good mathematical calculation and reasoning, intersects with notions of healthy consumption and familiar happiness. Such articulation contributes to the making of students as subjects current financial capitalism.

THEORETICAL AND METHODOLOGICAL ASPECTS

We have used a Foucaultian discourse analysis because it allow us to explore how the mathematics curriculum produce subjectivities. Foucault (1995) made it clear that his great objective in his research trajectory was "to create a history of the different ways in which, in our culture, the humans became subjects" (p. 231). The subjects are constituted in and by the discourse, that is, the subject is subject in the etymological sense of the word, "is an empty place - that when it being occupied reflects and is reflected by the relations of power that organize the discursive possibilities operated in this space" (Monteiro, Mendes, & Mascia, 2010, p. 54). In this sense, analyzing the instructions contained in topics of financial mathematics in high school textbooks is to describe discursive possibilities that make students become subjects of the discourses produced.

We have analyzed discourses articulated on the surface of mathematical content in textbooks. These textbooks' discourses instruct students in an educational institution with neoliberal characteristics. Instructions that consider the school not as a "place where ideologies are taught and learned, it [school], much more than this it is understood as an institution in charge of manufacturing new subjectivities" (Veiga-Neto, 2013, p. 38). These subjectivities are generated by processes created in our society, in our culture, making man a subject attached to an identity that is attributed to him as his own. The analysis material was six textbooks approved by Brazilian government. Financial mathematics was contained in all books. In five of them, the content appeared in a specific chapter. In one, financial mathematics was in a session inserted in algebra. In all, about 115 pages were analyzed. The examples we bring in this article are the most significant for articulating content to the constitution of family

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values. Like Foucault, we present some examples to illustrate the discursive regularities found in the analyzed material.

Before we present some analysis, it is important to present some points about neoliberalism, because this political model is embedded in our practices. Neoliberalism is a form of government that emerged in the mid-twentieth century with publication of Friedrich Hayek's book in 1944, which calls for an economic doctrine, restricting to state intervention in the economy and the free-market fundamentalism (Höfling, 2001). For Veiga-Neto (2013), one difference between neoliberalism and classical liberalism is that, in liberalism, the idea of market freedom was seen "as something natural and spontaneous" (p. 38). Already, in neoliberalism, this freedom "must be continually produced and exercised in the form of competition" (p. 38).

In this rationality, the state dissociates itself from some obligations, withdrawing its interventionist role and passing that responsibility on to the subject or large corporations, in which the state is "a set of institutions already established, of a whole set of realities already given" (Foucault, 2008b, p. 385). In this sense, it is considered plausible to think of a formation for the people who attain this type of society, being this the fundamental role of the curriculum and the schools. The mathematics curriculum enters the game of true and false, addressing economic instructions aligned with the neoliberal ideal, through financial mathematics instructions. In this way, the school becomes a place that manufactures governable subjects, produced in a neoliberal rationality. By governable subjects, we mean those that can be conducted - by themselves or by others - by actions established in power relations (Veiga-Neto & Lopes, 2010) that take place in the subtlety of discourses of a naturalized mathematics curriculum.

FAMILIES AND FINANCIAL MATHEMATICS

The purpose of presenting this brief description of neoliberalism is the intensity with which these ideas cross images analyzed, contributing to the formation of a type of identity aligned with these ideals, as we will see in the excerpts below.


Aplicações

Trabalhando, poupando e planejando o futuro

Um jovem casal sem filhos, cuja renda mensal conjunta é R$ 3000,00, decide organizar uma planilha de custos para equilibrar o orçamento doméstico. A análise dessa planilha nos primeiros meses revelou ao casal que, descontados os custos fixos, como pagamento da prestação do apartamento e de contas de consumo, transporte e alimentação, sobram ainda R$ 500,00.

O casal tomou, então, uma importante decisão: reservar R$ 250,00 desse excedente para gastos eventuais e aplicar, mensalmente, a quantia de R$ 250,00 na caderneta de poupança, pelos próximos dois anos, a fim de construir uma reserva financeira. Vamos admitir que o rendimento mensal da poupança seja de 0,7% ao mês nesse período.

O controle das despesas do lar é o primeiro passo para o equilíbrio do orçamento doméstico.

Working, saving and planning the future

A young couple without children, whose combined monthly income is R$3,000.00, decides to organize a spreadsheet of costs to balance the domestic budget. The analysis of this spreadsheet in the first months showed to the couple that, after deducting fixed costs, as payment for the provision of the apartment and of consumption, transportation and food bills, there were still R$500. The couple then made an important decision: save R$250 per month and to apply in the savings account for the next two years in order to build a financial reserve. Let us assume that the monthly income of savings is 0.7% per month in this period.

Figure 1: Family in textbooks (Iezzi, Dolce, Degenszajn, Périgo, & Almeida, 2013, p. 163, v.3).

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The text accompanying Figure 1 is entitled “Working, Saving and Planning the Future”; it points out that a young couple took “an important decision” - that of saving a monthly sum to have a financial reserve.

In our analysis, the figure shows “traditional” family model suggested by the image of a heterosexual couple without children. Many efforts have been made to normalize a plurality of ways to start a family in our society, even in other areas of knowledge, as in the work of Fiuza and Poli (2015), in which authors conceive the family in the “condition of nucleus of dynamic peculiarity, it can assume varied contours” (p. 151). However, seems to prevail in mathematical textbooks, fixed forms of what is considered a “true” family. What we want with this analysis, it is explicit that, along with the supposed objectivity contained in a properly math problem contextualized and the commendable role of teaching compound interest, the alchemy of school mathematics (Popkewitz, 2004) also teaches how to be a family, what is the right or acceptable way of living in society, among other apparently non-mathematical instructions.

The informational text contained in Figure 1 also seems to reinforce, for example family, one that makes family planning, which leads us to the idea that this doctrine task on the production of responsible subjects, which contribute to their own success to the success of your family and to that of the state economy. This analysis is based on what Foucault (2008a, p. 330) mentioned that in neoliberalism there may be “mercantile and non-mercantile” relations; in this case, the relationship between modern Western family model and financial mathematics of high school mathematics textbooks can be established by non-market relations.

Non-market relations are recognized as the social phenomena that influence the market, that is, the social influences that focus on the population, and that affect economic relations. Encouragement of this model of the modern Western family can provoke these non-market relations, which in turn can be reproducers of a certain type of capital, not necessarily material, but human capital (Valero, 2017).

Based on the ideas mentioned above, we can infer that this modern Western family model must also be seen as a company, composed of human beings who seek to manage their actions, decisions and produce goods and, consequently, capital.

We could glimpse evidence of how much the family model approaches a company, corroborated by Foucault (2008a): “it is about making the couple a unit of production at the same title than the classic firm” (p. 336). To be more detailed, we observe something similar to what Foucault called “a long-term contract” (Foucault, 2008a, p. 336) - marriage - a relationship between man and woman that can be considered as facilitator of some transactions, exchanges between them, working together, making decisions, recognizing the most advantageous options, and also acquiring physical and non-physical goods, but a way of maintaining a certain type of capital.

In this type of business, subjects are as machines, that is, individuals who create the skills to create and maintain a certain capital, such as the image - a couple seeking the

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accumulation of a certain capital, so that this factory (the couple) does not suffer from the consequences of the absence of a competent administration.

According to Sylvio Gadelha, "political economy has as its object the human behavior, or rather, the internal rationality that boost it". (Gadelha, 2013, p. 148). The work, or conduct, produces a mechanism that favours individual optimization, acting as a catalyst for their abilities.

In this way, we observe indications that the subject is "entrepreneur of himself, being for himself his own capital, being for himself his own producer, being for himself the source of [his] earnings" (Foucault, 2008a, p. 311). This subject does not value exchange, but production from its own practices, as a useful subject in its economic life, that is, the subject that is made from its decisions, be it investment, purchase, among other economic practices.

These processes of subjectivation occur because are in harmony with neoliberal form of government, in which "the state cannot be dissociated from the set of practices that have effectively made it a way of governing, a way of acting, a way of relating to the government" (Foucault, 2008b, p. 369), that is, a way governed by economic practices of control, in a very subtle way, in which every society should rule as the system governs.

These references corroborate that we consider the family as a company, the subjects as machines, that must generate capital through their competences, skills and aptitudes, producing subjectivities.

In this context, the subject must be responsible for its success, for its financial balance and for its productivity, through its skills and abilities. Financial mathematics gives instructions about economic practices of the market, therefore it is a technology to govern people: "the art of government is precisely to exercise power in the form, and according to the model, of economy" (Foucault, 2008b, p. 127).

But this model can only subjectivize capital accumulation for capital creation; the individual acts is himself the "human capital" through him decisions. This "human capital", according to Foucault:

> Is not a conception of labor power; it is a conception of capital-ability which, according to diverse variables, receives a certain income that is a wage, an income-wage, so that the worker himself appears as a sort of enterprise for himself (Foucault, 2008a, p. 310).

Figure 2 illustrates, with the image and text, what we are writing about the idea of human capital creation.

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Juros compostos

Introdução

Considere a seguinte situação:

Depois de um ano de economia, Miguel juntou R$ 500,00 e abriu uma caderneta de poupança para seu filho, como presente pelo 10º aniversário do menino.

Vamos supor que o rendimento dessa caderneta de poupança seja de 0,8% ao mês e que não será feita nenhuma retirada de dinheiro nem depósito nos próximos anos.

Quando o filho de Miguel completar 18 anos, que valor ele terá disponível em sua caderneta?

O mecanismo pelo qual o saldo dessa poupança irá crescer, mês a mês, é conhecido como regime de **capitalização acumulada** ou regime de **juros compostos**.

Qual é o princípio básico desse sistema de capitalização?

Pais e filhos podem conversar sobre a importância de poupar, a necessidade de consumir conscientemente e outros temas de educação financeira.

Compound interest - Introduction
Consider the following situation:
After a year of savings, Miguel joined R$500 and opened a savings account for his son, as a gift for the boy's 10th birthday.
Let us assume that the income from this savings account is 0.8% per month and that no withdrawal of money or deposits will be made in the coming months.
When Miguel's son turns 18, what value will he have deposited in his saving account?
The mechanism by which the balance of this savings will grow, month by month, is known as a cumulative capitalization regime or compound interest regime.
What is the basic principle of this capitalization system?

Figure 2: Family in textbooks (Iezzi et al., 2013, p. 58, v.3).

Figure 2 presents a situation in which a parent opens a savings account for the child. Next to the text, a image of father and son, accompanied by the caption that suggests "parents and children can talk about the importance of saving, the need to consciously consume and other subjects of financial education". The text mentions that the compound of interest capitalization system is a scheme of the cumulative monthly.

The meanings aroused by the image are associated with the formation of a subject who becomes responsible for his enterprise and financial success.

In addition, image also refers to an aspect that we identified in the conclusion of Oliveira's thesis. Problematizing financial pedagogies, she concluded that masculinity is linked to the "perspective of prediction, positioning it (masculine element) as an investor" (Oliveira, 2009, p. 181). Figure 2 presents the father, man, instructs the son, also a man, and the father gifts his son with a savings account. It is possible find some similarity with the Oliveira's conclusions: masculine gender has been associated with investment practices, reinforcing the idea that economic and financial practices are more common for men.

Figure 2 also seems to suggest and reaffirm, once again, the pattern of the traditional family in which the male figure is linked to the liability faced related to enterprises.

Foucault (2008a) wrote that when parents spend more time to instruct their children, the tendency is that they become more adaptable subjects, largest generators of capital, and human capital generators.

In this way, we relate the image to Foucault's (2008a) statement that parents pass on to their children the idea that it is necessary to produce human capital and, consequently, be more productive, which can generate entrepreneurial subjects. This fact is related to what has already been discussed in relation to the modern Western family model as an example of a traditional family.

The problematizations that we have brought to led us to the notion of *homo oeconomicus* "as entrepreneur of himself, being for himself his own capital, being for

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himself his own producer, being for himself the source of [his] earnings” (Foucault, 2008a, p. 311).

We could verify that financial mathematics links its practices of capital accumulation, creation of the entrepreneur itself to consumption practices , as can be seen in Fig. 3:


No terceiro milênio d.C., em uma sociedade organizada e consumista, as pessoas necessitam cada vez mais otimizar os gastos para obter um equilíbrio no orçamento doméstico. Esse equilíbrio, embora pessoal ou familiar, em seu conjunto favorece a estabilidade da própria economia do país.

Agora, seu grupo vai elaborar um orçamento domiciliar mensal.

Justificativa

A preparação laboriosa de um orçamento, com a ajuda da Matemática, contribui para a organização pessoal, para o exercício da cidadania e para a inclusão social.

In the third millennium (d.C), in an organized and consumer society, people increasingly need to optimize spending to achieve a balance in the domestic budget. This balance, although personal or family, as a whole favors the stability of the country's own economy.

Now, your group will draw up a monthly household budget.

Justification: The laborious preparation of a budget, with the help of Mathematics, contributes to personal organization, to the exercise of citizenship and to social inclusion.

Figure 3: Families in textbooks (Paiva, 2013, p. 62, v.1)

Figure 3 seems to corroborate with the conception of traditional normalized family as an instance inserted in organized and consistent society. In this sense, to contribute to economy state, subjects must be standardized, they must be instructed to optimize their spending and thus maintain a domestic balance. We note that, to justify proposals such as these, the exercise of citizenship was mentioned.

From these statements, we seek to think of “other ways” because the term *citizenship* does not have a single meaning. Thus, depending on the social context, culture, , citizenship is understood differently. So, would it be a citizenship based on the responsibility of generating the state economy or generating the personal economy? What kind of citizenship have math textbooks, financial math, and math curriculum sought to instruct?

It should be emphasized that the concept of citizenship was created by the ancient Greeks in order to evidence the individuals who inhabited the same *polis*, that is, the citizen (Gallo & Aspis, 2010). The citizenship meant an individual belonging to a community. However, it was not all those who lived in a city who were citizens, but those who had certain conditions.

Citizenship is linked to the form of “free and independent life”. In a neoliberal way of life, citizenship is treated with a focus on the market, that is, being a citizen is, above all, being a consumer. This citizen of consumption seeks to “maximize competition to produce freedom and that everyone may be in the economic game. In this way, neoliberalism constantly produces and consumes freedom” (Veiga-Neto, 2013, p. 39).

Thus, the ideas presented indicate freedom as an object of consumption. This is an economic game, in which the mathematics curriculum is linked to society focused on

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consumption to produce free citizens. Despite a disciplining for consumption, it is only possible to govern free bodies (Foucault, 1998), a guarded and induced freedom.

Neoliberal discourse, when it crosses the mathematics curriculum, produces liberties for a consumption world, to capture the attention of students and teachers, as subjects of an important piece in the neoliberal process of production and consumption. Discursive weaves that govern to establish free subjects - free yourselves and caught in a net consumption. But happy subjects! It is the that you can see in the image below:

In the figure beside we can observe photos of happy families, in the contents of financial mathematics in high school textbooks. These images address socio-educational economic relations that inserted students in this competitive game of the false and true. In this perspective, the role of the school and the curriculum of mathematics is also compacts with neoliberal policies to produce normalized subjects and to maintain a policy of economic development.

Figure 4: Families in textbooks (Leonardo, 2013, p. 23, v.3).

The problematizations that we bring here led us to discourses in textbooks and curriculum, in which each subject of these happy families is an entrepreneur himself/herself. These subjects “are proactive, innovative, investors, flexible, with a sense of opportunity, with a remarkable ability to bring about changes, etc.” (Gadelha, 2013, p. 156). In other words, the neoliberal free citizen.

Thus, based on our analysis, high school mathematics textbooks constitute a type of instruction aimed at the formation of a subject that compacts with the form of neoliberal government, first of all, a subject that is shaped to enter the order of these discourse to be adjusting to their economic situation and, by itself, to seek be undertaken, using their skills and abilities. Following these instructions, he will be a responsible subject, free, that is, will be citizen.

CONCLUDING REMARKS

The collections of books used for this study were deployed to all Brazilian public schools, which means that reached many Brazilian students. From our analysis we can infer that the financial mathematics, in a subtle way, facilitates specific forms of governing actions, but also freedom. The textbooks present the freedom that citizens have to consume in a market-oriented society, in which the citizen is, above all, a consumer. This is a partially free subject, but trapped in a consumer network, a network that is exercised by a form of government, and where mathematics and financial mathematics are important tools to know how to optimize consumption.

We identify instructions based on discursive practices, socially constituted truths, that materialize in the mathematics curriculum, contributing to subjects who will have

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capacity to address consumption with responsibility. Through the analyzes, we conclude that there are regularities in the figures and tasks contained in these textbooks, in which there is a link between the ideas of “good” economic practices, financial “health” and familiar “happiness”. What these figures and tasks seem to suggest is that, if the instructions contained in the images are followed, a “healthy” life will be achieved, passing through the family vision as a human capital-producing enterprise in which each member is an entrepreneur, responsible for success or failure of this company-family. These discourses are also linked to the idea of economic success for happiness. These activities subjective students, who are educated to fit in a society ruled by neoliberal policies.

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THE MATH ED COLLECTIVE: COLLABORATIVE ACTION IN AN ERA OF CYBERBULLYING AND HATE

The MathEdCollective

Abstract: Critical mathematics educators and organizations in the United States have become the target of cyberbullying and hate-based attacks. These attacks usually involve the same pattern, in which numerous hate-based organizations echo shallow soundbites about an educator's publication or an organization's policy document, inviting readers to attack with hate mail demanding that specific individuals be terminated from their position. In this paper, we present the MathEdCollective, which came together in the last year to create a unified voice from the U.S. mathematics education community to document these attacks, call for collective action, and create an organized online network as a line of defense.

BACKGROUND

Recent years have seen the rise of far-right movements across the globe. In the United States, this has manifested in the election of an explicitly xenophobic, white supremacist, homophobic, and sexist president, whose platform has emboldened many forms of hate speech. Critical scholars in mathematics education have been among the targets of such hate speech, especially those who speak about the violence of white privilege and white supremacy, and who identify as womxn, people of color, or queer (Gutiérrez, 2017; 2018). These attacks come from an alt-right [1] mobilized around whiteness in the form of emails, voicemails, messages posted on social media sites and blogs (often using stolen IP addresses), conservative news and media sites, as well as websites masquerading as journalism (Flaherty, 2017; Scheurich, 2017). Unlike critiques that focus on the rigor of scholarship (e.g., examining researchers' methodologies, grounding in prior studies, or theoretical perspectives), these attacks rely on vicious name-calling and frequently involve explicit threats of physical violence. We summarize these attacks against mathematics education scholars and organizations and then detail the responses of various sectors of the U.S. mathematics education community. Our goal is to contribute to an international conversation so that we can learn with and from colleagues in other countries and collectively strategize about ways to unite and support one another in the face of organized hate.

THE ATTACKS

The recent wave of attacks on scholars in mathematics education can be traced to August 2017, when a group called CampusReform wrote a series of reports about (1) an article by Luis Leyva (2017) in the *Journal for Research in Mathematics Education*, titled "Unpacking the Male Superiority Myth and Masculinization of Mathematics at the Intersections," (2) a joint statement from the National Council of Supervisors of Mathematics and TODOS: Mathematics for All titled "Mathematics Education Through the Lens of Social Justice" (NCSM/TODOS, 2016), and (3) a job posting at Texas State University for a mathematics educator in which one of the preferred

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qualifications was "knowledge of or engagement with issues of social justice." CampusReform, funded and organized by Turning Point USA, brands itself as a leading site for college news, a "watchdog" that "exposes bias and abuse on the nation's college campuses." But as their reports about scholars and scholarship in mathematics education indicate, what concerns them most are calls for social justice that challenge the status quo. Their modus operandi is to hire college students to find instances of critical scholarship on their campuses, write "news" stories about them, and circulate these stories to their conservative base.

Next, a CampusReform report in October 2017 featured a book chapter by Rochelle Gutiérrez (2017a), "Political Conocimiento for Teaching Mathematics: Why Teachers Need It and How to Develop It." The report highlighted Gutiérrez's argument that mathematics, as it is traditionally taught, perpetuates white supremacy. This report was picked up and echoed by Fox News, one of the largest news outlets in the United States. Subsequently, Gutiérrez began to receive a flood of hate via email, voicemail, and Twitter, much of it threatening her with termination from her position as a professor at the University of Illinois at Urbana-Champaign, sexual assault, and other violence. Little of it reflected familiarity with her scholarship, apart from what CampusReform and Fox had reported. Additional spinoff articles proliferated, each starting a new cycle of hate and threats. Members of a closed Facebook group called "Mathematics Education Researchers" started a Facebook discussion about Gutiérrez's research, which quickly filled with comments that were explicitly and implicitly racist and misogynistic, suggesting that ignorance, discomfort, and anger with regard to critiques of social injustice in mathematics education are problems that exist within our field as well.

CampusReform sparked a similar cycle in January 2018 with a piece about Laurie Rubel's (2017) *Journal of Urban Mathematics Education* article, "Equity-Directed Instructional Practices: Beyond the Dominant Perspective," taking issue with Rubel's pointing to ideological "tools of whiteness" in mathematics education. Again, conservative news and hate sites with large followings (such as Fox News, BreitBart, and Infowars) were quick to repost the CampusReform piece. This led to a massive wave of attacks on Rubel through email and Twitter, defending white supremacy and using language rife with misogynistic, anti-semitic, and homophobic violence.

A number of high-profile scholars and organizations in the U.S. voiced support for Gutiérrez and Rubel personally and their work through newspaper editorials, position statements, blog posts, and letters to their institutions. In the next section, we describe the MathEdCollective —a groundswell response to these attacks, highlighting its activities to date, and outlining its guiding principles.

THE MATHEDCOLLECTIVE

Initially following the attacks detailed above on Leyva, TODOS: Mathematics For All/NCSM, and Texas State University, a number of scholars (many of whom had already been in communication about social justice issues in mathematics education)

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began to engage in email conversations about the attacks. When the attacks on Gutiérrez started, these groups came together for weekly organizing phone calls. These initial calls started as a place for scholars to connect, to come together in solidarity against these attacks, and to offer support for Gutiérrez and others who were under attack. These organizing calls snowballed as more individuals were contacted by CampusReform for interviews. Through the calls, the MathEdCollective developed resources/guides about what to do if you or a colleague were contacted by CampusReform or other hate sites.

The group generated a number of artifacts and tools for the mathematics education field including an anonymous website, now at http://mathedcollective.wordpress.com/, an anonymous Twitter account, pop-up sessions at conferences, and resources to support colleagues under attack. The website began as a way to document the attacks, archive the many statements of support, and to offer resources. But as the attacks continued to grow, particularly as the attacks moved from Gutiérrez to Rubel, the MathEdCollective evolved in its role. Now the MathEdCollective started to generate more sophisticated tools, such as a set of pre-written response cards for educators, teachers, and researchers who might need to speak up about the attacks or defend critical issues within our field, a website template for protecting the intellectual freedom of K-16 educators used to monitor and report attacks, detailed plans and action steps for future pop-up sessions and meetings, and a cleaner and more useful website.

ORGANIZING PRINCIPLES

The MathEdCollective came together quickly to express and exhibit solidarity with U.S. mathematics educators and organizations under attack. The MathEdCollective has used various sources of inspiration in its organizing, including U.S. Civil Rights Era activism, works by Paulo Freire and bell hooks, and the hacker-activist group Anonymous. As the group grew in number and increased its activities, a series of implicit organizing principles began to evolve. For this paper, we present an analysis of these principles and demonstrate how the organizing principles correspond to the Collective's actions.

1. Shared Ownership of Ideas.

The MathEdCollective practices shared ownership of ideas, which creates a community that can shield individuals from further harm by "anonymizing" their ideas through the MathEdCollective's voice. The shared ownership means that the MathEdCollective is not an organization that one can truly "belong" to or "serve." Rather, it is a collective in which individuals can participate as they want or need. All participants are connected by a shared purpose, which guides how ideas and actions evolve and flow over time. Additionally, the MathEdCollective represents a geographically diverse group of mathematics education researchers at multiple points in their careers. The collective strives to practice consensus-driven decision making, radical acceptance, and a shared vision.

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2. Heterarchical and Open Membership.

The MathEdCollective is a heterarchical collective, meaning that it is without hierarchy. There are no official "leaders" or "representatives." The MathEdCollective practices open membership, which means that there is no defined membership or email list. Anyone who claims to be a part of the MathEdCollective simply becomes part of the MathEdCollective. This is part of a broader aim of inclusivity in the work of challenging social injustice. Our goal is not to elevate our own status through an exclusive club but to call even those who have not previously seen themselves as advocates for social justice to find a role in this work (whether or not they become active in the MathEdCollective). The heterarchical and open membership means that, for artifacts such as the website, the MathEdCollective can allow the sharing of information without attaching anyone's name to the content. In this way, the MathEdCollective operates as an entity, with all of its members and none of its members. And, the focus on openness allows anyone to contribute ideas without judgment, even if those ideas are still emerging.

3. Collective Action.

The MathEdCollective exists to create solidarity within the field of mathematics education, created in the wake of attacks on individual scholars and organizations. No decisions or action by the MathEdCollective reflect individuals; they reflect the consensus of the moment with whoever happens to be participating at the time. Actions taken by the MathEdCollective always reflect this notion that the collective is "by and for" mathematics educators. So, even if individuals do not feel they might "belong" or even engage in the actions of the MathEdCollective, collective action is taken with all mathematics educators/humans in mind. Again, there are not different ways to "belong" to the MathEdCollective, one belongs by virtue of acknowledging the politics and power dynamics involved in mathematics education.

4. Taking the High Road.

Because of the origins of the MathEdCollective in dealing with hate-fueled attacks, one of its main guiding principles is to take that negative hate and energy and turn it into something positive and productive. This can be seen in the public tweets that Gutiérrez engaged in, never arguing or fighting with her attackers, but showing love and explaining her ideas further. This can also be seen in Rubel's public sharing of hate email she was sent, juxtaposed with her calls for inclusion and love within the mathematics education community. Additionally, the MathEdCollective embraces community love, which involves taking each person's physical, emotional, and social needs just as seriously as their academic needs. The collective seeks to sustain a community of scholars, and this only comes from loving them rather than exploiting them for their intellectual energy.

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5. Creative Insubordination.

Creative Insubordination is a term used frequently to speak of ways to disrupt oppressive practices in mathematics teaching (Gutiérrez, 2013). The MathEdCollective engages in creative insubordination in how it operates within the field. For example, the collective has been involved in numerous pop-up sessions at every major mathematics education-related conference in the last year. The idea for pop-up sessions came from the desire to host brave dialogues about politics that did not necessarily reflect the will of the professional organizations. For some of these conferences, the MathEdCollective was specifically invited to run a session. For other conferences, the MathEdCollective created the pop-up session first, then informed the conference organizers that the session was happening only after it was planned. This delicate dance, working with and around leadership in the field of mathematics education to create spaces for discussion, is creative insubordination.

UNDERLYING BELIEFS AND GOALS

The MathEdCollective believes that mathematics education, particularly in the U.S. is a function of and perpetuates whiteness. The MathEdCollective recognizes that all humans perform mathematics, yet it is only one particular form of mathematics, layered in whiteness and socioeconomic privilege, that is valued in schools.

The MathEdCollective's goals are to protect, support, and inspire mathematics educators (university professors or primary and secondary teachers), document a range of public engagement with our scholarship (and do so in a click-able and share-able format), press professional organizations for action in alignment with their policy statements, bring a moral voice to the mathematics education space, and create a forum to speak about attacks outwardly and within our field that is safe and humane.

THE MATHEDCOLLECTIVE'S WORK SO FAR

Through weekly and then bi-weekly organizing calls, email exchanges, pop-up conference sessions, and general discussion within the field, the MathEdCollective has engaged in numerous projects, listed here, that reflect the organizing principles above.

Outreach/Support to Mathematics Educators.

1. Collected (public) letters of support/solidarity, thereby encouraging others to take a stand (Organizing Principles 3 & 4)
2. Detailed resources for those under attack (OP 3 & 4)
3. Ran pop-up sessions at various conferences (OP 1, 3, 4, & 5), including: National Council of Teachers of Mathematics Innov8, Joint Mathematics Meetings (American Mathematical Society, Mathematics Association of America), Creating Balance in an Unjust World, Science for the People, Mathematical Sciences Research Institute, Association of Mathematics Teacher Educators, Latinx in the Mathematical Sciences, AERA Special Interest Group: Research in Mathematics Education, American Association of University

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Professors (Champaign-Urbana), and National Council of Teachers of Mathematics Annual.

4. Started Advocacy Center for Teachers Under Attack (OP 2, 3 & 4)
5. Create Go-to Phrases resource (OP 3 & 4)

Advocacy within Academia.

6. Demanded our universities create task forces and policies for addressing future attacks (OP 3 & 4)
7. Created alliances between mathematicians and mathematics education scholars, in contrast to previous relationships when such attacks against mathematics education scholars have occurred so as to reflect a new history in the making (OP 4)
8. Wrote journal articles that analyze the trend of attacks in our field (OP 4)
9. Developed joint writing projects (OP 1 & 4)

Dissemination to the Public.

10. Created a website that documents what is happening in real time, providing easy access to an online repository of information and support (OP 1, 4, & 5). Offered mirrored pages to conservative content so that readers do not give further traffic to those sites or need to read offensive comments. This online repository is immediately updatable, operating on a very different time scale than other means of articulation (i.e., journal articles, conference talks, op-ed pieces) and does not require permission to publish ideas.
11. Created a Twitter account (OP 1 & 2)

ONGOING TENSIONS

1. Making decisions about who is the "MathEdCollective": Who "decides" who is receives the emails? Who is a member?

According to Organizing Principles 1 (Shared Ownership of Ideas) and 2 (Open Membership), participation and membership become tensions of the MathEdCollective's work due to safety concerns given the severity of threats received by Gutiérrez and Rubel amongst others. There has been concern about informants accessing collaborative organizing documents in order to plan the next attack. Therefore, the MathEdCollective has been cautious with whom to collaborate and how to store information and communicate with each other. This runs counter to the first two Organizing Principles, and the MathEdCollective has consistently worked to navigate this dilemma. Attention to safety and a desire to be open also raises tensions about who has the "authority" to create and/or curate content for the website.

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2. Determining a decision-making process

The MathEdCollective has strived for decision making by consensus. But this ideal conflicts with Organizing Principle 2 (Open Membership), in which people are not expected to consistently participate with the Collective. Often, the consensus is the consensus of those who happen to be involved in the particular conversation at the time, rather than the consensus of the collective as a whole.

3. "Credit" for doing this work for the field

The MathEdCollective is engaged in work that seeks to benefit the field of mathematics education, not necessarily **the collective itself** or any particular organization (e.g., NCTM). This creates a tension, particularly for those in tenure-track positions who are trying to amass "credit" for intellectual work and national service for promotion and tenure. Who can claim to own intellectual work within the Collective? This tension also invokes the stark differences between academic or institutional credit, which operates more like bank credit, and unofficial credit, which operates more like street credit.

Academia as an institution values original ideas and thoughts, which runs counter to Organizing Principle 1 (Shared Ownership of Ideas). For instance, the MathEdCollective struggled to generate a list of contributors for this paper and even considered no author other than "MathEdCollective." The MathEdCollective engaged in discussions with a smaller group, then decided to reach out to the larger collective for feedback, then invited those who planned to attend MES10 for deeper feedback. Tensions exists with the way scholars are evaluated and valued within the institution of academia and how these scholars have come together to organize. The MathEdCollective is still discussing these tensions around authorship, order of authors, and even considering ways of using creative insubordination to create potential publishing opportunities for untenured scholars who need institutional "credit" for scholarship and publications. There is not yet agreement amongst the group about authorship that meets the Organizing Principles while also "playing the game" of academia. The MathEdCollective acknowledges that trying to do powerful activist work is, in many ways, at odds with how the academy defines and values work. In addition, the MathEdCollective still struggles with who gets invited to engage in these conversations (as noted in Tension 1), as the actual list of those involved in the MathEdCollective is quite large.

4. Whom to trust?

A closed discussion on the private Mathematics Educational Researchers Facebook group quickly became filled with racist, misogynistic attacks on Gutiérrez and was locked from further comments. This was the only apparent instance in which people within the field of mathematics education seemed to be attacking not just Gutiérrez' ideas, but Gutiérrez herself. Whether these attacks originated from actual mathematics educators, or from outsiders who "troll" groups in order to antagonize members, is something that the MathEdCollective has yet to decipher. However, such questions

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raised issues for how to remain open to membership while considering Organizing Principle 4 of taking the high road and producing positive outcomes.

5. Connecting to the media

The MathEdCollective has only recently achieved limited success in connecting with the mainstream media in the U.S. The MathEdCollective has struggled in getting attention within the mainstream media and found that the news media operated in a particularly politically-siloed way. While the alt-right media seemed to be attacking Gutiérrez in a coordinated way, relying upon algorithms, many of the outlets that slanted toward a more liberal viewpoint did not seem interested in picking up the story that the MathEdCollective was trying to get them to cover. This lack of interest raised issues for how the MathEdCollective could better connect with or use media to our benefit in the future. It also shows how the MathEdCollective needs to better use sophisticated social media algorithms and dissemination strategies such as the alt-right has used.

6. Danger of appearing cliquish or closed to other voices

The MathEdCollective acknowledges that the U.S. point of view is only a small part of a global conversation. As the MathEdCollective has evolved, tension exists in how to learn from other mathematics educators and organizations who have traveled this road in other countries. Although so much of the MathEdCollective's story is embedded in the current political situation in the U.S., the MathEdCollective strives to connect to similar struggles worldwide in order to be a part of global conversation.

Having said this, the MathEdCollective recognizes that scholars in authoritarian regimes across the globe face deportation, prison, and worse for their words. The MathEdCollective acknowledge its privilege, even in this situation, that it has a voice and a government regime that has not yet silenced through punishment or violence.

FUTURE WORK OF THE MATHEDCOLLECTIVE

Below is a brief list of some of the future work that the MathEdCollective will engage within, giving consideration to how this organization fits in within the existing global mathematics education community (including MES), and why this work will continue to be important.

1. Serve on Advisory Board / Task Force for NCTM Advocacy Center
2. Develop and share a model of how to respond to attacks
3. Help the individual institutions and organizations that members of the MathEdCollective belong to learn to respond better
4. Place demands on professional organizations (including learning from other organizations)

DISCUSSION POINTS

1. What might be the role of MathEdCollective, not just within the U.S., but also globally in supporting/connecting/empowering marginalized and attacked

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voices within the mathematics education community? How might this work reflect international conversations revolving around attacks on scholars (e.g., the Scholars At Risk network [https://www.scholarsatrisk.org] or conversations happening in India [http://www.siawi.org/article14181.html])
2. In the commentary, “Why Mathematics (Education) was Late to the Backlash Party: The Need for a Revolution”, Gutiérrez (2017b) detailed how the U.S. mathematics education community was “late” to the ongoing online attacks on critical scholars throughout academia. How can the MathEdCollective and others globally organize the entire community to create the revolution that Gutiérrez calls for?

NOTES

1. We distinguish the alt-right from the broader category of far-right. Alt-right refers to a subset of the far-right, specifically a subset who views themselves as an alternative to the mainstream far-right, rejecting mainstream conservatism. And although there is no identified consensus ideology, white nationalism seems to be fundamental (Daniszewski, 2016).

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OPENING AN EXERCISE: MATHEMATICS PROSPECTIVE TEACHERS ENTERING IN LANDSCAPES OF INVESTIGATION

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Abstract: This text presents two possibilities to transform exercises of the school mathematics tradition into investigative activities related to landscapes of investigation. The discussion is located in the context of Critical Mathematics Education (CME). A task related to such transformation was developed for prospective mathematics teachers in order to reflect on mathemacy, making mathematical discoveries by students at schools and possible changes in traditional teaching practices. This task and reflections represented a first step of these prospective teachers entering CME.

MILIEUS OF LEARNING IN MATHEMATICS CLASSES

With a group of mathematics prospective teachers, I discussed some notions of Critical Mathematics Education (CME) according to a Skovsmose's paper about *milieus of learning*. These notions will be presented in this text and formed the base to design a task for the prospective teachers to transform traditional exercises into investigative activities.

The mathematics classes present different configurations, either by the type of activity developed by the students, or by the kind of communication between the teacher and the students. In this sense, Skovsmose (2000) presents this diversity referring to milieus of learning, arranged according to a matrix (Figure 1).

	Paradigm of exercise	**Ladscapes of investigation**
References to pure mathematics	(1)	(2)
References to a semi-reality	(3)	(4)
Real references	(5)	(6)

Figure 1: Milieus of learning (Skovsmose, 2000)

This table presents six milieus of learning located in the *paradigm of exercise* or in the *landscapes of investigation*, with different references: *pure mathematics*, *semi-reality* and *reality* (Skovsmose, 2000). The first reference is about activities whose context is strictly mathematical. The reference to a semi-reality is related to a constructed reality, and not to an observed reality. The information presented in an activity related to a semi-reality refers to situations that may occur. In activities with reference to reality, students and teacher work with real-life situations.

Skovsmose characterizes the school mathematics tradition as that one which is set specially in the paradigm of exercise. In this context, the bureaucratic absolutism appears "to draw on unlimited resources for stating in absolute terms what is right

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and what is wrong" (Alrø & Skovsmose, 2004, p. 26). In these environments, the goal is to train in a technique and memorize concepts and procedures by repetition. The students usually face the blackboard. The teacher presents mathematical techniques, ideas and examples and, then, the students solve some exercises, selected from the textbooks, that have only one answer (Skovsmose, 2000, 2001).

In the paradigm of exercise, it is possible to find patterns of communication where the teacher usually asks questions which have a unique answer and which is known by her/him in advance. The students, in turn, try to guess what she/he wants in response. The teacher evaluates it as right or wrong, representing the authority in the classroom. The teacher's task is to explain the algorithms and correct the students' errors.

On the other hand, aiming for critical learning, Skovsmose (2000) presents the landscapes of investigation with activities that can provide the students with the discovery of mathematical facts, and the reflection, the understanding and the decision making on facts of the reality. In this context, the students' participation is active and, when working in groups, they develop dialogic acts with their colleagues and teacher, which it is important for learning.

If the students are actively engaged in an investigative activity it is because they have accepted the teacher's invitation to develop the investigation. There is an intention and an attitude of curiosity that moves the participants. A landscape of investigation is established to provide meaning to the activities in which the students are participating (Skovsmose, 2011). They are invited to explore hypotheses and make discoveries. Once they are engaged in the activity, the teacher cannot anticipate what the students will discover in their investigation. This is because they can choose the path to be followed in the investigation and act according their decisions. Some questions may be proposed by the teacher, but others may arise during the activity, leading to new possibilities of investigation. An investigative activity, therefore, is characterized by a high degree of unpredictability. One does not look for genuine results in mathematics, but rather for the students to make their own discoveries (Skovsmose, 2011).

The critical learning of the mathematics is related to *mathemacy* (Skovsmose, 2007, 2011), which has a proximal relationship with the notion of literacy as described by Paulo Freire and matheracy, by Ubiratan D'Ambrosio. The literacy includes more than the reading and the writing competences. It also refers to the competence to interpret social, cultural, politic and economic situations (life-world) and act actively to change these situations (Skovsmose, 2011). In a similar way, besides the competence in handling mathematical techniques, "the mathemacy can be seen as a way of reading the world in terms of numbers and figures, and of writing it as being open to change" (Skovsmose, 2011, p. 83). With the mathemacy, one can read critically the situations where the mathematical concepts appear in an explicit or implicit way. These situations are called *mathematics in action* by Skovsmose. One can evaluate the power positions, the risks involved and possible changes related in those situations. With mathemacy, the student can change the way he/she looks at the situation and it could help him/her to change their usual actions in the face of that situation.

In the context of CME, Skovsmose's text from 2000 is widely used in Brazilian research on mathematics classes. Marcone and Milani (forthcoming) state that 60% of the scientific papers, published in the 2017 thematic edition on CME of a Brazilian known journal (Revista Paranaense de Educação Matemática), referred directly or indirectly to Skovsmose's text. Regularly in my teaching practice, when I discuss this paper with school teachers and prospective teachers, the vast majority state their teaching practices are located in the cells (1) and (3) of the milieus of learning matrix, and the landscapes of investigation represent difficulty or challenge for them. If one wonders about changing the traditional teachers' practices into another one related to landscapes of investigation, a first step could be starting from what the teachers already do in their practices, that is, activities based on the paradigm of exercises.

But how to transform exercises into investigative activities? A possibility for this transformation was presented in a course of a mathematics teachers education programme with the aim of discussing with the prospective teachers about the possible changes in the school mathematics tradition. The intention was to generate discussions about how the teacher could promote activities focused on the development of the mathemacy, and to enable the students to make discoveries in the context of pure mathematics, characterizing the activities of the landscapes of investigation. The description of this work in the course is the focus of the next section.

PREPARING THE ENVIROMENT TO OPEN AN EXERCISE

The context of the research was a final year course of a mathematics teacher education programme. Until this moment, the prospective teachers have already attended both pedagogical and mathematical content courses. In a certain class, I discussed with the prospective teachers the milieus of learning presented by Skovsmose (2000). In a total of sixty students, distributed in two classes, only two expressed knowing something about CME. The notion landscapes of investigation represented a novelty to the overwhelming majority of the students. The reading and the discussion of this text, therefore, represented the gateway to think about CME.

At first, the prospective teachers read the text. The discussion in class was guided by a power point presentation, made by myself, exploring at least one example from each milieu of learning. To address an exercise with reference to pure mathematics, I presented the following example: "For the functions f and g from $\mathfrak{R}$ to $\mathfrak{R}$ defined through $f(x) = 2x + 6$ and $g(x) = -2x + 5$, find the equation that defines the functions f^{-1} and g^{-1}". The goal of this exercise is for students to practise a technique to find the inverse function of a linear function.

How to transform that exercise into an activity related to landscapes of investigation? One possibility for that is presented by Skovsmose (2011) when the author brings the

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action of *opening an exercise*, with reference to pure mathematics. The author himself asks: “What could it mean to open an exercise and try to enter a landscape of investigation through this opening?” (Skovsmose, 2011, p. 32). The author proposes a situation similar to the following one: “Let us consider two linear functions f and g from $\Re$ to $\Re$ defined through $f(x) = ax + b$ and $g(x) = cx + d$, where the parameters a, b, c, d are real values, a and b different of zero. What could we say about the intersection of the functions f and g? And of f and f^{-1}? And of f^{-1} and g^{-1}?”.

Unlike the exercise earlier presented, there is no unique answer expected by the teacher for this investigative activity with reference to pure mathematics. Of course, to solve the earlier situation, the students need to know how to find the equation of the inverse function, but the task does not stop there. They are invited to explore mathematical concepts and hypotheses. In this situation, three questions are presented and it indicates that there are others that can be formulated by the teacher or the students, depending on their intentions with the activity. This example of opening an exercise to enter in a landscape of investigation shows that other possibilities of work may be done with reference to the theme proposed in the exercise.

But what about the references to semi-reality and reality? How to make the transition from the paradigm of exercise to the landscapes of investigation? What could it be to open an exercise when the reference is to semi-reality or reality? Skovsmose (2000, 2011) does not address directly this issue. In the course with the prospective teachers, I presented a possibility.

As an example of an exercise referencing semi-reality, I presented the following one: “John went to a green market to buy 5 kg of apples. If the price of the apple kilogram was R$ 7.50, how much did John pay for the purchase?” This is a classic example in Brazilian textbooks that brings the context of a person at a vegetables and fruits market who buys a certain product at a certain cost, and asks for the total amount spent by that person.

One does not know who is John, but it is possible that John from the exercise exists. One does not know if that market exists, but it is probable that there is some green market that sells apples at this price. It is possible that some John at some market buys 5 kg of apples at R$ 7.50 per kilogram. These possibilities make this exercise to fit into a semi-reality reference. One possibility of transforming this exercise into an investigative activity could be to listen to the students’ comments for the exercise while they are reading, listening and thinking about the information presented in the exercise. Some of these comments could be: “How expensive is this apple! What will John do with all this apple? It never gives 5 kg of apple on the scale! At the market near my house, I can find apples much cheaper than that one! I wonder if one finds that price in all green markets. Why did not he search the price somewhere else? How much does it cost to the farmer who produced this apple? This apple must be made of gold!”.

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All these comments can be silenced by the teacher if the goal is to work only in the paradigm of exercise. “That is ok! Now, solve the exercise. How much did John pay for the apples?”, the teacher may say. However, starting from each one of those comments enunciated by the students, an investigative activity may emerge. For example, let us consider the sentence “It never gives 5 kg of apple on the scale!”.

In a green market, when fruits, for example, are placed on the scale, it is difficult to get the exact quantity intended of the product (5 kg, as in the exercise above). When that quantity is a little bigger than what was requested by the costumer (5.1 kg for example), the seller usually rounds down this amount and charges for that one requested by the costumer. From this situation, the teacher and the students may wonder: How much does the seller lose at the end of a working day by rounding down the quantity of the products sold? What are the benefits to the seller and the costumer with such rounding? In addition to these questions, some visits to green markets can be organized to talk to sellers and costumers about the practices that these two groups develop in this context.

To exemplify an exercise with reference to a reality, I presented to the prospective teachers the following situation: “The absence of 30% in the Exame Nacional do Ensino Médio (ENEM) [1] in 2016 was 2.4 higher than 2015, but it followed the average of the historical sequence of the exam. In 2014, the absence was 28.9%; in 2013, 29.7%; in 2012, 27.9%. In 2011, 26.4% of candidates failed to take the exam. What was the absence in 2015? Construct a column chart to represent these data”. With this exercise, the teacher wants the students to use mathematical concepts and techniques, such as rule of three, percentage, and proportion, to calculate the absence rate in the ENEM in 2015. In addition, they need to construct a column chart in order to realize that the absence rate of the exam was consistent in the last six years. When this exercise is developed in the high school classes, some comments and questions can be made by the students: “Hmm, I do not know what course I want do to! I do not even know if I am going to university. I feel pressure from society and my parents to go to university. Why does everyone have to go to university? In 2016, some schools were occupied. A friend of mine could not do the ENEM because his school was occupied! Why were they occupied?”.

If the teacher considers some of these comments, the students could enter in landscapes of investigation on different themes. One possibility is to discuss the need and pressure to take a higher level course. Are there vacancies in Brazilian universities for all adolescents? What other possibilities of jobs exist that do not require higher education? Which careers are most promising? What does a promising career mean? Another possibility is to reflect on the protests that occurred in Brazil against the approval of the Proposed Amendment to the Constitution (PEC) 241 or 55 in the second semester of 2016. Why were some schools occupied and some students were not able to take the ENEM? How does the content of that PEC affect Brazilian education? What do the rates and numerical information mean?

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Until this moment in the course, a first step had been taken in the direction of the prospective teachers to become acquainted with some issues of CME. I know, however, that to hear and to discuss these issues, know of examples of investigative activities, and hear about the qualities of critical learning are not enough for the prospective teachers to encourage them to think about implementing investigative activities in their future teaching practice. Then, a second step was taken. "Now, it is your turn to open an exercise!", I told them.

In groups, the prospective teachers had to choose an exercise from a textbook with reference to pure mathematics or semi-reality or reality. In a second step, they should open this exercise in order to transform it into an investigative activity, describing the process of transformation and indicating what they had considered to create that activity. Finally, they should write down the investigative activity created.

This task would be an opportunity for the prospective teachers in the course to imagine themselves as teachers in a moment of lesson planning. They would be experiencing the practice of turning an exercise into an investigative activity. In the next section, some reflections on prospective teachers' responses to the requested task will be made.

ABOUT THE MOVEMENT FROM THE PARADIGM OF EXERCISE TO THE LANDSCAPES OF INVESTIGATION

For the task of transforming an exercise into an investigative activity, 14 responses were given by the prospective teachers. I analysed these answers with some questions in mind: What were the exercises references chosen by the prospective teachers? In the process of transformation, what were the aspects they considered? What were the references chosen for the investigative activity? What were the characteristics the prospective teachers attribute to an investigative activity? The answers to these questions were organized in a table and the important sentences of each task were highlighted.

The exercises chosen by the prospective teachers were taken from textbooks that circulate in Brazilian schools. Of the total number of exercises, five (36%) referred to pure mathematics, seven (50%) to a semi-reality and two (14%) exercises referred to reality. One hypothesis to be considered for the choice of the majority of prospective teachers for exercises with reference to a semi-reality is the great presence of the contextualization of mathematical concepts in the textbooks, following the government's recommendations for the mathematics curriculum in the schools.

Some groups of prospective teachers presented justifications for their references choices according to Skovsmose's text read and the discussions taken in the class. An example of this is what Group A wrote: "The exercise [chosen] *just* asks for the calculation of the arithmetic mean of the ages and heights of a sample of 12 players" (Group A, 2018, my emphasis). The adverb "just" indicates that there is only a direct and immediate calculation to be made by the students, characterizing the exercise as an application of a technique. Also in this sense, another group pointed out that the

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exercise chosen had only one possible answer, "all the relevant data for resolution were present in the exercise and the rule to convert litre to millilitre was presented to the student throughout the pre-exercise text" (Group B, 2018).

Regarding the process of opening an exercise, that is, transforming it into an investigative activity, it is possible to note some common characteristics among the aspects pointed out by the prospective teachers in this process. At first, I will consider that the transformation of the exercise into an investigative activity was, in fact, accomplished, in order to characterize aspects of this process. Later, I will discuss the understanding of the prospective teachers of an investigative activity, looking at the outcome of the transformation.

Creating a landscape of investigation may have to do with showing the students how mathematics is related to other areas of knowledge and with discovering new mathematical concepts when addressing real facts. One group transformed an exercise about population of bacteria (a reference to semi-reality) into an activity in which students had to discuss how the medicines work in the human body. According to this group, a landscape of investigation was created to provide an environment for students to "discover new mathematical concepts in contexts related to our reality, involving a relationship with Biology and Chemistry" (Group D, 2108). The goal was to "show how mathematics relates to other sciences and how it helps in the calculus in those areas" (Group D, 2018).

In the same direction, another group of prospective teachers turned an exercise that referred to a semi-reality into an investigative activity, relating the theme of the exercise to the reality of the students. "To turn this exercise into an investigative activity, the students must experiment in their own reality" (Group C, 2018). The group also stated that

> For the activity to be investigative, the students must investigate and experiment, in order to be able to discuss and reflect on what happens. Then, the activity will not be a simple and mechanical mathematical exercise (Group C, 2018).

It is possible to see that this group knows some differences between an investigative activity and an exercise: on the one hand, the discussion and reflection on reality and, on the other hand, the mechanical activity, respectively.

Yes, reading Group C's statement, it is possible to see that the group knows some differences between an investigative activity and an exercise.

Many groups stressed the importance of questioning in an investigative activity. The questions may arise in the development of the activity, indicating new directions to be followed by the students. The questions which announce the investigative activity created by the prospective teachers show an opening for different possibilities of the students' answers. "What can we say about the possible ramp sizes? And about the values of the internal angles formed by the sides of the ramp?" (Group E, 2018, on urban accessibility). Another type of question created for the investigative activities

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was: “What if this quantity [of coins] is even? What if all the coins were 10 cents?” (Group F, 2018), indicating new challenges in the investigative activity. The questions that begin with “What can we say ...?” and “What if ...?”, created by the prospective teachers, are related in some way to the questions of the examples of the landscapes of investigation presented in the class of the course when the text of Skovsmose (2000) was discussed.

The author of the questions was an aspect highlighted by the prospective teachers. The exercise brought by Group C was about a basketball court. The questions of the author of the textbook were related to the measures presented in a figure of such court. In the investigative activity created by this group, these questions should be proposed by the students themselves, and not by the teacher or the author of the textbook. “These questions [of the paradigm of exercise] will not be explained by the teacher” (Group C, 2018). The answers to these questions will be used to solve broader questions of the investigative activity. “The students have to think and come to conclusions about what actions they should take to solve the task” (Group C, 2018, on estimating the cost of repainting the basketball court).

Regarding the references of the activities created by the prospective teachers, the distribution was as follows: three (21%) activities with reference to pure mathematics, four (29%) referring to a semi-reality, four (21%) with reference to reality and three (21%) activities that combined references to reality and semi-reality. The number of the activities with reference to pure mathematics decreased when the exercise became an activity considered investigative by the prospective teachers. This is due to the fact that some groups believe that the transformation process should result in some contextualization. The same happened with the semi-reality exercises. The reference to the real life figured in the investigative activities.

From the five exercises with reference to pure mathematics, three maintained the reference after the transformation to an investigative activity. The concern of these groups was that the students could make new mathematical discoveries, discuss issues among their colleagues, create hypotheses to later come to conclusions, make generalizations and apply what was discovered in particular cases.

With the transformation from an exercise to an activity related to the landscapes of investigations, a new type of reference emerged. An understanding for this was given by one of the groups: “The proposal is to transform it [the exercise with reference to a semi-reality] into a landscape of investigation mixing references to semi-reality and reality (milieus of learning (4) and (6))” (Group B, 2018). The group created a situation of selling some products from imaginary supermarkets, characterizing a semi-reality, posed questions about where a certain purchase was worthwhile making, and suggested that the students do a price research in the supermarkets in the city where they lived, indicating reference to reality. As the group said, it was a mixture of references.

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Another understanding for the new reference was brought by two other groups. The activities created began as follows: “The school needs to repaint the basketball court” (Group C, 2018) and “One school comes to know that in the next year some students with reduced mobility will join the school” (Group E, 2018). These schools, at the time the investigative activities were created, did not exist. It was an assumption, characteristic of the semi-reality. At the same time, the appeal for reality was very strong in these two activities. Group C stated that the students “must experiment in their own reality” and, therefore, decided that the best for the activity they created was “to measure the basketball court that is in the school area” (Group C, 2018). The prospective teachers considered the existence of a basketball court in the school where they would set the activity. Group E did not state explicitly the reference to reality, but brought images of ramps in real urban spaces and discussed urban mobility.

This type of reference was due to the very characteristic of imagination that the task had. The prospective teachers had to imagine how they would make the transformation. Thus, the teacher, the students, the school and the lessons were created in the context of imagination. The reference that emerged, therefore, can be considered as an imagined reality.

The adjective ‘investigative’ for the activity was clarified in different ways when the activity was presented. Sometimes the adjective was explicit by the authors, sometimes not. It seems that the prospective teachers knew that to be investigative, the activity could not only ask the students to do direct calculations, which is a strong feature of the paradigm of exercise. Some calculations may appear in an investigative activity, but it should involve the students in other actions.

Then, what would be an investigative activity to the prospective teachers of the course? Analysing their answers, the adjective investigative may be related to: creating empirical models to represent a situation, observing and taking notes about a situation, interviewing people, comparing, interpreting information, reflecting on the meaning of a concept, discussing and presenting ideas, making discoveries, rising hypothesis, making decisions, posing questions, and contextualizing. It seems that, for the prospective teachers, there are notable differences between tasks in the two columns of Skovsmose’s matrix.

FINAL REMARKS

The discussion of Skovsmose’s paper and the task of transforming an exercise into an investigative activity represented a first step in the prospective teachers’ education in the context of CME. The aim was to reflect with them on how to transform exercises from school mathematics tradition into investigative activities with the different references in order to develop the mathemacy and promote mathematical discoveries by the students in the schools.

This process of transformation was based on what Skovsmose (2011) calls opening an exercise. In order to open an exercise with reference to pure mathematics, one can

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explore the theme proposed in the exercise in different directions, following Skovsmose's orientation. In order to open an exercise with reference to semi-reality and reality, I have proposed the exploration of the comments produced by students as they work through the exercise. Through one procedure or another, the exercise changes. The result of the transformation process carried out by the prospective teachers in the course indicated that the investigative activity should have a strong appeal to reality and semi-reality, besides mobilizing other actions that are not developed when one solves an exercise, such as: interpreting information, discussing and presenting ideas, making discoveries, reflecting and making decisions, and posing questions. Due to the characteristic of the task, a new reference emerged: an imagined reality.

The activity developed was a first moment in the education of the prospective teachers in the context of the CME. This was a possible way for the prospective teachers to approach this theme. A next possible step could be to implement the created activities in the classrooms at schools. This is what I intend to do in another opportunity to contribute to the reflection on how the prospective teachers enter in the CME.

NOTES

1. The Exame Nacional do Ensino Médio (ENEM) is an evaluation instrument whose overcoming could be used by the student to be admitted in some Brazilian universities.

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THE CONSTANT COMPETITION, A WAY OF GOVERNING THE MATHEMATICS TEACHER

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Freelance

This paper seeks to problematize how competition is operating as a technology of government, conducting the mathematics teacher's ways of being and acting. Through a Deleuze-inspired intensive reading, it is sought to open a discussion regarding how the assembling of discourses, truths, and rationalities have constituted a network. A network in which the mathematics teacher comes to desire what control him/her—to compete in order to show that he/she is the best and the fittest. The mathematics teacher has become a subject that must be constantly competing, thinking about him/herself and following his/her personal interest, a neoliberal subject constituted between uniformity and individuality.

INTRODUCTION

Over the last several decades, the mathematics teacher has become a focus of attention for researchers (Sfard, 2005). In this vein, research on mathematics teachers has been positioned as a major strand of the field (Kilpatrick, 2014). By navigating in discourses that have been produced and reproduced by research about the mathematics teacher, it is possible to identify certain recurrences regarding what is said or enunciated about the teacher. In studying these recurrences, Montecino and Valero (submitted) show that the desired teacher is produced by the amalgamation of a wide range of features, such as, a vast repertoire of knowledge and techniques for teaching, (suitable) personal attributes that favor the teacher's practices, and the engagement with his/her social context. The constitution of this desired teacher shapes conditions of possibility, in which the mathematics teacher is fabricated; the mathematics teacher's conduct is conducted through the delineating and disposing of the desired ways of being and acting.

The mathematics teacher is constantly challenged to show that he/she is the *fittest* (Montecino, 2017). The teacher is, continually, compared with other teachers, as well as with a desired image of his/herself, in order to assess his/her competencies, skills and knowledge. It would seem that the teacher is always subjected to competing against other that is better than him/her. It is asserted, within research, that the mathematics teacher is characterized for having diverse deficits. For example, it is recurrent to find arguments asserting the teacher does not have a complete mastery of the mathematical knowledge that he/she teaches, it is imperative for "teachers to learn the content they will be expected to teach at a deeper level as well as have an understanding of the connections between the content [of diverse levels]" (Bleiler, 2015, p. 233). This, since the mathematics teacher's knowledge "has been identified as an important factor that influences the outcomes of teacher practice" (Van den Kieboom, Magiera & Moyer, 2014, p.430). It is asserted that "[e]ffective teachers are

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those who develop strong knowledge of teaching, content, and how their students learn" (Hegedus, Tapper, & Dalton, 2016, p. 8).

This paper raises a set of questions intended to make us reflect on how competition is established as a means for the government of the mathematics teacher, by naturalizing truths about who is and who must be the teacher, as well as power/knowledge relationships that shape a discursive network. Moreover, the paper's contention is about the need of a critical questioning of discourses about the desired mathematics teacher to open a problematization about social, cultural, political, and economic forces entangled in the fabrication of the mathematics teacher. In what follows, the analytical strategy deployed for *researching research* (Pais & Valero, 2012), an *intensive reading*, and the empirical material are presented. Then, a discussion about competition, the desired teacher and the fabrication of *the teacher in debt* (Montecino & Valero, 2017) is deployed. This paper proposes that the mathematics teacher is thought and constituted through uniformity and individuality, which are based in the idea that individuals must be constantly assessed, in order to control them through competition and accountability (Sirotnik, 2004).

The mathematics teacher studied in this paper is not a concrete subject or group of subjects that have as profession the teaching mathematics. Rather, this teacher is a discursive formation shaped by power/knowledge relationships, naturalized truths, and governing technologies. However, that the focus is not on concrete mathematics teachers does not mean that this paper has nothing to do with them. This is because discursive networks configure the desired mathematics teacher's ways of being and acting, in which the becoming of mathematics teachers is framed.

DEPLOYING AN INTENSIVE READING OF THE RESEARCH ON THE MATHEMATICS TEACHER

As analytical strategy, this paper deploys a *researching research* strategy (Pais & Valero, 2012), which is accomplished through a Deleuze-inspired *intensive reading* of research produced about the mathematics teacher within the last five years. This strategy enables to argue that the mathematics teacher and his/her fabrication require a problematization and to discuss how teachers' subjectivities emerge. Subjectivities that emerge in the entanglement of the individual and discursive formations within which the desired mathematics teacher is unfolded.

According to Deleuze (1995), there are two ways a book could be read. Firstly, the *orthodox reading*, one searches for signified and signifiers. To read becomes in a matter of interpretation and hermeneutic. Secondly, the *intensive reading*, one leaves out interpretation, considering the book as "a little non–signifying machine" (Deleuze 1995, p. 8) that can be related with what is Outside. Here, the question turns towards how the book works. Consequently, an *intensive reading* strategy seeks to disrupt current discourses of the mathematics teacher research, by opening a discussion in which the mathematics teacher—his/her fabrication—and competition are rethought from the exteriority, in order to "identify the emergence of new control strategies and

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the reconfiguration of old ones" (Rose, 1999, p. 240) and to reveal the productive side of power of the mathematics teacher research. The strategy is to trouble the common sense within which the mathematics teacher is, and has been thought, through a critical analysis of the discursive network about the mathematics teacher woven by research. In this study, the empirical material is composed of publications in the last five years of journals of fields (in specific, it considered the publication of *Journal of Mathematics Teacher Education, Zentralblatt für Didaktik der Mathematik* and *Educational Studies in Mathematics*) and of OECD's reports focused on mathematics education and the mathematics teacher. The selected material is due to the diversity of papers, authors, ways of arguing and perspectives, which allow to have a current view of what is said about the mathematics teacher.

COMPETITION AND THE MATHEMATICS TEACHER

Competition is established as a mean for controlling and governing the mathematics teacher. Governing the mathematics teacher does not imply to impose some ideas, truths or ways of being and acting, rather

> it is always a versatile equilibrium, with complementarity and conflicts between techniques which assure coercion and processes through which the self is constructed or modified by himself. (Foucault, 1993, p. 204)

Moreover, currently control "is short–term and of rapid rates of turnover, but also continuous and without limit" (Deleuze 1992, p. 6).

Competition generates the conditions for the teacher to be able to change and evolve in function of results that he/she obtains of such competition—or at least this is what it is believed. Currently, competition has become the incentive and the way for professional improvement of the mathematics teacher, by producing a flow of information (discourses and truths) regarding what is desired and undesired. It also defines problems and their solutions so as to delimit the possibilities that the teacher has of being and acting. Moreover, through competition, the mathematics teacher searches for validation. A validation from the other and a self-validation that shows that the teacher is better than others, or that he/she is an effective and productive teacher. It seems the goal becomes to fabricate a competitive (valuable) teacher, who can compete and come out victorious. However, the research about the mathematics teacher has drawn on how teachers constantly fail, by having deficits in their knowledge or techniques for teaching, for example. The teacher is far from becoming the desired teacher; he/she needs to improve more and more to answer the best way to social expectations and requirements—which are continually changing.

Competition is framed in mercantile ideas, in which the mathematics teacher must think about him/herself and must follow his/her personal interests for becoming a successful teacher, and in this way benefit the society. By basing on the idea: the individual ambition serves the common good. "Perhaps, just because of the alignment with the bedrock structure of capital, competition is seen not only as necessary but good for society" (Nelson & Dawson, 2017, p. 305), by circulates as truth that all

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subjects can become more than what he/she is. Competition shapes a system for guiding the decision-making process for social and personal growth in function of the outcomes obtained. In this context, mathematics teachers competing against each other during all their work life, in order to obtain the highest results in teaching evaluation and other performance indicators, and to become or to be recognized as the best. In this way the mathematics teacher can access to promotions, better woks and economic incentives. Thus, the teacher needs to be an entrepreneur, an entrepreneur of him/herself, ensuring the maximization of his/her human capital for becoming better through the investment in him/herself. In words of Cotoi (2011), teachers must be "individuals that self-regulate, self-direct and are continuously in a process of redefining their competences" (p. 116).

THE CONSTITUTION OF THE INEFFICIENT AND UNPRODUCTIVE MATHEMATICS TEACHER

The current value of mathematics is "a result of the formal place mathematics occupies within late capitalism" (Pais 2013, p. 20), by providing a language for science and technology; school–level mathematics is relevant since it "can enhance "personal and social capability" by providing opportunities for initiative taking, decision making, communicating processes and findings" (OECD 2015, p. 99). In modernity "mathematics education is now seen as indispensable for every citizen" (Smid, 2014, p. 590), since mathematical knowledge is shaping a valid and desired way of thinking and arguing. In this context, teachers play "a unique role as experts who provide opportunities for students to engage in the practices of the mathematics community" (Bleiler, Thompson, and Krajčevski 2013, p. 105), he/she has "the Promethean task of bringing light [mathematical knowledge] to children for the benefit and progress of humanity" (Montecino & Valero, 2015, p. 794). According to diverse approaches "teachers are key to increasing educational quality" (Luschei and Chudgar 2015, p. 3) and "bringing about improvement in students' outcomes" (Callingham, Beswick, and Ferme 2015, p. 552). However, the dominant narratives circulating within research about the mathematics teacher highlight the teacher's deficiencies—an inefficient and unproductive subject, due to the (always) existing gap between the teacher and what is desired—, but "despite widespread recognition that teachers need to learn more in order for students to learn more, there is little consensus about what it is that teachers should be learning" (Lewis, Fischman, and Riggs 2015, p. 448). The desired mathematics teacher is an unreachable teacher even for those considered as good teaches. Because the desired teacher is not a teacher of flesh and bones, rather this is a fabrication "that research calculates and measures to actualize the desired model of the teacher" (Martins, Popkewitz, & Yanmey, 2015, p. 12). Even more, this desired teacher depends largely on how—nationally or internationally—the good practices are conceptualized, as well as the expectations of the educational system.

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THE MATHEMATICS TEACHER BETWEEN UNIFORMITY AND INDIVIDUALITY

The mathematics teacher must aspirate to become the desired teacher—a good, successful, effective and productive teacher. This desired teacher is (re)shaped by the entanglement of social and political interests. At the same time, the teacher must differentiate from others, by showing that he/she has the tools and knowledge for being considered better than others. This duality frames the mathematics teacher in a process of uniformity and individuality, in which the teacher is fabricated and where continuous competition is a key element for the becoming and governing of teachers—by delineating what is desired and by unveiling the deficits. Competition turns the mathematics teacher's interest away from social and political aspects of the teaching and learning of the mathematics, which displaces the teacher's interest in winning or being well assessed. Here the objective is to do things better than others and distinguish oneself from the others. The mathematics teacher is labelled based on his/her performance in competing as efficient and productive teachers or inefficient and unproductive teachers.

Moreover, competition is used for outlining the next target of the teacher and conducting the conduct of the teacher. Since competition is used for having an image of who the mathematics teacher is and who the mathematics teacher must be, it recognizes the gaps between the teacher and the desired teacher through continuous testing—or monitoring—and comparing of the results gathered. Finally, competition governs the mathematics teacher to constitute him/herself as a subject that pursues becoming the desired teacher.

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ANTHROPOMORPHISM AS A PEDAGOGICAL DEVICE IN MATHEMATICS TEXTBOOKS FOR COUNTRYSIDE BRAZIL

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The use of animals in textbooks to contextualize mathematics and seduce countryside students to learn the contents is problematized. Our theoretical lenses bring together anthropomorphism— in line with animal studies— and governmentality — in a Foucaultian sense of conducting conducts and producing subjectivities. Based on an analysis of the use of animals in the two official mathematics textbook collections for rural education in Brazil, we show that the anthropomorphization of animals operates as a pedagogical device that articulates mathematics learning, with notions of animals and nature in rural forms of life, and with norms, habits, moralities and directions about who the rural child should be. This study contributes with a critique of how apparently innocent techniques in textbooks operate as political technologies.

INTRODUCTION

In Brazilian mathematics textbooks for countryside population, animals appear as a means to engage students in mathematical learning through affectivity, empathy and familiarity of context. In rural life forms children have greater contact with animals in comparison to children living in urban areas. Therefore, the presence of animals seems an adequate way of catching the students' attention and of creating a familiarity for the purpose of better mathematical learning. It seems natural to use this resource that belongs to the context and experience of children to promote their learning of the contents.

Furthermore, these particular textbooks were made as a result of the long political struggle of countryside populations and different social movements for the recognition of rural life forms and conditions. More concretely, the Landless Movement, one of the largest NGO in the world defending the rights to the ownership of land in a Brazilian countryside dominated by a structure of large privately-owned territories, was involved in pushing the government towards the production of textbooks for countryside population. The textbooks were meant to build on the principles embraced in movement, which are linked to how to improve peasants' life and work conditions. Ideas such as collectivity, traditional farming techniques, familiar agriculture, agroecology, land reform, among others are central to social justice in countryside Brazil.

The textbooks can be understood as the materialization of the conflicting directions that try to govern who countryside people should be. In this political scenario, the textbooks are not only a central resource for school because they guide the work of teachers and the presentation of mathematical content and activities, but they also

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frame particular notions of who the countryside child should become. Thus, the question of how and in which direction textbooks are part of the governing techniques to conduct countryside population becomes important. In this particular context, the use of animals in mathematics textbooks are not only a simple motivational tool. We contend that animal anthropomorphization in mathematics textbooks is a governmentality technique of the childhood in countryside context. It operates as a pedagogical device that articulates the mathematical knowledge with desired norms, habits and morality.

To unfold our argument, we start with a presentation of our theoretical tools and analytical strategy. Then, we present our analysis of the discourses around the anthropomorphized animals in relation to mathematics. We conclude with some remarks about how such anthropomorphization in textbooks constitute a pedagogical device. We also comment on the significance of this study to understand the cultural politics of countryside mathematics education in Brazil.

GOVERNING THE COUNTRYSIDE CHILD THROUGH SCHOOL TEXTBOOKS

School in society is an institution of subjectivation because it operates regimes of truth. These regimes of truth are composed by a set of dispositive of power that ensure the governing of a nation. In school children learn their rights and duties as well as knowledge of different sciences, mathematics included. But they also learn social rules, values and moral codes: "school has connected the scope and aspirations of public powers with the personal and subjective capacities of individuals" (Popkewitz, 2004, p. 7). Therefore, this institution teaches us who is the desired citizen and by this governs in certain directions who people are to become.

Foucault (2008) explains that governmentality techniques are sophisticated and complex, since governing

> […] is a question not of imposing law on men, but of disposing things: that is to say, of employing tactics rather than laws, and even of using laws themselves as tactics - to arrange things in such a way that, through a certain number of means, such and such ends may be achieved (p. 95).

Mathematics and science textbooks are important tools in the government of the population through education (Peñaloza & Valero, 2016). As part of the curriculum, they replicate and produce ideas about the desired citizen in modern society because "'citizens' mathematical knowledge and competence are considered fundamental for the maintenance of modern forms of life" (Valero & Knijnik, 2016, p. 1).

Our purpose in this paper is to produce a read about whom is the desirable countryside child in specific *place* and from specific *lenses*. The *place* is the mathematics textbooks written particularly for Brazilian countryside population. The two collections with five books each are part of a public program to improve the education system in rural areas. The textbooks are a special part of Nacional Textbook Program (PNLD) that since

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1985 the Brazilian government has been running. This program distributes officially approved textbooks in different compulsory school subjects to all students at public schools. The textbooks had been written and published by private companies; however, every year the government issues a public call for the private publishers where it outlines the subjects needed and the goals to be achieve in these textbooks. There are strict guidelines on what the textbooks should contain of content and of transversal topics such as citizenship, interdisciplinarity, etc. After the private publishers send their proposals, government experts assess their quality. Some proposals are approved and then the publishers can produce the textbooks. At the end of the process PNLD publishes a list with the approved textbooks so that teachers can choose the suitable textbook to be used in their subject.

In 2013 and 2016, the government issued a public call for primary school textbooks (grades 1-5) for countryside population. Two textbook collections resulted. Countryside teachers could choose which one is more suitable for their students.

Concerning the *lenses*, we explore the technologies through which school mathematics textbooks govern notions of the rural child. We focus on the uses of anthropomorphism that appear recurrently in the ten mathematics textbooks, in illustrations to mathematics tasks, in exercises and in suggested practices at large. As previously mentioned, animals are considered to be natural part of rural life. The images and uses of animals portray particular relations between children and animals. Recent animal studies (e.g., Guida, 2016; Pedersen, 2003) have problematized the relation between humans and animals and how animals are positioned as subalterns to humans (Perdersen, 2003). A strong critical position about the images and relationships to animals in society point to the school as a site where we learn to relate with animals:

> [the school] serves to sustain and reproduce a worldview of animal objectification in which the socialization of children and youth to uncritically embrace such a view as “normal”, “natural” or “inevitable” plays an important part. (Pedersen, 2014, p. 2)

Especially in textbooks for primary school, tasks and activities with animal are common. The characters are as a tentative to generate an affective relationship between children and mathematics. In most cases, animals are presented as characters performing human practices and they have been taken “to symbolize, dramatize, and illuminate aspects of [human] experience and fantasies” (Daston & Mitman, 2005, p. 2). In this sense, animals are used to talk about money, love and power. In children literature, for example, authors use animals to sensitize children to moral values and social-cultural rules (Serpell, 1996). Anthropomorphism has not been employed only to animals but also to other nonhuman creatures, such as nature or even numbers. Anthropomorphism “is the word used to describe the belief that animals are essentially like humans, and it is usually applied as a term of reproach, both intellectual and moral” (Daston & Mitman, 2005, p. 2). These approaches have been taken

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> […] as a tool by which a variety of discourses are simultaneously called into the interpretation of the animal and operate as a way in which any potential anxiety about animal otherness and difference might be potentially reconciled (Wells, 1961, p. 98).

For Guida (2016) paying attention to anthropomorphism allows to question the relation between species, as well as how ideas about who is the desired human are associated with divisions and classifications among humans such as racism and sexism. In other words, animal studies shed light on the cultural assumptions of anthropocentrism and how it provides a ranking of species where animals are placed below humans, as well as of which kinds of humans are better than others. In this sense, the famous statement that feminism is “the radical notion that women are human beings” (Kramarae et al., 1991) claims a position similar to the claim that animals are “experiencing subjects with their own lives, separate from any form of human intervention” (Pedersen, 2003, p. 15). This is why animal studies help us understanding how particular notions of the human are created with in relation to its cultural other: the animal.

Our analytical strategy consisted in finding the regularities in which animals are presented in the books, and in identifying how they are anthropomorphized, and which connection is established to both mathematical content and activities, and to indications of norms, moralities, values and behavior. In the overall corpus, there were a total of 390 appearances of animals in the images and the texts with explanations or tasks in differents situations. In this article, we examine the regularities in these appearances and in relation on how the anthropomorphism is present.

ANIMALS IN MATHEMATICS TEXTBOOKS

In order to contextualize mathematical activities the textbooks often appeal to (a supposed type of) students’ lifestyle. In Figure 1 and 2, there are scenes about life in the countryside. Figure 1 is part of an activity where students are invited to reflect about time in the farm. The content of time measure is compulsory for this grade. Figure 2 is used to make students aware of the presence of mathematics in everyday. The teachers’ instruction in small red letters suggest to make students count the animals, observe the size of the threes, notice the shape of the roof, etc.

Talk with your colleague about what you observe in the scene of this community

Mathematics in eveyday. To be a child is very nice. Play in the countryside is quite fun.

Figure 1 (right): Bonjorno et al (2014a, p. 115)
Figure 2 (left): Thadei et al (2014, p. 08)

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These are idyllic images of common countryside life. The animals are shown as a part of the environment (e.g., the swimming ducks, the dogs running) or as part of a cylce of life and production (e.g., the milk cow and the porks for meet). Here animals are elements in rural practices. Images like these are frequently used in activities that explore explore compulsory mathematics contents in primary school such as counting, units of measurement and localization. In one of the textbooks analysed there is even a chapter called "Animals" and, for this chapter the teacher's manual suggests: "In the chapter on animals, mathematics is involved in estimating and identifying quantities, ordering, counting, and locating them spatially" (Gomes et al. 2014a, p. 250). But there are also other types of references to animals in images and in tasks: the anthropomorphized animals. A first type of "light" anthropophormization is the drawing of nice looking animals such as smiling frogs or pigs with rosy cheecks. These appearances are the most frequent and have no distinct function from the one mentioned above. Now we look closer at the 165 clear anthropomorphized animals.

A wise male traveller

In one of the textbook collections, there is a character that appears throughout the five books. It is "Zé Sabiá":

Figure 3: Bonjorno et al (2014a, p. 03)

Figure 4: Bonjorno et al (2014a, p. 134)

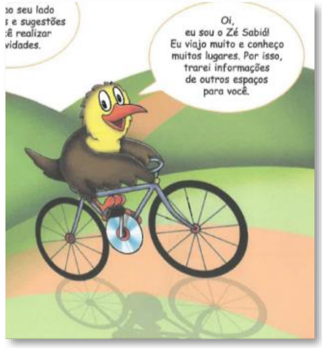

Figure 5: Bonjorno et al (2014b, p. 03)

"Zé Sábia" appears as an expert. This character always brings important information that helps the students to understand the context of the content approached in mathematics tasks. Its expert position is assigned because it has traveled around the world and it has had many experiences, after all, Zé Sabiá is a bird. In Figure 3, Zé Sabiá explain for the readers about its expert and traveler abilities that will serve to bring information about the world. In the figure 4, the character shows its abilities: it explains to the readers that tangram is a Chinese puzzle that can be used in geometry task. In sum, Zé Sabiá is wise because, by being a bird and flying to move around, it can know many things and help children with information. However, in other images as in the figure 5, Zé Sabiá appears riding a bike. That is, obviously, a human activity. Such anthropomorphic strategies seem only a legitimate way to engage the students in their school activities, in this case, mathematical activities. The name "Zé Sabiá itself is very interesting. Firstly, it is necessary to clarify that "sabiá" (rufous-bellied thrush)

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is a type of bird very common in the southern cone of South America, particularly the southeast of Brazil. But, the second point is that the word “sabiá” is very similar to the adjective “sábio” in Portuguese, that means “wise”. The name matches with the skills and roles of “Zé Sabiá” in this textbook collection. And third, Zé is the usual diminutive for the male name José. The character is a male wise, world-traveler.

More than contextualizing mathematics activities, the character is a tool to shows the world and to bring universal knowledge to the rural child.

Zé Sabiá: Presents information that provides students with knowledge from other distant or different realities contexts from those of the community. It intends to re-dimension the location to the global.

Figure 6: Bonjorno et al (2014c, p. 201)

Figure 6 is taken from the teacher’s manual. It provides information to teachers about the textbook authors’ pedagogical structuring of the textbook. It becomes clear that the explicit statement of connecting the local with the global implies a notion of the countryside child as being confined to a remote place (rural Brazil) and condition (rural lifeforms) that need to be re-dimensioned into the global.

Familiar female care

In this textbook collection, “Zé Sabiá” has a partner: Maria Sol, an anthropomorphized flower:

Hi, I am Maria Sol. I will be beside you, giving you tips and suggestions for you to do your activities.

Figure 7: Bonjorno et al (2014d, p. 3)

“Maria Sol” is a female character but, differently from the traveler and expert “Zé Sabiá”, she will always be assisting the students in the (mathematics) activities. “Maria Sol” is given a role of locality and care, almost a maternal position. The presence of these two characters together perform a gendering of roles and positions: the adventurous and expert man and the careful and delicate female. Such gendered performance of anthropomorphized characters appears in all ten textbooks in different ways. For example, Figure 8 shows the explicit message about values and moral in relation to the female body through an anthropomorphized dove. Using a traditional children’s song were a dove is given female attributes, the mathematical task merges

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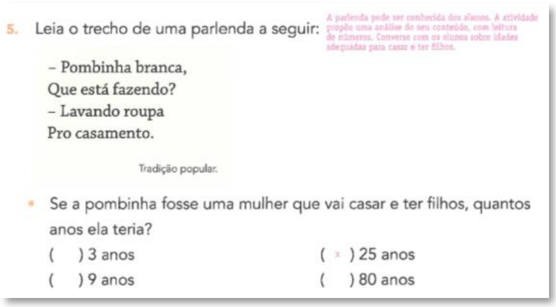
dove and woman and assign cultural rules and expectations to both (see Fig. 8). The folk song teach “the reading of numbers” (as suggested in the red text with a guide for teachers) to 2nd grade students. The questions and the choice of numbers connecting the marriage of women and the suitable age women to have children with the correct answer: 25 years old. The dove is taken to indirectly address the predicaments of child marriage, especially in countryside. In this way, the anthropomorphization softens the blunt social issues through a delicate tone: “Animals simplify the narrative to a point that would be found flat or at least allegorical if the same tales were recounted about humans” (Daston & Mitman, 2005, p. 9). But at the same time, the use of the animal ignores the features of the dove such as pigeon reproduction and life expectancy.

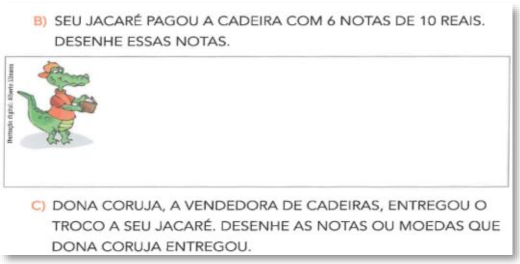


5. Read the following piece of music:
- Little white dove,
What are you doing?
-I am washing the clothes
To the wedding
If the little dove was a woman who is to marry and have children, how old should she be?

Figure 8: Gomes et al (2014b, p. 36)

Worker-animals

Animals also appear performing activities of work and economic exchange. In Figure 9, an alligator and an owl engage in a business. This is connected to the counting of money, addition and subtraction, compulsory mathematical contents. Furthermore, the situation illustrates the importance of teaching students to understand monetary exchanges and consumption in Western lifeforms.



B) Mr Alligator paid for a chair with six bills of 10 Reais. Draw these bills.
C) Mrs Owl, the saleswoman of the chairs, gave the change to the Mr Alligator. Draw the bills or coins that Mrs Owl returned.

Figure 9: Gomes et al (2014a, p. 115)

In Figure 10, the animals appear as a nice context to present quantitative information in a bar diagram form. However, the first question does a move of anthropomorphization when assigning the quality of “sleepy” to the animals. The word “dorminhoco” [sleepy] in Portuguese is associated to being lazy. Its opposite, sleeping less, is seen as a feature of diligence, and it is associated to an ethics of hard work. Implicitly in these tasks there is the idea of the countryside population as laid back; so children need to be directed towards similar habits as the hard-working animals in the farm: the donkey, the ox and the horse (those that sleep less).

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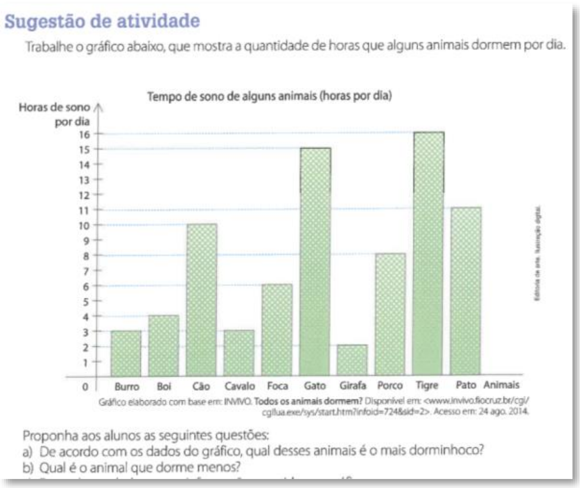
The types of exercises in Figures 9 and 10 direct countryside's students towards ideas of consumerism and work. An implicit message is that it is necessary to work more (sleep less) to consume more! Such an idea contrasts with the notions of economic solidarity which were present in the struggle of the Landless Movement for rural education; a notion that textbooks were supposed to embrace. See Fig. 10:



Suggested activity

Work with the graph below, which shows the number of hours that some animals sleep per day.

The animals are (in this sequence): donkey, ox, dog, horse, seal, cat, giraffe, pig, tiger duck.

Propose the following questions to the students:

a) According to data in the chart, which of these animals is the sleepiest?
b) Which one sleeps the least?

Figure 10: Bonjorno et al (2014d, p. 221)

ANTHOPOMORPHISM AS A PEDAGOGICAL DEVICE

We started our paper explaining our interest in making a political reading of mathematics education; in particular in understanding how the mathematics curriculum is part of the technologies of government in current modern societies. We argued for why looking at mathematics textbooks for countryside populations with the lenses of animal studies is a way of delving into the making of countryside children into desired citizens. We have not so far explained the notion of *pedagogical device*. Bernstein's notion of pedagogic device (Berstein, 2000) focused on the issue of the transformation of disciplinary knowledge into school knowledge. The notion of pedagogical device that we have implicitly adopted here is not concerned with a gaze that privileges knowledge. Rather, this notion allows us to think of the ways in which different elements of the curriculum, in particular time and spaces, when seen with particular lenses, can be recognized as materialized technologies of power. Following Friedrich (2010), a pedagogic device signals

> the production of an object within the particular rules and ordering principles of the pedagogical discourses. Pedagogical devices function in education as part of the regime of truth that dictates what is real and what is not, what is true and what is false, in the process of the intentional transmission of sets of values, knowledge and behaviors between subjects that is called education. (p. 661)

In the case of mathematics textbooks, a pedagogical device is the object of pedagogical discourse that articulates school mathematical knowledge, contents and activities, with

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the cultural norms, values and moralities that characterize the desired child-citizen that education fabricates. The pedagogical device uses the characterisitics of another object, familiarly seductive to the student, to directly and indirectly provide legitimizations through the authority invested in the mathematical knowledge and activity through which the former are brought to the child. In this way, pedagogical devices powerfully govern and direct subjectivities.

The analysis above evidenced how animals in these textbooks are used to create a familiar context for doing standard mathematical activities and learning usual contents. In this sense, the use of animals contibutes to generate motivation, closeness, contextualization and familiarity to facilitate learning. This is a very first function of animals as part of a pedagogical device. In these textbooks, animals are not different from the apples, toys, or any other kinds of objects that appear in any contemporary primary school mathematics textbook targitting other populations.

The concrete cases of anthropomorphization of animals as a pedagogical device in countryside mathematics textbooks articulate mathematical knowledge and activity with quite distinguishable cultural ideas of who the countryside child should be: mathematical facts and procedures are brought to countryside forms of life to improve them and connect them with a rich world of knowledge in distant localities. Knowledge as universal and masculine connects with positions of feminine local care, to deal with the ordinary tasks of mathematics. The pedagogical device also allows to educate farmes mathematically to optimize local farming production and to become a diligent consumer.

In other words, our analysis shows how (countryside) school mathematics is not simply about the learning of mathematical notions which should empower individuals; it is foremost a space of cultural politics where particular ideas are always in contentation and struggle, about who the desired citizens should be. For us understanding how animal anthropophormism functions as a pedagogical device alerts us not to assume the neutrality of compulsory school mathematics even when it is mobilized by lovely, friendly and cute animal characters.

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MAKING SENSE OF PERCENTAGES AND ITS IMPORTANCE IN UNPACKING INEQUALITY AND DISCRIMINATION

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Abstract: Percentage is an important concept as it exemplifies multiple levels of reasoning and more so in everyday personal and social life, where loans, discounts, societal trends of crime or inflation etc. determine our everyday existence. Yet middle school children are unable to grapple with it. This paper argues that a) the problem with learning percentages is due to an abstracted way of teaching b) Based on the experiences of Eklavya with middle school children in the Shahpur tribal block of M.P. - that if connected with real life, it not only improves the understanding of percentage but provides a powerful tool for social analysis. c) perhaps there is need to review mathematics curricula which privileges abstract mathematics and balance it with its connections to daily life.

INTRODUCTION

Percent in History:

It is well known that the history of percent dates back to the 3rd century B.C in India (1) Parker, M. & Leinhardt, G. (1995). The first context was that of taxation – the proportion of grain to be given as tax which dates back to 2100 B.C. in Babylon. This was in the form of say a third (1/3) of the grain produced – determined by dividing into equal parts and taking a part. In the 3rd century B.C., it was in terms of certain parts to a hundred – this idea lent itself to additive thinking. It remains till date the most widespread way in which interest on loan is calculated in India, but this is ignored by most textbooks. It was in 200 B.C in China and much later - 499 A.D. in India that percent became a multiplicative idea through the application of the 'Rule of Three'

Percent as a concept – its difficulties and its usefulness in understanding inequality and discrimination

One of the major problems with percent is that it is used in very many different contexts and draws its meaning from its context – interest rate percentage is very different from a literacy rate percentage, which in turn is quite different from a birth rate or discount context. It is precisely in dealing with each of these contexts and then abstracting from them that understanding and meaning is reached. The language is also complicating as it relates both rate and percent as well as has different terms in different languages. We will focus more on its relation to social reality. In a sense, percent, by bringing equivalence of the base, makes comparison possible and understandable. It is a useful handle to discuss inequality and discrimination in different contexts and ways and also discuss what could be fairer and more just ways of operating.

Percent as a powerful tool to understand socio- economic issues

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Today percent permeates our daily lives at multiple levels. In individual lives interest on loans and EMIs and on savings for different investments, taxation rates –whether through income tax or through GST, discounts of various types – buy one get one free – buy one get the second at half price – what is the difference? And so on. Equally important is the use and understanding of percent in understanding social and economic development or the lack of it.

In this paper

a) We will try to show that the contexts, if chosen carefully help to create a good understanding of the concept and also create a deeper understanding of social justice issues.
b) We will also try to argue through an analysis of prevalent textbooks that the way percent is taught, helps to create more confusions than clarity.
c) At the end we would like to argue that in the name of privileging 'the larger abstract goals' of mathematics, a rich opportunity to create a deeper understanding of mathematics as well as social justice issues is lost.

What we present here is a research in progress of trying to evolve a module or trajectory for teaching percentages in a socially meaningful way in an Indian tribal context. We share our journey as it is happening and would like your contributions to it.

EKLAVYA'S EXPERIENCES AND REFLECTIONS IN TEACHING PERCENTAGES AND RELATING TO SOCIAL JUSTICE ISSUES

The immediate context: Eklavya runs 12 middle school education support centres or Shiksha Protsahan Kendras(SPKs) Shahpur a tribal majority Block of Madhya Pradesh. Currently we are working with approximately 420 middle school children registered in our MS-SPKs. The SPKs provide out of school support to particularly at risk children so that they complete elementary education. The program has a system of 5 levels – 1. the students, 2. the Kendra sanchalaks (volunteer teachers) – who are local village youth, 3. anuvartankartas (regional youth who follow up and support the kendra sanchalaks) 4. the project team 5. Mentors/resource persons and reviewers. 4 and 5 along with 3 design and review the project elements. Ideas generated by resource persons are implemented and taken further by the Kendra sanchalaks and anuvartankartas through an iterative process. This iterative process is the object of research and reflection to evolve a critically rooted trajectory for the teaching of percentages.

Shahpur block has two large habitations Shahpur and Bhaunra – the SPKs are located in a radius of 5 to 20 km from either of these centres. While we worked on percentages in all the centres, student responses reported here are mainly from Handipani, a habitation about 8 k.m from Bhaunra.

The program tries to choose concepts in Maths, Science and Language that children find difficult and tries to make it interesting and meaningful by relating it to real life. Percentage, for all the reasons above, was an obvious chioice. We began by trying to

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make the concept simpler for children by trying to explain the idea and the procedures. We started teaching percentage in October 2016, in very much the same way as it is done in textbooks – introducing the idea through conversions from fractions and decimals and applying the rule of three to solve problems. After spending a little time on this approach we soon realized that children were quite confused and trying to attempt through trial and error. Asked to calculate 10% of 50, they would calculate 50% percent of 10. Or if told that someone spent Rs. 500 from their income of 1500 – what percentage was left, some would say 1000% - just substituting the % sign for the Rs. sign and so on. We were disturbed by this and reflected on why this could be happening.

As relating to context is part of our approach – we began discussing with children about the context which dealt with percentages in their daily lives. We were aware of discounts given to children who purchased notebooks from shops in nearby towns but not in the villages – a discount of 10% or 20%. Though these discounts were expressed in percentage terms, they were usually calculated by expanding the amount and adding what came about for every hundred or tens.

For example – given a problem - If the price of of a notebook is Rs. 20 and there is a 10% discount how much would you have to pay for 7 notebooks was solved as

-each notebook cost Rs. 20 and 7 notebooks would cost Rs. 140 – discount would be calculated – 10 on hundred 1 each on 4 tens so that's Rs. 10 + Rs. 4 = Rs. 14 discount. Net payable Rs. 140 – Rs. 14 = Rs. 126.

We asked children to solve the problem in both ways – the above and 140 x ~~10~~/~~100~~ = ~~140~~ x 1/~~10~~ = 14 – thus getting the same answer trying to see the link between additive and multiplicative thinking.

As discounts were not that widespread or in other forms like freebies, we looked for other local contexts. We found that the most widespread experience of households in the area is that of interest on loans particularly loans from the money lender. Most of the children who come to the SPKs are from marginalised tribal communities and from the financially weaker sections. Many families have taken loans from the moneylenders and continue to be in debt. The rate of interest levied on a monthly basis and is shockingly high. Interest is recovered every month! The rate of interest is different with or without mortgage – with mortgage it is 3-5% per month and without mortgage 10% per month. But it is expressed not in percent but in so many Rs. Per 100 per month. This is the same formulation as of 300 B.C. !!! Hardly anyone is able to pay back the Principal or part thereof but the interest is forcefully taken – if not in cash, in kind - things like – television, bike, cycle, grain.

The other widely experienced context in the rural economy is that of relative profit and loss when families sell their produce – relative to where they sell the produce and at what time. It is more a question of percentage difference between buyers and between times.So we tackled the twin problem of children understanding the concept of percentage and doing it through relevant contexts. We did this by concentrating on

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making children use estimation and additive ways to calculate the percentages. We encouraged children to use this method across all contexts – first on percentages in the loan context and later on percentage difference between amounts from sale to different buyers - problems from their own contexts. After a few months the expanded and additive ways seemed to be getting as mechanical as the other algorithms and the team thought again of how to go ahead? Comparisons were essential to relate to meaningful choice making in real contexts.

The summer camp of May 2018 gave us a very good opportunity to explore more meaningful and comparative contexts in which percentage plays a role – loans and relative difference between buyers – government support price and the trader.

Here are some of the problems and the responses of children.

On loans – we began with a simple contextual problem -

Problem 1.a. Munna bhaiya took a loan of Rs. 8000 at the rate of 10% per month from Kallu seth. How much interest would he pay in 6 months.

Children solved as follows –
Student 1.

8000 – 100 %	800 for one month 800 for one month
4000 – 50 %	800 for one month 800 for one month
2000 – 25 %	800 for one month 800 for one month
800 – 10 %	4800 for six month

Student 2

8000 – 100 %   800 – 10 % Rest as student 1.

As can be seen, different students showed different abilities of estimation – but it can be seen that they were making actual sense.

To the question “What would be the annual rate of interest?” Some calculated the amount of interest for 12 months and were surprised to see that it exceeds the Principal – adding 10%for 12 months discovered that it is 120%. Thus they were able to understand the meaning of more than 100% in real terms.

Children had been doing these problems as single issues for quite some time. Some discussions had also revealed the presence of other sources of loans with different rates of interest. We recently included these in our problems to be solved by children.

Part b. A self help group offers loans at 3% interest per month. If Munna Bhaiya had taken loan from them, then how much money would he save?

Student 2.

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8000 – 100 % 800 – 10 % 400 – 5 % 80 – 1 160 – 2 %

240 – 3 % and then added 240 for 6 months – coming to 1440

Student 1 for some reason also calculated 50%, 25% apart from 100%, 10%, 1% and 2% . It can be seen that in transition – children grapple with intermediary estimations too.

In another comparative problem children solved as follows and realised that

After introducing the comparison of two situations, one by one, we introduced with 3 simultaneous situations

Problem 2. Shersing took a loan of Rs. 8000 from a moneylender. Moneylender asked him to pay monthly interest on 10 Rs./hundred. After eight months, how much did Shersing pay to the moneylender. How much was the interest rate? If the monthly interest rate is 10 % then what will be the annual rate? Interest rate taken by SHG – 4% monthly ; interest rate of bank 24% annual. Compare the amount of interest on Rs. 8000 for8 months taken by bank, self help group and moneylender.

Moneylender – 10 % monthly

Self help group – 4 % monthly

Bank – 24 % yearly

Moneylender	Self Help Group	Bank
100 – Rs. 10	100 – Rs. 4	100 – Rs. 2
200 – Rs. 20	200 – Rs. 8	200 – Rs. 4
500 – Rs. 50	500 – Rs. 20	300 – Rs. 6
800 – Rs. 80	800 – Rs. 32	500 – Rs. 10
1000 – Rs. 100	1000 – Rs. 40	800 – Rs. 16
1500 – Rs. 150	1500 – Rs. 60	1000 – Rs. 20
2000 – Rs. 200	2000 – Rs. 80	20000 – Rs. 40
8000 – Rs. 6400	8000 – Rs. 320	8000 – Rs. 160
Interest for one month is Rs. 800 so the interest for 8 month will be **6400**	Interest for one month is Rs. 320 so the interest for 8 month will be **2560**	Interest for one month is Rs. 160 so the interest for 8 month will be **1280**

Here we can see beginnings of some multiplicative processes in calculating for 8 months. The tabular form gives a clear indication about the great differences in interest.

Another issue that we took up was of relative gain or loss in selling grain to different buyers. Earlier we had used the traditional sort of profit and loss problems which are there in textbooks. This is one of the problems set by us

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Problem 3. Raju sold his 10 quintals wheat to the merchant of Rs. 1200/quintal and got Rs. 12000**.** If he had registered in Govt. cooperative society he would get Rs. 2000/quintal rate for the same wheat. Did Raju make a profit or incurred a loss and by how much?

Student 1. Loss on Rs. 20000 – Rs. 8000 Loss on Rs. 10000 – Rs. 4000 Loss on Rs. 5000 – Rs. 2000 Loss on Rs. 2500 – Rs. 1000 Loss on Rs. 100 – Rs. 40

20000 – 100 % 10000 – 50 % 5000 – 25 % 2000 – 10 % 4000 – 20 % 8000 – 40 %

Raju's loss is 40 %

Outcomes from work on percentages

Working on rates of interest from different providers led to discussions with the students about who is able to access which source. It was a stark revelation that the sources with the least interest rates were more accessible to the better offs due to distances and paper work while the poor in most emergencies got caught with the moneylender.

Similarly in selling grain, children and sanchalaks realised that it is the better offs who can access the government support prices more easily..

As village projects were done in the, some of the above issues were related to the project on agriculture, tendu leaves etc.. For example, labour for collecting 50 tendu leaves works out to between Rs. 2 and 4 while the beedis made by those leaves cost Rs. 64 – 1600% or that the cost of labour for tendu is only 6.25% of the total product – a product that the very same labour buys!! This is a reflection from one of the anuvartankartas.

Children are also applying this to now analyse wrappers of goods and see what the mean 20%extra in a 5 gram pack of washing powder is just 1 gram!!

Our kendra sanchalaks andanuvartankartas are enthused by the deep relationship between mathematics and social inequity that are getting revealed. We will be working

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PERCENT IN SCHOOL EDUCATION

In the Indian School Curriculum

The perspective on middle school mathematics in India has been and is a negotiation between the practical demands of mathematics and then academic demands of mathematical reasoning and abstraction.

The NCF 2005, in its initial pages says draws guidance from the constitution

> Seeking guidance from the Constitutional vision of India ……. founded on the values of **social justice and equality**, certain broad aims of education have been identified in this document. (which include)
>
> predisposition towards participation in **democratic processes**, and the ability to work towards and contribute to economic processes and **social change**. (Emphasis ours)

It also lays down five guiding principles the first of which is as follows

> "the present NCF proposes five guiding principles for curriculum development: (i) connecting knowledge to life outside the school…

However, it is ironical that the section on Mathematics and curriculum drawn from the focus group paper on The Teaching of Mathematics seems to take a somewhat contradictory perspective. The connection of Mathematics to social justice is totally missing. The NCF, privileges abstraction and states that –

> "Developing children's abilities for mathematics is the main goal of mathematics education. The **narrow aim** of school mathematics is to develop **' useful'** capabilities, particularly those relating to numeracy-numbers, number operations, measurement, decimals and percentages. The higher aim is to develop the child's resources to think and reason mathematically to pursue assumptions to their logical conclusion and to handle abstraction." (NCF, 2005). (emphasis ours)

It is obvious that the needs of mathematics as an abstract subject are privileged over the everyday needs of the growing child to negotiate her daily life. The need of applying mathematics to better understand social inequality is nowhere mentioned. It seems to be more concerned with the discipline of mathematics, as earlier curriculum developers have been, than with the role of Mathematics in life.

> "Children use abstractions to perceive relationships, to see structures, to reason out things, to argue the truth or falsity of statements" (ibid.)

is all there is by way of critical reasoning. We have argued above that percent is one of the most important topics which can help understand and critique inequality and discrimination.

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In School Textbooks

Madhya Pradesh State Textbooks till a few years ago had substantial chapters on percent in their class 5, 6 and 7 textbooks. In class 5, percent was introduced in class 5, as hundredths – shading parts just like in fractions, and then attempting to establish the relationship between fractions, decimals and percentages – students are asked to fill tables like these

Fraction	Decimal	Percent
60/100	0.60	60%
	0.75	

The pedagogy consists of explaining the concept, explicating how to convert from one to the other, and introducing some 'real life' problems and again outlining a few steps to solve them. The class 6 chapter introduced the concept of ratios (in addition to fractions and decimals and showed the application of percentage to profit and loss and interest problems and the class 7 chapter extends this application to compound interest and complicated problems applying and extending the rule of three – viz.

> If a piece of German Silver (alloy) has 50% copper, 35% zinc and the rest nickel then if a piece of German Silver has 7.5 grams of nickel what is the weight of the piece?

A contextual problem? Seemingly so but the context is alien to most Indian children. This is followed by other contexts closer to home but in a random manner - for example a problem of % age of children present in a classroom on a particular day , is followed by a child reading a certain %age of the pages of a book with a third on percentage of plants that didn't germinate!! In such a scenario there is no scope to discuss any of the problem in its real meaning.

What is most surprising is that after NCF 2005, which expressed progressive and equitable aims, instead of making the teaching of percentages more coherent and connection to social justice issues stronger in school textbooks, the topic of percentages itself has been more or less removed from them except for a few exercises of conversion and application in the earlier strain!! Is it because the NCF has chosen to privilege 'the higher goal' of 'mathematical abilities' and 'abstractions'? Nowhere does the NCF focus group say that mathematics should be used to analyse of everyday lives in order to improve it. This is in contrast to the main goals of Education of the NCF.

Relating percentage to social issues in mathematics education literature

In the quick literature review that we did for this paper, we found some examples of using percentages along with statistics, to analyse both local classroom issues like the correlation of children who come to school on time and poverty and issues such as representation of women in the senate. (Michelle Allman and Almanzia Opeyo; Flannery Denny both in Eric (Rico) Gutstein & Bob Peterson ed. Rethinking

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Mathematics Teaching Social Justice by the Numbers 2013) there is dire need to collect and relate these experiences so that this powerful tool of mathematics can be used to bring about a better world.

IN CONCLUSION

Through our experiences some of which is shared above, we feel that

- Percentage, though a complex concept in abstraction can be very meaningfully dealt with if related to the child's own context.
- The problems in understanding percentages are because of the pedagogy, works through out at an abstract and mechanical level.
- On the other hand, it is a very powerful tool to analyse and understand hierarchy and discrimination. In order to do so the contexts and sequence have to be very carefully developed.

- **Our experience shows that if we develop a trajectory integrating social justice issues with the teaching of percentages and other such topics actually makes the understanding of the concept much stronger and also sharpens the understanding of social inequity.**
- There is often a polarisation between those mathematicians who privilege abstraction and those who work for social justice. What is reflected in the focus group paper on the teaching of mathematics is the former There needs to be an engagement to correct the balance.

NOTES

1. Rule of three - if any three of four unknowns from the principle, time, rate of interest and amount, the fourth can be calculated by cross multiplication.

2. MS-SPK- (*Middle School Shiksha Protsahahn Kendra*) is a non-formal education centre run by the joint effort of community and Eklavya Foundation and funded by the Jamshet Ji Tata Trust Mumbai. It's not a tuition centre nor the substitute of Govt. schools. It is a place to support at risk children, where children work on subject concepts, creativity, with their pace, it runs five days in a week for two hours daily before or after the school ours.

3. Handipani – a small village situated 18 K.M. far from block headquarter Shahpur in Betul district in MadhyaPradesh.

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CAPITALISM, MATHEMATICS AND BIOSOCIAL RESEARCH

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This paper addresses de Freitas' plenary speech in the previous MES as an excuse to continue the discussion initiated in Pais (2017a), where I explore the analogies between capitalism, mathematics and education, through engagement with elements from Lacanian psychoanalysis. My purpose here is to analyse how the workings of capital are concomitant with the ones of modern science, through a critique of de Freitas' engagement with biosocial research in mathematics education. The necessary obliteration of the subject that both capitalism and modern science require in order to thrive is also the defining trace of modern mathematics. I will argue that this obliteration is already at play as a real abstraction in the capitalist mode of production. I finish with a critique of biosocial research, and offer a hyperbolic speculation on the impact that such research could have in mathematics education.

INTRODUCTION

Elizabeth de Freitas' plenary during the last MES in Greece was an important moment for the community. She presented *biosocial research* as a "radical reconfiguring of education research" (2017, p. 56). Biosocial research, within the realm of education, consists mostly in tracking bodily movement and activity, together with a "rapid take-up of machine learning and new computational approaches" (p. 56) to processing data. This data is conceived as being "below the level of human perception" (p.57), and has been used with children and adults to track any behaviour amenable to measurement and modification. The paper reports research in mathematics education within this new trend, arguing, "this kind of research is changing the way we study mathematics education" (p. 56, 57). In particular, de Freitas focus on the recent research into *number sense* (Dehaene, 1997), involving brain imaging technology detecting parts of the brain that are activated when people and animals do calculations: "scientists are searching for the 'number neuron' on which they believe number sense is based" (p. 57)[1]. This "number sense" seems to be biologically determined (Chinn, 2015, quoted in de Freitas, 2017, p. 56), so that a better knowledge about it could allow for the fixing of many of the problems that students find when dealing with numbers in school.

From what I could noticed, and thanks to some illuminating comments I received from one of the reviewers of this paper, the MES audience received de Freitas' presentation with a combination of puzzlement, scepticism and a wish to attempt to engage with the

[1] It is truly remarkable that neuroscientists were able to register what mathematicians have not been able to do for centuries.

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unfamiliar. This hesitancy in engaging with de Freitas' work was not unrelated with the fact that she was very clear that drawbacks may occur when applying biosocial research to education, namely the production of highly conventional and reductionist models of learning due to the failure to capture the complexity of lived experience (p. 58). She wants to break "with the reductive scientism that often fuels biosocial research in education" (p. 58), and to reframe biosocial research as an affirmation of "bodily forces and materiality" (p. 58). This move is "adequate to the accelerated flows of advanced capitalism" (p. 70), and she urges the MES community to reconsider what constitutes the "social" by studying "how the digital sensors track the limit of human perception and turn it into the source for technical innovation and new configurations of control and governance" (p. 70).

de Freitas' paper offers us a privileged insight into the way in which late capitalism frames current educational research. In this paper, I also assume that capitalism is the *concrete universal* of our times, that is, "while it remains a particular formation, it overdetermines all alternative formations, as well as all noneconomic strata of social life" (Žižek, 2004, p. 3). However, I do not see my role as one of improving it, or making our work more adequate to it. Contrarily, I struggle to analyse the means by which our lives are entangled in capitalist economics, in ways that we might not be aware of or, what is even more problematic, in ways that makes us think that we are actually fighting against it. Notwithstanding our intentions and better knowledge, there are mechanisms at work that do not depend of our idea of them. That is, notwithstanding de Freitas' cautions and intent to appropriate biosocial research from an "inclusive materialistic" position (2017, p. 58), this research trend is always already embedded in broader societal arrangements that colour the way it is used in educational research. This condition is recognised by de Freitas and Sinclair (2013): "[t]he human body is, ultimately, a social entanglement, and any theoretical approach attempting to grasp its role in the classroom will have to address these larger socio-political issues" (p. 456). The crucial question to be asked about research is: what is the background against which biosocial research is emerging? What kind of society and subject does it presupposes?

de Freitas' intervention serves as an excuse for continuing the reflection initiated in Pais (2017a), where I explore the analogies between science, society, mathematics and education, through engagement with elements from Lacanian psychoanalysis. In particular, I show how mathematics presents an exemplary case of science and society's dreams of totality, in the way it seeks in its endeavours to tame the subject of its investigations. It is this untamed student – that talks, screws teacher's plans, refuses to learn, etc. – that becomes obliterated in biosocial research. In this paper, I will argue that this obliteration is already at play as a *real abstraction* in the capitalist mode of production (Sohn-Rethel, 1978). Something happens in the daily life of people that makes them prone not to consider such *un-sutured* subject. It is thus not surprising that

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the development of capitalist economies and M20[2] occurs concomitantly throughout modernity. Which also coincides with the implementation of mass education worldwide through programmes of scholarisation.

In what follows I will pin down three aspects that characterise capitalism, at the light of Lacanian psychoanalysis. I then extend the analysis initiated in Pais (2017a), of mathematics as a *fly-by-wire* science. My aim is to understand the genesis of modern mathematics as an essential part of the capitalist relations of production. Finally I return back to biosocial research, and explore these new research trends in mathematics education as examples of scientific approaches attuned with capital's drive towards automation and totality; and offer a hyperbolic speculation on the impact that such research could have in mathematics education.

CAPITAL: A VITAL FORCE

I will focus on three aspects that characterise capitalist economy, which I find important for my analysis of mathematics and its education. These steam from a Lacanian reading of Marx's theory (a thrilling research enterprise carried by Lacan himself, Louis Althusser, Sohn-Rethel and, more recently, by Slavoj Žižek and Samo Tomšič), which, far from moving away from Marxism, lead deeper into it.

Marx first expressed the view that *abstraction* was not the exclusive property of the mind, but arises in commodity exchange, in the beginning of *Capital* and earlier in the *Critique of Political Economy*, where he speaks of an abstraction other than that of thought. Sohn-Rethel (1978) further developed this idea by exploring how the essence of commodity abstraction is not thought induced: "it does not originate in men's minds but in their actions" (p. 43). That is, independently of what we consciously think to be the meaning and the outcomes of actions, these originate their own system of knowledge, a knowledge, as Lacan (2007) puts it, that does not know itself, but that, nonetheless, conditions our activity in the world. One can say that while we may be rational in our thoughts, in our actions, we are irrational.[3]

At stake here is the separation between the *exchange-value* and the *use-value* of a commodity which, in capitalism, assumes the necessity of an *objective social law* (Marx, 1976). As expressed by Sohn-Rethel (1978): "wherever commodity exchange takes place, it does in effective 'abstraction' from use", and he adds, "this is an abstraction not in mind, but in fact" (p. 25). That is, no matter how people think of commodities as objects of use, during the exchange the use of the commodity is

[2] M20 is a term coined by Roberto Baldino to designate the developments in mathematics that resulted from the work of mathematicians such as Cauchy, Weierstrass, Dedekind, Cantor, Hilbert, Frege and Russell, in their attempts to substantiate mathematics with a secure axiomatic system, whereby previous faulty notions of limit, continuity, infinity and infinitesimals can be elaborated in a secure way. That is, in a way that conventional meanings surpass ambiguities of common language (as small as we wish, infinitely close, etc.).

[3] Our daily lives are rich with examples of this dialectic. For instance, apropos of the recent paranoia concerning surveillance and personal data, while we rationally may know that the use of platforms such as Facebook exposes us to unforeseen troubles, in our actions, and against our better knowledge, we keep using them.

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insignificant, and what counts is its exchange-value.[4] The action of exchange is reduced to strict uniformity, eliminating the differences of people, commodities, locality and date (p. 30). In this way, it functions as a "social synthesis", where all commodities, including labour power, can be equated, despite their different uses, colours, flavours, or any other particular characteristics. It is precisely because it rends itself free from this tie-up with human activity that it functions as a social synthesis, providing a unified field of exchange. One can say that capitalism functions better without humans.

This drive towards automation and the technological development that encompasses it, is a result of the revolutionizing nature of capital. Marx himself was fascinated by the great material potential that a capitalist economy unleashes, as well as by its inherent contradictions. One relates with the fact that, while the tendency of production is always to increase, markets do get saturated. However, instead of reducing production, capital increases it, diversifies it, and transforms the market, so that production does not stop increasing. *All is possible* under the auspices of capitalism, which creates new consumer populations, new products, new needs; and this is also its *irrational* character, in the way it prioritises production over human needs.

As recently explored by Lacanian philosopher Samo Tomšič (2015), one way of describing capitalism is that it is life without negativity, that is, the efficiency and the logic of capitalism is supported by a fantasy/ideology of a subjectivity and a society without negativity (p. 7), where everything is conceived as "possible". In this sense, capital is creative potential, a specific form of vitalism, where any new senses and meanings are a priori foreseen in the real abstraction of commodity exchange (Sohn-Rethel, 1978).[5]

M20, CAPITALISM AND *PATTERNISM*

The enormous scientific development of the last two hundred years, particularly in M20, is concomitant with the establishment of capitalism as a global economic system. Lacan (2007) identifies the *logic of accumulation* as characteristic of modern science under the auspices of capitalist economy. Within capitalism, any measure has to produce surplus-value, otherwise it is discarded as obsolete, against the rules of the market, even immoral or unethical.[6] And the same with science. Any scientific result that threatens the homogeneity of science, its corpus of truth, results in a crisis. Modern science is built as an accumulative regime of knowledge, inasmuch as capitalist economy has at its core, the accumulation of capital. Any threat to this cycle of accumulation is seen as irrational, retrograde, and even impossible. In the same way that science's motto is "knowledge at all costs", capitalism's motto can be said to be "growing at all costs", whereby the market is left unchained in its power of assuring a

[4] Although people perceive commodities as multiple (every commodity has a different use, and even the same commodity can have different uses for different people or in different situations), its existence is *one*, defined by its exchange value in the market.
[5] For an application of this logic to mathematics education, see Baldino and Cabral (2018).
[6] Suffice to think about the frantic reaction every time someone suggests an increase in social benefits, a reduction of the working hours, or a public investment in public healthcare and education.

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natural social balance. Growing at all costs means that the logic of social organisation does not rest in individual needs, but rather in the needs of the market to sustain itself as a growing entity. The same can be said of science, where many of the scientific enterprises do not derive from individuals' needs, but the needs of knowledge itself, which, framed as "knowledge society", "knowledge production" or "human capital", are becoming indistinguishably ruled by the needs of capital.

In Pais (2017a) I explore the idea that modern mathematics thrived so vigorously through the XX century because it lost any contact with reality, its reality became endogenous with the signified, with the "meaning" of its symbols. The meaning of its symbols became conventions understood only by the initiated. Mathematics can be said to be the science where the foreclosure of subjectivity is most severe (see also Baldino & Cabral, 2018; Lensing, 2017). The process of mathematical formalisation seeks to assure that mathematics can function without a subject, making sure that nowhere one can find a trace of what is usually called the "subjective error". As previously explored, the same logic is at stake in capitalist production. The value of a commodity is not only defined by its use (by the "meaning" it has for the people who use it), but by the place it occupies in the set of all commodities, which defines its exchange value. Here resides the secret of capital's mesmerising growth: by reducing people to commodities, it gets rid of the burden of "subjectivity", reducing people to the value of their qualified labour power. It forms part of the very nature of modern science to foreclose the subject, inasmuch in the same way as it forms part of capitalism to register and manage people by numbers.

Particularly important for the discussion of biosocial research are those sciences whose methods are based on pattern recognition and pattern matching. Borovik (2017) uses the term *patternism* to designate the increasing saturation of patterns that characterises the new information environment. Particularly with mathematics, there is a push to abandon the formulation of mathematical models of real-world objects and processes, in favour of pattern-matching algorithms over large data sets. Google is a typical example, in the way it focuses on identifying patterns of behaviour, of information seeking, to predict the actions (intelligent writing, for instance) or the thoughts (intelligent search) of the user. The problem with this scheme is that one tends to only have access to what one already knows; if one is always within the same cosmos of information, built only out of our current interests, we will hardly experience the new. As noticed by Fochi (2013, p. 40), the paradox of apparatus like Google is that the world of seemingly extreme personalisation that Google creates (each user has its own personalised information, according to his or her recorded choices), "can only function by flattening the subject onto its identity with itself" (p. 41). That is, by limiting the subject to the information one has of it.

Pattern recognition and pattern matching does not lead to the formulation of any new thoughts. The subject is flattened. As mentioned by O'Neil (2017) when discussing her inciting book *Weapons of Math Destruction*, "we have all this data and we have patterns, and the machine learning algorithms are very good at pattern matching – but

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what they do is propagate historical practices" (quoted in Tarran, 2016, p. 42). These endeavours rest on the underlying logic that one can elaborate with precision what society needs and purpose solutions accordingly. Nowhere was this trend more noticeable than in the recent political affair involving Cambridge Analytica; a company that plays a major role in developing and deploying government-funded behavioural technologies, in close association with the World Well-Being Project, a group at the University of Pennsylvania's Positive Psychology Center that specializes in the use of big data to measure health and happiness in order to improve well-being; and with Aleksandr Kogan, who also works in the field of positive psychology and has written papers on happiness, kindness, and love. He ran the Prosociality and Well-being Laboratory, under the auspices of Cambridge University's Well-Being Institute. Although it might be surprising to associate a company accused of manipulating people with happiness and well-being, the relation between the two is not new nor unusual (see Shaw, 2018). It seems that surveillance, intelligence military programmes, and the manipulation of public opinion can go hand in hand with the major aims of fomenting love and happiness to the general population.

THE LURE OF BIOSOCIAL RESEARCH

If, as mentioned by François (2017) in her reaction to de Freitas' paper, biosocial research findings claim that the future inheres in the actual present, "then the danger is that this can lead to a determinism, a teleology, a prescriptive destine and a pre-given trajectory" (p. 98). Computation, and the quantification that encompasses it, becomes the mean by which one gauges his or her presence in the world. As analysed by Lensing (2017), this happens even in areas historically less prone to mathematical elucubrations, such as love and self-knowledge:

> phenomena can be processed by a computer exactly then, when they are reduced to their quantifiable aspects: "Love", for example, becomes a matching of two persons qua psycho-metrics; "health" becomes the result of the combination of singular physiological characteristics, etc. In this way, human practice –and with it the subject itself– becomes reduced to those aspects that can be quantified in an apparently objective way, and the regulation of even larger social areas –hitherto contingent on human interaction– becomes replaced by mathematically formalized complements. (p. 678)

Love, a sentiment historically resistant to any sorts of understanding, becomes possible through quantification. This "becoming" is not only a contingency, but a necessary feature: love *must become* a matching of two persons qua psychometrics to satisfy the needs of the market. What these endeavours render visible is capital as a real abstraction, where, in order to be, every inch of reality needs to be registered in the space of commodity exchange.

I am aware that this position is in the antipodes of de Freitas'. Her goal, rather than dismiss all neuroscience research, is "to invest in the possibility of neuroscience research that does not serve the computational dream of a control society" (de Freitas

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& Sinclair, 2016, p. 229). She says that she wants to rescue biosocial research from the scientism wherein it is situated. The problem is that both science and education do not occur in the political vacuum of inclusive materialism, but are in conformity with the exchange abstraction and commodity production. No matter de Freitas' thoughts on the matter, her actions – the very fact that she is highlighting biosocial research as a new reconfiguration for educational sciences, and the effect that her presentation had in MES – serves the purposes of industrialised education. In this respect, and notwithstanding the will to "decentre human agency" (de Freitas & Sinclair, 2014, p. 43) and moving away from humanism, de Freitas' position is still too human: she believes that through our thoughtful actions we can thwart the educational effects of biosocial research.

This assumption misses some of the philosophical insights of Deleuze's theory concerning capitalism – a philosopher central to de Freitas' theorisations. Deleuze is the philosopher of the virtual, of the bodies without organs, of flux and intensities, of percepts and affects.[7] In the words of Žižek (2004), Deleuze is the philosopher of late capitalism: with his lexicon and logic he analysed the crucial changes allowing capitalism to enter in its current phase. Contemporary capitalist ideology no longer functions towards centralization, consolidation, homogenization, and against diversity. Contrary, the latest trend in corporate management is to diversify, devolve power, and try to mobilize local creativity and self-organization, instigating decentralization as the flag of the 'new' digitalized capitalism. On the other hand, the old Foucauldian notion of normalization—that power/knowledge relations mould the subject towards the Norm—so dear to industrial capitalism, seems today no longer holding its power. Instead of the logic of 'totalizing normality' today's capitalism adopted the logic of the 'erratic excess':

> the more varied, and even erratic, the better. Normalcy starts to lose its hold. The regularities start to loosen. This loosening of normalcy is part of capitalism's dynamic. It's not a simple liberation. It's capitalism's own form of power. It's no longer disciplinary power that defines everything, it's capitalism's power to produce variety—because markets get saturated. Produce variety and you have a niche market. The oddest of affective tendencies are okay—as long as they pay. Capitalism starts intensifying or diversifying affect, but only in order to extract surplus-value. It hijacks affect in order to intensify profit potential. It literally valorises affect. The capitalist logic of surplus-value production starts to take over the relational field that is also the domain of political ecology, the ethical field of resistance to identity and predictable paths. It's very troubling and confusing, because it seems to me that there's been a certain kind of

[7] Affects in Deleuze are those pre-conscious 'processes' that are beyond signification or coding, but which, nonetheless, structure our sense of reality (Deleuze, 1990). They belong to the field of the Virtual, the "'real but abstract' incorporeality of the body" (Massumi, 2002a, p. 21). Brian Massumi and Patricia Clough have been developing Deleuze's theory to analyse political economy in terms of what they call the "capitalization of affects", which can be seen as one of the crucial extensions of late capitalism.

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> convergence between the dynamic of capitalist power and the dynamic of resistance. (Massumi, 2002b, p. 224, quoted in Žižek, 2004, p. 184, 185)

As Massumi suggests, the power of Capital—a strong machine of deterritorialization that generates new modes of reterritorialization, as Deleuze and Guattari (2004) put it—to produce variety is coupled with its power to co-opt what in principle are resistant forces against it. The capitalist *machinic* (to use a Deleuzian term) integrates in itself the different and fragmented local forms of resistance, by, for instance, creating new research trends, innovative educational approaches, new methodologies, giving the impression that things are being done, that education is possible, that people are engaged in making things better. Because knowledge has to grow and we cannot stop doing research, one has to look for the spaces where it can be done. After the poststructuralist train lost its steam, new materialism, more than human and biosocial research appeared as the new vanguard in social sciences and cultural studies. Biosocial research, especially cognitive and educational neuroscience, is today an area of increasing interest and funding opportunities. How long its steam would last depends entirely on how its results resonate with the real abstraction of capital. That is, it will depend on how lucrative and silent biosocial research can be.

Seventeen years after Baldino (2000) presented in this same conference his paper on the Neurone-Z, where he predicted the “number neuron” and the identification of the part of our brain responsible for mathematical learning. de Freitas’s paper showed us how this is no longer a dream, but a tangible possibility. On should not be surprised if in some years, a rigorous mechanism capable of identifying the part of our brains responsible for mathematical failure is available. We will be able to “see”, register, and measure the mathematical activity of our students by doing a simple brain scanner. This will make the lives of teachers so much easier, because it will create an efficient mechanism to assess students learning, thus providing a scientific way to answer the question of: When does a student learn? No more long tests, hard to mark and depending on teachers’ subjective moods. Just a quick and inexpensive brain check, and a grade will materialise. No more expensive and logistically complex international exams like PISA, but a simple brain scan made to all students in their schools will allow to scientifically measuring all human beings’ mathematical capacity. Moreover, these mechanisms will allow not only to diagnose failure, but, more incredibly, to offer a remedy: by doing a small brain surgery it will be possible to fix mathematical failure.[8] The community should be happy. We will soon have the so-desired total pedagogy and mathematics will finally be for all. Economies will flourish. Love and happiness will reign.

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[8] Although this might seem a film from the past, consider for instance the research made by Krause and Kadosh (2013) where they use electrical brain stimulation to improve the arithmetical skills of students.

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FACTORS RESPONSIBLE FOR SOCIALLY JUST PEDAGOGY IN MATHEMATICS EDUCATION

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Abstract: This study aims to explore the teachers' attribution about the elements responsible for social in/justice in mathematics education. It was a narrative inquiry with in-depth interviews of two mathematics teachers to explore their experience and opinions on socially in/just pedagogy. It was an iterative process of data construction and analysis side by side. The analysis of qualitative data revealed role of family, students' interest, the diverse context of students, economic status, and social structure as the responsible elements for social in/justice in mathematics education. The policymakers and curriculum designers along with mathematics teachers and students should understand how these elements are responsible for social in/justice mathematics classroom.

Keywords: Social justice, family roles, students' interest, social structure, and diversity

INTRODUCTION

In Nepalese context, mathematics teachers usually follow teacher-centered pedagogy. The teachers and students have different hierarchical positions suggesting that there exists a power relation between them. Teachers usually have a dominant role in teaching and learning and students are usually passive learners. But a social just pedagogy focuses on the active roles of learners giving them equal opportunities and access to learn mathematics. Social in/justice is one of the major issues in mathematics education in Nepal (Panthi & Belbase, 2017). Therefore, a socially just pedagogy is getting popularity day by day. First, I discussed my personal inspiration on this study followed by context of teaching mathematics in Nepal. Then I introduced theoretical framework for the study, method, results and discussion, implication, and conclusion.

My Inspiration

When I was a grade 10 student, it is difficult for me to understand my mathematics teacher's language because he was from a different community speaking different language and he often mixed up his mother language while teaching. He always followed teacher-centred pedagogy and I was only a receiver of his mathematical knowledge. It was as if he filled up my mind with mathematical content. When he entered in the classroom, he took a book from me; turned the paper. He told us, "listen to me". So, we are all silent in the classroom without an interaction. My method of learning also affected by teaching with rote memorization that I used to follow at the beginning of my teaching career. During the years, I would just focus

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on exam contents and follow traditional methods. Later, I realized that my teaching techniques did not value students' views and did not promote critical reflective thinking. So, I was inspired these to carry out this study. Due to the fact that students do not get opportunity to learn mathematics from equitably their social and cultural context creates more negative image toward the subject, have higher anxiety level and thus leading to negative attitudes (Belbase, 2013).

During my M.Phil. in mathematics education study, I read Freire (1970) that helped me understand problem-posing education is an alternative to banking pedagogy. I realized that through constructive dialogue, students may generate and examine problems from their own lives, and work collaboratively to construct solutions. Further I learned from Gutstein (2006) that mathematics educators are expected to address the issues of inequity in their classroom. Cotton and Hardy(2004) inspired my thinking about social justice as ''a way of working that accounts for, and works with, the links between oppressions, inequalities and exploitations that we see inside and outside our schools and classrooms'' (p. 90). Similarly, Gutierrez's (2007) view of justice or fairness as blurred with equality, I came to know that "students need different (not the same) resources, and treatment in order to achieve fairness(Gutierrez, 2007, pp. 40-41).

I think social justice means getting equal opportunities and access and treating according to the ability of the students in the mathematics classroom. Stocker (2007) describes the reason for constructing social justice lessons as a means to teach and practice key mathematical skills while developing student interest to injustices, fairness, and kindness by replacing purposeless content that furthers no student's ability to engage with their social reality(Nolan, 2009). In doing so, Gutstein and Peterson (2006), pointed that students respond positively when math is relevant to their life. Although equal educational opportunity may be valuable to some degree in gaining equity among different groups of students, it is insufficient for a just mathematics education (Fennema, 1990; Gutierrez, 2007).

I feel that students' interests were different that also created an in/justice in mathematics classroom. For this, teachers' skill affects in maintaining social justice in classroom. Five key elements are useful in teaching for social justicein mathematics education-- content mastery, critical thinking, action and social change, personal reflection, and awareness of multicultural group dynamics (Hackman, 2006). Taylor and Luitel (2005) concluded in a study that our mathematics can be contextualized by adding ethnomathematics. As a result, students can share the cultural experience in the classroom and how they can apply mathematics in their daily life. However, many teachers in Nepal use decontextualized pedagogy that influences fairness in the classroom. Moreover, Panthi, Luitel, and Belbase(2018)stated that teacher should care his/her students for maintaining social justice classroom. Similarly, mathematics teachers should connect mathematics to daily life while teaching(Panthi, Luitel& Belbase, 2018). Then only students get motivated to learn mathematics.

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To the best of my knowledge, very few studies have explored social justice issues in the Nepalese context. This study aimed to explore two mathematics teachers' attribution to the elements responsible for social in/justice in mathematics education. The research question which guided the study was--How do teachers' attribute elements responsible for social in/justice in mathematics education? I discussed context, diversity, and culture as bases to formulate a theoretical frame of this study.

Context of Teaching Mathematics

To me, the context of teaching mathematics is one of the key elements for maintaining social justice classroom. Teaching mathematics with a social just pedagogy refers to the context of lessons that explore critical (and oftentimes controversial) social issues using mathematics (Gustein, 2006). It applies pedagogical practices that encourage a co-created classroom and provides a classroom culture that encourages opportunities for equal participation and status. Valero (2004) suggested the techniques in which the series of layers of context surrounding the mathematics classroom play a role in the possibilities for teachers and students to engage in mathematics education. Therefore, it assists them in exploring the value of setting of the school and of the students.

Diversity

Generally, we see diversity in our mathematics classroom. There is a question of teaching with an acceptable balance of mathematical notion and non-mathematical notion. In this reference, Gutierrez (2009) suggests that instructors should grasp the tension of teaching mathematics that allows teachers to improve their own authentic practices and political clarity around issues of equity. Besides representing essential pedagogical and philosophical notion, the argument is that transforming curricula and cultures of school insufficient social and structural realities faced by marginalized students outside of school (Martin, 2003). These are the factors as race, class, ethnicity, gender, beliefs, and language proficiency (Gutierrez, 2007) responsible for socially just pedagogy in mathematics education.

Home and Classroom Cultures

NCTM (2000) states, "The mathematics classroom, then, is a place where all students should be encouraged to actively participate, contribute to discussions, share new ideas, and develop solutions to problems that are real to their respective cultural backgrounds and communities." Such a classroom may require teachers to view mathematics education from a social justice perspective. Gutstein and Peterson (2006) state, “Teachers should view students’ home cultures and languages as strengths upon which to build, rather than deficits for which to compensate” (p.3).They view that the teachers make effort to determine, grasp and include students' lived experience into mathematics curriculum and provide meaningful learning opportunities to students for success. I realized that students' and teachers' home and classroom cultures influence social justice in the mathematics classroom.

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THEORETICAL FRAMEWORK

I applied (Gustein, 2006) for theoretical framework based on understanding of mathematics for social justice. He divided his framework into two wider goals: mathematical pedagogical goals(his categorization as reading the mathematical word, succeeding academically in the traditional sense, and changing one's orientation to mathematics) and social justice pedagogical goals (his subdivision as reading the world with mathematics (i.e. more reflective process), writing the world with mathematics (i.e. more active process), and developing positive cultural and social identities.

Reading the mathematical word consists of having a strong mathematical foundation and understanding. Reading the world with mathematics means the use of mathematics to comprehend, examine, and research issues that affect students' direct lives and the broader social world. Writing the world with mathematics involves social agency. Writing the world with mathematics happens when students use mathematics as their voice to change the world. Lastly, developing positive cultural identities requires educators to honour and value students' various cultural backgrounds while also practicing them to change and achieved in dominant culture. Within this reference to the social justice pedagogical goals, this theory supports teachers' experience for responsible element for using socially just pedagogy in mathematics classroom.

RESEARCH METHOD

I used narrative inquiry to explore the stories of teachers. The narrative is also a means of developing and nurturing the skills of critical reflection and reflexivity that are necessary for anyone conducting research into their own practice (Bold, 2012). The narrative inquiry has been increasingly used as a major approach in qualitative research and its 'primary occupation has been the project of "capturing experience" […] and developing modes of analysis and interpretation that provide explanatory power for understanding "experience" (Hendry,2007, p.492-493). Polkinghorne (2007) notes that it is the "study of stories […] and narrative researchers study stories they solicit from others-- oral stories obtained through interviews and written stories through requests" (p. 471). Moreover, narrative inquiry represents the practical transformation of teachers' experience (Clandinin& Connelly, 2000). Connelly and Clandinin (2006) propose three commonplaces – temporality, sociality, and place.

The participants in this study were two public secondary level mathematics teachers in Kathmandu. They were selected purposively based on socio-economic and cultural backgrounds. The main cause for choosing Kathmandu as the research site was based on access to schools and teachers for the author. Among various way of making the narrative inquiry, the purpose of the study was to employ in-depth interviews with the teachers. I informed the head teacher of each selected school before my visit. I visited schools and met the participants (teachers) to set the interviews. I took informed consent from each of them to participate in the study. I conducted the interview with

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teachers in the Nepali language with a focus on social just pedagogy in the math classrooms. I recorded the interviews for transcribing and analyzing.

At first, I translated and transcribed the recorded data verbatim in English. I used the transcribed data for interpretation by generating meanings and notion from the interviews to construct narratives (Bold, 2012). Based on the meanings in the narratives of teachers, the researcher focused on ‘what's' of the stories (rather than the structure) and identified "common elements to theorize across cases" (Reissman, 2008, p.58). The thematic analysis continued with several steps such as "reading the transcripts several times, inductive coding, developing themes and subthemes and seeking to identify core narrative elements associated with each theme" (Ronkainen, Watkins, &Ryba, 2016, p.16). I compared various themes based on meaningful texts data from critical theoretical perspective. I analyzed and re-analyzed the data until five final themes emerged out of it.I used pseudonyms of the participants as Karma and Dharma for the anonymity.

RESULT AND DISCUSSION

Five themes emerged from text data -- the role of family, students' interest, students' and teachers' status, the diverse context of the learner, and social structure. I discussed each theme by connectingto praxis, as the interplay between theory and practice (Panthi, Luitel & Belbase, 2018) as follows:

The Role of Family

Dharma argued, "I feel the children of the parents who do not take care and watch their children for studying mathematics, are weak and passive in the classroom". I considered that they might neglect the learning and doing mathematics. Karma added, "I realize the parents who take care and guidance to their children, they are good and actively participate in the classroom learning. This affects most of the students' better performance. The teachers should focus more on marginal, average and weak students who have the poor performance in learning mathematics. However, teachers' counseling is necessary to parents for social justice. It shows that the family had a greater role in improving the performance of their children. The privileged parents have the higher incomes, greater knowledge to get more academic benefits for their children (Chiu & Walker, 2007).

Students' Interest

Karma said," I think the learning activities and outcome of mathematics depend on the intention of students' who take different discipline related with mathematics (otherwise) for further studies in college level". He highlighted that different interest of the students' future orientation influenced socially just classroom. For instance, he feels that students have various interests in the futures studies such as science, medicine, engineering, Nepali, English, social, accountant, law, etc. Students who studied the subjects (i.e. mathematics, physics, engineering, management, etc.) in the future are curious to learn school mathematics. They seemed very active in the

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classroom. Further, he added that the intention of students that they may not require detailed mathematics knowledge for studying law, journalism, social study, etc. in the future. This type of students gives less emphasis on mathematics in the classroom. Some of the students' intention was just to pass the examination. I felt that the preferences of students' practice in the subject according to their future orientation. Dharma argued, "I might be difficult to uplift working-class, disadvantaged, and weak students in learning, who have the poor economic and social background". He focused on the challenges to teach mathematics to different interest groups of students in a multicultural classroom context.Inequalities relating to social class, in particular, the low performance of students from homes with less economic resources remain a challenging and a common problem (Noyes, 2009).I felt that the students devoted towards mathematics regarding their intention of a different subject at the higher level. Wonnacott (2011) addressed the diverse needs of a multicultural student population and the diverse nature of students must be addressed to achieve equity.

Students' and Teachers' Status

Dharma said, "I realize most of the teacher and students have different economics and home, linguistic, cultural status". He focused various economic background of the home such as poor socio-economy, rich economy and so on. This influenced on learning of students. I realized that the students and teacher might have different indigenous languages. So, the status of the students and teacher affected the classroom learning and teaching mathematics. At first, they might become culturally different in the classroom. The various native languages and culture of students make the classroom inequity. Especially students did not understand topic due to the language of the students. In this context, it was difficult to understand mathematics in a meaningful way.

Dharma viewed, "I experience the students who have the high academic and economic background become superior in the classroom. They might be slightly dominating other colleagues who have the poor economy and working class". He pointed out that high economic and academic students dominated they are low economic and poor students. The difficulties that all students might tackle, students of different culture, females, other language, students from low-income families, and others tackle more demanding difficulties than their majority peers (Herzig, 2005).

The Diverse Context of Learners

Karma said:

> I look there is a diversity among students in the classroom. I want to give an example, as the students are the Brahmin, Kshetri, Newar, Magar,Tamang, Terain, Muslim, Rai, Gurung, etc. They are from different part of the region. Many of them are working class students.

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Dharma added, "I think the teachers might address the inequity issues in the diverse context". Social justice is associated only with the notions of ethnic root or male-female. However, those ideas are negligent the valuable part of elements of diversity between humans. Concerning the settlement of social justice, it is necessary to promote the right techniques for analyzing the distinction between people (Turhan, 2010). According to the Dharma, different castes and cultural representation of students exist in the classroom and so teachers face those issues of inequity in the classroom. Colquitt(2014) found in his study that the teachers who instruct in the context of social justice to the students have a higher level of reasoning and the performance score those with less social justice context. Further, it was necessary that the teacher prepared the course regarding its purpose for the diverse learners that expect to instruct and get opportunities for professional development to help the teachers of diverse learners. According to Karma, "However, it is challenging to address the inequity in mathematics teaching and learning; it might seem that there is a gap between different ethnics, castes, genders, and race. There was a group of divergent learners that influence the socially just classroom context. It might be inequity, injustice and unfair teaching and learning. So, the teacher should treat each and every student regarding their divergent context. The issue of diversity is the precursor of understanding social justice issues (Garii& Rule, 2009)

Karma said, "I opine the teacher must be facilitator, cooperator and knowledgeable person on the subject. Some of the learners have the innate capacity and so excellent in performance and others have less capacity to learn mathematics". The various group of learners must learn mathematics along each of the three strands of learning such as acquiring knowledge, participating in practices, and developing a sense of belonging. However, some learners found it difficult to engage with problems that are set in unfamiliar contexts or that are not sufficiently relevant to their lives (Herzig, 2005).

Social Structure

I thought home is the first school for the children and parents are the first teachers. In the words of Dharma, "there are the gender disparities and male domination exists in the mathematics classroom and even in our society." The belief of the parent and teacher was not positive towards females. Generally, females might not get equal opportunity in the home and even in school due to the domination from the parents and teachers towards them.

Dharma *said*, "I see the teachers' and the parents' belief is not positive towards the female students and female students did extra work in their home due to this they might never complete homework than male students". I clarify his views that they give the daughter works more on the home than a son. The parents invest more for studying mathematics to their sons. They give their sons more facility than daughters. In school, the daughter might not complete the homework of mathematics. Due to this, the study shows that the male has the better performance

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than the girls (Nolan, 2009). The school culture and social structure are powerful determinants of how students learn to perceive themselves. These factors influence the social interactions that take place between students and teachers and among students, both inside and outside the classroom. As Cotton and Hardy (2004) suggest: we build our course socially justifiable, students are being critical about practices, we are also being aware of the social and political structure and contexts within which operate.

IMPLICATIONS

This study might be helpful for the teachers, experts, curriculum designer, and policymakers to make the policy for empowering the disadvantaged, average and marginal learners. The teachers understand the responsible elements for social in/justice in the mathematics classroom and these assist them to think for making a plan of different strategies for equitable classroom. The study brings awareness to all stake holders for addressing these elements. I recommend that the similar studies need to be conducted at the university level.

CONCLUSION

I found that role of family, students' interest, the status of learners and teachers, the diverse context of learners, social structure are the major themes as responsible elements for socially in/justice mathematics classroom. The family has a valuable role to develop their children a positive attitude towards mathematics. The traditional belief system as "mathematics is difficult" to influence their children mathematical learning. Not all students motivated to learn mathematics because they have different future interest as lawyers, doctors, engineers, and teachers with or without mathematics etc. I found that students and teachers have different status affect students' mathematics learning. Students have diverse cultural backgrounds in the mathematics classroom. Our society is male-dominated society, as parents might give more facility to their sons than daughters. The high-quality teachers, standard curriculum, high expectation of the students are responsible for social justice classroom.

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AN ACTIVITY BASED ON AN EVERYDAY LIFE CATEGORY AND THE MOVEMENTS IN A MATHEMATICS TEACHER WORKING GROUP

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Abstract: The intention of these writings is to share the experience of an activity developed in a working group with mathematics teachers as a part of a project developed in five different cities in Brazil. The activity (one of a series) based on the everyday life category called decision-making brings to the group different/other points of view and makes possible the movement of different ways of life in a classroom. The power of this kind of activity is enhanced when notions from the Model of Semantic Fields (MSF) like meaning production and non-deficit reading engender an unusual discussion for the classroom and for mathematics teacher education.

A QUESTION, A THEORIZATION, MOVEMENTS

A quite common practice among Brazilian teachers, especially mathematics teachers, is the use of contextualized everyday life situations to discuss activities in the classroom throughout the 6th to the 9th year (11 to 15 years old). Right after introducing a mathematical concept, the context is ignored and the mathematical discussion prevails. Teacher professional development courses are frequently structured to present alternative ways to teach Mathematics in a classroom making use of technology, manipulative materials, problem solving methodologies and mathematical modelling. However, we should not ignore the fact that cultural, political and economic variables have their roles and affect the processes of teaching and learning Mathematics.

Would it be possible to work with teachers and develop activities that go beyond the mathematical content? Believing in this, researchers from the Sigma-t[3] network research group have conducted research thinking about and producing other ways of organizing educational practices. The research mentioned before and this work have, as theoretical background, the Model of Semantic Fields (MSF) developed by Romulo Campos Lins (2001, 2006, 2012). Lins says that the MSF is not a theory to be studied and followed step-by-step, but a theorization where the movement of its notions allows us ways of reading *worlds,* and also (re)inventing ourselves in these *worlds*.

In this paper, we analyse an activity based on the everyday life category called *decision making* that was implemented in a mathematics teacher working group and which is part of a research project on mathematics teachers' professional development. This working group happened in the city of Campo Grande, state of Mato Grosso do Sul, Brazil, during the second semester of 2016.

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Model of Semantic Fields (MSF)

In order to continue, we present some notions of the MSF so we can develop the discussions about the activity and what we came to understand when developing it with the teachers. The *meaning production* is the main aspect of the model because Lins assumes that this is essential, not only for the learning but also for the human cognition. By *meaning*, the MSF considers it as being what a person *actually says* about an object in a situation or activity and, *not everything a person could have* said about it. An *object* is anything one can say something about. The constitution of an object happens when we put into movement the meaning production for this object anywhere and anytime this process takes place, in our everyday lives, at home, in the Mathematics class and so on.

The MSF characterizes knowledge as a pair "[…] constituted by the stated proposition which one believes to be true (the *statement-belief*), together with a *justification* the subject has for holding that belief" (Lins, 2001, p. 43). The justification has to be acceptable for some *interlocutor* because one believes that this interlocutor would also say/adopt this justification. Knowledge is produced as it is enunciated.

Another notion in the MSF that we use is the *non-deficit reading* that, according to Lins (2012), applies in a general way to production processes both for meaning and knowledge. The non-deficit reading indicates a process where I believe in everything that someone says (if that is plausible) and it makes sense. The non-deficit reading reads the other by what this "other" is, understanding his/her legitimacies, and not by what is missing from (or what is wrong with) this "other". It is a positive way of reading. The non-deficit reading applies here in all the productions during an activity: writing, oral, gestures, expressions, movements, body language, etc. From now on, in this article, we assume that every time we use the term "read" or "reading" we are talking about a non-deficit reading.

ACTIVITES BASED ON EVERYDAY LIFE CATEGORIES

During one day, our most ordinary actions require us to make many decisions. Some of these are done automatically, and in most we do not need an elaborated or scientific knowledge. This is also the case when using Mathematics in our everyday lives. The Handbook of Mathematics Teacher Education Volume 1 (Sullivan and Wood, 2008) presents some researches and pedagogical perspectives where diversity, equity and culture can foster practices in teaching mathematics. Tatto, Lerman and Novotna (2010) present results of an investigation carried out across many countries into different aspects of the preparation and development of mathematics teachers that highlight the importance of broadening the learning process towards the mathematical activity and the many strategies available. Everyday life mathematics is directly

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relevant to Mathematics Education, but we just cannot transport it directly to the classroom (Lins and Gimenez (1997), Carraher and Schliemann (2002)).

The meaning productions on the streets, at school and even in Mathematics itself are very different from each other and, sometimes, one does not legitimate the other. We can identify the Mathematics of the mathematician, the Mathematics of the streets, the Mathematics of the mathematics teacher (LINS, 2006), and so on. We believe that activities not directly related to mathematical school contents, the *everyday life category* activities, are powerful elements to organize mathematics teacher education and, on account of their familiarities for teachers as well their pupils.

When dealing with activities based on everyday life categories, it is important to keep in mind that there is no control of the directions a discussion can possibly have. Life is a group of elements operating together: culture, social life, media, economy, religion, beliefs, family values, with each one influencing the other. Different ways of life lead to different ways of thinking and producing meanings, or rather, other ways of life lead to other ways of thinking and producing meanings. If we do not accept this variety of life forms, we are reinforcing one life form at the cost of others. It is possible that an activity is based on an everyday life form which makes no sense to others. The activity can be plausible for a group and not plausible for another group. Teachers and students can bring a mathematical or a completely non-mathematical discussion during an activity. Lins (2005) offers a perspective where an activity that discuss the idea of space presents two situations: (i) the centre of the category is the mathematics of the mathematician presenting ideas of Linear Algebra (basis, vector, plane, dimension, subspace, linear transformation), and (ii) the centre of the category is the everyday life presenting the natural space (surfaces and paths, distance, proximity, visual-geometrical, coordinates, plane (naturalised)).

During her doctorate course, Oliveira (2011) analysed a module of a course, trying to understand how an in-service professional development process based on an everyday life category called *decision making*, unfolded. The use of an everyday life category to create and develop the course made it unique. It is obvious that during the course, mathematical ideas were discussed and questioned, but, in the centre of this scenario, the mathematical content was not the only focus.

In 2014, a group of five Brazilian researchers proposed the research project "The use of everyday life categories in the professional development of teachers who teach Mathematics" (our translation) (Viola dos Santos, 2014). The main objective is to investigate the professional development of teachers who teach mathematics in spaces where activities involving the category of everyday life are used as problems.

In this project, working groups with teachers who teach mathematics were developed in five different cities in different states of Brazil: Bagé-RS, Campo Grande-MS, Diadema-SP, São João del Rei-MG and Sinop-RS. Assuming the contextual differences across Brazil, some different discussions were expected. Throughout the first semester of 2016, the researchers responsible for each city prepared activities

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based on everyday life that, together, created a set of activities. To validate these activities, each city set up a pilot working group and the meetings were recorded in audio and video. During the second semester of 2016 each city set up new working groups, with new participants, following the same methodology. We believe that these working groups are a space for in-service mathematics teacher education where they share not only their productions about the activities but their professional lives as well.

AN ACTIVITY AND SOME POSSIBILITIES

Here we present one activity and some of the teachers' productions about it that happened in the city of Campo Grande, in the state of Mato Grosso do Sul, as mentioned before. The working group talked about the activity during three meetings and in the first day of discussion, there were eight mathematics teachers from different segments: teachers, postgraduate students (some of them are also teachers) and lecturers. The activity called *"Thomaz Lanches"* was named after the diner that inspired it. The following transcription is the translation of the activity:

> Thomaz Lanches is a diner that charges its customers in a different way. The snacks are on a counter and the soft drinks are in refrigerators both at the disposal of the clients. The customers serve themselves at will and when they are going to pay, the cashier just asks what they ate and drank. Do you think that the owner of Thomaz Lanches loses money by charging customers this way? [4]

In this meeting, there were eight teachers and three of them knew the place. For the other five teachers, it seemed impossible to imagine such a place and they got surprised when the others told them it is a place in their city. Below we present some transcriptions of what these teachers said about the activity. Our intention is not to identify the teachers so we use fictional names.

After reading and thinking about the place and the activity, some teachers started talking in the group. The transcriptions do not represent the full dialogue but they are in the sequence they were brought to the group.

Marcelo: I don't think so. He takes advantage from the honest customers not to lose money. It is a sale strategy.

Márcia: The food there is very good, well prepared, but, it is more expensive and the portion size is smaller, compared to other places. However, some customers go there for ages, they are loyal to the place.

André: The owner surely makes some calculations based on what is produced during the day. And if at the end of the day the money is not what he expected, he immediately knows he is losing money. If he did not profit with the place, he would never keep on selling that way.

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At first, Marcelo, Márcia and André compare the food at Thomaz Lanches to other places (size and price) and try to give the activity an idea that could put Mathematics into motion. None of the teachers in the group questioned them about their opinions because for all their answers were plausible. They were enunciating what they believe it was true.

Márcia: The weirdest thing is not leaving the place without paying, but telling the cashier what you ate. It is strange, at first, because my husband and I went there and my husband kept telling me, “Do you think he will understand that what you are paying is referred to what you and I ate? The cashier will be looking at me and trying to guess what I ate and drank. If I leave the place with you, will he/she know what I ate? What he/she will think of me?”

João: When I was there, I didn’t feel comfortable getting the food or the beverage by myself. I felt strange doing it.

César: The first time I went there I didn’t know how it works, so I waited a long time for someone to get the food for me. The attendants simply put the food on the balcony and returned to the kitchen. I waited for some time until another customer told me that I had to serve myself. That was weird to me.

Marcelo: It is different because the usual way is to have an attendant that serves and takes note of everything we order. It is a kind of control that we don’t have at Thomaz Lanches.

Márcia knew the place before the activity and explained what she and her husband felt uncomfortable about, that is, for the lack of control by an employee. The discussion guideline then changed to a different subject. João and César talked about their experiences at the diner. They suggested it was an unusual way of behaving at a diner that can be experienced as totally normal, but in their culture, something totally unexpected. Each one experienced it their way and built an image of the place. The experience of going there a second time was not the same, because they recognized it as “being normal” (to that place).

Marcelo: If it is more expensive than in other places, the owner recovers the money loss with the customers that are going to tell the truth. The owner has to trust people.

Paulo: Talking about business, just by presenting a different way of getting and paying for the products you want draws people’s attention. In fact, I have never been there and I am curious to know the place. If I don’t tell the cashier what I really ate and drank I am going to get very embarrassed.

Paulo: But when you describe this place, I imagine that it is impossible to leave it without paying because there is always someone watching. There are other

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	places in the city where your table is on the sidewalk and you have to enter the place to pay your bill. It is possible to leave without paying, especially when the place is crowded. I trust the Brazilian people. A place like that may sound awkward to us because we are not used to it.
Márcia:	Places like that are completely possible in Brazil.
João:	You can pay just using money. In my opinion, it is another strategy. The owner doesn't pay bank fees for receiving with credit card.
Amanda:	I believe it is a way of relying on customers' honesty but, if he lost money all the time, he wouldn't keep on with this practice.
Paulo:	If the place worked the usual way, by attendants serving and controlling our orders, do you think he would have so many customers like he has now? I don't think so.

While the teachers continued talking about Thomaz Lanches a new perspective on the activity emerged: honesty, reliability. Márcia that pointed out that the size and the price were ways of not losing money, and started talking about the possibility of places like Thomaz Lanches being more common than the group thought at the beginning. She did not change her mind about what she said at the beginning but, as we read, a new meaning was produced and she was internalized by the other teachers' opinions. Nobody doubted what someone said in the working group because it was possible of happening or plausible.

After discussing the activity during the group meeting, each teacher decided to create an activity based on Thomaz Lanches to implement with their students. A common point/idea was that each chosen class would visit the place, but for bureaucratic and financial reasons it was not possible to carry this out.

One of the teachers presented the activity during a school pedagogical meeting and it caught the principal's attention, who asked the teacher to develop it with his students and helped him by providing a bus to take two groups of students to Thomaz Lanches diner. The first step was to talk to his 26 students from the seventh and ninth grades about trust and honesty. After that, they watched two videos. The first video was about a university in Brazil where the students could take popsicles from a fridge and pay without anyone watching them. The second video was about an experiment where a hidden camera recorded some watches in display on the sidewalk and a box to put the money into. The first part of the experiment took place in a city in the USA and the second one in Brazil (but it was obvious at the end of the second video the intention of the producer – it was an agency convincing people to go to the USA).

Initially, the teacher presented the activity by considering Thomaz Lanches a fictitious place and did not mention its name. Twenty-one students said that it was obvious that the business would have losses and five students said that the owner not only would have losses but it would go bankrupt. When asked if they would tell the

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truth about what they ate and drank, twenty-five students wrote they would do so, expressing moral or religious reasons for it. One of the students wrote that he would not tell the truth because, being the oldest son, he wouldn't have money to pay for everything his eight siblings would consume. Most of the students agreed that a place like that wouldn't exist. Only one student knew the real place and the teacher asked her no to tell them anything at that time. The teacher brought the students notes to the working group and we discussed.

Some days later, the students and the teacher visited the place. Just as they arrived at the bus the teacher told the students they were going to visit the place that most of them thought imaginary. At Thomaz Lanches they didn't show any surprise by the way things worked. The teacher paid for the beverages of all the students so they could use their money to pay only for the food. They talked to the owner and he told them that the philosophy of the place is to trust customers. After the visit, in the classroom, the teacher asked the students to think about what they had written before the visit and what they think about the place after visiting it. All of the students wrote that it was possible for a place like Thomaz Lanches exist and most of them indicated that they told the cashier what they ate because they felt uncomfortable not doing so. All this happened by the end of the second semester of 2016. Throughout the year 2017, the teacher was responsible for the project "Honesty" that involved all the students, their families, and the teachers in the school.

SOME CONSIDERATIONS

An in-service mathematics teachers education course based on activities of everyday life categories is not a traditional way of working with mathematics teachers. There is no way of preventing what directions the discussions are going to take. Such a course does not give the mathematics teachers a map to follow to accomplish the task. This kind of course leaves the teachers open to everything that can happen and invites them to take part in conducting the discussions (not being told by the course itself). Mathematical and non-mathematical productions are possible of being presented and they can be related to cultural, economic and social aspects of different forms of life. Teachers in our working groups are invited to give their opinions, talk about the activities and build with others possible ways of inhabiting other worlds. The activity described in this paper had so much impact on the teachers that it generated a project that involved all the students, teachers and the school staff during one year. When working with mathematics teachers we believe that it is not about being different; but it is about only being possible this way.

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NOTES

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3. Sigma-t is a network of research and development in Mathematics Education that brings together teachers and researchers interested in the Model of Semantic Fields (MSF). For more information visit www.sigma-t.org.

4. Original statement: *"Thomaz Lanches é uma lanchonete que cobra de seus clientes de uma maneira diferenciada. Os salgados ficam em gôndolas e os refrigerantes ficam em geladeiras a disposição dos clientes. Estes se servem à vontade e quando vão pagar o caixa pergunta quanto eles comeram e beberam. Você acha que o dono do Thomaz Lanches tem prejuízo por cobrar dessa maneira?"*

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GENDER ISSUE IN TEACHING AND LEARNING MATHEMATICS

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Abstract: Gender is a social construct with a power relationship between women and men who are distinguished on the basis of their biological differences. In terms of educational settings, gender is a mingled, spontaneous force that affects every social communication. The main purpose of this study was to explore gender issues in teaching and learning mathematics. I applied a qualitative approach working with one female and one male mathematics teacher and one female and one male student. The analysis of interview data revealed three major themes. To address these issues the schools, society, and families should give equal access, encouragement, empowerment and motivation to their daughters for enrollment in teaching and learning mathematics.

BACKGROUND

Gender issues have been discussed in many areas, for example, management and administrations, social works, development, politics, education, etc. "Gender is defined as a network of beliefs, personality, traits values, behavior and activities differentiating women and men through a process of social construction that has a number of distinctive features" (UNDP 2006, p. 82). Women and men are biologically different, but their roles, status, positions, responsibilities, and relationships are equal. Despite the reality, there is a discrimination that is still in existence between male and female concerning the gender-based factors in our society. The society treats males as superior and women are given weaker or inferior positions, although some changes can be felt with women in the position of president, chief justice, speaker of the house, the situation is not favorable for women in many aspects.

The patriarchal family structure has been common in Nepal, where females are dominated in every aspect of life considering men as superior to female (Gyawali, 2006, as cited in Bhandary, 2017). "The socialization of girls and boys shape their understanding of gender role and worldview. The deep-rooted socio-cultural norms and practices of the patriarchal system determine the roles of boys and girls."(Thapa, 2012, p. 37). It seems that society has marked the gender-based roles of males and females. The difference between their physical and biological aspects has been concerned without social phenomena. It has created a big gap in relation to male and female.

The principle of equality is related to providing equal rights to both male and female. Gender equality is considered to be achieved when women and men enjoy the same rights and opportunities across all sectors of society that should include economic participation and decision-making (Maxwell, n. d.). The dealing with both

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male and female should not be different as both do have equal aspirations to lead the society and the needs of women and men should be equally valued, favored and addressed too. Although people talk about equal rights between male and female and constitution also guarantees such rights, the equalities has not been realized yet in practice. It is also a case in the area of teaching-learning mathematics in Nepal. There are various gender issues related to the realm of mathematics teaching and learning at school and in the college as well. Among these issues, in this paper, I have especially focused on a few female related issues about teaching and learning mathematics at the university level.

In my journey of learning mathematics from school to university, there were a minimal number of girl students in the classroom. There is still a misconception that mathematics is a difficult subject among students and parents (Gafoor, & Kurukkan, 2015). Such misconception has severely affected women enrolment and participation in mathematics education in Nepal. Mathematics is normally favored and controlled by men (Chipman, 2005). It is taken as a complicated subject for women. I have experienced and seen in the capacity of female mathematics educator that learning mathematics is more challenging for girls not because it is difficult by nature but by the social stigma of mathematics in general. As a result, there was a very smaller number of female students compared to males.

The participation of women all over the world is very low in economic, intellectual, social and political opportunity. One of the reasons could be due to the less involvement of women in education in comparison to men. Even if they are involved in education, they were not permitted or simply discouraged to become intellectuals in this sophisticated and complex subject. Amelink and Tech (2012, p. 2) described that “the views of mathematics by females are shaped in part by gender-based stereotypes which convey misconceptions that differently innate mathematical abilities existing between males and female.” In Nepal, mathematics is regarded as one of the tough subjects and less interesting ones. Participation of females in higher education in the mathematics department is very low as compared to males. Girls prefer to study "easier" subjects such as biological science, education, sociology, economics, home science, culture, Nepali and history. Even if many of the girls who are interested in science and mathematics, they get married before completing bachelor’s and master’s degree and leave their further studies. They don’t tend to continue and make their career in those subjects. Most of them do not get good family support and environment for pursuing mathematics in higher education. They are often compelled to shift from mathematics and science to other easier subjects. Therefore, the number of women completing masters and Ph.D. in mathematics and mathematics education is very low as compared to the male counterparts.

The overall female literacy rate of Nepal is low in comparison to the male literacy rate. The male literacy rate is 75.1% compared to a female literacy rate of 57.4% (CBS, 2011). The female literacy rate has increased from 42.8% (CBS, 2001) to 57.4% (CBS, 2011). Though there is a slight increment in literacy rate of females

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through the period of 10 years, I have found very less percentage of females who have enrolled in different educational programs. Out of 1,652,624 students involved in several fields of study after completion of SLC, only 622, 012 are females i.e. only 37.64% (CBS, 2011). In the field of mathematics and statistics, there was an enrollment of only 2,820 females out of 17,260 students i.e. 16.34% in Tribhuvan University. It clearly shows that the participation of females in mathematics is low. From these reports, I discovered that there are fewer enrollments of the females in university education, particularly in the technical field.

When I started my professional life as a teacher from Balkumari College of Nepal, I found no girls in the mathematics classroom. In the college where I taught there were no female mathematics teachers teaching mathematics. So, I realized that it is a big issue in our teaching and learning context. I used to wonder why the number of girl students taking mathematics was less than that of boys or even nil. Why some of them left mathematics education without completing the course? Why do girl students take mathematics as a complicated one? Why is mathematics said to be irrelevant for girls? How could we empower girl students to learn mathematics? Therefore, I became interested to explore the issues behind those problems.

Various social, cultural and organizational barriers hinder women to be mathematics students and teachers at the university level. There are a great variation and inequality between men and women to participate in teaching and learning and joining a profession in mathematics, including other disciplines. In my experience, the involvement of females in mathematics teaching and learning at college and university levels have not been changed significantly in the last decade in Nepal. However, I have seen some improvements in teaching and learning, but it’s not improving significantly as is expected to be. “Just as gender equality in education and women in the teaching-learning have strong linkages, so the issues of women, teaching, learning and the feminization debate also have a place within the broader context of gender equality in society as a whole” (Kelleher, 2011, p. 5). Actual scenarios of the females participating in teaching and learning mathematics are the main issues to be analyzed. In this paper, I am going to find out the gender-related issues, especially focused on women in teaching and learning mathematics.

METHOD

Following a qualitative approach, I have explored university-level teachers' lived experiences of being a mathematics education faculty member. I applied in-depth interviews as a tool to collect experiential anecdotes of female mathematics teachers and students. I used a convenience sampling technique with one female and one male mathematics teacher. They had more than 15 years of teaching experience at the university level. I also selected one male and a female student studying mathematics at the undergraduate level. I took an interview with the participants separately in their place of convenience; I did the audio recording of their interview without harming their ethics. I used a guideline to conduct in-depth interviews with the participants. The guidelines included a common set of questions to get the

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information about gender-related issues in teaching and learning mathematics. Each interview lasted for about 30 to 40 minutes and it recorded using an audio tape recorder without harming their ethics. The interview data were transcribed for analysis and interpretation. The interview data were analyzed thematically, and three major findings were drawn. In order not to reveal the identity of the participants, pseudonyms were used.

ANALYSIS AND DISCUSSION

The qualitative analysis of interview data helped me to construct four themes associated with gender issues in teaching and learning mathematics. They are- lack of access and participation, lack of mathematical empowerment and low achievement of women in mathematics. Each theme has been analyzed and interpreted below.

Lack of Equal Access and Equal Participation

In relation to the theme associated with lack of equal access and participation, I asked the teacher participants about their views, *"How are the access and participation of female students in your college?"* The male teacher participant said, *"There is access but female student and teacher participation is low as compared to male in teaching-learning mathematics."* but the female teacher's answer was a bit different than that of the male teacher. She said, *"The ratio of female teacher and a female student is very low, the infrastructure like a classroom, toilet, etc. are not female friendly*". Then, I asked the same question to the female student and she answered, "*We have low access and there is not much female participation in class*". The male student said, *"There is the same access, but female student and teachers' participation is low."* Again, I asked, "*How are the female students and teachers treated?" The* male teacher and male student said, "*Good treatment*." The female student said, "*We respect our female teachers. But our male friends try to tease, rag and make fun of us and the female mathematics teachers as well*." The female teacher said, "*Students behave a bit funny and silly in the class. I must say, it's not that easy to motivate and bring them on track for the female teacher. The students show two different attitudes to the female and male teacher. One thing is that they underestimate and harass the female teachers giving them no respect at all. Another one is that some students are found giving due respect to female teachers as there is hardly any number of female teachers teaching mathematics".*

From the interview data and discussion in the literature, female teachers and students are not treated like male teachers. Although Iwu and Azoro (2017) believe that gender mainstream has led the women participation in science, mathematics, technology, and advanced studies, they are not provided opportunity to participate in teaching-learning mathematics whether it is in classroom activities for students or training for teachers. They are not getting equal access to and participation in teaching and learning mathematics. Girls are not active participants in mathematics due to the societal expectation (Ramu, 2014). Further, society does not have to

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believe that female mathematics teacher and the student has a caliber in mathematics as mathematics is a historically male-dominated field. Even if equal access is provided, the environment is not female friendly which makes them demotivated for participation.

Lack of Mathematical Empowerment

Empowerment is a form of gaining power in particular domains of activity by individuals or groups and the processes of giving power to them, or processes that foster and facilitate their taking of power (Ernest, 2002). In this section, I questioned the teacher participant that, *"How do the family, society and the college management empower the female students to study mathematics?"* Both the male as well as female teacher's reaction was similar. She mentioned, *"From the side of the college management, there is an empowerment to students as the college focuses on bright future of the students, but there is no special orientation program about the importance of mathematics for female students. But, it is not positive in the case of family and society. Male dominant society and family does not encourage females to study mathematics."* I raised the question to the male student, "How well does your family, society, college management encourage you to study mathematics?" The male student said, *"We are encouraged to study mathematics but our female friends are not much encouraged".* The female student said, *"All of them said that mathematics is not subject for female, but I came here to study mathematics ignoring what they have said. It was my personal decision and self-empowerment to study mathematics".*

In this context, Sivakumar (2017) explained that in order to provide equal status in the society women need to be empowered. If women are empowered, then they can get equal opportunities as men. Paudel (2017) agreed that if women are empowered, they become independent to take decisions about their life. They can analyze the value of mathematics. In the discussion, it is seen that the empowerment for the girls is lacking. Even the male teachers and students agree that there are few empowerments for girls in teaching and learning mathematics. Thus, the issue regarding the lack of empowerment is seen in the case of mathematics teaching and learning from the interview participants.

Mathematical Achievement gap

The main focus in this section is in comparing male and female achievement gap in mathematics. For this, my questions to teacher participants were, "*What is the attendance condition of the girl students in a mathematics class? What are the pass percentage of girls' student and the scoring scenario? Do you perceive any differences between male and female capabilities in math?*" The male teacher replied, "*Attendance condition is good in female students, but their pass percentage is lower than the male students. I think the perceiving capability of the female is a little bit lower than that of males.*" The female teacher replied, "*Female students' attendance condition is good, but their pass percentage is lower than the male student. I think the perceiving capability of male as well as female is the same but due to the lack of*

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encouragement, self-confidence, feminine problems, the females have lagged behind in perception." Then the questions to the student participants were, "*How regular do you attend your class? How is your academic performance? What is your further planning for the future?*" The male student replied, "*I am regular in my classes. My academic performance is quite sound. My future plan is to study masters, and then after, I want to apply for a job of mathematics teacher in a university.*" The female student replied, "*I try to attend my regular classes, but due to my personal causes I am not able to attend some classes. Sometimes, biological causes hamper my studies and academic performance. I have a plan to study masters, but there is still a question mark whether my family will allow me to do it or not.*"

Relating the issues that came from the interviews with literature, Pajares (2005) said girls report lower confidence than boys do in their math and science abilities. Similarly, Watt (2007) agrees that the gender difference in mathematics participation lies in the fact that girls have less confidence in their mathematical abilities than boys. In some ways, it may or may not be because of their birth or biological factors. Hall (2012) found that "females' attitudes toward mathematics and their mathematical self-concept was statistically significantly more negative than males', although males and females reported similar levels of understanding of the mathematics that was taught in class." These two reviews match with female participant's view. Gender gaps in relation to achievement and attitudes existed in the past and still exist in the present. While girls tend to score lower on standardized tests in mathematics than boys, the gap is not due to biological differences but because of socially constructed factors such as gender roles.

In the context of Nepal, NASA report (MOE, 2015), it is clearly seen that boys mean mathematics score is higher than girls. It is worth mentioning that all content areas of mathematics are dominated by boys. Similarly, NASA report (MOE, 2018) stated that the mean score of boys has been higher compared to mean score of a girl student in grade 8. The boy's students have achieved the mean score of 505 for boys and 495 for girls out of 550. Likewise, myself being a female mathematics teacher, I have experienced and coped with these sorts of challenges. Thus, the issue regarding the achievement gap is due to personal, family and the biological problem faced by a female in teaching-learning mathematics.

CONCLUSIONS

Women and men are biologically different but their roles, status, positions, responsibility are somewhat the same. In our male dominant society, there is a different way to see the female from the point of view of mathematics education. There is a belief that a female isn't able to grasp technical subjects such as mathematics, science and therefore is unable to teach and learn. While interacting with male and female teachers and male and female students in the university and summing up their view, I found this subject to be male dominant in the society. So, for the upliftment of women in society, we should focus on the issues that I have raised in this paper.

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PLANET OF THE APES AND MATHEMATICS EDUCATION (OR: FICTION AS THEORIZATION MOVEMENT IN MATHEMATICS EDUCATION)

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Abstract: In this paper we present a discussion about the fiction as theorization movement in Mathematics Education. This discussion is triggered by a movement that uses fiction to problematize initial mathematics teacher education and professional practice in the current social context in Brazil. We were inspired by the movie "Planet of the Apes" directed by F. J. Schaffner in 1968. We consider the use of fiction as a possible way of denaturalizing naturalized issues and also of operating other logics in relation to social-political issues of research in Mathematics Education.

INTRODUCTION

In Mathematics Education there are many recent discussions about its methods, procedures, themes and objectives. Notably, the "social turn" (Lerman, 2000) and the "sociopolitical turn" (Gutiérrez, 2013) invite us to discuss and problematize other spaces in research in Mathematics Education, going beyond the processes of teaching and learning mathematics. However, we believe there is a need to construct other invitations to problematize our own processes of producing discussions and theorisations.

Our discourse about the *other*, which puts us sometimes in unquestioned situations, as inhabitants of ivory towers, and the way we construct these discourses, must be problematized. How can I practice an inclusion movement if I am the one that is indicating who should be included? How can I practice a social justice movement if I assume an ideal of justice (and I do not give up on it), where I judge what should and/or should not be considered as justice? To what extent do we put our identities at risk in order to problematize our research assumptions? For what reasons do the dichotomies (oppressed and oppressive, dominant and dominated, the teacher (who knows) and the student (who does not know)) still stay present in our speeches, no matter how we try to question them? We do not expect to find answers for these questions. Rather, we raise them with the intention of producing some effects, without generalisations, and to spark other ways of producing meanings, different from those already produced.

These, among other issues, inhabit our discussions in this paper, not as a central focus but as a possibility to constitute fiction as a theorisation movement in Mathematics Education. A movement marked by the desire to be and produce with the *other* in

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such a way as to put aspects and elements of our identities (always local, multiple, in movements of transformation) at risk.

We present a discussion about fiction as a theorization movement in Mathematics Education. We highlight the potentiality of fiction as a way of problematizing some naturalised ideas in the spaces of initial mathematics teacher education and professional practice of mathematics teachers in the Brazilian context.

In the next section we present a fictional newspaper report on teacher education in Brazil. In the following sections, we discuss the potentialities of fiction as a theorization movement in Mathematics Education, and return to the example of the approach.

A SHORT FICTION

FOLHA DO BRASIL

Wednesday, February 23rd, 2011 – 67th year – nº 20666 www.folhadobrasil.br Director: J R V Santos

TEACHERS GET ANOTHER SALARY RAISE THIS YEAR. THEIR SALARY AND A FEDERAL JUDGE'S ARE THE SAME.

The raise in salary is one of the reasons to change the criteria for admission of teachers to the public education system. The new salaries and the admission process directly affect a large number of people that want to work as elementary teachers. This week we visit one of the courses most sought after and get in touch with this process.

The occupation of elementary school mathematics teacher has become one of the main positions in the national scenario. Note the influence that the Society of Mathematics Teachers exercises in the Ministry of Education, Science, and Technology, the respect that teachers receive from professionals of other areas, and the prestige they are accorded in the media. It is not strange for students from elementary and public schools in Brazil to feel the desire to engage with the careers of their teachers and become teachers like them.

Educating students through mathematics is a highly valued practice. However, being a mathematics teacher at the public elementary school is not an easy task, since it requires dedication, developmental courses, and, most importantly, the production of investigations about his/her professional practice. Teachers are researchers of their classes, of their schools, of the themes that involve the act of educating their students. The occupation requires a high level of training in the most varied of contexts. A teacher needs to know about World and Brazilian History, know the work of many

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writers, to understand the current economic situation of the country. The school, that is, her/his place of work, is not a preparation for life, it is life itself.

The initial teacher training course is held over four years in the Training Schools. There are several schools in several states and the level of quality is always very high. The infrastructure always is aligned with technological changes, with computers and didactic materials of the latest generation. The place, Training School, is the object of desire of the inhabitants of the cities. At least twice a year, elementary school students make excursions to these schools to learn about innovations.

The training teachers in the Training Schools are elementary teachers. While they work for a period in the Schools they do not stop teaching in their classes. They have reduced working hours and extra time to prepare their classes. A principle always emphasized in the Training Schools and experienced in the daily practices of teachers of elementary education is to prepare, discuss, and present their planned classes before giving them to the students. This is an extremely important step. In the teacher's workload, these activities are counted.

The themes discussed in these four years are basically evolving projects and discussions about classroom experiences. In small groups, the teachers dedicate themselves for a time, with an advisor teacher, to the study of one of the themes. They elaborate didactic proposals, study research from other teachers, watch videos of teachers teaching classes on the subject, they observe teachers and students, until they work with their own students.

After a project closure, there are always seminars and discussions with various teacher trainers and course colleagues. These are essential in both initial and continuing training, because trainee teachers provide other perspectives on the practices discussed, and in-service teachers reflect and share experiences.

There are projects on topics of general formation, in which the teachers in formation discuss arts, politics, literature, economics, among other subjects that are of interest at that moment for the population. Teachers are always attentive to what is happening in the country, at all levels. They know about the discussions and decisions that are taken in the Chamber of Deputies and in the National Senate, about the future proposals to expand the most varied sectors of the development agencies.

Prospective teachers attend at least four schools during their training. These schools are chosen by the teacher trainers with the purpose of giving them experiences in the most varied of contexts: schools with a great number of students and teachers, who have social projects with the community; small schools with few teachers with a focus on scientific research; schools with great difficulties with indiscipline of the students; schools in which the students are well behaved and have few disciplinary problems.

After four years of training in these spaces, future teachers become in-service teachers in the municipal and state elementary education systems. The training continues and in the first year the teachers always have the company of a senior teacher in their classes. This accompaniment favors the continuity of the discussions of the Training School.

In the second year, the in-service teacher becomes an elementary teacher, having her/his access to the teacher union guaranteed. This is an intermediate stage of the profession where he/she is no longer accompanied by a senior teacher, but takes part every week in working groups at school headquarters. His/her classroom also becomes a welcoming space for future teachers in initial formation.

Career progression goes through several stages until he/she reaches the position of head teacher, known as a senior teacher, and then as principal of a school. The last level of training of the professional in education is reached in these stages, directed to two distinct actions. In the first, senior teacher, the action turns to the Training Schools where the professional becomes responsible for accompanying the future teachers in initial formation. In the second, the activity is directed to the administrative career, where the teachers besides taking the direction of the schools are eligible to positions in the teachers' unions.

Because of the importance that teachers have in the most varied sectors of the country, it is not surprising that some important positions are occupied by them. By two consecutive mandates, the post of Minister of Education, Science, and Technology is occupied by mathematics teachers. The National Agency for the Promotion of Scientific Research is also presided over by a teacher, who works together with several others from many states of the federation. In the most diverse committees that discuss the most diverse subjects of national interest, there is always a mathematics teacher. In fact, in addition to the prestige and importance in the national scenario, teachers are active and involved in government actions.

The recent salary raise that has resulted in a record number of enrollments in the selection for initial training courses is coupled with a number of benefits received by these professionals. Thus, even with the raise, the salary may be, at the end of the month, the smallest part of the income they receive.

Elementary school teachers receive funding for their research projects in development. These payments are negotiated directly with the Ministry of Education, being annually readjusted. In addition to the funding, teachers are entitled to compensation for expenses related to travel for scientific dissemination in national and international events.

Other resources include the granting of scholarships to teachers in initial training and professionals from other areas who contribute to the development of projects in Training Schools. In addition to the equalization of the top salary of teachers to the top salary of a federal judge, the benefits received by educational professionals, which are not accounted for in the maximum wage, can increase the salary of these professionals in career development above those of federal judges.

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In addition to the salary benefits, elementary education professionals have the right to periodic licenses that aim at professional qualification and updating. Every three years these professionals can request four months of training, which allows them to take courses nationally or internationally without effects on their salaries. And three times during their career they can also apply for a sabbatical year that can be used for both longer-term training courses and specialization in areas other than their own. Sabbatical years are important for the more general training of these professionals.

The financial perspectives associated with the benefits and social prestige of these professionals make this career so coveted today. Salary equalization with the career of the Federal Judges is just another draw of this career that surely represents the profession of the future.

Mathematics teachers in elementary education have a lot of political strength and society supports their actions ever more. It is not easy to dribble a desire of this class and they are always present in any and every environment in which a new public policy for education and for the country is discussed.

J P A Paulo, Folha do Brasil elementary school reporter.

FICTION AND MATHEMATICS EDUCATION

The use of fiction in research in Mathematics Education in Brazil has been gaining ground. Vianna (2000) uses fiction to think the Mathematics Education as a research field where the scientific rationality is apart. In his doctoral dissertation, Vianna (2000) discusses the lives of mathematics educators working in mathematics departments. It highlights the confrontations that these professionals experience with professionals in the Mathematics area. In order to deal with the complex entanglement that daily life is, the author uses fiction to create four characters that analyze and discuss his dissertation, contributing to the writing of Vianna's work.

In a similar direction, Silva (2006) discusses the identity constitution of the Centro de Educação Matemática, an educational group that worked in Brazil between the 1980s and 1990s. Silva uses fiction as a resource to constitute the possible different identities from different theoretical perspectives. The researcher states that when creating characters for our interlocutors, making use of fiction, is "a way of making explicit the exercise of the difference involved in the constitution of identities" (Silva, 2006, p. 430). This exercise with fiction was the way used by the researcher to highlight the conception of identity adopted in her research: fragmented, heterogeneous, and always related to the perspective of the one that writes the history.

Another use of fiction in academic research that we would like to highlight is the work of De Freitas (2008), who constitutes three fictions in her article in order to analyse:

> … our reliance on the concept of intention. I believe that intention remains a crucial and problematic concept in the justification of social science research, and that many

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> researchers, when pushed to the edge of the "so what" question, will appeal to the moral goodness of their intentions (p. 2).

The researcher states that in using fiction she hopes to have:

> … moved tentatively toward one possible goal of educational research, as identified by Maggie Maclure and Ian Stronach in "Educational Research Undone", in which ambivalence is in the foreground, and the power relations between researcher and researched are repeatedly troubled (p. 3)

Pinto (2013) uses fiction to constitute the multiplicity of preparations existing in the Minerva Project (our translation) that was a mathematics teacher education project that occurred in the 1970s and 1990s in Brazil. The fiction makes it possible to bring to the work structural characteristics present in the collaborators' narratives and in the materials used in the production of the data of the research. This resource used by Pinto (2013) makes use of a potentiality of fiction in academic research that is to generate other formats, modes, and possibilities. The debates created by the author are possible worlds taken from a real one, because they are based on legitimacies constituted from his research data.

These investigations show some uses of fiction in Mathematics Education research, as well as constituting the background for our considerations of the work presented here.

A THEORETICAL FRAMEWORK

For us, fiction is a way of producing knowledge, with its peculiar characteristics, its forms, manners, limits, and possibilities. One way of characterizing the knowledge that we assume in this work is according to the Model of Semantic Fields (MSF), in which knowledge is defined as a "*statement-belief*" (the subject enunciates something in which she/he believes) together with a justification (what the subject understands as authorizing him/her to say what he says) (Lins, 2012, p. 12). Assuming justification as a constituent part of a knowledge differentiates MSF from other epistemologies. This juxtaposition makes it possible, for example, to differentiate the knowledge of different subjects when they make the "same" statement.

As highlighted by Lins (2012, p. 13, our translation):

> … any knowledge comes into the world naively. The one who produces it, who enunciates it, already speaks in a direction (the interlocutor) in which what he says, and with the justification he has, can be said. This direction represents a legitimacy that internalized the subject.

When we speak of legitimacy, we are not making a value judgment on the meaning, or knowledge, produced. Legitimate applies to modes of meaning production when it is considered in contrasted with some culture. Legitimate applies to ways of meaning production when it is considered contrasted with some culture. It only makes sense

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saying that an enunciation is, or is not, legitimate when we look at the cultural framework in which is embedded. The struggle to determine the legitimacy of a mode of meaning production is what defines cultures.

From MSF we understand that in producing meaning/knowledge the subject of knowledge constitutes the object about which he speaks about. Thus, there is no such thing as the object-subject dichotomy, the world of ideas, and the world of things. There are processes of producing meanings and constituting objects in the direction of an interlocutor.

Fiction as a movement of theorization has a firm ground in the notions of MSF, since it is characterized as a process of production of meaning/knowledge.

A SHORT FACTUAL ACCOUNT

As we have said, the fictional newspaper presented here reverses the social structure of the Brazilian educational system in a similar way to what was done in the film Planet of the Apes, in order to allow denaturalization of certain situations. In fact, the education of mathematics teachers in Brazil is done in undergraduate courses at universities – in person or at a distance. Some universities still separate training into two parts: mathematical content and pedagogical knowledge. The first is offered by centers of mathematics and the second by educational centers, which almost always offer general courses, not tailored to specificities of mathematics.

In Brazil not all states pay the salary floor established by the Ministry of Education. In the last three years, the real increase – greater than the rate of inflation – of the salary floor was 3.86%. When compared to the salary of Federal Substitute Judge – the lowest in the judicial career according to data from the National Council of Justice – the salary floor of the teacher is only 8.9% of the salary of a Federal Judge.

In 2014, the Ministry of Education launched the National Education Plan (our translation) that stipulated 20 goals to be reached in the next 10 years. The goals range from the expansion of investment, through the universalization of education, and qualitative improvement in teacher training and performance. Four years after the establishment of the goals, thought out as developed progressively, that is, if the structural goals are not reached the final targets will be hardly achieved, only 30% of the expected has been achieved. Goal 17, that determines that by 2020 the salary of public education professionals be equated with that of professionals with the same schooling, runs into the recently implemented Proposal of Constitutional Amendment n.55 that establishes a federal government spending ceiling for the next 20 years. Not only goal 17, but also goal 20 that predicted an increase in education investment that would receive 10% of the Brazilian Gross Domestic Product by the year 2024, is far from being achieved.

By fictioning a reality where this social structure is reversed, where training conditions become the object of desire of young Brazilians, and the professionals

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have higher salaries and attractive working conditions, we want to highlight that in our country the situation is diametrically opposed.

Fiction is a mode of meaning production, with potentiality to denaturalize the naturalized. The fictional mode of meaning production produces belief-statement in a direction of interlocution that does not belong to that culture, causing a defamiliarization, and possible decentralization of the members belonging to this culture.

Fiction as a mode of meaning production can be a practice of subversion of order that calls into question the relevance of the interlocutor considered legitimate in a culture by enabling interlocutors.

The fictional mode of producing knowledge is always related to culture. In this way, we can only speak in legitimate and non-legitimate modes if we are referring to a particular culture. Therefore, the fictional in one culture may not be fictional in another. For example, producing meaning for equations such as a scale balance that must be in equilibrium is a fictional mode in the mathematics of the mathematician (because it subverts the order of the dominant discourse). For ideological reasons, this mode is not legitimized within that culture. For some time, until the early sixteenth century, to say that the earth was round was a fictional way of referring to the earth's surface.

By including in this paper the fiction about mathematics teacher education in Brazil, we hope to denaturalize some of its characteristics. Such denaturalization, from our perspective, enhances other ways of operating, and some identities may be put at risk. This is made possible because fiction puts at stake the legitimacy of some ways of thinking about our profession. Perhaps fiction may also operate to denaturalize much of Brazilian research that focuses on aspects of mathematics teaching and learning, and where political and economic issues are not still explicitly discussed, moved, or problematized.

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EXAMINING DISCOURSES OF RACE, GENDER AND CLASS ON MATHEMATICS TEXT: A CASE OF THE BRAZILIAN NATIONAL EXAM OF SECONDARY EDUCATION - ENEM

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Abstract: Pedagogical texts play a crucial role in the legitimization of dominant knowledges as well as social and cultural realities. Throughout the world, scholars have been interested in the influence of mathematics texts on the maintenance of educational inequities. The purpose of this study is to examine the ways in which racialized, gendered, and class discourses (and imaginaries) are reified in mathematics texts in Brazil. The author analyzed discourses (and representations) of gender, race and class on mathematical questions from the Brazilian National Exam of Secondary Education (ENEM) from 2009 to 2017. Through critical textual analysis, the author explored the research questions: 'what are the racialized, gendered and classed discourses put forth on math questions?' and 'how do these discourses reify or resist whiteness and patriarchy?'. The findings indicate that mathematics questions on ENEM are deeply rooted in racist and sexist ideologies that perpetuate oppressive notions of who is capable of doing mathematics and allowed to be in mathematics spaces.

INTRODUCTION

Pedagogical texts play a crucial role in the legitimization of particular knowledges as well as shaping social and cultural realities. As sociopolitical productions, these texts reflect the interests of those involved in their production and dissemination. Furthermore, they help structure the ways in which individuals see themselves and others as well as imagine future possibilities (Apple, 1992; Rossini et al, 1998; Kress & Van Leeuven, 2001). Mathematical texts, a primary tool of socialization often reify discourses that portray women and people of color as lacking mathematical competence via oppressive gendered and racialized stereotypes while normalizing white men as successful doers of mathematics (Oyedeji, 1996, Macintyre & Hamilton, 2010; Tang et al, 2010). Thus, the purpose of this study is to examine the ways in which gendered and racialized ideologies are embedded in mathematics texts on the Brazilian National Exam of Secondary Education and how they reproduce of reify dominant discourses about women and people of color.

CONTEXT – WHAT ENEM IS? HOW DOES IT WORK? AND WHY IS IT IMPORTANT?

The *Exame Nacional do Ensino Médio* – ENEM (National Exam of Secondary Education) has been part of the Brazilian national education assessment since 1998. The exam was created by the *Instituto Nacional de Estudos e Pesquisas Educacionais*

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Anísio Teixeira – INEP (National Institute of Educational Research Anísio Teixeira) in partnership with the *Ministério Nacional da Educação* – MEC (Department of Education) with the intent of assessing Brazilian youth academic performance at the end of high school (INEP, 2018).

Since its initial implementation, the format of the ENEM has changed significantly. From 1998 to 2008, the ENEM was an interdisciplinary exam without clear splits between areas of content knowledge. Each exam had a total of 63 multiple choice questions based on academic subject Brazilian students were supposedly exposed to in secondary education. However, since 2009 the exam became compartmentalized into content knowledge areas with 45 objective questions each. Thus, currently ENEM is divided into four areas: Mathematics and its Technologies; Languages, Codes and its Technologies; Natural Sciences and its Technologies; and Social Sciences and its Technologies. Additionally, students are expected to write an essay. In different years, more than one version of the exam was produced to attend to the large number of examinees while minimizing the possibilities of fraud.

Questions included on the ENEM undergo a long validation process. INEP, after sending a public call for collaborators, selects a group of individuals to write potential items. These individuals are trained on the exam's rubric and work from a government-protected space. After elaboration, questions are revised by an editor who then decides the adequacy of items and possible revisions. Content-area specialists oversee the work of collaborators and editors and validate items based on a revision rubric. A pre-test is run to evaluate the "level of difficulty, degree of discrimination" and other parameters (INEP, 2018). Once items pass the validation process, they are added to a national database from which the ENEM questions are selected.

Lastly, since its implementation, ENEM has become an important measure for policy makers and university administrations interested in promoting access to higher education. Currently, ENEM scores allow students to apply for different college funding opportunities as well as enhance their chances of entering public universities. Students who obtain high scores on ENEM may be eligible to receive full funding to attend private universities.

FRAMING THE STUDY

The present study is framed on existing scholarship which affirms that a) the selection and production of educational texts are not neutral, b) texts participate in the legitimization and perpetuation of particular ways of constructing sociopolitical realities as well as in framing what is acceptable, credible, and even thinkable, c) legitimate knowledge in mainstream education serves to protect and reproduce inequalities and to undermine less powerful groups, and d) mathematical texts – as important components of the sociopolitical processes of mathematics learning and participation – promote oppressive gendered and racialized discourses that limits who is perceived as a successful doers of mathematics.

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Research has shown that the selection and production of educational texts are not neutral phenomena (Apple, 1992; Kress & Van Leeuven, 2001). As products of complex social practices undertaken by real people with real interests, educational texts represent particular ways of constructing reality and promote particular social ideologies. More importantly, they influence the ways in which one makes sense of the social order in which they exist, one's role in society as well as how one imagines future possibilities (Apple, 1992; Rossini et al, 1997; Kress & Van Leeuven, 2001; Marcuschi & Ledo, 2015). In *Text and Cultural Politics*, Apple (1992) argued that

> [texts] participate in creating what a society has recognized as legitimate and truthful. They help set the canons of truthfulness and, in that way, also help recreate a major reference point to what knowledge, culture, belief, and morality really are. p.5

Apple's argument sheds light on the role of texts not only in legitimizing knowledge but also in regulating ideologies. As the product of a selected group, texts not only normalize specific cultural, social, and political imaginaries and ideologies but also participate in setting limits on what is conceived as trustworthy, reasonable, believable, indeed sayable or thinkable (Apple, 1992; Marcuschi & Ledo, 2015).

Research has also shown that knowledge legitimized in mainstream education is embedded in social political ideologies that serves to perpetuate oppressive power hierarchies as well as social structures that benefit those in charge while disenfranchising others (Apple, 1992; Shi, 2004; Gutierrez, 2013). As noted by Apple (1992), all too often, "legitimate" knowledge does not include the historical experiences and cultural expressions of labor, women, people of color, and others who have been less powerful. p.7

Indeed, studies have shown that educational texts have been historically flooded with racist, sexist, classicist, heteronormative, xenophobic and other forms of violent discourses regardless of school subject (Weitzaman & Rizzo, 1974; Rossini et al,1997; Zittleman & Sadker, 2002; Good et al, 2010; Macintyre & Hamilton, 2010; Marcuschi & Ledo, 2015). Mathematics educational texts – which often support a sociopolitical structuring of mathematics education (Gutiérrez, 2013) which normalizes whiteness and heteropatriarcal ideologies – are no exception. A number of empirical studies have exposed how mathematical texts promote oppressive racialized and gendered discourses as well as reproduce implicit and explicit messages about who is allowed to be and succeed in mathematical spaces (Oyedeji, 1996, Macintyre & Hamilton, 2010; Tang et al, 2010; Piatek-Jimenez, 2014). Taking place in different parts of the world, these studies provide robust evidence that the selection and production of mathematics texts legitimize oppressive stereotypes about women, people of color and others while normalizing white men as always already more capable of doing mathematics. These studies also illuminate the role of mathematical texts in the framing of possible social imaginaries and limiting less powerful groups from seeing themselves as successful doers of mathematics.

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Drawing from the aforementioned scholarship, I take up the following questions: *What are the racialized, gendered and classed discourses put forth by mathematical text presented on ENEM?* And, *how do these discourses promote or resist oppressive ideologies?* Here, racialized, gendered and classed discourses refer to the ways in which gender, race and class are conceptualized, represented and framed through explicit and implicit messages in mathematical texts. Kress & Van Leeuwen (2001) have argued that educational texts are multimodal, and the combination of different semiotic resources allows for the construction of different meanings. Thus, in this study mathematical texts refers not only to written text, but also other semiotic resources used to convey meaning such as pictures, drawings, graphs and others.

DATA AND METHODS

For the purpose of this paper, all mathematics problems from the Brazilian National Exam of Secondary Education from 2009 to 2017 were analyzed (*N=810*). The period was chosen based on the relevance of the problems to the current format of the exam. Each problem was coded and organized into one of the following main categories:

a) *non-human subject problem*: refers to all the problems in which the presence of human beings is not made explicit. It also refers to problems in which actions are referred to be taken by organizations, companies, factories, etc (e.g., a factory that produces toilet paper…);

b)*gender*[1]: *non-specific problem*: refers to problems in which the presence of human beings is made explicit, yet their gender identities are not specified by gender makers such as names, pronouns and others. This category also includes problems in which actions were taken by a group of people in which gender was not made explicit (e.g., *uma equipe*/ a team, *uma familia*/ a family);

c) *female-centered problem*: refers to the problems in which human subjects are explicitly referred to by feminine pronouns, names and other indicators such as female related nouns (e.g., *mãe*/mother, *alunas*/ female students);

d) *male-centered problem*: refers to the problems in which human subjects are explicitly referred to by masculine pronouns, names and other indicators such as male related nouns (e.g., *pai*/father, *alunos*/ male students). This category also includes the problems in which masculine pronouns and other indicators represent a rather gender diverse population (e.g., when *brasileiros* (male Brazilians) is used to represent the entire Brazilian population);

e) *Female-male centered problem*: refers to problems in which both female and male subjects are mentioned under the conditions described in the last two categories. Images, characters actions, markers of social identities, and problems contexts were also analyzed to assess positionality and social discourses. A count of names -both fictitious and real - was also recorded. Whenever a real person was cited in the exam,

[1] [1]In this text, gender is considered a product of sociopolitical and cultural practices instead of a biological determination.

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a web search was conducted to gather further information (e.g., country of origin, job description, perceived gender identity, perceived racial categorization, others). Job descriptions were also coded and categorized and served as indicators of social prestige. Drawing from Macintyre & Hamilton (2010) the following job categories were used: Manual, Service/Care, Administrative, Professional, and Non- specific (when not enough information was provided). Categories were also created to classify problems in which adult characters were positioned inside the private family sphere - mothers, fathers, housewives, etc. – as well as to organize problems that were related to body and physical appearance.

DISCOURSES OF GENDER, RACE AND CLASS ON THE ENEM: WHO IS ALLOWED TO BE IN MATHEMATICS SPACES?

The distribution of ENEM questions in the main categories are represented in table 1. An analysis of the data reveals a striking difference in gender distribution with male-centered problems dominating the exam. They accounted for 46.8% of the questions while female centered problems accounted for 5.1%, and female-male centered problems for only 1.8%. Eighty-six fictitious names were used on ENEM. Here again, socially perceived male names clearly outnumber female – 77.9% compared to 22.1% respectively.

Sixteen references were made to real persons. These references were established by the citation of names and work (e.g., "Isaac Newton's law of universal gravitation…" or "The French physiologist Jean Poiseuille established…"). From these, only two referred to women – a Brazilian athlete (Fabiana Murer) and a U.S. based mathematician (Denise Kirschner). References were predominantly made by the use of first and last name as in the examples provided above, except for two: Kirschner and another mathematician referred to in the same problem with her –Glenn F. Webb. Given that first names may serve as social indicators of gender, the omission of the Kirschner's first name made it difficult if not impossible for examinees and others unfamiliar with her work to identify the scientist as a female character.

In *Intersectional Analysis in Critical Mathematics Education Research: a Response to Figure Hiding*, Bullock (2018) presented the concept of figure hiding in mathematics education to refer to the processes through which some identities and ways of thinking are rendered invisible in the field. As pointed out, apart from the work of educators committed to CME, mathematics education has promoted the invisibility of marginalized groups in different ways (e.g., through the legitimization of Eurocentric knowledge as well as policies, practices and methodologies of exclusion). From this perspective, figure hiding enables the perpetuation of dominant representations of mathematics as exclusively created by and for white men ignoring historical and current participation of women and people of color in mathematics education and research.

Drawing from Bullock, I argue that the erasure of certain groups from institutionalized mathematics texts or the omission of social indicators that would allow for their

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recognition as creators of mathematics knowledge serve as processes of figure hiding. In this sense, the (un)intentional scarcity of female characters on ENEM questions, especially in contexts in which they outnumber their male counterparts (e.g., schools) serves as a mechanism of figure hiding as it perpetuates the invisibility and exclusion of women from mathematical spaces. Similarly, the omission of social indicators that would allow for the recognition of women scientists' contribution to the mathematics community perpetuates patriarchal processes of knowledge legitimization that obfuscate the role of women in the creation and development of mathematics and science. Thus, by hiding the identities of women and female characters, the mathematics texts on the ENEM reify oppressive notions that mathematical spaces are not occupied by girls and women nor are mathematical knowledge created by them.

Grounded on scholarship focused on the conceptualization of race in Brazil that highlights the role of phenotype in the racial categorization of individuals (Guimarães & Huntley, 2000; Caldwell, 2007), all except two real characters would be socially perceived as white in Brazil. The two exceptions are Emanuel Araújo - an Afro-Brazilian artist, and Hiroo Kanamori -a U.S. based Japanese scientist. All mathematicians and scientists referred to on the ENEM are either from or based in European countries or the United States. None of the four Brazilians cited on ENEM are portrayed as scientists. Socially-coded indicators of race were also found. For instance, a particular problem represented a group of students from *favelas* struggling with mathematics. This is against the backdrop that the overwhelming majority of favelas' residents in Brazil are Black and brown populations (Amparo Alves, 2012).

It also relevant to note that this particular problem was one of very few in which students were explicitly portrayed as struggling with mathematics. Additionally, the analysis of job descriptions showed that most of the high-prestige positions represented in the ENEM questions have been historically occupied by white men (Artes, 2015; Guimarães and Huntley, 2000).

Recent research provides robust evidence of the racialized nature of mathematics education. This body of research reveals that mathematics education is a white institutional space in which whiteness is protected and promoted through the exclusion and dehumanization of Black and brown people (Martin, 2009, 2011, 2013; Martin, Anderson, & Shah, 2017; McGee, 2015) as well as the othering of people of color's cultural formations and ways of knowing and doing mathematics (D'Ambrósio, 1985; Joseph, 1987; Gerdes, 2001; Gutiérrez, 2013). Drawing from this literature, I argue that the exclusion of mathematicians and scientists of color, the focus on bodies that represent U.S. and Eurocentric knowledge production as well as the perpetuation of deficit-oriented stereotypes about students of color on mathematics texts on the ENEM demonstrate the racialized nature of mathematics education in Brazil. The invisibility and marginalization of people of color in the ENEM indicate, whether conscious or unconscious, attempts to preserve and protect the normalization of whiteness and Eurocentric intellectual domination in mathematical spaces as well as to limit the possibilities of seeing people of color as successful doers of mathematics.

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Overall, female characters were represented in 56 roles and males in 430 (Table 1). A critical analysis of these roles revealed stereotypical representations of female and male actors. While female characters were less likely to be portrayed in the workplace, males were less likely to be evoked in questions related to the body and physical appearance. Gender salient roles related to employment represented 39.3% and 53.7% of female and male roles respectively. The bottom portion of table 1 shows a comparison of employment by gender.

	Number of Occurrences	Percentage
Non-human subject	284	35.1
Gender Neutral	91	11.2
Female centered	41	5.1
Male centered	379	46.8
Female-Male centered	15	1.8
Total	810	100
	Male	*Female*
Fictitious Names	(n=67, 77.9%)	(n=19, 22.1%)
References to real personage	(n=14)	(n=2)
Gendered roles	(n=430)	(n=56)
Home/family sphere	4.65%	12.50%
Body/Physical Appearance	.70%	17.86%
Workplace	53.72%	39.29%
Learners/Students	8.14%	10.71%
Job description	(n=231)	(n=22)
Non-specific	6.9%	9.1%
Manual	11.7%	9.1%
Service/Care	22.9%	22.7%
Administrative	19.5%	4.6%
Professional	39.0%	54.6%

Table 1: Summary of problems distribution

Among employment-related contexts, both female and male characters were more likely to be portrayed on the *Professional* category (positions that requires some post-secondary education such as associate and college degrees). Yet, female characters were overwhelmingly portrayed in professions that are socially perceived as low-paying female occupations such as teaching, which accounted for 54.6% of the female professional contexts. Additionally, when female characters were described outside of stereotypical occupations, other indicators connected them to gender stereotypical portraits (e.g., a female project designer is said to be designing a round lid for a cooking pan). More importantly, when male and female characters were represented in the same professional position, the content of work undertaken by them conveyed gendered messages about women's abilities to perform mathematics and the types of knowledges available for them. For instance, both female and male math teachers are described in

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the ENEM questions, yet female math teachers were more likely to be described as conducting educational games with students whereas male teachers were portrayed developing complex mathematics concepts. Overall, regardless of gendered context, female characters were less likely to be portrayed as active subjects engaged in the active doing of mathematics and more likely to be evoke on passive roles. The same is not true for male characters.

Research on gender and mathematics education reveals that perceptions of girls and women as not good in mathematics and, therefore, not worthy of belonging in mathematical spaces, are not new to mathematics educational settings (Fennema, 1981; Grasset,1991; Walshaw, 2001; Ong, 2005; Lubienski & Ganley, 2017). Such perceptions are often rooted on oppressive gender stereotypes that position women and girls as unequipped for complex thinking, unsuited for leadership roles, and always already expected to fulfill patriarchal notions of 'the women's place' in society (e.g., caretakers, docile citizens, objects of heteronormative desires). This scholarship has also demonstrated the negative impacts of such perceptions on the learning and participation of girls and women in mathematics such as being subjected to academic and professional lower expectations, punishment for displaying assertiveness instead of 'lady-like' behaviors, and exposure to self-efficacy- and performance- inhibiting stereotype threats. Drawing from this scholarship, I argue that, by portraying women in gender stereotypical roles or as passive actors almost incapable of developing complex mathematics, mathematics texts on the ENEM helps to perpetuate inaccurate representations of the many roles women undertake in society, reinforces oppressive biases that portray girls' and women' as unfitted for mathematics and limits the opportunities of others (women or else) to see girls and women as fully capable of succeeding in school and professional mathematics.

Lastly, a total of 8 images portrayed gendered characters: two would be socially perceived as female - a cropped face of a model and an image a popular cartoon character; four images represented male characters made explicit in the written text, and the last two were male figures used to illustrate a gender non- specific problem (a drawing of boy illustrated *uma criança*/a child) and a female centered problem (an image of a male body on a problem in which a woman is said to be measuring her body fat index). Besides the use of images of male bodies to represent 'neutrality', male marks were also used in the written text to represent a rather gender diverse population. Historically, mathematics education has been presented as objective and neutral, yet research shows that such claim is grounded on ideologies that mark males and whiteness as objective and neutral. Glenn (1999) argued that in the social construction and institutionalization of gendered and racial dichotomies "the dominant category is rendered "normal" and therefore "transparent"…thus White appears to be raceless…and man appears to be genderless" (p.10). Thus, by evoking male markers as representative of a false gender neutrality and social 'normality', mathematical texts on the ENEM participate on the re-production of dominant assumptions and practices that obscure the experiences of others, their ways of seeing and being in the world as

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well as protect heteronormative patriarchal power that establishes white men's construction of reality as naturally inherent to the social order.

DISCUSSION AND CONCLUDING THOUGHTS

In 1996, the *Universidade de São Paulo* (USP) in partnership with the Brazilian Department of Education published a guide focused on the promotion of gender equity in Brazilian education. In this guide, the authors highlighted the educational, social and cultural dangers of perpetuating oppressive discriminatory discourses in educational spaces. Moreover, they offered suggestions in how to address sexism and challenge gender stereotypes in educational texts. This guide, published a few years before ENEM was first established, received great attention in the educational community as well as on channels of communication (Rossini et al, 1997). In 2006, a new edition of the guide was published to account for new scholarship on gender equity. Note that the new edition was also published before the time period of the ENEM exams analyzed in this study. Yet, based on this analysis, the suggestions on the guide were not taken into consideration in the creation and selection of mathematical texts on the ENEM.

Apple (1992) argued that texts can participate in oppression or liberation. As this analysis shows, mathematical texts on the ENEM promote oppressive beliefs about women and people of color. The racialized and gendered discourses reproduced on the ENEM reinforce heteropatriarchal and racist ideologies that normalizes not only whiteness but also the subjugation of women and people of color in mathematical spaces. Thus, we are left to wonder: what is INEP's definition of "*levels of discrimination*"? Discrimination against whom? And how is it assessed through their "long process of items validation" (INEP, 2018)? Apple (1992) also argued that texts are created by real people with real interests. Then I ask, what are the conscious and unconscious interests of all involved (individuals and institutions) in the development and validation of the questions I analyzed in this study? And, whose interests are these mathematics texts protecting?

Mathematics education in Brazil can only reach its liberatory potential once it has gone through a major transformation. Perhaps, challenging the reproduction of oppressive discourses in mathematical texts is just a small step, but in the right direction nonetheless.

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MATHEMATICS FOR ALL (RESPONSIBLE) STUDENTS: A DISCOURSE ANALYSIS OF THE ONTARIO CURRICULUM

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Abstract: In this paper, I use Critical Discourse Analysis to examine how the Grade 11–12 Ontario curriculum constructs the notion of mathematics learning and what identity categories it makes available for students. Although the document draws upon a "mathematics for all" discourse, it constructs and reflects mathematics as the gatekeeper for postsecondary education, it (re)produces an individualistic discourse for success, and it provides hierarchically related descriptions for different ways of learning mathematics and different mathematics learners. I conclude by discussing the curriculum's workforce-orientation along with its prototypical structure as being key features that are in tension with the project of an equitable mathematics education.

INTRODUCTION

Numerous studies have shown that mathematics education is an inequitable space with regards to gender, race, class, and culture. As a response to this reality, a "mathematics for all" slogan has emerged, under which a significant amount of research and practice operates. Nevertheless, "mathematics for all" has been criticized for being a device that effects inclusions and exclusions of individuals and countries (Valero, 2017), while "conceal[ing] the obscenity of a school system that year after year throws thousands of people into the garbage bin of society under the official discourse of an inclusionary and democratic school" (Pais, 2012, p. 58).

Starting from the viewpoint that exclusion at the macro-level is maintained by and creates instances of exclusion at the micro-level, this paper explores how the "mathematics for all" discourse unfolds itself in the Grade 11–12 Ontario mathematics curriculum. Although the curriculum document draws upon a "mathematics for all" discourse, my analysis shows that mathematics learning is discursively constructed in hierarchical ways for different categories of students. Moreover, success is described in individualistic terms, while the role of mathematics as the gatekeeper for postsecondary education is constructed and reflected by the document. I argue that this contradiction supports Pais' (2012) criticism about the "mathematics for all" discourse and I suggest that two features of the curriculum play a key role in giving rise to the contradiction: curriculum's workforce orientation and curriculum's prototypical structure.

VIEWING CURRICULUM DOCUMENTS AS DISCOURSES

Research on mathematics curriculum has taken several directions, including analyzing the mathematical content of curriculum documents and textbooks (Cai, Nie, Moyer, Wang, 2014), evaluating the clarity of curriculum reforms' messages (Bergqvist

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& Bergqvist, 2017), and exploring teachers' interaction with the curriculum (Agodini & Harris, 2016). Nevertheless, only a few studies have drawn attention to how the language of curriculum documents manifests particular ideological views in mathematics education (Herbel-Eisenmann, 2007).

Some researchers have suggested that curriculum and curriculum documents can be considered as discourses and have conducted discourse analyses, focusing on curriculum's underpinning ideologies (Doğan & Haser, 2014), the construction of the ideal learner (Morgan, 2009; Oughton, 2007), and the ways that learners are positioned in relation to the text (Herbel-Eisenmann & Wagner, 2007). These analyses highlight the fact that math curricula have a profound political dimension. For example, Doğan and Haser (2014) show that the elementary mathematics curriculum in Turkey reflects neoliberal and nationalist discourses. Similarly, focusing on inclusion and exclusion of subjectivities, Morgan (2009) explores how curriculum guidance for teachers in Britain constructs a distinction between "ideal" and "deviant" students.

Within this view, curricula are treated as artifacts that do not simply frame the materials that they intend to teach. Instead, a curriculum document transfers certain ideologies, constructs its objects (including mathematics) in specific ways, and includes certain subjectivities while excluding others. In this paper, I present a Critical Discourse Analysis (CDA) (Fairclough, 1995, 2003) of the Grade 11–12 mathematics curriculum document (Ontario Ministry of Education, 2007), which has been guided by the following research questions: How does the document construct "learning mathematics" as a concept? What identity categories does the document make available to students?

THEORETICAL–ANALYTICAL FRAMEWORK AND METHODS

This study uses the theoretical and analytical approach of CDA developed by Fairclough (1995, 2003). Within this view, discourse is "a particular way of representing some part of the (physical, social, psychological) world" (Fairclough, 2003, p. 17), usually through language. Fairclough's discourse analysis approach tries to bridge traditions which focus on a text's linguistic aspects with those inspired by social theory (Fairclough, 2003). This approach is exemplified in his three-dimensional model, according to which discourse analysis is conducted at three levels: the analysis of the linguistic features of the text, the examination of the processes through which the text has been produced and is consumed (i.e., the discursive practice), and the social practice which surrounds the communicative event (Fairclough, 1995).

Following Fairclough's (1995) three-dimensional model, I started by analyzing the curriculum document as discursive practice, focusing on its production. In this phase, I was interested in identifying how the document was produced, by whom, and from what discourse genres it draws upon. The analysis of the social practice and of the text were viewed as complimentary and were completed through a "back

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and forth" process. For the analysis of the social practice, I drew upon research in mathematics education, which focuses on the ways in which mathematics learning is conceptualized and identity categories are made available to students. For the textual analysis, I used some of the linguistic tools employed by Fairclough (1995, 2003) including the choice of words, the use of pronouns, modality (modal verbs and hedges), nominalisation, and the syntactical structure of the sentences (e.g., active versus passive voice).

RESULTS

Context and Structure of the Document

In Ontario, the Ministry of Education publishes curriculum documents for all school subjects, which describe the philosophy and the objectives of the offered courses. The mathematics curriculum is outlined in three documents, for Grades 1–8, Grades 9–10, and Grades 11–12. The textbooks are published by private companies and are approved by the Ministry of Education, while school boards choose textbooks among the ones available. Therefore, the Ontarian curriculum documents are the primary texts that communicate the intended mathematics curriculum to teachers and at the same time they play a role in shaping the structure and content of textbooks.

The Grade 11–12 mathematics curriculum document is structured in six chapters. The introductory chapter describes the role of schools and mathematics curricula. The second chapter provides an overview of the program, the curriculum expectations, and a brief outline of the offered courses. The third chapter describes seven mathematical processes which students are exposed to through the curriculum: problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating. The fourth chapter addresses issues of assessment and evaluation, while the fifth chapter discusses program planning issues. Finally, in the last chapter, the ten offered courses are described, including detailed expectations and sample problems for each of them.

The Invisible Author

The curriculum document is issued by the Ontario Ministry of Education and its authors are not specified. Instead at the end of the document, it is mentioned that: "The Ministry of Education wishes to acknowledge the contribution of the many individuals, groups, and organizations that participated in the development and refinement of this curriculum policy document". (p. 157)

This acknowledgement signifies two things: first, that the document is the result of a collective work and second that no individual or group of individuals is to be held accountable for it. This practice gives authority to the document, since it appears to be the product of widespread consensus, as guaranteed by the Ministry of Education.

This practice is reflected in the text body as well. Drawing upon a formal, official genre, the text almost gives the sense that it has no author. Apart from the sample

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problems, no personal pronouns are used throughout the text, referring to either the writers or the readers. Moreover, there is excessive use of passive voice (e.g., "The curriculum has been designed"), use of abstract nouns as subjects in active voice syntax (e.g., "goal", "principles", etc.), and use of the "curriculum" itself as subject in active voice syntax (e.g., "this document", "the Ontario mathematics curriculum", etc.). These textual choices highlight the technical and official facet of the curriculum over its political character.

The technical and official character of the text obscures that—like any curriculum document—the Grade 11–12 curriculum has a political orientation. Nevertheless, the document echoes a "mathematics for all" voice, as made clear in the next sentence: "The principles underlying this curriculum are shared by educators dedicated to the success of all students in learning mathematics." (p. 3)

This sentence urges the readers, who are primarily educators, to align with the document's perspective mainly through two linguistic features. First, the choice of the words "educators" (instead of, e.g., "teachers"), "dedicated" (instead of, e.g., "aiming to"), and "success" draw upon a romantic idea about education which is often treated as commonsensical in Western societies. In other words, the document does not only describe the teaching and learning of mathematics but a greater process of education through which students can succeed. Opposing the document means opposing the idea of "mathematics for all" and, thus, supporting mathematics only for some students. Second, the phrase "shared by educators" plays a double role. On the one hand, it appeals to the reader's identity as an educator, thus sketching the discourse of inclusivity as familiar and commonsensical. On the other hand, the absence of the definitive article (an alternative phrasing could be: "is shared by the educators who are dedicated") frames these educators in a vague and impersonal way. This vagueness makes it easier for the reader to identify with the offered role of the dedicated educator, since they are only required to theoretically commit to the discourse rather than judge their actual level of commitment to the practice of supporting all students' success.

Two Kinds of Mathematics–Two Kinds of Learners

The curriculum document creates a distinction between *mathematical facts and procedures* versus *concepts to understand and procedures to apply*. It is stated: "The required knowledge and skills include not only important mathematical facts and procedures but also the mathematical concepts students need to understand and the mathematical processes they must learn to apply". (p. 3)

The phrase "the required knowledge and skills" reflects the prototypical paradigm of learning assumed by the curriculum. Indeed, the document sets specific expectations in each course which should be met by all students and suggests specific ways through which this can be achieved. Through the use of the adjective "required" and the modal verbs "need to" and "must", meeting these goals is treated as a necessity. Although often treated as commonsensical, structuring a curriculum in terms of goals that *must*

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be met favors a certain mathematical practice (usually academic mathematics) as important, devalues alternative forms of mathematical practice, while at the same time it ignores students' non-school past experiences. In this sense, this practice is related to maintaining pre-existing inequalities because it engages and assesses different students in a predetermined discipline. Furthermore, the above sentence constructs two ways of knowing mathematics, and hence two kinds of mathematics learners, in a hierarchically related way: the passive and the active learner. The first way includes knowledge of *facts* and *procedures*, whereas the second way involves *understanding* the *concepts* and learning to *apply* the *processes*. The "not only... but also" syntactical structure implies that these two ways are hierarchical. By entailing no verb, the "not only" part of the sentence constructs the first way of knowing mathematics as passive, while the use of two verbs ("understand" and "apply") in the "but also" part of the sentence constructs this way of knowing mathematics as active. This dichotomy is prevalent throughout the text in various ways, including the contrast between "content" versus "thinking processes" (p. 6) and the connection between "applying" and "having a solid conceptual foundation" (p. 32).

This distinction is related to a prevalent hierarchical distinction in mathematics education between "procedural" and "propositional knowledge" (Walkerdine, 1998). Walkerdine (1998) challenges this distinction, by arguing that it has significant effects on the ways in which mathematics is taught. Procedural knowledge is advocated for equipping students to complete every-day or non-academic tasks related to mathematics, while propositional knowledge is viewed as necessary for students who are going to enter math related disciplines. In this sense, the dichotomy built by the curriculum is not innocent since it reflects and reproduces the existing divisions of labor at the societal level.

The connection between different ways of knowing mathematics and different professional prospects is also evident in the way that the curriculum describes students' competencies in the various courses. For example, the Grade 11 university preparation course "introduces the mathematical concept of the function by extending students' experiences with linear and quadratic relations" through a process in which "students investigate, represent, solve, develop, reason, and communicate" (p. 43). On the contrary, the Grade 11 mathematics for workplace preparation course prepares students to "broaden their understanding of mathematics" through "solving, applying, calculating, and consolidating their mathematical skills" (p. 77). The verbs used in the two cases require students to perform different mathematical actions, reinforcing the idea that active and passive knowledge of mathematics are required for university education and the workplace respectively.

Mathematics for All (Responsible) Students

Although the document draws upon discourses of inclusivity and sets the goal of providing quality mathematics education for *all* students, some students are still excluded from the curriculum. In the document, it is stated:

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> It [the curriculum] must engage all students in mathematics and equip them to thrive in a society where mathematics is increasingly relevant in the workplace. It must engage and motivate as broad a group of students as possible, because early abandonment of the study of mathematics cuts students off from many career paths and postsecondary options. (p. 4)

At first glance, the second sentence seems to be a repetition or elaboration of the first one. This is reinforced by the similarity of the two sentences' syntaxes, along with the repetition of the phrase "It must engage". However, the first sentence states the need for the curriculum to engage "all students", while the second sentence refers to "as broad a group of students as possible". This difference indicates that the curriculum considers it impossible to engage *all* students with mathematics. Furthermore, the reference to "all" or "most" students along with the reason provided for students' need to learn mathematics ("early abandonment of the study of mathematics cuts students off from many career paths and postsecondary options") draws upon the discourse that school is an institution which provides individuals with opportunities. School's role is captured as giving students the necessary qualifications so that they can make an informed career choice, rather than being excluded from an early age.

The use of the modal verb "must" in the above quote establishes both a function of necessity and of potentiality. Although the curriculum *must* perform a certain action, this might not happen. The modality in describing the curriculum's goal here is in contradiction with other extracts of the document, in which a more definite description is attempted. For example, when describing the overall and specific expectations for each course, the document always adopts the phrase "By the end of this course, students will […]" (e.g., p. 45). This contradiction can be accounted for as an acceptance that the goal of "mathematics for all", albeit important, will not be achieved. At the same time, the specific courses are designed in a way in which specific expectations can and *will* be met for all categories of students.

With regards to students who do not engage in mathematics, the document depicts this situation as the students' own fault, by creating a connection between students taking on the responsibility of their learning and being successful in school mathematics:

> Students have many responsibilities with regard to their learning. Students who make the effort required to succeed in school and who are able to apply themselves will soon discover that there is a direct relationship between this effort and their achievement and will therefore be more motivated to work. There will be some students, however, who will find it more difficult to take responsibility for their learning because of special challenges they face. The attention, patience, and encouragement of teachers and family can be extremely important to these students' success. However, taking responsibility for their own progress and learning is an important part of education for all students, regardless of their circumstances. (p. 5)

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Two categories of students are constructed here: the students who take responsibility for their learning and those who fail to do so. The document constructs the former category as more able to exercise agency than the latter. Indeed, the first two sentences employ the active voice and have the word "students" as their subject, while the third sentence makes use of the vague phrase "There will be some students". Teachers' and family's responsibility, although mentioned, is linguistically mitigated through the nominalisation of their actions (attention, patience, and encouragement) and the use of no article in front of the words "teachers" and "family".

It is significant that the success story of some students, the special challenges faced by some (other) students, and the curriculum's requirement (taking responsibility for their *own* learning) are all described in individualistic terms. Although the above extract recognizes that students might face special challenges, these are not treated in terms of structural obstacles in relation to, e.g., class, gender, or race. Therefore, success or failure in mathematics are constructed as the result of individual action.

Different Courses

The curriculum offers four Grade 11 courses (university preparation, university/college preparation, college preparation, and workplace preparation) and six Grade 12 courses (three courses for university preparation, two for college preparation, and one for workplace preparation). In the text, the courses for university preparation always come first, followed by the college preparation, and then by the workplace preparation courses. This can be viewed as signifying that the curriculum document is more invested in framing the academic courses as opposed to the non-academic ones.

Each course has as prerequisites one or more Grade 9–12 courses, which are schematically represented in the document, in a way that some of the possible transitions between the courses are depicted. It is significant to note that according to these transitions, a student's choice in Grade 12 is not very different than their choice in Grade 9 with regards to its academic orientation. For example, choosing academic mathematics in Grade 9 does not lead to the workplace preparation course in Grade 12, nor vice versa. Therefore, the curriculum can be viewed as a device which aims at stabilizing the academic or non-academic orientation of students' educational paths.

Let us focus on the Grade 12 courses. Six courses are offered, namely: *Advanced Functions, Calculus and Vectors, Mathematics of Data Management* (university preparation), *Mathematics for College Technology, Foundations for College Mathematics* (college preparation), and *Mathematics for Work and Everyday Life* (workplace preparation). The more academic a student's orientation, the more courses the student must take; preparing for university demands participation in three courses, preparing for college in two, and preparing for the workplace in one course.

Moreover, the names of these courses are significantly different. Starting with the university preparation courses, all three names employ formal mathematics

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jargon (functions, calculus, vectors, data management). This is not the case for any of the other courses, in which the only mathematics-related word is the word “mathematics” itself. Another interesting linguistic feature is the use of the prepositions “of” and “for”. While “Mathematics of Data Management” signifies that data management is a field which entails mathematics, the course names “Mathematics for College Technology” and “Mathematics for Work and Everyday Life” suggest that mathematics is exterior and imposed on college technology, work and everyday life. Moreover, the students of these courses are positioned in very different ways. In the case of “Mathematics of Data Management”, the main concept of the clause, in relation to which the reader is positioned, is mathematics. In other words, the reader is expected to be enculturated in the mathematics as it plays out in the field of data management and not in the field of data management itself. On the contrary, in “Mathematics for College Technology” and “Mathematics for Work and Everyday Life”, the reader is positioned in relation to the practices of college technology, work, and everyday life and is invited to be enculturated on how mathematics is used in these fields.

Finally, the analysis of the courses revealed that the workplace preparation courses not only draw upon a non-academic perception of mathematics, but they also construct the students of these courses as having been unsuccessful in mathematics. For example, the following problem is from the Grade 12 workplace preparation course: “Estimate the change from the $20 offered to pay a charge of $13.87.” (p. 80)

This problem does not resemble the high-school mathematics genre but is instead similar to elementary school mathematics. This signifies that the work-place course is intended for those students who did not do well in mathematics in previous years. This micro-level situation reflects and constructs the role of mathematics as a gatekeeper to postsecondary education: if a student does well in math, they are expected to enrol in academic mathematics courses and continue their studies at a postsecondary level.

DISCUSSION AND CONCLUSIONS

Fairclough (1995) maintains that there is a dialectic relation between social formation, school as an institution, and the level of concrete action. Social formation ultimately determines school as an institution, which in turn determines the actions and events that might occur within the school environment. At the same time, however, changes in actions might reshape the institution, which could in turn result in transformation of the social formation. My reading of Fairclough’s view suggests that no fully equitable curriculum can be achieved in a capitalist society. Nevertheless, the process of reflecting and acting upon curriculum documents with the goal of addressing and dealing with its inequitable functions is worth pursuing since it can potentially transform education and social formation more generally.

My analysis showed that the Grade 11–12 Ontario curriculum document draws upon inclusive discourses that advocate in favor of mathematics education for *all* students,

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while constructing schools as institutions that provide individuals with opportunities. However, the document prioritizes academic preparation and it excludes students who "do not take up the responsibility of their learning". Through the focus on student's responsibility, performance in mathematics is viewed as the individual's success or failure, while structural factors affecting a student's relationship with mathematics are not highlighted. At the same time, different categories of students, such as the passive and the active learner as well as the responsible and the non-responsible learner, are constructed in hierarchical ways. Overall, the document reflects and reproduces the role of mathematics as the gatekeeper for postsecondary education, by inextricably connecting performance in mathematics and postsecondary goals: the student who performs well in math is expected to continue their studies at the university level while at the same time being successful in math is a requirement for the pursuit of university studies. I consider this reading of the document to be an example of the limitations that the "mathematics for all" discourse has for the project of an equitable mathematics education.

Pais (2012) understands exclusion in mathematics education as being a structural characteristic of an inequitable system rather than a marginal problem that can be addressed by a technical solution. I argue that there are two characteristics of the Ontarian curriculum directly support this function: its workforce-orientation and its prototypical structure. Given the central role of mathematics in neoliberal economies, prioritizing students' preparation for the workforce means requiring all students to learn some mathematics, but only a few students to learn advanced mathematics and proceed into mathematics-related disciplines. In this sense, inequitable learning outcomes become a requirement of a successful workforce-oriented education. The prototypical structure of the curriculum becomes the vehicle through which this function is achieved. Indeed, the document sets specific expectations and suggests specific ways through which these can be met. These expectations are diverse, aiming to cover the different needs of students oriented to different career paths, thus addressing the reality that not all students will use mathematics in the same contexts or in the same way in their jobs and adult lives. Nevertheless, the needs that the curriculum aims to cover initiate from the demands of the workforce, as opposed to, e.g., from the students' interests or experiences. In this sense, the curriculum forms and assesses students' learning through expectations and methods which are not relevant to all students and have been specified externally according to the needs of an inequitable economic system.

CDA can be a valuable tool for the study of curriculum documents, since it enables a focus on the power relations that give rise to and are maintained by discourse (Oughton,2007) and it is useful for envisioning how this relationship could be different (Herbel- Eisenmann & Wagner, 2007). Within this perspective, this analysis of the Ontarian curriculum raises the question of what alternatives to workforce-oriented and prototypically structured curricula could be pursued. Such an endeavour should not assume that an alternative curriculum could act

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independently of inequitable social and educational structures, but rather understand the project of an equitable curriculum as an everlasting process of critiquing and overcoming pre-existing aspects of inequity as well as critically examining itself.

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SYMBIOSIS BETWEEN MATHEMATICS AND CULTURAL GAME: A NEPALESE EXPERIENCE

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Abstract: *Generally, every human society has some sort of games or game like activities associated with recreation, enjoyment and pastimes. The intention of this paper is to see how the game like Bagha Chal (Tiger Move), played in Nepalese culture, can be consolidated into everyday teaching-learning activities in order to enhance students' mathematical understanding. The methodological procedures included observation, in-depth interview and examination of students' worksheets based on interview guidelines and observation checklist. The mathematical knowledge hidden in the various sectors of gameplay have been analyzed and found as an effective tool to develop mathematical concepts and explore ideas they practiced during the game.*

Keywords: *Bagha chal*, Cultural game, Ethnomathematics, School mathematics, Pedagogy.

INTRODUCTION

Games are one of the environmental activities of human culture. It is usually associated with competition, sportsmanship, winning, losing, amusement, refreshment and many other similar and related notions (Ascher, 1991). Most of the children are not taken as a competitor against their peers in case of learning the gameplay. However in a game, there is a challenge between the players. Each player plays the game to win bounded within the same rules as for their opponent (Nkopodi & Mosimege, 2009). Generally, games are taken as recreational activities, but the case might not always be the same. While performing a game, there is always competition to win, rule to follow and enjoyment to experience. Children have been learning mathematical concepts through different ways at home, in the playground, while working in a team, interacting with their colleagues and so on. They have been using various mathematical concepts while playing different games. For instance, Snakes and Ladders (*Ludo*) is a popular game for children in many countries of the world. It is easy to make from basic materials and can be adapted to suit many learning situations. This game can help the children in developing concepts of fundamentals operations.

Game is one of the major environmental activities of every culture (Bishop, 1991). Various mathematical concepts, ideas and knowledge are practiced implicitly by playing varieties of games associated in each culture. Mathematics has structure and rules, which are similar in case of games. Dienes (1960) emphasizes that the learning mathematics can occur in maximum if we relate it with games. His approach to mathematics learning uses games, songs and dance

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to make it more appealing to children. Thus, games and game like activities could be the effective tool for the motivation to learn mathematics and develop the logical thinking to solve mathematical problems. My study was intended to uncover the mathematical ideas embedded in *Bagha Chal* game and analyze its contribution in the process of teaching of school mathematics.

RESEARCH QUESTIONS

This paper was intended to seek the answer of the following questions:

1. What are the mathematical ideas embedded in *Bagha Chal* game?
2. How do teacher use cultural game to teach mathematics in the classroom?

CONCEPTUAL FRAMEWORK

The primary goal of my research was to uncover mathematical ideas embedded in the *Bagha Chal* game and connect it with teaching and learning of school mathematics. The conceptual framework developed in my study shows how the cultural game acts as a mediated tool to connect learner's cultural activities with teaching and learning of school mathematics based on a constructivist-learning environment. The mathematical understanding is important for everyone to perform everyday work. In this framework, physical and mathematical world are two different domains where the children live and learn simultaneously. Children possess different cultural activities in their physical world where playing is one of the common activity. Students' out-of-school knowledge is mostly ignored in the teaching and learning of school mathematics and the school pedagogy is alien to the children's home culture (Pradhan, 2017).

The mathematics practiced in different cultures implicitly is known as the ethnomathematics. Ethnomathematics refers to a form of mathematics (D'Ambrosio, 2006) that varies as a consequence of being embedded in cultural activities whose purpose is other than doing mathematics. People practice mathematical ideas in the process of playing cultural game without knowing formal mathematics. My argument in this study is that the learners' culture and everyday activities regarding ethnomathematical ideas is an integral part of education in general, and learning mathematics in particular. These

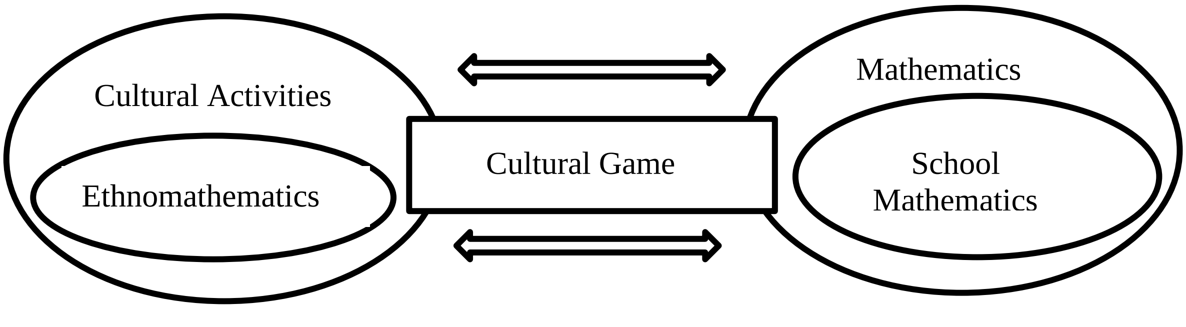

Figure 1: Cultural Game: Connecting Cultural Activities and Mathematics (Pradhan, 2018)

ethnomathematical ideas would act as vehicles for understanding school

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mathematics (Adam, 2004). This framework highlighted the cultural game as a pedagogical tool to integrate the ethnomathematical ideas and practices originating in the learner's culture with those of school mathematics (Pradhan, 2018). It believes that classrooms and other learning environments cannot be isolated from the communities in which they are embedded, and students come to school with their own values, norms and concepts they have acquired from their culture and environment (Adam, 2004). The students' home culture and out-of-school mathematical ideas and knowledge provides opportunities to connect with the formal mathematics (Pradhan, 2017). Above figure shows a conceptual framework to connect the students' out-of-school knowledge and embedded mathematical ideas in their cultural context and to school mathematics.

METHODS AND PROCEDURES

Qualitative research relies primarily on the collection of qualitative data. It is a field of inquiry that crosscuts disciplines and subject matters (Denzin & Lincoln, 2005). In my study, I chose qualitative research design, as I wanted to make sense of the complex world of the cultural games, mathematical ideas embedded in these games, and the implications of these games in the teaching and learning of school mathematics. It would not be possible for me to quantify such ideas and knowledge in figures and numbers. The qualitative research design is considered appropriate since I wish to obtain an in-depth understanding of how and what learners thought about the games as they play them.

My research was intended to observe ethnomathematical ideas and knowledge of the students in cultural games that the students playing in their community. A total of 5 teachers and 12 students of different grades were participated in this study from a public school of Kathmandu district. The different grade level students in a study provides an opportunity for collaborative learning environment. It is necessary to make a plan in their approach to data recording before entering to the field. I prepared an interview guidelines and observation protocols for teachers and students so that it would be easier for me to generate the data in the fields (Creswell, 2009). I collected data through examining documents, observing behavior and interviewing participants. I carefully recorded all the possible conservations with the help of the video camera and take field notes as much as I could. My data generated from the students' community reflect how they are rich in ethnomathematical ideas and knowledge. After observing the data, I linked them with many possible theories to derive meaning making them. In my study, the cultural games in the students' community and their ways of understanding the natural phenomena, and their ethnomathematical knowledge were analyzed with the notions of pluralism.

BAGHA CHAL: A DIDACTICAL GAME

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Bagha Chal is a strategic, two-player board game that originated in Nepal. There is taken to be twenty goats and four tigers. One player controls four tigers and another player controls twenty goats. The tigers hunt the goats while the goats attempt to block the tigers' movements. Pieces are positioned at the intersection of the lines and not inside the areas delimited by them. Directions of valid movement between these points are connected by lines. At the start of the game, all four tigers are placed on the four corners of the grid, facing the center. All goats start off the board. The pieces must be put at the intersections of the board lines and moves following these lines.

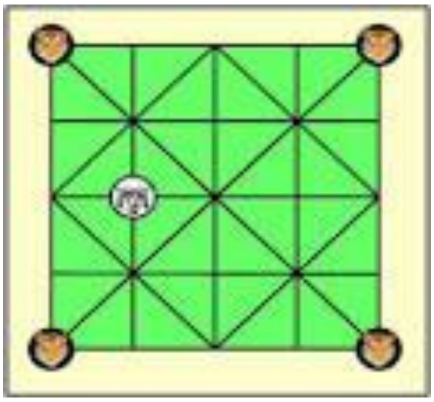

Figure 2: Board of *Bagha Chal*

The player controlling the goats moves first by placing a goat onto a free intersection on the board. Then it is the tigers' turn. One tiger is then moved to an adjacent position along the lines that indicate the valid move. Tigers capture goats by jumping over them to an adjacent free position. Goats cannot move until all twenty goats have been put on the board. The tigers win once they have captured five goats. Goats try to avoid being captured (jumped over) and they win by blocking the tigers' moves till they are unable to move. Sometimes the game can fall into a repetitive cycle of positions. Goats especially may use this resort to defend themselves against being captured. To avoid this situation, an additional rule has been established: when all the goats have been placed, no move may return the board to a situation that has already occurred during the game. Although no historical proof exists, *Bagha Chal* is supposed to have been played first by goatherds. Times have changed now. The *Bagha Chal* boards can be found in bronze, wood and metal. However, this can be used as a didactical game which support teaching and learning certain mathematical concepts and knowledge.

It is obvious to say that every human activity involves mathematical knowledge, ideas and concepts implicitly or explicitly. Mathematics is found everywhere from its own perspectives. Various studies have shown that mathematics is one of the difficult subjects in school. Through the use of games, very difficult concepts and ideas can turn easy and simple, as children are very interested in playing games. In this regards, Doumbia (1989) has pointed out that the simplest game could include the difficult mathematical calculation if it is seriously analyzed. Further, Pulos and Sneider (1994) found that appropriate didactical game provides an opportunity to children to learn mathematical concepts and ideas. The active participation and engagement of children is considered necessary for effective teaching and learning of mathematics. This study explored the mathematical ideas embedded in the board of *Bagha Chal*

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game and also observed its contribution to create an opportunity for students in effective learning of mathematical ideas.

MATHEMATICAL IDEAS IN *BAGHA CHAL* GAME

Every human activity has some sort of mathematical ideas. Bishop (1991) viewed that mathematical practices exist in all human activities. Playing is one of such activities that exists in every culture. During my data collection period, I discussed with a mathematics teacher if the games played by students could be linked up with the teaching and learning of school mathematics. The methods involved during the game play made me to think that there could be numerous such concepts that could be linked up with the classroom teaching. Therefore, I asked him if we could observe the students constructing the board and playing the *Bagha Chal* game. The teacher enthusiastically asked “I was looking forward to learn how to use the game in mathematics classroom. My concern about the game was whether the game could incorporate the teaching learning of mathematics in classroom”. After the observation, I asked him if any such mathematical concepts could be associated with the method of game play. The analysis of the *Bagha Chal* revealed a number of mathematical concepts. It is important to indicate that the list of mathematical concepts specified below is not exhaustive. The following mathematical concepts can be conceptualized in the analysis of board of *Bagha Chal* game and in the process of playing.

Bagha Chal is a board game and the board is square in shape. While drawing a net of *Bagha Chal* game, a player practices a lot of geometrical knowledge. During the phase of data collection, I gathered 3 groups of students, 2 members in each, and asked them to construct a frame of *Bagha Chal*. I observed the ways of making board of three groups and noticed a group to have a peculiar way. The ways of drawing the net of board of the players exhibited their own geometrical thinking. The styles of different players to draw the net are the outcomes of their connection of geometrical knowledge they have.

Group I & II: This group divided each sides of square into four equal parts and formed parallel lines (Red Pen and Brown Pen). Then they got 16 small squares. After that, the two diagonals were drawn (Black Pen) and the midpoints of adjacent sides were joined to give lines parallel to one of the diagonals.

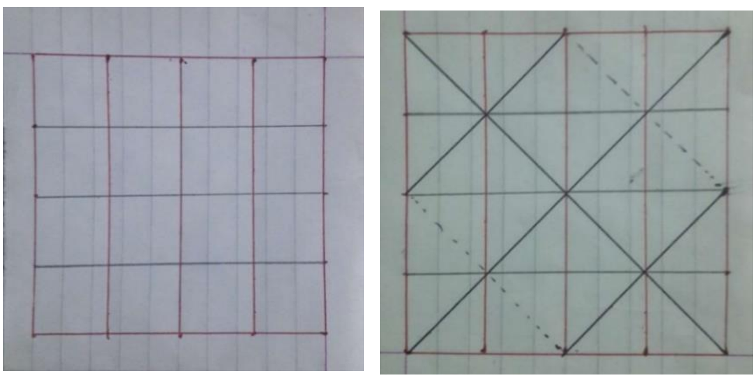

Figure 3: Player’s II Methods of Constructing Board

Group III: This group drew a square at first and then drew two diagonals. The point of intersection of two diagonals is the center of the board (Red Pen). Two mutually perpendicular lines were drawn through the

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center (Brown Pen) whose end points came to meet at the midpoints of the sides of the square. Then they drew the lines that joins the midpoints of the adjacent sides of the square (Blue Pen). The line joining the midpoints of the adjacent sides of a square also formed a square.

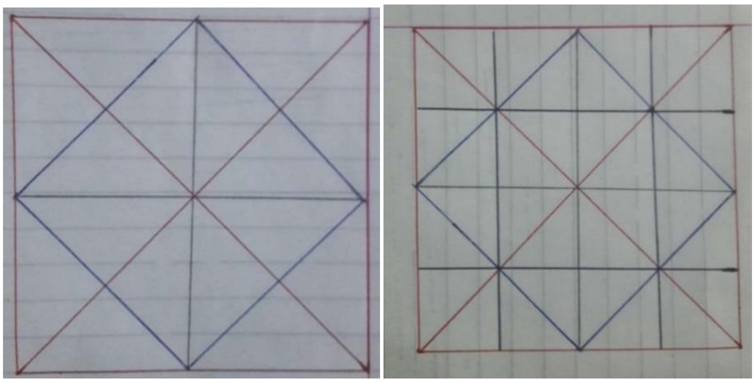

Figure 4: Player's I Methods of Constructing Board

The frame of the game is resulted due to the geometrical ability of the students. After the observation of the frame development, I discussed with a mathematics teacher if the mathematical concepts were embedded in the game, which could be linked up with the classroom teaching. The teacher enthusiastically replied the possibility of incorporation such concepts in classroom teaching and learning. Then he asked that he could apply the ethnomathematical concepts in the classroom teaching.

Teaching of Verification of Pythagorean Theorem

During the teaching, the teacher primarily mentioned the statement Pythagorean Theorem: In a right angled triangle, the square of hypotenuse is equal to the sum of squares of other two sides. This statement was simply to be memorized by the students just for solving the problems without any visualization.

The concept of right angled triangle and verification of Pythagorean Theorem could be conceptualized through the board of *Bagha Chal*. The board of *Bagha Chal* is square in shape. Each side of square are divided into four large equal parts and then each larger square will be divided into 4 equal small squares forming a total of 16 squares, as shown in the diagram. In every small square, it gives rise to two right angled triangle when a diagonal is drawn. In the adjoining figure, ABC is right angled triangle. The side AB, BC and CA are the altitude, the base and the hypotenuse of the triangle ABC respectively. Here,

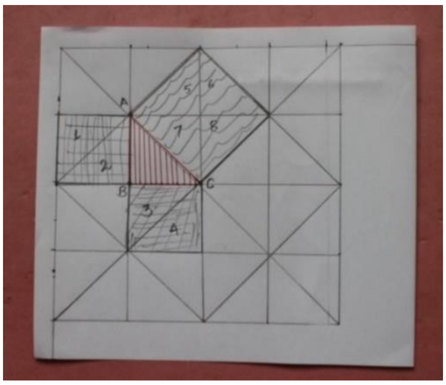

Figure 5: Verification of Pythagorean Theorem

$(AB)^2$ = Area of square whose one side is AB
= Area of triangle 1 + Area of triangle 2
= One small square

$(BC)^2$ = Area of square whose one side is BC
= Area of triangle 3 + Area of triangle 4
= One small square

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$(AC)^2$ = Area of square whose one side is AC
= Area of triangle 5 + Area of triangle 6 + Area of triangle 7 + Area of triangle 8

= Two small square

i.e. $(AB)^2 + (BC)^2 = (AC)^2$

The teacher and I believed that this concept could act as a bridge for visualization of the theorem. The teacher connected this concept from the board of game with the classroom teaching of the theorem. Then I took a post-class interview with the students. Then I asked the students about how they felt about the teaching with this model. One of the students replied “Before the use of this teaching model, we were just made to learn the theorem for purpose of solving common problems. Later, when this model was taken in action for teaching, we could easily visualize the theorem.” Another student viewed “After this lecture, I was surprised to know that mathematics could even be linked with the games that we were playing just for fun. Now, I can say that everything linked with the game is just a fun, even these boring mathematics classes. This lecture really proves it.” The use of game in classroom teaching seems to be really appealing and the outcome of the understanding as per the students was quite appreciable.

Teaching of Centroid of Triangle

The board of the *Bagha Chal* game has exhibited rich mathematical contents. The teaching of abstract concept of centroid of triangle can also be objectified through the use of the board of *Bagha Chal* game. The centroid of a triangle is the point of the intersection of the medians. It is a point which divides in the ratio of 2:1. In the observation of the board of the *Bagha Chal* game, E and F are the midpoints of the sides AB and AC of triangle ABC. The quadrilateral BCFE is a trapezoid and the diagonals BF and CE of Tpz BCFE intersect at a point G. Here the line BF and CE are the medians of triangle ABC and hence G is the centroid of triangle ABC. Regarding the effectiveness of the board of *Bagha Chal* game, one of the student participants viewed “I learnt most of the mathematical concepts through rote and memorization with the aim to solve problems. By using the board of *Bagha Chal* game, the abstract concept of centroid of triangle was

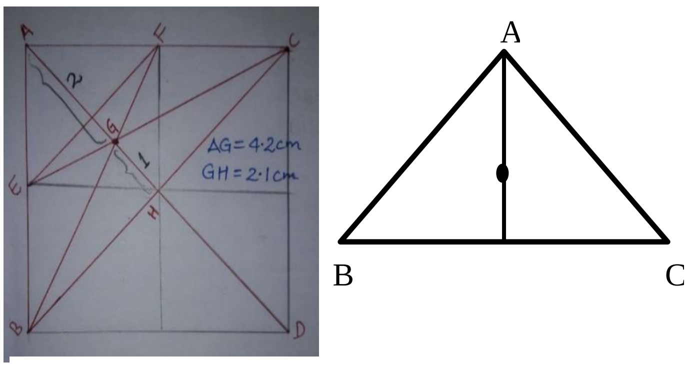

Figure 6: Centroid of Triangle in Board of *Bagha Chal*

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visualized". From the interaction of my student participants, I came to know that, the students' everyday activities and experience paly vital role for the development of mathematical concepts. The teacher participant also viewed that the cultural artefacts, games and daily activities of the students were useful for the teaching and learning of school mathematics. The question regarding the use of cultural games in the teaching and learning of school mathematics, one of the teacher participants T_1 viewed that "I never felt that mathematical ideas can be found in the cultural games. After the interaction with you, I have started to see the objects from the mathematical perspective". Most of my research participants agreed with the view of T_1. They all accepted the importance of cultural artefacts and game like activities for the conceptual understanding of the mathematical concepts. The incorporation of cultural games in the process of teaching and learning of mathematics enhance mathematical understanding of the students.

Teaching of Infinite Series

Identification of varieties of geometric figures in the construction of the board for *Bagha Chal* game such as squares, rectangles, triangles, parallel lines, transversal line and associated angles. The board of the *Bagha Chal* also reveals the properties of various geometrical figures viz. the sum of the internal angles of the triangles, of the square/ rectangle; vertically opposite angles, sum of the adjacent angles, corresponding angles, alternate angles and so on. The concept of symmetry was also found in the board of *Bagha Chaal*. The *Bagha Chal* board can be obtained by folding the square plane lamina in possible symmetric shape in square and triangle. From the observation of the board, it can be uncovered that the quadrilateral joining the midpoint of the corresponding sides of a square is again a square. It also helps to conceptualize the infinite geometric sequences by joining the middle points of the squares.

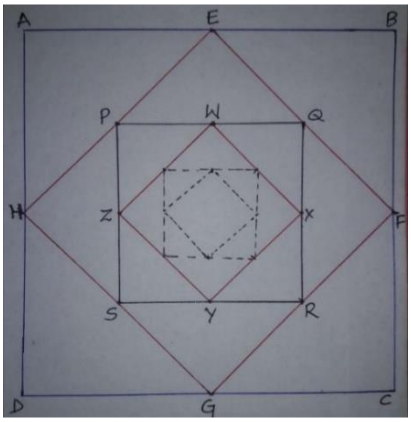

Figure 7: Model of Infinite Sequence

If 'a' is the length of a side of square ABCD, and E, F, G, H be the midpoints of the sides AB, BC, CD and DA respectively then AE = EB = ½ a = AH. The second square EFGH is obtained by joining the midpoints of the sides of the square ABCD, so the length of one of the sides EH of the square EFGH= $\sqrt{AE^2 + AH^2}$= a/√2. Similarly, the side of the third square PQRS is a/2 and so on and we get the infinite number of squares. The sides of the length of the squares possess infinite geometric sequences with common ratio 1/√2.

From the observation of the classroom, students were encouraged to explore the mathematical concepts of the transformation geometry and the properties triangles, rectangles and verification of mid-point theorem. I found that the board of *Bagha Chal* game mediated for the development of the concepts of

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a lot of mathematical concepts. The logical reasoning and its development of the children and mathematical thinking about the phenomena are the heart for developing mathematical understanding (Nkopodi & Mosimege, 2009). From the observation of the board game, the different pattern and structure found in the board are the metaphors that contributed for the development of mathematical reasoning.

CLASSROOM IMPLICATIONS OF THE GAME

Enjoyment and fun play an important role in learning mathematics and are some indicators of attitudes (Ernest, 1986). Using culturally specific games in classrooms show that the learners bring various levels of knowledge in respect to the games. In fact, when asked about their feelings regarding their experience in learning mathematics with use of game, the students responded that they enjoyed the lessons very much, and analyzing the issues raised in the lessons were highly entertained. Cultural games involved participatory and communal strategies in which generation of knowledge is the result of a continuous process of experimentation, innovation, and adaptation, as Vygotsky (1978) maintained that "learning is a necessary and universal aspect of the process of developing culturally organized, specifically human, psychological functions"(p.90). In this connection, most of my research participant would draw the figure of the board of *Bagha Chal* game. Through the games played in students' home, community and even in school, various mathematical concepts and ideas are practiced implicitly. Thus, it is beneficial to connect the students' ethnomathematical knowledge to school mathematics that create environment for stimulating and encouraging mathematical discussion between groups of children and between pupil and teacher (Ernest, 1986). Nkopodi and Mosimege (2009) viewed that the use of indigenous games in mathematics classroom provides an opportunity to relate students' experiences to the learning of mathematical concepts, ideas and skills encountered in their classroom.

CONCLUDING REMARKS

Every game has different mathematical ideas if it is deeply analyzed. In this paper, I have analyzed one of the popular cultural game *Bagha Chal* from the mathematical perspective and observed the various mathematical concepts found inside it. Nevertheless, games provide a variety of mathematical language, problem-solving skills such as creating strategies, collecting, organizing and analyzing information. By means of game, students not only develop mathematical ideas, but also sharpen mental coordination, concentration levels, memory and their ability to communicate. It also provides an atmosphere for children to share ideas with their peers. The change in students' motivation can be linked to the importance of giving students, opportunities for mathematical explorations and communication by relating to the real-world for meaningful mathematics learning.

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MATHEMATICS FOR THE ADULT DAILY-WAGE EARNER, REVISITED

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Abstract: The mathematics taught in adult education seems to be largely based on school mathematics, presumably to make up for what adult learners missed at school. However, neither the livelihood needs nor the lived everyday experiences of the self-employed daily-wage earner are addressed by a curriculum that is largely based on arithmetical ability. We suggest that topics that are traditionally considered to be `higher mathematics' are more suitable, but this requires us to ``ease up'' a bit on what we regard as ***mathematics*** *in itself.*

BACKGROUND

There seems to be a consensus world over, not only on the need for mathematics in adult education, but also on what such content should comprise of. Broadly, often referred to as *the numeracy curriculum*, it includes the arithmetic of whole numbers, with the four operations of addition, multiplication, subtraction and division assuming centrality. Fractions and decimals are introduced, as also measurement of length, area, volume and time. Some financial literacy, data handling and use of maps / charts is common. These are accompanied by word problems that attempt to highlight the use of such mathematical knowledge in everyday situations[1]. Thus there is aconcurrence with the content of mathematics taught at the primary stage in schools. In terms of curricular expectation of outcomes, again we see a concurrence, though the arithmetical competence expected from adults may often be less. Pedagogy attempts to be significantly different since learners have life experience, but assessment is again similar to that in children's mathematics classrooms.

In this essay, we ask: *What mathematics is* ***appropriate*** *for the adult learner ?* Here, by adult learners, we specifically mean those from the poorest sections of Indian society, who are daily wage earners, marginal farmers and self-employed sellers of small goods. Such learners come to mathematics with important and specific needs. Does the numeracy curriculum address these needs ? Is it effective, empowering ?

In brief, the argument we wish to present here is the following. We argue that the numeracy curriculum largely ignores both the real needs of learners and their learning potential. We suggest that topics from `higher mathematics' such as optimization,

[1] The consensus we refer to here is what is reflected in policy documents on adult education such as (NCAE, 2011). Education researchers like David Kaye have questioned this definition of numeracy (Kaye, 2018)

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probability, interpolation and extrapolation, transformation and symmetry are likely to be more successful in engaging and empowering the mature adult learner. But for this to be possible, the very nature of mathematical learning requires redefinition. The tools of mathematics are built for universal applicability and hence mastering them requires a great deal of effort (as children world over would attest!). But simple *familiarity* with notions and use of heuristics (with limited validity) may be yet better than struggles to achieve *proficiency* in `elementary' mathematics.

Admittedly, the argument is polemical, and based on (surely inadequate) personal experience. Yet, the presentation is based firmly on the conviction that there are some underlying principles worthy of discussion. While we do not present a rigorous theoretical framework, we try and place the discussion in education research context[2].

WHAT IS TAUGHT

How is the content of the adult mathematics classroom determined ? How are the goals of mathematics education articulated in the context of adult neo-literates ? In the **concept note** prepared by the Ministry of the Human Resources Development of the Government of India (MHRD, 2016), the mention of mathematics is confined to `functional numeracy'. The **National Curriculum Framework for Adult Education** document (NCAE, 2011) offers a carefully constructed organizational framework for adult education in the country. It insists on a participatory curricular process in the adult classroom. When it comes to mathematics, the document offers a piece of critical insight:

> It is assumed that mathematics, numeracy in particular, has universal methods of learning. This contrasts with the evidence that illiterate adults transact mathematics in their everyday life in market and productive situations with ease, but use different algorithms, that can change from place to place. Therefore just like language, teaching of mathematics, including shapes and geometry, must bridge the ethno-mathematical algorithms with standard methods.

This at once stresses on the need to respect socially and culturally rooted mathematical practice but also seeks to bridge these with standard mathematics.

However, the mention of *mathematical algorithms* alerts us to the procedural and computational view of mathematics based on arithmetical ability. Despite the insistence on rootedness in cultural practices and participatory classrooms (which is in consonance with the broad literature on *ethnomathematics*) the view of mathematics **as given** (and determined by school curricula) dominates these and other official statements on adult education.

At this point, I would like to highlight a problematic feature hidden in demands for participatory processes in adult classroom. When it comes to literacy, there is a clear **contract** in existence between the adult learner and the teacher: typically, the learner entering the course can read and write at best with difficulty and often not at all. The

[2] This paper has a significant overlap with (Ramanujam, 2018) though the central argument is different.

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goal then is clear: on completion, the learner should be fluent in reading and proficient at writing. In mathematics, this is hardly clear. Neither teacher nor learner is very clear on the entry-level ability of the learner. On the other hand, the learner who had been at best only to primary school, equates mathematics with arithmetic based on her memory. Hence there is a happy *equilibrium* between the arithmetical expectation of the learner and the arithmetical offering of the curriculum. This equilibrium is surely not what is meant by participatory processes in curricular documents. But the adult learner's participation is *goal setting* is important for a participatory process.

The need for culturally and contextually rooted content has been widely emphasized all over the world. The educational research area of **Adults Learning Mathematics** and its *journal* discuss many of the questions addressed here. David Kaye has built a strong critique of the definition of numeracy and mathematics for adults (Kaye, 2018). Tine Wedege (Wedege 2016) points us to a “two grand stories” picture of adult math education: the school subject curriculum model and the ethno curriculum model. Contemporary research is said to privilege the latter. The notion of `bridging' in (NCAE, 2011) is thus in line with educational research.

The primacy accorded to the `subject curriculum' is evident in the international surveys (IALSS, 2003), on which many national / state adult education curricula are based. For instance, the Canadian Saskatchewan curriculum for Adult Basic Education (ABE, 2006) lists the topics as: Numbers and number sense, Algebra, Ratio / Rate / Proportion, Measurement, Geometry, Statistics and Probability. While there are extensive attempts to incorporate process strands and meta-cognitive thinking, what are called `content skills' accord primacy to the `subject curriculum'.

Is it then possible to conceive of a **different mathematics** for adults at all? Gelsa Knijnik (Knijik, 2007) shows how the mathematics produced by the landless peasant society differs from school mathematics, especially in its oral and `forgetful' forms and its immanence. This has a parallel to what we will discuss later.

What we focus on is the question: if an adult learner were to seek of mathematics tangible help in her professional practice, would it suffice to offer her the `subject curriculum' of number sense, algebra etc ? If not, what mathematics can be offered to her and how ? In a democratic society, does the learner have no right to demand that her investment in mathematics learning be of “benefit” that is seen as a benefit in her own terms ?

If this work were to be placed in a theoretical framework, it would best fit into that of *Critical Mathematics* (Freire 1994) and Reading and Writing the World with Mathematics (Gutstein, 2016), a framework developed extensively by (Frankenstein, 1995) and (Skovsmose, 2004). When learners start using mathematics to study reality, they come to view mathematics as useful and prepare to shape society. Acting in the world using mathematical means empowers adult learners in ways that not only improve their standing in the world, but eases their relationship with knowledge

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systems, and thus power structures in society. This has relevance to the discussion on the politics of mathematics education and its relation to democracy (Valero, 2017). From another perspective, Frankenstein (Frankenstein, 1995) argues for an emphasis on developing the ability in the learner to interrogate and present data, with mathematics acting as a tool to interpret and challenge inequities in society

AN EXPERIENTIAL ACCOUNT

In what follows, I will share some of my personal experiences in adult education in the broad context of Total Literacy Campaigns (TLCs) in India in the 1990s (Rao, 1993).

The context

In 1989-1990, a voluntary mass literacy programme in Ernakulam, Kerala, with the help of India's National Literacy Mission, got approximately 1000 volunteers to teach close to a 100,000 learners. This was followed the next year by a similar programme in Pondicherry and soon a number of districts took up TLCs. During 1990-1991, a voluntary group called Tamil Nadu Science Forum took up such a literacy exercise in Chennai (or Madras as the city was then called). Without any governmental support, this effort ran classes for nearly 1200 learners in six slums of South Madras, with about 100 volunteers running the classes. The experiences I report are from some of these classes that I taught as a volunteer. The learners I worked with were women, daily wage earners, from the poorest sections of Indian society, and yet they enthusiastically participated in the exercises described below.

In particular, I wish to tell the story of Velamma, an itinerant vegetable seller, carrying vegetables in bags slung over shoulders and on an overhead basket. She once asked if attending adult education classes would help her earn ten rupees more daily. Given that her daily earnings varied between Rs 50 and 100, she was asking for a big value addition due to education. And yet, why not, indeed ?

The problem

Thus began a rather interesting journey. We started classes that were initially one-on-one, meeting roughly 3 times a week. Very soon, Velamma brought a few other friends, all of them itinerant vegetable sellers like her. In five to six months, there were 20 to 30 learners, appearing in class at different frequencies. We entirely ignored the mathematics content of the official texts (which was principally teaching arithmetical operations). What we did was different, and had as its sole aim the goal of increasing daily earnings. The content was not fixed beforehand, nor was learning assessed systematically. In my opinion, what sustained the exercise was an articulated goal shared by learner and teacher alike. While there was no guarantee of goal fulfilment, a sense of making progress was sufficient for the adult learner to commit to further exploration (and indeed, to invite others to join in the experience).

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To explain what we did, it is important to get an idea of the daily routine of someone like Velamma. Typically, she would start her day at dawn in the wholesale vegetable market, where she would buy a mix of vegetables for, say Rs 200. These would be packed into her big basket and bags. Then she would choose between one of five or six localities where she operated, take a bus to that place, and spend much of the day alternately walking the streets and settling down at some fixed places (like one near a temple). At the end of the day, if she had unsold vegetables, she would sell the perishable ones to one of the bigger shops (for a loss) and carry over the rest to the next day.

Thus Velamma's choice and decision points consisted of: what vegetable mix to buy at what rates, how much of each, and where to ply her wares. These decisions were subject to drastic constraints: weight (she had to carry them under the hot sun), volume (only three containers), perishability and expected sales. There was also a matter of packing her basket and bags, since each time she made a sale, she had to potentially unpack everything, and this might not always be possible. Her pricing was variable, and hence she could also choose to raise / lower prices as she wished. There were further considerations: a low-margin item such as tomato might sell well anywhere, whereas a high-margin cauliflower might sell better in an upscale neighbourhood; but then she might not be allowed inside those houses at all.

The solution

One can easily set this up as a multi-dimensional combinatorial optimization problem and prove that this is a computationally hard problem to solve ! At least, this was how I approached the problem initially and realised that my mathematical training (from some of the top academic institutes of the country) would not help directly. But then it was clear that what we needed at that time was not a **universal** solution, one that would yield the best answer to every such potential situation, but a solution that was **adequate**, simple and could well be sub-optimal.

What we did in class was to **tabulate** (more generally, represent) the data. We met in the evening and wrote down what Velamma had bought on that day, at what rates, and what she sold where, at what rates. Initially I was eager to record everything, and we noted attributes like perishability, weight, volume etc. Soon I realised that this was futile for several reasons: for one, the data became too vast to process; for another, these attributes were constant and the learners found it silly to write these down every day. This last point is important to the story here: Velamma had a pretty good idea of weights, volumes and perishability of the goods she was dealing with, and saw no utility in writing them down. The particular combination used on a particular day was relevant to decision making, but she could not be brought to see this relevance, whereas price variability was important to her. She was not interested in minimizing her physical discomforts, but only in increasing her earnings, keeping discomfort to her own manageable levels. Dropping these variables turned out to be wise as we progressed, both because the learners were automatically correcting for

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them anyway. For instance, if we came up with a combination based on pricing that was difficult to handle physically, rather than discard it and seek another, they would modify the solution redistributing the mix appropriately. Once again, this might yield suboptimal outcomes but the simplicity achieved was well worth the `loss'.

Thus grew a set of tables, and graphs (`pictures' to the learners), and soon patterns began to appear. By the end of a month, we could infer functional variations and formulate many *heuristic "rules of thumb"* on product mix to buy from wholesale market. This led to a **rule book** (a filled and annotated notebook) that suggested different combinations at different prices. Icons were used to represent vegetables, units were left implicit so that 2 meant 2 Kilograms whereas 500 meant 500 grams, and so on. Initially we named rules by people (mainly due to my training that privileges X's theorem, Y's definition etc), but this led only to meaningless squabbles and everyone was happy to drop the convention. The result was an organic naming policy (though I could not have thought such phrasing those days); to an outsider names like *Thursday rule* or *Brown tuber rule* would make no sense, but they carried meaning to us by way of reference to a discussion that took place in class.

At this point I should emphasize the fact that Velamma was a remarkably intelligent woman. She entered into the `game' with great gusto, made fun of my obvious inability to predict prices, and contributed to rule making in a big way. The entire exercise acquired significant depth when more learners joined in it. We could now compare different buying and selling patterns, make up more heuristic rules. In my opinion, the essential component in their learning was this process, of systematic reflection on their own practices, aided by *symbolic rule making activity*. They could also assess the effectiveness of rules themselves, though I had to urge patience on them, not give up on a rule on one failure, but *amortize over time*. Once the notion of amortization (definitely a higher mathematics idea) was intuitively understood by the learners, progress was rapid.

During the same period, we also went through many packing exercises so that the learners could minimize physical effort. The effort required for unpacking when a buyer asked for an item that lay deep inside needed to be taken into account. Discussion on this turned out to be very useful.

Higher mathematics in neo-literate classroom

Thus, the classes comprised a form of problem centred learning, involving many sophisticated mathematical notions like *optimization, functional variation, expectation, amortization, estimation, approximation* etc. However the engagement of the learners with the notions themselves was highly informal, lacking in rigour and with no universal validity. Nonetheless, the engagement was sincere and committed, and more importantly, perceived as successful, its validity empirically verified within the restricted domain of practice.

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In the end, the class never had a formal conclusion either, but simply petered out, meeting over longer intervals. The techniques were surely limited and met their maxima; when the learners felt they were not getting more out of the rule book. Elsewhere, in (Ramanujam, 2018), I have taken up more examples and illustrated the many processes in our adult education mathematics classes, including the development of stylized jargon in the classroom and the use of symmetries and transformations. The Velamma story is recounted at length here merely to make the point that engagement with mathematical themes typically considered to be higher mathematics might be relevant, feasible, participatory and empowering for adult neo-literate learners.

MATHEMATICS, REVISITED

Are there any curricular / pedagogic lessons in an experiential account such as this ? Much of the human population is involved in economic activity that is privately run, albeit part of larger industrial or market processes. The autonomy of decision making that Velamma and her colleagues have comes with a strong mathematical need that is scarcely addressed by school education. What kind of mathematics addresses such a need? Broadly speaking, this is the realm of commercial mathematics, profit and loss, ratio and proportion, data handling etc included in school curricula, but also probability, expectations, optimization, risk assessment, amortization etc that are typically taught in business schools. Unfortunately, the latter aim at proficiency, and mathematical methods that guarantee complete answers with universal applicability, and the preparation for such mathematics takes years of learning. Perhaps it is possible to revisit these curricular domains for engaging with such notions only to achieve acquaintance, a familiarity that suffices for creating and using heuristics, and seeking help when needed.

Paolo Freire talked of reading the **world** and a political objective of adult education has always been to empower learners with ways to understand the world. Can the mathematics classroom attempt to reason about the learner’s life experiences ? It is then important to have a flexible curriculum that is alive for such opportunities, and attempts to find problems to solve from learner's lives.

Tools of local validity

On the other hand, such experiences offer another hypothesis worth examining. These learners did explore, did infer `theorems' but were interested in them in the specific setting they were posed in and not interested in their generalizations. Statements of universal validity, dear to mathematicians, seemed to be considered rather irrelevant. This perhaps also means that there are many mathematical journeys possible for adult learners if we approach the content in a different way, not in its generality, but in whatever specificity it can be embedded in. (Note that I am not conflating specificity with concreteness; the content may well be abstract.)

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Some general observations on mathematics and its learning may be relevant here. The discipline of mathematics concerns itself with the creation of mathematical knowledge, and engages with the world in terms of the questions posed for mathematical consideration from the sciences and knowledge systems, and in the answers it provides to them, which in turn leads to an enrichment of mathematical knowledge, more questions and answers, and so on. Clearly, socio-economic-political forces both sustain and influence such engagement. However, the authentication of knowledge by the process of **proof**, both by *social* and *deductive* means, serve to **universalise** the content. The employment of higher and higher levels of abstraction is conscious, stylised and has historically led to applications one may have never dreamed of (had one stayed close to the application and formulated the need). But this very distance also makes it hard to solve specific problems from everyday life that are complex, non-linear, semi-structured , involve partially specified conditions, and a large number of variables whose behaviour is partially known. However, if we are willing to dilute the notion of "solving", these problems may perhaps be addressed in a less rigorous, and yet meaningful, way that enlarges our capabilities. This calls for giving up on universal validity and development of tools that are of limited, local validity.

Accessible mathematics

The trouble, of course, lies in the validation of such ``knowledge", but what validation do we seek ? Methods that improve lives of one set of learners are not *necessarily* helpful for another set of learners, but are very *likely* to be useful for them, at least in serving as starting point. What is more important is the accessibility of techniques used. If mathematics education is seen as offering highly reliable and applicable techniques that exclude much of the learning population, an alternative that offers less reliable but yet useful and easily universally accessible techniques seems to be worth exploring.

Currently, when any topic is introduced to the learner, then we demand proficiency in it. We could settle instead for *acquaintance* with the topic, whereby the learner is exposed to the mathematical core of the concept in experiential ways, sees applications illustrated and gets sufficiently familiar with it to be able to recognise the need for the concept and its use later on, and perhaps be able to look up or seek help as needed. This is typically what is expected of skilled workers in factories, we could fashion a learning environment on such principles as well. ICT could help too: if Velamma's class were to meet today, mobile phones would play a central role in data and tabulation, and the internet would also be used to collect information and make predictions, strengthening the central activity of an optimization study project. In our vision, the curricular areas of study for the adult mathematics learner would not be structured as Arithmetic, Geometry etc but as Perceiving patterns, Dealing with uncertainty, Optimizing our resources, Processes, Symmetry, Form and content, Aesthetics, Modes of reasoning, and so on. Problem solving in groups would be the

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medium for engaging with such content, rather than as a skill in isolation. Such a structure would emphasize process rather than content, appeal to the maturity of the adult learner, and be more purposive as well as participatory.

The big question remains: how can such proposals for mathematics curriculum be sustained and defended in a global context that sees mathematics education as gearing people for participation in a global, competitive economy (Valero, 2017) ?

CONCLUDING REMARKS

We have argued that a demand on adult mathematics education that comes from learners' productive lives that needs to be addressed carefully; this typically involves mathematics taught at the university, but whose underlying notions may be accessible to adult learners, in a manner that suffices for their needs. But we point out that this necessitates a re-think on what we consider to be mathematics in itself. Much like art forms that have no permanence, such mathematical creations in classrooms may have no lasting value, except in having offered mathematical experience for the learner, *educating* her.

Given the significant difficulty of achieving among adult neo-literate learners even those competencies found at the elementary school, it seems utopian to speak of ambitious goals for mathematics in adult classrooms. And yet, without the articulation of such goals, the socio-political agenda of mathematics education will remain one of exclusion[3].

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AN AFRICA-CENTRED KNOWLEDGES APPROACH TO THEORY USE IN RESEARCH ABOUT MATHEMATICS EDUCATION AND SOCIETY

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The choice of theory for studying the social, political and ethical dimensions of mathematics education is an ongoing concern of the community. This paper offers an Africa-centred knowledges approach to theory use – located in Southern theory – to explore relations between on the one hand how and where we know, and on the other, what we know about mathematics education and society. The approach is applied to two cases of educational research conducted in southern Africa; teaching and learning in multilingual classrooms, and academic "success" in contexts of extreme poverty. The discussion shows what theory of different origins, used by researchers with varied lived experiences, becomes when put to use in African realities and the resulting contribution to knowledge about mathematics and society.

INTRODUCTION

This paper is about theory use in research on mathematics education and society. Systemic, operable use of theory that builds on previous work and talks across contexts is often regarded as a sign of the maturing of an academic research field (Jablonka & Bergsten, 2010). The word "theory" has origins in the notion of "seeing", and hence points to "understand[ing]" of the world (Lemons, 2017, p. x). We use theory to make "visible something that cannot be captured without mediation by the theoretical concepts" (Jablonka & Bergsten, 2010, p. 27). For the Mathematics Education and Society (MES) community, an ongoing debate is about what theory is appropriate for both understanding mathematics education and society and for changing the status quo. Hence we are drawn to theories located in "emancipatory" or "deconstructive" paradigms (see Stinson et al., 2015). We seek not just to get our descriptions "right", but also to make our research "count" in society (Adler & Lerman, 2003).

My concern is with relations between, on the one hand *how* and *where* we know, and on the other, *what* we know about mathematics education and society. For *how* we know is not just about method, but is also about "history, culture and for most of us, modernity itself" (Ashcroft, 2014, p. 65). I seek to continue the conversation about the extent to which we as mathematics education researchers engage critically with how our theoretical choices "name" the world of mathematics education and society (e.g. Pais & Valero, 2014; Ernest, 2016). For this discussion I position myself as a white, female, middle class, English-home language researcher in South Africa, one of a number of postcolonial contexts represented in the MES community. In these contexts, coloniality is not just about power over space, but the colonisation of being, knowledge and discourse (Ramugondo, 2017). The naming of Africa is "filtered"

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through colonial and postcolonial pasts (including apartheid in South Africa) thus "silently naming the knowledge outcomes" (Cooper & Morrell, 2014, p. 2).

To explore these relations I bring to this conversation an Africa-centred knowledges approach to theory use (e.g. Cooper & Morrell, 2014), an approach that is located more broadly in Southern theory (e.g. Connell, 2007). I begin with a brief description of this approach, initially looking beyond mathematics education to the field of social HIV research for understanding. As will be shown, an Africa-centred knowledges approach and social HIV research share many concerns and interests with the MES community. With this understanding, I then turn to two areas of concern in mathematics education; mathematics education in contexts of language diversity, and understanding academic "success". I use examples of research on these concerns conducted in southern Africa to conceptualise how theory is put to use to produce African-centred knowledge about mathematics education and society.

AFRICA-CENTRED KNOWLEDGES

Africa-centred knowledges, like the Southern theory in which it is located, is an emerging, heterogeneous body of work united by a common recognition of the geopolitics of knowledge production and critical engagement with inequities between the "centre" and related "peripheries" (Connell, 2007; Morrell, 2016). This work strives to be "emancipatory" and to "contribute to the democratisation of knowledge" (Morrell, 2016, p. 3). As suggested by the names *Africa-centred* and *Southern*, place is visible as we ask questions about what theory is appropriate for understanding a context, what are the politics and origins of our theoretical concepts, and how is the researcher who uses the theory positioned in the context. Yet this approach is not restricted by fixed boundaries and knowledges and by the location of theories or researchers. Rather terms such as *global North*/*South* and *centre*/*periphery* "emphasise relations – of authority, exclusion and inclusion, hegemony, partnership, sponsorship, appropriation – between intellectuals, institutions in the metropole and those in the world periphery" (Connell, 2007, viii-ix). As suggested by Cooper and Morrell (2014), the "African continent is multiple, global, and dynamic" (p. 2), and also united by its geopolitics and history. Emphasis is placed on relationality, flexibility, and a dialectic relationship between Africa and the metropole (Morrell, 2016). An Africa-centred knowledges approach seeks to avoid the reification that comes with binary descriptions of theory, place and identity such as "Eurocentric vs. Afrocentric", and "traditional vs. modern" (Cooper & Morrell, 2014). Instead, theory becomes Africa-centred when "entangled" in African realities and contexts (p. 3), identities are fluid, and researchers are agentic and reflexive.

This description suggests that an Africa-centred knowledges approach to theory is a "messy", created "third space" (Cooper & Morrell, 2014, p. 3), working in the "grey area that marks the edges of North and South" (Morrell, 2016, p. 3). To help us understand this "messiness" I turn to an example of this approach in practice – social HIV research in South Africa – as analysed by Hodes and Morrell (2018).

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Social HIV research conducted in South Africa

Social – as distinct from clinical – HIV research is concerned with the social, political, ethical and economic aspects of the pandemic. Analysing the archive of social HIV research conducted in South Africa, Hodes and Morrell (2018) note that traditional metrics of knowledge production identify the centre as dominant in terms of production and as a funding source, with contexts such as South Africa in the periphery a site of "unprocessed data" (p. 27). Thus, centre-periphery power asymmetries are reproduced. However, Hodes and Morrell (2018) apply Africa-centred knowledge concepts to give a nuanced account of both the *what* and *how* of the South African contribution to knowledge, which they describe as "a knowledge 'incursion' from the epicentre of pandemic" (p. 23).

Firstly, regarding the *what* and *where* of the contribution, South African HIV research contributes a context that is not just a setting, but has "power and meaning" (Hodes & Morrell, 2014, p. 24). The scale and socio-political history of the pandemic makes South Africa a complex context, leading researchers to ask questions not asked elsewhere. The focus on certain populations in this context has produced findings that challenge "calcified" concepts of *otherness* and *risk* and their relations to gender and race (p. 26). By bringing notions of *power*, *position*, and the *structure-agency interplay* to understand the problem, the researchers have contributed to changing ideas about the social determinants of HIV transmission.

Secondly, regarding the *how* of the contribution, Hodes and Morrell (2018) argue that these two contributions were achieved using recognised academic approaches to research methods, collaborations, and publications. Yet, this process was not passive, unidirectional and uncritical, but active, relational and strategic. For example, the researchers in South Africa explored the complex context using centre epistemologies, theoretical concepts originating and known elsewhere, and recognised research methods. They were careful not to make sweeping claims but placed emphasis on the "contingent, complex and elusive nature of findings" (p. 25). These choices, Hodes and Morrell (2018) argue, help the HIV researchers to avoid both essentialising and othering the South Africa context and claiming universality, thus positioning themselves to give value locally (e.g. to policy), and globally.

MATHEMATICS EDUCATION IN CONTEXTS OF LANGUAGE DIVERSITY

With this understanding of Africa-centred knowledges, I now turn to the relatively small body of research (Setati, Chitera & Essien, 2009) on mathematics education in multilingual classrooms in South Africa. I show how theory from the centre has become "entangled" in South African realities (Cooper & Morrell, 2014, p. 3). The theory use has not simply been from the centre to the periphery, but the researchers have reflexively contributed to the centre in three ways; context, new perspectives on conceptions of language, learners and teachers, and research methods.

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For the purposes of this conference paper my discussion is informed by reviews of this research (e.g. le Roux et al., 2016; Phakeng & Essien, 2016; Setati, Chitera & Essien, 2009). This research is concerned with access to mathematics in multilingual classrooms, focusing on describing and explaining language practices in these classrooms and the implications for practice. Although research on the relationship between language and mathematics learning emerged in the 1970s (Phakeng & Essien, 2016), the South African research discussed here emerged in the 1990s. This was a time when key reforms at the centre – such as *relevance*, *learner-centredness*, and *communication* – were being taken up in the periphery (le Roux et al., 2016; Phakeng & Essien, 2016). This research agenda was driven by Jill Adler and Mamokgethi Phakeng (formerly Setati), but developed and strengthened in collaborations. These researchers' varied lived experiences of postcolonial Africa (e.g. Graven, Phakeng & Nyabanyaba, 2016; Lerman, 2016) have meant "conscious engagement with context" (Morrell, 2016, p. 5) and a commitment to practice.

For these researchers the context of a complex South African language landscape, the *where* of the research, matters and has led them to ask particular questions about language diversity. South Africa has eleven official languages, with immigration from elsewhere in Africa adding to this diversity. In theory, schools and learners can choose the Language of Learning and Teaching (LoLT). The socio-political history of the country means that these languages are not just mediums of communication but take on particular political meanings; English holds symbolic power, and language continues to be used as a proxy for race and class. Thus, in practice English is the dominant LoLT, with most learners learning mathematics in an additional language while sharing their home and other languages with one another, and in some cases, the teacher. In South Africa it is a majority – not a minority – of mathematics learners who are not fluent in English as the LoLT (Phakeng & Moschkovich, 2013).

Regarding the *what* of the research conducted in this context, researchers have challenged understandings of language diversity that may have become "commonsense" in other contexts (le Roux et al., 2016, p. 93). Adler's research brought the notion of *multilingualism* – as the norm – to wider conversations about bilingualism (Phakeng & Essien, 2016). The notion of language as a *resource* for learning has challenged deficit views of bi/multilingual learners (Phakeng & Essien, 2016). This notion also turns deficit views of teachers' practices such as revoicing and code-switching as *teacher-centred* into talk about teachers' *knowledgeability* (le Roux et al., 2016; Phakeng & Essien, 2016). Adler uses Lampert's *dilemmas* to represent the personal, contextual work of teachers as they manage inherent tensions, thus avoiding binaries (le Roux et al., 2016). Not only has Adler used these concepts to understand her context, but she has demonstrated how they can be used by teachers to talk and think about their own practice (le Roux et al., 2016). In addition, Phakeng has brought to explanations of language practices in multilingual classrooms the perspective of language use in mathematics education as *political* (Phakeng & Essien, 2016). Her focus on teachers' deliberate use of learners' home language in these

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classrooms demonstrates how sociocultural and cognitive perspectives can be brought together (Setati, Chitera & Essien, 2009). Finally, in terms of research methods, Phakeng's notion of *"re"-presentation* of data has challenged data collection practices in contexts of language diversity (Setati, Chitera & Essien, 2009).

Adler and Lerman (2003) note that, given the historical dominance of the centre – empirically by defining "good" practices and theoretically through a focus on the *mathematics* of these practices – investigating practices in other contexts runs the risk of essentialising the periphery and producing descriptions of deficit. Yet in terms of the *how* of this language research, Adler, Phakeng and their collaborators have, like the HIV researchers in South Africa, interacted reflexively and strategically with the context but also with the centre to make their contributions count both locally and globally. Firstly, they have used multiple data sources such as interviews and observations to provide rich descriptions of language practices in settings that represent the language diversity of South African classrooms, such as rural vs. urban schools, and schools with varied socio-political histories (le Roux et al., 2016). Secondly, they have utilised theory originating in the centre, for example, Vygotsky's socio-cultural theory, Lave and Wenger's social practice theory (le Roux et al., 2016), and Gee's sociolinguistics (Phakeng & Moschkovich, 2013). These are political choices; not only do they allow a comprehensive view of the local, but they permit a respectful, non-deficit account of this context and allow the researchers to talk back to the metropole. Thirdly, these researchers have published in the dominant international English-language journals, but paid explicit attention in these publications to how they operationalise the theory from the centre in their context (le Roux et al., 2016). They have also brought their research in the South African context to discussions with researchers in the centre (le Roux et al., 2016; Setati, Chitera & Essien, 2009). Finally, while using socio-cultural and socio-political perspectives, researchers have consciously put this theory to use to investigate access to *mathematics* in multilingual classrooms (le Roux et al., 2016).

UNDERSTANDING ACADEMIC "SUCCESS" IN CONTEXTS OF POVERTY

This section focuses on a study by Zimbabwean researcher Leadus Madzima (2014) of academic "success" in a context of extreme poverty in her country. While the Africa-centred knowledge perspective on this case shows similar strategies and contributions as the previous example, crucially we see here how theory from within the periphery – specifically African philosophy – is put to use to supplement theory originating in the centre. Madzima's research is not specific to mathematics education, but it speaks to a key concern of the MES community, that is, what theory might help us to understand differential educational performance. A growing number of mathematics education researchers are using theoretical perspectives – for example, from poststructuralism, critical theory, and critical race theory – to view student participation in mathematics as an interplay between individual action (often referred to as *identity* or *agency*) and wider social structure. Some researchers have directed attention to understand how some (but not all) students succeed in education

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institutions that structurally are regarded as reproducing the power and privilege of certain groups (e.g. McGee & Martin, 2011; Stinson, 2008).

As in the previous case, Madzima's (2014) study contributes a complex context, the where of the research, one that is a product of its colonial and postcolonial history. The learners in focus attend a secondary school located a high-density township of Mbare, in the Zimbabwean capital city of Harare. Madzima notes that this context is characterised by extreme poverty, illness and hunger. School attendance and performance is a particular problem for orphans and girls. Yet, some learners do succeed at school. Thus Madzima (2014) has asked questions about how these learners "develop strong identities and powerful senses of agency" (p. 192). Madzima herself was born and grew up in Mbare, practised as a teacher in Harare, and speaks Shona, the dominant language.

Madzima's (2014) second contribution to knowledge is her choice of a concept from African philosophy – *hunhuism* (personhood) – to supplement theories from the centre. She argues that for her context, theories from the centre "individually and collectively approach an explanation without providing a full answer" (p. 192). For example, and I refer the reader to Madzima (2014) for the full argument, she notes that Bourdieu's *cultural habitus* has been used in a variety of contexts, including Zimbabwe, to show how class binaries are reproduced in school, but this does not explain success for marginalised learners. Gidden's *structuration* also brings into view the work of structures, but it adds a productive view of subjects as reflexive, and having some choice. Finally, she sees Butler's *performativity* as having potential to help her understand how learners overcome the constraints of class, gender, ethnicity, and so on. In this third view, power relations are produced in subjects' action as they work with multiple positions in time and space, and identity is something to be achieved. Butler, in Madzima's view (2014), focuses on how resilient identities are built through oppositional cultures. Yet her local context leads her to ask a different question about the structure-agency relationship: "… from which structural properties should [subjects] draw resources – strategies, knowledge, attitudes, beliefs, motivation and practice – to construct successful identities, especially if, as in Zimbabwe, most systems and structures have fallen apart?" (pp. 195-196).

Thus "to enhance, extend and transform the precision of other interpretative tools" (p. 196), Madzima (2014) uses the notion of *hunhuism*. This concept summarises the Shona social system, and she uses it as a model for identity development that provides structural resources for academic success. For this conference paper I briefly describe the concept and its use in the study (further details in Madzima, 2014), before applying our understanding of Africa-centred knowledges to explore further the *what* of her contribution to knowledge and *how* this is achieved.

Madzima (2014) writes that hunhuism is rooted in stories of the origins of the Shona and belief in the spirit-gods. Indeed, an individual's identity (*hunhu*) is believed to be created by the Great Shona Spirit, named Mwari, and to imitate Mwari's own identity which summarises all that is good about Shona society. The family plays an important role in socialising the individual into the valued knowledge, behaviour and beliefs of the Shona. In this rootedness in family and history, "identity formation […] is both a relational and multiple dialectical process of being structured and structuring" (p. 196). The collective is important:

> …learners predispose themselves to work with their nuclear families, their extended families, as well as friends and other groups whose members reciprocate in building pools of communal resources from which communal selves select the necessary products to create their personal or collective identities. (pp. 196-197)

Within this Shona philosophy, notions such as *hard work*, *motivation* and *competition* take on particular meanings. For example, hard work prompts blessings for both rich and poor. For learners who face seemingly insurmountable difficulties on a daily basis, Madzima (2014) argues, this perspective inspires hope, trust and humility. Failure of any form is seen to reflect an absence of "strong, coherent subjectivity" (p. 195). Kinship and the collective inform views on motivation and competition; the Shona work hard to reserve the lineages of their immediate and extended families, and success is not only a sign of individual strength, but for the good of the community. This encourages values such as responsibility, reciprocity, dignity, respect and compassion.

Madzima (2014) argues that learners in her study employed a number of hunhu identities and strategies to succeed academically, thus contributing to our understanding of the *structure-agency interplay*. For example, since a person of substance is disciplined, learners sought to demonstrate discipline in their choice of friends, lifestyle, speech, dress, and attitude to schooling. Although they faced suffering in the form of "extreme discipline" (p. 200), hunger, and homelessness, they worked hard to create spaces where they could identify as school learners. So they worked into the night after completing home duties, and studied while standing in queues or hawking. Key to their success, in Madzima's (2014) view, is the notion of *groupworks*; learners participated in a network of efficient, self-selected study communities in which they "built a rich pool of intellectual wealth and resources" (p. 202) in an absence of material and human resources at school. Most importantly, attempting to be a successful school learner did not mean forsaking the values of home and the community, and assimilating into dominant school values. Rather, "their new identities were products of their family, class and economic woes, woven together with the *optimism of hunhuism*" (p. 199, emphasis in the original). Their poverty motivated them to succeed academically, as they strived to become "persons of substance" for themselves and the community.

From an Africa-centred knowledges perspective, Madzima's (2014) contributions to knowledge – context, theory and conceptual understanding – are achieved through reflexive, strategic interaction with the centre. For example, as suggested by my brief review, she acknowledges the contribution of theory from the centre to our

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understandings of the structure-agency relations. Secondly, she does not claim superiority of hunhuism over other concepts or that the concept in itself is the full solution. Thirdly, like the multilingualism research in South Africa, Madzima (2014) uses multiple data collection methods – such as learners' diaries and writing on topical issues, fieldnotes, semi-structured interviews, and critical dialogues – to provide rich descriptions of individual learner's identity work. Since she shares language resources with the learners, they contributed in English or Shona. In addition, she aimed not to "appropriate" the research space, but to "share" it, "by treating learners not as poverty-stricken victims to be researched but rather as co-authors, to be respectfully regarded as capable of articulating their phenomenal lived circumstances, and of contributing to new knowledge" (p. 198). Thus, she addresses the danger that, by focusing on local concepts and groups, identities are reified.

CONCLUSIONS

This paper has used an Africa-centred knowledges approach to theory use to explore relations between, on the one hand *how* and *where* we know, and on the other *what* we know about mathematics education and society. As a member of the MES community located on the periphery in a postcolonial context, working in this space constitutes a political move (Cooper & Morrell, 2014), and one that is both "precariously pertinent and dangerously important" (Mudimbe, 1988, p. 5, cited in Cooper & Morrell, 2014, p. 2). The application of this approach within the constraints of a conference paper surfaces the nuances of what becomes of theory when it is put to work on particular practical problems in southern Africa. The research discussed here matters locally, but also disrupts unidirectional relations with the centre by producing global knowledge. To achieve the latter, the researchers have acted agentically and strategically to contribute context, local theoretical concepts, new ways of using centre concepts, and particular research methods.

Analysing the impact of the global knowledge economy on mathematics education, Ernest (2009) identifies *ideological*, *dominance*, *appropriation* and *recruitment* effects that reproduce asymmetries between the centre and periphery. Yet he argues that such structural analyses do not "make space for the personal" (p. 74), that is, the roles, motivations and agency of individuals who navigate these structures. The Africa-centred knowledges approach to theory use – and Southern theory more broadly – necessarily focuses on place. We are interested in using theory to solve local problems, and are concerned with both the origins of that theory and our positioning in the local. Yet the discussion of the research by Madzima, Adler, Phakeng and their collaborators in this paper suggests that this approach does not reify locations and identities. Rather, our interest is in what happens to theory when it is used by researchers who share a common critical engagement with local and global inequities. This paper demonstrates how this focus surfaces the nuances of the relations between the *what*, *how* and *where* of knowledge production in our field. Certainly, there are many examples of research about mathematics and society in Africa and elsewhere in the periphery that deserve such a nuanced understanding.

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Such understandings are "important" as we strive to build an ethical community that values and builds on the contributions of its diverse participants.

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AN AMERICAN GAZE AT EQUITY IN MATHEMATICS EDUCATION: WOMEN ARAB CITIZENS OF ISRAEL

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Abstract: In this paper, I explore consonance and dissonance between equity narratives about mathematics education in the United States with the context of Arab Israeli women in Israel. I highlight three constructs -- power relations, education from a neoliberal orientation, and gender equity. I share results of empirical research around life-story interviews with 14 Arab Israeli women studying mathematics education in Israel. I conducted grounded theory coding of these interviews and explore three themes that emerged from the data: sense of power relations, influence of family, and contradictions around gender equity.

INTRODUCTION

In this paper, I explore the socio-political context of equity in mathematics education in Israel. As a researcher from the U.S., I come to this exploration using an American lens, which underlies this study, its data collection, analysis, and interpretation of findings. Therefore, I begin with a concise overview from the U.S. context, and then proceed to the research context of interest, in an attempt to make this lens explicit.

A critical narrative about equity in the U.S. is that mathematics education is a capital-building resource and is not allocated equitably. Historically marginalized groups in the U.S., like African Americans and Latinx, have been granted differentially less access to resources like qualified teachers, quality mathematics curricula, advanced course offerings, and instructional technology (Lipman, 2016). Inequitable access to mathematics becomes consequential because of how it is used as a gatekeeper in the U.S. for access to higher education, employment, and broader capital resources.

There is unofficial but de-facto racial segregation across the U.S., including and often more extreme in those cities that present as progressive. Schools and teacher certification are state regulated, and attempts to secure community level control for African American and Latin@ communities have been repeatedly denied. Across this landscape of unofficial segregation, Black learners, in particular, experience a mathematics education that is dehumanizing and violent (Martin, 2018). National teacher unions have prioritized protecting white teachers who teach in communities of color, leveraging white supremacists' fears that community control of schools will further politicize African Americans and foster greater solidarity among them. Schools in underserved communities are largely staffed with white teachers "as missionaries" (Martin, 2007) and are being co-opted by a non-unionized charter-school movement funded by national business corporations that advocate "no-excuses" orientations to learning.

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Equity research in the U.S. tends to use either a race or a gender lens but not their intersection (Bullock, 2018). Mathematics ability is seen to correspond at once with normative masculinity and with whiteness (Mendick, 2006; Stinson, 2006). Emerging studies report how women of color in the U.S. face structural racism and unequal opportunities to learn as well as sexism and gendered racism known to be prevalent in mathematics learning settings (Levya, 2017; Joseph, Viesca, & Bianco, 2016). Black and Latina girls as learners of mathematics are left to challenge not only how mathematics is used to uphold white supremacy (Gutiérrez, 2017), but also how it is used to reinscribe the patriarchy's power and domination. In this paper, I address the question as to how this equity narrative from U.S. corresponds to the sociopolitical context of Israel, with a focus on Arab Israeli women.

RESEARCH CONTEXT

Israel was founded in 1948, and its majority (~75%) population and ruling majority are Jews, a racially, religiously, and socioeconomically diverse socially-constructed category, encompassing secondary ethnicities derived from cycles of migration in response to persecution. Israeli Arabs (~21%) are citizens of Israel who are descendants of those who remained (or were allowed to remain) in their villages during Israel's war of 1948, a war fought between Israel and the Palestinians with Jordan, Egypt, Syria and Iraq. Israel is a democracy and its Arab residents are citizens, though the fullness of this citizenship is often called to question, as a citizenship that is incidental as opposed to essential (Bishara, 2017) or as "citizens without citizenship" (Sultany, 2003). For this paper, I alternate between identifiers "Arab citizens of Israel" and "Arab Israelis." I caution readers that this group is also not monolithic but multicultural and multiracial, as well as geographically, socioeconomically, and religiously diverse. I avoid the identifier Palestinian here because this will generate confusion with the Palestinians in the West Bank and Gaza, who are not citizens of Israel, but this choice is not meant to disidentify these people from a Palestinian identity.

Israel has three public school sectors: Arab, Jewish secular, and Jewish religious. Arabic is the language of instruction in the former, whereas Hebrew is the language of instruction in the latter two. There is residential separation among all three sectors, with some instances of "mixed" cities and fewer examples of "mixed" schools. As a consequence of their marginalization, Arab villages tend to be in the lower half of socioeconomic rankings. There remains a gap between rates of Arab (46%) and Jewish youth (71%) who achieve a high school matriculation certificate making them eligible for higher education, but this gap is rapidly shrinking (46% up from 27% in 2012) (Hai, 2012). Similarly, the rate of increase of scores on the mathematics matriculation exam for students in Arab sector schools is three times higher than for students in the other sectors (Forgasz & Mittelburg, 2008).

On the one hand, Arab women face multiple aspects of marginalization, as members of an Arab minority in Israel; as women in a largely conservative, patriarchal society; and for many, as residents of Israel's geographic periphery (Haj-Yahya, Schnell, & Khattab, 2016). Indeed, around half of Arab women in Israel between the ages of 18-

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22 are neither employed nor in school or training (Haj-Yahya et al., 2016), and the work participation rate for Arab women of all ages is far below 50%, though increasing each year (Hai, 2012). On the other hand, Arab women currently exceed Arab men in terms of participation in higher education in Israel, there is higher participation among Arab girls than boys in advanced mathematics and science courses in high school, and this rate of participation exceeds that of Jewish girls (Ayalon, 2002). More broadly, gender inequality in school mathematics performance is largely non-existent in the Arab school sector, and favors girls when present, reversing the trend in Jewish secular schools (Ayalon, 2002). Exploring these contradictions forms the core rationale of the current study.

FRAMEWORKS

Power relations

In general, perspectives that stress minority-majority relationships run the risk of normalizing unequal power relations among and obfuscating actual relative sizes of groups. For example, today, youth of color form the numerical majority in most American cities and are not numerical minorities. A fundamental and enduring organizing principle in American society is racial segregation, which engenders fear and ultimately, violence. In educational research and practice, this segregation tradition expresses itself through classification of students primarily according to race, whereby school performance of so-called “minorities” are compared between (and not within) certain socially constructed racial groups and, usually unnamed as such, the white ruling-majority. There is a legacy of cultural deficit theories that then connect school performance with ascribed cultural characteristics. A twist on the oppositional and dysfunctional culture theories from the 1990s is the most current, in which mathematics success is explained by “grit” or by “growth mindset,” theories that downplay or ignore systematic inequities and unequal opportunities to learn.

Israeli society is organized primarily according to a broad socially constructed ethnic affiliation, with any racial or religious distinctions secondary. Although this distribution renders Jews as a population majority, there is a contradiction in this relationship. Jews possess a historical legacy of being marginalized, persecuted, and terrorized, with a fresh trauma of surviving an attempted genocide. Even though Jews are the ruling majority group in Israel, their historical legacy of being marginalized as Jews remains salient. Arab Israelis, in contrast, are not immigrants to the region, but are rooted with a long history to this place and are absent a legacy of persecution (prior to the current conflict) or a historical self-concept as a minority group. As Ma’ari (1978 as cited by Eiskovitz, 1997) explains, “both Arab and Jewish Israelis' self-images contradict their present demographic status, and each group's self-concept is mirror-opposite to the other's.”

As in the U.S., educational research in Israel is prone to taking a “gap-gazing” position (Gutiérrez, 2008) that focuses on school performance between (and less so, within) these broad groups, as evident in the above context description. Unequal power relations are inherent in how students in the Arab school sector are required to study

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Hebrew starting in elementary school, but in all three of the Jewish school sectors, learning Arabic is an elective, offered in only some schools. This has created an imbalance in which only a small proportion of Jewish Israelis are able to speak and understand Arabic whereas Arab Israelis tend to be proficient, at least, and oftentimes fluent, in Hebrew as a second language. In addition, whereas the Jewish school sectors focus on the teaching of English as a second language; for the Arab school sector, English is the students' third language. This has created a situation where English proficiency (instead of mathematics) functions as a gatekeeper to higher education: only 30% of eligible Arab students in 2012 continued on to higher education compared to 49% among Jews, explained in terms of English proficiency (Feniger & Ayalon, 2016).

Neoliberalism and education

Similar to the U.S., the neoliberal orientation to education in Israel has aligned education with job preparation and is organized around stratification of individual students using standardized test score measurements. A meritocracy ideology is fundamental in mathematics education in both societies, whereby success is interpreted in terms of effort or superiority and any lack of achievement in terms of lack of ability or effort and not in terms of systemic structures that differentially pose obstacles to success (Rubel, 2017). Such an outlook on education converts schools and learning into competitive arenas for individual students, depoliticizes education, and thereby weakens solidarity among students. In the face of structural marginalization, a widespread belief in meritocracy through education as a mechanism to social mobility persists among Arab Israelis, which perhaps explains their consistently high educational aspirations (Agbaria, 2017; Khattab, 2003). Even though schools are officially segregated via language of instruction, curricular content and teacher preparation in the Arab school sector are controlled and regulated by the Israeli government, meaning that education that might address their geo-and socio-political identities is limited. Thus, Arab education in Israel is a mechanism for social mobility, but at the same time, a vehicle for state control (Mazawi, 1994).

Gender equity

Different from their Jewish counterparts in Israel, gender inequality in mathematics achievement in the Arab sector schools favors girls. Zedan (2010) explains this trend in terms of increasing feminism among teachers, Islam's encouragement of education of girls, and as part of a feedback loop from the consistent increase in numbers of girls from Arab sector schools who continue to higher education. Ayalon (2002) and Nasser and Birenbaum (2005) explain girls' participation in mathematics as a paradoxical result of a set of *constricted* opportunities -- as girls in a conservative, patriarchal society and as students in Arab sector schools with more limited curricular options, in which mathematics is one of few paths to individual excellence. Despite the gender conservatism in Arab Israeli society (and in Israeli society at large), girls in the Arab school sector have greater self-confidence in mathematics and are more likely to continue towards professions around in mathematics than boys (Mittelberg & Lev-Ari,

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1999). The most common higher education path for Arab women in mathematics is teacher education, with this choice of career most often selected for them by their family (Eilyan, Zedan, & Toran, 2007).

RESEARCH METHODS

I conducted interviews with 14 Arab Israeli women, ranging in age from 20 to 45. Seven are masters students in mathematics education at two universities, 4 are undergraduate students in mathematics education at two teachers college, 2 are doctoral students at two universities, and 1 is a mathematics teacher educator. 11 of the women identify as Muslim, with 8 wearing traditional hijab; 2 identify as Christian; and 1 as Bedouin Muslim. Five of them live in a cluster of towns along the border in central Israel (the "triangle"), three in mixed Arab-Jewish cities in central Israel, four in more rural, northern Israel, and two in desert, southern Israel. Each interview lasted approximately an hour and was conducted either in the participant's home (5), public spot in their village (2), a nearby location of their choice (1), or at their university/college (6), in a common second language of Hebrew. The interviews included questions about the participant's educational biography, the geography of that biography, experiences learning mathematics in schools and in colleges/universities, and their thoughts about the role of their identity as Arab women in their educational experiences. I translated the audio-recordings into English, used grounded theory to conduct open, focused coding (Charmaz, 1996). This process generated a set of codes that I then organized into three thematic clusters.

I identify as Jewish and am a citizen of Israel, but was born and raised in the United States. I exercised my Right of Return as a Jew to be able to establish this citizenship, which is itself under contention by Palestinians who are not citizens of Israel but wish to return to their pre-1948 villages but remain unable to do so as part of the ongoing conflict. I was trained as a mathematics teacher in Israel, but have significant experience as a mathematics teacher educator in the U.S. As an immigrant who speaks Hebrew as a second language, I hold a social position of outsider, to Jewish Israelis and also to Arab Israelis, which in this case, positioned me as somewhat of a neutral or third-party to the existing tensions. I speak beginner Arabic and have limited but expanding experience in Arab sector schools or communities.

RESULTS

Sense of power relations

In general, the participants shared (or were comfortable sharing) limited experiences of experiences of discrimination in Israeli society. All of them indicated that they felt that they were treated equally at higher education institutions and by Jewish facult and peers, though mostly remaining socially separate. At the same time, at points in their interviews, several (3) of the women described painful humiliation in regional travel

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on public transportation of being stopped and frisked because of their appearance as Muslim women. When I pursued why their expressions of social injustice seemed so tempered, one interviewee explained that while Jews "can easily pass criticism, we think more than once before we pass criticism." She explained that she fears that if she makes critical evaluations about social injustices in Israel, it will be taken up as "threatening" and not as social activism. Despite their limited accounts of past experiences of discrimination, nearly all of the women explicitly named fears of potential discrimination, racism, or violence. More specifically, they expressed fear of: leaving their villages, traveling alone, being alone with Jews, exposure to verbal or physical harassment or attack, and job discrimination. In many ways, their statements of fear mirrored my fears of potential violence as a Jewish visitor to their villages, with a key difference being that their evaluation of risks was not tempered by the presence of military or other state "defense" forces.

Family influences

All of the participants live with or in close proximity to large extended family, and notably, their families had lived in those villages for many generations. Nearly all talked about their family's influence on their selection of mathematics as an educational pursuit and teaching as a career. For most, this was a result of the supposed appropriateness of teaching for a woman or a mother, in terms of the perceived daily demands. For instance, one woman explained: "Our parents tell us that it's better, more comfortable for your future as a woman to be a teacher, because you finish work at the same time as your kids finish school, and you can go home together. I went with this. Our parents, our grandparents all always told us that it was best for a daughter." In some cases, interviewees reported that they were geographically constricted by their fathers or husbands to study in their own villages, and this constraint limited higher education options to teacher preparation. In other cases, interviewees' families selected mathematics teaching as a result of the perception that teaching is a career that has job availability and security, and that through teaching, one can remain professionally within the Arab sector. As one interviewee explained, "My mom told me to become a teacher. I wanted to study Economics or business or other things, but they (my parents) told me that there would be no jobs for me as an Arab in those things but there are jobs for teachers."

Nearly all of the women described their selection of degree programs in mathematics education in terms of the connection to the teaching profession. Half of the participants expressed regrets about complying with their families' requirements that they become teachers. For example, one interviewee indicated that she thought that she "wanted to be something bigger," and three other interviewees described that they found classroom teaching challenging and not good matches for their personal strengths and weaknesses. Three others reported that they had chosen teaching simply because their psychometric test scores did not qualify them for their first choice of various medical professions.

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Contradictions around gender equity

All of the interviewees emphasized that they excelled in school mathematics and attributed their excellence to sheer effort. All described self-discipline of arduous and consistent drilling of mathematics exercises, at home, alone. For example, one interviewee explained that she had gathered several different textbooks and would routinely practice exercises from the range of sources. Most explained their sense that girls in their society are more compliant about homework, "listen better," and "try harder." They explained that girls spend more time at home than boys, because of family and societal customs around girls and restrictions about what spaces they are allowed to occupy or pass through. On one hand, they emphasized the greater spatial freedom and wider range of career options afforded boys. Many of the women talked about how boys are freer to be outside, have a greater variety of career options, many of which are more lucrative than teaching. They also noted that boys are free to travel outside of Israel and can continue to higher education in other countries, where entrance requirements are less competitive. In contrast, they explained that for the most part, girls are not allowed by their families to travel abroad, and therefore must meet the more stringent Israeli higher-education entrance requirements. On the other hand, interviewees also described the extra burdens on boys of pressures to begin accumulating financial capital for a future family, and how they felt that being able to focus on school mathematics was actually their privilege.

All of the women expressed that higher education in mathematics education is a means of self-realization and of acquiring social capital. Interviewees reported a sense of social value that accompanies school achievement in their society and the high status of success in mathematics. As one interviewee explained, "She says -- among Arabs, mathematics is "wow"." Similarly, another explained, "I felt a kind of power because I succeeded. People wanted my help. Social power. Pride. People say, wow, who is that?" A third interviewee explained that even though there is an excess of mathematics teachers in the Arab sector, being a certified teacher lends a woman more status in society, in terms of social status for arranged marriages.

DISCUSSION

My exploration of equity in mathematics education for Arab women in Israel has clarified consonance and dissonance with the U.S. equity narrative as outlined in the paper's outset. As in the U.S., one way that the ruling majority in Israel maintains its power is through control and marginalization of its Arab minority. Marginalization yields differential access to a wide range of resources, including mathematics education. Success in mathematics through education is a means of social mobility in both societies, but in Israel, this social mobility for Arab Israeli women seems especially tempered by its routing toward "feminized" professions, like teaching and nursing. State control of curriculum and teacher certification in the U.S. and in Israel make little space for teaching and learning of mathematics to include using mathematics to understand and respond to socio-political injustices.

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The U.S. and Israel are both characterized by segregated schools, although different from the unofficial segregation in U.S. schools, in Israel the separation is both explicit and comprehensive. On the one hand, the comprehensive separation in Israel clearly supports continuation of bi-directional fear and does little for shared understandings. Developing or encouraging socio-political consciousness among mathematics teachers in Israel, in all school sectors, is necessary, to challenge the neoliberal direction in Israeli society oriented around individualism and to instead, cultivate solidarity by improving the teaching of mathematics by connecting it to the lives of students. Arab sector schools in Israel are staffed by Arab Israelis, not necessarily local to the community, but at the minimum are Arabic speakers. This differs from the U.S, where schools in communities of color are typically staffed by white teachers subsumed in "no-excuses" rhetoric or teaching as a kind of missionary work. This particular point of dissonance would seem to highlight possibilities for politicization in mathematics education in Arab sector schools in Israel, because of potential coherence and solidarity across teachers and students in terms of social justice issues.

Girls and women from underserved groups in the U.S. and in Israel face sets of challenges derived from their intersectionality as members of marginalized groups and as women. Yet mathematics, as a space, does not seem to have been masculinized in Arab Israeli society, and women seem to face fewer obstacles around their participation in mathematics. This finding has been noted elsewhere (e.g., Forgasz & Mittelberg, 2008) but needs further exploration. And yet, paradoxically, while mathematics in Arab Israeli society is seen as a place for girls and women to excel, their tendency to excel is encapsulated in restrictions around other kinds of participation and constrictions of movement. Arab Israeli girls outperform Arab boys in mathematics, but this is explained as an outcome of their marginalization in facets of life other than school and mathematics. In the U.S., we often argue for the importance of girls and women having access to mathematics because of how mathematics is used as a gatekeeper in society. In the Arab Israeli context, the role of mathematics as a gatekeeper is far less clear, which suggests that we find ways to add additional significance to or extend opportunities that build on these women's existing success in mathematics.

Cross-cultural explorations are useful in how they illuminate aspects of each culture that might otherwise remain unnoticed. Many readers are likely drawing on this exploration to better understand equity in mathematics education in the U.S. or elsewhere around the globe. A potential implication for the Israeli context are career education interventions in Arab Israeli communities, particularly around careers that draw on girls' excellence in mathematics, so that families and young women can select from among a range of options. However, a different implication, one that does not put the burden on those who are marginalized, would be for workplace interventions in high-tech, business, and scientific sectors to do outreach and create workplace structures and cultures that invite and honor the participation of Arab women.

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NOTES

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A REINTERPRETATION OF OBSTACLES TO TEACHING

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Abstract: Reinterpretation describes a process of combining different theories and integrating findings that have been produced within different research paradigms. The process is illustrated by the theoretical approaches within which the author team situates their research: the subject-scientific approach and the Anthropological Theory of the Didactic (ATD). In taking the subject-scientific approach as a starting point, we reinterpret findings from an ATD-analysis on the teaching of limits of functions on school-level. Obstacles due to the specific organisation of the mathematical knowledge to be taught are examined and related to more general societal conditions. The reinterpretation leads to further questions and an extended perspective from which both theoretical approaches can profit.

INTRODUCTION

In mathematics education research, a huge variety of theories from different disciplines and different research paradigms is employed. We, the team of authors, also have different disciplinary backgrounds. The disciplines of psychology and mathematics shaped our educational biographies.

In addressing the obstacles in teaching and learning mathematics, psychological theories play a prominent role for this specific area of research (e.g., represented by the "International Group for the Psychology of Mathematics Education" (PME)). This strong focus on psychological theories that does not take into account the social, societal and political dimension of teaching and learning is criticised within the mathematics education community (cf. Valero & Zevenberg, 2004). In psychology, the denial of these dimensions has been criticised as well, and here this critique has led to the development of alternative theoretical approaches. The subject-scientific approach is one within several critical psychologies. This theoretical approach originates from a Marxist-oriented critique of psychology and the role of research in human and social sciences and provides categories for analysing and understanding human actions within capitalist societies (Holzkamp, 1985; Markard, 2010; Tolman, 2013). It is certainly not specific for the content-dimension of mathematics, so additional theories have to be consulted. In this paper, we consult the Anthropological Theory of the Didactic (ATD) and its methodology of praxeological analyses. It allows conceptualising mathematical knowledge in a way that takes both the practical part of mathematical activity and the corresponding justifications into account. Here, the praxis and justification of mathematical activities are conceived in their institutional constitution.

To relate the subject-scientific approach with ATD, we propose the specific process of reinterpretation (cf. Hochmuth, in press; Hochmuth & Schreiber, 2015). For this contribution, we decided to take the subject-scientific approach as a starting point to reinterpret an ATD-analysis on the teaching of limits of functions on school-level.

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Obstacles due to the specific organisation of the mathematical knowledge to be taught at school are examined from a subject-specific perspective and are related to more general societal conditions that go beyond the purely mathematical perspective. The reinterpretation process allows us, as a team of authors coming from different backgrounds, to develop a common discourse for collaboration. It should be noted that we granted the subject-scientific approach the privilege of reinterpretation.

THE SUBJECT-SCIENTIFIC APPROACH

A Marxist approach to an educational psychology

The identification and understanding of obstacles that occur in teaching and learning is an important aspect of educational research. This endeavour is shared with the discipline of psychology. Psychological theories, in their self-conception, seek to provide explanations for human actions involved in teaching-learning processes. The German tradition of Critical Psychology offers a specific lens for understanding human actions and has its roots in a Marxist critique of 'bourgeois' psychology [1] (Markard, 2010; Tolman, 2013). Through this critique, the subject-scientific approach has been developed with the aspiration of a psychological research programme that

A. relates psychology to a critique of society.
B. provides a theoretical link between the individual and societal conditions.
C. establishes a psychology from the standpoint of the subject which focuses on meaning-reason-relations instead of following a behaviour-determination paradigm (Markard, 2000; Schraube & Osterkamp, 2013).

(A) The integration of social theory and its critique of society into psychology brings the political dimension to the fore. The categories and theoretical considerations in "Das Kapital" (Marx, 1979/ 1890) are considered relevant for recognising and understanding contemporary societal interrelations that mediate between individual existence and societal (re-)production. The historic-specific moments of production and reproduction are often hidden in structural configurations that are not visible in the immediate appearance of a phenomenon in question [2]. This led to two considerations: 1. The value of academic work was phrased in terms of its emancipatory objective. It was perceived to be essentially possible to overcome restrictive conditions through social action, which requires an active positioning to contradictory societal demands (Holzkamp, 1970). 2. The entanglement of academic work in current power relations needs to be reflected (Markard, 2000). *Objective thought forms* (objektive Gedankenformen) permeate academic thinking . For example, the thought forms of the 'circulation sphere' can be found in educational research [3]. Therefore, academic knowledge production is not neutral and shaped by the current social formation.

(B) In "Die Grundlegung der Psychologie" (Foundation of psychology), Holzkamp (1985) presents an elaboration of analytical categories [4]. Within these basic categories, relations between individual existence and its reproduction in view of societal production relations are reflected. All human actions are perceived to be societal-mediated. The assertions made by these categories refer to the structure of the

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relation between human beings and their life conditions: Human beings are both (re-)producers of and subjected to life conditions and societal demands. This *twofold possibility* (doppelte Möglichkeit) is the central characteristic of all basic categories. Therefore, subjectivity cannot be reduced to the mere intersection of economic conditions. It always entails the (however small) possibility to extend established practices and alter societal conditions.

The analytical core-category *action potence* (Handlungsfähigkeit) [5] describes the individual's opportunities and constraints to act from her or his specific position within societal conditions (Holzkamp, 1986). The category captures the individual and societal level at once. It furthermore follows the basic assumption that in antagonistic class conditions, the attempt to gain more control over conditions is always accompanied by the risk of getting in conflict with the agents of power and provoking restrictions . This is reflected in the analytical split of the category into two alternatives: First, *restricted action potence* (restriktive Handlungsfähigkeit) stands for a modality of alignment with or subjection to given power structures. It describes the safekeeping of one's own action potence at the cost of the (re-)production of restrictive conditions. Second, *generalised action potence* (verallgemeinerte Handlungsfähigkeit) is directed towards extending one's own control over restrictive conditions, and thus entails the possibility of overcoming existing power relations, in alliance with others.

Learning and teaching are specific forms of human actions (Holzkamp, 1995). It is assumed that all human actions are reasonably grounded in one's own specific life conditions. Learning activities are initiated by the attempt of overcoming or regaining control over an action problem. Learning is not necessarily initiated by teaching. The teacher can shape the potential learning object in a specific form, but the individual learner still sets her or his own goals according to her or his perception.

(C) Subject-scientific considerations are essentially given by *meaning-reasoning-relations* [6] (Holzkamp, 1985). The research programme does not aim to classify and evaluate individuals but to understand their activities from their subjective perspective. This subjective perspective is not detached from the subject's life conditions. Conditions are given in the form of societal-mediated meanings that constitute a space of *action possibilities* (Handlungsmöglichkeiten). From one's specific position, only a part of the *generalised societal action possibilities* is available, which is termed *subjective action possibilities* (subjektive Handlungsmöglichkeiten). The reasons for one's actions are based on one's own subjective space [7]. Extending one's space of subjective action possibilities can ensure or even expand one's own action potence.

Research can help to elucidate opportunities and constraints to act in their entanglement within specific conditions and the way those manifest in meaning-reasoning-patterns. An intersubjective understanding is targeted through the discursive form of a reasoning discourse. It is directed towards making human activities understandable, including those that might at first glance seem incomprehensible.

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For understanding human actions in teaching-learning-processes, an adequate reconstruction of *generalised societal action possibilities* is necessary, which also reflect the content dimension of the learning object in question. However, the subject-scientific approach alone does not offer a terminology for analysing the specifics of mathematics. External theories and instruments have to be consulted. It cannot be assumed that any theory or tool offering specific insights into mathematics shares the quite specific constitution of subjectivities and the relation between human beings and society of the subject-scientific approach. In order to integrate theories and findings from a different research paradigm and at the same time retain the political dimension, these have to undergo the process of reinterpretation.

THE ANTHROPOLOGICAL THEORY OF THE DIDACTIC (ATD)

In ATD (Chevallard, 1992; Bosch, 2015) [8], knowledge is theoretically specified via the concept of *praxeology*. This basic epistemological model describes knowledge in the form of two inseparable, interrelated blocks: the praxis block (know-how) consists of *types of problems* or *tasks* and a set of relevant *techniques* used to solve them. The logos block (know-why) consists of a two-levelled reasoning discourse. On the first level, the *technology* describes, justifies and explains the techniques and on the second level the *theory* organises, supports and explains the technology. Praxeologies sharing the same technological discourse could be integrated into a *local organisation*. The integration of different local organisations sharing the same theoretical discourse gives rise to a *regional organisation*. Another assumption of ATD is that institutional conditions constitute the technological-theoretical discourse and the practices available. This assumption has two aspects. First, within ATD, persons are seen as the *subject* of the *institutions* (Douglas, 1986) they enter during their lives [9]. So a subject is not an individual acting freely but

> an emergent of a complex web of institutional subjections. What we call the ‘liberty’ of the person thus appears as the effect obtained by playing off one or more institutional subjections against each other (Chevallard, 1992, p. 147).

Second, in considering one specific piece of mathematical knowledge, the institutional conditions of one specific institution determine the praxeologies involving this piece of knowledge within this institution. So, different institutions give rise to different praxeologies. This institutional dependence of knowledge is at the core of ATD. *The didactic transposition process* (Chevallard, 1985) is a theoretical instrument to explicitly take the process of production, development, diffusion and modification of knowledge through different institutions into account. Regarding teaching and learning of mathematics at school, institutions that should be considered are: the mathematical community where the scholarly mathematical knowledge is produced, the educational system, where decisions about educational processes are made, and the classroom, where the teacher makes choices considering the actual teaching process (Barbé, Bosch, Espinoza & Gascón, 2005). The institutional constraints on knowledge in general also affect the research on teaching and learning processes itself.

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Because researchers in didactics deal with a reality that takes place in social institutions, and because they often participate at these institutions (as researchers, teachers, students, or in several positions at the same time), we need to protect ourselves—to emancipate—from the institutional points of view about this reality, that is, from the common-sense models used to understand it. (Bosch, 2015, p. 52)

Therefore, ATD proposes a "gesture of detachment" (Bosch, 2015, p. 52). This gesture is performed by building an *epistemological reference model* that explicitly states the point of view from which to look at didactic phenomena and whose aspects of the didactic transposition processes are taken into account. The reference model also determines the phenomena that are visible to the researcher and the corresponding conclusions drawn from the analysis. In addition to the concepts of praxeology and the didactic transposition processes, ATD provides another important tool to characterise institutional constraints on different societal levels. The study of these constraints is organised in a hierarchy of *levels of didactic codetermination* (Chevallard, 2002). The scale is structured as follows

Civilization ↔ Society ↔ School ↔ Pedagogy ↔ Discipline ↔ Domain ↔ Sector ↔ Theme ↔ Issue

In total, research questions in ATD concern the institutional level, so the research focus lies beyond the individual [10]. From this institutional point of view, it follows that there is no precise concept of learning in ATD. Teaching and learning processes are modelled externally as a sequence of six different didactic moments that help to create or re-create mathematical praxeologies [11].

Restrictions on the teacher's practice – An ATD analysis of the teaching of limits

To illustrate the theoretical concepts explained above and to specify the material we use for the reinterpretation, this subsection summarises the analysis by Barbé et al. (2005) with a limitation to the following aspects: (1) The epistemic reference model used by Barbé et al. (2005) to analyse (2) the knowledge to be taught about limits of functions and (3) their conclusions from an analysis of the mathematical knowledge actually taught to a class of Spanish secondary school students.

(1) The epistemic reference model focusses on the corresponding scholarly knowledge "*that legitimates the knowledge to be taught*" (Barbé et al., 2005, p. 241). The regional reference mathematical organization (MO) is, besides others, composed of two closely related local MOs (MO_1, MO_2). Both share the same theory of real numbers. Technology elements of MO_1 can be considered as techniques of MO_2. The praxis block of MO_2 is needed to justify the logos block of MO_1.

- In MO_1 (*algebra of limits*), the types of tasks are of the form "Calculate the limit of a function $f(x)$ as $x \to a$, where a is real or $\pm\infty$". The function is supposed to be given by its algebraic expression, so the techniques rest on algebraic manipulations. The technological discourse is composed of algebraic rules such as $\lim\limits_{x \to a}\big(f(x)+g(x)\big) = \lim\limits_{x \to a} f(x) + \lim\limits_{x \to a} g(x)$.

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- In MO_2 (*topology of limits*), the types of tasks are of the form "Show the existence (or non-existence) of the limit of a function $f(x)$ as $x \to a$, where a is real or $\pm\infty$" or "Show the property $\lim_{x\to a}\big(f(x)+g(x)\big) = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$". The necessary techniques involve $\varepsilon - \delta$-inequalities or the consideration of certain convergent sequences. The technological discourse involves the $\varepsilon - \delta$-definition of convergence.

(2) Whereas the reference mathematical organizations MO_1 and MO_2 are strongly related, the knowledge to be taught is composed of disjoint parts of both reference MOs: The curricular tasks and techniques that form the praxis block of the knowledge to be taught correspond to the praxis block of MO_1 but there are no elements corresponding to the logos block of MO_1. This is denoted by a mathematical organisation $MO_1' = [T\ /\ \tau\ /\ \ /\ \ ]$, where T and τ stand for the types of tasks and techniques of MO_1. The blank spaces in the 4-tuple indicate the absent logos block of MO_1 [12]. The technological-theoretical discourse proposed by the curricular documents is made up of parts of MO_2, focussing on the existence of limits without considering corresponding practices. This is denoted by $MO_2' = [\ /\ \ /\ \theta\ /\ \Theta\ ]$, where θ and Θ indicate parts of the technology and theory of MO_2, respectively.

(3) On the basis of the reference model, Barbé et al. (2005) conclude that restrictions on the knowledge actually taught arise. This leads to an incomplete and unstructured organization of the mathematical knowledge. Due to the lack of connection between the $MO's$, the teacher lacks a mathematically adequate justification discourse and the didactic process only focusses on a few of the didactic moments.

> In general terms, we can postulate that if the knowledge to be taught is made of a collection of punctual mathematical organisations that are not linked to each other through an operative technological-theoretical discourse, then the possible corresponding spontaneous didactic organisations that the teacher can use will not be able to really integrate the six different moments of the didactic process (Barbé et al., 2005, p. 261).

REINTERPRETATION

Reinterpretation is a process that can be used for combining different theories and integrating findings that have been produced within different research paradigms. It is not a fixed method but rather shifts the attention to two aspects of the production of scientific knowledge about human beings – including their actions and social relations - as the object of research. These aspects are represented in the differentiation between categorical assumptions and theories. Both point to a different level to the object of research. Categorical assumptions shape the research process and the interpretation of findings. They contain assumptions about subjectivity and human life conditions. Within the subject-scientific approach, these assumptions are explicitly stated by the basic categories. Theories offer a specific understanding of a phenomenon in question. In combination with their used methodology, they always consist (though sometimes implicitly) of categorical assumptions (Markard, 2010).

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One focus of reinterpretation addresses the impact of the specific research formation of the study or theoretical approach in question. A comparison with the basic categories informs the revaluing of the theoretical concepts regarding their content and scope. It aims at identifying implicit presuppositions regarding the constitution of subjectivity and human life conditions within the study in question. This can lead to the revelation of mystifications and curtailments which certainly represents an overlap with other forms of ideology critique . It can also offer a characterisation of the research findings according to their emancipatory potential: What aspects of the relation between human beings and their life conditions are addressed, and what issues could be addressed by human beings to extend their action potence? (Markard, 2005, 2010).

Within another focus, the research outcomes are reformulated from the standpoint of the subject's specific position (e.g., mathematics high-school teacher). It involves a reconstruction of how the human actions observed are reasonable. This involves a certain degree of speculation: What premises need to be assumed to make an empirically observed action an intersubjectively understandable action? What other action possibilities does an individual have within the described conditions and how might these face restrictions? What kinds of actions are suggested by the specific situation without getting in conflict with agents of power? This reformulation explicitly opposes a dislocation of societal contradictions as characteristics of an individual (Markard, 2005, 2010).

In our conviction, reinterpretation is not a one-way affair that works only within the subject-scientific approach. The research in question can profit from further reflection and theory development, as well. However, mutual benefit can only be created if both sides acknowledge the ideas behind the basic categories. This goes along with positioning these ideas to be the reigning interpretative stance. If the categorical assumptions of the subject-scientific approach are rejected, the findings of the reinterpretation process or even the process in general would certainly be considered non-productive.

Reinterpretation of the ATD analysis

ATD offers a clarification of the mathematical knowledge structure to the subject-scientific approach. The praxeological analyses provide a description of mathematical organisations that allows capturing substantial aspects of mathematical actions in a specific institutional context. Mathematical knowledge as such is not taken for granted but is, in principle, conceptualised as institutionally dependent. Reformulated within the categories of the subject-scientific approach, these descriptions can be interpreted as a reconstruction of a specific part of generalised societal action possibilities. Certain action possibilities are highlighted that reflect interrelations between the individual teacher's position and institutional aspects. The specific part elucidated is characterised by the epistemological reference model. The underlying scholarly knowledge perpetuates a certain mathematical perspective that is oriented toward western academic mathematics classifications and values of coherence and consistency.

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Our reinterpretation of the findings addresses two different sides within the subject-scientific research programme: assumptions concerning generalised societal action possibilities in the described educational setting (I), and the teachers' perspective (II).

(I)The empty blocks in the knowledge to be taught limit teacher's action possibilities. They make certain teaching activities (e.g., mathematical justifications for specific techniques) impossible, at least within the contemporary institutional setting. However, the mathematical perspective alone does not constitute action possibilities that are available for the teacher. The idea that teachers' actions should be directed towards complete mathematical organisations mirrors societal demands on mathematics teachers that derive from the sphere of academic mathematics. Interestingly, the voids in the knowledge structure have been identified in curricular documents and actual teaching. Shifting the focus to the (re-)production of knowledge structures as an aspect of the (re-)production of societal conditions, it could be speculated that capitalist schooling generates a specific knowledge structure that shrinks the mathematical logos-block. Pursuing this line of thought questions the neutrality of the content-dimension.

(II) Only under the premise that the teacher strives to extend her or his possibilities of presenting knowledge to be taught in alignment with the reference MOs, the analysis provides the voids that need to be filled from a teacher's perspective. But simply adopting such a reference model that serves research purposes for understanding teacher actions comes with the risks of reproducing a conventional thought form: Academic knowledge sets the standards for extramural practices and teacher's reasoning should be based on this knowledge.

If teachers do not follow teaching actions suggested by research or explained by a theoretical model of teaching activities, it does not mean that the teacher actions are "unreasonable". These teaching actions are just reasonable grounded in other parts of the teacher's space of subjective action possibilities that we as researchers are unable to grasp (yet). To give an example, the teacher in class could unfold that the aspects represented by the two MOs cannot be connected in a coherent manner with school mathematical arguments. We still do not know about meaning-reason-patterns behind such teaching actions. Further questions that consider the teacher's standpoint could deepen the understanding of teaching obstacles: How do the restrictions in generalised societal action possibilities influence the teacher's actions and what are typical meaning-reason-patterns?

DISCUSSION

Reinterpretation offers an opportunity to deal with theoretical approaches that are situated within different research paradigms and to integrate their findings within one's own. Interdisciplinary academic work that stems from different research paradigms could profit from and be guided by a collaborative reinterpretation process. In our concrete example of bringing together ATD and the subject-scientific approach, both theories profited from the perspectives of the other. It has to be acknowledged that both

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approaches already align quite well in their stand against personalising societal aspects and in their acknowledgement of the importance of taking into account the specific institutional context or the specific subject position to understand human actions.

ATD can profit from the categorical assumptions of the subject-scientific approach that hold the potential of elucidating institutional conditions from different levels of codetermination, exceeding the content dimension. This also allows for analysing the interplay of conditions coming from different institutions. Within ATD, the subject is “an emergent of a complex web of institutional subjections” (Chevallard, 1992, p. 147). The subject-scientific perspective could extend this notion of subjectivity. It provides an integration of the subject’s possibilities to alter institutional conditions.

The subject-scientific approach can profit from ATD’s lens on mathematical knowledge structures and explications of its institutional dependence. This further differentiation of the knowledge dimension allows for describing the mediation between societal (re-)production and individuals more precisely.

In general, common standpoints are not an indispensable prerequisite for a subject-scientific reinterpretation of a theoretical approach. Theoretical concepts and research findings produced within a research paradigm that denies the socio-political dimension of mathematics education can be considered, as well. It allows consolidating those findings with one’s own research perspective without having to reproduce a restrictive paradigm that masks societal conditions and personalises “failed” teaching-learning-process. In this case, it offers to maintain a certain positioning of the researcher but not a mutual furthering of theoretical understanding of phenomena within mathematics education. Still, reinterpretation cannot dissolve the power relations it is entangled in itself.

NOTES

1. Psychological research has been criticised for serving the interest of the dominant class (marked by the term ‘bourgeois’). This has been characterised by the production of knowledge that obscures class conditions and societal contradictions by explaining human actions within capitalist conditions as solely being part of human nature.

2. A reduction or disregard of specific aspects – for example the reduction of commodities to their use-value, or the division between private and societal work – lead to an invisibility of certain relations of societal (re-)production.

3. In the thought form of the ‘circulation sphere’, the ideas of freedom and equality are dominating. Marx’s analyses have shown, however, that they are corrupted by an underlying process of exploitation. Education itself is part of societal (re-)production and subjected to this corruption. Educational research (at least partly) reproduces mentioned thought form by highlighting that education serves freedom and equality while neglecting that education could serve exploitation as well.

4. The basic categories were developed in a historical-empirical investigation of general historical-specific characteristics of relations between societal and individual reproduction. The main part of Holzkamp’s analyses considers psychological issues like cognition, emotion, and motivation, taking into account the general situation of social mediateness of individual existence without (explicitly) taking into account the historic-specificity of human societies. Concretisations for capitalistic conditions are indicated and are further informed by theoretical and empirical research of the community.

5. Different translations of ‘Handlungsfähigkeit’ exist. Translations as ‘capacity to act’ or ‘agency’ can be found, as well.

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6. The individual leaner is not transfigured to be fully autonomous, rationally acting on purely cognitive reasoning. The individual's reasoning takes place in the cognitive and affective-motivational sphere.

7. Reasons do not have to be perceived as rational from an external perspective. Furthermore, reasons are often difficult to put into words and may not be accessible for conscious reflection.

8. An overview of research problems and results can be found in Bosch et al. (2011).

9. It is important to note, that this subjection is not understood as a loss of freedom but seen as the "idea of being empowered (both cognitively and practically) through the subjection to institutions" (Bosch, 2015, p. 52f.).

10. The ATD understanding of didactics emerges from this institutional point of view : Didactics (of mathematics) is the science of studying the conditions and possibilities of production, development, change and dissemination of (mathematical) knowledge in society, in persons and in institutions.

11. "This process presents a nonhomogeneous structure and is organized into six distinct moments, […] Each moment has a specific function to fulfil which is essential for a successful completion of the didactic process. These six moments are: the moment of the first encounter, the exploratory moment, the technical moment, the technological–theoretical moment, the institutionalization moment, and the evaluation moment." (Barbé et al., 2005, p. 238)

12. In ATD, praxis- and logos blocks are inseparable and neither of them is empty. There is no praxis without logos and no technological-theoretical discourse without practical elements. The empty spaces in MO_1' and MO_2' indicate that no elements of the corresponding reference MOs were found in the curricular documents.

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STUDENTS' CRITICAL PERCEPTIONS ABOUT MATHEMATICS EDUCATION

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This paper presents excerpts from 10 semi-structured interviews conducted with 8th and 9th grade students in a Norwegian countryside school. Interviews were conducted to gain an insight into students' potential to critically perceive three aspects of mathematics education (ME), namely, subject content, mode of instruction and perceived relevance. Results indicate that pupils hold diverging views regarding these aspects of ME and hence find it difficult to reflect coherently upon them. This reveals students' unfamiliarity with their role as critical thinkers and learners. However, when encouraged, pupils demonstrated the potential to think critically about above-mentioned aspects of ME along with suggesting changes to mathematics teaching they are used to.

INTRODUCTION

The capability to think and reflect critically on one's surroundings, to comprehend one's reality and how one is participating in a particular situation has been an important issue of discussion in the literature concerning education (McPeck, 1981). Critical thinking (CT) is mentioned as a fundamental ability 21st century learners are expected to gain through education, both in Norwegian (Ludvigsen et al., 2014; Ludvigsen et al., 2015; Opplæringsloven, 1998/2018) and international education policies (McComas, 2014; National Research Council (NRC), 2010; Partnership for 21st Century Skills (P21), 2009; Saavedra & Opfer, 2012).

Besides being an aim of education, CT is also considered to be vital for solving mathematical problems, hence being equally important for ME. The subject mathematics, occupying much time in students' education process, should play an important role when it comes to developing pupils' CT skills. To cater for this role, D'Ambrosio (1999) proposes to insert the concept of *matheracy*, with its critical focus, in elementary curriculum. In *matheracy*, comes "the capabilities of drawing conclusions from data, making inferences, proposing hypotheses and drawing conclusions from the results of calculations …" as first steps to develop students' intellectual and critical posture towards use of mathematics within the society. This will help promoting a sense of equity, dignity and democracy within every citizen, using ME in this highly mathematized society (ibid, p. 133). Further, Skovsmose (2014b) suggests the notion of *mathemacy* while discussing the *critical* aspect of ME. The idea of *mathemacy* within Critical Mathematics Education (CME), calls for moving ahead from considering CT in mathematics as a mere toolkit for solving

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existing mathematical tasks (like, reading and interpreting data, drawing conclusions and making inferences), to employing a *critical* outlook towards the role mathematics and ME play both in one's own life and in socio-political issues (Skovsmose, 2014b). *Mathemacy*, is aspired to be the mathematical equivalent of *Literacy* as introduced by Paulo Freire within Critical Pedagogy and language learning (ibid). Therefore, *mathemacy* emphasises *critically analysing* not only mathematical data and information, but also how mathematics and ME plays a part in peoples' personal lives, and in socio-political domains of society. Simultaneously, CME emphasises the need to divert attention towards students' perspectives and interests to map how *mathemacy* can help students to become more critically aware of both mathematics', i.e. the curriculum's, and ME's role in their personal lives and socio-political issues in society.

First section of chapter one, Purpose, Scope and Adaption of Education, of the Norwegian Education Act (NEA), states that, "[Through education] the students and apprentices must learn to *think critically*, and act in an ethically and environmentally conscious way. They *must* have *co-responsibility and the right to influence* [their education]" (italics added) (Opplæringsloven, 1998/2018)[i]. This means that education should help learners develop into critical citizens who are aware of their rights and, are capable of collaborating with each other and taking decisions about their lives independently.

Little research has been done to elucidate student's CT in mathematics (Jablonka, 2014) and if learners can critically assess the ME they receive (Lindenskov, 2010). Jablonka (2014, p. 121), mentions CT being discussed as a "by-product of mathematical learning, as an explicit goal of ME, as a condition for mathematical problem solving…", i.e. as an intellectual tool to learn mathematical skills better. However, she concurrently highlights the "under-theorization" and a lack of explicit mention of CT in ME literature and various programs related to CME (ibid). Moreover, she contends that students' "critical thinking does not automatically emerge as a by-product of any mathematics curriculum but only with a pedagogy that draws on students' contributions and affords processes of reasoning and questioning when students collectively engage in intellectually challenging tasks" (ibid, p. 122). Hence, if learners are, at some point in life, expected to use mathematical knowledge acquired at school, then reflecting critically about what they learn in mathematics, its relevance and how to apply it on situation at hand are skills they need to feel self-confident. The purpose of introducing cognitive processes of CT in education, as stated in NEA is also to help transform pupils into critically competent, co-operative and responsible citizens of tomorrow's global society. Consequently, it was interesting to probe if Norwegian learners possessed the competency to critically comprehend their current mathematics learning and how are they being prepared to lead their lives in near future. Hence, this paper explores the research question, "Can young learners' *critically* perceive three aspects of their ME (*subject content, mode of instruction* and *perceived relevance*) and what are their critical thoughts *about* these aspects?"

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BACKGROUND OF THE PROJECT

This study is a part of a bigger project called Local Culture for Understanding Mathematics and Science (LOCUMS, 2016). The focus of the project is to achieve an insight into learners' outlook on how the use of practical activities rooted in learners' own culture can benefit them in learning mathematics and science at lower secondary school level. Emphasis was laid on *youth culture*. A pre-project questionnaire was designed, having some open and closed ended questions about themes such as: learner's general views about education, perceptions about mathematics and science education, their activities of interest, what they desire to learn about at the school, their thoughts on culture etc. After collecting the questionnaire responses, four common youth interests within the class were selected and students were grouped into four or five groups. Each group got a practical task to do, where they had to apply the knowledge of mathematics and/or science. Following the project, semi-structured interviews were conducted with selected learners to get a vision of the experience they had that day, their general outlook towards learning mathematics and science, the extent of activities used in the mathematics and science teaching they receive and its relevance. Students' responses on questions *about* their ME collected in first 10 semi-structured interviews (from abundant amount of data material collected for the whole project) form a basis for data analysis and discussion in this paper.

THEORETICAL FRAMEWORK

Critical Mathematics Education (CME)

CME aims to address mathematics *critically* in all its forms and application. Such an open statement allows the research in CME to flourish in many possible directions since no limits have been specified regarding *who* should be critical and *about what* in relation to mathematics and ME. Among other things, scope of CME includes both the concern for enhancing learners' autonomy (Skovsmose, 2014a), and the need to acknowledge students' interests, and the requirement for adopting a critical stance towards ME. To cater for learners' interests, hopes, aspirations and motives in ME, Skovsmose introduced the notion of *students' foregrounds* as, "the opportunities which the social, political, economic and cultural situation provides for the person" (Skovsmose, 2011, pp. 21-22). Further, the definition of a *foreground* was extended to include as well "… the person's experiences of possibilities and obstructions". In addition, Skovmose states that "It is a preoccupation of critical mathematics education to acknowledge the *variety* of students' foregrounds and *to develop a mathematics education that might provide new possibilities for the students*." (Skovsmose, 2014a, p. 117) (italics added). CME emphasizes students' interests and urges to provide learners with a chance to *evaluate* what is happening in their mathematics classroom (Skovsmose & Nielsen, 1996). Therefore, in this paper we provided young students with a new possibility to *critically* reflect upon three aspects of their ME and expect to better understand students' *[critical]* foregrounds. Adding the word *critical* to the

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concept of students' foregrounds is in alignment with our intention in this paper. By students' *critical* foregrounds, we mean *students' capability to understand, reason for, critically reflect upon, suggest alternatives and also criticize* the *subject content*, *mode of instruction* and *perceived relevance* of their ME and its significance in their own lives. We believe that like other types of students' foregrounds, the capacity to critically perceive their ME can reveal interesting information regarding students' hopes, expectations and motives of studying mathematics.

There have been studies discussing the importance of CT and reflection as an intellectual tool to better learn mathematical skills, solve mathematical problems (Wheatley, 1992), and in inquiry and dialogue based mathematics learning (Alrø & Skovsmose, 2002). These investigations have also mentioned a positive relation between learners' potential to critically analyse their mathematics learning situations and their ability to learn mathematics as a consequence. However, we found relatively few studies asking learners themselves for their *critical* perspectives *about* different aspects of their ME. Some of the studies making such attempts include, Gebremichael (2014), Kacerja (2012), Kollosche (2017) and Lindenskov (2010), however, enquiring if students can *critically perceive* their ME was not their main focus. Since exploring students' interests (as in CME) was driving force for this project, we provided learners with opportunities to express their *critical* foregrounds about three aspects of their ME.

METHOD

Semi-Structured interviews

Interviewing is considered to be an appropriate method for collecting qualitative data, where the interviewer engages in a conversation with the interviewee, keeping in mind the intention "to elicit views and opinions of the participants" (Creswell, 2014, p. 190). We wanted to get informed regarding the above-mentioned research question(s), in line with my observations, through the help of individual opinions, perceptions and thoughts of the learners participating in the study, interviewing turned out to be the optimal way to proceed. However, not all types of interviews allow the freedom to delve deeply and engage in a flexible individual-centred conversation with the interviewees based on researcher's observations, we chose semi-structured interviews. One learner per group was chosen based on the level of their activity on the project day. Keeping in mind the principal of representativeness, attempt was made to select students with different interests, level of activity (high, medium and low), cultural backgrounds and achievement level in mathematics (high, moderate and low achievers). Ethical concerns were taken care of by applying for permission to collect personal data to the Norwegian Center for Research Data (Norsk senter for forskningsdata, NSD).

Preliminary analysis of pre-project questionnaires served as the basis to design interview questions, where students' conflicting opinions to statements concerning study of mathematics and science disciplines were observed. For instance, students

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agreeing to the statement, "studying mathematics and science is important for them as it will improve their career opportunities", but simultaneously disagreeing that "I am interested in what I learn in mathematics and science" is an example of diverging viewpoints. Such contrasting opinions triggered the curiosity of authors to dig deeper into students' *critical* understanding *about subject content*, *mode of instruction* and *perceived relevance* of ME. Therefore, interview was designed to know learners' opinions and experiences regarding these aspects and how they observe their coherence in their lives. Since ME is a domain consisting wide and complex range of curriculum issues, teaching-learning theories, strategies, education policies etc., it was important for us to focus on selected aspects of ME we could enquire about from learners. Therefore, we chose these three aspects to concentrate on things concerning ME which learners get a *daily interaction* with in their mathematics classrooms and personal lives.

INTERPRETATION OF DATA

The interviews were conducted in Norwegian, translated to English and interview transcripts analysed to identify learners' critical perceptions about ME. Following is the descriptive analysis of learners' critical perceptions with respect to the – *subject content of mathematics, mode of instruction in mathematics* and *perceived relevance of classroom mathematics in their everyday lives/real-life situations* from the interviews with first 10 informants. These excerpts also illuminate contradictions observed in questionnaire responses where learners despite finding mathematics uninteresting and not-so-relevant to learn for their personal lives, cannot give up the idea that it is important to learn.

Learners' critical perceptions towards the subject content of mathematics:

Thinking critically about the subject they learn at school may be difficult for the learners if this is not a part of their 'didactical contract' (Brousseau, 2002). However, when stimulated to think critically some of the learners (four out of 10) said that *not everything* they learn about mathematics at the school is important but they had difficulties in articulating particular examples of such subject content.

I (Interviewer):	Is it something in the subject mathematics you think is in vain/waste of time to learn?
S_2 ($Student_{Number}$):	no…I don't think so actually…you are obliged to know little mathematics to manage yourself and to do something…

Here S_2, regards learning mathematics at school as an obligation and hence finds it difficult to imagine that anything in mathematics, a subject that occupies so much of school's time can be in vain. There have been several other examples in the interviews where we could observe that though learners face difficulties in identifying relevance of mathematics' subject matter for real-lives (see perceptions on relevance), they cannot imagine that this 'stuff' is useless for them. Such responses were six out of 10 in number indicating that keeping complete *trust* in importance of school curriculum can hinder the development of learners' critical perceptions regarding it. Similarly, in

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the following extract, learner struggles to understand the rationale for cramming all the textbook formulas when she/he has the technology at hand to get help with that.

I: is it something in mathematics which you think is in vain to learn?

S_5: eh… to cram all the formulas and like how to find volume of geometrical figures…it's completely…eh… like it is useful but it's like… one needs not to learn all the formulas because now we have smartphones and we can look it up for them if we need …that you use one week or two just to teach us the formulas when we can use that time to learn other things which are more important …

This shows that, given an opportunity to have their voice heard, these learners can express valid opinions about what they think is important to learn in the mathematics classroom, and can even explain their choice(s). This supports the argument made by Lindenskov (2010) that children have the potential to make decisions *about student curriculum* and also have set objectives they want to attain through their choices.

Learners' critical perceptions towards the mode of instruction in mathematics:

Regarding the mode of instruction in ME, it was not hard to recognize that the learners do not have an optimal experience in mathematics classes. Most of them describe the mathematics classes to be *long, difficult* and *quite boring*. But in addition to being difficult or boring, some of them also mention the lack of practical activities as a factor influencing their experiences in mathematics class.

I: for example, you said that Gym is what you like the best to do…but what would you say is the difference between that you do in gym and that you do in mathematics hours which makes you say that you don't understand everything…

S_1: Eh…like in gym then I can kind of…it's like the effort which counts very much and so…eh…that I don't use or one has to use his/her brain a bit but it is very much physical also…and eh…if you understand…

I: yeah… yeah…okay… so you mean to say that it's not so much physical work which you get in mathematics….?

S_1: yeah…one must think a lot!

Here the learner expresses the lack of making any physical effort in mathematics class as a reason of not liking mathematics as a subject. This section also raises another major issue for discussion, i.e., in mathematics, often, reaching the correct answer counts as an effort to solve mathematical problems, whilst any other attempts at a solution are not rewarded. This dissatisfaction brings forward both, the way mathematics is taught, where no physical activities are included, and the way it is assessed, no other effort than arriving at the one right solution is evaluated and rewarded. Similar opinions are also expressed by other learners when asked explicitly about the issue of having practical activities as a part of mathematics class.

I: How would you describe the situation in the classroom when it comes to mathematics and science teaching?

S_8: … but in mathematics it's not a lot… it's just calculations…

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I: but if you had a chance…had you changed the teaching in mathematics and science?

S_4: no…

I: nothing?

S_4: maybe a bit more [practical] activities in mathematics but otherwise I don't think so of anything…

These episodes illustrate learners' critiques of traditional teaching (Yackel & Cobb, 1996), task/exercise-based instruction (Skovsmose, 2001, 2011) or conventional explain-practice methods (Wheatley, 1992) of teaching mathematics, as we like to call it. Simultaneously, after some encouragement, *S_4's* reply in these excerpts favour a more practical, team-oriented and dialogical approach (Alrø & Skovsmose, 2002) to learn mathematics. Students' replies indicate that it would be better to improve their interest in learning mathematics and satisfaction with mathematics classes.

Learners' critical perceptions towards the perceived relevance of classroom mathematics in their everyday lives:

This section is more a reflection on the lack of learners' potential for thinking critically about the relevance of mathematics, rather than a manifestation of it, emphasizing simultaneously the necessity of imparting this capacity to them. With respect to the perceived relevance of mathematics, it seems that learners, despite of being uninterested in learning mathematics, think of it as a subject of immense importance. Moreover, the much-discussed importance of mathematics in the media, and by the adults surrounding these pupils, contributes to this conflict and it's 'what they *say'* that wins the game instead of *experienced relevance*. Such influences can also act as barriers to learner's attempts to think critically about relevance of mathematics in their personal lives.

I: but do you think that what you learn about mathematics and science would be important for you later in your life?

S_2: yea…they say like that at least…

I: who?

S_2: the teachers…they say that at least…

I: yea…but do you think it yourself? You should answer for yourself…

S_2: yea…I maybe think that too…like if you will have children then you can help them with their studies at home…

I: yea…but like in everyday life like…in real situations…where are you going to see mathematics and science actually?

S_2: eh…it's difficult! (pause…)

I: it was a nice example you gave that you can help your children with their homework and so but otherwise?

S_2: no…I don't know… no…<u>I have not thought a lot about it…</u>

In this context, students' responses echo Kollosche (2017, p. 637) in stating that, "This restriction of the relevance of mathematics to elementary skills after the assertion of a

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general relevance of mathematics appears frequently in the interviews of the students". This restriction of relevance of mathematics to elementary skills is a critique of the kind of ME in which pupils cannot recognize real-life aspects of mathematics pointing out simultaneously underdeveloped CT skills of students to reason *about* ME.

CONCLUSION

The ability to think critically is a skill of vital importance to be imparted in responsible future citizens through education. Based on this study, we argue that learners can think critically *about* their ME, but these skills are neither demanded of them in mathematics classrooms, nor are they cultivated for their further development in future. Hence, training for these skills does not seem to be a part of mathematics classroom culture. Based on the interviews, it can be observed that these processes usually take place implicitly and individually within each learner and therefore are easily forgotten due to the lack of attention they receive in mathematics teaching-learning. Consequently, one can observe the incoherence between learners' critical perceptions regarding subject content and perceived relevance of their ME. Therefore, even if there is no direct recipe for developing critical learners, it does not mean that we should refrain from beginning this process in schools. This study suggests that students *possess* the potential to critically *evaluate* and recommend modifications in their mathematics classroom, provided they are given the opportunity, time, space to do so, are heard and encouraged to participate in discussions regarding their ME in accordance with the NEA. CT, hence, cannot be thought of as emerging spontaneously within learners as a result of education they receive, but needs to be explicitly prompted and nurtured in the classroom, and incorporated in school culture Jablonka (2014).

Observing learners' potential to suggest, cooperate and improve ME, it does not seem sufficient to teach them mathematics so that they can understand it in a better way (project-based, inquiry-based, problem-solving etc.) (Cobb & Yackel, 1998; Wheatley, 1992). In addition, this study suggests that it can be helpful to ask the learners and listen to them, so they can co-operate and share the responsibility of developing learner-centred approaches for learning mathematics together with their teachers. Accordingly, learners could receive central attention and learn to *be persistently critical* (Little, 2003) about *what*, *why* and *how* they are receiving ME to become critical, cooperative and responsible mathematics learners.

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[i] In Norwegian, "Elevane og lærlingane skal lære å tenkje kritisk og handle etisk og miljøbevisst. Dei skal ha medansvar og rett til medverknad" (Opplæringsloven, 1998/2018).

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THE ROLE MATHEMATICAL THINKING PLAYS IN MASONS' WORKPLACE IN WEST BENGAL: A PRELIMINARY EXAMINATION

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This paper investigates the pivotal role played by mathematical ideas behind masons' profession in India, where formal learning for this profession is practically absent. How informal learning of mathematics occurs and how cognition comes into action, are some of the important aspects of this study. The theoretical framework relies on Ethnomathematics as these mathematical ideas are practiced among identifiable laborer groups and can be regarded as essential component of socio-cultural knowledge. The objective of this work-in-progress is to recommend strong critical inputs into studying adult learning processes and to school mathematics education curricula especially in third world countries.

রাজমিস্ত্রি পেশাবর্গটির সাথে সংযুক্ত মানুষেরা কর্মক্ষেত্রে কীভাবে গাণিতিক ধারণা কাজে লাগান তা ভারতবর্ষের প্রেক্ষিতে অনুসন্ধান করার চেষ্টা করবে বর্তমান গবেষণাপত্রটি। এদেশে তাঁদের কোনো প্রাতিষ্ঠানিক শিক্ষালাভের ব্যবস্থা প্রায় নেই বললেই চলে। তাঁরা কীভাবে নানান অপ্রাতিষ্ঠানিক কায়দায় গণিত শেখেন আর তাঁদের চেতনা (cognition) এক্ষেত্রে কীভাবে কাজে আসে— মূলত এই দুটি দিক আমরা বিচার করতে চাই। গবেষণাটির তাত্ত্বিক প্রেক্ষিত হল এথনোম্যাথেমেটিক্স কারণ এক্ষেত্রে এই গাণিতিক ধারণাগুলির প্রয়োগ ঘটাচ্ছেন নির্দিষ্ট একধরণের পেশার সাথে যুক্ত শ্রমিকদের দল এবং এগুলি সমাজ-সাংস্কৃতিক জ্ঞানভাণ্ডারের অপরিহার্য অংশ। চলমান এই অনুসন্ধানমূলক গবেষণাটির লক্ষ্য বয়স্কদের শিখণপদ্ধতি পর্যবেক্ষণ এবং স্কুলের গণিত শিক্ষার পাঠক্রমে কিছু গুরুত্বপূর্ণ সমালোচনামূলক সংযোজন।

INTRODUCTION

Recently we have undertaken studying the work of a group of five masons in semi-urban West Bengal, India. These men are Bangla speakers, live and work locally, and are experts in the masonry of reinforced concrete works— such as construction of pillars, beams, roofs and any other part of building that needs reinforced concretization. In a construction site the reinforced concrete foundation is first prepared by construction workers who install vertical pillars, or columns, to the ceiling level. Next, another group of construction workers, the bricklayers, start their work of filling the concrete structure of the brick walls and when the time for the reinforced concrete roof arrives, the same initial group of masons is again engaged. After the completion of the first story of a building, similar collaboration among the masons who are experts in reinforced concrete works and the bricklayers follows to build the second story.

Our goal for this paper is to investigate the mathematical ideas and techniques that are embedded in the process of masonry and also to decipher the individual perspectives of the masons in each step of their actions.

The people with whom we worked are mostly 'illiterate' from the point of view of formal education; those few who had schooling had to abandon it at a very early age due to their financial constraints. Yet their mathematical ideas and the application of those in the construction work made various structures stand for a long time, defying

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cyclones, rains, floods, droughts, and tropical humidity. Therefore, their informal learning in terms of everyday problem solving, and teaching their novice co-workers, must have played a very important role in the process of becoming successful in their trade. One of our tasks in this project is to analyse this learning process which may help to make alternative curricula in the field of school education, especially mathematics education, and also open up a new methodology for studying adult learning processes. In depth studies of the mathematical ideas and the informal learning process of the masons in West Bengal in the cultural context are pertinent for comprehensive research in this field. Inadequate historical data and texts regarding masons or their working techniques and perspectives, particularly in respect to the old historical monuments and structures (like the famous Bengal Terracotta) [1], compels us to go for personal interactions with the few remaining masons or their offspring in the districts of Murshidabad, Malda, Bankura, Hooghly, Cooch Behar— the seats of old Bengal rulers. Further work that will deepen our knowledge is required but is beyond the scope of the present paper.

The primary impetus of this study may be credited to our interest regarding the unrecognized and uncategorized edifice of knowledge carried on by many people who successfully implement their skills— beginning from the tiny kitchen run by our mothers to the masons, the blacksmiths, the carpenters around us, or the native boat-builders (*Nouka Karigar*) (see Mukhopadhyay, 2013). These people are skilled without any training and not aware of 'academic mathematics'. Secondly, an unfortunate lack of literature recognizing this edifice of social knowledge in the context of Indian subcontinent makes us think critically given the studies already done on different continents in exploring Ethnomathematics (for an overview, see Mukhopadhyay & Greer, 2012). Our study may gradually expose some areas of understanding that could counteract or even endorse the ideas of contemporary western epistemology.

This study attempts to recognize the unconventional knowledge resources embodied in this community of workers so that they can live with dignity and experience a boost in socio-economic and socio-political status to a more equitable standard.

THEORETICAL FRAMEWORK

The theoretical perspective for this study relies on recognition of the mathematical ideas and practices of masons as professionals in their everyday lives. This means recognizing their cognitive abilities, in contrast to the Mathematical (with 'capital-M', as stated by Barton (1996)) system. The latter system actually has been developed with a particular direction and aristocracy to prepare the elite and allows the elite to assume effective management of the productive sector (D'Ambrosio, 1985). This makes our attempt a political one and places the study in the theoretical framework of what is called Ethnomathematics (D'Ambrosio, 1985).

This framework is more relevant in the third world countries with a long history of colonizers expropriating the century old wisdom of native cultural/occupational

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groups with a suppression of mathematical knowledge and practices and the imposition of those of the colonizers (Mukhopadhyay & Greer, 2012). This same thing happened to the traditional masons in India with the advent of the British colonizers and the western perspective of education. During the last half of the 19th century, a new breed of technocrats called 'Civil Engineers' emerged as products of newly found technology institutions in India (Sanyal, 1994, pp. 1-7), and the legacy of the masons representing builders of the awe inspiring Taj Mahal got lost with it.

In coherence with this concept of Ethnomathematics, at first we need to look into the very root of the definitions of mathematics, which despite undergoing changes over the years, remained to have maintained the western perspective only. "To avoid being constrained by this western connotation of the word mathematics, we speak, instead," as reminded by Marcia Ascher (1991), "about mathematical ideas. The particular ideas, the way they are expressed, the context and ideational complex of which they are a part, vary depending on the culture" (p. 3). We approach, with an intention to place our findings regarding the mathematical ideas used by the masons, in a model proposed by Alan J. Bishop (1988). He describes mathematics as a cultural product that emerged from humans engaging in six universal activities (counting, locating, measuring, designing, playing, and explaining) in a sustained and conscious manner.

However, while studying these activities, our personal observer subjectivity will be involved. We are accustomed only to the 'capital-M' Mathematics and this cannot be averted without attending to the spontaneous responses and interpretations of the people involved in these activities. Nonetheless, we possess a willingness "... to redefine the working concepts (in this case 'mathematics') in response to the perspectives of the culture(s) being considered" (Barton, 1996, p. 1037), keeping us on a righteous path.

By the phrase 'masons of Bengal' we refer to a 'cultural group' where the concept of culture is derived from the definitive consideration of D'Ambrosio, as mentioned by Barton (1996, p. 1036):

> His [*D'Ambrosio's*] definition [*of culture/cultural group*] … is specific to mathematics: a cultural group is a group which has developed practices, knowledge, jargons and codes (in particular to encompass the way they mathematise) (D'Ambrosio, 1984).

Cognition also deserves a special attention as a part of the theoretical perspective because it plays a unique role in giving the activities successfully transformed into a final material-cultural shape of the work.

METHODOLOGY

The fieldwork for this research was conducted by the first author and reported solely (in first person) from his observations. Generally speaking, the masons in West Bengal are historically from a background of not only a vast diversity in terms of gender, caste, race, religion, language and economic level but often are victims of various forms of discrimination. All being males and being able to speak the same

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dialect of Bangla, they made my work easier in the first step. They worked earlier in a construction project for a friend, Saifuddin Ahmed, who, despite having no formal education in civil engineering, has a deep interest and knowledge in masonry works. He acted as a link between the masons and myself, which eased my study.

I progressed through participant observation and conducted informal interviews (all in Bangla). My appearance of being ‘well-educated’, ‘modern’, ‘economically stable’ with a digital camera and a smartphone in hand, might have created a distance within this group (which perhaps acted as a reminder of their own financial conditions), but may have also been mitigated through an intimacy with them in various ways. During my first field visit in the first week of March, 2018, at two places named Khanyan and Boinchi in Hooghly district, I took part in their personal conversations, jokes and their woes and worries regarding their families. The national and/or state level issues regarding different political parties, recent government policies like demonetization [2], GST (Goods and Services Tax), and their own personal experiences about those, also figured in our discourse.

An existing hierarchy among these masons at their workplace could be sensed from different levels of their daily wages [3]. Shankar Malo (age: 34), coming from a fishing community (‘*Dalit*’; scheduled caste [4]), who never joined his ancestral profession; and Bablu Soren (age: 40), from the scheduled tribe *Santhal* [4], together lead the team. Second-in-command Gopal (age: 52), also from a scheduled caste [4], joined the group following Shankar’s invitation, leaving the job as a fish-seller at a considerably later age. Madhu (age: 30), another *Santhal*, came through his contact with Bablu to work as an assistant along with Kishore (age not known). Kishore is from the Hindi-speaking state of Bihar but is able to speak Bangla. They wanted their stories told in their own terms using their real names too.

The purpose of my visit was yet to be clear to them and this matured during my last visit in the latter half of April. Having an occasional acquaintance with some institutional or state-level workshops, they initially mistook me to be an engineering student. Ultimately I apprised them of the actual purpose and nature of my work; my studies about their unique ways of understanding and applying mathematical ideas, thereby finding a means to help make curricula for mathematics education in a new way. Inspired by my appraisal and perhaps realising that this work was meant for the good of mankind, they not only promised to help me, but also helped me with their unique explanation to an onlooker who was inquisitive about the nature of my role at the site. They said:

> “Ask an illiterate beggar to return the balance of a 20 rupee note keeping 5 rupees for herself. She returns exactly 15 after her own way of counting. This gentleman here intends to study something like this— finding [*the perspective*] from our way of work”.

The entire course of my work aims to keep attentive to the mason’s perspective of their own practices and understanding in their field of action, as is argued by Barton (1996, p. 1040).

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MATHEMATICAL IDEAS AND TECHNIQUES IN MASON'S WORK

In this section, I will address some of the key cognitive issues of masons regarding mathematical thinking and techniques, which are pertinent here. See endnote [5] for some of the unedited videos directly from the field.

Numeracy

Numbers play a vital role in this profession and most of the workers are fluent in computing addition and subtraction of two or three-digit integers mentally. Additionally, Shankar is expert in multiplying mentally and he has to take the main responsibility for the numerical calculations required in the construction work. Regarding leadership at work, I noticed that those who belong to the higher levels of the occupation get to do more complex numerical operations. Yet perhaps it is this computational fluency that determines one's position in the hierarchy? Such an instance follows—

Eleven vertical columns of reinforced concrete, each having two parts, four feet below the ground level and around 10 feet above it, have to be made for the foundation in a building. Each column needs four steel rods (of diameter 16mm) run through it in such a manner that they constitute four vertices of a square (of sides 170mm) inscribed within, and keeping sides parallel with the sides of, the bigger square i.e. the cross section (250mm x 250mm) of the vertical concrete column (**Fig 1**). Square iron rings (dimension 170mm x 170mm, diameter eight mm) are prepared manually to put 150mm apart from each other, around the vertical rods to maintain them in a square position (**INSET 1**, **Fig 1**). The ends of the four vertical rods, bent in 'L' shape, are connected to a square grid (made of 16 still rods of same length and of diameter 10mm) at the base of a pit dug (**INSET 1**, **Fig 5**). From these preliminary

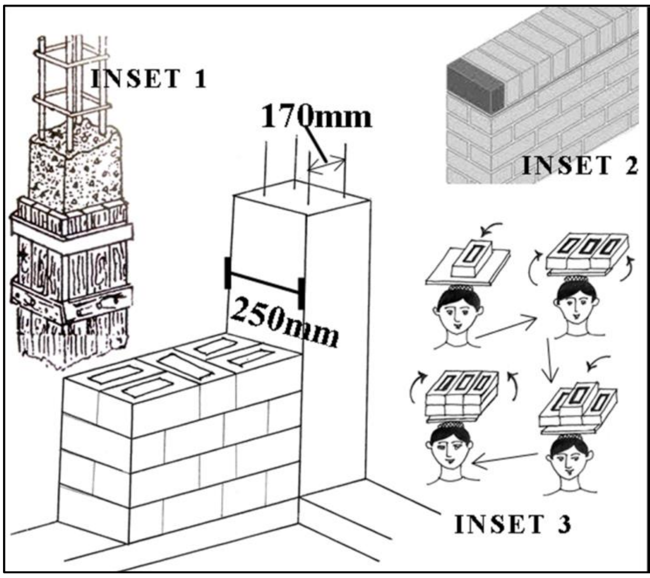

Figure 1

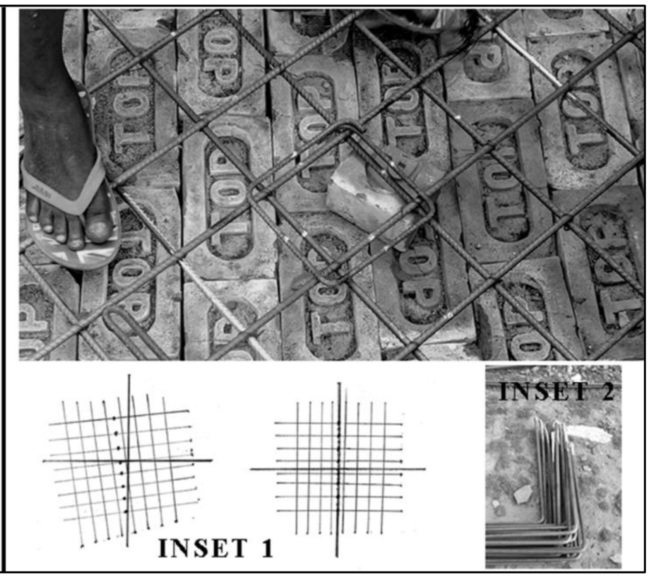

Figure 2

data, keeping in mind that the standard industrial length of any rod is 12m, Shankar takes only one-two minutes to calculate in his head how many pieces of rods of

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different diameters are needed in total to construct these 11 columns. Similar computations are done for the reinforced ceiling.

Although specialized mostly in reinforced concrete, this group often must take on other occasional work-related emergencies such as bricklaying. The standard bricks in West Bengal are approximately 10in x 5in x 3in after the mortar is applied. The walls are mostly 10in and 5in in breadth. One of the most commonly followed brick-laying patterns is shown in **Fig 1**. Below is an excerpt from a conversation between me and Bablu whilst he lays bricks to make a wall of 10 inches width:

Me: About how many bricks are needed for a wall of particular dimension if this pattern is followed? What if the length is not a multiple of 10in or 5 in?

Bablu: First I calculate the number of bricks needed for the first layer and then I multiply that with the total number of layers needed. When the length of a wall is not a multiple of ten or five, we divide and use a brick's ¼ or ½ or ¾ portion cut across the width [6], and make up the rest by applying mortar. If height becomes a "problem", that is, not a multiple of three, this has to be managed by laying the bricks on their sides to make up the required height [*usual 3in thickness is augmented to 5in in this case*] (**INSET 2**, **Fig 1**).

To weigh a steel rod of a particular diameter, Shankar squares the diameter (in mm) of a rod and he divides the resulting number by 162 using the calculator in his mobile phone. What he gets is the weight (in kg) of 1m length of the respective rod. Considering the density of any steel rod being a particular constant, a number very close to 162 is easily obtained from the formula for the volume of a cylindrical solid and using the fact that weight = volume x density. Shankar says, "I don't know why, but at stores, simultaneously weighing the rods in the weighing machine and using this formula, I've got approximately the same results. There must be some mathematics behind this."

The masons are fluent in using a combined system of units in metric as well as imperial. **Table 1** depicts their conversion rules.

Serial No.	Length in inches	Corresponding length in 'mm' that masons use	Standard international length in 'mm'	Error in 'mm'
1	1	25	25.4	0.4
2	1.5	40	38.1	1.9
3	2	50	50.8	0.8
4	10	250	254	4
5	12 (1 ft.)	300	304.8	4.8

Table 1: Conversion rule used by the masons

Use of imperial measurements are evident since the colonial era. Metric is used in accordance with the current market standards. Justifying use of millimetres, Shankar

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says, “While measuring, going wrong by 1-1½ inches matters a lot. But 2-5mm? That doesn’t matter at all”. It is evident that the errors within the $\pm$5mm limit seem negligible to them [7].

Placing small squares at perfect centre of bigger squares

This job is done twice per column. They have to place the square grid at the dead centre of the pit, four sides of the grid maintaining a gap of two inches equally from the pit wall. Then a square ring of dimension 170mm x 170mm is placed and fixed exactly on the middle of the grid with its four vertices providing points for the attachment of the four vertical rods with their ‘L’ shaped ends. The accuracy of putting the grid at the central position of the pit is maintained only with the help of threads and plumb-ball [5]. Two threads (generally called ‘centre-line threads’), horizontal to the ground and perpendicular to each other, are attached slightly above the ground level by pegs, the intersecting point of which gives a virtual position of the exact central point of the square pit (**Fig 4**). Then the midpoints of the four outer most grid-rods are chalk-marked with the help of an ordinary metre tape. The midpoints of the remaining rods also are marked accordingly. The plumb-ball thread is hung down to the grid level, touching the centre-line-threads laid above. To ascertain the required grid position, the grid is manually adjusted by moving it to match all the four outer-rod-midpoints with the plumb-ball tip (**Fig 4**). The second job is done by marking the midpoints of the sides of a ring and placing it in such a way that the two opposite sides’ midpoints of the ring come in a straightline with the

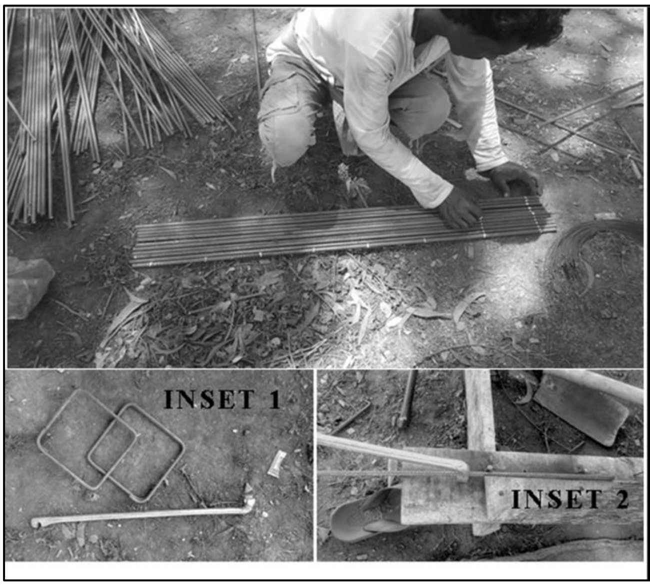

Figure 3

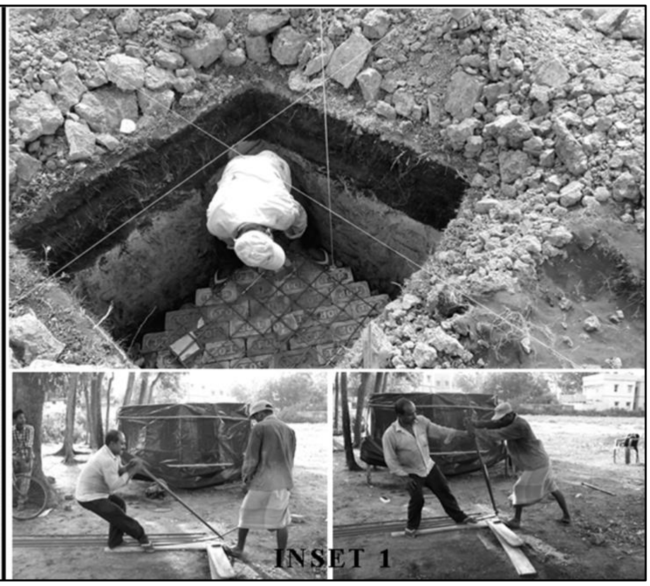

Figure 4

pre-marked midpoints of the grid-rods (**Fig 2**).

Using of templates

While making a grid, at first Shankar gives a grid-rod some chalk-marks that are at a fixed distance apart from each other. Using that marked rod (as a template) Kishore then marks the others (**Fig 3**) and then 16 marked-rods are tied to each other keeping

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those marked points one above another, to make one grid [5]. A temporary table with wooden planks is prepared where a small piece of rod is pierced through a plank into the earth and an iron spike is nailed on it at such a distance from that rod that depicts the sides (170mm) of the square ring. Using this table and a lever-like tool (colloquially 'dice'), rings are made [5] one after another (**INSET 1 & 2**, **Fig 3**).

Combinatorics

After the completion of the skeleton (as shown in **INSET 1**, **Fig 5**), the next stage is that of making temporary molds with wooden planks to put around these skeletons for shuttering for pouring concrete for casting. This step is of much importance since the cross section of this mold decides the ultimate cross section of the reinforced columns (**INSET 1**, **Fig 1**). Inside these molds, concrete is poured. The team has in possession a pile of wooden planks of varying sizes and particularly the breadths of them varying from 3in to 6in. To make a column of cross section 12in x 12in, four shutters of 14in each in breadth are required to be made. It is because a rebate of 2in per plank is necessary for fixing the planks. Out of the available stock of planks among 3in, 4in, 5in, 6in breadths, to make a 14in shutter, they attach wooden planks sideways along their varying breadth, generally choosing one of the following equations: 5 + 5 + 4 = 14; 5 + 3 + 3 + 3 = 14; 6 + 4 + 4 = 14. These options are exercised by the spontaneous decision of the mason in charge [5] (**INSET 1**, **Fig 6**).

MATHEMATICS OF THE SUBALTERN

In this section, I describe, in Ethnomathematical terms, the mathematical ideas of the masons as expressed in their own terms. These ideas, and the explanations of the formulations by the masons themselves, perhaps intersect some portion of the ideas usually described by 'cognition'. As an outsider to their profession and with formal knowledge of mathematics, I became a learner of mathematics from the so-called uneducated masons. This is what I refer to as mathematics of the 'subaltern' [8].

Time

Depending on the distance between home and the worksite and the means of transport, the masons have to make an itinerary which involves a balance among the time in transport, carrying out the job and balancing the degree of labor involved in the job itself. In most of the cases, while working, their time management is done without the help of any wristwatch as it hinders this type of job.

Pattern

Patterns are displayed in various activities, such as knitting wire [5], in adjusting the gap between iron rings for the columns/beams, in laying bricks for the walls [9], as well as in the layer below the foundation [9], in arranging bricks on head to carry it around [5] (**INSET 3**, **Fig 1**), in carrying the fabricated components (**Fig 5**), and in adding finishing touches to designing the sun sheds. These patterns are actually followed with a view to ensuring the factors like shape/size, carrying capacity, load

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distribution, longevity, physical/material safety and above all the aesthetic appeal of the finished structure.

Volume and Weight

Although in preparing concrete mixture for casting reinforced components or the cement mortar for plastering walls, the prescribed method follows the notion of volume as the norm, the masons, in contrast, seem to depend on concepts of both volume and weight or perhaps a complex entanglement of both. Kishore, entrusted to manually compile the concrete mixture consisting of sand+cement+stone-chips or cement mortar consisting of sand+cement, decides the required proportions using his cognitive ability of calculating (more appropriately, 'sensing') weight and volume by bare eyes and hands. Bablu claims, often with certainty, to guess the weight of a piece of rod simply by handling it. Those who carry the bricks on their head from the pile to the site, have to have an embodied cognition to accommodate the combined factors of distance, the laden weight carried, distribution of weight (evident from the number and pattern of arranging the bricks on head [5] (**INSET 3**, **Fig 1**)) and time required for the task. An intensive study of these scenarios is required in a multiple setup, but is not yet done in the Indian context.

Arithmetic and geometry

Cognitive efficiency of these types can be traced in their manually parting and using the quartered (¼, ½, ¾) bricks with a bare-eyed estimation and in bending the rods in right angles [5] (**INSET 1**, **Fig 4**) relying only on visual cues. Shankar justifies his understanding of a right angle by putting himself on a point on the straight line joining myself and a piece of brick put apart, and then moving at a right angle from that position, without realizing the 'geometrical explanation' behind it. To maintain the central position of the grid inside the pit, Shankar stands with his legs apart just above one centre-line-thread and visually verifies whether the chalk-markings of the midpoints of the grid-rods are in line with that thread [10] (**INSET 1**, **Fig 2**).

Although mentioned above, the cognitive aspects for the masons are in fact intertwined in a very complex way, as can be observed in each activity that they perform. This can be seen, for example, to a great extent in some of the work of pouring concrete. This involves a specific group of workers with a concrete mixer, who simultaneously adjust the resultant quantity of required materials (cement, sand, stone chips, water), their distances from the machine, and the number of workers, in different work sites. I have observed that the proportion of the materials used for a single load of compound is maintained by adjusting the relative pace of the material carriers in such a way that the carriers do not need to wait, nor does the machine need to be stopped for a single moment [5]. Another cognitive feature is evident in the adaptive expertise of the masons. Not only do they use various non-standard tools and instruments (either they instruct local blacksmiths or they make these themselves in many cases), but also they adapt to the environment of the worksite. The instruments used to tie knots with the binding wire and to bend rods into the 'L'

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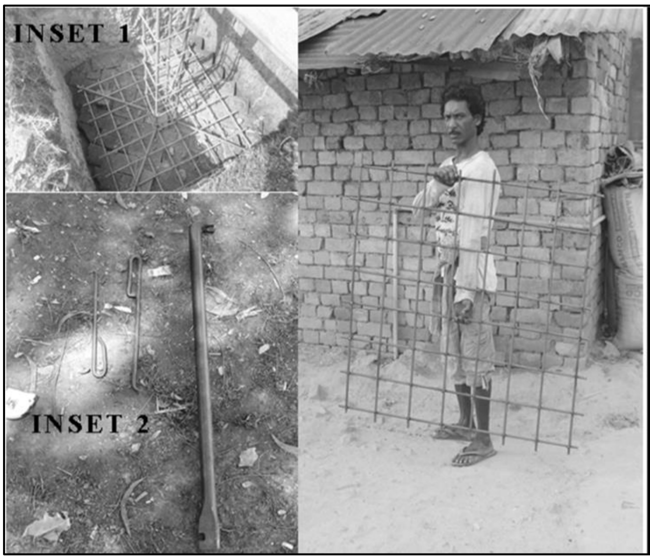
shape are (**INSET 2**, **Fig 5**) such examples. The tabletop used for making the square shaped ring (**INSET 2**, **Fig 3**) and the one they use to give support while bending the rod [5] are examples of adapting tools they assemble at the worksite itself. The temporary tabletop is made by tying rods to two adjacent trees (**INSET 2**, **Fig 6**), use of an adjacent swamp to collect water for concrete compiling [5], or making temporary tents by choosing shadowy area near the site, are some examples of adaptation with the local environment. **Fig 6** shows, how in perceiving reality, the masons make strategies for their action, how that action modifies the reality and in turn produces new information which in turn generates a new set of strategies for further action (D'Ambrosio, 1985). These are components of an endless cyclical process that is the basis for the theoretical framework of Ethnomathematics (D'Ambrosio, 1985). As shown above, the cognitive abilities of the masons fit well within this framework.



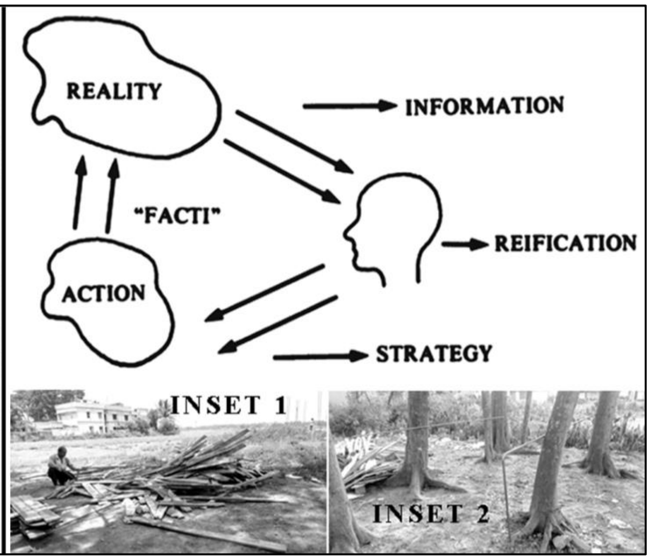


Figure 5 **Figure 6**

ON FORMAL SCHOOLING AND INFORMAL LEARNING

Twenty years ago Shankar had studied up to the secondary level. When I asked him about his memories of his school days, particularly his experience and knowledge of mathematics and how he utilizes them in his current work, his response was uncertain. Stressing to have almost no memory of his school days, he admits that mathematics must be involved in all the activities he undertakes, but he doesn't realise in which way, or doesn't want to put much importance to it (refer to his views on weighing rods). It is interesting to see that Moreira and Pardal (2012) faced almost similar situations while interviewing masons in Lisbon about their school experiences in relation with the work they do. What deserves to be mentioned are Shankar's observations of 'cantilever-concept' in the branches of trees and his conviction that this phenomenon in nature has influenced human learning. He evaluates his co-workers' abilities/interests with respect to specific components of the job and thus arranges the whole programme accordingly. From personal experience during his tenure as a novice in Mumbai, he stresses allocating the suitable jobs to match the

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ability of each worker and thus building proficiency for the team through repeated practices. Shankar also admits to have noticed that Kishore is often reluctant to learn new tasks other than what he is typically assigned to. He has also noticed that Kishore avoids tasks that involve intricate and complex measurements, and is often unable to remember complex techniques while trying to complete his assigned tasks to the level of utmost perfection. However, Kishore is diligent in his work of binding knots, mixing cement mortar by maintaining perfect proportion, and particularly mindful of his personal safety as well as that of the material (**Fig 5**).

According to Shankar and Bablu, they prefer their jobs to be less physically taxing and more of managerial quality. Incidentally, Bablu narrated a story about how he elevated himself from a mere earth-digging labourer to his present leadership position. In the same way Gopal also managed to raise his position within the group hierarchy. Shankar and Bablu admit that this pursuit is essential for promotion of a worker in this sector.

It is worth mentioning that Shankar stresses on his realisation of the importance of cooperation among the workers. He strongly values his co-workers' advice and implements the ongoing changes of labour patterns as a whole. He believes that earlier caste/community-based occupations are no longer valid: a farmer's son need not necessarily be a farmer anymore. Shankar therefore mentions that it is through mutual understanding and cooperation that the society as a whole can move ahead. Regarding his own profession of 'Raj Mistri' (Raj = Royal; Mistri = Mason) he emphasises going deeper in knowing the historical aspects of knowledge about the masonry and the masons who built the royal mansions in much earlier times.

DISCUSSION

One limitation of this study is that only a group of specialized reinforced concrete workers (male) have been included, leaving out many other specialized groups and female workers associated in the field. It also lacks multi-location observations.

However, we can identify some areas where new implications may be suggested not only for an alternative and more successful method for studying adult learning processes, but also for an alternative process of elementary classroom teaching. These suggestions, pertinent for mathematics learning, are: **1.** To consider the interest and cognitive ability of the learners rather than the universally imposed mechanical method of induction for learning; **2.** To consider the importance of situational perspectives (natural environment, socio-political environment); **3.** To consider the input of culture/cultural group (occupational group). These inputs strengthen the re-visioning of alternatives, as suggested by Subramanian (2015), in the context of multicultural-multilingual classroom curricula across India. This raises, however, an important question for us regarding how could these mathematics be taught formally without compromising the contextual factors.

Masons and construction workers are a major part of the unorganized sector of labourers in India today— which constitutes 92% of all labourers (Verma, 2013). Not

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only deprived of their due recognition, they also get a paltry daily wage. This sector, both private and public, is mostly ruled by the current capitalistic middlemen-culture, that keeps the workers at bay while making secret deals with the contractor (Verma, 2013). These middlemen extract money from both ends and, in the process, negate knowledge. We do not wish to engage in the current fan-fanaticism about an invented past depicted skilfully and proposed politically in front of us (which argues everything about masonry came from *Vishwakarma* [11], everything came from *Vedas*, aeroplanes came from *Pushpak Ratha*, etc.). Yet to stand for the historical contribution of the working class to the society and the societal knowledge system in India, is a challenging task. It is challenging in the sense that the earlier proposition tends to delude minds of those in this field of study who, despite being honest in every way possible and despite being keen to find solution of the societal issues in a culturally-historically responsive context, lack political insight. Paulo Freire declared, as mentioned by Mukhopadhyay (2013, p. 94), that “The intellectual activity of those without power is always characterized as non-intellectual”. This is what empowers our political standpoint in this regard, to search for “quality of life and dignity for the entire mankind” (D’Ambrosio, 1998).

ACKNOWLEDGEMENT

Special thanks go to Mr. Amal Kumar Santra and Ms. Arundhati Chakraborty for their cordial help in translating parts of this paper from Bangla to English and in drawing some of the schematic diagrams respectively.

NOTES

1. Most of the texts about terracotta constructions lack the perspectives of the masons building it.

2. See https://gribaconfo.com/wp-content/uploads/2017/05/205-Sriram.pdf

3. In this work, ₹ 450 (Shankar, Bablu), ₹ 425 (Gopal), ₹ 275 (Kishore, Madhu).

4. See http://in.one.un.org/task-teams/scheduled-castes-and-scheduled-tribes/

5. https://drive.google.com/open?id=1AxiSsdQXwrsY_-vfc7tCJ9G2FfXl2Pte

6. ‘Half bat’, ‘quarter bat’ and ‘three quarter bat’ of a brick; ‘academic terminology’.

7. Millimetre is used to nullify errors of unwanted displacement/omission of decimal points (d.p.) [3048mm (no d.p.) = 3.048m (d.p. used)]; ‘Perspective of a civil engineer’ (Sanyal, 1994, p. 13).

8. After first used by Antonio Gramsci, this intellectual discourse/term/concept has undergone several theoretical debates over the last couple of decades. However, for the purpose of this paper, the term would be used from postcolonial theory perspective and would refer to lower classes and the social groups who are at the margins of a society: a subaltern is a person rendered without agency by social status. See https://en.wikipedia.org/wiki/Subaltern_(postcolonialism)

9. To avoid Straight Joint Error; ‘academic terminology’ (Sanyal, 1994, p. 28).

10. Parallax error detection; ‘academic terminology’.

11. The god of architecture; Indian Mythology.

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MATHEMATICS TEACHER IDENTITY, TEACHER IDENTITY, AND INTERSECTIONAL IDENTITIES:

AN OVERVIEW AND RECOMMENDATIONS FOR FURTHER RESEARCH

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This literature review examines n=18 articles in the area of intersectionality and teacher identity, with 6 articles specifically covering mathematics teacher identity. Recommendations include bringing the general teacher identity literature and mathematics teacher identity literature closer together, covering intersectional identities when doing reviews of the literature, and focusing on the role of reflection and reflective practices.

Identity is not a unidimensional construct. Mishler (1992, 1999) proposed that identity "be defined as a collective term referring to a set of sub-identities (among them a work identity)" (1999, p. xv). Despite the clear existence of racial identities and sexual orientation identities, there is not much research into how they are co-constructed along with teacher identities, particularly in regards to mathematics education. In this paper, I review 18 articles that attempt to address these intersectional aspects of teacher identity.

RESEARCH QUESTIONS

My research questions for this literature review are: 1) What literature exists in terms of intersecting other identities, such as race, gender, sexual orientation, and disability, with teacher identities? 2) What are the strengths and limitations of the existing literature that exists on this intersection? 3) What can teacher educators do to support teachers with these identities in developing their teacher identities?

METHODOLOGY

In this study, I conducted searches on Google Scholar, JSTOR, and ERIC using terms such as "mathematics teacher identity", "mathematics teachers", "mathematics related teacher identity" and "teacher identity" both by themselves and in conjunction with various combinations of "race", "sexual orientation", "LGBTQ", "intersectionality". This review considered only articles written in English; as a result, the retrieved articles primarily address teachers in the United States and England, although one article (Samuel & Stephens, 2000) deals with a South African context. I included articles that were broad reviews of the literature in both mathematics teacher identity and teacher identity even if they did not cover intersectional themes. The inclusion criteria was empirical research studies with a significant component of other identities (such as race and sexual orientation) or if theories about multiple identities appeared in their theoretical framework. For each article that I found, I scanned the references list and

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the "cited-by" lists in the various databases to look for other relevant articles, which due to time limitations, were selected by looking at the titles only. This process yielded a database of 19 articles; I excluded one article that used a qualitative fiction methodology (de Freitas, 2004). Three additional literature reviews and five additional articles were utilized to help formulate the argument even though they did not meet the inclusion criteria.

THE RELATIONSHIP BETWEEN MATHEMATICS EDUCATION RESEARCH AND MORE GENERAL EDUCATION RESEARCH

I had initially fallen into the trap of believing that I could confine the scope of this research to mathematics-related teacher identity. Many mathematics education researchers fail to consider the broader teacher identity research; Lutovac and Kaasila (2017) observed that researchers writing about mathematics-related teacher identity do not frequently cite articles from the broader field on the topic of teacher identity, particularly when these articles do not specifically deal with mathematics education. They pointed out that "mathematics education is as much about education as it is about mathematics" (p.10) and that when mathematics educators fail to consider theories and ideas from the broader educational literature, they position mathematics education as being separate from other disciplines and prevents their research from being considered within the wider discourse on teacher identity. One of my goals in this review, therefore, is to bring the literature on mathematics-related teacher identity into dialogue with the more general research on teacher identity; therefore, I interweave both sets of literature, notating (as needed) when a study only deals with mathematics teachers.

INTERSECTIONAL IDENTITIES MISSING FROM LITERATURE REVIEWS

In the first part of this investigation, I started by consulting two well-known literature reviews on the topic of teacher identity (Beauchamp & Thomas, 2009; Izadinia, 2013) and one more recent literature review on mathematics-related teacher identity (Lutovac & Kaasila, 2017) to see what they said about teachers' identities in terms of race, gender, and sexual orientation.

Beauchamp and Thomas (2009) had no mentions of race, gender, or sexual orientation in their narrative or the descriptions of any of the studies that they cite. Izadinia (2013) included in their review one study in which preservice teachers wrote autoethnographies deconstructing their sexual orientation and gender identity (Vavrus, 2009), but Izadinia subsumed it under a strangely ideologically neutral category of "reflection" and left out any mention of gender identity or sexual orientation, noting only that the students in the study demonstrated "critical consciousness." Izadinia also included a study on gender and teaching, but subsumed this under a category of "prior experiences." Lutovac and Kaasila (2017) included a category in their review entitled "Power, Social Justice, Gendered Discourses and Race in Identity and its Development." Surprisingly, though, the studies they review in this category are all about poststructuralist analysis of power and discourse; although two studies address

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gender through a post-structuralist lens (Llewellyn, 2009; Walshaw, 2013), none of the studies from this section address race. The one study that Lutovac and Kaasila cite that addresses race, Clark, Badertscher and Napp (2013), is prefaced by a note that it covers "racial and cultural communities" and then only the methodology of the study is discussed, without any reference to it being about African American teachers or any exploration of its relationship to race. Furthermore, Clark et al. was not mentioned at all by Lutovac and Kassila in the category about race.

NARRATIVES OF TEACHERS OF COLOR ELIDED BY RESEACHERS

Teacher identity is often studied in terms of the construct of agency. At first glance, it might seem that agency is merely a reframing of teacher efficacy. Teacher efficacy as a construct comprises dimensions such as locus of control (believing that one's actions have a significant effect, particularly in regards to student achievement) and self-efficacy (the belief that a teacher will be able to carry out their responsibilities successfully (Tschannen-Moran & Hoy, 2001).

Teacher efficacy is usually measured quantitatively using efficacy scales and is about measuring a characteristic that teachers possess. By contrast, agency is "an emergent phenomenon of actor-situation transaction" (Biesta, Priestly, and Robinson, 2015) meaning that it is something a teacher does rather than something they have. This does not mean that agency only resides in the present moment, however; Biesta et al., cited Emirbayer & Mische (1998), who argued that agency reactivates "past patterns of thought and action" (Emirbayer & Mische, p. 971), relies on imagined "future trajectories of action" (p. 971), and involves making "practical and normative judgments among alternative possible trajectories of action" (p. 971). Teacher agency is much more dynamic than teacher efficacy, and requires more of a qualitative approach to really understand its phenomenological dimensions.

Beauchamp and Thomas (2009), by contrast, defined agency as the "empowerment to move ideas forward, to reach goals or even to transform the context" (p. 183). In their conception, agency is about defining and carrying out one's own goals, choosing whether to accept or reject the goals others think you should have. These definitions are not incompatible, however, since one's ability to move ideas forward requires having had certain experiences in the past, and is almost certainly also about imagining how the ideas might unfold in the future.

Studies of teacher identity generally center more around agency than efficacy, given that agency provides a much richer understanding of a teacher's identity. However, when teachers of color are the teachers under consideration, researchers tend to focus less on the teachers' agency and focus more on their perceived ability to reach students of color (e.g Basit & McNamara, 2004; Su, 1997). Here the focus shifts from agency and efficacy to a less teacher centered idea of effectiveness. Although ethnic minority teachers, when asked about the advantages of being an ethnic minority teacher, tend to focus on the importance of being a role model or on their ability to motivate ethnic minority students (Basit & McNamara, 2004), this may be an artifact of the way in

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which the questions are asked; researchers tend to ask more about the students of teachers of color instead of about the teachers of color themselves. This focus on effectiveness limits researchers' ability to focus on agency and identity by shifting the focus away from the teachers and towards the students.

In this schema, researchers care about teachers of color primarily insofar as this caring can translate into equity within the classroom for students of color. A naïve version of this theory is called "cultural match", where having a teacher from the same racial group automatically means reaching students from that group more effectively. Although there is extensive research literature on the strengths that black teachers bring to the enterprise of teaching black students, much of it is predicated on the enactment of normative presentations of sexuality and gender, as Brockenbrough (2012) found in his study of eleven Black queer male (BQM) teachers.

Achinstein and Aguirre (2008) criticized this notion of cultural match, arguing that cultural knowledge needs to be explicitly cultivated amongst preservice teachers of color rather than assuming that teachers of color will automatically be able to teach students of color (p. 1553). Cultivating the resources of teachers of color is rarely discussed in the literature, however, which tends to focus more on the need to carefully manage the fragility of young white male preservice teachers (e.g. Carson & Johnston, 2001) or on the development amongst primarily white teachers of an orientation towards social justice (e.g. Boylan & Woolsey, 2015).

Moving beyond cultural match, Clark, Badertscher, and Napp (2013) bring a more nuanced perspective; they studied two African American mathematics teachers, Madison Morgan (referred to in the study with "she" pronouns) and Floyd Lee (identified in the study as male), focusing on how their identities as teachers support African American students' mathematical identity. Reworking Wenger's (1998) proposal that teachers represent their communities of practice and bring them to their students, Clark et al. focus on the meaning that the term "represent" has within an African American context, a term reflecting pride in one's actions that reflect well on their community. In other words, African American teachers that are in the classroom doing mathematics helps to bring their students along into the mathematical communities of practice.

What is missing from Clark et al.'s (2013) research, however, is a closer investigation of how the teachers of color's identities as mathematics learners were formed. Given the importance of mathematics teachers experiences as learners to their identities as teachers, it is crucial to understand teachers of color both in terms of their histories as mathematics learners (Drake, Spillane & Hufferd-Ackles, 2001) and as learners learning about subject-matter pedagogy (Spillane, 2000). Researchers need to learn about the narratives and trajectories of the teachers' learning, not just the way in which teachers cultivate the identities of their students.

This centering of teacher of colors' identities is similarly missing from Vomvoridi-Ivanović's (2012) study, which focused on the ways in which Latino/a preservice

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teachers teach Latino/a students in culturally responsive ways. Although the article proposed that preservice teachers need to learn how to select tasks to draw on students' experiences and to learn how to make meaningful cultural connections, it did not address the role of the preservice teachers' identity formation or narratives in making sense of how they teach students from similar cultural backgrounds.

Teachers of color also occupy a liminal space in terms of identity; as Samuel and Stephens (2000) argued about two South African teachers of color, "their own university training has perhaps caused them to be misfits in the cultural contexts from which they came", but at the same time they do not easily fit into the existing structure in the schools, and "cannot proceed into the school environment without seriously challenging the status quo" (p. 488).

Thus, building off of Achinstein and Aguirre's (2008) argument about cultivating the resources of teachers of color, teacher educators need to provide a space for preservice and inservice teachers of color to reflect on their racialized identities. Chao's (2014) study of inservice teachers demonstrates the consequences of not having this training and support. Chao provided an example of a Latino teacher, Mr. Leche, who sought to center issues of race, gender, sexual orientation and culture in his mathematics classroom but found that he had been inadequately prepared. Chao observed that the teacher felt that their issues were "hidden topic in his professional development, spoken about only in shallow ways or glossed over completely" (p. 103). Teachers of color need to be guided in ways in which to integrate their racial and cultural identities into their classroom practice.

If teachers of color are not given support and professional development that fosters this integration, they might "adopt different teacher identities because these new identities allow more power and opportunities to actually teach" (p. 109). An example of this is another Latino mathematics teacher studied by Chao, Mr. Ginobili, who failed to integrate his Latino background into the classroom and had instead developed a much stronger identity as a soccer coach. Mr Ginobili, however, lamented not being able to "inspire and engage his mathematics students" (p. 108). As Chao stressed, teacher educators "must be better at supporting mathematics teachers through all the various aspects of their identity, beyond just knowledge of mathematics, and to include specifically sociocultural and community-based identities" (p. 110).

Williams (2014) explained the importance of having opportunities for reflection with others on one's multiple identities. She argued that "when teachers communicate their identities through reflecting on their experiences, they are able to cope with their own experiences by knowing the experiences of others. Also, they might use a conglomerate of positive teacher identities to build and expand identities of their own" (p. 158).

Reflection is one of the most-studied aspects of teacher education, but reflection in and of itself is not enough; future research should examine what reflection looks like for teachers of color, closely following their progress through both the preservice and induction phases of teaching.

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IDENTITY MANAGEMENT, DISCLOSURE, AND LGBTQ TEACHERS

This section addresses LGBTQ (Lesbian, Gay, Bi, Trans, Queer) teacher identity and the ways in which the literature on LGBTQ teachers differs from the literature on teachers of color. Given the paucity of research on LGBTQ mathematics teachers, this section relies heavily on the general education literature, however, given that one of the overarching themes of this review is to argue that the general education literature applies to mathematics teachers, this research should still be relevant to LGBTQ mathematics teachers.

Studies about LGBTQ teacher identity often focus on questions of identity management and disclosure. For example, Paparo and Sweet (2014) studied two preservice music teachers, a gay man (Brett) and a lesbian woman (Nicole), and their experiences interacting with their cooperating teachers and with their students. Whereas with teachers of color the research question is often restricted to how they can effectively reach students of color, with gay and lesbian teachers the research tends to focus almost exclusively on whether and how they come out to their colleagues and students. The research on LGBTQ teachers, unlike the research about teachers of color, is heavily focused on teacher narratives and little emphasis is placed on the impact LGBTQ teachers have on students' academic performance. (e.g., Brockenbrough, 2012, p. 748).

Endo, Reece-Miller, and Santavicca (2010) observed that when their participating teachers "chose to disclose their sexual identity to others [in the school setting], those individuals made our teachers' homosexuality their core identity" (p. 1027). A parallel situation, I argue here, is that when researchers are studying LGBTQ subjects , they seem to focus only on their LGBTQ identity. Paparo and Sweet (2014), for example, do not discuss the racial or cultural backgrounds of the two preservice teachers they studied. Brockenbrough (2012) lamented that many studies on queer teachers' experiences include only white subjects or fail to analyze the role of race in the narratives of the non-white subjects (p. 745). This tendency to focus primarily on sexual orientation likely comes from only studying white subjects; in Brockenbrough's study, the teachers were reluctant to discuss being gay and were far more willing to discuss their blackness and their maleness (p. 751). Given the ways in which whiteness is assumed and unmarked in educational research, it is all too easy to reduce LGBTQ subjects to merely being about their sexual orientation and gender identity.

Returning to the theme of the previous section about having opportunities to reflect on one's identities in preservice teaching, Vavrus (2009) had a group of preservice teachers write autoethnographies that deconstructed their sexual orientation and gender identity and then used categorical analyses to analyze the autoethnographies. Despite the essay prompts not including race, race came up in half of the preservice teachers of color's papers and in two white students' papers (p. 388). Vavrus' method thus seems like a promising method for bringing intersectionality into the teacher education classroom.

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Although there is some research on LGBTQ identity and students' mathematical identity (see Fischer, 2013) and on creating transgender-inclusive mathematics curricula for social justice (Rands, 2013), there is not any research specifically on LGBTQ mathematics teachers. Although I was not surprised to discover this, I was surprised to discover that there are not any scholars who have remarked upon this absence.

RECOMMENDATIONS FOR FURTHER RESEARCH

Gaps in Existing Literature Reviews

I want to come back to what I started with, talking about literature reviews. Those writing the literature reviews in the field of teacher identity essentially ignore intersectional identities, and even when they encounter them in the literature it still tends not to register as being about those identities. Recall, for example, Izadinia's (2013) subsuming of gender identity, sexual orientation, and gender under the categories of reflection and prior experiences, and Lutovac and Kaasila (2017)'s neglecting to put the one study they reviewed that actually studied race and teachers (Clark, Badertscher and Napp, 2013) under the "Power, Social Justice, Gendered Discourses and Race in Identity and its Development" category in their review while none of the studies in that category addressed race. Those writing the literature reviews fail to properly consider intersectional identities, and as a result miss an opportunity to really write in a thorough way about the rather significant gaps in the existing literature. This is perhaps a problem with using literature reviews to identify holes in the literature; one does not necessarily know what they do not know, and so it can be challenging to figure out directions for future research if others have not been down that path before. This makes it difficult for others to contemplate doing research in these areas; if it has never been discussed as a hole in the research literature, it is harder to justify to funders and reviewers the need for the research. Part of the goal of this paper is to shine more light on the need for research in this area.

The Importance of Reflection

Given the importance of both reflection to teacher identity development (Beauchamp and Thomas, 2009) and reflective practices (Izadinia, 2013) in developing student teachers' identities, it is noteworthy that only one study in this review (Vavrus, 2009) focused on reflective practices. Further research should explore both the role of reflection in developing teachers' other identities and the ways in which reflective practices might be harnessed for use with mathematics preservice teachers to help them reflect on said identities.

Bringing the Mathematics Education and non-Mathematics Education Literature Closer Together

Echoing the recommendations of Lutovac and Kaasila (2017), there needs to be more attention placed in mathematics education research to what is going on in the less specialized education literature. More than just citing this general literature,

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mathematics educators also need to attend and present at more general conferences and to publish in journals that are not specific to mathematics education. Particularly when the research on sexual orientation and mathematics teacher identity are completely absent, it becomes incumbent upon those doing research in mathematics teacher identity to branch out and consider the more general literature when doing literature reviews and framing their own studies.

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RE-MYTHOLOGIZING MATHEMATICS? LESSONS FROM A SACRED TEXT

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This paper seeks to elaborate the metaphor of math as mythology through consideration of a genre of mythic texts drawn from indigenous Kapon communities in Guyana, South America. These maiyin "memory verses" arrange a series of poetic images into a textual structure that resembles a vertical axis of a discrete, ordered set. Understanding the historical and cultural context of indigenous ordinality extends our understanding of mathematical mythologies as social critique.

INTRODUCTION

Mathematical activity sometimes seems to take place in a world of imagination, within a landscape of relationships and situations just a little out of reach of the immanent world of the classroom. This sense of distance, mystery or Otherness has been expressed through the metaphor of mythology (Andersson & Wagner, 2018; Wagner & Herbel-Eisenmann, 2009; Khan, 2011). In all these works, the metaphors that link mythology and mathematics call for discursive transformations—acts of mythopoesis—that uncover mathematical experiences buried under dominant practices in education and research. These critical perspectives through mythopoesis seek wider recognition of the way students experience mathematical activities (Andersson & Wagner, 2018; Wagner & Herbel-Eisenmann, 2009), and the engagement of mathematics with processes of disempowerment (Khan, 2011). These works of critical mythopoesis open mathematics education research to new ways of connecting histories and epistemologies to mathematical imaginations (Gutiérrez, 2012).

In English, the word "myth" refers to a story, a category that is freighted with many concepts of text, time, place, and purpose. A myth is typically considered as a specific genre, different from a lesson, a song, a medical prescription or diagnosis, and other ways of explaining life; it refers to things that happened in a different time and place; and telling myths might sometimes seem more recreational or artful than purposeful. In communities where myths are a part of daily life, though, these distinctions often collapse. During a time in my life when I lived temporarily in these kinds of communities, I learned that the important feature of myth is that it invokes a powerful world that's regarded as real but that lives in the interstices of our more manifest world. Mythic worlds, mythic modes of action, and mythic stories across multiple genres are all much closer together than the English word "myth" can express.

This paper shares a mythic text drawn from my doctoral research in cultural anthropology in indigenous Kapon communities in Guyana, South America. I share an example of a sacred text called *maiyin*, which Kapon translate as *memory verse*. *Maiyin* texts have a strong ordinal structure reminiscent of a number line. These verses are part of a spiritual and social movement among indigenous people that began in mid-19th

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century British Guiana and that reclaim indigenous identities despite devastating economic and religious colonial policies. Understanding some of the historical and cultural meanings associated with locations on the *maiyin* axis shows how an indigenous ordinal mythopoesis took place. The case of *maiyin* helps demonstrate why mythopoeis is an inherently critical act.

RE-MYTHOLOGIZING MYTHOLOGY…AND THEN, MATHEMATICS

Levi-Strauss' structuralist myth analysis was greatly influential even by virtue of being highly critiqued. His approach involves abstracting fundamental binary oppositions or "mythemes" from a texts, creating proportionality statements along the lines of *raw:cooked::unmarried man:married man with marriageable daughters*. A structuralist analysis of myth involves noticing the ways in which the constituents of these proportionality statements permute themselves through available positions across the narrative dimension of the myth. While Levi-Strauss' mythemes often touch on significant tensions in the texts that he studied, his approach positioned indigenous people in negative ways and failed to focus on some of the most significant teachings of mythic traditions. Because structuralist analysis purports to uncover pan-human principles of logical thought that are frozen in the minds of indigenous peoples, it positions indigenous people outside of history and as people without agency. Furthermore, structuralist analysis missed the central point of some myths and related genres that tell about processes of social reproduction, and sometimes serve as the basis of social critique or social action. It's necessary, then, to re-mythologize mythology by noticing the involvement of myths in explaining and promoting social transformations:

> …not only static aspects of the social and cultural world…but the process through which these aspects are produced or maintained (or, as the case may be, transformed or destroyed) (Turner, 2017, p. 4).

When we compare myth—as story, knowledge, or mode of action—to math, some elements to keep in mind are that myth-telling sometimes brings a powerful world into engagement, literally, with the merely sensory world; that myths tell about processes of social reproduction; that myths guide action during social change; and that historical experiences can be incorporated into the mythic landscape.

Our current comparisons between myth and math acknowledge many of these dimensions. Andersson and Wagner noticed that "myth" may refer to a variety of genres, not specifically narrative ways of sharing knowledge (2018). Myths can change with new tellings or with historical events (Wagner & Herbel-Eisenmann, 2009). The idea of bringing worlds with different ways of knowing and different forms of action together is represented in the themes of concealment, boundlessness, control and seeking (Andersson & Wagner, 2018). Importantly, we've noted that mythic worlds highlight processes of becoming, empowerment and social critique (Khan, 2011). While classical scholars of myth analysis—Jung, Campbell, Eliade, Levi-Strauss—tended towards political conservativism, "(l)eft-wingers entranced by the power of myth have been far more likely to put their hands to creating new myths than

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interpreting old ones' (Graber, 2017 p. xxxiv). It is not surprising then, that mythopoesis in mathematics education also takes critical perspectives towards our field of study. But what is it about myths and myth-making that lend themselves to these critical perspectives?

ETHNOGRAPHIC AND HISTORICAL SETTING

I learned about *maiyin* texts and the sacred ordinal scale through fieldwork on a syncretic or constructed religion known as Areruya or Alleluia that is practiced by some of the Carib-speaking indigenous groups in triple frontier region of Guyana, Venezuela and Brazil. Areruya combines aspects of Christianity with indigenous concepts of spirituality, that many entities have souls or motivating forces; that these forces can separate from the body; and that sometime soon, at the end of the world, all these spirits will leave their present vessels. Understanding this spiritual tradition and its expression through the ordered character of the *maiyin* axis requires substantial attention to details of history and social organization from colonial British Guiana.

Some Areruya practices probably trace to the mid-1850s (a longer commentary and historical sources for this section can be found in Staats, 2003). The original Dutch occupiers of the region engaged indigenous communities to patrol the forests for African people who had escaped enslavement on coastal sugar plantations. In exchange for returning people to plantation enslavement, the Dutch enacted a policy of annual gift distributions to indigenous communities. Gift lists in historical documents included manufactured items like knives, combs, razors, mirrors, scissors, and trade cloth. England gained administrative control of the region in the early 1800s. The end of African slavery across the British Empire during 1833 to 1838 also ended the distribution of gifts which were no longer necessary from the point of view of English administrators. Some Kapon communities were especially affected because they acted more as middle men in indigenous trade networks than through specialized production of indigenous products that were traded by other groups in the area. Though Kapon don't seem to have direct historical stories from the time of Emancipation, *maiyin* texts seem to preserve fairly faithful descriptions of the trade goods withheld by the English.

Anglican missionization increased around this same time, and in the mid to late 19th century, indigenous men in Southern and Central British Guiana began to have visionary experiences in which they received hymns, prayers and other teachings, they said, through their souls' direct contact with God in heaven. Indigenous stories of the origin of Areruya say that these prophets realized that the English were withholding sacred knowledge from them, seemingly, as they withheld trade goods, so prophets sought sacred knowledge directly through spirit travel. These stories also record social discord about the religion within indigenous communities, because the first prophet, Bichiwung, was attacked and dismembered by indigenous sorcerers, *piaichang*. Several times Bichiwung's wife rubbed his body with medicine that he had received from God, and was able to restore his body and bring him back to life. In the final attack, *piaichang* scattered the many small pieces of his body so widely that they could not all be found, and he could not be revived. The term *piaichang* is commonly

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translated in anthropological literature as *shamans*, indigenous people who were (and to limited extent in this region, still are) trained in methods of spiritual action that can cure people who have lost their souls, but who also terrify people because they can cause illness and death.

Kapon prophets' souls traveled to heaven to receive several kinds of texts from God: *eremu* (hymns), *esegungang* (prayers) and *maiyin* (memory verses). Some people occasionally receive new sacred texts in contemporary times, too. Cultural anthropologists would see this belief system as a transformation of prior indigenous mythic and spiritual knowledge rendered in English as *shamanism* and in Kapon as *piai*, but Kapon radically reject the association of Areruya and shamanism, because they see their sacred practice as an advancement over shamanism.

CONVOCATION OF A SACRED ORDINAL AXIS

Maiyin are long, poetic texts based on some 50 images that, for a particular village, are spoken in a rigidly consistent order. Because *maiyin* are quite long, I will indicate how to construct one *maiyin* accurately, without transcribing each line. *Maiyin* have a "repeating" stanza that alternates with a "naming" stanza (these are my own terms, not indigenous ones). In the *maiyin* text that I describe in this paper, the repeating stanza is:

Areruya serü	This Areruya
Mingazak	Chosen (words)
Pada ewa	Heaven's cord
Pada winong	From heaven
Serü bïk uaburemang bïk	On this sacred knowledge
Pada iyebü	Heaven is coming
Wagi abong	Goodness' throne

This repeating stanza is spoken as the first stanza of the *maiyin*, as the last stanza, and in between each of the naming stanzas. The repeating stanza expresses a sense of continuity along a line. *Pada ewa* / *Heaven's cord* has resonance with *dewa*, the cord that affixes one's sleeping hammock to the beams of the house. In this sense, *maiyin* narrates a continuous pathway from the lived world to the transcendent world. People use a similar, vertical, continuous gesture when the describe souls leaving their vessels at the end of time.

The second stanza of this *maiyin*, which is the first naming stanza, lists the most basic foods of life in a phrase that links them to the food's spiritual seat of power in heaven, *abong*. In this way, the *maiyin* text described here focuses on the *abonock*, a bench or throne that is a soul's seat in the afterlife. In ordinary life, *abong* refers to a low bench. Academic training in anthropology also brings to mind the bench that a shaman sits on during curing sessions, but Kapon completely reject comparisons between Areruya and shamanic activities. Glossing *abong* as a "place where something resides" is closer to

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the spiritual sense, because several other *maiyin* images focus on the container of things—the *wai* bowl that holds cassava drink that gets passed from mouth to mouth at every collective meal, the *wai* that is a musical instrument that projects sound and words into people singing or saying Areruya texts, and the sense that at the end of time, the motivating spirit of everything will leave it's current seat and return to heaven.

Tuna abong	Water's throne
Morok abong	Fish's throne
Torong abong	Bird's throne
Ok abong	Meat's throne
Tuma abong	Meat broth's throne
Egi abong	Dry cassava bread's throne

From here, readers can construct the *maiyin* text accurately by reading the repeating stanza, reading nine naming stanzas created as above with the sacred words in a possessive relationship with *abong*. Each naming stanza is followed by the repeating stanza, so that the repeating stanza is also the final stanza (Table 1).

Naming Stanza 1		**Naming Stanza 2**	
Tuna	Water	*Pïzau*	Cassiri/cassava drink bowl
Morok	Fish	*Wai*	Cassiri/cassava drink bowl
Torong	Bird	*Wa'nok*	Drink
Ok	Meat	*Kiari*	Food
Tuma	Meat broth		
Egi	Dry cassava bread		
Naming Stanza 3		**Naming Stanza 4**	
Maming	Apparel	*Sabado*	Shoe
Arika'	Scissor	*Puküru*	Trousers
Sarai jarai	Hair comb	*Pokiri*	Belt
Wanamari	Mirror	*Awüni*	Beaded pectoral
Mawasa	Razor	*Arok*	Hat
		Keraba	Hair oil
Naming Stanza 5		**Naming Stanza 6**	
Wai	Musical instrument	*Maming*	Apparel

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Iga riga	Musical instrument	*Kazurü*	Bead
Arawera	Musical instrument	*Kamichang*	Lap cloth
Sedumba	Musical instrument	*Maria*	Knive
Sambana	Musical instrument	*Waka*	Axe
Kurak	Clock	*Kurupara*	Gunpowder
Aibiribing	God's power	*Piroto*	Shotgun shell
Kompas	Compass	*Keyapo*	Cap
		Arakapiza	Shotgun
Naming Stanza 7		**Naming Stanza 8**	
Jise Kulai	Jesus Christ	*Apaji*	Sister
Pregomang	Pregoman/Elijah	*Megabaji*	Megabaji
Mochi	Moses'	*Anjijiria*	Anjijiria
Epurï	Abel	*Richabek*	Elizabeth
Enjeru amuk	Angels	*Remangbaji*	Mary
Naming Stanza 9			
Pada	Heaven		
Proroi	Church yard		
Sochi	Church		
Kad	God		
Wïi	Sun		
Abonok	Throne		

Table 1: Maiyin naming stanzas.

I hesitate to label *maiyin* texts as an ethnomathematical case, but rather, propose that both Kapon *maiyin* and mathematicians' axes are similar impalpable representations that express some of their curators' central ways of knowing the world (Gerofsky, 2011). The structure of *maiyin* texts suggests several comparisons to mathematical ordinal scales: the explicit sense that there is a cord or axis connecting the earthly world with realms of greater power; the segmentation into minor and major "tick marks" formed from the 54 names and the repeating stanzas; and the sense of movement towards increasingly potent and esoteric images. In *maiyin*, the hierarchy of an ordinal scale arranges elements of life into a restorative vision of the Kapon world. I argue that any discrete, ordered set (for example, the ordering of alphabets) has some mathematical character, which can be made more explicit within a particular cultural

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tradition, such as the association of number and letter in *gematria* traditions (Olson, 2017).

SACRED ORDINALITY AS SOCIAL REPRODUCTION

Mythic texts often explain how the social and natural world is regenerated, usually by describing linkages across a variety of forms of spiritual, social and natural power. The ordinality of *maiyin* texts express social reproduction in several ways.

The most fundamental of these is the way the text literally constructs the image of a Kapon person, a sheath that any Kapon person could inhabit, and locates this person within an organized, indigenous universe. This person possesses all the most beloved foods, the manufactured and local products that make one beautiful, and explosively powerful artifacts like shotguns and ammunition that are listed textually near heaven's edge. The image of a person gives way in stanzas seven and eight to the names of historical individuals who played important roles in the early decades of the religion—the Areruya prophets and the revered women who cooked for them. The ordinal structure of the text describes the place of a person in the universe as well as their pathway into the afterlife towards the center of universal power.

Another central image of social reproduction is found in the musical instruments in stanza five, *wai*, *iga riga*, *arawera*. Anytime anyone sings or speaks an Areruya text, these instruments project the words and the music through the person, ensuring an authentic performance of the Kapon universe. This implies that Areruya knowledge should be reproduced with faithful exactitude.

In a sense, *maiyin* texts are directly the kind of mythopoetic curriculum that Khan (2011) describes. Learning the 54 *maiyin* words contributes to social reproduction because, leaders say, these are the words that people will speak in heaven, "the ABCs of heaven," citing another nominal ordinal scale, though no one on earth knows yet what these words will mean in the afterlife. This feature of *maiyin* is semiotically a little like the distinction between cardinal and ordinal numbers, because the labels can be disassembled and used again in a different context.

When we speak of re-mythologizing mathematics, then, the metaphor of mythology guides us to consider how mathematics is concerned with social reproduction. We would consider how mathematics is associated with the construction of robust personhood; how mathematics is intended to guide persons through the passages of life; and how these constructions identify and value particular forms of spiritual, epistemological, natural, or social power.

SACRED ORDINALITY AS SOCIAL TRANSFORMATION

Sometimes violent or disruptive experiences mean preclude the possibility of social reproduction. Because myths commonly describe means of social regeneration, when there are dramatic ruptures in ways of life, myths offer the possibility of mapping a new way of living in a changed world. In these situations, myths can become the basis for social transformation, for political action, and restoring a sense of identity or

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empowerment. As a response to colonial and religious domination, Areruya recaptures the sources of power, privilege and wellbeing that Europeans withheld from them and provides a vision and aspiration of economic and spiritual independence. *Maiyin* is the Areruya genre that names this response most clearly: a re-possession of material goods and spiritual power on indigenous terms. The case of *maiyin* demonstrates the depth of purpose underlying the act of mythopoesis. Mythopoesis is a deeply critical process.

Still, any social transformation creates new fractures and areas of contestation within the new form of knowledge. In *maiyin*, a historical feature that stands out is the similarity in stanzas three and six to the trade goods that Kapon communities received in exchange for policing the interior against people who escaped slavery. In contemporary days, no one I spoke to made this connection. *Maiyin* seems to refer to historical events that have been forgotten. Nation-building often places disempowered people into conflict with each other, but this historical awareness is not available for consideration within the Areruya cultural tradition.

Another feature of *maiyin* is quietly contested by some women in Kapon communities. The women of stanza seven are formally referred to as people who helped prepare ritually pure food for the prophets, but that were not wives or sexual partners of the prophets. Some contemporary women maintain genealogies that dispute this, that a living person in their community is a descendent of one of these partnerships, but these alternative histories are very closely held, and would be considered "mistakes" by contemporary religious leaders.

RE-MYTHOLOGIZING MATHEMATICS ?

Mythic texts provide access to powerful co-present places, times and social relationships, and in this way, they usually are concerned with issues of social reproduction and social transformation. Understanding these processes through the case of *maiyin* memory verses helps to affirm the connection between mythopoesis and critical perspectives on social life in current mathematics education literature (Andersson & Wagner, 2018; Wagner & Herbel-Eisenmann, 2009; Khan, 2011).

When we speak of re-mythologizing mathematics, then, we should be able to describe the ways mathematics establishes linkages across various forms of power in order to reproduce society. Re-mythologizing mathematical social reproduction will require attention, as in *maiyin* texts, to processes that constitute personhood. These generative processes allow people to use mythic knowledge to navigate times of change. Re-mythologizing mathematics may require us to follow the historical and cultural meanings of mathematics substantially outside of the boundaries of what is currently relevant in our research community. At the same time, we will need to remember that mythic discourses have a strong legitimizing function—in *maiyin*, this is strongly asserted through the ordinal structure of the text—that creates new possibilities for forgetting or disputing the historical conditions of change.

Learning about *maiyin* texts can help mathematics educators reflect on the radical limitations of our own mathematical representations of personhood. An example of this

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kind of critical mythopoesis was offered in Elizabeth de Freitas' MES9 plenary (2017, p. 70):

> the student's body is being reconfigured and reassembled. The conventional understanding of how a body "knows" mathematics is changing, and this change entails a simultaneous change in what constitutes number.

Neurocognitive measurement is its own transcendent field of knowledge, a mythopoetic action that is establishing new ways of socially ordering students.

By contrast, the ordinality of *maiyin* builds a model of indigenous personhood-in-the-world that is empowering and protective, even today as the extractive industries of logging precious woods and of gold and diamond mining continue centuries old colonial processes. In *maiyin*, ordinality becomes a trope of cultural empowerment and of social critique, both of the colonial relationship and of fault lines within indigenous communities.

A PERSONAL MEMORY VERSE

Discussing cultural knowledges and activities that are not one's own is fraught with asymmetries of power, economics and opportunity. In cultural anthropology since the 1980s, researchers have been expected to be willing to make their selves vulnerable within their research writing, so that their personal position is subject to the same process of critical analysis as the life experiences of their research participants (see, for example, Darragh, 2018).

I feel comfortable writing about this *maiyin* text, because many people discussed its meanings and translation with me. However, I'm sure that Kapon people who supported my research only told me surface elements about Areruya. They explained things at a level that I was ready to understand, and I believe that they protected deeper explanations that I wasn't ready for or that they wished to hold private. For this reason, readers shouldn't mistake my description for an authoritative exegesis.

I orient myself differently towards *maiyin* compared to the *eremu* (hymns) that I recorded. Even though I worked with Areruya leaders to translate and interpret a number of hymns, some told me that they were "afraid I might make a mistake," that I might translate them incorrectly. This is why, after I completed my dissertation, I wasn't able to continue in my intended career as an anthropologist of lowland South American communities. I could not cope with the idea that my life's work would be built from other people's fear. This led me to return to an earlier stage of my life in which I taught math at the college level, and eventually to become an educational researcher, and to participate in communities like MES. Writing this paper is a way for me to collect some pieces of my life together and to restore them to my personal continuum.

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ARCHIMEDES COMES TO SCHOOL: A SELF-INITIATED CURRICULUM PROJECT AROUND BIG IDEAS

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An interdisciplinary and integrative project based on the book, The Sand Reckoner, *was designed and implemented in an upper secondary class in a Greek School by a team of teachers, around 'big ideas' associated with ethical dilemmas rather than mathematical relationships. Story-telling with other genres and modalities created a curricular experience for both teachers and students, characterized by active involvement of students in the learning process. Two university researchers and a teacher-researcher from the team reflect on the role of a self-initiated curriculum project, the relationships between ethics, story-telling, drama and interdisciplinary curriculum.*

INTRODUCTION

Magical, one-size-fits-all solutions to the constructed 'problem' of a failing mathematics curriculum seems universal regardless of geographic location, level of schooling, or specific content strand. We discuss here a recent instructional design project that allowed for collaborations among teachers in an upper secondary school in Greece, and among the teachers and mathematics education researchers involved in the project. The ensuing dialogue offers a glimpse into specific ways that the important characteristics of a 'successful curricular experiment' are not on the surface level, but are found rather in the ways that the teachers approach their work as professionals, their roles, and the possibilities for student participation.

In our analysis, what seems most important is that the teachers organized the curriculum around big ideas and themes. Yet even this statement is in a sense missing the point, since it is not as if one could simply tell educators to organize curriculum around 'big ideas.' There is a wealth of research on such a notion of curricular design (Wiggins & McTighe 2005; Alleman, Knighton, & Brophy, 2010; McTighe & Wiggins, 2012; Ritchart, 1999; Charles, 2005). Most of the ongoing discussion of 'big ideas' in curricular design focuses on strategies for planning and assessment that attend to student understanding of conceptual relationships that are developed over long periods of time. The emphasis in this approach is the training of teachers to think purposefully about curricular planning, instead of implementing a supplied curriculum, and to plan for development and deepening of student understanding of the disciplinary content.

In contrast with the backward-design approach, the teachers of the Archimedes project began not with big idea objectives, but with questions *about* big ideas (What sort of responsibility does a mathematician or scientist have for the later application of the ideas they create? How can we popularize mathematics and its connections to society for the broader community?). Also, rather than leading students to a ‘better’ and ‘deeper’ understanding of a list of mathematical big ideas, the curriculum *used* mathematical big ideas to *gain a deeper understanding of these ‘Serious Questions’*.

THE PROJECT

The year-long, interdisciplinary project unfolded in a class of twenty-six 10th grade students, and was organized around the integration of several disciplinary subjects through the reading of a literary work, *The Sand Reckoner*, by Gillian Bradshaw (2001). Taking its title from a work by the ancient Greek mathematician /scientist/engineer Archimedes, this book explores key moments in his life in ways that make seemingly natural connections among mathematics, history, philosophy, physics and technology. *The Sand Reckoner’s* chapters are structured around moral and ethical dilemmas that possibly confronted Archimedes at key points in his life, and in this way, have the potential to engage readers in the examination of relationships between mathematics and these moral and ethical dilemmas.

The Archimedes project was designed primarily by three teachers, the mathematics and language teachers of the class with the teacher-librarian, who was also a physics teacher. Then these three collaborators discussed further lessons that might or should be involved in the project—depending on the story, and proposed cooperation with additional teachers of the same class in order to plan the activities together.

•To enliven the literary text, which spoke about music and dance, the project teachers cooperated with the music teacher who chose the soundtrack of the reading aloud of the text by the language teacher. Then this teacher explained the music terms of text while two students performed them with the help of a cello and a violin. The mathematics teacher, supported by a PowerPoint presentation, referred to the relationship of Mathematics and Physics with Music, creating opportunities for further discussion with the students.

•The need of discussing Aristarchus’ contribution to astronomy emerged through the references to him in the book. Through activities developed with the students, they learned that Aristarchus determined the relationship between the Sun-Earth and the Moon-Earth distances, using simple geometrical concepts, and that he was the first scientist to propose the heliocentric system. The group also decided to enact an expressive reading of the dialogues from Luminet’s book, "The Rod of Euclid" (2003), creating the opportunity for the students to see and discuss how his contemporaries treated Aristarchus.

•Furthermore, the teacher-librarian, in collaboration with some students, discussed Copernicus, who restored the heliocentric system in the 16th century, as well as Galileo, through a digital presentation entitled, 'From Aristarchus to Galileo, " regarding the evolution of worldviews about the shape, size and movements of the Earth and the stars.

•The religion-teacher of the class, who was invited to participate, developed discussion with the students about the reaction of the Catholic Church to the scientific discoveries of that time while a student read Galileo’s public abjuration of Heliocentric system.

•The mathematician and physicist used and discussed with the students excerpts from several books, *Little Science for All*, by Claude Allègre, (2004), and *Story of Mathematics*, by Richard Mankiewicz (2002) (Image 1). In collaboration with the language teacher, the teachers decided to enact a theatrical cold reading of the dialogue *Sagredo-Galileo* from Brecht's theatrical play, *The Life of Galileo* (1966).

Image 1: Reading excerpts of the books

Exploiting *The Sand Reckoner’s* references to mechanics, the physics teacher introduced the students to the law of levers and the finding of the gravity center of a flat body, utilizing experimental verification. The mathematics teacher developed the activity, ‘Balancing Mobiles,’ which was described in detail in the book, *Mathematics from History, The Greeks,* of M. Brading (2000), in order for the students to find a pattern for the balance of a specific construction. So the need for cooperation with the English language teacher emerged in order for the students to interact with other selections referring to Archimedes in this book. She created an acronym with the concepts of mathematics and physics cited in the text, and because she felt insecure with the many scientific concepts, the mathematics teacher co-taught the English lessons in order to explain the terms.

•The technology teacher was also inspired to participate in the activities. She collaborated with the teacher-librarian, and together they designed a digital presentation concerning the most significant technological achievements in the era of Archimedes. They also decided to present through experiments simple machines such as the lever, the pulley and the winch, enabling the students to discuss the role of machines in Physics. The teacher- librarian also chose to read excerpts from Denis Guedj’s book, *The*

Parrot's Theorem, which referred to technological achievements of Archimedes. These two teachers also prepared a list of topics on the technological achievements of Archimedes and his time, to support the students' choice for developing their own synthetic work required in the technology lesson.

•On the occasion of the excerpt from the book describing how shocked Archimedes was from the disaster caused by the catapults he invented, the three teachers organized with the students of the class a roundtable on the "moral and political responsibility of the scientist" (image 2). Each teacher had to find texts on this theme related to their own discipline, to be given to the groups of students. They also asked of the students that were members of the school debate team to support the other students of the class in the development of arguments for the round table discussion.

Image 2: The roundtable

ANALYSIS OF THE PROJECT

According to Alleman, Knighton, & Brophy (2010, p. 25), the most macro layer of 'big ideas' includes overarching cross-curricular and yearlong content. These ideas pop up frequently during planning and implementing of units and lessons. Often, Alleman et al. (2010) assert, big ideas at this level exist without teachers being aware of how they influence their teaching. A key commonality between this way of understanding the work of teachers and the work carried out by the Archimedes Project teachers is that the teachers in the Archimedes Project also were not always explicitly aware of the big ideas that they were working with. This is because they were focused more on facilitating engagement with moral and ethical questions, introduced by the book. There are, however, three critical differences: First, the Archimedes teachers did explicitly think about and discuss the big ideas in the midst of the project, as they became aware of them, and then in turn worked to amplify attention to these ideas with the students.

Second, the teachers did not focus on mathematical big ideas in the foreground of the activities, but instead used mathematics with their students as appropriate to explore and attempt to resolve the moral and ethical dilemmas. Third, the teachers did not work directly to teach students the mathematical big ideas as if these students were ignorant of them, but instead created the space for their students to use and extend the mathematical ideas, as apprentices who already brought understandings with them to the new material. This is evident in the list of themes that emerged with the students throughout the unit: mathematics and war; women mathematicians; mathematics and nature; mathematics in society; the wonderful world of fractals.

A common approach in mathematics education is to use tools such as literature, artistic works, websites, and other resources, for representing mathematical concepts. Such an approach expects units of instruction to link otherwise disconnected sets of lessons through the 'big ideas', and to justify any particular activity within a lesson with its eventual contributions to the development of the linking big ideas. In contrast with this approach, the Archimedes teachers developed activities to support the book's ideas; the activities were not designed independent of the book as efficient routes to conceptual development. For example, there were no activities designed as stand-alone introductions to the systems of counting and measuring of enormous numbers in ancient Greece; no activities about the area of a parabola or volume of a sphere independent of the stories about using physics to accomplish these tasks found in the book. Discussion of torque and centers of gravity, unsolved problems and the implications of a lack of solution, and so on, took place in the context of the book and its associated moral questions, rather than as individual lessons or isolated lessons. Instead of lessons organized around examples that illustrate concepts for the students, the Archimedes teachers established opportunities for the students to work through the dilemmas themselves. For example, students organized a thematic exhibition of the material surrounding the changing perceptions of planetary motion through history; students dramatized the ethical and political responsibility of scientists via a role-played roundtable of scientists; they organized broadcast radio programs on the themes mentioned above (mathematics and war; women mathematicians; mathematics and nature; mathematics in society; the wonderful world of fractals). Such opportunities reverse the direction of big ideas from acting as the foundation of a carefully constructed sequence of lessons to being the ideas that students bring with them from their lives to the examination of broad questions for humanity.

> *The round table was one that had the most experiential character ... it was the thing that introduced us more within this philosophy and it put me in the position of the scientist; put me in the process of handling such a process where all sciences meet ... I felt that I was the presenter ...* (Student comment)

Bottom Up Growth Stemming from Personal Assessment of One's Own Teaching

Activities were not limited to reading and discussing the book. The Project incorporated other resources to extend the questions and ideas – other reading materials. In the examination of unsolved problems, the students were encouraged to realize that the unsolvability was only within a particular context of straightedge and compass, and to find solutions with other methods – for example, the trisection of an angle using other tools. Teachers offered a broader historical comprehension of the changing perception of the sun, Earth, and planets from ancient Greece to Copernicus, Brahe, Kepler and Galileo. As Alleman, Knighton, & Brophy, (2010) point out, it can be challenging for teachers to keep multiple levels of big ideas in mind as they plan, implement, and assess lessons. They claim that it is nevertheless crucial in order to enhance meaning, to strive for efficient use of curricular resources, and to provide the most opportunities for powerful instruction in the time available. In contrast, the Archimedes teachers circumvented the complexities of planning and implementing curriculum with so many layers of big ideas at hand. Yes, a retroactive analysis of a strong curriculum can later identify ways that such big ideas, multiply-layered, manifested themselves in the project. But important for our analysis is to recognize how these teachers relinquished significant features of the multi-layered nature of the big ideas to the book, the students, and the arts-based performances that extended the curricular experiences, rather than feeling an obligation to directly explain these ideas for the students.

Where did this approach come from, and why is it so significantly different from what appears in mainstream research about big ideas in mathematics education? The Archimedes teachers created a bottom-up reconstruction of their curriculum out of a perceived sense that their work was not fully serving the needs of their students.

> '*We did all these activities due to our own need to create a learning environment that would be: meaningful for students, compatible with their nature, and that would motivate them to learn … and the same time would fulfill our common vision for education, different from the goals of the official and 'narrow' curriculum'* (Pota, the third author)

While teachers in research on big ideas are guided in the creation of curriculum at multiple levels, and gain increased confidence in this approach over time through consistent guidance from researchers (Alleman, et al., 2010), the teachers in our investigation made their own choices in an action-research context, seeking to understand and to look for how to exploit their innovations as a curriculum experience.

The bottom-up action research led these teachers to arts-based experiential learning rather than to focus on analytic construction of a unit consisting of small pieces that combine in a path to content-based 'big ideas'. It is interesting to note that the teachers did not rely on common notions of multiple representations of concepts to construct procedural skills growing out of these representations. Instead, the mathematical representations were embedded in complex inquiries. '*We learned several things that we did not know that were connected with mathematics and geometry.*' (Student Comment)

Here we see how the teachers enacted a curriculum that avoided direct modeling of mathematics concepts in favor of students comparing models, employing models toward specific purposes other than to learn mathematics – such as to convince others of a philosophical position, to explain their thinking within a discussion, or to open up new possibilities to explore (Appelbaum, 2012). This is an example of how direct modeling is sometimes thought to violate aesthetics, whereas indirect modeling, used as tools for other ends, can be powerful in evoking engagement with both the ideas of the model and the issues at hand. Sometimes, as with an architect who uses a scale model of a design to convince people to invest in a large project, a student might analogously use a mathematical model as a part of a larger persuasive argument – in this project, for example, about mercenaries in the army, or about their interpretation on historical shifts in social understandings of planetary motion. At other times, students might use a model as part of a picture of the relationships in a system that they are trying to understand with others, as a scientist might use a model of the water cycle to explain photosynthesis; in this project, students used models from Copernicus, Kepler, Brahe, and so on, to measure the distance from the Earth to the Sun. At yet other times, students might use models to imagine connections across previously disconnected topics and questions, such as when the students in this project placed unsolved mathematical problems in a broader, historical context, recognizing the role of the broader context for the development of particular mathematical ideas.

This less overt use of multiple representations in mathematics enabled students more readily to transfer knowledge across contexts. For example, students used mathematical notions to solve problems of physics, and vice versa: equal weights in equal distances balance, and students created mathematical models to guide their constructed mobiles while using their models to explore the physical forces involved in balance; action line graphs helped with understanding changing elements of a text through time, in turn used to discuss properties of functions and the changes in variables. The roundtable focusing on Archimedes' facing the catastrophe resulting from the use of his catapult, and the broader discussion of social dimensions of science, led students in the panel –role playing a Scientist, Theologian, Biologist, a Representative of a Peace Organization; also a Panel Coordinator, A TV and Radio Reporter that covered the events of the catastrophe; the remaining students played the audience and addressed questions to the panel—led in turn to further questions about the role of mathematics in everyone's everyday lives, and thus to conversations of where and how mathematics and culture are related. Students created five radio programs on the themes of women, war, society, and the wonder of fractals. This back and forth across apparent disciplinary boundaries became a central feature of the curricular structure. For example, in order to further prepare their radio programs, students visited an exhibition with text and various materials, including art, poetry, studied several articles on their themes; they created roles similar to those in the earlier roundtable to create their radio programs.

Backstories

What we find more helpful in understanding the use of the stories in this project is the way that the students and teachers together engaged in a Platonic, Socratic dialogue about moral questions having to do with love, justice, responsibility, authenticity, and so on. In the storytelling community, one strategy is that storytellers tell more than one story: there is the explicit story that involves plots, characters, scenes, circumstances, and points of view, and so on; but more significant is what one might term the ‘backstories’: Backstories are the stories behind the stories, the morals, lessons, and nuanced, complex investigations of personal and social crises that one can investigate through the case examples in the surface story. Specifically, according to Egan (1989), storytelling can dramatize binary opposites with the disciplinary content, in order to create dramatic tensions in the backstory that are resolved clearly or with ambiguity by the storytelling-curriculum. We found the backstories to be powerful elements of the Archimedes Project. Some of the binary opposites were just/unjust, narrative/summary, progress/irresponsibility, pure versus applied mathematics; and even mathematics/ non-mathematics.

In our study of the Archimedes Project, what we observed is that the teachers chose a work of literature, which provided such a set of backstory themes that were particularly engaging for the students. We suggest that a sincere interest in learning about the everyday life of one’s students, and the themes with which they routinely engage outside of school, can help teachers to dramatize the content with engaging themes; we also note that a curriculum such as the one under study allows for students themselves to inject themes and moral questions from their own lives, and that this is a way that teachers can use such backstories, that is, that teachers can loosely structure only a minimal set of enough components of their curriculum so that the backstory is co-created with their students through the unfolding curriculum. In the Archimedes Project, the mathematics teacher (our third author) in particular mentioned returning again and again with the students to the following tensions as they continued to blossom in ongoing work: men/women in mathematics; mathematics as an absolutistic subject out of our reality/ as a part of our lives; teaching for the content of curriculum/ teaching around big ideas sublating mathematics teaching; mathematics as culture free/ as connected to culture; using only the mind for mathematics/ using the whole body; listening/ doing; doing mathematics/ playing with mathematics.

What we can sift out of the analysis of the Archimedes Project is that there has been far too much focus on the explicit surface stories, and the production of mathematical literature that features mathematical content. What this well-intended –but, from our standpoint on mathematics education, possibly misguided –effort has produced is an industry of interdisciplinary literature, some well-written and highly engaging, some less so, but most of which sell their value on the inclusion of mathematical concepts. What would be more valuable, we propose, is that the content of the assigned curriculum be used to dramatize important moral questions and complex resolutions or ambiguous acceptance of life tensions. This enables the students to engage with the mathematics in a way that is more powerful because the mathematics curriculum is a surface story that leads the teachers and students to engage with what really matters: the *backstories*. Because stories are inherently multilayered, with multiple backstories, multiple interpretations, and interpretations that differ from individual to individual, they create opportunities for students and teachers to engage in conversations that bring these different layers together in ways that generate clarification, complexification, synthesis, analysis, differentiation, and further forms of intellectual development.

Concluding

In the Archimedes project teachers created a curriculum—an integrative curriculum—around the reading of a literary work, sharing the experience with their students in ways distinctly counter to mathematics as a set of goals, topics, concepts, skills, and procedures that students have to 'learn'. Literacy, storytelling, the use of art, theater, and other genres facilitated a dynamic interaction among students and teachers: to approach mathematical ideas from different perspectives; to focus on big ideas; and, in our case, to emphasize ethical and moral ideas interwoven with mathematical ideas. The use of literature for mathematics strongly encouraged an interdisciplinary approach to knowledge, facilitating the consideration of the curriculum as an entity rather than a set of small, discrete concept and skill objectives.

Paris (1993) writes of teachers constructed as consumers of curriculum; here we can speak about teachers who developed their own curricular experience around the story, developing in the process an innovative, integrative curriculum that also met the needs of the assigned, formal curriculum. What could motivate other teachers to explore this kind of professional encounter and curricular experience? As we can see in a work such as that of Smith & Girod (2003), it is assumed that teachers must be led to feel a strong need even to engage in such challenging work: "If they do not see their existing curricula as failing their students, they have no motivation to engage in this demanding process." Yet, the Archimedes teachers express this very concern as a primary motivator of their undertaking. What unfolded was an ongoing, long-term, creative effort on the part of the group, both social and professional.

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CLIMATE CHANGE CONTROVERSIES IN THE MATHEMATICS CLASSROOM

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In this paper, we investigate the challenges teachers expressed in relation to the inclusion in mathematic classrooms of politically controversial topics like climate change. Based on critical mathematics education, we consider mathematics education to be political and not neutral. Data from two empirical studies, an online survey and a research partnership, were analysed. We found that some teachers saw maintaining neutrality as a challenge, that curriculum allowed little room for controversies, and that they found it difficult to deal with students' opinions and their lack of motivation. However, some stressed the importance of a pedagogy for hope, motivating students to take a stance and to act. We suggest that researchers take an active role by including controversies in the education of mathematics teachers.

INTRODUCTION

"Global warming is a total, and very expensive, hoax!" (Trump, 2013) and "No challenge poses a greater threat to future generations than climate change" (Obama, 2015). The first quote is Donald Trump's tweet from 2013, the second is Barack Obama's tweet from 2015, and these two quotes illustrate different poles of opinion in the climate change debate. Climate change is one of society's most complex challenges for several reasons: it involves physical science, economics, ethics, and political aspects. Levin, Cashore, Bernstein, and Auld (2012) referred to climate change as a super wicked problem, and identified four unique features related to such problems: time is running out; those seeking to solve the problem are also those causing it; there is no central authority; and present policies discount the future irrationally. Barwell (2013) argued that mathematics is crucial to understand, to describe, to predict and to communicate about climate change, and that mathematics education needs to be involved. He suggested that ideas from critical mathematics education are relevant when considering how teaching and learning can engage with issues of climate change.

Climate change controversies entail scientific, social and political aspects. In this paper, we understand political controversy as Hess (2009) defined it; an issue that spark significant disagreement. Hess argued that schools need to face controversies, and raised the important question of what determines whether a topic is a controversy. Depending on context, she defined a tipping point at which a topic move from being a controversial issue to a settled case (and vice versa). There is scientific consensus that climate change exists, and we see this as a settled case. However, there are many political controversies connected to climate change, related to such issues as the effects of climate change, what should be done about it, and who should bear the burden. Therefore, we consider the issues of climate change to be (potential) political controversies. In these disagreements, arguments cased on mathematics are often used.

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For instance, when Trump announced the withdrawal from the Paris Agreement, he argued that China should contribute more because they are the world leader in emissions. Measured in absolute values in 2016, China has 10.4 Gt CO_2e (no. 1) while USA has 5 Gt CO_2e (no. 2). However, measured per capita, China has 8.5 t CO_2e (no. 36) and USA has 20 t CO_2e (no. 14) (Knoema, n.d.). These two different perspectives (absolute values vs per capita) illustrate how numbers can be employed strategically, and challenge the idea that numbers are non-political. Mathematics researchers such as Atweh (2012), Ernest (2009), Mellin-Olsen (1987), Skovsmose (1994) emphasized that mathematics is not objective, value-free or non-political. Public debates about climate change can be based on biased use of mathematics; for example, in the selective use of graphs by interest-organizations. Steffensen, Herheim, and Rangnes (2018, p. 1) stressed that, if citizens view such mathematics as neutral and value-free, they "could be exposed to a political standpoint without being aware of it".

The Norwegian Education Act (1998) emphasized that students "shall learn to think critically and act ethically and with environmental awareness". The general part of the curricula emphasized that climate change is one of the greatest environmental threats globally, and stressed that subjects should facilitated students in gaining insight into, and critically reflecting on, the challenges and dilemmas related to climate change (Ministry of Education and Research, 2017). Although several researchers in mathematics education argue for including controversies in the classroom, and have described some of the challenges this involves for teachers (e.g. Gutstein & Peterson, 2005), few have investigated teachers' perspectives related to including climate change in mathematics education. In this paper, we draw upon data from two empirical studies to investigate the question: *What challenges do teachers express related to the inclusion of political controversies about climate change when teaching mathematics*. The purpose of this paper is to reveal challenges and, by doing so, to develop knowledge about what can support in- and pre-service teachers when including controversial topics like climate change in mathematics teaching.

THEORETHICAL PERSPECTIVE AND EARLIER RESEACH

Gutstein and Peterson (2005, p. 6) problematized the notion that it is possible to teach mathematics in a neutral manner. They exemplified this with a mathematical problem concerning multiplication, with two different context; one involving consumerism; the other including global awareness and empathy. They argued that teachers, who do not bring out the underlying implications of e.g. consumerism, or help students to confront the role of mathematic in the context of important global issues, are making a political choice "whether the teachers recognize them as such or not". However, Monroe, Plate, Oxarart, Bowers, and Chaves (2017, p. 2) stress that many teachers think "their job is limited to conveying factual information", while some facilitated critical thinking and address ethical considerations. Climate change controversies often arise from what is considered to be factual information, and public perceptions of mathematical models as an exact science that is driven by experts with correct answers can cause public dis-interest. Ernest (2009, p. 207) argued that "human interest and values plays a

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significant part in the choices of mathematical problems, methods of solution, the concepts and notations constructed in the process”. An absolutist views on mathematics could facilitate an antidemocratic exploitation; for instance, through careful use of mathematical data by politicians or interest organizations. Ernest claimed, therefore, that a socially responsible mathematics education should create an awareness of this in schooling. Similarly, Steffensen et al. (2018) argued that climate change could serve as a powerful topic to teach citizens to recognize interest-driven use of mathematics.

Simic-Muller, Fernandes, and Felton-Koestler (2015) stressed and problematized the fact that the real-life contexts implemented in policy documents and teaching tend to be “neutral” topics such as shopping. Furthermore, their research indicated that pre-service teachers were less willing to teach controversial topics than other real-life contexts, and that, when the teachers talk about real-life tasks, they “equate real-world contexts with any contextualized or story problems”, e.g. pizza cutting (2015, p. 69). Hess (2009) emphasized some barriers blocking the inclusion of controversies in school, including the fear of indoctrination of students into a particular political viewpoint. Hess and McAvoy (2014) raised the issue that many Americans dispute the fact that climate change exists, and although there is a strong consensus in the scientific community, many teachers choose to teach climate change as a controversy, not as a settled case. Furthermore, Hess (2009) found that teachers used different approaches teaching controversial topics, such as denial, avoidance, and balance, as well as privileging of a particular perspective. For instance, a balanced approach would be if teachers’ deal with the existence of climate change as if there were two equally valid sides; and an avoidance approach would be to exclude the topic. Several studies, as reported by Hess (2009), found that many teachers avoid controversial topics and do not confront political interests and values. Similarly, Atweh and Brady (2009) reported “resistance by many mathematics teachers and curricula developers to deal with controversial social issues as a source of examples of mathematical problems” (p. 271). They further argued for a “willingness to deal with controversial topics in which debate and difference of opinion and interests are part of the equation rather than nuisance variables” (p. 272). Furthermore, Atweh (2012) emphasized that mathematics education should shift towards an ethical and responsible approach, including changes in the curriculum, and more problem solving, modelling and real-world activities. Similarly, Jurdak (2018, p. 30) argued for changes because present curriculum could “constrain the ability to interrupt inequality because of its structure and orientation”, and suggested that curriculum should facilitate the learning of mathematics in rich social contexts.

Abtahi, Gøtze, Steffensen, Hauge, and Barwell (2017) addressed the issue of how the inclusion of climate change in mathematics education can be a moral and ethical act. With regard to whether it is right to expose students for problems like climate change in the classroom, they argued that students already face these issues, and that the real question is whether they should deal with them alone or in dialogue with others. Similarly, Simic-Muller et al. (2015) reported that pre-service teachers believed that,

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if they protected the students from difficult problems, they were teaching with care. A related issue was raised by Wals (2011), who problematized the way that education for sustainability could lead to a sense of hopelessness, which in turn could lead to paralysis. It was considered that such problems of great magnitude and complexity could easily overwhelm citizens. To address such hopelessness, Freire (1992) introduced a pedagogy for hope, arguing that practice should always be rooted in hope; otherwise, students could feel powerless in relation to the possibility for change.

Nicholls (2017) found that it is important to consider teachers' beliefs when teaching climate change because political views influenced decisions about inclusion/exclusion of topics in classrooms. Similarly, Steffensen et al. (2018) stressed how the controversies related to climate change and teachers' values could influence teaching. For example, two teachers designed different questions to emphasize certain issues; while one emphasized the increased rate at which Arctic ice was melting, the other stressed how small the rise in global temperature had been. Both teachers used mathematics to support their claims. Namdar (2018) found that when pre-service mathematics and science teachers included climate change in their teaching, they considered it a benefit that they could encourage students to act, raise awareness, increase content knowledge, and influence future decision-makers. The challenges they perceived included students' disinterest, teachers' preparedness, technical difficulties related to teaching, the controversial nature of the subject, and existing misconceptions. The above-mentioned research on controversies and climate change concerns mainly pre-service teachers. In this paper, we discuss findings related to the challenges raised by in-service teachers and these findings are linked to theory and previous research.

METHODS

The data for this paper was collected from two independent studies. The first was an online survey. This was conducted to gain insight into teachers' thoughts on teaching mathematics in the context of climate change, their aims, what they emphasised, and any challenges they experienced (see also Abtahi et al., 2017). The survey was posted in a Facebook group for mathematics teachers, and the total number of respondents was 72: with 15% at primary school level, 43% at lower-secondary school and 47% at upper-secondary school; some worked several places. All questions were voluntary, which meant that some questions had fewer than 72 respondents. Both closed and open questions were included, but in this paper only the answers to the open questions have been analysed. The second data was from a one-year research partnership with three mathematics and natural science teachers in lower secondary school. This was inspired by action research (see also Steffensen et al., 2018) and researched into how climate change could facilitate students' development of critical perspectives. A strategic sample was obtained based on previous knowledge of teachers' engagement in both climate change and mathematics. The data collected included video recordings of seven partnership meetings and 42 classroom interactions, audio recordings of interviews, written materials from teachers and students, and field notes.

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In the analysis, utterances from the partnership meetings and answers to three survey questions provided the basis for answering the research question. The three questions where we identified challenges were: “What challenges do you experience when teaching climate change?” (Question 1 with 41 responders); “We would be grateful if you would elaborate on your thoughts on climate change in mathematics education” (Q2, 38); and “If you had sufficient resources, how would you like to work with climate change in teaching?” (Q3, 35). All data was transcribed, coded and categorized into themes by using NVIVO. Six central themes emerged from the data (number of utterances in parenthesis): No challenges (3), Content knowledge (25), Teaching resources (16), Practical issues (10), Objectivity (10), and Values (14). These were partly overlapping; for instance, utterances related to objectivity could also take up teaching resources. We excluded challenges of a more general nature and those with weak links to political controversies. For instance, although the number of utterances indicated that content knowledge was regarded as an important topic, the teachers did not connect this to the political controversy, and explicitly concerned teachers and students content knowledge. We then had four themes left: Teaching resources, Practical issues, Objectivity and Values. The utterances from the three teachers were matched with categories established for the online survey for consistency of perspectives. In the results and discussions section, we present various challenges included in the four themes that structure the text. We have chosen to analyze some representative utterances from each theme to exemplify these challenges.

RESULTS AND DISCUSSIONS

The theme *teaching resources* involved both those designed specifically for teaching (such as textbooks with tasks, ICT, and video) and those available in the media (such as web sites and news). An example of the former was an utterance from the survey in which the teacher problematized the types of task used in mathematics education: “Fewer tasks with "Per and Kari should pick strawberries" and more "Emissions of greenhouse gases makes up..."” (a respond to Q2). By comparing a task concerning greenhouse-emissions with one involving old-fashioned and typical Norwegian names like Per and Kari and a common Norwegian activity like picking strawberries, the teacher problematizes this rather indifferent semi-realistic reference and calls for a more meaningful real-life reference. Although both tasks refer to reality, there is an important difference between these two real-life contexts: while one context often simply exemplifies mathematics processes such as division and fractions, the other context can involve complex issues as well. For instance, the very nature of the greenhouse gases issue means that it has the potential for political controversy and, if students disagree with measures such as taxes on greenhouse-gases, this could spark disagreement in the mathematics classroom. This teacher’s problematization of the use of different real-life contexts was in contrast to the findings from Simic-Muller et al. (2015), where the pre-service teachers did not differentiate between the different contexts real-life task could have. Another teacher from the survey stated that it was a challenge to find “Good and up-to-date resources that are not political” (a respond to

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Q1). This utterance calls for "non-political" resources, hence a more neutral perspective, and it is not recognized that, as research has revealed, neutral teaching of mathematics is impossible, including mathematical task and teaching resources (cf. Gutstein and Peterson, 2005; Simic-Muller et al., 2015).

The theme *practical issues* included challenges related to lack of time, available ICT and other resources, large classes, and the curricula. Here we focus on concerns related to curricula. The teachers from the research partnership found it challenging to balance the extensive list of competence aims in mathematics, the pressure to prepare students for exams, and the desire to include climate change in mathematics teaching. Teachers from the survey, gave following utterances: "Unfortunately, we must prioritize getting through the curriculum" (Q3) and "Do not expand subjects with topics not explicitly mentioned" (Q2). Thus, the teachers hesitated to expand subjects with topics not mentioned and prioritized in curricula, since these are less likely to be tested on exams. This can be interpreted as a conflict between their responsibility for student's grades and their desire to include important real-life problems. Subject curricula and competence aims guide the teacher's choice of problems in mathematics education and, if an absolutist view of mathematics has influenced the curricula (cf. Ernest, 2009), then the curricula may become an obstacle hindering the inclusion of important social issues, as the above mentioned utterances exemplified. A curriculum should not hinder the inclusion controversial socio-political problems (cf. Atweh, 2012; Jurdak, 2018), and changes are necessary so that curricula can provide space for teachers and students to engage in mathematics related to social problems. Although the teachers in the research partnership expressed concerns about including climate change and balancing competence aims and exams, they chose to do so by taking an interdisciplinary approach (cf. Atweh & Brady, 2009). Curricula changes take time and, if we think it is important to include controversial topics like climate change, then we have to find ways to support teachers in their efforts to balance between competence aims, interpreted, enacted and experienced curriculum; to prepare students for exams; to prepare students to be(come) engaged citizens capable of discussing real-life controversial topics.

The theme *objectivity* focused on how teachers' and students' political standpoints are perceived to be a challenge, and how the ideals of objectivity and neutrality are maintained. One teacher from the survey considered it a challenge to "be objective" (Q1). One way to interpret this is to be unbiased when presenting a topic or when choosing specific tasks (cf. Steffensen et al., 2018), another way can refer to not influencing (or indoctrinating) students (cf. Hess, 2009). Another teacher from the survey said that it was a challenge "to be neutral in the debate" (Q1). This teacher's concern about remaining neutral (not taking a part or supporting a position) in the climate change debate could indicate that their aim is to present themselves as neutral in classroom debates. A relevant question then becomes whether it is possible to teach climate change or mathematics in a neutral manner (cf. Gutstein & Peterson, 2005). A related utterance from the survey is: "There are many views in the climate debate. Teachers are not exceptions and I am one of them. If we are too eager to present our

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own views, it will be easily mistaken" (Q2). This teacher recognizes that there are many views on climate change among teachers, and expressed the opinion that to present one's own views is challenging because it can be "mistaken" (it does not say whether by the students, their parents or the school administration). Underlying this utterance may be a fear of doing harm, thrusting opinions, or being forced into conflicts with students (or others) who disagree. It is safer to simply convey the factual information that many teachers see their job as being limited to (cf. Monroe et al., 2017). Presenting controversial topics and, in particular private views, can be challenging for teachers for fear of being accused of indoctrinating students with political viewpoints (cf. Hess, 2009). The teachers in the research partnership emphasized not only presenting factual knowledge, but also engaging students in critical thinking and ethical consideration of climate change through discussions and debates. In these teaching activities, students' opinions were facilitated and were recognized as a productive contribution to learning. Contrary to this recognition to students' opinions, some teachers considered it a challenge that students had opinions, as this utterance from the survey exemplifies: "Students have strong opinions from home or from their own "research" online" (Q1). The teacher recognized that students can hold "strong opinions", which they might have obtained from parents and/or from online resources. Why this is seen as a challenge, is not explained. If a student's contribution is not what the teacher considers appropriate, this could potentially give rise to political controversies in the classroom. To facilitate debates, critical discussions and that airing of different opinion (cf. Atweh & Brady, 2009), can challenge the understanding of mathematics education as neutral, objective and value free (cf. Ernest, 2009; Mellin-Olsen, 1987).

The theme *values* included teachers' concerns about motivation and engagement, as well as ethical considerations. One example related to teacher motivation is the following utterance from the survey: "This is an important area, but the teacher must be personally involved to prioritize it" (Q2). The teacher identified climate change as an important topic, but express that teachers have to be engaged at a personal level in order to make space for the topic. Given the demanding curriculum with no explicit competence aim on climate change, this highlights the teacher's role in bringing climate change into the mathematics classroom. Two utterances concerning student motivation were: "Some of the students care absolutely zero" (Q1), and "You hear too much about it and that nothing helps, it gets a bit weary" (Q1). The first utterance recognized that some students do not care about climate change. The wording "absolutely zero" stresses that this lack of engagement is perceived as a challenge for the teacher. The second utterance offered perceived reasons for students' apparent lack of motivation, namely too much focus, in combination with a feeling of helplessness (cf. Freire, 1992; Wals, 2010). The magnitude and complexity of climate change can easily overwhelm citizens and lead to paralysis. It seems to worry the teachers that students do not care, even though the topic is so serious and can potentially have a huge negative impact on our future. The teachers from the research partnership also expressed concern about student motivation, as one utterance exemplifies: "It does not really mean anything, 180 ppm [parts per million], oh well, yes, and then these girls

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think, can we do our make up now?" The reference to 180 ppm illustrates an example of how students are indifferent to figures from climate change publications and, from the teacher perspective; do not mean anything to them. A second utterance picks up this theme: "Everyone agrees to do something about it, but they [students] don't do anything about it themselves". This expresses a concern that the students "don't do anything". The teachers from the research partnership discussed how they could motivate and engage students in climate change, and make them believe that individual action does indeed contribute. This utterance illustrates this:

> "And I think that in one way it is important that they make conscious choices, for instance with respect to consumption as we discussed. Somehow do something to stop climate change… and not just sit there playing PlayStation. So I think in a way it is important to also show that, if everyone contributes, it is actually possible to do something".

The teacher stressed two key actions here, "make conscious choices", and "do something", and emphasized how important it is that "everyone contributes". The focus is on providing students with hope that actions matter, hence a pedagogy of hope. Another challenge raised was allowing students to face the gravity of climate change. One teacher from the survey described it as a challenge "To explain the severity combined with the lack of action". The teacher does not explain why this was perceived as a challenge, but it could be interpreted as a reluctance to worry the students, and a desire to shelter them from a serious problem (cf. Simic-Muller et al., 2015). The weakness with this kind of argument is that students probably are already aware of the severity of the problem and the "lack of action", and then the question becomes whether they should face this without the guidance of teachers (cf. Abtahi et al., 2017).

CONCLUDING REMARKS

The challenges teachers identified in relation to including climate change controversies in the mathematics classroom were associated with teaching resources and practical issues, and they problematized the fact that the curriculum allowed little room for teaching complex issues. They considered the curricula to be their guidelines, and when controversial topics were not mentioned explicitly, some said that these would not be included in their teaching plans. Some described it as a challenge that students lacked motivation, while others problematized the fact that some students had already formed opinions outside school. In contrast, some teachers stressed that tasks connected to climate changes could be meaningful, and others that it was important to encourage students to take a stance, act and learn from experience that we all can do something. With regard to the objectivity, we found that this appeared within several themes (explicit and implicit), although with different perspectives. For instance, some teachers expressed concerns regarding teacher objectivity in the classroom and called for non-political teaching resources. We maintain, like Gutstein and Peterson (2005), that when mathematics teachers choose to strive for neutrality and objectivity in mathematics education, or choose to include/exclude controversial issues like climate change, this is already a political standpoint. Many of the challenges described by the in-service teachers in the online survey and research partnership are similar to those

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pre-service teachers have highlighted in other research. This indicates the importance of an increased focus on raising the awareness of teachers, teacher educators and researchers regarding controversies such as climate change. Since teachers may consider an approach based on controversies and critical mathematics education to be very demanding, we suggest that a critical focus on this issue be implemented in teacher education. The aim of such an addition would be to give pre- and in-service teachers the opportunity to practice and gain experience in teaching controversial topics, and to cope with challenging issues such as ethical and objectivity concerns. Furthermore, more research is needed on controversies in the curricula, teaching resources and the question of how to implement controversy themes in classrooms.

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UNDERSTANDING SEGREGATION: UPPER SECONDARY SCHOOL STUDENTS' WORK WITH THE SCHELLING MODEL

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There are few research studies focusing on how humans can better understand segregation using mathematical models. In this paper, we explore how upper secondary school students work with the Schelling model using a computer game that was purposely designed for this study. The students were then allowed to run the model themselves. The results show that it was difficult to anticipate the degree to which segregation is generated within the model. The students mainly gave two types of explanations for the results. The first one was based on human psychology and the other was based on the mathematical principle that underlie the utility function of the model. The results are discussed from a perspective that illustrates the complexity of the subject, rather than as a measure of the teaching intervention.

INTRODUCTION

Segregation is a strong and a widespread phenomenon in society. There is segregation by, for instance, sex, age, income, or religion (Schelling, 1971). A common research area to study patterns of segregation is in choice of residential area, but the core mechanism works for other areas as well, such as school choice (Spaiser et al, 2018). We can see the results of segregation for instance in students choosing between different schools and how it affects teachers' choices of workplace (e.g. Bertilsson, 2014). Sweden is an interesting case since it is a country that is part of the so called Nordic model with a century of policy making based on equity and equality:

> Structurally, the Nordic model consisted of a public, comprehensive school for all children with no streaming from the age of seven to sixteen years. The overarching values were social justice, equity, equal opportunities, inclusion, nation building, and democratic participation for all students, regardless of social and cultural background and abilities. (Imsen, Blossing & Moos, 2017, p. 568).

The history and these norms are visible in the curriculum and the various syllabi for Swedish schools. For instance, in the curriculum for the upper secondary school, it is stipulated that,

> According to the Education Act, the education should be carried out in accordance with fundamental democratic values and human rights, covering the inviolability of people, the freedom and integrity of the individual, the equal value of all people, gender equality and solidarity between people. (Skolverket, 2013, p. 10)

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Furthermore, we can read that all students should "consciously determine their views based on knowledge of human rights and fundamental democratic values, as well as personal experiences" (Skolverket, 2013, p. 10) and that all who work in the school should "actively promote equality of individuals and groups" (Skolverket, 2013, p. 10). Teachers have the responsibility to educate the students about these norms and they should

> "make clear the fundamental democratic values of Swedish society and human rights, and together with the students discuss conflicts that can occur between these values and rights and actual events" (Skolverket, 2013, p. 11).

Even in mathematics education the social perspective is visible; for instance, it says that one of the aims of mathematics is to help students to understand society with the help of mathematics. There is an increasing emphasis in Swedish policies saying that education should focus on diversity and inclusion (Imsen et al, 2017). At the same time, segregation is increasing in society (Spaiser et al, 2018).

The mechanisms generating segregation make it a far trickier subject to tackle than just educating people about equity and equality. Several modelling studies have shown that segregation can occur even when people prefer diversity (Pancs and Vriend, 2007; Tsvetkova, Nilsson, Öhman, Sumpter & Sumpter, 2016). Although sharing the same values and norms as the other countries described in the Nordic model, Sweden has a high level of school segregation (Spaiser et al, 2018). One conclusion is that public policies are not enough to improve integration (Pancs & Vriend, 2007). Hence, we need to study individuals and how they understand segregation, especially if they have had a chance to understand the mechanism behind it. At the same time, most policy making is based on the predictions of a segregation model, the Schelling model. Segregation is then viewed as a mathematical phenomenon, in the sense that individuals follow predefined rules about where to live based on their neighbours. These rules are then implemented in the form of a mathematical model (Schelling, 1971).

Looking at research, not many studies have focused on the predictions of the Schelling model from an empirical point of view (Tsvetkova et al, 2016), and so far, there appears to be only one study, by Tsvetkova et al (2016), looking at students' interaction with the Schelling model. Here, we present further analysis of data, and the focus is to study in what ways students, after interaction with this particular mathematical model through a computer game and a short lecture about segregation, will be able to make reasonable guesses about the levels of segregation and perceived happiness. This is the aim of the study. The research questions posed are: (1) How do upper secondary school students' guesses differ from the modelling results when working with the Schelling model? and, (2) What are their explanations of these differences based on?

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THE SCHELLING MODEL

The starting point for looking at segregation is the study of collective behaviour: the idea that there "is a curious mathematical consistency among certain human activities" (Davis & Sumara, 2008, p. 168). It has been concluded, based on the work of Schelling, that segregation is not necessarily the result of one particular individual's actions and choices, or desires and attitudes (Davis & Sumara, 2008). It is more about a chain of reaction in a collective group. The Schelling model is a mathematical model that translates unorganized individual behaviour into collective behaviour. By using the squares on a checkerboard to represent, for instance, individual houses in a neighbourhood and tokens of two colours to represent two different groups of people, the model starts with a fully integrated society with the tokens randomly distributed over the board (Schelling, 1971). A series of simulations can then be made using various assumptions. The basic idea is that each individual of one group has a set tolerance level for the share of the other colour token in a neighbourhood. If the number of the other colour is in excess of the tolerance level then the individual moves to a new location. The model shows that even when groups are reasonably tolerant, segregation happens quickly and inevitably.

In Tsvetkova et al (2016), we used four different assumptions for utility functions, all representing different preferences. Here, we analyse one of them – the Same and Diverse game (c.f. Pancs & Vriend, 2007). In this game, individuals strive for similarity up to a certain level in combination with a preference for some diversity. It has been concluded that such a function has practical implications regarding equity and equality (seen as preference for some level of diversity) and can be taught in school and implemented in policies (Tsvetkova et al, 2016). This is in line with the idea of the Nordic education model where schooling was thought of one important part of educating the members of the society into openness and flexibility (c.f. Imsen et al, 2017).

METHODS

First, we describe the data collection including how the computer game was designed, and then how data were analysed.

Data collection

One conclusion that was made in Tsvetkova et al (2016) was that most game experiments found in our literature review did not allow communication. Since we wanted the students to be active and engaged, we set up a different design. The aim was to have a game that was simple, intuitive, and engaging. The solution was a two-dimensional Schelling game using a six-by-six square grid using two colours of tokens, blue and yellow. The colours were chosen as neutral colours (that also happen to be the colours of the Swedish flag).

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In total, 20 upper secondary school classes participated with a total of 399 participants. They were from all three grades meaning that the students age span was 16-19. Each class had between 13 to 25 students and they came from different educational programmes, both vocational and programmes preparing for university studies. The classes came from three different regions in Sweden (east, middle and west). Each student was assigned a number and colour and they borrowed a surf tablet to use as game controller. The game was then projected on a screen so everyone could see their own movement on the checkerboard as well as their neighbours' movement. They could move their avatars to an empty spot, but only up, down, left, or right, not diagonally. The students played four games with different scoring rules that were equivalent to the four utility functions. (For a more detailed description of the interpretation of the Schelling model into computer game, see Tsvetkova et al, 2016.)

Utility functions	Simulation Outcomes	Experiment Outcome
Same	84.2	100
Diverse	51.9	41.7
Same and Diverse	72.9	62.2
Same or Different	8.3	63.4

Table 1: The different functions and outcomes, results are proportion of same type neighbours in percentages (%).

The students also answered one questionnaire before the game and one after the game, both questionnaires were about background information such as gender, residential status and grades. After playing the game the students listened to a short lecture about segregation, how it works and how the game they just have played describes different aspects of segregation. As the final step, the students were asked to run the model themselves and write down answers to a series of questions on a worksheet. In total, the lesson was planned and timed to be one hour long. As it was planned, the students were considered both as learners but also future communicators about segregation, as well as research colleagues since they participated in generating results for the modelling. They participated in not only the games, but also running the modelling themselves and in both situations they needed to communicate and negotiate the meaning of their movement on the board but also interpreting the results when running the model. This is in line with previous work described by Lerman (2001).

As discussed in Tsvetkova et al (2016), during the game itself, very few groups managed to achieve a global coordination although most groups had high level of

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communication. The game was very dynamic with an average of 3.8 moves per second.

In this paper, we analyse a subset of the data generated by 14 of the 20 classes. The reason for this was that for the first five groups, we tried a different worksheet that focused more on the use of, rather than the understanding of the model. One group was excluded since they did not have the time to complete the worksheet. In the end, 272 of the total 399 students' worksheets were analysed. Of these 272 responses, 267 had filled in all the information which gives a response rate of 98%. Although running the models themselves on the surf tablet, they were allowed and encouraged to discuss the matters with their neighbours. The reason for allowing collective reasoning was the principle of wisdom of the crowd (c.f. King, Cheng, Starke & Myatt, 2011).

In the present paper, the focus is on the first question. The information given to the students was the following:

> To run the simulation, first you need to adjust the parameters and then select the model. Then you can start the actual simulation. When the simulation has stopped, you write down the final average percentages for similar neighbours and the final perceived happiness/ content. Repeat the simulation 3 times for each model in order to arrive with reliable results.
>
> **Task 1.** Which level of segregation do you get when individuals prefer both mono-culture and diversity, i.e. you are happy both with neighbours that are similar to yourself and neighbours that are different?
>
> Before you run the simulation, guess the results.

At this point in the intervention the students had experienced playing the game, which involved interacting according to the utility function, and experiencing seeing their own points changed as their numbers of neighbours changed.

The students were now asked to guess both the average of similar neighbours and the average percentage of happiness. Then they were instructed to run the models (see Figure 1).

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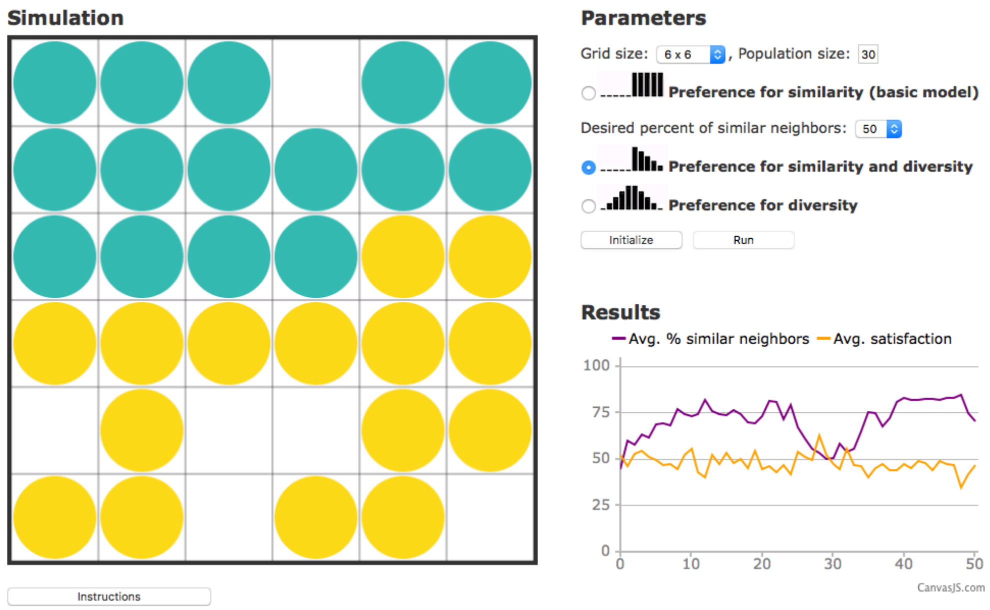

Figure 1: The Schelling model as presented to the students.

Lastly, they were asked to comment on their guesses in relation to results from the simulation. The formulation was the following:

> Now you compare your guesses with your results. Any difference? In what way? What do you think is the reason for the results of the simulation?

The questions were aimed to focus on how quickly segregation is created even though people prefer both colours, i.e. the Same and Diverse game.

Method of analysis

The data were analysed by summarising all the guesses from the students and calculating the average. The guesses were compared to the modelling outcome. The simulations were run for 100,000 time periods and replicated 1,000 times. Since the research question is about how students' guesses differ and not a question whether they are significantly different, we use only descriptive statistics such as mean and mode. The written comments from the students were analysed using thematic analysis (c.f. Braun & Clarke, 2006) looking for patterns in the responses. A first analysis showed that there were mainly two types of comments: comments on how close/ far away their guesses were from the simulation outcomes, and comments that contained some information about how the students interpreted the situation. Only the latter group was analysed using thematic analysis. The responses were grouped looking for patterns (similarity and difference) and as a second step, the themes were controlled against each other but also back to the original data. The reason for the second step is because themes had to "cohere together meaningfully, while there should be clear and identifiable distinctions between themes" (Braun & Clarke, 2006, p. 91). Here, the focus is not how many responses falling into the different categories,

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but to present the themes. The data were collected and handled according to the ethics stipulated by the CODEX given by the Swedish Research Council.

RESULTS

The results are divided into two parts. In the first part, we have the descriptive statistics of the guesses made by the students and the outcome from the modelling. The second part is the results from the thematic analysis.

Students' guesses and modelling outcome

The results from the students' guesses and the simulation are presented in Table 2:

	Students' average guess	Modeling outcome
Similar neighbour	65.76	79 ± 1.3
Happiness	64.08	47 ± 2.6

Table 2: Students' guesses and simulation results given in percentages (%). Numbers in parentheses indicate standard deviations in simulation outcomes.

As we can see in Table 2, the average guess differs as the students underestimated the degree of segregation and overestimated the degree of happiness. However, looking at the mode of students' guessing about similar neighbour, the most common guess was 70, which is closer to the modelling outcome. The range of the responses was 90. The mode for happiness was 50, which is close to the outcome from the model, and the range was 94. This variation is reflected in the students' comments.

Students' explanations

When analysing the students' responses, some students commented on how different the results were compared to what they expected. Some students just stated 'it was different' or 'it was x % off'. Such responses were left without further analysis. One example was the following (where S and number indicate the response):

> it was opposite what I thought it should be. [S264].

The responses that had some information about how the students interpreted the situation were grouped into themes. Two themes were the result of this analysis. One theme was explanations based in the psychology of people:

> Perhaps it is because people are never content. [S54]; Slightly less average percentages (similar neighbours) and a big difference in average happiness. You rather want to be

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> surrounded with people that are similar then different. [S85]; People want more segregation than I thought. [S107].

The argumentation does not separate between human behaviour and the mathematical modelling, as if the students see the tokens as humans just as when they themselves played the game. Some students even gave the tokens human feelings, here illustrated by the following response:

> The degree of happiness regarding culture should be 100% since you have no problems. The model is based on parts more than how complex feelings are. [S224]

Some students anticipated the high degree of segregation and the low level of happiness and provided some explanations using human behaviour:

> Not such mega difference compared to my guess. You don't want to be someone who is against the stream, most people don't anyway [S261]

It is not clear whether these students had understood the difference between the mathematical model and the application of the model. Since we do not have further information about how to interpret these statements, whether there is a deeper understanding that lies behind these comments, the conclusion drawn here is that it appears that some students did not make the distinction between the simulation and when they played the computer game, the latter being the case when the tokens did represent a specific person.

The second theme appeared in students' responses that referred to the mathematical principle behind the utility function:

> The players want to have a security to still have more than 50% of similar neighbours even though one neighbour is moving and then you could have lower degree of happiness in order to gain more security. [S258]; "The tokens" want to be sure to have more than 50% of their own colour. [S224]; the neighbours are more similar to each other in the simulation compared to my guesses and the degree of happiness is lower. Everyone could probably not get 50% independent how they move as long as they are not collaborating. [S187].

These students appear to have understood the fundamental principle of the model and the predicted outcome. Some commented on the complexity of segregation as described by the computer game the students just played. Here, this is illustrated by the following two responses:

> Even if they wanted diversity, they were attracted to each other. Domino effect. |S13]; When someone is dissatisfied, they move and that ruins it for their neighbours. When everyone is ok happy, it stops. [S47]

Several students mentioned the domino effect, the chain of actions that is created when one token moves. Based on these comments, we see that some students appear to understand the model and how quickly segregation is created even though preferences allow both similarity and diversity.

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DISCUSSION

The aim of the paper was to study upper secondary school students' guesses about the level of segregation after participating in a class designed to stimulate understanding of segregation using the Schelling model. As described earlier, the Same and Diverse game which is in focus here, is of special interest since it has practical implications and is thought of as being able to be taught in school and through policies (Tsvetkova et al, 2016). In regard to the curriculum and norms and values such as inclusion, equity and equality, the design did allow the students to "discuss conflicts that can occur between these values and rights and actual events" (Skolverket, 2013, p. 11). Looking closer at the students' explanations to outcomes in relation to their guesses, there are two main themes besides the answers that just on a superficial level commented on how far/ close they were in their guesses. These two themes were 'psychology of people' and 'the mathematical principle of the model', where the first indicates a struggle to separate between the model and the computer game and the other indicates that some students understood how the game worked, including the chain of actions (c.f. Schelling, 1971).

Although using the possibility of collective reasoning, in line with the idea of wisdom of the crowds (King et al, 2011), the results illustrate how hard it is to understand exactly how quickly segregation is created even in the most beneficial circumstances such as in the Same and Diverse game. However, using similar reasoning as Lerman (2001), the written texts are not end results but more tools for learning. Therefore, we don't see the guesses as a measure, although relatively close, of how effective the lesson design was, but more an indication of how complex this area is. Davis and Sumara (2008) use Schelling's work as an example of sociological work that is also connected to psychology and human behaviour. We think the results from our study function as a further illustration, now with an educational perspective. Some of the comments from the students support this conclusion: although the mathematical model and its tokens do not have any emotions, they describe and model on group level individuals who do have emotions and desires.

One possible implication would then be that this is a complex topic, but one way to fulfil the norms described in the Nordic education model (c.f. Imsen et al, 2017), norms that are embedded in the Swedish curriculum (c.f. Skolverket, 2013), is to talk about this also from a mathematical point of view, using the computer game that we designed. Although the results from our study show that some students do struggle to transfer from the computer game situation to the modelling situation, we think the tool offers a starting point: the incredible activity of the students that was measured (Tsvetkova et al, 2016) suggests that it is a good way of initiating a conversation. However, we also understand that although we designed a lesson where the students participated in many different educational tasks, one hour of active work appears not enough to tackle such a complex topic. Also, we did not offer what a teacher could: a follow up class where potential misunderstandings can be dealt with. Therefore, we

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think this is a good topic for further elaboration, especially when considering that this is a research area that has just started.

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SOCIAL ASPECTS OF SHIFTING STANDARDS OF VĀKYA ALMANAC COMPUTATIONS

Venkateswaran TV

Vigyan Prasar

Abstract: Vākya algorithms consisting of versified mnemonic sentences, encoding the 'tables' of the periodically recurring true positions of the celestial bodies were used by the public official called 'panchangin'(or panjangan) in south India to compute the almanac from about 1100 CE. The paper examines the reform of the Vākya pañcāngkam, during the 1860s, as the computed planetary positions did not match with the observed results, and shows that the demand for 'accuracy' resulted from a complex interplay of science and the social. 'Public(s)' were recruited, public communication/ astronomy-mathematics popularisation undertaken to result in the epistemological standards. To the extent 'standards of accuracy' are social, the computational methods could be seen to be changing in line with shifting necessities of the society.

INTRODUCTION

Temporal order of the religious and social culture of the South Indian medieval society was based upon the cosmic confluence of the nine-luminaries (navgrahas- five visible planets, Sun, Moon and two invisible shadow planets rāhu and ketu). A unique mode of computing pañcāngkam (native almanac), called Vākya method, consisting of series of algorithms to compute positions of 'navagraha ', inspired by the epicycle siddhānthic tradition of astronomy, was followed from around 1100 CE to compute the confluence and find the appointed time for various civic and cultural activities (see Sarma, K.V., 2008, 2008b and Sriram, M.S., 2014 for overview of Vākya system). These activities were performed by a public official called 'panchangin' (or panjangan), a practice, perhaps commenced around 1100 CEs, continued unhindered during the Mughal rule in medieval south Indian regions, endured the Company Raj, and came to an end with the rise of modernity during the early nineteenth century (e.g. see Buchanan, 1807: 632 and 645; Ellis, 1818; Maitland, 1843: 105; and Warren, 1825). Using the versified mnemonic sentences which encoded the 'tables' of the periodically recurring true positions of the celestial bodies, the panchangin interpolated the position of the celestial object for any desired time. For the ease of memorisation, the numbers were converted into 'words' and 'sentences' (Vākya) with each alphabets denoting a 'numeral'[1]. 'Correction' and 'improvements' were made in the Vākya pañcāngkam computation system many times, once around 1300s, then during 1500s and later around 1860s primarily because the computed planetary positions do not match with the observed results.

Scholars posit internal demand for 'accuracy' as the main or sole source of motivation for these reforms. However, a careful examination of the reforms of the 1860s initiated at the behest of Ragoonatha Charry, a native astronomer who lived during 1860s (see Shylaja, B.S., 2009 &2012) indicate a complex interplay of science and the social. The paper explores the social context of the Vākya almanac reform, in particular during the mid-nineteenth century and argues that to the extent 'standards of accuracy' are social; the computational methods could be seen to be changing in line with shifting necessities of the society.

TIRUKGAṆITHA PAÑCĀṄKAM

The ultimate goal of finding appropriate time depended on working out Pañca (five) + [a]ñga (limbs/pars) aspects of pañcāṅkam, vis vāra (weekday), tithi(lunation), nakṣatra (position of the Moon in the ecliptic), yoga (sum of the angle of Moon and Sun) and Karaṇa (half of a Tithi). The Pañca- añga, depend upon the true positions of the nine celestial luminaries. The true positions were computed using the Vākya manual, while the procedure for fixing the Pañca- añga and making prognosis from the same depended partly on conventions of a particular sect within the traditional society and partly on dharmasastras[2].

However, during early nineteenth century the discrepancy between the predicted circumstance of eclipses parameters like the timing of various contacts, magnitude and so on by Vākya manual and the observation of actual occurrence was noticeable (see Venkateswaran, T. V. 2018 and Young, R.F., and Jebanesan, S., 1995 for such an event). An astronomer in the service of the Madras Observatory, Chinthamani Ragoonatha Charry and a traditional pañcāṅkam maker Sundaresa Srautigal initiated a radically different way of casting the pañcāṅkam using the data from modern astronomical nautical almanac rather than following the tables, data and procedures stipulated in the Vākya text. Colloquially this was called as Tirukgaṇitha pañcāṅkam, a corruption of Dṛk+gaṇitha, meaning that 'which agrees with the observation'.

While a section of the Tamil society welcomed it as timely and much-needed reform of the ancient system, another section viewed it as a blasphemous capitulation to 'modernity' symbolised by the 'western rule'[3]. Ragoonatha Charry lamented "many do not even listen to the arguments; they immediately retort.. 'stop...stop... stop… eclipses are caused by demonic agents rāhu and ketu... you appear to have joined with English and are speaking against our scared purāṇas... in few days you are going to immerse in English religion and going to get excommunicated from our caste..." (Charry 1871:10).

Arguments in support and dissonance spilled into the pages of newspapers[4], pamphlets[5] and numerous 'sádas' conducted by various religious institutions and palace administration. While the conservatives argued, by providing intricate interpretations of quotes from reputed ancient āccariyās[6], that if the ancient texts were not followed for observing the rituals, disasters would result, the progressives demonstrated their computation with visual corroboration. The key points of the debate were around what (all) counts as evidence, whom all constituted the 'erudite experts', and proper conduct of academic debate to arrive at the proper modalities of social order.

PASSIVE CONSUMER TO DEMANDING CUSTOMER

Hitherto, religious and political institutions such as maṭam[7] and ātīṉam[8] camastāṉam[9], provided patronage to in-house pañcāṅkam maker, who computed and circulated pañcāṅkam for their own following. The village level panchangin held one or more such hand copied pañcāṅkam circulated by these institutions. A contemporary source says, in Bengal province around "1820[s],.. Almanacs were in manuscript, and were copied and sold by the Daivagya Brahmans" (Calcutta Review 1850:153) who were astrologers with skill in signing. They had "under their arm an Almanac wrapped in cloth" (ibid) and received "contributions from the poorest, and are admitted even into the recesses of the female apartments"(ibid) The Almanacs issued by the Court of Raja Kristna Ray of Nadiya was "held in highest repute; next to that, the Bali one" (ibid).

In Tamil region, pañcāṅkams issued by the courts of Thanjavur and Pudukkottai were popular (on Pudukkottai pañcāṅkam see, Ayyar 1938: 220). The colonial rulers too added a 'native almanac' in their 'Madras almanac and compendium of intelligence'. Sree Rama Sheshan Shastri, native astronomer in the employment of the Madras college (see Madras Male Orphan Asylum 1839) computed the native almanac. The missionaries too came out with their own, for example, Tranquebar Mission Almanac and Jaffna almanac (see for example Dashiell, G., 1835).

As long as the pañcāṅkams were produced by the feudal patronage, circulated through the village panchangin, the audience was mostly just a passive consumer having little role in the production or circulation. However, with the emergence of a marketplace for the pañcāṅkams, the relation between the audience and the producers of pañcāṅkams significantly changed. The emergence of the printed pañcāṅkam caused a rift in the traditional arrangements of production, circulation and consequently authentication. The consumers, who now were not just village panchangin, but also ordinary 'public', could directly purchase a printed pañcāṅkam without needing the intermediaries. Further, although legitimisation by the religious institutions was still imperative, as the economics was based on the market and sales, the royal and religious patronage became redundant.

Even while the colonial administration published its version of pañcāṅkam, as part of the yearly almanac, the compiler had been a traditional jyotiśi who computed using the traditional modes. The Dṛk pañcāṅkam was radical not only from the mode of publication but also of the compiler, computation and the mode of authentication. Although hailing from a pañcāṅkam computing family, Ragoonatha Charry's fame was mostly due to his forays in modern astronomy, the discovery of a minor planet, variable star studies, eclipse expeditions and finally the membership as a fellow of the Royal Astronomical Society. In contrast to the traditional pañcāṅkam, which required unwavering faith on the computations, he recruited the readers to verify his predictions of certain celestial phenomena. Thus the new 'patrons', in principle every reader, could observe the predicted events and satisfy for themselves the authenticity of the computations. With this end in mind, a list of observable celestial phenomena, such as eclipse or lunar occultation, was given, with a pictorial guide, in the Dṛk pañcāṅkam for ease of observation by the general public.

The traditional institutions were shaken. The clamour for the Dṛk pañcāṅkam among the general public[10] alarmed the traditional pañcāṅkam computers. They argued that Dṛk, although agreed with visual observation, is 'aśāstriya' and only the computations made from traditional Vākyas would be śāstriya[11]. The conservatives resorted to using of the traditional mode of academic dispute resolution- sádas[12] and organised a slew of them between 1870-73 at various places- Kanchipuram, Kumbakonam, Kallakurichi, Thanjavur, Coimbatore, Mannarkudi and so on, with the aim to declare Dṛk as aśāstriya. While many sádas were organised by the various maṭams, and camastāṉams, a few of them were organized by 'citizen committees'. Careful examination of the composition of the sádas clearly indicate that while traditional scholars of Dharmaśāstras and rituals dominated the ones organised by the camastāṉams, those organised by 'citizen committees' and by many of the maṭams went beyond and included modern scholars in mathematics, such as school teachers, and 'native' officials wielding social status in the then emerging modern society.

The modern professionals replacing the traditional scholars, at times even in the sádas organised by the maṭams did not go unnoticed by the conservatives. "Those who take up the position of chair of an assembly should be well versed in the topic of the discussion or at least be able to give satisfactory reply to objection to his ruling..." rued Kiruṣṇā jōciyar (1872; 64), a traditional panchangin and sneered "..are we bound by the ruling of such an assembly which is chaired by a person without such necessary qualification?" (ibid: 64). Nevertheless, even Kiruṣṇā jōciyar and others had to appeal to the 'public(s)' in the then emerging modern society. Even they could not afford to talk within the ivory towers of sádas and had to convince their customers. Obviously the direct ocular evidence provided by the Dṛk could not match the intricate and inane interpretations of scriptures.

OBSERVATION AS THE KEY CRITERIA

The traditional Brahmin had to perform nitya (daily / routine) naimitika (occasional, special) kāmya (optional) karmas (rites/rituals) as stipulated by the dharmaśāstras. One of the essential features of any rite is the act of 'sankalpa'(resolve/ commitment) to complete the pious, holy act in its entirety. The doer of the sankalpa begins with the words "Now..., at this time, I resolve/ commit do this...". This necessarily implies the doer should be clear about what is 'now' and 'at this time'. Dharmaśāstras stipulate fifteen parameters to identify 'now' and 'at this time', being Mahā-Kalpa, Kalpa, Manvantara, Yuga, Yugapāda, year, ayana, ritu, month, paksha, tithi, vāra, nakṣatra, yoga and karna. It is believed that the rites and rituals will yield intended results only if the sankalpa is appropriately done. Thus, for religious purposes, the correct assignment of the fifteen factors for every moment becomes imperative. The first five does not change for thousands of years, the next five related to year and month, and the last five are what is generally computed and presented in the pañcāṅkam. Hence the veracity of a pañcāṅkam is crucial for the proper conduct of rites and rituals as enjoined by the dharmaśāstras. How to verify if the pañcāṅkam is correct?

The progressive group emphasised the 'exact' nature of the science of astronomy and argued that 'observation' is the primary criteria.

> *"..Jothisha is used for determining the appointed correct time for rituals, and hence it is more pertinent than any other śāstras and in fact equivalent to Veda...[as it is an] exact science, it is amenable to test by Pratyakṣa [observation] and not based upon the opinion or interpretations..."* (Charry 1874; 4)

Ragoonatha Charry said "all the existing pañcāṅkam provided information only on Lunar and Solar eclipses and do not speak about nakshatra samagam or nakshatra grahanam (occultation of planets by stars or occultation of stars by the Moon)..." (Charry 1874; 2). The pañcāṅkam published by Ragoonatha Charry not only contained the usual tables but also 'pictorial representation occultation of Venus by Moon [that took place in 1872] to many rural hinterland' as well as a rare occultation of Jupiter and Venus that took place at the wee hours of Oct 16, 1872 (ibid:7). Further, he computed the circumstances of the 1872 eclipse and gave a pictorial comparison of what to expect on the eclipse day from both Dṛk and vākya computations. Pictorial representation of naked-eye celestial events made it easy for non-expert public, just with the use of the mechanical clock, to use them and verify for themselves which of these predictions are matching the actual observation.

For anyone who had access to a decent mechanical clock, it was clear that the Dṛk computations were far superior to the traditional Vākya predictions. One of the readers of the Dṛk pañcāṅkam, Pa Rangachariyar from Kanchipuram in his letter dated 1872 June 6 complimented, "the timing of contacts were precise" (Charry 1874; 2). U Ve Anathachariyar, a Sub magistrate of Poonamalli observed the event and wrote back 'the dazzling Venus went behind the Moon's disc and we could appreciate your expertise' (ibid; 2). We come to know that quite a few people gathered together and tested the predictions made in the Ragoonatha Charry's new pañcāṅkam. The readers were satisfied as they could test for themselves the correctness of the predictions computed using the Dṛk method. Pacchaperumalkoil Vidwan Gomadam Thirumalachariar another patron of the Dṛk, in his letter dated 6 June 1872 stated that "We were wonder stuck seeing the concordance between the observed progress of eclipse with the pictorial guide you had published in your pañcāṅkam" (Charry 1874; 2). Easy availability of clocks and wristwatches enabled the verification by the public. A sivite scholar Mavoor Shrithira Chellappa Odayan along with Brahman scholars Venkatesa Dikshitar and Sri Govindapuram Sundareshe Pourathi from Nagapattinam region said that they used the wrist watches that were first corrected to the railway time and used to verify the events predicted by Charry (Charry 1874; 4).

Karuṅkuḷam Kiruṣṇā jōciyar, a reputed jyotiśi, wrote many pamphlets to discredit the Dṛk pañcāṅkam (Kiruṣṇā jōciyar 1868, 1872). Another contemporary, traditional almanac computer, maruthuvakudi jyotiśi, was dismissive of the use of modern astronomical instruments like telescopes and asserted that 'they are needed only for those who use modern modes of computations' and not for those who follow the traditional Vākya procedures' (Charry 1874: 4). The conservatives conceded that the Dṛk pañcāṅkam provided 'much precise prediction of the phenomena like eclipses' (Kiruṣṇā jōciyar 1872: 3-4) however, 'as there is no sanction from ancient scriptures for the procedure adopted for computations', it is not 'suitable for observance of religious rituals' and hence at best it could be 'used for predictive astrology' (ibid: 3-4). Kiruṣṇā jōciyar argued that while it is possible to confirm the predictions of observable phenomena like eclipses observationally, the concept like say, 'nakṣatra', is purely an ephemeral phenomenon and hence can only computed. In the Indian astronomical traditions, nakṣatra implies an arc of the ecliptic with length 13° 20' and the interval of time during which the Moon traverses an arc of 13° 20' is called as nakṣatradivasa or nakṣatradina (sidereal day). However, in the pañcāngkam, each civil day is ascribed uniquely with a nakṣatra based upon specific convention. He argued that "although the computations of circumstance of eclipses are precise [in Dṛk pañcāṅkam], it would not be appropriate for computation of tithi, nakṣatra" for "one may use modern [Dṛk] computations for observable phenomena like eclipses but for computing ephemeral tithi, nakṣatra [etc] we should only use the ancient [Vākya] procedures" (ibid:6).

Ragoonatha Charry retorted

> "How can compute the motion of the Sun and Moon one way for computing the tithi, nakṣatra and so on and another way for computation of the eclipses? Is there any authoritative text that stipulates such a practice? The suggestion to use one method of computation for eclipses and another method for tithi, nakṣatra is made only with the ultimate aim of confusing the lay public" (Charry 1874: 10)

If a procedure yielded precise predictions for the eclipses, the same movement of luminaries encoded by the mathematics must give correct interpolation of the intermediate points, tithi, nakṣatra and so on. Further correction factors (turuvam-bija corrections added to original Vākya algorithm from time to time to match the predictions with the actual occurrence) are factored even in the Vākya to update the predictions to match with the observed values (Charry 1874: 4).

Some of the traditional Jyothis too saw merit in the new pañcāṅkam; Venkateswara Dikshadar Sunderesa Pourathi a reputed traditional almanac computer from Govindapuram a village near Kumbakonam admitted that although they "have been [hitherto] computing out Panchang based upon the Soorya Siddhantham, Ariyapadigam Siddhantha Siromani .. [their] computations have been incorrect, due to kālakrīya dosha..." (ibid; 8). Even seers of religious sects were curious, ventured to test the claims of Ragoonatha Charry. The head of the Mutt at Pacchaperumalkoil observed the eclipse 1872 found it to to be precise and declared that "..while other pañcāṅkams were in error [and hence] we have commenced using [Dṛk] pañcāṅkam for inferring tithi and other pañcāṅka information for correct observance of rituals."

DISCUSSION

Elsewhere we have argued how the 'ocular evidence' and the 'novel instruments like globe, telescope, clock, orrery, eclipse diagram' enabled the native literati encountering modern astronomy and the European missionaries encountering Vākya method to 'behaved as if they were members of one community of rational beings, despite power hierarchies' (Venkateswaran, T. V. 2018) and that the that the "calendar reform was not only a colonial project of mastery over the 'native cosmos' through the instrument of standardization, but also a necessary condition for the 'native society', in particular, aspiring native elites to adjust to the then emerging modern capitalist society" (Venkateswaran, T. V. 2009 & forthcoming 2018).

The shift in the way the pañcāṅkams were produced, circulated and consumed, with the arrival of the print culture, enabled the enlargement of the readership (see, for example, Darnton, R. 2002 and Ghosh, 2006) and their relationship with the pañcāṅkams. The new patrons of the printed pañcāṅkams, educated elites, were encountering the debate in a cultural context of colonial contempt for the 'native' culture. The modern elites were at pains to reclaim the rational and 'scientific' elements in the past, and in particular in the siddhāantic astronomical tradition. At the same time by accommodating the claims of purāṇa with the mathematical siddhāantic traditions, the elites attempted to soften the tension between the two (these aspects will be explored in another paper).

Pañcāṅkam computations often have predictive computations of pratyakṣa (observable) celestial events like eclipses and apratyakṣa (unobservable) like samkranti, tithi and so on. Even observable events like eclipses require accurate timekeeping instruments to know if the actual occurrence has been at the time predicted by the Pañcāṅkam. The shadow of a gnomon or water clock to check used by the astronomers for verifying the 'accuracy' of their predictions, before the advent of the mechanical clocks, are tedious and can be used only by experts. Hence if there was indeed a discordant, say an eclipse was predicted as visible from a place but found not occurring, often the blame was placed on the door of the computer and not on the method. The arrival of the clock and the spread of personal wristwatches made the time telling easy and ubiquitous. With the spread of mechanical clocks, the common users of the Pañcāṅkam were in a position to demand the 'accuracy' for themselves the correctness of the predictions made in the Pañcāṅkam.

Although still 'haunted by the ghosts of dead religious beliefs', in the disenchanted modern world, celestial movement of luminaries was seen as a continuum in which one can infer the 'apratyakṣa' from the 'pratyakṣa'. Thus if a pañcāṅkam failed to predict pratyakṣa phenomena 'accurately', then one cannot have faith in the same to give us accurate apratyakṣa predictions. The call for 'better accuracy' in pañcāṅkam was initiated by modern native astronomers and received support from the then-emerging professional elites. The maṭams, the religious institutions patronised by the emerging elites had to lend their authority to the modernisation and reform of the pañcāṅkam. The astronomical data was taken from the best possible 'scientific' source, nautical almanac, and the computations of the pañcāṅkam aspects such as tithi and so on were made according to the rules laid down in dharmaśāstras; a hybrid pañcāṅkam was concocted.

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Endnotes:

1 This was distinct from the Tamil numeral notations and the panchangins were to learn and become proficient in both these numeral notations.

2 For example in Tamil calendar the solar month will begin on a particular day, if the samkranti, the moment when the Sun enters an Indian zodiac sign or rasi, occurs before the sunset, whereas, in adjoining Kerala, the month begins on the day of samkranti, only if the occurrence is aparahna, i.e., three-fifths of a day; otherwise the next day is considered as the beginning of the month. In both convention, one need to know the precise astronomical timing of the entry of the true sun onto a zodiac, whereas the implication for the calendar may differ.

3 We would call them respectively 'progressive' and 'conservative', although one is aware that these words are loaded implications

4 Letters and counters were published in the newspapers, 'The native public opinion' and 'The Hindu' published from Madras.

5 While Ragoonatha Charry wrote extensively in defence in his pañcāṅkam, the opponent, Kiruṣṇā jōciyar (1872) wrote a book titled 'tirukkaṇita pañcāṅka kaṇṭaṉam' to deride the Ragoonatha Charry's efforts.

6 The astronomers like Aryabhata Bhaskara Bhramagupta were elevated as āccariyās, seers, who obtained their knowledge through revelation.

7 Derived from the Sanskrit maṭha, a Brahminical religious institution, with a focus on the preceptor-disciple relationship and lineage, often solely consisting of Brahman ascetics.

8 Maṭha catering to Śaiva upper cast Hindu sects, often excluding Brahmans.

9 Court of local kings and rulers; During the Colonial rule, number of Indian rulers were permitted to retain their court, but under the suzerainty of the queen of England.

10 Although we talk of 'general public' as if it is an undifferentiated entity, in a caste and class ridden society such as the South India at the time of colonial period, indeed the 'public' were fragmented in to various social and economic classes. Most of the non-Brahmin, upper caste sections used the pañcāṅkam through Brahmin intermediaries. Thus the main social class that took part in the debate were Brahmin and to an extent Śaivite religious scholars.

11 = scriptural, classical and academic with aśāstriya as the antonym.

12 Assembly of learned scholars, often organised by the ruler or a religious sect to settle a dispute of the interpretation of scripture.

THE CURRICULAR-TOY, MATHEMATICS AND THE PRODUCTION OF GENDERED SUBJECTIVITIES[1]

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In Brazilian mathematics textbooks for primary school, toys that appear in images and text connect mathematical contents and activity with notions of gender. Based on a Foucault-inspired analysis of a corpus of 103 textbooks, it is argued that the use of toys creates a pedagogical device called a curricular-toy. This object, that emerges in the discursive practice of textbooks in the curriculum, insert children into particular binary, heteronormative notions of being boy and being girl. The gendering is legitimized by the power of the mathematical contents and activities within which gendering takes place. The curricular-toy serves as a device in the making of children subjectivities with mathematics in the school curriculum. The study contributes to current research on the cultural politics of mathematics education.

PROBLEMATIZING TOYS IN TEXTBOOKS

Capturing the attention of primary school children to learn mathematics has been a longstanding preoccupation of mathematics educators. The artefacts to do so have also been varied: mathematical manipulatives like centi-cubes, stories that relate to children's lives, games and playing, and of course toys. Toys are present in concrete activities, at home, and in the images of textbooks that appeal to a familiarity to captivate children and attract them to the world of school mathematics. A quick browse through textbooks will show that there are frequently images of children playing with toys. A more detailed scrutiny would show how particular ideas about child-toy interaction articulate the practices of school mathematics with notions of gender and processes of gendering.

The analysis of the images in Brazilian primary school textbooks allows us to problematize how a "curricular-toy" becomes a pedagogical device that effects gendering through the establishment of relationships child–toy–mathematics. We contend that images and related problems in textbooks are not only seeking to teach mathematical knowledge. The images and texts mobilize attractive scenarios for the mathematics curriculum to captivate the attention of children, seducing them to learning, through a context that has, at least hypothetically, a direct relationship to child play. In these scenarios, particular patterns of the relationships toys–child–mathematics perform gendering in children. A curriculum-toy operates an *alchemy* (Popkewitz, 2004) not only of mathematics into school mathematics, but also of

[1] Short after submitting this paper, Deise Souza passed away. Marcio and Paola, we want to thank her for her careful empirical work, for her sharpness, and for the work that we did together for this analysis. It was Deises' decision that the order of authors be this one to reflect how we engaged in the construction of this particular paper.

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children's toys into curricular-toys that insert in children certain heteronormative notions of being boy and girl.

We unfold our argument presenting some of the theoretical tools that help us in our analysis of the corpus of textbooks that were considered. We then explore some of the recurrent statements and truths that consistently appear in the textbooks and finalize with a reflection on how curricular-toys perform the gendering of children through mathematics.

CURRICULUM AND GENDERING

Mathematics education as a matter of curriculum (Appelbaum & Stathopoulou, 2016) is not about the best way of transmitting mathematical contents, but about understanding how mathematics education is a terrain of cultural politics where, in the everyday practices of teaching and learning, notions of mathematics and the production of subjectivities in schooling are negotiated. In the curriculum, textbooks play an important role since, more often than not, they become an important artefact that guides the teacher and materializes the official curriculum. Since in Brazil a first official national curriculum was only published in 2017, textbooks have indeed been such a guide and materialization of the curriculum. The curriculum and its textbooks are a profitable space to produce subjectivities.

In mathematics textbooks for the early years of school, there appear images and texts that explicitly mention boys and girls and connect them with toys. As mentioned above, these are often perceived as the motivational and contextual element to seduce children into learning mathematics. From our perspective, these are part of the discursive practices though which subjectivation takes place. The discursive practice —"a set of anonymous, historical rules, always determined in time and space" (Foucault, 1987, p. 144)— that we interrogate here is the gendering of the subject in Brazilian primary school mathematics textbooks.

In our time, the gendering of subjects is governed by a political economy with historically constructed characteristics. First, biological binaries of the female and masculine are repeated and essentialized in notions of being a girl or a boy (Butler, 2003). Second, these notions in the math curriculum are linked to the mathematical content and activities. So the truth of being gendered as a boy or a girl appear "centered on the form of scientific discourse and the institutions that produce it" (Foucault, 1998, p. 52). Third, artefacts such as toys in the curriculum —what we call a *curricular-toy*, which is a particular object emerging in the discursive practices of the mathematics curriculum— articulate mathematical elements with notions of gender.

Our analysis follows the insight of recent research that adopts a post-structural, performative perspective on gendering (e.g., Mendick, 2005; Llewellyn, 2009). This research problematizes binarisms and conceptualizes gender as a construction that "takes into account ways of thinking and acting made available and generated within the physical, social, cultural, discursive and historical practices of the communities

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organised around fixed identity categories” (Walshaw, Chronaki, Leyva, & Stinson, 2017, p. 185). We show how notions of gender that children meet are articulated in the particular patterns of relationships mathematics–toy–child that are part of the discursive practices of primary school textbooks. Thinking of gendering, understood as the generation of particular notions of what it means to be and act as a gendered body and subject, and as the constant performing of the notions in the practices that makes us subjects, allows us to identify both the subtle and explicit ways in which these notions and performances of gender are recurrently inscribed in textbooks and their pedagogical devices.

GENDERING DISCOURSES AND THE CURRICULAR-TOY

The corpus of textbooks analysed is composed of 103 textbooks approved by the Brazilian National Textbook Program (BNTP), distributed in 69 manuals of mathematical literacy for the 1st, 2nd and 3rd grades of primary education, and 34 books for the 4th and 5th grades. In Brazil, the curricular guidelines are very open suggestion for teachers in relation to the organization of school mathematics. On the contrary, textbooks are state regulated: the BNTP makes a public run for publishing companies to propose textbooks. These are examined and evaluated according to criteria for quality in each subject. Early years school textbooks assessments occur every three years. If a proposal is approved, the Federal Government guarantees the distribution of the textbooks to all Brazilian public schools. Teachers receive a summary of all the approved books so that teachers can choose which collection they want to use in their classes. All the students in the country receive the books chosen by their teacher. There is a large public investment in textbooks. For this, publishers and authors make a big effort to have their textbooks approved and to enter the order of the discourse.

The discourse analysis performed allowed to identify the regularities contained in 539 images where children’s toys and games appeared together with boys and girls. The questions guiding the analysis were: 1. Which toys and activities are presented in the images? 2. Which notions of gender are portrayed in the images? 3. Which mathematical activities are present? 4. How do these connect in the images and text? As a result of the analysis, different statements and its associated truths became visible. Here we will discuss how the set of images analysed produces a truth about being a boy or a girl. In other words, we analyse how curricular–toys, articulated with the intention to teach mathematics, produce a sophisticated technique that transforms children into boys and girls.

The following table shows the result of the classification made of the 528 figures.

Types of toys	Images with girls	Images with boys	Images with girls and boys
Dolls	83	0	3
Soft toys (animals)	35	0	0
Balls	35	155	1

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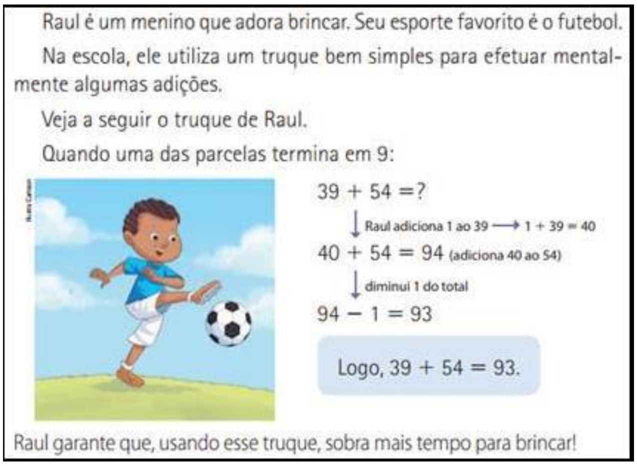
Types of toys	Images with girls	Images with boys	Images with girls and boys
Bowling	14	11	0
Soap ball	2	0	0
Cars	3	117	0
Games (cards, darts, etc.)	0	5	0
Marbles	8	34	1
Spinning top	0	4	0
Superheroes	0	16	1
Video games	2	4	0

Table 1: Quantity of images relating toys with girls or boys.

Dolls and soft animals appeared mainly in images with girls, while balls, cars and marbles appeared mainly in images with boys. Very few images depict situations of interaction between boys and girls (only 6 figures). The images portray everyday life scenes, which become naturalized and materialized through the tasks and math contents. In these scenes, the toys connect to particular ideas of how to be girl or boy; and how to have feminine or masculine behaviours.

Soccer, a game of cultural significance in Brazil, is often the activity where balls and boys appear. Figure 1 shows is about a strategy for mental when there is a sum and one of the addends is a number that ends in 9. The text in the image makes an implicit connection between the child's like for playing and something he uses in school. A trick that saves time that can be used to play. The sentence is implicitly stating different ideas: Playing is important because it is what a "boy adores". Mathematical knowledge is left in the background and is subjected to the smart "trick" of the male who can gain more time to play. Smartness in the use of mathematics for achieving one's purpose is put forward as a quality of learning and using mathematics.


Raul é um menino que adora brincar. Seu esporte favorito é o futebol.
Na escola, ele utiliza um truque bem simples para efetuar mentalmente algumas adições.
Veja a seguir o truque de Raul.
Quando uma das parcelas termina em 9:
39 + 54 = ?
Raul adiciona 1 ao 39 → 1 + 39 = 40
40 + 54 = 94 (adiciona 40 ao 54)
diminui 1 do total
94 − 1 = 93
Logo, 39 + 54 = 93.
Raul garante que, usando esse truque, sobra mais tempo para brincar!

Raul is a boy who adores to play. His favourite sport is soccer. At school, he uses a very simple trick to do some sums mentally.

Try to follow Raul's trick.

When one of the addends ends in 9:

[…]

Raul guarantees that using this trick there is more time to play!

Figure 1: More time to play (Longen, 2014, p. 151).

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Despite being in the background, the association of mathematical knowledge to playing football makes the discursive construction of playing work as a truth, as “the set of rules according to which the true is distinguished from the false and attributes to the true specific effects of power” (Foucault, 1998, p. 53). The term “power” is used here to distinguish a set of actions that determine the position of the subject who plays with the ball. The ball becomes a masculinized object that belong to a mathematically smart male. In the 191 images with balls and balls, there is a predominance of masculinized images for soccer. Furthermore, the phrase “Raul is a boy who loves to play” does not only include boys, it also excludes girls: “every instance of a masculine image or name or pronoun invokes its absent other which is feminine” (Dowling, 1991, p. 3). So, the appearance of the curricular-toy of a ball builds a notion of the desirable woman: she does not play soccer and she does not learn and use “tricks” with mathematical knowledge.

In the textbooks, there are 35 images of girls related to a ball. These are quite different. Figure 2 is a task of correspondence and comparison. Mathematically, this action can be done in any possible way: unless specified by an explicit rule, the objects from one set can be connected with the objects of another set. The task provides no further instruction on criteria for matching the two sets of objects. This image is taken from the teachers’ manual where a suggested solution for the task is proposed in blue. The blue lines between the children on the top and the toys at the bottom show a possible answer. The girl to whom the ball is assigned stands in a non-sporty posture. The ball is not in motion and it is not necessarily a football. The girl and ball are passive. The mathematical demand of correspondence is also basic. The female child seems not to be gendered in the same way as the subject of masculine identity linked to the dynamic of soccer, team uniforms, soccer shoes, and athletic postures shown in Figure 1. The curricular-toy of a ball functions as a cultural pedagogical device which regulates expected behaviours of boys and girls, generating a notion of both the female subject in relation to posture, physical activity and mathematical demand.

Comparison

Link each child with a toy.

a) How many children are there?
   5 children.
b) How many toys are there?
   5 toys.

The quantity of children is iqual to the quantity of toys. 5 is equal 5

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Figure 2: Correspondence child and toy (Pessôa, Vieira, & Ribeiro, 2014, p. 49).

Figure 2 also shown a gendered distribution of the other toys to boys and girls: spin top and truck for Danilo and Mateus; and doll and turtle to Sofia. The consistent distribution of types of toys are connected to the implicit naturality of the mathematical activity of correspondence: the child on the top with the toy at the bottom puts forward the idea that children and these objects belong together, as naturally as the number of children is equal to the number of toys.

Such an implicit naturality assigns particular gendered roles to female and male bodies. In Figure 3, the image presents a task that alludes to measures of time, the two possible ways of expressing hours and an implicit calculation. The girl is sitting and has a doll in her hand, close to her face, and a pink tea set on the floor.

It is 19 hours, or 7 hours at night. I still have 1 hour to play.

Figure 3: Time to play (Nani, 2014, p. 227).

The curricular-toy of a doll is associated to girls as subjects who play with objects that require passive and delicate posture of care, in a context of home activity. The girl is thinking —not talking— and the sentence she thinks implicitly states a time to change activity (20:00 hours). This image is part of a sequence about time in daily routines such as waking up, eating, going to school, doing homework and going to bed. By learning how to read the clock and calculate time, she can take advantage of one hour more of play. Even though the activity is justified by the desire of the child to maximize play time that already appeared in figure 1, the discursive construction around the girl in Figure 3 is different from the one around the boy in Figure 1. There is no smart trick that demands ability to operate with the tenths. The girl converts between two systems of time measurement, and she performs a simple subtraction of one hour.

Figure 4 is connected to the learning of fractions, more concretely how to calculate a third of a natural number. An image of two girls sitting playing with dolls is used.

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Ana gave Isabela a third of her dolls. To calculate a third, just divide by three. How many dolls did Isabela receive?

Figure 4: A third part of the dolls (Silveira, 2014, p. 244).

The curricular-toy of a doll is systematically linked to the cultural tradition of girls playing with dolls. A doll as an objectivation of humans in miniature appears associated to activities of "care for the other". The mathematical operation of division is instantiated here through the generosity of a girl to her friend sharing dolls. The feminized care for the other is justified by the mathematical legitimacy of dividing the number of dolls. All these are part of a pedagogy that involves techniques and development of neutral abilities internal to mathematical knowledge. Thus, the child learns to be mother playing with doll; the girl learns to take care of the family by playing with the pan, the teapot and the cup. The girls also learn to distribute time for the home routines, after having enjoyed paying. In such an articulation, the curricular-toy doll perpetuates social functions of women for benefit of a fixed social structure (Foucault, 1987).

In the textbooks, we also find child games that are socially and culturally linked to boys, but that become associated with girls, as in the example: "Sofia, Lucas' sister, collects car toys. She has 20 car toys and receives 5 more on her birthday. How many car toys does she have now?" (Matricardi, 2014, p. 110). The subject "Sofia", female, collector of car toys, appears in the textbooks as one of the few girls doing a boy activity. Her collecting of cars seems to be justified by the fact that she is "Lucas' sister". This case works as a discursive strategy that it be understood as "a ritual genuflexion to the discourse of equal opportunities and so defuses antisexist petards" (Dowling, 1991, p. 4).

Toys also appear in situations of purchase and exchange for the learning of addition, subtraction and dealing with money. In Figure 5, we bring an example of the seductive discourse of consumption, in which a shop window is presented to students. The text proposes:

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Look at the toy store window to answer the questions: a) What is the most expensive toy? b) Juliana wants to buy the doll. Look at the money she has. [A note of 20 reais, a note of 10 reais, a note of 5 reais, a note of 2 reais and 3 coins of 1 real each] and decide if it is enough. Is there any money left or missing? How much? c) Ramon bought the soccer ball and paid with three notes of 10 reais. How much will he get for change?

Figure 5: Toys in the shop window (Padovan & Milan, 2014, p. 129)

Boys buy boy-toys, and girls buy girl-toys. Thus, the phrases "Ramon bought a soccer ball and paid with 3 notes of 10 reais [Brazilian currency]" (Padovan & Milan, 2014, p. 129) and "Lúcia bought a doll for R$ 55,00 and used only two notes of real to pay for them" (Matricardi, 2014, p. 136) are part of a discursive network linking mathematical operations to gendered ways of purchasing objects. Furthermore, these can also be read as a discourse of consumption, in this case of toys. Therefore, the toy curriculum teaches more than content, it teaches more than being a boy or a girl, as it also teaches what legitimate desires are, as if everyone could reach them.

THE CURRICULAR-TOY AS A PEDAGOGICAL DEVICE

The mathematics curricular-toys produce visible normativity and constitutes itself as scientifically legitimized authority that imposes modes of lives and ways of being in our society. In the curriculum-toy there is an incessant inclusion/exclusion movement located in the discursive binary: boys do not play with dolls, girls must learn to take care of the other. In these complex discursive relationships that govern the interaction of children and toys, children who play learn to lead and to conduct themselves, a "way of behaving in a more or less open field of possibilities" (Foucault, 1995, p. 244). The images in this article and many others we have analyzed produce norms about who are the desirable girl and boy. The discourses appear along with mathematical content, teaching children through mathematics how to perform themselves into one of two socially possible categories.

The analysis allowed us to identify the curriculum-toy as an important element in the discursive practice, a kind of *pedagogical device* to effect gendering in its articulation with the truths of school mathematical knowledge and activity. We follow Friedrich's notion of pedagogical device as

> part of the regime of truth that dictates what is real and what is not, what is true and what is false, in the process of the intentional transmission of sets of values, knowledge and behaviours between subjects that is called education (Friedrich, 2010, p. 661).

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The curricular-toy is an object, that emerges in the discursivity of the mathematics textbooks. It is not a toy for playing; it is a toy for seducing children into doing some kinds of mathematics. Curricular-toys appear as a discursive regularity presented to children who must relate quantities to numbers, handle and administer a monetary system and to compare and order quantities. But the curricular-toy also inserts children into particular heteronormative binary notions of objects, roles and values that boys and girls should have.

The pedagogical device of the curricular-toy has “a strategic and dominant function” (Foucault, 1998, p. 365). It allows to produce and reproduces discursive positions such as the demarcation of gender as a binary because it legitimizes such positions through their articulation with the production of mathematical concepts, results and activities. This makes the pedagogical device of the curricular-toy a device of control and domination of gender in the social field. Gendered positions become imperative strategies that function as the matrix of possible life in the math curriculum. Moreover, the pedagogical device allows to reproduce gender assignments, excluding other corporal images that do not fit into the discursive binary (Butler, 2003) and thus are not part of the curriculum.

In this network of discursive practices articulated around the pedagogical device of the curricular-toy, it is possible to demarcate the predominance of the strategic objective to awaken in children the interest for mathematical knowledge and, in the moment that constitutes as truth in the field of mathematical education, it encompasses a double process:

In the first place, the discursive constitution around the toy and the games generates a production around this subject in the manuals of orientation of didactic work with the mathematical activities. There are recommendations for reading to teachers and students about the role of toys in the curriculum. This discursive construction generates so-called "negative" and "positive" effects: teachers who are able to diversify learning activities and those who cannot. Discursive movement that implies an architecture for the constitution of a desirable teacher.

Second, this goal has effects on the lives of children who play —the constitution of a life imprisoned in the images of gender moved in the curriculum. A reuse of the ways of inhabiting the human. The gender - how we feel and how we are - has come to be reused not only in religious discourses but also in the political aims of the field of education traversed by the economy. The toy curriculum sells objects and lives.

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THE MATHEMATICALLY COMPETENT CITIZEN IN BRAZILIAN AND SWEDISH MATHEMATICS CURRICULUM AND TEXTBOOKS

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Through the comparative analysis of the curricular policies and textbooks in Brazil and Sweden since 1980, we trace the notions of the mathematically competent citizen that articulate the direction for the governing of students' subjectivities through mathematics education. We conducted a content analysis of curriculum documents and selected textbooks for grade 9. Our results point that while in Sweden the official curriculum has played a clearer steering of notions of mathematical competence, in Brazil the nationally assessed and approved textbooks have been central in producing ideas of what should characterize the mathematically competent citizen. In both countries, the link between mathematics and citizenship seems to privilege mathematical competence and knowledge as necessary for participation in economic activities.

INTRODUCTION

Mathematics education has long been linked with scientific progress and thus economic welfare for individuals, communities and nations. More recently, the mathematics education of citizens is argued to be a promoter of diversity and equity to overcome different types of social injustice. The voices that claim the necessity of an increase in mathematical performance of the population are many, nationally and internationally. However, it seems that the more the desire for mathematical achievement for all, the more access to higher achievement becomes reserved for the few. In other words, the desire of inclusion with and through mathematics simultaneously produces exclusion.

The overall aim of our joint research is to show how this double gesture of inclusion and exclusion is played out in the notions of the mathematically competent citizen. Contrary to the nowadays apparent international unification of the discourse on the need for mathematics mentioned above, we follow Tröhler's (2017) argument about the importance of unpacking claims on the globalization of curricula and education to privilege the study of how notions of education and citizenship are articulated in particular national histories and contexts. Thus, we contrast curricula and selected textbooks in Brazil and Sweden in an attempt of illuminating the question of what characterizes the desired mathematically

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competent citizen (as well as the feared ones) that appear in the regularities of official curricular documents and selected textbooks in both countries.

CURRICULUM, TEXTBOOKS AND GOVERNMENTALITY

Foucault's discussion of governmentality has been fruitful to think about education in modern times. The term captures the idea that the production of subjectivities as an effect of power occurs through the deployment of interconnected knowledge-based techniques that steer people's conduct by rational means (Foucault, Burchell, Gordon, & Miller, 1991). The creation of rationalities or ways of thinking about who one should strive to be is connected with the production of truths about the self, which conducts are desirable and what it takes to behave in such way. Power operates not only in the conduction of conduct in certain directions, but also in the differentiation of people according to the norms of conduct set in operation through the technologies of government.

The school curriculum is a tool that has historically been used to form and change the direction of education. In this sense, it can be understood as a technology of government to fabricate notions of citizenship that shape which kinds of people are desirable and those who are feared (Popkewitz, Diaz, & Kirchgasler, 2017). Political changes in views of society have encompassed changes in notions of citizenship. The latter include not only who is considered to be a member of the political body, but also what should characterize the desired citizen. These are not only the moral characteristics, but also the forms of knowledge and competence that citizens should possess. Education in modern times has been the privileged tool to address social problems and direct population politically. This what has been called the "educalization" of modern societies (Tröhler, 2011).

When used to think about mathematics education, the notion of governmentality can be fruitful to study the ways in which diverse technologies of mathematics education generate particular forms of reasoning about what counts as mathematical competence, thus governing the conduct of populations and individuals towards becoming certain kinds of people (Valero & Knijnik, 2016). From this point of view, mathematics education and curricula are not simple expressions of a mathematical plan of study devised by a mathematical interest; they are cultural artefacts that articulate particular ideas of how school mathematics and notions of citizenship entangle in the process of generating political subjects with and through the pedagogical practices of school mathematics. Suffice to say that such artefacts take form in history, in the confluence between societal changes and subject directed intentions.

Furthermore, curricular intentions and frames for action are not simply inscribed in the words of curricular texts; they also materialize in other artefacts and technologies that make part of the dispositive of mathematics education. Textbooks are one of the most salient devices in pedagogical practice (Fan et al., 2013). Textbooks unfold curricular intentions in content and in how it should

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contribute to citizenship. The curriculum is a large dispositive of citizenship that through a series of interconnected pedagogical technologies direct the conduct of the population towards desired forms of being. From this perspective, the making of mathematically educated citizens is a matter of governing political subjectivities with and through the power dispositive of mathematics pedagogies.

Some studies have shown how the mathematics curriculum in different times and spaces has effected rational, scientific-minded, economically-functional subjectivities (e.g., Diaz, 2017; Llewellyn, 2018). More recently, some studies have argued that, while appealing to the goodness and desire for mathematics, the mathematics curriculum also inserts students into a strong economic logic (Valero & Knijnik, 2015). This operates inclusions and exclusions of different types of people according to how useful to current financial rationalities (Valero, 2017). In a new neoliberal mentality, in which economic value is highly desired, the mathematics curriculum may be contributing to overemphasize humans as economic beings. This has been shown to be the case in different types of education in Brazil (de Toledo e Toledo et al., 2018; Neto & Valero, 2018; Silva & Valero, 2018).

In this paper we explore the notions of the desired mathematically competent citizen that are inscribed in the mathematics curriculum and related textbooks. The contrasting analysis between Sweden and Brazil allows us to ask what seems to be privileged and what seems to be left aside or feared in these two different contexts.

METHODS AND DATA SOURCES

The analysis was carried out on the mathematics curricular policies since the 1980’s in the two countries, and the most used mathematics textbooks for 9th grade in schools in Brazil and in Sweden. The analysis was comparative to highlight the similarities and differences in which these constructions take place in each country. For Sweden we examined the mathematics curriculum Lgr1980 (Regeringen, 1980) Lpo1994 (Utbildingsdepartamentet, 1994) and Lgr2011 (Skolverket, 2011); as well as the textbook *Matematikboken Z* (Undvall, Johnson, & Welén, 2013). For Brazil we looked at the Curricular Guidelines of 1998 (Brazil, 1998) and the very recent national curriculum of 2018 (Brazil, 2017); as well as the textbook *Praticando Matemática* (Andrini & Vasconcelos, 2015).

We identified the regularities in the enunciations that appear in these documents about the implicit or explicit notions of citizenship and their connection to mathematics. From the enunciations, we identified the statements that seems to characterize the desired mathematically competent citizen, as well as what seems to characterized the feared type of person. The contrast between the two contexts allowed us to sharpen the characterization of each country, as well as to see the way in which the school mathematics curriculum is steered towards the fabrication of certain type of people.

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ANALYSIS

Sweden

From the Lgr80 to Lpo94 and to Lgr11, the mathematics curriculum has changed from presenting mathematics as central to the "role of adults as citizens" (Lgr80, p. 98) in a democracy, to a subject of communication (Lgr94), to a culturally important area of scientific knowledge of utility in different connections and situations (Lgr11, p. 7). Aspects such as aesthetics and beauty in mathematics are also highlighted in Lgr11. The explicit connection of mathematics as a key type of knowledge for democratic citizenship appeared in Lgr80; but it is not present anymore in the mathematics curriculum in Lgr11. The broad mention of education for democracy remains in the formulation of the aims of the general law of education and in the general introduction of the curriculum. In contrast, problem solving has been a constant topic and an increasingly central activity connected to mathematics. In Lgr11 it is even more highlighted than in previous curricular formulations. In Lgr11, the connection of mathematics with technology is made more explicit, not only in terms of its use but also in terms of its production.

The increased focus on the use of technology in tight connection with mathematics is dominant in the new curriculum and in the political steering of education for achieving national goals of digitalization in society. The emphasis on technology has allowed for a new connection between education policy makers and the curriculum. Just consider the following recent example. *Ny Teknik* is a branch journal for engineers and alike. Based on the increase of activity in computer programming as a new ground area of development of new technologies and of capital, a big problem is formulated: There is a great lack of programmers in Sweden. At the end of 2016 there was a lack of 30,000 IT experts and by the end of 2020, 60,000 programmers will be needed (Ny Teknik, 2017). Such problem should of course be addressed through education. The Swedish government's overall digitalization strategy for the school system sets the vision that the Swedish school system should be exemplary in the best possible use of digitalization to achieve high digital skills in children and students and to promote knowledge development and equity (Regeringen, 2017, p. 4). Programming is the most recent amendment to the Swedish national curriculum. It has become articulated in the course plan for mathematics and technology since 2017 and it is now a compulsory area of teaching to be offered by teachers from July 1st 2018. The school curriculum in mathematics seen as an area of educational policy is clearly intertwined with private actors, interests and expertise in very different areas. In an open letter from Spotify —a recently indexed IT service company in the stock market— to the Swedish government, the founders of this global Swedish success urge the government to support business start-ups, including early efforts of programming in schools "so that we can get hold of the talent that there is, as well as to avoid missing women programmers. (https://www.va.se/nyheter/2016/04/12/oppet-brev-fran-spotify/, our translation)

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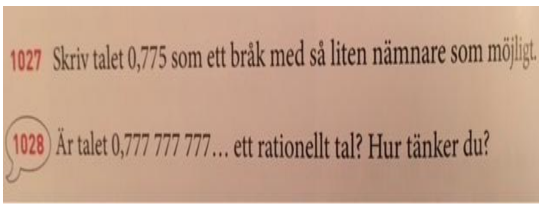
In very short time a whole wave of documents, teacher support courses, materials, educational offers by municipal and national actors was set in motion. Programming should be seen in a broader perspective, as a capacity for children, future citizen adults, that covers creativity, control and regulation, and simulation. This wider perspective of programming thus includes all aspects of digital skills (National Agency for Education, 2018, p 10). The fast adoption of programming as an element in the curriculum and the effort to facilitate implementation are connected with clear economic interests; and this becomes evident not only through the influence of companies, but also in the arguments about the centrality of digital competences for individuals and social economic growth and the future of equity and democracy in Sweden (Damberg & Fridolin, 2016).

In Sweden, the curriculum not only regulates the syllabus, but also the evaluation system and the definition of the levels of knowledge that students have to achieve. These are clear accountability mechanisms for teachers and schools. The textbook industry, however, is not State regulated, and teachers and schools can freely choose among a wide range of textbooks produced by commercial publishers. Textbooks of course provide a basic interpretation of the curricular framework and provide the compulsory topics; but they can vary in their pedagogical approach. Since Lgr80, textbooks have changed significantly, not only due to new forms of printing and the expansion of the textbook industry, even now towards digitalization. Textbooks reflect the views of mathematics education in the curriculum and notions of the desired mathematically competent citizen.

The mathematics textbook Z (Undvall, Johnson & Welén, 2013) for 9th grade connects to some of these characteristics. There is a use of second person and direct sentences that make an active reader who is addressed as the target of the book. In the introductions to the chapters, where the aims are explained, the students are approached with "You are going to learn" as an independent and responsible student. This form of addressing the student appears systematically in only certain types of tasks where students have to formulate their own ideas and build an argument. For example, the question 1028, in a bubble that indicates the presence of a "talk task" or discussion task to be solved with a co-student, uses the direct second person form. This contrasts with the normal indirect and passive forms of language, and with the formulations of instructions (as in question 1027).


1027 Skriv talet 0,775 som ett bråk med så liten nämnare som möjligt.

1028 Är talet 0,777 777 777… ett rationellt tal? Hur tänker du?

1027. Write the number 0,775 as a fraction with a nominator as little as possible.

128. Is the number 0,777 777 777… a rational number? How do you think?

Figure 1: *Z* (2013, p. 14)

Besides the typical mathematical explanations of content, a number of word problems using real information and contexts for the application of concepts

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appear. The contexts of problem solving relate much to sports, outdoor activities and nature. They are accompanied by illustrations where people play football, swim, or hike. These are supposed to portray not only the contexts known by the students, but also the idea that it is important to be a physically active person. Here we have composed a collage of many of the sports illustrations in the book.

1193 Du får veta att $3^9 = 19\,683$. Hur mycket är då

a) 3^8 b) 3^{10}

1194 a) Hur många cigaretter röks i Sverige på ett år? Svara i grundpotensform. Avrunda faktorn före tiopotensen till en decimal.

b) Hur lång är en cigarett? Avrunda till hela centimeter.

c) Hur mycket kostar en cigarett?

d) Hur mycket koldioxid bildas när en cigarett brinner upp? Avrunda till tiotal milligram.

e) Att röka en cigarett tar ungefär 5 minuter. Hur lång tid skulle det ta att röka jättecigaretten? Avrunda till tiotal år.

Det röks i Sverige

Låt oss lägga alla de 15 miljoner cigaretter som röks i Sverige just idag i en enda lång rad med början i Stockholm. Den skulle i så fall hamna någonstans i södra Europa.

Tänk dig att vi tänder eld på den nästan 130 mil långa och 30 miljoner kr dyra cigaretten. Bland de ungefär 4 000 olika ämnen som då släpps ut i luften finns det 2,5 ton koldioxid, 700 kg tjära och 60 kg nikotin.

Figure 2: Collage of pictures in Z Figure 3: *Z* (2013, p. 51)

There are also a number of problems referring directly to a healthy life and adequate behavior in contexts such as sustainable development, consumption of organic food, use of pesticides, viruses, bacteria and health, soft drinks, world military expenses, use of oil in the world, cigarettes and non-smoking, car handling on cold winter days, walking on ice, electricity costs, summer jobs and salaries, water, population and birth of children. The following problem exemplifies this (Figure 3):

> **Sweden smokes**. Let's put all the 15 million cigarettes smoked in Sweden just today in a single long row starting in Stockholm. It would end up somewhere in southern Europe. Imagine that we light the nearly 1,300 km long and 30 million SEK expensive cigarette. Among the approximately 4,000 different substances released into the air there are 2.5 tons of carbon dioxide, 700 kilos of tar and 60 kilos of nicotine.1194. a) How many cigarettes are smoked in Sweden in a year? Reply in basic form. Round off the factor to the decimal point to one decimal place. b) How long is a cigarette? Round off to full centimeter. c) How much does a cigarette cost? d) How much carbon dioxide is formed when a cigarette burns up? Round off to tens of milligrams. e) Smoking a cigarette takes about 5 minutes. How long would it take to smoke the giant cigarette? Round off to ten years (Z, 2013, p. 51).

The contexts in the problems do not only serve as a motivational device or a basis for meaning-making. They offer a set of norms and values about what good citizens should do and how mathematical knowledge is part of it.

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Brazil

The Curricular Guidelines (Brazil, 1998) and the recent National Curriculum (Brazil, 2017) recognize mathematics as a human cultural creation that is important for democracy as much as for solving problems and as a tool for solving problems in life. Communication, ethics and responsibility are important elements of mathematics education. The Brazilian governing of school education has been based on a broad curricular guideline which the different states, educational institutions and teachers had the responsibility of taking in consideration. Besides the curricular guidelines, a national evaluation system and a regulated textbook production were the key elements of policy to steer education. It is only in 2018 that a new National Curriculum, that specifies levels of achievement, has been formulated and has been set as a binding guideline for the provision of education. The process of enactment of the new curriculum is only starting at the moment. Therefore, the governing of mathematics education is currently under transformation.

In Brazil there is not explicit reference to companies influencing the curriculum in basic school; however, the new national curriculum has been supported by big companies interested in improving specific skills in the Brazilian population, especially in mathematics. For example, the *Lemann Foundation* and the *Itaú Social Foundation* are part of large business conglomerates in Brazil and they had an important participation in mobilizing the new curriculum in Brazil. The support of the private sector in the mathematics curriculum can be seen through the interest in supporting contexts for trade and consumption, as discussed in (Silva & Valero, 2018).

In Brazilian textbooks there appear many problems about the necessity of preserving natural resources and saving them. Water use is a recurrent topic that is not only measured and mathematized, but also around which advice and norms of use are presented. In contrast to Swedish textbooks, where water seems to exist in excess, in Brazilian textbooks water has to be saved.

Connecting knowledge:

For personal hygiene, cooking, cleaning the house, washing the clothes, etc, each person uses, in average, 200 liters of water per day. A reservoir as this would be able to supply a group of 500 people for about how many days? Remember $1m^3 = 1000L$

Figure 4: *Praticando Matemática* (2015, p. 25)

In the following example, there is an argument about the necessity of decreasing the emission of polluting gases with the use the less polluting fuels. This only makes sense because there is a mathematical argument that show us that this

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choice is cheaper than the use of fossil fuel. So, the environmental question must be aligned with consumption and money savings.


Uma agência de aluguel de automóveis colocou um anúncio que dizia:

CONECTANDO SABERES

POLUA MENOS E ECONOMIZE MAIS!

Para incentivar o uso do carro a gás, que po
menos, essa agência apresentou uma prom
ção, de acordo com a tabela abaixo.

Categoria	Gasolina (R$)	Gás (R$)
popular	diária: 80,00 km: 1,00	diária: 50,00 km: 0,80
semiluxo	diária: 120,00 km: 2,00	diária: 80,00 km: 1,00
luxo	diária: 150,00 km: 3,00	diária: 100,00 km: 1,80

A car rental store advertised:

Pollute less and economize more

To encourage the use of gas cars that pollute less, this store showed a promotion according to the table below:

Category	Gasoline (R$)	Gas (R$)
Popular	Daily: 80,0 Km: 1,00	Daily: 50,0 Km: 0,80
Semi luxury	Daily: 120,0 Km: 2,00	Daily: 80,0 Km: 1,00
Luxury	Daily: 150,0 Km: 3,00	Daily: 100,0 Km: 1,80

Figure 5: *Praticando Matemática* (2015, p. 133)

The notion of a citizen that does not produce "waste" and can calculate savings with mathematics has gone through every textbook. The desirable citizen in the textbook is the person who can secure all resources for himself and for the public. And the mathematics is the tool to legitimize these arguments in line with recommends official document: "To make systematic observations to investigate, organize and present convincing arguments" (Brazil, 2018)

DIFFERENCES IN GOVERNING STRATEGIES

Despite the apparent similarity in the contents listed for 9th grade as well in the ideas such as problem solving, reasoning and communication, the notions of the desirable mathematically competent citizens and the technologies to govern their conduct are different in both countries. The curriculum in Sweden seems to be more closed. In particular, the grid of levels of attainment and performance in mathematics that emerges out of the connections between curriculum, assessment and textbooks keep a tighter control over the progression of the student's mathematical development, both with respect to the development of knowledge and of competences (in Swedish, *förmågor*).

In Brazil, the broadness of the guidelines leaves the space for the textbook to be a strong instrument for governing the students' conduct. The problems in textbooks articulate closely mathematical competence and knowledge with morally right forms of being. The values and norms of good conduct are more evident in Brazilian textbooks, and subtler in Swedish textbooks. This is seen in the use of problems with direct instructions for right behavior both in mathematics and in moral terms in Brazilian textbooks, and in the appeal to contexts that capture a sense of Swedishness such as enjoying sports and outdoors activities.

Both systems have tended to specify more and more what is to be taken as highly valued mathematical behavior and performance. The emphasis on problem

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solving privileges a functional view of knowledge for use, application and generation of profit. The connection with digitalization and consumption of technology. What is evident in both cases is that school mathematics is given an increasingly function in with respect to the making of economically active people who are good (even responsible) consumers; the link to democracy seems to be geared towards the extent to which mathematical competence and knowledge are necessary for participation in economic activities.

Our analysis does not intend to show that this is indeed what will happen with all students in Brazil or Sweden. We acknowledge that an analysis of the regularities in the curricular formulations and in the textbooks does not grasp the unfolding of those formulations in practice. It cannot either account for various ways of unfolding citizenship. However, it is important to highlight the discursive effect of those formulations in generating particular ways of thinking about what characterizes the subjectivities that are desired. Not only are they a frame; they also entail that those children whose forms of life do not align to these particular ways are, by definition, set in an excluded position and are made ready to be converted and saved through the strong narrative of school mathematics. The ethical question of the direction in which school mathematics intends to govern people remain as an issue of discussion for educators and researchers alike.

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BILDUNG, PROFESSIONALIZATION AND COMPETENCIES: CLASHING NARRATIVES IN GERMAN MATHEMATICS TEACHER EDUCATION RESEARCH AND PRACTICE

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Abstract: In Germany and Austria, the so-called "PISA shock" not only led to rethinking the aims of mathematics instruction in schools, but also to rethinking mathematics teacher education – both traditionally thought of as processes of "Bildung". Since then, research and reform efforts in mathematics teacher education are increasingly focussed on an understanding of "teaching as a profession" for which "competencies" play a central role. I will argue that while on a surface level "Bildung", "professionalization" and "competencies" may seem compatible, the underlying concepts and related narratives show significant incongruencies. This is also a question of power structures in the educational system: While "Bildung" and "professionalization" are essentially emancipative narratives aimed at a strong teacher personality, "competencies" aim much more strongly at adapting to predetermined norms of action.

INTRODUCTION

Since the final establishment of compulsory education and a secular educational system in Austria (1774), Prussia[1] (1794) and Bavaria (1804) until the German (and, although to a lesser extent, Austrian) "PISA shock" at the beginning of the 21th century, there has been an ever-recurring feeling of inadequacy of (not only but especially *mathematics*) education in schools and concerns about the proper qualification of (mathematics) teachers. Likewise, there is a recurring need for new, strong narratives capable of channelling the somewhat vague and inconsistent public and/or governmental discomfort with mathematics instruction and hence acting as a consensual rationale for rather authoritarian reforms of mathematics instruction as well as mathematics teacher education (see Führer, 2000). In this regard, "Bildung", "professionalization" and "competencies" act as functional equivalents of each other. But I would argue that the introduction of new "regulatory ideas" does *not* mark "paradigm shifts" in a Kuhnian sense. Mathematics education research rather tries to reinterpret and integrate each of those intentionally broad or even fuzzy concepts – all the while accepting consciously or unconsciously that the specific understanding of these concepts among mathematics educators might veer away considerably from their more traditional meaning and common use in the public and in educational policy. If this was correct, mathematics education researchers and/or mathematics teacher educators would presumably either be consciously grounding their activities on an ideological superstructure effectively hiding their self-interests and/or unconsciously and unintentionally contributing to educational policies they might not really agree with, even though they use the same words.

[1] For the 19th century, the development is discussed below with a focus on Prussia, since it is best documented for this state and the development in Prussia is, by and large, typical for the other states.

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To substantiate my hypothesis, I will present three narratives evolving in the German-speaking countries over the approximately 200-year history of mathematics teacher education:

1. The narrative of “Bildung” as originally established in Neo-Humanism in parallel with mathematics teaching becoming a “proper job” in the 19th century.
2. The narrative of “professionalization of teachers” developed in post-1968 German sociology and educational sciences.
3. The narrative of “competencies” which becomes dominant in educational sciences and educational policy in conjunction with PISA and the Bologna Process in the European Union at the beginning of the 21st century.

FIRST NARRATIVE: “BILDUNG” AND THE OCCUPATIONALIZATION OF MATHEMATICS TEACHING

“Bildung” as a specifically German interpretation of the aims of education evolved during the 18th century. Wilhelm von Humboldt established “Bildung” as the guiding formula for educational reform in both schools and universities in Prussia in the 19th century. As a highly empathetic category, “Bildung” in Humboldt's sense is nothing short of “man's true purpose”, namely “forming all his powers in the highest and most proportionate manner”[2] (Humboldt, 1851, p. 9). There is also a more pragmatic interpretation of a socially generalizable “core” of “Bildung” as “Allgemeinbildung” (translates to “common” or “general” Bildung) inherent in Humboldt’s writings. “Allgemeinbildung” is what anyone should know and be able to do in order to become, “by himself and regardless of his occupation, a good, decent and – minding his rank– enlightened human and citizen” (Humboldt, 1971, pp. 147–148). Yet, according to von Humboldt, the characteristic difference between any *true* form of “(Allgemein-)Bildung” and “mere” (vocational, pragmatic) education is that “Bildung” always affects personal development, while education may be restricted to the aquisition of practical skills. For “Bildung”, Humboldt considers “any knowledge, any skill which does not elevate the faculty of thinking and imagination by a complete insight into the rigorously enumerated reasons or by elevation to a universal intuition (like the mathematical or aesthetical) [is] dead and sterile” (Humboldt, 2017, p. 134). Over the course of the 19th century, the ambivalence of the emphatic meaning of “Bildung” and the much more pragmatic meaning of “Allgemeinbildung” is mirrored in an inherently inconsistent understanding of what “Bildung” means in teacher education for different groups of teachers.

At the beginning of the 19th century, “Bildung” began to become a leading narrative in the field of school education and teacher training. As such, “Bildung” had to act as an orientation, “which could make the distance between the actual and the desired state argumentatively accessible” (Führer, 2000, p. 2) and should be consensual among decision-makers. Such a narrative should also contextualize interventions in the educational system which should not appear as arbitrary but as “plausible consequences from

[2] This and all other originally German quotations have been translated by the author.

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plausible principles" (op. cit.). The consequences of the call for more "Bildung" in teacher education then vary greatly, depending on whether one looks at teachers for the lower, non-academic track of the public school system ("Elementarschulen", "Volksschulen"), or at the teachers of the "Gymnasium" (academic track of the secondary school system).

Let us begin with teachers of the "Gymnasium". Those teachers started to receive academic training at universities in Prussia as early as the mid-19th century. In fact, mathematics at German universities largely owes its status as an established academic discipline at German universities to the newly introduced task of teacher education. Previously, mathematics at universities was taught nearly exclusively as part of the "studium generale" of traditional studies (theology, law studies, medicine). With the introduction of regular teacher education studies, mathematics became a proper subject of study. At the same time, the number of mathematicians working at German universities as well as the body of scientific knowledge in mathematics grew considerably. With the improved education of teachers, the mathematical content that had traditionally been part of the "studium generale" was increasingly being shifted away from the university to the upper years of "Gymnasium" (Schubring, 1990). In turn, the standard of mathematical studies at universities was increasing so strongly that already at the end of the 19th century an unbridgeable "gap" between school mathematics and university mathematics became a common complaint (Schubring, 2007). For university teacher education, the narrative of "Bildung" was basically interpreted in the sense of the neo-humanistic motto "Bildung through science". It represents the idea that one gains the best mathematics teachers precisely by enabling them to encounter "mathematics as a science" as pure and unbiasedly as possible without having to take into account one's occupational goal in any way whatsoever. Likewise, instruction in general or the teaching of mathematics as objects of scientific investigation are largely foreign to the Neo-Humanists of the 19th century. For a long time, teacher education studies at universities contained little or no educational sciences or subject didactics. From the onset and throughout the history of the German universities, the motto "Bildung through science" has been closely linked to another motto: "unity of research and teaching". In Wilhelm von Humboldt's somewhat romantically transfigured ideas, university teaching should consist above all of students living and working together with experienced researchers for several years and thus gradually becoming part of a culture of research and science (Humboldt, 2017). However, more recent higher education research has been calling "unity of research and teaching" a myth or even a "lifelong lie of the professorship" (Schimank, 2010, p. 52), which serves as an excuse for not having to pay special attention to one's own teaching and, in essence, not being interested in the students' learning progress or their occupational goals. According to Schimank (2010), this somehow worked until the 1960s because university studies were largely reserved for the educated upper class, for whom such a blind "esteem for science" (op. cit., p. 52) was formative. Thus, questions of meaning could not gain a lasting foothold. According to Schimank (2010), "Bildung through science" is highly ambivalent, especially for prospective teachers: On the one hand, school teaching is not research. It is

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not a search for new insights, but a decisive mediation of “yesterday's state of research” (Schimank, 2010, p. 51) – because only that state is sufficiently secure and thus suitable for everyday and professional use. On the other hand, the “nimbus of scientific excellence” (op cit.) legitimizes firstly the high professional prestige and secondly the high salary of the teachers at the *Gymnasium*. And thirdly, this nimbus ensures that political or economic influences stay out of the classroom thus ensuring the autonomy of teaching. Looking at teacher education in STEM subjects, especially in mathematics, Merzyn (2004) notes that mathematics and science teacher associations had argued as early as 1920 that academic teaching of mathematics and science at universities could be both scientifically sound and yet only moderately useful for the later work of teachers. Merzyn (2004) also assumes that, to this day, among mathematics and science professors at universities, it is still a widely held view that “a good teacher is the result of a little common sense and a few methodological tricks being added to the study of science and/or mathematics” (op. cit., pp. 406/407).

If we turn our attention to the teachers of primary and lower secondary schools, those were still educated at special teacher colleges without typical university rights (e.g., the right to award doctorates) until well after the Second World War at least in some parts of Germany and in Austria. The narrative of “Bildung” is *not* the narrative of “Bildung through science” for most of the history of teacher education for primary and lower secondary schools. It is, on the one hand, the narrative of “Bildung through Allgemeinbildung” and it is, on the other hand, the much more open admission that “Bildung” in teacher education is in competition with very practical, down-to-earth aspects of vocational training. It should be noted that the training of teachers for these teaching posts in the 19th century is much more diverse regionally. Until 1854, they were mainly trained at so-called “seminars” whose curricula varied greatly from region to region. In more rural areas, they were often confined to teaching prospective teachers the basics of reading, writing and arithmetic. In more urban areas, there are also significantly more sophisticated seminar concepts, such as that of Friedrich Adolph Diesterweg. In addition to a solid “Bildung”, Diesterweg’s seminarians were also expected to learn the basics of the “still evolving disciplines of pedagogy, psychology, anthropology and didactics” (Schütze, 2014, p. 328). These more sophisticated seminar concepts aim at “Bildung” of the seminarians in two respects: Firstly, their subject related studies very much follow Humboldt’s ideas of “Allgemeinbildung” as something everyone should try to acquire regardless of the needs of everyday life and one’s job. Secondly, for Diesterweg and proponents of similar seminar concepts, the work of teachers is considered to be something that should be understood and reflected upon within the scope of teacher education on the grounds of scientific theories and cannot simply be learned on-the-job within the framework of a kind of “master apprenticeship”. So, teacher education at these seminars aimed at “insight into the rigorously enumerated reasons” underlying both the subjects (e.g., mathematics) and *the teaching of those subjects* – which is a goal that corresponds much better to more recent ideas of teacher professionalization than the motto “Bildung through science” does. However, progressive concepts of teacher education at the seminars faced a considerable

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backlash in the aftermath of 1848s "March Revolution" in Prussia. The Stiehl Regulations of 1854 restricted teaching at the seminars to subject matter "which is absolutely necessary for school-keeping in the ordinary elementary schools" and explicitly warned against "any attempt of scientific treatment of the subjects" (Stiehl, 1854, p. 6). There is an even stronger aversion towards educational knowledge, pedagogy and psychology. On the one hand, this is a clear setback considering the more elaborate seminar concepts of Diesterweg and the like. On the other hand, even today students "often still demand the acquisition of exactly such routines of action as the Stiehl Regulations promised" (Schütze, 2014, p. 338). The "notions of what theory should and can achieve, namely to provide directly applicable knowledge for teaching, which can be found in the Regulations" (op. cit.), are still widespread even today. Nevertheless, at the beginning of the 20th century, more progressive concepts of teacher education had been fully restored and the new question regarding the "Bildung" of primary and lower secondary school teachers was whether it should take place at "proper", general universities or at specific academic institutions for teacher education / teacher colleges. In many respects, the role of Eduard Spranger for the idea of these teacher colleges corresponds to the role of Wilhelm von Humboldt for the idea of the university. Spranger (1920) argues that proper "Bildung" of prospective teachers at "schools for (common) people" needs to take into account *all of men's powers* (be it intellectual-scientific, technical-economical, aesthetical, social, or religious), not just intellectual ones, and is therefore not in good hands at the university, which is one-sidedly oriented towards intellectual-scientific powers. As a contemporary critic, Johannes Kühnel (1920) already pointed out the inconsistency of this assertion with the motto "Bildung through science" used for the education of teachers of "Gymnasium". Modern critics of Spranger's plea for teacher colleges consider it as a thinly veiled attempt to maintain the desired status differences between the teachers of both types of schools by means of the different educational pathways.

SECOND NARRATIVE: "TEACHING AS A PROFESSION" AND THE "TECHNOLOGY DEFICIT IN EDUCATION"

One of the main differences between teacher education for teachers of the "Gymnasium" at universities and teacher education for teachers of "lower" schools at the more progressive seminars and later at teacher colleges is the question to what extent teacher education benefits from a *scientific understanding of pedagogy, psychology and didactics*. Traditionally, universities have focused on the scientific study of subjects (e.g., mathematics), while seminars and teacher colleges have focused on the scientific study of pedagogy, psychology and didactics. The question of how *scientific pedagogical and didactic knowledge* relates to work routines of experienced and successful teachers is at the very heart of the second narrative to be examined here: the *professionalization* of teachers. "Teaching as a profession", "professionalization of teachers" or "teachers' professional development" are used here synonymously as a collective term for such approaches which assume that pedagogical objectives such as "education" or "Bildung" refer to processes which (a) are not exclusively aimed at imparting knowledge

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and norms, but always address the person of the educated subject as a whole; (b) are embedded in classroom practices, which take place in complex social arrangements that are only theoretically permeable and predictable to a certain degree; (c) therefore, require a reflexive-adaptive approach to teaching that does not simply consist in the quasi-technical application of previously achieved scientific results.

Perhaps the most prominent justification for such a position can be found in the widely received article “Pedagogy and the Technology Deficit in Education” by Luhmann and Schorr (1982). They argue that, since there are no sufficient laws of causality for social systems as complex as a classroom, “there is also no objectively correct technology that one would only have to recognize and then apply” (op. cit., p. 19). “In view of the complexity of the interaction system” instruction, teachers could hardly name or determine “the factual prerequisites for purposeful action” (op. cit., p. 19). Therefore, *uncertain* “causal plans” would have to take the place of *certain* laws of causality in processes of educational and “Bildung”. Such “causal plans” are always “assumptions about causal connections, which are only incompletely supported by empirical evidence” (Fromm 2017, p. 87) and “causal plans” necessarily “deviate from reality, on which one must nonetheless trust in order to obtain a basis for one's own experience and action that is readily available and sufficiently clear” (Luhmann and Schorr 1982, p. 18). Scientific knowledge about education and “Bildung” can therefore contribute to “enabling the teacher to have a more promising and justified orientation towards action with higher prospects of success” (Fromm, 2017, pp. 87–88). However, this is only possible at the price that scientific findings are “tentatively overstretched and supplemented by more or less systematic and reflected craft knowledge” (op. cit.) of educators.

The notion that scientific knowledge can neither completely control nor theoretically anticipate concrete everyday actions of teachers and that professional action can therefore ultimately only be learned through the accumulation and reflection of experiences in the concrete client relationship between teachers and students can also be found in Ulrich Oevermann's (2016) “revised theory of professionalization”. In the scope of this paper, it is not possible to present this theory in greater detail. However, it seems essential to point out that Oevermann is particularly critical of teacher education at universities. According to Overmann, an induction to the “art of teaching” should essentially consist of a “combination of case-oriented, case-reconstructive exemplary material exploration” (op. cit., p. 149) and in “forms of practical ‘learning by doing’ under the guidance of experienced teachers” (op. cit.). In reality, however, the “teaching of proven and relevant methodological and theoretical knowledge” (op. cit.) dominates at the university. Methodological and theoretical knowledge about education is a *necessary* but by no means *sufficient* basis for professional teaching. In addition, Oevermann sees the danger of a fragmentation of theoretical knowledge, which threatens to become self-sufficient in the form of “self-licensing knowledge territories” (op. cit., p. 149), due to the subdivision of the university's disciplines. If we look at the practice of

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teacher education in Germany, one could think that the *two-fold need for professionalization* (academic studies on the one hand, practical introduction to the “art of teaching” on the other hand) called for by Luhman, Schorr and Oevermann would be harmoniously realised with the predominant two-phase structure of teacher education in Germany (first phase at the university, second phase at seminars and in schools under the supervision of experienced teachers). However, there is hardly any aspect of teacher education that is more controversially discussed in German teacher education research than its two-phase structure. While, in theory, a fruitful and inescapable complementarity of the knowledge and skills to be acquired in both academic and practical confrontation with the work of teachers is presumed, in practice it is often the case that both phases of teacher education assume that the knowledge to be imparted in the other phase can be either abandoned or else transferred to ones’ own phase.

Comparing “Bildung” and “professionalization” as possibly clashing narratives in teacher education, we can, on the one hand, state that “professionalization” in contrast to Humboldt’s emphatic notion of “Bildung” implies a more pronounced acknowledgement that there are limits in trying to gain “complete insight into the rigorously enumerated reasons” (Humboldt, 2017, p. 134) of educational action and the teachers’ trade. On the other hand, “Bildung” as well as “professionalization” imply that teacher education is not simply the transfer of scientific knowledge which can later on be applied in a mere technical manner. Both “Bildung” and “professionalization” aim at the formation of a teacher *personality* which is capable of independent judgement and situational-reflective adaptation and transformation of his or her own practice. If we turn our attention to *mathematics teacher education*, it should first of all be noted that the discourse on professionalization and the talk of the “technology deficit in education” has played a prominent role in sociology and educational sciences since the 1980s, whereas it has played a marginal role at best in mathematics education research in the German-speaking countries.

THIRD NARRATIVE: THE POST-PISA TAKE ON “COMPETENCIES” AND “PAEDAGOGICAL CONTENT KNOWLEDGE”

Professionalization is only becoming an increasingly discussed topic with the reforms at all levels of the German (and, by and large, also the Austrian) education system that were set in motion by the (presumably) unsatisfactory PISA results and accelerated by the Bologna Process in the EU. But in the course of these reforms, another understanding of teacher professionality finds its way into mathematics education discourse. This interpretation largely does away with the structuralist interpretations of professionality and the concerns about a possible “technology deficit in education” discussed in the section above and focusses on a rather pragmatic understanding of professionality which is rooted in the identification and assessment of knowledge and, more specifically, *competencies* of successful teachers – their success, in turn, measured more or less exclusively on grounds of the competencies their students possess according to (more or less standardized) assessments (Terhart, 2011). According to the almost exclusively referenced definition of Franz E. Weinert (2001), competencies are defined

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as “the readily available or learnable cognitive abilities and skills which are needed for solving problems as well as the associated motivational, volitional and social capabilities and skills which are in turn necessary for successful and responsible problem solving in variable situations” (pp. 27–28). If we apply such a definition to the competencies of teachers, there can be little doubt that the “problems” mentioned in the definition are problems of teaching, learning, education and “Bildung”. If we then assume the development of professional competencies as a requirement for teacher education, we are, again, confronted with the questions (a) whether systematic, academic instruction at universities can already lead to the development of such competencies and (b) to what extent such competences are actually based on knowledge elements that can be detached from the integration into practical action, which can then be taught within the framework of a course and could be assessed in standardized pen-and-paper tests. In my opinion, despite the current increase in research efforts in the field of mathematics teacher education, these questions play a rather subordinate role in mathematics education research. On the one hand, almost all major studies of professional competence of (prospective) mathematics teachers conducted in Germany refer to the work of Lee Shulman (1987), for which the (not yet theorized, often tacit) “craft knowledge” of the teachers plays a major role. A significant difference to the theories of professionalization discussed above lies in the fact that these studies do not regard the implicit “wisdom of practice” as a necessary local-situational complement in concrete practice, but rather as something that, in the end, has to be surveyed, codified and systematized by mathematics education research with the goal of including it within formal education and the associated systems of examination. Kolbe (1997, p. 132) has argued that the essence of “pedagogical content knowledge” as “craft knowledge”, which a certain teacher might apply to a certain situation, cannot simply be made instructive for other practitioners and for other situations of use by researchers attempting a universally valid, context-free survey and reformulation of such knowledge. In large-scale professional competence surveys such as TEDS-M (Döhrmann, Kaiser, & Blömeke, 2012) or COACTIV (Krauss, Baumert, & Blum, 2008) such concerns are largely ignored or do not play a role at all because “pedagogical content knowledge” is essentially identified with traditional knowledge inventories of mathematics education research. In fact, these knowledge stocks are simply *declared* professional knowledge, but the special features of the pedagogical client relationship and the “craft knowledge/wisdom of practice” of mathematics teachers are once again ignored, even devalued.

The decisive difference between “competencies” and “professionalization” in the sense of the previous section is thus named: Only the special features of the pedagogical client relationship and the “craft knowledge/wisdom of practice”, *determines* the professionality of teacher behaviour in the sense of the professionalization theories mentioned there. But this notion is effectively abandoned within the research on “professional competencies” in mathematics education – which makes it questionable whether a productive resolution of the long-lamented opposition between theory and practice in teacher education is desired at all.

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CONCLUSIONS

I started this paper with the hypothesis that the use of narratives "Bildung", "professionalization" and "competencies" has an ideological character and potentially contributes to the consolidation of problematic power structures in schools and teacher education. Comparing an emancipative idea such as "Bildung" and "competencies", the differences are rather stark: A core element of "Bildung" has always been that "Bildung" is concerned with personal development, that is processes of developing an independent, well-rounded personality. This is supported by the structuralist theories of professionalization, which assume a limited "teachability" of teaching in academic settings. In contrast, "competency orientation" suggests for both instruction in schools and teacher education that one could essentially limit oneself to teaching skills and abilities for problem solving. Such a position is simply not compatible with the traditional understanding of "Bildung" and it raises serious doubts regarding the need for professionalization according to established theories of professionalization. So far, mathematics education research in Germany has done little to counter the impression that under the label "professional competencies" it is ultimately only pursuing the preservation of its vested rights in teacher education.

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TENSIONS IN THE SWEDISH FRITIDSHEM MATHEMATICS CURRICULUM: A POLICY ENACTMENT PERSPECTIVE

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In 2016 the Swedish fritidshem got its own curriculum where mathematics is formally introduced. The space where students can experience informal forms of mathematics in activities derived out of their own interest risks being slowly transformed into a schoolified form of mathematics, steered by teachers and striving for learning effectiveness. A policy enactment perspective was used to investigate the material, interpretive and discursive dimensions of the enactment process. Based on document analysis, observations and interviews in two cases, tensions between two different and competing discourses were identified: one driven by student's interests and one driven by teacher's mathematical agenda. The meaning of fritidshem math will configure in the tensions about what counts as desirable forms of mathematical activity in practice.

INTRODUCTION

Fritidshem — literally translated "freetime home"— is a special type of institutional offer for children in school age, for activity after the end of the school day, based on play and socialization. The character of this offer has changed through time and different policy documents have tended to make this institution come closer and closer to school. In 2009, for example, the National Agency of Education changed the name of the *activity* at the *fritidshem* to be called *education* (Prop. 2009/10:165). In 2016, a new curriculum for both preschool and the *fritidshem* was issued where specific subjects were introduced. One political argument for this change is that all institutional offers to young people in Sweden should support their school development in order to achieve the ambition of improving the overall quality of education and pupils' achievement in Sweden, monitored nationally through tests and internationally through comparative studies of school achievement such as PISA (Regeringskansliet, 2016). In the curricula, mathematics expressed in terms of problem-solving in everyday life is explicitly mentioned as one of the new knowledge areas that is to be made visible in *fritidshem* education.

Nowadays it has become naturalized to think that children have to start earlier dealing with various formalized forms of knowledge such as mathematics in order to increase their learning capacity. It has also become natural to think that all spaces of life should support the core activity of children, which is performing well in school. This is what research in *extended education* has highlighted (Holmberg, 2018). But, who would dare to question the political intention of making mathematics visible and explicit in *fritidshem*? Notwithstanding its apparent goodness, a critical investigation of this change in policy calls for problematizing since changes in the governing of children's life and free time are being made for the sake of better mathematics. As part of this, the concept of *schoolification,* the process of becoming like school, in this case the

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tendency of the activity and content in *fritidshem* turning similar to the ones in school, needs to be discussed.

The new curriculum poses two interconnected challenges to the people involved in *fritidshem*. First, is a schoolification of *fritidshem* desirable? Second, what could be the meaning of *fritidshem* mathematics? These are important issues to debate because the strong narrative of the necessity of high performance in school mathematics to develop Sweden in the future overshadows the value of free spaces for children to socialize, play and be creative in, and at the same time engage in informal mathematical activity. Furthermore, since school mathematics has a very strong logic and tradition, such logic can be easily imposed on *fritidshem* mathematics, making the latter just an artificial version of the former. What is at stake is the possibility of the emergence of forms of informal mathematical activity within the frame of a *fritidshem* where student's interests and free space can be a grounding principle.

The tension between these challenges is part of the everyday life of the people working at *fritidshem*. In this paper, we delve into the tension with the intention of exploring how people at *fritidshem* enact the new curriculum directed to the area of mathematics. This is important because the meaning that fritidshem mathematics will get in practice will emerge out of the policy enactment process of this new area for mathematics education in Sweden.

RESEARCHING CURRICULUM CHANGE AS POLICY ENACTMENT

We conceive of the new curriculum as an educational policy that poses problems that must be solved in the context of the people and institutions that enact them (Ball, 2000). A change in the official curriculum put forward by a governmental agency and what teachers have to do to "implement" it can be understood in terms of policy enactment. "Enactment" refers to the understandings and interpretations of the policy document that unfold in practice (Braun, Maguire & Ball, 2012). Ball et al. (2012) challenge the idea of policy implementation as a linear "top-down" process. They criticize the assumption that educational institutions have to respond to policy demands and other expectations, as if the people in school were not part of the process itself: "Teachers, and an increasingly diverse cast of 'other adults' working in and around schools, not to mention students, are written out of the policy process or rendered simply as ciphers who 'implement'" (Ball et al., 2012, p. 10). Instead, they conceive putting a policy document into practice as an active process were people appropriate and reconfigure the meaning of policies in the context of their institutional practices. This requires translation and interpretation within the ongoing process of education. As a result, new configurations of practice will emerge, not as right or wrong "implementations", but as the result of what people actually can do given the characteristic of their institutional arrangements. In this sense, policy enactment theory allows to understand how people perform curriculum and to research moving away from a deficit perspective of failure in implementing the curriculum. This is an important issue for mathematics education research. Policy enactment theory interweaves three aspects of the policy process: the material, the interpretive and the discursive (Ball et al., 2012). These aspects are all

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relevant in putting policy into action. The *material aspect* refers to physical contexts and the way different artefacts emerge and are used to materialize the new curriculum They are described as "instruments and effects of discourse" (Ball 2015, s. 307). The material aspects are researched by paying attention to the ways in which different resources and artefacts are used to construct and express meaning. The *interpretive aspect* covers the different ways of communicating, articulating and understanding the policy to make sense of it (Ball et al., 2012). This is researched by attending to how the actors and voices involved elaborate ideas of the curricular change, documented in different sources such as the teacher's words and the policy document changes. The *discursive aspect* highlights how the process of meaning making relates to a history and a context. Discursive strategies are about events, productions and social processes (Ball, 2015). The discursive aspect is researched by paying attention to the positioning of the actors such as students and teachers in texts and practices.

When studying policy enactment, Ball et al., (2012) suggest a method of process-oriented interaction between empirical data and theoretical framework focusing on the material, interpretative and discursive aspects. The first author in this paper, Anna Wallin, carried out a case study in two *fritidshem*, where she is supporting *fritidshem* teachers, school teachers and staff in developing *fritidshem* mathematics. The case studies consisted of eleven participant observations, where teachers and staff were followed in their interaction with students; and semi-structured interviews with five practitioners; teachers, staff and a headmaster, regarding their interpretation of the new curriculum and how they translate these ideas into their practice. The observations and interviews were video-recorded, transcribed and analysed to identify how teachers, staff and headmaster express about *fritidshem* mathematics, and how material, interpretative and discursive elements play out in their enactment. Furthermore, an analysis of the curriculum and other related documents has been carried out. Additional interviews with two recognized teachers who have been active in shaping *fritidshem* have been conducted as a way to highlight generative insights emerging from the cases. The material, discursive and interpretive aspects of policy enactment (Ball et al., 2012) traced in the interviews and observations are seen in the light of each other in the results.

THE TENDENCY TOWARDS SCHOOLIFICATION

The new curriculum of *fritidshem* appeared as part of historical changes. The initial purpose of *fritidshem* was fostering working class students and helping guardians with childcare (Rohlin, 2001). Nowadays, *fritidshem* is available to all students up to the age of 13, under the circumstance that the student's guardians are employed. The *fritidshem* is subject to a fee of maximum 800 SEK per month (Skolverket, 2012). More than 80% of Swedish younger students attend the *fritidshem* (Skolverket, 2017). From being institutions administered by the authorities of social affairs, in 1990 the *fritidshem* were set under the same management as elementary schools, so that almost each school would have its *fritidshem* section (Rohlin, 2001). Besides saving money, the change in the 1990s was supposed to generate collaboration and integration

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between school and *fritidshem* (Hippinen, 2011). In the 2000s, the earlier focus on socialization and childcare has been replaced by a focus on educational and learning aspects, with the aim of explicitly complementing the elementary school by enabling students to develop knowledge (Rohlin, 2013; Hippinen, 2011). Such changes are visible in the language use at *fritidshem*: today, the academically educated staff in *fritidshem* are called *teachers* (directed to education at fritidshem), and the time students spend there is called *education*; while the earlier expression used for educated staff was *pedagogue* and the students participated in *activities*.

Since 2016, the *fritidshem* has moved closer to school with the promulgation of the curriculum. The core content is divided in four different subject areas: 1) language and communications, 2) creative and aesthetic forms of expression, 3) games, physical activities and outside activities, and 4) nature and society. The latter highlights "Mathematics as a tool to describe everyday situations and solving everyday problems" (Skolverket, 2016 a, p. 25). This formulation of mathematics has similarities to the one directed to mathematics of elementary school, for years 1-3 and 4-6, promoting students to develop "strategies for mathematical problem solving in everyday situations, and to mathematical formulation of questions based on everyday situations" (Skolverket, 2011, p. 62). The *fritidshem* curriculum also highlights that its role is not to emphasize assessments but to complement the education provided in elementary school (Skolverket, 2016b). In the commentary material to the *fritidshem* curriculum, the National Agency of Education further explains intended directions for practice: "In the education, it is possible to capture the opportunities for learning that arise in everyday life, but also to develop situations that allow students to use mathematics" (Skolverket, 2016b, p. 22).

The envisioned connection between *fritidshem* and school became materialized in *artefacts* such as the matrix and the pedagogical planning. The matrix is an evaluation device that allows to establish goals and match them with activities to reach the goals. It became a common instrument in elementary school practice since the alignment of goals, activities and evaluations in levels of achievement that matrices usually express is central in the assessment of pupils' academic skills. Kane (personal communication, 20180503) tells that the matrix is being adopted in *fritidshem*, but it has been used as an evaluation tool for the overall pedagogical activity, not individual students. The pedagogical plans, another artefact belonging to school, is being used to frame the students' needs and interest as ways to design activities that enable them to develop in *fritidshem* education. Indeed, in one of the researched *fritidshem*, a matrix was introduced by the headmaster with the intention of helping staff working with the new curriculum. This matrix defined three possible levels of achievement in six different areas: language and communication, creativity, norms and values, sustainable lifestyles, the students' responsibility and influence, and confidence in their own ability, nature and society, school and the outside world. For the practice of *fritidshem* to reach the highest level in the matrix, the staff should secure that "the activities are

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well planned and evaluated through the pedagogical planning and the analysis contributes to constant quality enhancement" (example from the matrix).

The working of artefacts like this in the new context have a steering effect on *fritidshem* teachers and students. In this way, the *fritidshem* as a space for students play, free activity and creativity which embodied the value of gaining experience through engagement in activities outside the structured framework of school is somehow being slowly transformed. This generates tensions around what *fritidshem* teachers and staff should do and which meaning *fritidshem* mathematics could adopt. This tension is illustrated by one of the teachers who said: "When one discusses school, one always considers mathematics education. It is so obvious. It is so extremely much school!". Teachers at *fritidshem* distance themselves from formal teaching (Hippinen, 2011). The strong tradition of school mathematics is based on formal teaching and is often justified in relation to an externally defined curriculum. So, mathematics in *fritidshem* stands in contrast with the school mathematics as it is embedded in activities and games derived from the student's interests.

"Schoolification" as an international tendency to expand the logic of school to areas of children's life that had remained outside traditional school has been discussed for early childhood education and preschool (Gunnarsdottir 2014; Lager, 2015; Lembrér, 2015). The notion of schoolification "highlights anxiety regarding preschool and the *fritidshem* becoming too similar to school in terms of content and character" (Lager, 2015 p. 18). Analyzing the curriculum, the commentary material, matrix and pedagogical plan through the lens of policy enactment theory, the tendency of schoolification emerged. Explicit attention to subject areas in the curriculum and school-oriented explanations were visible. Aims of the *fritidshem* curriculum states that: "The education of the *fritidshem* complements pre-school class and school in the implementation and fulfilment of the curriculum goals" (Skolverket, 2016b p. 5). Schoolification is prominent in mathematics, in comparison to, for example, art or physical training, because of the entrenched discourses of mathematics as a core school subject and its high status in society. As the *fritidshem* teacher said: mathematics "is so extremely much school!". The tendency of schoolification appeared even more clearly in the analysis of the teachers' practices, as we will see below.

TENSIONS IN *FRITIDSHEM* PRACTICE

Two main tensions became evident in how teachers give meaning to *fritidshem* in the interviews within a tendency to schoolification, one in the relation to school norms and one in relation to mathematics.

School norms

> We have moved into an incredibly strong culture and that is school. It has been difficult, and it is still difficult for many *fritidshem* to claim and show the aim of their practice. We will adjust even though the year is 2017, 2018 is coming, and it has to be on the table much more, much further up in the hierarchy all the way up to the National Agency of Education and politics. (Interview with fritidshem-teacher, 20171215)

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This statement, exemplifies the tension between *fritidshem* and school. The *fritidshem* was positioned as in need of development to gain recognition in the educational system. Evaluation as a way to improve activities was in focus in what teachers said. At the same time, the opportunities for interpreting, appropriating and evaluating the curriculum were perceived as limited: “The control has become larger and clearer through the steering document, but unfortunately work conditions have deteriorated”, said a teacher. Despite this, all the interviewed practitioners viewed the formulation of the curriculum in a positive way, since it may give them access to further qualifications and thus the possibility of improving the status of their profession in the education system. The commentary material supplied to the curriculum was perceived as more useful, containing nuances and suggestions more adaptable to the practice of *fritidshem*.

New uses of materials such as the matrix and pedagogical planning were produced as part of the enactment process. In one of the *fritidshem* it became particularly evident how the use of these artefacts generated tensions between *fritidshem* and the school norms. The headmaster expected teachers, directed to *fritidshem*, to do the pedagogical planning of their activities. However, “that is not going so well”, expressed the teachers. This tension became evident in the “production of visual materials” (Ball et al., 2012, p. 121), as mentioned before. “We have this matrix, it’s good to have something to check at; you need a map so you know where to go… But if we are going to measure, like the school, then it will be the wrong way for me”. Anxiety regarding the school norms, measuring like the school and what to do with the policy document were in focus. A teacher explained clearly that the new curriculum “needs to develop, but not becoming schoolified, absolutely not”. “I am afraid… hum… it will become school, because we have to start from the school norms”. One teacher had read some previous proposals of curriculum directed to the *fritidshem*, and the teacher described the present one as “less schoolified than the original proposal”. The concept of schoolification was mentioned and discussed in almost all interviews. The voices were varied. Some questioned “why is the school the one to decide”, while others expressed that the school and the *fritidshem* have become closer during the history, “the school is not either the same as 20 years ago”. Some teachers even opposed the notion: “I don’t like that word, schoolification… we are here for the children, to teach them…”. Issues regarding what will happen to the students’ “free time” and the student’s own ideas if the discourse of *fritidshem* will attune to school were highlighted.

Mathematics

Artefacts and contexts were discussed in the interviews when talking about the meaning of *fritidshem* mathematics. Important ideas were using concepts, encouraging students to play games, and participating and developing in activities. All the *fritidshem* teachers, staff and headmaster framed the role of the material aspects in relation to the mathematical areas in the curriculum and the importance of involvement of students and their interests: “We never use the math books”, “We never say, now we will work

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with math, so now should we play this game". All of the interviewed practitioners emphasized the role of students' interests and the importance of meaningful activities in the practice of *fritidshem*. Situated and informal teaching were in focus, to engage the student in activities and "to make them curious". The headmaster clarified that the purpose of *fritidshem* is to put the students, their needs and interest in the foreground. Concerns were framed about the way the curriculum separated the discourse of *fritidshem* into subject areas. The description of *fritidshem* as a space where students "have the opportunity to make subjects fit together" was at stake. One teacher clarified that the subject-divided-discourse was not suitable for the practice of *fritidshem*: "there is a risk that practitioners will make a check-list out of it, to make it look like school". The same teacher problematized "if we want, we can see math in everything, but we should not limit ourselves to destroy baking with math". The teacher also framed that the educators positions the role of mathematics as if it is something merely good: "Why should we always talk about baking and mathematics… running in the corridor is also mathematics". The concern of directing the children's attention explicitly to the mathematics was to be questioned: "It will become some kind of moral mathematics". Mathematics was described as a well-known area in discussions, activities and games. The role of mathematics in the discourse of *fritidshem* was framed in a positive way under the circumstances that the mathematics had come out of the student's interests, emerged out of the situated activity and not appeared as the subject area of mathematics in school. The tension of, to which extent mathematics should become visible was evident and got expressed in the relationships between teachers and students. A teacher was telling an episode to make the point: "the students were playing with clay and thought it was very exciting. Then a teacher came and said 'it's actually math we're dealing with'. Then everyone dropped it and left, because the math is so tensional". The discussions about mathematics also concern the change in the *fritidshem* teachers' profession: "We need to be conscious about what we already do, to get the focus in the curriculum and to become stronger as profession". The fact that mathematics is introduced in the curriculum of *fritidshem* is a historical event, which needs to be enacted and discussed in practice.

STUDENT OR TEACHER-DRIVEN ACTIVITIES?

Here we go closer to the enactment of *fritidshem* teachers and staff in doing *fritidshem* mathematics. To analyse the role of contexts, interactions and to investigate how student and teachers were using artefacts in practice, the material, interpretative and discursive aspects of policy enactment were used. The observations in the two *fritidshem* were limited to investigate the activities: "Sara's Café" and "Our city". Students, school teacher, *fritidshem* teachers and staff were active in interactions and in activities. Two mathematical discourses appeared when the observations were analyzed through the aspects. In the 11 observations, five situations are analysed as mathematical, out of the interpretations and material aspects. The discourses that emerged out of the analysis were articulated through the contexts, positions, interpretations and actions in the situations. It was possible to distinguish two

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directions, one in which the student's mathematical interest was the core of the activity: *the student-driven discourse,* and one where the teachers' focus and direction led the mathematical activity: *the teacher-driven initiative*. Three of the mathematical situations were analysed as student-driven and two as driven by the teacher's initiative.

In the student-driven discourse, students are positioned as interested and in charge of the activity where problems that are tacked with mathematical tools emerge. The students advanced the mathematical perspective in interactions and in activities. For example, in the recurrent activity "Sara's Café", a society emerged. Offices, a bank, a veterinary and Sara's Café were arranged by the students. The students interacted and played. The teacher played a role as Sara and acted in the background of the activity, staging and problematizing situations. The mathematical actions and dialogues came out of the student's imaginations. The students took responsibility of the situated activity.

In the teacher's driven discourse, the teacher had a prominent role in processes, communications and actions. The school teachers and *fritidshem* staff were steered the activity and the students were following. The discourse emerged in the activity "Our city" where the *fritidshem* and the school were cooperating. The students received instructions on how to create a map to afterwards build a city based on it, with the intention of programming robots to navigate it. The students interacted mainly with the teacher and answered questions coming from the teacher. The students were active when they were prompted to do so, like measuring, when the teacher held the ruler.

CONCLUSIONS

The importance of strengthening the profession of the practice of *fritidshem* was evident in the results. *Fritidshem* has been neglected for a long time (Rohlin, 2013). However, the results indicate that participants stand fast in their way to frame subjects in their practice. They continue to emphasize their perspective centred on activity. In that sense, artefacts such as the matrix and the pedagogic planning were used in other manners than in school. *Fritidshem* practitioners value the specific role of *fritidshem*. They are aware of the importance of their work to foreground and prioritize the beauty of an activity-based educational space through play and games. The tensions regarding school norms and mathematics intertwine with the teachers' desire of professional recognition. Is it at all possible for *fritidshem* to be accepted in the educational field when valuing a creative and informal discourse derived out of the student's interests? Will that kind of discourse be perceived as effective and useful for society? The tendency towards schoolification in *fritidshem* is the result of the school setting the norm, and of the increasing desire of regulating people's lives effectively through for instance policy document. In the cases studied, when the school and the *fritidshem* were in co-operation, the logic of school took a major part. The mathematics came out of the school teachers' ideas and a more schoolified discourse appeared, a *teacher-driven initiative*. Questions and the interaction was more targeted to the content. The strong logic of school mathematics and the absence of a consolidated *fritidshem* mathematical discourse made it possible for a teacher-driven discourse to control the situation. The

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results also show that it is possible to articulate a discourse that stands in contrast to the teacher-driven initiative. In a *student-driven discourse*, the teachers of *fritidshem* are professionals who find ways and situations to engage the students in informal mathematical activities, departing from student's interests and needs. In this discourse the school norms were not present. When the student's interest involves mathematics and if the situation invites to problem solving with tools from mathematics, then the activities become rich situations for mathematical activity. In the student-driven discourse, mathematical interactions were evident in spontaneous and playful activities, derived from the students' interest. The teacher staged the game but did not steer the mathematics interactions or actions, the students did. The problem solving mathematical activity became a part of the game and the problem needed to be solved for the play to continue.

The enactment of the curriculum is a tension loaded territory. On the one hand, the new curriculum seems to be important to *fritidshem* teachers for obtaining more legitimacy and recognition in their practice. However, the results indicate that the direction in which *fritidshem* mathematics might be developed is a fragile field of tension between an informal, activity based and student-driven mathematics, and more teacher-driven, formalized and schoolified type of mathematics. In the midst of the tensions, practitioners express a strong desire to discuss the curriculum in forums, to support each other, and to generate situations with opportunities for mathematical activities adapted for the practice of *fritidshem*. Since mathematics is such a loaded area of the school curriculum, the entrance of the area in the *fritidshem* curriculum makes it even more important to discuss. "Math is so tensional". It is political and therefore mathematical aspects in the curriculum need to be enacted and interpreted among different networks, chains and actors. It is evident that the new curriculum document is nothing in itself. A curriculum needs people, teachers and students to survive: "Enactments are collective and collaborative, but not just simply in the warm fuzzy sense of teamwork…" (Ball et al., 2012, p. 3). This case study shows how policy enactment tools allow us researchers to engage in the meanings of *fritidshem* mathematics articulated in practice out of a policy enactment process. Through the aspects of policy enactment theory, we could grasp a mathematical discourse that was adapted to this new area for mathematics education in Sweden, the practice of *fritidshem*. A discourse, where the mathematics came out of the student's interests as an instrument to maintain a free space for playing, far away from society's desire of effectiveness and a forced learning capacity.

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EXPLORING THE IDENTITIES LEARNERS ARE DEVELOPING: A CASE OF ELEMENTARY MATHEMATICS CLASSROOM

Anchal Arora

Kendriya Vidyalaya Sangathan

Abstract: This paper attempts to understand mathematics learning with respect to the identities learners are developing as part of their elementary mathematics classroom. Based on the observations of an elementary mathematics classroom, it explores the relation between students' identities, their perception of 'ability' as mathematics learners and the discipline of mathematics. It was found the nature of tasks and norms along with aspects like grades and marks influence the identities learners develop.

INTRODUCTION

The lack of achievement and learners participation is often guarded of in terms of cognitive abilities (Ruthven, 1987; Cobb, 2011). These attributes like intelligence are considered of individuals and can be carried across contexts (Gresalfi, Barab & Sommerfeld, 2012). However, studies like Nunes, Schliemann & Carraher (1993) has helped understand how knowledge and intelligence are relational phenomena distributed over time and space. It is the particular systems shaped by norms and values that make a particular individual appear as intelligent and knowledgeable or not (Gresalfi, Barab & Sommerfeld, 2012). Intelligence is considered as 'the kinds of dispositions we develop to act in particular ways and consider how these dispositions develop in relation to learning opportunities with which learners are presented over the time' (ibid., p. 41). Learning of mathematics is not only restricted to procedural or conceptual fluency but becoming a certain kind of person (Kilpatrick, Swafford and Findell, 2001). In other words, the emphasis is on developing identities where learners see themselves as mathematically able, develop dispositions towards mathematics as sensible and useful, and engage in challenging mathematical situations in a persistent manner (Cobb & Hodge, 2011).

UNDERSTANDING IDENTITY

Contemporary research in mathematics education has seen an emergent body of studies focusing on identity in mathematics (Boaler & Greeno, 2000; Gresalfi, 2004; Anderson, 2007; Sfard & Prusak, 2005; Hodgen & Marks, 2009; Martin, 2007). The manner they have conceptualized and operationalized the concept has varied. According to interpretative scheme delineated by Cobb and his colleagues, normative identity and personal identity can be the two core constructs to explore identity enactment in mathematics classrooms activity (Cobb, Gresalfi & Hodge, 2009; Cobb& Hodge, 2011). Normative identity as a construct concerns with the identity students need to develop to become mathematical persons in the particular classroom context. It indicates the general as well as socio-mathematical norms students need to fulfil and identify with to become a successful mathematics learner in the given

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classroom. Cobb & Hodge (2011) elaborate the socio-mathematical norms as i) norms for what count as mathematics, ii)normative ways of reasoning with the tools and written symbols, iii) norms for what count as mathematical understanding and lastly iv) the normative purpose of engaging in a mathematical activity. The students and teacher jointly constitute these norms and obligations. On the other hand, the construct of personal identity help understands whether and how the student engages in the mathematical activity. Personal identity is an ongoing process of being a particular kind of person in relation to the local social world of the classroom (p.190). Students' accounts of what counts as effectiveness and mathematical competence and how and to what extent they relate to it or their stories about themselves and others mathematical competence are the means to understand learners' personal identities. Personal identity is similar to what Sfard & Prusak (2005) calls as "collection of stories about persons" (p.16) defining who one is in the given context defined both individually as well as collectively (Bishop, 2012). Such an interpretative scheme of identity conceptualization can help get situated accounts of how identities are developing as doers of mathematics in particular settings (Cobb, Gresalfi & Hodge, 2009). This paper based on data collected from an Indian mathematics classroom explores the mathematical identity formation among elementary learners with a focus on the nature of tasks and norms.

METHODOLOGY

The students in this study were initially studying in grade 2 of an Indian government school situated in Delhi and gradually moved to grade 3 in the next academic year. The school followed the CBSE curriculum and NCERT textbooks. The teacher was a trained elementary teacher with an experience of around 8 years. The paper here presents the observations of their mathematics classroom and interaction the researcher had with the students informally through the case of 3 students.

ZOOMING IN THE ELEMENTARY MATHEMATICS CLASSROOM ACTIVITY

According to the distributed view of intelligence as Gresalfi, Barab & Sommerfeld (2012) suggests, ' intelligence would be the joint accomplishment of what tasks affords, how resources support students to perceive and act on the task, and, ultimately, which affordances of the tasks student realize ' (pg. 47). The following interaction (translated from Hindi) is regarding the view children had on open-ended tasks like 45= ___ + ____ as part of their mathematical classroom:

Researcher : How was today's class? How were the sums? How did you find them?

Child 2 does not respond and remains silent.

Researcher: Share. Were they good? Or difficult? Or boring? How did you find them?

Child 2 (slowly): They were good. But I like the other ones more.

Researcher: Other ones? Which?

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Child 2: The ones where we have to find out the answer.

Child 3: Ma'am She is talking about the ones in which we have to keep number one below the other and take carryover.

Researcher (to child 2): Which? The carry over ones?

Child 2: Yeah.

Researcher (to child 2): Okay. Why? Why do you like them?

Child 2: They are easy.

Researcher (to child 2): What is easy about them?

Child 2: They are just easy. It's easy to find if it has been done right or wrong.

Researcher (to child 2): How? How to find if it has been done right or wrong?

Child 2: Ma'am tells the answer.

Researcher (referring to child 2): ***** likes the carry over ones. You? (looking towards child 1 and 3) Which ones do you like?

Child 3: I like both of them.

Child 1: Me too. But I like the today ones more. There are many answers. Have to use the brain.

Child 3: Yes there are many answers.

The open-ended tasks were part of this mathematics classroom. However, they were not a regular feature. The classroom observations and responses helped understand how children responded differently to the nature of these tasks and the varying norms. Child 3 and 1 enjoy the mathematical challenge open-ended tasks brings and showed persistence while solving them. On the other hand child 2, was particularly silent in the given class. She later shares her comfort with the traditional close-ended addition problems. Intriguingly, the reason she shares is the convenience it provides in knowing if the problem is solved correctly or not. Moreover, she also shares her comfort with the norm where the authority is with the teacher to tell if the answer is correct or not. In another such conversation, it was found for child 3, the space to work along with a peer in a mathematics classroom is a valued aspect to enjoy and like mathematics. While for child 2, ability to get good grades was an important determinant in defining her comfort and liking of the subject. Thus the tasks and norms emerged as crucial aspects to understand the enactment of the identities these students are developing.

REFLECTIONS

The dynamic and fluid nature of identity can be a useful tool to track and analyse how the nature of tasks and norms of mathematics classroom or the pedagogical approach contribute in learners engagement with mathematics as a discipline, the

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attitude and dispositions they develop towards the discipline and how they perceive themselves and other participants with respect to it.

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CHRONOTOPE AS AN ANALYTICAL TOOL: A TEACHER'S DILEMMA MANAGEMENT IN A MATHEMATICS CLASSROOM

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Teachers in Korean primary classrooms often feel a "dilemma of mediation" as they respond to their own actions and students' actions during math lessons (Adler, 2001). We trace how a teacher manages this dilemma and, in particular, the tensions arising in classroom discourse, using Bakhtin's notion of chronotope. We identified two chronotopes that allow us to describe the students' and the teacher's participation as well as how the curriculum affects a teacher's decision making. The chronotope concept is a useful analytical tool because it reveals key dilemmas experienced by the teacher at a time of curriculum reform.

INTRODUCTION

Recent Korean mathematics education encourages teachers to develop their own curriculum based on the themes and larger objectives provided by the national curriculum (The Korean Ministry of Education, 2014). This reflects a shift in the role of the curriculum to a general guideline for mathematics teaching practice instead of being a prescriptive description of mathematics lessons. However, teachers often feel a tension between their expanded authority in curriculum design and implementation and the conventional status of the curriculum and the textbook; they experience a "dilemma of mediation" in which they must balance knowledge of their students' mathematical backgrounds and interests with the broader social expectations of math education in Korea (Adler, 2001; Barwell & Pimm, 2016).

Teachers' dilemma management is not only involved in their lesson planning phase, but also occurs during classroom conversation in both explicit and subtle ways. Especially when a teacher and her students are collaboratively participating in a math discussion, a teacher needs to make decisions about the direction of the conversation. In this sense, it is important to deeply explore teachers' decision-making process in their mathematics teaching practice in order to understand their experience of curriculum reform and to develop resources for teacher support.

This study takes Bakhtin's notion of chronotope (1981) as an analytical tool to examine how different temporal and spatial dimensions of students' and the teacher's experience are involved in a mathematics teaching practice. No matter the extent to which a classroom discourse is confined by the curriculum, the participants of the discourse still shape and reshape it through the diverse, multidimensional, and multi-layered characteristics of language in a classroom. Although there are various factors that influence classroom repertoire, this study focuses on the ways in which chronotopes describe how the teacher negotiates the tensions across curricular expectations, students' actions, and her own actions in class.

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THEORETICAL FRAMEWORK

Bakhtin (1981) developed the chronotope concept to analyse how time and space are represented in novels. The chronotopic approach helps us grapple with the idea of temporal and spatial dimensions affecting a teacher's decision-making, including planning for a time trajectory into the future. In this study, the spatial dimension of the chronotope concept refers to visual diagrams, material places, and conceptual spaces that are referenced or represented in the math class discourse. Blommaert (2015) developed the notion of chronotope as a way to reduce the dichotomy of "micro" and "macro" levels in sociolinguistic analysis in order to acknowledge the complex and multi-layered nature of discourse. Van Eijik and Roth (2010) explain that chronotopes help us in understanding a space as a lived entity which includes sociohistorical stories of the people lived and involved therein, instead of viewing it as "objective and isolated" space. In this sense, taking the notion of chronotope as a tool to analyse classroom discourse discourages simplifying the contexts and participants' identities involved in a classroom interaction (Chronaki, 2017). The chronotope concept provides a perspective to explore speakers' utterances through how they are situated in relation to other people and contexts (Blommaert & De Fina, 2017).

METHODS

Data for this paper draws from a classroom discourse study of students' and their teacher's collaborative participation in the math class. Since we are interested in their classroom language use, the main data was collected as video and audio recordings of mathematics lessons from May 2017 to July 2017. The first author observed the math class two or three times a week for 11 weeks total and 26 lessons. The first author also kept a field notebook, focusing on the significant or interesting aspects of the classroom conversations from her perspective, drawings and writing on the board, and informal teacher interviews before and after each lesson. The participants of this research project were 19 third graders and their homeroom teacher in a primary school in one of the biggest cities in South Korea.

DATA SOURCE AND ANALYSIS

Building on a previous paper about the linguistic dimensions of the teachers' dilemma of mediation, we wanted to know more about what influenced the teacher's dilemma management. Teacher interviews revealed that she intensely wanted to promote students' mathematical interest, but she felt a great burden of covering material dictated in the curriculum document.

Based on data sources and analysis, we have identified two preliminary chronotopes based on two future orientations which we call (1) the promoting mathematical interest chronotope and (2) the curriculum and exam focus chronotope. Sample excerpts of the

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transcribed and translated data are offered here, but more excerpts and images from curriculum documents will be offered in the presentation.

#1: Promoting mathematical interest chronotope

The learning objectives in this unit included topics about building rectangles, right-angled triangles, and squares. The excerpt below was a part of the discussion following the learning of basic concepts and definitions of the two-dimensional shapes.

1 Teacher:	Alright, what if a window is not rectangular but is another shape, then…
5 Jisun:	Airplane! The airplane windows are round shaped.
6 Teacher:	You're right. The airplane window is round. How about the windows of this classroom?
7 Hyosuk:	There might be spaces in that way. *[Referring to the spaces in between round airplane windows.]*
8 Teacher:	That's a good idea. A window is round, and so is this window. *[Drawing two circles next to one another on the board.]* Then what might come through the space between these windows? *[In comparison to the two rectangular windows which do not have a space between them.]*
9 Students:	Shin-chan's butt! *[Referring to a Japanese cartoon character.]*
10 Teacher:	Shin-chan's butt! *[Giggle.]*

In line 1, the initial chronotope was closer to the curriculum and exam-focused chronotope because the teacher initiated and attempted to redirect conversation related to the textbook's suggestion, but it soon shifted towards the promoting mathematical interest chronotope. The students brought their memories of windows from times and spaces outside of their immediate classroom, such as the shape of airplane windows as well as the body part of a cartoon character (lines 5, 9). The teacher encouraged the mathematical interest chronotope by accepting and praising students' utterances instead of redirecting the conversation immediately.

#2: Curriculum and exam focus chronotope

This selection was a part of a conversation in which the teacher introduced a new curricular diagram intended to show rotation, because this diagram would possibly be used in examination task instructions.

49 Teacher:	Absolutely! If I want to make this in this way…*[Rotating a sheet of paper with a shape drawn at 90 degrees.]*
50 Hyungyu:	90 degrees!
51 Jisun:	Rotating at a right angle.
52 Teacher:	Good job! Rotate at a right angle. *[Pointing at the screen, which shows a new sign they are going to learn.]* It looks like this. As I can't tell you "Let's rotate the triangle at a right angle" while you are taking a test, we will appoint a sign like this. If you see an angle made when the clock moves from 12 o'clock to 3 o'clock, then how many times do you rotate at a right angle?

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We consider this as the teacher's effort to reference a time and a space different from the immediate classroom because she was focused on test preparation. She reminds the students that she can't assist them during the test. Her comments involve reminding students of a previous lesson—a different time and space—but helping them learn printed notations that they need to interpret for the test in the future.

CONCLUSION

This paper suggests that the concept of chronotope helps to describe the teacher's negotiation of two often-conflicting goals: preparing students to be successful in the future and promoting students' mathematical interest. While data analysis is still underway in many cases, even though the teacher attempts to invite different temporal and spatial dimensions into the conversation, the chronotope doesn't shift much from what is recommended in the teacher guidebook. As we develop this study, we would like to further identify the ways in which the curriculum affects the classroom chronotopes which emerge through the teacher's teaching practice.

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SCIENCE TECHNOLOGY ENGINEERING MATHEMATICS (STEM) LAND: DEEP LEARNING OF MATHEMATICAL CONCEPTS THROUGH EBD AND BY USING MATERIALS

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Abstract: This is an action research project in which we describe deep learning of Mathematical concepts in children through using materials and by Education by Design (EBD). For this paper we define deep learning as: a) ability to apply what they learn in different contexts, b) ability to retain the concepts they learn from term to term or across grades, c) ability to connect what they learn with what they deeply care about.

The two methods described are a) use of physical materials to learn abstract concepts, b) creation of projects through Education By Design (EBD).

EBD is a process of creating projects for learning or to demonstrate learning of a concept. In this paper we will focus on real life problems. We also describe how these EBDs have altered the norms and functioning of the school.

CONTEXT

Education by Design (EBD): An EBD (Mobilia, 1998)

> is a classroom dynamic that guides the thoughtful design of learning experiences for students. In an EBD classroom, students frequently work collaboratively to achieve desired results (for example solutions to real life problems) as they develop knowledge and understanding, critical skills, and vital habits of mind.

In EBDs inquiry methods and self-evaluation are frequently employed.

STEM land comprises of a team of youth engineers who teach in rural STEM centers run in two outreach schools of Auroville – Udavi School and Isai Ambalam School. The engineers were born and brought up in and around Auroville and volunteer part time in both the schools. Both schools aspire towards the holistic development of the child and cater to children from villages surrounding Auroville.

This paper primarily focuses on the younger group of 68 children from 3rd to 7th grade at Isai Ambalam school (age group 8-13 years). Isai Ambalam School follows the central board syllabus and we work intensively with the children for 6 hrs/week during the Environmental Sciences (EVS)/Science and Mathematics classes. In demographics, the occupation of parents at the school is unskilled labor (35%), skilled labor (55%) and salaried workers (10%). The predominant community (45%) accessing the school is SC (Scheduled Caste) which are communities that are socially disadvantaged. Most parents of these children have not completed the 8th standard.

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PHILOSOPHIES OF STEM LAND

At STEM land we follow the three principles of true teaching by Sri Aurobindo (Aurobindo, 1910). One, **Nothing can be taught** and the teacher is not an instructor or task-master, but a helper and a guide who shows the child how to acquire knowledge for him(/her)self. Second, Mind **needs to be consulted in its own development**: working with student initiated projects or student questions are good examples. Third, to **Work from near to far**: from the concrete to the abstract, from the known to the unknown. These principles are well aligned with a constructivist philosophy. The name STEM land is in reference to Mathland (Papert, 2002) as spaces where children would learn Mathematics through the provision of appropriate materials these include materials that children can access on their own, as well as tools to create their own projects. Our progression towards meeting these principles is described in this paper.

LITERATURE ON MATHEMATICS EDUCATION

The dislike and fear of Mathematics in children is well documented in literature (Daniel, 1969). The underachievement of children especially from rural and disadvantaged backgrounds is further documented in literature both nationally (Banerjee, 2016) and internationally (Howley and Gunn, 2003).

This is an action research project, as the research has been conducted by the teachers themselves. The purpose of this work is not to analyze children's limitations, but to describe what helped us become more effective in the hope that it will be useful for others. The paper also briefly notes how the functioning of the school itself has transformed through working on real life challenges using philosophies of EBDs.

Our aim is close to the NCF (National curriculum framework) 2005 (Pal Y, et al., 2005) that describes how Mathematics education should address the "higher goals" of broadening the child's mind to help Mathematize (or think Mathematically) and build critical skills like problem solving and logical thinking while addressing narrow goals of knowing skills in Mathematics. The NCF 2005 further states that "Learning should be made enjoyable and should relate to real life experiences. Learning should involve concepts and deeper understanding." In further references (Mayer, 2002) we find that the goal of instruction should emphasize objectives that include cognitive processes associated with "Understand, Apply, Analyze, Evaluate, and Create". These goals are similar to those of Education By Design.

In this paper we describe the interventions that led us towards real life EBDs that helped us learn more about how children learn deeply - retain, apply concepts and also connect with what they care about, and fundamentally alter some of the norms at the school.

LEARNING WITH TEACHING MATERIAL

High School Survey of Student Engagement (HSSSE) found that creation of learning materials could possibly engage children more filling the engagement gap (Yazzie-

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Mintz, 2009). Further research suggests that engaged students are better able to make an effort to comprehend complex ideas or master difficult skills throughout their education (Fredricks, et al., 2011).

We noticed the difficulty that children were having with learning in a didactic setting and introduced materials. In specific, we tracked the use of Ganit Rack to automatize addition and subtraction up to 20, Vaughn to master the multiplication tables. In both cases, we were able to see an improvement in the interest of children to engage, and ability work independently or with peers without direct engagement with teachers. With the Ganit rack 8 out of 12 children from the 3rd grade class wanted more problems from the teacher and later solved their own problems. They started seeing patterns of 5 they had not noticed before. We worked with 16 children (4th and 5th grade) who had not been able to master multiplication tables and they appeared to be able to master these with the use of Vaughn Cube. On re-assessment, after 6 months, the children asked for a quick refresh (1 day) after which 14 children had retained the tables. Few of the children were able to answer in the context of the Vaughn cube but had to be supported to correlate this method with their multiplication tables and its application. Deep learning was only partially met in retention supported (with scaffolding).

MAKING LEARNING MATERIALS

We looked at children making their own learning materials rather than using readymade ones. In specific we looked at three, making their own 1m scale from wood and 1m^2 square from chart to understand length and area and creating materials for comparing triangle and square areas with ratios. A 6th grade girl said that she was thrilled to use the hacksaw and cut the wood for the first time. The concepts of conversion from 100 cm to 1m and 1000 cm^2 fit into a 1m^2 were understood and retained over time. When creating scaled rectangle and triangles they noted that a (right) triangle is half the area of the rectangle with the same base and height and the areas scaled as squares of the sides. 18 of 22 children said learning with materials (creating and making) helped them understand and retain concepts better. We noted that on being asked where they could apply mathematics in their daily life the responses of children were in most cases still limited to purchasing from shops. We needed to do more to connect what they did with what they cared about.

REAL LIFE EDUCATION BY DESIGN (EBD)

When faced with acute water shortage at school children began to want to what happened to it. They started to learn about ground water, its depth and discuss in groups the reasons behind the shortage of water, came up with hypotheses and looked for ways to confirm them. They designed an instrument to measure the depth of water. This required some electronics and real life measurements and they started connecting what they learn in school with real life. They studied the water cycle (science) and different kinds of soil to understand ground water and created bunds across the school to increase water recharge.

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They also built a pond to have a sense on abundance even in times of difficulty. They found that they needed to learn to work as teams to be successful at such projects and the need for more time to accomplish them. They students asked to come to school on Saturdays for projects and started staying at the school once a week. The school was primarily meant as a day school and lighting at night was limited, children built torches and set up light fixtures. They found that the when the bore well runs sometimes water fills and overflows, they created a sensor for overflow detection. They also looked at recycling the kitchen waste water and created a kitchen garden. The children look at the issues in school as an opportunity to be creative and solve them and take up field work such as painting, digging, masonry, plumbing, planting along with the teachers.

Measurements whether of wires and cables or for the pond they built were made with intent. Conversations on ratios of sand with cement became normal. English was written to describe their observations meticulously. The younger children who were not able to write in English wrote in Tamil first and then translated it into English and learned to present bilingually. The learning was thus integrated over many subjects.

In a survey of 22 children in Isai Ambalam school who were asked where they learn more (1 – class, 5 – both, 10 –EBDs) the average was 6.6. After 6 months of completing an EBD when asked about what they remember children were able to remember materials used for building the pond, many were able to connect mathematics they learned – ‘measurements, ratios, multiplication, division, addition, subtraction'. They were also able to describe skills like ‘process of painting in one direction to get a good finish’, 'how to make a raised bed’, ‘how to grow plants without using chemicals'. The response of how accomplished they feel after EBDs averaged at 8.4 on a scale of 1 to 10.

With stay overs and Sat school the didactic set up of the school has been significantly altered. This increased interactions between children the teachers on Saturday and in the evenings. Even though there was resistance especially for girls to stay over, this has more or less become a norm at the school. Due to the increased inputs from the children many unused area of the school have come to life. An overall improved perception of the school has led to an increase in enrolment of 40% of children with more diversity and a better mix of communities. The survey of the new parents pointed towards them noticing improved confidence and communication skills in the current children.

CONCLUSIONS

We found that while children tend to be more engaged in learning when they use materials, yet the learning is not necessarily deep. However, real life EBDs allow children to take up something they care about, apply their learning, and retain it longer leading to deep learning. Creating their own learning materials seems an intermediate between the two. Engagement with real life EBDs transformed the school from a more didactic arrangement to student-centred living learning

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environment. The children's ability to work in teams and their communication skills have increased evidence of deep learning and have been acknowledged and appreciated by the communities served by the school.

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THE ART OF SELLING MATHEMATICS

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Abstract: The paper tries to bring out various dimensions of presenting mathematics as a product. The author explores these through various mediums such as advertisements on hoarding, billboards, posters, books and digital media. The field of teaching mathematics and the need of mathematics have turned mathematics as a product in India. It has given space to various ways of selling it to students, parents, public and other stakeholders. An analysis of advertising and depicting mathematics by these mediums shows how the social, cultural and political aspects of mathematics are being used to sell it and the challenges it possesses to mathematics education.

INTRODUCTION

Mathematics is part of everyday situations, professional contexts, technological enterprises and research procedures. According to Christensen, Skovsmose & Yasukawa (as cited in Vithal & Skovsmose, 2012), mathematics operates in a variety of cultural and socio political practices. Explicitly or implicitly, mathematics always plays a key role be it socially, politically or economically as it is one of the important skills required to crack entrance examinations for joining various government and non –government services in many countries. In India also, quantitative skills are key to join such services. Almost all such examinations have 'Mathematics', especially 'quantitative aptitude' as a major portion of the papers. Even, non – US students across the globe have to crack exams like GRE, GMAT to be eligible for admissions in US. Demand for relevant computational skills (at least for the sake of cracking entrance examinations) has provided space for coaching centres to grow and teach computational skills to school students as well as aspiring job seekers (mostly graduates). With the rise in such informal sectors of education, Mathematics seems to boom as an industry. Mathematics is often referred to as the 'killer' subject in India (Rampal, 2003; Ramanujam, 2012). The emotional and attitudinal issues are especially important in mathematics education, since Mathematics functions as a gatekeeper, both as a qualification for further study and desirable jobs, and as a prerequisite for certain types of cultural participation (Evans, Tsatsaroni, & Staub, 2007). Even such negative notions and perceptions of mathematics among public, students and fresh graduates act as a catalyst for coaching industry. These sectors target the persistent problem of mathematics education across the globe i.e. the 'Fear of Mathematics' and grab the attention of students and public in general towards such fear and claim to offer remedial services related to it.

RATIONALE

The paper titled 'The Art of Selling Mathematics' tries to provide a glimpse into the 'Commodification of Mathematics'. Vithal & Skovsmose (2012) assert that poverty and its related issues (e.g. youth unemployment), which have many implicit and

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explicit connections to mathematics education, do not seem to feature strongly in mainstream mathematics education research, literature, conferences and in theorising mathematics teaching and learning, even though they have major policy implications. Thus, one of the purposes of the paper is also to present an implication of such issues which is reflected in selling ‘mathematics’. The author argues that such commodification leads to unfair depictions of mathematics in public space. It makes mathematics synonymous with lot of computations, shortcuts, tricks and a formula based subject which is unfair to the image of mathematics.

DEPICTION AND ADVERTISEMENT OF ‘MATHEMATICS’ IN PUBLIC SPACES

In the paper some samples of photographs (clicked by author) depicting mathematics as a product have been presented. The photographs (in the conference poster) cover *coaching centres, books (in local market), social media and digital platforms and promotions/events in public places.*

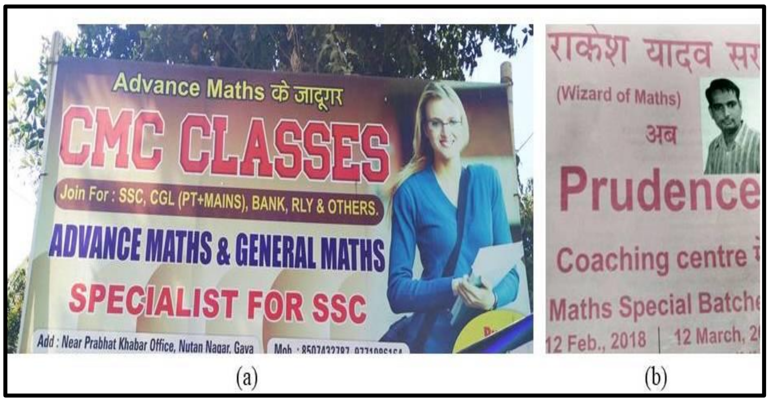

Image 1: Billboard (a) and Poster (b) of a Coaching Institute

Image 1(a) is a billboard of a coaching. It uses a tagline in Hindi - ‘*Advance Math ke Jadugar’* [*A Magician of Advance Mathematics*]. Image 1(b) is an advertisement of mathematics classes by an individual. It presents him as *‘Wizard of Maths’*. The billboard (Image 1(a)) uses a lady holding books (perhaps to draw attention of males). It boldly emphasises mathematics as ‘Advance Mathematics & General Mathematics’ which is quite popular in the coaching world. The billboard (coaching institute) is located in Gaya, Bihar where the sex ratio is not ideal - 918 (Bihar, 2011). The use of female figure on billboard, in a way also reflects the existence of gender bias notions in the market of mathematics. The owner of the institute perhaps expects greater number in enrolment of boys than girls in institute, uses a female figure in the billboards to draw male attention. There are many coaching institutes for competitive mathematics where these present the teacher or the trainer as ‘King’. Such institutes also claim to offer remedial services for mathematics exams, luring students with catchy phrases like *‘Learn mathematics from the beginning – 0,1,2,3...*’. These target college students and fresh graduates who look for employment opportunities. Generally, the mathematics syllabus of such competitive exams comprises school level arithmetic, algebra, and mensuration, geometry and data interpretation. There

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are many coaching institutes also, which target school level students. These advertise time to time on the basis of topics relevant to school level mathematics and entrance exams of universities and prestigious engineering institutions.

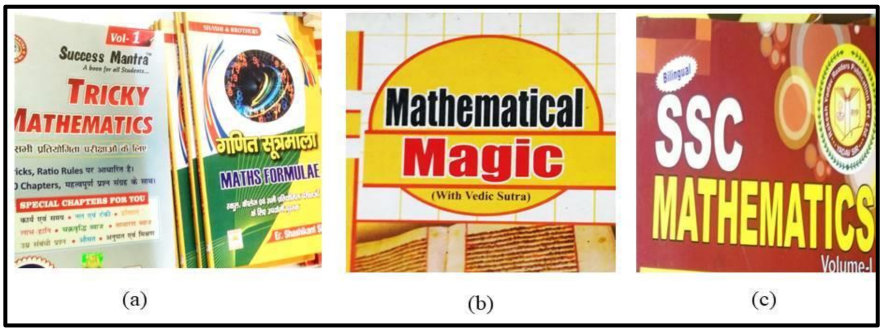

Image 2: Books of mathematics in a daily market

Even the market is flooded with mathematics books centred on formulas and tricks for short and fast calculations. Some books make an ‘emotional’ appeal to the target group viz. *‘Mathematical magic with Vedic sutras’*, *‘fast calculations with Vedic maths’*(Image 2(b)). The word *‘Vedic’* reflects cultural and historical aspect of India. There is a lot of debate on the title ‘*Vedic Maths’* among mathematicians and academicians. Raju (2014) asserts that there is nothing *Vedic* in *Vedic* mathematics, while (Glover, 2014) defends the relevance of Vedic mathematics. An analysis of mathematical content of the book shows that they cannot be from the Vedas (Dani, 1993). Irrespective of the debate, still the use of term *‘Vedic’* exists in market. Arousal of emotions of cultural and historical connection with *‘Vedic’* is used to sell and propagate such mathematics books. Dani (1993) finds it shocking to see how much people driven by misguided notions are able to exploit the Indian society's urge for self- assertion. Generally, on book stalls at railway stations, such other ‘*tricky*’ books are displayed along with magazines of politics, current affairs and adult magazines. It also depicts mathematics as a strong part of the adult market. In her study on portrayals of mathematics in young adult fictions, Darragh (2018) also points out mathematics as ubiquitous part of it .With time, digital platforms such as social media (Facebook) and YouTube have provided space and name to individuals who claim to provide remedies for mathematics problems. These trainers (teachers) publish video contents on tricks for fast calculations. Several posts with captions like ‘*Calculate within seconds’, ‘Throw away your calculator’* etc. are sponsored on social media to target youth. These examples and mediums show how the widespread ‘fear of mathematics’ is strongly (mis)used by the enterprises and also show how mathematics is being made synonymous with computations, formulas, shortcuts and tricks in public space.

CONCLUSION

The various dimensions of selling mathematics are reflected in the various ways social, cultural and political aspects of mathematics are used. Use of elements like ‘emotion’, ‘fear’ and ‘comfort’ and use of terms like ‘remedy’, ‘fast calculation’, ‘solutions without formulas’, ‘tricky mathematics’, ‘solution within seconds’, ‘speed

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with *Vedic* mathematics' find an important place in taglines of these mediums which also reflect the current need and demand for faster calculation in mathematics. To claim oneself as a *'Magician, 'King' or 'Wizard'* of mathematics' seems to be a trend now. It also reflects the trainers' utilitarian mentality and disrespect for the vast field mathematics. Such adjectives portray mathematics as something supernatural or elusive, which can be conquered by magicians or kings only. Enterprises also seem to use the market on the basis of local and regional language, for example: publishing bilingual materials of mathematics (Image 2(c)). On the other hand, the same students at college level (who study mathematics) face the lack of resources in local languages and also suffer from language transition (Mishra & Sharma, 2017). Such practices can also be implemented by university and colleges for disadvantaged students. *The more one calculates fast, the more chance one has to crack entrance examinations* seems to be the key. The heavy rush for employment and the dependency of selection in jobs on fast calculation techniques has created a scenario where there is a boom in enterprises related to mathematics as well as content creation related to tricks and fast calculations. The effect can't also be ignored as coaching industry contributes a major portion to economy. Growth of such mathematics industry focused on short tricks also brings a threat to teaching practices which overshadow the broader notions & ideas of mathematics. Public can be misled by such image of mathematics. Sam & Ernest (2008) studied about the image of mathematics among British adult public. Most of their images seemed to relate to mathematics learning experience in schools (Sam & Ernest, 2008). The depiction of mathematics through these mediums and advertisements reflect a situation where there is focus on portraying mathematics as a tough subject (for garnering and relating to school experiences) and attracting students, public and others in return for remedies. It has led to depiction of mathematics as a subject which is mostly concerned with calculations and computations. Ravish Kumar one of the journalists and anchors at a news channel NDTV India, in one of his blogs writes on the theme of children who turn into criminals due to fear of Mathematics and English. He writes:

> किसी भी शहर की दीवार पर मरदाना कमजोरी के दवा बेचने वाले के साथ गणित और अंग्रेजी के ही मास्टर पास होने की दवा बेचते नजर आते हैं. (Kumar, 2015)

> In other words, he highlights the fear of Mathematics and English in society and satires on the extent to which it has spread - "A time has come when even the remedies for these subjects are being sold in the same manner in which the locals (self-claimed doctors) advertise across the whole town to sell drugs for curing masculinity weakness".

It is an irony that Mathematics accounts for a greater part of economy of this country by selling and presenting a narrow aspect of it to the market. It has the power to popularise such unfair perceptions of mathematics. If perceptions of mathematics are constructed outside of the school setting, perhaps we need to be combating those perceptions in a similar arena (Darragh, 2018). Such perceptions need to be changed and countered on a large scale by all concerned for mathematics education.

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THE ACADEMY, THE WORLD OUT THERE AND THE SHARING OF THIS DIFFERENCE

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Abstract: In this paper I present a view on meaning production which highlights the difference between two worlds: the academy and the everyday life. I indicate that the difference between these two worlds is based on accepted legitimacies in each one of them. My intention is to present both theoretical and empirical support to a position on the benefit of sharing the difference. This support is taken from the Model of Semantic Fields, based on the works of Romulo Lins (Lins 2005, 2012).

THE ACADEMY WAY OF MEANING PRODUCTION

Romulo Lins, a Brazilian mathematics educator, provides us with a framework that makes it possible to think about meaning production and the communication process beyond the objectivist approach to these concepts.

From the point of view of Lins' theorisation, meaning is not transmissible from one interlocutor to another. Instead, each one of the subjects in the communication process produces meaning in the interlocutor direction, saying things he believe would be said by that interlocutor.

> In the Model of Semantic Fields the notion of communication is replaced by the notion of *communicative space*, which is an interaction process in which interlocutors are shared. In a conceptual inversion, "communication" no longer corresponds to something like "two people talking to each other", but "two cognitive subjects talking in the direction of the same interlocutor" (Lins, 2012, p.24, our translation)

That demarcation is a result of the process that constitutes culture. In each culture some production modes of meaning are accepted, some others are not. This acceptance is a result of struggle for the control and determination about whom cannot speak like those who form part of that culture.

In academy, for example, some modes of meaning production are not lawful. Particularly in mathematics courses (or within the research area), to say that one equation is like a scale balance and solve it in terms of putting in or removing weight from both sides, is not accepted.

In mathematics, since the age of Enlightenment, any relation with the material world was banished from its theories. So, when a mathematician says things about equations, number, space, isomorphism and rings, he does not talk about things that can seen in the material world. Mathematics is a definitional, "internalist" and symbolic science. That is say, its not accept (or is necessary) things from outside itself. In others words, in the academy there exist meaning production modes that do not legitimise other modes from outside of the power relations which determinate

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each area. **These** academic modes are continually linked with the ideal/idea of objectivity.

EVERYDAY LIFE MEANING PRODUCTION MODE

On the other hand, when people are outside of the academy, even the ones peoples which can produce meaning in those terms previously mentioned, other modes of meaning production are mobilised.

In everyday life, many modes of meaning production are natural and naturalised. For example, when we are in the bakery buying some bread, we hardly think that communication is an accident, as discussed by Derrida and Austin. Lins (2012, p.28) says "the cartesian subject is able to buy meat at the butcher's shop" (our translation). When children are building kites, they no longer think about perpendicular straight lines, angles or planes, they just mobilise some legitimacies about combinations of materials that are light enough to fly.

The legitimacies mobilised in everyday life constitute culture too, like the legitimacies in the academy, but in everyday life more modes of meaning production are accepted. So, a larger group of legitimacies can be mobilised by the subject during the meaning production process.

This is the fundamental difference between everyday life and academic ways of meaning production. In everyday life one object can be a lot of things. Numbers, for example, can be quantities, can be things, a post code, a house address, can be even mathematics related. But in Mathematics, numbers, based in Peano axioms, are only positions in a sequence.

CATEGORIES OF EVERYDAY LIFE AS ELEMENTS ORGANISING EDUCATION

How we can put these two worlds together? Why do that? To answer these questions, Lins (2005, 2012) has developed a proposal for mathematics teacher education. This propose I extend to the any level of education.

The proposal of Lins (2005) consists:

> first, in adopting a new set of categories to organise the mathematical education of mathematics teachers. Instead of Linear Algebra or Metric Spaces or Geometry, courses are structured around notions such as Space or Measurement or Decision Making. The key idea is that those are everyday categories, well familiar — in their own everyday ways — both to future teachers and to their future students, so they can function as a firm ground from which to proceed, at the same time they are already framing much of what will be present in school mathematics classrooms. (Lins, 2005, p.4)

Lins (2005) proposal for a course structured around space, for example, from legitimate meanings in everyday life. This may include familiar ideas such as space as meaning an empty place. The teacher can make the following questions:

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What changes if we decide to introduce, for instance, a way of comparing the proximity between things? And what if we introduce a system to locate things in this space?" (Lins, 2005, p.4)

From the answers to these questions the teacher may go on to discussions beyond the

naturalised meanings of space. In Lins' (2005) proposal, the teacher training would be focused on increasing the repertoire of meaning production modes of the teachers. This is would be possible by discussing the legitimacy of several modes of meaning production during the teacher education.

Pedagogy and mathematics are presented as categories for the analysis of the process, but not as categories that actually drive the production of meaning, knowledge and understanding — as much as this also does not happen in actual classrooms. (Lins, 2005, p.5)

That way, the teachers, and the students, would see that the mathematics knowledge is not a substitute for the knowledge mobilised in everyday life, but only another way to produce meanings. A mode of meaning production that mobilise others legitimacies. In this case, education would be understudy as a way to increase the repertoire of modes of meaning production and, increase the ways of talking about the world.

CONCLUSION

Cultures are defined by which meaning production modes are considered legitimate and which are not. The educational process should make it possible for the teacher to access different modes of producing meaning and make him able to produce meaning according to each one of them.

The reflexive experience of difference afforded by sharing meanings produce modes in teacher education (and in school too) that foster at the same time the mathematical knowledge and understanding of the teacher.

Seen from this point of view, education is not a colonisation process but a way to increase the repertoire of meaning production modes, and to increase the capacity to see and operate on the world.

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TEACHING MATHEMATICS THEMATICALLY THROUGH SIGNIFICANT CULTURAL CONTEXTS

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Abstract: Through the ages, the historical monuments have been a means of visual expression used to convey a wide spectrum of tangible and intangible ideas. They can often transcend languages and have the ability to take the spectator at different times and into different worlds. By integrating mathematics with such cultural context into formal teaching and learning through employing a thematic approach, we can provide learners with a strong perspective towards the mathematical concept in focus and we will equip students with right amount engagement to develop high order thinking, challenge the intellectual capacity, scaffold instructions, foster creativity and historical empathy, and deepen their understanding of their own and others' human experience. This paper aims to create engaging and a constructivist learning environment by providing culturally relevant context in classrooms, specifically discussing how the Konark Sun Temple, which has been identified as a World Heritage Site by UNESCO, can be inculcated into a thematic approach to discuss multiple mathematical ideas and concepts related to the school curriculum at various levels.

OBJECTIVES

This research was done in order to explore some territories regarding the cultural aspect of mathematics teaching and trying to provide some relatable mathematical experiences to students of different grades. While there have been some examples of culturally relevant and ethnographic mathematics teaching for students, this research was unique in itself because of the examples it presents and the Indian specific context it provides. This research tried to answer and evaluate some questions like:

1. Does teaching mathematics in combination with traditional cultural elements and values help students understand various mathematical topics?
2. Can including elements from the cultural context in their mathematics education encourage more meaningful learning and provide more effective achievements?

RATIONALE & PROCEDURE

The National Council of Teachers of Mathematics (NCTM, 1989), strongly advocates that students should be exposed to a variety of diversified experiences that are related to the cultural, historical and scientific evolution of mathematics. On the same note, Davidson (1989) accentuates on harnessing cultural values as a means of conveying mathematical content helps to concretize, visualize and emphasize the relevance of mathematics to the learners' lives, which turns the sense-making process appeal to the utilitarian and aesthetic aspect of the discipline; and at the same time makes the

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lesson more engaging, interesting and enjoyable. Similarly, Civil (2002) emphasizes the need for the creation of mathematical experiences that are related to the cultural experiences of students as these experiences are often rich in mathematical concepts; where mathematics would be evolved as a cultural product to perform various activities such as counting, measuring, designing, locating and playing (Alan Bishop 1986, 1988). In fact, Gay (2010, p. 164) advocate that the academic success is improved by accepting the fact that mathematical and scientific knowledge is present in all cultural groups, extracting math and science knowledge and skills embedded in the everyday activities and cultural heritages of different ethnic groups … and connecting school mathematics with the funds of knowledge present in such different existing cultural heritage. These kinds of learning environment can stimulate student interest and motivation while developing important mathematics concepts, processes, and skills. When teachers design integrated and contextualized experiences that are aligned with curriculum standards, mathematics becomes more relevant and meaningful to students; hence providing different perspective along with facilitating collaborative, competitive and peer learning environment for students to build their concepts and explore mathematics in a constructive way to create a more sustainable teaching-learning environment.

The researcher developed lesson plans to help visualize the different concept in focus; every lesson plan was supposed to be economic with the content while keeping a check on the pacing of the rhythm of the lesson. The researcher mapped the concepts of numeracy, time and circles with the cultural heritage site the Konark Sun Temple, which has been identified as a World Heritage Site by UNESCO. The following table states the three concepts mapped to the wheel of the Konark Sun Temple:

Concepts	Grade
Numeracy	2nd
Time	8th
Circle	9th

Table: Concepts and their grades.

The following are the excerpts of the lesson plan of the concept in focus:

Numeracy

As Nickson (1994) pointed out, “one of the major shifts in thinking in relation to the teaching and learning of mathematics in recent years has been with respect to the adoption of differing views of the nature of mathematics as a discipline” (p. 10). Basic counting is the first in a series of concepts in math classes students need to succeed in college and life. What’s more, when students make the connection between the arithmetic and the real-life applications, they develop abstract reasoning

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skills. Teachers can also help children reason and problem solve at this stage by providing them with opportunities to interact with the cultural product such as the Sundial (the figure given above helps visualize and exercise the sense making process while learning numeracy) to facilitate counting exercise.

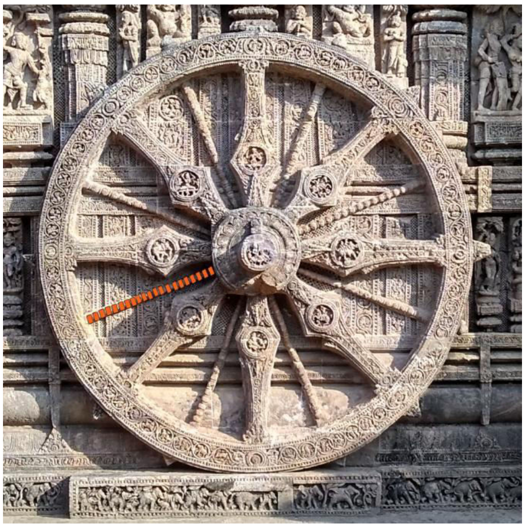

Figure 1: Numeracy

Time

For students, learning to tell time can be difficult; many students suffer even in middle grade with calculating the time. Although we can teach students to tell time in hours and half-hours by various means either by the analog clocks or even with digital clock. But what about if we don't have either of them or neither of them is reliable at some scenario or we want to explore the ways to calculate the time in a more organic manner such as sundial. Not only, it will last longer than any analog and digital clock available but is a magnificent architectural work and a very powerful educational tool to scaffold critical inquiry and analytical skills of learners.

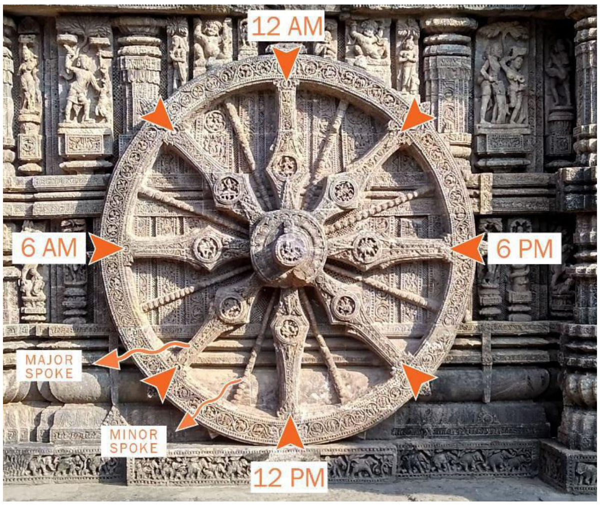

Figure 1: Numeracy

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Circle

For students, visualization of area of sector, arc length etc., could be a little overwhelming often a time, many students suffer even in higher grades with such topics. Having real life examples not only help foster historical empathy (given relevant interventions for the same integrated in the lesson plan) but also mapped to visualize such concepts will encourage students to engage, explore and build upon their existing knowledge of the concepts with a strong foundation.

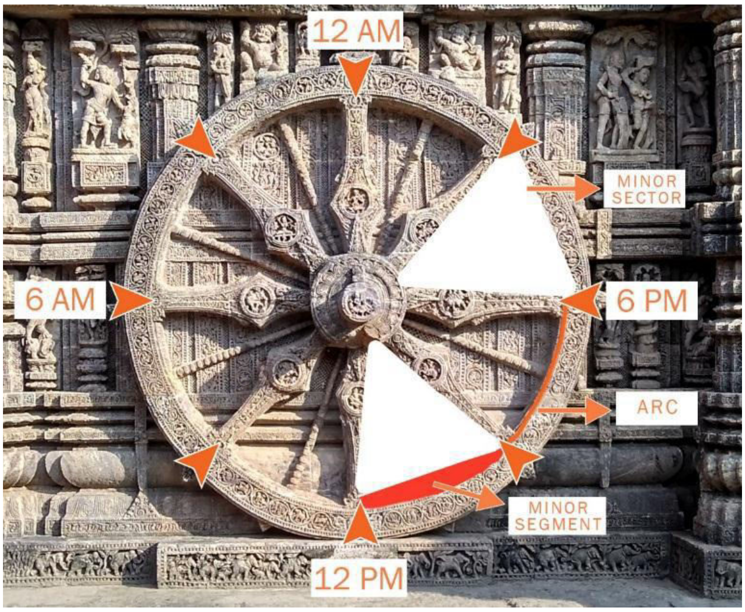

Figure 3: Circle

RELEVANCE TO THE CONFERENCE

The mere tapping in the potential of such architectural marvel helped illustrate three crucial concepts in mathematics which were mapped with the wheels of the Sun Temple. Imagine how much understanding of astronomy, geometry, architecture, material, construction and aesthetics along with time coordination would have happened between astronomers, engineers, sculptors to create something like this 750 year ago. This paper provides a framework and an example of how a culturally significant and historical monument can be used as a pedagogical tool and proves to be a source of motivation and engagement which seamlessly aligns with the theme "Cultural and Social Aspects of Teaching and Learning Mathematics".

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TEACHING MATHEMATICS FOR SOCIAL JUSTICE: A DIALOGIC REFLECTION

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Abstract: This study explores the challenges of learning to teach mathematics for social justice by actively engaging six elementary school teachers in a critical research process. Wherein the researcher, an elementary teacher herself, collaborates with the others to explore critical mathematics education through a task based programme. The goal is to blur the distinction between research, learning, and action, by providing the researcher and the participants opportunities to collectively engage in reflective dialogue and in the process enabling each one to actualise ways of knowing and growing on their own.

INTRODUCTION

As teachers of mathematics, we have taught our students to solve pseudo problems that exist in textbooks, but not real problems which exist in their lives (Fasheh, 2015). These problems are part of their existence; rooted in their culture, caste, gender, and socio- economic background, making them vulnerable to social injustice. Mediating classroom learning through everyday mathematics and critical pedagogy (Rampal, 2015) brought such problems in the realm of the national primary school textbooks. The books titled Math Magic for classes I-V were developed during the period 2006-2008, and reprinted every year since then (NCERT, 2018). However, teachers have called for "spaces to share and discuss their problems and struggles" (Takker, 2011, p.38) regarding the philosophy of the National Curriculum Framework 2005 (NCERT, 2005) and the corresponding textbooks. Yet in all these years, need of a teacher to understand the challenges of critical pedagogy has been ignored (Batra, 2005). Critical pedagogy is a challenging process which may not come 'naturally' to any teacher as it requires a specific ideological orientation (Ernest, 2009).Teachers need to assist each other in learning to learn new ways of knowing, exploring, analysing, reflecting and becoming critical researchers (Kincheloe, 2012) as they take the path of being critical teachers - as has been explored in this study.

THEORETICAL PERSPECTIVES AND PROCESSES

This study has drawn from Freire's (1997) critical theoretical research, where the people he studied were significant partners in the process. Freire suggests that teachers' critical consciousness can emerge only through dialogical, problem posing education that moves past reflection towards actual action. The goal of this study was thus to blur the distinction between research, learning, and action by turning participants into co-researchers and providing them the opportunity to engage in critical pedagogy. The study was conducted through collaborative 'teacher researchers' (Kincheloe, 2012) , by a formal study group (referred to as Group) of six elementary school teachers and the researcher (also an elementary school teacher), to

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develop knowledge and skills that allow them to connect their educational practice with larger social visions, which may be shared with students.

The Group met in the primary wing of a government school in Delhi and it worked together for two and a half years from April 2014 to December 2016. Out of the different forms of interaction, the two important forms were- Meeting sessions and Study sessions. Narrative analysis was used to focus on the themes that emerged from these sessions (Clandinin & Connelly, 2000).

Meeting sessions: Identifying critical issues

In these sessions, the researcher accompanied the teachers to their classes as a collaborator, not as an observer, and engaged with the children in whatever task they were busy in. While doing that, they discussed the students and their stories which was always an enriching experience for both. These exchanges provided cues for future subjects for critical reflection and also introduced the researcher to their social landscape. In one of the meetings with Teacher 6 (T6), she shared that she was tired of her students picking up fights on the caste issue. The group of 'gujjars' call others 'chamaar', 'naai' (derogatory names for castes considered lower in status). The teacher confessed she found this difficult to handle and resorted to beating them when they fought.

Frequent interactions helped the Group to bring their attention to the issues they were either missing out in their own teaching practice or were suppressing with their anger and authority. The Group discussed why they distanced themselves from issues of class, caste or gender when their students were grappling with these, even as their identities were forging notions of dominance of one group over the other. They also reflected on how their actions and reactions could contribute adversely to these issues.

Study Sessions: Learning and reflection

These sessions were planned, time bound and led by the researcher as a mentor. It was a structured circle time where they all gathered on a regular basis after school hours. Members expressed views and concerns, could argue, take risk and speak their minds and be heard without any fear of being judged by the others in the Group. The aim was to engage in 'problematisation' (Dewey, 2007) through a collective study of selected literature in critical mathematics education. Readings were taken up in the Group for a process of dialogic reflection; we expressed our thoughts, shared experiences, with agreements-disagreements. Each session was the site of further nodes of inquiry, taking the dialogue ahead. More readings were added, making the Study sessions more fluid and open.

Academic readings and the corresponding tasks encouraged the Group towards exploring the opportunities to reach out to their students in a more responsive way by engaging them in critical pedagogy.

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TEACHERS ENGAGING IN CRITICAL PEDAGOGY

T6 in an attempt to engage her students of Class IV (age 9-10) in critical pedagogy did a small unit, on the refreshments to be served in the school 'Mega Parent Teacher Meeting'. The school tea stall run by 'Santosh aunty', sold tea for Rs 10 a cup, but when they calculated the cost for the expected parents, they realised it was exceeding their budget, to which T6 suggested they could ask Santosh aunty to give them some discount. She offered to give them the tea at Rs 3 for the mega PTM. They had an animated discussion with their teacher (T6), but she finally dismissed the unit saying she could not handle the noise created by the students. When the Group, collectively reflected on the narrative shared by the teacher, they discerned that it was not just 'noise' that stopped her but many other complex social issues which needed to be identified.

> T6: A student said, "this is unfair, aunty charges extra money from our teachers, if she can sell the tea for Rs 3 then why does she sell at Rs 7 more?"
>
> A girl responded, "yes why not, she has to single-handedly marry off her three daughters."

The teacher, after discussion in the Group, accepted her reluctance for further discussion as she was unable to resolve the idea of 'dowry' in her own consciousness and also wanted to protect children from discussing a serious issue. But this meant loosing the opportunity to bring in sensitivity and respect towards the woman entrepreneur; also to address the contested shaping identities of young girls considered a burden because of the 'dowry' their parents have to accumulate to marry them.

One of the teachers found herself spontaneously invoking critical pedagogy, while attending to her Class V student's observations. To avoid the queue for drinking water, one student filled his bottle from the pipe going in the drain and showed her saying, "Madam look all of this clean water is going down the drain (translated from Hindi)". Her first reaction was to scold the child thinking he must have done some mischief. However, after looking at the bottle she too was intrigued by the child's observation and learnt that the bottle had been filled from the drain pipe of the Reverse Osmosis (RO) filter. She was equally concerned by the water going waste and therefore did her homework to understand the machine and the process of RO and decided to take it up as a planned study with her students.

She divided her Class V students in groups and asked them to collect and record how much water was being wasted and how much came out as purified water. As they collected water for the next 4 days, they concluded that for every one bottle filled for drinking, three or sometimes even four bottles got wasted. She didn't stop there. She made them scale up further, as there were 4 RO purifiers in the school. They estimated that in this way their school produces approximately 240 to 250 litres of waste water in one month. She said she wanted them to capture the seriousness of this

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matter, therefore she asked - if one school produces 250 litres of waste water then how much water will be wasted in a month by all the schools of Delhi?

Children were completely shocked and worried as they realised the magnitude of water scarcity in their city and compared it to the amount of water being wasted. There shouldn't be any RO filters, students proposed; as they also learnt about the alternatives of water purifying mechanisms like using alum, boiling, bio-sand filters and the effectiveness of these methods depending on the nature and extent of water impurity. Children could understand that ROs are useful to treat hard water, yet they have been installed in all schools. The teacher said, they inquired that their school gets the regular supply of water (not brackish) which does not need an RO, but both the water supply and the ROs are provided by the same Government. The teacher was able to nurture her students' agency to question, yet found herself inadequately prepared to challenge the Government, her employer, to bring about any change in this regard.

CONCLUDING THOUGHTS

It was not easy for the Group to extend their roles to incorporate the issues of inequities and unfairness in their practice. There was reluctance towards critical discussions, often assuming children are too young to understand the complexities of the social world. It was challenging for them to negotiate hierarchies and take a political stand about various social issues. There was resistance in coming out of the comfort zone, as their belief about their role as a teacher was that of 'neutrality' and their image about mathematics was 'context free'. For them teaching mathematics meant enabling students to become efficient with procedural knowledge of standard algorithms irrespective of any contextual engagement. Their expectation from the learner was to receive and memorise the given knowledge passively without any critical dialogue or discussion. Freedom of observations and asking critical questions was strongly contended and were perceived as a loss of teacher's authority in the classroom. Making the classroom a democratic space by allowing students to take decisions and make observations was a difficult step to take, as we were all habituated to function like 'meek dictators' (Kumar, 2005).

Collective dialogue on various readings and tasks associated with critical mathematics education, allowed the Group to explore mathematics beyond procedures and formulas. Teachers on various occasions recognised the learner's agency in making sense of this inequitable world. They could see how even young learners could identify and differentiate between fair and unfair means when given access to the tools of mathematics, like for instance by measuring and comparing volumes of water wasted in schools. In fact, the students' quest for fair use of water and their passionate advocacy against the government provision of RO filters encouraged the Group to continue to practice critical pedagogy; they were convinced that this would enable them to nurture their students' pursuit for a socially just society. In the course of time, each teacher has taken on greater levels of responsibility – some have become incharge of their school curriculum, while others

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have taken on the role of zonal coordinators. All of them enthusiastically await the culmination of 'their' thesis, a result of their collective study.

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MATHEMATICS AND SUSTAINABLE DEVELOPMENT GOALS: AN EXAMPLE OF DIGITAL GAMES BASED LEARNING

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Abstract: Twenty-first century learners must be motivated and prepared to address the problems facing humanity. Despite years of advocacy, alternate teaching and learning practices for a sustainable future, we are not meeting this challenge. United Nation's Sustainable Development Goal Target 4.7 is an acknowledgment of the critical importance of education for sustainable development, global citizenship and other transformational education movements for a sustainable and peaceful future for all. Mathematics is regarded highly in society for its utility and being humanity's go-to tool for solving its most complex problems. Also, video games which are highly popular in today's culture and can prove to be effective learning tools and medium for popularization of mathematics. This poster will present a tool, which is a digital learning game designed for secondary graders. It helps the player to visualize the application of coordinate geometry and learning about the Sustainable Development Goals of the United Nations at the same time.

AIMS OF THE STUDY

If not more than learning about the facts from science and the formulas of mathematics, socio emotional skills and peace and sustainability education are equally important to be fostered and developed in present-day school-goers (Darvasi, 2016; Farber and Schrier, 2017). But findings from one of the Public Opinion Surveys compiled by the Organisation for Economic Co-operation and Development (OECD) in June of 2017 indicate that there is very little awareness about the Sustainable Development Goals (SDGs) among citizens and the youth. The SDGs, often called the people's goals, can only be achieved when people are engaged, made aware and mobilized into action. As future global citizens, school students must also know about the SDGs, concepts of sustainable development and what a global body like the United Nations (UN) is doing about sustaining the health, availability of resources and about the wellbeing of our planet. At the same time, the discipline of mathematics is also known to generate a sense of fear and stress among students. Students are hardly able to visualize math in action and often question its applications in their real lives.

This makes us question; how do we go about spreading awareness and embedding the concepts of sustainability into curricula and make mathematics more meaningful as well? One of the possible solutions as proposed by the 'Textbooks for Sustainable Development – A Guide to Embedding (UNESCO MGIEP, 2017)' is to use examples and contexts from sustainability to teach the 'core disciplines' like mathematics,

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science, language and geography. The guide which is specifically designed for textbook authors, and is about writing textbooks which embed principles of sustainability into core disciplines provides a very novel idea for curriculum integration and development of tools for such an endeavour. The author designed a digital game to work towards this problem's possible solution. This was done keeping in mind, the proven effectiveness of well-designed digital games in the interactive age (Gee 2003; Cazden 1981) and gaming environments working as learning environments at the same time (Shaffer and Gee, 2006). An additional advantage of using a digital video game as a resource is that it catches the attention of learners very quickly and has positive effect on motivation (Ketamo et al., 2013). The objective of this project was to develop a digital, windows based, computer video game to spread awareness about the SDGs embedded with a concept from mathematics.

THEORETICAL CHARACTER

About the game

The game titled 'Dimension Destination' was developed specifically to reinforce understanding of coordinate geometry skills and inculcate a sense of spatial awareness in the player. 'Applying mathematical knowledge and skills to familiar and unfamiliar situations' has been identified as one of the many overall goals of secondary school mathematics curriculum in curriculum frameworks of many states (Department of Education, 2013; Faculty of Education and Department of Mathematics, 2015; UNESCO 1984; European Commission, 2011). At the same time, these curriculum frameworks for mathematics education also mention inculcating spatial awareness skills as one of the goals of teaching geometry to secondary school students. The game employing these two principles, requires the players to apply their understanding of the 'Cartesian Plane' and the coordinate system, and spatial reasoning skills to move forward and progress in the game. As the player moves forward, spatial reasoning is also being developed gradually. This was one of the results, observed while the game was tested with the sample. 'Dimension Destination' only runs on computers with Windows platform. Figure 1 shows a screenshot from actual gameplay.

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Figure 1: Dimension Destination Gameplay

While interdisciplinarity has become relevant for emergent sciences and social sciences of the 21st century, mathematics is still rooted in rudimentary computational processes and problems (Sriraman). Students developing an enjoyment and fascination with mathematical concepts has been identified as one of the goals of mathematics education in India (National Council of Educational Research and Training, 2005). The game was developed by the authors themselves using Unity's platform. The researchers consulted secondary students to learn about what could be an appropriate story and design for the game. The learning objectives of the game were listed and then specific tasks and in-game activities were designed and mapped to the learning outcomes the game is intended to achieve. The overall objectives of the game are:

1. To demonstrate understanding of principles and concepts of coordinate geometry.
2. To identify and understand that mathematics can be used and has applications in real life scenarios.
3. To identify and be aware about the United Nations' Sustainable Development Goals.

The overall objectives of using the game as a learning tool mapped to various activities inside and outside of the game described in Table 1.

S. No.	Overall Objective	Activity
1	To demonstrate understanding of principles and concepts of coordinate geometry.	Finding location of collectibles and resources with help of their coordinates.
2	To identify and understand that mathematics can be used and has applications in real life scenarios.	Discussing learnings from the game.

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3	To identify and be aware about the United Nations' Sustainable Development Goals.	Associating collectible with a UN SDG to help the Earth.

Table 1: Mapping overall objectives to lesson activities.

RATIONALE TO CONFERENCE THEME

Using digital games for teaching can enable the teacher to quickly catch attention of modern day learners and have positive impact on motivation and retention. Indian National Curriculum Framework (2005) talks about providing enjoyable experiences to students as one of the goals of mathematics education and also lays emphasis on enriching teachers with a variety of mathematical resources. There is a dire need of alternate, engaging and fun learning resources for popularization of mathematics in society and among students.

When it comes to gaming, there are hardly any digital games out there, mobile or PC, which are specifically designed for mathematics education and trying to inculcate mathematical ideas among students. The game is designed as such that it enables the player to use mathematical skills and learn about the SDGs at the same time. This presentation will have high synergies with the 'Mathematics Education and Society' conference and the theme 'Sociology of Mathematics Education' as it will talk about using a new approach of teaching mathematics, of using gaming as a pedagogy to address the practices and lifestyle of twenty-first century learners of modern society. The participants will have hands-on opportunity to play the game during the presentation.

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KARETAO PĀNGARAU: USING PUPPETRY TO COMMUNICATE MATHEMATICALLY

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Abstract: The learning of the Maori language and cultural knowledge have been severely disrupted by European colonisation. Puppetry has been used extensively in English-medium education to encourage and motivate students to practice speaking a new language. This poster reports on a classroom-based project to encourage the acquisition of mathematics language and cultural knowledge using traditional puppets called karetao. The use of karetao is culturally significant being an art form practiced by Māori people prior to colonisation. The students' interactions with the karetao are captured on video by the student presenters themselves.?

INTRODUCTION

Usiskin, (2012) argues that in order to understand mathematics, students must understand the language of maths. This is a challenge, linguistically and pedagogically, for Māori-medium maths classrooms, a unique space within the wider educational context of maths teaching and learning. There is a substantial collection of research that supports the argument that the explicit teaching of mathematical language can simultaneously support the acquisition of mathematical knowledge (Dowker, Bala, & Lloyd, 2008; Hunter, 2005; Pimm, 1987; Pitvorec, Willey, & Khisty, 2011; Schleppegrell, 2007). Encouraging communication is one of the main aspects to supporting students' learning in key subjects like mathematics Pimm (1991). To communicate mathematically, students need an understanding of the register of mathematics language, (Cummins, 2000; Pimm, 1987).

In classes where Māori language (te reo Maori) is the medium of communication, a number of linguistic and pedagogical issues arise. In Māori-medium schooling most students and teachers are second language (L2) learners of te reo Māori, as a result of overt policies including English only schooling and covert policies such as English only workplaces. As a consequence, te reo Māori was endangered (Spolsky, 2003).

Despite 35 years of schooling innovation and Māori language revitalization, including the elaboration of the language to teach pāngarau/mathematics, there are still pedagogical and cultural challenges. Students not performing at the expected national level for pāngarau / mathematics is one. This can be attributed to a number of interconnected variables such as teacher competence and students learning pāngarau/mathematics in their second language - Māori (Trinick, 2015). Despite language revitalization efforts in schooling, English remains the dominant language of homes, communities and in Māori-medium education.

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METHODOLOGY

The project focuses on two ethnomathematical ideas, supporting language and cultural knowledge revitalization. This project provides a chance to revise the traditions associated with Karetao usage while simultaneously being used as a pedagogical tool to support communicating mathematically. The use of Karetao and story-telling have a rich history in Māori culture..

Lee (2009) contends that Pūrākau (Myths and legends) story telling have evolved over the centuries. The pre-European craft of storytelling was adapted with the introduction of reading and writing. Storytelling underwent further adaptation in other mediums including the arts, poetry, and film. Historically Karetao were used as a medium for the telling of Pūrākau, to instruct young people in tribal history (World Encyclopedia of Puppetry) and they too have been adapted.

The national museum of New Zealand, Te Papa, gives this description of karetao: "Karetao are ceremonial marionettes in the form of men. The body, legs, and head are usually carved from a single piece of wood. The arms and, occasionally, the legs are articulated. They are operated by tightening and releasing attached cords. In this manner karetao were made to imitate the haka (fierce rhythmic dance) by the operator to the accompaniment of waiata (chant)."

Traditional Karetao require complex skills to make and operate so for this project the students use hand puppets.

As a pedagogic tool Sam Patterson and Cheryl Morris (2015) contend that using puppets makes the classroom a less threatening place. "Students share with less risk, and the puppet makes the situation a lot more like a performance." 8 July 2015

Susan Sirigatti (2014) contends that: "they're not only extremely useful as an educational toy that connects play with learning but also in supporting children's development in several important areas."

Those important areas include language and literacy development, behavioural/social development, conflict resolution, thinking skills, and motor skills.

TRANSCRIPT

This transcript is in the Māori language, with English translations provided in brackets. All the language is spoken by Year 4 students (7 – 8 years old). The goal is to provide a puppet show to help Year 1 and 2 students (5 – 6 years old) to count in Māori. The puppets names are Mua (black t-shirt) meaning front, and Muri (white t-shirt) meaning back.

Mua & Muri: Kia ora Nau mai, haere mai ki te hotaka Karetao Pāngarau. (Hello. Welcome to the Math Puppets Show.)

Muri: Kei te aha tatou i tēnei rā e Mua? (What are we doing today Mua?)

Mua: I tēnei wā kei te ako tatou ki te tatau whakamua mai i te kotahi ki te tekau. (Today we are learning to count forwards from one to 10.)

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Muri: Whuuu kua rite mai koutou? (Fuu, are you ready?)

Mua: Whai mai i ā ahau. Tahi!	Muri: 1	Mua: Rua!	Muri: 2!
Mua: Toru!	Muri: 3!	Mua: Whā!	Muri: 4!
Mua: Rima!	Muri: 5!	Mua: Ono!	Muri: 6!
Mua: Whitu!	Muri: 7!	Mua: Waru!	Muri: 8!
Mua: Iwa!	Muri: 9!	Mua: Tekau!	Muri: 10!

Mua: Wii, tino kino te pai! (Wii, you are too good!) Inaianei, me kōrero tahi. Kua rite? (Now we will count together. Are you ready?)

Muri: Āe (Yes)

Mua: Toru, whā (3, 4)

Mua/Muri: tahi, rua, toru, whā, rima, ono, whitu, waru, iwa, tekau! (1, 2, 3, 4, 5, 6, 7, 8, 9, 10!)

Mua: Ka pai e Muri. Kei runga rawa atu koutou! (Good one Muri. You are all so amazing!).
Inaianei māku te tau e whakaatu, mā koutou e korero. Kua rite mai koutou? (Now I will show a number and you will say it. Are you ready?)
He aha tēnei tau? – Āe, tika ko te rua. (What's this number? – Yes, you are right, it's a 2.)
He aha tēnei tau? – Tika anō ko te whā. (What's this number? – Right, it's a 4.)
He aha tēnei tau? – Āe, ko te whitu. (What's this number? – Yes it's a 7.)
He aha tēnei tau? – Ka pai hoki koutou ko te iwa. (What's this number? – Well done, it's a 9.)
Hoki mai ākuanei e hoa mā. Ka kite. (Return soon friends. See you again.)

DATA COLLECTION

The study was conducted alongside a Ministry of Education funded intervention programme designed to accelerate learning in pāngarau/mathematics. The students selected for participation in the Karetao Pāngarau intervention programme were Year 6 to 8 students who were given the responsibility to create a series of math videos. These students were not chosen because they were having difficulty in maths, but were chosen because they wanted to be involved. Maths content for the videos follows the progressions set out in the Māori language Mathematics curriculum called Te Poutama Tau.

The unit is in a mainstream school with approximately 70 students ranging from Year 1 to Year 8. The Year 7 and 8 students created and filmed a series of videos using puppets to encourage the speaking of the Māori language within the school for Māori Language Week held in July 2017. It proved to be popular with all staff and students

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at the school. After the success of this puppet series the New Entrants teacher started using hand puppets in her math programme to engage the students in the learning of the Māori language and math concepts.

The facilitator viewed the New Entrants teacher using puppets to teach number to her class. The use of puppets grabbed the attention of the students, allowed them to be less self-conscious, and gave them a voice, an avenue to practice their counting in Māori through the puppet, thus lessening the whakamā (shyness, embarrassment, shame) they might feel. Based on the facilitator's observations puppets offered an opportunity for students to communicate mathematically in the Māori language. After discussions with the teachers about the positive impacts that were happening as a result of the use of the puppets, it was agreed that a more coordinated approach to the use of the puppets and the gathering of data was needed.

Beginning in 2018 the Year 7 and 8 class developed puppet characters and scripts based on the Māori language math curriculum. The initial videos were produced without an audience to help students hone their production skills. After this initial stage live presentations were presented and recorded. All recordings will be made available on the school website so that the lessons can be accessed by all teachers, students and their families.

This provides two sets of data; the impacts on learning that a live show has and the impacts of recorded presentations with no audience. The data will measure pre and post attitudes and motivation towards math learning, engagement and interaction. The research will analyse students' progress in math and assess if any improvements can be attributed to the use of puppets.

CONCLUSION

Puppets help overcome the linguistic and pedagogical challenges that Māori medium education faces. Students are driving most aspects of the Karetao Pāngarau (Maths Puppets) series. They will be heavily involved in the production of the videos and, crucially, the students will create the scripts, with guidance from teachers at a minimum. Thus, we get a student voice in the indigenous language that other students will be able to relate to. Students will be the teachers so they will approach the teaching through their unique perspectives. Using karetao allow students to create and deliver scripts using their knowledge of the Māori language, while being aligned to specific math learning outcomes.

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CHALLENGES AT THE BORDER OF NORMALITY: STUDENTS IN SPECIAL EDUCATIONAL NEEDS IN AN INCLUSIVE MATHEMATICS CLASSROOM

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This research describes how students perceived as being in special educational needs in mathematics (SEM), either as students in access to mathematics, or as students in struggle to get access, are challenged in their participation in mathematics education. Discourse analysis is used as a tool and a theory to construe discourses from students own stories of participation in an inclusive mathematics classroom. Distinguishing between (d)iscourse as stories in texts, and (D)iscourses as social and political recognisable units, the result shows the same, yet different, discourses; tasks, the importance of the teacher, to be (un)valued and math is boring, all indicating a Discourse of accessibility in mathematics education. The accessibility is challenged in two ways, the students are challenged in their participation since they do not fit into the 'normal' education, and the mathematics education is challenged to meet every students' need to promote equity.

INTRODUCTION

This research takes off in the intersection of two research paradigms, mathematics education and special education. In this intersection issues of diversity, equity, participation and normalisation in terms of mathematics (not) for all becomes visible and is often framed by the notion of inclusion (Roos, accepted). Inclusion is used in many different ways in educational research, and have different directions, towards rights and ethics, efficiency, politics or pragmatic use in practice (Dyson, 1999). In this paper inclusion is seen as a process of participation in the mathematics education with a direction towards both rights and ethics and use in practice. In mathematics education inclusion is often connected to special educational needs in mathematics (SEM) (Secher-Schmidt, 2016), meaning the education needs to pay attention to specific needs of the students, and provide solutions within the education in order to include all students in the mathematics. This in order to make the mathematics accessible for the individual student. Often the student in mind when talking about SEM is a low achiever, struggling to get access to mathematics. Nevertheless, even a high achiever can be in SEM even though she or he is in access to mathematics, because she or he might need specific solutions in the mathematics education in order to get included and have optimal opportunities to learn. Consequently, the notion of SEM can be seen as a spectra, showing a diversity in access to mathematics. Inclusion in mathematics for both ends of the SEM-spectra can be provided through the construction of learning situations to promote participation with focus on teaching practices and intervention strategies enhancing learning, where the learning situations permit meetings among differences (Scherer et al., 2016). How do we design these teaching practices promoting participation within an inclusive classroom, where we

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find SEM-student in both ends of the spectra? And, most importantly, how do the students perceive their participation in the inclusive mathematics classroom? Hence, the aim of this paper is to describe how students perceived as being in SEM, either as students in access to mathematics, or in struggle to get access, perceive inclusion in mathematics education.

PROCESSES OF NORMALISATION IN RELATION TO DIVERSITY AND EQUITY

People want to be included in the societal discourse. To be included, there is an amount of mathematical knowledge needed to be able to have a critical agency (Greer, 2009) and to be included in society at large (Valero, 2012). Consequently, the mathematics education in every society needs to respond to the knowledge needed, and educate as such to meet this need and to develop mathematics education engaging every student. This kind of education is often referred to as inclusive education (e.g. Sullivan, 2015a) and is about how to get students to be active participants in mathematics classrooms (Roos, accepted). When inclusive mathematics education is mentioned, the challenges to this kind of education is often highlighted, such as diversity and equity (Askew, 2015). If looking into research focusing on mathematics education and diversity, there is a commonly strive to challenge the current views in educational policies initiatives, which rather perpetuate diversity as something difficult than use diversity as an asset. The core message in the research is that by embracing diversity and take it as a point of departure, we can actually enhance learning for all. This can be done by looking at classrooms as learning communities and instead of having individuals as the learning unit in the classroom, considering the group as the learning unit (Askew, 2015). This does not imply neglecting differences, but rather see them as natural and not define them as deficiencies (Moschkovich, 2007 discusses this in relation to bilingual learners). However, this is challenged by the traditional way of western-mathematics teaching, where one teacher is supposed to have a one-size-fits-all education which “turns diversity into a problem” (Askew, 2015, p. 129).

Not seldom the issue of equity is being discussed in relation to diversity (e.g Nasir & Cobb, 2007). Though, here the point of departure often is on the societal level investigating different aspects of society from an (in)equity and access perspective (Bishop & Forgaz, 2007). Gutiérrez (2007) discusses the importance of taking a critical perspective embracing equity in research. Also Valero (2011) discusses equity in mathematics education and how it is strongly connected to “larger processes of inclusion and exclusion in society”. This implies issues of equity in mathematics education gets political and some researchers highlight this by using terms like social justice and democratic access (Pais & Valero, 2011). A focus on how ideological views of equity transcend into mathematics education in school is hereby implied. Under the surface of both diversity and equity, processes of normalisation become visible. This because there are national curriculums to be followed, and in them there is a direction towards what is normal to know in mathematics at a certain age. It also

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becomes visible by the fact that researchers have identified diversity and (in)equity as important factors for inclusion. Hence, the researchers implicitly (and sometimes explicitly) make processes of normalisations visible. Someone, or something (policy makers, scholars, mathematics teachers, the community), sets the agenda for what is normal in mathematics learning and teaching. Not seldom processes of normalisation clashes with equity and processes of inclusion sets out of play. This suggests inclusion is enhanced by identifying processes of normalisation, and move beyond them by seeing diversity as natural. Accordingly, in order to address the process of normalisation in relation to diversity in the inclusive mathematics classroom and issues of equity, there are challenges in the mathematics education (research) to find ways to get every student to be an active participant in the mathematics classroom.

THEORETICAL AND METHODICAL APPROACH

The aim of this study is to describe how students identified as being in SEM perceive inclusion in mathematics education. To investigate this, students' stories of participation in mathematics education needs to be identified. This identification is made by using Discourse Analysis (DA). The focus of DA is the study of language and text, what we actually can hear, read and see *and* particularly what is beyond the text. That is, DA helps construing discourses by analysing the use of a certain type of language in a certain type of situation. Thereby you can say something about the social world (in terms of discourses). In this study, when going beyond the text, this something becomes visible in construed discourses of what influences student participation in mathematics education. DA is chosen because of the explanatory power of social contexts and meaning making. In this research, the perspective of Gee (2014a, 2014b) is used, since this focus on DA is descriptive, and I intend to describe how students perceive their participation in an inclusive mathematics classroom to be able to have optimal opportunities to learn.

Gee (2014a, 2014b) use two theoretical notions when explaining DA, *big and small discourses* (henceforth Discourse with capital D and discourse with lowercase d), where Discourse is looking at a wider context, social and political. Discourses are always embedded in many various social institutions at the same time, involving various sorts of properties and objects. For example, a Discourse can be "assessment in mathematics." Discourses are always language *plus* other stuff, such as actions, interactions, values, beliefs, symbols, objects, tools and places. Small d discourse is focused on language in use, what *stretches of languages* we can see in the conversations or stories we investigate (Gee, 2014a) meaning what small conversations appear within the greater story. In this research, big and small discourses will be the theoretical perspective. Gee provides tools for analysing different forms of interaction, both spoken and written. These tools focus communication and ask questions in order to go beyond the text. In this research the tools are used methodologically and has been adopted according to the aim. For example, tools that has been used is the *Topic and theme tool* with the questions: what is the topic and theme for each part in the students texts? When the theme is not

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the topic and has deviated from the first choice, why was it chosen? *The topic flow, topic chaining tool* has been used with the questions: how are the topics linked to each other to create (or not) a chain creating an overall topic of coherent sense? Similarly is the *Big "D" discourse tool* has been used with the questions: what Discourse is this language a part of? What sort of values, beliefs, objects, tools, and environments are associated with this language within this Discourse?

To conclude, DA is used both as a theory and a tool and provides a set of methodological and theoretical lenses in this study.

Setting the scene

During one semester (January to June 2016) I observed two classes (grade 7 and 8) at a public lower secondary school in an urban area in the south of Sweden that has set out to implement inclusive work as a way of teaching (implemented by the principal). After each observation student interviews were made. I observed at least one mathematics lesson each week for each class. Since both ethical and organisational issues had to be taken into consideration, the selection of students for the interviews were made in cooperation with the teachers, students and parents. The special teacher in mathematics and the mathematics teachers suggested students they perceived as being in some kind of SEM and if the students and parents gave their consent they were selected to take part in the study. The interviews were conducted when the students had time, and the teachers and students allowed it (they did not want interruptions of the ordinary lessons). The interviews took place in a room next to the classroom when the students had "class time" once a week. The interviews were based on close in time observations. I asked questions about situations and tasks and showed photos of tasks on the blackboard. We also looked at tasks in their textbooks. The first and the last interview were based on a questionnaire about how they perceived their mathematics education and their own mathematical knowledge. In this paper the focus is on two students in the same class in grade 8, Ronaldo, who struggles to get access to mathematics, and Edward, who is in access to mathematics. Both students attend this school because it is the one closest to their home. Ronaldo describes himself as a student with learning difficulties "I have difficulties within all subject, and it's like concentration and all that." He perceives himself as someone in more difficulties than others and as easy distracted "if someone drops a pen I focus on that immediately". He experiences that he keep forgetting stuff "I don't remember, I have to repeat a lot". Edward describes himself as a person that thinks mathematics is easy and don't need much help at all. He does not have to put in any effort, mathematics works "automatically" for him and he "already knows" most of what they are doing in math class. This is described by Edward: "I see it in front of me, and then just … buzzz… (points a finger to the head and spins it) the answer just comes, basically". Remembering is easy and "everybody says I am good at maths, but I don't like it".

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Data analysis

In this paper twelve interviews, six with Edward and six with Ronaldo, and four observations have been used in the analysis. The observations were used as contextualisation for the interviews as well as for supporting construction of big Discourses. When analysing the data by asking questions to the text, both small and big discourses appeared. That is, the questions were asked to the text and the answers made stretches of language(s) visible, signalling for small discourses. When adding analysis of the data from the observations, such as text on the blackboard and the actions of the teachers, big Discourses could be construed. In the analysis three Discourses were construed, the discourse of assessment (described in Roos, 2018), the Discourse of mathematics classroom setting and the Discourse of accessibility in mathematics education. In this paper I will focus and describe one of the three Discourses, the Discourse of accessibility in mathematics education.

THE DISCOURSE OF ACCESSIBILITY IN MATHEMATICS EDUCATION

In the analysis, stretches of languages concerning access in relation to participation of the students in the mathematics education were visible. From the stretches of language, four (d)iscourses were construed, indicating a (D)iscourse of accessibility. The discourses are described below.

The discourse of tasks

Both Edward and Ronaldo talked about tasks and implicitly how the tasks influence their participation in the mathematics education.

Edward talks about not understanding a task during a mathematics lesson: "It's not really that often that occur, I dare to say. Often I already know it, it feels like a lot of repetition, what we do." He also talks about how the tasks on the black board do not challenge him:

Edward: Yea, well, it might be that they (the teachers) don't do super challenging tasks on the black board. They don't. You can figure it out before…

Interviewer: Would you like more challenging tasks on the blackboard?

Edward: Yes, yes.. then you can learn something from it. If you have something a little little harder, then you can learn from it. Then you can sit down and begin to think.

Here Edward describes the tasks used as examples by the teacher in the introductions[i] as easy, and he does not get any stimulation to think, get challenged and learn by the tasks.

When Edward describes his feelings when working with tasks in the textbook he says: "It is just painful .. almost boring. […] But then it also [depends on] what kind of task it is, some of them just sucks, yea!" […] "Some of them just **doesn´t**[ii] work."

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Here he describes how the tasks influence his participation. When he describes tasks that are fun he connects it to calculate the resistance.

Edward:	I think it is exciting, an exciting area to see how much resistance, to find out the answer.
Interviewer:	So it is not the calculation?
Edward:	No, the calculation is just a tool and a way. It is fun to calculate with power and stuff like that, it's fun.
Interviewer:	Why do you think that is fun, and not math's overall?
Edward:	Yea, no but it because I am interested of [it], to calculate it. I am kind of not that super interested to calculate how much the water-park earns. It pushes [me] more.

Here Edward explains why the theme of the task is important, he needs to be engaged in the subject to an active participant.

Ronaldo talks about tasks as often complicated: "Some are more complicated. Like calculate the percentage of and then calculate, the percentage of, like, a number. No, well, I don't know, I think percentage is difficult, and to calculate the percentage." Also algebra tasks are challenging for Ronaldo, for instance when he attempts to solve 5x = 4X+8:

Ronaldo:	I would start by looking at what side has the most X:es.
Interviewer:	Yea, and where is that?
Ronaldo:	The five.
Interviewer:	You can scrabble if you want to.
Ronaldo:	No, but I think.. I don't remember, first you sort of take minus five and change signs… minus four maybe ah… minus four… hell, I don't know...

Ronaldo tries to remember a procedure to solve the equation. When he doesn't remember he gets frustrated. He also talks about algebra tasks as boring, "with X and Y and everything". When Ronaldo describes his feelings when working with tasks, insecurity about how to approach the task is an issue: "when getting a task, to find out when to use multiplication or minus or plus or such - I feel insecure". Ronaldo describes hard tasks as: "Problem-solving and text. Then I usually get help by talking to the peer next to me about how to do it, and, in any case Edward, he is really good at math, and then I like to sit next to him also." Ronaldo gets teaching in reading comprehension to help him with those type of tasks since "it is often those kind of tasks I fail at, at tests."

Here Ronaldo expresses insecurity in how to approach tasks. He also describes how hard it is for him to comprehend problem-solving tasks involving text. Then he usually uses a peer to help him out. Edward is a peer he likes to get help from. The

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observations show that the tasks used in the education is most often tasks from the text-book.

The discourse of importance of the teacher

Both Edward and Ronaldo talk about the importance of the mathematics teacher and how the teacher can enhance or diminish their participation in the mathematics education.

Edward states that "it does!" make a difference what mathematics teacher you have. "I think the whole class thought that when we had another [math teacher]." Edward describes that his feelings about mathematics to some extent depends on the teacher: "I have never liked math's. No, I don't think it is fun. I think it is quite easy. It depends a little on the teacher, you can have a boring teacher. […] they could tell a joke sometimes, not be so serious."

Edward also talks about the importance of the teacher in order to challenge him at the mathematics lesson: "Then the lesson gets somewhat different, like when we had a substitute teacher who was a secondary teacher, I think that was great. […] I learned a lot, because it was kind of on another level. It feels like it was a much higher level than… the regular teachers. […] Because, we entered stuff I **think** the ordinary teachers wouldn't have picked up, because it was easy to go one step further, because it was a little higher maths, which I don't think we had gone through." He felt that he learned a lot from the secondary teacher: "I thought it was good when we had that secondary teacher, then I learned a lot in the introductions." When he talks about his ordinary mathematics teachers he says;

Edward: They present a little too basic stuff I think.

Interviewer: But you want it more difficult?

Edward: Well a little more challenge, so you learn something from it.

Ronaldo describes the importance of the pace of the teacher for his possibility to participate: "if they take it nice and easy, like this. Sometimes they speak a little too fast and stuff like that." For him, in order to understand, it is important "to take it nice and easy, so I usually ask after the lesson, if we could repeat it once more if they have the time." He likes it when "she [the special teacher] does it really slow and methodical."

For Ronaldo to participate it is also of importance to get help from the teacher with reading. "Karen [the special teacher] also helps me quite good [with the reading].

Also, for Ronaldo, it is important that the teacher listens to the students, what they want regarding tasks and ways to work. "because she [the math teacher] took us outside and did something called active math, then you are outside and do math's instead […] it should be better if we were out sometimes … it would be more fun. […] She [the math teacher] was more like more like she understood what everybody

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said, really good. It was a lot of us who wanted to go out and have active maths like this, and then she did it.

The discourse to be (un)valued

In both Edward and Ronaldo's interviews stretches of languages of how they are valued. In Edwards case it is about how he perceives himself as sometimes not valued. This is visible when Edward in the end of two different interviews says the following:

Interviewer:	Anything to add?
Edward:	Well, it could be, that if you raise your hand in the introduction it can happen, it often happens that they [the teachers] let the ones that have difficulties answer, because they know… well, they know that he [meaning himself] probably knows the answer, so.. yea.. Sometimes you get a little frustrated when you are not allowed to say anything, but often it's okay. But then, when it is always the same person that raises the hand, it gets tedious.
Interviewer:	Is it something special you thought of during maths today?
Edward:	I think it may be that I didn't get to answer one single time
Interviewer:	You didn't [get to answer]?
Edward:	No, I didn´t.
Interviewer:	How did that feel?
Edward:	How did that feel?

To be unvalued is also visible when he reflects on when he answers a question in the classroom, what does he do? "well, then I answer it. But it's not that often they [the teachers] throw a question.. to me.. because, well they often throw it to the ones that do not listen." The observations support that during the lessons Edward often raise his hand, but do not get to respond often.

In Ronaldo's case it is not about being unvalued, it is about being valued and feel secure, which he doesn't feel in the classroom. "I am kind of nervous to say the wrong thing, - that somebody giggles kind of all the time or stuff like that" But in the small group he sometimes attends he feels secure and "dare to says stuff to, it feels like I am developing more [in the small group]". Hence, Ronaldo wants to be valued as a SEM-student and get special education in a smaller group sometimes. "It has become a lot better now, we have started to walk out in small groups [outside the classroom], which we didn't do before, and it is much better now. I concentrate better, and it is peaceful and quiet. "He feels valued and gets the support he wants, but sometimes he would like more guidance about when to participate in the small group.

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Interviewer: Do you think it is good or bad to go out of the classroom with the special teacher?

Ronaldo: Good

Interviewer: Do you do it often?

Ronaldo: The times we can do it... no, but when I feel really secure I stay [in the classroom]. But if I feel in between, or a bit insecure, like don't really know, then I go

Interviewer: Do you decide by yourself? [if you should go away or stay]

Ronaldo: Yes

Interviewer: And how do you feel about that, that you decide on your own?

Ronaldo: Both good and bad. Sometimes maybe they know better how much I am able to do and such...

Interviewer: Yea, right, do you listen to them when they say something about it?

Ronaldo: They usually don't say anything about it.

Although, sometimes he does not get to decide by himself, like during test taking. "It is just decided that I do it in another room, mum and the others have decided that I shall go to another room because I concentrate better", and sometimes he gets "tips about when to join [the small group] or not". The observation notes show that during the semester it becomes more common that the special teacher has a little group in a small room next to the ordinary classroom during mathematics lessons.

The discourse of math is boring

Both Edward and Ronaldo talk about mathematics as something boring. Edward has quite negative feelings towards the subject and describes mathematics as "the most boring subject, because when we are going to math's class, then it feels like you just dig yourself down into the sand. You want it to pass. So you can get out of there". So, even though he finds mathematics "quite easy" he has "never liked math" and "don't think it is fun". For him, "math is more like staple food, without math you get nowhere, but then you can develop math in different areas." Meaning he uses math to find out other, for him more interesting stuff in for example physics. If he gets an excuse for leaving he is happy. "like now for instance, I got excited because I got away". A clue for why he thinks mathematics is boring can be seen in his answer to the question what is fun in math. "I think that when you are doing more practical stuff, then it is fun, instead of having the nose in the text book all the time." Also in his description of his ideal mathematics lesson. "Then you would have those whiteboards in front of you, and been sitting and sketching and experimenting. Because then it's much faster. Instead of keep turning pages, draw lines and write the number of the task and all that, these things take such a long time. I want to spend the

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time on math.” Hence, to administrate mathematics challenges Edwards participation in the mathematics education.

Ronaldo expresses boredom in relation to listen for a long time.

Ronaldo:	It was just a blank in my head.
Interviewer:	It was a blank in the head? Does it happen a lot?
Ronaldo:	Yes, it is so bloody much of introductions now. It is so boring, so you can’t stand listening.

From Ronaldo’s description of how he wants a lesson to be we can get a clue of what is boring.

Ronaldo:	An introduction when you start a new [content], so, you should have an introduction starting something new, then when you come [the next time] you work with that instead of having an introduction each lesson.
Interviewer:	What happens when they always have introductions? […]
Ronaldo:	I listen, but it’s hard, it is introductions all the time.
Interviewer:	What is hard?
Ronaldo:	Like listen...
Interviewer:	Is it hard to understand?
Ronaldo:	Yea, well not that much, but it just becomes messy when they have all these introductions all the time.
Interviewer:	Do you think that you mix stuff, or what is it that becomes messy?
Ronaldo:	No, but it is just that, it gets too much!

What Ronaldo is saying is supported by the observation notes where his body language (with a lowered head) and oral expressions with swear words and sighs signals that he is discontent.

When Ronaldo describes how he learns mathematics best, he contrasts it with when he doesn’t learn.

Ronaldo:	mmm... well like... not just sit down and work, but like be more active also, you might do some math outdoors, or like do math games or something, not just sit down with the text book all the time, it gets so bloody tedious or like boring like hell in the end. So vary things. […]
Ronaldo:	So, not just an introduction for half an hour and then work in the text book until the end [of the lesson]. Some lessons can be like that, but it’s kind of like that all the time…
Interviewer:	Is it your experience, that it is too much [of that]?

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Ronaldo: Yes, yes…It gets so boring at the end, that you don't cope when it is too boring.

Thus, Edward and Ronaldo's participation is challenged by the fact that they perceive the mathematics education and how it is set up boring.

DISCUSSION – CHALLENGES AT THE BORDER OF NORMALITY

What is it about the four above described discourses (the discourse of tasks, the discourse of the teacher, the discourse to be (un)valued and the discourse of boring) which calls for a Discourse of accessibility in mathematics education? Well, all the construed discourses pinpoints issues in the education creating obstructions for participation of the SEM-students, implying obstructions also for access, or enhanced access, to mathematics. Hence, these students are challenged since they somehow are on the border of normal in the inclusive mathematics classroom.

The discourse of tasks show that the students are hindered by the choice of tasks, but in different ways. In Edward's case, the tasks do not challenge him, which hinders him from enhancing his learning in mathematics and doing unchallenging tasks is almost painful for him. Tasks that he values are tasks involving other subjects such as physics and tasks that are on a higher mathematical level. In Ronaldo's case he struggles to try to remember a procedure, and it seems the tasks do not help him to understand concepts. He also expresses insecurity in how to approach tasks and how tasks involving text hinder him in his struggle to get access. Tasks that are valued by Ronaldo are tasks that are not too difficult for him and tasks that involves prior knowledge he is familiar with. The discourse of importance of the teacher highlights how vital the teacher is in the mathematics classroom. To be able to see every student as a human being, to engage them to participate and to make them feel as parts of something they create together. This is seen in how Edward wants something more of the teacher than the teaching, he wants the teacher to "tell a joke sometime" suggesting there is something else important than just the teaching. Edward also sees how different teachers can challenge him in different ways, for him it is important to be challenged. For Ronaldo it is important that the teacher listens to him and is vigilant about what he wants regarding tasks and ways to work. Also, that the teacher is "really slow and methodical". The discourse to be (un)valued highlight the fact that Edward perceives himself unvalued in the classroom and feels discontent. He is not challenged, and he is not valued, which seem to create frustration. This might also be a reason for his negative feelings for mathematics. In Ronaldo's case it is not about being unvalued, but the importance of being valued as a SEM-student, getting special education in a smaller group sometimes. He needs to feel secure when doing mathematics. This security is hard for him to get in the ordinary mathematics classroom, and he sees the small group as an opportunity and a secure calm space. The discourse of math is boring is partly explained by the three other discourses. If you as a student often get tasks that doesn't challenge you, or challenge you to much, it gets boring. If you have a teacher who doesn't see you as a whole person, but just as a student passing by, it gets boring. If you feel unvalued, or constantly have to

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struggle to be valued, it gets boring. For Edward one of the boring things is to never be challenged, and to spend the time on doing stuff he does not learn from, which can be seen in his statement "I want to spend my time on maths". For Ronaldo it is to never really fully understand.

These discourses together paint a picture of challenges for accessibility for the SEM-students on the boarder of normality, which is manifested in Ronaldo and Edwards texts. The western way of teaching mathematics, the "one-size-fits-all"-education (Askew, 2015) does not fit these SEM-students, hence diversity is not embraced even though the school is set out to work inclusively. It seems that there are important issues to address regarding; the choice of tasks; the relations between teacher and students; the valuing of students voices in the classroom and the attitudes towards mathematics, to be able to enhance accessibility in mathematics education. This implies that it is not only the participation of the SEM-students who is challenged, also the actual mathematics education at the school is challenged. Meaning, in order to include every students in the mathematics education, and to promote equity, the education needs to address these challenges.

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[i] In Swedish mathematics classrooms it is very common that each lesson starts with an introduction, called "genomgång". These introductions can even occur in the middle or in the end of the lesson.

[ii] Bold text indicate an emphasize by the informant.

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